ESSAYS IN ECONOMIC MEASUREMENT WITH APPLICATION TO JAPAN

by

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Abstract

The objective of this dissertation is to improve our understanding of economic measurement, in particular, index number theory and its application to the Japanese economy. Two different approaches exist in index number theory: the first decomposes a value ratio of costs or profits into the product of a price index times a quantity index. The second approach decomposes a value difference in costs or profits into the sum of a price indicator plus a quantity indicator. This thesis will examine both approaches. Following the ratio approach, our first essay will investigate the origins of the growth of the Japanese standard of living. This growth is attributed to technical progress, changes in output prices and input quantities. In many ways, improvements in the terms of trade are synonymous with technical progress, because they make it possible to obtain more for less. We compare two distinct impacts of changes in the terms of trade and technical progress on the Japanese standard of living, and will show that while technical progress made the most contribution, the impact of the changes in the terms of trade were negligible. The second and third essays follow the difference approach in index number theory. The second essay deals with the producer model and proposes a productivity analysis based on the difference approach in index number theory. We show that following the difference approach, change in real income per unit primary input can be additively decomposed into explanatory factors such as technical progress, changes in relative output prices and deflated input quantities. The third essay deals with the consumer model, introducing the concept of the exact and superlative indicator into the difference approach; we will show that the Bennet indicator is an exact and superlative indicator. The second essay applies the new decomposition result to the Japanese market sector for the years 1955–2006. The third essay uses Japanese aggregate consumption data and compares different real expenditures based on various distinct indexes and indicators.

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Co-Authorship Statement

Chapter 2 includes joint work with Erwin Diewert and Koji Nomura. I started working on this chapter in order to improve Diewert, Mizobuchi and Nomura (2005). Erwin Diewert and I discussed the direction the chapter should be improved. I collected all the data which is necessary to construct input and output data for the Japanese market sector. Exceptions are the investment data and initial capital stocks which Koji Nomura provided. The novelty of our database is to extract the residential structures and land used by the market sector from the economy-wide residential structure and land. Since it is very difficult to divide residential structures and land between the market sector and the non-market sector, even the study on the market sector often include the entire part of residential structure and land in the list of the inputs of the market sector or exclude it from the list of the input of the market sector. I collected annual data of newly constructed owner-occupied houses and land formation in order to estimates of the residential structure and land used by the market sector. This chapter was initially drafted by me and revised by Erwin Diewert.

Chapter 3 includes joint work with Erwin Diewert. I proposed the idea of productivity analysis based on the difference approach in index number theory and wrote a paper with proofs of justifying the productivity analysis based on Bennet indicators. However, there is a problem that a part of my analysis is based on the income function which is not linearly homogeneous with respect to prices. Introducing a new functional form and looking at real prices, Erwin Diewert showed that even if we assume the usual income function, which is linearly homogeneous with respect to prices, we can still justify the productivity analysis based on Bennet indicator. I collected the input and output data for Japanese market sector and calculated the origins of the Japanese labour productivity growth for the period 1955-2006. My initial manuscript has been revised by Erwin Diewert.

Chapter 4 includes joint work with Erwin Diewert. I proposed the idea of introducing the concept of exact and superlative indicators into the difference approach in index number theory and wrote a paper with proofs of Bennet indicators as superlative indicators. However, there is a problem that a part of my analysis is based on the cost function which is not linearly homogeneous with respect to prices. Introducing a new functional form and looking at real prices, Erwin Diewert showed that even if we assume

the usual cost function, which is linearly homogeneous with respect to prices, we can still show that Bennet indicators are exact and superlative indicators. I collected the Japanese aggregate consumption data and calculated real expenditure by applying several indexes and indicators to collected data. My initial manuscript has been revised by Erwin Diewert.

Chapter 1. Objective and Overview

1.1 Objective

Economic analysis is concerned with modelling the supply and demand for individual goods and services (commodities) by individual economic agents. However, given the enormous numbers of commodities in real life economies, we must use aggregated data. How can this aggregation best be accomplished? Index number theory studies this type of problem and asks how individual price data should be summarised, or aggregated, in a single aggregate price level (price index or price indicator), and how individual data on quantities should be summarised in a single aggregate quantity level (quantity index or quantity indicator). Of the two approaches in index number theory, the ratio approach decomposes the value ratio of expenditures (in the case of consumer), or costs and profits (in the case of producer), between the current and the reference periods into the product of a price index and a quantity index. On the other hand, the difference approach decomposes the value difference in expenditures, costs and profits between the current and the reference periods into the sum of a price indicator and a quantity indicator. In this thesis, we examine the theoretical and empirical problems of these two approaches. In practice, the Laspeyres and Paasche indexes, which are based on the ratio approach, are widely used by statistical agencies all over the world. However, index number theory recommends superlative indexes, such as the Fisher and Törnqvist indexes, because they are free of the substitution bias from which the Laspeyres and Paasche indexes suffer. In the difference approach, the Bennet indicator plays a central role. We recommend its use in the context of consumer as well as producer.

Chapter 2 looks at the contribution of the market sector to changes in Japanese living standards between 1955 and 2006. Initially, we take a conventional Total Factor Productivity (TFP) growth approach, in which TFP growth is measured as a year-to-year Fisher quantity index of gross outputs, divided by a Fisher quantity index of primary inputs. We document the slowdown in Japanese TFP performance in the post-bubble period. This chapter also examines what happens when inventories and land are omitted from the list of primary inputs. The remainder of this chapter looks at the market sector's contribution to the growth in Japanese living standards, and decomposes this growth into three contribution factors: factors for technical progress, changes in real output prices (including changes in the terms of trade) and growth in primary input

quantities. We adapt the decomposition results developed by Diewert and Morrison (1987), as well as Kohli (1990) (2004), to this real income context, and observe that technical progress and the growth in capital services were the main contributors to real income growth; changes in the terms of trade, on average, had a minimal effect on real income. Finally, the chapter changes gears from a gross output approach to a theoretically preferable net output approach. In the net output context, we discover that the role of capital deepening as a contributor to higher living standards diminished, and the role of technical progress and labour growth became more important. The index number approach makes a weaker assumption on production technology and preferences, but its data requirements are more stringent. We then construct prices and quantities for inputs and outputs of the Japanese market sector for the period between 1955 and 2006. This database will be used in Chapter 3.

While Chapter 2 follows the traditional ratio approach in index number theory, wherein TFP is defined as the ratio of the output and input quantity indexes, Chapter 3 takes an alternative difference approach in index number theory: the approach decomposes a value difference into the sum of a price difference plus a quantity difference. Applying this difference approach, we decompose the growth of a new measure of labour productivity into additive contribution factors: factors for technical progress, changes in real output prices (including changes in the terms of trade) and growth in relative input quantities. This new measure of labour productivity takes into account changes in the terms of trade. We then use this methodology to investigate the growth in real income per unit labour in the Japanese economy from 1955 to 2006. Chapter 3 also introduces a new flexible functional form for an income function based on the normalised quadratic functional form pioneered by Diewert and Wales (1987).

Chapter 4 discusses the ratio approach in index number theory in the context of the consumer problem. As previously explained, the traditional economic approach to index number theory is based on a ratio concept. The Konüs true cost of living index is a ratio of cost functions evaluated at the same utility level, but with the prices of the current period in the cost function appearing in the numerator, and the prices of the base period in the cost function in the denominator. The Allen quantity index is also a ratio of cost functions in which the utility levels vary, but the price vector is held constant in the numerator and denominator. A corresponding theory initiated by Hicks (1946) exists for differences in cost functions; this chapter develops this approach. Diewert

(1976) has defined superlative price and quantity indexes as observable indexes that were exact for a ratio of unit cost functions, or for a ratio of linearly homogeneous utility functions. Chapter 4 looks for counterparts of his results in the difference context, for both flexible homothetic and flexible nonhomothetic preferences. The Bennet indicators of price and quantity change turn out to be superlative for the nonhomothetic case; the underlying preferences are of the translation homothetic form, as discussed by Balk, Färe and Grosskopf (2004), Chambers (2001) and Dickenson (1980). Using Japanese aggregate consumption data from national accounts, we calculate real expenditures based on different types of indexes and indicators. We show that real expenditures based on superlative indexes and superlative indicators are very close in magnitude.

1.2 Overview

This thesis is organized as follows. Chapter 2 conducts the decomposition of the change in Japanese standard of living into several components for the years between 1955 and 2006. Chapter 3 proposes the new measure for labour productivity and applies it to the Japanese economy for the years between 1955 and 2006. Chapter 4 examines the ratio approach to index number theory in the consumer context; here we apply different indexes and indicators to Japanese aggregate consumption data. Chapter 5 concludes this thesis. A detailed description of Chapter 2's data construction for the Japanese market sector between 1955 and 2006 is given in Appendix A. The price series and the quantity series for 13 aggregated inputs and outputs are listed in Appendix B. New functional forms are introduced in Chapters 3 and 4. Appendixes C and F include the proofs that they are also flexible functional form.

Chapter 2. On Measuring the Productivity and the Standard of Living in Japan, 1955-2006[1]

2.1 Introduction

The Japanese economy has experienced tremendous growth in the past half century, but there has been a pronounced slowdown for more than 10 years since the bubble economy collapsed.[2] This unique experience of the post-war Japanese economy has been attracting the interest of numerous researchers. However, most of these studies have been concerned with the growth of production and the productivity of the Japanese economy. These studies quantify the various sources of Japanese economic growth and measure the productive efficiency of the Japanese economy.[3]

In this chapter, we are primarily concerned with increases in the standard of living in the post-war Japanese economy. In particular, we focus on the contribution of Japan's market sector to improvements in Japanese living standards over the period 1955–2006. We have excluded the general government sector from our analysis because this sector may provide its outputs without charging for them, or it may sell them for prices that are not economically significant. Our analysis will rely on the assumptions of revenue-maximising and cost-minimising behaviour, assumptions that are generally not applicable for the government sector. There are other measurement difficulties associated with the government sector. Because it is difficult to measure multiple outputs in this sector, the *System of National Accounts 1993*[4] (see the UN (1993)) recommends measuring the value of these difficult-to-measure outputs of the general government as the cost of producing the outputs, and recommends setting the price of these outputs as equal to an input cost index. This is not a proper way of measuring

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[1] It includes joint work with W. Erwin Diewert and Koji Nomura.

[2] Although the Japanese economy has been in a period of economic recovery since 2002, its growth is still modest.

[3] See Hayashi and Nomura (2005), Hayashi and Prescott (2002), Jorgenson and Nishimizu (1978), Jorgenson, Kuroda and Nishimizu (1987), Jorgenson and Nomura (2005)(2007), Nishimizu and Hulten (1978) and Nomura (2004).

[4] Hereafter called 1993 SNA.

output from a theoretical point of view.[5] The household sector is also a producer in the *1993 SNA*. In particular, this sector produces imputed rent from owner-occupied houses. However, this is a special type of production where productivity improvements cannot take place, and so we have decided to exclude this sector from our concept of the market production sector. Therefore, we completely net out the general government and the household sector from our framework in order to avoid the above-mentioned difficulties.

Real Gross Domestic Product (GDP) is often used as a proxy for a country's living standard. However, real GDP is a measure of the level of production, and it is well known that real GDP can be a very misleading indicator of a country's welfare in the face of changing terms of trade.[6] Real (Gross) Income, which deflates nominal GDP by the price of domestic consumption goods, is more appropriate as a measure of economic welfare. Economic welfare comes from consumption. Real income captures how much consumption people can purchase for their income. For example, if the nominal output is constant over two consecutive periods, an improvement in the terms of trade should increase economic welfare by enhancing the purchasing power of domestic households. Real Income always increases in this situation. However, the conventional Laspeyres type real GDP may decrease in this case.

We shed new light on the measurement of the standard of living in Japan for the years 1955–2006 by focusing on the real income generated by the market sector. We attempt to measure the determinants of *real income growth* in Japan by adapting the analytic framework for productivity measurement, developed by Diewert and Morrison (1986) and Kohli (1990) (2004), to the real income growth context. We thus find that the main determinants of growth in real income generated by the market sector of the economy are:

- Technical progress or improvements in Total Factor Productivity (TFP);

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[5] There is an additional difficulty with the SNA treatment of input cost in the government sector: the user cost of capital in the government sector is just depreciation, with no imputation for the financial cost of capital. Thus, if a government building is sold to the private sector, GDP will increase!

[6] Hamada and Iwata (1984) show that real GNP (equivalent to real GDP in this chapter) may overestimate national welfare when the terms of trade deteriorate for a country. Kohli (2004) demonstrated that an improvement in the terms of trade can actually lead to a fall in real GDP.

- Growth in non-consumption real domestic output prices, growth in the real prices of exports and falls in the real prices of imports; and
- Growth in primary inputs.

Using our adapted decomposition formula, all three of these determinants can be calculated using only the observable price and quantity data over the years 1955–2006. A difficulty in applying this decomposition formula is uncertainty as to what is the "correct" user cost of capital. In particular, should an exogenous rate of return be used in the formula, or should an endogenous balancing rate of return that makes the value of inputs equal to the value of outputs be used? Schreyer (2007) surveys several different procedures for estimating rates of return. We use ex post balancing (real) rates of return. These balancing real rates of return seem to capture business conditions in the post-war Japanese economy.

GDP in a closed economy with no government is simply consumption plus gross investment plus changes in inventory. Economists have argued for a long time that Net Domestic Product (NDP), which equals consumption and net investment, is a more appropriate welfare measure than GDP.[7] The problem with the gross concept is that it gives us a measure of output that is not sustainable. In order to move to a more theoretically appropriate NDP concept using our framework for the determinants of real income growth, it is necessary to treat depreciation as an offset to gross investment. Thus, we take the depreciation term in the user cost formula out of costs and treat it as a negative output that will act as an offset to gross investment. Our theoretical results, which explain the growth of gross real income generated by the market sector, can still be applied to the net real income concept.

Our dataset of the market sector has been constructed from different data sources. For net output data, we heavily depend on data in Japanese national accounts.[8] For capital stocks and investment, we also made extensive use of the investment and asset data in the Keio Economic Observatory (KEO) database. For labour input, we constructed total

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[7] See Marshall (1890), Pigou (1924), Samuelson (1961) and Weitzman (1976). More recent papers that argue for the net product framework are Diewert and Fox (2005), Oulton (2004), and Weitzman (2003).

[8] In 1978, the Japanese system of national accounts was revised to comply with the guidelines proposed by the United Nations System of National Accounts (1968 SNA). In 2000, it was revised to comply with the guidelines newly proposed by the United Nations System of National Accounts (1993 SNA).

hours of work for three different types of workers: employees, the self-employed, and family workers from the Japanese *Labour Force Survey* and the Japanese national accounts. One limitation of our study which must be noted is that we have no industry detail in our database, and thus we cannot locate the contributions of TFP growth in individual industries to the aggregate market sector TFP growth.[9]

The remainder of this chapter is organized as follows. In section 2.2, we outline a framework for measuring productivity and decomposing real income growth into several explanatory factors. This framework is an adaptation of a methodology introduced by Diewert and Morrison (1987). In section 2.3, we explain our data construction. The rate of return is endogenously determined from observed data. The procedure for determining the rate of return is explained there. In section 2.4 of the chapter, we present a conventional TFP growth accounting for the market sector of the Japanese economy for the years 1955–2006. There have been several recent studies that do more or less the same thing, so one might question the value of yet another study of Japanese TFP growth.[10] However, all of these alternative studies cover a much shorter period (with the exception of Nomura (2004)), and there are other significant differences. In the present study, we consider only the market sector of the Japanese economy, where productivity improvements are possible under current national income accounting conventions. Moreover, our focus is not on TFP growth rates per se but rather the contribution of TFP growth to real income growth, i.e. the existing studies do not follow our treatment of changes in the terms of trade. In section 2.4.2, the gross income methodology developed in section 2.2 is implemented using our Japanese market sector database for the years 1955–2006, and the net income methodology is implemented in section 2.4.3. In section 2.5, we conclude the chapter.

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[9] Moreover, we cannot measure the contributions to aggregate TFP growth of shifts in labour resources from less productive to more productive sectors. This difficulty comes from the aggregate nature of our labour data. Since we consider three types of labour (employees, the self-employed, family workers), we can only capture the contribution of shifts in labour resources within these three types. On the other hand, industry price and quantity data are notoriously unreliable because of the lack of detailed surveys on gross outputs and intermediate inputs, particularly for service industries. A final reason for our use of aggregate market sector data is that reliable breakdowns of exports and imports by industry are not available.

[10] Some of the recent studies are Hayashi and Nomura (2005), Hayashi and Prescott (2002), Jorgenson and Nomura (2005) (2007) and Miyagawa, Ito and Harada (2004).

2.2 The Theoretical Framework

In this section, we present the production theory framework which will be used in the remainder of the chapter. The main references are Diewert and Morrison (1986) and Kohli (1990).[11]

Initially, we assume that the market sector of the economy produces quantities of M (net)[12] outputs, $y \equiv [y_1,...,y_M]$, which are sold at the positive producer prices $P \equiv [P_1,...,P_M]$. We further assume that the market sector of the economy uses positive quantities of N (primary)[13] inputs, $x \equiv [x_1,...,x_N]$ which are purchased at the positive primary input prices $W \equiv [W_1,...,W_N]$. In period t, we assume that there is a feasible set of output vectors y that can be produced by the market sector if the vector of primary inputs x is utilized by the market sector of the economy; denote this period t production possibilities set by S^t. We assume that S^t is a closed convex cone that exhibits a free disposal property.[14]

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[11] The theory also draws on Samuelson (1953), Diewert (1974; 133-141) (1980) (1983; 1077-1100), Fox and Kohli (1998), Kohli (1978) (1991) (2003) (2004a) (2004b), Morrison and Diewert (1990), Samuelson (1953) and Sato (1976). This chapter is essentially an extended version of Diewert, Mizobuchi and Nomura (2005) using newly available data. The theoretical framework explained in this section was recently used by Diewert and Lawrence (2006).

[12] If the mth commodity is an import (or other produced input) into the market sector of the economy, then the corresponding quantity y_m is indexed with a negative sign. We will follow Kohli (1978) (1991) and Woodland (1982) in assuming that imports flow through the domestic production sector and are "transformed" (perhaps only by adding transportation, wholesaling and retailing margins) by the domestic production sector. The recent textbook by Feenstra (2004; 76) also uses this approach.

[13] Primary inputs only include labour and capital services from reproducible assets, inventories and land. Intermediate inputs fall into the category of (net) outputs.

[14] For a more explanation for the meaning of these properties, see Diewert (1973) (1974; 134), Woodland (1982) or Kohli (1978) (1991). The assumption that S^t is a cone means that the technology is subject to constant returns to scale. This is an important assumption since it implies that the value of outputs should equal the value of inputs in equilibrium. In our empirical work, we use an ex post rate of return in our user costs of capital, which forces the value of inputs to equal the value of outputs for each period. The function g^t is known as the *GDP function* or the *national product function* in the international trade literature (see Kohli (1978) (1991), Woodland (1982) and Feenstra (2004; 76). It was introduced into the economics literature by Samuelson (1953). Alternative terms for this function include: (i) the *gross profit*

Given a vector of output prices P and a vector of available inputs x, we define *the period t market sector GDP function*, $g^t(P,x)$, as follows:[15]

(2.1) $g^t(P,x) \equiv \max_y \{P \cdot y : (y,x) \text{ belongs to } S^t\}$; $t = 0,1,2, \ldots$.

Thus, market sector GDP depends on t (which represents the period t technology set S^t), on the vector of output prices P that the market sector faces and on x, the vector of inputs that is available to the market sector.

If P^t is the period t output price vector and x^t is the vector of inputs used by the market sector during period t and if the GDP function is differentiable with respect to the components of P at the point P^t,x^t, then the period t vector of market sector outputs y^t will be equal to the vector of first order partial derivatives of $g^t(P^t,x^t)$ with respect to the components of P; i.e., we will have the following equations for each period t:[16]

(2.2) $y^t = \nabla_P g^t(P^t,x^t)$; $t = 0,1,2, \ldots$.

If the GDP function is differentiable with respect to the components of x at the point P^t,x^t, then the period t vector of input prices W^t will be equal to the vector of first order partial derivatives of $g^t(P^t,x^t)$ with respect to the components of x; i.e., we will have the following equations for each period t:[17]

(2.3) $W^t = \nabla_x g^t(P^t,x^t)$; $t = 0,1,2, \ldots$.

The constant return to scale assumption on the technology sets S^t implies that the value of

---

function; see Gorman (1968); (ii) the *restricted profit function*; see Lau (1976) and McFadden (1978); and (iii) the *variable profit function*; see Diewert (1973) (1974) (1993).

[15] The function $g^t(P,x)$ will be linearly homogeneous and convex in the components of P and linearly homogeneous and concave in the components of x; see Diewert (1973) (1974; 136). Notation: $P \cdot y \equiv \sum_{m=1}^{M} P_m y_m$.

[16] These relationships are due to Hotelling (1932; 594). Note that $\nabla_P g^t(P^t,x^t) \equiv [\partial g^t(P^t,x^t)/\partial P_1, \ldots, \partial g^t(P^t,x^t)/\partial P_M]$.

[17] These relationships are due to Samuelson (1953) and Diewert (1974; 140). Note that $\nabla_x g^t(P^t,x^t) \equiv [\partial g^t(P^t,x^t)/\partial x_1, \ldots, \partial g^t(P^t,x^t)/\partial x_N]$.

outputs will equal the value of inputs in period t; i.e., we have the following relationships:

(2.4) $g^t(P^t,x^t) = P^t \cdot y^t = W^t \cdot x^t$; $t = 0,1,2, \ldots$.

The above material will be useful in what follows. However, our focus is not on GDP; instead, our focus is on the income generated by the market sector or more precisely, on *the real income generated by the market sector*. However, since market sector GDP (the value of market sector production) is distributed to the factors of production used by the market sector, nominal market sector GDP will be equal to nominal income of the market sector. As an approximate welfare measure that can be associated with market sector,[18] we will choose to measure the *real income generated by the market sector in period t* in terms of the number of consumption bundles that the nominal income could purchase in period t. Therefore, we define ρ^t as follows:

(2.5) $\rho^t \equiv W^t \cdot x^t / P_C{}^t$; $t = 0,1,2, \ldots$

$= w^t \cdot x^t$

$= p^t \cdot y^t$

$= g^t(p^t,x^t)$

where $P_C{}^t > 0$ is the *period t consumption expenditures deflator* and the market sector period t *real output price* p^t and *real input price* w^t vectors are defined as the corresponding nominal price vectors deflated by the consumption expenditures price index; i.e., we have the following definitions:[19]

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[18] We focus on the income produced by the market sector. Thus, we do not take into account domestic or national incomes from other sources. For example, we dismiss the net income received from abroad (e.g. the balance of primary incomes and current transfer in relation to the rest of the world) and social benefits from the government. Moreover, our suggested welfare measure has a problem as welfare measure such as it is not sensitive to the distribution of the income that is generated by the market sector. Our measure of *domestic* welfare generated by the market sector is only an approximate one. Thus, our analysis in this chapter is a first step toward the more comprehensive analysis on Japanese living standard.

[19] Our approach is similar to the approach advocated by Kohli (2004b; 92), except he essentially deflates nominal GDP by the domestic expenditures deflator rather than just the domestic (household) expenditures deflator; i.e., he deflates by the deflator for C+G+I, whereas we suggest deflating by the deflator for C. Another difference in his approach compared to the present approach is that we restrict our analysis to the

(2.6) $p^t \equiv P^t/P_C{}^t$; $w^t \equiv W^t/P_C{}^t$; $t = 0,1,2, \ldots$.

The first and last equalities in (2.5) imply that period t real income, ρ^t, is equal to the period t market sector GDP function, evaluated at the period t real output price vector p^t and the period t input vector x^t, $g^t(p^t,x^t)$. Thus *the growth in real income over time can be explained by three main factor; t (Technical progress or Total Factor Productivity growth), the changes in real output prices and the growth of primary input quantities*. We will shortly give formal definitions for these three growth factors.

Using the linear homogeneity properties of the GDP functions $g^t(P,x)$ in P and x separately, we can show that the following counterparts to the relations (2.2) and (2.3) hold using the deflated prices p and w:[20]

(2.7) $y^t = \nabla_p g^t(p^t,x^t)$; $t = 0,1,2, \ldots$
(2.8) $w^t = \nabla_x g^t(p^t,x^t)$; $t = 0,1,2, \ldots$.

Now we are ready to define a family of *period t productivity growth factors* $\tau(p,x,t)$:

(2.9) $\tau(p,x,t) \equiv g^t(p,x)/g^{t-1}(p,x)$; $t = 1,2, \ldots$.

Thus, $\tau(p,x,t)$ measures the proportional change in the real income produced by the market sector that is induced by the technical change going from period t – 1 to t, facing the reference real output prices p and using reference input quantities x. We can choose the reference vectors for the measure of technical progress defined by (2.9) from the current year or the previous year: a *Laspeyres type measure* $\tau_L{}^t$ that chooses the period t–1 reference vectors p^{t-1} and x^{t-1} and a *Paasche type measure* $\tau_P{}^t$ that chooses the period t reference vectors p^t and x^t:

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market sector GDP, whereas Kohli deflates all of GDP (probably due to data limitations). Our treatment of the balance of trade surplus or deficit is also different.

[20] If producers in the market sector of the economy are solving the profit maximization problem that is associated with $g^t(P,x)$, which uses the original output prices P, then they equivalently solve the profit maximization problem that is associated with $g^t(p,x)$, which uses the normalized output prices $p \equiv P/P_C$; i.e., Therefore, their behaviour can be described by using either $g^t(P,x)$ or $g^t(p,x)$.

$$(2.10)\ \tau_L^t \equiv \tau(p^{t-1},x^{t-1},t) = g^t(p^{t-1},x^{t-1})/g^{t-1}(p^{t-1},x^{t-1})\ ; \qquad t = 1,2, \ldots ;$$

$$(2.11)\ \tau_P^t \equiv \tau(p^t,x^t,t) \qquad = g^t(p^t,x^t)/g^{t-1}(p^t,x^t)\ ; \qquad t = 1,2, \ldots .$$

Since both measures of technical progress are equally valid, it is natural to average them to obtain an overall measure of productivity growth. If we want to treat the two measures in a symmetric manner and we want the measure to satisfy the time reversal property from index number theory[21] (so that the estimate going backwards is equal to the reciprocal of the estimate going forwards), then the geometric mean will be the best simple average to take.[22] Thus we define the geometric mean of (2.10) and (2.11) as follows:[23]

$$(2.12)\ \tau^t \equiv [\tau_L^t\, \tau_P^t]^{1/2}\ ; \qquad t = 1,2, \ldots .$$

At this point, it is not clear how we will obtain empirical estimates for the theoretical productivity growth indexes defined by (2.10)–(2.12). One obvious way would be to assume a functional form for the GDP function $g^t(p,x)$, collect data on output and input prices and quantities for the market sector for a number of years (and for the consumption expenditures deflator), add error terms to equations (2.7) and (2.8) and use econometric techniques to estimate the unknown parameters in the assumed functional form. However, econometric techniques are generally not completely straightforward: different econometricians will make different stochastic specifications and will choose different functional forms.[24] Moreover, as the number of outputs and inputs grows, it will be impossible to estimate a flexible functional form. Thus we will suggest methods for implementing measures like (2.12) in this chapter that are based on exact index number techniques.

We turn now to the problem of defining theoretical indexes for the effects on real income

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[21] See Fisher (1922; 64).

[22] See the discussion in Diewert (1997) on choosing the "best" symmetric average of Laspeyres and Paasche indexes that will lead to the satisfaction of the time reversal test by the resulting average index.

[23] The theoretical productivity change indexes defined by (2.10)–(2.12) were first defined by Diewert and Morrison (1986; 662-663). See Diewert (1993) for properties of symmetric means.

[24] "The estimation of GDP functions such as (2.19) can be controversial, however, since it raises issues such as estimation technique and stochastic specification. ... We therefore prefer to opt for a more straightforward index number approach." Ulrich Kohli (2004a; 344).

due to changes in real output prices. Define a family of *period t real output price change factors* $\alpha(p^{t-1},p^t,x,s)$:[25]

$$(2.13)\ \alpha(p^{t-1},p^t,x,s) \equiv g^s(p^t,x)/g^s(p^{t-1},x)\ ; \qquad s = 1,2, \ldots .$$

Thus $\alpha(p^{t-1},p^t,x,s)$ measures the proportional change in the real income produced by the market sector that is induced by the change in real output prices going from period t–1 to t, using the reference technology that is available during period s and using the reference input quantities x. Thus, each choice of the reference period for technology s and the reference input vector x will generate a possibly different measure of the effect on real income of a change in real output prices going from period t–1 to period t.

Again, we can choose the reference vectors for the measure of output price change defined by (2.13) from the current year or the previous year: a *Laspeyres type measure* $\alpha_L{}^t$ that chooses the period t–1 reference technology and reference input vector x^{t-1} and a *Paasche type measure* $\alpha_P{}^t$ that chooses the period t reference technology and reference input vector x^t:

$$(2.14)\ \alpha_L{}^t \equiv \alpha(p^{t-1},p^t,x^{t-1},t-1) = g^{t-1}(p^t,x^{t-1})/g^{t-1}(p^{t-1},x^{t-1})\ ; \qquad t = 1,2, \ldots ;$$
$$(2.15)\ \alpha_P{}^t \equiv \alpha(p^{t-1},p^t,x^t,t) \quad = g^t(p^t,x^t)/g^t(p^{t-1},x^t)\ ; \qquad t = 1,2, \ldots .$$

Since both measures of real output price change are equally valid, it is natural to average them to obtain an overall measure of the effects on real income of the change in real output prices:[26] The simple geometric mean is justified from the same reason as that for the technical progress shift function.

$$(2.16)\ \alpha^t \equiv [\alpha_L{}^t\, \alpha_P{}^t]^{1/2}\ ; \qquad t = 1,2, \ldots .$$

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[25] This measure of real output price change was essentially defined by Fisher and Shell (1972; 56–58), Samuelson and Swamy (1974; 588–592), Archibald (1977; 60–61), Diewert (1980; 460–461) (1983; 1055) and Balk (1998; 83-89). Readers who are familiar with the theory of the true cost of living index will note that the real output price index defined by (2.13) is analogous to the Konüs (1924) *true cost of living index* which is a ratio of cost functions, say $C(u,p^t)/C(u,p^{t-1})$ where u is a reference utility level: g^s replaces C and the reference utility level u is replaced by the vector of reference variables x.

[26] The indexes defined by (2.13)–(2.16) were defined by Diewert and Morrison (1986; 664) in the nominal GDP function context.

Finally, we look at the problem of defining theoretical indexes for the effects on real income due to changes in real output prices. Define a family of *period t input quantity growth factors* $\beta(x^{t-1},x^t,p,s)$:[27]

(2.17) $\beta(x^{t-1},x^t,p,s) \equiv g^s(p,x^t)/g^s(p,x^{t-1})$; $s = 1,2, \ldots$.

Thus $\beta(x^{t-1},x^t,p,s)$ measures the proportional change in the real income produced by the market sector that is induced by the change in input quantities going from period t–1 to t, using the technology that is available during period s and facing the reference real output prices p. Thus, each choice of the reference period for technology s and the reference real output price vector p will generate a possibly different measure of the effect on real income of a change in input quantities going from period t–1 to period t.

Again, We can choose the reference vectors for the measure of input quantity change defined by (2.13) from the current year or the previous year: a *Laspeyres type measure* $\beta_L{}^t$ that chooses the period t–1 reference technology and reference real output price vector p^{t-1} and a *Paasche type measure* $\beta_P{}^t$ that chooses the period t reference technology and reference real output price vector p^t:

(2.18) $\beta_L{}^t \equiv \beta(x^{t-1},x^t,p^{t-1},t-1) = g^{t-1}(p^{t-1},x^t)/g^{t-1}(p^{t-1},x^{t-1})$; $t = 1,2, \ldots$;
(2.19) $\beta_P{}^t \equiv \beta(x^{t-1},x^t,p^t,t) \quad = g^t(p^t,x^t)/g^t(p^t,x^{t-1})$; $t = 1,2, \ldots$.

Since both measures of real input growth are equally valid, it is natural to average them to obtain an overall measure of the effects of input growth on real income:[28] The simple geometric mean is justified from the same reason as that for the technical progress shift function.

(2.20) $\beta^t \equiv [\beta_L{}^t \beta_P{}^t]^{1/2}$; $t = 1,2, \ldots$.

We provide all the theoretically motivated measures such as $\tau_L{}^t$, $\alpha_L{}^t$, $\beta_L{}^t$, $\tau_P{}^t$, $\alpha_P{}^t$, and $\beta_P{}^t$.

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[27] This type of index was defined as a true index of value added by Sato (1976; 438) and as a real input index by Diewert (1980; 456).

[28] The theoretical indexes defined by (2.17)–(2.20) were defined in Diewert and Morrison (1986; 665) in the nominal GDP context.

Now, we consider how to calculate these measures from the observed data. On the first sight, the effort is hopeless. Under the constant returns to scale assumption on the technology set S^t, we have

(2.21) $p^t y^t = g^t(p^t,x^t)$

It means that we observe values for the denominator of Laspeyres type measure such as τ_L^t, α_L^t, β_L^t and for the numerator of Paasche type measure such as τ_P^t, α_P^t, β_P^t. However, since we only know the denominator or the numerator of indicators, we cannot calculate even any indicators of τ_L^t, α_L^t, β_L^t, τ_P^t, α_P^t, and β_P^t.

For example, the numerator of Laspeyres type measure technical progress $g^t(p^{t-1},x^{t-1})$ is the hypothetical revenue that would result from using the period t technology with the period t−1 input quantities and real output prices. Fortunately, these hypothetical revenues can be inferred from observed data if the period t revenue function follows the Translog functional form. The following Translog functional form is known as the flexible functional form which approximates the arbitrary revenue function by the second order:[29]

(2.22) $$\ln g^t(p,x) \equiv a_0^t + \sum_{m=1}^M a_m^t \ln p_m^t + (1/2) \sum_{m=1}^M\sum_{k=1}^M a_{mk} \ln p_m^t \ln p_k^t + \sum_{n=1}^N b_n^t \ln x_n^t + (1/2)\sum_{n=1}^N\sum_{j=1}^N b_{nj} \ln x_n^t \ln x_j^t + \sum_{m=1}^M\sum_{n=1}^M c_{mn} \ln p_m^t \ln x_n^t ; \quad t = 1,2, \ldots .$$

Note that the coefficients for the quadratic terms are assumed to be constant over time. The coefficients must satisfy the following restrictions in order for g^t to satisfy the linear homogeneity properties that we have assumed in section 2.2 above:[30]

(2.23) $\sum_{m=1}^M a_m^t = 1$ for t = 0,1,2, ...;
(2.24) $\sum_{n=1}^N b_n^t = 1$ for t = 0,1,2, ...;
(2.25) $a_{mk} = a_{km}$ for all k,m ;

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[29] This functional form was first suggested by Diewert (1974; 139) as a generalization of the translog functional form introduced by Christensen, Jorgenson and Lau (1971). Diewert (1974; 139) indicated that this functional form was flexible.

[30] There are additional restrictions on the parameters which are necessary to ensure that $g^t(p,x)$ is convex in p and concave in x.

(2.26) $b_{nj} = b_{jn}$ for all n,j ;
(2.27) $\sum_{k=1}^{M} a_{mk} = 0$ for m = 1,...,M ;
(2.28) $\sum_{j=1}^{N} b_{nj} = 0$ for n = 1,...,N ;
(2.29) $\sum_{n=1}^{N} c_{mn} = 0$ for m = 1,...,M ;
(2.30) $\sum_{m=1}^{M} c_{mn} = 0$ for n = 1,...,N .

Theorem 1: Adaptation of Diewert and Morrison (1986)[31]: If g^{t-1} and g^t are defined by (2.22)–(2.30) above and there is competitive profit maximizing behaviour on the part of all market sector producers for all period t, we have

(2.31) $\tau^t = p^t y^t/(\alpha^t \beta^t p^{t-1} y^{t-1})$; t = 1,2, ...

where
(2.32) $\ln\alpha^t = \Sigma_{m=1}(1/2)[(p_m^{t-1} y_m^{t-1}/p^{t-1}\cdot y^{t-1}) + (p_m^t y_m^t/p^t\cdot y^t)] \ln(p_m^t/p_m^{t-1})$;
(2.33) $\ln\beta^t = \Sigma_{n=1}(1/2)[(w_n^{t-1} x_n^{t-1}/w^{t-1}\cdot x^{t-1}) + (w_n^t x_n^t/w^t\cdot x^t)] \ln(x_n^t/x_n^{t-1})$;

The above theorem shows how we can calculate the theoretically motivated measures such as τ^t, α^t, and β^t from observed data. Another contribution of the theorem is to show that the real income growth can be exactly decomposed into the three explanatory factors: productivity growth, real output price change, and input quantity change.

For some purposes, it is convenient to decompose the aggregate period t contribution to real income growth due to changes in all real output prices α^t into separate effects for the change in each real output price. Similarly, it can sometimes be useful to decompose the aggregate period t contribution to real income growth due to changes in all input quantities β^t into separate effects for the change in each input quantity. We indicate how this can be done, making the same assumptions on the technology that we have made so far.

We first model the effect of the change in the real output price of output m, say p_m, going from period t–1 to t or *the period t real price change factor for output m*. Counterparts

[31] Diewert and Morrison (1986) established their proof using the nominal GDP function $g^t(P,x)$. However, it is easy to rework their proof using the deflated GDP function $g^t(p,x)$ using the fact that $g^t(p,x) = g^t(P/P_C,x) = g^t(P,x)/P_C$ using the linear homogeneity property of $g^t(P,x)$ in P. This argument is also true for theorem 2 and 3.

to the theoretical Laspeyres and Paasche type price indexes defined by (2.14) and (2.15) above for all the outputs are the following *Laspeyres type measure* α_{Lm}^t that chooses the period t–1 reference technology, holds other real output prices (other than output m) constant at their period t–1 levels, and holds input quantities constant at their period t–1 levels and a *Paasche type measure* α_{Pm}^t that chooses the period t reference technology, holds other real output prices constant at their period t levels, and holds input quantities constant at their period t levels;

$$(2.34)\ \alpha_{Lm}^t \equiv g^{t-1}(p_1^{t-1},...,p_{m-1}^{t-1},p_m^t,p_{m+1}^{t-1},..., p_M^{t-1},x^{t-1})/g^{t-1}(p^{t-1},x^{t-1})\ ; \qquad m = 1,...,M;\ t = 1,2, ... ;$$

$$(2.35)\ \alpha_{Pm}^t \equiv g^t(p^t,x^t)/g^t(p_1^t ,...,p_{m-1}^t,p_m^{t-1},p_{m+1}^t,..., p_M^t,x^t)\ ; \qquad m = 1,...,M;\ t = 1,2, ...$$

Since both measures of real output price changes are equally valid, it is natural to average them to obtain an overall measure of the effects on real income of the change in the real price of output m:[32]

$$(2.36)\ \alpha_m^t \equiv [\alpha_{Lm}^t\, \alpha_{Pm}^t]^{1/2}\ ; \qquad m = 1,...,M\ ;\ t = 1,2, ...$$

Under the assumption that the deflated GDP functions $g^t(p,x)$ have the Translog functional forms as defined by (2.22)–(2.30), the arguments of Diewert and Morrison (1986; 666) provide the following exact decomposition of the period t aggregate real output price contribution factor α^t into a product of separate price contribution factors α_m^t:

Theorem 2: Adaptation of Diewert and Morrison (1986): If g^{t-1} and g^t are defined by (2.22)-(2.33) above and there is competitive profit maximizing behaviour on the part of all market sector producers for all periods t, then we have

$$(2.37)\ \alpha^t = \alpha_1^t\alpha_2^t... \alpha_M^t\ ; \qquad t = 1,2, ...$$

where

[32] The indexes defined by (2.34)–(2.36) were defined by Diewert and Morrison (1986; 666) in the nominal GDP function context.

(2.38) $\ln\alpha_m^t = (1/2)[(p_m^{t-1}y_m^{t-1}/p^{t-1}\cdot y^{t-1}) + (p_m^t y_m^t/p^t\cdot y^t)]\ \ln(p_m^t/p_m^{t-1})$; $\quad m = 1,...,M.$

For example, we consider the case where there are four net outputs:

- 1: Domestic sales;
- 2: Investment;
- 3: Exports and;
- 4: Imports.

Since commodities 1, 2, and 3 are outputs, y_1, y_2, and y_3 will be positive but since commodity 4 is an input into the market sector, y_4 will be negative. Hence an increase in the real price of exports will *increase* real income but an increase in the real price of imports will *decrease* the real income generated by the market sector, as is evident by looking at the contribution terms defined by (2.38) for m = 3 (where $y_m^t > 0$) and for m = 4 (where $y_m^t < 0$). The above decomposition (2.38) is useful for analyzing the impacts that the changes in the real price of exports (i.e., a change in the price of exports relative to the price of domestic consumption) and in the real price of imports make on the real income generated by the market sector.

We now model the effects of the change in the quantity of input n, say x_n, going from period t–1 to t or *the period t quantity growth factor for input n*. Individual counterparts to the overall theoretical Laspeyres and Paasche type quantity indexes defined by (2.18) and (2.19) above for all the inputs are the following *Laspeyres type measures* for input n β_{Ln}^t that chooses the period t–1 reference technology and holds constant real output prices at their period t–1 levels and holds other input quantities (other than input n) constant at their period t–1 levels and the *Paasche type measures* for input n β_{Pn}^t that chooses the period t reference technology and hold constant real output prices at their period t levels and hold other input quantities constant at their period t levels;

(2.39) $\beta_{Ln}^t \equiv g^{t-1}(p^{t-1},x_1^{t-1},...,x_{n-1}^{t-1},x_n^t,x_{n+1}^{t-1},..., x_N^{t-1})/g^{t-1}(p^{t-1},x^{t-1})$; $\quad n = 1,...,N;\ t = 1,2, ...$;

(2.40) $\beta_{Pn}^t \equiv g^t(p^t,x^t)/g^t(p^t,x_1^t,...,x_{n-1}^t,x_n^{t-1},x_{n+1}^t,..., x_N^t)$; $\quad n = 1,...,N;\ t = 1,2, ...$.

Since both measures of input change are equally valid, as usual, we average them to obtain an overall measure of the effects on real income of the change in the quantity of

input n:[33]

(2.41) $\beta_n^t \equiv [\beta_{Ln}^t \beta_{Pn}^t]^{1/2}$; $n = 1,...,N$; $t = 1,2, ...$.

Under the assumption that the deflated GDP functions $g^t(p,x)$ have the Translog functional forms as defined by (2.22)–(2.30), the arguments of Diewert and Morrison (1986; 666) can be adapted to provide the following exact decomposition of the period t aggregate input growth contribution factor β^t into a product of separate input contribution factors β_n^t:

Theorem 3: Adaptation of Diewert and Morrison (1986): If g^{t-1} and g^t are defined by (2.22)–(2.30) above and there is competitive profit maximizing behaviour on the part of all market sector producers for all periods t, then we have

(2.42) $\beta^t = \beta_1^t \beta_2^t ... \beta_N^t$; $t = 1,2, ...$.

where

(2.43) $\ln\beta_n^t = (1/2)[(w_n^{t-1} x_n^{t-1}/w^{t-1} \cdot x^{t-1}) + (w_n^t x_n^t/w^t \cdot x^t)] \ln(x_n^t/x_n^{t-1})$; $n = 1,...,N$.

Substituting the results in Theorems 2 and 3 into the decomposition in Theorem 1 and rearranging the terms, we obtain the following formula;

(2.44) $\rho^t/\rho^{t-1} = \tau^t \prod_{m=1}^M \alpha_m^t \prod_{n=1}^N \beta_n^t$; $t = 1,2, ...$.

Thus *the growth in real income over time can be explained by three main factors: productivity growth, the growth of real output prices, and the growth of input quantities*.

Rather than look at explanatory factors for the growth in real income, it is sometimes convenient to express the level of real income in period t in terms of cumulated growth factors. Thus (2.45) below defines a period t index of the technology level or of Total Factor Productivity in period t relative to period 0, T^t; (2.46) defines a period t cumulative index of the real price change factor for output m since period 0, A_m^t; and (2.47) defines a period t cumulated index of the quantity growth factor for input n since

[33] The indexes defined by (2.33)–(2.41) were defined by Diewert and Morrison (1986; 667) in the nominal GDP function context.

period 0, B_n^t:

(2.45) $T^0 \equiv 1; T^t \equiv T^{t-1}\tau^t;$ $t = 1,2,\ldots,;$
(2.46) $A_m^0 \equiv 1; A_m^t \equiv A_m^{t-1}\alpha_m^t;$ $m = 1,2,\ldots,M$ and $t = 1,2,\ldots,;$
(2.47) $B_n^0 \equiv 1; B_n^{t-1} \equiv B_n^0\beta_n^t;$ $n = 1,2,\ldots,N$ and $t = 1,2,\ldots,;$

Using the appropriate equalities (2.44) for the chain links that appear in (2.45)–(2.47), we can establish the following exact relationship for the level of real income in period t, ρ^t, relative to its counterpart in period 0, ρ^0, and the cumulated growth factors defined by (2.45)–(2.47):

(2.48) $\rho^t/\rho^0 = T^t\prod_{m=1}^{M}A_m^t\prod_{n=1}^{N}B_n^t;$ $t = 1,2, \ldots .$

2.3 Data Construction

2.3.1 The Definition of the Market Sector

In this chapter, we focus on the production of the market sector within the entire economy. This is the sector that plays a critical role for economic growth. Following the *1993 SNA*, we classify the producers for the entire economy into the following five mutually exclusive institutional sectors:

- IU1; Non-financial corporations;
- IU2; Financial corporations;
- IU3; General government;
- IU4; Households[34];
- IU5; Non-profit institutions serving households.

In our definition, the market sector consists of IU1, IU2, and IU5. Thus, we subtract the government's and the households' outputs and inputs from aggregate output, capital services and labour input in order to perform the growth accounting exercise explained in

[34] According to the UN (1993), the unincorporated enterprises are parts of the household sectors. However, we added the unincorporated enterprises to IU1 or IU2 based on the characteristics of the enterprise. Therefore, in our classification, the household sector only includes household own-account producers.

the previous section. There are two types of transactions between the market sector and the general government sector: (1) the sales of goods and services of the general government sector to the market sector and (2) the purchases of intermediate inputs by the general government sector from the market sector. Note that the sales of goods and services by the general government sector are considered as inputs for the market sector, while the purchases of intermediate inputs by the general government sector are regarded as outputs by the market sector. Thus, the difference between (1) and (2) is the net output by the market sector, which has been purchased by the general government sector. In the Japanese national accounts, the imputed rent of the owner-occupied dwellings is the major output by the household sector.[35] The households sector produces imputed rent by using capital services from the stock of owner-occupied houses and land services from the residential land under the owned houses. Therefore, we need to subtract the imputed rent from the consumption output, the stock of owner-occupied houses from the total stock of residential buildings, and the residential land under the owned houses from the total residential land.[36]

2.3.2 The Database for Inputs and Outputs

Our final dataset consists of price and quantity series for the variables listed below. They are the inputs and outputs for the market sector. We also followed the conventions on the treatment of indirect taxes that Jorgenson and Griliches introduced, i.e. we adjusted prices for tax wedges whenever possible, so that the adjusted prices reflect the prices that producers face.[37] We also included the services of inventories and land (utilized by the market sector) as primary inputs.

The net outputs in our most disaggregated database are as follows:

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[35] We ignore other own-account production by households.

[36] The household sector purchases goods and services for the maintenance of owner-occupied houses. The expenses on these goods and services are produced by the market sector and thus are part of our market sector output.

[37] Thus our suggested treatment of indirect commodity taxes in an accounting framework that is suitable for productivity analysis follows the example set by Jorgenson and Griliches, who advocated the following treatment of indirect taxes: "In our original estimates, we used gross product at market prices; we now employ gross product from the producers' point of view, which includes indirect taxes levied on factor outlay, but excludes indirect taxes levied on output." Jorgenson and Griliches (1972; 85).

- C; Domestic final consumption expenditure of households (excluding the imputed rent of owner-occupied dwellings);
- N; Final consumption expenditure of private non-profit institutions serving households (NPISHs);
- G; Net purchases of goods and services by the general government from the market sector;
- X; Exports of goods and services (including direct purchases in the domestic market by non-resident households since this is a source of revenue to the market sector);
- M; Imports of goods and services (excluding direct purchases abroad by resident households, since this is not a source of revenue to the Japanese market sector);
- I1 to I95; Gross investments for 95 asset categories;
- IV1 to IV4; Change in inventories for 4 types of inventory asset.

The primary inputs in our most disaggregated database are as follows:

- K1 to K95; Capital services from 95 classes of fixed assets;
- KIV1 to KIV4; Inventory services for 4 classes of inventory assets;
- LD1 to LD4; Market sector land services and
- LB1 to LB3; Labour input for 3 types of labour.

Dividing the prices of inputs and outputs by the price of domestic final consumption expenditure of households P_C, we constructed the corresponding real input and output prices. The net outputs C, N, G, X, and M have been constructed as aggregates of the net output components listed above, and are essentially based on the data of the Japanese national accounts. Since the Japanese national accounts experienced several revisions, there are different data series even for the same variable. We chose the reference data series from the most recent publication and extended these series by using the growth rates of other data series from earlier publications.[38] A more detailed explanation of our data construction methods is provided in Appendix A.

Quantities of capital services, inventory services, and land services are proportional to

[38] The data from the Japanese national accounts are taken from publications such as the *Annual Report on National Income Statistics 1975*, *Report on National Accounts from 1955 to 1998*, *Annual Report on National Accounts of 2000* and *Annual Report on National Accounts of 2004-2008*. See for further explanation.

capital stocks of fixed assets, inventories, and land at the beginning of the year. Capital stocks have been constructed by applying the perpetual inventory method, using the gross investment data (I1 to I95) to the initial stocks of 1955, based on asset-specific depreciation rates suggested by Nomura (2004). Changes in Inventories IV1–IV4 are calculated by using the difference between the inventory stock in the current period and the inventory stock in the previous period. The initial stocks for fixed assets, inventory stocks and land, and the price and quantity series for investments have been taken from the KEO database. This is a comprehensive productivity database for the Japanese economy constructed by KEO at Keio University.[39] The detailed procedures used to construct prices and quantities for capital stocks and services are explained in Nomura (2004). For residential structures and residential land, we use other data sources in order to net out the stocks that the household sector holds, which correspond to the stocks of residential land and structures used by owner-occupied households. Our procedures will be explained in the following subsection. Prices of capital services for the 95 types of reproducible capital services K1–K95, for the 4 types of inventory services KIV1–KIV4 and the 4 types of land services LD1–LD4 are calculated by applying a user cost formula that we will explain in the next section.

Estimates of the quantities of labour services LB1–LB3 are based on hours of work. There are three different types of workers: employees, the self-employed, and family workers. After we calculate the average labour income and the hours of work for each type of worker, we aggregate these three types of labour into a labour aggregate using a superlative index number formula.

Investments I1–I95 correspond to the capital services K1–K95. Changes in inventories IV1–IV4 correspond to the inventory services KIV1–KIV4. Table 2-1, which follows, lists the names of all capital services, inventory services, and land services.

[39] This database also includes labour inputs. Kuroda, Shimpo, Nomura, and Kobayashi (1997) explain the data construction of labour inputs in detail. In this chapter, we use only their relative wages for employees, the self-employed and family workers from the KEO database.

Table 2-1: List of Capital Services, Inventory Services, and Land Services

Name			
K1	Trees (growth)	K53	Other ships
K2	Livestock (growth)	K54	Railway vehicles
K3	Textile products	K55	Aircraft
K4	Wooded products	K56	Bicycles
K5	Wooden furniture and fixtures	K57	Transport equipment for industrial use
K6	Metallic furniture and fixtures	K58	Other transport equipment
K7	Nuclear fuel rods	K59	Camera
K8	Metallic products	K60	Other photographic and optical instruments
K9	Boilers and turbines	K61	Watches and clocks
K0	Engines	K62	Physics and chemistry instruments
K11	Conveyors	K63	Analytical and measuring instruments and testing machines
K12	Refrigerators and air conditioning apparatus	K64	Medical instruments
K13	Pumps and compressors	K65	Miscellaneous manufacturing products
K14	Sewing machines	K66	Residential construction (wooden)
K15	Other general industrial machinery and equipment	K67	Residential construction (non-wooden)
K16	Mining, civil engineering and construction machinery	K68	Non-residential construction (wooden)
K17	Chemical machinery	K69	Non-residential construction (non-wooden)
K18	Industrial robots	K70	Road construction
K19	Metal machine tools	K71	Street construction
K20	Metal processing machinery	K72	Bridge construction
K21	Agricultural machinery	K73	Toll road construction
K22	Textile machinery	K74	River improvement
K23	Food processing machinery	K75	Erosion control
K24	Sawmill, wood working, veneer and plywood machinery	K76	Seashore improvement
K25	Pulp equipment and paper machinery	K77	Park construction
K26	Printing, bookbinding and paper processing machinery	K78	Sewer construction
K27	Casting equipment	K79	Sewage disposal facilities
K28	Plastic processing machinery	K80	Waste disposal facilities
K29	Other special industrial machinery, nec	K81	Harbor construction
K30	Other general machines and parts	K82	Fishing port construction
K31	Office machines	K83	Airport construction
K32	Vending, amusement and other service machinery	K84	Agricultural construction
K33	Electric audio equipment	K85	Forest road construction
K34	Radio and television sets	K86	Forestry protection
K35	Video recording and playback equipment	K87	Railway construction
K36	Household electric appliance	K88	Electric power facilities
K37	Electronic computer and peripheral equipment	K89	Telecommunication facilities
K38	Wired communication equipment	K90	Other civil engineering and construction
K39	Radio communication equipment	K91	Plant engineering
K40	Other communication equipment	K92	Mineral exploration
K41	Applied electronic equipment	K93	Custom software
K42	Electric measuring instruments	K94	Pre-packaged software
K43	Generators	K95	Own-account software
K44	Electric motors	KIV1	Finished-goods inventory
K45	Relay switches and switchboards	KIV2	Work-in-process inventory
K46	Other industrial heavy electrical equipment	KIV3	Work-in-process inventory for cultivated assets
K47	Electric lighting fixtures and apparatus	KIV4	Material inventory
K48	Passenger motor vehicles	LD1	Land for agricultural use
K49	Trucks, buses and other vehicles	LD2	Land for industrial use
K50	Two-wheel motor vehicles	LD3	Land for commercial use
K51	Motor vehicle parts	LD4	Land for residential use
K52	Steel ships		

The following subsection explains how we constructed the price, quantity and value

series for the capital inputs. The details on how our labour inputs were constructed are found in Appendix A.

2.3.3 User Costs and Real Rates of Return

Prices of capital services, inventory services and land services are estimated as user costs of capital. User costs are constructed for each of our capital stock components. The general formula for a user cost for a capital stock component in year t, u^t, is as follows:

(2.49) $u^t = P_K^t - (1 - \delta)P_K^t(1 + i^t)/(1 + r^t)$,

where P_K^t is the beginning of year t asset price for the capital stock component, δ is the geometric depreciation rate that applies to the asset,[40] i^t is the amount of asset-specific price change that is expected to occur over the course of year t and r^t is the nominal rate of return (or opportunity cost of capital) that producers face at the beginning of year t. Thus, the user cost of a durable input is equal to its purchase cost or opportunity cost at the beginning of the year, P_K^t, less the discounted expected value of the depreciated asset at the end of the year.[41] The nominal rate of return r^t can be decomposed into a real rate of return component r^{t*} and an expected general inflation component, i^{t*}, by using the following formula:

(2.50) $1 + r^t = (1 + r^{t*})(1 + i^{t*})$.

The practical problem for the economic statistician with a user cost formula of the type defined by (2.49) and (2.50) is that there is *uncertainty* about how exactly to estimate the depreciation rate δ, the relevant real rate of return r^{t*} and the two anticipated inflation rates, i^{t*} (general inflation rate) and i^t (asset-specific inflation rate). If we estimate i^{t*} and i^t as actual ex post inflation rates, we will almost certainly generate user costs u^t that are negative for some years, which is not sensible in our context since we want our user costs to closely approximate market rental rates for the assets and since these rates would not be negative. Even if we estimate i^{t*} and i^t by smoothing the ex post values for these variables or by using a forecasting model, with Japanese data, we will inevitably generate some negative user costs for land components because of the very rapid land price

[40] We assume that the depreciation rates for the inventory stocks and land components are 0.

[41] We have temporarily neglected tax factors for the sake of simplicity.

inflation that occurred in Japan during the 1980s. We decided to avoid these negative user cost problems by assuming that producers expect the asset-specific inflation rate i^t to equal the general inflation rate i^{t*}, i.e. we make the following assumption:

(2.51) $i^t = i^{t*}$.

If we substitute (2.50) and (2.51) into (2.49), we find that our user cost formula simplifies as follows:[42]

(2.52) $u^t = P_K{}^t - (1 - \delta)P_K{}^t/(1 + r^{t*}) = [r^{t*} + \delta]P_K{}^t/(1 + r^{t*})$.

We approximate the beginning of the year asset price $P_K{}^t$ by the corresponding year t investment price $P_I{}^t$. Thus, our final user cost formula has the following generic form, where τ^t is the business tax rate:[43]

(2.53) $u^t \equiv [r^{t*} + \tau^t + \delta]P_I{}^t/(1 + r^{t*})$.

There remains the problem of finding a suitable real interest rate series, r^{t*}, which will be discussed below. Once r^{t*} has been determined, the user cost defined by (2.53) can be calculated for each of our 95 types of capital services, 4 types of inventory services, and 4 types of land services. The corresponding beginning-of-year capital stocks have already been described and tabled. We normalize the resulting user costs, so that they are 1 in 1955, and make offsetting normalizations are made to our capital quantity series. The resulting input prices are $W_{K1}{}^t$–$W_{K95}{}^t$, $W_{KIV1}{}^t$–$W_{KIV4}{}^t$ and $W_{LD1}{}^t$–$W_{LD4}{}^t$, and the resulting quantities are $x_{K1}{}^t$–$x_{K95}{}^t$, $x_{KIV1}{}^t$–$x_{KIV4}{}^t$ and $x_{LD1}{}^t$–$x_{LD4}{}^t$. The price series and the quantity series for 13 aggregated inputs and outputs are listed in Appendix B.[44]

Finally, we discuss how we obtained a suitable real interest rate series r^{t*}. The rate we chose is an economy-wide ex post real rate of return. If we use the user cost formula (2.53) to form prices for capital services, we can rearrange the *value of outputs equals the*

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[42] For discussions about alternative assumptions for user cost formulae, see Schreyer (2001) (2007) (2008) and Diewert (1980) (2004) (2005b).

[43] For business structures and land, we need to add the appropriate specific property tax rates to the general tax rate τ^t.

[44] More detailed data will be provided to the interested reader.

value of primary inputs equation into a single (linear) equation involving an unknown real rate of return. The solutions to these equations (one for each year) are the series of ex post real interest rates, r^{t*}, listed below in Figure 2-1.

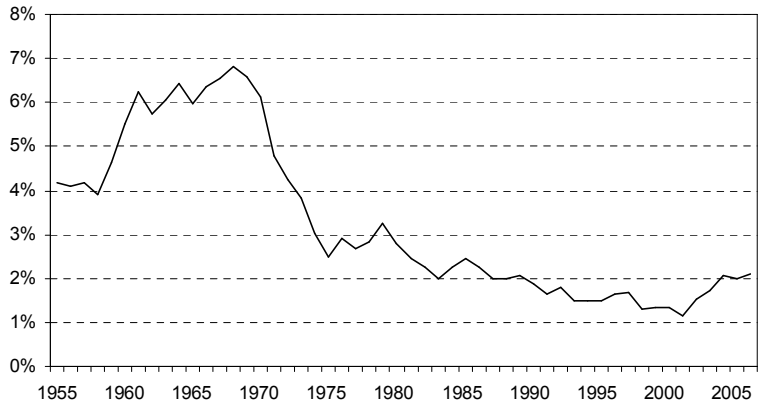

Figure 2-1: The Real Rate of Return for the Years 1955-2006

The average rate for the years 1955–2006 is 3.25%. This rate is consistent with "the long run observed economy-wide real rates of return for most OECD countries which fall into the 3 to 5 percent range" (Diewert (2004)). The performance of the Japanese economy over the years 1955–2006 can be divided into 5 periods:

- Period 1: 1955–1973: Rapid economic growth;
- Period 2: 1974–1979: Stagnation between oil shocks;
- Period 3: 1980–1990: Stable economic growth;
- Period 4: 1991–2001: Long recession and stagnation;
- Period 5: 2002–2006: Modest economic recovery.

The average real rates of return for the above periods are 5.379% (Period 1), 2.86% (Period 2), 2.214% (Period 3), 1.483% (Period 4) and 1.885% (Period 5). Even though the Japanese economy is currently in a period of economic recovery, its average real rate of return is still significantly smaller than the average real rate of return of the period of rapid economic growth (Period 1).

2.4 Results

2.4.1 Japanese Productivity Growth: A Conventional Approach

In this section, we measure the productivity growth of the market sector of the Japanese economy using a conventional chained Fisher index number approach. Depending on whether inventory services or land services are included in the list of primary inputs, we can consider three different cases:

- Case 1: inventory services and land services are in the list of primary inputs;
- Case 2: land services are not in the list of primary inputs; and
- Case 3: inventory services and land services are both excluded from the list of primary inputs.

Since we endogenously determine the real rate of return from the zero-profit condition, the composition of primary inputs affects the real rate of return. Basically, the conventional measure of productivity growth is set equal to a chained Fisher output quantity index divided by a chained Fisher input quantity index. Thus, the composition of primary inputs also affects the productivity growth. Thus in Table 2, under the above three assumptions, we list three sets of the Fisher year to year output and input growth factors, y^t/y^{t-1}, x^t/x^{t-1}, respectively, along with their ratios, $\tau^t \equiv [y^t/y^{t-1}]/[x^t/x^{t-1}]$ and the balancing year t real interest rates r^{t*}. Figure 2 plots three sets of the real rate of return, and Figure 3 plots three sets of Total Factor Productivity.[45]

[45] Total Factor Productivity T^t is calculated from the ratios of output growth factors and input growth factors τ^t using equation (2.45).

Table 2-2: Chained Fisher Indexes of Output, Input and Productivity Growth and the Balancing Real Rate of Return in the Japanese Economy, 1956-2006.

	Case 1				Case 2				Case 3			
	y^t/y^{t-1}	x^t/x^{t-1}	τ^t	r^{t*}	y^t/y^{t-1}	x^t/x^{t-1}	τ^t	r^{t*}	y^t/y^{t-1}	x^t/x^{t-1}	τ^t	r^{t*}
1956	1.11124	1.04614	1.06223	0.04108	1.11130	1.05025	1.05812	0.10607	1.11130	1.04580	1.06263	0.14230
1957	1.09559	1.04684	1.04657	0.04185	1.09565	1.05395	1.03957	0.11285	1.09565	1.04972	1.04375	0.15657
1958	1.05114	1.03596	1.01466	0.03895	1.05135	1.04498	1.00610	0.11178	1.05135	1.04278	1.00823	0.15762
1959	1.13732	1.03339	1.10057	0.04630	1.13733	1.03941	1.09421	0.14250	1.13733	1.04126	1.09227	0.20277
1960	1.17782	1.04742	1.12450	0.05528	1.17783	1.06034	1.11080	0.18621	1.17783	1.05894	1.11228	0.27052
1961	1.15667	1.04835	1.10332	0.06227	1.15657	1.06481	1.08617	0.20866	1.15657	1.06578	1.08518	0.30588
1962	1.07165	1.06155	1.00951	0.05742	1.07167	1.08370	0.98890	0.19320	1.07167	1.08216	0.99031	0.28022
1963	1.10261	1.04217	1.05800	0.06053	1.10258	1.05847	1.04168	0.20565	1.10258	1.06334	1.03690	0.29516
1964	1.13972	1.04800	1.08753	0.06418	1.13972	1.06508	1.07008	0.21995	1.13972	1.06744	1.06772	0.31324
1965	1.05337	1.05219	1.00113	0.05963	1.05338	1.06821	0.98611	0.20023	1.05338	1.07048	0.98403	0.27948
1966	1.13090	1.06055	1.06634	0.06342	1.13091	1.06332	1.06356	0.21862	1.13091	1.06617	1.06072	0.30559
1967	1.13551	1.05547	1.07583	0.06543	1.13552	1.06088	1.07035	0.23938	1.13552	1.06338	1.06784	0.33723
1968	1.13640	1.04436	1.08813	0.06830	1.13642	1.06490	1.06716	0.25819	1.13642	1.06546	1.06660	0.36665
1969	1.14133	1.05589	1.08092	0.06596	1.14127	1.06961	1.06699	0.26210	1.14127	1.07158	1.06503	0.36999
1970	1.10251	1.05400	1.04602	0.06122	1.10246	1.07714	1.02351	0.24603	1.10246	1.07974	1.02104	0.33981
1971	1.03639	1.04883	0.98814	0.04787	1.03592	1.07085	0.96738	0.19496	1.03592	1.07397	0.96457	0.26161
1972	1.08462	1.03835	1.04457	0.04237	1.08456	1.05161	1.03133	0.18738	1.08456	1.05762	1.02548	0.24757
1973	1.10733	1.05480	1.04980	0.03816	1.10783	1.05966	1.04546	0.17564	1.10783	1.06523	1.03999	0.22985
1974	0.98945	1.02220	0.96796	0.03022	0.98892	1.03300	0.95733	0.12274	0.98892	1.03444	0.95600	0.15718
1975	1.00794	1.00881	0.99914	0.02485	1.00775	1.01496	0.99290	0.09538	1.00775	1.01541	0.99246	0.12091
1976	1.06272	1.04473	1.01722	0.02923	1.06273	1.04880	1.01328	0.10476	1.06273	1.05231	1.00990	0.13064
1977	1.03519	1.03237	1.00273	0.02665	1.03508	1.03481	1.00026	0.09700	1.03508	1.03634	0.99878	0.11946
1978	1.05398	1.02280	1.03048	0.02824	1.05394	1.02587	1.02736	0.10472	1.05394	1.02822	1.02501	0.12730
1979	1.08746	1.02771	1.05814	0.03241	1.08740	1.03195	1.05373	0.11403	1.08740	1.03460	1.05104	0.13687
1980	1.02811	1.03101	0.99719	0.02798	1.02829	1.03747	0.99115	0.09910	1.02829	1.03642	0.99215	0.11903
1981	1.02026	1.02825	0.99224	0.02463	1.02022	1.03468	0.98602	0.09013	1.02022	1.03458	0.98612	0.10787
1982	1.02889	1.02404	1.00474	0.02265	1.02889	1.02819	1.00068	0.08504	1.02889	1.02921	0.99969	0.10114
1983	1.01990	1.03434	0.98604	0.01999	1.01990	1.03807	0.98250	0.07831	1.01990	1.03906	0.98156	0.09257
1984	1.04942	1.01686	1.03202	0.02257	1.04940	1.02096	1.02786	0.08678	1.04940	1.02248	1.02633	0.10153
1985	1.05245	1.03022	1.02158	0.02432	1.05234	1.03605	1.01572	0.09176	1.05234	1.03720	1.01459	0.10619
1986	1.02548	1.02563	0.99985	0.02258	1.02588	1.03199	0.99408	0.08965	1.02588	1.03286	0.99324	0.10290
1987	1.04535	1.02963	1.01528	0.01992	1.04535	1.03581	1.00921	0.08714	1.04535	1.03763	1.00744	0.09902
1988	1.06680	1.03324	1.03249	0.01981	1.06678	1.03803	1.02770	0.09075	1.06678	1.03888	1.02686	0.10267
1989	1.06722	1.03107	1.03507	0.02050	1.06723	1.03660	1.02956	0.09508	1.06723	1.03738	1.02878	0.10683
1990	1.05502	1.02909	1.02520	0.01859	1.05505	1.03562	1.01876	0.09000	1.05505	1.03581	1.01857	0.10086
1991	1.03178	1.02859	1.00310	0.01636	1.03175	1.03598	0.99592	0.07772	1.03175	1.03654	0.99539	0.08660
1992	1.01006	1.01805	0.99215	0.01782	1.01007	1.02277	0.98758	0.07115	1.01007	1.02342	0.98696	0.07857
1993	0.99101	1.01111	0.98012	0.01506	0.99101	1.01390	0.97742	0.05873	0.99101	1.01450	0.97685	0.06449
1994	1.01093	1.00959	1.00133	0.01479	1.01093	1.01095	0.99999	0.05543	1.01093	1.01161	0.99933	0.06052
1995	1.01713	1.01180	1.00526	0.01474	1.01716	1.01322	1.00388	0.05368	1.01716	1.01374	1.00337	0.05836
1996	1.02595	1.01054	1.01525	0.01637	1.02596	1.01163	1.01416	0.05584	1.02596	1.01160	1.01419	0.06072
1997	1.01392	1.00811	1.00576	0.01668	1.01389	1.00926	1.00458	0.05302	1.01389	1.00913	1.00471	0.05759
1998	0.97028	1.00632	0.96418	0.01289	0.97030	1.00708	0.96348	0.04171	0.97030	1.00699	0.96356	0.04533
1999	0.99587	1.00156	0.99431	0.01343	0.99592	1.00232	0.99362	0.04096	0.99592	1.00248	0.99345	0.04446
2000	1.02453	1.01694	1.00746	0.01351	1.02452	1.01671	1.00768	0.04145	1.02452	1.01731	1.00709	0.04484
2001	0.99262	0.99708	0.99553	0.01150	0.99263	0.99741	0.99520	0.03631	0.99263	0.99748	0.99514	0.03918
2002	1.00694	0.99907	1.00788	0.01548	1.00693	0.99939	1.00754	0.04101	1.00693	0.99981	1.00712	0.04391
2003	1.00865	1.00271	1.00593	0.01721	1.00868	1.00235	1.00631	0.04266	1.00868	1.00259	1.00607	0.04559
2004	1.03540	1.00793	1.02726	0.02056	1.03538	1.00847	1.02669	0.04775	1.03538	1.00881	1.02634	0.05092
2005	1.03493	1.00521	1.02957	0.01977	1.03491	1.00565	1.02909	0.04688	1.03491	1.00601	1.02872	0.04998
2006	1.03959	1.01539	1.02383	0.02121	1.03949	1.01556	1.02356	0.04833	1.03949	1.01503	1.02410	0.05163
Average	y^t/y^{t-1}	x^t/x^{t-1}	τ^t	r^{t*}	y^t/y^{t-1}	x^t/x^{t-1}	τ^t	r^{t*}	y^t/y^{t-1}	x^t/x^{t-1}	τ^t	r^{t*}
1956-2006	1.05642	1.02934	1.02596	0.03241	1.05641	1.03613	1.01926	0.11577	1.05641	1.03707	1.01834	0.15054
1956-1973	1.10956	1.04857	1.05821	0.05446	1.10957	1.06151	1.04542	0.19274	1.10957	1.06282	1.04414	0.27011
1974-1979	1.03946	1.02644	1.01261	0.02860	1.03930	1.03156	1.00748	0.10644	1.03930	1.03355	1.00553	0.13206
1980-1990	1.04172	1.02849	1.01288	0.02214	1.04176	1.03395	1.00757	0.08943	1.04176	1.03468	1.00685	0.10369
1991-2001	1.00764	1.01088	0.99677	0.01483	1.00765	1.01284	0.99486	0.05327	1.00765	1.01316	0.99455	0.05824
2002-2006	1.02510	1.00606	1.01889	0.01885	1.02507	1.00628	1.01864	0.04533	1.02507	1.00645	1.01847	0.04841

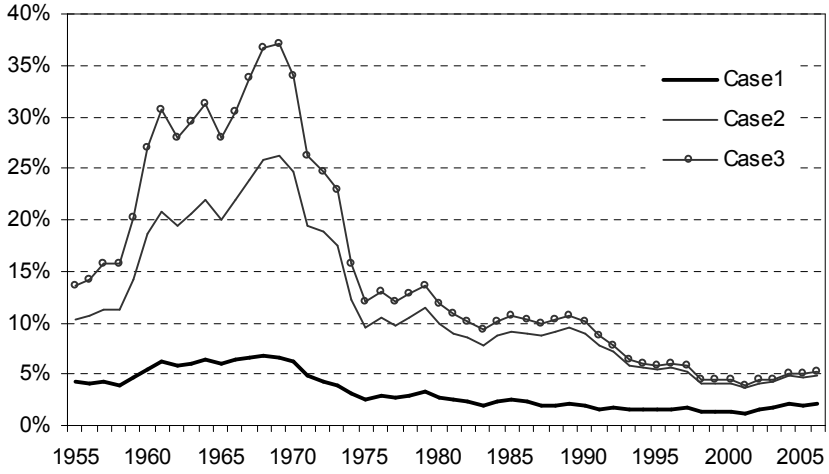

Figure 2-2: Comparison of Real Rate of Return for the Years 1955-2006

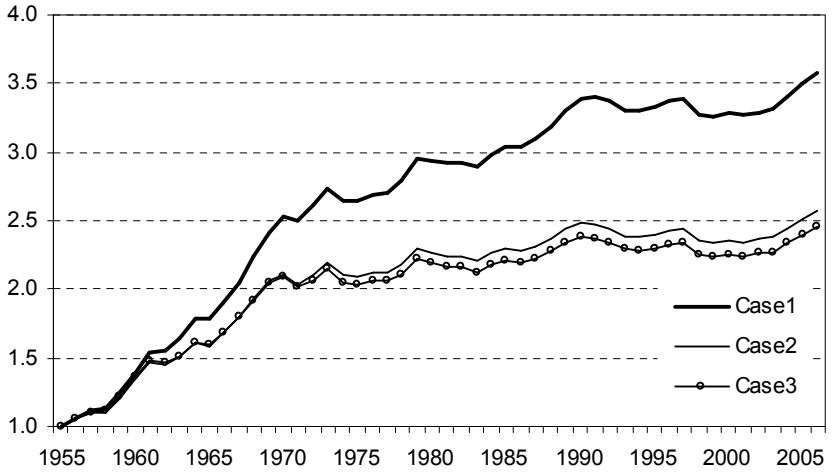

Figure 2-3: Comparison of Total Factor Productivity for the Years 1955-2006

The above table and figures show that it is extremely important to include land and inventory services in the list of primary inputs. If they are omitted from a productivity analysis, the results will be distorted. The growth rates for primary inputs will be excessively high, and thus productivity growth will be underestimated. The exclusion of land services from the primary inputs significantly changes our estimate. This exclusion bias becomes more serious during periods when the price of land is very high. Thus, productivity growth when land is included (Case 1) is almost 50% greater than when it is excluded during the years 1980–1990 (Cases 2 and 3). This period includes the bubble era at the end of the 1980s.

Now, we turn to the theoretical framework presented in section 2.2 and determine the factors that explain real income growth in the Japanese economy using a traditional value added framework as opposed to the net framework that will be discussed later in section 2.4.3.

2.4.2 The Decomposition of Deflated GDP Growth

We implement the Diewert and Morrison type approach using our Japanese database. The chain link information on period by period changes in real income that corresponds to (2.44) is listed in Table 2-3.

Table 2-3: The Decomposition of Market Sector Real Income Growth into Translog Productivity, Real Output Price Change and Input Quantity Contribution Factors

	ρ^t	τ^t	α_N^t	α_G^t	α_X^t	α_M^t	α_I^t	α_{IV}^t	β_K^t	β_{KIV}^t	β_{LD}^t	β_{LB}^t	α_T^t
1956	1.13349	1.06212	1.00065	1.00110	1.00404	0.99244	1.01873	1.00309	1.00105	1.00232	1.00037	1.04235	0.99644
1957	1.09829	1.04655	1.00033	1.00032	0.99667	0.99924	1.00742	0.99853	1.00899	1.00279	1.00086	1.03376	0.99591
1958	1.05396	1.01464	1.00004	0.99874	0.99183	1.02114	0.99164	0.99958	1.01509	1.00245	1.00105	1.01703	1.01280
1959	1.12932	1.10058	1.00006	0.99919	0.99872	1.00485	0.99045	0.99973	1.01405	1.00074	1.00179	1.01651	1.00356
1960	1.17609	1.12450	1.00024	0.99918	0.99979	1.00329	0.99757	0.99847	1.01589	1.00248	1.00130	1.02714	1.00308
1961	1.15450	1.10336	1.00011	0.99889	0.99297	1.00584	1.00429	0.99608	1.02537	1.00288	1.00514	1.01424	0.99876
1962	1.04941	1.00952	1.00000	0.99811	0.99005	1.00977	0.98409	0.99725	1.03126	1.00431	1.00515	1.01969	0.99972
1963	1.07386	1.05803	0.99995	0.99764	0.99478	1.00610	0.97719	0.99821	1.02886	1.00170	1.00495	1.00621	1.00085
1964	1.13395	1.08754	1.00052	0.99889	0.99828	1.00149	0.99622	0.99954	1.02677	1.00238	1.00521	1.01297	0.99977
1965	1.03110	1.00114	0.99985	0.99816	0.99082	1.00920	0.98204	0.99881	1.02821	1.00242	1.00700	1.01375	0.99994
1966	1.12652	1.06633	1.00024	0.99941	0.99500	1.00369	0.99855	0.99925	1.02267	1.00161	1.01541	1.01967	0.99867
1967	1.13975	1.07582	1.00031	0.99990	0.99643	1.00537	1.00280	0.99893	1.02417	1.00188	1.01505	1.01338	1.00179
1968	1.12644	1.08816	1.00009	0.99905	0.99443	1.00567	0.99305	0.99897	1.02842	1.00300	1.00627	1.00614	1.00007
1969	1.14055	1.08095	1.00031	0.99920	0.99730	1.00066	1.00166	1.00019	1.03111	1.00278	1.01529	1.00580	0.99796
1970	1.09177	1.04605	1.00058	0.99883	0.99558	1.00529	0.99146	0.99856	1.03248	1.00256	1.00760	1.01054	1.00085
1971	1.02189	0.98815	1.00042	0.99810	0.99451	1.01105	0.98340	0.99910	1.03310	1.00200	1.00624	1.00692	1.00550
1972	1.08106	1.04460	1.00052	0.99821	0.99212	1.01061	0.99523	1.00020	1.02578	1.00033	1.00664	1.00522	1.00265
1973	1.11654	1.04975	1.00039	0.99937	0.99809	0.99228	1.01663	1.00128	1.02340	1.00021	1.01344	1.01685	0.99039
1974	0.95963	0.96786	1.00021	1.00525	1.00805	0.96430	0.99443	0.99792	1.02382	1.00127	1.00826	0.98908	0.97206
1975	0.96594	0.99916	0.99999	0.99241	0.99079	1.00263	0.97304	0.99944	1.01696	1.00108	1.00648	0.98451	0.99340
1976	1.03629	1.01722	1.00012	0.99908	0.98907	1.00535	0.98143	1.00000	1.01332	0.99973	1.00374	1.02743	0.99437
1977	1.02215	1.00273	1.00006	0.99884	0.98364	1.01502	0.99028	0.99992	1.01237	1.00030	1.00573	1.01363	0.99841
1978	1.05758	1.03049	0.99996	0.99813	0.98503	1.02459	0.99612	1.00012	1.01230	0.99988	1.00386	1.00660	1.00925
1979	1.07995	1.05812	1.00022	1.00047	1.00610	0.97331	1.01299	1.00035	1.01476	0.99979	1.00324	1.00972	0.97925
1980	0.99400	0.99719	0.99997	1.00187	1.00201	0.96453	0.99932	0.99909	1.01497	1.00116	1.00347	1.01112	0.96646
1981	1.01436	0.99224	0.99983	1.00031	0.99792	1.00624	0.99041	0.99965	1.01418	1.00072	1.00178	1.01134	1.00414
1982	1.02273	1.00473	0.99999	0.99900	1.00205	0.99970	0.99334	0.99998	1.01316	1.00020	1.00243	1.00810	1.00175
1983	1.01495	0.98603	0.99995	0.99958	0.99164	1.01139	0.99268	1.00005	1.01180	1.00012	1.00202	1.02010	1.00293
1984	1.04747	1.03202	1.00000	0.99933	0.99876	1.00703	0.99313	0.99997	1.01107	0.99984	1.00049	1.00539	1.00578
1985	1.04710	1.02152	1.00003	1.00130	0.99178	1.01037	0.99195	0.99970	1.01902	1.00022	1.00296	1.00785	1.00207
1986	1.03404	0.99987	1.00004	0.99747	0.97995	1.03974	0.99217	1.00001	1.01695	1.00015	1.00052	1.00786	1.01890
1987	1.04380	1.01526	1.00004	0.99866	0.99459	1.00739	0.99790	0.99999	1.01835	0.99986	1.00182	1.00939	1.00193
1988	1.06827	1.03249	1.00005	0.99896	0.99760	1.00282	1.00197	1.00001	1.01614	1.00009	1.00224	1.01446	1.00041
1989	1.06921	1.03507	1.00005	1.00011	1.00178	0.99552	1.00447	0.99996	1.01766	1.00010	1.00183	1.01122	0.99729
1990	1.05126	1.02520	1.00024	1.00026	0.99912	0.99558	1.00130	0.99990	1.01948	1.00033	1.00226	1.00681	0.99470
1991	1.02965	1.00311	1.00009	0.99935	0.99462	1.00677	0.99725	0.99992	1.01904	1.00019	1.00106	1.00811	1.00136
1992	1.00706	0.99215	0.99997	0.99945	0.99537	1.00639	0.99587	1.00001	1.01729	1.00009	1.00223	0.99843	1.00173
1993	0.98677	0.98012	0.99990	0.99951	0.99131	1.00801	0.99698	1.00009	1.01235	1.00001	1.00194	0.99683	0.99925
1994	1.00652	1.00133	1.00004	0.99920	0.99630	1.00393	0.99610	1.00009	1.00732	0.99989	1.00166	1.00070	1.00022
1995	1.01522	1.00526	1.00006	0.99957	0.99810	1.00152	0.99887	0.99999	1.00543	0.99991	1.00098	1.00544	0.99961
1996	1.01852	1.01525	1.00004	0.99968	1.00399	0.99364	0.99539	0.99999	1.00697	1.00012	1.00139	1.00202	0.99761
1997	1.00570	1.00576	1.00005	1.00019	1.00090	0.99481	0.99600	0.99991	1.00884	1.00017	1.00130	0.99781	0.99571
1998	0.97011	0.96418	1.00008	0.99991	1.00235	1.00247	0.99502	0.99999	1.00896	1.00015	1.00145	0.99580	1.00482
1999	0.98762	0.99431	0.99993	0.99966	0.98883	1.00867	0.99475	0.99996	1.00571	1.00000	1.00049	0.99539	0.99739
2000	1.01821	1.00746	1.00000	1.00004	0.99535	0.99765	1.00087	0.99995	1.00482	0.99978	1.00091	1.01136	0.99302
2001	0.98803	0.99554	1.00009	1.00009	1.00440	0.99634	0.99437	1.00014	1.00656	1.00002	1.00066	0.98990	1.00072
2002	1.00473	1.00788	0.99983	1.00009	1.00063	0.99950	0.99780	0.99997	1.00595	0.99984	1.00035	0.99296	1.00013
2003	1.00378	1.00593	0.99993	0.99977	0.99627	1.00023	0.99903	0.99994	1.00334	0.99991	1.00080	0.99866	0.99651
2004	1.03283	1.02726	1.00007	1.00019	1.00032	0.99439	1.00264	0.99995	1.00428	0.99986	0.99989	1.00389	0.99471
2005	1.02784	1.02958	1.00029	1.00053	1.00462	0.98584	1.00186	1.00006	1.00473	0.99985	1.00012	1.00051	0.99040
2006	1.04007	1.02384	1.00040	1.00084	1.01021	0.98415	1.00489	1.00027	1.00691	1.00035	1.00098	1.00709	0.99420

Average	ρ^t	τ^t	α_N^t	α_G^t	α_X^t	α_M^t	α_I^t	α_{IV}^t	β_K^t	β_{KIV}^t	β_{LD}^t	β_{LB}^t	α_T^t
1956-2006	1.05000	1.02596	1.00012	0.99944	0.99657	1.00192	0.99622	0.99965	1.01591	1.00086	1.00369	1.00858	0.99842
1956-1973	1.10436	1.05821	1.00026	0.99902	0.99563	1.00489	0.99625	0.99921	1.02315	1.00216	1.00660	1.01601	1.00048
1974-1979	1.02026	1.01260	1.00009	0.99903	0.99378	0.99753	0.99138	0.99962	1.01559	1.00034	1.00522	1.00516	0.99112
1980-1990	1.03702	1.01287	1.00002	0.99971	0.99611	1.00366	0.99624	0.99984	1.01571	1.00025	1.00198	1.01033	0.99967
1991-2001	1.00304	0.99677	1.00002	0.99970	0.99741	1.00184	0.99650	1.00000	1.00939	1.00003	1.00128	1.00016	0.99922
2002-2006	1.02185	1.01890	1.00010	1.00028	1.00241	0.99282	1.00124	1.00004	1.00504	0.99996	1.00043	1.00062	0.99519

The growth rate of real income ρ^t is decomposed into the product of a productivity growth factor τ^t, several factors for changes in real output prices α_C^t (domestic final consumption), α_N^t (non-profit institution final consumption), α_G^t (net government purchases from the market sector), α_X^t (exports), α_M^t (imports), α_I^t (investments in reproducible capital) and α_{IV}^t (inventory changes); and several factors for growth in input quantities β_K^t (capital services), β_{KIV}^t (inventory services), β_{LD}^t (land services) and β_{LB}^t (labour input). The last column in Table 3, α_T^t, is the effect of changes in the terms of trade, and is simply the product of the real price change factors of exports and imports α_X^t and α_M^t. Based on the average contribution factors, real income ρ^t grew at a very high average annual rate of 5%. On average, productivity growth τ^t accounted for 2.596% of

the real income growth, labour input growth β_{LB}^t for 0.858% and capital services growth β_K^t for 1.591%, while the contribution factors for changes in real output prices were –0.378% due to investment prices falling faster than consumption prices α_I^t; –0.343% per year due to export prices falling faster than consumption prices α_X^t; and 0.192% per year due to import prices falling more rapidly than consumption prices α_M^t. Thus, the effect of changes in the terms of trade on living standards α_T^t was very small for Japan on average over the entire period 1955–2006: an overall negative contribution of –0.151% per year. However, during shorter periods of time, it had a greater impact. It accounted for –0.888% of the average real income growth rate of 2.026% during the stagnation period between the oil shocks (1973–1979), and it also accounted for –0.481% of the average real income growth rate of 2.185% during the period of economic recovery (2002–2006). Productivity growth contributed the most to the overall growth in real income, and the growth in capital services made the second largest contribution on average over the years 1955–2006. However, the average contribution after 1973 of the capital services growth factor (1.196% per year) was larger than that of the productivity growth factor (0.837% per year). We can see that the importance of the contribution of capital services growth relative to the contribution of productivity growth increases over time. However, this is not the end of the story. During the period of economic recovery 2002–2006, the average contribution factor for productivity growth of 1.889% became greater than the average contribution factor of capital services growth, which was 0.504%. Thus, we can observe that the recent economic recovery was mostly boosted by productivity growth rather than by the deepening of capital.

The annual change information in the previous table can be converted into cumulative changes using equations (2.48). Table 2-4 and Figure 2-4, which follow, give this cumulative growth information.

Table 2-4: The Decomposition of Market Sector Real Income Level into Cumulative Productivity, Real Output Price Change and Input Quantity Contribution Factors

	ρ^t/ρ^{1955}	T^t	A_N^t	A_G^t	A_X^t	A_M^t	A_I^t	A_{IV}^t	B_K^t	B_{KIV}^t	B_{LD}^t	B_{LB}^t	A_T^t
1955	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1956	1.13349	1.06212	1.00065	1.00110	1.00404	0.99244	1.01873	1.00309	1.00105	1.00232	1.00037	1.04235	0.99644
1957	1.24491	1.11155	1.00098	1.00142	1.00069	0.99168	1.02629	1.00161	1.01005	1.00512	1.00123	1.07754	0.99236
1958	1.31208	1.12782	1.00101	1.00015	0.99252	1.01265	1.01772	1.00119	1.02528	1.00758	1.00227	1.09589	1.00507
1959	1.48175	1.24126	1.00107	0.99934	0.99124	1.01756	1.00799	1.00091	1.03968	1.00832	1.00406	1.11398	1.00865
1960	1.74267	1.39579	1.00131	0.99852	0.99103	1.02091	1.00555	0.99938	1.05620	1.01083	1.00537	1.14422	1.01175
1961	2.01192	1.54005	1.00142	0.99742	0.98406	1.02687	1.00987	0.99547	1.08300	1.01373	1.01053	1.16051	1.01050
1962	2.11133	1.55472	1.00142	0.99553	0.97427	1.03690	0.99379	0.99273	1.11686	1.01810	1.01574	1.18336	1.01022
1963	2.26726	1.64493	1.00137	0.99319	0.96918	1.04323	0.97113	0.99095	1.14909	1.01983	1.02076	1.19071	1.01108
1964	2.57097	1.78893	1.00188	0.99209	0.96751	1.04479	0.96746	0.99049	1.17985	1.02225	1.02608	1.20615	1.01084
1965	2.65093	1.79097	1.00173	0.99026	0.95863	1.05440	0.95009	0.98931	1.21313	1.02472	1.03326	1.22273	1.01078
1966	2.98631	1.90976	1.00198	0.98967	0.95383	1.05829	0.94871	0.98857	1.24063	1.02637	1.04918	1.24678	1.00943
1967	3.40364	2.05456	1.00229	0.98958	0.95043	1.06397	0.95137	0.98751	1.27062	1.02831	1.06497	1.26347	1.01123
1968	3.83398	2.23569	1.00239	0.98864	0.94514	1.07000	0.94475	0.98649	1.30672	1.03139	1.07165	1.27122	1.01130
1969	4.37284	2.41666	1.00269	0.98784	0.94259	1.07071	0.94632	0.98668	1.34737	1.03425	1.08803	1.27859	1.00924
1970	4.77412	2.52795	1.00328	0.98669	0.93842	1.07638	0.93824	0.98525	1.39113	1.03689	1.09630	1.29207	1.01009
1971	4.87863	2.49798	1.00370	0.98481	0.93327	1.08827	0.92267	0.98437	1.43717	1.03896	1.10313	1.30101	1.01565
1972	5.27410	2.60939	1.00422	0.98305	0.92592	1.09982	0.91827	0.98456	1.47422	1.03930	1.11045	1.30780	1.01834
1973	5.88872	2.73920	1.00461	0.98243	0.92415	1.09133	0.93354	0.98583	1.50872	1.03952	1.12538	1.32983	1.00855
1974	5.65097	2.65117	1.00483	0.98759	0.93159	1.05236	0.92834	0.98377	1.54466	1.04084	1.13467	1.31531	0.98037
1975	5.45848	2.64895	1.00481	0.98009	0.92301	1.05513	0.90331	0.98322	1.57086	1.04196	1.14203	1.29494	0.97390
1976	5.65659	2.69456	1.00494	0.97919	0.91293	1.06078	0.88654	0.98322	1.59177	1.04168	1.14631	1.33047	0.96841
1977	5.78190	2.70193	1.00500	0.97806	0.89799	1.07671	0.87792	0.98314	1.61147	1.04199	1.15288	1.34860	0.96687
1978	6.11481	2.78430	1.00496	0.97623	0.88455	1.10318	0.87451	0.98326	1.63129	1.04187	1.15733	1.35751	0.97582
1979	6.60372	2.94613	1.00518	0.97669	0.88994	1.07374	0.88588	0.98361	1.65537	1.04165	1.16107	1.37070	0.95557
1980	6.56409	2.93783	1.00515	0.97852	0.89172	1.03565	0.88527	0.98271	1.68014	1.04286	1.16510	1.38595	0.92352
1981	6.65838	2.91503	1.00498	0.97883	0.88987	1.04212	0.87678	0.98236	1.70397	1.04361	1.16717	1.40167	0.92735
1982	6.80973	2.92883	1.00497	0.97785	0.89169	1.04181	0.87094	0.98234	1.72640	1.04382	1.17001	1.41302	0.92897
1983	6.91155	2.88792	1.00492	0.97743	0.88424	1.05367	0.86457	0.98239	1.74676	1.04394	1.17237	1.44142	0.93169
1984	7.23964	2.98041	1.00492	0.97678	0.88314	1.06108	0.85863	0.98235	1.76610	1.04377	1.17295	1.44920	0.93708
1985	7.58065	3.04453	1.00495	0.97805	0.87588	1.07209	0.85172	0.98206	1.79968	1.04400	1.17642	1.46057	0.93902
1986	7.83873	3.04412	1.00498	0.97557	0.85832	1.11469	0.84506	0.98207	1.83018	1.04415	1.17702	1.47205	0.95677
1987	8.18206	3.09058	1.00503	0.97427	0.85368	1.12293	0.84328	0.98206	1.86376	1.04401	1.17916	1.48587	0.95862
1988	8.74062	3.19099	1.00508	0.97326	0.85162	1.12609	0.84494	0.98207	1.89384	1.04410	1.18180	1.50736	0.95901
1989	9.34557	3.30290	1.00513	0.97336	0.85314	1.12104	0.84872	0.98203	1.92728	1.04421	1.18396	1.52427	0.95641
1990	9.82460	3.38613	1.00537	0.97361	0.85239	1.11609	0.84982	0.98192	1.96483	1.04456	1.18663	1.53466	0.95134
1991	10.11592	3.39665	1.00546	0.97298	0.84780	1.12364	0.84749	0.98184	2.00223	1.04476	1.18789	1.54710	0.95263
1992	10.18736	3.36999	1.00543	0.97245	0.84388	1.13082	0.84398	0.98185	2.03686	1.04486	1.19053	1.54466	0.95427
1993	10.05262	3.30300	1.00533	0.97197	0.83654	1.13988	0.84144	0.98194	2.06202	1.04486	1.19284	1.53977	0.95356
1994	10.11815	3.30738	1.00537	0.97119	0.83345	1.14436	0.83816	0.98202	2.07711	1.04475	1.19482	1.54085	0.95377
1995	10.27214	3.32479	1.00543	0.97078	0.83187	1.14609	0.83721	0.98201	2.08840	1.04466	1.19599	1.54923	0.95340
1996	10.46240	3.37550	1.00547	0.97047	0.83519	1.13880	0.83335	0.98200	2.10296	1.04479	1.19766	1.55236	0.95112
1997	10.52201	3.39494	1.00552	0.97065	0.83594	1.13289	0.83002	0.98191	2.12154	1.04497	1.19921	1.54896	0.94703
1998	10.20747	3.27334	1.00559	0.97056	0.83791	1.13569	0.82589	0.98190	2.14055	1.04512	1.20094	1.54245	0.95160
1999	10.08113	3.25472	1.00552	0.97024	0.82854	1.14553	0.82155	0.98187	2.15278	1.04512	1.20152	1.53535	0.94912
2000	10.26470	3.27901	1.00552	0.97028	0.82469	1.14284	0.82227	0.98182	2.16316	1.04489	1.20262	1.55279	0.94249
2001	10.14186	3.26437	1.00561	0.97037	0.82832	1.13866	0.81764	0.98195	2.17735	1.04491	1.20341	1.53711	0.94317
2002	10.18978	3.29009	1.00543	0.97046	0.82884	1.13809	0.81583	0.98192	2.19032	1.04475	1.20384	1.52629	0.94329
2003	10.22826	3.30958	1.00536	0.97023	0.82575	1.13836	0.81504	0.98186	2.19764	1.04465	1.20480	1.52425	0.94000
2004	10.56405	3.39979	1.00543	0.97042	0.82601	1.13197	0.81719	0.98181	2.20705	1.04450	1.20466	1.53017	0.93503
2005	10.85811	3.50035	1.00573	0.97093	0.82983	1.11595	0.81871	0.98187	2.21749	1.04434	1.20480	1.53095	0.92605
2006	11.29320	3.58378	1.00613	0.97174	0.83831	1.09826	0.82271	0.98213	2.23280	1.04470	1.20598	1.54180	0.92068

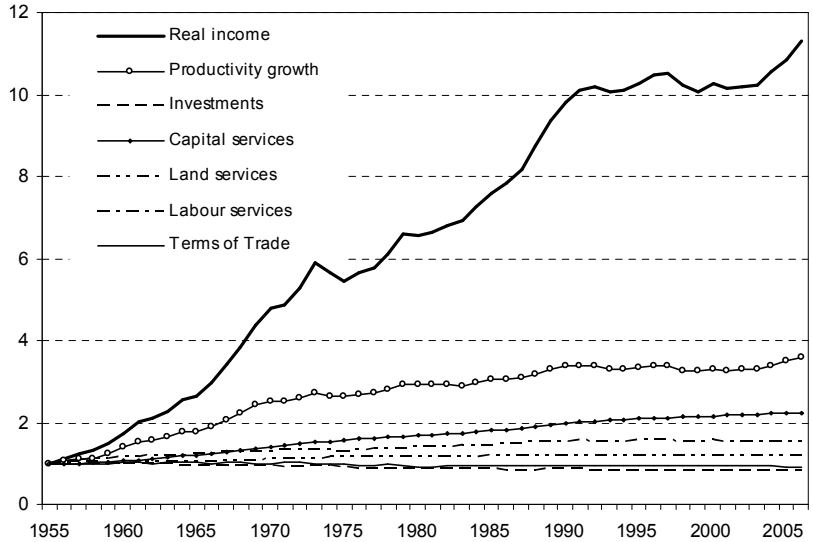

Figure 2-4: Decomposition of Market Sector Real Income Level into Factors

The level of real income ρ^t/ρ^{1955} is decomposed into the product of the level of productivity T^t; the levels of several real output prices A_C^t, A_N^t, A_G^t, A_X^t, A_M^t, A_I^t and A_{IV}^t; and the levels of several input quantities B_K^t, B_{KIV}^t, B_{LD}^t and B_{LB}^t. The last column in Table 2-4, A_T^t, is the cumulative effect of changes in the terms of trade, and is simply the product of the levels of real export and import prices A_X^t and A_M^t. Over the 52 year period, real income (from the gross domestic product point of view) grew over eleven times ($\rho^{2006}/\rho^{1955} = 11.2932$), which was spectacular.[46] From Table 2-4 above, it can be seen that productivity growth contributed the most to the overall growth in real income ($T^{2006} = 3.58378$), and that the growth in capital services made the next largest contribution ($B_K^{2006} = 2.23281$), followed by the growth in labour input ($B_{LB}^{2006} = 1.5418$). The change in real investment prices made a negative contribution ($A_I^{2006} = 0.82271$), as did the change in real export price ($A_X^{2006} = 0.83831$), and the change in real import prices made a modest positive contribution ($A_M^{2006} = 1.09826$). Thus, the change in the terms of trade made a modest negative contribution ($A_T^{2006} = 0.92068$). Figure 2-4 plots real income and the main factors contributing to its growth.

2.4.3 The Decomposition of Deflated NDP Growth

There is a severe flaw with all of the analysis presented in the previous sections. The problem is that depreciation payments are part of the user cost of capital for each asset, but depreciation does not provide households with any sustainable purchasing power. Hence our real income measure defined by (2.5) above is overstated.

To see why Gross Domestic Product overstates income, consider the model of production that is described by the following researchers:

"We must look at the production process during a period of time, with a beginning and an end. It starts, at the commencement of the Period, with an Initial Capital Stock; to this there is applied a Flow Input of labour, and from it there emerges a Flow Output called Consumption; then there is a Closing Stock of Capital left over at the end. If Inputs are the things that are put in, the Outputs are the things that are got out, and the production of the Period is considered in isolation, then the Initial Capital Stock is an Input. A Stock Input to the Flow Input of labour; and further (what is less well recognized in the tradition, but is equally clear when we are strict with translation), the Closing Capital Stock is an Output, a Stock Output to

[46] However, note that real income grew just over ten times by 1991, and in the 15 years since then, it has grown by only 11.6%.

match the Flow Output of Consumption Goods. Both input and output have stock and flow components; capital appears both as input and as output" John R. Hicks (1961; 23).

"The business firm can be viewed as a receptacle into which factors of production, or inputs, flow and out of which outputs flow...The total of the inputs with which the firm can work within the time period specified includes those inherited from the previous period and those acquired during the current period. The total of the outputs of the business firm in the same period includes the amounts of outputs currently sold and the amounts of inputs which are bequeathed to the firm in its succeeding period of activity." Edgar O. Edwards and Philip W. Bell (1961; 71–72).

Hicks and Edwards and Bell obviously had the same model of production in mind: in each accounting period, the business unit combines the capital stocks and goods in process that it has inherited from the previous period with "flow" inputs purchased in the current period (such as labour, materials, services and additional durable inputs) to produce current period "flow" outputs as well as end of the period depreciated capital stock components, which are regarded as outputs from the perspective of the current period (but will be regarded as inputs from the perspective of the next period).[47]

All of the "flow" inputs that are purchased during the period and all of the "flow" outputs that are sold during the period are the inputs and outputs that appear in the usual definition of cash flow. These are the flow inputs and outputs that are very familiar to national income accountants. But this is not the end of the story: the firm inherits an endowment of assets at the beginning of the production period and at the end of the period, the firm will have the net profit or loss that has occurred because of its sales of outputs and its purchases of inputs during the period. In addition, *it will have a stock of assets that it can use when it starts production in the following period*. Just focusing on the flow transactions that occur within the production period will not give a complete picture of the firm's productive activities. Hence, to get a complete picture of the firm's production activities over the course of a period, it is necessary to add the value of the closing stock of assets less the beginning of the period stock of assets to the cash flow that accrued to the firm from its sales and purchases of market goods and services during the accounting period.

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[47] For more on this model of production and additional references to the literature, see the Appendices in Diewert (1977) (1980). The usual user cost of capital can be derived from this framework if depreciation is independent of use.

We illustrate the theory described above by considering a very simple two output, two input model of the market sector. One of the outputs is output in year t, Y^t, and the other output is an investment good, I^t. One of the inputs is the flow of non-capital primary input X^t and the other input is K^t, capital services. Suppose that the average prices during period t of a unit of Y^t, X^t and I^t are $P_Y{}^t$, $P_X{}^t$ and $P_I{}^t$, respectively. Suppose further that the interest rate prevailing at the beginning of period t is r^t. The value of the beginning of period t capital stock is assumed to be $P_I{}^t$, the investment price for period t. The user cost of capital is calculated as $u^t = (r^t + \delta^t + \tau^t)P_I{}^t/(1 + r^t)$. As usual, it represents the price of capital services input. Thus, the period t profit of the market sector is expressed as follows:

$$(2.54)\ \Pi^t = P_Y{}^t Y^t + P_I{}^t I^t - P_X{}^t X^t - [(r^{t*} + \delta)P_I{}^t/(1 + r^{t*})]K^t.$$

Under the assumption of constant returns to scale, a zero profit condition should be satisfied such as $\Pi^t = 0$. Using this condition, we obtain the following *value of output equals value of input* equation:

$$(2.55)\ P_Y{}^t Y^t + P_I{}^t I^t = P_X{}^t X^t + [(r^{t*} + \delta)P_I{}^t/(1 + r^{t*})]K^t.$$

Equation (2.55) is essentially the closed economy counterpart to the (*gross*) *value of outputs equals* (*gross*) *value of primary inputs* equation (2.4), $P^t \cdot y^t = W^t \cdot x^t$, that we have been using thus far in this study. We now come to the point of this rather long digression: *the (gross) payments to primary inputs that is defined by the right hand side of (2.55) is not income*, in the sense of Hicks.[48] The owner of a unit of capital cannot spend the entire period t gross rental income $(r^{t*} + \delta)P_I{}^t/(1+r^{t*})$ on consumption during period t because the depreciation portion of the rental, $\delta P_I{}^t/(1+r^{t*})$, is required in order to keep his or her capital intact. Thus the owner of a new unit of capital at the beginning of period t loans the unit to the market sector and gets the gross return $(r^{t*} + \delta)P_I{}^t$ at the end of the period, plus the depreciated unit of the initial capital stock, which is worth only $(1 - \delta)P_I{}^t$. Thus, $\delta P_I{}^t$ of this gross return must be set aside in order to restore the lender

[48] We will use Hicks' third concept of income here: "Income No. 3 must be defined as the maximum amount of money which the individual can spend this week, and still be able to expect to spend this week, and still be able to expect to spend the same amount *in real terms* in each ensuing week." J.R. Hicks (1946; 174).

of the capital services to his or her original wealth position at the beginning of period t. This means that *period t Hicksian market sector income* is not the value of payments to primary inputs, $P_X^t X^t + [(r^{t*} + \delta)P_I^t/(1 + r^{t*})]K^t$; instead it is the value of payments to labour $P_X^t X^t$, plus the reward for waiting, $[r^{t*} P_I^t/(1 + r^{t*})]K^t$. Using this definition of market sector (net) Hicksian income, we can rearrange equation (2.55) as follows:

(2.56) Hicksian market sector income $\equiv P_X^t X^t + [r^{t*}P_I^t/(1 + r^{t*})]K^t$

$= P_Y^t Y^t + P_I^t I^t - [\delta P_I^t/(1 + r^{t*})]K^t$

= Value of consumption + value of gross investment − value of depreciation.

Thus, in this Hicksian net income framework, our new output concept is equal to our old output concept less the value of depreciation. We take the price of depreciation to be the corresponding investment price $P_I^t/(1 + r^{t*})$, and the quantity of depreciation is taken to be the depreciation rate times the beginning of the period stock, δK^t.

Hence the overstatement of income problem that is implicit in the approaches used in the previous sections can readily be remedied: all we need to do is to take the user cost formula for an asset and decompose it into two parts:

- One part that represents depreciation and foreseen obsolescence, $[\delta P_I^t/(1 + r^{t*})]$ and
- The remaining part that is the reward for postponing consumption, $[r^{t*}P_I^t/(1 + r^{t*})]$.

Thus, in this section, we split up each user cost times the beginning of the period stock K^t into the depreciation component $[\delta^t P_I^t/(1 + r^{t*})]K^t$ and the remaining term $[r^{t*}P_I^t/(1 + r^{t*})]$ K^t. We regard the second term as a genuine income component, but the first term is treated as an intermediate input cost for the market sector and is an offset to gross investment made by the market sector during the period under consideration. *Thus, in this section, we use a net product approach instead of a gross product approach*. Using the chained Törnqvist indexes, we construct prices and quantities of net investment P_{NI} and y_{NI} and "reward for waiting" capital service W_{KW} and x_{KW}. Waiting capital services is compared with the gross user cost concept that was used in the previous section, and net investment is also compared with the gross investment that was used in the previous section in Table 2-5 and Figures 2-5 and 2-6.

Table 2-5: The Quantity and Price of Gross Investment, Depreciation, Net Investment, Capital Services, Waiting Capital Services (Billion Yen)

	Price					Quantity				
	P_I^t	P_{DEP}^t	P_{NI}^t	W_K^t	W_{KW}^t	y_I^t	y_{DEP}^t	y_{NI}^t	x_K^t	x_{KW}^t
1955	1.00000	1.00000	1.00000	1.00000	1.00000	1618.6	1014.0	604.6	2070.8	1056.8
1956	1.07799	1.08491	1.06922	1.08191	1.07906	1988.9	1010.7	979.7	2082.1	1071.4
1957	1.14107	1.14181	1.13803	1.15677	1.17091	2412.8	1048.4	1367.4	2153.2	1104.9
1958	1.10683	1.11037	1.10165	1.08155	1.05376	2535.9	1122.9	1416.0	2283.2	1160.3
1959	1.10406	1.09317	1.10949	1.15227	1.21048	3008.9	1201.9	1810.0	2417.9	1216.2
1960	1.12828	1.09662	1.14554	1.27205	1.44628	4052.4	1298.5	2748.4	2584.6	1288.7
1961	1.21002	1.14280	1.24282	1.43460	1.72758	5030.0	1475.0	3540.9	2873.6	1410.5
1962	1.23850	1.16964	1.27208	1.41665	1.66188	5640.8	1725.0	3905.8	3267.5	1571.3
1963	1.24410	1.16251	1.28334	1.44748	1.73453	6310.8	1985.3	4319.5	3679.8	1740.2
1964	1.26346	1.17169	1.30732	1.50767	1.85201	7337.8	2255.2	5070.4	4109.1	1918.4
1965	1.28731	1.19228	1.33273	1.48527	1.77918	7589.2	2574.3	5027.5	4615.6	2128.0
1966	1.33289	1.21074	1.39186	1.56819	1.93720	8685.0	2855.8	5832.9	5071.3	2320.5
1967	1.38861	1.24330	1.45860	1.64780	2.07154	10288.5	3200.8	7066.5	5612.7	2543.6
1968	1.43391	1.27105	1.51184	1.72926	2.21700	12292.3	3669.4	8573.8	6327.7	2831.9
1969	1.49307	1.32124	1.57474	1.77701	2.25834	14428.7	4263.0	10103.6	7218.9	3186.3
1970	1.56173	1.37852	1.64856	1.80976	2.25524	16754.7	4976.3	11711.1	8287.8	3609.0
1971	1.59771	1.41885	1.68260	1.69396	1.93093	17458.3	5821.7	11668.4	9543.7	4094.7
1972	1.66686	1.45957	1.76613	1.68648	1.85512	19078.7	6557.8	12586.9	10678.6	4548.3
1973	1.93381	1.64631	2.07349	1.88426	2.05008	21349.8	7317.4	14101.7	11840.4	5006.6
1974	2.35584	2.00876	2.52401	2.16568	2.18505	20170.2	8134.3	12352.5	13105.6	5511.4
1975	2.45112	2.09258	2.62432	2.12267	1.95938	19936.6	8722.9	11665.3	14071.4	5928.3
1976	2.55621	2.14654	2.76375	2.28986	2.27639	20780.6	9209.9	12067.0	14889.3	6292.8
1977	2.66758	2.23783	2.88593	2.36134	2.31200	21330.6	9690.8	12202.2	15693.4	6648.4
1978	2.75138	2.27503	3.00240	2.50163	2.58908	23135.9	10218.0	13459.0	16534.2	6997.2
1979	2.94163	2.37953	3.25045	2.72584	2.96687	24388.7	10897.7	14093.8	17572.5	7404.6
1980	3.17559	2.53657	3.53519	2.82205	2.96381	24266.1	11596.0	13477.4	18658.7	7841.9
1981	3.23328	2.58179	3.60019	2.79355	2.82885	24825.2	12297.3	13476.4	19745.2	8275.5
1982	3.24736	2.60505	3.60435	2.76155	2.71767	24953.0	13006.2	13081.3	20819.3	8688.3
1983	3.23912	2.60486	3.58846	2.70685	2.58656	24683.6	13686.6	12345.5	21837.9	9070.1
1984	3.24600	2.59707	3.61110	2.80907	2.84527	25793.8	14373.0	12849.0	22836.7	9427.0
1985	3.22555	2.58060	3.58866	2.85633	2.98583	28001.8	15621.3	13935.3	24610.0	10041.4
1986	3.16648	2.53286	3.52345	2.79678	2.91406	29879.8	16865.7	14728.6	26285.3	10568.1
1987	3.14775	2.51094	3.51065	2.75369	2.84163	32170.8	18221.6	15812.4	28206.7	11232.7
1988	3.17448	2.52204	3.55162	2.79600	2.93024	36216.4	19513.5	18513.9	29998.1	11828.6
1989	3.26989	2.57824	3.67878	2.87215	3.03092	39761.6	21067.9	20576.9	32068.4	12467.2
1990	3.36079	2.63630	3.79504	2.87945	2.95053	43024.0	22885.1	22203.4	34502.6	13223.6
1991	3.40881	2.67085	3.85260	2.83715	2.77940	44379.5	24716.1	22132.6	37050.8	14069.8
1992	3.41944	2.66346	3.88374	2.80031	2.69507	43490.6	26456.4	20147.5	39485.2	14880.8
1993	3.41916	2.66361	3.88287	2.70041	2.42947	42076.7	27694.6	18053.4	41286.1	15526.7
1994	3.39038	2.63181	3.86525	2.66357	2.38827	41384.3	28380.9	16975.6	42385.3	15996.1
1995	3.36617	2.60287	3.85509	2.65553	2.41817	41473.8	28871.3	16720.6	43217.6	16383.1
1996	3.32070	2.55734	3.82112	2.67333	2.54540	43423.7	29593.2	17931.2	44299.0	16793.5
1997	3.31842	2.55981	3.81126	2.65758	2.49934	43709.9	30619.2	17492.4	45691.6	17224.4
1998	3.24115	2.53578	3.65180	2.50101	2.12572	40835.5	31679.7	14245.4	47134.2	17664.8
1999	3.16839	2.47610	3.57614	2.45188	2.10171	40556.0	32380.2	13511.9	48066.2	17926.4
2000	3.15573	2.45212	3.59639	2.47580	2.20944	40715.9	32980.4	13240.1	48854.5	18141.2
2001	3.06595	2.38618	3.48448	2.37012	2.04429	40621.5	33842.7	12566.8	49938.8	18395.7
2002	2.99292	2.32119	3.42590	2.36613	2.15363	38912.1	34627.7	10532.6	50928.0	18631.3
2003	2.95534	2.27919	3.42691	2.36658	2.23309	38698.7	35050.2	10062.1	51479.1	18782.7
2004	2.94031	2.26154	3.43047	2.46403	2.53391	39369.2	35619.2	10262.0	52176.3	18947.0
2005	2.92203	2.24905	3.40409	2.48954	2.62787	40884.9	36236.7	11153.9	52938.7	19138.9
2006	2.92197	2.25761	3.37541	2.51931	2.69424	41477.0	37180.6	11037.2	54059.6	19394.4

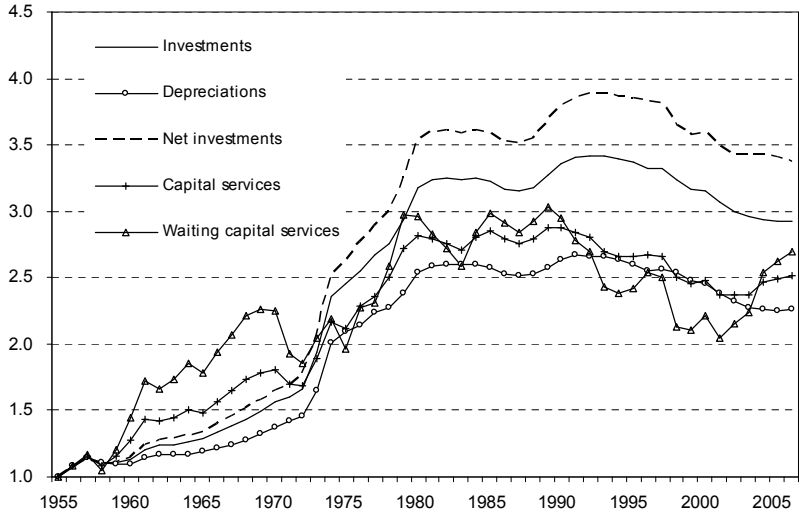

Figure 2-5: Price of Gross Investment, Depreciation, Net Investment, Capital Services and Waiting Capital Services

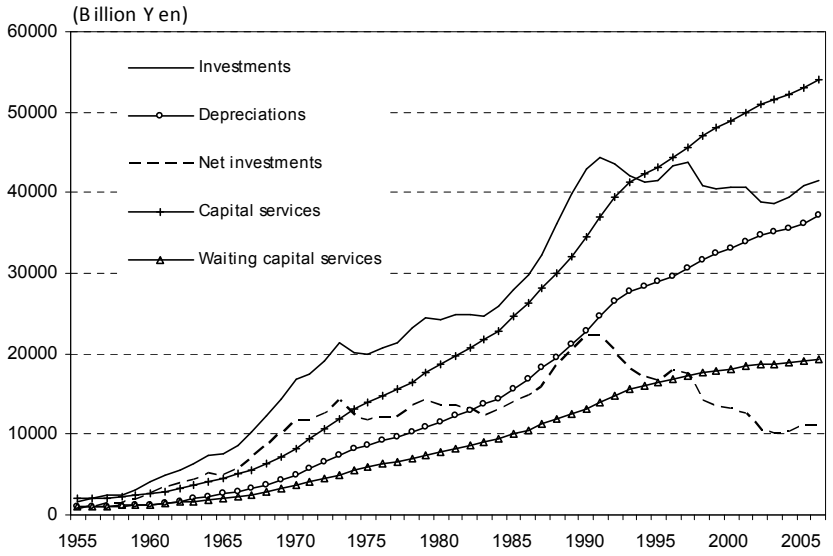

Figure 2-6: Quantity of Gross Investment, Depreciation, Net Investment, Capital Services and Waiting Capital Services (Billion Yen)

Note that the price of net investment increases much more rapidly than that of the gross investment. The explanation for this fact is as follows. Machinery and equipment investment has increased much more than structures investment in Japan, similar to the situation in most countries. But due to the computer chip revolution, the price of machinery and equipment investment declined much more rapidly than the price of structures (which tended to increase). But structures have a low depreciation rate, and machinery and equipment items have high depreciation rates. Therefore, when we subtract depreciation from gross investment, the weight of machinery and equipment is reduced in the net investment aggregate relative to the weight of structures, thus leading to a much higher rate of overall price increase in the net investment aggregate. Note that the price of waiting capital services increases much more rapidly than the other investment prices. This is mainly due to the fact that land services are included in the capital services, but that there is no investment in land. Hence, the situation is explained by the fact that land prices in Japan have been increasing much more rapidly than the prices of investment goods over most of the sample period. Note that gross investment grew 25.625 times over the entire period 1955–2006, whereas net investment grew only 17.667 times. Note that (traditional) capital services input grew 26.15 times over the entire period 1955–2006, whereas waiting capital services grew only 18.343 times.

All of the analysis presented in Section 2.4.2 above applies to the new situation with

obvious modifications. The counterpart to Table 2-3 in the previous section, which uses the new framework, is Table 2-6 below.

Table 2-6: The Decomposition of Market Sector Net Real Income Growth into Translog Productivity, Real Output Price Change, and Input Quantity Contribution Factors

	ρ^t	τ^t	α_N^t	α_G^t	α_X^t	α_M^t	α_I^t	α_{DEP}^t	α_{IV}^t	β_{KW}^t	β_{KIV}^t	β_{LD}^t	β_{LB}^t	α_T^t
1956	1.14124	1.07188	1.00075	1.00127	1.00466	0.99128	1.02165	0.98704	1.00357	1.00187	1.00268	1.00043	1.04905	0.99590
1957	1.10415	1.05360	1.00038	1.00037	0.99618	0.99912	1.00852	0.99689	0.99831	1.00489	1.00321	1.00098	1.03884	0.99530
1958	1.05559	1.01679	1.00004	0.99856	0.99066	1.02422	0.99045	1.00386	0.99951	1.00726	1.00280	1.00120	1.01950	1.01466
1959	1.14475	1.11487	1.00007	0.99908	0.99854	1.00551	0.98917	1.00637	0.99969	1.00669	1.00084	1.00203	1.01876	1.00404
1960	1.19189	1.14038	1.00027	0.99908	0.99976	1.00369	0.99728	1.00319	0.99829	1.00844	1.00278	1.00145	1.03045	1.00345
1961	1.15857	1.11541	1.00012	0.99877	0.99219	1.00648	1.00477	1.00173	0.99565	1.01385	1.00320	1.00570	1.01582	0.99863
1962	1.04195	1.01063	1.00000	0.99790	0.98893	1.01088	0.98231	1.00507	0.99695	1.01674	1.00479	1.00573	1.02194	0.99969
1963	1.07459	1.06498	0.99994	0.99737	0.99418	1.00681	0.97458	1.00877	0.99800	1.01555	1.00189	1.00553	1.00694	1.00095
1964	1.13610	1.09806	1.00057	0.99877	0.99808	1.00167	0.99579	1.00209	0.99948	1.01494	1.00265	1.00580	1.01446	0.99974
1965	1.02527	1.00128	0.99984	0.99794	0.98975	1.01028	0.97997	1.00623	0.99867	1.01572	1.00270	1.00782	1.01537	0.99993
1966	1.13169	1.07439	1.00027	0.99934	0.99441	1.00412	0.99838	1.00279	0.99916	1.01295	1.00180	1.01723	1.02200	0.99851
1967	1.14275	1.08478	1.00035	0.99989	0.99603	1.00598	1.00312	1.00073	0.99881	1.01375	1.00210	1.01677	1.01491	1.00199
1968	1.12786	1.09843	1.00010	0.99894	0.99381	1.00630	0.99228	1.00322	0.99885	1.01607	1.00333	1.00697	1.00682	1.00007
1969	1.13784	1.09042	1.00034	0.99911	0.99700	1.00074	1.00185	0.99967	1.00021	1.01744	1.00309	1.01701	1.00645	0.99773
1970	1.08637	1.05145	1.00065	0.99870	0.99508	1.00589	0.99050	1.00279	0.99839	1.01792	1.00286	1.00848	1.01177	1.00094
1971	1.00920	0.98620	1.00047	0.99786	0.99382	1.01245	0.98134	1.00451	0.99899	1.01704	1.00225	1.00702	1.00780	1.00619
1972	1.07891	1.05062	1.00059	0.99798	0.99107	1.01204	0.99459	1.00352	1.00023	1.01301	1.00037	1.00753	1.00592	1.00300
1973	1.11472	1.05726	1.00044	0.99929	0.99784	0.99124	1.01892	0.99828	1.00146	1.01138	1.00024	1.01529	1.01916	0.98909
1974	0.94066	0.96333	1.00024	1.00603	1.00926	0.95904	0.99362	1.00179	0.99761	1.01133	1.00145	1.00948	0.98747	0.96792
1975	0.96030	0.99854	0.99999	0.99117	0.98930	1.00306	0.96871	1.01126	0.99935	1.00819	1.00125	1.00754	0.98201	0.99233
1976	1.04438	1.02005	1.00014	0.99894	0.98731	1.00622	0.97845	1.01086	1.00000	1.00660	0.99969	1.00435	1.03195	0.99346
1977	1.02200	1.00301	1.00007	0.99866	0.98108	1.01740	0.98875	1.00443	0.99991	1.00623	1.00035	1.00664	1.01580	0.99816
1978	1.06224	1.03517	0.99995	0.99784	0.98273	1.02846	0.99550	1.00393	1.00014	1.00592	0.99986	1.00446	1.00763	1.01070
1979	1.07981	1.06743	1.00026	1.00054	1.00703	0.96931	1.01498	0.99795	1.00041	1.00706	0.99976	1.00373	1.01122	0.97612
1980	0.98564	0.99692	0.99997	1.00217	1.00233	0.95905	0.99918	1.00229	0.99895	1.00743	1.00135	1.00402	1.01289	0.96128
1981	1.01146	0.99092	0.99980	1.00036	0.99758	1.00727	0.98880	1.00444	0.99959	1.00677	1.00083	1.00207	1.01322	1.00482
1982	1.01944	1.00553	0.99999	0.99883	1.00239	0.99965	0.99219	1.00241	0.99997	1.00587	1.00023	1.00284	1.00946	1.00204
1983	1.01201	0.98365	0.99994	0.99951	0.99021	1.01336	0.99143	1.00330	1.00006	1.00501	1.00014	1.00237	1.02359	1.00344
1984	1.05171	1.03755	1.00000	0.99921	0.99855	1.00823	0.99197	1.00448	0.99996	1.00454	0.99981	1.00058	1.00632	1.00677
1985	1.04489	1.02509	1.00003	1.00151	0.99045	1.01207	0.99064	1.00421	0.99965	1.00791	1.00026	1.00346	1.00918	1.00240
1986	1.03065	0.99980	1.00005	0.99704	0.97655	1.04671	0.99083	1.00417	1.00002	1.00649	1.00017	1.00061	1.00922	1.02216
1987	1.03911	1.01800	1.00005	0.99843	0.99363	1.00871	0.99753	1.00161	0.99998	1.00771	0.99984	1.00214	1.01107	1.00228
1988	1.06749	1.03844	1.00006	0.99878	0.99717	1.00332	1.00232	0.99971	1.00001	1.00656	1.00011	1.00265	1.01709	1.00048
1989	1.06643	1.04162	1.00006	1.00013	1.00209	0.99474	1.00525	0.99917	0.99995	1.00674	1.00012	1.00216	1.01328	0.99682
1990	1.04517	1.03003	1.00028	1.00030	0.99896	0.99479	1.00153	1.00033	0.99988	1.00743	1.00039	1.00268	1.00809	0.99376
1991	1.02175	1.00368	1.00010	0.99923	0.99361	1.00806	0.99673	1.00163	0.99990	1.00753	1.00023	1.00127	1.00970	1.00161
1992	0.99798	0.99056	0.99996	0.99934	0.99443	1.00770	0.99503	1.00348	1.00001	1.00671	1.00011	1.00268	0.99810	1.00208
1993	0.97592	0.97585	0.99988	0.99940	0.98941	1.00978	0.99632	1.00179	1.00011	1.00500	1.00001	1.00236	0.99615	0.99908
1994	1.00578	1.00163	1.00005	0.99901	0.99546	1.00484	0.99521	1.00331	1.00010	1.00343	0.99987	1.00203	1.00086	1.00027
1995	1.01641	1.00647	1.00007	0.99948	0.99768	1.00186	0.99862	1.00162	0.99999	1.00279	0.99990	1.00120	1.00666	0.99953
1996	1.02106	1.01867	1.00005	0.99961	1.00486	0.99228	0.99441	1.00390	0.99998	1.00301	1.00015	1.00170	1.00247	0.99709
1997	1.00156	1.00702	1.00006	1.00023	1.00110	0.99369	0.99514	1.00223	0.99989	1.00316	1.00021	1.00159	0.99732	0.99478
1998	0.95572	0.95599	1.00009	0.99989	1.00289	1.00304	0.99388	1.00020	0.99999	1.00301	1.00018	1.00178	0.99481	1.00594
1999	0.98354	0.99298	0.99991	0.99958	0.98602	1.01087	0.99342	1.00435	0.99996	1.00170	1.00000	1.00060	0.99426	0.99674
2000	1.01889	1.00931	1.00000	1.00006	0.99417	0.99706	1.00109	1.00075	0.99994	1.00144	0.99973	1.00114	1.01421	0.99125
2001	0.98271	0.99441	1.00011	1.00011	1.00552	0.99540	0.99293	1.00414	1.00017	1.00169	1.00003	1.00082	0.98737	1.00090
2002	1.00280	1.00991	0.99978	1.00012	1.00079	0.99937	0.99722	1.00276	0.99996	1.00158	0.99980	1.00044	0.99117	1.00017
2003	1.00394	1.00747	0.99991	0.99971	0.99529	1.00030	0.99877	1.00232	0.99992	1.00107	0.99988	1.00101	0.99832	0.99559
2004	1.03544	1.03433	1.00009	1.00024	1.00040	0.99294	1.00332	0.99837	0.99993	1.00125	0.99982	0.99986	1.00489	0.99334
2005	1.02869	1.03721	1.00037	1.00066	1.00578	0.98235	1.00233	0.99823	1.00008	1.00155	0.99981	1.00014	1.00064	0.98803
2006	1.03834	1.02988	1.00050	1.00103	1.01266	0.98041	1.00605	0.99489	1.00033	1.00210	1.00043	1.00123	1.00890	0.99282

Average	ρ^t	τ^t	α_N^t	α_G^t	α_X^t	α_M^t	α_I^t	α_{DEP}^t	α_{IV}^t	β_{KW}^t	β_{KIV}^t	β_{LD}^t	β_{LB}^t	α_T^t
1956-2006	1.04857	1.02964	1.00014	0.99936	0.99605	1.00216	0.99564	1.00229	0.99961	1.00765	1.00097	1.00421	1.00983	0.99812
1956-1973	1.10575	1.06563	1.00029	0.99890	0.99511	1.00548	0.99586	1.00204	0.99912	1.01253	1.00242	1.00739	1.01811	1.00054
1974-1979	1.01823	1.01459	1.00011	0.99886	0.99279	0.99725	0.99000	1.00504	0.99957	1.00755	1.00039	1.00603	1.00601	0.98978
1980-1990	1.03400	1.01523	1.00002	0.99966	0.99544	1.00435	0.99561	1.00238	0.99982	1.00659	1.00030	1.00232	1.01213	0.99966
1991-2001	0.99830	0.99605	1.00003	0.99963	0.99683	1.00223	0.99571	1.00249	1.00000	1.00359	1.00004	1.00156	1.00017	0.99902
2002-2006	1.02184	1.02376	1.00013	1.00035	1.00298	0.99108	1.00154	0.99931	1.00005	1.00151	0.99995	1.00054	1.00078	0.99399

The growth rate of net real income ρ^t is decomposed into the product of a productivity growth factor τ^t; several factors for changes in real output prices α_C^t, α_N^t, α_G^t, α_X^t, α_M^t, α_I^t, α_{DEP}^t and α_{IV}^t; and several factors for growth in input quantities β_{KW}^t, β_{KIV}^t, β_{LD}^t and β_{LB}^t. The new results are quite interesting. While the average growth rate of real income ρ^t was 5% per year in the gross product approach, the average growth rate of net real income ρ^t now decreased by 0.143 percentage points per year to 4.857%. More importantly, there were some big shifts in the explanatory factors. Productivity growth

τ^t now accounts for 2.964% of the average net real income growth, compared to 2.596% of the average real income growth, an increase of 0.368 percentage points per year. Capital services growth $\beta_K{}^t$ accounted for 1.591% of the average real income growth, while waiting capital services growth $\beta_{KW}{}^t$ accounted for 0.765% of the average net real income, a decrease of 0.826 percentage points per year. The average contribution of labour input growth $\beta_{LB}{}^t$ has marginally increased from 0.858% per year to 0.983% per year. The average contributions of changes in real export prices $\alpha_X{}^t$ and real import prices $\alpha_M{}^t$ remain quite similar estimates in the previous product approach. Thus, as we stated in the previous analysis, the effect of changes in the terms of trade on living standards $\alpha_T{}^t$ was negligible for Japan on average over the entire period 1955–2006: an overall negative contribution of −0.188%. However, during shorter periods of time, it had greater impacts. It accounted for −1.022% of the average growth rate of net real income of 1.823% during the stagnation period between oil shocks (1973–1979), and it also accounted for −0.601% of the average growth rate of net real income 2.184% during the period of economic recovery (2002–2006). The negative contribution of the change in real investment prices $\alpha_I{}^t$ equal to −0.436% was offset by the positive contribution of the change in real depreciation prices $\alpha_{DEP}{}^t$ equal to 0.231%. Finally, we note that the productivity recovery in the period 2002–2006 is quite striking. Using the previous gross product approach, the average contribution of productivity growth during this period was 1.89% per year, and using the current net output model, its average contribution increases to a very respectable 2.376% per year.

The annual change information in the previous table can be converted into cumulative changes using equations (2.48). Table 2-7 and Figure 2-7 below give this cumulative growth information.

Table 2-7: The Decomposition of Market Sector Net Real Income Cumulative Growth into Productivity, Real Output Price Change, and Input Quantity Contribution Factors using the Translog Net Product Approach

	p^t/p^{1955}	T^t	A_N^t	A_G^t	A_X^t	A_M^t	A_I^t	A_{DEP}^t	A_{IV}^t	B_{KW}^t	B_{KIV}^t	B_{LD}^t	B_{LB}^t	A_T^t
1955	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1956	1.14124	1.07188	1.00075	1.00127	1.00466	0.99128	1.02165	0.98704	1.00357	1.00187	1.00268	1.00043	1.04905	0.99590
1957	1.26010	1.12932	1.00112	1.00164	1.00082	0.99041	1.03035	0.98397	1.00187	1.00677	1.00589	1.00141	1.08979	0.99122
1958	1.33015	1.14828	1.00117	1.00019	0.99147	1.01440	1.02051	0.98777	1.00138	1.01408	1.00871	1.00261	1.11104	1.00575
1959	1.52268	1.28019	1.00123	0.99927	0.99003	1.01998	1.00945	0.99406	1.00108	1.02086	1.00955	1.00464	1.13188	1.00981
1960	1.81487	1.45990	1.00150	0.99835	0.98979	1.02374	1.00671	0.99723	0.99936	1.02948	1.01236	1.00610	1.16635	1.01329
1961	2.10265	1.62838	1.00163	0.99713	0.98207	1.03038	1.01151	0.99895	0.99502	1.04374	1.01560	1.01184	1.18481	1.01190
1962	2.19086	1.64569	1.00162	0.99503	0.97119	1.04159	0.99362	1.00401	0.99198	1.06121	1.02046	1.01764	1.21080	1.01159
1963	2.35427	1.75262	1.00156	0.99241	0.96554	1.04868	0.96836	1.01281	0.99000	1.07771	1.02239	1.02326	1.21920	1.01254
1964	2.67468	1.92449	1.00214	0.99119	0.96369	1.05043	0.96428	1.01493	0.98949	1.09381	1.02511	1.02920	1.23683	1.01228
1965	2.74226	1.92695	1.00197	0.98915	0.95381	1.06123	0.94496	1.02125	0.98817	1.11100	1.02788	1.03725	1.25584	1.01221
1966	3.10339	2.07031	1.00224	0.98850	0.94848	1.06560	0.94343	1.02410	0.98734	1.12539	1.02972	1.05512	1.28347	1.01070
1967	3.54640	2.24582	1.00259	0.98839	0.94471	1.07197	0.94637	1.02485	0.98617	1.14087	1.03188	1.07282	1.30261	1.01271
1968	3.99986	2.46688	1.00270	0.98734	0.93887	1.07873	0.93906	1.02815	0.98503	1.15920	1.03532	1.08030	1.31149	1.01278
1969	4.55120	2.68994	1.00304	0.98646	0.93605	1.07952	0.94080	1.02781	0.98524	1.17941	1.03851	1.09867	1.31995	1.01049
1970	4.94427	2.82834	1.00369	0.98518	0.93145	1.08588	0.93186	1.03068	0.98366	1.20055	1.04148	1.10799	1.33549	1.01144
1971	4.98977	2.78932	1.00416	0.98307	0.92569	1.09940	0.91447	1.03532	0.98266	1.22101	1.04382	1.11577	1.34590	1.01770
1972	5.38352	2.93053	1.00476	0.98108	0.91743	1.11263	0.90952	1.03896	0.98289	1.23689	1.04420	1.12417	1.35387	1.02076
1973	6.00110	3.09832	1.00520	0.98038	0.91544	1.10288	0.92673	1.03717	0.98432	1.25096	1.04445	1.14135	1.37980	1.00962
1974	5.64498	2.98470	1.00545	0.98629	0.92392	1.05771	0.92081	1.03902	0.98197	1.26513	1.04597	1.15217	1.36251	0.97724
1975	5.42086	2.98035	1.00543	0.97759	0.91403	1.06095	0.89200	1.05072	0.98133	1.27550	1.04728	1.16086	1.33800	0.96974
1976	5.66141	3.04010	1.00558	0.97655	0.90244	1.06755	0.87277	1.06213	0.98133	1.28391	1.04695	1.16591	1.38075	0.96340
1977	5.78595	3.04926	1.00565	0.97524	0.88536	1.08613	0.86296	1.06683	0.98124	1.29191	1.04732	1.17365	1.40256	0.96162
1978	6.14605	3.15650	1.00560	0.97313	0.87007	1.11704	0.85907	1.07103	0.98137	1.29956	1.04717	1.17889	1.41326	0.97190
1979	6.63654	3.36935	1.00586	0.97366	0.87619	1.08275	0.87194	1.06883	0.98177	1.30873	1.04692	1.18328	1.42912	0.94870
1980	6.54124	3.35898	1.00583	0.97577	0.87823	1.03841	0.87123	1.07128	0.98074	1.31845	1.04833	1.18803	1.44754	0.91196
1981	6.61617	3.32848	1.00563	0.97613	0.87610	1.04595	0.86147	1.07604	0.98034	1.32738	1.04921	1.19049	1.46667	0.91636
1982	6.74479	3.34690	1.00562	0.97498	0.87819	1.04559	0.85475	1.07863	0.98031	1.33518	1.04945	1.19387	1.48054	0.91823
1983	6.82582	3.29217	1.00555	0.97450	0.86959	1.05956	0.84742	1.08219	0.98037	1.34186	1.04959	1.19670	1.51547	0.92138
1984	7.17875	3.41580	1.00555	0.97374	0.86833	1.06828	0.84062	1.08705	0.98033	1.34796	1.04939	1.19739	1.52504	0.92762
1985	7.50102	3.50150	1.00558	0.97521	0.86004	1.08118	0.83275	1.09163	0.97999	1.35861	1.04967	1.20153	1.53904	0.92985
1986	7.73095	3.50081	1.00563	0.97232	0.83987	1.13168	0.82511	1.09618	0.98001	1.36743	1.04985	1.20226	1.55324	0.95046
1987	8.03333	3.56383	1.00568	0.97079	0.83451	1.14153	0.82308	1.09794	0.97999	1.37797	1.04968	1.20483	1.57043	0.95263
1988	8.57546	3.70083	1.00574	0.96960	0.83215	1.14532	0.82498	1.09763	0.98000	1.38701	1.04979	1.20802	1.59728	0.95308
1989	9.14513	3.85485	1.00580	0.96972	0.83389	1.13929	0.82931	1.09671	0.97995	1.39635	1.04992	1.21063	1.61849	0.95005
1990	9.55820	3.97062	1.00609	0.97002	0.83302	1.13336	0.83058	1.09707	0.97983	1.40672	1.05033	1.21387	1.63159	0.94411
1991	9.76607	3.98523	1.00619	0.96927	0.82770	1.14249	0.82787	1.09886	0.97973	1.41732	1.05057	1.21541	1.64741	0.94564
1992	9.74634	3.94760	1.00616	0.96863	0.82309	1.15128	0.82376	1.10268	0.97974	1.42682	1.05069	1.21867	1.64429	0.94760
1993	9.51160	3.85227	1.00603	0.96805	0.81437	1.16253	0.82073	1.10465	0.97985	1.43396	1.05070	1.22155	1.63795	0.94673
1994	9.56654	3.85854	1.00609	0.96709	0.81067	1.16816	0.81679	1.10831	0.97995	1.43888	1.05056	1.22403	1.63936	0.94699
1995	9.72349	3.88351	1.00616	0.96659	0.80878	1.17032	0.81567	1.11011	0.97994	1.44290	1.05045	1.22549	1.65028	0.94654
1996	9.92824	3.95602	1.00620	0.96621	0.81271	1.16128	0.81110	1.11444	0.97993	1.44724	1.05061	1.22758	1.65435	0.94379
1997	9.94376	3.98379	1.00626	0.96644	0.81360	1.15396	0.80716	1.11692	0.97982	1.45181	1.05083	1.22953	1.64992	0.93887
1998	9.50347	3.80847	1.00636	0.96633	0.81596	1.15747	0.80222	1.11714	0.97981	1.45618	1.05102	1.23172	1.64137	0.94445
1999	9.34707	3.78172	1.00626	0.96592	0.80455	1.17005	0.79694	1.12200	0.97976	1.45866	1.05102	1.23247	1.63194	0.94136
2000	9.52364	3.81693	1.00626	0.96597	0.79986	1.16661	0.79781	1.12284	0.97970	1.46075	1.05073	1.23387	1.65513	0.93313
2001	9.35896	3.79559	1.00638	0.96608	0.80428	1.16124	0.79217	1.12749	0.97987	1.46322	1.05075	1.23489	1.63423	0.93396
2002	9.38517	3.83320	1.00616	0.96620	0.80492	1.16051	0.78997	1.13060	0.97983	1.46553	1.05055	1.23543	1.61979	0.93412
2003	9.42213	3.86183	1.00607	0.96591	0.80113	1.16086	0.78900	1.13322	0.97976	1.46710	1.05042	1.23668	1.61707	0.93000
2004	9.75600	3.99442	1.00616	0.96615	0.80145	1.15266	0.79162	1.13137	0.97969	1.46893	1.05023	1.23650	1.62497	0.92380
2005	10.03593	4.14307	1.00653	0.96678	0.80608	1.13232	0.79346	1.12937	0.97977	1.47121	1.05003	1.23668	1.62600	0.91274
2006	10.42070	4.26684	1.00702	0.96778	0.81628	1.11015	0.79826	1.12360	0.98009	1.47429	1.05049	1.23820	1.64047	0.90619

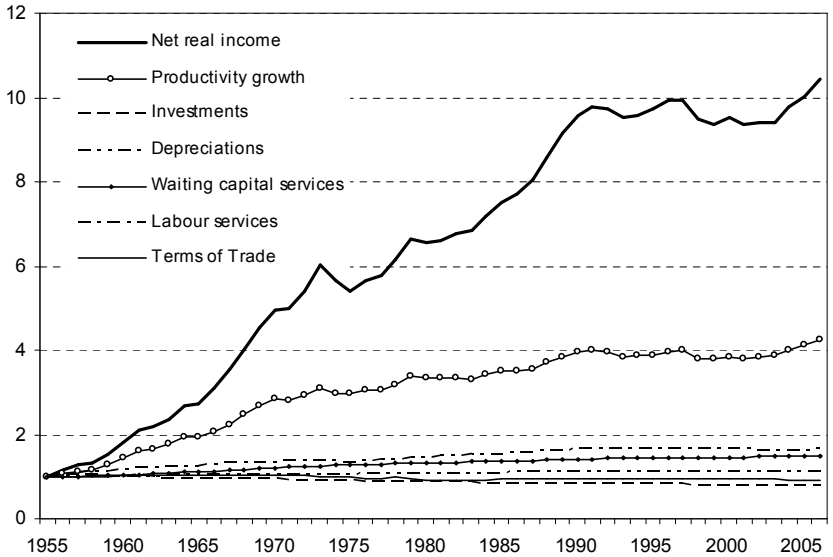

Figure 2-7: Decomposition of Market Sector Net Real Income Level into Factors

The current level of net real income compared to its level in 1955, ρ^t/ρ^{1955}, is decomposed into the product of the level of productivity factor T^t; the levels of several real output price factors A_C^t, A_N^t, A_G^t, A_X^t, A_M^t, A_I^t, A_{DEP}^t and A_{IV}^t; and the levels of several input quantity factors B_{KW}^t, B_{KIV}^t, B_{LD}^t and B_{LB}^t. Over the 52 year period, net real income grew about 10.4 times (ρ^{2006}/ρ^{1955} = 10.42067). From Table 2.7 above, it can be seen that productivity growth contributed the most to the overall growth in real income (T^{2006} = 4.26684), and that the growth in labour input made the next largest contribution (B_{LB}^{2006} = 1.64047), followed by the growth in waiting capital services (B_{KW}^{2006} = 1.05049) and the growth in land input (B_{LD}^{2006} = 1.2382). There were smaller effects due to the changes in real output prices, such as the contributions of the changes in depreciation prices (A_{DEP}^{2006} = 1.12359), real export prices (A_X^{2006} = 0.81628) and real import prices (A_M^{2006} = 1.11014). The combined effects of cumulative changes in the output prices relative to the price of household consumption were negligible (0.94832) over the entire sample period. Figure 2-7 plots net real income and the main factors contributing to its growth.

2.5 Conclusion

On a theoretical level, the results of Diewert and Morrison (1986) were modified to give an exact decomposition of the growth in real incomes generated by the market sector. Empirically, we analyzed the contribution to Japanese living standards according to the income generated by the market sector. First, we constructed a database of inputs and outputs of the market sector. Second, we took a measured conventional TFP growth approach by applying Fisher quantity indexes. We calculated traditional TFP in the case in which land and inventories were included as primary inputs, and two other cases in which at least one of these two inputs was excluded. Our results showed the importance of the inclusion of land and inventories when computing TFP growth rates.[49] Third, we applied the exact decomposition results to decompose the growth of Japanese real income into the contributions of changes in real output prices and changes in primary input quantities. We observed that productivity growth and the growth of capital services are the main contributors to real income growth when a traditional user cost approach to the pricing of primary inputs is used. We also observed that changes in the terms of trade had very small effects on real income on average. Fourth, we moved to our theoretically

[49] Nomura (2004) also made this observation. We noted also that balancing real rates of return were greatly exaggerated when inventory and land inputs were omitted.

preferred measure of real net income. We applied our theoretical results to decompose the growth of Japanese real net income into the contributions of changes in real output prices and changes in input quantities. We observed that in this net approach, productivity growth was still the largest contributor (and was an even more important factor than before). However, the contribution of capital services was greatly reduced in this net product approach, becoming smaller than the contribution of labour input.

A few problems with our approach should be mentioned:

- Our labour aggregate has not been sufficiently disaggregated to capture changes in the average quality of labour input over time. Characteristics such as education, sex, age and experience should be taken into account when constructing a measure of aggregate labour input.
- We included non-profit institutes serving households (NPISHs) as part of our market sector. However, goods and services produced by NPISHs are traded free or at prices that are not economically significant. Since our theoretical approach relies on competitive profit maximizing behaviour, NPISHs should be netted out of the market sector.
- We have not dealt with intangible assets and the problems associated with accounting for R&D investments.
- We were not able to provide sectoral contributions analysis. Since a primary focus of our chapter was to look at the effects of changes in the prices of exports and imports on living standards, we could not extend our analysis to industrial sectors because reliable data on exports by industry and imports used by industry are not available.

However, it would be straightforward to extend our analysis to datasets in which a more detailed breakdown of exports and imports by commodity classification is available. This would enable researchers to give more precise estimates of the effects on the income produced by the market sector of an oil shock or any other unusual movement in the prices of internationally traded goods.

Chapter 3. An Economic Approach to the Measurement of Productivity Growth Using Differences Instead of Ratios[50]

3.1 Introduction

The recent boom in the prices of natural resources and the low prices of manufactured goods produced by some developing countries has stimulated interest in the effects of changes in the prices of exports and imports on living standards for a country. An improvement in a country's terms of trade has much the same effect as an improvement in a country's productivity growth. Diewert (1983), Diewert and Morrison (1986), Morrison and Diewert (1990), Kohli (1990) (1991) (2003) (2004a) (2004b) (2006) (2007), Diewert, Mizobuchi and Nomura (2005) and Diewert and Lawrence (2006) have all developed production theory methodologies which enable one to obtain exact index number estimates of the contributions of productivity growth and changes in a country's terms of trade. The present chapter is yet another contribution to this exact index number literature.

Many observers use labour productivity (real output divided by labour input) as an approximate welfare measure. However, improvements in a country's terms of trade (an increase in the price of exports relative to the price of imports) do not show up in labour productivity measures, because the effects of changes in output and intermediate input prices are removed from the measure of output growth. The primary contribution of the present chapter is a proposed improved measure of labour productivity that will allow us to assess the relative contributions to welfare of an improvement in Total Factor Productivity and of changes in real international prices. Our proposed measure is equal to the nominal income generated by the market sector of the economy divided by the product of the price of consumption times the quantity of labour input.[51] This proposed measure can be modeled using production theory and exact index number techniques. This approach will be implemented for the Japanese business sector in section 3.5 of this chapter. The main determinants of growth for this measure are:

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[50] It includes joint work with W. Erwin Diewert.

[51] The analysis presented in sections 3.2–3.4 below is somewhat more general. Instead of deflating nominal income by a single price, we deflate by a fixed weight price index of outputs and instead of deflating nominal income by labour input, we deflate by a fixed weight quantity index of inputs. However, in our empirical work in section 3.5, we will specialize these indexes as indicated.

- Technical progress or improvements in the Total Factor Productivity of the market sector of the economy;
- Changes in domestic output prices or the prices of internationally traded goods and services relative to the price of consumption; and
- The effects of capital deepening; i.e., growth in market sector capital input relative to the growth of market sector labour input.

Section 3.2 introduces the market sector nominal output function. With a constant returns to scale technology, the value of output is distributed to the primary inputs that produced the market sector outputs. Thus the nominal output function can also be interpreted as a nominal income function. In section 3.3, this income function is used to provide theoretical definitions of the effects of real output price and relative input quantity changes on deflated market sector real income. A formal definition of productivity change is also provided. Section 3.4 introduces the normalized quadratic income function. Using this functional form to represent the technology of the market sector in each period enables us to obtain empirically observable exact measures of the effects of real output price and relative input quantity changes on deflated market sector real income. Appendix C shows that this functional form is a flexible functional form.

It turns out that our empirically observable measures of price, quantity and productivity change are equal to measures of price and quantity change that were originally suggested by Bennet (1920). Thus our chapter is also a contribution to the recent literature on the Bennet indicators of price and quantity change.[52]

In section 3.4, our decomposition of real income per unit labour input change into explanatory factors largely parallels the corresponding decomposition of nominal income change obtained by Diewert and Morrison (1986) and Kohli (1990), who derived exact results using the Translog functional form. An advantage of the present approach over the Translog approach is that our present approach is valid even if some individual prices or quantities are zero whereas the Translog approach fails if an exogenous price or quantity approaches zero. Since zero prices and quantities do occur empirically, applied

[52] Our results in section 3.4 are similar in part to the results obtained by Balk, Färe and Grosskopf (2004). See Diewert (1992) (2005c), Chambers (2001) (2002) and Balk (2003) (2007) for additional material on the Bennet indicators of price and quantity change.

welfare economists may find our present approach useful in these situations. Another advantage of our suggested difference approach is that it is valid even if value subaggregates (such as net exports or inventory change) change sign over the two periods being compared where as the traditional ratio approach to index number theory breaks down under these conditions.

Section 3.5 applies our methodology based on the difference approach to the analysis of the market sector in Japanese economy. We decompose changes in the (gross) real income per unit labour and the net real income per unit labour input for the years 1955–2006. We found that productivity growth and the growth in capital services were the two main contributors to the growth in real income per unit labour and net real income per unit labour. It is also shown that changes in terms of trade had smaller effects on them on average.

3.2 The Production Theory Framework

In this section, we outline the economic approach to production theory which will be used in the remainder of the chapter.[53] The main reference is Diewert and Morrison (1986).[54] The economic approach to production theory relies on the assumption of (competitive) *optimizing behaviour* on the part of producers. In our empirical work, we will apply the economic approach to the market sector of the Japanese economy. Thus we will consider only that part of the Japanese economy that is motivated by profit maximizing behaviour.[55]

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[53] This material is drawn from Diewert, Mizobuchi and Nomura (2005) and Diewert and Lawrence (2006).

[54] The theory also draws on Samuelson (1953), Fisher and Shell (1972), Diewert (1974; 133-141) (1980) (1983; 1077-1100), Archibald (1977), Fox and Kohli (1998), Kohli (1978) (1990) (1991) (2003) (2004a) (2004b) (2006) (2007) and Morrison and Diewert (1990).

[55] The Japanese market sector excludes all of the general government sectors such as schools, hospitals, universities, defence and public administration where no independent measures of output can be obtained. For owner occupied housing, output is equal to input and hence no productivity improvements can be generated by this sector according to SNA conventions. However, we do include the consumption of residential housing services in our model.

We assume that the market sector of the economy produces quantities of M (net)[56] outputs, $Y \equiv [Y_1,...,Y_M]$, which are sold at the positive producer prices, $P \equiv [P_1,...,P_M]$. We further assume that the market sector uses positive quantities of N primary inputs, $X \equiv [X_1,...,X_N]$, which are purchased at the positive primary input prices $W \equiv [W_1,...,W_N]$. In period t, we assume that there is a feasible set of output vectors Y that can be produced by the market sector if the vector of primary inputs X is utilized by the market sector; denote this period t production possibilities set by S^t. We assume that S^t is a closed convex cone that exhibits a free disposal property.[57]

Given a vector of output prices P and a vector of available primary inputs X, we define *the period t market sector income function*, $g^t(P,X)$, as follows:[58]

$$(3.1)\ g^t(P,X) \equiv \max_Y \{P\cdot Y : (Y,X) \text{ belongs to } S^t\}\ ; \qquad t = 0,1,2, \ldots .$$

Thus market sector nominal income depends on t (which represents the period t

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[56] If the mth commodity is an import (or other produced input) into the market sector of the economy, then the corresponding quantity y_m is indexed with a negative sign. We will follow Kohli (1978) (1991) and Woodland (1982) in assuming that imports flow through the domestic production sector and are "transformed" (perhaps only by adding transportation, wholesaling and retailing margins) by the domestic production sector. The recent textbook by Feenstra (2004; 76) also uses this approach.

[57] For a more explanation for the meaning of these properties, see Diewert (1973) (1974; 134) or Woodland (1982) or Kohli (1978) (1991). The assumption that S^t is a cone means that the technology is subject to constant returns to scale. This is an important assumption since it implies that the value of outputs should equal the value of inputs in equilibrium. In our empirical work, we use an ex post rate of return in our user costs of capital, which forces the value of inputs to equal the value of outputs for each period.

[58] The function g^t is known as the *GDP function* or the *national product function* in the international trade literature (see Kohli (1978) (1991), Woodland (1982) and Feenstra (2004; 76)). It was introduced into the economics literature by Samuelson (1953). Alternative terms for this function include: (i) the *gross profit function*; see Gorman (1968); (ii) the *restricted profit function*; see Lau (1976) and McFadden (1978); and (iii) the *variable profit function*; see Diewert (1973) (1974). However, we will call it the (nominal) income function, since it also defines the amount of income that is distributed to the vector of primary inputs that is used by the market sector. The function $g^t(P,X)$ will be linearly homogeneous and convex in the components of P and linearly homogeneous and concave in the components of X; see Diewert (1973) (1974; 136). Notation: $P\cdot Y \equiv \sum_{m=1}^{M} P_m Y_m$.

technology set S^t), on the vector of output prices P that the market sector faces and on X, the vector of primary inputs that is available to the market sector.

If P^t is the period t output price vector and X^t is the vector of inputs used by the market sector during period t and if the income function is differentiable with respect to the components of P at the point P^t,X^t, then the period t vector of market sector outputs Y^t will be equal to the vector of first order partial derivatives of $g^t(P^t,X^t)$ with respect to the components of P; i.e., we will have the following equations for each period t:[59]

(3.2) $Y^t = \nabla_P g^t(P^t,X^t)$; $\qquad t = 0,1,2, \ldots$.

Thus the period t market sector supply vector Y^t can be obtained by differentiating the period t market sector income function with respect to the components of the period t output price vector P^t.

If the income function is differentiable with respect to the components of X at the point P^t,X^t, then the period t vector of input prices W^t will be equal to the vector of first order partial derivatives of $g^t(P^t,X^t)$ with respect to the components of X; i.e., we will have the following equations for each period t:[60]

(3.3) $W^t = \nabla_X g^t(P^t,X^t)$; $\qquad t = 0,1,2, \ldots$.

Thus the period t market sector input prices W^t paid to primary inputs can be obtained by differentiating the period t market sector income function with respect to the components of the period t input quantity vector X^t.

The constant returns to scale assumption on the technology sets S^t implies that the value of outputs will equal the value of inputs in period t; i.e., we have the following relationships:

(3.4) $g^t(P^t,X^t) = P^t \cdot Y^t = W^t \cdot X^t$; $\qquad t = 0,1,2, \ldots$.

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[59] These relationships are due to Hotelling (1932; 594). Note that $\nabla_P g^t(P^t,X^t) \equiv [\partial g^t(P^t,X^t)/\partial P_1, \ldots, \partial g^t(P^t,x^t)/\partial P_M]$.

[60] These relationships are due to Samuelson (1953) and Diewert (1974; 140). Note that $\nabla_X g^t(P^t,X^t) \equiv [\partial g^t(P^t,X^t)/\partial X_1, \ldots, \partial g^t(P^t,X^t)/\partial X_N]$.

The above material will be useful in what follows. Note that our focus is not on the value of outputs generated by the market sector; instead our focus is on the amount of nominal income generated by the market sector. Since the value of market sector production is distributed to the factors of production used by the market sector, nominal market sector output will be equal to nominal market sector income; i.e., from (3.4), we have $g(P^t,X^t) = P^t \cdot Y^t = W^t \cdot X^t$. We will choose to measure the *real income generated by the market sector in period t*, r^t, in terms of the number of fixed basket consumption bundles (with weights represented by the nonnegative, nonzero vector $\mu > 0_M$) that the nominal income generated by the market sector could purchase in period t[61]; i.e., we define r^t as follows:

$$
\begin{aligned}
(3.5)\ r^t &\equiv W^t \cdot X^t / P^t \cdot \mu && t = 0,1,2, \ldots \\
&= (1/P^t \cdot \mu) g^t(P^t,X^t) && \text{using (3.4)} \\
&= g^t(P^t/P^t \cdot \mu, X^t) && \text{using the linear homogeneity of } g^t(P,X) \text{ in } P \\
&= g^t(p^t,X^t) && \text{using definition (3.6) below} \\
&= p^t \cdot Y^t && \text{using (3.4) and (3.6)}
\end{aligned}
$$

where $P^t \cdot \mu > 0$ is a *period t consumption expenditures deflator*[62] and the market sector period t *real output price* p^t and *real input price* w^t vectors are defined as the corresponding nominal price vectors deflated by the consumption expenditures deflator; i.e., we have the following definitions:[63]

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[61] This measure can be interepreted as an approximate welfare measure associated with market sector production. Since some of the primary inputs used by the market sector can be owned by foreigners, our measure of *domestic* welfare generated by the market production sector is only an approximate one. Moreover, our suggested approximate welfare measure is not sensitive to the distribution of the income that is generated by the market sector.

[62] In our empirical work, we will form 7 subaggregates of Japanese net outputs where the first subaggregate is consumption. We will choose our 7 dimensional μ vector to be the first unit vector. Thus we simply deflate the period t P^t and W^t price vectors by P_1^t, the price of consumption in period t.

[63] Our approach to measuring real income is similar to the approach advocated by Kohli (2004b; 92), except he essentially deflates nominal GDP by the domestic expenditures deflator rather than just the domestic (household) expenditures deflator; i.e., he deflates by the deflator for C+G+I, whereas we suggest deflating by the deflator for C. Another difference in his approach compared to the present approach is

(3.6) $p^t \equiv P^t/P^t\cdot\mu$; $w^t \equiv W^t/P^t\cdot\mu$; $t = 0,1,2, \ldots$.

The first and last equality in (3.5) imply that period t real income, r^t, is equal to the period t income function, evaluated at the period t real output price vector p^t and the period t input vector X^t, $G^t(p^t,X^t)$. Thus *the growth in real income over time can be explained by three main factors: t or technical progress or Total Factor Productivity growth* (shifts in g^t)*, growth in real output prices* (changes in p^t) *and growth in primary input quantities* (changes in X^t).

However, rather than find an exact decomposition for the change in real income over time into explanatory factors, the methodology to be developed in the following section only allows us to find an exact decomposition for the change in real income divided by an index of primary inputs. Thus define the *real income generated by the market sector in period t per unit primary input*, ρ^t, as our previous real income measure r^t divided by a period t index of primary inputs used, $X^t\cdot\theta$ (with weights represented by the nonnegative, nonzero vector $\theta > 0_N$); i.e., define ρ^t as follows:

(3.7) $\rho^t \equiv W^t\cdot X^t/[(P^t\cdot\mu)(X^t\cdot\theta)]$ $t = 0,1,2, \ldots$

$= (W^t/P^t\cdot\mu)\cdot(X^t/X^t\cdot\theta)$ rearranging terms

$= w^t\cdot x^t$ using definitions (3.6) and (3.8)

$= p^t\cdot y^t$ using (3.5) and (3.8)

$= g^t(p^t,x^t)$

where the last equality follows using (3.4) and the linearly homogeneity of $g^t(P,X)$ in both P and X and where the market sector period t *relative output quantity and input quantity* vectors y^t and x^t are defined as the corresponding quantity vectors Y^t and X^t deflated by the primary input index deflator $X^t\cdot\theta$ [64]; i.e., we have the following definitions:

---

that we restrict our analysis to the market sector GDP, whereas Kohli deflates all of GDP (probably due to data limitations). Our treatment of the balance of trade surplus or deficit is also different.

[64] In our empirical work, we will form the following aggregates of Japanese primary inputs: K (capital services), KIV (inventory services), LD (land services) and LB (labour input). We will choose our 4 dimensional θ vector to be the first unit vector. Thus we simply deflate the period t quantity vectors Y^t and X^t by $X_L{}^t$, the quantity of labour in period t. We regard the resulting measure ρ^t defined by (3.7) as *an improved measure of labour productivity* since it takes into account changes in the prices of investment,

(3.8) $y^t \equiv Y^t/X^t \cdot \theta$; $x^t \equiv X^t/X^t \cdot \theta$; $t = 0,1,2, \ldots$.

Using the linear homogeneity properties of the income function $g^t(P,X)$ in P and X separately, we can show that the following counterparts to the relations (3.2) and (3.3) hold using the real prices p^t and w^t defined by (6) and the deflated quantities y^t and x^t defined by (3.8):[65]

(3.9) $y^t = \nabla_p g^t(p^t,x^t)$; $t = 0,1,2, \ldots$
(3.10) $w^t = \nabla_x g^t(p^t,x^t)$; $t = 0,1,2, \ldots$.

In the following section, we will define various explanatory factors that will be used to explain the *change* in the real income generated by the market sector in period t per unit primary input over the previous period, $\rho^t - \rho^{t-1}$, into explanatory factors which are also differences. There will be three sets of explanatory factors that are associated with:

- Changes in real output prices p^t;
- Changes in relative primary input quantities x^t and
- Changes in technology; i.e., shifts in the income functions g^t.

3.3 The Theoretical Explanation of Per Unit Primary Input Real Income Growth using Differences

Now we are ready to define a family of *period t productivity growth factors or technical progress shift factors* $\tau(p,x,t)$ using the difference approach as opposed to the usual ratio

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exports and imports relative to the price of consumption. Traditional measures of labour productivity cannot take into account such price changes; in particular, they cannot take into account changes in the country's terms of trade.

[65] If producers in the market sector of the economy are solving the profit maximization problem that is associated with $g^t(P,X)$, which uses the original output prices P and the original primary input vector X, then they will also solve the profit maximization problem that uses the deflated output prices $p \equiv P/P \cdot \mu$ and the deflated primary input vector $x \equiv X/X \cdot \theta$; i.e., they will also solve the revenue maximization problem defined by $g^t(p,x)$.

approach:[66]

$$(3.11)\ \tau(p,x,t) \equiv g^t(p,x) - g^{t-1}(p,x)\ ; \qquad t = 1,2, \ldots .$$

Thus $\tau(p,x,t)$ measures the change in the real income per unit primary input produced by the market sector at the reference real output prices p and reference relative input quantities used by the market sector x where the first term in the right hand side of (3.11) uses the period t technology represented by g^t and the second term in (3.11) uses the period t–1 technology g^{t-1}. Thus each choice of reference vectors p and x will generate a possibly different measure of the shift in technology going from period t–1 to period t. Note that we are using the chain system to measure the shift in technology.

It is natural to choose special reference vectors for the measure of technical progress defined by (3.11): a *Laspeyres type measure* $\tau_L{}^t$ that chooses the period t–1 reference vectors p^{t-1} and x^{t-1} and a *Paasche type measure* $\tau_P{}^t$ that chooses the period t reference vectors p^t and x^t:[67]

$$(3.12)\ \tau_L{}^t \equiv \tau(p^{t-1},x^{t-1},t) = g^t(p^{t-1},x^{t-1}) - g^{t-1}(p^{t-1},x^{t-1})\ ; \qquad t = 1,2, \ldots ;$$
$$(3.13)\ \tau_P{}^t \equiv \tau(p^t,x^t,t) \qquad = g^t(p^t,x^t) - g^{t-1}(p^t,x^t)\ ; \qquad t = 1,2, \ldots .$$

Since both measures of technical progress are equally valid, it is natural to average them to obtain an overall measure of technical change. If we want to treat the two measures in a symmetric manner and we want the measure to satisfy the time reversal property from the difference approach to index number theory[68] (so that the estimate going backwards is equal to the negative of the estimate going forwards), then the arithmetic mean will be the best simple average to take in this context. Thus we define the arithmetic mean of (3.12) and (3.13) as follows:

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[66] The corresponding ratio type measure, $\tau(p,x,t) \equiv g^t(p,x)/g^{t-1}(p,x)$ is due to Diewert and Morrison (1986; 662). A special case of it was defined earlier by Diewert (1983; 1063).

[67] Diewert and Morrison (1986; 662-663) introduced the ratio counterparts to (3.12) and (3.13) in the nominal GDP context

[68] Diewert (2005c; 366) developed the axiomatic approach to index number theory using differences and introduced this time reversal test, which is the counterpart to the usual time reversal test that can be found in Fisher (1922; 64). Balk (2003; 29) also emphasized the importance of a symmetric treatment of time. Balk (2007) further developed the axiomatic approach using differences.

(3.14) $\tau^t \equiv (1/2)[\tau_L{}^t + \tau_P{}^t]$; $t = 1,2, \ldots$.

At this point, it is not clear how we will obtain empirical estimates for the theoretical productivity growth indexes defined by (3.12)–(3.14). One obvious way would be to assume a functional form for the nominal income function $g^t(P,X)$, collect data on output and input prices and quantities for the market sector for a number of years, add error terms to equations (3.2) and (3.3) and use econometric techniques to estimate the unknown parameters in the assumed functional form. However, econometric techniques are generally not completely straightforward: different econometricians will make different stochastic specifications and will choose different functional forms.[69] Moreover, as the number of outputs and inputs grows, it will be impossible to estimate a flexible functional form. Thus in the following section, we will suggest methods for estimating productivity change measures like (3.14) that are based on exact index number techniques.

We turn now to the problem of defining theoretical indexes for the effects on real income per unit primary input due to changes in real output prices. Define a family of *period t real output price change factors* $\alpha(p^{t-1},p^t,x,s)$:[70]

(3.15) $\alpha(p^{t-1},p^t,x,s) \equiv g^s(p^t,x) - g^s(p^{t-1},x)$; $s = 1,2, \ldots$.

Thus $\alpha(p^{t-1},p^t,x,s)$ measures the difference in the real income per unit primary input produced by the market sector that is induced by the change in real output prices going from period t–1 to t, using the technology that is available during period s and using the reference input quantities x. Thus each choice of the reference technology s and the reference input vector x will generate a possibly different measure of the effect on real income per unit primary input of a change in real output prices going from period t–1 to period t.

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[69] "The estimation of GDP functions such as (3.19) can be controversial, however, since it raises issues such as estimation technique and stochastic specification. ... We therefore prefer to opt for a more straightforward index number approach." Ulrich Kohli (2004a; 344).

[70] This measure of real output price change is the difference version of the usual ratio concept due to Fisher and Shell (1972; 56-58), Samuelson and Swamy (1974; 588-592), Archibald (1977; 60-61), Diewert (1980; 460-461) (1983; 1055) and Balk (1998; 83-89).

Again, it is natural to choose special reference vectors for the measures defined by (3.15): a *Laspeyres type measure* α_L^t that chooses the period t–1 reference technology and reference input vector x^{t-1} and a *Paasche type measure* α_P^t that chooses the period t reference technology and reference input vector x^t:

(3.16) $\alpha_L^t \equiv \alpha(p^{t-1},p^t,x^{t-1},t-1) = g^{t-1}(p^t,x^{t-1}) - g^{t-1}(p^{t-1},x^{t-1})$; $t = 1,2, \ldots$;

(3.17) $\alpha_P^t \equiv \alpha(p^{t-1},p^t,x^t,t) \quad = g^t(p^t,x^t) - g^t(p^{t-1},x^t)$; $t = 1,2, \ldots$.

Since both measures of real output price change are equally valid, it is natural to average them to obtain an overall measure of the effects on real income per unit primary input of the change in real output prices:

(3.18) $\alpha^t \equiv (1/2)[\alpha_L^t + \alpha_P^t]$; $t = 1,2, \ldots$.

Finally, we look at the problem of defining theoretical indexes for the effects on real income per unit primary input due to growth in relative input quantities going from period t–1 to t. Define a family of *period t relative input quantity growth factors* $\beta(x^{t-1},x^t,p,s)$:

(3.19) $\beta(x^{t-1},x^t,p,s) \equiv g^s(p,x^t) - g^s(p,x^{t-1})$; $s = 1,2, \ldots$.

Thus $\beta(x^{t-1},x^t,p,s)$ measures the difference in the real income per unit primary input produced by the market sector that is induced by the change in input quantities relative to the index of primary inputs used by the market sector going from period t–1 to t, using the technology that is available during period s and using the reference real output prices p. Thus each choice of the reference technology s and the reference real output price vector p will generate a possibly different measure of the effect on real income of a change in relative input quantities going from period t–1 to period t.

Again, it is natural to choose special reference vectors for the measures defined by (3.19): a *Laspeyres type measure* β_L^t that chooses the period t–1 reference technology and reference real output price vector p^{t-1} and a *Paasche type measure* β_P^t that chooses the period t reference technology and reference real output price vector p^t:

(3.20) $\beta_L^t \equiv \beta(x^{t-1},x^t,p^{t-1},t-1) = g^{t-1}(p^{t-1},x^t) - g^{t-1}(p^{t-1},x^{t-1})$; $t = 1,2, \ldots$;

(3.21) $\beta_P{}^t \equiv \beta(x^{t-1},x^t,p^t,t) \qquad = g^t(p^t,x^t) - g^t(p^t,x^{t-1})$; $\qquad t = 1,2, \ldots$.

Since both measures of (relative) input quantity change are equally valid, it is natural to average them to obtain an overall measure of the effects of relative input change on real income:

(3.22) $\beta^t \equiv (1/2)[\beta_L{}^t + \beta_P{}^t]$; $\qquad t = 1,2, \ldots$.

Recall that market sector real income for period t was defined by (3.5) as r^t which is equal to nominal period t factor payments $W^t \cdot X^t$ deflated by an index of fixed weight consumption prices, $P^t \cdot \mu$. Recall also that r^t was further deflated by the fixed weight index of primary inputs, $X^t \cdot \theta$, in order to obtain the period t real income per unit primary input measure ρ^t defined by (3.7). Recall also that using the linear homogeneity properties of the income function $g^t(P,X)$ in P and X, we showed that ρ^t is equal to $g^t(p^t,x^t)$. It is convenient to define γ^t as the *absolute amount of growth in real income per unit primary input* going from period t–1 to t:

(3.23) $\gamma^t \equiv \rho^t - \rho^{t-1}$; $\qquad t = 1,2, \ldots$.

In the following section, we will show that under certain functional form assumptions on the income functions g^t, γ^t is exactly equal to the sum of τ^t, α^t and β^t defined above by (3.14), (3.18) and (3.22) respectively.

3.4 The Normalized Quadratic Income Function and Bennet Indicators of Price, Quantity and Productivity Change

Suppose that the period t nominal net revenue or income function g^t has the following *normalized quadratic functional form*:[71]

(3.24) $g^t(P,X) \equiv a^t \cdot P\theta \cdot X + c^t \cdot X\mu \cdot P + (1/2)\, P \cdot AP\, [\theta \cdot X/\mu \cdot P]$

[71] This functional form is a generalization to many primary inputs of the normalized quadratic unit profit function introduced by Diewert and Wales (1992; 707) which in turn is an adaptation of the normalized quadratic functional form used by Diewert and Wales (1987) (1988a) (1988b) in a variety of contexts. It is also a generalization of the normalized quadratic profit function introduced by Diewert and Ostensoe (1988; 44).

$$+ P\cdot BX + (1/2)\, X\cdot CX\, [\mu\cdot P/\theta\cdot X]\ ; \qquad t = 0,1,2,...$$

where a^t and c^t are M and N dimensional vectors of unknown parameters which can be different for each time period t, $A = [a_{mk}]$ is an M by M positive semidefinite symmetric matrix of unknown parameters, $B = [b_{mn}]$ is an M by N matrix of unknown parameters, $C = [c_{ni}]$ is an N by N negative semidefinite symmetric matrix of unknown parameters and μ and θ are the same known vectors of parameters that appeared in definitions (3.6) and (3.8) above. Note that with our curvature restrictions on the matrices A and C, $g^t(P,X)$ will be convex (and linearly homogeneous) in the components of P and concave (and linearly homogeneous) in the components of X. Note that by allowing the parameter vectors a^t and c^t to change arbitrarily with time, we are allowing for very general forms of technical progress. However, the theory to be developed below does require that the parameter matrices A, B and C to be fixed over time. In Appendix C below, we show that the g^t defined by (3.24) is a flexible functional form.

Now evaluate (3.24) at the data for period t, P^t, X^t. Dividing both sides of the resulting equation by $\theta\cdot X^t$ times $\mu\cdot P^t$ gives us the following equation for period t real income per unit primary input, ρ^t:

$$(3.25)\ \rho^t = g^t(p^t,x^t) = a^t\cdot p^t + c^t\cdot x^t + (1/2)\, p^t\cdot Ap^t + p^t\cdot Bx^t + (1/2)\, x^t\cdot Cx^t\ ; \qquad t = 0,1,2,...$$

where $p^t \equiv P^t/\mu\cdot P^t$ and $x^t \equiv X^t/\theta\cdot P^t$.

Differentiating (3.24) with respect to the components of P and evaluating the resulting derivatives at the data pertaining to period t leads to the following equations using (3.2):

$$(3.26)\ Y^t = \nabla_P g^t(P^t,X^t) = a^t\theta\cdot X^t + c^t\cdot X^t\mu + AP^t[\theta\cdot X^t/\mu\cdot P^t] - (1/2)P^t\cdot AP^t\theta\cdot X^t[\mu\cdot P^t]^{-2}\mu + BX^t + (1/2)[\theta\cdot X^t]^{-1}X^t\cdot CX^t\,\mu\ ; \qquad t = 0,1,2,...\ .$$

Now divide both sides of (3.26) by $\theta\cdot X^t$, define $p^t \equiv P^t/\mu\cdot P^t$, $x^t \equiv X^t/\theta\cdot X^t$ and $y^t \equiv Y^t/\theta\cdot X^t$ and equations (3.26) become the following equations:

$$(3.27)\ y^t = \nabla_P g^t(p^t,x^t) = a^t + c^t\cdot x^t\mu + Ap^t - (1/2)p^t\cdot Ap^t\mu + Bx^t + (1/2)x^t\cdot Cx^t\mu\ ;\ t = 0,1,2,...\ .$$

Now premultiply both sides of equation t in (3.27) by the transpose of p^t. Using $p^t\cdot\mu = P^t\cdot\mu/\mu\cdot P^t = 1$, the resulting equations become:

$$(3.28)\ p^t\cdot y^t = a^t\cdot p^t + c^t\cdot x^t + (1/2)\, p^t\cdot Ap^t + p^t\cdot Bx^t + (1/2)\, x^t\cdot Cx^t = \rho^t = \quad g^t(p^t,x^t)\ ;t = 0,1,2,\ldots\ .$$

Recall definition (3.16) for the Laspeyres period t real output price change factor, $\alpha_L{}^t$. Using the g^t functions defined by (3.25), we have:

$$\begin{aligned}(3.29)\ \alpha_L{}^t &\equiv g^{t-1}(p^t,x^{t-1}) - g^{t-1}(p^{t-1},x^{t-1}) \qquad\qquad t = 1,2,\ldots\\ &= [a^{t-1}\cdot p^t + c^{t-1}\cdot x^{t-1} + (1/2)\, p^t\cdot Ap^t + p^t\cdot Bx^{t-1} + (1/2)\, x^{t-1}\cdot Cx^{t-1}] - p^{t-1}\cdot y^{t-1}\end{aligned}$$

where we have used definition (3.24) in order to evaluate $g^{t-1}(p^t,x^{t-1})$ and we have also used equation t–1 in (3.28) to obtain $g^{t-1}(p^{t-1},x^{t-1})$ equal to $p^{t-1}\cdot y^{t-1}$.

Similarly, using definition (3.17) for the Paasche period t real output price change factor, $\alpha_P{}^t$, we have:

$$\begin{aligned}(3.30)\ \alpha_P{}^t &\equiv g^t(p^t,x^t) - g^t(p^{t-1},x^t) \qquad\qquad t = 1,2,\ldots\\ &= p^t\cdot y^t - [a^t\cdot p^{t-1} + c^t\cdot x^t + (1/2)\, p^{t-1}\cdot Ap^{t-1} + p^{t-1}\cdot Bx^t + (1/2)\, x^t\cdot Cx^t].\end{aligned}$$

Using (3.29) and (3.30), it can be seen that the sum of the above two real output price change factors is equal to the following expression:

$$\begin{aligned}(3.31)\ 2\alpha^t &= \alpha_L{}^t + \alpha_P{}^t \qquad\qquad t = 1,2,\ldots\\ &= p^t\cdot y^t - p^{t-1}\cdot y^{t-1} + [a^{t-1}\cdot p^t + c^{t-1}\cdot x^{t-1} + (1/2)\, p^t\cdot Ap^t + p^t\cdot Bx^{t-1} + (1/2)\, x^{t-1}\cdot Cx^{t-1}]\\ &\qquad - [a^t\cdot p^{t-1} + c^t\cdot x^t + (1/2)\, p^{t-1}\cdot Ap^{t-1} + p^{t-1}\cdot Bx^t + (1/2)\, x^t\cdot Cx^t].\end{aligned}$$

Unfortunately, the expressions on the right hand sides of (3.29)–(3.31) are not observable without a knowledge of the unknown parameters in the g^t functions defined by (3.24). However, the *Bennet* (1920)[72] *indicator of real output price change*, $P^B(p^{t-1},p^t,y^{t-1},y^t)$,

[72] Bennet noticed that the value aggregate difference $p^t\cdot y^t - p^{t-1}\cdot y^{t-1}$ is exactly equal to the sum of the price change term, $P^B(p^{t-1},p^t,y^{t-1},y^t)$ defined by (3.32) and the corresponding quantity change term, $Q^B(p^{t-1},p^t,y^{t-1},y^t)$ defined as $(1/2)[p^{t-1} + p^t]\cdot[y^t - y^{t-1}]$. Diewert (1992) termed P^B and Q^B the Bennet *indicators* of price and quantity change for the value aggregate; i.e., he introduced the term *indicator* as the difference counterpart to the price and quantity *index* concepts in traditional ratio type index number theory. Diewert (2005c) developed the axiomatic or test approach to price and quantity indicators and showed that

defined by (3.32) is empirically observable:

$$(3.32)\ P^B(p^{t-1},p^t,y^{t-1},y^t) \equiv (1/2)[y^{t-1} + y^t]\cdot[p^t - p^{t-1}]\ ; \qquad t = 1,2, ...$$

where the observable output quantity vectors (deflated by the index of primary inputs) $y^t \equiv Y^t/X^t\cdot\theta$ were defined earlier by (3.8) and the deflated output price vectors $p^t \equiv P^t/P^t\cdot\mu$ were defined earlier by (3.6).

We now show that if there is competitive net revenue maximizing behaviour in the market sector in each period t and the market sector income functions g^t are defined by (3.24) above, then the Bennet indicator of real price change, P^B defined by (3.32), is exactly equal to α^t defined by (3.18), the arithmetic average of the Laspeyres and Paasche real output price change factors. Using definition (3.32), we have:

$$\begin{aligned}
(3.33)\ 2P^B(p^{t-1},p^t,y^{t-1},y^t) &= [y^{t-1} + y^t]\cdot[p^t - p^{t-1}]\ ; \qquad t = 1,2, ... \\
&= p^t\cdot y^t - p^{t-1}\cdot y^{t-1} + p^t\cdot y^{t-1} - p^{t-1}\cdot y^t \\
&= p^t\cdot y^t - p^{t-1}\cdot y^{t-1} \\
&\quad + p^t\cdot[a^{t-1} + c^{t-1}\cdot x^{t-1}\mu + Ap^{t-1} - (1/2)p^{t-1}\cdot Ap^{t-1}\mu + Bx^{t-1} + (1/2)x^{t-1}\cdot Cx^{t-1}\mu] \\
&\quad - p^{t-1}\cdot[a^t + c^t\cdot x^t\mu + Ap^t - (1/2)p^t\cdot Ap^t\mu + Bx^t + (1/2)x^t\cdot Cx^t\mu] \qquad \text{using (3.27)} \\
&= p^t\cdot y^t - p^{t-1}\cdot y^{t-1} \\
&\quad + [p^t\cdot a^{t-1} + c^{t-1}\cdot x^{t-1} + p^t\cdot Ap^{t-1} - (1/2)p^{t-1}\cdot Ap^{t-1} + p^t\cdot Bx^{t-1} + (1/2)x^{t-1}\cdot Cx^{t-1}] \\
&\quad - [p^{t-1}\cdot a^t + c^t\cdot x^t + p^{t-1}\cdot Ap^t - (1/2)p^t\cdot Ap^t + p^{t-1}\cdot Bx^t + (1/2)x^t\cdot Cx^t] \text{ using } p^t\cdot\mu = 1 \\
&= p^t\cdot y^t - p^{t-1}\cdot y^{t-1} + [a^{t-1}\cdot p^t + c^{t-1}\cdot x^{t-1} + (1/2)\ p^t\cdot Ap^t + p^t\cdot Bx^{t-1} + (1/2)\ x^{t-1}\cdot Cx^{t-1}] \\
&\quad - [a^t\cdot p^{t-1} + c^t\cdot x^t + (1/2)\ p^{t-1}\cdot Ap^{t-1} + p^{t-1}\cdot Bx^t + (1/2)\ x^t\cdot Cx^t] \qquad \text{simplifying} \\
&= \alpha_L{}^t + \alpha_P{}^t \qquad \text{using (3.31)} \\
&= 2\alpha^t \qquad \text{using definition (3.18).}
\end{aligned}$$

Thus under our assumptions on technology, the theoretical measure of period t per unit primary input market sector real income change due to change in real output prices, α^t defined by (3.18), is exactly equal to the observable Bennet indicator of real price change, P^B defined by (3.32).[73]

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the Bennet indicators were the difference counterparts to the Fisher price and quantity indexes in terms of their axiomatic properties. Balk (2007) also looked at the axiomatic properties of the Bennet indicators.

[73] Equations (3.33) show that the theoretical measure of change in g^t due to changes in real output prices p^t, α^t defined by (3.18), is equal to $\sum_{m=1}^{M} (1/2)[y_m{}^{t-1} + y_m{}^t][p_m{}^t - p_m{}^{t-1}]$. In Appendix D below, we show that

The above analysis can be modified to give us an observable estimator for the theoretical measure of the effects of relative input quantity change on market sector real income per unit primary input, β^t defined by (3.22) above. However, it is first necessary to differentiate the normalized quadratic functions $g^t(P,X)$ defined above by (3.24) with respect to the components of X and then use Samuelson's Lemma (3.3) in order to obtain expressions for the period t input price vectors W^t.

Differentiating (3.24) with respect to the components of X and evaluating the resulting derivatives at the data pertaining to period t leads to the following equations using (3.3):

$$(3.34)\quad W^t = \nabla_X g^t(P^t,X^t) \qquad t = 0,1,2,...$$
$$= a^t\cdot P^t\theta + \mu\cdot P^t c^t + (1/2)[\mu\cdot P^t]^{-1}P^t\cdot AP^t\theta$$
$$+ B^T P^t + CX^t\,[\mu\cdot P^t/\theta\cdot X^t] - (1/2)[\theta\cdot X^t]^{-2}X^t\cdot CX^t\mu\cdot P^t\theta.$$

Now divide both sides of (3.34) by $\mu\cdot P^t$, define $p^t \equiv P^t/\mu\cdot P^t$, $x^t \equiv X^t/\theta\cdot X^t$ and $w^t \equiv W^t/\mu\cdot P^t$ and equations (3.34) become the following equations:

$$(3.35)\quad w^t = a^t\cdot p^t\theta + c^t + (1/2)p^t\cdot Ap^t\theta + B^T p^t + Cx^t - (1/2)x^t\cdot Cx^t\theta\ ; \qquad t = 0,1,2,..\ .$$

Now premultiply both sides of equation t in (3.35) by the transpose of x^t. Using $x^t\cdot\theta = X^t\cdot\theta/\theta\cdot X^t = 1$, the resulting equations become:

$$(3.36)\quad w^t\cdot x^t = a^t\cdot p^t + c^t\cdot x^t + (1/2)\ p^t\cdot Ap^t + p^t\cdot Bx^t + (1/2)\ x^t\cdot Cx^t\ ; \qquad t = 0,1,2,...$$
$$= g^t(p^t,x^t) \qquad \text{using (3.28).}$$

Now recall definition (3.20) for the Laspeyres period t relative input quantity growth factor, $\beta_L{}^t$. Using the g^t functions defined by (3.24), we have:

$$(3.37)\quad \beta_L{}^t \equiv g^{t-1}(p^{t-1},x^t) - g^{t-1}(p^{t-1},x^{t-1}) \qquad t = 1,2, ...$$
$$= [a^{t-1}\cdot p^{t-1} + c^{t-1}\cdot x^t + (1/2)\ p^{t-1}\cdot Ap^{t-1} + p^{t-1}\cdot Bx^t + (1/2)\ x^t\cdot Cx^t] - p^{t-1}\cdot y^{t-1}.$$

Similarly, using definition (3.21) for the Paasche period t relative input quantity growth

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each term in this summation can be interpreted as an approximate theoretical measure of the change in g^t due to the change in a single real price $p_m{}^t$.

factor, β_P^t, we have, using (3.25) and (3.28):

$$(3.38)\ \beta_P^t \equiv g^t(p^t,x^t) - g^t(p^t,x^{t-1}) \qquad t = 1,2, \ldots$$
$$= p^t\cdot y^t - [a^t\cdot p^t + c^t\cdot x^{t-1} + (1/2)\, p^t\cdot Ap^t + p^t\cdot Bx^{t-1} + (1/2)\, x^{t-1}\cdot Cx^{t-1}].$$

Thus using (3.37) and (3.38), it can be seen that the sum of the above two relative input quantity growth factors is equal to the following expression:

$$(3.39)\ 2\beta^t = \beta_L^t + \beta_P^t \qquad t = 1,2, \ldots$$
$$= p^t\cdot y^t - p^{t-1}\cdot y^{t-1} + [a^{t-1}\cdot p^{t-1} + c^{t-1}\cdot x^t + (1/2)\, p^{t-1}\cdot Ap^{t-1} + p^{t-1}\cdot Bx^t + (1/2)\, x^t\cdot Cx^t]$$
$$- [a^t\cdot p^t + c^t\cdot x^{t-1} + (1/2)\, p^t\cdot Ap^t + p^t\cdot Bx^{t-1} + (1/2)\, x^{t-1}\cdot Cx^{t-1}].$$

Unfortunately, the expressions on the right hand sides of (3.37)–(3.39) are not observable without a knowledge of the unknown parameters in the g^t functions defined by (3.24). However, the *Bennet* (1920) *indicator of relative input quantity change*, $Q^B(w^{t-1},w^t,x^{t-1},x^t)$, defined by (3.40) is empirically observable:

$$(3.40)\ Q^B(w^{t-1},w^t,x^{t-1},x^t) \equiv (1/2)[w^{t-1} + w^t]\cdot[x^t - x^{t-1}]\ ; \qquad t = 1,2, \ldots$$

where the observable real input price vectors $w^t \equiv W^t/\, P^t\cdot\mu$ were defined earlier by (3.6) and the input quantity vectors (deflated by the index of primary inputs) $x^t \equiv X^t/X^t\cdot\theta$ were defined earlier by (3.8).

We now show that if there is competitive profit maximizing behaviour in the market sector in each period t and the market sector income functions g^t are defined by (3.24) above, then the Bennet indicator of relative input quantity change, Q^B defined by (3.40), is exactly equal to β^t defined by (3.22), the arithmetic average of the Laspeyres and Paasche relative input quantity growth factors. Using definition (3.40), we have:

$$(3.41)\ 2Q^B(w^{t-1},w^t,x^{t-1},x^t) = [w^{t-1} + w^t]\cdot[x^t - x^{t-1}]\ ; \qquad t = 1,2, \ldots$$
$$= w^t\cdot x^t - w^{t-1}\cdot x^{t-1} + w^{t-1}\cdot x^t - w^t\cdot x^{t-1}$$
$$= p^t\cdot y^t - p^{t-1}\cdot y^{t-1} + w^{t-1}\cdot x^t - w^t\cdot x^{t-1} \qquad \text{using (3.38) and (3.36)}$$
$$= p^t\cdot y^t - p^{t-1}\cdot y^{t-1}$$
$$+ x^t\cdot[a^{t-1}\cdot p^{t-1}\theta + c^{t-1} + (1/2)p^{t-1}\cdot Ap^{t-1}\theta + B^Tp^{t-1} + Cx^{t-1} - (1/2)x^{t-1}\cdot Cx^{t-1}\theta]$$
$$- x^{t-1}\cdot[\, a^t\cdot p^t\theta + c^t + (1/2)p^t\cdot Ap^t\theta + B^Tp^t + Cx^t - (1/2)x^t\cdot Cx^t\theta] \text{ using (3.35)}$$
$$= p^t\cdot y^t - p^{t-1}\cdot y^{t-1}$$

$$+ [a^{t-1}\cdot p^{t-1} + c^{t-1}\cdot x^t + (1/2)p^{t-1}\cdot Ap^{t-1} + p^{t-1}Bx^t + x^t\cdot Cx^{t-1} - (1/2)x^{t-1}\cdot Cx^{t-1}]$$
$$- [a^t\cdot p^t + c^t\cdot x^{t-1} + (1/2)p^t\cdot Ap^t + p^t\cdot Bx^{t-1} + x^{t-1}\cdot Cx^t - (1/2)x^t\cdot Cx^t] \qquad \text{using } x^{t-1}\cdot\theta = 1$$
$$= p^t\cdot y^t - p^{t-1}\cdot y^{t-1}$$
$$+ [a^{t-1}\cdot p^{t-1} + c^{t-1}\cdot x^t + (1/2)p^{t-1}\cdot Ap^{t-1} + p^{t-1}Bx^t - (1/2)x^{t-1}\cdot Cx^{t-1}]$$
$$- [a^t\cdot p^t + c^t\cdot x^{t-1} + (1/2)p^t\cdot Ap^t + p^t\cdot Bx^{t-1} - (1/2)x^t\cdot Cx^t] \qquad \text{using } x^t\cdot Cx^{t-1} = x^{t-1}\cdot Cx^t$$
$$= (1/2)[\beta_L{}^t + \beta_P{}^t] \qquad \text{using (3.39)}$$
$$= \beta^t \qquad \text{using definition (3.22).}$$

Thus under our assumptions on technology, the theoretical measure of period t per unit primary input market sector real income change due to change in relative input quantities, β^t defined by (3.22), is exactly equal to the observable Bennet indicator of relative input quantity change, Q^B defined by (3.40).[74]

We now turn our attention to developing an observable measure of technical progress. Recall the (unobservable) theoretical measures of technical progress $\tau_L{}^t$, $\tau_P{}^t$ and τ^t defined by (3.12)–(3.14) respectively. If the income functions g^t are defined by (3.24), then by substituting these definitions for the g^t into definitions (3.12)–(3.14), we obtain the following (unobservable) expressions for these technical progress measures:

$$(3.42)\quad \tau_L{}^t \equiv g^t(p^{t-1},x^{t-1}) - g^{t-1}(p^{t-1},x^{t-1}) ; \qquad t = 1,2, ...$$
$$= [a^t - a^{t-1}]\cdot p^{t-1} + [c^t - c^{t-1}]\cdot x^{t-1} ;$$
$$(3.43)\quad \tau_P{}^t \equiv g^t(p^t,x^t) - g^{t-1}(p^t,x^t) ; \qquad t = 1,2, ...$$
$$= [a^t - a^{t-1}]\cdot p^t + [c^t - c^{t-1}]\cdot x^t ;$$
$$(3.44)\quad \tau^t \equiv (1/2)[\tau_L{}^t + \tau_P{}^t] ; \qquad t = 1,2, ...$$
$$= (1/2)[p^{t-1} + p^t]\cdot[a^t - a^{t-1}] + (1/2)[x^{t-1} + x^t]\cdot[c^t - c^{t-1}] .$$

Now look at the period t change in real income per unit primary input over the previous period, $\rho^t - \rho^{t-1}$ equal to $p^t\cdot y^t - p^{t-1}\cdot y^{t-1}$. Subtract the Bennet indicator of real price change $P^B(p^{t-1},p^t,y^{t-1},y^t)$ defined by (3.32) and subtract the Bennet indicator of relative input quantity change $Q^B(w^{t-1},w^t,x^{t-1},x^t)$ defined by (3.40) from this income difference and evaluate the resulting expression using the g^t defined by (3.24). We obtain the

[74] Equations (3.41) show that the theoretical measure of change in g^t due to changes in relative input quantities x^t, β^t defined by (3.22), is equal to $\sum_{n=1}^N (1/2)[w_n{}^{t-1} + w_n{}^t][x_n{}^t - x_n{}^{t-1}]$. In Appendix D below, we show that each term in this summation can be interpreted as an approximation to a theoretical measure of the change in g^t due to the change in a single deflated input quantity $x_n{}^t$.

following identity:

$$
\begin{aligned}
(3.45)\ & p^t\cdot y^t - p^{t-1}\cdot y^{t-1} - (1/2)[y^{t-1} + y^t]\cdot[p^t - p^{t-1}] - (1/2)[w^{t-1} + w^t]\cdot[x^t - x^{t-1}] \qquad t = 1,2, \ldots \\
&= -(1/2)[a^{t-1}\cdot p^t + c^{t-1}\cdot x^{t-1} + (1/2)\, p^t\cdot Ap^t + p^t\cdot Bx^{t-1} + (1/2)\, x^{t-1}\cdot Cx^{t-1}] \\
&\quad + (1/2)[a^t\cdot p^{t-1} + c^t\cdot x^t + (1/2)\, p^{t-1}\cdot Ap^{t-1} + p^{t-1}\cdot Bx^t + (1/2)\, x^t\cdot Cx^t] \\
&\quad - (1/2)[a^{t-1}\cdot p^{t-1} + c^{t-1}\cdot x^t + (1/2)p^{t-1}\cdot Ap^{t-1} + p^{t-1}Bx^t - (1/2)x^{t-1}\cdot Cx^{t-1}] \\
&\quad + (1/2)[a^t\cdot p^t + c^t\cdot x^{t-1} + (1/2)p^t\cdot Ap^t + p^t\cdot Bx^{t-1} - (1/2)x^t\cdot Cx^t] \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{using (3.33) and (3.41)} \\
&= (1/2)[p^{t-1} + p^t]\cdot[a^t - a^{t-1}] + (1/2)[x^{t-1} + x^t]\cdot[c^t - c^{t-1}] \qquad \text{cancelling terms} \\
&= \tau^t \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{using (3.44).}
\end{aligned}
$$

Thus the first line in (3.45) gives us an observable exact estimator for the theoretical technical progress measure τ^t. Using (3.32), (3.40) and (3.45), it can be seen that under the assumption that the income functions g^t are defined by (3.24), we have the following exact decomposition for the change in real income per unit primary input, $\rho^t - \rho^{t-1}$:

$$(3.46)\ \gamma^t \equiv p^t\cdot y^t - p^{t-1}\cdot y^{t-1} = \rho^t - \rho^{t-1} = \alpha^t + \beta^t + \tau^t\ ; \qquad t = 1,2, \ldots .$$

The above equation says the change in real income per unit primary input is equal to the sum of a change in real output prices factor α^t plus a change in relative primary input quantities factor β^t plus a change in technical efficiency term τ^t where all three explanatory factors can be estimated using observable price and quantity data pertaining to periods t and t–1.[75]

Rather than look at explanatory factors for the difference in real income per unit primary input between the adjacent periods, it is sometimes convenient to express the difference in real income between the current period t and the reference year 0 in terms of the difference in the indicator of the technology level T^t, of the change in the level of real output prices in period t, A^t, and of the difference in the level of primary input quantities in period t, B^t. Thus, we use the growth factors τ^t, α^t, and β^t as follows to define the

$$(3.47)\ T^0 \equiv 0;\ T^t = T^{t-1} + \tau^t; \qquad t = 1,2, \ldots$$

$$(3.48)\ A^0 \equiv 0;\ A^t = A^{t-1} + \alpha^t; \qquad t = 1,2, \ldots$$

[75] The decomposition (3.46) is a difference counterpart to the ratio decomposition of nominal income growth obtained by Diewert and Morrison (1986; 663–665) and Kohli (1990).

(3.49) $B^0 \equiv 0$; $B^t = B^{t-1} + \beta^t$; $\quad t = 1,2, \ldots$.

Using the chain links that appear in (3.47)–(3.49), we can establish the following exact relationship for the *cumulative change in real income per unit primary input* going from period 0 to period t:

(3.50) $\rho^t - \rho^0 = A^t + B^t + T^t$; $\quad t = 1,2, \ldots$.

Instead of using the first line in (3.45) to define the technical progress term that turns out to be equal to τ^t, we can obtain some alternative expressions for technical progress (or the change in Total Factor Productivity) using Bennet (1920) identity for the decomposition of the change of a value aggregate into price and quantity components; i.e., Bennet showed that the following exact identity holds:

(3.51) $p^t \cdot y^t - p^{t-1} \cdot y^{t-1} = (1/2)[y^{t-1} + y^t] \cdot [p^t - p^{t-1}] + (1/2)[p^{t-1} + p^t] \cdot [y^t - y^{t-1}]$.

Substituting (3.51) into (3.45) leads to the following alternative exact expression for the technical progress term τ^t:

(3.52) $\tau^t = \quad (1/2)[p^{t-1} + p^t] \cdot [y^t - y^{t-1}] - (1/2)[w^{t-1} + w^t] \cdot [x^t - x^{t-1}]$; $\quad t = 1,2, \ldots$.

Thus the absolute changes in the (deflated) output quantities, $y^t - y^{t-1}$, are weighted by the average of the real output prices for periods t–1 and t, $(1/2)[p^{t-1} + p^t]$, and then we subtract the absolute changes in the (deflated) primary input quantities, $x^t - x^{t-1}$, weighted by the average of the real input prices for periods t–1 and t, $(1/2)[w^{t-1} + w^t]$. We call the right hand side of (52) the *primal Bennet measure of technical progress*.[76] It is a difference counterpart to the following *primal Fisher index of productivity growth* or index of technical progress:[77]

(3.53) $T^t \equiv [p^t \cdot y^t \; p^{t-1} \cdot y^t / p^t \cdot y^{t-1} \; p^{t-1} \cdot y^{t-1}]^{1/2} / [w^t \cdot x^t \; w^{t-1} \cdot x^t / w^t \cdot x^{t-1} \; w^{t-1} \cdot x^{t-1}]^{1/2}$.

---

[76] Balk (2003; 29) (2007), Diewert (2005c; 353) and Diewert and Fox (2005; 8) all suggested variants of this Bennet indicator of real profit change as a measure of efficiency improvement.

[77] See Diewert and Nakamura (2003) for material on the Fisher (1922) index and its use in the traditional ratio approach to productivity measurement.

However, we can obtain a third expression for τ^t, which is also instructive. Under our assumptions (3.4), it can be seen that $p^t \cdot y^t$ is equal to $w^t \cdot x^t$ for each t. Thus we have:

$$(3.54)\ p^t \cdot y^t - p^{t-1} \cdot y^{t-1} = w^t \cdot x^t - w^{t-1} \cdot x^{t-1}$$
$$= (1/2)[x^{t-1} + x^t] \cdot [w^t - w^{t-1}] + (1/2)[w^{t-1} + w^t] \cdot [x^t - x^{t-1}]$$

where we have applied the Bennet value difference decomposition to obtain the second equality in (3.54). Substituting (3.54) into the first line of (3.45) leads to the following exact expression for τ^t:

$$(3.55)\ \tau^t = (1/2)[x^{t-1} + x^t] \cdot [w^t - w^{t-1}] - (1/2)[y^{t-1} + y^t] \cdot [p^t - p^{t-1}]\ .$$

Thus if real input prices increase faster than real output prices, there will be positive technical progress. We call the right hand side of (3.55) the *dual Bennet measure of technical progress*. It is a difference counterpart to the following *dual Fisher index of productivity growth* or index of technical progress:[78]

$$(3.56)\ \tau_F{}^t \equiv [w^t \cdot x^t\ w^t \cdot x^{t-1} / w^{t-1} \cdot x^t\ w^{t-1} \cdot x^{t-1}]^{1/2} / [p^t \cdot y^t\ p^t \cdot y^{t-1} / p^{t-1} \cdot y^t\ p^{t-1} \cdot y^{t-1}]^{1/2}\ .$$

The above difference approach can be converted into a more traditional growth rate approach. The period t rate of growth of real income per unit primary input is equal to the period t change in real income generated by the market sector, $\rho^t - \rho^{t-1}$, divided by last period's real income, ρ^{t-1}. Using (3.40), we have:

$$(3.57)\ [\rho^t - \rho^{t-1}]/\rho^{t-1} = [\alpha^t + \beta^t + \tau^t]/\rho^{t-1}; \qquad t = 1,2, \ldots\ .$$

Thus the rate of growth of market sector real income per unit primary input is explained by a sum of three additional explanatory factors, α^t/ρ^{t-1} (the contribution of real output price change), plus β^t/ρ^{t-1} (the contribution of input quantity growth relative to the

[78] The fact that primal indexes of Total Factor Productivity Growth (an index of output quantity growth divided by an index of input quantity growth) could be also written in dual form (an index of input prices divided by an index of output prices) dates back to the pioneering contributions of Jorgenson and Griliches (1967).

average growth of primary inputs), plus τ^t/ρ^{t-1} (the contribution of technical change).[79]

Using the results derived in Appendix D, the overall period t real price change term α^t is equal to the sum of M individual real price change terms, $\sum_{m=1}^{M} \alpha_m^t$, and the overall relative input quantity change term β^t is equal to the sum of N individual relative input quantity change terms, $\sum_{n=1}^{N} \beta_n^t$. Moreover, these individual price and quantity terms can be calculated empirically without econometric estimation. Using these results from Appendix D, (3.57) can be rewritten as follows:

$$(3.58)\ [\rho^t - \rho^{t-1}]/\rho^{t-1} = [\textstyle\sum_{m=1}^{M} \alpha_m^t + \sum_{n=1}^{N} \beta_n^t + \tau^t]/\rho^{t-1}\ ; \qquad t = 1,2, \ldots .$$

The decomposition of market sector real income growth (per unit primary input) given by (3.58) is comparable to the decomposition of real income growth that was derived by Diewert, Mizobuchi and Nomura (2005) and Diewert and Lawrence (2006) using the Translog methodology that was originally developed by Diewert and Morrison (1986) and Kohli (1990). However, the present normalized quadratic methodological approach has an advantage over the earlier Translog approach in that *the present approach allows individual prices and quantities to be zero*, whereas the Translog approach fails if any (exogenous) price or quantity becomes zero. Since a great deal of R&D effort is devoted to the development of *new* goods and services, it is useful to have a methodology that is able to deal with the creation of new products. A second advantage of the present approach is that when we specialize the fixed weight input index to be labour input, a "better" decomposition of labour productivity into explanatory factors is obtained; i.e., in our methodological approach, real output is replaced by real income and hence the effects on real income per unit labour input of changes in the terms of trade can be modeled using our present approach.

3.5 An Application to the Japanese Economy for 1955–2006

3.5.1 The Japanese Data

We apply our methodology to a modified version of the Japanese productivity database

[79] If there are only two primary inputs, labour and capital, and the primary input weighting vector θ is the unit vector (1,0) so that the input aggregate collapses down to labour input, then the term β^t/ρ^{t-1} is the contribution of capital deepening; i.e., of the growth of capital input relative to the growth of labour input.

developed by Chapter 2. This database consists of the quantity, price, and value series for eleven main classes of outputs and inputs for the Japanese market sector, with some further detailed breakdowns for the investment outputs and capital service inputs; see Appendix E for a detailed listing of the data. We briefly mention the data construction procedure of Chapter 2. We followed the conventions introduced by Jorgenson and Griliches on the treatment of taxes; i.e., we adjusted prices for tax wedges whenever possible so that the adjusted prices reflect the prices that producers face.[80] We also included the services of inventories and land as additional capital inputs. A listing of the outputs and inputs follows.

The 7 main classes of net outputs are:

- C; Domestic final consumption expenditure of households (excluding imputed rent for owner-occupied houses);
- N; Final consumption expenditure of private non-profit institutions serving households (NPISHs);
- G; Net sales of goods and services by the market sector to the general government sector; i.e., the value aggregate is equal to minus government sales of goods and services to the market production sector plus purchases of intermediate inputs from the market sector;
- X; Exports of goods and services (excluding direct purchases in the domestic market by non-resident households) and
- M; Imports of goods and services (excluding direct purchases abroad by resident households);
- I; Investment which consists of the following 12 subaggregates: I1: Animals and plants; I2: Construction; I3: Textile products; I4: Wood products; I5: Furniture and fixtures; I6: Metallic products; I7: General machinery; I8: Electric machinery; I9:

[80] Thus our suggested treatment of indirect commodity taxes in an accounting framework that is suitable for productivity analysis follows the example set by Jorgenson and Griliches who advocated the following treatment of indirect taxes: "In our original estimates, we used gross product at market prices; we now employ gross product from the producers' point of view, which includes indirect taxes levied on factor outlay, but excludes indirect taxes levied on output." Dale W. Jorgenson and Zvi Griliches (1972; 85). All other taxes such as taxes on financial assets and poll taxes which do not affect the producers' behaviour are ignored in this study.

Automobiles; I10: Other transportation; I11: Precision machinery; I12: Other investment products;

- IV; Change in Inventories which consists of the following 4 subaggregates: IV1: Finished goods inventory change; IV2: Work in progress inventory change; IV3: Work in progress inventory change for cultivated assets; IV4: Change in materials inventory.

The 4 main classes of primary inputs are:

- K; Capital service which consists of the following 12 subaggregates: K1: Animals and plants; K2: Construction; K3: Textile products; K4: Wood products; K5: Furniture and fixtures; K6: Metallic products; K7: General machinery; K8: Electric machinery; K9: Automobiles; K10: Other transportation; K11: Precision machinery; K12: Other investment products;
- KIV; Inventory service which consists of the following 4 components: KIV1: Finished good inventory services; KIV2: Work in progress inventory services; KIV3: Work in progress inventory services for cultivated assets; KIV4: Materials inventory services;
- LD; Land service which consists of the following 4 subaggregates: LD1: Agricultural land services; LD2: Industrial land services; LD3: Commercial land services; LD4: Residential land services for renters;
- LB; Labour input which consists of the following 3 subaggregates: LB1: Labour input of the self-employed; LB2: Labour input of family workers; LB3: Labour input of employees in the market sector;

Prices and quantities for the net output aggregates Y_C, Y_N, Y_G, Y_X, Y_M have been constructed using data in the national accounts based on 1968 SNA and 1993 SNA[81] and National Income Statistics which was the national accounting system prior to the introduction of 1968 SNA. We took the numbers constructed on the basis of 1993 JSNA as our standard. We extended the data series backwards by using data from 1968 JSNA and the earlier national income statistics.

The capital stocks are stocks at the beginning of the year. Estimates for the reproducible capital stocks during 1955–2006 have been constructed by applying the perpetual

[81] We call Japanese national accounts based on 1968 (1993) SNA simply 1968 (1993) JSNA, hereafter.

inventory method to the initial stocks in 1955 and using the investment data and asset specific depreciation rates. The initial capital stocks, the investment data and the change in inventory data have been taken from capital and investment data in the KEO database. This is a comprehensive productivity database for the Japanese economy constructed at Keio University. The detailed procedures used to construct these capital data are explained in Nomura (2004). It should be noted that the KEO data base of price and quantity data for investments and the corresponding capital stocks for 95 classes of asset. This chapter has aggregated these 95 asset classes into 12 classes for reproducible capital. Estimates of the quantities of labour services X_{LB} are based on hours of work. There are three different types of workers; the self employed, family workers and employees. Hours of works for each type of worker are aggregated into the quantity of aggregate labour input by applying a Fisher (1922) price index.

Price and quantity data for market sector net outputs and primary inputs are listed in Tables E1 (prices) and E2 (quantities) below. The detailed data on investments, changes in inventory, capital stocks, inventory stocks and land are listed in Tables E3–E7 (prices) and E8–E12 (quantities).

3.5.2 The Decomposition of Real Income Growth into Explanatory Factors

Substituting the estimates defined in (3.32), (3.41) and (3.45) into equation (3.52), we can decompose the growth rate of real income per unit labour $(\rho^t - \rho^{t-1})/\rho^{t-1} = \gamma^t/\rho^{t-1}$ into the contribution of technical progress τ^t/ρ^{t-1}, changes in real output prices $\alpha_C{}^t/\rho^{t-1}$ (domestic final consumption),[82] $\alpha_N{}^t/\rho^{t-1}$ (non-profit institution final consumption), $\alpha_G{}^t/\rho^{t-1}$ (net government purchases from the market sector), $\alpha_X{}^t/\rho^{t-1}$ (exports), $\alpha_M{}^t/\rho^{t-1}$ (imports), $\alpha_I{}^t/\rho^{t-1}$ (investments in reproducible capital) and $\alpha_{IV}{}^t/\rho^{t-1}$ (inventory changes) and growth in relative input quantities $\beta_K{}^t/\rho^{t-1}$ (capital services), $\beta_{KIV}{}^t/\rho^{t-1}$ (inventory services), $\beta_{LD}{}^t/\rho^{t-1}$ (land services) and $\beta_{LB}{}^t/\rho^{t-1}$ (labour input).[83] The chain link information on period by period changes in real income per unit labour input that corresponds to (3.52) is given in Table 3-1. The effect of changes in the terms of trade is $\alpha_{XM}{}^t/\rho^{t-1}$ and is simply the sum of the contributions of real price changes in exports and imports $\alpha_X{}^t/\rho^{t-1}$ and $\alpha_M{}^t/\rho^{t-1}$.

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[82] Since we divided market sector nominal income by the price of consumption, $\alpha_C{}^t/\rho^{t-1}$ will be identically equal to zero and hence it is not listed.

[83] Since we divided market sector nominal income by the quantity of labour input, $\beta_{LB}{}^t/\rho^{t-1}$ will be identically equal to zero and hence it is not listed.

Table 3-1: Decomposition of Growth Rate of Real Income Per Unit Labour (%)

	γ^{τ}/ρ^{t-1}	τ^{t}/ρ^{t-1}	$\alpha_N{}^{t}/\rho^{t-1}$	$\alpha_G{}^{t}/\rho^{t-1}$	$\alpha_X{}^{t}/\rho^{t-1}$	$\alpha_M{}^{t}/\rho^{t-1}$	$\alpha_{XM}{}^{t}/\rho^{t-1}$	$\alpha_I{}^{t}/\rho^{t-1}$	$\alpha_{IV}{}^{t}/\rho^{t-1}$	$\beta_K{}^{t}/\rho^{t-1}$	$\beta_{KIV}{}^{t}/\rho^{t-1}$	$\beta_{LD}{}^{t}/\rho^{t-1}$
1956	4.44867	6.16182	0.06632	0.11226	0.41197	-0.77573	-0.36376	1.89131	0.31518	-2.17691	-0.04565	-1.51192
1957	2.87446	4.57677	0.03315	0.03240	-0.33890	-0.07741	-0.41631	0.74566	-0.14922	-0.81207	0.04929	-1.18521
1958	2.00107	1.40635	0.00374	-0.12786	-0.82820	2.09237	1.26417	-0.84716	-0.04260	0.74947	0.13084	-0.53588
1959	9.40710	10.00776	0.00620	-0.08480	-0.13447	0.50801	0.37354	-1.00889	-0.02934	0.67686	-0.03759	-0.49664
1960	11.39564	12.25692	0.02537	-0.08627	-0.02266	0.34772	0.32506	-0.26007	-0.16189	0.45406	0.05441	-1.21196
1961	12.02089	9.87093	0.01186	-0.11711	-0.74467	0.61915	-0.12553	0.45644	-0.42145	2.43849	0.18021	-0.27296
1962	0.65867	1.02717	-0.00037	-0.19034	-1.00636	0.97419	-0.03217	-1.61067	-0.27202	2.01110	0.27188	-0.54592
1963	6.00309	5.81813	-0.00565	-0.24375	-0.53866	0.62781	0.08915	-2.37507	-0.18648	2.60020	0.12646	0.18011
1964	10.38152	8.83887	0.05422	-0.11601	-0.18209	0.15700	-0.02509	-0.39846	-0.04880	2.08023	0.15132	-0.15475
1965	0.26884	0.14921	-0.01467	-0.18490	-0.92682	0.91727	-0.00955	-1.81352	-0.11846	2.08152	0.15052	0.02870
1966	8.30842	6.67815	0.02531	-0.06157	-0.52277	0.38307	-0.13970	-0.15269	-0.07812	1.36975	0.03927	0.62803
1967	10.90512	7.79238	0.03298	-0.01017	-0.37574	0.56626	0.19052	0.29416	-0.11398	1.74651	0.10816	0.86455
1968	11.20952	8.93386	0.00975	-0.10101	-0.59204	0.59657	0.00453	-0.73902	-0.10851	2.62481	0.27313	0.31197
1969	12.66910	8.31488	0.03252	-0.08557	-0.28794	0.07037	-0.21756	0.18416	0.01995	2.89693	0.25461	1.26919
1970	6.81164	4.69959	0.06019	-0.12126	-0.45890	0.54747	0.08857	-0.88636	-0.14864	2.72774	0.20001	0.19178
1971	0.80235	-1.13843	0.04221	-0.19201	-0.55358	1.10380	0.55021	-1.68052	-0.08855	2.84638	0.16764	0.29540
1972	7.04619	4.57045	0.05415	-0.18580	-0.81746	1.09228	0.27482	-0.49672	0.02079	2.34018	0.01274	0.45557
1973	8.21062	5.16147	0.04033	-0.06564	-0.19834	-0.80638	-1.00472	1.71536	0.13194	1.61309	-0.03921	0.65799
1974	-2.10599	-3.30126	0.02096	0.52592	0.78894	-3.55585	-2.76691	-0.55100	-0.20618	2.79465	0.16055	1.21729
1975	-0.80101	-0.05185	-0.00123	-0.78528	-0.92131	0.26167	-0.65964	-2.72388	-0.05333	2.24790	0.14923	1.07707
1976	-0.91568	1.70858	0.01237	-0.09118	-1.09543	0.53152	-0.56391	-1.86509	-0.00081	0.26413	-0.09106	-0.28872
1977	-0.04042	0.28968	0.00604	-0.11593	-1.65183	1.48929	-0.16254	-0.97772	-0.00704	0.68083	-0.00065	0.24692
1978	4.61772	3.07762	-0.00407	-0.19174	-1.53785	2.47558	0.93773	-0.40146	0.01203	0.98133	-0.02657	0.23283
1979	6.24388	5.79355	0.02299	0.04859	0.62635	-2.77360	-2.14725	1.32970	0.03592	1.13219	-0.04378	0.07197
1980	-2.46584	-0.23679	-0.00283	0.18431	0.19778	-3.56576	-3.36798	-0.06985	-0.08939	0.97508	0.09021	0.05138
1981	-0.47804	-0.75036	-0.01721	0.03119	-0.20801	0.62034	0.41233	-0.96505	-0.03501	0.90058	0.04771	-0.10223
1982	0.91102	0.50508	-0.00083	-0.10075	0.20521	-0.02997	0.17524	-0.67400	-0.00256	0.94956	0.00435	0.05493
1983	-1.76589	-1.32994	-0.00522	-0.04176	-0.83239	1.12161	0.28922	-0.72845	0.00494	0.30348	-0.02194	-0.23621
1984	3.82543	3.21652	-0.00034	-0.06847	-0.12652	0.71447	0.58795	-0.70254	-0.00412	0.89396	-0.02571	-0.07181
1985	3.34933	2.14525	0.00298	0.13122	-0.83450	1.04168	0.20719	-0.81838	-0.02988	1.58949	0.00965	0.11181
1986	2.04506	0.05804	0.00398	-0.25710	-2.03370	3.88014	1.84644	-0.79428	0.00312	1.32266	0.00234	-0.14014
1987	2.73782	1.58341	0.00412	-0.13556	-0.54988	0.74751	0.19763	-0.21337	-0.00167	1.37929	-0.02728	-0.04875
1988	4.22853	3.25248	0.00553	-0.10609	-0.24517	0.28680	0.04163	0.20014	0.00079	0.99066	-0.01045	-0.14616
1989	4.86549	3.54667	0.00518	0.01110	0.18092	-0.45688	-0.27596	0.45343	-0.00443	1.25601	-0.00436	-0.12214
1990	3.88612	2.57180	0.02425	0.02607	-0.08930	-0.44836	-0.53766	0.13151	-0.01065	1.61561	0.02519	0.04000
1991	1.54626	0.35054	0.00875	-0.06525	-0.54072	0.67575	0.13503	-0.27589	-0.00826	1.48972	0.01028	-0.09865
1992	0.97480	-0.83497	-0.00305	-0.05479	-0.46585	0.63862	0.17277	-0.41528	0.00096	1.83890	0.01108	0.25918
1993	-0.80061	-1.92132	-0.00987	-0.04899	-0.87092	0.79541	-0.07552	-0.30133	0.00918	1.28832	0.00345	0.25547
1994	0.53597	0.11707	0.00443	-0.08087	-0.37287	0.39526	0.02239	-0.39280	0.00852	0.71471	-0.01116	0.15367
1995	0.62974	0.52496	0.00554	-0.04283	-0.19063	0.15194	-0.03869	-0.11381	-0.00090	0.30076	-0.01266	0.00738
1996	1.51755	1.53595	0.00386	-0.03194	0.39919	-0.63814	-0.23895	-0.46359	-0.00133	0.59592	0.01089	0.10674
1997	0.93056	0.54779	0.00482	0.01910	0.09015	-0.51950	-0.42934	-0.40024	-0.00896	1.01264	0.01890	0.16585
1998	-2.32846	-3.52030	0.00747	-0.00915	0.23108	0.24313	0.47422	-0.49102	-0.00100	0.99101	0.01754	0.20277
1999	-0.50530	-0.55889	-0.00740	-0.03375	-1.12645	0.86508	-0.26138	-0.52810	-0.00355	0.77766	0.00263	0.10747
2000	-0.01529	0.69245	-0.00019	0.00444	-0.46803	-0.23583	-0.70386	0.08725	-0.00511	-0.00730	-0.02959	-0.05339
2001	0.42896	-0.40427	0.00900	0.00880	0.44108	-0.36885	0.07223	-0.56832	0.01433	1.09798	0.00850	0.19071
2002	1.62364	0.78536	-0.01750	0.00953	0.06361	-0.05021	0.01340	-0.22275	-0.00316	0.95067	-0.01162	0.11970
2003	0.59777	0.53901	-0.00713	-0.02316	-0.37656	0.02356	-0.35300	-0.09819	-0.00613	0.45800	-0.00841	0.09678
2004	2.61774	2.66161	0.00727	0.01929	0.03238	-0.57112	-0.53874	0.26765	-0.00546	0.28555	-0.01758	-0.06185
2005	2.69552	2.91441	0.02956	0.05311	0.46561	-1.43705	-0.97143	0.18762	0.00639	0.48674	-0.01571	0.00481
2006	2.76460	2.28553	0.03999	0.08351	1.01583	-1.59688	-0.58104	0.48778	0.02672	0.38950	0.02927	0.00336

Average	γ^{τ}/ρ^{t-1}	τ^{t}/ρ^{t-1}	$\alpha_N{}^{t}/\rho^{t-1}$	$\alpha_G{}^{t}/\rho^{t-1}$	$\alpha_X{}^{t}/\rho^{t-1}$	$\alpha_M{}^{t}/\rho^{t-1}$	$\alpha_{XM}{}^{t}/\rho^{t-1}$	$\alpha_I{}^{t}/\rho^{t-1}$	$\alpha_{IV}{}^{t}/\rho^{t-1}$	$\beta_K{}^{t}/\rho^{t-1}$	$\beta_{KIV}{}^{t}/\rho^{t-1}$	$\beta_{LD}{}^{t}/\rho^{t-1}$
1956-2006	3.27008	2.60627	0.01235	-0.05800	-0.35132	0.18932	-0.16200	-0.38418	-0.03620	1.20032	0.04493	0.04659
1956-1973	6.96794	5.84035	0.02653	-0.10163	-0.45098	0.49688	0.04590	-0.38789	-0.08223	1.57046	0.11378	-0.05733
1974-1979	1.16642	1.25272	0.00951	-0.10160	-0.63185	-0.26190	-0.89375	-0.86491	-0.03657	1.35017	0.02462	0.42623
1980-1990	1.92173	1.32383	0.00178	-0.02962	-0.39414	0.35560	-0.03854	-0.38008	-0.01535	1.10694	0.00816	-0.05539
1991-2001	0.26493	-0.31555	0.00212	-0.03047	-0.26127	0.18208	-0.07919	-0.35119	0.00035	0.91821	0.00271	0.11793
2002-2006	2.05985	1.83719	0.01044	0.02846	0.24018	-0.72634	-0.48616	0.12442	0.00367	0.51409	-0.00481	0.03256

Looking at Table 3-1, it can be seen that there are five different periods: the rapid economic growth for 1955–1973, the slowdown of economic growth between two oil shocks for 1974–1979, the revival of steady economic growth for 1980–1990, the long recession for 1991–2001 and the modest economic recovery for 2002–2006. Over the 52 period, real income per unit labour ρ^t grew on average by 3.27008% annually. Productivity growth τ^t contributed the most to the overall annual growth in real income per unit labour (2.60627%), the growth in the capital services $\beta_K{}^t$ contributed the second largest amount to the overall annual growth in real income per unit labour (1.20032%),

declining real import prices α_M^t contributed 0.18932% per year and the changes in land services β_{LD}^t contributed 0.04659% per year. The largest negative contributing factor to the growth of real income per unit labour over the sample period was the fall in real investment prices α_I^t (–0.38418% per year) while falls in real export prices α_X^t contributed –0.35132% per year. The remaining contributions were very small. Thus, the effect of changes in the terms of trade on real income per unit labour α_{XM}^t was very small for Japan on average over the entire period 1955–2006: an overall negative contribution of –0.162% per year. We can see that the importance of the contribution of capital services growth relative to the contribution of productivity growth increases over time. The average contribution after 1973 of capital services growth β_K^t (1.1964% per year) was larger than that of productivity growth τ^t (0.8368% per year). However, this is not the end of story. During the period of economic recovery 2002-2006, the average contribution of productivity growth τ^t of 1.83719% became bigger than the average contribution of capital services growth β_K^t, which was 0.51409%. Thus we can observe that the recent increase of real income per unit labour was mostly boosted by productivity growth τ^t rather than by capital deepening β_K^t.

The annual change information in the previous table can be converted into cumulative changes using equations (3.47)–(3.49). The difference between the current level of net real income per unit labour and its level in 1955, $\rho^t - \rho^{1955}$ is decomposed into the sum of the level of productivity factor T^t, the levels of several real output price factors A_C^t, A_N^t, A_G^t, A_X^t, A_M^t, A_I^t and A_{IV}^t, and the levels of several input quantity factors B_K^t, B_{KIV}^t, B_{LD}^t and B_{LB}^t. The cumulative effect of changes in the terms of trade is A_T^t and is simply the product of the levels of real export and import prices A_X^t and A_M^t. Following Table 3-2 and Figure 3-1 give this cumulative growth information.

Table 3-2: Decomposition of the Cumulative Change in Real Income Per Unit Labour (in 1955 yen)

	$\rho^t - \rho^{1955}$	T^t	$A_N{}^t$	$A_G{}^t$	$A_X{}^t$	$A_M{}^t$	$A_{XM}{}^t$	$A_I{}^t$	$A_{IV}{}^t$	$B_K{}^t$	$B_{KIV}{}^t$	$B_{LD}{}^t$
1956	0.08816	0.12211	0.00131	0.00222	0.00816	-0.01537	-0.00721	0.03748	0.00625	-0.04314	-0.00090	-0.02996
1957	0.14765	0.21684	0.00200	0.00290	0.00115	-0.01697	-0.01583	0.05291	0.00316	-0.05995	0.00012	-0.05449
1958	0.19026	0.24678	0.00208	0.00017	-0.01649	0.02758	0.01109	0.03487	0.00225	-0.04399	0.00290	-0.06590
1959	0.39458	0.46414	0.00221	-0.00167	-0.01941	0.03861	0.01921	0.01296	0.00161	-0.02929	0.00209	-0.07669
1960	0.66536	0.75539	0.00282	-0.00372	-0.01995	0.04687	0.02693	0.00678	-0.00223	-0.01850	0.00338	-0.10549
1961	0.98356	1.01668	0.00313	-0.00682	-0.03966	0.06326	0.02361	0.01886	-0.01339	0.04605	0.00815	-0.11271
1962	1.00309	1.04714	0.00312	-0.01246	-0.06950	0.09215	0.02265	-0.02890	-0.02146	0.10568	0.01621	-0.12890
1963	1.18226	1.22079	0.00295	-0.01974	-0.08557	0.11089	0.02531	-0.09978	-0.02702	0.18329	0.01998	-0.12353
1964	1.51073	1.50045	0.00467	-0.02341	-0.09134	0.11586	0.02452	-0.11239	-0.02857	0.24911	0.02477	-0.12842
1965	1.52012	1.50566	0.00415	-0.02987	-0.12370	0.14789	0.02419	-0.17573	-0.03270	0.32180	0.03003	-0.12742
1966	1.81106	1.73951	0.00504	-0.03202	-0.14201	0.16130	0.01929	-0.18107	-0.03544	0.36977	0.03140	-0.10543
1967	2.22466	2.03505	0.00629	-0.03241	-0.15626	0.18278	0.02652	-0.16992	-0.03976	0.43601	0.03551	-0.07264
1968	2.69616	2.41084	0.00670	-0.03666	-0.18116	0.20788	0.02671	-0.20100	-0.04433	0.54642	0.04700	-0.05952
1969	3.28880	2.79979	0.00822	-0.04066	-0.19463	0.21117	0.01653	-0.19239	-0.04339	0.68193	0.05891	-0.00014
1970	3.64780	3.04748	0.01140	-0.04705	-0.21882	0.24002	0.02120	-0.23910	-0.05123	0.82569	0.06945	0.00996
1971	3.69297	2.98340	0.01377	-0.05786	-0.24998	0.30216	0.05218	-0.33371	-0.05621	0.98593	0.07888	0.02659
1972	4.09282	3.24275	0.01685	-0.06840	-0.29637	0.36414	0.06777	-0.36189	-0.05503	1.11872	0.07961	0.05244
1973	4.59157	3.55628	0.01929	-0.07239	-0.30842	0.31516	0.00674	-0.25769	-0.04702	1.21671	0.07723	0.09241
1974	4.45314	3.33928	0.02067	-0.03782	-0.25656	0.08142	-0.17514	-0.29391	-0.06057	1.40041	0.08778	0.17243
1975	4.40159	3.33595	0.02059	-0.08835	-0.31584	0.09826	-0.21758	-0.46919	-0.06400	1.54506	0.09738	0.24174
1976	4.34314	3.44501	0.02138	-0.09417	-0.38577	0.13219	-0.25358	-0.58824	-0.06405	1.56192	0.09157	0.22331
1977	4.34059	3.46333	0.02176	-0.10150	-0.49024	0.22639	-0.26386	-0.65008	-0.06450	1.60498	0.09153	0.23892
1978	4.63253	3.65791	0.02151	-0.11362	-0.58747	0.38290	-0.20457	-0.67546	-0.06374	1.66702	0.08985	0.25364
1979	5.04551	4.04110	0.02303	-0.11041	-0.54604	0.19945	-0.34659	-0.58751	-0.06136	1.74191	0.08695	0.25840
1980	4.87223	4.02446	0.02283	-0.09746	-0.53214	-0.05113	-0.58327	-0.59242	-0.06764	1.81043	0.09329	0.26201
1981	4.83947	3.97303	0.02165	-0.09532	-0.54640	-0.00861	-0.55501	-0.65857	-0.07004	1.87215	0.09656	0.25501
1982	4.90161	4.00749	0.02159	-0.10219	-0.53240	-0.01065	-0.54305	-0.70454	-0.07022	1.93692	0.09686	0.25875
1983	4.78006	3.91594	0.02123	-0.10507	-0.58970	0.06655	-0.52315	-0.75468	-0.06988	1.95781	0.09535	0.24250
1984	5.03872	4.13344	0.02121	-0.10970	-0.59825	0.11486	-0.48339	-0.80219	-0.07015	2.01826	0.09361	0.23764
1985	5.27386	4.28404	0.02142	-0.10049	-0.65684	0.18799	-0.46885	-0.85964	-0.07225	2.12985	0.09429	0.24549
1986	5.42224	4.28825	0.02171	-0.11914	-0.80439	0.46952	-0.33488	-0.91727	-0.07203	2.22581	0.09446	0.23532
1987	5.62495	4.40548	0.02201	-0.12918	-0.84511	0.52486	-0.32024	-0.93307	-0.07215	2.32793	0.09244	0.23171
1988	5.94659	4.65289	0.02243	-0.13725	-0.86375	0.54668	-0.31708	-0.91784	-0.07209	2.40329	0.09164	0.22059
1989	6.33234	4.93408	0.02284	-0.13637	-0.84941	0.51045	-0.33896	-0.88189	-0.07244	2.50287	0.09130	0.21091
1990	6.65543	5.14790	0.02486	-0.13420	-0.85684	0.47318	-0.38366	-0.87096	-0.07333	2.63719	0.09339	0.21424
1991	6.78898	5.17817	0.02562	-0.13984	-0.90354	0.53154	-0.37200	-0.89479	-0.07404	2.76586	0.09428	0.20572
1992	6.87448	5.10494	0.02535	-0.14464	-0.94440	0.58755	-0.35684	-0.93121	-0.07396	2.92714	0.09525	0.22845
1993	6.80358	4.93478	0.02448	-0.14898	-1.02153	0.65800	-0.36353	-0.95790	-0.07314	3.04124	0.09556	0.25107
1994	6.85066	4.94507	0.02486	-0.15608	-1.05428	0.69272	-0.36156	-0.99240	-0.07239	3.10403	0.09458	0.26457
1995	6.90628	4.99144	0.02535	-0.15987	-1.07112	0.70614	-0.36498	-1.00246	-0.07247	3.13059	0.09346	0.26522
1996	7.04116	5.12795	0.02570	-0.16271	-1.03564	0.64942	-0.38622	-1.04366	-0.07259	3.18355	0.09443	0.27471
1997	7.12513	5.17738	0.02613	-0.16098	-1.02751	0.60255	-0.42496	-1.07977	-0.07340	3.27492	0.09613	0.28968
1998	6.91308	4.85679	0.02681	-0.16182	-1.00646	0.62469	-0.38177	-1.12449	-0.07349	3.36517	0.09773	0.30814
1999	6.86813	4.80708	0.02615	-0.16482	-1.10666	0.70164	-0.40502	-1.17146	-0.07381	3.43434	0.09796	0.31770
2000	6.86678	4.86836	0.02614	-0.16442	-1.14808	0.68077	-0.46731	-1.16374	-0.07426	3.43370	0.09535	0.31298
2001	6.90474	4.83259	0.02693	-0.16364	-1.10905	0.64813	-0.46092	-1.21403	-0.07299	3.53085	0.09610	0.32985
2002	7.04902	4.90238	0.02538	-0.16280	-1.10340	0.64367	-0.45973	-1.23382	-0.07327	3.61533	0.09506	0.34049
2003	7.10300	4.95105	0.02474	-0.16489	-1.13740	0.64579	-0.49161	-1.24269	-0.07383	3.65669	0.09431	0.34923
2004	7.34081	5.19285	0.02540	-0.16314	-1.13446	0.59391	-0.54055	-1.21837	-0.07432	3.68263	0.09271	0.34361
2005	7.59210	5.46455	0.02815	-0.15819	-1.09105	0.45994	-0.63111	-1.20088	-0.07373	3.72801	0.09124	0.34406
2006	7.85678	5.68336	0.03198	-0.15019	-0.99380	0.30706	-0.68674	-1.15419	-0.07117	3.76530	0.09405	0.34438

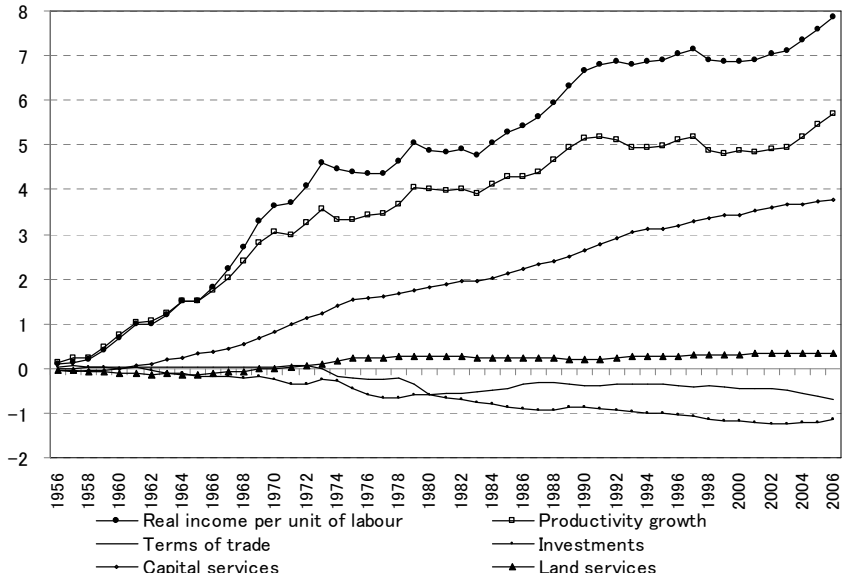

Figure 3-1: Real Income Change Per Unit Labour (in 1955 yen)

Over the 52 year period, real income per unit labour grew by 7.85678 yen at the prices of 1955. From Table 3-2, it can be seen that productivity growth contributed the most to the overall growth in real income per unit labour (T^{2006} = 5.68336 yen, 72.34 % of the overall change in real income per unit labour), the growth in capital services made the second largest contribution (B_K^{2006} = 3.7653 yen, 47.92 % of the overall change in real income per unit labour), the change in real investment prices made the third largest contribution in magnitude, (A_I^{2006} = –1.15419 yen, –14.69 % of the overall change in real income per unit labour), the change in terms of trade made the forth largest contribution (A_{XM}^{2006} = –0.68674 yen, –8.74 % of the overall change in real income per unit labour) followed closely by the growth in land services (B_{LD}^{2006} = 0.34438 yen, 4.38 % of the overall change in real income per unit labour). The change in the real price of the consumption of NPISHs and the growth in inventory services have very small impact on the growth in the real income per unit labour (less than one percent of the overall change in real income per unit labour). Figure 3-1 plots real income per unit labour and main factors contributing to its growth.

3.5.3 The Decomposition of Net Real Income Growth into Explanatory Factors

In the previous subsection 3.5.2, we focus on the real income per unit labour. Deflating the income of the market sector by the price of household consumption, we obtain the (gross) real income. Real income captures how much consumption people can purchase for their income. Since economic welfare comes from consumption, while real GDP is the measure of output, real income is the measure of welfare. However, it is well known that net real income is the better measure of welfare, because it captures the sustainable level of welfare. Net income is the gross income net of the value of depreciated assets in the production period. By deducting depreciation from the income, we come closer to a measure of income that could be consumed in the present period without impairing production possibilities in future period. Deflating the net income of the market sector by the price of household consumption, we obtain the net real income. This subsection analyses the net real income per unit labour. By applying our methodology to the Japanese economy, we can decompose the change in net real income per unit labour into explanatory factors.

In this section, we consider the production model based on net output concept. We illustrate the theory by considering a very simple two output, two input model of the market sector. One of the outputs is output in year t, Y^t and the other output is an

investment good, I^t. One of the inputs is the flow of noncapital primary input X^t and the other input is K^t, capital services. Suppose that the average prices during period t of a unit of Y^t, X^t and I^t are P_Y^t, P_X^t and P_I^t respectively. Suppose further that the real interest rate prevailing at the beginning of period t is r^{t*}. The value of the beginning of period t capital stock is assumed to be P_I^t, the investment price for period t. The user cost of capital is calculated such as $u^t = (r^{t*} + \delta^t + \tau^t)P_I^t/(1 + r^{t*})$. As usual, it represents price of capital services input. Thus, the period t profit of the market sector is expressed as follows:[84]

$$(3.59)\ \Pi^t = P_Y^t Y^t + P_I^t I^t - P_X^t X^t - [(r^{t*} + \delta)P_I^t/(1 + r^{t*})]K^t$$

Under the assumption of constant returns to scale, a zero profit condition should be satisfied such as $\Pi^t = 0$. Using this condition, we obtain the following value of output equals value of input equation:

$$(3.60)\ P_Y^t Y^t + P_I^t I^t = P_X^t X^t + [(r^{t*} + \delta)P_I^t/(1 + r^{t*})]K^t.$$

Equation (3.60) is essentially the closed economy counterpart to the (gross) value of outputs equals (gross) value of primary inputs equation (3.4), $P^t \cdot Y^t = W^t \cdot X^t$. The (gross) payment to primary inputs that is defined by the right hand side of (3.60) is not income, in the sense of Hicks. Net income in this chapter is the same concept as Hicks' third concept of income: "Income No. 3 must be defined as the maximum amount of money which the individual can spend this week, and still be able to expect to spend this week, and still be able to expect to spend the same amount in real terms in each ensuing week." (Hicks, 1946).

The owner of a unit of capital cannot spend the entire period t gross rental income $(r^{t*} + \delta)P_I^t/(1 + r^{t*})$ on consumption during period t because the depreciation portion of the rental, $\delta P_I^t/(1 + r^{t*})$, is required in order to keep his or her capital intact. Thus the owner of a new unit of capital at the beginning of period t loans the unit to the market sector and gets the gross return $(r^{t*} + \delta)P_I^t$ at the end of the period plus the depreciated unit of the initial capital, which is worth only $(1 - \delta)P_I^t$. Thus δP_I^t of this gross return must be set aside in order to restore the lender of the capital services to his or her original wealth position at the beginning of period t. This means that *period t Hicksian market sector*

[84] We have temporarily neglected tax factors for the sake of simplicity.

income is not the value of payments to primary inputs, $P_X^t X^t + [(r^{t*} + \delta)P_I^t/(1 + r^{t*})]K^t$; instead it is the value of payments to labour $P_X^t X^t$ plus the reward for waiting, $[r^{t*}P_I^t/(1 + r^{t*})]K^t$. Using this definition of market sector net income, we can rearrange equation (3.60) as follows:

(3.61) Hicksian market sector income $\equiv P_X^t X^t + [r^{t*}P_I^t/(1 + r^{t*})]K^t$

$= P_Y^t Y^t + P_I^t I^t - [\delta P_I^t/(1 + r^{t*})]K^t$

= Value of consumption + value of gross investment – value of depreciation.

Thus in this net income framework, our new output concept is equal to our old output concept less the value of depreciation. Hence the overstatement of income problem that is implicit in the approaches used in previous subsections can readily be remedied: all we need to do is to take the user cost formula for an asset and decompose it into two parts:

- One part that represents depreciation and foreseen obsolescence, $\delta P_I^t/(1 + r^{t*})$ and
- The remaining part that is the reward for postponing consumption, $r^{t*}P_I^t/(1 + r^{t*})$.

Thus, in this subsection, we split up each user cost times the beginning of the period stock K^t into the depreciation component $[\delta^t P_I^t/(1 + r^{t*})]K^t$ and the remaining term $[r^{t*}P_I^t/(1 + r^{t*})]K^t$. The first term is considered as an intermediate input cost for the market sector and is an offset to gross investment made by the market sector during the period under consideration. We regard the second term as a genuine income component and call it *waiting capital services*. We take the price of depreciation P_{DEP}^t to be the corresponding investment price and the quantity of depreciation Y_{DEP}^t is taken to be the depreciation rate times the beginning of the period stock such as $P_{DEP}^t=P_I^t/(1 + r^{t*})$ and $Y_{DEP}^t=\delta_n^t K^t$. We take the price of waiting capital services W_{KW}^t to be the corresponding investment price times the real rate of return and the quantity of waiting capital services X_{KW}^t is taken to be the beginning of the period stock such as $W_{KW}^t=r^{t*}P_I^t/(1 + r^{t*})$, and $X_{KW}^t=K^t$.[85]

[85] These data have already been constructed in Chapter 2.

Table 3-3: Decomposition of Growth Rate of Net Real Income Per Unit Labour (%)

	γ^{t}/ρ^{t-1}	τ^{t}/ρ^{t-1}	α_N^{t}/ρ^{t-1}	α_G^{t}/ρ^{t-1}	α_X^{t}/ρ^{t-1}	α_M^{t}/ρ^{t-1}	$\alpha_{XM}^{t}/\rho^{t-1}$	α_I^{t}/ρ^{t-1}	$\alpha_{DEP}^{t}/\rho^{t-1}$	$\alpha_{IV}^{t}/\rho^{t-1}$	$\beta_{KW}^{t}/\rho^{t-1}$	$\beta_{KIV}^{t}/\rho^{t-1}$	$\beta_{LD}^{t}/\rho^{t-1}$
1956	5.16234	7.09905	0.07684	0.13006	0.47728	-0.89871	-0.42143	2.19117	-1.34163	0.36516	-1.13236	-0.05289	-1.75163
1957	3.42361	5.31072	0.03815	0.03728	-0.38997	-0.08907	-0.47904	0.85802	-0.31773	-0.17171	-0.54500	0.05672	-1.36380
1958	2.15837	1.71104	0.00428	-0.14635	-0.94794	2.39487	1.44694	-0.96964	0.38901	-0.04875	0.23544	0.14976	-0.61336
1959	10.90201	11.44312	0.00709	-0.09691	-0.15368	0.58056	0.42688	-1.15297	0.66740	-0.03353	0.25145	-0.04296	-0.56757
1960	12.89222	13.95568	0.02860	-0.09726	-0.02555	0.39202	0.36647	-0.29321	0.33740	-0.18251	0.08207	0.06134	-1.36637
1961	12.41574	11.56860	0.01319	-0.13028	-0.82842	0.68878	-0.13964	0.50777	0.18408	-0.46884	0.98403	0.20048	-0.30365
1962	-0.05697	1.06168	-0.00041	-0.21100	-1.11560	1.07994	-0.03566	-1.78551	0.50639	-0.30155	1.01286	0.30139	-0.60518
1963	6.07559	6.47743	-0.00631	-0.27214	-0.60141	0.70094	0.09953	-2.65175	0.90162	-0.20820	1.39313	0.14119	0.20109
1964	10.59010	9.83337	0.06050	-0.12943	-0.20317	0.17517	-0.02799	-0.44457	0.22028	-0.05445	1.13624	0.16883	-0.17266
1965	-0.29847	0.13606	-0.01634	-0.20590	-1.03213	1.02149	-0.01064	-2.01958	0.62043	-0.13192	1.12983	0.16762	0.03196
1966	8.80586	7.47759	0.02834	-0.06895	-0.58548	0.42902	-0.15646	-0.17101	0.29033	-0.08750	0.74617	0.04399	0.70337
1967	11.19742	8.56801	0.03677	-0.01134	-0.41889	0.63129	0.21240	0.32795	0.07765	-0.12706	1.02863	0.12058	0.96383
1968	11.35070	9.90224	0.01085	-0.11232	-0.65829	0.66333	0.00504	-0.82172	0.34066	-0.12065	1.49602	0.30370	0.34688
1969	12.40148	9.17208	0.03611	-0.09503	-0.31975	0.07815	-0.24160	0.20451	-0.03339	0.02215	1.64448	0.28275	1.40943
1970	6.28300	5.16961	0.06701	-0.13498	-0.51082	0.60941	0.09860	-0.98664	0.28873	-0.16545	1.51001	0.22264	0.21348
1971	-0.44911	-1.37424	0.04722	-0.21479	-0.61928	1.23479	0.61551	-1.87996	0.44914	-0.09906	1.48908	0.18754	0.33046
1972	6.83319	5.11441	0.06134	-0.21046	-0.92597	1.23728	0.31130	-0.56265	0.36379	0.02355	1.20144	0.01443	0.51604
1973	8.03421	5.79734	0.04577	-0.07450	-0.22512	-0.91524	-1.14036	1.94693	-0.17785	0.14975	0.78480	-0.04450	0.74682
1974	-4.04094	-3.71652	0.02382	0.59790	0.89691	-4.04248	-3.14557	-0.62641	0.17587	-0.23439	1.31797	0.18253	1.38388
1975	-1.38038	-0.10221	-0.00142	-0.91075	-1.06851	0.30348	-0.76504	-3.15909	1.11363	-0.06186	1.08412	0.17308	1.24916
1976	-0.14292	1.96380	0.01443	-0.10636	-1.27792	0.62007	-0.65785	-2.17580	1.07874	-0.00094	0.18410	-0.10623	-0.33681
1977	-0.05550	0.30838	0.00699	-0.13420	-1.91210	1.72395	-0.18815	-1.13178	0.44191	-0.00814	0.36441	-0.00075	0.28582
1978	5.07852	3.52999	-0.00471	-0.22198	-1.78042	2.86607	1.08565	-0.46479	0.40251	0.01393	0.49912	-0.03076	0.26956
1979	6.22923	6.69302	0.02650	0.05600	0.72197	-3.19702	-2.47505	1.53269	-0.21115	0.04141	0.53332	-0.05046	0.08295
1980	-3.28620	-0.29272	-0.00327	0.21248	0.22800	-4.11068	-3.88267	-0.08052	0.22556	-0.10305	0.47476	0.10400	0.05923
1981	-0.76339	-0.89638	-0.02001	0.03627	-0.24183	0.72121	0.47938	-1.12197	0.44163	-0.04070	0.42176	0.05547	-0.11885
1982	0.58638	0.56313	-0.00097	-0.11747	0.23926	-0.03494	0.20432	-0.78584	0.24192	-0.00298	0.41516	0.00508	0.06404
1983	-2.05034	-1.61699	-0.00610	-0.04885	-0.97365	1.31195	0.33830	-0.85207	0.32618	0.00578	0.10538	-0.02567	-0.27630
1984	4.24524	3.74457	-0.00040	-0.08033	-0.14842	0.83815	0.68972	-0.82416	0.45662	-0.00484	0.37844	-0.03016	-0.08424
1985	3.13115	2.52050	0.00348	0.15332	-0.97501	1.21708	0.24207	-0.95617	0.42739	-0.03491	0.63356	0.01127	0.13064
1986	1.71035	0.03128	0.00466	-0.30103	-2.38115	4.54306	2.16191	-0.92998	0.42064	0.00365	0.48056	0.00274	-0.16408
1987	2.27648	1.81145	0.00484	-0.15924	-0.64595	0.87810	0.23215	-0.25064	0.16284	-0.00196	0.56634	-0.03204	-0.05727
1988	4.15239	3.84301	0.00653	-0.12519	-0.28930	0.33842	0.04912	0.23617	-0.02917	0.00093	0.35578	-0.01234	-0.17247
1989	4.59268	4.16851	0.00611	0.01310	0.21364	-0.53952	-0.32587	0.53544	-0.08498	-0.00523	0.43499	-0.00515	-0.14423
1990	3.28436	3.02015	0.02871	0.03086	-0.10573	-0.53083	-0.63656	0.15570	0.03319	-0.01261	0.58774	0.02982	0.04735
1991	0.76672	0.38625	0.01042	-0.07770	-0.64391	0.80471	0.16080	-0.32854	0.16323	-0.00984	0.56734	0.01224	-0.11747
1992	0.06414	-0.94522	-0.00366	-0.06576	-0.55905	0.76638	0.20733	-0.49836	0.34787	0.00115	0.69645	0.01330	0.31103
1993	-1.89217	-2.39426	-0.01195	-0.05932	-1.05467	0.96322	-0.09145	-0.36490	0.17705	0.01111	0.52801	0.00418	0.30937
1994	0.46194	0.16668	0.00542	-0.09902	-0.45656	0.48398	0.02742	-0.48096	0.33154	0.01044	0.32593	-0.01367	0.18816
1995	0.74731	0.64555	0.00679	-0.05249	-0.23359	0.18617	-0.04741	-0.13945	0.16285	-0.00110	0.17906	-0.01552	0.00904
1996	1.77005	1.85391	0.00473	-0.03909	0.48858	-0.78103	-0.29245	-0.56740	0.39290	-0.00163	0.27511	0.01333	0.13065
1997	0.51581	0.70358	0.00588	0.02332	0.11007	-0.63425	-0.52418	-0.48864	0.22306	-0.01094	0.35816	0.02308	0.20248
1998	-3.77675	-4.38280	0.00916	-0.01121	0.28329	0.29806	0.58135	-0.60195	0.01909	-0.00122	0.34076	0.02150	0.24858
1999	-0.91622	-0.70282	-0.00920	-0.04199	-1.40173	1.07648	-0.32525	-0.65716	0.43183	-0.00441	0.25578	0.00327	0.13373
2000	0.05146	0.91206	-0.00024	0.00555	-0.58482	-0.29467	-0.87950	0.10902	0.07476	-0.00639	-0.06015	-0.03697	-0.06671
2001	-0.11205	-0.54598	0.01124	0.01099	0.55078	-0.46058	0.09020	-0.70966	0.41341	0.01789	0.35110	0.01061	0.23814
2002	1.42882	0.97646	-0.02197	0.01196	0.07986	-0.06304	0.01682	-0.27966	0.27758	-0.00396	0.31590	-0.01459	0.15028
2003	0.61393	0.73102	-0.00897	-0.02914	-0.47366	0.02963	-0.44403	-0.12351	0.23225	-0.00772	0.15285	-0.01058	0.12174
2004	2.87675	3.39291	0.00914	0.02427	0.04073	-0.71828	-0.67756	0.33661	-0.16542	-0.00686	0.06357	-0.02211	-0.07779
2005	2.78109	3.69571	0.03708	0.06663	0.58412	-1.80279	-1.21867	0.23537	-0.18021	0.00802	0.15082	-0.01970	0.00603
2006	2.59345	2.98263	0.05012	0.10467	1.27331	-2.00163	-0.72832	0.61141	-0.51884	0.03349	0.01738	0.03669	0.00421

Average	γ^{t}/ρ^{t-1}	τ^{t}/ρ^{t-1}	α_N^{t}/ρ^{t-1}	α_G^{t}/ρ^{t-1}	α_X^{t}/ρ^{t-1}	α_M^{t}/ρ^{t-1}	$\alpha_{XM}^{t}/\rho^{t-1}$	α_I^{t}/ρ^{t-1}	$\alpha_{DEP}^{t}/\rho^{t-1}$	$\alpha_{IV}^{t}/\rho^{t-1}$	$\beta_{KW}^{t}/\rho^{t-1}$	$\beta_{KIV}^{t}/\rho^{t-1}$	$\beta_{LD}^{t}/\rho^{t-1}$
1956-2006	3.12280	2.97005	0.01416	-0.06624	-0.40356	0.21122	-0.19234	-0.44215	0.23084	-0.04036	0.54517	0.05016	0.05351
1956-1973	7.09563	6.57910	0.02994	-0.11357	-0.50468	0.55633	0.05166	-0.42794	0.20924	-0.09114	0.80269	0.12681	-0.07116
1974-1979	0.94800	1.44608	0.01094	-0.11990	-0.73668	-0.28766	-1.02434	-1.00420	0.50025	-0.04167	0.66384	0.02790	0.48909
1980-1990	1.62537	1.53605	0.00214	-0.03510	-0.46183	0.42109	-0.04074	-0.44310	0.23835	-0.01781	0.44132	0.00937	-0.06511
1991-2001	-0.21089	-0.39119	0.00260	-0.03697	-0.31833	0.21895	-0.09938	-0.42982	0.24887	0.00046	0.34705	0.00321	0.14427
2002-2006	2.05881	2.35575	0.01308	0.03568	0.30087	-0.91122	-0.61035	0.15604	-0.07093	0.00459	0.14010	-0.00606	0.04090

Thus in the net production model of this subsection, we add depreciations to the original list of net outputs and use waiting capital services instead of capital services among the original list of primary inputs. Substituting the estimates defined in (3.32), (3.41) and (3.45) into equation (3.52), we can decompose the growth rate of real income per unit labour $(\rho^{t} - \rho^{t-1})/\rho^{t-1} = \gamma^{t}/\rho^{t-1}$ into the contribution of technical progress τ^{t}/ρ^{t-1} changes in real output prices α_C^{t}/ρ^{t-1}, α_N^{t}/ρ^{t-1}, α_G^{t}/ρ^{t-1}, α_X^{t}/ρ^{t-1}, α_M^{t}/ρ^{t-1}, α_I^{t}/ρ^{t-1}, $\alpha_{DEP}^{t}/\rho^{t-1}$ and

α_{IV}^t/ρ^{t-1},[86] and growth in relative input quantity changes β_{KW}^t/ρ^{t-1}, β_{KIV}^t/ρ^{t-1}, β_{LD}^t/ρ^{t-1} and β_{LB}^t/ρ^{t-1}.[87] The chain link information on period by period changes in net real income per unit labour input that corresponds to (3.52) is given in Table 3-3.

The new results are quite interesting. While the average growth rate of real income per unit labour ρ^t was 3.27008% per year in the gross output model, the average growth rate of net real income per unit labour ρ^t has now decreased by 0.14728 percent points per year to 3.1228%. More importantly, there are some big shifts in the explanatory factors. Productivity growth τ^t now accounts for 2.97005% of the overall annual growth in net real income per unit labour compared to 2.60627% of the overall annual growth in real income per unit labour, an increase of 0.36378 percentage points per year. The growth in capital services β_K^t accounted for 1.20032% of the overall annual growth in real income per unit labour while the growth in waiting capital services β_{KW}^t accounts for 0.54517% of the overall annual growth in net real income per unit labour, a decrease of 0.65515 percentage points per year. The average contributions of changes in real export prices α_X^t and real import prices α_M^t remain quite similar estimates in the previous gross output model. Thus, as we stated in the previous analysis, the effect of changes in the terms of trade on living standards α_T^t was negligible for Japan on average over the entire periods 1955–2006: an overall negative contribution of -0.19234%. The negative contribution of the change in real investment prices α_I^t equal to –0.44215% was offset by the positive contribution of the change in real depreciation prices α_{DEP}^t equal to 0.23084%. Finally, we note that the productivity recovery in the period 2002-2006 is quite striking. Using the previous gross output model, the average contribution of productivity growth during this period was 1.83719% per year and using the current net output model, its average contribution increases to a very respectable 2.35575% per year.

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[86] Since we divided market sector nominal income by the price of consumption, α_C^t/ρ^{t-1} will be identically equal to zero and hence it is not listed.

[87] Since we divided market sector nominal income by the quantity of labour services, β_{LB}^t/ρ^{t-1} will be identically equal to zero and hence it is not listed.

Table 3-4: Decomposition of the Cumulative Change in Net Real Income Per Unit Labour (in 1955 yen)

	$\rho^t - \rho^{1955}$	T^t	$A_N{}^t$	$A_G{}^t$	$A_X{}^t$	$A_M{}^t$	$A_{XM}{}^t$	$A_I{}^t$	$A_{DEP}{}^t$	$A_{PI}{}^t$	$B_{KW}{}^t$	$B_{KIV}{}^t$	$B_{LD}{}^t$
1956	0.08830	0.12143	0.00131	0.00222	0.00816	-0.01537	-0.00721	0.03748	-0.02295	0.00625	-0.01937	-0.00090	-0.02996
1957	0.14988	0.21695	0.00200	0.00290	0.00115	-0.01697	-0.01583	0.05291	-0.02866	0.00316	-0.02917	0.00012	-0.05449
1958	0.19004	0.24879	0.00208	0.00017	-0.01649	0.02758	0.01109	0.03487	-0.02143	0.00225	-0.02479	0.00290	-0.06590
1959	0.39723	0.46626	0.00221	-0.00167	-0.01941	0.03861	0.01921	0.01296	-0.00874	0.00161	-0.02001	0.00209	-0.07669
1960	0.66896	0.76041	0.00282	-0.00372	-0.01995	0.04687	0.02693	0.00678	-0.00163	-0.00223	-0.01828	0.00338	-0.10549
1961	0.96438	1.03567	0.00313	-0.00682	-0.03966	0.06326	0.02361	0.01886	0.00275	-0.01339	0.00513	0.00815	-0.11271
1962	0.96286	1.06407	0.00312	-0.01246	-0.06950	0.09215	0.02265	-0.02890	0.01629	-0.02146	0.03222	0.01621	-0.12890
1963	1.12528	1.23723	0.00295	-0.01974	-0.08557	0.11089	0.02531	-0.09978	0.04040	-0.02702	0.06947	0.01998	-0.12353
1964	1.42559	1.51608	0.00467	-0.02341	-0.09134	0.11586	0.02452	-0.11239	0.04664	-0.02857	0.10169	0.02477	-0.12842
1965	1.41622	1.52035	0.00415	-0.02987	-0.12370	0.14789	0.02419	-0.17573	0.06610	-0.03270	0.13712	0.03003	-0.12742
1966	1.69156	1.75415	0.00504	-0.03202	-0.14201	0.16130	0.01929	-0.18107	0.07518	-0.03544	0.16045	0.03140	-0.10543
1967	2.07250	2.04564	0.00629	-0.03241	-0.15626	0.18278	0.02652	-0.16992	0.07782	-0.03976	0.19544	0.03551	-0.07264
1968	2.50189	2.42023	0.00670	-0.03666	-0.18116	0.20788	0.02671	-0.20100	0.09071	-0.04433	0.25204	0.04700	-0.05952
1969	3.02428	2.80659	0.00822	-0.04066	-0.19463	0.21117	0.01653	-0.19239	0.08930	-0.04339	0.32131	0.05891	-0.00014
1970	3.32177	3.05136	0.01140	-0.04705	-0.21882	0.24002	0.02120	-0.23910	0.10297	-0.05123	0.39280	0.06945	0.00996
1971	3.29917	2.98221	0.01377	-0.05786	-0.24998	0.30216	0.05218	-0.33371	0.12557	-0.05621	0.46774	0.07888	0.02659
1972	3.64148	3.23842	0.01685	-0.06840	-0.29637	0.36414	0.06777	-0.36189	0.14380	-0.05503	0.52793	0.07961	0.05244
1973	4.07147	3.54869	0.01929	-0.07239	-0.30842	0.31516	0.00674	-0.25769	0.13428	-0.04702	0.56993	0.07723	0.09241
1974	3.83783	3.33380	0.02067	-0.03782	-0.25656	0.08142	-0.17514	-0.29391	0.14445	-0.06057	0.64613	0.08778	0.17243
1975	3.76124	3.32813	0.02059	-0.08835	-0.31584	0.09826	-0.21758	-0.46919	0.20623	-0.06400	0.70628	0.09738	0.24174
1976	3.75342	3.43559	0.02138	-0.09417	-0.38577	0.13219	-0.25358	-0.58824	0.26526	-0.06405	0.71636	0.09157	0.22331
1977	3.75039	3.45244	0.02176	-0.10150	-0.49024	0.22639	-0.26386	-0.65008	0.28941	-0.06450	0.73627	0.09153	0.23892
1978	4.02772	3.64520	0.02151	-0.11362	-0.58747	0.38290	-0.20457	-0.67546	0.31139	-0.06374	0.76352	0.08985	0.25364
1979	4.38516	4.02926	0.02303	-0.11041	-0.54604	0.19945	-0.34659	-0.58751	0.29927	-0.06136	0.79413	0.08695	0.25840
1980	4.18485	4.01142	0.02283	-0.09746	-0.53214	-0.05113	-0.58327	-0.59242	0.31302	-0.06764	0.82307	0.09329	0.26201
1981	4.13984	3.95857	0.02165	-0.09532	-0.54640	-0.00861	-0.55501	-0.65857	0.33906	-0.07004	0.84793	0.09656	0.25501
1982	4.17415	3.99152	0.02159	-0.10219	-0.53240	-0.01065	-0.54305	-0.70454	0.35321	-0.07022	0.87222	0.09686	0.25875
1983	4.05349	3.89637	0.02123	-0.10507	-0.58970	0.06655	-0.52315	-0.75468	0.37240	-0.06988	0.87842	0.09535	0.24250
1984	4.29819	4.11220	0.02121	-0.10970	-0.59825	0.11486	-0.48339	-0.80219	0.39872	-0.07015	0.90023	0.09361	0.23764
1985	4.48633	4.26365	0.02142	-0.10049	-0.65684	0.18799	-0.46885	-0.85964	0.42440	-0.07225	0.93830	0.09429	0.24549
1986	4.59231	4.26559	0.02171	-0.11914	-0.80439	0.46952	-0.33488	-0.91727	0.45047	-0.07203	0.96808	0.09446	0.23532
1987	4.73580	4.37976	0.02201	-0.12918	-0.84511	0.52486	-0.32024	-0.93307	0.46073	-0.07215	1.00378	0.09244	0.23171
1988	5.00347	4.62749	0.02243	-0.13725	-0.86375	0.54668	-0.31708	-0.91784	0.45885	-0.07209	1.02671	0.09164	0.22059
1989	5.31182	4.90736	0.02284	-0.13637	-0.84941	0.51045	-0.33896	-0.88189	0.45315	-0.07244	1.05592	0.09130	0.21091
1990	5.54246	5.11944	0.02486	-0.13420	-0.85684	0.47318	-0.38366	-0.87096	0.45548	-0.07333	1.09719	0.09339	0.21424
1991	5.59807	5.14746	0.02562	-0.13984	-0.90354	0.53154	-0.37200	-0.89479	0.46732	-0.07404	1.13834	0.09428	0.20572
1992	5.60275	5.07838	0.02535	-0.14464	-0.94440	0.58755	-0.35684	-0.93121	0.49274	-0.07396	1.18924	0.09525	0.22845
1993	5.46438	4.90328	0.02448	-0.14898	-1.02153	0.65800	-0.36353	-0.95790	0.50569	-0.07314	1.22785	0.09556	0.25107
1994	5.49752	4.91524	0.02486	-0.15608	-1.05428	0.69272	-0.36156	-0.99240	0.52948	-0.07239	1.25124	0.09458	0.26457
1995	5.55138	4.96177	0.02535	-0.15987	-1.07112	0.70614	-0.36498	-1.00246	0.54121	-0.07247	1.26414	0.09346	0.26522
1996	5.67992	5.09640	0.02570	-0.16271	-1.03564	0.64942	-0.38622	-1.04366	0.56975	-0.07259	1.28412	0.09443	0.27471
1997	5.71804	5.14840	0.02613	-0.16098	-1.02751	0.60255	-0.42496	-1.07977	0.58623	-0.07340	1.31059	0.09613	0.28968
1998	5.43749	4.82282	0.02681	-0.16182	-1.00646	0.62469	-0.38177	-1.12449	0.58765	-0.07349	1.33590	0.09773	0.30814
1999	5.37200	4.77258	0.02615	-0.16482	-1.10666	0.70164	-0.40502	-1.17146	0.61852	-0.07381	1.35419	0.09796	0.31770
2000	5.37564	4.83718	0.02614	-0.16442	-1.14808	0.68077	-0.46731	-1.16374	0.62381	-0.07426	1.34993	0.09535	0.31298
2001	5.36770	4.79849	0.02693	-0.16364	-1.10905	0.64813	-0.46092	-1.21403	0.65311	-0.07299	1.37481	0.09610	0.32985
2002	5.46883	4.86760	0.02538	-0.16280	-1.10340	0.64367	-0.45973	-1.23382	0.67275	-0.07327	1.39717	0.09506	0.34049
2003	5.51291	4.92009	0.02474	-0.16489	-1.13740	0.64579	-0.49161	-1.24269	0.68943	-0.07383	1.40814	0.09431	0.34923
2004	5.72071	5.16517	0.02540	-0.16314	-1.13446	0.59391	-0.54055	-1.21837	0.67748	-0.07432	1.41273	0.09271	0.34361
2005	5.92738	5.43980	0.02815	-0.15819	-1.09105	0.45994	-0.63111	-1.20088	0.66409	-0.07373	1.42394	0.09124	0.34406
2006	6.12546	5.66761	0.03198	-0.15019	-0.99380	0.30706	-0.68674	-1.15419	0.62446	-0.07117	1.42527	0.09405	0.34438

Figure 3-2: Net Real Income Change Per Unit Labour (in 1955 yen)

The annual change information in the previous table can be converted into cumulative changes using equations (3.47)–(3.49). The difference between the current level of net real income per unit labour and its level in 1955, $\rho^t - \rho^{1955}$ is decomposed into the sum of the level of productivity factor T^t, the levels of several real output price factors A_C^t, A_N^t, A_G^t, A_X^t, A_M^t, A_I^t, A_{DEP}^t, and A_{IV}^t, and the levels of several input quantity factors B_{KW}^t, B_{KIV}^t, B_{LD}^t, and B_{LB}^t. Table 3-4 and Figure 3-2 give this cumulative growth information.

Over the 52 year period, net real income per unit labour grew by about 6.12546 yen at the price of 1955. From the above Table 3-7, it can be seen that productivity growth contributed the most to the overall growth in net real income per unit labour (T^{2006} = 5.66761, 92.53% of the overall growth in net real income per unit labour) and the growth in waiting capital services made the next largest contribution (B_{KW}^{2006} = 1.42527, 23.27% of the overall growth in net real income per unit labour) followed by the change in real investment price in magnitude (A_I^{2006} = –1.15419, –18.84% of the overall growth in net real income per unit labour). There were smaller effects due to the changes in real output prices such as the contributions of the changes in real depreciation prices (A_{DEP}^{2006} = 0.62446, 10.19% of the overall growth in net real income per unit labour), real export prices (A_X^{2006} = –0.9938, –16.22% of the overall growth in net real income per unit labour), and real import prices (A_M^{2006} = 0.30706, 5.01% of the overall growth in net real income per unit labour). The change in the real price of the consumption of NPISHs and the growth in inventory services had very small impact on the growth in net real income per unit labour (less than one percent of the overall change in real income per unit labour). Figure 3-2 plots net real income per unit labour and main factors contributing to its growth.

3.6 Conclusion

In this chapter, we derived a decomposition for changes in real income per unit primary input into explanatory factors that is exact for a flexible functional form; i.e., we showed that the change in real income generated by the market sector per unit primary input is equal to the sum of a productivity growth term, plus terms due to changes in real output prices, plus terms due to changes in relative primary input quantities where all three sets of explanatory factors can be calculated using observable price and quantity data. The above difference approach can be converted into a growth rate approach. However, the present approach has an advantage over earlier exact Translog approaches in that the present approach allows individual prices and quantities to be zero. Our present

approach also allows value subaggregates (such as inventory change or net exports) to change sign from period to period. If prices or quantities are zero or a value aggregate changes sign, the Translog approach fails, so our present approach offers some clear advantages in these situations.

We applied our methodology to analyze changes in the amount of real income per unit labour generated by the market sector of the Japanese economy for the years 1955-2006. The main findings emerging from this application is that, taken over the entire time period of 52 years, productivity growth and the growth of reproducible capital stocks and their resulting services are the two main contributors to the growth of real income per unit labour. We also observed that changes in the terms of trade had very small effects on real income per unit labour on average. We moved to our theoretically preferred measure of net real income per unit labour. We applied our methodology to analyze changes in the amount of net real income per unit labour generated by the market sector of the Japanese economy. We observed that in the net approach, productivity growth was still the largest contributor (and was an even more important factor than before). However, the contribution of capital services was greatly reduced.

Chapter 4. Exact and Superlative Price and Quantity Indicators[88]

4.1 Introduction

Traditional index number theory adopts a theoretical framework based on a ratio concept. In this approach, the ratio of the value aggregate between two periods is decomposed into the product of a price index and a quantity index. The price index, a function of the price and quantity data pertaining to the two periods under consideration, is interpreted as the ratio of the current price of the aggregate to the aggregate price in the base period. The quantity index, another function of the price and quantity data pertaining to the two periods, is interpreted as the ratio of the current period quantity aggregate to the base period quantity aggregate. In the economic approach to index number theory, it is assumed that the consumer has preferences over the individual quantities in the aggregate that can be represented by a utility function which has a dual cost function. This cost function is used to define consumer's family of Konüs (1939) price indexes or true cost of living indexes and the consumer's family of Allen (1949) quantity indexes.

If the consumer's preferences are homothetic (so that they can be represented by a linearly homogeneous utility function), then the family of Konüs price indexes collapses to a ratio of unit cost functions and the family of Allen quantity indexes collapses to a ratio of utility functions, where these functions are evaluated at the data of say period 1 in the numerator and the data of period 0 in the denominator. If preferences are homothetic, then Konüs and Byushgens (1926), Afriat (1972) and Pollak (1983) showed that certain numerical index number formula were exactly equal to the underlying theoretical economic indexes, provided that the consumer's utility function or dual unit cost function had certain functional forms. Diewert (1976) took this theory of exact indexes one step further and looked for indexes that were exact for flexible functional forms, for either the linearly homogeneous utility function or for the dual unit cost function and he called such indexes that were exact for flexible functional forms superlative. However, empirically, it has been shown that consumer preferences are generally not homothetic and hence the relevance of Diewert's concept of a superlative index is somewhat doubtful, at least in the consumer context. But Diewert (1976; 122)

[88] It includes joint work with W. Erwin Diewert. A version of this chapter has been accepted for publication. Diewert W.E. and Mizobuchi H. (2009) Exact and Superlative Price and Quantity Indicators. Macroeconomic Dynamics, Volume 13.

did implicitly develop a stronger concept for a superlative index in the context of general nonhomothetic preferences and we will formalize his idea in the present chapter in section 4.2 below where we will define strongly superlative indexes. Section 4.2 will also review the standard definitions for exact and superlative indexes in the case of homothetic preferences.

In section 4.3, we switch from the traditional economic approach to index number theory, which is based on ratios, to an economic approach pioneered by Hicks (1942) (1943) (1945–46) which is based on differences. In the traditional approach to index number theory, a value ratio is decomposed into the product of a price index times a quantity index whereas in the difference approach, a value difference is decomposed into the sum of a price indicator (which is a measure of aggregate price change) plus a quantity indicator (which is a measure of aggregate quantity change). The difference analogue to a theoretical Konüs price index is a Hicksian price variation and the difference analogue to an Allen quantity index is a Hicksian quantity variation such as the equivalent or compensating variation. For normal index number theory, the theoretical Konüs and Allen indexes are defined using ratios of cost functions but in the difference approach to index number theory, the theoretical price and quantity variation functions are defined in terms of differences of cost functions. In the difference approach, the counterparts to price and quantity index number formulae are price and quantity indicator functions.[89] Both index number formulae and indicator functions are known functions of the price and quantity data pertaining to the two periods under consideration. In section 4.3, we provide a definition for an exact price or quantity indicator function.

In sections 4.4 and 4.5, we develop further the difference approach to index number theory. In section 4.4, we will define a given price or quantity indicator function to be superlative if it is exactly equal to a corresponding theoretical price or quantity variation under the assumption that the consumer has homothetic preferences that are represented by a flexible linearly homogeneous utility function or which are dual to a flexible unit cost function. We draw on the theory of superlative price and quantity indexes to exhibit many superlative indicator functions. The theory that we develop in section 4.4 for the case of homothetic preferences turns out to be a variant of the theory of superlative indicators developed earlier by Diewert (2005c).

[89] This indicator terminology was introduced by Diewert (1992a) (2005).

In section 4.5, we will define a given price or quantity indicator function to be strongly superlative if it is exactly equal to a corresponding theoretical price or quantity variation, under the assumption that the consumer has (general) preferences which are dual to a flexible cost function that is subject to money metric utility scaling. The term money metric utility scaling is due to Samuelson (1974) and it is simply a convenient way of cardinalizing a utility function. It proves to be much more difficult to find strongly superlative price or quantity indicator functions but in section 4.5, we show that the Bennet (1920) indicator functions are strongly superlative. Our results require that the consumer's preferences be represented by a certain translation homothetic cost function that is a variant of the normalized quadratic cost function introduced by Diewert and Wales (1987) (1988a) (1988b). The flexibility of this functional form is shown in Appendix A. Our work draws on the earlier work on translation homothetic preferences (or linear parallel preferences) by Blackorby, Boyce and Russell (1978), Dickinson (1980), Chambers and Färe (1998), Chambers (2001; 111) and Balk, Färe and Grosskopf (2004).

The practical usefulness of the difference approach to the measurement of price and quantity change is illustrated at the end of section 4.5 where we show that under certain conditions including the assumption that each household faces the same prices in each period, it is possible to exactly measure the arithmetic average of the economy's sum of the individual household equivalent and compensating variations using only aggregate data since this aggregate measure of welfare change is exactly equal to the Bennet quantity indicator using aggregate quantity data. In other words, the difference approach to the measurement of aggregate price and quantity change has better aggregation properties than the traditional ratio approach.

In section 4.6, we provide economic interpretations for each term in the sum of terms that make up the Bennet price and quantity indicators. The decomposition results developed here are analogues to similar results obtained by Diewert and Morrison (1986) and Kohli (1990) in the traditional approach to index number theory.

In section 4.7, we illustrate the use of the difference approach to measure aggregate Japanese consumption and we contrast the traditional ratio approach to the measurement of real consumption to our difference approach.

Section 4.8 concludes.

4.2 Exact and Superlative Price and Quantity Indexes

In preparation for the difference approach to aggregate price and quantity measurement, in this section, we review the standard ratio approach to the measurement of price and quantity change. Thus we will define exact price and quantity indexes and present two definitions for a superlative price index. In the following sections, we will attempt to adapt these standard index number theory concepts to the difference context.

The starting point for the economic approach to index number theory is the consumer's cost or expenditure function C. Thus suppose that the consumer has preferences that are defined by the *utility function* f(q) over all nonnegative N dimensional quantity vectors $q \equiv [q_1,...,q_N] \geq 0_N$.[90] In addition, suppose that f is a nonnegative, increasing,[91] continuous and quasiconcave function over the nonnegative orthant $\Omega \equiv \{q : q \geq 0_N\}$. Now suppose that the consumer faces the positive vector of commodity prices $p >> 0_N$ and suppose that the consumer wishes to attain the utility level u belonging to the range of f as cheaply as possible. Then the consumer will solve the following *cost minimization problem* and the consumer's *cost function*, C(u,p), will be the minimum cost of achieving the target utility level u:

(4.1) $C(u,p) \equiv \min_q \{p \cdot q : f(q) \geq u ; q \geq 0_N\}$.

It can be shown[92] that C(u,p) will have the following properties: (i) C(u,p) is jointly continuous in u,p for $p >> 0_N$ and $u \in U$ where U is the range of f and is a nonnegative function over this domain of definition set; (ii) C(u,p) is increasing in u for each fixed p and (iii) C(u,p) is nondecreasing, linearly homogeneous and concave function of p for each $u \in U$.[93] Conversely, if a cost function is given and satisfies the above properties, then the utility function f that is dual to C can be recovered as follows.[94] For $u \in U$ and $q >> 0_N$, define the function F(u,q) as follows:

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[90] Notation: $q \geq 0_N$ means each component of q is nonnegative; $q >> 0_N$ means each component of q is positive and $q > 0_N$ means $q \geq 0_N$ but $q \neq 0_N$ where 0_N denotes an N dimensional vector of zeros. Also $p \cdot q$ denotes the inner product of the vectors p and q; i.e., $p \cdot q = p^T q \equiv \sum_{n=1}^N p_n q_n$.

[91] Thus if $q^2 >> q^1 \geq 0_N$, then $f(q^2) > f(q^1)$.

[92] See Diewert (1993; 124).

[93] Call these conditions on the cost function Conditions I.

[94] See Diewert (1974; 119) (1993; 129) for the details and for references to various duality theorems.

(4.2) $F(u,q) \equiv \max_p \{C(u,p) : p \cdot q \leq 1 ; p \geq 0_N\}$.

Now solve the equation:

(4.3) $F(u,q) = 1$

for u^* and this solution u^* will equal $f(q)$.

The utility function $f(q)$ and the dual cost function $C(u,p)$ are used in order to define the consumer's *family of Konüs* (1939) *true cost of living indexes*, $P_K(p^0.p^1,f(q))$, where p^0 and p^1 are the vectors of positive commodity prices that the consumer faces in periods 0 and 1 respectively and $u = f(q)$ is a positive reference level of utility:

(4.4) $P_K(p^0.p^1,f(q)) \equiv C(f(q),p^1)/C(f(q),p^0)$.

Thus for each reference quantity vector q that gives rise to a positive utility level, $u = f(q) > 0$, the consumer's aggregate price index for that reference level of utility is the ratio of $C(u,p^1)$ to $C(u,p^0)$.

The consumer's utility and cost functions can be used in order to define the consumer's *family of Allen* (1949) *quantity indexes*, $Q_A(q^0.q^1,p)$, where q^0 and q^1 are the observed consumption vectors that the consumer chose in periods 0 and 1 respectively and $p >> 0_N$ is a strictly positive vector of reference prices:

(4.5) $Q_A(q^0,q^1,p) \equiv C(f(q^1),p)/C(f(q^0),p)$.

The meaning of (4.5) is that if the consumer faces the reference price vector p, then his or her period t utility, $f(q^t)$, is set equal to the minimum cost of achieving this utility level using the reference prices p, $C(f(q^t),p)$, for $t = 0,1$ and the consumer's quantity index is set equal to the ratio $C(f(q^1),p)/C(f(q^0),p)$. Samuelson (1974) called this type of cardinalization of utility, *money metric utility*.[95] However, note that different choices of p will generate different cardinalizations of utility and different Allen quantity indexes.

[95] The basic idea can be traced back to Hicks (1942).

It is useful to specialize the above definitions for price and quantity indexes for the case where the consumer's preferences are *homothetic*[96] or *neoclassical*. We say that a utility function is *neoclassical* if it satisfies the following properties over the positive orthant: (i) f is a positive function; i.e., $f(q) > 0$ if $q >> 0_N$; (ii) f is positively linearly homogeneous; i.e., $f(\lambda q) = \lambda f(q)$ for all $\lambda > 0$ and $q >> 0_N$ and (iii) f is concave; i.e., for $0 < \lambda < 1$, $q^0 >> 0_N$ and $q^1 >> 0_N$, we have $f(\lambda q^0 + (1-\lambda)q^1) \geq \lambda f(q^0) + (1-\lambda)f(q^1)$. It turns out that a concave function defined over the positive orthant is also continuous over this domain of definition. Furthermore, f defined over the positive orthant has a continuous extension to the nonnegative orthant[97] and this extended f will also satisfy properties (ii) and (iii) above. The extended f(q) will also be nondecreasing in its variables q over the nonnegative orthant.[98]

If the consumer's preferences are neoclassical, then it turns out that the corresponding cost function defined by (4.1) above has the following representation:

(4.6) $C(u,p) = c(p)u$

where $c(p) \equiv C(1,p)$ is the consumer's *unit cost function*. It also turns out that the unit cost function, c(p), is also a neoclassical function, i.e., it is a positive, nondecreasing, continuous, concave and linearly homogeneous function of p over the positive orthant. Finally, the consumer's utility function f can be recovered from a knowledge of the unit cost function as follows:[99] for $q >> 0_N$,

(4.7) $f(q) = 1/\max_p \{c(p) : p \cdot q = 1; p \geq 0_N\}$.

The assumption that the consumer has neoclassical (or homothetic) preferences greatly

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[96] Preferences are homothetic if the consumer's utility function can be written as G[f(q)] where f is neoclassical and G is a continuous increasing function of one variable. Note that the homothetic preferences G[f(q)] can be represented by the neoclassical utility function f. Thus, at times in what follows, we will sometimes refer to neoclassical preferences as homothetic preferences. The concept of homotheticity is due to Shephard (1953).

[97] See Fenchel (1953; 78) or Rockafellar (1970; 85).

[98] See Diewert (1974; 111).

[99] This is a version of the Samuelson (1953) Shephard (1953) duality theorem; see also Diewert (1974; 110-112) and Samuelson and Swamy (1974).

simplifies index number theory. Under the assumption of neoclassical preferences, for each reference q such that f(q) is positive, we have[100]

$$(4.8)\ P_K(p^0,p^1,f(q)) \equiv C(f(q),p^1)/C(f(q),p^0) \qquad \text{using definition (4.4)}$$
$$= c(p^1)f(q)/c(p^0)f(q) \qquad \text{using (4.6)}$$
$$= c(p^1)/c(p^0).$$

Thus under the assumption of neoclassical preferences, the Konüs price index is equal to the unit cost ratio, $c(p^1)/c(p^0)$, and is independent of the reference utility level.

Similarly, under the assumption of neoclassical preferences, for each positive reference price vector p, we have

$$(4.9)\ Q_A(q^0,q^1,p) \equiv C(f(q^1),p)/C(f(q^0),p). \qquad \text{using definition (4.5)}$$
$$= c(p)f(q^1)/c(p)f(q^0) \qquad \text{using (4.6)}$$
$$= f(q^1)/f(q^0).$$

Thus under the assumption of neoclassical preferences, the Allen quantity index is equal to the utility ratio, $f(q^1)/f(q^0)$, and is independent of the reference price vector p.

Now suppose that the consumer has homothetic preferences (which we represent by a neoclassical utility function f(q) or the dual unit cost function c(p)) and he or she faces prices $p^t >> 0_N$ in period t and minimizes the cost of achieving the utility level u^t in period t for t = 0,1. Let q^t be the consumer's observed quantity vector for period t so that $u^t = f(q^t)$ for t = 0,1. Then the consumer's observed period t cost, $p^t \cdot q^t$ can be written as follows:

$$(4.10)\ p^t \cdot q^t = C(f(q^t),p^t) = c(p^t)f(q^t)\ ; \qquad t = 0,1.$$

Under these assumptions, the consumer's ratio of period 1 expenditures to period 0 expenditures satisfies the following equations:

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[100] See Shephard (1953) (1970), Pollak (1983) and Samuelson and Swamy (1974). Shephard in particular realized the importance of the homotheticity assumption in conjunction with separability assumptions in justifying the existence of subindexes of the overall cost of living index.

(4.11) $p^1 \cdot q^1/p^0 \cdot q^0 = [c(p^1)f(q^1)]/[c(p^0)f(q^0)]$ using (4.10)

$= [c(p^1)/c(p^0)][f(q^1)/f(q^0)]$

$= P_K(p^0,p^1,f(q))Q_A(q^0,q^1,p)$

for arbitrary reference q and p using (4.8) and (4.9).

Thus under the assumption of homothetic preferences and cost minimizing behavior on the part of the consumer for the two periods under consideration, the consumer's observed expenditure ratio is equal to the product of the Konüs price index for arbitrary reference vector q and the Allen quantity index for arbitrary reference vector p.

Note that in general, without a knowledge of the consumer's preferences, the Konüs price index and the Allen quantity index are not directly observable; i.e., they are *theoretical* indexes as opposed to the "*practical*" bilateral price and quantity *formulae*, say $P(p^0,p^1,q^0,q^1)$ and $Q(p^0,p^1,q^0,q^1)$, that are known functions of the observed consumer data pertaining to the two periods being compared. We assume that the bilateral index number formulae P and Q satisfy the following *product test* for all strictly positive price and quantity vectors:[101]

(4.12) $p^1 \cdot q^1/p^0 \cdot q^0 = P(p^0,p^1,q^0,q^1)Q(p^0,p^1,q^0,q^1)$.

Diewert (1976; 117) defined a quantity index $Q(p^0,p^1,q^0,q^1)$ to be *exact* for a neoclassical utility function f if under the assumption that the consumer minimizes the cost of achieving the utility level $u^t = f(q^t)$ in period t for t = 0,1, we have

(4.13) $Q(p^0,p^1,q^0,q^1) = f(q^1)/f(q^0)$;

i.e., the quantity index $Q(p^0,p^1,q^0,q^1)$ is exactly equal to the utility ratio which in turn is equal to the theoretical Allen quantity index under the assumption of neoclassical preferences.[102] Under the same assumptions of cost minimizing behavior and assuming that the preferences of the consumer can be represented by the dual unit cost function c(p), then Diewert (1976; 134) defined a price index $P(p^0,p^1,q^0,q^1)$ to be *exact* for c(p) if we have

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[101] This is Fisher's (1922) weak factor reversal test.

[102] Diewert (1976) gave many examples of exact index number formulae drawing on the earlier work of Konüs and Byushgens (1926), Pollak (1983) (originally written in 1971) and Afriat (1972).

(4.14) $P(p^0,p^1,q^0,q^1) = c(p^1)/c(p^0)$;

i.e., the price index $P(p^0,p^1,q^0,q^1)$ is exactly equal to the ratio of unit costs which in turn is equal to the theoretical Konüs price index under the assumption of neoclassical preferences.

Suppose the index number pair $P(p^0,p^1,q^0,q^1)$ and $Q(p^0,p^1,q^0,q^1)$ satisfy the product test (4.12) and either P is exact for c(p) or Q is exact for f(q).[103] Then Diewert (1976) defined P and Q to be *superlative* indexes if either c or f could provide a second order approximation to an arbitrary twice continuously differentiable neoclassical unit cost function $c^*(p)$ or to an arbitrary twice continuously differentiable neoclassical utility function $f^*(q)$.[104] Thus the advantage of superlative price and quantity indexes is that they can generate reasonably accurate price and quantity aggregates without having to undertake any econometric estimation of preferences, which becomes difficult or impossible as the number of commodities in the aggregate increases.

Examples of superlative price index formulae[105] are the Fisher (1922) ideal price index P_F and the Törnqvist (1936) (1937) Theil (1967) index P_T defined as follows:

(4.15) $P_F(p^0,p^1,q^0,q^1) \equiv [p^1\cdot q^0/p^0\cdot q^0]^{1/2}\,[p^1\cdot q^1/p^0\cdot q^1]^{1/2}$;

(4.16) $\ln P_T(p^0,p^1,q^0,q^1) \equiv \sum_{n=1}^{N} (1/2)[s_n^0 + s_n^1] \ln [p_n^1/p_n^0]$

where the period t expenditure share on commodity n is defined as $s_n^t \equiv p_n^t q_n^t/p^t\cdot q^t$ for n = 1,...,N and t = 0,1.

This completes our summary of the existing theory for superlative indexes in the case of homothetic preferences. Unfortunately, if the consumer's preferences are homothetic, then all income elasticities of demand are equal to unity but Engel's Law and other econometric evidence strongly suggests that income elasticities are not equal to one and

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[103] Of course, if P is exact for c, P and Q satisfy (4.12) and f is dual to c, then Q is exact for f (and vice versa).

[104] Blackorby and Diewert (1979) showed that if c is a differentiable flexible functional form and has a differentiable dual f(q), then f is also flexible in the class of neoclassical utility functions and vice versa.

[105] See Diewert (1976) for the details.

hence consumer preferences are *not* homothetic. Thus while the theory of exact and superlative indexes may be very useful when we wish to construct subaggregate prices and quantities, it seems that superlative indexes may not be appropriate when constructing overall aggregate consumer price and quantity indexes. Thus we need to determine whether we can find indexes which are exact for more general nonhomothetic preferences. Fortunately, this can be done.

Suppose the consumer has general preferences defined by the utility function f(q) and the general cost function C(u,p) is dual to f. As usual, let p^t and q^t be the observed price and quantity data pertaining to period t and define the period t level of utility $u^t \equiv f(q^t)$ for t = 0,1. We assume that the consumer is minimizing the cost of achieving the utility level u^t in period t so we have:

(4.17) $p^t \cdot q^t = C(f(q^t),p^t)$; $\qquad t = 0,1.$

Under the above assumptions, we say that the bilateral price index number formula, $P(p^0,p^1,q^0,q^1)$, is *exact* for the cost function C if there exists a u^* such that u^* is between u^0 and u^1 so that

(4.18) either $u^0 \leq u^* \leq u^1$ or $u^1 \leq u^* \leq u^0$ and
(4.19) $P(p^0,p^1,q^0,q^1) = C(u^*,p^1)/C(u^*,p^0) \equiv P_K(p^0,p^1,u^*)$.

Thus P is an exact index number formula if under the assumption of cost minimizing behavior, $P(p^0,p^1,q^0,q^1)$ is exactly equal to the Konüs theoretical price index $P_K(p^0,p^1,u^*)$ where u^* is an intermediate reference level of utility. The requirement that the reference level of utility be between the period 0 and 1 utility levels (or possibly equal to one of these levels) is a natural one: we do not want the reference utility level to be too far from the two levels actually experienced by the consumer during the two periods under consideration.

Initially, we define P to be a *strongly superlative index number formula* if it is exact according to the definition immediately above and in addition, the cost function C(u,p) that P is exact for can approximate an arbitrary cost function to the second order.

Diewert (1976; 122) showed that the Törnqvist Theil index P_T defined by (4.16) is exact for a general Translog cost function where the reference level of utility u^* is equal to $[u^0$

$u^1]^{1/2}$, the square root of the product of the period 0 and 1 utility levels. Since the general translog cost function is a fully flexible functional form, this shows that P_T is a strongly superlative price index.

Since the scaling of utility is arbitrary up to an increasing transformation of an initial representation of the utility function, we will find it convenient to impose money metric utility scaling on the underlying utility function f and its dual cost function C. Thus let $p^* >> 0_N$ be an arbitrary positive price vector. We will assume that the consumer's utility is scaled so that the dual cost function C satisfies the following equation:[106]

(4.20) $C(u,p^*) = u$ for all $u \in U$.

Thus our final definition for a *strongly superlative index number formula* P is that it is exact according to the above definition (4.18) and (4.19) and in addition, the cost function C(u,p) that P is exact for can approximate an arbitrary cost function (that satisfies the money metric utility scaling property (4.20)) to the second order.

An analogous definition of exactness can be made for a quantity index. Thus we say that the bilateral quantity index number formula, $Q(p^0,p^1,q^0,q^1)$, is *exact* for the cost function C if there exists a reference price vector $p^* \equiv [p_1^*,...,p_N^*]$ such that p^* is between p^0 and p^1 so that

(4.21) either $p_n^0 \leq p_n^* \leq p_n^1$ or $p_n^1 \leq p_n^* \leq p_n^0$ for n = 1,...,N and
(4.22) $Q(p^0,p^1,q^0,q^1) = C(f(q^1),p^*)/C(f(q^0),p^*) \equiv Q_A(q^0,q^1,p^*)$.

Thus Q is an exact index number formula if under the assumption of cost minimizing behavior, $Q(p^0,p^1,q^0,q^1)$ is exactly equal to the Allen theoretical quantity index $Q_A(q^0,q^1,p^*)$ where p^* is a vector of intermediate reference prices. The requirement that the reference price vector p^* be between the period 0 and 1 price vectors that the consumer faced (or possibly equal to one of these two vectors) is again a natural one: we do not want the money metric cardinalizing vector of reference prices to be too far from the two price vectors actually faced by the consumer during the two periods under consideration.

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[106] If the cost function C(u,p) satisfies Conditions I and in addition, satisfies the money metric utility scaling conditions (4.20), then we will say that C satisfies Conditions II.

Finally, we define Q to be a *strongly superlative index number formula* if it is exact according to the definition immediately above and in addition, the cost function C(u,p) that is dual to the utility function f can approximate an arbitrary cost function (that has the money metric utility scaling property (4.20)) to the second order.

The above material summarizes the theory of exact and superlative indexes which is based on decompositions of the *value ratio* into price and quantity components that *multiply* together. In the following section, we will review and extend the companion theory that is based on decompositions of the *value difference* into a *sum* of a price change component and a quantity change component.

4.3 Value Differences, Variations and Indicators of Price and Quantity Change

Assume that the consumer's cost function, C(u,p), satisfies Conditions I and the dual utility function is f(q) as usual. Throughout this section, we will assume that p^t and q^t are the observed price and quantity data pertaining to period t and we define the consumer's period t observed level of utility $u^t \equiv f(q^t)$ for t = 0,1. We assume that the consumer is minimizing the cost of achieving the utility level u^t in period t so that conditions (4.17) hold; i.e., we have $p^t \cdot q^t = C(f(q^t),p^t)$ for t = 0,1. Our task in the present section is to decompose the consumer's observed value change over the two periods under consideration, $p^1 \cdot q^1 - p^0 \cdot q^0$, into the sum of two terms, one of which is the part of the value change that is due to price change and the other part due to quantity change. This is the *difference approach* to explaining a change in a value aggregate as opposed to the usual *ratio approach* used in index number theory.[107]

The difference counterpart to the Allen (1949) quantity index explained in the previous section is the following *Hicks Samuelson quantity variation* Q_S: for each strictly positive reference price vector $p >> 0_N$, define $Q_S(q^0,q^1,p)$ as follows:[108]

(4.23) $Q_S(q^0,q^1,p) \equiv C(f(q^1),p) - C(f(q^0),p)$.

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[107] Hicks (1942) seems to have been the first to explore the similarities between the two approaches.

[108] Samuelson (1974) recognized that C(f(q),p) was a valid cardinalization of utility for any reference price vector p and thus (4.23) is a valid cardinal measure of the utility difference between periods 0 and 1. Hicks on the other hand only considered the special cases (4.24) and (4.25) defined below.

Just as the Allen quantity index $Q_A(q^0,q^1,p)$ defined by (4.5) was an entire family of indexes (one for each reference price vector p), so too is the family of quantity variations, Q_S. Two special cases of (4.23) are of particular importance, the *equivalent and compensating variations*, Q_E and Q_C, defined as follows:[109]

(4.24) $Q_E(q^0,q^1,p^0) \equiv Q_S(q^0,q^1,p^0) = C(f(q^1),p^0) - C(f(q^0),p^0)$;
(4.25) $Q_C(q^0,q^1,p^1) \equiv Q_S(q^0,q^1,p^1) = C(f(q^1),p^1) - C(f(q^0),p^1)$.

Thus the equivalent variation uses the period 0 price vector p^0 as the reference price vector while the compensating variation uses the period 1 price vector p^1 as the reference price vector.

Generalizing Hicks (1939; 40–41) (1946; 331–332), we will define a family of *Hicksian price variation functions* $P_H(p^0,p^1,f(q))$ as follows: for each nonnegative reference quantity vector q, define $P_H(p^0,p^1,f(q))$ as follows:

(4.26) $P_H(p^0,p^1,f(q)) \equiv C(f(q),p^1) - C(f(q),p^0)$.

Just as the Konüs price index, $P_K(p^0.p^1,f(q))$, defined by (4.4) was an entire family of indexes (one for each reference quantity vector or reference utility level $u \equiv f(q)$), so too is the family of Hicksian price variations. Two special cases of (4.26) are of particular importance, the *Laspeyres and Paasche price variation functions*, P_{HL} and P_{HP},

[109] Henderson (1941; 120) introduced these variations in the N = 2 case and Hicks (1942) introduced them in the general case, although his exposition is difficult to follow. The term compensating variation is due to Henderson (1941; 118) and the term equivalent variation is due to Hicks (1942; 128). Hicks (1939; 40–41) initially defined the compensating variation as a measure of price change: "As we have seen, the best way of looking at consumer's surplus is to regard it as a means of expressing, in terms of money income, the gain which accrues to the consumer as a result of a fall in price. Or better, it is the *compensating variation* in income, whose loss would just offset the fall in price and leave the consumer no better off than before." However, later, Hicks (1942; 127-128), following Henderson (1941; 120) defined (geometrically) the compensating variation as $C(u^1,p^1) - C(u^0,p^1)$ and the equivalent variation as $C(u^1,p^0) - C(u^0,p^0)$, which are measures of welfare (or quantity) change.

defined as follows:[110]

(4.27) $P_{HL}(p^0,p^1,f(q^0)) \equiv P_H(p^0,p^1,f(q^0)) = C(f(q^0),p^1) - C(f(q^0),p^0)$;[111]
(4.28) $P_{HP}(p^0,p^1,f(q^1)) \equiv P_H(p^0,p^1,f(q^1)) = C(f(q^1),p^1) - C(f(q^1),p^0)$.[112]

Thus the Laspeyres price variation uses the period 0 quantity vector q^0 as the reference quantity vector while the Paasche price variation uses the period 1 quantity vector q^1 as the reference quantity vector.

Let $M^0 \equiv p^0 \cdot q^0$ be the consumer's nominal "income" or expenditure on the N commodities in period 0. Then $P_{HL}(p^0,p^1,f(q^0))$ is the amount of nominal income that must be added to the period 0 income M^0 in order to allow the consumer, facing period 1 prices p^1, to achieve the same utility level as was achieved in period 0, which is $u^0 \equiv f(q^0)$. Similarly, let $M^1 \equiv p^1 \cdot q^1$ be the consumer's nominal "income" in period 1. Then $P_{HP}(p^0,p^1,f(q^1))$ is the amount of nominal income that must be subtracted from the period 1 income M^1 in order to allow the consumer, facing period 0 prices p^0, to achieve the same utility level as was achieved in period 1, which is $u^1 \equiv f(q^1)$.

Note that the equivalent quantity variation defined by (4.24) matches up with the Paasche price variation defined by (4.28) in order to provide an exact decomposition of the value change going from period 0 to 1; i.e., using these definitions and assumptions (4.17), it can be seen that:

(4.29) $p^1 \cdot q^1 - p^0 \cdot q^0 = C(f(q^1),p^1) - C(f(q^0),p^0) = Q_E(q^0,q^1,p^0) + P_{HP}(p^0,p^1,f(q^1))$.

Similarly, the compensating quantity variation defined by (4.25) matches up with the Laspeyres price variation defined by (4.27) in order to provide another exact

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[110] In the index number literature, $C(u^0,p^1)/C(u^0,p^0)$ is known as the Laspeyres Konüs (1939; 17) *true cost of living index* or price index and $C(u^1,p^1)/C(u^1,p^0)$ is known as the Paasche Konüs theoretical price index; see Pollak (1983). It can be seen that (4.27) and (4.28) are the difference counterparts to these ratio type indexes.

[111] Hicks (1945–46; 68) called this measure the 'price compensating variation' and distinguished this measure from a quantity compensating variation, which he did not define in a very clear manner. Hicks also considered price and quantity variations in Hicks (1943).

[112] Hicks (1945–46; 69) called this measure the "price equivalent variation".

decomposition of the value change going from period 0 to 1:

$$(4.30)\ p^1\cdot q^1 - p^0\cdot q^0 = C(f(q^1),p^1) - C(f(q^0),p^0) = Q_C(q^0,q^1,p^1) + P_{HL}(p^0,p^1,f(q^0)).$$

A problem with the quantity variations defined by (4.24) and (4.25) and the price variations defined by (4.27) and (4.28) is that they asymmetrically single out a reference price or quantity vector that pertains to a single period. Since both measures are equally valid and if a single measure of price or quantity change is required, then for some purposes, it may be useful to take an arithmetic average of the equivalent and compensating variations defined by (4.24) and (4.25) (denote the resulting average quantity variation as $Q_A(q^0,q^1,p^0,p^1)$) and to take an arithmetic average of the price variations defined by (4.27) and (4.28) (denote the resulting average price variation as $P_{HA}(p^0,p^1,q^0,q^1)$). It can be seen that these average price and quantity variations will also provide an additive decomposition of the value change; i.e., we have:

$$(4.31)\ p^1\cdot q^1 - p^0\cdot q^0 = C(f(q^1),p^1) - C(f(q^0),p^0) = Q_A(q^0,q^1,p^0,p^1) + P_{HA}(p^0,p^1,f(q^0),f(q^1)).$$

All of the price and quantity variations defined above cannot be evaluated in general using observed price and quantity data pertaining to the two periods under consideration. Thus we now turn our attention to the problem of finding *observable approximations* to the above theoretical variation functions.

Looking at definition (4.24) for the equivalent variation, it can be seen that the term $C(f(q^0),p^0)$ is equal to period 0 expenditure on the N commodities, $p^0\cdot q^0$, and hence this term is observable. The remaining term, $C(f(q^1),p^0)$, is not observable but we can use Shephard's (1953; 11) Lemma in order to obtain the following first order approximation to this term:

$$\begin{aligned}(4.32)\ C(f(q^1),p^0) &\approx C(f(q^1),p^1) + \nabla_p C(f(q^1),p^1)\cdot[p^0 - p^1] \\ &= C(f(q^1),p^1) + q^1\cdot[p^0 - p^1] && \text{using Shephard's Lemma} \\ &= p^1\cdot q^1 + p^0\cdot q^1 - p^1\cdot q^1 && \text{using (4.17) for } t = 1 \\ &= p^0\cdot q^1.\end{aligned}$$

Using (4.17) for t = 0, (4.32) and definition (4.24), we obtain the following first order approximation to the equivalent variation:

$$(4.33)\ Q_E(q^0,q^1,p^0) \approx p^0\cdot q^1 - p^0\cdot q^0$$
$$= p^0\cdot[q^1 - q^0]$$
$$\equiv V_L(p^0,p^1,q^0,q^1)$$

where the observable *Laspeyres indicator of quantity change*, $V_L(p^0,p^1,q^0,q^1)$, is defined as $p^0\cdot[q^1 - q^0]$, the inner product of the base period prices p^0 with the quantity change vector, $q^1 - q^0$. In a similar fashion, it can be shown that a first order approximation to the term $C(f(q^0),p^1)$ is $p^1\cdot q^0$ and so a first order approximation to the compensating variation $Q_C(q^0,q^1,p^1)$ defined by (4.25) is:[113]

$$(4.34)\ Q_C(q^0,q^1,p^1) \approx p^1\cdot q^1 - p^1\cdot q^0$$
$$= p^1\cdot[q^1 - q^0]$$
$$\equiv V_P(p^0,p^1,q^0,q^1)$$

where the observable *Paasche indicator of quantity change*, $V_P(p^0,p^1,q^0,q^1)$, is defined as $p^1\cdot[q^1 - q^0]$, the inner product of the current period prices p^1 with the quantity change vector, $q^1 - q^0$.

Note that V_L and V_P are the difference counterparts to the ordinary Laspeyres and Paasche quantity indexes, Q_L, and Q_P, defined as follows:

$$(4.35)\ Q_L(p^0,p^1,q^0,q^1) \equiv p^0\cdot q^1/p^0\cdot q^0\ ; \quad Q_P(p^0,p^1,q^0,q^1) \equiv p^1\cdot q^1/p^1\cdot q^0.$$

We now turn our attention to the problem of finding observable approximations for the Laspeyres and Paasche price variation functions defined by (4.27) and (4.28) above. An observable first order approximation to the term $C(f(q^0),p^1)$ in (4.27) is

$$(4.36)\ C(f(q^0),p^1) \approx C(f(q^0),p^0) + \nabla_p C(f(q^0),p^0)\cdot[p^1 - p^0]$$
$$= C(f(q^0),p^0) + q^0\cdot[p^1 - p^0] \qquad \text{using Shephard's Lemma}$$
$$= p^0\cdot q^0 + p^1\cdot q^0 - p^0\cdot q^0 \qquad \text{using (4.17) for } t = 0$$
$$= p^1\cdot q^0.$$

Using (4.17) for t = 0, (4.36) and definition (4.27), we obtain the following first order

[113] The first order approximations (4.33) and (4.34) were obtained by Hicks (1942; 127–134); see also Diewert (1992a; 568).

approximation to the Laspeyres price variation:

$$(4.37)\ P_{HL}(p^0,p^1,f(q^0)) \approx p^1\cdot q^0 - p^0\cdot q^0$$
$$= q^0\cdot[p^1 - p^0]$$
$$\equiv I_L(p^0,p^1,q^0,q^1)$$

where the observable *Laspeyres indicator of price change*, $I_L(p^0,p^1,q^0,q^1)$, is defined as $q^0\cdot[p^1 - p^0]$, the inner product of the base period quantity vector q^0 with the price change vector, $p^1 - p^0$. In a similar fashion, it can be shown that a first order approximation to the term $C(f(q^1),p^0)$ is $p^0\cdot q^1$ and so a first order approximation to the Paasche price variation $P_{HP}(p^0,p^1,f(q^1))$ defined by (4.28) is:

$$(4.38)\ P_{HP}(p^0,p^1,f(q^1)) \approx p^1\cdot q^1 - p^0\cdot q^1$$
$$= q^1\cdot[p^1 - p^0]$$
$$\equiv I_P(p^0,p^1,q^0,q^1)$$

where the observable *Paasche indicator of price change*, $I_P(p^0,p^1,q^0,q^1)$, is defined as $q^1\cdot[p^1 - p^0]$, the inner product of the current period quantity vector q^1 with the price change vector, $p^1 - p^0$.[114]

Note that I_L and I_P[115] are the difference counterparts to the ordinary Laspeyres and Paasche price indexes, P_L, and P_P, defined as follows:

$$(4.39)\ P_L(p^0,p^1,q^0,q^1) \equiv p^1\cdot q^0/p^0\cdot q^0\ ;\quad P_P(p^0,p^1,q^0,q^1) \equiv p^1\cdot q^1/p^0\cdot q^1.$$

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[114] The first order approximations (4.37) and (4.38) were obtained by Hicks (1945–46; 72–73) (1946; 331).

[115] Hicks (1942; 128) (1945–46; 71) called I_L and I_P the Laspeyres and Paasche variations but we will reserve the term "variation" for the (unobservable) theoretical measures of price and quantity change defined by (4.23) for changes in quantities and by (4.26) for changes in prices. We will follow Diewert (1992a; 556) (2005; 313) and use the term "indicator" to denote a given function of the price and quantity data pertaining to the two periods under consideration so that the term *indicator* becomes the difference theory counterpart to an *index number formula* in the ratio approach to the measurement of price and quantity change. Since P and Q are usually used to denote price and quantity indexes, a different notation is required to denote price and quantity indicators. Using I to denote a price indicator and V to denote a quantity (or volume) indicator follows the conventions used by Diewert (2005c). Note that national income accountants use the term "volume index" to denote a quantity index.

In the usual approach to index number theory, it proves to be useful to take the *geometric* average of the Laspeyres and Paasche price indexes, leading to the Fisher price index P_F defined by (4.15), since the Fisher index has very good properties from the viewpoint of the test or axiomatic approach to index number theory; see Diewert (1992b) and Balk (1995). However, in the axiomatic approach[116] to price and quantity measurement in the difference context, it proves to be better to take the *arithmetic* average of the Paasche and Laspeyres indicators. This leads to the *Bennet* (1920) *indicators of price and quantity change* defined as follows:

(4.40) $I_B(p^0,p^1,q^0,q^1) \equiv (1/2)I_L(p^0,p^1,q^0,q^1) + (1/2)I_P(p^0,p^1,q^0,q^1) = (1/2)[q^0+q^1]\cdot[p^1-p^0]$;
(4.41) $V_B(p^0,p^1,q^0,q^1) \equiv (1/2)V_L(p^0,p^1,q^0,q^1) + (1/2)V_P(p^0,p^1,q^0,q^1) = (1/2)[p^0+p^1]\cdot[q^1-q^0]$.

Note that Hicks (1942; 134) (1945-46; 73) obtained the Bennet quantity indicator V_B as an approximation to the arithmetic average of the equivalent and compensating variations and he also identified V_B as a generalization to many markets of Marshall's consumer surplus concept.

It can be verified that the Laspeyres, Paasche and Bennet price and quantity indicators can be used in order to obtain the following exact decompositions of the value change in the aggregate over the two periods under consideration:

(4.42) $p^1\cdot q^1 - p^0\cdot q^0 = I_L(p^0,p^1,q^0,q^1) + V_P(p^0,p^1,q^0,q^1)$;
(4.43) $p^1\cdot q^1 - p^0\cdot q^0 = I_P(p^0,p^1,q^0,q^1) + V_L(p^0,p^1,q^0,q^1)$;
(4.44) $p^1\cdot q^1 - p^0\cdot q^0 = I_B(p^0,p^1,q^0,q^1) + V_B(p^0,p^1,q^0,q^1)$.

We conclude this section by defining indicator counterparts to our index number definitions of exactness in the case of nonhomothetic preferences. As usual, we assume that the consumer minimizes cost in periods 0 and 1 so that the consumer has the utility function f(q) that satisfies the usual regularity Conditions I and has the dual cost function C(u,p) so that equations (4.17) are satisfied. Recall that the price index number formula $P(p^0,p^1,q^0,q^1)$ was defined to be exact for the cost function C if conditions (4.18) and (4.19) were satisfied. The price indicator counterpart to this definition is as follows:

[116] See Diewert (2005c) and Balk (2007) on the axiomatic approach to measures of price and quantity change using differences.

$I(p^0,p^1,q^0,q^1)$ is *exact* for the cost function C if there exists a u^* such that u^* is between $u^0 \equiv f(q^0)$ and $u^1 \equiv f(q^1)$ so that

(4.45) either $u^0 \leq u^* \leq u^1$ or $u^1 \leq u^* \leq u^0$ and
(4.46) $I(p^0,p^1,q^0,q^1) = C(u^*,p^1) - C(u^*,p^0) = P_H(p^0,p^1,u^*)$.

Thus $I(p^0,p^1,q^0,q^1)$ is exact for the preferences that are dual to C(u,p) if under the assumption of cost minimizing behavior on the part of the consumer, $I(p^0,p^1,q^0,q^1)$ is exactly equal to the theoretical Hicksian price variation function $P_H(p^0,p^1,u^*)$ defined by (4.26) for a reference utility level u^* that is between the period 0 and 1 utility levels attained by the consumer.

Recall that the quantity index number formula $Q(p^0,p^1,q^0,q^1)$ was defined to be exact for the cost function C if conditions (4.21) and (4.22) were satisfied. The quantity indicator counterpart to this definition is as follows: $V(p^0,p^1,q^0,q^1)$ is *exact* for the cost function C if there exists a reference price vector $p^* \equiv [p_1^*,...,p_N^*]$ such that p^* is between p^0 and p^1 so that

(4.47) either $p_n^0 \leq p_n^* \leq p_n^1$ or $p_n^1 \leq p_n^* \leq p_n^0$ for n = 1,...,N and
(4.48) $V(p^0,p^1,q^0,q^1) = C(f(q^1),p^*) - C(f(q^0),p^*) = Q_S(q^0,q^1,p^*)$.

Thus $V(p^0,p^1,q^0,q^1)$ is exact for the preferences that are dual to C(u,p) if under the assumption of cost minimizing behavior on the part of the consumer, $V(p^0,p^1,q^0,q^1)$ is exactly equal to the theoretical Hicks Samuelson quantity variation function $Q_S(q^0,q^1,p^*)$ defined by (4.23) for a reference price vector p^* that is between the period 0 and 1 price vectors faced by the consumer..

In the following section, we will assume that the consumer has homothetic preferences and we will attempt to find price and quantity indicators that are exact and superlative in this case. In section 4.5, we will drop the assumption of homothetic preferences and we will attempt to find superlative indicators in this more general context.

4.4 Superlative Price and Quantity Indicators in the Homothetic Preferences Case

We now suppose that the consumer's utility function f(q) is neoclassical and the dual unit cost function is c(p). Under these conditions, using (4.6), we have

(4.49) $C(f(q),p) = c(p)f(q)$.

Thus the family of Hicks Samuelson quantity variations Q_S defined by (4.23) and the family of Hicksian price variations P_H defined by (4.26) have the following structures under the assumption of neoclassical preferences:

(4.50) $Q_S(q^0,q^1,p) \equiv C(f(q^1),p) - C(f(q^0),p) = [f(q^1) - f(q^0)]c(p)$;
(4.51) $P_H(p^0,p^1,f(q)) \equiv C(f(q),p^1) - C(f(q),p^0) = [c(p^1) - c(p^0)]f(q)$.

It turns out that if we choose the vector of reference prices p in (4.50) to be equal to p^0 or p^1, then we can find exact quantity indicator functions $V(p^0,p^1,q^0,q^1)$ and if we choose the reference quantity vector q in (4.51) to be equal to q^0 or q^1, then we can find exact price indicator functions $I(p^0,p^1,q^0,q^1)$, by drawing on exact index number theory in the case of homothetic preferences. Thus let $P(p^0,p^1,q^0,q^1)$ and $Q(p^0,p^1,q^0,q^1)$ be an exact pair of price and quantity indexes; i.e., they satisfy (4.12), (4.13) and (4.14) in section 4.2. Now let the reference price vector p in (4.50) above equal the period 0 price vector, p^0. Then $Q_S(q^0,q^1,p^0)$ becomes the equivalent variation $Q_E(q^0,q^1,p^0)$ and thus (4.50) becomes the following equation:

$$
\begin{aligned}
(4.52)\ Q_E(q^0,q^1,p^0) &= [f(q^1) - f(q^0)]c(p^0) \\
&= [\{f(q^1)/f(q^0)\} - 1]c(p^0)f(q^0) \\
&= [Q(p^0,p^1,q^0,q^1) - 1]p^0\cdot q^0 \qquad \text{using (4.13) and (4.10) for } t = 0 \\
&\equiv V_E(p^0,p^1,q^0,q^1).
\end{aligned}
$$

Thus the observable function of the data, $V_E(p^0,p^1,q^0,q^1)$, defined to be equal to $[Q(p^0,p^1,q^0,q^1) - 1]p^0\cdot q^0$, is *exactly equal to the equivalent variation*, $Q_E(q^0,q^1,p^0)$, and hence is an exact quantity indicator function. If in addition, Q is exact for a *flexible* neoclassical utility function f, then we say that the corresponding $V_E(p^0,p^1,q^0,q^1)$ is a *superlative quantity indicator*.

Now let the reference price vector p in (4.50) above equal the period 1 price vector, p^1. Then $Q_S(q^0,q^1,p^1)$ becomes the compensating variation $Q_C(q^0,q^1,p^1)$ and thus (4.50) becomes the following equation:

(4.53) $Q_C(q^0,q^1,p^1) = [f(q^1) - f(q^0)]c(p^1)$

$$= [1 - \{f(q^0)/f(q^1)\}]c(p^1)f(q^1)$$
$$= [1 - Q(p^0,p^1,q^0,q^1)^{-1}]p^1 \cdot q^1 \qquad \text{using (4.13) and (4.10) for } t = 1$$
$$\equiv V_C(p^0,p^1,q^0,q^1).$$

Thus the observable function of the data, $V_C(p^0,p^1,q^0,q^1)$, is *exactly equal to the compensation variation*, $Q_C(q^0,q^1,p^0)$, and hence is an exact quantity indicator function. If in addition, Q is exact for a *flexible* neoclassical utility function f, then we say that the corresponding $V_C(p^0,p^1,q^0,q^1)$ is a *superlative quantity indicator*.

Thus each superlative quantity index function, $Q(p^0,p^1,q^0,q^1)$, generates two superlative quantity indicator functions, $V_E(p^0,p^1,q^0,q^1)$ defined in (4.52) which is exact for the theoretical equivalent variation, and $V_C(p^0,p^1,q^0,q^1)$ defined in (4.53) which is exact for the theoretical compensating variation. Since there are an infinite number of superlative quantity indexes[117], there are an infinite number of superlative quantity indicators.

The above analysis can be repeated with some modifications in order to find superlative price indicator functions. Thus again let $P(p^0,p^1,q^0,q^1)$ and $Q(p^0,p^1,q^0,q^1)$ be an exact pair of price and quantity indexes. Now let the reference quantity vector q in (4.51) above equal the period 0 quantity vector, q^0. Then $P_H(p^0,p^1,f(q^0))$ becomes the Laspeyres price variation $P_{HL}(p^0,p^1,f(q^0))$ defined by (4.27) and thus (4.51) becomes the following equation:

$$(4.54)\quad P_{HL}(p^0,p^1,f(q^0)) = [c(p^1) - c(p^0)]f(q^0)$$
$$= [\{c(p^1)/c(p^0)\} - 1]c(p^0)f(q^0)$$
$$= [P(p^0,p^1,q^0,q^1) - 1]p^0 \cdot q^0 \qquad \text{using (4.14) and (4.10) for } t = 0$$
$$\equiv I_{HL}(p^0,p^1,q^0,q^1).$$

Thus the observable function of the data, $I_{HL}(p^0,p^1,q^0,q^1)$, defined to be equal to $[P(p^0,p^1,q^0,q^1) - 1]p^0 \cdot q^0$, is *exactly equal to the Laspeyres price variation*, $P_{HL}(p^0,p^1,f(q^0))$, and hence is an exact price indicator function. If in addition, P is exact for a *flexible* unit cost function c, then we say that $I_{HL}(p^0,p^1,q^0,q^1)$ is a *superlative price indicator*.

Now let the reference quantity vector q in (4.51) above equal the period 1 quantity vector, q^1. Then $P_H(p^0,p^1,f(q^1))$ becomes the Paasche price variation $P_{HP}(p^0,p^1,f(q^1)$ and thus

[117] See Diewert (1976).

(4.51) becomes the following equation:

$$
\begin{aligned}
(4.55)\ P_{HP}(p^0,p^1,f(q^1)) &= [c(p^1) - c(p^0)]f(q^1) \\
&= [1 - \{c(p^0)/c(p^1)\}]c(p^1)f(q^1) \\
&= [1 - P(p^0,p^1,q^0,q^1)^{-1}]p^1\cdot q^1 \qquad \text{using (4.14) and (4.10) for } t = 1 \\
&\equiv I_{HP}(p^0,p^1,q^0,q^1).
\end{aligned}
$$

Thus the observable function of the data, $I_{HP}(p^0,p^1,q^0,q^1)$, defined to be equal to $[1 - P(p^0,p^1,q^0,q^1)^{-1}]p^1\cdot q^1$, is *exactly equal to the Paasche price variation*, $P_{HP}(p^0,p^1,f(q^1))$, and hence is an exact price indicator function. If in addition, P is exact for a *flexible* unit cost function c, then we say that $I_{HP}(p^0,p^1,q^0,q^1)$ is a *superlative price indicator*. Again, since there are many superlative price index functions $P(p^0,p^1,q^0,q^1)$, there will be many superlative price indicator functions.[118]

There is one more detail to be settled in this analysis of superlative price and quantity indicator functions that are generated by traditional index number formulae: we want the sum of the price indicator and quantity indicator to be exactly equal to the value difference. Thus suppose that we are given bilateral index number formulae P and Q that satisfy the product test (4.12) and we use these indexes to define the quantity indicators $V_E(p^0,p^1,q^0,q^1)$ by (4.52) and $V_C(p^0,p^1,q^0,q^1)$ by (4.53) and the price indicators $I_{HL}(p^0,p^1,q^0,q^1)$ by (54) and $I_{HP}(p^0,p^1,q^0,q^1)$ by (4.55). Then using (4.12), it can be shown that numerically, the following equations will hold:

$$(4.56)\ p^1\cdot q^1 - p^0\cdot q^0 = I_{HP}(p^0,p^1,q^0,q^1) + V_E(p^0,p^1,q^0,q^1)\ ;$$
$$(4.57)\ p^1\cdot q^1 - p^0\cdot q^0 = I_{HL}(p^0,p^1,q^0,q^1) + V_C(p^0,p^1,q^0,q^1)\ .$$

Thus the equivalent variation indicator $V_E(p^0,p^1,q^0,q^1)$ generated by Q needs to be matched up with the Paasche price variation indicator $I_{HP}(p^0,p^1,q^0,q^1)$ generated by P and

[118] Diewert (2005c; 333–337) also defined superlative price and quantity indicators in the case where consumer preferences were homothetic. Diewert's (2005; 336) *superlative economic indicator of price change* was defined as $I_E(p^0,p^1,q^0,q^1) = (1/2)I_{HL}(p^0,p^1,q^0,q^1) + (1/2)I_{HP}(p^0,p^1,q^0,q^1)$ where I_{HL} and I_{HP} are defined by the third equation in (4.54) and (4.55) respectively where the index number formula $P(p^0,p^1,q^0,q^1)$ is superlative. Thus our present definition of a superlative price or quantity indicator is a variation of Diewert's earlier definition. It should be noted that Fox (2006) generalized Diewert's (2005) bilateral approach to multilateral comparisons.

the compensating variation indicator $V_C(p^0,p^1,q^0,q^1)$ generated by Q needs to be matched up with the Laspeyres price variation indicator $I_{HL}(p^0,p^1,q^0,q^1)$ generated by P in order for the value difference to equal the sum of a price and quantity indicator.

This completes our discussion of superlative indicators when the consumer's preferences are homothetic. In the following section, we address the much more difficult task of finding superlative indicators in the nonhomothetic case.

4.5 Strongly Superlative Price and Quantity Indicators

The holy grail of applied welfare economics is to obtain a quantity variation indicator that is exact for fully flexible preferences. To our knowledge, no one yet has succeeded in this quest.[119] In this section, we will show that the Bennet quantity indicator is exact for fully flexible preferences, subject to the money metric cardinalization of utility defined by (20), except that normalized prices that are adjusted for general inflation between the two periods must be used in place of the original prices facing the consumer. Since our focus is on quantity variations, this scaling of prices does not seem to be too serious a drawback to our suggested indicator of quantity change.

We now distinguish the original (unscaled) price vector $P^t \equiv [P_1^t,...,P_N^t] >> 0_N$ that the consumer faces in period t for t = 0,1 from the scaled or normalized price vector p^t which is proportional to P^t and will be defined shortly. As in previous sections, the consumer's observed quantity vector in period t is q^t for t = 0,1. Let the consumer's utility function f(q) satisfy Conditions II (which are the usual nonhomothetic assumptions plus the assumption of money metric utility scaling (4.20) for some strictly positive reference prince vector $P^* >> 0_N$) and let the corresponding dual cost function be C(u,P). We assume that the consumer's cost function has the following *translation homothetic*

[119] Recent attempts by Weitzman (1988) and Diewert (1992a) ended up making homotheticity assumptions or in the case of Diewert's (1992a) Theorems 2 and 4, unrealistic assumptions relating the parameters of preferences to utility levels were made. Chambers and Färe (1998) and Chambers (2001) also came close but their preference classes fell short of being fully flexible; Chambers (2001; 111) explained the problem with his class of preferences. Diewert (1976; 123-124) had a fully flexible result but his result was exact for a Malmquist (1953) quantity index which is not an exact result for a quantity variation and moreover, the Malmquist index does not have the convenient aggregation properties that a Hicks-Samuelson quantity variation possesses.

normalized quadratic functional form,[120] which is a special case of *translation homothetic preferences*:[121]

(4.58) $C(u,P) \equiv b \cdot P + (1/2)\,(\alpha \cdot P)^{-1} P \cdot BP + c \cdot Pu$

where $\alpha > 0_N$, b and c are N dimensional parameter vectors and B is parameter matrix. These parameter vectors and matrix satisfy the following restrictions, where $P^* >> 0_N$ is the reference vector which appears in (4.20), the definition for C to satisfy money metric utility scaling at the reference prices P^*:

(4.59) $B = B^T$ so that B is symmetric and B is negative semidefinite;
(4.60) $BP^* = 0_N$;
(4.61) $b \cdot P^* = 0$ and
(4.62) $c \cdot P^* = 1$.

Using the techniques in Diewert and Wales (1987), it can be shown that (4.59) implies that the C defined by (4.58) is globally concave. In the Appendix F, we show that this functional form is flexible in the class of preferences satisfying the money metric utility scaling restrictions in (4.20) for any predetermined parameter vector $\alpha > 0_N$; i.e., given any $\alpha > 0_N$, we can find vectors b and c and a matrix of parameters B such that the restrictions (4.59)–(4.62) are satisfied and the resulting C defined by (4.58) is flexible at the arbitrary point (u^*,P^*). However, in general, this flexible functional form may not satisfy Conditions II for all $u > 0$ and all $P >> 0_N$. In the Appendix F, we will define the

---

[120] Diewert and Wales (1987) (1988a) (1988b) introduced the normalized quadratic cost function which can be defined as $C(u,P) \equiv b \cdot P + [(1/2)\,(\alpha \cdot P)^{-1} P \cdot BP + c \cdot P]u$ where b, c and B satisfy (59)-(62) and they showed that this functional form was flexible in the class of cost functions that satisfy the money metric utility scaling restrictions (20) for any predetermined parameter vector $\alpha > 0_N$. The advantage of this functional form is that it contains a flexible unit cost function as a special case (just set $b = 0_N$). However, since preferences are generally nonhomothetic, this advantage is not necessarily a huge one.

[121] Chambers and Färe (1998; 640) and Chambers (2001; 111) introduced the term "translation homothetic preferences" and studied these preferences in some detail and noted their importance for the measurement of welfare change; see also Balk, Färe and Grosskopf (2004). Blackorby, Boyce and Russell (1978; 348) introduced this class of preferences and Dickinson (1980; 1713) referred to this class of preferences as linear parallel preferences. Dickinson (1980; 1715–1717) exhibited several examples of this class of preferences that were flexible.

region of prices and utility levels where the functional form satisfies the required regularity conditions for a cost function.

Assuming that the consumer's preferences can be represented by the cost function defined by (4.58)–(4.62) for the two periods under consideration, then assuming cost minimizing behavior on the part of the consumer, the following equations will hold:

$$(4.63)\ P^t \cdot q^t = C(f(q^t),P^t) = b \cdot P^t + (1/2)\,(\alpha \cdot P^t)^{-1} P^t \cdot BP^t + c \cdot P^t\, f(q^t)\ ; \qquad t = 0,1.$$

Using Shephard's Lemma, the consumer's observed period t demand vector q^t is equal to the following expression:

$$(4.64)\ q^t = \nabla_P C(f(q^t),P^t) = b + (\alpha \cdot P^t)^{-1}\, BP^t - (1/2)(\alpha \cdot P^t)^{-2}\, P^t \cdot BP^t\, \alpha + c\, f(q^t)\ ; \qquad t = 0,1.$$

If there is a great deal of general inflation between periods 0 and 1, then the compensating variation will be much larger than the equivalent variation simply due to this general inflation and taking an average of these two variations will be difficult to interpret due to the change in the scale of prices. In order to eliminate the effects of general inflation between the two periods being compared, it will be useful to *scale* the prices in each period by a fixed basket price index of the form $\alpha \cdot P$ where $\alpha \equiv [\alpha_1,...,\alpha_N] > 0_N$ is a nonnegative, nonzero vector of price weights.[122] Thus, having chosen the price

[122] A reasonable "standard" choice for the weighting vector α is $\alpha \equiv q^0/P^0 \cdot q^0$. For this choice of α, the vector of period t normalized prices, $p^t \equiv P^t/P^t \cdot \alpha$, can be interpreted as a period t vector of "real" prices using a fixed base Laspeyres price index to do the deflation of nominal prices. Diewert (2005c; 340-341) commented on the general inflation problem as follows: "The above quotation alerts us to a potential problem with our treatment of value changes; namely, if there is a great change in the general purchasing power of money between the two periods being compared, then our indicators of volume change may be "excessively" heavily weighted by the prices of the period that has the highest general price level. Put another way, the units that quantities are measured in do not require any comparisons with other quantities but the dollar price of a quantity is the valuation of a unit of a commodity *relative to a numeraire commodity, money*. Thus the indicators of price change that we have discussed in this chapter encompass both *general changes* in the purchasing power of money as well as *changes in inflation adjusted* prices. Thus if there is high inflation between periods 0 and 1 and quantities have increased, then the use of symmetric in prices and quantities indicators (like the Bennet and Montgomery indicators) will shift some of the inflationary increase in values over to the indicator of volume change." Diewert (2005c; 341)

weighting vector α, the period t *real prices* that the consumer faces p^t are defined as follows:

(4.65) $p^0 \equiv P^0/\alpha\cdot P^0$; $p^1 \equiv P^1/\alpha\cdot P^1$.

Note that these real price vectors will satisfy the following restrictions:

(4.66) $\alpha\cdot p^t = \alpha\cdot P^t/\alpha\cdot P^t = 1$; t = 0,1.

Divide both sides of equation (4.63) by $\alpha\cdot P^t$ and using definitions (4.65), the resulting equations become:

(4.67) $p^t\cdot q^t = C(f(q^t),p^t) = b\cdot p^t + (1/2)p^t\cdot Bp^t + c\cdot p^t f(q^t)$; t = 0,1.

Similarly, substituting equations (4.65) and (4.66) into equations (4.64) leads to the following equations relating the consumer's period t quantity vectors q^t to the real price vectors p^t:

(4.68) $q^t = \nabla_P C(f(q^t),p^t) = b + Bp^t - (1/2)p^t\cdot Bp^t\alpha + cf(q^t)$; t = 0,1.

With the above preliminaries out of the way, we are ready to state our first Proposition which relates the Bennet quantity indicator defined earlier by (4.41), $V_B(p^0,p^1,q^0,q^1) \equiv (1/2)[p^0+p^1]\cdot[q^1-q^0]$, to the theoretical equivalent and compensating variations defined by (4.24) and (4.25), $Q_E(q^0,q^1,p^0) \equiv C(f(q^1),p^0) - C(f(q^0),p^0)$ and $Q_C(q^0,q^1,p^1) \equiv C(f(q^1),p^1) - C(f(q^0),p^1)$, where we are using the scaled real price vectors p^t defined by (4.65) as reference price vectors in place of the original nominal price vectors P^t.

Proposition 1: Let the consumer's observed period t data be (P^t,q^t) and suppose that the consumer minimizes the cost of achieving the period t utility level for each period t = 0,1. Let $\alpha > 0_N$ be a given vector of price weights that are used in order to construct the period

---

suggested deflating the prices of the second period by a general index of inflation going from period 0 to 1 whereas our solution is more specific in that we choose a Laspeyres type index to do the deflation. Diewert (1992a; 566) discussed other normalizations that have been used historically by various authors in order to construct suitable real prices for use in the measurement of welfare change by volume or quantity indicators.

t real price vectors, $p^t \equiv P^t/\alpha \cdot P^t$ for t = 0,1. Suppose a consumer has preferences f(q) which are dual to the translation homothetic normalized quadratic cost function C(u,P) defined by (4.58)–(4.62) and define $u^t \equiv f(q^t)$ for t = 0,1. Then the Bennet quantity indicator defined by (4.41) using the real prices defined by (4.65) is exactly equal to the arithmetic average of the equivalent and compensating variations defined by (4.24) and (4.25) using the real price vectors as reference prices rather than the original nominal price vectors; i.e., we have

(4.69) $V_B(p^0,p^1,q^0,q^1) = (1/2)Q_E(q^0,q^1,p^0) + (1/2)Q_C(q^0,q^1,p^1)$.

Proof:

$$
\begin{aligned}
(4.70)\ 2V_B(p^0,p^1,q^0,q^1) &= [p^0 + p^1]\cdot[q^1 - q^0] && \text{using definition (4.41)}\\
&= p^0\cdot q^1 - C(f(q^0),p^0) + C(f(q^1),p^1) - p^1\cdot q^0 && \text{using (4.67)}\\
&= p^0\cdot[b + Bp^1 - (1/2)p^1\cdot Bp^1\alpha + cf(q^1)] - C(f(q^0),p^0)\\
&\quad + C(f(q^1),p^1) - p^1\cdot[b + Bp^0 - (1/2)p^0\cdot Bp^0\alpha + cf(q^0)] && \text{using (4.68)}\\
&= p^0\cdot b + p^0\cdot Bp^1 - (1/2)p^1\cdot Bp^1 + p^0\cdot cf(q^1) - C(f(q^0),p^0)\\
&\quad + C(f(q^1),p^1) - [p^1\cdot b + p^1\cdot Bp^0 - (1/2)p^0\cdot Bp^0 + p^1\cdot cf(q^0)] && \text{using (4.66)}\\
&= [p^0\cdot b + (1/2)p^0\cdot Bp^0 + p^0\cdot cf(q^1)] - C(f(q^0),p^0)\\
&\quad + C(f(q^1),p^1) - [p^1\cdot b + (1/2)p^1\cdot Bp^1 + p^1\cdot cf(q^0)] && \text{using (4.59)}\\
&= C(f(q^1),p^0) - C(f(q^0),p^0) + C(f(q^1),p^1) - C(f(q^0),p^1) && \text{using (4.67)}\\
&= Q_E(q^0,q^1,p^0) + Q_C(q^0,q^1,p^1) && \text{using definitions (4.24) and (4.25)}
\end{aligned}
$$

which is equivalent to (4.69). Q.E.D.

Corollary 1: Under the conditions of the above Proposition, the following equality holds:

$$
\begin{aligned}
(4.71)\ V_B(p^0,p^1,q^0,q^1) &= C(f(q^1),(1/2)[p^0+p^1]) - C(f(q^0),(1/2)[p^0+p^1])\\
&\equiv Q_S(q^0,q^1,(1/2)[p^0+p^1]).
\end{aligned}
$$

Proof: From (4.70), we have the following equality:

$$
\begin{aligned}
(4.72)\ 2V_B(p^0,p^1,q^0,q^1) &= C(f(q^1),p^0) - C(f(q^0),p^0) + C(f(q^1),p^1) - C(f(q^0),p^1)\\
&= p^0\cdot c[f(q^1) - f(q^0)] + p^1\cdot c[f(q^1) - f(q^0)] && \text{using definition (4.67)}\\
&= [p^0 + p^1]\cdot c[f(q^1) - f(q^0)]\\
&= C(f(q^1),p^0+p^1) - C(f(q^0),p^0+p^1)
\end{aligned}
$$

where the last equality follows adding and subtracting terms and using definition (4.67) for C. Using the linear homogeneity property of C(u,p) in p, it can be seen that (4.72) implies (4.71). Q.E.D.

The equality (4.71) shows that *the Bennet quantity indicator*, $V_B(p^0,p^1,q^0,q^1)$, *is a strongly superlative indicator*, since it is exact for the theoretical quantity variation, $Q_S(q^0,q^1,(1/2)p^0 + (1/2)p^1)$, using reference prices that are between p^0 and p^1, namely the arithmetic average reference prices $(1/2)p^0 + (1/2)p^1$.

There is a counterpart to Proposition 1 for the Bennet price indicator. Proposition 2 relates the Bennet price indicator defined earlier by (4.40), $I_B(p^0,p^1,q^0,q^1) \equiv (1/2)[q^0+q^1]\cdot[p^1-p^0]$, to the theoretical Laspeyres and Paasche price variation functions defined by (4.27) and (4.28), $P_{HL}(p^0,p^1,f(q^0)) \equiv C(f(q^0),p^1) - C(f(q^0),p^0)$ and $P_{HP}(p^0,p^1,f(q^1)) \equiv C(f(q^1),p^1) - C(f(q^1),p^0)$, where again we use the scaled real price vectors p^t defined by (4.65) as reference price vectors in place of the original nominal price vectors P^t.

Proposition 2: Under the hypotheses listed in Proposition 1, the Bennet price indicator defined by (4.40) using the real prices defined by (4.65) is exactly equal to the arithmetic average of the Laspeyres and Paasche price variations defined by (4.27) and (4.28) using the real price vectors as reference prices rather than the original nominal price vectors; i.e., we have

(4.73) $I_B(p^0,p^1,q^0,q^1) = (1/2)\, P_{HL}(p^0,p^1,f(q^0)) + (1/2)P_{HP}(p^0,p^1,f(q^1))$.

Proof:[123]

$$
\begin{aligned}
(4.74)\ 2I_B(p^0,p^1,q^0,q^1) &= [q^0 + q^1]\cdot[p^1 - p^0] && \text{using definition (4.40)}\\
&= p^1\cdot q^0 - C(f(q^0),p^0) + C(f(q^1),p^1) - p^0\cdot q^1 && \text{using (4.67)}\\
&= p^1\cdot[b + Bp^0 - (1/2)p^0\cdot Bp^0\alpha + cf(q^0)] - C(f(q^0),p^0)\\
&\quad + C(f(q^1),p^1) - p^0\cdot[b + Bp^1 - (1/2)p^1\cdot Bp^1\alpha + cf(q^1)] && \text{using (4.68)}
\end{aligned}
$$

[123] Our technique of proof is closely related to the techniques used by Balk, Färe and Grosskopf (2004; 160-161) but our functional form assumptions are different and they do not establish a flexibility result for the class of functional forms that they use in their proofs.

$$= p^1\cdot b + p^1\cdot Bp^0 - (1/2)p^0\cdot Bp^0 + p^1\cdot cf(q^0) - C(f(q^0),p^0)$$
$$+ C(f(q^1),p^1) - [p^0\cdot b + p^0\cdot Bp^1 - (1/2)p^1\cdot Bp^1 + p^0\cdot cf(q^1)] \text{using (4.66)}$$
$$= [p^1\cdot b + (1/2)p^1\cdot Bp^1 + p^1\cdot cf(q^0)] - C(f(q^0),p^0)$$
$$+ C(f(q^1),p^1) - [p^0\cdot b + (1/2)p^0\cdot Bp^0 + p^0\cdot cf(q^1)] \quad \text{using (4.59)}$$
$$= C(f(q^0),p^1) - C(f(q^0),p^0) + C(f(q^1),p^1) - C(f(q^1),p^0) \quad \text{using (4.67)}$$
$$= P_{HL}(p^0,p^1,f(q^0)) + P_{HP}(p^0,p^1,f(q^1))$$
using definitions (4.27) and (4.28)

which is equivalent to (4.73). Q.E.D.

Corollary 2: Under the conditions of the above Proposition, the following equality holds:

$$(4.75)\ I_B(p^0,p^1,q^0,q^1) = C((1/2)f(q^0)+(1/2)f(q^1),p^1) - C((1/2)f(q^0)+(1/2)f(q^1),p^0)$$
$$\equiv P_H(p^0,p^1,(1/2)f(q^0)+(1/2)f(q^1)) .$$

Proof: From (4.74), we have the following equality:

$$(4.76)\ 2I_B(p^0,p^1,q^0,q^1) = C(f(q^0),p^1) - C(f(q^0),p^0) + C(f(q^1),p^1) - C(f(q^1),p^0)$$
$$= [p^1\cdot b + (1/2)p^1\cdot Bp^1 + p^1\cdot cf(q^0)] - [p^0\cdot b + (1/2)p^0\cdot Bp^0 + p^0\cdot cf(q^0)]$$
$$+ [p^1\cdot b + (1/2)p^1\cdot Bp^1 + p^1\cdot cf(q^1)] - [p^0\cdot b + (1/2)p^0\cdot Bp^0 + p^0\cdot cf(q^1)] \text{ using (4.58)}$$
$$= 2[p^1\cdot b + (1/2)p^1\cdot Bp^1 + p^1\cdot c(1/2)\{f(q^0)+f(q^1)\}]$$
$$- 2[p^1\cdot b + (1/2)p^1\cdot Bp^1 + p^1\cdot c(1/2)\{f(q^0)+f(q^1)\}] \quad \text{rearranging terms}$$
$$= 2C((1/2)f(q^0)+(1/2)f(q^1),p^1) - 2C((1/2)f(q^0)+(1/2)f(q^1),p^0)$$
using definition (4.58)
$$= 2\ P_H(p^0,p^1,(1/2)f(q^0)+(1/2)f(q^1)) \quad \text{using definition (4.26)}$$

which is equivalent to (4.75). Q.E.D.

The equality (4.75) shows that *the Bennet price indicator*, $I_B(p^0,p^1,q^0,q^1)$, *is a strongly superlative indicator*, since it is exact for the theoretical Hicksian price variation, $P_H(p^0,p^1,(1/2)u^0+(1/2)u^1)$, using the arithmetic average of the period 0 and 1 utility levels, u^0 and u^1, as the reference utility level.

Bennet (1920) showed that the sum of the Bennet price and quantity indicators, $I_B(p^0,p^1,q^0,q^1)$ plus $V_B(p^0,p^1,q^0,q^1)$, is numerically equal to the value difference, $p^1\cdot q^1 - p^0\cdot q^0$; recall (4.44) above. The above two Propositions show that the Bennet indicators have strong economic interpretations if we use real prices instead of nominal prices when

calculating these indicators; i.e., they are both strongly superlative indexes.[124]

Another advantage of the Bennet quantity indicator is that it has a nice aggregation over households property. Thus let $\alpha > 0_N$ and suppose that there are H households in the economy and household h has normalized quadratic translation homothetic preferences $f^h(q)$ that are dual to the following cost function C^h for h = 1,...,H:

$$(4.77)\ C^h(u^h,P) \equiv b^h\cdot P + (1/2)\,(\alpha\cdot P)^{-1}P\cdot B^hP + c^h\cdot Pu^h$$

where b^h, c^h and B^h satisfy the restrictions (4.59)–(4.62) for h = 1,...,H. Let q^{ht} be household h's observed consumption vector for period t and let household h face the price vector P^{ht} in period t for h = 1,...,H and t = 0,1. Define the vector of real prices that household h faces in period t, p^{ht}, as follows:

$$(4.78)\ p^{ht} \equiv P^{ht}/\alpha\cdot P^{ht}\ ; \qquad t = 0,1\ ;\ h = 1,...,H.$$

Now make the hypotheses in Proposition 1 for each household and we find that the sum over households of the Bennet quantity indicators $V_B(p^{h0},p^{h1},q^{h0},q^{h1})$ for each household h is equal to the average of the sum of the household h equivalent and compensating variations, $Q_E{}^h(q^{h0},q^{h1},p^{h0})$ and $Q_C{}^h(q^{h0},q^{h1},p^{h1})$; i.e., using Proposition 1, we have:

$$\begin{aligned}(4.79)\ \textstyle\sum_{h=1}^H V_B(p^{h0},p^{h1},q^{h0},q^{h1}) &\equiv \textstyle\sum_{h=1}^H (1/2)[p^{h0} + p^{h1}]\cdot[q^{h1} - q^{h0}]\\ &= (1/2)\textstyle\sum_{h=1}^H Q_E{}^h(q^{h0},q^{h1},p^{h0}) + (1/2)\textstyle\sum_{h=1}^H Q_C{}^h(q^{h0},q^{h1},p^{h0})\\ &= \textstyle\sum_{h=1}^H Q_S{}^h(q^{h0},q^{h1},(1/2)[p^{h0}+p^{h1}]) \qquad \text{using Corollary 1}\end{aligned}$$

where for h = 1,...,H, $Q_S{}^h(q^{h0},q^{h1},(1/2)[p^{h0}+p^{h1}])$ is the Hicks Samuelson theoretical quantity variation for household h using the vector of average real prices facing household h for the two periods under consideration, $(1/2)p^{h0} + (1/2)p^{h1}$, as the reference price vector. Thus if individual household price and quantity data are available, the sum of these theoretical quantity variations can be calculated as the sum of the observable Bennet quantity indicators.

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[124] Diewert (2005c) and Balk (2007) indicated that the Bennet indicators had excellent axiomatic properties as well. Thus the Bennet indicators seem to be the difference counterparts to the Fisher indexes in normal ratio index number theory, since the Fisher indexes also have strong economic and axiomatic properties.

If in addition, each household faces the same vector of prices p^0 in period 0 and p^1 in period 1, then (4.79) simplifies as follows:

$$(4.80)\ V_B(p^0,p^1,q^0,q^1) \equiv (1/2)[p^0 + p^1]\cdot[q^1 - q^0]$$
$$= (1/2)\textstyle\sum_{h=1}^{H} Q_E^h(q^{h0},q^{h1},p^0) + (1/2)\sum_{h=1}^{H} Q_C^h(q^{h0},q^{h1},p^0)$$
$$= \textstyle\sum_{h=1}^{H} Q_S^h(q^{h0},q^{h1},(1/2)[p^0+p^1])$$

where the aggregate period t quantity vectors q^t are defined as the sum of the individual household quantity vectors:

$$(4.81)\ q^0 \equiv \textstyle\sum_{h=1}^{H} q^{h0}\ ;\ q^1 \equiv \sum_{h=1}^{H} q^{h1}\ .$$

Thus under the assumptions of Proposition 1 and the assumption that each household faces the same prices in each period, the aggregate Bennet indicator of quantity change, $V_B(p^0,p^1,q^0,q^1)$ defined by the first line in (4.80), is *exactly* equal to the arithmetic average of the sum of the individual household equivalent variations, $\sum_{h=1}^{H} Q_E^h(q^{h0},q^{h1},p^0)$, plus the sum of the individual compensating variations, $\sum_{h=1}^{H} Q_C^h(q^{h0},q^{h1},p^0)$. Under these hypotheses, the aggregate Bennet indicator of quantity change is also *exactly* equal to the sum over households of the Hicks Samuelson theoretical quantity variations using the vector of average real prices facing household h for the two periods under consideration, $\sum_{h=1}^{H} Q_S^h(q^{h0},q^{h1},(1/2)[p^0+p^1])$.[125]

4.6 The Decomposition Properties of the Bennet Indicators

In the production context, Diewert and Morrison (1986) and Kohli (1990) (1991) developed a methodology that enables one to obtain exact decompositions of various Törnqvist indexes into explanatory factors for each price or quantity change using the assumption of a translog technology.[126] It would be useful if we could provide a similar decomposition result for the Bennet indicators but we are not able to accomplish this task. However, Diewert and Morrison (1986; 674-676) developed an *average of first order*

[125] This result is analogous to Chambers' (2001; 114) exact result for an aggregate normalized Bennet quantity indicator in the context of Chambers' benefit function framework.

[126] Diewert (2002) also developed some decomposition results for the Fisher indexes but these results lack the simplicity of the Törnqvist decomposition results.

approximations methodology which gave very similar results to their translog methodology[127] and so we will use this second approach below in order to provide economic interpretations for each separate term in the Bennet indicators.

In this section, we will not make any specific parametric assumptions; we will assume only that the consumer's cost function C(u,P) satisfies conditions I and in addition, C(u,P) and the dual f(q) are once differentiable in a neighbourhood around the observed period t real price and quantity vectors, $p^t \equiv P^t/\alpha \cdot P^t$ and q^t, and around the observed period t utility levels, $u^t \equiv f(q^t)$, for t = 0,1. Hence the following equations will be satisfied by the data under the assumption that the consumer minimizes costs in each period:

(4.82) $p^t \cdot q^t = C(f(q^t),p^t)$; $\quad t = 0,1;$
(4.83) $q^t = \nabla_P C(f(q^t),P^t) = \nabla_p C(f(q^t),p^t)$; $\quad t = 0,1.$

The first set of equalities in (4.83) follows from Shephard's Lemma and the second set follows from the proportionality of the real prices p^t to the corresponding nominal prices P^t and the linear homogeneity of the cost function C(u,P) in the components of P so that the partial derivative functions $\partial C(u,P)/\partial P_n$ are homogeneous of degree 0 in their price variables.

Define the *nth partial Bennet price and quantity indicators*, $I_n(p_n^0,p_n^1,q_n^0,q_n^1)$ and $V_n(p_n^0,p_n^1,q_n^0,q_n^1)$, as follows:

(4.84) $I_{Bn}(p_n^0,p_n^1,q_n^0,q_n^1) \equiv (1/2)[q_n^0 + q_n^1][p_n^1 - p_n^0]$; $\quad n = 1,...,N;$
(4.85) $V_{Bn}(p_n^0,p_n^1,q_n^0,q_n^1) \equiv (1/2)[p_n^0 + p_n^1][q_n^1 - q_n^0]$; $\quad n = 1,...,N.$

Note that the above partial indicators using real prices sum up to the overall Bennet indicators using real prices; i.e., we have:

(4.86) $I_B(p^0,p^1,q^0,q^1) = \sum_{n=1}^N I_{Bn}(p_n^0,p_n^1,q_n^0,q_n^1)$;
(4.87) $V_B(p^0,p^1,q^0,q^1) = \sum_{n=1}^N V_{Bn}(p_n^0,p_n^1,q_n^0,q_n^1)$.

We will relate the above observable partial indicators to *theoretical partial indicators*: for

[127] See Morrison and Diewert (1990) and Diewert and Lawrence (2006).

each n, define the *Laspeyres and Paasche partial price variations*, α_{Ln} and α_{Pn}, as follows:[128]

(4.88) $\alpha_{Ln} \equiv C(f(q^0),p_1^0,...,p_{n-1}^0, p_n^1, p_{n+1}^0,...,p_N^0) - C(f(q^0),p^0)$; $\quad n = 1,...,N$;
(4.89) $\alpha_{Pn} \equiv C(f(q^1),p^1) - C(f(q^1),p_1^1,...,p_{n-1}^1, p_n^0, p_{n+1}^1,...,p_N^1)$; $\quad n = 1,...,N$.

Thus the nth partial price Laspeyres variation, α_{Ln}, is the difference in real expenditure that would result if the standard of living of the consumer were held constant at the period 0 utility level, $u^0 \equiv f(q^0)$, and all real prices are also held constant at their period 0 levels except that we allow the nth real price to increase from the period 0 level, p_n^0, to the period 1 level, p_n^1. The nth partial price Paasche variation, α_{Pn}, has a similar interpretation except that the reference utility level is held constant at the period 1 level, $u^1 \equiv f(q^1)$, and all real prices are held constant at their period 1 levels and as before, we allow the nth real price to increase from the period 0 level, p_n^0, to the period 1 level, p_n^1.

It is possible to adapt the first order approximation methods used to derive the approximations (4.36) and (4.38) in the present context. Thus first order approximations to the unobservable terms in (4.88) and (4.89) can be obtained as follows: for n = 1,...,N, we have:

(4.90) $C(f(q^0),p_1^0,...,p_{n-1}^0, p_n^1, p_{n+1}^0,...,p_N^0) \approx C(f(q^0),p^0) + [\partial C(f(q^0),p^0)/\partial p_n][p_n^1 - p_n^0]$
$= C(f(q^0),p^0) + q_n^0[p_n^1 - p_n^0]$ using Shephard's Lemma (4.83) ;

(4.91) $C(f(q^1),p_1^1,...,p_{n-1}^1, p_n^0, p_{n+1}^1,...,p_N^1) \approx C(f(q^1),p^1) + [\partial C(f(q^1),p^1)/\partial p_n][p_n^0 - p_n^1]$
$= C(f(q^1),p^1) + q_n^1[p_n^0 - p_n^1]$ using Shephard's Lemma (4.83).

Substituting (4.90) and (4.91) into (4.88) and (4.89) leads to the following *observable first order approximations*, a_{Ln} and a_{Pn} to the Laspeyres and Paasche partial price variations, α_{Ln} and α_{Pn}:[129]

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[128] These variations are difference counterparts to the partial indexes defined in Diewert and Morrison (1986) and Kohli (1990).

[129] Using simple feasibility arguments for the cost minimization problems defined by the left hand sides of (4.90) and (4.91), it can be shown that $a_{Ln} \geq \alpha_{Ln}$ and $a_{Pn} \leq \alpha_{Pn}$ so that the Laspeyres partial price indicators a_{Ln} generally biased upwards for the true partial Laspeyres price indexes α_{Ln} and the Paasche partial price indicators a_{Pn} generally biased downwards for the true partial Paasche price indexes α_{Pn}; i.e., these partial

(4.92) $\alpha_{Ln} \approx q_n^0[p_n^1 - p_n^0] \equiv a_{Ln}$; $n = 1,...,N$;
(4.93) $\alpha_{Pn} \approx q_n^1[p_n^1 - p_n^0] \equiv a_{Pn}$; $n = 1,...,N$.

Thus using definitions (4.84) for the Bennet partial price indicators, $I_{Bn}(p_n^0,p_n^1,q_n^0,q_n^1)$, it can be seen that they are exactly equal to the arithmetic average of the Laspeyres and Paasche partial price indicators, $(1/2)a_{Ln} + (1/2)a_{Pn}$, which in turn approximate the average of theoretical Laspeyres and Paasche partial price variations, $(1/2)\alpha_{Ln} + (1/2)\alpha_{Pn}$, to the first order; i.e., we have:

Proposition 3:

(4.94) $I_{Bn}(p_n^0,p_n^1,q_n^0,q_n^1) = (1/2)a_{Ln} + (1/2)a_{Pn} \approx (1/2)\alpha_{Ln} + (1/2)\alpha_{Pn}$; $n = 1,...,N$.

The above results are nonparametric; i.e., the approximations given by (4.92)–(4.94) are first order Taylor series approximations that are valid no matter what (once differentiable) preferences the consumer holds. However, if we assume that the consumer has preferences that can be represented by the translation homothetic normalized quadratic cost function C(u,P) defined by (4.58)–(4.62), then we can obtain an exact expression for the gap between the Bennet partial indicator on the left hand side of (4.94) and the average of the theoretical partial variations on the right hand side of (4.94); i.e., we can obtain the following expression for the *bias* BBP_n in the nth Bennet partial price indicator; i.e., we have:

(4.95) $I_{Bn}(p_n^0,p_n^1,q_n^0,q_n^1) = (1/2)\alpha_{Ln} + (1/2)\alpha_{Pn} + BBP_n$; $n = 1,...,N$;
(4.96) $BBP_n \equiv -(1/2)\alpha_n[p^0 \cdot Bp^0 + p^1 \cdot Bp^1][p_n^1 - p_n^0]$

where α_n is the nth component in the weighting vector α that is used to form real prices. Since $\sum_{n=1}^N \alpha_n p_n^t = 1$ for t = 0,1, it can be seen that:

(4.97) $\sum_{n=1}^N BBP_n = 0$

so that the sum of the bias terms in the Bennet partial indicators $I_{Bn}(p_n^0,p_n^1,q_n^0,q_n^1)$ sums

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price indicators will generally have some substitution bias, which will tend to cancel out when we take their averages.

to zero.[130] Let P^* be the money metric utility scaling vector which appears in (60)-(62) and define its real counterpart by $p^* \equiv P^*/P^* \cdot \alpha$. If p^0 is proportional to p^*, then $p^0 \cdot Bp^0$ is equal to 0 and if p^1 is proportional to p^*, then $p^1 \cdot Bp^1$ is equal to 0 and under these conditions, it can be seen that all of the bias terms BBP_n will be equal to 0 as well. Hence if p^0 and p^1 are close to each other, then we can choose the reference price vector p^* to be close to p^0 and p^1 and the bias terms will all be close to 0.

Finding economic interpretations for the Bennet partial quantity indicators, $V_{Bn}(p_n^0,p_n^1,q_n^0,q_n^1)$, is more difficult. For each n, we first define the theoretical *Laspeyres and Paasche partial quantity variations*, β_{Ln} and β_{Pn}, as follows:

(4.98) $\beta_{Ln} \equiv C(f(q_1^0,...,q_{n-1}^0, q_n^1, q_{n+1}^0,...,q_N^0),p^0) - C(f(q^0),p^0)$; $n = 1,...,N$;

(4.99) $\beta_{Pn} \equiv C(f(q^1),p^1) - C(f(q_1^1,...,q_{n-1}^1,q_n^0,q_{n+1}^1,...,q_N^1),p^1)$; $n = 1,...,N$.

Thus the nth partial quantity Laspeyres variation, β_{Ln}, is the difference in real expenditure that would result if the real prices of the consumer are held constant at their period 0 levels p^0 and all quantities are also held constant at their period 0 except that we allow the nth quantity to increase from the period 0 level, q_n^0, to the period 1 level, q_n^1. The nth partial quantity Paasche variation, β_{Pn}, has a similar interpretation except that the reference prices are held constant at their period 1 levels p^1 and all quantities are also held constant at their period 1 levels except that we allow the nth quantity to increase from the period 0 level, q_n^0, to the period 1 level, q_n^1.

In order to obtain observable first order approximations to the theoretical quantity variations defined by (4.98) and (4.98), it is first necessary to develop some preliminary material. Define the function $h^t(q)$ for q's in a neighborhood of q^t as follows:

(4.100) $h^t(q) \equiv C(f(q),p^t)$; $t = 0,1$.

Under our assumptions, $h^t(q)$ is once differentiable at q^t and we can calculate the vector of first order partial derivatives as follows:

(4.101) $\nabla_q h^t(q^t) = [\partial C(f(q^t),p^t)/\partial u]\ \nabla_q f(q^t)$; $t = 0,1$.

[130] This must be the case in order for Proposition 2 to hold.

Under our assumptions, q^t solves the cost minimization problem defined by $C(f(q^t),p^t)$ for $t = 0,1$ and since $f(q)$ is differentiable at q^t, there exists a nonnegative Lagrange multiplier λ^t such that the following first order necessary conditions for the period t cost minimization problem are satisfied:[131]

$$(4.102)\quad p^t = \lambda^t \nabla_q f(q^t)\ ; \qquad t = 0,1.$$

But Samuelson (1947) showed that the period t Lagrange multiplier λ^t which appears in (4.102) is also equal to the period t marginal cost around the equilibrium point so that we have:

$$(4.103)\quad \lambda^t = \partial C(f(q^t),p^t)/\partial u\ ; \qquad t = 0,1.$$

Substituting (4.102) and (4.103) into (4.101) gives us the following simple expression for the derivatives of the function $h^t(q)$ defined by (4.100):

$$(4.104)\quad [\partial C(f(q^t),p^t)/\partial u]\nabla_q f(q^t) \equiv \nabla_q h^t(q^t) = p^t\ ; \qquad t = 0,1.$$

Equations (4.104) seem to have been first derived by Balk (1989; 166) so we can call these relationships *Balk's Lemma*. With the above preliminary material out of the way, we can now proceed to the task of finding first order approximations to the theoretical partial quantity variations β_{Ln} and β_{Pn} defined by (4.98) and (4.99). Thus a first order approximation to the unobservable term $C(f(q_1^0,...,q_{n-1}^0, q_n^1, q_{n+1}^0,...,q_N^0),p^0)$ in (4.98) is:

$$\begin{aligned}(4.105)\quad & C(f(q_1^0,...,q_{n-1}^0, q_n^1, q_{n+1}^0,...,q_N^0),p^0) && n = 1,...,N\\ &\approx C(f(q^0),p^0) + [\partial h^0(q^0)/\partial q_n][q_n^1 - q_n^0] && \text{using (4.100) for } t = 0\\ &= C(f(q^0),p^0) + p_n^0[q_n^1 - q_n^0] && \text{using (4.104) for } t = 0.\end{aligned}$$

Similarly, a first order approximation to the unobservable term $C(f(q_1^1,...,q_{n-1}^1,q_n^0,q_{n+1}^1,...,q_N^1),p^1)$ in (4.99) is:

$$\begin{aligned}(4.106)\quad & C(f(q_1^1,...,q_{n-1}^1,q_n^0,q_{n+1}^1,...,q_N^1),p^1) && n = 1,...,N\\ &\approx C(f(q^1),p^1) + [\partial h^1(q^1)/\partial q_n][q_n^0 - q_n^1] && \text{using (4.100) for } t = 1\end{aligned}$$

[131] Strictly speaking, we require $q^t >> 0_N$ to ensure that conditions (102) are satisfied and later we will also require that marginal cost be positive so that $\partial C(f(q^t),p^t)/\partial u > 0$ for $t = 0,1$.

$$= C(f(q^1),p^1) + p_n^1[q_n^0 - q_n^1] \qquad \text{using (4.104) for } t = 1.$$

Substituting (4.105) and (4.106) into (4.98) and (4.99) leads to the following *observable first order approximations*, b_{Ln} and b_{Pn} to the Laspeyres and Paasche partial quantity variations, β_{Ln} and β_{Pn}:[132]

(4.107) $\beta_{Ln} \approx p_n^0[q_n^1 - q_n^0] \equiv b_{Ln}$; $\qquad n = 1,...,N$;

(4.108) $\beta_{Pn} \approx p_n^1[q_n^1 - q_n^0] \equiv b_{Pn}$; $\qquad n = 1,...,N$.

Thus using definitions (4.85) for the Bennet partial quantity indicators, $V_{Bn}(p_n^0,p_n^1,q_n^0,q_n^1)$, it can be seen that they are exactly equal to the arithmetic average of the Laspeyres and Paasche partial quantity indicators, $(1/2)b_{Ln} + (1/2)b_{Pn}$, which in turn approximate the average of the theoretical Laspeyres and Paasche partial price variations, $(1/2)\beta_{Ln} + (1/2)\beta_{Pn}$, to the first order; i.e., we have:

Proposition 4:

(4.109) $V_{Bn}(p_n^0,p_n^1,q_n^0,q_n^1) = (1/2)b_{Ln} + (1/2)b_{Pn} \approx (1/2)\beta_{Ln} + (1/2)\beta_{Pn}$; $\qquad n = 1,...,N.$

This completes our theoretical discussion of the properties of the Bennet indicators. In the following section, we illustrate the use of these indicators for a Japanese data set.

4.7 The Bennet Indicators using Japanese Data

In this section, we apply our methodology to Japanese consumption data. These data were constructed from the Japanese national accounts for 12 classes of expenditure for the period 1980-2006. The prices for each commodity class were normalized to equal one in 1980; see Tables G-1 and G-2 in Appendix G for a listing of the data. We chose

[132] Using simple feasibility arguments for the cost minimization problems defined by the left hand sides of (4.102) and (4.103), it can be shown that $b_{Ln} \geq \beta_{Ln}$ and $b_{Pn} \leq \beta_{Pn}$ so that the Laspeyres partial quantity indicators b_{Ln} generally biased upwards for the true partial Laspeyres quantity indexes β_{Ln} and the Paasche partial quantity indicators b_{Pn} generally biased downwards for the true partial Paasche quantity indexes β_{Pn}; i.e., these partial quantity indicators will generally have some substitution bias, which will tend to cancel out when we take their averages.

food and non-alcoholic beverages to be our numeraire commodity[133] Aggregate expenditures evaluated in terms of real prices are 127753 billion yen in 1980 and 232679 in 2006. Therefore, household expenditures evaluated in real prices increased by 104927 billion yen over the last 27 years. We calculate Bennet indicators of quantity changes and real price changes to decompose the expenditure difference for every year. Table 4-1 lists value the real expenditure differences and the Bennet indicators for the period 1981–2006. Table 4-2 lists their annual averages. It tells us that the effects of real price changes are much smaller than the effects of quantity changes. However, the impact of real price changes has been significant for the last decade.

Table 4-1: Real Expenditure Differences and Bennet Indicators, 1981-2006

Year	Difference	Bennet Quantity Indicator	Bennet Price Indicator
1981	2657.4	1966.8	690.7
1982	8980.9	6252.6	2728.3
1983	3191.4	4054.7	-863.3
1984	2352.7	3456.0	-1103.2
1985	6652.0	6002.3	649.7
1986	6622.4	4996.6	1625.7
1987	9040.9	6538.3	2502.6
1988	8122.2	7847.4	274.8
1989	8089.1	8398.1	-309.0
1990	5992.3	8497.3	-2505.0
1991	1489.5	5275.9	-3786.4
1992	6674.9	4586.8	2088.1
1993	2324.7	2438.6	-113.9
1994	5214.8	5253.6	-38.8
1995	6675.9	3224.2	3451.7
1996	4329.8	5314.7	-984.9
1997	1297.4	1683.8	-386.3
1998	-4934.2	-2125.0	-2809.2
1999	1424.7	1188.5	236.3
2000	4120.6	1851.2	2269.3
2001	3656.7	3969.5	-312.8
2002	1932.1	2188.8	-256.7
2003	4.3	1433.0	-1428.7
2004	480.4	3708.4	-3228.0
2005	4579.9	3944.9	635.1
2006	3953.4	6167.5	-2214.1

Table 4-2: Annual Averages of Real Expenditure Differences and Bennet Indicators

Year	Difference	Bennet Quantity Indicator	Bennet Price Indicator
1980-2006	4035.6	4158.3	-122.6
1981-1990	6170.1	5801.0	369.1
1991-2000	2861.8	2869.2	-7.4
2001-2006	2434.5	3568.7	-1134.2

[133] This choice of deflator means that we used the weighting vector $\alpha = (1,0,\ldots 0)^T$.

Our focus is on real consumption that measures the overall utility or volume of aggregate consumption. Real consumption can be computed throughout either the traditional ratio approach to quantity indexes or by the difference approach as outlined in this chapter. However, if we use the ratio approach, the choice of specific index number formula could matter for the value of real consumption. Therefore, we use the difference approach as well as alternative index number formulae in order to evaluate the performance of the difference approach relative to that of the ratio approach.

Real consumption coincides with the corresponding nominal value at the reference year. Setting 1980 to be the reference year, we calculate different versions of real consumption for all years using the ratio approach and the difference approach. Fixed base and chained quantity indexes were computed using the Laspeyres, Paasche, Fisher and Törnqvist-Theil formulae.[134] The results are listed in Table 4-3 below. The last column of Table 4-3 lists the corresponding Bennet estimate of total consumption. The first entry in this column is simply the 1980 measure of Japanese total consumption expenditures divided by the price of food; i.e., the first entry in the second column of the Table. The next entry in the Bennet column just adds the Bennet measure of quantity change or volume change V_B defined by (4.41) above where the real price vectors and quantity vectors pertaining to the years 1980 and 1981 are used in the formula. The 1982 entry in the Bennet column is just the 1981 entry plus the Bennet measure of quantity change going from 1981 to 1982 and so on.

Looking at Table 4-3, it can be seen that all of the index number estimates of real Japanese consumption are very close to each other with the exception of the fixed base Laspeyres and Paasche estimates. This lack of correspondence is normal since these indexes are known to differ from their superlative counterparts when a fixed base is used. The superlative chained indexes are particularly close to each other. But how do these chained superlative indexes compare to the corresponding Bennet estimates of real consumption listed in the last column of Table 4-3? It can be seen that the Bennet measures are always equal to or greater than their chained superlative counterparts but

[134] Two versions of the Törnqvist-Theil were computed for both fixed base and chained indexes: one that constructed the price index first using the usual formula (and then the quantity index was defined by dividing real expenditures by this direct price index) and the other was constructed by directly comparing a share weighted average of log quantity changes and exponentiating. These two quantity indexes are superlative as is the Fisher index; see Diewert (1976) for the formula details.

the differences are not very large: on average, the Bennet estimate exceeds its chained Fisher counterpart by 0.74% per year, with a maximum deviation of 1.1%.

Table 4-3: Comparison of Japanese Real Consumption, 1980-2006

Year	Total Expenditure	Ratio Approach										Difference Approach
		Fixed Base Index					Chained Index					
		Laspeyres Quantity Index	Paasche Quantity Index	Fisher Quantity Index	Tornqvist Quantity Index	Implicit Tornqvist Quantity Index	Laspeyres Quantity Index	Paasche Quantity Index	Fisher Quantity Index	Tornqvist Quantity Index	Implicit Tornqvist Quantity Index	Bennet Quantity Indicator
1980	127752.7	127752.7	127752.7	127752.7	127752.7	127752.7	127752.7	127752.7	127752.7	127752.7	127752.7	127752.7
1981	130410.1	129723.2	129705.2	129714.2	129714.5	129714.1	129723.2	129705.2	129714.2	129714.5	129714.1	129719.5
1982	139391.1	135942.2	135820.7	135881.4	135885.0	135881.7	135909.6	135830.6	135870.1	135871.1	135870.4	135972.1
1983	142582.5	139930.0	139723.9	139826.9	139829.8	139825.9	139890.4	139778.2	139834.3	139835.3	139834.5	140026.9
1984	144935.2	143397.3	143032.0	143214.5	143216.7	143211.9	143312.0	143160.8	143236.4	143237.4	143236.6	143482.8
1985	151587.2	149444.7	148913.2	149178.7	149179.0	149173.6	149246.3	149063.8	149155.0	149156.0	149155.2	149485.1
1986	158209.6	154316.2	153786.8	154051.3	154058.6	154048.9	154151.3	153940.0	154045.6	154047.0	154045.8	154481.7
1987	167250.5	160639.7	160067.6	160353.4	160365.5	160354.6	160474.4	160251.2	160362.8	160364.3	160363.1	161020.0
1988	175372.7	168160.1	167493.2	167826.3	167841.9	167828.7	168006.2	167755.6	167880.8	167882.7	167881.1	168867.4
1989	183461.8	176183.0	175390.8	175786.5	175810.7	175789.6	176072.5	175781.1	175926.7	175928.8	175927.1	177265.5
1990	189454.1	184481.0	183247.0	183863.0	183935.2	183864.1	184306.9	183951.3	184129.0	184132.2	184129.5	185762.8
1991	190943.7	189745.3	188051.9	188896.7	188939.3	188886.1	189504.2	189110.7	189307.3	189310.6	189307.8	191038.7
1992	197618.6	194323.3	192481.4	193400.1	193406.1	193384.4	194039.0	193622.0	193830.4	193833.8	193830.9	195625.6
1993	199943.2	196770.9	194760.7	195763.2	195728.0	195727	196436.0	196010.0	196222.9	196226.7	196223.3	198064.1
1994	205158.0	202159.1	199719.7	200935.7	200849.2	200883	201593.5	201165.2	201379.3	201383.0	201379.6	203317.7
1995	211834.0	205423.2	202857.5	204136.3	204059.7	204090	204745.7	204290.4	204517.9	204521.8	204518.2	206541.9
1996	216163.8	210890.5	207390.9	209133.4	209019.5	209048.8	209921.7	209399.4	209660.4	209665.1	209660.4	211856.6
1997	217461.2	212945.5	208639.4	210781.4	210627.0	210645.4	211585.7	211004.4	211294.8	211298.6	211294.5	213540.4
1998	212527.0	210785.1	206006.4	208382.1	208221.2	208186.6	209497.1	208936.5	209216.6	209219.0	209216.1	211415.4
1999	213951.7	212406.8	206864.1	209617.1	209381.3	209366.7	210687.9	210084.2	210385.9	210386.6	210385.3	212603.8
2000	218072.3	214655.4	208394.7	211501.9	211186.4	211206.1	212530.9	211863.0	212196.7	212196.6	212195.9	214455.0
2001	221729.0	219124.7	211575.8	215317.2	214843.1	214905.3	216449.2	215674.6	216061.6	216061.0	216060.5	218424.6
2002	223661.1	221564.5	213242.1	217363.5	216757.6	216910.3	218589.8	217802.2	218195.6	218194.5	218194.6	220613.4
2003	223665.4	223647.4	213901.5	218720.2	218000.7	218185.9	220041.5	219154.9	219597.8	219596.3	219596.3	222046.3
2004	224145.8	228274.4	216395.4	222255.5	221340.7	221580.2	223775.6	222753.6	223264.0	223261.8	223261.8	225754.8
2005	228725.7	233194.6	219024.9	225998.7	224784.3	225100.2	227751.1	226625.5	227187.6	227184.5	227184.7	229699.6
2006	232679.2	241317.8	223461.4	232218.0	230683.0	230972	234019.0	232665.4	233341.2	233336.2	233336.5	235867.2

What are we to conclude from the above results? For the Japanese data, it seems that a standard superlative index number approach to measuring aggregate real consumption will be fairly close to the results generated by the theoretically preferable Bennet approach, which has better aggregation over consumer properties and is consistent with nonhomothetic preferences. However, there seem to be small but significant differences between the Bennet estimates and those generated by chained superlative indexes.

4.8 Conclusion

This chapter has established satisfactory difference theory counterparts to the standard results on exact and superlative indexes in the ratio approach to the aggregation over commodities problem. The counterpart to a superlative index number formula is a superlative indicator formula. We found that the Bennet indicators of price and quantity change were (strongly) superlative and thus we recommend their use in practical applications of cost benefit analysis when ex post variations must be calculated.

In section 4.7 above, we found that, somewhat surprisingly, the results using the Bennet indicator of quantity change are rather close to the quantity aggregates generated by a

superlative quantity index. This is somewhat reassuring in that the ratio and difference approaches to economic aggregation seem to give more or less the same answer, at least for our Japanese data set.

Finally, we mention one strong advantage of the difference approach over the ratio approach: the ratio approach fails if the quantity aggregate has a value equal to zero in the base period whereas the difference approach is unaffected by this complication. This observation is important if labour supply enters the consumer's utility function (negatively rather than positively) since in this case, zero or negative value aggregates can readily occur. Although we did not formally model this situation, we are confident that our techniques can be generalized to cover this situation.

Chapter 5. Summary and Conclusion

5.1 Summary of Contributions

This thesis has addressed several important theoretical and empirical problems in economic measurement. Each of these problems was addressed with reference to both of the existing approaches in index number theory: the ratio approach and the difference approach.

In our analysis of the Japanese living standard, we focused on real income and net real income in the Japanese market sector as appropriate measures. We adopted the decomposition formula developed by Diewert and Morrison (1986), as well as Kohli (1990) (2004), based on the ratio approach in index number theory. Applying their decomposition formula, we decomposed the growth in real income and net real income into contribution factors such as technical progress, changes in real output prices and growth in input quantities. The key roles of technical progress and capital services growth were observed in an analysis of real income as well as net real income, and the role of capital services significantly decreased in the net approach. Construction of a new data set of price and quantity of input and output for Japanese market sector is a notable feature of this chapter.

We applied the difference approach to a producer model as well as a consumer model; in the producer model, we were concerned with a new approach to productivity measurement based on the difference approach. In this new approach, productivity growth is measured by the difference between output quantity differences and the input quantity differences instead of the ratio of output quantity ratio to input quantity ratio. First, we proposed theoretical measures of contribution factors for technical progress, changes in real output prices and growth in relative input quantities. Second, we showed that Bennet indicators coincided with these theoretical measures by introducing an income function based on the normalized quadratic functional form pioneered by Diewert and Wales (1987). Bennet indicators add up to the change in the real income per unit primary input. Thus, our theoretical result would enable us to decompose the growth in real income per unit primary input into additive contribution factors. We applied our result to investigate the growth in real income and net real income per unit labour in the Japanese economy from 1955 to 2006, and observed that productivity growth and capital services were the two main contributors to growth in real income and

net real income per unit labour.

In the consumer model, we addressed the price and quantity aggregation problem with multiple consumption goods in the difference approach, and investigated the best method for calculating representative price and quantity difference. Diewert (1976) has defined superlative price and quantity indexes as observable indexes that are exact for a ratio of unit cost functions or for a ratio of linearly homogeneous utility functions. We looked for counterparts of his results in the difference context of index number theory for both flexible homothetic and nonhomothetic preferences. First, we proposed theoretical measures for the aggregate price and quantity indicator. Second, we showed that Bennet indicators coincided with these theoretical measures by introducing a cost function with the translation homothetic normalized quadratic functional form discussed by Balk, Färe and Grosskopf (2004), Chambers (2001) and Dickenson (1980). Using Japanese aggregate consumption data from the national accounts, we calculated real expenditures based on different types of indexes and indicator

5.2 Directions for Further Research

We conclude this thesis by noting three unsolved problems remaining for future research: (1) an industrial analysis of the Japanese standard of living, (2) a productivity measurement based on alternative types of the rate of return and (3) an investigation of the theoretical foundation of the arithmetic mean of the sum of compensating variations and that of equivalent variations.

In our analysis of real income and net real income, we provided an economy-wide analysis but did not provide an industry contributions analysis. The primary focus of our analysis was on the effects of changes in the terms of trade on living standards. We would like to extend our analysis by introducing two new components into our database. First, we will introduce the industry production accounts consisting of inputs and outputs. Second, we will introduce a more detailed breakdown of exports and imports by commodity classification. Changes in the terms of trade reflect price changes in international commodities, and industries respond to price changes in different ways. Thus, the extended database would enable us to decompose the impact of changes in the terms of trade on living standards into industries and commodities. We are especially interested in how changes in crude oil prices have affected the industries' incomes. Increased oil price would more severely affect the incomes of the industries that use oil

heavily than those of other industries. Through such a study, we could investigate the channels through which changes in crude oil price affect a country's standard of living.

The measurement of prices and quantities of capital services is indispensable for growth accounting, which this thesis' analysis adopts. The quantity of capital services is proportional to the productive capital stock, and their growth rates coincide with each other. Capital stock is estimated based on the perpetual inventory method. On the other hand, the price of capital services from a fixed asset, which is called a user cost or a rental price, consists of three main components: rate of return, depreciation rate and change in its asset's price. Among them, the choice of the rate of return is an important element in the construction of the price of capital services. Two types of rate of return exist: the *ex-post* rate of return and the *ex-ante* rate of return. Under an *ex-post* approach, the rate of return is computed by imposing the condition that estimated capital services exactly correspond to gross operating surplus and the capital element of gross mixed income. Under an *ex-ante* approach, the rate of return is chosen as to best reflect economic agents' expectations of the investment's required return. A typical example is a government one-year bond rate. It is thus unlikely that there would be exact equality between the value of capital services and gross operating surplus, and the capital element of gross mixed income.

The *ex-post* approach has stood the test of time and is still in use by many researchers; this thesis also used the *ex-post* rate of return. However, the *ex-ante* approach has several advantages, two of which are as follows: (1) the theoretical assumptions needed are less restrictive than for the ex-post approach and (2) the *ex-ante* approach provides a means of splitting mixed income between income and labour, and income and capital. By following the ex-ante approach, we avoid the problem of imputing a wage for the self-employed and family workers. We would like to conduct growth accounting by constructing the price of capital services based on an *ex-ante* rate of return. Our concerns are the differences in productivity growth and wages for the self-employed and family workers between estimates based on an *ex-ante* rate of return and an *ex-post* rate of return.

We show that the Bennet quantity indicator coincides with the arithmetic mean of the sum of compensating variations and that of equivalent variations. Positive sums of compensating variations and equivalent variations must enable the gainers to compensate the losers and have something left over for themselves. Using the sum of compensating

variations and equivalent variations for a cost–benefit test is often justified by appealing to this line of reasoning. We would like to investigate the possibility of using the arithmetic mean of the sum of compensating variations, as well as that of equivalent variations, for a cost–benefit test. But there are practical and theoretical problems associated with the sums of compensating variations and equivalent variations. The practical problem is that we need to estimate consumers' cost functions or indirect utility functions in order to calculate compensating and equivalent variations. If these functions vary across consumers, we need to estimate the distinct cost functions and distinct indirect utility functions for each consumer, and this is a practically impossible task. The theoretical problem is the so-called Boadway paradox (Blackorby and Donaldson (1990)). A move from one Walrasian equilibrium to another Walrasian equilibrium always yields a positive sum of compensating variations and a non-positive sum of equivalent variations. This leads to the violation of transitivity in social decision making. On the other hand, the arithmetic means of the sum of compensating variations and the sum of equivalent variations clearly has the advantage of easy implementation over the sums of compensating variations and the sums of equivalent variations. As this thesis shows, it can be calculated from observable prices and quantities by using a Bennet indicator formula. However, we have no clear answer for whether or not the arithmetic mean of the sum of compensating variations and the sum of equivalent variations violates transitivity. When we are able to show that it is free from the Boadway paradox, we will be able to clearly justify its use in a cost–benefit analysis.

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Appendix

Appendix A. Data Construction

In this appendix, we explain our methods for data construction in detail. The main data sources are the Japanese national accounts and the KEO database and these sources will often not be explicitly acknowledged in what follows.

1. Final Demand Components other than Gross Capital Formation

There are five price and quantity series for final demand components other than investments and inventory changes in our database:

- C; Domestic final consumption expenditure of households (excluding the imputed rent of owner-occupied dwellings);
- N; Final consumption expenditure of private non-profit institutions serving households;
- G; Net purchases of goods and services by the general government;
- X; Exports of goods and services (including direct purchases in the domestic market by non-resident households);
- M; Imports of goods and services (excluding direct purchases abroad by resident households).

1.1 Data Sources

Our output data series (other than for investments and inventory changes) are based on Japanese official national accounts (JSNA), which has been published by the Economic and Social Research Institute of the Cabinet of Office and the Economic Research Institute of the Economic Planning Agency of the Japanese Government. We call Japanese national accounts JSNA. In 1978, the Japanese system of national accounts was revised to comply with the guidelines proposed by the United Nations System of National Accounts (1968 SNA). In 2000, it was revised to comply with the guidelines newly proposed by the United Nations System of National Accounts (1993 SNA). The Economic and Social Research Institute and Economic Research Institute published separate historical data series based on 1968 SNA and then later based on the 1993 SNA (but not extending back to 1955). Our basic strategy is to start with the reference data

series in the JSNA based on the 1993 SNA[135] and extend this series backwards by using the growth rates of other data series in the 1968 JSNA. Before the 1968 JSNA was introduced, the Japanese national accounts were called National Income Statistics. Data from the National Income Statistics are used only for constructing the imputed rent of owner-occupied houses. Publications we used are as follows;

National Income Statistics

- *Annual Report on National Report on National Income Statistics 1975*, Economic Planning Agency

1968 JSNA

- *Report on National Accounts from 1955 to 1998*, Economic and Social Research Institute, Cabinet Office
- *Annual Report on National Accounts of 2000*, Economic Planning Agency

1993 JSNA

- *Annual Report on National Accounts of 2004-2008*, Economic and Social Research Institute, Cabinet Office

1.2 Laspeyres Quantity Indexes and Subindexes

The Laspeyres quantity index (and therefore, a corresponding Paasche price index) is applied for constructing quantity indexes everywhere in the Japanese national accounts. Therefore, we can implicitly derive the price and quantity of a component from data series for the aggregate and all other components. Suppose that 0 is the base year and there are two products, B and C. Further suppose that the statistical agency forms a fixed base Laspeyres quantity index that aggregates these two products into the aggregate A say. The base period expenditure shares of products B and C are

$$\text{(A1)}\ s_B^0 \equiv P_B^0 Q_B^0/[P_B^0 Q_B^0 + P_C^0 Q_C^0] = V_B^0/[V_B^0 + V_C^0]\ ;$$
$$s_C^0 \equiv P_C^0 Q_C^0/[P_B^0 Q_B^0 + P_C^0 Q_C^0] = V_C^0/[V_B^0 + V_C^0] \quad = 1 - s_B^0$$

where $V_B^0 = P_B^0 Q_B^0$ is the value of expenditures on product B in period 0 and P_B^0 and Q_B^0 are the corresponding price, etc. The price of A in period 0 can be set equal to 1 and the quantity of A in period 0 can be set equal to the expenditure on B and C in period 0.

[135] In the future, we will abbreviate "the JSNA based on the 1993 SNA" to "the 1993 JSNA" and "the JSNA based on the 1968 SNA" to "the 1968 JSNA".

The value of the aggregate A in period t is set equal to the sum of the expenditure values on B and C, $V_B^t + V_C^t$, so that putting this all together, we have:

(A2) $P_A^0 = 1$; $V_A^0 \equiv V_B^0 + V_C^0 \equiv Q_A^0$; $V_A^t \equiv V_B^t + V_C^t$.

Use the Laspeyres quantity index formula to determine the period t quantity aggregate Q_A^t and the corresponding price P_A^t as follows:

(A3) $Q_A^t/Q_A^0 = s_B^0(Q_B^t/Q_B^0) + s_C^0(Q_C^t/Q_C^0)$; $P_A^t = (V_A^t/V_A^0)/(Q_A^t/Q_A^0)$.

Now suppose that we have value series for A, B and C for periods 0 and t (so that in particular we can calculate the shares defined by (A1) above), we know the statistical agency has used the equations in (A3) in order to calculate the price and quantity of A in period t, and we know the prices and quantities for product A and B for periods 0 and t. Our problem is to calculate prices and quantities for product C. We can set the price of product C in period 0 to 1 so that $P_C^0 = 1$ and set the quantity of product C in period 0 to the period 0 value, so that $Q_C^0 = V_C^0$. Now use the first equation in (A3) to solve for Q_C^t, which is a straightforward linear equation in one unknown. Once Q_C^t has been determined, the corresponding price P_C^t can be set equal to $(V_C^t/V_C^0)/(Q_C^t/Q_C^0)$. We found this data recovery technique to be extremely useful in practice.

1.3 Domestic Final Consumption Expenditure of Households (C)

For the years 1955-1998, current and constant yen series for Domestic Final Consumption Expenditure of Households are found in the *Report on National Accounts from 1955 to 1998* (1968 JSNA); see Part 1 Flows, [1] Figures of Calendar Year, I Time Series Tables, 1 Arranged for Main Figures; Calendar Year, in billions of yen. The constant yen series is at the market prices of 1990. We use these series to construct price and quantity series for Domestic Final Consumption Expenditure of Households. However, this aggregate consumption is the sum of market sector sales to households (our C) plus direct purchases abroad by resident households. Therefore, we need to use the technique described in section 1.2 above to remove the latter series.

Our target consumption aggregate also excludes the imputed expenditures on owner-occupied housing and so we have to deduct these expenditures from the national accounts consumption aggregate as well. We follow two steps in order to accomplish

this task. First, we construct price and quantity series for total housing consumption (rental expenses plus imputed owner-occupied expenses) from the data of the 1968 JSNA series. Second, for the years prior to 1980, we divided the total housing expenses between the owner-occupied houses and the rental houses based on their relative floor space.[136] We will explain how we estimated these relative floor spaces in the following section. For the years 1970-1998, we use the current and constant yen series for the sum of rental and owner occupied housing using unpublished data from the 1968 JSNA.[137] For the years 1955-1973, current and constant yen series for total housing expenses can be found in the *Annual Report on National Income Statistics 1975*; see Part 1 Basic Accounts and Main Tables, 2 Figures of Calendar Year, Account 1 Gross National Product and Expenditure. From these two sources, we can construct price and quantity series for the years 1955-1998. We linked these two data series at the time when data series based on the 1968 JSNA started which was 1970.[138] Thus we extended backwards the series in the 1968 JSNA using the growth rates of housing expenditures tabled in the National Income Statistics for the years 1955-1970.[139] As explained in section 2.3 below, we constructed estimates for relative floor spaces of owner-occupied houses and rental houses for the years 1955-2006. Thus, we could decompose our already constructed data series for the total quantity of housing services into quantity components for imputed rent and market rent by using the ratios of floor spaces. Thus we have obtained price and quantity series for the imputed rent of the owner-occupied houses. We then constructed price and quantity series for C for the years 1955-1998 by applying chained Fisher indexes to the price and quantity series for Domestic Final Consumption Expenditure of Households of the 1968 JSNA and our price and quantity series for the imputed expenditures on owner-occupied housing, with the quantity of owner occupied housing entering the index number formula with a negative sign. The resulting series will be linked to the more recent 1993 JSNA household consumption

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[136] This is the procedure which had been adopted by Japanese national accounts until 2006.

[137] Shuji Hasegawa at the Japanese Cabinet Office has made available to us these data series.

[138] Some variables in JSNA based on 1968 SNA are revised back to 1955. However, total housing expenses in the 1968 JSNA are available only after 1970.

[139] There is another database consisting of gross outputs, exports, and imports for the years 1951-1968. This database was constructed at the joint project between Japan Center of Economic Research and Keio Economic Observatory in the early 1970s, headed by Professor Iwao Ozaki, Keio University. We used the estimates of households' housing expenditure from this database in order to extrapolate the JSNA data based on the 1968 SNA backwards for the years 1955-1968.

series to be described in the next paragraph.

For the years 1980-2002, current and constant yen series for (1) Final Consumption Expenditures of Household (excluding imputed rents for the services of owner-occupied dwellings), (2) Direct Purchases Abroad by Resident Households, and (3) Direct Purchases in the Domestic Market by Non-Resident Households can be found in the *Annual Report on National Accounts of 2004* (1993 JSNA), and these data series are further updated for the years 1980-2003 (constant yen series only for the years 1994-2003) using the *Annual Report on National Accounts of 2005* (1993 JSNA) and for the years 1994-2006, using the information posted on the Cabinet Office website: *Annual Report on National Accounts of 2008* (1993 JSNA); see Part 1 Flow Accounts; 1.Integrated Accounts; (1) Gross Domestic Product Account (Production and Expenditure Approach); Calendar Year, in billions of yen. The constant yen series in the *Annual Report on National Accounts of 2004* is at the market prices of 1995. The constant yen series in the *Annual Report on National Accounts of 2005* is at the market prices of 2000. The constant yen series in the *Annual Report on National Accounts of 2008* is at the market prices of 2000 (chained).[140] From these data series, we constructed price and quantity series for each of the three variables listed above for the years 1980-2006.

One difference between the 1968 JSNA and the 1993 JSNA is in the treatment of social benefits in kind.[141] It was a part of Domestic Final Consumption Expenditure of Households instead of a part of Government Final Consumption Expenditure in the 1968 JSNA. However, it became a part of Government Final Consumption Expenditure instead of a part of Domestic Final Consumption Expenditure of Households in the 1993 JSNA. We consider social benefits in kind as a part of Domestic Final Consumption Expenditure of Households following the treatment of the 1968 JSNA.[142] For the years

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[140] Constant yen series based on chained indexes have been introduced in the *Annual Report on National Accounts of 2006*. Since a chained index avoids the substitution bias that the usual fixed based index is likely to show, we use constant yen series based on chained indexes as much as possible.

[141] Social benefits in kind consist of transfers made by government units to households. It consists of subsidies for medical treatment and for text books which are supposed to be delivered to all the students without any cost. However, these goods are also produced by the production sector.

[142] Following the same treatment of social benefits in kind as in the 1968 JSNA, we can construct price and quantity series for C for a longer period than we can construct following the same treatment in the 1993 SNA.

1990-2003, current and constant yen series for Social Benefits in Kind etc. are found in the *Annual Report on National Accounts of 2005* (1993 JSNA), and these data series are further updated for the years 1996-2006 in the *Annual Report on National Accounts of 2008* (1993 JSNA); see Part 1 Flow Accounts; 5. Supporting Tables, (8) Final Consumption Expenditure of General Government classified by Purpose; Fiscal Year, in billions of yen. The constant yen series in the *Annual Report on National Accounts of 2005* are at the market prices of 1995. The constant yen series of the updated data series in the *Annual Report on National Accounts of 2008* is at the market prices of 2000. Since these annual data series are based on a fiscal year, we transformed these data series of Social Benefits in Kind etc. based on fiscal years into those based on calendar years by linear interpolation.[143] From these data series, we constructed price and quantity series for Social Benefits in Kind etc. based on calendar years for the years 1991-2006.

For the years 1991-2006, we constructed price and quantity series for our consumption aggregate C by applying chained Fisher indexes to the above data series, which were essentially equal to an aggregate of: (1) Final Consumption Expenditures of Household (excluding imputed rent services of owner-occupied dwellings), (2) (less) Direct Purchases Abroad by Resident Households, plus (3) Direct Purchases in the Domestic Market by Non-Resident Households and plus (4) Social Benefits in Kind.

We constructed two data series for C: one data series constructed from the data of the 1968 JSNA for the years 1955-1998 and the other data series for C constructed from the data of the 1993 JSNA for the years 1991-2006. We linked these two data series at 1991, the time when the data series for social benefits in kind became available in the 1993 JSNA.

There is a final adjustment to our concept for the Domestic Final Consumption Expenditures of Households, C. Goods and services purchased for the maintenance of owner-occupied houses (in other words, maintenance expenses) are part of the imputed rent of owner-occupied houses. Since they are produced by the market sector, we must

[143] The fiscal year in Japan is from April for the current year to May for the next year. Suppose that data series of fiscal year t are v_t. Then, data series of calendar year t, V_t is calculated as follows: $V_t = (1/4) \times v_{t-1} + (3/4) \times v_t$.

add these maintenance expenses to our aggregate C.[144] We aggregated these two variables by applying chained Fisher indexes. The operating surplus from the owner-occupied houses is the imputed rent minus the maintenance expenses for owner-occupied houses. Thus, once we obtain the imputed rent and the operating surplus from the owner-occupied houses, we can implicitly construct maintenance expenses.[145] For the years 1980-2003, current yen series for Operating surplus (imputed services of owner-occupied dwellings) can be found in the *Annual Report on National Accounts of 2005* (1993 JSNA) and this data is updated for the years 1996-2006 using the *Annual Report on National Accounts of 2008* (1993 JSNA); see Part 1 FLOWS, 2. Income and Outlay Accounts classified by Institutional Sectors, (5) Households (Including Private Unincorporated Enterprises). For the years 1955-1979, we assume the ratio of the imputed rent to maintenance expenses is constant and is the same as the average ratio over the five years 1980-1984. By utilizing this ratio and the imputed rent of owner-occupied houses, we can extrapolate the maintenance expenses backwards.

1.4 Consumption Expenditures of Non-Profit Institutes (N)

For the years 1955-1998, current and constant yen series for Final Consumption Expenditure of Private Non-Profit Institutions Serving Households (N) is found in the *Report on National Accounts from 1955 to 1998* (1968 JSNA); see Part 1 Flows, [1] Figures of Calendar Year, I Time Series Tables, 1 Arranged for Main Figures; Calendar Year, in billions of yen. The constant yen series is at the market prices of 1990.

For the years 1980-2003, current and constant yen series for Final Consumption Expenditure of Private Non-Profit Institutions Serving Households N is found in the *Annual Report on National Accounts of 2004* (1993 JSNA), and these data series are further updated for the years 1980-2003 (constant yen series only for the years 1994-2003) in the *Annual Report on National Accounts of 2005* (1993 JSNA), the years 1994-2006 on the Cabinet Office website, *Annual Report on National Accounts for 2008* (1993 JSNA); see Part 1 Flow Accounts; 1.Integrated Accounts; (1) Gross Domestic

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[144] It is possible to make a separate data series of the goods and services for the maintenance. However, since its value is relatively small (about 5 % of the imputed rent), we treat it as a part of household consumption.

[145] We assume that the deflators for the maintenance expenses and the operating surplus of the owner-occupied houses are the same as that of the imputed rent of the owner-occupied houses.

Product Account (Production and Expenditure Approach); Calendar Year, in billions of yen.

Thus we have two data series for N. We link these two data series at 1980, the time when data series for the 1993 JSNA starts.

1.5 Net Purchases of Goods and Services by the Government (G)

We define net purchases of goods and services by the general government (G) as the purchases of intermediate inputs by the general government (G1) minus the sales of goods and services of the general government to the market sector (G2):

- G (= G1 – G2): Net purchases of goods and services by the general government;
- G1: Purchases of intermediate inputs by the general government;
- G2: Sales of goods and services of the general government.

By definition, sales of goods and services of the general government (G1) is equal to government final consumption expenditures (G3) minus the gross output of general government (G4).

- G2: (= G3 − G4) Sales of goods and services by the general government;
- G3: Gross output of the general government;
- G4: Government final consumption expenditures.

For the years 1955-1998, current and constant yen series for (1) Purchases of intermediate inputs by the general government (G1), (2) Gross output of the general government (G3), and (3) Government final consumption expenditures (G4) are found in the *Report on National Accounts from 1955 to 1998* (1968 JSNA); see Part 1 Flows, [2] Figures of Calendar Year, I Time Series Tables 1, Arranged for Main Figures. The constant yen series is at the market prices of 1990. From these data series, we constructed price and quantity series for G1, G3, and G4. Thus, we can implicitly construct price and quantity series for G2 by using the data series of G3, and G4. In the end, we constructed data series of G for the years 1955-1998 by applying chained Fisher indexes to the price and quantity series of G1 and G2.[146]

[146] When we aggregated these two data series, we put a negative sign in front of the quantity series G2.

In the 1993 JSNA, current and constant yen series for G1 and G2 are available for the years 1990-2006. For the years 1990-2003, current and constant yen series for Commodity and Non-Commodity Sales of the General Government (G2) are found in the *Annual Report on National Accounts of 2005* (1968 JSNA); see Part 1 Flows, 5. Supporting Tables, Table 8, Final Consumption Expenditure of General Government classified by Purpose, Fiscal Year, in billions of yen. These data series are further updated for the years 1996-2006 in the *Annual Report of National Accounts of 2008* (1993 JSNA). The constant yen series in the *Annual Report on National Accounts of 2005* is at the market prices of 1995. The constant yen series of the updated data series in the *Annual Report on National Accounts of 2008* is at the market prices of 2000 (chained). We transform the data series for G2 based on fiscal years into those based on calendar years by linear interpolation.[147] We construct price and quantity series for G1 from the data of the 1993 JSNA for the years 1990-2006. For the years 1990-2003, current and constant yen series for Purchases of intermediate inputs by the general government (G1) are found in the *Annual Report on National Accounts of 2005* (1993 JSNA), and these data are updated for the years 1996-2006 in the *Annual Report on National Accounts of 2008* (1993 JSNA); see Part 1 Flow, 5. Supporting Tables, (2) Gross Domestic Product and Factor Income classified by Economic Activities; Calendar year, in billions of yen. The constant yen series in *Annual Report on National Accounts of 2005* is at the market prices of 1995. The constant yen series of the updated data series in *Annual Report on National Accounts of 2008* is at the market prices of 2000 (chained). From these data series, we construct price and quantity series for G1 for the years 1990-2006.

Thus we have two data series for G: one data series constructed using the data of the 1968 JSNA for the years 1955-1998 and the other data series for G constructed from the data of the 1993 JSNA for the years 1991-2006. We linked these two data series at 1991.

[147] G2 based on fiscal years is available for the years 1990-2006. However, since we transform data based on fiscal yeas to data based on calendar years by using linear interpolation as we explained before, G2 based on calendar years is available only for the years 1991-2006.

1.6 Exports and Imports of Goods and Services (X and M)

For the years 1955-1998, current and constant yen series on the 1968 JSNA basis are available for the following four variables; (1) Exports of goods and services (S1), (2) Imports of goods and services (S2), (3) Direct purchases abroad by resident households (S3), and (4) Direct purchases in the domestic market by non-resident households, (S4). These series are found in the *Report on National Accounts from 1955 to 1998* (1968 JSNA); see Part 1 Flows, [1] Figures of Calendar Year, I Time Series Tables, 1 Arranged for Main Figures; Calendar Year, in billions of yen. The constant yen series are at the market prices of 1990. For the years 1980-2003, current and constant yen series for the above variables such as S1, S2, S5, and S6 can be found in the *Annual Report on National Accounts of 2005* (1993 JSNA), and these data are further updated for the years 1994-2006 in the *Annual Report on National Accounts of 2008*; see Part 1 Flow Accounts; 1.Integrated Accounts, (1) Gross Domestic Product Account (Production and Expenditure Approach), Calendar Year, in billions of yen. The constant yen series in the *Annual Report on National Accounts of 2005* is at the market prices of 1995. The constant yen series of the updated data series in the *Annual Report on National Accounts of 2008* is at the market prices of 2000 (chained).

We have two data series for S1, S2, S3, and S4 individually. As usual, we linked these two data series at 1980, the time when the data of the 1993 JSNA start.

Finally, we constructed data series of X by applying chained Fisher indexes to the price and quantity series of S1 and S4 and constructed data series for M by applying chained Fisher indexes to the price and quantity series for S2 and S3.[148]

2. Capital Services and Investments

There are price and quantity series for investments and capital stocks for 95 asset classes; 90 tangible assets and 5 intangible assets. These data are taken from the KEO data base.[149]

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[148] When we constructed these aggregates, we put negative sign in front of the quantity series for S3 and S4.

[149] Nomura (2004) gives the detailed explanation of the construction of capital data of KEO data base.

- I1 to I95: Investments for asset classes;
- K1 to K95: Capital services for the asset classes.

We have already listed the 95 asset classes in Table 1. The numbering of investments corresponds to that of capital services. Quantities of capital services are proportional to capital stocks at the beginning of the year. Thus, we first describe how we constructed the capital stocks.

2.1 Capital Stocks

For constructing capital services, we make use of the following data from the KEO database:

- Gross fixed capital formation by asset class for the whole country for the years 1955-2006;
- Gross fixed capital formation by asset class for the market sector for the years 1955-2006;[150]
- Asset price indexes for the years 1955-2007;
- Capital stocks by asset class at the end of 1955;
- Depreciation rates.

Time series for Gross Fixed Capital Formation (GFCF) and for asset price indexes for each asset class are available in the latest version of the KEO database. By subtracting the investment in 1955 from the corresponding capital stock in 1955, we can construct capital stocks at the beginning of 1955. If the initial constructed capital stock of a good becomes negative, we set the resulting capital stock to be zero. These constructed capital stocks at the beginning of 1955 are our initial benchmark estimate of the capital stocks. Now, all the requirements to apply the Perpetual Inventory Method (PIM) are ready and we can apply the method to estimate the capital stocks. Since asset price indexes are available for the years 1955-2006, we obtain nominal and real capital stocks for 95 asset classes for the years 1955-2006. By applying the user cost formula (53) in the main text, we can construct prices and quantities for the capital services of these 95 asset classes.

[150] Nominal and real GFCF for the market sector are available at KEO database.

2.2 Investments

All investment goods are produced by the market sector. However, the bit of GFCF that is used by the government sector must be subtracted from the total economy estimates for GFCF by asset. Fortunately, the required information is available for constructing market sector investment by asset class and is available on the KEO data base. Thus we could obtain price and quantity data on investments for 95 asset classes for the market sector over the years 1955-2006 based on the KEO database.[151] However, the stock of residential structures includes the stocks held by owner occupiers of houses. We need to extract the residential capital held by owner occupiers from the total stock of residential structures in order to obtain the market sector's residential capital that is used to produce market rental housing. We now turn to the problem of estimating the owner's portion of the residential housing capital stock.

2.3 The Decomposition of Residential Structures Capital into Two Components

Overall residential structures can be classified into owner-occupied houses and rental houses. The owner-occupied houses are attributed to household sector. The household sector earns imputed rent by providing the services of its owned houses to itself. Therefore, the residential structures held by owner occupiers should be subtracted from the total stock of residential housing. In order to accomplish this task, we first calculate the total floor space of owner-occupied houses and rental houses. Second, we divide the total stock of residential structures into owner-occupied houses and rental houses according to their relative total floor spaces.

2.3.1 Data sources

- *Housing Survey of Japan Volume 1,* R*esults for the Whole of Japan* (1968) (1973) (1978) (1983) (1988) (1993), Statistics Bureau;
- *Revised Report on National Income Statistics* (1951-1973), Economic Planning Agency;
- *Housing Survey of Japan, volume 1, results for whole Japan* (1968)(1973)(1978)(1983)(1988), Statistics Bureau;

[151] These data are used for constructing capital services for the market sector.

- *Housing and Land Survey of Japan volume 1, results for whole Japan* (1993)(1998)(2003), Statistics Bureau;
- *Monthly of Construction Statistics* (1955-2006), Ministry of Land, Infrastructure and Transport

2.3.2 Total Floor Space Estimates for Owner-Occupied Houses and Rental Houses

Point estimates for total floor space for owned and rented houses are available every five years from 1968 to 2003 in the *Housing survey of Japan* (1968) (1973) (1978) (1983) (1988) (1993) and the *Housing and land survey of Japan* (1998) (2003). We use these data as benchmarks. We extend these benchmarks by utilizing the information on the net investment of the housing stock every year. Annual data for residential investment and loss of residential buildings are available from *Monthly of Construction Statistics* (for the years 1960-2006). Annual investment (loss) is the floor space which has been newly added to (subtracted from) the existing stock of housing.

The following two data series from *Monthly of Construction Statistics* are used:

- New dwellings started: new construction starts of dwellings by owner occupant relation (floor area) for the years 1955-2006.
- Loss of residential buildings: changes in total area and in dwelling units, by cause of loss for the years 1955-2006.

New dwellings started provide estimates for the area of new construction of the following four different kinds of houses:

- (1) Owned houses; Dwellings which are owned by the households occupying them.
- (2) Rented houses; Rented houses which are owned and administrated by the local government or public corporations.
- (3) Issued houses; Dwellings which are owned or administered by private companies, public bodies, etc and rented to their employees or officials in order to meet the needs of their work or issued as a part of salaries and wages regardless of rent being paid.
- (4) Houses built for sales; Dwellings constructed by the public or private sectors in order to be sold with the site under the house included in the sale.

We classify the types of overall houses as owned houses or rental houses. Since the data in the *Monthly of Construction Statistics* have a more detailed classification than we are using, we regroup the four different kinds of overall houses listed there into two categories; owned houses and rental houses. We regard the floor area of new construction of owned houses (1) and houses built for sales (4) as additions (new investment) to the floor stock of the owned houses. We regard the floor area of new construction of rental houses (2) and issued houses (3) as additions (new investment) to the floor stock of the rented houses.[152]

Statistics on the loss of residential buildings show the destroyed floor space of total residential building by aging, natural disaster, and fires. These statistics include the loss of the owner-occupied houses and the loss of rental houses. We distribute the area of the floor space losses between the owner-occupied houses and the rental houses in proportion to the areas of their stocks and we obtain net investment series (in terms of floor space) for the two types of house. We adjust these net investments every 5 years between benchmarks by a linear interpolation method so that the accumulated floor spaces coincide with the benchmark floor spaces. The earliest benchmark floor space is the one in 1968. For the years 1955-1967, we use the same adjustment coefficient as the one used between 1968 and 1973. We apply the Perpetual Inventory Method to estimate floor space of housing for the years 1955-1967.

Now, we calculate floor spaces of overall residential structures, owner-occupied houses, and rental houses. The last step is to make this database of floor space more consistent with the 1993 JSNA data. Until 2003, housing expenses have been divided between owner-occupied houses and rental houses according to their floor space in JSNA. The 1993 JSNA based data for imputed rents for owner-occupied houses and for market rents gives us another estimate for the ratio of the floor space of owner-occupied houses and the floor space of rental houses for the years 1981-2006. These 1993 JSNA based estimates are quite similar to our estimates which have just been described. However, for the years 1981-2006, we use the 1993 JSNA based estimates as the primary estimate

[152] This assumption will only be approximately correct since some of the houses built for sale can be sold to home owners who lend their houses for tenants. As will be seen shortly, our assumptions here do not have to be precisely correct. We just need them to be approximately correct so that we can construct reasonable estimates of floor space by type of house between censuses.

and link our earlier series to the 1993 JSNA data. This enables us to provide a breakdown of the total stock of residential structures into owner-occupied stocks and market sector rental stocks.

3. Inventory Services and Changes in Inventories

We regard the change in inventories as outputs and the stock of inventories as inputs to production of the market sector.[153] There are price and quantity series for 4 types of changes in inventory and 4 types of inventory services in our database.

- IV1: Changes in finished-goods inventories;
- IV2: Changes in work-in-process inventories;
- IV3: Changes in work-in-process inventories for cultivated assets;
- IV4: Changes in materials inventories.

- KIV1: Finished-goods inventory services;
- KIV2: Work-in-process inventory services;
- KIV3: Work-in-process inventory services for cultivated assets;
- KIV4: Material inventory services.

The quantities of inventory services are proportional to inventory stocks held at the beginning of the year. Thus, first of all, it is necessary to construct prices and quantities for the inventory stocks. For constructing inventory stocks, we make use of the following data from the KEO database. The following nominal and real inventory stocks are available at the KEO database:

- (1) Finished-goods inventories at the end of the years 1955-2004;
- (2) Work-in-process inventories at the end of the years 1955-2004;
- (3) Work-in-process inventories for cultivated assets at the end of the years 1955-2004;
- (4) Material inventories at the end of the years 1955-2004.

Since they are stock at the end of the years 1955-2004, we use them as stocks at the beginning of the years 1956-2005. We extend these data series backward to 1955 and

[153] This follows the treatment advocated by Diewert (2005a).

forward to 2007. For the years 1997-2007, current yen series for inventory stocks (1), (2), and (4) are found in the *Annual Report on National Accounts of 2008* (1993 JSNA); Part 2 STOCKS, 5. Supporting Tables, (1) Closing Stocks of Assets/Liabilities for the Nation. For the years 2006-2007, we extrapolate the data series in the KEO data base by using the growth rate in the corresponding data series of the JSNA. Since there is no information on the inventory stocks (3), we assumed that the growth rate of the nominal inventory stocks (3) is the same as that of the nominal inventory stocks (4). For the same periods 1955 and 2005-2006, we extend prices of these four types of inventory stocks using the average growth rate over the recent 5 years.

The change in inventories over a year for each of our four types of inventory is regarded as an (investment) output of the market sector and it is equal to the difference between the stock of the inventory class under consideration at the beginning of current year and the stock of the same inventory class at the beginning of the next year. Since we have already prepared estimates for our 4 types of inventory stock, it is straightforward estimates of inventory change for the years 1956-2006. For 1955, we extrapolate the change in inventory backward by using the average growth rate over the following 5 years. Adding change in inventory in 1955 to the inventory stock at the beginning of 1956, we obtain inventory stock at the beginning of 1955.

In the end, we obtain estimates for the 4 types of changes in inventories IV1-IV4 for the years 1955-2006. We also obtain estimates for the four types of nominal and real inventory stocks (1)-(4). By applying the user cost formula (53), we can construct prices and quantities of four types of inventory services KIV1-KIV4 for the years 1955-2006.

4. Land Services

There are price and quantity series for four types of land in our database:

- LD1: Agricultural land services;[154]
- LD2: Industrial land services;
- LD3: Commercial land services;

[154] Our estimates for agricultural land only include the land used for agricultural production. Other types of land such as waste and fields are excluded from land for agricultural use.

- LD4: Residential land services.

Quantities of land services are proportional to land stocks at the beginning of the year. Thus, first of all, we have to construct land stocks.

4.1 Four Types of Land

For measuring land stocks, we make use of the following data from the KEO database, where estimates for nominal and real land stocks are available for the following types of land:

- (1): Land for agricultural use;
- (2): Land for industrial use;
- (3): Land for commercial use;
- (4): Land for residential use;
- (5): Land for general government.

KEO estimates are supplemented by data taken from the following websites.

- Website of *Land Use Survey*, http://www.mlit.go.jp/hakusyo/tochi/tochi_.html, Ministry of Land, Infrastructure, Transport and Tourism;
- Website of *Prefectural Land Price Survey*, http://tochi.mlit.go.jp/english/index.html, Ministry of Land, Infrastructure, Transport and Tourism

Since the KEO estimates are for stocks at the end of the years 1955-2004, we use them as estimates for the land stocks at the beginning of the years 1956-2005. We extend these data series backward to 1955 and forward to 2006. For the stocks of 1955, we assume that they are equal to the corresponding stocks of 1956. For stocks of 2006, we estimate the real land stocks (1)-(4) by using the growth rates for the area space information found in the website of the *Land Use Survey* and we also extrapolate nominal land stock (1)-(4) by using the growth rate of nominal land stocks calculated from the real land stocks and land prices that are found in the website of the *Prefectural Land Price Survey*. For information on land for the general government sector, current yen estimates for land for general government for the years 1996-2006 can be found in the *Annual Report on National Accounts of 2008* (1993 JSNA); see Part 2 Stocks, 2 Accounts classified by

Institutional Sectors, (3) General Government; Calendar Year in billion yen. We obtain an estimate for the nominal land stock (5) in 2006 by using the growth rate of nominal land for general government using the JSNA just described. The price of land for residential use (4) in 2006 is obtained by using the rate of growth in the land price for general government (5) over 2005-2006.

We attribute land for general government as land that is being used for commercial uses. It is not quite true but it is a reasonable approximation. Thus, by subtracting land for general government from land for commercial use, we obtain final estimates for commercial land. Since the stocks of land in the KEO database are stocks at the end of the period, we convert these stocks to beginning of period stocks. We assume that between 1955 and 1956, the area of each type of land is constant and the price of each land changes at the average growth rate over the years 1956-1960. There remains one problem that land for residential use also includes land being used for owner-occupied houses. Since the imputed rent of owner-occupied houses is an output by household sector, it is excluded from the output produced by the market sector. Therefore, the services of the land and residential building stocks for owner-occupied houses should also be excluded from the input of market sector. We will explain how to subtract land for owner-occupied houses from land for residential use in the following section.

4.2 Residential Land

Residential land is the land that lies under residential structures. We divide the residential land into the owned residential land and the rental residential land in proportion to the land areas utilized by the two types of residential structures. We calculate the area of the site under owner occupied and rental houses.[155] The nominal and real residential land is divided into owned residential land and rental residential land according to their areas of use. The owned residential land will be excluded from land for residential use of the market sector.

4.2.1 Data Sources

- *Housing Survey of Japan, Volume 1, Results for the Whole of Japan* (1968) (1973) (1978) (1983) (1988) (1993), Statistics Bureau;

[155] Unfortunately, the land under owner-occupied houses is not always owned land.

- *Housing and Land Survey of Japan, Volume 1, Results for the Whole of Japan* (1998) (2003), Statistics Bureau;
- *Monthly of Construction Statistics* (1955 to 2004), Ministry of Land, Infrastructure and Transport;

4.2.2 The Land under the Owned Houses and the Site under the Rental Houses

For the years 1968-2006, we use benchmark land areas under owned houses and rental houses, which are available every five years; see *Housing and Land Survey of Japan* and *Housing Survey of Japan*. We interpolate these areas between benchmarks using annual "Areas of finished housing sites" reported annually by the *Monthly Statistics of Construction*. The area of a finished housing site is the area of the site which is currently transformed into residential land. There are four types of site:

- (1) Housing sites for a public housing complex;
- (2) Housing sites for individual houses;
- (3) Housing sites for re-development; and
- (4) Sites for villas.

We regard (2) plus (3) as the newly added area under owned houses. We regard (1) as the newly added area of under rental houses.

Initially, we assume the change in land areas under owned houses is proportional to the newly added areas for owned houses and the change in the areas under rental houses is proportional to the newly added areas for rental houses. We then proportionally adjust these initial estimates for land areas by type of housing so that the accumulated area of residential land coincides with their benchmark values in the 5 year surveys.

The earliest benchmark survey for land areas is the one in 1968. However, there is no benchmark and annual data to impute the areas under owned houses and rental houses for prior years. We extend backward the area of land for owner-occupied houses and rental houses by using their average growth rates over the 10 years 1968-1978.

In the end, we obtain estimates for four types of nominal and real land stocks (1)-(4). By applying the user cost formula in (53), we can construct prices and quantities of four types of land services LB1-LB4 for the years 1955-2006.

5. Labour Inputs

There are price and quantity series for 3 types of labour input in our database;

- LB1: Labour input of employees in the market sector[156];
- LB2: Labour input of the self-employed;
- LB3: Labour input of family workers;

First, we calculate the number of workers who are actually working and the average yearly hours of works. Second, we multiply the number of workers by the average yearly hours worked. Thus we can get the yearly hours of work for the three types of workers.

5.1 Data Sources

The Japanese *Labour Force Survey* plays the key role in the construction of our labour input database. There are six types of workers in *Labour Force Survey*. We construct the number of workers and the average weekly hours of works for these six types of workers using the data in the *Labour Force Survey*; see the following publications:

- *Annual Report on the Labour Force Survey* (1970, 1972-2006), Statistics Bureau;
- Website for the *Labour Force Survey* http://www.stat.go.jp/data/roudou/index.htm, Statistics Bureau

There were two major revisions of the *Labour Force Survey* in 1961 and 1967:

- Revision of 1961; population coverage is changed from 14 years old to 15 years old and
- Revision of 1967; the definition of "employed persons not at work" is changed. The coverage of population is smaller after the revision than before the revision.

We ignore the revision of 1961. Education is compulsory for children between the ages of 6 and 15 in Japan. Thus, we expect that the revision of 1961 did not affect the result

[156] Employees include paid family workers.

of the survey. However, there is clear evidence that the revision of 1967 has had a large impact on the survey results.[157] The *Annual Report on the Labour Force Survey* issued in 1970 contains old data series for the years 1955-1967 and new data series for the years 1967-1970. Since the old data series follows the definition before the revision of 1967, we need to adjust the old data series so that it is consistent with the new data series. We extend the new data series by using the growth rate of old data series for the years 1955-1966.

5.2 Three Types of Workers in Labour Force Survey

In the *Labour Force Survey*, all the workers are classified into three types of worker according to their "characteristics of employment" and then they are further divided into two types of "labour force status". Therefore, there are data on six types of worker and the total number of workers by type and their average hours of work can be found in the *Labour Force Survey*.

Labour Force Status

According to the definitions in the *Labour Force Survey*, employed persons are the workers in the labour force who are not unemployed. Employed persons are classified into the following two types according to their labour force status:

- Employed persons at work: all persons who worked for pay or profit or worked as unpaid family workers for at least one hour during the survey week
- Employed persons not at work: employees who were absent from work but received or expected to receive wages or salaries or self-employed workers whose absence from work has not exceeded 30 days.

Characteristics of Employment

All the employed persons are classified into following three types of worker according to

[157] According to *Annual Report on the Labour Force of Survey* issued at 1970, the number of total employed persons in 1967 is 4940 (before the revision) and 4920 (after the revision) in ten thousands of persons. The average weekly hours worked of total employed persons is 46.4 hours (before the revision) and 48.8 hours after the revision.

the following characteristics of employment:

- Self-employed workers: persons who own and operate unincorporated enterprises;
- Family workers: persons who work in unincorporated enterprises operated by family members;
- Employees: persons who work for wages or salaries as employees of unincorporated enterprises, companies, corporations and associations or levels of government.

Six Types of Workers in the Labour Force Survey

There are data on the following six types of worker in the *Labour Force Survey*:

- L1: Employees at work;
- L2: Employees not at work;
- L3: Self-employed workers at work;
- L4: Self-employed workers not at work;
- L5: Family workers at work;
- L6: Family workers not at work.

Average Yearly Hours Worked

Average weekly hours worked for three types of workers (self-employed workers, family workers and employees) are available in the *Labour Force Survey*. Multiplying the weekly hours worked by fifty two, we can calculate the yearly hours worked. They are the average yearly hours of works for employed persons at work; those who are actually working during the week of survey investigation.

- AH1: Average yearly hours worked for employees at work;
- AH2: Average yearly hours worked for self-employed workers at work;
- AH3: Average yearly hours worked for family workers at work.

5.3 Three Types of Worker in the National Accounts

The numbers of the three types of worker are available in the National Accounts. There are labour data in the 1968 JSNA for the years 1955-1998. There are labour data in the

1993 JSNA for the years 1980-2006. We extend the data of the 1993 JSNA backwards by using the growth rates of the 1968 JSNA data. The three types of worker in the JSNA are:

- L7: Employees;
- L8: Family workers and the self-employed workers;
- L9: Government workers.

In comparison with the data series of the *Labour Force Survey*, the number of workers in JSNA includes the number of employed persons not at work such as employees who were absent from work but received or expected to receive wages or salaries or self-employed workers whose absence from work has not exceeded thirty days. By using the ratio of the number of the employed persons at work and the number of the employed persons not at work in *Labour Force Survey*, we constructed the number of three types of the employed persons at work as follows:

- L10 = L1−L9: Employees in market sector;
- L11 = L3: The self-employed;
- L12 = L5: Family workers.

Multiplying the number of employed persons by category by the corresponding average hours worked, we can calculate total hours worked by category of worker:

- TH1 = L10×AH1: Total hours worked by employees in the market sector;
- TH2 = L11×AH2: Total hours worked by the self-employed;
- TH3 = L12×AH3: Total hours worked by family workers.

5.4 Compensation of Employees

For the years 1955-1998, current yen series for the compensation of employees is found in the *Report on National Accounts from 1955 to 1998* (1968 JSNA); Part 1 Flows, [1] Figures of Calendar Year, II Time Series Tables 2 Arranged for Main Figures, Distribution of National Income and National Disposable Income, in Calendar Year (Billion Yen) at current prices. For the years 1980-2003, current yen series for compensation of employees can be found in the *Annual Report on National Accounts of 2005* (1993 JSNA), and these data are further updated for the years 1996-2006 in the

Annual Report on National Accounts of 2008 (1993 JSNA); see Part 1 Flows, 1. Integrated Accounts, (2) Distribution of National Income and National Disposable Income, in Calendar Year (billion yen) at current prices. We link the data of the 1993 JSNA to the data of the 1968 JSNA at 1980, the time when data based on the 1993 JSNA start. Thus we have constructed a data series for the compensation of employees for the whole economy for the years 1955-2006.

For the years 1955-1998, compensation of employees for the general government is found in the *Report on National Accounts from 1955 to 1998* (1968 JSNA); see Part 3 Long Time Series of Production, II Gross Domestic Product and Factor Income by Kind of Economic Activity (1) at current prices. For the years 1990-2003, current yen series for compensation of employees for the government sector is found in the *Annual Report on National Accounts of 2005* (1993 JSNA) and these data are further updated for the years 1996-2006 in the *Annual Report on National Accounts of 2008* (1993 JSNA); see Part 1 Flows, 1. Supporting tables, (2) Gross Domestic Product and Factor Income classified by Economic Activities, in Calendar Year (billion yen) at current prices. We link the data of 1993 JSNA to the earlier data of the 1968 JSNA at 1990, the year when data of the JSNA 1993 start. Therefore, we have constructed the data series of compensation of employees for the government sector for the years 1955-2006.

The difference between the total compensation of employees and compensation of employees for the government sector is considered to be the compensation of employees of the market sector and period t, we denote this market sector labour compensation by $V_{LB1}{}^t$. The period t hourly wage for employees in the market sector $W_{LB1}{}^t$ is obtained by dividing $V_{LB1}{}^t$ by the period t yearly hours worked for employees in the market sector $TH1^t$ as follows:

(A4) $W_{LB1}{}^t = V_{LB1}{}^t/TH1^t$

There remains the problem of determining appropriate wage for the self-employed and family workers. We certainly expect the wage for a family worker to be very much less than the wage for an employee and we would expect the wage for a self-employed person to be somewhat less than the wage for an employee. However, it is difficult to know what exactly is the appropriate wage for self-employed and family workers.

We hypothesize that the wage for the self-employed $W_{LB2}{}^t$ is a fraction f_2 of the wage for

employees in market sector W_{LB1}^t and the wage for family workers W_{LB3}^t is a fraction f_3 of the wage for employees in market sector W_{LB1}^t.

(A5) $W_{LB2}^t = f_2 W_{LB1}^t$; $W_{LB3}^t = f_3 W_{LB1}^t$ for t = 1955,⋯,2006

For the ratio between the wage per employee and the average wage per the self-employed, we use the estimate in the KEO database of 32.8%. For the average ratio between the wage per employee and the wage per family worker, we use the estimate in the KEO database of 11.3%. We assume that the average wage per worker coincides with the mean of "the average yearly hours worked times hourly wages" for the years 1955-2006:

(A6) $0.328 = (1/52)\sum_{t=1955}^{2006}[(AH_2^t \times f_2 W_{LB1}^t)/(AH_1^t \times W_{LB1}^t)]$
(A7) $0.113 = (1/52)\sum_{t=1955}^{2006}[(AH_3^t \times f_3 W_{LB1}^t)/(AH_1^t \times W_{LB1}^t)]$

Thus, we can derive estimates for the two fractions f_2 and f_3 from equations (A6) and (A7). Solving these equations, we obtain f_2= 0.3252 and f_3= 0.1014.

6. Taxes and Subsidies

There are 8 types of taxes in our database. We allocated all Japanese direct and indirect taxes and subsidies into the following categories:[158]

- TA1 Consumption taxes: Taxes that fall on final demands such as H, I1-I95 (these taxes must be deducted from the final demand consumption prices, since producers do not get the revenue from these taxes);
- TA2 Labour taxes: Taxes that fall on labour input plus contributions to social securities (we do not deduct these taxes from the SNA estimates of labour income since firms must pay these taxes but we require estimates of labour taxes as an aid in determining capital taxes);
- TA3 Capital taxes[159]: Taxes that fall on capital stock components such as the corporation tax (these taxes are added to the user cost of capital);

[158] Some categories have subcategories. The subcategories are explained in footnotes.

[159] Capital taxes are decomposed into the following categories: (1) General capital taxes; (2) Capital taxes on tangible capital; (3) Capital taxes on tangible capital excluding land; (4) Capital taxes on tangible capital

- TA4 Property taxes[160]: Taxes that fall on specific capital stock components (these taxes are added to the user cost of capital for the relevant specific capital stock);
- TA5 Household capital taxes: Taxes on household non productive physical assets like durable goods;
- TA6 Poll taxes: Taxes that households pay, regardless of their incomes;
- TA7 Tariffs: Taxes imposed on imported goods[161];
- TA8 Subsidies on products: (these subsidies act as an offset to the indirect taxes that fall on final consumption).

Household capital taxes and poll taxes do not change any prices which the production sector faces. Therefore, we ignore them in our model of production. Other taxes such as TA1-TA4, TA7, and TA8 have been reflected in the prices of net outputs and primary inputs in our database.

There are several taxes which can be attributed more than one category. We will explain how to allocate these taxes into different categories later in this section.

6.1 Data Sources for Taxes

We use tax revenue data of central and local government.

- *National Tax Agency Annual Statistics Report* (1955 to 2006), National Tax Agency;
- *Annual Statistical Report on Local Government Finance* (1955 to 2006), Ministry of Internal Affairs and Communications;

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excluding land, structures, and inventories; (5) Capital taxes on tangible capital including land and excluding inventories and (6) Financial capital.

[160] Different rates of property tax are imposed on the following specific types of property capital stock: (1) Structures; (2) Structures and land in general; (3) Transportation equipment; (4) Forest wood; (5) Land in general; (6) Land for agricultural use; (7) Non agricultural land and (8) Forest lands, waste lands and land used for purposes other than agricultural use.

[161] The total tax on an imported good can consists of consumption taxes, liquor taxes, tobacco taxes, gasoline taxes, etc. Therefore, a different level of tariffs may be imposed on each imported good. We group tariffs into the following four categories: (1) Tariffs for imported goods in general; (2) Tariffs for foodstuffs; (3) Tariffs for mineral fuels and (4) Tariffs for miscellaneous goods.

- *1970 National Wealth Survey of Japan, Volume 1, Summary Report* (1973), Economic Planning Agency.

6.2 Adjustments of Tax Data from Fiscal Years to Calendar Years

Tax revenues in the various data sources are the amounts of tax revenue settled during a fiscal year. However, many taxes are imposed on economic activity using *different* fiscal years. Therefore, tax revenues in a fiscal year are not necessarily revenues from economic activity during the "regular" fiscal year, which is April 1 to March 30. By linear interpolation, we adjust the tax revenues of each tax so that tax revenues are imposed on economic activities during calendar years. For example, the tax revenue from the tobacco tax in a "tobacco fiscal year" is imposed on tobacco sales from March 1 in the current year to February 28 in the next year. Thus, using the following linear interpolation, we can adjust tax revenue from tobacco tax so that the adjusted tax revenue is from tobacco sales during a calendar year (from January to December in the current year).

(A8) $T_{2006} = (1/6)\times t_{2005}+(5/6)\times t_{2006}$

where t is tax revenue from March in the current year to February in the next year and T is adjusted tax revenue of the current calendar year.

Prefectural and municipal inhabitants' taxes also require special attention. As we explain later, these inhabitants' taxes consist of two parts: one is a poll tax and the other one is proportional to income in the previous year. The poll tax is attributed to the current year. However, the income part of the inhabitants' tax is attributed to the previous year.

6.3.1 Income Tax

A careful treatment is needed for modelling the income tax. Since households' incomes can be considered as returns to labour or capital or more specific property, we divide total income tax revenue between labour taxes, capital taxes, and property taxes.

For the national income tax, the tax collection agency adopts two tax collection methods: (a) withholding at source, and (b) self-assessment on a tax return basis. Income tax is

paid by self-assessment unless it is withheld at the income source. Income taxes on wages, salaries, interest, dividends, etc. are withheld by the payers of such income who remit the withheld tax to the government on the behalf of taxpayers. Under the self-assessment tax system, each taxpayer is required to file a return and pay the appropriate amount of tax at the same time. The withholding tax system is firmly built into the basic structure of the individual income tax. The amount withheld is treated as an advance payment of the income tax payable by the recipient of the income.

Withholding Income Tax

The amounts of withholding taxes on the different sources of income are listed in *National Tax Agency Annual Statistics Report*. Based on the characteristics of the sources of income, all withholding income taxes are classified into the following categories of taxes:

- (1) Interest income, etc. (treated as a general capital tax);
- (2) Dividends (capital tax);
- (3) Capital gains of listed stocks, etc. (capital tax);
- (4) Wage and salaries (labour tax);
- (5) Retirement income (labour tax);
- (6) Remuneration, fees, etc. (labour tax);
- (7) Income of non residents, etc. (ignored).

Thus we attribute (1) Interest income, (2) Dividends, and (3) Capital gains of listed stocks, etc. to financial assets and (4) Wage and salaries, (5) Retirement income and (6) Remuneration, fee, etc. to labour. We ignore (7) Income of non residents, etc.

Self-Assessed Income Tax

With respect to the taxpayers of self-assessed income tax, the statistics of the amounts of incomes is listed by type of income in *National Tax Agency Annual Statistics Report*. First, self-assessed income tax is split between the different types of income on the basis of the relative sizes of the types of income. Then, we attribute taxes on different types of income into the categories of taxes based on the characteristics of the source of

income:[162]

- Operating income:
  - Business income (labour tax and capital tax)
  - Farm income (labour tax and capital tax)
- Rental income (capital tax);
- Comprehensive capital gains (capital tax);
- Sporadic income (labour tax);
- Miscellaneous income (labour tax);
- Difference by aggregation of profit and loss (capital tax);
- Sales of wood in forest (property tax);
- Long term separate capital gains (capital tax);
- Short term separate capital gains (capital tax)[163];
- Capital gains of stocks, etc. (capital tax)

Operating income refers to the incomes of farmers and the self-employed. Therefore, operating income is considered as a return to labour and capital. We attribute tax on operating income in proportion to labour input and capital service.[164] The majority of all employees pay income tax for their salaries through the withholding tax system.[165]

Prefectural and Municipal Inhabitants' Tax

Prefectures and municipals impose an income tax called the individual inhabitants' tax. Individual inhabitants' tax consists of two parts: (a) individual inhabitants' tax as a lump sum payment that households pay regardless of their incomes and (b) individual inhabitants' tax with an income rate that households pay based on their income. There

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[162] The tax category to which each income tax is attributed is in a parenthesis.

[163] Long and short term separate capital gains are imposed when capital stocks are transferred from one agent to the other.

[164] Output and labour input at current prices can be calculated independent from tax on operating income. The differences between output at current prices and labour input at current prices can be considered as capital service at current prices.

[165] The employees who earn more than certain amount (for example, more than 20 million yen in 2003) have to pay more income tax in addition to income tax paid under the withholding tax system. However, the number of those earners is very small.

is no doubt that we can attribute tax (a) as a poll tax. However, since we do not know the source of income for tax (b), we encounter the same problem as we do in allocating self-assessed income tax into different categories. We split tax (b) on the basis of the relative sizes of the types of income. Then, we attribute taxes on different types of income into the categories of taxes based on the characteristics of the source of income. Since we know the types of incomes of those who pay withholding income tax and those who pay self-assessed income tax, we can calculate the type of income for total households. Corporate inhabitants' tax is imposed on corporations. Since it is imposed on the financial capital and the profits of corporations, we regard these taxes as capital taxes.

6.3.2 Inheritance Tax

An inheritance tax is tax imposed on the acquisition of the types of property listed below. We split the inheritance tax between these properties on the basis of the relative sizes of the values of properties. Then, we attribute the inheritance taxes to the categories of taxes based on the characteristics of properties. The types of property are listed below and our attribution of the associated tax is noted in parentheses:

- Housing land including leasehold (property tax);
- Rice fields, farm fields, forest lands and other lands (property tax);
- Total land (property tax);
- Houses, structures (property tax);
- Business (agriculture) property (property tax);
- Securities (capital tax);
- Cash, deposit (capital tax);
- Household properties (household capital tax);
- Other properties (capital tax);
- Obligations and funeral expenses (poll tax).

6.3.3 Taxes on the Holdings of Transportation Equipment

Motor vehicle weight taxes, motor vehicle taxes and light motor vehicle taxes are imposed on the holdings of motor vehicles. Since households and the production sector own motor vehicles, these taxes are paid by both households and producers. The part of these taxes paid by households is attributed to household capital tax (and is not relevant

for the purposes of this study). The other part of these taxes is attributed to the production sector and should be regarded as a specific tax on the services of the taxed vehicles, just as property taxes on structures used by the production sector are part of the user costs of business structures. We need to construct the total stock of automobile vehicles and divide this stock between households and the production sector. Following the convention of JSNA, we divide the stock in half. We observe the automobile vehicle stock of households from the *National Wealth Survey* in 1970. The survey tells us the relative sizes of the value of motor vehicles owned by households and producers. Cutting the stock in half is consistent with this data source.[166]

6.3.4. Fuel Tax

There are taxes on the use of fuels such as electricity, gas, gasoline, and light oil.

National Fuel Taxes:

- Gasoline taxes;
- Local road taxes (fuel taxes on gasoline);
- Promotion of power resources development tax (fuel taxes on electricity).

Local Fuel Taxes:

- Light oil delivery taxes (fuel taxes on light oil);
- Electricity and gas taxes;
- Electricity taxes;
- Gas taxes.

These taxes are charged when these fuels such as electricity, gas, gasoline, and light oil are purchased. These fuels are consumed both by households and the market sector. Therefore, it is not clear about how to categorize fuel taxes. We attributed the payment of these taxes by households as a consumer tax. We attributed the payment of these taxes by the market sector as a property tax which is necessary for using a specific capital stock.

[166] The *National Wealth Survey* in 1970 is a comprehensive survey of assets for all sectors in the Japanese economy. The total transportation equipment wealth for all sectors less the household sector was 2678.4 billion yen. The household sector owned transportation equipment valued by 2471.9 billions yen.

Total values for the above energy sources purchased within the whole country and the values of these purchases by the household sector are available in the *Input Output Tables* for Japan and this enables us to apportion the above taxes into household and market sector components. Since Input Output tables are constructed only every five years, there are no data available between years. We interpolate the data between the 5 year censuses using the assumption of constant growth rates.

6.4 Subsidies

For the years 1955-1998, the amount of subsidies is found in the *Report on National Accounts from 1955 to 1998* (1968 JSNA); see Part 1 Flows, 2. Income and Outlay Accounts classified by Institutional Sectors, (1) Total Economy, 1. Generation of Income Account, Calendar Year (Billion Yen). We transformed this data series into that based on calendar years by linear interpolation. For the years 1980-2003, the amount of subsidies can be found in the *Annual Report on National Accounts of 2005* (1993 JSNA) and this data series is updated for the years 1996-2006 in the *Annual Report on National Accounts of 2008* (1993 JSNA); see Part 1 Flows, 1. Consolidated Accounts for the Nation, (1) Gross Domestic Product Account (Production and Expenditure Approach), Fiscal Year (Billion Yen). Hence, we linked the data of the 1993 JSNA to the data of the 1968 JSNA at 1980, the time when the 1993 JSNA data starts. We treat the amount of subsidies as a deduction from consumption taxes.

Appendix B. Tables of Prices, Quantities, and Value for Inputs and Outputs

Table B-1: Market Sector Output and Input Prices for Japan 1955-2006

	P_C^t	P_N^t	P_G^t	P_X^t	P_M^t	P_I^t	P_{IV}^t	W_K^t	W_{KIV}^t	W_{LD}^t	W_{LB}^t	P_{DEP}^t	W_{KW}^t
1955	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
1956	0.997030	1.046150	1.033880	1.029650	1.053760	1.077990	1.047120	1.081910	1.052520	1.162470	1.051630	1.084910	1.079060
1957	1.027380	1.104460	1.078260	1.034260	1.091090	1.141070	1.052240	1.156770	1.192040	1.361140	1.106040	1.141810	1.170910
1958	1.026250	1.106340	1.026590	0.968780	0.941460	1.106830	1.038450	1.081550	1.090750	1.409410	1.171500	1.110370	1.053760
1959	1.058940	1.146820	1.026600	0.989370	0.933430	1.104060	1.063200	1.152270	1.283080	1.840230	1.276470	1.093170	1.210480
1960	1.090840	1.203860	1.021790	1.017380	0.936580	1.128280	1.061450	1.272050	1.541940	2.538670	1.385160	1.096620	1.446280
1961	1.154690	1.285830	1.027970	1.010180	0.947610	1.210020	1.050610	1.434600	1.877510	3.192780	1.569720	1.142800	1.727580
1962	1.236120	1.376100	1.005770	0.981420	0.937460	1.238500	1.063230	1.416650	1.737270	3.301640	1.762160	1.169640	1.661880
1963	1.324490	1.467960	0.964130	0.999170	0.954060	1.244100	1.078100	1.447480	1.828620	3.733480	2.005080	1.162510	1.734530
1964	1.359500	1.567040	0.935380	1.008560	0.967250	1.263460	1.092800	1.507670	1.930700	4.281610	2.270120	1.171690	1.852010
1965	1.458860	1.664440	0.903830	0.997200	0.959750	1.287310	1.129580	1.485270	1.808260	4.274190	2.548200	1.192280	1.779180
1966	1.516950	1.760620	0.907170	0.994030	0.965610	1.332890	1.142600	1.568190	2.008820	4.874150	2.831780	1.210740	1.937200
1967	1.567860	1.864450	0.931760	0.995450	0.950970	1.388610	1.144880	1.647800	2.227060	5.716390	3.174850	1.243300	2.071540
1968	1.649960	1.977760	0.918190	0.995730	0.950360	1.433910	1.175620	1.729260	2.422450	6.820760	3.655630	1.271050	2.217000
1969	1.710270	2.109430	0.900760	1.008100	0.978950	1.493070	1.224460	1.777010	2.473840	7.761170	4.256680	1.321240	2.258340
1970	1.827940	2.400920	0.890240	1.038050	0.996770	1.561730	1.256900	1.809760	2.397440	8.440790	4.993850	1.378520	2.255240
1971	1.949920	2.700550	0.841400	1.060330	0.961500	1.597710	1.286390	1.693960	1.959910	7.871140	5.799770	1.418850	1.930930
1972	2.059210	3.056210	0.795480	1.052060	0.915090	1.666860	1.411500	1.686480	1.923710	8.744840	6.654600	1.459570	1.855120
1973	2.293510	3.598060	0.851860	1.152710	1.096240	1.933810	1.663190	1.884260	2.241680	10.207760	8.061310	1.646310	2.050080
1974	2.832640	4.587410	1.450220	1.512360	1.756610	2.355840	1.909370	2.165680	2.276260	9.169440	10.334070	2.008760	2.185050
1975	3.160420	5.108800	1.024940	1.586770	1.927220	2.451120	2.008040	2.122670	1.942790	7.836040	12.225790	2.092580	1.959380
1976	3.465260	5.709340	1.061470	1.617900	2.038490	2.556210	2.111660	2.289860	2.290330	8.911060	13.168570	2.146540	2.276390
1977	3.715340	6.174370	1.056740	1.557680	1.966810	2.667580	2.153110	2.361340	2.214140	8.857750	14.391590	2.237830	2.312000
1978	3.873910	6.403510	0.977400	1.458740	1.681340	2.751380	2.180310	2.501630	2.444300	10.079840	15.412520	2.275030	2.589080
1979	3.997830	6.807150	1.039560	1.577670	2.150860	2.941630	2.390120	2.725840	2.855770	11.720960	16.353280	2.379530	2.966870
1980	4.324130	7.337000	1.256800	1.730550	2.937710	3.175590	2.487100	2.822050	2.741050	11.660220	17.353530	2.536570	2.963810
1981	4.521580	7.520150	1.336600	1.786490	2.958080	3.233280	2.531950	2.793550	2.509730	11.548060	18.445300	2.581790	2.828850
1982	4.628340	7.690270	1.299130	1.851010	3.033520	3.247360	2.575510	2.761550	2.369060	11.493750	19.267130	2.605050	2.717670
1983	4.717860	7.791750	1.296640	1.792800	2.869880	3.239120	2.579670	2.706850	2.182940	11.147980	19.589510	2.604860	2.586560
1984	4.828580	7.971620	1.284070	1.821300	2.796780	3.246000	2.559260	2.809070	2.396290	12.596350	20.454430	2.597070	2.845270
1985	4.918420	8.146100	1.396140	1.767550	2.641360	3.225550	2.466960	2.856330	2.455580	13.682780	21.099150	2.580600	2.985830
1986	4.945450	8.225600	1.236200	1.551800	1.847600	3.166480	2.327550	2.796780	2.279880	14.049020	21.674870	2.532860	2.914060
1987	4.947370	8.264210	1.155100	1.486870	1.696490	3.147750	2.266760	2.753690	2.160910	14.759250	22.097420	2.510940	2.841630
1988	4.961360	8.335650	1.095340	1.460980	1.647590	3.174480	2.279210	2.796000	2.238680	16.225280	22.782910	2.522040	2.930240
1989	5.048840	8.529050	1.121160	1.509190	1.756420	3.269890	2.298600	2.872150	2.302370	18.225000	23.940750	2.578240	3.030920
1990	5.171530	8.962910	1.163740	1.534660	1.875460	3.360790	2.323190	2.879450	2.193120	18.891880	25.645820	2.636300	2.950530
1991	5.283030	9.239720	1.152410	1.498570	1.796120	3.408810	2.338150	2.837150	1.989660	17.662450	27.319490	2.670850	2.779400
1992	5.358280	9.343340	1.140510	1.460650	1.703170	3.419440	2.741140	2.800310	1.896910	16.157210	28.206060	2.663460	2.695070
1993	5.402760	9.334670	1.125730	1.362610	1.565730	3.419160	2.976970	2.700410	1.618010	13.767730	28.854180	2.663610	2.429470
1994	5.417350	9.396900	1.091400	1.320330	1.497590	3.390380	2.920080	2.663570	1.558300	12.943610	29.342760	2.631810	2.388270
1995	5.396620	9.405670	1.068760	1.292550	1.466560	3.366170	2.894290	2.655530	1.546450	12.567620	29.507420	2.602870	2.418170
1996	5.395910	9.434970	1.055420	1.338230	1.560540	3.320700	2.886630	2.673330	1.650710	12.773090	29.696380	2.557340	2.545400
1997	5.455230	9.577770	1.075030	1.362740	1.650650	3.318420	2.877270	2.657580	1.607780	12.041860	30.492160	2.559810	2.499340
1998	5.408500	9.553270	1.062110	1.375150	1.601710	3.241150	2.837860	2.501010	1.285240	9.664080	30.131830	2.535780	2.125720
1999	5.374790	9.443580	1.043110	1.253040	1.470210	3.168390	3.178060	2.451880	1.276670	9.168170	29.642000	2.476100	2.101710
2000	5.338730	9.378940	1.037690	1.201000	1.491250	3.155730	3.071270	2.475800	1.370380	9.257340	29.195500	2.452120	2.209440
2001	5.281180	9.339730	1.029590	1.227830	1.520400	3.065950	2.992370	2.370120	1.173960	8.139020	29.358540	2.386180	2.044290
2002	5.192820	9.070420	1.015560	1.212870	1.500880	2.992920	2.959490	2.366130	1.270450	8.230890	28.803100	2.321190	2.153630
2003	5.144860	8.943230	0.998580	1.171080	1.484310	2.955340	2.972450	2.366580	1.368130	8.134240	28.443330	2.279190	2.233090
2004	5.072680	8.859370	0.990770	1.157000	1.525140	2.940310	2.944410	2.464030	1.616600	8.750360	28.024610	2.261540	2.533910
2005	5.009530	8.913120	0.996010	1.173800	1.653540	2.922030	2.956980	2.489540	1.684710	8.827350	28.286970	2.249050	2.627870
2006	4.928420	8.986470	1.009670	1.218630	1.783590	2.921970	2.968640	2.519310	1.737280	8.972510	28.400300	2.257610	2.694240

Table B-2: Market Sector Output and Input Quantities for Japan 1955-2006 (Billion Yen)

	y_C^t	y_N^t	y_G^t	y_X^t	y_M^t	y_I^t	y_{IV}^t	x_K^t	x_{KIV}^t	x_{LD}^t	x_{LB}^t	y_{DEP}^t	x_{KW}^t
1955	5021.9	99.7	246.2	887.1	931.2	1618.6	467.3	2070.8	250.4	1349.4	3739.1	1014.0	1056.8
1956	5530.3	108.3	221.3	1062.3	1182.0	1988.9	501.7	2082.1	268.1	1352.1	4057.7	1010.7	1071.4
1957	6012.1	115.2	226.2	1201.5	1453.0	2412.8	492.7	2153.2	290.3	1358.1	4332.0	1048.4	1104.9
1958	6386.9	117.2	261.7	1276.6	1256.4	2535.9	157.5	2283.2	311.2	1365.4	4476.2	1122.9	1160.3
1959	6887.8	130.5	281.1	1465.4	1543.0	3008.9	529.2	2417.9	317.9	1377.3	4620.4	1201.9	1216.2
1960	7717.1	147.4	313.7	1647.3	1892.7	4052.4	653.6	2584.6	340.4	1384.9	4878.1	1298.5	1288.7
1961	8572.7	164.8	347.9	1730.9	2394.6	5030.0	1122.7	2873.6	366.7	1412.9	5027.5	1475.0	1410.5
1962	9191.6	174.0	409.6	2062.2	2366.2	5640.8	501.1	3267.5	410.2	1441.8	5241.3	1725.0	1571.3
1963	10044.5	187.8	486.2	2225.5	2830.4	6310.8	767.8	3679.8	429.9	1471.0	5309.7	1985.3	1740.2
1964	11177.3	229.2	504.2	2733.8	3217.2	7337.8	874.4	4109.1	460.3	1502.1	5454.7	2255.2	1918.4
1965	11716.1	254.0	539.3	3415.8	3396.7	7589.2	652.6	4615.6	495.6	1546.6	5609.2	2574.3	2128.0
1966	12984.0	260.4	607.9	4004.3	3805.3	8685.0	855.7	5071.3	521.7	1650.3	5834.1	2855.8	2320.5
1967	14445.6	258.1	613.6	4273.1	4671.9	10288.5	1521.2	5612.7	554.1	1753.3	5995.6	3200.8	2543.6
1968	15681.1	263.0	744.0	5297.6	5244.2	12292.3	1597.9	6327.7	609.6	1796.4	6072.9	3669.4	2831.9
1969	17369.9	272.4	897.6	6425.7	5950.8	14428.7	1738.5	7218.9	667.3	1902.7	6147.7	4263.0	3186.3
1970	18768.9	219.5	1107.8	7530.7	7299.4	16754.7	1696.7	8287.8	730.1	1958.6	6283.8	4976.3	3609.0
1971	19777.2	200.9	1357.6	8841.4	7766.2	17458.3	339.7	9543.7	792.1	2010.1	6370.2	5821.7	4094.7
1972	21619.9	195.7	1630.1	9225.3	8530.1	19078.7	251.8	10678.6	804.8	2070.2	6433.3	6557.8	4548.3
1973	23557.2	182.4	1875.0	9733.5	10560.7	21349.8	1720.4	11840.4	813.9	2197.0	6638.0	7317.4	5006.6
1974	23359.7	175.0	1363.1	12014.8	11045.2	20170.2	1765.4	13105.6	874.9	2288.9	6507.0	8134.3	5511.4
1975	24406.4	164.7	2084.9	11903.8	9894.3	19936.6	-486.3	14071.4	937.8	2382.4	6336.1	8722.9	5928.3
1976	25034.8	164.1	2107.7	13889.8	10533.7	20780.6	565.5	14889.3	920.4	2443.2	6626.8	9209.9	6292.8
1977	25928.1	195.1	2384.0	15520.9	10906.3	21330.6	-238.1	15693.4	940.8	2541.1	6776.3	9690.8	6648.4
1978	27211.6	199.6	2760.1	15504.0	11461.0	23135.9	-400.6	16534.2	932.3	2609.2	6850.2	10218.0	6997.2
1979	29010.0	222.9	2985.7	16156.4	12917.4	24388.7	2330.4	17572.5	917.7	2664.2	6963.1	10897.7	7404.6
1980	29175.7	247.4	2961.1	18909.4	11991.4	24266.1	1632.9	18658.7	1001.9	2724.3	7096.3	11596.0	7841.9
1981	29574.4	255.8	3221.6	21127.0	12220.9	24825.2	499.1	19745.2	1061.0	2757.4	7232.8	12297.3	8275.5
1982	31045.2	257.5	3490.6	21105.8	12401.6	24953.0	331.2	20819.3	1079.5	2805.5	7330.4	13006.2	8688.3
1983	32055.8	277.1	3832.2	21747.5	12004.1	24683.6	-482.8	21837.9	1091.8	2847.8	7573.8	13686.6	9070.1
1984	32799.6	295.0	4079.0	24565.9	13270.7	25793.8	664.3	22836.7	1073.8	2858.2	7641.0	14373.0	9427.0
1985	33817.0	309.2	3823.3	26018.5	13399.1	28001.8	471.7	24610.0	1098.5	2918.4	7741.6	15621.3	10041.4
1986	34804.3	331.6	4761.2	24414.8	13447.7	29879.8	-494.4	26285.3	1115.9	2928.9	7844.8	16865.7	10568.1
1987	36189.6	342.7	4864.9	24189.0	15084.7	32170.8	354.2	28206.7	1097.9	2965.8	7970.2	18221.6	11232.7
1988	37910.8	358.7	5392.2	25437.1	17470.0	36216.4	432.5	29998.1	1110.6	3010.7	8168.8	19513.5	11828.6
1989	39642.6	377.6	5872.1	27800.0	20373.4	39761.6	1511.8	32068.4	1126.0	3046.2	8328.9	21067.9	12467.2
1990	41428.8	393.6	6377.3	29635.8	21940.3	43024.0	1007.8	34502.6	1179.6	3090.3	8428.3	22885.1	13223.6
1991	42338.8	417.6	7506.2	30911.4	21754.5	44379.5	543.8	37050.8	1215.0	3112.6	8546.1	24716.1	14069.8
1992	43275.0	455.3	8017.2	32142.5	21465.2	43490.6	30.7	39485.2	1234.4	3165.3	8523.4	26456.4	14880.8
1993	43637.1	475.9	8417.2	32169.2	21223.4	42076.7	-578.0	41286.1	1235.9	3217.5	8478.5	27694.6	15526.7
1994	44856.8	490.1	9057.3	33343.9	22881.5	41384.3	-485.1	42385.3	1209.0	3267.7	8488.3	28380.9	15996.1
1995	45742.8	516.3	9800.6	34833.0	25941.8	41473.8	682.7	43217.6	1186.3	3299.0	8563.5	28871.3	16383.1
1996	46895.8	524.7	10109.7	36801.8	29815.4	43423.7	941.3	44299.0	1218.3	3344.6	8591.8	29593.2	16793.5
1997	46975.7	516.5	10034.4	40874.6	30304.4	43709.9	898.4	45691.6	1262.6	3388.6	8561.1	30619.2	17224.4
1998	46367.6	574.2	10676.1	39773.4	28421.3	40835.5	-24.1	47134.2	1305.1	3444.3	8503.1	31679.7	17664.8
1999	46760.0	620.4	11401.7	40588.1	29241.0	40556.0	-1246.7	48066.2	1303.8	3465.1	8440.5	32380.2	17926.4
2000	47416.2	575.0	11257.3	45789.5	32202.5	40715.9	121.2	48854.5	1236.7	3505.2	8595.6	32980.4	18141.2
2001	48359.5	584.1	11463.2	42592.6	32777.7	40621.5	-933.6	49938.8	1243.2	3535.5	8456.4	33842.7	18395.7
2002	48857.9	617.1	11668.2	45801.9	33060.5	38912.1	-501.1	50928.0	1192.2	3552.4	8360.6	34627.7	18631.3
2003	49114.5	657.1	11982.7	49728.8	34448.3	38698.7	-702.5	51479.1	1164.7	3590.7	8342.3	35050.2	18782.7
2004	50179.5	690.6	12023.3	56596.4	37014.4	39369.2	-101.3	52176.3	1126.4	3585.4	8396.4	35619.2	18947.0
2005	51190.1	723.2	11688.8	60478.9	39484.7	40884.9	1130.4	52938.7	1089.8	3590.7	8403.6	36236.7	19138.9
2006	52727.9	742.5	10671.3	66512.1	41765.2	41477.0	2469.5	54059.6	1172.2	3635.6	8505.2	37180.6	19394.4

Appendix C. On the Flexibility of the Normalized Quadratic Income Function

Recall that the period t normalized quadratic nominal net revenue or income function $g^t(P,X)$ was defined by (3.24) in Chapter 3. In this Appendix, we will establish the flexibility of a special case of this functional form where there is no technical progress so that the parameter vectors a^t and c^t which appeared in (3.24) in Chapter 3 are simply the constant vectors a and c respectively. Thus in this Appendix, we consider the following functional form for g(P,X):

(C1) $g(P,X) \equiv a\cdot P\theta\cdot X + c\cdot X\mu\cdot P + (1/2)\, P\cdot AP\, [\theta\cdot X/\mu\cdot P] + P\cdot BX + (1/2)\, X\cdot CX\, [\mu\cdot P/\theta\cdot X]$.

We will show that the above functional form is flexible at the arbitrary positive vector of net output prices, $P^* >> 0_M$, and primary input quantities, $X^* >> 0_N$. The nonnegative, nonzero weighting vectors $\mu > 0_M$ and $\theta > 0_N$ are assumed to be known and we scale these vectors so that the following restrictions are satisfied:

(C2) $\mu\cdot P^* = \mu^T P^* = 1$; $\theta\cdot X^* = \theta^T X^* = 1$.

The M by M parameter matrix A and the N by N parameter matrix C are assumed to be symmetric;[167] i.e.:

(C3) $A = A^T$; $C = C^T$.

Thus the normalized quadratic function g(P,X) defined by (C1) has M unknown a_m parameters in the a vector, N unknown c_n parameters in the c vector, M(M+1)/2 unknown a_{mj} parameters in the A matrix, MN unknown b_{mn} parameters in the B matrix and N(N+1)/2 unknown c_{nk} parameters in the C matrix.

Let $g^*(P,X)$ be an arbitrary twice continuously differentiable income function (at the point

[167] We assume that A is positive semidefinite and that C is negative semidefinite. With these definiteness restrictions, $g^t(P,X)$ will be globally convex in P and globally concave in X. These curvature properties can be imposed econometrically without destroying the flexibility of the functional form using the techniques explained in Diewert and Wales (1987) (1992).

P^*,X^*) which satisfies the appropriate regularity conditions for an income function.[168] In order for g(P,X) defined by (C1) to be a flexible functional form[169] at the point (P^*,X^*), the following equations must be satisfied:

(C4) $$g(P^*,X^*) = g^*(P^*,X^*) ;$$
(C5) $$\nabla_P g(P^*,X^*) = \nabla_P g^*(P^*,X^*) ;$$
(C6) $$\nabla_X g(P^*,X^*) = \nabla_X g^*(P^*,X^*) ;$$
(C7) $$\nabla_{PP}^2 g(P^*,X^*) = \nabla_{PP}^2 g^*(P^*,X^*) ;$$
(C8) $$\nabla_{XX}^2 g(P^*,X^*) = \nabla_{XX}^2 g^*(P^*,X^*) ;$$
(C9) $$\nabla_{PX}^2 g(P^*,X^*) = \nabla_{PX}^2 g^*(P^*,X^*) .$$

The linear homogeneity of $g^*(P,X)$ in P and Euler's Theorem on homogeneous functions will imply the following restrictions on the level and first and second order derivatives of g^* evaluated at P^*,X^*:

(C10) $$g^*(P^*,X^*) = P^{*T}\nabla_P g^*(P^*,X^*) ;$$
(C11) $$\nabla_{PP}^2 g^*(P^*,X^*)P^* = 0_M ;$$
(C12) $$P^{*T}\nabla_{PX}^2 g^*(P^*,X^*) = \nabla_X g^*(P^*,X^*)^T .$$

The linear homogeneity of $g^*(P,X)$ in X and Euler's Theorem on homogeneous functions will also imply the following restrictions on the level and first and second order derivatives of g^* evaluated at P^*,X^*:

(C13) $$g^*(P^*,X^*) = X^{*T}\nabla_P g^*(P^*,X^*) ;$$
(C14) $$\nabla_{XX}^2 g^*(P^*,X^*)X^* = 0_N ;$$
(C15) $$\nabla_{PX}^2 g^*(P^*,X^*)X^* = \nabla_P g^*(P^*,X^*) .$$

Since g(P,X) is also linearly homogeneous in P and X separately, g will also satisfy the restrictions (C10)-(C15) with g replacing g^*.

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[168] See Diewert (1973) (1974; 136) for a listing of these regularity conditions. The important properties for our purposes are that g(P,X) is linearly homogeneous and convex in the components of P and linearly homogeneous and concave in the components of X. See also Samuelson (1953; 20) and Gorman (1968).

[169] Diewert (1974; 113) introduced the concept of a flexible functional form. Diewert (1974; 137-139) also gave some examples of flexible functional forms for income functions (or variable profit functions using his terminology), including the translog functional form.

The restrictions (C10)–(C15) mean that we do not require all of the parameters in the a and c vectors and in the A, B and C matrices in order for the normalized quadratic g(P,X) to be a flexible functional form. Thus we impose the following linear restrictions on the unknown parameters of g:

(C16) $c^T X^* = 0$;
(C17) $AP^* = 0_M$;
(C18) $CX^* = 0_N$;
(C19) $P^{*T}B = 0_N^T$;
(C20) $BX^* = 0_M$.

The N plus M linear restrictions are not all independent; any N+M−1 of these restrictions will imply the remaining restriction.[170]

Using (C1)–(C3) along with the restrictions (C16)–(C20) leads to the following expressions for the first and second derivatives of the normalized quadratic income function g evaluated at (P^*,X^*), which we set equal to the corresponding derivatives of g^*:

(C21) $\nabla_P g(P^*,X^*) = a \qquad = \nabla_P g^*(P^*,X^*)$;
(C22) $\nabla_X g(P^*,X^*) = c + a^T P^* \theta \qquad = \nabla_X g^*(P^*,X^*)$;
(C23) $\nabla_{PP}^2 g(P^*,X^*) = A \qquad = \nabla_{PP}^2 g^*(P^*,X^*)$;
(C24) $\nabla_{XX}^2 g(P^*,X^*) = C \qquad = \nabla_{XX}^2 g^*(P^*,X^*)$;
(C25) $\nabla_{PX}^2 g(P^*,X^*) = B + a\theta^T + \mu c^T \qquad = \nabla_{PX}^2 g^*(P^*,X^*)$.

It is immediately evident that we can set the parameter vector a equal to $\nabla_P g^*(P^*,X^*)$ and we can set the parameter matrices A and C equal to $\nabla_{PP}^2 g^*(P^*,X^*)$ and $\nabla_{XX}^2 g^*(P^*,X^*)$ respectively. The restrictions (C11) and (C14) on the derivatives of g^* imply that A and C satisfy the restrictions (C17) and (C18) respectively. Given that the parameter vector a is determined by (C21), we can use (C22) in order to determine the parameter vector c:

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[170] The restrictions (C16)–(C20) imply that the normalized quadratic functional form defined by (C1) has M + N − 1 + (M−1)(N−1) + (M(M−1)/2) + (N(N−1)/2) linearly independent free parameters, which is the minimal number required for a flexible functional form in this context. Thus the normalized quadratic functional form with the restrictions (C16)–(C20) imposed is a *parsimonious flexible functional form*.

$$\text{(C26)}\ c \equiv \nabla_X g^*(P^*,X^*) - a^T P^* \theta$$
$$= \nabla_X g^*(P^*,X^*) - \nabla_P g^*(P^*,X^*)^T P^* \theta \qquad \text{using (C21)}$$
$$= \nabla_X g^*(P^*,X^*) - g^*(P^*,X^*)\theta \qquad \text{using (C10).}$$

We need to check whether the c defined by (C26) satisfies the restriction (C16):

$$\text{(C27)}\ X^{*T}c = X^{*T}[\nabla_X g^*(P^*,X^*) - g^*(P^*,X^*)\theta] \qquad \text{using (C26)}$$
$$= g^*(P^*,X^*) - g^*(P^*,X^*) \qquad \text{using (C13) and (C2)}$$
$$= 0.$$

Thus the restriction (C16) is satisfied by c.

Now that a and c have been defined, use (C25) to define the matrix B:

$$\text{(C28)}\ B \equiv \nabla_{PX}^2 g^*(P^*,X^*) - \nabla_P g^*(P^*,X^*)\theta^T - \mu[\nabla_X g^*(P^*,X^*) - g^*(P^*,X^*)\theta]^T$$
$$\text{using (C21) and (C26)}$$
$$= \nabla_{PX}^2 g^*(P^*,X^*) - \nabla_P g^*(P^*,X^*)\theta^T - \mu\nabla_X g^*(P^*,X^*)^T + g^*(P^*,X^*)\mu\theta^T.$$

Now check whether the B defined by (C28) satisfies the restrictions (C19):

$$\text{(C29)}\ P^{*T}B = P^{*T}[\nabla_{PX}^2 g^*(P^*,X^*) - \nabla_P g^*(P^*,X^*)\theta^T - \mu\nabla_X g^*(P^*,X^*)^T + g^*(P^*,X^*)\mu\theta^T]$$
$$= \nabla_X g^*(P^*,X^*)^T - g^*(P^*,X^*)\theta^T - \nabla_X g^*(P^*,X^*)^T + g^*(P^*,X^*)\theta^T$$
$$\text{using (C12), (C10) and (C2)}$$
$$= 0_N^T .$$

Finally, check whether the B defined by (C28) satisfies the restrictions (C20):

$$\text{(C30)}\ BX^* = [\nabla_{PX}^2 g^*(P^*,X^*) - \nabla_P g^*(P^*,X^*)\theta^T - \mu\nabla_X g^*(P^*,X^*)^T + g^*(P^*,X^*)\mu\theta^T]X^*$$
$$= \nabla_P g^*(P^*,X^*) - \nabla_P g^*(P^*,X^*) - \mu\, g^*(P^*,X^*) + g^*(P^*,X^*)\mu$$
$$\text{using (C15), (C2) and (C13)}$$
$$= 0_M .$$

Thus the normalized quadratic income function is a parsimonious flexible functional form.

Appendix D. Measures for the Effects of Individual Price and Quantity Changes

For many purposes, it is convenient to decompose the aggregate period t contribution factor due to changes in all deflated output prices α^t into separate effects for a change in each output price. Similarly, it can sometimes be useful to decompose the aggregate period t contribution factor due to changes in all deflated market sector primary input quantities β^t into separate effects for a change in each input quantity. In this Appendix, we indicate how this can be done.

We first model the effects of a change on per unit primary input real income of a single real output price, say p_m, going from period t–1 to t. Recall the definitions of the overall theoretical Laspeyres and Paasche type price indexes defined by (3.16) and (3.17). We adapt these definitions to the case where only a single real output price changes. Thus the mth *Laspeyres measure of real output price change* α_{Lm}^t chooses the period t–1 reference technology and holds constant other real output prices at their period t–1 levels and holds deflated inputs constant at their period t–1 levels x^{t-1} and the mth *Paasche measure of real output price change* α_{Pm}^t chooses the period t reference technology and reference deflated input vector x^t and holds constant other real output prices at their period t levels:

(D1) $\alpha_{Lm}^t \equiv g^{t-1}(p_1^{t-1},...,p_{m-1}^{t-1},p_m^t,p_{m+1}^{t-1},..., p_M^{t-1},x^{t-1}) - g^{t-1}(p^{t-1},x^{t-1})$; $\quad m = 1,...,M;$

(D2) $\alpha_{Pm}^t \equiv g^t(p^t,x^t) - g^t(p_1^t,...,p_{m-1}^t,p_m^{t-1},p_{m+1}^t,..., p_M^t,x^t)$; $\quad m = 1,...,M.$

Since both measures of real output price change are equally valid, it is natural to average them to obtain an *overall measure of the effects on deflated real income (per unit primary input) of the change in the real price of output m*:

(D3) $\alpha_m^t \equiv (1/2)[\alpha_{Lm}^t + \alpha_{Pm}^t]$; $\quad m = 1,...,M$; $t = 1,2, ...$.

We are not able to obtain observable exact measures for the theoretical measures defined by (D1)–(D3) but we are able to obtain observable first order approximations to these theoretical measures. Note that $g^t(p^t,x^t)$ which appears in (D2) is equal to $p^t \cdot y^t$ which is observable and $g^{t-1}(p^{t-1},x^{t-1})$ which appears in (D1) is equal to $p^{t-1} \cdot y^{t-1}$ which is also observable. Using Hotelling's Lemma (3.9), it is straightforward to obtain the following first order approximations to the unobservable terms in (D1) and (D2):

$$\text{(D4)}\ g^{t-1}(p_1^{t-1},\ldots,p_{m-1}^{t-1},p_m^{t},p_{m+1}^{t-1},\ldots,p_M^{t-1},x^{t-1}) \qquad m = 1,\ldots,M\ ;\ t = 1,2,\ldots$$
$$\approx g^{t-1}(p^{t-1},x^{t-1}) + [\partial g^{t-1}(p^{t-1},x^{t-1})/\partial p_m][p_m^{t} - p_m^{t-1}]$$
$$= p^{t-1}\cdot y^{t-1} + y_m^{t-1}[p_m^{t} - p_m^{t-1}] \qquad \text{using (3.9);}$$
$$= [p_1^{t-1},\ldots,p_{m-1}^{t-1},p_m^{t},p_{m+1}^{t-1},\ldots,p_M^{t-1}]\cdot y^{t-1} \qquad \text{rearranging terms.}$$

$$\text{(D5)}\ g^{t}(p_1^{t},\ldots,p_{m-1}^{t},p_m^{t-1},p_{m+1}^{t},\ldots,p_M^{t},x^{t}) \qquad m = 1,\ldots,M\ ;\ t = 1,2,\ldots$$
$$\approx g^{t}(p^{t},x^{t}) + [\partial g^{t}(p^{t},x^{t})/\partial p_m][p_m^{t-1} - p_m^{t}]$$
$$= p^{t}\cdot y^{t} + y_m^{t}[p_m^{t-1} - p_m^{t}] \qquad \text{using (3.9)}$$
$$= [p_1^{t},\ldots,p_{m-1}^{t},p_m^{t-1},p_{m+1}^{t},\ldots,p_M^{t}]\cdot y^{t}\ .$$

Substituting (D4) and (D5) into (D1) and (D2) leads to the following first order approximations to the theoretical price change measures α_{Lm}^{t} and α_{Pm}^{t}:

$$\text{(D6)}\ \alpha_{Lm}^{t} \approx y_m^{t-1}[p_m^{t} - p_m^{t-1}] \qquad m = 1,\ldots,M\ ;\ t = 1,2,\ldots$$
$$\equiv a_{Lm}^{t}\ ;$$
$$\text{(D7)}\ \alpha_{Pm}^{t} \approx y_m^{t}[p_m^{t-1} - p_m^{t}] \qquad m = 1,\ldots,M\ ;\ t = 1,2,\ldots$$
$$\equiv a_{Pm}^{t}$$

where we have defined the mth observable *Laspeyres and Paasche partial indicators of real output price change*, a_{Lm}^{t} and a_{Pm}^{t}, in (D6) and (D7) respectively. These partial indicators are first order approximations to the theoretical measures of price change defined by (D1) and (D2).

Note that y^{t-1} is a feasible solution to the revenue maximization problem defined by $g^{t-1}(p_1^{t-1},\ldots,p_{m-1}^{t-1},p_m^{t},p_{m+1}^{t-1},\ldots,p_M^{t-1},x^{t-1})$ and thus the following inequality will hold:

$$\text{(D8)}\ g^{t-1}(p_1^{t-1},\ldots,p_{m-1}^{t-1},p_m^{t},p_{m+1}^{t-1},\ldots,p_M^{t-1},x^{t-1}) \geq [p_1^{t-1},\ldots,p_{m-1}^{t-1},p_m^{t},p_{m+1}^{t-1},\ldots,p_M^{t-1}]\cdot y^{t-1}.$$

Thus using (D1), (D4), (D6) and (D8), we have

$$\text{(D9)}\ \alpha_{Lm}^{t} \geq a_{Lm}^{t}\ ; \qquad m = 1,\ldots,M\ ;\ t = 1,2,\ldots\ ;$$

i.e., the observable first order approximation indicator of real price change a_{Lm}^{t} will always be equal to or less than its theoretical counterpart α_{Lm}^{t}. The difference between α_{Lm}^{t} and a_{Lm}^{t} is thus due to substitution bias.

In a similar fashion, we can show that y^t is a feasible solution to the revenue maximization problem defined by $g^t(p_1^t,...,p_{m-1}^t,p_m^{t-1},p_{m+1}^t,...,p_M^t,x^t)$ and thus the following inequality will hold:

(D10) $g^t(p_1^t,...,p_{m-1}^t,p_m^{t-1},p_{m+1}^t,...,p_M^t,x^t) \geq [p_1^t,...,p_{m-1}^t,p_m^{t-1},p_{m+1}^t,..., p_M^t]\cdot y^t$.

Thus using (D2), (D5), (D7) and (D10), we have

(D11) $\alpha_{Pm}^t \leq a_{Pm}^t$; $\qquad$ m = 1,...,M ; t = 1,2, ... ;

i.e., the observable first order approximation indicator of real price change a_{Pm}^t will always be equal to or greater than its theoretical counterpart α_{Pm}^t.

Since the substitution bias for our observable partial price indicators goes in opposite directions, this suggests that taking an average of these two indicators should lead to a closer approximation to the average of the underlying theoretical partial indicators. Thus define the period t *Bennet* (1920) *mth partial indicator of real price change for output price p_m*, $P_m^B(p^{t-1},p^t,y^{t-1},y^t)$, as the arithmetic average of the Laspeyres and Paasche partial indicators:

(D12) $P_m^B(p_m^{t-1},p_m^t,y_m^{t-1},y_m^t) \equiv (1/2)[y_m^{t-1} + y_m^t][p_m^t - p_m^{t-1}]$; $\qquad$ m =1,...,M ; t = 1,2, ...

$$= (1/2)\, a_{Lm}^t + (1/2)\, a_{Pm}^t$$

$$\approx (1/2)\, \alpha_{Lm}^t + (1/2)\, \alpha_{Pm}^t$$

where y_m^t is the mth component of the observable period t output quantity vector (deflated by the index of primary inputs) $y^t \equiv Y^t/X^t\cdot\theta$ which was defined by (3.8) and p_m^t is the mth component of the period t deflated output price vector $p^t \equiv P^t/P^t\cdot\mu$ which was defined earlier by (3.6). Note that the sum over m of the individual price change Bennet indicators, $\sum_{m=1}^M P_m^B(p_m^{t-1},p_m^t,y_m^{t-1},y_m^t)$, is equal to the overall Bennet indicator of real price change, $P^B(p^{t-1},p^t,y^{t-1},y^t)$, defined in section 3.4 by (3.32).

Thus the Bennet partial indicators of real price change, $P_m^B(p_m^{t-1},p_m^t,y_m^{t-1},y_m^t)$, will be at least a first order approximations to the theoretical measures of real price change, $(1/2)\, \alpha_{Lm}^t + (1/2)\, \alpha_{Pm}^t$, and we would normally expect this approximation to be better than a first order approximation.

The approximations derived thus far are completely nonparametric. However, if we assume that the functions g^t are of the normalized quadratic type defined by (3.24) in Chapter 3 where the parameters satisfy counterparts to the restrictions (C20)-(C24) in Appendix C, then we can obtain an explicit expression for the bias in the approximations given by (D12); i.e., it can be shown that

$$\text{(D13)}\ P_m^B(p_m^{t-1},p_m^t,y_m^{t-1},y_m^t) = (1/2)\ \alpha_{Lm}^t + (1/2)\ \alpha_{Pm}^t + \text{Bias}_m^t \qquad m = 1,...,M\ ;\ t = 1,2,...$$

where the Bias between the mth Bennet partial indicator for period t is equal to:

$$\text{(D14)}\ \text{Bias}_m^t = (1/2)\mu_m[p_m^t - p_m^{t-1}]\{c^{t-1}\cdot x^{t-1} + c^t\cdot x^t - (1/2)[p^{t-1}\cdot Ap^{t-1} + p^t\cdot Ap^t] + (1/2)[x^{t-1}\cdot Cx^{t-1} + x^t\cdot Cx^t]\}.$$

The normalized prices p_m^t weighted by the μ_m sum to unity; i.e., we have:

$$\text{(D15)}\ \textstyle\sum_{m=1}^M \mu_m p_m^t = \sum_{m=1}^M \mu_m p_m^{t-1}\ ; \qquad t = 1,2, ...$$

Using (D15), it can be seen that

$$\text{(D16)}\ \textstyle\sum_{m=1}^M \text{Bias}_m^t = 0\ ; \qquad t = 1,2,... .$$

Recall the normalizations (C20)–(C24) in Appendix C. If the price vectors P^t are all proportional to the reference price vector P^* and if the primary input vectors X^t are all proportional to X^*, then it can be seen that all of the terms in the curly brackets in (D14) will be equal to zero and hence all of the bias terms Bias_m^t will also be equal to zero. This observation indicates that if the price and quantity variations in our data set are not too far from being proportional and the technology functions g^t have the normalized quadratic functional form, then the bias terms will be small.[171]

It is also useful to have a decomposition of the overall contribution of deflated input growth to the growth of real income into separate contributions for each deflated primary

[171] Our results can be regarded as approximate difference counterparts to the ratio type results obtained by Diewert and Morrison (1986; 672) for the translog functional form. As noted earlier, an advantage of the present approach is that it is well defined even if some prices are zero whereas the Diewert-Morrison approach breaks down as any price approaches zero.

input that is used by the market sector. Recall definitions (3.20) and (3.21) for the overall theoretical Laspeyres and Paasche type measures of quantity change. We now want to adapt these definitions to the case where only a single deflated quantity changes going from one period to the next. Thus the nth *Laspeyres measure of deflated input quantity change* β_{Ln}^t chooses the period t–1 reference technology and holds constant other deflated input quantities at their period t–1 levels and holds real output prices at their period t–1 levels p^{t-1} and the nth *Paasche measure of deflated input quantity change* β_{Pn}^t chooses the period t reference technology and reference real output price vector p^t and holds constant other deflated input quantities at their period t levels:

(D17) $\beta_{Ln}^t \equiv g^{t-1}(p^{t-1},x_1^{t-1},...,x_{n-1}^{t-1},x_n^t,x_{n+1}^{t-1},..., x_N^{t-1}) - g^{t-1}(p^{t-1},x^{t-1})$; $\quad n = 1,...,N;$

(D18) $\beta_{Pn}^t \equiv g^t(p^t,x^t) - g^t(p^t,x_1^t,...,x_{n-1}^t,x_n^{t-1},x_{n+1}^t,..., p_N^t)$; $\quad n = 1,...,N.$

Since both measures of input change are equally valid, as usual, we average them to obtain *an overall measure of the effects on real income (per unit primary input) of the change in the quantity of deflated input n*:

(D19) $\beta_n^t \equiv (1/2)[\beta_{Pn}^t + \beta_{Pn}^t]$; $\quad n = 1,...,N$; $\quad t = 1,2, ...$

As was the case for the partial price change measures, we are not able to obtain observable exact measures for the theoretical measures defined by (D17)–(D19) but we are able to obtain observable first order approximations to these theoretical measures. Note that $g^t(p^t,x^t)$ which appears in (D18) is equal to $p^t \cdot y^t$ (which is equal to $w^t \cdot x^t$) which is observable and $g^{t-1}(p^{t-1},x^{t-1})$ which appears in (D17) is equal to $p^{t-1} \cdot y^{t-1}$ (which is equal to $w^{t-1} \cdot x^{t-1}$) which is also observable. Using Samuelson's Lemma (3.10), it is straightforward to obtain the following first order approximations to the unobservable terms in (D17) and (D18):

(D20) $g^{t-1}(p^{t-1},x_1^{t-1},...,x_{n-1}^{t-1},x_n^t,x_{n+1}^{t-1},..., x_N^{t-1})$ $\quad n = 1,...,N$; $t = 1,2, ...$

$$\approx g^{t-1}(p^{t-1},x^{t-1}) + [\partial g^{t-1}(p^{t-1},x^{t-1})/\partial x_n][x_n^t - x_n^{t-1}]$$

$$= w^{t-1} \cdot x^{t-1} + w_n^{t-1}[x_n^t - x_n^{t-1}] \quad \text{using (3.7) and (3.10)}$$

$$= [x_1^{t-1},...,x_{n-1}^{t-1},x_n^t,x_{n+1}^{t-1},..., x_N^{t-1}] \cdot w^{t-1} \quad \text{rearranging terms.}$$

(D21) $g^t(p^t,x_1^t,...,x_{n-1}^t,x_n^{t-1},x_{n+1}^t,..., p_N^t)$ $\quad n = 1,...,N$; $t = 1,2, ...$

$$\approx g^t(p^t,x^t) + [\partial g^t(p^t,x^t)/\partial x_n][x_n^{t-1} - x_n^t]$$

$$= w^t \cdot x^t + w_n^t[x_n^{t-1} - x_n^t] \quad \text{using (3.7) and (3.10)}$$

$= [x_1^t,...,x_{n-1}^t,x_n^{t-1},x_{n+1}^t,..., x_N^t]\cdot w^t$ rearranging terms.

Substituting (D20) and (D21) into (D17) and (D18) leads to the following first order approximations to the theoretical quantity change measures β_{Ln}^t and β_{Pn}^t:

(D22) $\beta_{Ln}^t \approx w_n^{t-1}[x_n^t - x_n^{t-1}]$ $\quad$ $n = 1,...,N$; $t = 1,2, ...$

$\equiv b_{Ln}^t$;

(D23) $\beta_{Pn}^t \approx w_n^t[x_n^{t-1} - x_n^t]$ $\quad$ $n = 1,...,N$; $t = 1,2, ...$

$\equiv b_{Pn}^t$

where we have defined the mth observable *Laspeyres and Paasche partial indicators of deflated input quantity change*, b_{Ln}^t and b_{Pn}^t, in (D22) and (D23) respectively. These partial indicators are first order approximations to the theoretical measures of quantity change defined by (D17) and (D18).

Using the fact that each function $g^t(p,x)$ is concave in x, it can be seen that the first line in (D20) is equal to or less than the third line and the first line in (D21) is equal to or less than the third line. Using these inequalities and (D17)-(D23), it can be seen that the following inequalities will be satisfied:

(D24) $\beta_{Ln}^t \leq b_{Ln}^t$; $\quad$ $n = 1,...,N$; $t = 1,2, ...$;

(D25) $\beta_{Pn}^t \geq b_{Pn}^t$; $\quad$ $n = 1,...,N$; $t = 1,2, ...$;

i.e., the observable Laspeyres partial indicator of deflated quantity change b_{Ln}^t will always be equal to or greater than its theoretical counterpart β_{Ln}^t while these inequalities will be reversed for the Paasche measures. These differences between β_{Ln}^t and b_{Ln}^t and β_{Ln}^t and b_{Ln}^t are due to substitution bias.

Since the substitution bias for our observable partial quantity indicators goes in opposite directions, this suggests that taking an average of these two indicators should lead to a closer approximation to the average of the underlying theoretical partial indicators. Thus define the period t *Bennet* (1920) *indicator of relative input quantity change for deflated input n*, $Q_n^B(w^{t-1},w^t,x^{t-1},x^t)$, as follows:

(D26) $Q_n^B(w_n^{t-1},w_n^t,x_n^{t-1},x_n^t) \equiv (1/2)[w_n^{t-1} + w_n^t][x_n^t - x_n^{t-1}]$; $\quad$ $n = 1,...,N$; $t = 1,2, ...$

where w_n^t is the nth component of the observable real input price vector $w^t \equiv W^t/P^t \cdot \mu$ defined earlier by (3.6) and x_n^t is the nth component of the input quantity vector (deflated by the index of primary inputs) $x^t \equiv X^t/X^t \cdot \theta$ defined earlier by (3.8).

The Bennet partial indicators of deflated input quantity change, $Q_n^B(w_n^{t-1},w_n^t,x_n^{t-1},x_n^t)$, will be at least a first order approximations to the corresponding theoretical measures of input quantity change, $(1/2)\beta_{Lm}^t + (1/2)\beta_{Pm}^t$, and we would normally expect these approximations to be better than a first order approximation. Note that these approximations are nonparametric.[172]

Note that the sum over n of the individual input quantity change Bennet indicators, $\sum_{n=1}^N Q_n^B(w_n^{t-1},w_n^t,x_n^{t-1},x_n^t)$, is equal to the overall Bennet indicator of relative input quantity change , $Q^B(w^{t-1},w^t,x^{t-1},x^t)$, defined in section 3.4 by (3.40).

[172] These are difference counterparts to the ratio type results obtained by Diewert and Morrison (1986; 672) for the translog functional form. As noted earlier, an advantage of the present approach is that it is well defined even if some inputs are zero whereas the Diewert-Morrison approach breaks down as any input quantity approaches zero..

Appendix E. Japanese Price and Quantity Data

Table E-1: Prices of Main Aggregates

	P_C^t	P_N^t	P_G^t	P_X^t	P_M^t	P_I^t	P_{IV}^t	W_K^t	W_{KIV}^t	W_{LD}^t	W_{LB}^t	P_{DEP}^t	W_{KW}^t
1955	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
1956	0.997030	1.046150	1.033880	1.029650	1.053760	1.077990	1.047120	1.081910	1.052520	1.162470	1.051630	1.084910	1.079060
1957	1.027380	1.104460	1.078260	1.034260	1.091090	1.141070	1.052240	1.156770	1.192040	1.361140	1.106040	1.141810	1.170910
1958	1.026250	1.106340	1.026590	0.968780	0.941460	1.106830	1.038450	1.081550	1.090750	1.409410	1.171500	1.110370	1.053760
1959	1.058940	1.146820	1.026600	0.989370	0.933430	1.104060	1.063200	1.152270	1.283080	1.840230	1.276470	1.093170	1.210480
1960	1.090840	1.203860	1.021790	1.017380	0.936580	1.128280	1.061450	1.272050	1.541940	2.538670	1.385160	1.096620	1.446280
1961	1.154690	1.285830	1.027970	1.010180	0.947610	1.210020	1.050610	1.434600	1.877510	3.192780	1.569720	1.142800	1.727580
1962	1.236120	1.376100	1.005770	0.981420	0.937460	1.238500	1.063230	1.416650	1.737270	3.301640	1.762160	1.169640	1.661880
1963	1.324490	1.467960	0.964130	0.999170	0.954060	1.244100	1.078100	1.447480	1.828620	3.733480	2.005080	1.162510	1.734530
1964	1.359500	1.567040	0.935380	1.008560	0.967250	1.263460	1.092800	1.507670	1.930700	4.281610	2.270120	1.171690	1.852010
1965	1.458860	1.664440	0.903830	0.997200	0.959750	1.287310	1.129580	1.485270	1.808260	4.274190	2.548200	1.192280	1.779180
1966	1.516950	1.760620	0.907170	0.994030	0.965610	1.332890	1.142600	1.568190	2.008820	4.874150	2.831780	1.210740	1.937200
1967	1.567860	1.864450	0.931760	0.995450	0.950970	1.388610	1.144880	1.647800	2.227060	5.716390	3.174850	1.243300	2.071540
1968	1.649960	1.977760	0.918190	0.995730	0.950360	1.433910	1.175620	1.729260	2.422450	6.820760	3.655630	1.271050	2.217000
1969	1.710270	2.109430	0.900760	1.008100	0.978950	1.493070	1.224460	1.777010	2.473840	7.761170	4.256680	1.321240	2.258340
1970	1.827940	2.400920	0.890240	1.038050	0.996770	1.561730	1.256900	1.809760	2.397440	8.440790	4.993850	1.378520	2.255240
1971	1.949920	2.700550	0.841400	1.060330	0.961500	1.597710	1.286390	1.693960	1.959910	7.871140	5.799770	1.418850	1.930930
1972	2.059210	3.056210	0.795480	1.052060	0.915090	1.666860	1.411500	1.686480	1.923710	8.744840	6.654600	1.459570	1.855120
1973	2.293510	3.598060	0.851860	1.152710	1.096240	1.933810	1.663190	1.884260	2.241680	10.207760	8.061310	1.646310	2.050080
1974	2.832640	4.587410	1.450220	1.512360	1.756610	2.355840	1.909370	2.165680	2.276260	9.169440	10.334070	2.008760	2.185050
1975	3.160420	5.108800	1.024940	1.586770	1.927220	2.451120	2.008040	2.122670	1.942790	7.836040	12.225790	2.092580	1.959380
1976	3.465260	5.709340	1.061470	1.617900	2.038490	2.556210	2.111660	2.289860	2.290330	8.911060	13.168570	2.146540	2.276390
1977	3.715340	6.174370	1.056740	1.557680	1.966810	2.667580	2.153110	2.361340	2.214140	8.857750	14.391590	2.237830	2.312000
1978	3.873910	6.403510	0.977400	1.458740	1.681340	2.751380	2.180310	2.501630	2.444300	10.079840	15.412520	2.275030	2.589080
1979	3.997830	6.807150	1.039560	1.577670	2.150860	2.941630	2.390120	2.725840	2.855770	11.720960	16.353280	2.379530	2.966870
1980	4.324130	7.337000	1.256800	1.730550	2.937710	3.175590	2.487100	2.822050	2.741050	11.660220	17.353530	2.536570	2.963810
1981	4.521580	7.520150	1.336600	1.786490	2.958080	3.233280	2.531950	2.793550	2.509730	11.548060	18.445300	2.581790	2.828850
1982	4.628340	7.690270	1.299130	1.851010	3.033520	3.247360	2.575510	2.761550	2.369060	11.493750	19.267130	2.605050	2.717670
1983	4.717860	7.791750	1.296640	1.792800	2.869880	3.239120	2.579670	2.706850	2.182940	11.147980	19.589510	2.604860	2.586560
1984	4.828580	7.971620	1.284070	1.821300	2.796780	3.246000	2.559260	2.809070	2.396290	12.596350	20.454430	2.597070	2.845270
1985	4.918420	8.146100	1.396140	1.767550	2.641360	3.225550	2.466960	2.856330	2.455580	13.682780	21.099150	2.580600	2.985830
1986	4.945450	8.225600	1.236200	1.551800	1.847600	3.166480	2.327550	2.796780	2.279880	14.049020	21.674870	2.532860	2.914060
1987	4.947370	8.264210	1.155100	1.486870	1.696490	3.147750	2.266760	2.753690	2.160910	14.759250	22.097420	2.510940	2.841630
1988	4.961360	8.335650	1.095340	1.460980	1.647590	3.174480	2.279210	2.796000	2.238680	16.225280	22.782910	2.522040	2.930240
1989	5.048840	8.529050	1.121160	1.509190	1.756420	3.269890	2.298600	2.872150	2.302370	18.225000	23.940750	2.578240	3.030920
1990	5.171530	8.962910	1.163740	1.534660	1.875460	3.360790	2.323190	2.879450	2.193120	18.891880	25.645820	2.636300	2.950530
1991	5.283030	9.239720	1.152410	1.498570	1.796120	3.408810	2.338150	2.837150	1.989660	17.662450	27.319490	2.670850	2.779400
1992	5.358280	9.343340	1.140510	1.460650	1.703170	3.419440	2.741140	2.800310	1.896910	16.157210	28.206060	2.663460	2.695070
1993	5.402760	9.334670	1.125730	1.362610	1.565730	3.419160	2.976970	2.700410	1.618010	13.767730	28.854180	2.663610	2.429470
1994	5.417350	9.396900	1.091400	1.320330	1.497590	3.390380	2.920080	2.663570	1.558300	12.943610	29.342760	2.631810	2.388270
1995	5.396620	9.405670	1.068760	1.292550	1.466560	3.366170	2.894290	2.655530	1.546450	12.567620	29.507420	2.602870	2.418170
1996	5.395910	9.434970	1.055420	1.338230	1.560540	3.320700	2.886630	2.673330	1.650710	12.773090	29.696380	2.557340	2.545400
1997	5.455230	9.577770	1.075030	1.362740	1.650650	3.318420	2.877270	2.657580	1.607780	12.041860	30.492160	2.559810	2.499340
1998	5.408500	9.553270	1.062110	1.375150	1.601710	3.241150	2.837860	2.501010	1.285240	9.664080	30.131830	2.535780	2.125720
1999	5.374790	9.443580	1.043110	1.253040	1.470210	3.168390	3.178060	2.451880	1.276670	9.168170	29.642000	2.476100	2.101710
2000	5.338730	9.378940	1.037690	1.201000	1.491250	3.155730	3.071270	2.475800	1.370380	9.257340	29.195500	2.452120	2.209440
2001	5.281180	9.339730	1.029590	1.227830	1.520400	3.065950	2.992370	2.370120	1.173960	8.139020	29.358540	2.386180	2.044290
2002	5.192820	9.070420	1.015560	1.212870	1.500880	2.992920	2.959490	2.366130	1.270450	8.230890	28.803100	2.321190	2.153630
2003	5.144860	8.943230	0.998580	1.171080	1.484310	2.955340	2.972450	2.366580	1.368130	8.134240	28.443330	2.279190	2.233090
2004	5.072680	8.859370	0.990770	1.157000	1.525140	2.940310	2.944410	2.464030	1.616600	8.750360	28.024610	2.261540	2.533910
2005	5.009530	8.913120	0.996010	1.173800	1.653540	2.922030	2.956980	2.489540	1.684710	8.827350	28.286970	2.249050	2.627870
2006	4.928420	8.986470	1.009670	1.218630	1.783590	2.921970	2.968640	2.519310	1.737280	8.972510	28.400300	2.257610	2.694240

Table E-2: Quantities of Main Aggregates

	Y_C^t	Y_N^t	Y_G^t	Y_X^t	Y_M^t	Y_I^t	Y_{IV}^t	X_K^t	X_{KIV}^t	X_{LD}^t	X_{LB}^t	Y_{DEP}^t	X_{KW}^t
1955	5021.93	99.7	246.2	887.1	-931.19	1618.55	467.26	2070.78	250.35	1349.35	3739.08	-1013.98	1056.79
1956	5530.33	108.3	221.3	1062.31	-1181.97	1988.94	501.74	2082.05	268.12	1352.06	4057.71	-1010.72	1071.35
1957	6012.13	115.17	226.2	1201.54	-1452.99	2412.77	492.66	2153.18	290.27	1358.11	4332.02	-1048.35	1104.88
1958	6386.92	117.23	261.74	1276.55	-1256.36	2535.86	157.46	2283.23	311.15	1365.44	4476.21	-1122.85	1160.28
1959	6887.8	130.53	281.12	1465.38	-1543.03	3008.9	529.22	2417.85	317.86	1377.29	4620.40	-1201.87	1216.19
1960	7717.12	147.44	313.67	1647.27	-1892.68	4052.44	653.59	2584.59	340.40	1384.91	4878.13	-1298.45	1288.70
1961	8572.65	164.8	347.87	1730.88	-2394.61	5029.95	1122.73	2873.57	366.67	1412.86	5027.46	-1474.96	1410.54
1962	9191.56	173.97	409.64	2062.21	-2366.16	5640.77	501.09	3267.51	410.20	1441.78	5241.33	-1725.00	1571.28
1963	10044.54	187.81	486.24	2225.54	-2830.42	6310.78	767.75	3679.77	429.94	1470.95	5309.69	-1985.28	1740.23
1964	11177.28	229.16	504.18	2733.79	-3217.16	7337.75	874.4	4109.11	460.32	1502.12	5454.66	-2255.18	1918.35
1965	11716.05	254.02	539.26	3415.77	-3396.70	7589.15	652.63	4615.61	495.59	1546.55	5609.24	-2574.29	2128.03
1966	12983.96	260.36	607.93	4004.31	-3805.25	8685.01	855.72	5071.31	521.74	1650.33	5834.14	-2855.77	2320.45
1967	14445.61	258.09	613.57	4273.06	-4671.89	10288.48	1521.19	5612.72	554.12	1753.33	5995.64	-3200.82	2543.56
1968	15681.11	263.03	743.96	5297.63	-5244.24	12292.33	1597.88	6327.71	609.57	1796.40	6072.94	-3669.37	2831.89
1969	17369.93	272.4	897.58	6425.65	-5950.82	14428.67	1738.5	7218.94	667.30	1902.65	6147.65	-4263.00	3186.26
1970	18768.86	219.54	1107.79	7530.68	-7299.40	16754.68	1696.7	8287.84	730.14	1958.63	6283.77	-4976.28	3608.98
1971	19777.23	200.85	1357.62	8841.4	-7766.17	17458.28	339.73	9543.67	792.13	2010.11	6370.20	-5821.68	4094.67
1972	21619.87	195.7	1630.08	9225.33	-8530.08	19078.74	251.81	10678.56	804.77	2070.15	6433.29	-6557.78	4548.30
1973	23557.21	182.35	1874.96	9733.5	-10560.73	21349.75	1720.37	11840.44	813.85	2197.03	6637.98	-7317.38	5006.56
1974	23359.72	174.96	1363.1	12014.83	-11045.24	20170.17	1765.38	13105.60	874.89	2288.91	6507.01	-8134.31	5511.41
1975	24406.43	164.74	2084.9	11903.76	-9894.33	19936.55	-486.32	14071.44	937.80	2382.35	6336.13	-8722.92	5928.25
1976	25034.78	164.12	2107.73	13889.81	-10533.70	20780.55	565.45	14889.30	920.42	2443.24	6626.77	-9209.92	6292.83
1977	25928.14	195.11	2384.03	15520.87	-10906.27	21330.58	-238.05	15693.36	940.76	2541.07	6776.30	-9690.84	6648.35
1978	27211.57	199.64	2760.08	15504.03	-11461.00	23135.89	-400.64	16534.23	932.26	2609.20	6850.16	-10218.00	6997.19
1979	29010	222.93	2985.7	16156.37	-12917.35	24388.68	2330.44	17572.47	917.67	2664.22	6963.09	-10897.68	7404.59
1980	29175.73	247.44	2961.08	18909.38	-11991.39	24266.08	1632.92	18658.69	1001.93	2724.27	7096.29	-11595.97	7841.87
1981	29574.38	255.82	3221.61	21126.97	-12220.92	24825.15	499.1	19745.21	1061.03	2757.40	7232.80	-12297.31	8275.47
1982	31045.18	257.46	3490.64	21105.75	-12401.61	24952.99	331.17	20819.31	1079.45	2805.51	7330.42	-13006.19	8688.31
1983	32055.76	277.14	3832.22	21747.47	-12004.07	24683.58	-482.75	21837.91	1091.80	2847.83	7573.77	-13686.58	9070.12
1984	32799.64	294.98	4078.99	24565.89	-13270.73	25793.75	664.29	22836.71	1073.80	2858.20	7641.00	-14372.98	9426.97
1985	33816.99	309.17	3823.26	26018.54	-13399.11	28001.84	471.72	24609.99	1098.48	2918.38	7741.62	-15621.28	10041.42
1986	34804.25	331.6	4761.21	24414.83	-13447.72	29879.8	-494.43	26285.34	1115.88	2928.86	7844.75	-16865.70	10568.07
1987	36189.58	342.67	4864.93	24188.97	-15084.68	32170.76	354.18	28206.74	1097.86	2965.80	7970.15	-18221.64	11232.70
1988	37910.76	358.7	5392.22	25437.14	-17470.04	36216.42	432.53	29998.10	1110.60	3010.66	8168.81	-19513.54	11828.58
1989	39642.56	377.64	5872.14	27800	-20373.36	39761.58	1511.75	32068.35	1126.02	3046.20	8328.94	-21067.91	12467.18
1990	41428.75	393.58	6377.27	29635.75	-21940.26	43023.96	1007.8	34502.60	1179.56	3090.27	8428.33	-22885.06	13223.59
1991	42338.79	417.64	7506.15	30911.36	-21754.50	44379.49	543.81	37050.83	1214.98	3112.64	8546.10	-24716.06	14069.82
1992	43274.97	455.31	8017.22	32142.49	-21465.20	43490.55	30.73	39485.17	1234.44	3165.26	8523.37	-26456.44	14880.78
1993	43637.05	475.91	8417.17	32169.17	-21223.42	42076.69	-578.01	41286.14	1235.88	3217.54	8478.52	-27694.61	15526.74
1994	44856.84	490.11	9057.28	33343.86	-22881.54	41384.3	-485.12	42385.26	1208.95	3267.65	8488.29	-28380.94	15996.10
1995	45742.75	516.3	9800.58	34832.96	-25941.84	41473.75	682.7	43217.56	1186.30	3298.99	8563.54	-28871.26	16383.14
1996	46895.81	524.71	10109.66	36801.75	-29815.42	43423.66	941.34	44299.00	1218.30	3344.56	8591.77	-29593.21	16793.51
1997	46975.66	516.52	10034.35	40874.61	-30304.37	43709.9	898.42	45691.61	1262.57	3388.62	8561.06	-30619.22	17224.38
1998	46367.62	574.19	10676.12	39773.36	-28421.29	40835.54	-24.09	47134.23	1305.07	3444.28	8503.13	-31679.72	17664.83
1999	46760.01	620.35	11401.73	40588.1	-29240.97	40555.97	-1246.7	48066.24	1303.77	3465.14	8440.53	-32380.15	17926.38
2000	47416.24	574.99	11257.3	45789.53	-32202.49	40715.85	121.16	48854.51	1236.66	3505.16	8595.55	-32980.39	18141.21
2001	48359.52	584.1	11463.2	42592.6	-32777.73	40621.5	-933.62	49938.82	1243.22	3535.54	8456.40	-33842.68	18395.67
2002	48857.91	617.14	11668.24	45801.94	-33060.49	38912.12	-501.05	50927.95	1192.16	3552.41	8360.61	-34627.69	18631.25
2003	49114.48	657.09	11982.72	49728.8	-34448.27	38698.68	-702.5	51479.05	1164.74	3590.69	8342.31	-35050.16	18782.74
2004	50179.52	690.57	12023.29	56596.35	-37014.35	39369.2	-101.29	52176.34	1126.40	3585.42	8396.39	-35619.17	18946.98
2005	51190.08	723.24	11688.76	60478.87	-39484.71	40884.87	1130.43	52938.69	1089.79	3590.65	8403.60	-36236.69	19138.86
2006	52727.91	742.48	10671.31	66512.12	-41765.17	41476.98	2469.53	54059.57	1172.18	3635.58	8505.20	-37180.62	19394.42

Table E-3: Prices of Investment Goods

	P_{I1}^t	P_{I2}^t	P_{I3}^t	P_{I4}^t	P_{I5}^t	P_{I6}^t	P_{I7}^t	P_{I8}^t	P_{I9}^t	P_{I10}^t	P_{I11}^t	P_{I12}^t
1955	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
1956	1.013960	1.076640	1.006890	1.025680	1.039070	1.103920	1.111840	0.995050	1.063490	1.196700	1.047690	1.002980
1957	1.080960	1.108720	1.081150	1.126050	1.099140	1.210150	1.334730	1.033280	1.114810	1.313380	1.058980	0.999740
1958	1.039700	1.052270	1.078030	1.067620	1.047520	1.113850	1.366490	1.082550	1.109340	1.192400	1.060670	0.987220
1959	1.061940	1.058460	1.072890	1.117600	1.085140	1.080550	1.325200	1.009920	1.157220	1.189420	1.027890	0.986490
1960	1.170090	1.084550	1.033570	1.176150	1.126280	1.141930	1.387940	1.011260	1.135630	1.177050	1.048870	1.002990
1961	1.271830	1.212600	1.107270	1.374640	1.222520	1.171780	1.419660	1.012840	1.131200	1.166290	1.030430	1.043140
1962	1.283050	1.250300	1.121380	1.376590	1.291040	1.132420	1.472200	1.018640	1.122300	1.137570	1.046120	1.067480
1963	1.392270	1.276650	1.155190	1.422010	1.301660	1.119300	1.440100	0.960180	1.124600	1.119170	1.064250	1.092740
1964	1.394690	1.312550	1.166080	1.426540	1.385530	1.149880	1.443800	0.931880	1.111750	1.121010	1.138170	1.128690
1965	1.523840	1.349260	1.204320	1.414380	1.414300	1.177330	1.448240	0.944150	1.112520	1.119800	1.134280	1.134290
1966	1.822960	1.429770	1.004380	1.524080	1.404440	1.197440	1.446440	0.926330	1.114520	1.124980	1.143920	1.151880
1967	2.171790	1.515940	1.233400	1.664090	1.402840	1.242930	1.482450	0.911510	1.112220	1.192510	1.153410	1.164540
1968	2.551500	1.577280	1.283510	1.754590	1.418160	1.258000	1.550190	0.929670	1.103700	1.193470	1.158150	1.204550
1969	2.826740	1.664830	1.416990	1.825200	1.418540	1.293050	1.603910	0.950400	1.092270	1.268130	1.225660	1.218920
1970	2.956160	1.769140	1.338410	1.909170	1.538020	1.354870	1.648180	0.969220	1.100890	1.344780	1.285960	1.236670
1971	2.906870	1.818690	1.425620	1.918090	1.568150	1.379620	1.688930	0.955250	1.114280	1.414590	1.330410	1.309630
1972	2.922680	1.948420	1.411250	2.203380	1.637650	1.390760	1.718050	0.922610	1.131220	1.425120	1.340760	1.373480
1973	3.379110	2.374400	1.602980	3.107440	2.002810	1.551900	1.889760	0.938260	1.166320	1.824390	1.477850	1.482730
1974	3.680360	2.862390	1.835910	3.425690	2.597900	2.156160	2.410360	1.118460	1.443260	1.884620	1.904720	1.891850
1975	4.186620	2.996910	1.923020	3.378120	2.591000	2.163640	2.453290	1.128530	1.488720	2.261930	1.973130	2.006550
1976	4.586530	3.179360	2.077470	3.582320	2.659050	2.241420	2.469560	1.122230	1.521240	2.305070	2.027470	2.041120
1977	4.906660	3.332960	2.163310	3.609510	2.850350	2.362540	2.558420	1.138570	1.562110	2.595450	2.127180	2.094250
1978	5.036890	3.497150	2.135720	3.357750	2.832040	2.421600	2.580640	1.108140	1.577450	2.577170	2.107920	2.156250
1979	5.149150	3.825380	2.363480	4.088120	3.093030	2.491710	2.662150	1.122710	1.606320	2.536380	2.192150	2.270590
1980	5.372870	4.195140	2.439750	4.354170	3.322060	2.710570	2.789050	1.158250	1.631930	2.879040	2.333480	2.466950
1981	5.717980	4.296200	2.492800	4.031610	3.335350	2.752170	2.829740	1.151250	1.643400	2.938310	2.363120	2.486140
1982	5.947020	4.312120	2.514700	4.303630	3.331890	2.723780	2.831030	1.137130	1.642560	3.248480	2.397090	2.533930
1983	6.094470	4.315810	2.492790	4.305520	3.311860	2.742610	2.811790	1.107430	1.630370	3.419340	2.360120	2.531910
1984	6.287140	4.371940	2.498700	4.440060	3.342740	2.811900	2.810440	1.082000	1.623540	3.015210	2.350210	2.531630
1985	6.392240	4.372990	2.498290	4.583920	3.337870	2.808680	2.808240	1.034230	1.605130	2.985130	2.350940	2.528720
1986	6.368370	4.344430	2.437090	4.261590	3.324890	2.765360	2.773380	0.963970	1.592580	2.816950	2.275550	2.470260
1987	6.146010	4.389660	2.421480	4.540350	3.313510	2.690430	2.725670	0.905460	1.573860	2.737830	2.289130	2.452040
1988	6.154320	4.469690	2.426240	4.375910	3.343560	2.743440	2.738000	0.885840	1.570470	2.721350	2.315690	2.451640
1989	6.305180	4.670140	2.481040	4.592570	3.420720	2.848770	2.802670	0.887970	1.577650	2.760760	2.337570	2.456000
1990	6.420740	4.857770	2.548100	4.666190	3.517260	2.910200	2.870330	0.888020	1.577920	2.810990	2.387530	2.477360
1991	6.190660	4.975180	2.569040	4.584930	3.572650	2.960160	2.906220	0.865850	1.572320	2.916740	2.439920	2.501760
1992	5.948050	5.048310	2.525920	4.556200	3.625700	2.961700	2.900560	0.845750	1.577050	2.872270	2.429750	2.434270
1993	5.666390	5.079810	2.560560	4.984620	3.604260	2.947700	2.880620	0.827700	1.568860	2.999960	2.424740	2.415900
1994	5.423990	5.086130	2.467130	4.782250	3.538880	2.894820	2.850640	0.799820	1.554450	2.879680	2.382140	2.343930
1995	4.805810	5.106470	2.443430	4.614750	3.516910	2.901790	2.825860	0.755750	1.542220	2.866900	2.365060	2.311520
1996	5.068370	5.102960	2.494960	4.768820	3.488730	2.863760	2.800280	0.696560	1.521700	2.842940	2.334700	2.288520
1997	5.238470	5.141800	2.544490	4.887030	3.491090	2.874880	2.815980	0.666090	1.520340	2.894040	2.342460	2.280060
1998	5.219020	5.016590	2.520910	4.511860	3.473840	2.828210	2.772050	0.637920	1.496530	2.915610	2.324650	2.241050
1999	5.497600	4.927600	2.424140	4.535090	3.425900	2.768720	2.714390	0.604340	1.479770	2.774340	2.293180	2.198360
2000	5.579630	4.962480	2.406320	4.553340	3.433250	2.764470	2.706840	0.574070	1.480040	2.719610	2.322340	2.173680
2001	5.540430	4.877160	2.365110	4.489180	3.411480	2.731070	2.664630	0.514400	1.449730	2.606870	2.315480	2.123380
2002	5.477710	4.801400	2.331370	4.411300	3.379950	2.700530	2.616250	0.474780	1.423230	2.651800	2.284410	2.065220
2003	5.698420	4.825420	2.325400	4.425600	3.375280	2.707690	2.587950	0.434590	1.409490	2.751410	2.262830	1.991220
2004	6.055530	4.873480	2.326260	4.467240	3.370950	2.748040	2.573640	0.403960	1.405250	2.841450	2.255820	1.951780
2005	6.037300	4.917170	2.341050	4.434590	3.402800	2.802230	2.561750	0.373860	1.397110	2.890480	2.245130	1.916640
2006	5.908520	4.983170	2.362840	4.552560	3.447370	2.821160	2.549440	0.358830	1.391600	2.903540	2.227290	1.893860

Table E-4: Prices of Capital Services

	W_{K1}^t	W_{K2}^t	W_{K3}^t	W_{K4}^t	W_{K5}^t	W_{K6}^t	W_{K7}^t	W_{K8}^t	W_{K9}^t	W_{K10}^t	W_{K11}^t	W_{K12}^t
1955	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
1956	1.012050	1.077430	1.005060	1.023650	1.036750	1.101050	1.105140	0.972690	1.033170	1.244750	1.024550	0.953070
1957	1.082300	1.119180	1.081150	1.128260	1.105460	1.221940	1.343230	1.032100	1.081750	1.370320	1.040920	0.922020
1958	1.036350	1.008900	1.075690	1.063690	1.040010	1.103100	1.373200	1.075690	1.075080	1.241840	1.070270	0.872360
1959	1.077010	1.093970	1.083080	1.135560	1.117800	1.126010	1.409080	1.045390	1.148570	1.305500	1.074560	0.895500
1960	1.203150	1.218890	1.058440	1.224930	1.216120	1.267880	1.558900	1.101150	1.164690	1.362870	1.134330	0.949120
1961	1.330910	1.387420	1.146730	1.458200	1.365470	1.359460	1.656420	1.141420	1.186980	1.424550	1.136320	1.021840
1962	1.324890	1.367720	1.150330	1.440740	1.409610	1.278160	1.683790	1.123650	1.154810	1.428090	1.142210	1.002910
1963	1.447030	1.428390	1.189100	1.497860	1.440720	1.285080	1.662370	1.067200	1.162630	1.444680	1.174130	1.036810
1964	1.462730	1.514700	1.207090	1.515950	1.559180	1.346950	1.709260	1.051720	1.165100	1.495720	1.318690	1.094400
1965	1.572900	1.492980	1.233270	1.480210	1.549860	1.336840	1.674740	1.038150	1.152250	1.509730	1.290400	1.056480
1966	1.881800	1.618340	1.031570	1.605060	1.565760	1.384320	1.708460	1.029830	1.180130	1.588210	1.313730	1.091330
1967	2.248610	1.741130	1.271520	1.763020	1.585790	1.457060	1.775520	1.024040	1.189470	1.698130	1.342410	1.120500
1968	2.657290	1.849670	1.328170	1.871460	1.629310	1.499940	1.879140	1.055910	1.200070	1.710980	1.375430	1.176490
1969	2.927180	1.924470	1.458650	1.931520	1.603950	1.515810	1.933460	1.072060	1.185430	1.791720	1.455620	1.172070
1970	3.046350	1.977810	1.366690	1.996570	1.694320	1.548840	1.975060	1.079840	1.178160	1.859480	1.527050	1.157930
1971	2.824270	1.807930	1.423220	1.936800	1.618540	1.455450	1.907550	1.012030	1.149940	1.841510	1.525790	1.097220
1972	2.740030	1.820380	1.395810	2.192560	1.645310	1.417090	1.897240	0.961770	1.166220	1.824920	1.534750	1.093020
1973	3.078940	2.147640	1.575690	3.061930	1.976590	1.543770	2.054100	0.962450	1.209740	2.301960	1.618810	1.148230
1974	3.245840	2.364510	1.775050	3.297970	2.466230	2.040550	2.510100	1.103250	1.485980	2.284500	1.956530	1.373060
1975	3.582610	2.264900	1.831500	3.177460	2.356980	1.934150	2.468460	1.081880	1.518380	2.643720	1.995580	1.335690
1976	3.963840	2.536440	1.996120	3.415180	2.473300	2.070410	2.576560	1.102220	1.623090	2.789440	2.078130	1.406050
1977	4.222730	2.603100	2.073640	3.425670	2.622330	2.153650	2.666290	1.116500	1.690080	3.140220	2.145390	1.416290
1978	4.381570	2.856680	2.064060	3.223960	2.654570	2.264860	2.761330	1.113010	1.751790	3.222250	2.170050	1.480380
1979	4.507120	3.229380	2.296310	3.960250	2.947250	2.385620	2.922340	1.149320	1.814480	3.306750	2.267230	1.593590
1980	4.629820	3.361650	2.348560	4.161370	3.090950	2.516620	3.023000	1.168260	1.818290	3.687790	2.338690	1.687360
1981	4.897570	3.315220	2.387560	3.820410	3.053940	2.497370	3.034880	1.152790	1.804050	3.711510	2.321440	1.662290
1982	5.063120	3.256280	2.401880	4.057290	3.020360	2.434700	3.010630	1.132990	1.788470	4.034760	2.335070	1.663930
1983	5.161810	3.182600	2.370400	4.030210	2.964510	2.406330	2.969630	1.099600	1.779340	4.208090	2.289180	1.631130
1984	5.396870	3.394960	2.396840	4.209080	3.059780	2.540550	3.042230	1.100480	1.817140	3.826880	2.304600	1.669640
1985	5.548270	3.504910	2.413670	4.386810	3.102460	2.586630	3.079140	1.074140	1.808960	3.842860	2.321890	1.680970
1986	5.563380	3.466120	2.356050	4.077840	3.086050	2.538310	3.031630	1.014030	1.803670	3.588160	2.292400	1.628410
1987	5.402230	3.471320	2.341440	4.338350	3.059810	2.449140	2.965490	0.958280	1.782560	3.444300	2.293980	1.602940
1988	5.470500	3.577000	2.352960	4.197370	3.105530	2.516640	2.993940	0.941050	1.784180	3.400520	2.315760	1.611700
1989	5.650760	3.764730	2.406990	4.408540	3.182880	2.619500	3.032410	0.938360	1.769470	3.378120	2.335350	1.610510
1990	5.742780	3.804640	2.458140	4.442400	3.230190	2.625070	3.053280	0.926120	1.749770	3.369270	2.365620	1.596720
1991	5.520460	3.752770	2.470550	4.336430	3.236490	2.614190	3.047600	0.894070	1.710640	3.465960	2.386030	1.576300
1992	5.294250	3.705370	2.425190	4.298630	3.269270	2.599370	3.023300	0.869210	1.705660	3.401700	2.375420	1.516260
1993	4.993690	3.549070	2.440840	4.648790	3.183790	2.508930	2.946570	0.837700	1.669510	3.493200	2.345220	1.474790
1994	4.786800	3.508130	2.350170	4.454260	3.124680	2.454030	2.929910	0.812120	1.664170	3.397830	2.309870	1.437440
1995	4.267640	3.542170	2.328860	4.301660	3.113400	2.464630	2.914430	0.773300	1.664370	3.421450	2.291760	1.431600
1996	4.510910	3.651830	2.386500	4.470490	3.121960	2.468630	2.915450	0.728620	1.649980	3.435220	2.281130	1.432150
1997	4.654280	3.634830	2.437210	4.584900	3.121700	2.473760	2.927520	0.703160	1.630260	3.493230	2.297450	1.419600
1998	4.607190	3.349300	2.413980	4.207790	3.047550	2.358460	2.836220	0.668440	1.579560	3.460420	2.251170	1.359580
1999	4.852360	3.300830	2.323920	4.235000	3.012780	2.313840	2.787150	0.636730	1.569080	3.299830	2.233120	1.334600
2000	4.934550	3.385540	2.311170	4.268620	3.041830	2.340500	2.797650	0.612830	1.573730	3.254070	2.258180	1.333210
2001	4.880130	3.249700	2.265460	4.187620	2.993430	2.277340	2.729250	0.556500	1.533910	3.090160	2.223150	1.287900
2002	4.846020	3.314760	2.239720	4.137190	2.995220	2.289080	2.702340	0.525110	1.508660	3.179660	2.203310	1.268810
2003	5.052020	3.378150	2.236370	4.162620	3.009580	2.320640	2.688440	0.493220	1.492050	3.323880	2.193470	1.234080
2004	5.438420	3.638170	2.259960	4.264880	3.078460	2.441150	2.734320	0.476890	1.497340	3.508700	2.213520	1.245540
2005	5.450470	3.736330	2.284620	4.258030	3.134320	2.516730	2.749200	0.454490	1.493440	3.593580	2.212090	1.235870
2006	5.372410	3.814800	2.321220	4.404170	3.209360	2.562170	2.759480	0.443600	1.488780	3.626360	2.207230	1.238990

Table E-5: Prices of Inventory Services, Land Services and Labour Inputs

	W_{KIV1}^t	W_{KIV2}^t	W_{KIV3}^t	W_{KIV4}^t	W_{LD1}^t	W_{LD2}^t	W_{LD3}^t	W_{LD4}^t	W_{LB1}^t	W_{LB2}^t	W_{LB3}^t
1955	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
1956	1.043440	1.043440	1.061550	1.043440	1.135470	1.270140	1.244560	1.230670	1.051630	1.051630	1.051630
1957	1.104180	1.104190	1.293860	1.104180	1.288190	1.631390	1.571420	1.583180	1.106040	1.106040	1.106040
1958	1.008100	1.008100	1.187050	1.008100	1.278250	1.871490	1.785990	1.812570	1.171500	1.171500	1.171500
1959	1.196480	1.196480	1.379930	1.196480	1.566800	2.784440	2.656850	2.573610	1.276470	1.276470	1.276470
1960	1.425300	1.425300	1.679750	1.425300	1.931170	4.889860	4.252750	4.101770	1.385160	1.385160	1.385160
1961	1.599870	1.574240	2.327070	1.585280	2.166240	7.303600	5.966420	5.854340	1.569720	1.569720	1.569720
1962	1.522540	1.465500	2.079910	1.486780	2.040700	8.362630	6.661360	6.492260	1.762160	1.762160	1.762160
1963	1.613500	1.530380	2.179840	1.560980	2.160570	10.128740	7.818580	7.725070	2.005080	2.005080	2.005080
1964	1.717140	1.619760	2.270420	1.654510	2.325780	12.016850	9.306510	9.297390	2.270120	2.270120	2.270120
1965	1.647410	1.540450	2.006850	1.580010	2.287510	11.832880	9.412880	9.436570	2.548200	2.548200	2.548200
1966	1.781710	1.658730	2.430620	1.699360	2.630230	12.915870	10.826490	10.761300	2.831780	2.831780	2.831780
1967	1.889920	1.750660	3.091250	1.785290	3.112180	14.536570	12.654850	12.867060	3.174850	3.174850	3.174850
1968	2.045230	1.870940	3.486630	1.891400	3.553510	17.232300	15.372530	15.973710	3.655630	3.655630	3.655630
1969	2.083160	1.898010	3.637120	1.900690	3.738480	19.732820	17.992710	19.083260	4.256680	4.256680	4.256680
1970	2.037990	1.850400	3.439130	1.839060	3.792670	21.853770	19.841280	21.687540	4.993850	4.993850	4.993850
1971	1.689130	1.523130	2.693710	1.501920	3.427970	20.778490	18.427540	20.803210	5.799770	5.799770	5.799770
1972	1.657300	1.526290	2.585020	1.482420	3.871070	23.079710	19.926660	23.835990	6.654600	6.654600	6.654600
1973	1.886700	1.706180	3.338750	1.740610	4.454890	27.259960	22.794220	28.875430	8.061310	8.061310	8.061310
1974	1.907800	1.693230	3.480970	1.778920	4.406940	23.623040	19.854000	25.636530	10.334070	10.334070	10.334070
1975	1.647420	1.469720	2.749260	1.541890	4.002770	19.682780	16.648130	21.702340	12.225790	12.225790	12.225790
1976	1.948290	1.727020	3.233080	1.809120	4.808830	21.819870	18.496890	24.682640	13.168570	13.168570	13.168570
1977	1.909690	1.691140	2.960590	1.727370	4.917040	21.186930	18.063160	24.849900	14.391590	14.391590	14.391590
1978	2.128160	1.858560	3.113320	1.922920	5.762680	23.525680	20.116500	28.776380	15.412520	15.412520	15.412520
1979	2.423090	2.129990	3.896530	2.355190	6.719480	26.715940	22.977900	34.554990	16.353280	16.353280	16.353280
1980	2.263380	2.036210	3.998700	2.345790	6.619480	26.085410	22.493770	35.622920	17.353530	17.353530	17.353530
1981	2.099820	1.895890	3.255290	2.191290	6.514820	25.445460	22.030920	36.129500	18.445300	18.445300	18.445300
1982	1.997340	1.806670	2.958250	2.048130	6.404970	25.134460	21.849260	36.504720	19.267130	19.267130	19.267130
1983	1.859660	1.657540	2.657310	1.858210	6.178410	24.255490	21.157290	35.658400	19.589510	19.589510	19.589510
1984	2.057540	1.798990	2.850640	2.031680	6.962820	27.329720	23.994040	40.212750	20.454430	20.454430	20.454430
1985	2.134220	1.839390	3.035610	1.958820	7.468590	29.542980	26.364940	43.459370	21.099150	21.099150	21.099150
1986	2.018290	1.713900	2.928790	1.651870	7.366850	29.941610	27.711480	44.516390	21.674870	21.674870	21.674870
1987	1.914660	1.615940	2.905170	1.521990	7.100350	31.150430	30.270820	46.693680	22.097420	22.097420	22.097420
1988	1.983820	1.670870	3.026100	1.573340	7.226160	34.306810	34.428140	50.844950	22.782910	22.782910	22.782910
1989	2.028980	1.702280	3.184420	1.646110	7.308000	38.864760	39.915930	57.033460	23.940750	23.940750	23.940750
1990	1.925880	1.607040	3.170560	1.560640	6.903300	40.492950	42.289090	59.310810	25.645820	25.645820	25.645820
1991	1.755410	1.457820	2.866360	1.391640	6.503750	38.182040	39.506590	55.201480	27.319490	27.319490	27.319490
1992	1.688670	1.398690	2.583530	1.310020	6.498810	35.382610	35.194900	50.527510	28.206060	28.206060	28.206060
1993	1.449980	1.194810	2.144730	1.098350	5.791880	30.781750	29.114040	43.851060	28.854180	28.854180	28.854180
1994	1.403520	1.150490	1.997840	1.052880	5.642240	29.634820	26.604400	41.967690	29.342760	29.342760	29.342760
1995	1.398960	1.138530	1.867270	1.065900	5.601670	29.534330	24.922760	42.033650	29.507420	29.507420	29.507420
1996	1.496620	1.204350	1.927880	1.165540	5.797960	30.872150	24.405080	44.063770	29.696380	29.696380	29.696380
1997	1.462750	1.169530	1.835550	1.132920	5.640450	29.694680	22.168570	42.613240	30.492160	30.492160	30.492160
1998	1.179070	0.932520	1.382110	0.892890	4.640500	24.246230	17.113960	35.183780	30.131830	30.131830	30.131830
1999	1.174830	0.916650	1.320130	0.904520	4.590610	23.464550	15.641170	33.935310	29.642000	29.642000	29.642000
2000	1.263410	0.970420	1.374550	1.001190	4.789800	24.058540	15.208670	34.946080	29.195500	29.195500	29.195500
2001	1.121900	0.858110	1.130760	0.623190	4.400390	21.310100	12.830460	31.314010	29.358540	29.358540	29.358540
2002	1.219940	0.929910	1.135470	0.677610	4.640480	21.655060	12.476130	32.190200	28.803100	28.803100	28.803100
2003	1.310760	0.993370	1.278020	0.737270	4.856630	21.156920	11.909550	32.077210	28.443330	28.443330	28.443330
2004	1.548720	1.169200	1.439040	0.909110	5.487130	22.369050	12.506160	34.599480	28.024610	28.024610	28.024610
2005	1.616290	1.214560	1.494870	0.944380	5.753020	21.938540	12.575080	34.622810	28.286970	28.286970	28.286970
2006	1.669090	1.248430	1.536550	0.970710	5.798470	21.336310	13.187400	34.798120	28.400300	28.400300	28.400300

Table E-6: Prices of Depreciations

	P_{DEP1}^{t}	P_{DEP2}^{t}	P_{DEP3}^{t}	P_{DEP4}^{t}	P_{DEP5}^{t}	P_{DEP6}^{t}	P_{DEP7}^{t}	P_{DEP8}^{t}	P_{DEP9}^{t}	P_{DEP10}^{t}	P_{DEP11}^{t}	P_{DEP12}^{t}
1955	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
1956	1.012540	1.077900	1.005470	1.024220	1.037650	1.102360	1.103620	0.973900	1.065050	1.235480	1.025110	0.948160
1957	1.075650	1.091770	1.075670	1.120340	1.093440	1.204020	1.305920	0.993290	1.111480	1.310730	1.033150	0.900770
1958	1.041520	1.030710	1.079880	1.069450	1.049060	1.115760	1.345570	1.041480	1.114470	1.220350	1.076740	0.865700
1959	1.054040	1.017810	1.064890	1.109270	1.076850	1.072490	1.300630	0.967800	1.152040	1.206240	1.050440	0.836640
1960	1.134610	1.017590	1.012050	1.151670	1.102500	1.118160	1.346710	0.951990	1.115010	1.179570	1.066270	0.810010
1961	1.222830	1.106210	1.074980	1.334550	1.186110	1.137610	1.366590	0.942980	1.101090	1.183690	1.039100	0.815730
1962	1.235310	1.133890	1.090300	1.338440	1.254180	1.101040	1.425320	0.951820	1.092980	1.232970	1.060250	0.817290
1963	1.334030	1.147580	1.117710	1.375880	1.258460	1.082990	1.385300	0.900060	1.089840	1.234250	1.076730	0.815260
1964	1.331800	1.172960	1.124440	1.375590	1.334870	1.108810	1.390880	0.874740	1.073480	1.254650	1.194120	0.830880
1965	1.457870	1.207020	1.163430	1.366360	1.365040	1.137360	1.400230	0.878990	1.076860	1.311960	1.188950	0.826310
1966	1.719320	1.264690	0.964150	1.463040	1.352050	1.149480	1.388890	0.858300	1.072700	1.342830	1.194180	0.818060
1967	2.033800	1.322240	1.180770	1.593090	1.351260	1.189900	1.421410	0.844420	1.068350	1.419850	1.208800	0.816060
1968	2.372390	1.360590	1.222950	1.671800	1.363790	1.198640	1.484920	0.853040	1.056100	1.411420	1.224130	0.826190
1969	2.641530	1.436730	1.353300	1.743150	1.367160	1.234930	1.556670	0.872640	1.048870	1.500390	1.309340	0.833620
1970	2.791870	1.524390	1.281700	1.828270	1.481360	1.297460	1.631160	0.891490	1.061370	1.593860	1.394280	0.846800
1971	2.724000	1.573590	1.381190	1.858320	1.528890	1.336630	1.693200	0.888540	1.088280	1.696460	1.461410	0.866430
1972	2.700430	1.682480	1.373860	2.145010	1.605000	1.353920	1.732220	0.860030	1.110360	1.721860	1.499520	0.883520
1973	3.080060	2.052020	1.565850	3.035460	1.970390	1.515950	1.920580	0.874440	1.149870	2.212220	1.604430	0.952510
1974	3.329620	2.469610	1.794600	3.348610	2.558530	2.107650	2.457810	1.033680	1.424800	2.290580	1.988690	1.146970
1975	3.784020	2.592300	1.890020	3.320150	2.566820	2.126510	2.521840	1.046660	1.480330	2.782010	2.088380	1.167850
1976	4.123850	2.743330	2.036270	3.511280	2.623010	2.196970	2.538620	1.036720	1.509610	2.833170	2.139430	1.178570
1977	4.425620	2.874430	2.126520	3.548130	2.818240	2.322370	2.640110	1.054360	1.555630	3.206470	2.225760	1.195710
1978	4.542370	3.011400	2.098710	3.299570	2.799320	2.379640	2.666090	1.024750	1.570550	3.199620	2.223550	1.189060
1979	4.618700	3.270220	2.313720	4.002050	3.046810	2.439250	2.746910	1.033680	1.593930	3.151150	2.293310	1.247690
1980	4.813760	3.575080	2.391950	4.268870	3.278250	2.657470	2.888340	1.065500	1.620720	3.582660	2.403220	1.355250
1981	5.147220	3.668540	2.452900	3.967090	3.303930	2.708130	2.947780	1.062470	1.637700	3.671750	2.416060	1.372830
1982	5.359320	3.697660	2.482050	4.247750	3.311590	2.688420	2.966550	1.053740	1.642300	4.062040	2.450940	1.394930
1983	5.509280	3.718020	2.465780	4.258860	3.300330	2.712890	2.957010	1.030810	1.633450	4.289910	2.425830	1.392140
1984	5.684700	3.768130	2.469310	4.387830	3.329390	2.778820	2.956490	1.009500	1.626160	3.808370	2.408160	1.383930
1985	5.797680	3.781160	2.472410	4.536430	3.330400	2.779590	2.960750	0.971180	1.611660	3.792120	2.406190	1.354450
1986	5.823630	3.760630	2.418010	4.228220	3.326640	2.743710	2.933880	0.914340	1.602450	3.590090	2.381550	1.314900
1987	5.681080	3.810870	2.412800	4.524080	3.329350	2.680790	2.899250	0.865750	1.591110	3.502070	2.397490	1.306960
1988	5.732290	3.883720	2.419200	4.363200	3.361990	2.735470	2.918020	0.845190	1.590540	3.466040	2.412670	1.303470
1989	5.910210	4.055390	2.472360	4.576510	3.440520	2.838800	2.986900	0.845100	1.598220	3.492650	2.430030	1.289330
1990	6.052380	4.220370	2.541240	4.653630	3.546790	2.902360	3.064640	0.844450	1.602710	3.556730	2.484700	1.291110
1991	5.875480	4.339070	2.575550	4.596550	3.622740	2.967660	3.120240	0.826190	1.606970	3.725850	2.537150	1.294960
1992	5.645630	4.401090	2.533580	4.570020	3.675280	2.970680	3.116070	0.805300	1.616940	3.679010	2.533720	1.245950
1993	5.393090	4.436340	2.576890	5.016420	3.666660	2.966510	3.103460	0.789470	1.615220	3.854190	2.540680	1.246530
1994	5.175650	4.439310	2.484800	4.816510	3.608760	2.915560	3.070990	0.763550	1.602780	3.714920	2.507750	1.223200
1995	4.607230	4.456350	2.460790	4.647550	3.589890	2.922410	3.041240	0.722340	1.591470	3.707580	2.485980	1.220550
1996	4.841410	4.446780	2.509200	4.796040	3.558000	2.880110	3.010340	0.669960	1.569100	3.672760	2.456230	1.204530
1997	5.006680	4.494470	2.566140	4.928620	3.569510	2.899350	3.038440	0.646310	1.573940	3.756050	2.478940	1.195080
1998	5.043500	4.444300	2.576230	4.610880	3.596360	2.890280	3.032570	0.628530	1.570380	3.841270	2.478720	1.184780
1999	5.310730	4.365580	2.479030	4.637770	3.551260	2.831410	2.975160	0.595910	1.553810	3.664570	2.457200	1.160100
2000	5.370230	4.382820	2.453700	4.642980	3.546550	2.818890	2.960560	0.566250	1.550790	3.579210	2.467160	1.142180
2001	5.346700	4.319450	2.418090	4.589740	3.533600	2.792250	2.922370	0.515200	1.524590	3.434580	2.448600	1.118300
2002	5.271300	4.244090	2.376880	4.497430	3.489400	2.753260	2.862470	0.478200	1.492950	3.494610	2.405850	1.084100
2003	5.465680	4.252820	2.363010	4.497180	3.471790	2.751480	2.824660	0.442490	1.474210	3.628940	2.379480	1.041710
2004	5.800240	4.295870	2.360650	4.533270	3.460920	2.788660	2.808280	0.416080	1.467550	3.752950	2.360170	1.020550
2005	5.791830	4.342500	2.379370	4.507180	3.500240	2.848110	2.807690	0.391420	1.461490	3.828880	2.348200	1.004110
2006	5.694080	4.428860	2.412450	4.648140	3.565960	2.880390	2.810330	0.379420	1.462480	3.861720	2.335950	1.000780

Table E-7: Prices of Waiting Capital Services

	W_{KW1}^{t}	W_{KW2}^{t}	W_{KW3}^{t}	W_{KW4}^{t}	W_{KW5}^{t}	W_{KW6}^{t}	W_{KW7}^{t}	W_{KW8}^{t}	W_{KW9}^{t}	W_{KW10}^{t}	W_{KW11}^{t}	W_{KW12}^{t}
1955	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
1956	1.009270	1.074030	1.002220	1.020920	1.034280	1.098800	1.108370	0.970160	0.949820	1.265030	1.022010	0.971160
1957	1.119030	1.137240	1.119080	1.165550	1.137620	1.252610	1.422200	1.114870	1.004040	1.496780	1.076280	0.990410
1958	1.009500	1.004300	1.046670	1.036570	1.016920	1.081450	1.431740	1.148200	0.972120	1.289630	1.041000	0.901990
1959	1.196660	1.153220	1.208980	1.259350	1.222650	1.217600	1.637870	1.217760	1.139260	1.510440	1.184050	1.044060
1960	1.553490	1.374880	1.379490	1.569800	1.502990	1.524120	2.006180	1.445030	1.293610	1.736460	1.442920	1.260090
1961	1.877770	1.677120	1.643410	2.040240	1.813870	1.739160	2.267970	1.607460	1.409780	1.914120	1.576970	1.459520
1962	1.782140	1.614370	1.565880	1.922260	1.802020	1.581300	2.228260	1.522000	1.315460	1.825860	1.513670	1.403610
1963	2.018020	1.706720	1.683240	2.072030	1.895910	1.630950	2.246990	1.455940	1.351480	1.872840	1.615750	1.500240
1964	2.116840	1.833640	1.779250	2.176660	2.113190	1.754530	2.382740	1.468120	1.402220	1.984310	1.883680	1.631570
1965	2.160770	1.769120	1.716710	2.016140	2.015150	1.678240	2.253430	1.405980	1.348000	1.915460	1.750050	1.535780
1966	2.693750	1.945860	1.498260	2.273520	2.095550	1.786250	2.384570	1.433330	1.457120	2.085030	1.856000	1.644090
1967	3.307520	2.095120	1.899610	2.562930	2.161970	1.914300	2.525980	1.451580	1.500970	2.258770	1.948930	1.728410
1968	4.040590	2.247880	2.056490	2.811280	2.274520	2.015620	2.715920	1.549450	1.568840	2.311170	2.062940	1.870830
1969	4.331910	2.310820	2.187910	2.818200	2.186330	1.996540	2.730750	1.553040	1.535670	2.378750	2.119420	1.843920
1970	4.326150	2.327930	1.955030	2.788750	2.228820	1.979070	2.698260	1.524660	1.479730	2.399540	2.128110	1.776880
1971	3.419380	1.983810	1.714130	2.306260	1.872920	1.658820	2.341660	1.255140	1.317240	2.151390	1.812110	1.552680
1972	3.058020	1.916400	1.547680	2.416390	1.786170	1.525210	2.219870	1.140420	1.319850	2.054380	1.686860	1.502530
1973	3.224920	2.198120	1.643780	3.186530	2.046210	1.591400	2.300470	1.093890	1.373670	2.514430	1.674360	1.516840
1974	3.006490	2.291750	1.639690	3.059560	2.314470	1.925720	2.554760	1.144460	1.658140	2.314220	1.795680	1.791000
1975	2.793790	2.030420	1.426450	2.505810	1.919350	1.604940	2.267240	1.005890	1.636760	2.429840	1.556410	1.589150
1976	3.394860	2.370630	1.718200	2.962800	2.185050	1.853790	2.586700	1.130570	1.920170	2.758140	1.782470	1.824760
1977	3.457990	2.384320	1.707630	2.849200	2.226040	1.864890	2.646010	1.129970	2.037390	3.072670	1.762480	1.810800
1978	3.831670	2.700130	1.824240	2.868050	2.384440	2.068430	2.900970	1.223490	2.209320	3.325780	1.908290	2.105700
1979	4.203990	3.146980	2.175790	3.763480	2.797090	2.293840	3.247080	1.355670	2.365200	3.671890	2.128260	2.389680
1980	3.979260	3.155060	2.048180	3.655360	2.730410	2.275540	3.247170	1.321320	2.315120	3.959180	2.025910	2.392040
1981	3.936760	3.007910	1.935300	3.129970	2.538500	2.136670	3.144070	1.260660	2.227160	3.853320	1.873610	2.201420
1982	3.880660	2.882400	1.846920	3.160800	2.402420	2.000480	3.015900	1.201270	2.163530	4.047820	1.791960	2.117700
1983	3.731760	2.725550	1.710250	2.953920	2.234500	1.881640	2.898840	1.132840	2.153580	4.114890	1.653950	1.969500
1984	4.275050	3.027710	1.895200	3.367680	2.497630	2.132750	3.151000	1.226220	2.299340	3.926910	1.816510	2.202840
1985	4.625730	3.203300	2.007030	3.682540	2.646040	2.256380	3.268910	1.258350	2.306150	4.007300	1.919300	2.404090
1986	4.587530	3.140110	1.927190	3.369960	2.597200	2.186780	3.170450	1.199320	2.310220	3.643090	1.868600	2.314670
1987	4.322260	3.097370	1.847470	3.464060	2.496680	2.052660	3.025570	1.125480	2.265260	3.386640	1.806190	2.209210
1988	4.482800	3.222160	1.894450	3.416780	2.577510	2.142120	3.078080	1.129550	2.272180	3.327140	1.857070	2.284140
1989	4.687900	3.405020	1.954490	3.617890	2.655020	2.244170	3.034940	1.114650	2.203820	3.208340	1.886310	2.364300
1990	4.519230	3.350720	1.882920	3.448090	2.553560	2.150500	2.902870	1.046330	2.126250	3.054090	1.808290	2.260750
1991	4.033760	3.176150	1.743760	3.112060	2.381980	2.009230	2.731490	0.951880	1.983700	3.007390	1.687040	2.096370
1992	3.809250	3.080720	1.674910	3.021160	2.365680	1.963870	2.653200	0.914160	1.943490	2.906990	1.644960	2.014470
1993	3.224350	2.779630	1.499130	2.918350	2.081000	1.725790	2.403370	0.809200	1.826020	2.832930	1.451570	1.725290
1994	3.058270	2.720030	1.418250	2.749120	2.016320	1.664120	2.430840	0.794160	1.837090	2.822710	1.406030	1.626270
1995	2.765130	2.764450	1.415600	2.673550	2.024060	1.681150	2.455190	0.781460	1.863870	2.906920	1.404340	1.604610
1996	3.078410	2.938130	1.537160	2.938110	2.137520	1.764390	2.544090	0.793970	1.867510	3.015140	1.477810	1.707510
1997	3.113620	2.891510	1.544790	2.966980	2.106670	1.745380	2.511420	0.767560	1.791640	3.024730	1.465830	1.685990
1998	2.599540	2.443380	1.290890	2.310410	1.765170	1.448260	2.186720	0.648720	1.633870	2.762140	1.218940	1.378340
1999	2.744340	2.415160	1.250310	2.339080	1.755700	1.428030	2.162620	0.634120	1.636760	2.630940	1.216340	1.370440
2000	2.959490	2.554150	1.324600	2.506460	1.875230	1.521740	2.242920	0.652440	1.658560	2.660260	1.306690	1.473670
2001	2.729000	2.365540	1.209010	2.294810	1.730540	1.396080	2.092160	0.586630	1.587790	2.458140	1.200760	1.326620
2002	2.921200	2.516170	1.290310	2.441460	1.854360	1.494620	2.158680	0.601160	1.576010	2.605150	1.280710	1.425320
2003	3.210640	2.627160	1.359730	2.587790	1.954700	1.583270	2.212280	0.604950	1.563690	2.770080	1.342620	1.474870
2004	3.920430	3.028170	1.563010	3.001520	2.240540	1.846400	2.431560	0.657070	1.595140	3.076050	1.532290	1.710530
2005	4.047260	3.166480	1.628730	3.085260	2.344040	1.949590	2.489530	0.658330	1.595870	3.179830	1.576360	1.760010
2006	4.071540	3.270770	1.689780	3.255760	2.448640	2.017550	2.521140	0.659410	1.578860	3.213010	1.603460	1.808370

Table E-8: Quantities of Investment Goods

	Y_{I1}^{t}	Y_{I2}^{t}	Y_{I3}^{t}	Y_{I4}^{t}	Y_{I5}^{t}	Y_{I6}^{t}	Y_{I7}^{t}	Y_{I8}^{t}	Y_{I9}^{t}	Y_{I10}^{t}	Y_{I11}^{t}	Y_{I12}^{t}
1955	13.31	1127.17	1.36	0.33	3.61	7.84	191.6	77.64	123.58	46.57	19	6.53
1956	15.41	1321.52	1.82	0.43	4.72	9.91	273.11	115.65	160.94	54.84	22.91	7.97
1957	17.87	1556.83	2.33	0.55	6.23	12.18	329.58	156.46	217.02	74.66	29.88	9.8
1958	19.62	1594.4	2.5	0.72	7.39	11.51	329.5	170.66	258.94	99.6	30.78	11.07
1959	20.26	1957.15	3.09	0.86	8.82	13.97	395.72	234.4	237.09	91.92	39.05	15.22
1960	15.2	2523.88	4.88	1.26	12.71	18.79	559.97	330.79	376.38	137.31	53.16	18.35
1961	17.89	2896.81	6.76	1.55	17.54	31.2	774.78	434.83	536.51	196.97	75.7	27.16
1962	23.97	3211.32	8.99	2.04	22.24	41.7	854.5	509.06	611.19	222.5	86.76	42.22
1963	25.63	3616.49	11.46	2.41	28.86	50.13	917.57	543.91	701.46	257.25	111.14	45.34
1964	34.83	4182.81	15.55	3.44	36.38	66.28	996.48	657.55	878.3	316.36	123.23	52.93
1965	39.48	4352.3	17.28	4.05	41.47	83.41	950.8	700.54	895.1	338.7	138.06	73.64
1966	36.23	4856.22	25.14	4.76	59.83	86.52	1099.18	837.89	1130.29	392.39	160.22	83.56
1967	36.97	5485.82	24.26	5.64	84.32	92.38	1471.96	1121.52	1374.32	416.32	205.22	130.29
1968	32.65	6282.25	26.46	6.49	117.38	108.57	1878.51	1437.94	1708.52	492.27	243.8	233.38
1969	30.72	7200.59	27.28	7.47	172.08	163.21	2250.56	1857.71	2046.25	513.89	278.62	293.24
1970	31.34	8149.89	30.91	8.6	216.46	212.76	2739.22	2439.49	2334.42	520.56	310.39	346.22
1971	33.15	8599.94	29.85	8.32	218.45	198.58	2749.95	2586.09	2318.14	503.48	356.33	455.95
1972	37.43	9509.51	33.25	7.99	215.37	213.03	2683.29	3050.3	2590.81	566.52	418.25	529.31
1973	39.36	10524.4	35.01	6.96	205.48	271.97	3221.84	3766.99	2741.18	475.2	516.41	541.41
1974	32.02	10284.79	34.48	7.05	171.3	238.19	2919.41	3716.83	2024.94	431.2	528.76	517.19
1975	26.89	10477.63	30.8	6.58	154.7	222.55	2588.4	3568.89	2050.73	367.61	529.78	539.45
1976	45.41	10745.5	33.05	7.32	177.79	213.96	2685.96	3882.86	2035.17	374.64	562.15	788.96
1977	38.51	11020.3	34.04	7.78	176.59	219.3	2689.41	4337.08	2125.31	371.05	576.46	730.96
1978	23.34	11616.12	35.72	7.96	183.22	233.76	2774.56	5061.44	2548.48	467.36	614.17	1130.96
1979	32.13	12033.37	40.76	8.15	211.02	278.79	2986.79	5460.94	2793.01	558.19	687.16	1134.87
1980	33.03	11899.16	41.58	7.69	207.86	248.55	3013.99	5919.69	2754.88	497.41	698.73	1071.33
1981	32	11837.65	50.89	7.41	205.09	260.33	3361.1	6358.72	2748.64	574.22	707.87	1190.94
1982	33.02	11767.59	60.88	6.36	210.02	242.12	3487.19	6940.57	2490.05	507.33	707.72	1323.1
1983	30.6	11300.33	66.07	5.4	197.73	231.32	3536.13	7430.42	2478.86	555.78	723.41	1460.81
1984	31.42	11232.55	77.72	4.74	202.1	242.29	4087.09	8942.98	2326.53	684.74	766.38	1596.98
1985	35.3	11720.86	96.1	4.26	224.54	244.36	4418.71	11244.79	2597.56	810.69	869.08	1769.21
1986	35.35	12415.71	107	4.62	248.4	236.66	4453.08	12393.72	2804.9	844.32	924.34	2203.22
1987	37.33	13364.85	118.96	4.65	287.82	211.41	4532.37	13732.43	3231.38	945.51	984.46	2374.03
1988	38.87	14658.46	146.5	5.32	349.36	229.07	5385.86	15780.72	3596.63	893.23	1088.22	3144.82
1989	42.95	15698.06	170.35	5.64	418.15	226.45	6171.23	17293.44	4122.26	882.45	1198.65	3874.34
1990	43.28	16771.48	198.34	5.99	485.88	218.2	7100	18611.7	4772.73	847.61	1312.6	4041.83
1991	42.04	17186.52	195.66	6.36	457.69	208.95	7348.48	19389.53	4591.9	757.88	1380.23	4711.3
1992	40.61	16808.47	213.06	6.41	426.8	187.45	6869.43	19047.24	4595.64	700.76	1402.5	5030.3
1993	42.18	16597.09	197.34	5.39	364.59	166.74	6052.29	18846.8	4435.1	589.87	1274.54	5020.06
1994	42.33	16483.61	190.4	5.09	312.53	147.87	5703.26	18527.61	4376.74	556.7	1200.02	5064.57
1995	42	15477.06	186.58	5.01	274.93	142.24	6309.77	21813.88	4997.97	578.16	1226.66	5161.61
1996	39.24	15583.72	169.28	4.63	263.43	153.47	6801.42	26559.08	5273.38	591.51	1404.73	5304.91
1997	36.57	15449.48	172.65	4.8	270.47	155.39	7259.99	27212.56	5145.99	573.85	1503.32	5257.9
1998	34.81	14220.84	144.04	4.73	241.06	150.98	6831.66	25124.37	4237.95	454.88	1525.15	5802.81
1999	32.14	13862.71	147.62	4.41	225.24	157.14	6659.38	25962.21	4641.87	517.11	1536.68	5837.14
2000	35.12	13423.6	142.34	4.4	217.26	164.5	6889.77	27384.46	4921.56	539.34	1516.06	6198.2
2001	35.41	13017.41	133.44	4.35	209.43	164.48	6923.07	28403.62	5020.97	605.61	1534.74	6541.86
2002	34.08	12642.58	132.24	4.18	202.73	138.22	6124.06	25101.46	5274.53	636.25	1453.39	6649.7
2003	31.05	12187.11	134.47	3.91	194.36	131.54	6167.31	26961.09	5550.62	640.41	1466.72	6758.98
2004	28.54	12182.13	132.88	3.84	191.59	127.37	6821.78	28356.53	5289.34	580.96	1502.48	6869.98
2005	29.88	11994.6	137.88	4.03	198.46	132.24	7721.56	31976.57	5716.31	581.64	1587.16	7280.61
2006	29.67	12061.59	141.15	4.08	203.19	137.75	7965.23	33396.38	5612.65	569.32	1663.2	7399.95

Table E-9: Quantities of Capital Services

	X_{K1}^{t}	X_{K2}^{t}	X_{K3}^{t}	X_{K4}^{t}	X_{K5}^{t}	X_{K6}^{t}	X_{K7}^{t}	X_{K8}^{t}	X_{K9}^{t}	X_{K10}^{t}	X_{K11}^{t}	X_{K12}^{t}
1955	85.33	1430.70	3.20	0.16	8.80	20.57	256.78	60.04	44.95	134.66	10.30	15.28
1956	64.05	1448.85	2.60	0.21	8.10	19.87	256.39	66.34	70.44	120.05	12.01	11.79
1957	50.01	1485.96	2.38	0.29	7.71	19.51	273.48	79.34	102.99	111.96	14.42	10.00
1958	40.99	1544.64	2.45	0.38	7.65	19.49	298.88	98.78	144.60	110.19	17.72	9.31
1959	35.36	1597.51	2.58	0.49	7.78	19.38	318.23	119.43	187.54	113.39	20.10	9.12
1960	31.84	1687.78	2.87	0.62	8.18	19.62	348.36	150.42	215.48	115.92	23.60	9.64
1961	28.09	1865.08	3.77	0.84	9.16	20.49	406.50	195.32	279.13	126.32	28.95	10.82
1962	26.12	1994.37	5.08	1.08	10.78	22.97	495.97	254.14	375.53	143.32	37.24	13.02
1963	26.37	2142.88	6.72	1.41	12.92	26.66	585.83	319.87	471.20	160.34	44.62	16.69
1964	27.08	2307.36	8.64	1.77	15.73	31.18	674.42	382.53	568.57	180.11	54.12	20.72
1965	30.10	2499.17	11.63	2.33	19.49	37.51	765.58	459.22	698.14	204.01	62.30	25.55
1966	33.80	2691.31	14.23	2.94	23.56	45.62	835.33	535.91	802.73	226.48	70.86	32.20
1967	36.08	2882.99	18.78	3.61	29.77	53.46	922.60	628.76	946.31	254.44	79.37	41.35
1968	38.23	3130.47	21.45	4.37	39.05	61.42	1072.04	767.02	1128.33	284.20	92.85	55.62
1969	38.89	3405.93	23.96	5.20	52.18	70.90	1276.36	952.17	1356.62	321.78	107.98	79.38
1970	39.50	3733.37	26.02	6.11	74.00	86.98	1515.02	1197.16	1625.77	359.82	123.05	111.88
1971	40.54	4082.44	28.64	7.12	101.05	108.42	1814.09	1529.95	1904.14	390.97	138.01	151.69
1972	42.06	4448.46	29.88	7.82	125.49	126.09	2072.56	1826.99	2100.64	411.45	156.07	203.57
1973	44.69	4845.87	31.53	8.26	146.39	144.21	2281.19	2187.84	2273.98	438.17	175.22	299.64
1974	47.38	5332.15	32.70	8.30	161.59	168.78	2566.99	2596.38	2406.15	444.03	201.14	365.09
1975	46.04	5774.01	33.11	8.36	170.34	186.66	2743.31	2953.54	2376.95	455.32	225.12	410.02
1976	43.83	6228.43	32.04	8.27	175.31	200.88	2842.01	3238.77	2378.60	456.51	240.20	465.98
1977	48.49	6656.65	32.03	8.41	183.12	212.71	2926.11	3527.56	2346.29	451.06	254.87	580.47
1978	48.70	7081.64	32.27	8.65	189.27	224.24	3012.82	3874.41	2343.05	453.93	273.68	690.15
1979	44.27	7519.16	32.97	8.88	196.33	236.70	3101.39	4324.16	2495.89	475.38	295.82	852.96
1980	44.06	7959.03	35.41	9.12	206.02	254.25	3214.64	4721.83	2638.96	500.25	318.67	993.21
1981	44.67	8388.15	39.63	9.16	214.84	266.20	3324.61	5184.01	2838.72	513.76	347.16	1071.70
1982	43.63	8796.46	46.05	9.12	222.41	278.73	3476.82	5616.34	2939.23	541.16	372.87	1195.66
1983	43.71	9166.97	54.17	8.79	230.58	287.72	3626.21	6076.23	2974.63	546.89	392.65	1366.45
1984	43.11	9534.95	61.54	8.26	236.35	294.47	3765.12	6576.93	3019.63	563.09	416.51	1556.60
1985	42.80	10193.62	71.02	7.97	247.44	302.33	4008.37	7781.29	3084.00	597.51	447.41	1760.07
1986	44.23	10634.37	84.47	7.38	258.11	309.78	4258.57	9072.51	3200.68	647.97	491.14	2003.33
1987	45.31	11212.70	97.76	6.96	270.35	326.25	4478.57	10189.07	3341.75	740.73	539.18	2388.96
1988	46.81	11733.75	111.05	6.65	287.83	330.32	4684.14	11386.40	3568.61	808.48	592.13	2733.69
1989	48.39	12220.39	130.63	6.60	312.84	333.11	5023.30	12795.04	3845.83	855.51	654.54	3198.15
1990	50.93	12784.89	152.88	6.66	347.49	335.31	5449.35	14218.29	4229.34	884.78	727.17	3819.23
1991	52.35	13398.23	178.52	6.80	388.27	336.11	5970.94	15615.26	4685.29	900.41	811.33	4259.11
1992	52.52	14107.92	194.22	7.02	417.84	335.68	6456.13	16914.28	4998.04	899.85	893.84	4845.83
1993	52.24	14581.28	211.37	7.19	439.99	332.36	6770.31	17509.78	5240.49	882.37	957.21	5425.35
1994	52.49	15063.94	216.34	7.04	447.56	326.52	6861.65	17982.41	5373.15	846.41	978.31	5755.96
1995	53.12	15478.78	216.84	6.83	446.01	318.62	6880.58	18492.73	5486.88	815.87	977.79	5926.21
1996	53.36	15785.76	215.65	6.65	437.55	310.54	7015.93	19593.80	5749.04	795.60	986.88	6046.92
1997	52.46	16090.43	208.01	6.41	427.58	304.64	7220.87	21483.86	6030.31	783.00	1034.02	6259.92
1998	51.02	16306.47	204.36	6.27	420.07	299.63	7485.04	23057.45	6227.03	769.44	1082.30	6582.59
1999	49.25	16535.69	190.64	6.14	408.75	294.46	7641.50	23623.55	6150.79	736.82	1130.58	7270.61
2000	46.91	16770.98	183.09	5.96	394.50	290.56	7742.13	23971.79	6198.08	719.13	1163.38	7908.00
2001	46.35	16874.29	176.09	5.81	382.76	287.98	7860.95	25441.19	6316.06	709.12	1183.49	8502.46
2002	46.04	17023.40	167.99	5.68	370.70	285.59	7950.02	26557.09	6436.69	711.73	1208.75	9158.55
2003	45.29	17225.37	162.20	5.54	358.18	279.82	7901.27	26406.99	6605.18	722.29	1208.23	9824.96
2004	43.62	17415.24	159.31	5.35	345.93	273.66	7875.96	27190.85	6834.61	732.41	1214.45	10322.71
2005	41.54	17615.07	156.79	5.19	334.60	267.46	7978.86	27933.67	6959.51	731.65	1233.24	10734.51
2006	40.61	17867.91	157.13	5.12	326.23	262.47	8232.61	29280.86	7183.83	731.96	1263.74	11186.19

Table E-10: Quantities of Inventory Services, Land Services and Labour Inputs

	X_{KIV1}^t	X_{KIV2}^t	X_{KIV3}^t	X_{KIV4}^t	X_{LD1}^t	X_{LD2}^t	X_{LD3}^t	X_{LD4}^t	X_{LB1}^t	X_{LB2}^t	X_{LB3}^t
1955	75.44	14.53	130.5	29.89	1020.84	59.87	194.3	74.34	637.74	224.27	2877.08
1956	85.55	16.95	128.93	36.79	1020.84	59.87	194.3	76.98	647.11	215.8	3194.8
1957	95.53	22.45	127.28	46.25	1019.82	60.39	196.25	80.81	666.08	214.66	3451.28
1958	106.8	26.49	125.43	55.68	1019.32	61.07	198.58	84.17	647.68	209.95	3618.58
1959	115.9	24.22	124.22	57.48	1017.92	62.23	202.89	88.33	649.17	203.86	3767.36
1960	128.51	28.71	122.57	66.59	1018.5	62.24	204.18	91.73	656.23	198.58	4023.32
1961	147.67	34.44	120.69	73.87	1018.36	64.33	211.85	97.56	628.18	191.59	4207.68
1962	178.03	44.65	118.54	87.16	1016.4	66.39	219.68	102.81	579.27	181.19	4480.87
1963	195.39	48.46	116.77	91.12	1016.33	68.13	226.05	108.54	569.23	171.99	4568.48
1964	215.25	54.68	114.82	102.68	1016.38	69.87	232.5	114.47	568.61	167.62	4718.43
1965	241.18	64.19	111.75	111.2	1016.8	78.6	234.35	121.69	555.17	159.28	4894.8
1966	266.37	66.64	107.94	118.04	1017.93	78.6	273.82	128.87	561.19	155.07	5117.88
1967	292.88	74.68	104.5	127.03	1017.63	78.6	313.61	135.57	575.24	154.92	5265.48
1968	334.28	90.32	101.67	142.49	1019.48	87.33	314.66	143.24	579.44	150.93	5342.57
1969	385.33	100.98	100.02	153.97	1019.94	87.33	353.45	150.44	584.91	147.88	5414.86
1970	428.77	119.64	98.65	171.81	1020.4	96.07	354.52	162.71	573.35	140.36	5570.05
1971	477.08	139.41	96.62	182.48	1021.88	104.8	354.26	173.68	555.38	129.47	5685.35
1972	492.58	146.21	94.91	177.63	1022.93	113.54	354.95	186.69	557.72	123.79	5751.78
1973	503.83	150.31	92.4	177.46	1022.29	113.54	394.64	200.24	567.35	114.85	5955.78
1974	531.64	174.59	90.73	205.61	1021.81	118.78	415.5	211.8	550.49	108.09	5848.43
1975	585.47	188.26	90.21	215.79	1022.04	123.14	437.29	224.55	531.23	105.48	5699.43
1976	578.26	179.63	89.4	211.26	1022.24	124.02	453.07	233.76	533.72	104.19	5988.87
1977	595.1	185.11	89.21	213.82	1022.08	126.64	487.55	241.16	540.82	104.96	6130.52
1978	592.25	181.12	87.83	212.3	1021.35	126.64	510.47	249.05	556.28	107.16	6186.72
1979	597.29	171.72	87.13	198.75	1020.83	126.64	526.67	257.06	553.53	103.93	6305.62
1980	660.7	181.27	86.37	228.69	1020.2	127.51	546.13	263.93	541.27	98.69	6456.34
1981	704.32	206.48	85.49	234.66	1019.43	127.51	553.09	270.46	533.05	96.55	6603.2
1982	720.17	220.29	84.52	229.88	1018.59	128.38	568.35	276.05	534.65	95.58	6700.19
1983	738.43	215.17	83.88	231.71	1017.88	130.13	579.01	281.9	526.45	93.51	6953.82
1984	727.86	214.67	83.71	221.84	1018.24	130.13	578.57	285.34	514.58	91.76	7034.67
1985	739.41	230.08	83.52	225.94	1019.67	130.13	607.49	286.59	512.6	89.83	7139.19
1986	752.1	235.93	82.62	229.91	1018.6	130.13	606.81	290.5	512.68	88.12	7243.95
1987	747.67	225.91	81.4	222.94	1018.24	130.13	622.01	292.56	513.33	87.94	7368.89
1988	754.07	225.34	80.25	235.79	1017.88	138.8	636.24	291.61	508.61	86.16	7574.04
1989	769.2	234.58	78.63	232.13	1017.16	138.8	649.41	293.99	499.87	84.93	7744.14
1990	815.56	250.54	76.37	238.03	1017.52	138.8	662.92	298.46	484.44	82.3	7861.59
1991	823.16	281.1	74.21	251.44	1017.88	138.8	662.51	305.85	467.64	76.85	8001.62
1992	841.84	300.37	73.75	236.16	1018.06	143.14	676.78	309.57	452.15	71.77	7999.44
1993	851.58	307.09	72.09	221.58	1018.24	147.48	690.38	313.82	428.95	66.17	7983.4
1994	840.34	306.1	71.14	199.63	1018.96	147.48	710.18	316.47	420.61	64.11	8003.58
1995	837.66	298.06	70.27	180.27	1019.67	147.48	716.23	322.18	414.01	62.28	8087.25
1996	858.69	313.76	69.17	184.2	1020.03	147.48	730.3	327.49	402.31	59.6	8129.86
1997	896.01	324.54	67.62	190.31	1020.39	147.48	744.08	332.65	402.32	57.78	8100.96
1998	938.9	325.22	66.48	196.15	1020.39	147.48	758.97	340.67	388.04	55.76	8059.33
1999	931.18	343	66.73	185.75	1021.47	147.48	759.91	345.76	383.34	53.96	8003.24
2000	858.92	355.67	65.97	173.49	1021.83	147.48	774.69	349.8	372.88	51.36	8171.31
2001	843.34	385.73	66.45	172.02	1021.47	147.48	790.17	351.28	353.88	48.82	8053.7
2002	846.27	313.05	67.48	168.9	1021.47	147.48	790.88	355.34	343	46.33	7971.28
2003	827.87	303.81	67.1	163.88	1021.83	147.48	805.39	359.53	334.11	43.96	7964.25
2004	816.72	269.22	66.78	159.71	1022.55	138.8	805.61	363.67	332.73	42.79	8020.87
2005	776.4	263.59	66.93	170.37	1023.26	138.8	805.68	364.86	329.32	41.29	8032.98
2006	863.51	251.54	63.87	188.48	1024.93	138.8	817.52	371.71	317.59	35.07	8152.53

Table E-11: Quantities of Depreciations

	Y_{DEP1}^{t}	Y_{DEP2}^{t}	Y_{DEP3}^{t}	Y_{DEP4}^{t}	Y_{DEP5}^{t}	Y_{DEP6}^{t}	Y_{DEP7}^{t}	Y_{DEP8}^{t}	Y_{DEP9}^{t}	Y_{DEP10}^{t}	Y_{DEP11}^{t}	Y_{DEP12}^{t}
1955	-72.66	-557.04	-2.8	-0.13	-6.45	-12.98	-175.24	-40.43	-32.51	-92.99	-8.46	-12.28
1956	-54.36	-564.94	-2.27	-0.18	-5.91	-12.54	-174.22	-44.95	-50.96	-81.8	-9.85	-9.1
1957	-42.27	-578.03	-2.08	-0.24	-5.6	-12.31	-185.62	-54.26	-74.54	-75.55	-11.82	-7.37
1958	-34.47	-598.33	-2.14	-0.31	-5.52	-12.3	-202.75	-68.47	-104.57	-73.77	-14.53	-6.57
1959	-29.57	-618.25	-2.25	-0.41	-5.57	-12.23	-215.62	-84.05	-135.52	-75.52	-16.47	-6.15
1960	-26.49	-648.18	-2.51	-0.51	-5.82	-12.38	-236.32	-107.61	-155.58	-77.21	-19.33	-6.19
1961	-23.26	-693.53	-3.29	-0.69	-6.45	-12.93	-276.68	-141.44	-201.29	-84.37	-23.72	-6.74
1962	-21.52	-747.86	-4.43	-0.89	-7.5	-14.5	-338.85	-185.78	-270.44	-95.24	-30.52	-7.93
1963	-21.59	-806.43	-5.88	-1.16	-8.92	-16.83	-401.13	-236.26	-339.13	-105.85	-36.58	-9.94
1964	-22.04	-871.75	-7.55	-1.46	-10.79	-19.68	-462.85	-285.92	-408.27	-118.27	-44.39	-12.31
1965	-24.33	-951.35	-10.16	-1.92	-13.31	-23.68	-525.88	-345.25	-500.28	-132.62	-51.14	-15.25
1966	-27.15	-1026.89	-12.44	-2.43	-16.01	-28.8	-573.56	-406.09	-574.11	-145.79	-58.17	-19.15
1967	-28.88	-1109.92	-16.41	-2.98	-20.14	-33.74	-634.39	-481.46	-675.68	-163.2	-65.2	-25.81
1968	-30.54	-1209.37	-18.74	-3.6	-26.31	-38.77	-740.29	-592.91	-803.75	-182.53	-76.35	-36.05
1969	-31.02	-1324.33	-20.93	-4.29	-34.99	-44.75	-885.54	-744.81	-962.69	-207.68	-88.87	-51.26
1970	-31.67	-1461.43	-22.73	-5.04	-50.37	-54.9	-1053.39	-947.06	-1148.93	-234.33	-101.21	-74.27
1971	-32.75	-1618.26	-25.02	-5.88	-69.72	-68.43	-1267.39	-1224.27	-1340.09	-255.12	-113.51	-103.96
1972	-34.2	-1776.83	-26.1	-6.45	-87.18	-79.59	-1451.56	-1466.65	-1473.81	-268.13	-128.35	-142.52
1973	-36.54	-1945.91	-27.55	-6.81	-102.21	-91.03	-1601.38	-1771.25	-1590.7	-286.3	-144.15	-230.64
1974	-38.9	-2136.05	-28.57	-6.85	-113.03	-106.53	-1814.2	-2106.71	-1679.56	-288.57	-165.39	-285.69
1975	-37.79	-2325.95	-28.93	-6.9	-119.47	-117.82	-1940.56	-2401.74	-1657.67	-296.13	-184.99	-320.62
1976	-36.03	-2510.19	-27.99	-6.82	-123.15	-126.79	-2014.14	-2642.35	-1655.78	-297.78	-197.38	-368.28
1977	-39.93	-2688.29	-27.99	-6.94	-128.8	-134.26	-2072.15	-2887.95	-1631.02	-293.12	-209.04	-472.92
1978	-40.05	-2859.3	-28.19	-7.13	-133.14	-141.54	-2136.92	-3186.57	-1630.23	-296.87	-224.44	-580.07
1979	-36.41	-3025.83	-28.81	-7.33	-138.16	-149.41	-2203.67	-3576.15	-1740.12	-314.31	-242.53	-734.58
1980	-36.27	-3208.46	-30.94	-7.52	-144.95	-160.48	-2290.34	-3918.18	-1840.58	-330.94	-261.17	-866.77
1981	-36.86	-3388.45	-34.63	-7.56	-151.24	-168.02	-2375.75	-4320.82	-1984.5	-339.97	-284.43	-932.08
1982	-35.95	-3560.57	-40.24	-7.52	-156.72	-175.93	-2493.39	-4699.9	-2058.35	-359.62	-305.26	-1047.25
1983	-36.01	-3727.45	-47.33	-7.25	-162.65	-181.61	-2608.75	-5117.62	-2085.24	-362.01	-321.33	-1213.49
1984	-35.54	-3877.73	-53.77	-6.81	-166.96	-185.87	-2717.12	-5570.84	-2117.66	-372.49	-340.83	-1400.92
1985	-35.29	-4148.66	-62.05	-6.57	-175.01	-190.83	-2905.42	-6664.87	-2167.27	-395.81	-366.15	-1605.41
1986	-36.53	-4342.47	-73.81	-6.09	-182.86	-195.53	-3096.71	-7856.59	-2255.69	-429.97	-401.94	-1856.97
1987	-37.5	-4591.72	-85.42	-5.74	-191.77	-205.93	-3265.03	-8877.46	-2354.91	-486.43	-441.19	-2264.31
1988	-38.82	-4797.35	-97.03	-5.49	-204.44	-208.5	-3424.63	-9977.8	-2508.39	-530.38	-484.36	-2624.26
1989	-40.22	-4998.48	-114.14	-5.45	-222.21	-210.26	-3688.82	-11293.44	-2695.04	-565.45	-535.16	-3099.43
1990	-42.44	-5238.85	-133.58	-5.49	-247.01	-211.65	-4016.74	-12620.26	-2964.23	-586.43	-594.36	-3740.52
1991	-43.67	-5502.06	-155.99	-5.61	-275.89	-212.15	-4412.19	-13909.58	-3274.32	-597.34	-663.01	-4167.92
1992	-43.81	-5769.98	-169.7	-5.79	-296.6	-211.88	-4778.39	-15089.08	-3486.61	-597.09	-730.35	-4753.36
1993	-43.57	-5995.68	-184.69	-5.93	-312.54	-209.79	-5016.41	-15591.52	-3652.05	-583.79	-782.07	-5332.22
1994	-43.77	-6180.28	-189.03	-5.81	-318.11	-206.1	-5084.04	-16008.81	-3730.75	-557.58	-799.27	-5634.65
1995	-44.33	-6342.11	-189.47	-5.64	-317.35	-201.11	-5097.41	-16476.8	-3804.6	-536.15	-798.71	-5759.14
1996	-44.55	-6465.93	-188.43	-5.49	-311.68	-196.01	-5200.58	-17508.7	-3973	-523.02	-805.96	-5826.89
1997	-43.81	-6569.83	-181.76	-5.29	-304.86	-192.29	-5356.96	-19319.18	-4152.62	-515.04	-844.24	-5997.71
1998	-42.63	-6695.59	-178.57	-5.17	-299.76	-189.13	-5557.52	-20812.77	-4276.43	-505.78	-883.51	-6303.77
1999	-41.16	-6789.93	-166.58	-5.07	-291.94	-185.86	-5677.76	-21315.96	-4216.2	-482.06	-922.75	-7013.72
2000	-39.2	-6858.6	-159.98	-4.91	-281.84	-183.4	-5759.71	-21620.75	-4241.07	-470.22	-949.22	-7680.68
2001	-38.75	-6920.64	-153.86	-4.79	-273.62	-181.77	-5849.54	-23050.23	-4312.47	-464.63	-965.5	-8299.28
2002	-38.49	-6977.54	-146.78	-4.69	-265.17	-180.27	-5912.65	-24146.82	-4386.41	-466.88	-985.92	-8989.7
2003	-37.87	-7032.14	-141.72	-4.57	-256.36	-176.62	-5879.47	-24009.63	-4497.48	-475.04	-985.2	-9706.27
2004	-36.46	-7080.48	-139.2	-4.41	-247.71	-172.74	-5864.29	-24842.29	-4653.83	-483.17	-990.07	-10243.19
2005	-34.72	-7146.74	-137	-4.28	-239.71	-168.82	-5947.14	-25639.42	-4731.35	-484.28	-1005.3	-10690.22
2006	-33.95	-7210.91	-137.3	-4.22	-233.81	-165.67	-6144.88	-27048.64	-4883.98	-486.5	-1029.98	-11179.98

Table E-12: Quantities of Waiting Capital Services

	X_{KW1}^t	X_{KW2}^t	X_{KW3}^t	X_{KW4}^t	X_{KW5}^t	X_{KW6}^t	X_{KW7}^t	X_{KW8}^t	X_{KW9}^t	X_{KW10}^t	X_{KW11}^t	X_{KW12}^t
1955	12.66	873.66	0.40	0.03	2.35	7.59	81.54	19.62	12.44	41.68	1.84	3.00
1956	9.69	886.46	0.33	0.04	2.18	7.33	82.17	21.39	19.48	38.24	2.16	2.69
1957	7.74	907.44	0.30	0.05	2.11	7.19	87.85	25.11	28.45	36.34	2.60	2.61
1958	6.52	937.64	0.31	0.07	2.13	7.19	96.11	30.44	40.04	36.30	3.20	2.70
1959	5.77	969.78	0.33	0.09	2.20	7.15	102.55	35.72	52.03	37.69	3.63	2.89
1960	5.31	1016.55	0.36	0.11	2.35	7.23	112.06	43.73	59.91	38.54	4.27	3.28
1961	4.76	1085.46	0.48	0.15	2.68	7.56	130.17	55.72	77.81	41.84	5.23	3.81
1962	4.50	1164.39	0.64	0.19	3.21	8.47	158.03	71.45	104.97	47.79	6.72	4.68
1963	4.63	1251.19	0.85	0.25	3.90	9.83	186.11	88.41	131.88	53.93	8.05	6.13
1964	4.85	1348.38	1.09	0.31	4.79	11.50	213.62	103.67	159.87	60.98	9.75	7.63
1965	5.50	1460.00	1.47	0.41	5.98	13.83	242.20	123.24	197.10	69.96	11.20	9.37
1966	6.28	1570.90	1.80	0.52	7.27	16.83	264.41	141.87	227.49	78.62	12.73	11.84
1967	6.77	1695.40	2.37	0.63	9.25	19.72	291.52	163.49	268.99	88.70	14.23	14.62
1968	7.21	1843.90	2.71	0.77	12.20	22.65	336.99	196.28	322.04	98.93	16.60	19.06
1969	7.36	2013.10	3.02	0.91	16.40	26.15	398.90	238.78	389.69	111.38	19.26	27.28
1970	7.37	2214.88	3.28	1.07	22.77	32.08	472.16	294.13	470.34	123.19	21.99	37.52
1971	7.40	2436.87	3.61	1.25	30.41	39.99	561.36	366.93	555.15	133.49	24.66	49.18
1972	7.49	2665.63	3.77	1.37	37.25	46.50	638.65	434.75	616.25	140.77	27.90	64.29
1973	7.77	2918.01	3.98	1.45	42.98	53.18	699.96	509.03	671.06	149.25	31.27	82.00
1974	8.07	3199.62	4.13	1.45	47.24	62.25	776.76	600.08	713.12	152.71	35.99	96.94
1975	7.85	3471.20	4.18	1.46	49.41	68.84	828.31	677.58	705.80	156.34	40.43	109.00
1976	7.42	3759.24	4.04	1.45	50.60	74.08	854.17	734.56	708.85	155.81	43.14	121.19
1977	8.11	4026.57	4.04	1.47	52.66	78.45	881.02	790.81	700.98	155.09	46.26	141.73
1978	8.21	4303.32	4.07	1.52	54.41	82.70	903.90	855.61	698.93	154.19	49.70	157.64
1979	7.46	4571.73	4.16	1.56	56.38	87.30	926.99	939.21	742.06	158.37	53.80	185.27
1980	7.39	4844.60	4.47	1.60	59.19	93.77	955.47	1015.27	784.11	166.49	58.06	209.54
1981	7.38	5112.48	5.00	1.61	61.62	98.17	981.72	1098.89	840.17	170.90	63.36	227.99
1982	7.29	5369.82	5.81	1.60	63.58	102.80	1018.14	1174.42	867.24	178.53	68.37	249.64
1983	7.30	5619.46	6.84	1.54	65.67	106.11	1053.65	1241.22	876.09	181.87	72.16	273.93
1984	7.17	5865.47	7.77	1.45	66.98	108.60	1085.76	1316.22	888.70	187.50	76.58	299.69
1985	7.11	6256.33	8.96	1.40	69.84	111.50	1144.15	1498.32	904.51	198.44	82.22	326.19
1986	7.26	6537.86	10.66	1.29	72.47	114.25	1206.46	1681.08	934.26	214.48	90.26	354.48
1987	7.35	6916.96	12.34	1.22	75.60	120.32	1260.94	1846.64	975.58	250.33	99.16	393.81
1988	7.48	7243.58	14.02	1.17	80.13	121.82	1309.54	2020.29	1046.28	273.79	109.11	431.35
1989	7.61	7558.15	16.49	1.16	87.08	122.85	1388.70	2209.02	1133.41	285.22	120.93	488.28
1990	7.88	7918.30	19.30	1.17	96.48	123.66	1491.13	2399.52	1246.11	293.15	134.61	561.24
1991	8.04	8314.04	22.54	1.19	107.95	123.96	1621.80	2594.00	1387.85	297.66	150.38	627.90
1992	8.08	8725.55	24.52	1.23	116.64	123.80	1744.71	2790.22	1485.64	297.32	165.80	707.43
1993	8.03	9048.33	26.68	1.26	122.46	122.58	1822.85	2915.15	1560.87	293.78	177.66	785.09
1994	8.08	9341.86	27.31	1.23	124.24	120.42	1847.50	2997.28	1612.46	285.04	181.63	849.52
1995	8.12	9609.83	27.37	1.20	123.20	117.51	1853.42	3069.43	1651.01	276.45	181.79	906.54
1996	8.12	9834.31	27.22	1.17	120.26	114.53	1886.37	3207.21	1741.23	269.35	183.77	961.30
1997	7.96	10014.83	26.26	1.12	117.04	112.35	1936.13	3414.09	1839.08	264.71	192.91	1019.50
1998	7.71	10173.64	25.80	1.10	114.53	110.51	2001.03	3593.32	1909.79	260.59	202.20	1074.46
1999	7.43	10326.17	24.07	1.08	110.92	108.60	2037.23	3688.98	1893.93	252.69	211.58	1143.20
2000	7.08	10460.96	23.11	1.04	106.90	107.16	2054.37	3751.88	1915.61	247.00	218.30	1201.27
2001	6.98	10544.33	22.23	1.02	103.38	106.21	2083.96	3890.88	1960.91	242.25	222.32	1258.26
2002	6.92	10657.15	21.21	1.00	99.79	105.33	2111.89	3989.92	2006.39	242.41	227.44	1315.36
2003	6.80	10765.80	20.47	0.97	96.16	103.20	2094.93	3967.86	2062.46	244.37	227.88	1365.32
2004	6.56	10878.78	20.11	0.94	92.67	100.93	2083.78	4003.82	2133.98	245.93	229.38	1405.20
2005	6.25	10984.12	19.79	0.91	89.47	98.64	2103.90	4040.37	2179.87	243.72	233.06	1438.85
2006	6.10	11075.81	19.84	0.90	87.08	96.80	2161.16	4134.19	2250.02	241.39	239.09	1476.94

Appendix F. On the Flexibility of the Translation Homothetic Normalized Quadratic Cost Function.

Let $P^* >> 0_N$ be an arbitrary predetermined reference price vector and let $\alpha > 0_N$ be a predetermined weighting vector. Define the translation homothetic normalized quadratic cost function, C(u,P) by (4.58) where the two parameter vectors b and c and the parameter matrix B satisfy the restrictions (4.59)–(4.62). The restrictions (4.61) and (4.62) imply that the b and c vectors each have only N − 1 independent parameters, b_n and c_n respectively, while the restrictions (4.59) and (4.60) imply that the N by N matrix $B \equiv [b_{nm}]$ has only N(N−1)/2 independent parameters b_{mn}. Thus this functional form has 2N − 2 + N(N−1)/2 independent parameters in all.[173] In this Appendix, we will show that this C(u,P) is flexible in the class of cost functions satisfying Conditions II over a region of utility levels u and price vectors P.

Let $C^*(u,P)$ be an arbitrary cost function satisfying Conditions II and suppose that it is twice continuously differentiable at $u^* > 0$ and $P^* >> 0_N$. We assume that it satisfies *money metric utility scaling* at the reference prices P^* so that

(F1) $C^*(u,P^*) = u$ for all $u \geq 0$.

In order for C(u,P) defined by (4.58)–(4.62) to be flexible at (u^*,P^*), the following equations need to be satisfied for some choice of b, c and B:

(F2) $C^*(u^*,P^*) = C(u^*,P^*) = u^*$ using (4.58)–(4.62);
(F3) $\nabla_P C^*(u^*,P^*) = \nabla_P C(u^*,P^*) = b + cu^*$ using (4.58)–(4.62);
(F4) $\partial C^*(u^*,P^*)/\partial u = \partial C(u^*,P^*)/\partial u = 1$ using (4.58) and (4.62);
(F5) $\nabla^2_{PP} C^*(u^*,P^*) = \nabla^2_{PP} C(u^*,P^*) = (\alpha \cdot P^*)^{-1} B$ using (4.58)–(4.60);
(F6) $\partial^2 C^*(u^*,P^*)/\partial u^2 = \partial^2 C(u^*,P^*)/\partial u^2 = 0$ using (4.58);
(F7) $\nabla^2_{Pu} C^*(u^*,P^*) = \nabla^2_{Pu} C(u^*,P^*) = c$ using (4.58).
(F8) $\nabla^2_{uP} C^*(u^*,P^*) = \nabla^2_{uP} C(u^*,P^*) = c^T$

The number of equations in (F2)–(F8) is $1+N+1+N^2+1+N+N$. However, Young's Theorem on the equality of second order partial derivatives implies that there are only

[173] This turns out to be the minimal number of parameters required for a functional form to be flexible in the class of cost functions satisfying Conditions II; hence this flexible functional form is also *parsimonious*.

N(N+1)/2 independent equations in (F5) instead of N^2 and the N equations in (F8) are implied by the N equations in (F7). This leaves 3 + 2N + N(N+1)/2 equations to be satisfied. However, both $C^*(u,P)$ and $C(u,P)$ are positively linearly homogeneous in the prices P. Hence Euler's Theorem on homogeneous functions implies the following 3 sets of further restrictions on the derivatives of C^* and C:

(F9) $C^*(u^*,P^*) = P^*\cdot\nabla_P C^*(u^*,P^*)$; $C(u^*,P^*) = P^*\cdot\nabla_P C(u^*,P^*)$;
(F10) $\nabla^2_{PP}C^*(u^*,P^*)P^* = 0_N$; $\nabla^2_{PP}C(u^*,P^*)P^* = 0_N$;
(F11) $P^*\cdot\nabla^2_{Pu}C^*(u^*,P^*) = \partial C^*(u^*,P^*)/\partial u$; $P^*\cdot\nabla^2_{Pu}C(u^*,P^*) = \partial C(u^*,P^*)/\partial u$.

Thus there are 1+N+1 further equations which can be dropped which leaves 1 + 2N + N(N−1)/2 equations to be satisfied.

Finally, both C^* satisfies (F1) and C satisfies (4.20); i.e., both C and C^* satisfy money metric utility scaling at the reference prices P^*. Differentiation of (F1) and (4.20) gives us the following additional 3 restrictions on the levels and derivatives of C and C^*:

(F12) $u^* = C^*(u^*,P^*) = P^*\cdot\nabla_P C^*(u^*,P^*)$; $u^* = C(u^*,P^*) = P^*\cdot\nabla_P C(u^*,P^*)$ using (F9);
(F13) $\partial C^*(u^*,P^*)/\partial u = 1$; $\partial C(u^*,P^*)/\partial u = 1$;
(F14) $\partial^2 C^*(u^*,P^*)/\partial u^2 = 0$; $\partial^2 C(u^*,P^*)/\partial u^2 = 0$.

Thus we will require that C have at least 2N − 2 + N(N−1)/2 free parameters so that this number of independent equations can be satisfied. Using the above material, it can be seen that we will satisfy all of the equations (F2)–(F8) if we can find b, c and B which satisfy equations (F3), (F5) and (F7) where the chosen b, c and B must satisfy the restrictions (4.59)–(4.62). This can readily be done. Use equations (F5) in order to define B as follows:

(F15) $B \equiv \alpha\cdot P^*\ \nabla^2_{PP}C^*(u^*,P^*)$.

Since $\alpha > 0_N$ and $P^* >> 0_N$, $\alpha\cdot P^*$ is greater than 0. Since $C^*(u,P)$ is concave in P, $\nabla^2_{PP}C^*(u^*,P^*)$ is a negative semidefinite symmetric matrix and hence so is B. Since $C^*(u,P)$ is linearly homogeneous in P, (F10) holds and so $\nabla^2_{PP}C^*(u^*,P^*)P^* = 0_N$. Thus the B defined by (F15) satisfies the restrictions (4.59) and (4.60). Now use equations (F7) in order to define c:

(F16) $c \equiv \nabla^2_{Pu}C^*(u^*,P^*)$.

(F11) and (F13) will imply that the c defined by (F16) satisfies the restrictions (4.62). Now define u^* using (F2):

(F17) $u^* \equiv C^*(u^*,P^*)$.

Finally, define b using (F3) and definitions (F16) and (F17):

(F18) $b \equiv \nabla_P C^*(u^*,P^*) - \nabla^2_{Pu}C^*(u^*,P^*)C^*(u^*,P^*)$.

We need to verify that the b defined by (F18) satisfies the restriction (4.61). Using definition (F18), we have:

(F19) $P^*\cdot b = P^*\cdot[\nabla_P C^*(u^*,P^*) - \nabla^2_{Pu}C^*(u^*,P^*)C^*(u^*,P^*)]$
$= C^*(u^*,P^*) - [\partial C^*(u^*,P^*)/\partial u]C^*(u^*,P^*)]$ using (F9) and (F11)
$= C^*(u^*,P^*) - 1C^*(u^*,P^*)$ using (F13)
$= 0$.

Thus the parameter vector b defined by (F18) does indeed satisfy the restriction (4.61) and this completes our proof of the flexibility of the translation homothetic normalized quadratic functional form.

Suppose C(u,P) is defined by (4.58) where B, b and c satisfy the restrictions (4.59)–(4.62). The region of prices and utility levels where C satisfies the appropriate regularity conditions for a cost function are the set of P and u which satisfy the following inequalities:

(F20) $u \geq 0$; $P \geq 0_N$;
(F21) $\nabla_P C(u,P) = b + (\alpha\cdot P)^{-1}BP - (1/2)\,(\alpha\cdot P)^{-2}\,P\cdot BP\alpha + cu \geq 0_N$;
(F22) $\partial C(u,P)/\partial u = c\cdot P > 0$.

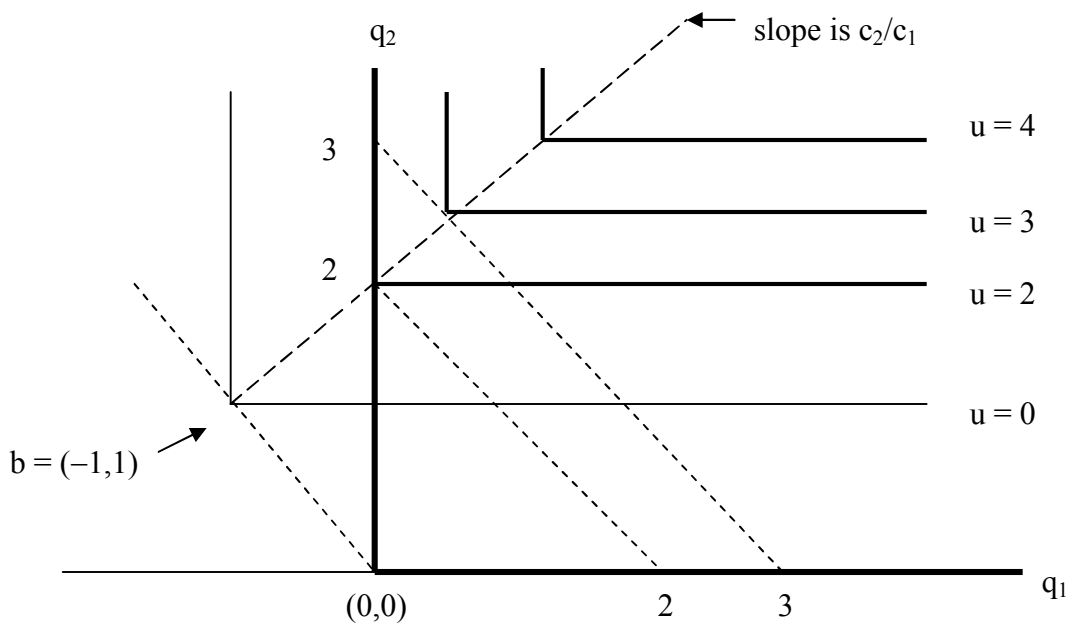
We will illustrate what the preferences dual to the normalized quadratic translation homothetic cost function look like for the case of two commodities. For all of the examples defined below, we choose the price weighting vector α which is used to form real prices as $\alpha^T = [\alpha_1,\alpha_2] \equiv [1,0]$ and the reference price vector for money metric utility

scaling P^* to be $P^{*T} = [P_1^*, P_2^*] \equiv [1,1]$. For *Example 1*, define the parameters in (4.58) as follows:

(F23) $b^T = [b_1, b_2] \equiv [-1,1]$; $c^T = [c_1, c_2] \equiv [1/2, 1/2]$; $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix} \equiv \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

The preferences corresponding to this functional form are graphed in Figure F-1.

Figure F-1: Leontief Translation Homothetic Preferences with No Inferior Goods



It can be seen that the regular region of utility levels and price vectors for this cost function is $u \geq 2$ and $P \geq 0_N$. The dual direct utility function is defined only over the set of quantity vectors such that $q_1 \geq 0$ and $q_2 \geq 2$. Note that all of the indifference curves are simply parallel shifts of a base Leontief or L shaped indifference curve that goes through the point $b = [-1,1]$. The consumer's income expansion path or Engel curve is the dashed line that passes through (0,2) and has slope $c_2/c_1 = 1$. Note also that $b \cdot P^* = (-1)(1) + (1)(1) = 0$. Any point $q \equiv [q_1, q_2]$ that lies along the dashed line through b and the origin will have the property that $q \cdot P^*$ will equal 0 and so as we vary b along this dashed line and vary the c vector, we will be able to approximate an arbitrary Engel curve locally by using this functional form.[174] Finally, to illustrate money metric utility scaling, shift the dashed line through the point b and the origin (0,0) in a parallel fashion

[174] This is a special case of the class of cost functions studied by Lau and Tamura (1972).

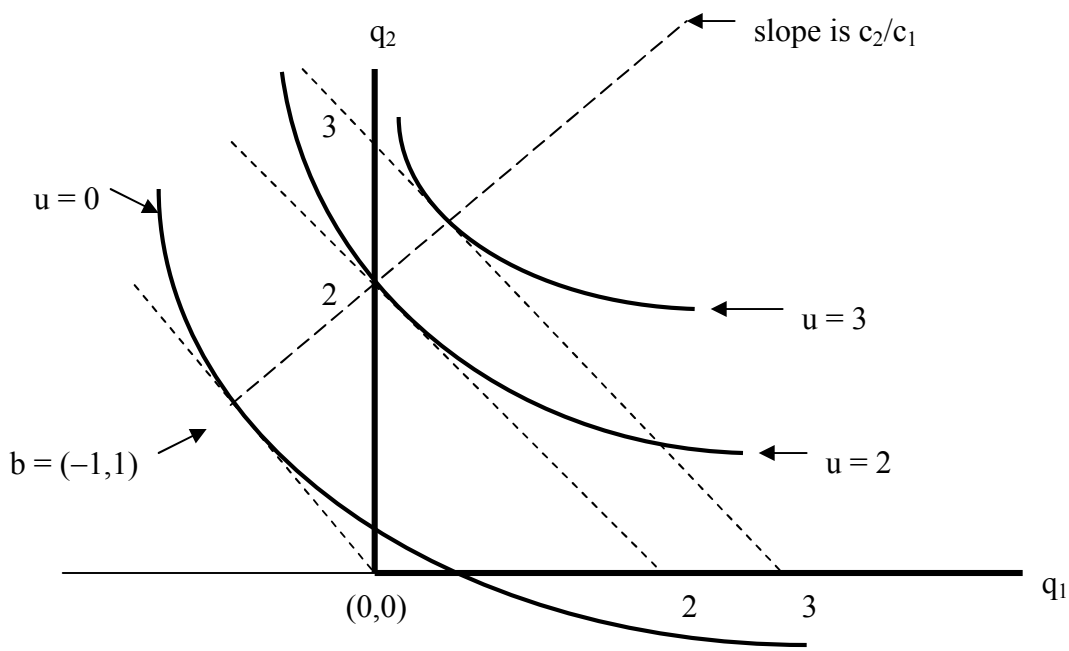
until it is just tangent to an indifference curve; we have drawn two of these parallel budget lines that are tangent to the u = 2 and u = 3 indifference curves. The distance of these lines from the origin serves to cardinalize utility.

Now consider *Example 2* and define the parameters in (4.58) as follows:

(F24) $b^T = [b_1,b_2] \equiv [-1,1]$; $c^T = [c_1,c_2] \equiv [1/2,1/2]$; $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix} \equiv \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$.

The preferences that correspond to this functional form are graphed in Figure F-2.

Figure F-2: Translation Homothetic Preferences with No Inferior Goods



The dashed line through the point b and the origin is the set of q_1 and q_2 that satisfy the equation

(F25) $P_1^* q_1 + P_2^* q_2 = P_1^* b_1 + P_2^* b_2 = 0$.

Note that the base indifference curve (which corresponds to the zero utility level u = 0) is tangent to this budget line defined by (F25) and the higher utility indifference curves are simply parallel shifts of this base indifference curve. It can be shown that the points (0,

$2-3^{1/2}$) and (1/2, 0) are on the base indifference curve. It can also be shown that as P_2/P_1 tends to zero, the upper limiting point on the base indifference curve tends to the point (–3/2, 2)[175] and as P_2/P_1 tends to plus infinity, the lower limiting point on the base indifference curve tends to (∞, –∞). As was the case with the Figure 1 preferences, The consumer's income expansion path or Engel curve is the dashed line that passes through (0,2) and has slope $c_2/c_1 = 1$. Again, to illustrate money metric utility scaling, shift the dashed line through the point b and the origin (0,0) in a parallel fashion until it is just tangent to an indifference curve; we have drawn two of these parallel budget lines that are tangent to the u = 2 and u = 3 indifference curves. The distance of these lines from the origin serves to cardinalize utility. Thus for example, all of the points on the u = 2 indifference curve are assigned the utility level 2 while all of the points on the u = 3 indifference curve are assigned the utility level 3.

The regular region of prices and utility levels is more difficult to describe in a succinct fashion. Basically, given u[176] and $P >> 0_N$, use Shephard's Lemma to generate $q \equiv \nabla_P C(u,P)$ and check whether this q is nonnegative. If so, then the given u and P belong to the regular region. In our example, the vector c had positive components so the regularity condition $\partial C(u,P)/\partial u = c \cdot P > 0$ will automatically be satisfied.

Note that the vector c is equal to $\nabla^2_{uP}\, C(u,P)$ which in turn is equal to the vector of derivatives of the consumer's Hicksian demand functions, $q(u,P) \equiv \nabla_P\, C(u,P)$, with respect to an increase in utility u and thus indicates how the household's consumption changes as utility increases. If $\partial q_n(u,P)/\partial u = \partial^2 C(u,P)/\partial P_n \partial u$ is negative, then we say that commodity n is an *inferior* commodity at u, P. Although it is easy to show that not all commodities can be inferior at a particular point, there is nothing to prevent one or more commodities from being inferior. Hence if the consumer's preferences are represented by a translation homothetic normalized quadratic cost function in a neighbourhood of a point where the consumer has one or more inferior commodities, then the parameter vector c will have one or more negative components. This means that there will exist positive price vectors P such that c·P is negative and hence C(u,P) defined

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[175] This indifference curve can be extended to cover higher levels of q_2 in the obvious way.

[176] The utility level u does not have to be nonnegative in this example. If we allow negative utility levels, then the dual preferences f(q) are well defined for all $q \geq 0_N$; i.e., if we restrict ourselves to $u \geq 0$, then we will not be able to define the consumer's preferences in the little triangle which is below the u = 0 indifference curve intersected with the nonnegative quadrant.

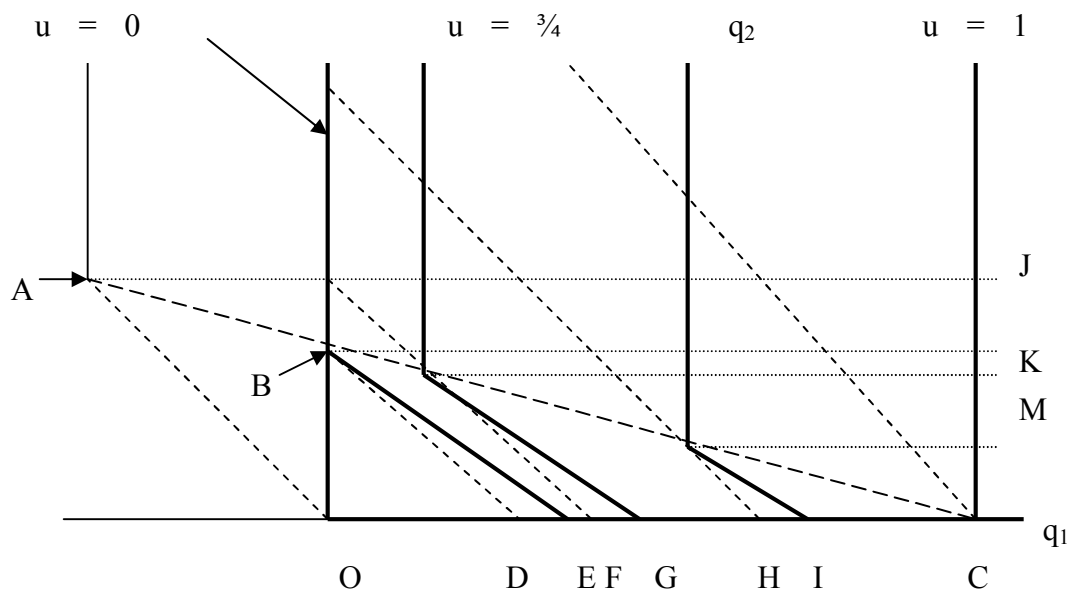
by (4.58) cannot be regular at these price vectors. Fortunately, it is possible to show that the cost function defined by (4.58) can still be locally regular and provide a valid representation of a consumer's nonhomothetic preferences in a neighbourhood of a point (u,P) where C(u,P) satisfies the required regularity conditions for a cost function locally.

We conclude this section by considering an example where the second commodity is inferior. For simplicity, we consider again the case of Leontief preferences. Thus for *Example 3*, define the parameters in (4.58) as follows:

(F26) $b^T = [b_1,b_2] \equiv [-1,1]$; $c^T = [c_1,c_2] \equiv [4/3,-1/3]$; $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix} \equiv \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

The preferences corresponding to this functional form are graphed in Figure F-3.

Figure F-3: Leontief Translation Homothetic Preferences with an Inferior Good



The u = 0 indifference curve is the L shaped curve that has its corner at the point A, which is the point $(q_1,q_2) = (-1,1) = (b_1,b_2) = b$. As usual, the point b lies on the line $P^*\cdot q = 0$ and the dashed line segment AO is part of this line. The dashed line segments ending in D, F, H and C are all parallel to the line segment AO. The consumer's Engel curve at the reference prices $P^{*T} = [P_1^*,P_2^*] = [1,1]$ is the intersection of the line segment AC with the nonnegative orthant which is the line segment BC. The first regular point on this line segment is the point B which corresponds to the point $(q_1,q_2) = (0,3/4)$ and the

corresponding money metric utility level is $u = 3/4$. L shaped indifference curves that are translations of the base indifference curve corresponding to $u = 0$ have been drawn for the utility levels $u = 3/4$, $u = 1$, $u = 2$ and $u = 3$. However, it can be seen that there is a problem with these indifference curves: they cross each other! Geometrically, it is easy to solve this problem: for the $u = 3/4$ indifference curve, replace the lower line segment BK by the line segment BE, where the point E on this line segment must be strictly between the points D and C. Similarly replace the bottom part of the $u = 1$ indifference curve that passes through M by the line segment that ends at the point G where this line segment is parallel to the line segment BE, replace the bottom part of the $u = 2$ indifference curve that passes through N by the line segment that ends at the point I where this line segment is also parallel to the line segment BE, and so on. The resulting system of indifference curves no longer cross and are well behaved. Algebraically, we need to restrict the prices P_1 and P_2 so that the restrictions (F22), $c \cdot P > 0$, are satisfied. Under assumptions (F26), (F22) becomes the following restriction on prices:

(F27) $P_2 < 4\, P_1$.

In order to satisfy the restrictions (F21), we require the following restrictions on u:

(F28) $\frac{3}{4} \leq u \leq 3$.

The inequalities (F27) and (F28) define the regular region for the Example 3 preferences.

For more information on how local information on a cost function can be used to form local approximations to utility functions, see Blackorby and Diewert (1979).

Viewing this last example, it can be seen that when there are inferior goods, the regular region may not be very large; i.e., our suggested functional form will not be able to provide an adequate global approximation to arbitrary preferences. However, with all of the examples, it can be seen that if relative prices do not change too much and the utility levels in the two periods being compared are fairly close, translation homothetic normalized quadratic preferences will be able to provide a good local approximation to arbitrary preferences. On the other hand, if relative prices differ markedly and/or utility levels differ considerably, then the approximation may not be very close. But having results that are exact for second order approximations to arbitrary preferences is better than having results that are exact for only first order approximations!

Appendix G. The Japanese Consumption Data

We use the household consumption data in the Japanese national accounts. Household consumption is classified into 12 categories; (1) Food and non-alcoholic beverages; (2) Alcoholic beverages and tobacco; (3) Clothing and footwear; (4) Housing, electricity, gas and water supply; (5) Furnishings, household equipment and household services; (6) Health; (7) Transport; (8) Communication; (9) Recreation and culture; (10) Education; (11) Restaurants and hotels; (12) Miscellaneous goods and services.

Expenditure series for each category are provided at current prices and constant prices in the national accounts. For the years 1980-2003, current and constant yen series for the 12 consumption goods are found in the *Annual Report on National Accounts of 2005*; see the Economic and Social Research Institute (2005); Part 1 Flows; 5. Supporting Tables; (13) Composition of Final Consumption Expenditure of Households classified by Purpose; Calendar Year, in billions of yen. The constant yen series are at the market prices of 1995. For the years 1996-2006, current and constant yen series for the 12 consumption goods are found on *Annual Report on National Accounts of 2008*; see the Economic and Social Research Institute (2007). The constant yen series are at the market prices of 2000. We link the current and constant yen series of these two reports to construct current and constant prices for the 12 goods for the period 1980-2006. For the years 1996-2006, we use current and constant yen series taken from the Annual Report on the National Accounts of 2008. We interpolate these series backward by using the growth rate of current and constant yen series taken from *Annual Report on the National Accounts of 2005*. The price of each good is implicitly derived from the current and constant yen series. We normalize all the prices so that prices in 1980 are one and adjust the corresponding quantity series in order to preserve values. We regard the constant yen series as the quantity series. We list the resulting price and quantity series for 1980-2006 in Table G-1 and Table G-2 respectively. We also list the real prices of 12 goods in Table G-3.

Table G-1: Prices of Consumption Goods

Year	Food and non-alcoholic beverages	Alcoholic beverages and tobacco	Clothing and footwear	Housing ,electricity ,gas and water supply	Furnishings ,household equipment and household services	Health	Transport	Communication	Recreation and culture	Education	Restaurants and hotels	Miscellaneous goods and services
1980	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1981	1.04162	1.07216	1.03888	1.05957	1.02152	1.02092	1.04887	1.04322	1.06233	1.07679	1.05108	1.03687
1982	1.04697	1.08538	1.06201	1.10067	1.02653	1.04762	1.12421	1.03614	1.07852	1.14704	1.08435	1.05841
1983	1.07685	1.13687	1.08312	1.13579	1.02254	1.05256	1.12714	1.03037	1.10095	1.20658	1.11704	1.07842
1984	1.11502	1.21343	1.10829	1.16822	1.03326	1.06930	1.14733	1.01435	1.13302	1.26061	1.14997	1.10152
1985	1.12865	1.22243	1.13071	1.19664	1.03306	1.10449	1.16704	1.00670	1.17007	1.31554	1.17421	1.09722
1986	1.12573	1.26110	1.15100	1.20559	1.04714	1.13071	1.15367	1.00081	1.18575	1.36529	1.20012	1.09963
1987	1.11440	1.27879	1.16626	1.22047	1.04753	1.15577	1.16040	1.00150	1.18393	1.41058	1.21627	1.10285
1988	1.11997	1.27435	1.17891	1.23856	1.04888	1.15943	1.16186	0.98687	1.18153	1.45914	1.23302	1.10879
1989	1.14457	1.27041	1.22581	1.26618	1.05149	1.17543	1.17658	0.99035	1.20139	1.51821	1.28285	1.12579
1990	1.18888	1.28750	1.28017	1.30016	1.05341	1.19413	1.18787	0.97857	1.23202	1.60228	1.32780	1.14023
1991	1.24622	1.30227	1.33633	1.33486	1.06092	1.19979	1.21371	0.95705	1.26447	1.66855	1.37310	1.15870
1992	1.25307	1.30740	1.37554	1.36658	1.06566	1.24310	1.22328	0.93804	1.29031	1.73603	1.40949	1.17392
1993	1.26774	1.30678	1.37512	1.39606	1.05435	1.26089	1.23661	0.91270	1.29919	1.80541	1.43427	1.18747
1994	1.27496	1.32120	1.36187	1.42254	1.02534	1.27442	1.23729	0.88941	1.28709	1.86929	1.44748	1.20909
1995	1.25026	1.32541	1.35712	1.44468	1.00444	1.28208	1.22791	0.92536	1.25411	1.92655	1.44297	1.21485
1996	1.25226	1.32096	1.36823	1.46079	0.97937	1.29761	1.21633	0.88694	1.19769	1.97597	1.44650	1.22327
1997	1.27192	1.33666	1.39597	1.50130	0.97486	1.31024	1.22771	0.86328	1.18973	2.01820	1.48891	1.23723
1998	1.29105	1.34005	1.41566	1.51774	0.95657	1.30974	1.21124	0.82059	1.17403	2.05449	1.49427	1.24117
1999	1.28641	1.38709	1.41436	1.50993	0.94419	1.29259	1.21288	0.79750	1.15311	2.08789	1.49443	1.25487
2000	1.26024	1.38261	1.40445	1.50566	0.90247	1.28611	1.22926	0.77160	1.11376	2.12104	1.48423	1.25409
2001	1.24679	1.37807	1.38329	1.49958	0.85999	1.28711	1.23033	0.73706	1.04903	2.13690	1.47162	1.27311
2002	1.23008	1.37027	1.35960	1.48518	0.81462	1.27004	1.21938	0.73040	1.00066	2.16401	1.46972	1.26947
2003	1.22405	1.39253	1.34381	1.47242	0.77259	1.26163	1.22346	0.73076	0.94145	2.18028	1.47433	1.26684
2004	1.23003	1.41678	1.34030	1.45903	0.72654	1.24966	1.23720	0.72653	0.88308	2.19088	1.48340	1.26366
2005	1.21146	1.41617	1.34573	1.44856	0.69341	1.24689	1.26614	0.69996	0.81847	2.21460	1.48515	1.26429
2006	1.21304	1.43922	1.35569	1.44506	0.66537	1.23537	1.28849	0.68186	0.75676	2.21964	1.49338	1.26790

Table G-2: Quantities of Consumption Goods

Year	Food and non-alcoholic beverages	Alcoholic beverages and tobacco	Clothing and footwear	Housing ,electricity ,gas and water supply	Furnishings ,household equipment and household services	Health	Transport	Communication	Recreation and culture	Education	Restaurants and hotels	Miscellaneous goods and services
1980	27979.8	5406.9	10906.6	22524	6773.2	5243.7	12836.1	1337.6	9851.1	2899	9279.5	12715.1
1981	28510.8	5253	10648.9	23231.6	6794	5989.5	12830.9	1519.3	9993.3	2900.1	9029.5	13022.1
1982	29430.6	5447.7	11260.5	23815.8	7622	6779.4	12701.3	1793.1	10970.9	2871	9412.9	13836.9
1983	29734.4	5486.4	11390.7	24493.4	8262	7002.9	13329.4	2013.9	11506.7	2895.2	9432.9	14382.1
1984	29726.9	5152.7	11562.3	25417.8	8863.7	7072.5	13368.6	2252.5	12186	2890.3	9751.7	15152.2
1985	30525.2	5111.2	11698.3	26347.2	9722.7	7448.2	13454.1	2418	12879.5	2865.2	10264.9	16710.1
1986	30637.9	5078.4	12083.5	26918.3	9555	7117.5	14553.8	2512.3	14328.7	2933.2	10415.1	18182.5
1987	30901.4	5153.8	12567	27922.6	9930.5	7009.9	15752.4	2534.7	15707.7	2987.1	10705	19467.8
1988	31245.2	5296.6	12889.7	29059	10414.3	6843	17832.4	2631.9	17404.3	3065	11013.4	20465.4
1989	31361	5580.8	13143	30659.8	10639.8	6614.8	20079.2	2690.6	19274.4	3167	11027.1	21945.4
1990	31661.6	5936.2	13194.3	32219.9	11520.2	6523.1	21872.1	2820.6	22051.8	3178.2	10896.8	22606.2
1991	31936.4	6075.1	13347.7	33795.6	12338.9	6547.8	22633.1	3095.2	22331.2	3231.9	11266.2	23146.1
1992	32724.6	6207.7	12904.5	35358.9	12584.2	6392.3	23059.4	3321.5	22187.6	3259	12007.8	24315.8
1993	32801.6	6247.8	12806.9	36709.1	12854.6	6338.2	23170.8	3956.9	22198.2	3263.7	12463.3	23959.9
1994	33203.4	6511.2	12823.5	37792	13528	6643.4	23599.9	4324	22050.8	3199.5	13140.9	25342.4
1995	34143.8	6710.4	13395.8	38563.1	14169.2	6587.9	23772.7	4586.7	22544.2	3134.6	13191.4	24623.6
1996	34131.8	6762.4	13509.2	39533.8	14231.2	6566.5	24697.8	5685.3	23595.5	3101.3	13681.5	25394.3
1997	33560.4	6729.2	12286.8	40206.5	14178.9	6911.9	24888.4	6564	25154	3054.5	13693	25717.9
1998	34117.3	7017.8	11293.5	40810.9	13574.4	7132.6	22911	7283.1	24820.6	2960.5	13794.4	25069.1
1999	34389.4	6973	9924.6	41648.9	13579.8	7798.1	23125.4	8342.6	25104	2901.3	13873.5	24746.1
2000	34508.9	6952.9	9027.5	42669.4	13236	8045.9	23367.4	8946.8	27752.1	2852	13658	23638.4
2001	34920.3	7002.2	8662.9	43505.4	13498.9	8424.1	23786.1	9856.6	29575.4	2836	14079.2	22977.7
2002	35032.3	7064.9	7988.3	44344.6	13301.9	8677	23854.9	10529.2	30515.1	2812.5	14169.5	23274.2
2003	34544.8	6660.2	7662.5	45170.2	13587.1	9231.2	23633	11030.7	32334.1	2773.9	14009.1	23010.8
2004	34388.1	6540	7383.2	46038.6	14163.5	9202.4	23726.1	11584.3	34894.6	2829	14029.6	23494.8
2005	33583.9	6277.2	7347.2	47045.9	14945.4	9563.4	23990.4	12139.4	36975.4	2801.8	14243.5	24281.1
2006	33424	6098.5	7582.7	48066.7	15915.5	9644.2	24167.1	12878.3	41188.1	2783.1	14401.1	25168.5

Table G-3: Real Prices of Consumption Goods

Year	Food and non-alcoholic beverages	Alcoholic beverages and tobacco	Clothing and footwear	Housing ,electricity ,gas and water supply	Furnishings ,household equipment and household services	Health	Transport	Communication	Recreation and culture	Education	Restaurants and hotels	Miscellaneous goods and services
1980	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1981	1.00000	1.02932	0.99737	1.01723	0.98070	0.98013	1.00696	1.00153	1.01988	1.03376	1.00908	0.99544
1982	1.00000	1.03668	1.01436	1.05129	0.98048	1.00061	1.07377	0.98965	1.03013	1.09558	1.03570	1.01092
1983	1.00000	1.05574	1.00582	1.05473	0.94956	0.97744	1.04670	0.95683	1.02237	1.12047	1.03732	1.00145
1984	1.00000	1.08826	0.99396	1.04771	0.92668	0.95899	1.02898	0.90971	1.01614	1.13057	1.03134	0.98789
1985	1.00000	1.08309	1.00182	1.06024	0.91530	0.97859	1.03401	0.89195	1.03669	1.16558	1.04036	0.97215
1986	1.00000	1.12025	1.02245	1.07094	0.93019	1.00442	1.02482	0.88903	1.05332	1.21281	1.06608	0.97682
1987	1.00000	1.14751	1.04654	1.09517	0.93999	1.03712	1.04127	0.89869	1.06239	1.26577	1.09141	0.98963
1988	1.00000	1.13785	1.05263	1.10590	0.93653	1.03524	1.03741	0.88117	1.05497	1.30284	1.10094	0.99002
1989	1.00000	1.10995	1.07097	1.10625	0.91868	1.02696	1.02797	0.86526	1.04964	1.32645	1.12081	0.98359
1990	1.00000	1.08295	1.07678	1.09360	0.88605	1.00441	0.99915	0.82310	1.03628	1.34772	1.11684	0.95908
1991	1.00000	1.04497	1.07231	1.07113	0.85131	0.96274	0.97391	0.76796	1.01464	1.33889	1.10181	0.92977
1992	1.00000	1.04336	1.09773	1.09058	0.85044	0.99204	0.97623	0.74859	1.02972	1.38542	1.12483	0.93683
1993	1.00000	1.03080	1.08470	1.10121	0.83168	0.99459	0.97544	0.71994	1.02481	1.42411	1.13136	0.93668
1994	1.00000	1.03627	1.06817	1.11575	0.80422	0.99958	0.97045	0.69760	1.00952	1.46616	1.13531	0.94834
1995	1.00000	1.06011	1.08547	1.15550	0.80338	1.02545	0.98212	0.74013	1.00308	1.54092	1.15413	0.97168
1996	1.00000	1.05486	1.09261	1.16652	0.78209	1.03621	0.97131	0.70827	0.95642	1.57793	1.15512	0.97685
1997	1.00000	1.05090	1.09753	1.18034	0.76644	1.03013	0.96524	0.67872	0.93538	1.58673	1.17060	0.97273
1998	1.00000	1.03795	1.09651	1.17558	0.74092	1.01447	0.93818	0.63560	0.90936	1.59133	1.15741	0.96136
1999	1.00000	1.07826	1.09946	1.17375	0.73397	1.00480	0.94284	0.61994	0.89638	1.62304	1.16171	0.97549
2000	1.00000	1.09711	1.11443	1.19475	0.71611	1.02053	0.97542	0.61227	0.88377	1.68304	1.17774	0.99512
2001	1.00000	1.10529	1.10947	1.20275	0.68976	1.03234	0.98680	0.59117	0.84138	1.71391	1.18032	1.02111
2002	1.00000	1.11397	1.10530	1.20739	0.66224	1.03248	0.99130	0.59378	0.81349	1.75924	1.19481	1.03202
2003	1.00000	1.13764	1.09784	1.20291	0.63117	1.03070	0.99951	0.59700	0.76912	1.78120	1.20447	1.03496
2004	1.00000	1.15183	1.08965	1.18618	0.59067	1.01596	1.00583	0.59066	0.71793	1.78116	1.20599	1.02734
2005	1.00000	1.16898	1.11084	1.19571	0.57238	1.02925	1.04514	0.57778	0.67561	1.82805	1.22592	1.04361
2006	1.00000	1.18645	1.11760	1.19127	0.54851	1.01841	1.06220	0.56211	0.62385	1.82982	1.23110	1.04523