HIGH EFFICIENCY WIRELESS POWER TRANSMISSION AT LOW FREQUENCY USING PERMANENT MAGNET COUPLING

by

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ABSTRACT

A new method of electrical wireless power transfer has been parameterized and experimentally verified for a variety of size-scales and applications. The main distinction between this and previous methods of wireless power transfer is the nature of the coupling mechanism, which is a magnetic interaction between synchronized, rotating, permanent magnets. Its main components can be viewed as equivalent to an electric motor, a magnetic gear, and an electric generator. Its performance parameters such as power, range and efficiency are within the same order of magnitude as previously known resonant inductive power transfer devices. However, it has the distinct benefit of operating at much lower operating frequencies.

A theoretical model of the new system has been developed with sufficient detail to characterize and predict experimental behavior of various sizes. The theoretical treatment has been divided into three main interactions: the motor, the generator and the magnetic gear. The mechanism for operation, as well as a model for efficiency and losses have been developed for each interaction.

The viability of this new method of wireless power transfer was experimentally verified for two size-scales. The larger size-scale achieved 1.6 kW of power transfer with 15 cm separation. The main target applications of this size-scale are for wireless charging of electric vehicles and industrial applications. The smaller size-scale achieved 60 W of power transfer with 10 cm separation. The main target applications of this size-scale are for powering medical implants and consumer electronics. Both size-scales achieved efficiencies in the range of 81%, and the operating frequency did not exceed 150 Hz. The design and construction of the devices are outlined for both size-scales.

Misalignment tolerance between the transmitting device and the receiver device was experimentally investigated, and related control schemes for managing the power transfer were implemented and tested.

Additionally, the potential risk to human health from the time-varying magnetic field produced by this system was evaluated using exposure limits set within two widely
adopted standards. For short-term exposure to the larger-scale device, the fields met the standards at a distance beyond 6 cm, and for long-term exposure, beyond 1 meter.
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1 INTRODUCTION

For the better part of industrialized history, transmitting electrical energy involved physically connecting the devices with current carrying wires. Although this method is perfectly acceptable for most uses of electrical energy, there are applications that would benefit greatly if electrical energy could be transferred without a physical connection.

The current methods of wireless power transmission can be categorized into near-field and far-field devices. Far-field devices often use collimated beams of electromagnetic radiation of various frequencies. These include radio, microwave, and laser devices. Far-field devices can support large separation distances due to the collimation of the radiation. However, they require accurate alignment, and often require direct lines of sight.

Near-field devices are currently dominated by inductive and resonant inductive transfer devices. These devices use the non-radiative portion of the electromagnetic field. They closely resemble a transformer with primary windings separated from the secondary by an air gap. The power transfer and efficiency of inductive devices decrease dramatically with increasing separation distances. The introduction of resonant inductive devices mitigates this decrease of efficiency to a certain extent. However, the advantage of resonant induction relies heavily on the quality factor of the resonator, which generally improves with higher frequency, at least up to a point. Therefore, resonant inductive devices usually operate in the frequency range of 10 kHz to 10 MHz.

The work presented in this thesis proposes an alternate method of power transfer which is similar to resonant inductive methods in power, range and size, but operates with a different principle. It has the advantage of operating at much lower frequencies, bypassing some of the disadvantages of inductive devices. These disadvantages include high frequency electronics, complex control, complex communication systems, and ambient radio frequency (RF) electromagnetic fields which may give rise to electromagnetic interference (EMI) and possible risks to human health.
The main coupling mechanism of the proposed method is the interaction between two permanent magnets (PM) in synchronized rotation. Due to the orientation of the PMs, one can apply torque to the other, allowing the transfer of mechanical energy. The mechanical energy of rotating PMs can be transformed to and from electrical energy using common machines such as motors and generators, or their equivalent, completing the electrical power transfer system.

The coupling of PMs in rotation is not a new idea. It is commonly referred to as a magnetic gear. However, this type of coupling has generally been viewed as a way to produce rotational coupling, not to transfer electrical power. It is used in applications where contactless rotational coupling is needed across small distances without a physical connection. Applications include pumps for hazardous materials and laboratory magnetic stirrers.

A somewhat counter-intuitive advantage of this type of coupling is that it has essentially no inherent losses. A time-varying magnetic field can be produced by rotating a single PM at a constant speed, with negligible power consumption to overcome bearing friction. This time-varying magnetic field can then be used to rotate a second PM. Rotating two synchronized PMs also consumes very little power, assuming there are no eddy current or hysteresis losses in the PM materials. This assumption is verified in Section 5.2.5.2. If the second magnet experiences a torque as it does work on an external element, the first magnet experiences an equal and opposite torque, drawing an equal amount of mechanical power from the drive mechanism. Therefore, by loading the receiver PM with a generator, and driving the transmitter PM with a motor, an efficient, simple wireless power transfer device is achieved. Also somewhat counter-intuitive is the fact, established in this work, that useful amounts of power can be transferred in reasonably compact devices.

This thesis explores the application of this coupling type for the purpose of electrical wireless power transfer. Background information provided in Chapter 2 briefly discusses resonant wireless power transfer technologies. Although they operate differently from the proposed method, they are presently the most similar method in the current field.
A complete characterization of the new system, arising from basic principles, is offered in Chapter 3. The characterization is organized into 3 sections according to the 3 main interactions: the motor, the generator and the magnetic gear. Numerous magnetic gears types are reviewed in Section 3.1 with parameterization of torque, power and losses. The two main ways of motoring in such systems are introduced in Section 3.2, and the two main ways of generating in such systems are introduced in Section 3.3. The most promising way of generating is analyzed in detail with parameterization of torque, speed, load, losses and efficiency in Section 3.3.2.

Since the parameters of this system depend on the physical size of the devices, the behavior of the system under the scaling of physical dimensions is explored in Chapter 4. This analysis shows that the power capability of the system can be widely adjusted by scaling the physical dimensions of the device. This suggests a highly flexible design process for arbitrary power requirements.

The validity of such parameterization was experimentally established at two size and power scales, as discussed. A larger device achieving 1600 W of transfer and a smaller device achieving 60 W of transfer were constructed and tested.

Finally, the time-varying magnetic field produced by the 1600 W transfer device was measured as a function of position and this data is used to provide a very positive conclusion regarding potential human health risks as discussed in Chapter 6. This evaluation is presented in reference to widely accepted standards for human exposure limits to magnetic fields.
2  BACKGROUND

The currently proposed wireless power transfer method resembles inductive wireless power transfer methods in many aspects including range (1-20 cm), power level (1-5000W) and efficiency (50-95%). Although the principles of operation differ quite significantly, the general topology is similar. Therefore, the principles of operation of inductive or resonant inductive wireless transfer devices are briefly discussed in this section. Following this discussion, unsolved issues which hinder the inductive technologies are identified. Appendix C contains a detailed case-study in which the currently proposed system was compared against the resonant inductive systems.

2.1  Resonant inductive wireless power transfer

Inductive power transfer devices can be understood as a separated transformer, as depicted in Figure 1.

![Inductive power transfer as separated transformer](image)

Figure 1:  Inductive power transfer as separated transformer

By separating the primary winding from the secondary winding with a non-magnetic gap (such as air), contact-less or wireless power transfer is possible. However, when the windings no longer share a magnetic closed loop circuit that is enhanced by high permeability material such as iron, and when the air-gap increases to more useful separations, the efficiency of the transformer dramatically decreases. Two common methods mitigate this drop in efficiency. First, the frequency of the AC fields is increased, which reduces the
relative importance of resistivity of the windings in comparison to inductive effects. Second, resonant gain is introduced in the primary and/or secondary circuits by introducing capacitance\(^2\). The quality factor of such electrical resonators increases with frequency, at least up to a point. For these reasons, the operating frequency is generally increased to the highest practical value. For most cases, this is in the 100 kHz range, allowing the efficiency to reach the 90% range, with transfer rates in the 5kW range\(^3\). In extreme cases, a frequency of 10MHz has been used to further improve efficiency and range\(^4\). A detailed discussion is included in Appendix C, where resonant inductive transfer systems were analyzed, including the mechanism in which efficiency and various losses are dependant on operating frequency.

\[\text{2.2 Inductive power transfer concerns}\]

There are several challenges for the inductive systems, for which there seems to be no solution in the near future. A first challenge is the possible risk to human health. There have been many studies which suggest negative effects of radio frequency (RF) fields on animals and humans\(^5\). Regardless of whether these concerns are valid, they are nevertheless an obstacle in the widespread application of inductive systems among the general public. A second challenge is the cost of a complex power supply operating at high frequency and at high power\(^6\). A third challenge is precision and potentially dynamic frequency control. Resonant inductive systems have a sharp resonant frequency peak. Operation outside this range will dramatically reduce efficiency. Moreover, the optimum frequency may change with load, separation, and presence of extraneous objects. This requires active frequency compensation in the transmitter side, with more complex control and communication between transmitter and receiver\(^7\). Finally, these systems produce considerable electromagnetic radiation which may cause electromagnetic interference for nearby electronics\(^8\).

\[\text{2.3 Introduction to the proposed method}\]

This thesis proposes an alternative to inductive transfer devices, one that operates with comparable power, efficiency and range, but at much lower frequencies, bypassing the difficulties of high frequency operation. Instead of using resonance to enhance the coupling,
a pair of permanent magnets, co-rotating in synchronization, is used, as illustrated in Figure 2.

Figure 2: Separated transformer enhanced by synchronized co-rotating PM cores

The mechanical rotation of the two permanent magnets is synchronized by their magnetic interactions, which provides the desired torsional coupling between the transmitter and receiver side.

Unlike the conventional inductive coupling, the purpose of the transmitter and receiver windings is not to interact directly with one another. In the new scheme, the interaction between the transmitter and receiver sides is dominated by the interaction between the two magnets. The purpose of the transmitter and receiver windings is to interact with their own respective magnets. Thus, the power transfer can be approximately categorized in three general steps illustrated in Figure 3.
Figure 3: Power flow of proposed method

In the 1\textsuperscript{st} step of Figure 3, the power is provided to the transmitter winding in the form of an electrical current. This current in the winding produces a mechanical torque on the transmitter PM causing it to rotate. This step converts electrical power in the transmitter winding to mechanical power in the transmitter PM. This interaction is analogous to a motor. It is discussed in detail in Section 3.2.

In the 2\textsuperscript{nd} step, the transmitter PM induces a torque on the receiver PM through magnetic interaction. This torque on the receiver magnet causes it to rotate synchronously with the transmitter magnet, and transfers mechanical power across the air-gap. This interaction is analogous to a magnetic gear. This is discussed in detail in Section 3.1.

In the 3\textsuperscript{rd} step, the rotating receiver PM spins inside the receiver windings. This produces a changing magnetic flux in those windings and gives rise to an induced electrical current. Thus mechanical power is transferred back into electrical power towards load. This interaction is analogous to a generator and discussed in detail in Section 3.3.
The proposed method allows the efficient wireless power transfer at mechanical frequencies, in the range of 150 Hz, or 9000 rpm. These operating frequencies are 2-3 orders of magnitude lower than those of inductive wireless systems.

This method thus addresses challenges facing resonant inductive transfer devices, in particular ones that are related to its high frequency operation. This method allows the system to operate with simple low frequency power electronics, without active frequency compensation, with negligible RF EMI, and with considerable improvement in human safety.
3 ANALYSIS OF PROPOSED METHOD

The theoretical treatment is divided into 3 sections corresponding to 3 main interactions: motoring, generation and magnetic gearing.

3.1 Magnetic gear stage

One of the three main steps in the proposed transfer method involves the transmission of mechanical power from the transmitter to the receiver side. This was done through the magnetic interaction of two spinning permanent magnets separated by an air gap. The axes of rotation of each permanent magnet were secured in place during transfer. The magnets were synchronized by the torque applied by one magnet to the other. This is known in many fields as a magnetic gear.

3.1.1 Types of magnetic gears

There are countless ways to configure a magnetic gear, each of which could be used to transfer wireless power. The main variables include: the orientation of the axes of rotation of the transmitter and receiver rotors with respect to one another, the orientation of the magnetization of the permanent magnet with respect to its respective axis of rotation, the number of magnetic poles in each rotor, and the magnet material properties. These variables determine important performance characteristics such as maximum torque transfer capability at any distance of separation.
3.1.1.1 Orientation

Figure 4: Orientation of 3 types of magnetic gears

There are 3 particularly popular orientations. The orientation depicted in Figure 4(a) shows co-axial axes of rotation with axially magnetized rotors. The rotors would have PMs magnetized in alternating directions around the circumference. This generally gives high torque transfer capacity, but low separation distances\(^{10}\).

The orientation depicted in Figure 4(b) shows co-axial axes of rotation, with radially magnetized rotors. Generally these magnetic gears have inner and outer rotors which are overlapping in the axial direction. The rotors would also have PMs magnetized in alternating radial directions. This generally also gives high torque transfer capacity, but offers next to no separation distance\(^{11}\).

The orientation depicted in Figure 4(c) shows parallel axes of rotation (dotted lines) with radially magnetized rotors. Generally these magnetic gears do not have a common axis of rotation; instead the axes are separated by a gap. The rotors are typically longer cylinders with PMs magnetized in the radial direction. This generally gives moderate torque transfer capacity and offers larger separation distances\(^{12}\).

From qualitative evaluation, the magnetic gear orientation in Figure 4(c) was chosen for the proposed method of power transfer, largely due to the greater separation distance allowed.
3.1.1.2 Number of poles

The number of poles is also an important factor that affects performance characteristics. The rotor can have any even, whole number of poles.

For applications which require stiffness, or close angular agreement between the rotors, it is generally advantageous to have greater number of poles. This is because the rotor stiffness is increasing with increasing number of poles\(^ {13}\). However, stiffness was neither beneficial nor detrimental to the proposed method of power transfer because the speed of rotation will be fairly constant.

For applications which require unequal rotor speeds, an unequal number of poles in the transmitter and receiver PM can be used. For example a transmitter rotor with 2 poles coupled to a receiver rotor with 4 poles will result in the receiver rotor spinning with half the speed of the transmitter\(^ {14}\). However, unequal rotor speed was neither beneficial nor detrimental to the proposed method of power transfer.

For applications which require large separation distances between the transmitter and the receiver, it is generally advantageous to have a smaller number of poles. This can be understood by thinking of the field produced by any PM arrangement in terms of a multi-pole expansion. The terms in the expansion start with the dipole term, then quadruple term, and onwards\(^ {15}\). The relative magnitude of each term expresses the portion of the magnetization dedicated to each type of field. The dipole field is strongest at large distances, \(r\), since it is proportional to \(r^{-3}\), whereas the higher pole fields are proportional to even more negative powers of \(r\). Since the strength of the field at large distances is most important, a strong dipole term is preferred. Therefore it was advantageous, in this application, to use the minimum number of poles (i.e. two) for each PM rotor.
3.1.1.3  Soft magnetic coupling enhancement

In some magnetic gearing arrangements, it has been shown that placing soft magnetic material such as iron at strategic places around and between the rotor can positively affect the coupling characteristics\textsuperscript{16}. This has especially been the case of the orientation in Figure 4(b).

However, for the purpose of wireless power transfer, to maintain the wireless gap, no iron can be placed between the rotors. Any iron placed around each rotor generally has the effect of shielding the field of one PM from the other PM\textsuperscript{17}. Therefore in the proposed transfer method, it was decided that no iron should be used.

3.1.1.4  Permanent Magnet Materials

There are many PM materials which could be used. The strength of the magnetization is important, characterized by the residual magnetic field. In recent years, the residual field of bonded and sintered Neodymium and Samarium Cobalt have increased dramatically to the order of 1.4 Tesla. When operating higher than a certain temperature, PMs lose their magnetization. In general, Samarium Cobalt PMs can operate at higher temperatures than Neodymium PMs\textsuperscript{18}. Finally, the cost of the PM can be a large driving factor. Due to the fact that the price of bonded Neodymium magnets is becoming more economical, and due to their high residual field, bonded Neodymium magnets were chosen for this application. The only drawback of bonded Neodymium magnets was that the operating temperature should not exceed 80\degree Celsius\textsuperscript{19}. However, this should not be a problem for most applications.

3.1.2  Theoretical treatment

For the purpose of wireless power transfer, these various considerations led to a magnetic gearing system design which has parallel axes of rotation, with both rotors magnetized across the diameter, with 2 poles per rotor, as illustrated in Figure 4(c). The 2-pole-rotor was constructed from a single piece of PM magnetized uniformly in magnitude and direction throughout. This was in contrast of typical magnetic gear construction where PMs of that application are of annular shapes, bonded to the outer surface of soft-magnetic shafts with high number of poles. The reason for this departure was that in typical applications, the
magnetic near-field is used, whereas for this particular application, the far-fields are also used. This is discussed in detail in Section 3.1.2.1. The shapes of the solid permanent magnet considered in this thesis are cylindrical and spherical PMs. The PM orientation is illustrated in Figure 5, which is the on-axis view of the magnetic gear illustrated in Figure 4(c).

![Figure 5: Magnetic gear orientation used for propose wireless power transfer system](image)

In the following section, this particular type of magnetic gear is analyzed. First, approximations were made to determine the magnetic field produced by each PM rotor. Second, using the magnetic field previously determined, the torque was calculated between the two PMs. Third, the time-averaged torque and power were calculated. Lastly, losses were estimated.

### 3.1.2.1 Introduction to far-field dipole approximation

When dealing with magnetic interactions, it was crucial to determine the magnetic field produced by each rotor. Since the rotors were made of a uniformly magnetized solid piece of permanent magnet of arbitrary shape, the exact solution of that field at all locations near and far from the magnet are usually prohibitively complex. Only for idealized shapes, such as infinite cylinders or perfect spheres, can the field be expressed analytically\cite{20,21}. For other shapes, numerical computation can lead to accurate results. However, intuitive understanding of the field as a function of design variables such as various dimensions is often illusive using numerical methods.
For the purpose of calculating the torque between two rotors of a magnetic gear, a popular method is used here, in which the near-field and far-field are considered separately. The near-field is defined as the field at a distance small compared to the largest dimension of the magnet. (It should be noted that this idea is completely different than the so-called electromagnetic near field, which is generally viewed as with about one wavelength of EM radiation at the frequency in question). The near-field is closely related to the surface fields of the PM and depends on the shape of the PM as well as the size. The far-field is defined as the field at a distance large compared to the largest dimension of the overall magnet shape. In the case of a uniformly magnetized PM, the far-field no longer depends on the particular shape of the magnet. Instead, it mainly depends on the total volume of the PM, independent of its shape. The characteristics of the far-field is very similar to an idealized dipole field, and is therefore called the far-field dipole approximation. This approximation will be used in the following sections to determine torque.

3.1.2.2 Calculation of the far-field

Under the dipole approximation, which is only valid for the far field, the field of the PM closely approaches that of an idealized dipole. The magnitude of the field of an idealized dipole is directly proportional to the magnitude of its dipole moment ($m_1$), inversely proportional to the cube of separation distance ($r$), and periodic with the magnetic latitude angle ($\lambda$) expressed in Equation (1), where $\mu_0$ is the permeability of free space22.

$$B(m_1, r, \lambda) = \frac{\mu_0 m_1}{4\pi r^3} \sqrt{1 + 3\sin^2(\lambda)}$$  \hspace{1cm} (1)

The definition of $\lambda$ is illustrated in Figure 6.

![Figure 6: Definition of coordinates and field magnitude contour plot](image-url)
The magnitude of the dipole moment \( (m_1) \) characterizes the strength of the magnetic dipole, and is proportional to the volume of the magnet \( (V) \) as well as the residual magnetic field \( (B_r) \) of the magnet\textsuperscript{23, 24}, expressed in Equation (2).

\[
m_1 = \frac{V B_r}{\mu_0}
\]  

(2)

The residual magnetic field is a measure of the strength of the magnetic material and usually depends on material properties and degree of magnetization during manufacturing.

There are a few important behaviors of the dipole field. The magnitude is greatest along the magnetization axis \( (\lambda = 90^\circ \text{ and } -90^\circ) \), and half that value along the perpendicular plane \( (\lambda =0^\circ) \). The overall magnitude is directly proportional to the volume of the magnet as well as the residual magnetic field.

![Figure 7: Rotating magnetic field \( (B_1) \) produced by a rotating PM with dipole moment vector \( (m_1) \)](image)

The most crucial behavior for the purpose of magnetic gearing is that if the direction of magnetic dipole \( (m_1) \) were to be rotated in the plane of the plot illustrated in Figure 7, it will produce a magnetic field at a set position a distance away \( (B_1) \) which rotates in the opposite direction with magnitude that varies between on-axis magnitude and half that value. The magnitude of the rotating magnetic field is always non-zero and positive, which allows the close coupling of two rotors at any angle in the rotation. In other words, there is 1 to 1 angle correspondence between transmitter and receiver rotors.
3.1.2.3 Torque

Due to the field created by each rotor, there is both a torque and a linear force felt by each rotor. For the purpose of a magnetic gear the attractive/repulsive linear forces between the rotors are irrelevant; it is the torque which is of primary concern.

The torque felt by a magnetic dipole in the presence of a magnetic field can be described as the twisting effect that which tends to align the dipole in the direction of that field. This torque is expressed as the cross product of the dipole moment and the magnetic field in Equation (3).

$$T = m_2 \times B_1 \quad (3)$$

In the present case, this equation can be understood as the torque ($T$) felt by the receiver PM with a magnetic moment ($m_2$), under the influence of a field ($B_1$) produced by the transmitter PM.

![Figure 8: Coordinate definition of magnetic gear](image)

The magnetic moment of the transmitter magnet ($m_1$), the magnetic field it produces at the location of the receiver magnet ($B_1$), and the magnetic moment of the receiver magnet ($m_2$) will be constrained in a common plane in the orientation illustrated by Figure 8. $\theta_1$ and $\theta_2$ are the angular positions of the transmitter and receiver magnets respectfully, and $\theta$ is the angle at which the receiver PM ($m_2$) lags behind the field produced by the transmitter PM ($B_1$). Here, $\theta$ will be called the lagging angle, as discussed below.

In this configuration, the direction of the cross product was normal to, and into or out of the page, so its scalar value provides a full description. The variables in Equation (4), $T$, $m_2$ are $B_1$, are the magnitudes of vectors $T$, $m_2$ and $B_1$, respectfully.
A first observation is that with no loading torque the angle $\theta$ between the moment ($m_2$) and the field ($B_1$) is zero. This is because if $\theta$ is not zero, the second rotor will feel a torque which tries to align the receiver magnetic moment ($m_2$) with that field to bring the value of $\theta$ to zero. Since the angle of the field ($B_1$) depends on the angle of the transmitter rotor ($\theta_1$), and since the direction of the receiver rotor ($\theta_2$) will follow the direction of ($B_1$), ($\theta_2$) will follow ($\theta_1$) for any angle. This maintains the previously mentioned one to one angular correspondence between transmitter and receiver rotors.

The coupling torque increases with increasing $\theta$ until the maximum operational value of 90°, expressed in Equation (4). Beyond 90°, the coupling torque will decrease. Therefore, the maximum coupling torque ($T_m$) is simply the product of the magnitude of the field and the magnetic moment, expressed in Equation (5).

$$T_m = m_2 B_1$$

The magnitude of the magnetic moment ($m_2$) is constant at any angle since it only depends on the PM volume and magnetization. However, the magnitude of the field ($B_1$) is not constant during the rotation. The magnitude of the field is periodic with the rotation angle ($\theta_1$) expressed in Equation (6).

$$B_1(m_1, r, \theta_1) = \frac{\mu_0 m_1}{4\pi r^3} \sqrt{1 + 3\sin^2(\theta_1)}$$
Figure 9: Transient behavior of $B_1$ as a function of $\theta_1$

The maximum time-averaged torque transferred then, is the product of the time-averaged magnitude of the field and the magnitude of magnetic dipole. Figure 9 illustrates that the normalized field (i.e. ratio of actual field to its maximum value) oscillates between 0.5 and 1.0. Therefore, time-averaged torque transferred is approximately $\frac{3}{4}$ of the maximum value (dotted line).

Equations (5) and (6) are combined to express time averaged maximum torque ($\overline{T}_m$).

$$\overline{T}_m = \frac{3}{4} \frac{\mu_0 m_1 m_2}{4\pi r^3}$$  \hspace{1cm} (7)

The maximum time-averaged torque, $\overline{T}_m$, is thus proportional to the scalar product of the two dipoles moments ($m_1$ and $m_2$) and inversely proportional to the $3^{rd}$ power of separation between the dipoles ($r$). Since the dipole moments are proportional to the volume of the two PMs, the time-averaged torque is proportional to the product of the volumes of the two permanent magnets.
3.1.2.4 Power

The time-averaged maximum mechanical power transferred ($P_m$) is the product of the time-averaged maximum torque ($\bar{T}_m$) and the angular frequency of rotation ($\omega$) expressed in Equation (8).

$$P_m = \omega \bar{T}_m$$

(8)

Power transfer between the transmitter and receiver, through the air-gap, is characterized by these expressions for torque and power.

It is clear that the power is also proportional to the speed of rotation.

3.1.2.5 Losses

Apart from, rolling, sliding and other types of friction, which can be minimized with correct bearing selection, there is only one type of loss inherent to the magnetic gear - eddy current loss. This arises when conductive materials are in the vicinity of the magnetic gear, possibly including the PM magnets themselves, if they are conductive as is often the case. Conductive materials interact with the changing magnetic fields created by the rotating PMs. Currents are created proportional to the time-derivative of the magnetic field, which results in resistive losses, as well as the creation of a counter-magnetic field that opposes the motion of the source magnet. This phenomenon is often called eddy-current damping.\textsuperscript{25}

Calculation of the power loss due to eddy current damping can only be done accurately by numerical means that take into account the shapes and fields in question. Such a determination was deemed unnecessary in this project since it was experimentally found to be a very small effect, as described in Section 5.2.5.2.

There are two main reasons for this. First, the eddy current damping depends on the conductivity of the material, which in this case, is not sufficiently high for the materials used in the rotor, namely Neodymium PM and stainless steel. Second, for the present case of two rotors spinning in synchronization, in steady-state operation, the direction of the magnetic field produced by the one rotor does not change in the perspective of the other rotor. This can
be seen in Figure 8 that $m_2$ and $B_1$ are always rotating at the same speed in the same
direction. Therefore, any time variance of the field at the perspective of the rotor is due to the
variance in the magnitude, not direction, of the field, which oscillates between 100% and
50% of the maximum. Therefore the amplitude of oscillation experienced by the rotors is
about $\frac{1}{4}$ that of the oscillations experienced by stationary objects.

It is for these reasons that the mechanical coupling of the magnetic gear is nearly 100%
efficient. This was experimentally confirmed as described in Section 5.2.5.2. While there was
non-trivial friction in the bearings, those are best described as part of the motor and generator
stages, as described in following sections.

3.2 Motor stage

The interaction between the transmitter winding and the transmitter PM is very similar to
those found in an electric motor. Current is supplied to the transmitter windings causing the
transmitter PM to turn$^{26}$. Therefore, electrical power in the transmitter windings is being
converted to mechanical power in the transmitter PM rotor.
There are two main options for converting electrical current into mechanical rotation of a permanent magnet. The first option uses any type of off-the-shelf electric motor, driving the PM rotor as illustrated in Figure 10(a). The second option involves integrating electrical windings around the PM rotor. The currents in the windings produce a magnetic field which will interact directly with the magnetic dipole of the permanent magnet, providing the torque that induces its rotational motion, as illustrated in Figure 10(b). The first option does not take advantage of the fact that the rotor in question has a magnetic dipole, whereas the second option does. Using a separate motor may require less customization, and using integrated windings may lead to smaller, more robust, and more efficient devices.

Both methods are viable for wireless power transmission, and each will be discussed in the following Sections 3.2.1 and 3.2.2.
3.2.1 Separate electric motor

When using a separate electric motor to drive the transmitter magnet, any motor that converts electrical power into mechanical power at the shaft can be used. Popular motors include AC induction motors, brushed DC motors and brushless DC motors (BLDC). Generally AC induction and BLDC motors are preferable as they have long life.

AC induction motors are inexpensive, efficient and quiet. Their efficiency increases with power level, reaching approximately 90% for common designs at 5hp\textsuperscript{27}.

However, without additional inverters, their speed is ultimately limited by the power-line frequency, which is at most 3600 rpm for most places in the world. Limiting the speed reduces the power transfer capability shown in previous Section 3.1.3.4. To operate at higher speeds require gearboxes and belting systems, which add frictional losses, noise and complexity. However, the AC induction motor is certainly one of the easiest and most robust methods.

A DC brush motor can also serve the purpose. However, they are inherently noisy and generally less efficient\textsuperscript{28}. The advantage of using DC motors is that the motor and control devices can be extremely cheap, and they do not have an inherent speed limitation.

A brushless DC motor is one of the most suitable motors for this application, they are efficient at any power level, capable of very high speeds, and quiet. The only disadvantage is that the motor and control electronics are relatively expensive\textsuperscript{29}.

No matter which kind of motor drives the transmitter rotor, there will be losses associated with it, including additional friction in the components as well as electrical losses in the copper and hysteresis and eddy current losses in the iron. These losses adversely affect the overall power transfer efficiency of the device, but this design is easy to construct and implement using off-the-self motors and other components. This is the method used for experimental verification work described in Chapter 5.
3.2.2 Custom BLDC motor

In an alternative drive design, the transmitter rotor can be driven by mounting coils directly around the permanent magnet of the rotor, as illustrated in cross-section in Figure 11. A current supplied in those coils produce a magnetic field that causes the permanent magnet to rotate. This is very similar to a BLDC motor, except the rotor permanent magnet serves both as part of a motor, as well as part of a magnetic gear transferring power to the receiver side.

Figure 11: Motoring by custom BLDC machine

In order for the PM in the rotor to serve both as a component inside the motor as well as a component in the magnetic gear to drive the receiver PM, the design of this motor must differ slightly from that of conventional BLDC motor design.

The most important deviation is the absence of any soft-magnetic material. It is conventional for most BLDC motors to place windings around slots made of iron or steel that surrounds the entire circumference of the motor as illustrated in Figure 12.
This has the effect of enhancing the field created by the current in those windings, by creating closed “magnetic circuits”. It also allows less precise placement of windings. However for the current application, such use of iron would substantially prevent the field created by the transmitter PM from reaching the receiver PM. A BLDC motor which does not use any soft-magnetic material is called ironless or coreless. An ironless BLDC motor is already used in some specialized applications. The main advantages they have in those specialized applications are high rotational speeds, and the absence of losses associated with iron, such as eddy current, leakage and hysteresis losses. To our knowledge an ironless BLDC motor has never before been used in a wireless power transfer device.

A second deviation from convention BLDC motor design is that the rotor for this device is practically made up entirely of a uniformly magnetized permanent magnet, rather than relatively thin annular shaped magnets bonded to an iron shaft. The reason conventional BLDC motors use annular shaped magnets is because it is compatible with high number of poles. Another reason, is that a conventional BLDC motor interacts with only the near-fields or surface fields of the magnets, whereas the currently proposed wireless power transfer device interacts with both the near and far-fields. To obtain a strong far-field, the total volume of the PM must be maximized and therefore a solid, uniformly magnetized PM rotor is used.
The principles of operation of such a BLDC motor is similar to most motors. In this case it is simpler due to the absence of soft magnetic materials. The basic principle is that a wire of length, \( L \), carrying a current, \( I \), in a magnetic field, \( B \), feels a Lorentz force, \( F \), according to Equation (9).

\[
F = LI \times B
\]  

(9)

To use this force, consider windings made of a single turn placed around the PM rotor in the fashion illustrated in the cross-sectional diagram in Figure 13.

**Figure 13: Force (F) felt by current carrying wire in magnetic field of PM**

Consider a cylindrical PM with length, \( L \) and axis of rotation directed into the page. A single turn of wire surrounds the PM centrally, with paths into and out of the page, left and right of the PM. The left and right side of the winding is placed within the magnetic field of the PM, \( B \). When a current, \( I \), is supplied to the winding, the winding feels the Lorentz force, \( F \), in the directions shown. This result in a torque in the windings of magnitude \( T \), in Equation (10), where \( B_x \) is the magnitude of the field felt by the wire in the \( x \)-direction, \( r \) is the distance illustrated in Figure 13, and \( L \) and \( I \) are magnitudes of \( L \) and \( I \), respectfully.

\[
T = 2(LIB_x)r
\]  

(10)

\[
P = 2(LIB_x)r\omega
\]  

(11)
Technically this torque is applied by the magnetic field on the wire, but due to Newton’s law of reciprocal action, this torque is also applied by the wire, on the magnet. Since for this BLDC motor, the windings are held stationary and the PM rotor is allowed to rotate, this torque causes the PM rotor to rotate\(^3\).

The mechanical power is therefore \(P\), shown in Equation (11).

Consider that the PM rotor is rotating at constant velocity. It is important to recognize in the equation for power (11), the values of \(L\), \(r\), and \(\omega\) are constant with respect to time, while \(B_x\), the magnetic field in the \(x\)-direction felt by the windings varies sinusoidally, and the current, \(I\), is yet undefined. To obtain a maximum time-averaged torque, \(I\) must have a time-varying waveform in phase with \(B_x\).

Therefore the current supplied to the windings are typically controlled with an inverting driver to produce an AC current \(I\) in phase with \(B_x\). The value of \(B_x\) is typically sensed by a Hall Effect sensor to ensure that \(I\) is in phase with \(B_x\). The specific shape of the waveform of \(I\) is typically either sinusoidal or trapezoidal.

Unlike typical BLDC motors, iron-losses are absent in this design. The main losses are resistive losses in the windings, frictional losses in the bearings, and switching losses in the driver.

### 3.3 Generator stage

The purpose of generator stage is to convert the mechanical power of the rotating receiver magnet back into electrical power in the windings. This interaction is therefore analogous to that of a conventional electrical generator.
Figure 14:  Generating by using a separate electric generator

Similar to the motoring interaction, there are two main ways to convert the mechanical power to electrical power. The first involves connecting the rotating PM rotor to a separate generator illustrated in Figure 14(a). This method does not make use of the fact the rotor contains a large PM. It generally adds additional frictional losses as well as other losses associated with the generator, but allows the assembly to be constructed of widely-available off-the-self generators and other components.

The second way involves mounting windings directly around the PM in the rotor as illustrated in Figure 14(b). The rotating magnetic field created by the rotating PM creates a time-changing flux linkage in the windings and generates back-EMF to be used as electrical power. This method takes advantage of the fact the rotor contains a PM. It requires the design of a custom PM generator, but allows higher efficiency.
Both are viable methods for the proposed power transfer device, and will be considered in more detail in the following sections. For the work described in this thesis, the custom BLDC generator was the selected method, as it produces a more compact design for the receiver.

3.3.1 Separate generator

Any type of electric generator that transforms mechanical power from a rotating shaft into electrical power can be used for this application. In addition, many kinds of electric motors can also be used in reverse as the generator for this application32.

Similar to the motoring case, generally using a separate generator adds additional losses and lower efficiency. It also increases the number of moving parts, vibration and noise. However, this method allows the construction of the receiver side using off-the-self components and requires little design and development time.

3.3.2 Custom BLDC generator

As an alternative to using a separate generator, windings could be mounted directly around the receiver PM rotor to act as a generator. This is very similar to the case of the custom BLDC motor previously described in Section 3.2.2. For the PM in the receiver rotor to serve both as a component in the magnetic gear, as well as a component in the generator, the same considerations are necessary as in the custom BLDC motor case. These are the absence of any soft magnetic materials which tend to shield the far-field, and the use of a solid uniformly magnetized PM which occupies practically the entire volume of the rotor, maximizing the sensitivity to far-field dipole fields.

Especially as the smaller size of this approach may be most beneficial in the receiver, here the operation of the custom BLDC generator is analyzed in some detail. The PM would rotate in the close vicinity of the winding and produce a time-changing flux. The change in flux gives rise to induced EMF and current, which is applied to an electrical load.
3.3.2.1 Power

To analyze the behavior, the components are simplified in Figure 15.

Figure 15: Simplified custom BLDC generator

A magnetic rotor of a circular cross-section will be considered. It is uniformly magnetized with $B_r$ residual magnetization. The rotation of the rotor forms an angle ($\phi$) between the magnetization vector and the winding plane. A single winding of $N$ turns and total resistance $R_C$, surrounds the PM centrally. Since the windings do not have a soft magnetic core, and because the rotation frequency is relatively low, it is reasonable to assume the self-inductance in the winding plays a negligible role. This assumption is experimentally confirmed in the following Section 5.1.5.1.

The friction of all receiver rotor parts is consolidated into a single torque, $T_{\text{fric}}$. The frictional torque in this case is defined to include both mechanical friction, as well as any eddy current damping. The mechanical friction component includes all bearing friction. Eddy current damping friction occurs when the rotating PM create currents in conductive materials in the vicinity. These currents produce a magnetic field which acts against the rotating PM, in the form of a torque proportional to frequency. The eddy current damping components include eddy currents in the metallic bearings, as well as eddy currents in the windings. Since the rolling frictional torque of the bearing also increase with frequency, it is reasonable to describe the sum of the frictional torque, $T_{\text{fric}}$, as approximately proportional to the speed expressed in Equation (12), where $b$ is the frictional constant.

$$T_{\text{fric}} = b \omega$$ (12)
When the windings are in open circuit, the voltage induced is proportional to the derivative of flux linkage ($\Phi$) in Equation (13).

\[ V = \frac{\partial \Phi}{\partial t} \quad \text{(13)} \]

It is clear in the geometrical configuration that the flux will oscillate at the rotation frequency, and so can be approximated, in terms of the angle of rotation ($\phi$) in the following Equation (14).

\[ \Phi = \Phi_0 \sin(\phi) \quad \text{(14)} \]

When the PM rotates with angular velocity $\omega$, $\phi = \omega t$, this produces an induced voltage, $V$ in Equation (15).

\[ V = \Phi_0 \omega \cos(\omega t) \quad \text{(15)} \]

When an electrical load of resistance $R_L$ is added and the circuit is closed, the circuit behaves as a voltage source in series with the winding resistance $R_C$ in series with the load resistance $R_L$ illustrated in circuit in Figure 16.

![Circuit Diagram](image)

**Figure 16:** Generator loaded by $R_L$

When operating at certain $R_L$ and $\omega$, the time-averaged power obtained by the load, $P_L$, is expressed in Equation (16).

\[ P_L = \frac{1}{2} (\Phi_0 \omega)^2 \frac{R_L}{(R_C + R_L)^2} \quad \text{(16)} \]
For any given BLDC generator design, \( \Phi_0 \) and \( R_C \) are constant during operation. However, speed \((\omega)\) and load \((R_L)\), are adjustable. Therefore in Figure 17 below, \( P_L \) is graphed with respect to \( R_L \) for a number of operating angular velocities. The values of \( \Phi_0 \) and \( R_C \) used in Figure 17 are 0.044 V\( \text{s} \) and 9.2 \( \Omega \) respectfully. These values are from a custom BLDC generator constructed and discussed in detail in Section 5.2, they serve merely as an example in this section.

![Figure 17: Power without synchronization or mechanical speed restriction](image)

Figure 17: Power without synchronization or mechanical speed restriction

The power values in Figure 17 assumes there is arbitrary freedom in \( \omega \) and \( R_L \). However, for the present case, the magnetic gear restricts the operation of the generator in certain regions of Figure 17. This region is identified in the following section.

For a given magnetic gear orientation and separation, there is a maximum time average torque sustainable, \( T_m \), as expressed in Equation (7) and described in Section 3.1.2.3.

Loading the receiver PM with a torque greater than \( T_m \) causes the loss of synchronization.
between the two PM rotors. Therefore it is required that the sum of all frictional and electromagnetic torque applied to the generator PM be smaller or equal to $T_m$. This restriction is expressed in Equation (17).

$$T_m \geq T_{\text{fric}} + \frac{P_L + P_C}{\omega}$$  \hspace{1cm} (17)

The restriction for synchronization is expressed in full in Equation (18). The previously assumed form of $T_{\text{fric}} = b\omega$ is used here. The expression of $P_C$ is identical to $P_L$ except with the reversal of $R_L$ and $R_C$.

$$T_m \geq b\omega + \left( \frac{1}{2} \Phi_0^2 \omega \frac{R_L}{(R_C + R_L)^2} \right) + \left( \frac{1}{2} \Phi_0^2 \omega \frac{R_C}{(R_C + R_L)^2} \right)$$  \hspace{1cm} (18)

Equation (18) is simplified and solved for $\omega$ in Equation (19).

$$\omega \leq \frac{T_m}{b + \frac{\Phi_0^2}{2 \cdot (R_C + R_L)}}$$  \hspace{1cm} (19)

This expression for $\omega$ in (19) is substituted back into the expression for power at the load in (16), obtaining the expression for power restriction under synchronization requirement in Equation (20).

$$P_L \leq \frac{1}{2} \left( \frac{\Phi_0 T_m}{b} \right)^2 \frac{R_L}{\left( R_C + R_L + \frac{\Phi_0^2}{2b} \right)^2}$$  \hspace{1cm} (20)

Therefore, the inequality of (20) has identified the regions of Figure 17 which are outside the operating limits dictated by the synchronization requirement of the magnetic gear. Figure 17 is re-plotted in Figure 18, along with the region which violates the synchronization requirement identified in Equation (20). An example value for $T_m$ of 0.065 Nm is used in Figure 18
Figure 18: Power with synchronization, without mechanical speed restriction

There is one last restriction, the mechanical restriction to operating speed. This final restriction identifies yet another region of Figure 18 in which operation cannot occur.

In most rotating structures, the speed of rotation is restricted by the tensile strength of the rotor material. Operating beyond a certain speed would cause the rotor to break apart under centrifugal forces. The tensile stress ($\sigma$) experienced by a solid rotating cylinder for example, is dependant on the density ($\rho$), the angular velocity ($\omega$), the outer radius ($r_0$), and the Poisson’s ratio ($\nu$) expressed in Equation (21)\textsuperscript{33}.

$$\sigma = \rho \omega^2 r_0^2 \frac{(3 + \nu)}{8}$$  \hspace{1cm} (21)

This stress experienced by the rotor must not exceed the tensile strength of the material with a considerable safety factor, ($sf$). This restriction is expressed in (22).
Expression (22) is solved for $\omega$ to obtain the Equation for mechanical speed restriction in (23).

$$\omega \leq \frac{1}{r_0} \sqrt{\frac{\sigma}{sf} \left( \frac{8}{3 + \nu} \right)} \frac{1}{\rho}$$  \hspace{1cm} (23)

Therefore, the inequality of (23) identifies a region in Figure 18 which violates mechanical speed restrictions. This region, along with the regions which violates synchronization requirement, is plotted in Figure 19.

![Graph showing synchronization and mechanical speed restriction](image)

**Figure 19:** Power with synchronization and mechanical speed restriction

The maximum power achievable under the restrictions is illustrated in Figure 19, identified by $P_{\text{max}}$. $P_{\text{max}}$ is achieved by operating at a load resistance of $R_{L,\text{max}}$ and a speed of $\omega_{\text{max}}$, also identified in Figure 19.
\( P_{\text{max}} \) is of particular interest and can be expressed analytically by solving for \( R_{L,\text{max}} \) in Equation (19) and substituting that \( R_{L,\text{max}} \) and \( \omega_{\text{max}} \) into Equation (16). The result is presented in Equation (24).

\[
\begin{align*}
P_{\text{max}} &= \left[ T_m \omega_{\text{max}} - b \omega_{\text{max}}^2 \right] \left[ 1 - \frac{2 R_c}{\Phi_0^2 \left( \frac{T_m}{\omega_{\text{max}}} - b \right)} \right] \\
&= \left[ T_m \omega_{\text{max}} - b \omega_{\text{max}}^2 \right] \left[ 1 - \frac{2 R_c}{\Phi_0^2 \left( \frac{T_m}{\omega_{\text{max}}} - b \right)} \right]
\end{align*}
\]

(24)

The content of the first brackets in Equation (24) is the maximum mechanical power that can be transferred by the magnetic gear under synchronization, minus the frictional power loss. The content of the second brackets can be interpreted as the efficiency of converting mechanical power into electrical power in the generator. There are two reasons for this interpretation. The first is that this term is unit-less and can vary between 0 and 1 for all reasonable situations, which are characteristics of an efficiency. The second is that multiplying this term with an expression for mechanical power yields an expression for electrical power, suggesting it is an efficiency to convert mechanical power to electrical power.

In particular, this efficiency term approaches 1 for large values of \( \Phi_0^2 / R_c \). Since \( \Phi_0 \) and \( R_c \) are magnetic flux amplitude and resistance of the winding, it can be reasonably concluded that the efficiency depends heavily on the winding of the BLDC generator. Therefore to achieve a high efficiency, the term, \( \Phi_0^2 / R_c \), must be maximized. The term, \( \Phi_0^2 / R_c \), has been named the “winding factor”. The winding factor is explored further in the following Section 3.3.2.3.

### 3.3.2.2 Efficiency

The efficiency of the custom BLDC generator is defined as the efficiency of converting mechanical power in the receiver PM rotor into electrical power in the load illustrated by the power flow diagram in Figure 20.
Figure 20: Power flow diagram of custom BLDC generator

Since there are no soft-magnetic materials in this system there is no iron losses characteristic of traditional generators. The main losses are therefore frictional losses in the mechanical rotation of the PM, and resistance and eddy current losses in the copper windings around the PM. $T_{\text{fric}}$ is the sum of many components previously mentioned at the beginning of Section 3.3.2.1. The dominant components of $T_{\text{fric}}$ are the mechanical friction in the bearings, the eddy current damping in the bearings, and the eddy current damping in the windings. Eddy current damping in the bearing can optionally be eliminated by using non-metallic bearings such as full ceramic bearings.

Under synchronization and mechanical speed restrictions, the expression for efficiency at full power is obtained by dividing the expression for power in Equation (24) by the input mechanical power, $\bar{T}_m \omega$. This gives an intuitive expression for efficiency ($\eta$) illustrated in Equation (25).

$$\eta = \left[ 1 - \frac{b \omega_{\text{max}}}{\bar{T}_m \omega_{\text{max}}} \right] \left[ 1 - \frac{2 R_c}{\Phi_0 \omega_{\text{max}}} \left( \frac{\bar{T}_m}{\omega_{\text{max}}} - b \right) \right]$$  \hspace{2cm} (25)

The expression shows that the total efficiency is the product of 2 efficiencies. The first term is the mechanical efficiency, the proportion of mechanical power remaining after subtracting...
frictional power. The second term is the electrical efficiency, depending on the winding quality factor $\Phi_0^2/R_c$.

The optimization of mechanical efficiency involves the optimization of bearings. The proper choice of type, size, lubrication, and material properties of the bearing can reduce the bearing frictional loss down to a negligible fraction compared to the torque transferred, $T_m$. The optimization of mechanical efficiency also involves reducing eddy current damping in the windings. This can be achieved by reducing the diameter of the wire used in the windings, which reduces eddy currents in the windings, much like the lamination of iron. These considerations can yield high efficiency for a range of sizes and power levels, as verified experimentally and described in Chapter 5.

The optimization of electrical efficiency mainly involves the maximization of winding factor $\Phi_0^2/R_c$. It is clear from the expression for winding factor that it is desirable to increase the magnetic flux while reducing the resistance of the winding. This involves proper design of the windings around the PM rotor as will be discussed in detail in the next section.

### 3.3.2.3 Winding optimization

Using the criteria of winding factor, $\Phi_0^2/R_c$ it is possible to determine the optimal location for each turn of wire and the optimal total number of turns. Numerical methods can be used for each particular shape of PM and general desired winding orientation. Numerical methods in this case can be very helpful to determine the winding parameters. However, analytical methods are also helpful to gather intuitive insight into the relationships for winding factor and various variables. Both numerical and analytical methods are discussed in this section.
3.3.2.3.1 Numerical method

**Figure 21:** Placement of turns of the winding around PM

The amplitude of magnetic flux of the winding, $\Phi_0$, is the flux when the magnet dipole vector is normal to the winding plane, illustrated in Figure 21. Starting with a single turn of wire, each additional turns of wire adds flux as well as resistance. It is optimal to add an additional turn as long as the benefit of the additional flux outweighs the detriment of additional resistance. This optimization is evaluated using Equation (26). Using a simple algorithm, the optimal shape, size and location of the wire can determined.

$$ \frac{\Phi_0^2}{R_C} = \frac{\left( \sum_{i=1}^{n} \Phi_{0i} \right)^2}{\sum_{i=1}^{n} R_{Ci}} \quad (26) $$

Equation (26) evaluates the total winding factor, $\Phi_0^2/R_C$, where $\Phi_{0i}$ is the magnetic flux, and $R_{Ci}$ is the resistance of the $i^{th}$ turn illustrated in Figure 21, and $n$ is the total number of turns of wire.

The first turn should be placed at a location where the winding factor is highest for the single turn. Then each additional turn should be added at a location not already covered, where the total winding factor experiences the greatest increase. This algorithm can then recursively select the next best locations for turns of wire.
Figure 22: Total winding factor ($\frac{\Phi_0^2}{R_c}$) is maximum for a specific number of turns (n)

Ultimately, with larger values of $n$, the winding factor will start to decrease as illustrated in Figure 22. At this point, it is no longer worthwhile to add any additional turns of wire since the flux gained at the outer reaches no longer outweighs the resistance added. In this way, the algorithm determines the optimal shape and size of winding which maximizes winding factor and electrical efficiency.

Using this approach, as long as the field of the PM rotor is known, and the flux for each winding can be calculated, it is possible to determine the optimal placing of the windings. For special shapes of PM such as a uniformly magnetized sphere, there is an analytical solution to the field, which could be used to transform the algorithm to a 2 dimensional problem, significantly simplifying the process. Specific optimization examples are described in Chapter 5.

A detailed example of this algorithm is provided in Appendix B for an infinite cylindrical PM, magnetized across the diameter.
3.3.2.3.2 Analytical method

In addition to the numeric solution, it is also helpful to develop an analytical treatment to gain some intuitive knowledge about the relationships. In particular, the effect of gauge or thickness of wire used, the packing factor of the wire, and resistivity of the wire, can be explored.

\[
R_C \propto N \frac{Rl}{A_C} \quad (27)
\]

Consider the situation in Figure 23 in which the total area, \(A_T\), allotted for the winding is given. The total resistance of the winding, \(R_C\), is proportional to the number of turns, \(N\), times the resistance of each turn. The resistance of each turn is proportional to the resistivity of the conductive material, \(R\), and the length of the turn, \(l\), and inversely proportional to the cross-sectional area of the wire, \(A_C\) (27).

\[
N = \frac{A_T}{A_C \cdot pf} \quad (28)
\]
(28) is substituted into (27) for \( A_c \) to give \( R_C \) in Equation (29).

\[
R_C \propto N^2 \frac{Rl}{\mu pf}
\]  

(29)

The total flux of the winding is proportional to \( N \) in Equation (30).

\[
\Phi_0 \propto N
\]  

(30)

Therefore the behavior of the total winding factor is expressed in (31).

\[
\frac{\Phi_0^2}{R_C} \propto N^2 \frac{A_t pf}{N^2 \frac{Rl}{l}}
\]  

(31)

The first observation is that the winding factor is independent of \( N \); the gauge of wire used does not affect the winding factor. This is because when increasing wire density, it increases magnetic flux, as well as the resistance. In the end these effects cancel one another. The second observation is that the winding factor is inversely proportional to the resistivity of the material. Therefore it is advantageous for example, to use copper instead of aluminum. Third, the winding factor is linearly proportional to the packing factor. Therefore it is advantageous to wind round wire as close as possible. A randomly wound winding, using wires with round cross-sections, has a packing factor of approximately 78%. A hexagonally close-packed winding, where round wires are placed in a hexagonal pattern in cross-section, has a packing factor of 91%.
4 SCALING

Performance parameters such as power and efficiency depend on physical size of the device. The nature of this dependency can allow the prediction of performance parameters of a wide range of size-scales. It can thus provide further understanding of the system.

4.1 Torque

As described in Chapter 4, the magnitude of the far-field of each PM is dependant on the magnitude of the magnetic dipole moment. The expression of the dipole moment (2) and far field of a dipole (1) shows that the far field is directly proportional to the volume of the PM.

The torque capability is the product of the field and the receiver dipole moment illustrated in Equation (7). Therefore the torque is proportional to the product of the volumes of the PMs, and inversely proportional to the $3^{rd}$ power of their separation distance.

Consider the case when a linear scaling factor, $l$, is applied to all the physical dimensions including the PMs as well as the separation distance. In the equation for the torque capability (7), each dipole moment, $m$, is proportional to $l^3$, while $r$ is proportional to $l$. Therefore the torque capability is proportional to $l^3$. Therefore a large torque capacity can be gained by a small linear scaling factor.

4.2 Operating speed

The mechanical speed limit arises due to the fact that centrifugal stresses cannot exceed the tensile strength of the PM material, covered in Section 3.3.2.1. Using the expression for the centrifugal stress (23), it is clear that when increasing the radius of the PM rotor, the angular velocity of rotation must be decreased to maintain the same centrifugal stress. It is clear that the angular velocity of the rotation is inversely proportional to radius. Therefore under a linear scaling factor of all physical dimensions, $l$, the speed of operation is proportional to $l^{-1}$. 
4.3 Mechanical power transfer

The mechanical power transferred is simply the product of torque and angular velocity. Therefore under a linear scaling factor, $l$, of all physical dimensions including separation distance, the power is proportional to $l^2$.

Alternatively, when the linear scaling factor is applied to the dimensions of the PMs only, leaving the separation distance unscaled, the power is proportional to $l^5$. From either perspective, it is clear that a large increase in power can be achieved by a small increase in scale.

4.4 Winding factor

To transform the mechanical power to electrical power and visa-versa, the efficiency is dependant on the winding factor. This is seen in expression for the efficiency of the generator (25). The winding factor is expressed in Equation (32).

$$\frac{\Phi_0^2}{R_C}$$  \hspace{1cm} (32)

The flux, $\Phi_0$, is proportional to the area of the winding. The winding resistance, $R_C$, is proportional to the length of the winding, and inversely proportional to the cross-sectional area of the wires.

Therefore under the linear scaling factor $l$, $\Phi_0$ is proportional to $l^2$, and $R_C$ is proportional to $l^1$ assuming the diameter of the wire is also scaled by $l$. Therefore the total winding factor is proportional to $l^5$.

4.5 Electrical efficiency

The electrical efficiency is expressed as the 2nd bracket in Equation (25), re-written below in (33).
\[ \eta_{elec} = \left[ 1 - \frac{2R_c}{\Phi_0^2} \left( \frac{\bar{T}_{m}}{\omega_{max}} - b \right) \right] \] (33)

Previously torque has been determined to be proportional to \( l^3 \), and operating speed has been determined to be proportional to \( l^1 \). Neglecting frictional constant, \( b \), electrical efficiency is then proportional to \((1 - l^1)\). It is clear from that expression that an increase in size will generally lead to an improvement in efficiency as well.

### 4.6 Scalability conclusion

Using the scaling behaviors described previously, a wide range of device sizes can be constructed with varying power capacity and high efficiency by scaling the physical dimensions of the components, enabling a wide range of potential applications.
5 EXPERIMENTAL VERIFICATION

This wireless power transfer device was constructed and tested at 2 size scales, a kilowatt scale model for industrial and automotive applications, and a 60 watt scale model for consumer electronics and biomedical applications.

5.1 Kilowatt scale application

A power transfer device was designed to be capable of transferring a few kilowatts and was then constructed and tested. The intended application at this scale is charging of electric automobiles and industrial power transfer.

5.1.1 Automotive application

In the automotive sector, there is currently widespread interest for electric and plug-in hybrid electric vehicles (EVs). For widespread adoption of these technologies, there are a few problems that need to be overcome related to charging of the batteries, described below. The currently proposed method of wireless power transfer addresses a number of them.

The first such charging-related problem is associated with the common contact-type electrical connection plug. Many standard electrical plugs such as the NEMA standard plugs perform very well indoors. However when the car needs to be charged outdoors, in wet and dirty environment, these contact-plugs suffer from problems. One is the chance of electrical shock in the rain. Another is the corrosion of the connectors arising from the outside environmental factors such as rain, road salt, and frost. These problems have been addressed by certain existing non-contact connector methods, such as the Magne-Charge system for the EV1\textsuperscript{34}. These use a type of inductive power transfer, at very close separation. Essentially the transmitter winding is in the plug and the receiver winding is in the socket on the vehicle. The power is transferred through inductive methods as previously described.

However, these technologies are not wireless because the plug must still be connected to a cord which must be fully inserted into a socket on the vehicle, just as with a contact-
connector. These wired technologies also suffer from a few additional problems. One relates to charging in public. The major obstacle facing the electric car proliferation is the short travel range on a single charge due to heavy, large and expensive batteries\textsuperscript{35}. Although many advances have been made to improve battery technology, the practice of charging both at home and the destination essentially doubles the range of the vehicle. Therefore it is important that electric cars can charge in public parking garages and on busy streets. Here, the threat of copper theft and vandalism of the cords become a significant concern. These problems can be addressed with various well known inductive wireless devices\textsuperscript{36} outlined in Section 2.1.

However, those wireless technologies suffer from the high frequency of operation. The most important concern of which is potential health risks of the high frequency electro-magnetic radiation generated from their operation, discussed in Section 2.2. Currently, there isn’t a well known method to transfer the required levels of electrical energy wirelessly at low frequency, except for the method proposed here.

By operating at much lower frequencies, the threat to human health is significantly reduced. This is especially important when charging at home or in a crowded public place such as a busy sidewalk where many people will be exposed. The evaluation of human health risks will be addressed in more detail in Chapter 6.

5.1.2 Industrial applications

Besides the automotive application, this power capacity is in a range that is relevant to many industrial applications, specifically autonomous robotics, where it may be advantageous to power devices wirelessly.

Autonomous robots often perform continuous tasks without human supervision. They are often battery powered to have a high degree of mobility. In order to operate truly without supervision, they need to autonomously re-charge their batteries\textsuperscript{37}. Contact-charging methods require high degree of autonomous alignment precision. The method proposed here would
allow very robust charging with comparatively modest alignment requirements and could thus decrease the cost and complexity of autonomously charging robots.

5.1.3 Design procedure

The design and implementation of the kilowatt model was largely motivated by the application of electric car charging. First, separation and power requirements were estimated for a typical electric car. Second, size, shape and rotation speed of the magnets were determined to satisfy the power requirement and the separation requirement. Finally, the method of motoring on the transmitter side and generating on the receiver side were determined. These steps are described below.

5.1.3.1 Magnet type selection

The magnet type is selected based on performance, cost, and availability, which led to the selection of bonded Neodymium magnets. They have very high performance, with residual fields in the range of 1.4 Tesla (T). One minor disadvantage is their lower operating temperature, which cannot exceed 80°C. The cost of these magnets is now relatively low. There are many manufacturers producing custom bonded Neodymium magnets to various shapes, sized and magnetizations, so it may be the optimal choice for commercial production.

It is likely that this is eventually the right economically viable choice for potential future production. Finally, there are many manufacturers producing custom bonded Neodymium magnets to various shapes, sizes and magnetizations.
5.1.3.2 Separation and power requirement

![Electric vehicle ride height and charging separation](image)

Figure 24: Electric vehicle ride height and charging separation

The separation requirement between the transmitter PM and receiver PM during the power transfer was estimated. In one of the possible charging orientations, the receiver is mounted on the bottom of the vehicle and the transmitter is mounted on the floor (Figure 24). In this case, the axis to axis separation is equal to the ride-height of the vehicle. A brief survey of a few of the most popular passenger cars in production today yields 15 cm for this separation requirement. Of course this is only approximate.

The power requirement was determined by considering the typical rate of charge of today’s all-electric and plug-in hybrid-electric cars. This was found to be in the 1 kW range, although more powerful system are of interest as well.

5.1.3.3 Size, shape and speed requirement

With power and range requirements determined, it was possible to determine the size of PMs and the speed of rotation that would satisfy those requirements.

First, a general shape of the PM was selected. Since these PM rotors experience rapid rotation, a shape was chosen that is consistent with a rotational symmetry to avoid unnecessary aerodynamic friction. Additionally, considering the rotor under cylindrical coordinates, the maximum radius was minimized, while the volume was maximized. This is because the maximum radius of the PM limits the ultimate mechanical rotation speed. It is for this reason the shape of the PM was chosen to have a radius constant along its length. In
other words, a cylindrical PM rotor has been designed, magnetized uniformly, in direction and magnitude, in an arbitrary direction perpendicular to the axis of rotation, across the rod (Figure 25). This PM was to be custom made, as discussed in section 5.1.4.1.

Figure 25: Shape and orientation of finalized PM rotors

For rotors magnetized across the diameter such as this, there is a restriction set by the manufacturers of Neodymium magnets. Typical maximum magnetization path length for most manufacturers today is 5 cm\(^3\), and this ended up determining the diameter of the cylinder. The length of the cylinder, however, was still to be determined.

Now that the shape of the magnet has been chosen to be cylindrical and the radius set to 2.5 cm, the maximum rotation speed was determined. This speed is limited by the tensile strength of the magnet material under centrifugal forces. The tensile strength of bonded Neodymium PMs is 37 MPa\(^3\). Equation (21) is an expression for this stress for cylindrical rotors. A failure of this type is catastrophic and potentially dangerous to the operator. Therefore it was decided to operate at a speed which produces a stress level no higher than \(1/20^\text{th}\) of the tensile strength of the PM rotor. Therefore using Equation (21) with material properties of bonded Neodymium magnets at 2.5 cm radius, it was determined that the rotation frequency should be limited to 150 Hz or 9000 rpm. According to the expression
(21), this will produce $1/20^{th}$ the stress it can withstand. The rotation frequency of 9000 rpm is practical speed for rotors, spindles and bearings.

With all the above information, the final dimensions can be determined using Equation (7). With $r$ set to 15 cm and $P$ set to 1 kW, the product of the dipole moments of each PM is determined to be 48,000 Joule$^2$/T$^2$. Using the expression for dipole moment (2), assuming a residual field of 1.45 T, and using the characteristics of typical N52 Neodymium magnets, the product of the volumes of the two PM required has been determined to be $3.58 \times 10^{-8}$ m$^6$.

Since the power depends on the product of the volumes of the 2 PMs, the total volume was minimized by setting the individual volumes to be equal to one another. This determined the total volume for each magnet to be $1.89 \times 10^{-4}$ m$^3$. Since the PM has already been determined to be a cylinder of radius 2.5 cm, the length must be 9.7 cm, set at 10 cm for simplicity in manufacturing. The resultant design of the PMs is summarized in Table 1.

<table>
<thead>
<tr>
<th>Power</th>
<th>1 kW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>15 cm</td>
</tr>
<tr>
<td>Magnet shape</td>
<td>Cylinder</td>
</tr>
<tr>
<td>magnet diameter</td>
<td>5 cm</td>
</tr>
<tr>
<td>Magnet length</td>
<td>10 cm</td>
</tr>
<tr>
<td>Speed of rotation</td>
<td>150 Hz</td>
</tr>
</tbody>
</table>

Table 1: Design parameters of kilowatt wireless power transfer model

5.1.3.4 Method of motoring and generating

As mentioned in Sections 3.2 and 3.3, there are several possible motoring and generating methods. In general, the choice of each method depends on the application. For the purpose of this thesis, several factors were considered, including short term issues such as the time
and cost required to develop a working model, and more significant long term issues such as efficiency, potential cost in volume production, weight, size, noise and vibration levels.

The outcome was a compromise between these short term and long term considerations. It was decided that the motor driving method would be used on the transmitter side, employing an off-the-self motor and widely available components such as belts, pulleys, shafts and bearings to provide mechanical power to the transmitter PM rotor. This saved time and resources which would otherwise have been required to design and build a custom BLDC motor, as described in Section 3.2.2.

In contrast, it was decided that the intrinsic generating method would be used on the receiver side (that is, a custom BLDC generator described in Section 3.3.2). This required more time and resources to design and build, but yielded better performance. These decisions had the additional benefit of enabling the separation of the performance results of the receiver side from the transmitter side. The results could then be extrapolated to the situation where both sides are custom BLDC machines.

5.1.3.5 Permanent magnet rotor design

It was necessary to overcome non-trivial design challenges in order to mount the PM to a shaft. Since the custom PM is only widely available in simple shapes, a solid cylinder PM, uniformly magnetized through the diameter, was employed. The interior of neodymium PMs have a powder consistency, and as a result cannot be machined easily or precisely, so machining operations such as drilling were avoided, to prevent fracturing the magnet. Instead, a different method was employed whereby the PM was surrounded by a stainless steel structure as illustrated in Figure 26.
First, the magnet was inserted into a fitted sleeve with a tight fit. The sleeve was extended at both sides, where “stubs” were inserted. These stubs were machined from a single piece of stainless steel, with the same diameter as the PM on one side, and the desired diameter of the shaft on the other side. The stubs were also fitted with a tight fit into the sleeve. The length of the sleeve protrudes slightly past the stub. This protrusion of the sleeve was then forced inwards using a hammer to secure both stubs and PM. Finally, the sleeve was machined to 1 mm thick to minimize the outer diameter of the rotor. The same design was used for construction of both transmitter and receiver PM rotor.

5.1.3.6 Motor design on the transmitter

The transmitter required a motor capable of producing a shaft power of at least 1 kW at a speed of 9000 rpm. An off-the-shelf AC induction motor was used mainly because it was readily available. One disadvantage of AC induction motors is that their speed is limited to roughly 3600 rpm or 60 Hz, which is the frequency of the power line, but to overcome this frequency limitation, the motor was mechanically geared up using a belting system with pulleys of appropriate diameters by a factor of 2.5 to reach the design speed of 150 Hz. The high speed pulley was mounted on a dedicated shaft held by two dedicated bearings, and the dedicated shaft was then coupled to the stub of the PM rotor illustrated in Figure 27. A variable frequency drive (VFD) was used to control the speed and acceleration of the motor.
The motor used in this experiment was manufactured by Baldor and was rated for 5 horsepower output at the shaft, and 3450 rpm at full-load. The motor was mounted on a sliding mount which was used to vary the tension of the belt, and the sliding mount was secured to a 5 cm thick wooden table top.

The poly-V-belt of type J with 6 ribs was chosen based on the criteria for power and speed. In cross-section, this belt resembles a series of small single-V-belts linked side by side. Low speed and high speed pulley diameters of 15 cm and 6.3 cm, respectfully, were chosen to achieve the desired speed multiplication factor. All belts and pulleys were standard parts, so the speed multiplication factor was slightly lower than the ideal design, resulting in a PM rotor speed of 144 Hz. The large pulley was mounted on the motor shaft with a quick-disconnect bushing and the small pulley was mounted on the dedicated shaft, which was separate but co-axial to the PM rotor. The belt was tensioned using the sliding mount of the motor.
The diameters of the dedicated shaft and the PM rotor were 1.5 cm. The dedicated shaft was separated from the PM rotor shaft because there was considerable radial force applied by the tension in the belt to the dedicated shaft and the bearings. It was not desirable for this radial force to be transferred to the PM rotor, so the shafts were separated, and coupled together with a shaft-coupler capable of handling the speed and torque as illustrated in Figure 27. The shaft coupler was an off-the-shelf device with sufficient tolerance for radial misalignment, and generally incapable of transferring radial loads to the PM rotor. As a result, the PM rotor experienced only the applied torque. All bearings were mounted in cast iron, and the shafts were made of stainless steel to cope with the torque and minimize risk of mechanical failure.

5.1.3.7  Generator design on the receiver

Since the shape and size of the PM rotor was already determined, the main design issues of the generator involved the nature of the windings. There exist countless winding designs with varying complexity\(^{40}\). Since the windings would be wound by hand in this case, it was important to limit the complexity of the design. For this reason, a single phase topology was used, the cross-section of which is illustrated in Figure 28.

![Cross-sectional diagram of winding design of BLDC generator](image)
For ease of manufacturing, the complete winding was separated into two individual windings, labeled 1 and 2. The area $I'$ in Figure 28 signifies for the return path of wires in area $I$, and similar for area $2'$ for 2. Windings 1 and 2 were then electrically connected in series to form a single phase winding. The inner diameter (ID) of the winding was designed to be as close as possible to the outer diameter of the PM rotor to minimize the air gap. A winding ID of 5.5 cm was determined to offer sufficient extra space for the rotating PM rotor.

The outer diameter (OD) and optimum angular gap ($\theta_{\text{gap}}$) were determined with winding optimization techniques discussed in Section 3.3.2.5.1. The OD determines the overall size of the device and therefore it is preferable to minimize it. The angular gap, $\theta_{\text{gap}}$, however, does not have much impact on device size, so an algorithm slightly different from the one described in Section 3.3.2.5.1 was used to determine $\theta_{\text{gap}}$. For various ODs, the turns in the windings are freely allocated, as long as the turn lies between the ID and OD, up to any $\theta_{\text{gap}}$. This algorithm determines the maximum possible winding factor given a certain OD. The algorithm was then repeated for various ODs in order to select an OD which maximizes winding factor and minimizes size of the device.

It was then necessary to calculate the magnetic flux of each turn of the wire. The magnetic field for a finite cylinder, magnetized across the diameter, cannot be easily expressed analytically, and performing the algorithm numerically would be computationally intensive. Instead, the magnetic flux of each turn was estimated using the known magnetic field of an infinitely long cylinder, which can be expressed analytically. This was a reasonable estimate since the field of an infinite cylinder is very similar to that of a finite cylinder in regions close the cylinder wall and far from the cylinder ends, and the majority of the windings satisfy these conditions. The details of this process is provided in Appendix B.
Using this method, the winding factor was calculated for each OD restriction. It was also possible to calculate the estimated electrical efficiency based on the second term of Equation (24) discussed in Section 3.3.2.4. $\tau_m$ and $\omega_{\text{max}}$ were both previously calculated, and the friction coefficient was considered zero for this estimate. The electrical efficiency estimated in this way is plotted as a function of OD of the windings in Figure 29.

![Figure 29: Estimated winding factor as a function of OD of the custom BLDC generator](image)

Figure 29 shows that only a relatively small OD of about 6.5 cm was required for a high efficiency. In order to be certain that the desired level of electrical efficiency will be attained, an OD of 7.5 cm was used, resulting in a total radial winding thickness of 1 cm. The angular gap at the OD of 7.5 cm was approximately 30°, but this value is not critical to the winding factor. The OD, ID and $\theta_{\text{gap}}$ has thus determined the total cross-sectional area of the winding.

The final design parameter required was the thickness of the wire. In the previous sections, it was shown that the thickness of wire does not affect winding factor, and this is true for this situation. However, the output voltage from the windings does depend on the thickness of the wire, because it depends on the total number of turns in the winding, which is inversely proportional to the wire cross-sectional area, when the total winding cross-sectional area is defined previously. 240 V$_{\text{rms}}$ (root-mean-square) was set as the target output voltage, consistent with the input requirements of typical electric car charging circuits. The magnetic
flux of each turn (which was calculated previously when calculating winding factor), combined with the value of the total cross-sectional area of the winding, has determined that 12 gauge magnet wire with 0.78 packing factor (consistent with random packing) would achieve the target output voltage.

The construction of the windings and stator proved to pose some challenges. Some small deviations from the design were necessary to overcome the challenges, discussed in the following sections.

5.1.4 Construction and assembly

Some practical challenges were overcome when fabricating the device, mainly related to construction and assembly, as described in this section.

5.1.4.1 Custom magnet construction

The custom fabricated neodymium magnet was manufactured in Asia and purchased from a North American distributor. The magnet was fabricated with the dimensions illustrated in Figure 30.

![Figure 30: Dimensions of custom Neodymium PM](image)

The PM was designed to be magnetized by the manufacturer to have a residual field of 1.45 Tesla. The actual PM received had a residual field of only 1.10 Tesla, which was by measuring the far field of the PM at several distances and comparing that field it to the
known field of a magnetic dipole in Equation (1). The cylindrical magnet was fitted with a sleeve and stubs illustrated in Figure 26, using 316 type stainless steel, of the non-magnetic type, with a relatively low conductivity and near unity permeability. The thickness of the sleeve was machined to 1 mm. Two of these PM rotors were constructed identically, one for the transmitter, one for the receiver.

5.1.4.2 Transmitter construction

The transmitter side of the power transfer device was constructed as designed. Pre-mounted bearings were used, which allowed the ease of assembly. The bearings were of the self-aligning type, which made them relatively tolerant to misalignment. All the components of the transmitter and receiver devices were mounted on a large 5 cm thick table top. The transmitter was mounted on one side, and the receiver mounted on the other side. The receiver device was mounted on a sliding base, which enabled the testing at a range of different separation distances between the transmitter and receiver.

5.1.4.3 Receiver construction

The generator design has determined the shape and size of the PM and the windings. The method in which the windings are constructed, supported, and assembled with the rotor to form the complete device is described here.
The winding was constructed in two identical halves, according to the design shown in Figure 28. One half is illustrated in Figure 31. Since this is an iron-free machine, the material supporting the windings, or the stator backing material, must be non-magnetic, and to minimize eddy current losses, the stator material must also be non-conductive. Additionally, the stator backing must be structurally strong. For these reasons, a Phenolic material, grade G-10 Garolite, was used. G-10 is a glass-cloth laminate bound with epoxy resin, commonly used in circuit boards. This material is non-conductive, non-magnetic, and has a tensile strength similar to aluminum. It was also relatively easy to machine and had the additional advantage of bonding extremely well to epoxy adhesives since the main component of its own construction is epoxy resin. A G-10 stator backing of thickness of 1 cm thickness was estimated to provide a sufficient structural strength, and so this was incorporated into the device.

The turns in the windings were bonded to each other, as well as to the G-10 backing, using epoxy. A common high-strength epoxy was used that was designed for potting electronics, with a long curing time to be compatible with the lengthy hand-winding process.
The G-10 backing required an obstruction, or “bobbin”, in the center to realize the angular gap requirements in the design (see Figure 31(b)). This also served as the bobbin to wind the first few turns around. This bobbin was constructed separately and bonded to the G-10 stator backing using epoxy.

The copper windings were designed to have an inner diameter of 5.6 cm. As illustrated in Figure 31(a), in order for the windings to satisfy the inner diameter requirement, it was necessary to add a cylindrical spacer of the appropriate diameter. This spacer would then be removed once the epoxy was cured, and therefore should not bond to the epoxy. Polytetrafluoroethylene (PTFE) was chosen to be the martial of this spacer, additionally wrapped in a film of polyethylene. Neither of these materials bond well to epoxy, ensuring the smooth removal of the spacer.

Copper magnet wire was used instead of aluminum wire because copper has higher conductivity, and is more malleable, leading to more accurate winding. Solid wire was used instead of stranded or litz wire because although stranded or litz wires are more flexible, they have a higher resistance due to the air gaps and insulation between the strands. Also, because the operating frequency was sufficiently low, the skin-effect advantage of litz wire was negligible. Finally, round cross-sectional wire was used instead of square wire. Square wire has the potential to achieve better packing factors, but with practical winding procedures, the curves of the designed winding cause the square wire to spin torsionally, over a distance of few turns. After approximately 5 turns, the square wire would have spun by 90°, negating the advantages of square wire. Therefore, with common winding techniques, square wires have no advantage over round wires. Therefore solid round magnet wires were used, insulated by a thin layer of enamel.

Finally, the thickness of the wire was selected. As mentioned previously, to obtain the desired output voltage of 240 Vrms, assuming a 0.78 packing factor, 12 gauge wire is required. However, since the fairly thick, solid, copper wires have a certain degree of rigidity, they have a tendency to cross-over one another, even with a relatively high tension applied by hand. As a result, repeated winding operations consistently yielded a packing factor of 0.5.
The winding factor was re-calculated using the lower packing factor, and was found to produce an estimated electrical efficiency still very close to the originally designed value. Taking into account the revised packing factor, a thinner wire of gauge of 14 was used to obtain the originally designed voltage output of $240\, \text{V}_\text{rms}$.

The two halves of the stator were built identically, and subsequently bonded to one another using epoxy to form a complete cylindrical stator, realizing the design of Figure 28.

![Diagram of rotor and stator assembly using G-10 “end caps”](image)

**Figure 32: Assembly of rotor to the stator using G-10 “end caps”**

To complete the construction, the PM rotor, as illustrated in Figure 26, was mounted into the center of the stator and allowed to rotate. For this purpose, end-caps were fitted to each end of the stator as illustrated in Figure 32. For the same reasons related to the stator backing material, the end caps were also made of G-10. The end-caps were securely fastened to the G-10 stator with bolts. There were eight bolts for each end cap, uniformly distributed around the circumference. Wire terminations exit through one of the end caps (not shown). On the inside of each end cap, a housing bore for the bearing was machined. The housings bores were blind holes, protecting the bearings from contaminants.

Bearings were attached on each end of the PM rotor stubs. Bearings were chosen according to size, material and lubrication. More detailed discussion of different bearings used, is discussed in the following section. The bearings are secured on the shaft with either an interference fit when possible, or with a loose fit combined with an adhesive specially
designed for bonding cylindrical fittings. The completed receiver assembly was mounted on a sliding base, which itself was mounted to the 5 cm thick table opposite to the transmitter PM rotor for testing. The completed power transfer device is illustrated in Figure 33.

**Figure 33:** Completed power transfer device (transmitter and receiver)

5.1.4.4 Bearings

There were special considerations when choosing the bearings for this application. The mechanical specifications such as speed, axial and radial load, were well within typical ratings of most ball bearings, but one aspect of this application was not conventional, which was the presence of a time-varying magnetic field. From the perspective of bearing, the PM to which they are mounted is the dominant source of the magnetic field it experiences. In addition, the inner race of the bearing does not rotate with respect the PM, so the inner race experiences a negligible time-varying magnetic field. Only the rolling balls and outer race
experience a substantial time-varying magnetic field, and these time-varying magnetic fields create eddy current losses as well as hysteresis losses in the balls and outer race, depending on the material.

This effect has a non-negligible effect on the performance. Since the eddy currents and hysteresis losses create frictional losses and lead to lower efficiency. To eliminate these effects, hybrid ceramic and full ceramic bearings were used. Experimental results gathered from using different types of bearings are discussed in the following sections.

5.1.5 Experimental verification

Using the completed device, winding factor, power output and other parameters were measured and compared to the designed values.

5.1.5.1 Coiling verification

The designed coiling parameters such as voltage, resistance, and winding factor were verified once the construction and assembly of the system was complete. First, the PM rotors were spun in synchronization up to the speed of 144.1 Hz. At that speed, the open circuit (OC) voltage of the two windings connected in series was measured to be 252.8 V\text{rms}. At zero speed, the DC resistance of the two windings in series was measured to be 0.6 Ω. The inductance was also measured and found to be negligible. The coiling verification results are tabulated in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>OC Voltage</td>
<td>252.8 ± 0.1 \text{Vrms}</td>
</tr>
<tr>
<td>At frequency</td>
<td>144.1 ± 0.1 \text{Hz}</td>
</tr>
<tr>
<td>DC resistance</td>
<td>0.6 ± 0.1 \Omega</td>
</tr>
<tr>
<td>Winding factor</td>
<td>0.250 ± 0.05 \text{T}^2 \text{m}^4\Omega^{-1}</td>
</tr>
<tr>
<td>Electrical efficiency</td>
<td>0.97 ± 0.01</td>
</tr>
</tbody>
</table>

Table 2: Winding factor verification
Since the coiling design was initially performed at the designed speed of 150 Hz, the measured OC voltage was extrapolated to that speed to verify coiling parameters. The extrapolation was based on the assumption that the voltage is linearly proportional to speed, which was also experimentally verified. This extrapolation resulted in an OC voltage of 263 V\textsubscript{rms} at 150Hz, 9% above the designed voltage of 240 V\textsubscript{rms}. This was largely due to the conservative estimates of the magnetic field produced by a finite cylinder using the known field of an infinite cylinder. Since this device was mainly used for experimental verification, the increased voltage was not a detriment. If it was required in the future to produce the originally designed voltage of 240 V\textsubscript{rms}, the frequency of operation could be reduced to reduce the voltage. This comes at the cost of reduced power capacity proportional to the speed reduction. Alternatively the stator could be re-wound using a slightly thicker wire.

Using the measured values of the OC voltage and the angular frequency, the total magnetic flux in the windings, $\Phi_0$, has been calculated to be 0.395 Vs using Equation (15). Combined with the resistance of the coil, the winding factor, $\Phi_0^2/R_c$ was calculated to be 0.259 V^2s^2/\Omega. Substituting this into the electrical efficiency term on the right hand side of Equation (25) yielded a predicted electrical efficiency 97%. This was well within acceptable range. It is concluded that the winding process described previously had produced a winding factor with sufficiently electrical efficiency.

5.1.5.2 Power transfer verification

The main variables that contributed to the power transferred were speed, load resistance and separation distance. The first set of measurements was performed at constant separation distance to explore the speed, load resistance and power relationships. A second set of measurements was performed to explore the relationship between power and separation distance.

At a constant separation distance between the PMs of 14.5 cm, measured from each axis of rotation, a variety of measurements were taken at different speeds and load resistances. First, the maximum speed of rotation that still maintained synchronization was measured for various load resistances, the results of which are illustrated in Figure 34(a). This test was
performed by slowly increasing speed until the synchronization was lost. These results were consistent with the anticipated behavior previously expressed in Equation (19).

Figure 34: Power transfer results with respect to $R_L$ at $d=14.5\text{cm}$

Uncertainties not visible: $R_L \pm 0.5 \ \Omega$, Power $\pm 25 \ \text{W}$, Rotation speed $\pm 0.1 \ \text{Hz}$

As expected, the maximum synchronous speed increases with greater load resistance as described in Equation (19), until $f_{\text{max}}$ was reached.

The power with respect to load resistances, at the synchronization limit is illustrated in Figure 34(b). The power is maximized when operating at maximum frequency of 139 Hz, with an optimum load resistance of 37.6 $\Omega$. Measurements for the load resistances greater than 37.6 $\Omega$ was not taken, since at higher resistances, the frequency was still limited to the
same value, and so was the voltage. Therefore at the higher resistances, the power drops with respect to the relationship $P = \frac{V^2}{R_L}$. The relationship of power and speed is illustrated in Figure 35.

Figure 35: Power transfer results with respect to frequency at $d=14.5$cm

Uncertainties not visible: Power ±25 W, f ±0.1 Hz

The maximum power before loss of synchronization was recorded for each respective operating speed in Figure 35. The relationship was evidently linear, as anticipated by Equation (8). Neglecting the small frictional torque, the maximum time-averaged torque was independent of speed. Therefore the power was linear with speed according to the $P = \overline{T_m} \omega$ relationship.

This procedure was repeated for various separation distances. The maximum attainable power characterized by $P_{\text{max}}$ was recorded for each separation distance in Figure 36.
Figure 36: Maximum power transfer results with respect to separation distance
Uncertainties not visible: Power ±25 W

These measurements were compared to theoretical predictions of Section 3.1 illustrated as the dotted line in Figure 36. The theoretical prediction (the dotted line) is based on Equations (34) and (35), where $D$ is the dipole moment of the receiver PM, and $B$ is the magnetic field produced by the transmitter PM, and $\omega$ is the angular frequency.

$$P = T\omega$$ (34)

$$P = |B \times D|\omega$$ (35)

The on-axis magnitude of $B$ (corresponding to $\lambda = 90^\circ$ in Equation (1)) was measured using a Gauss meter, as a function of separation distance. The maximum time-averaged power was calculated by using the time-averaged magnitude of $B$, which was 75% of the on-axis magnitude, expressed in Equation (7). The angle between $B$ and $D$ was assumed to be $90^\circ$ during maximum power transfer.

Comparing the measured power against the predicted power using the above method, they were in close agreement, suggesting the validity of the understanding of the system.
5.1.6 Experimental verification of efficiency

The losses were categorized into receiver (generator) losses, transmitter (motor) losses, and magnetic gear losses. The receiver losses were further categorized into frictional losses and generator losses. The transmitter losses were further categorized into AC induction motor losses and frictional losses of the rotating components. The magnetic gear losses were negligible compared to the receiver and transmitter losses and therefore not evaluated in this section. The magnetic gear losses are explored in Section 5.2.5.2.

5.1.6.1 Receiver frictional losses

The receiver frictional torque, $T_{\text{fric}}$, was defined to be the sum of many frictional and eddy current damping forces, including the mechanical frictional losses of the bearings, eddy current damping losses in the bearings, as well as the eddy current damping losses in the windings. These losses were measured using the technique described below. The contribution of each component was individually determined by measuring the difference between the total frictional torque when each contribution is first present and then removed.

5.1.6.1.1 Friction measurement technique

The frictional losses in the custom BLDC receiver were measured by recording the speed as the rotor slows to a zero from full speed rotation, under no other influence but friction torque. The behavior is governed by the basic kinematics Equation (36), where $I$ is the moment of inertia, $\dot{\omega}$ is the angular acceleration, and $T(\omega)$ is the friction torque, which is a function of angular velocity.

$$I\dot{\omega} = T(\omega)$$

(36)

The rotational moment of inertia was calculated using the expression of moment of inertia for a solid cylinder (37), where $M$ is total mass, and $r$ is the radius of the cylinder.

$$I = \frac{Mr^2}{2}$$

(37)
The total mass of the PM rotor in the receiver was measured, as well as the outer radius to determine moment of inertia. Since the stubs on either side of the rotor were relatively small, and since the density of stainless steel was similar to the density of the Neodymium PM, the moment of inertia of $7.4 \times 10^{-4}$ kgm$^2$ calculated in this way is a reasonable estimate.

Acceleration, $\ddot{\omega}$, was measured by measuring the speed as a function of time as the rotor slows to zero, as illustrated in Figure 37(a). This speed is then differentiated as a function of time, in order to obtain the acceleration shown in Figure 37(b).
Figure 37: Speed, acceleration and torque of an example deceleration curve for efficiency measurement

The torque as a function of time illustrated in Figure 37(c) was calculated as the product of the angular acceleration as a function of time in Figure 37(b) and the moment of inertia. However, it was reasonable to assume the frictional torque depends only on speed, not time. Therefore the frictional torque is graphed against speed, as illustrated in Figure 38. This determines the dependence of frictional torque as a function of speed.
It is apparent from the graph in Figure 38 that the frictional torque is linearly dependant on speed with a non-negligible $y$-intercept. This $y$-intercept can be interpreted to be the presence of speed independent, sliding friction. This was not surprising, since practical ball-bearings cannot be completely rid of sliding friction.
Figure 39: Frictional power loss as a function of rotation speed

Power loss was calculated and plotted in Figure 39 by multiplying the frictional torque of Figure 38 with the angular frequency.

5.1.6.1.2 Frictional loss components

This method of measuring frictional torque was repeated for various types of bearings for the purpose of separating the contribution of each frictional component. Using linear extrapolation of data such as in Figure 38, the frictional torque at 150 Hz was determined. Table 3 lists the power losses associated with various bearing types at 150 Hz.

<table>
<thead>
<tr>
<th>experiment #</th>
<th>bearing</th>
<th>lubrication</th>
<th>winding</th>
<th>total losses at 150Hz (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>full ceramic</td>
<td>oil</td>
<td>not installed</td>
<td>5.5 ± 0.5</td>
</tr>
<tr>
<td>2</td>
<td>full ceramic</td>
<td>oil</td>
<td>installed</td>
<td>98 ± 9</td>
</tr>
<tr>
<td>3</td>
<td>all steel</td>
<td>grease</td>
<td>installed</td>
<td>180 ± 20</td>
</tr>
</tbody>
</table>

Table 3: Frictional power loss measurement summary

In the first experiment, the losses of full ceramic bearings with oil lubrication were measured without the presence of the windings. The losses of this measurement reflect the mechanical friction losses of the bearings. Since there were no eddy current losses because the full
ceramic bearings contain no metallic components, and there are no eddy currents in the windings. Therefore the mechanical friction losses are in the range of 5.5 W.

In the second experiment, the same test was conducted with the same bearing, but this time with the winding installed. The losses of this experiment reflected the sum of the mechanical friction component in the bearings, as well as the eddy current damping component in the windings. Therefore it was deduced that the eddy current damping losses in the windings were the difference in losses between experiment # 1 and 2. This is calculated to be 95 ± 10 W.

In the third experiment, a magnetic stainless steel bearing, lubricated with grease, was used. The winding was also installed for this test. The losses of this measurement reflect the sum of the mechanical friction components in the stainless steel bearings, the eddy current damping and hysteresis losses in the bearings, as well as the eddy current damping component in the windings. Therefore it was deduced that the losses involved in using metallic bearings is the difference between experiment # 3 and 2, which is 90 ± 30 W. This value includes the eddy current and hysteresis losses in the bearing, but does not account for the difference in lubrication between experiment # 3 and 2, or the difference in mechanical frictional between full ceramic and steel bearings. Therefore this result has greater errors than the previous results.

It was concluded that the major contributions of loss was the eddy current damping in the winding and the bearings. The eddy current in the bearing could be easily eliminated by replacing metallic bearings with full ceramic bearings. For this reason, the eddy current losses in the bearings were not as accurately measured as the eddy current losses in the windings.

The eddy current losses in the windings were not as easily eliminated since the windings cannot simply be removed. One method of reducing this loss would be to use wires of smaller diameter. This reduces eddy currents, similar to the way that lamination in iron reduces eddy currents. The winding factor would be unaffected as long as the total cross sectional area of the winding was kept constant by using more turns. However, the output voltage would increase in this case. To keep the voltage constant, some of the turns could be
connected in parallel, but it would be necessary to ensure there were no recirculation currents.

5.1.6.2 Receiver generator losses

In most motors and generators, there are various copper and core losses. In the current case of a core-less BLDC device, there were no core losses due to the absence of soft-magnetic materials. In this case, the losses associated with the generator stage were purely copper losses in the windings.

It has previously been established in Section 5.1.5.1, through measurement, that the winding self-induction is negligible. Therefore the copper losses, $P_{\text{copper}}$, are dependant on the total coil resistance ($R_C$), and current ($I_{\text{rms}}$), expressed in Equation (38).

$$P_{\text{copper}} = I_{\text{rms}}^2 R_C$$  \hspace{1cm} (38)

At the maximum energy transfer rate, which correspond to maximum copper losses, the rms current is 6.42 Amps. With a measured coil resistance of 0.6 Ω, this equated to 68 Watts, 4.3% of power at the load. This was consistent with the predicted value according to the theoretical model developed in Section 3.3.2.5.

5.1.6.3 Receiver total losses

The total efficiency of the receiver was calculated, for a device using full ceramic bearings, including all the frictional losses such as mechanical and eddy current losses, as well as all generator losses. The total loss was the sum of $P_{\text{copper}}$, as described above, and $P_{\text{fric}}$, the loss of experiment # 2 in Table 3. The total efficiency was the fraction of the output power, $P_{\text{out}}$ over the sum of the output power plus all losses, expressed in Equation (39).

$$\eta = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{fric}} + P_{\text{copper}}}$$  \hspace{1cm} (39)

This total efficiency was calculated to be 90% ± 2% for operation at maximum $P_{\text{out}}$. 
5.1.6.4 Transmitter losses

On the transmitter side, the losses were more substantial. The motor had a rated efficiency of 85% when operating at full power. Additionally, there were also losses associated with the belting system, the shaft coupler, as well as the four mounted metallic bearings, two of which were in close proximity to the PM rotor. In addition to the traditional mechanical frictional losses of the aforementioned components, there were also considerable eddy current losses associated with the transmitter PM rotor spinning close to the metallic mounted bearings (see Figure 27). Each bearing was made from magnetic stainless steel and mounted in a cast-iron housing. The close proximity of both conductive and magnetic materials lead to considerable eddy current and hysteresis losses.

The frictional and magnetic losses of the driving components were substantial. However, the measurement of these losses was not necessary, since this transmitter design was a fast and convenient method to construct the transmitter, but would never be used in a real application. In a real application where efficiency is crucial, such as charging an electric vehicle (EV), the transmitter would most likely resemble a custom BLDC motor as discussed in Section 3.2.2. Therefore the efficiency of the current system using an AC induction motor was not measured.

5.1.6.5 Total system transfer efficiency

The total transfer efficiency is the product of the receiver efficiency and the transmitter efficiency. The receiver efficiency characterizes the ability of the receiver to transform mechanical power to electrical power. This has previously been calculated to be 90% in Section 5.1.6.3. The transmitter efficiency characterizes the ability of the transmitter to transform electrical input power to mechanical power. Although this efficiency was estimated to be low in the current implementation as described in the previous Section 5.1.6.4, a system optimized for total efficiency would use a custom BLDC motor topology and achieve much higher efficiency.

Assuming the custom BLDC motor implementation would achieve similar efficiency as the efficiency achieved by the current custom BLDC generator, the overall power transfer
efficiency including both transmitter and receiver would be roughly estimated to be in the range of 81%.

5.1.7 Control

Although no controls were implemented for the current kilo-watt scale device, they were implemented for the 60 W device, and are briefly discussed here. A complete description is offered Section 5.2.7.

Hall Effect sensors are useful to implement control schemes, since they offer a convenient and readily available method of sensing and measuring magnetic field intensity. These sensors can be mounted entirely on one side, the transmitter or the receiver, to accomplish their purpose, with no additional communication necessary between the transmitter and the receiver.

For example, Hall Effect sensors could be mounted in strategic locations on the transmitter side to detect the proximity of the receiver PM for the application of charging an EV. Presence and location of the incoming EV can be detected, and charging of the vehicle can be initiated without human intervention. Four oppositely mounted Hall sensors could be used to detect misalignment in two dimensions, and could serve as an indication for the driver to re-park the car towards a specific direction and distance.

In the same orientation of linear Hall sensors it was possible to measure the lagging rotation angle, $\theta$, between the two PMs, as illustrated in Figure 8. Once this angle exceeds 90°, synchronization was lost. Therefore using this technique, the loss of synchronization was avoided by monitoring the lagging angle between PMs and decreasing the speed when the lagging angle becomes too large.

Lastly, the magnitude of the current to the motor could be monitored, for example to detect whether the charge is complete for the EV. Once the current falls below a certain value, and the EV is fully charged, the transmitter motor could be stopped to conserve energy.
The implementation of these control schemes are discussed in more detail in the Section 5.2.5.

5.1.8 Discussion

The new method of wireless power transfer using permanent magnet enhancement has been experimentally verified, and the models developed in Chapter 3 to predict the behavior of the system have been experimentally confirmed and quantitatively verified. The values for power and efficiency have been accurately predicted using this model with acceptable accuracy. The experimental results show that this proposed method is a viable approach for wireless power transfer, and specifically that it is viable of the application of charging EVs.

5.1.9 Future work

There is substantial on-going work planned for the power transfer model at the kilo-watt scale for automotive and industrial purposes.

5.1.9.1 Electric vehicle demonstration

A key next step is to build a demonstration system that charges the battery of a production EV or plug-in hybrid vehicle. The receiver components will be mounted on the vehicle, and the transmitter components will be installed in a parking structure. For this demonstration, first, operation of the wireless charging system will be tested in real operating conditions. The main concern is associated with the long-term operation since an over-night charge of an electric vehicle will require approximately 5-8 hours of continuous operation. Although this is well within the operating limits of continuous-duty electric motors and generators such as ventilation fan motors, this performance ability must be confirmed. One possible concern is the generation of excess heat, and depending upon experiment results with long operating times, there may be a need for cooling. This could be achieved by introducing room-temperature air-flow through the motors and generators, cooling through the use of a heat-sink, or other methods. The bearings must also be adequate for operation throughout the anticipated temperature range, with adequate speed and life ratings.
Second, various mounting orientation of the wireless charging system will be tested under real operating conditions. Although countless orientations are possible, the main candidates are front-mounting and floor-mounting orientations depicted in Figure 40.

![Figure 40: Two possible EV charging orientations](image)

Each mounting orientation will be evaluated for performance such as clearance, range, alignment, ease of operation, and ease of installation.

Third, the control schemes will be tested under real operating conditions. The required level of intelligence will be evaluated, and the control implementations will be tested. Feedback from the daily operator of the vehicle will be also be used to improve the control schemes, especially user-interface.

### 5.1.9.2 Ongoing device optimization

In addition, improvements to the basic charging principles were also considered.

The first such improvement is to use a custom BLDC motor on the transmitter side (described in Section 3.2.2) rather than using the currently implemented, separate motor. The complete system is illustrated in Figure 41.
There would be many benefits to a system which uses a custom BLDC motor as well as a custom BLDC generator. Due to the lower number of mechanical parts, as well as the intrinsic high efficiency of BLDC machines, this system would likely have a much higher efficiency than the currently implemented device (estimated in Section 5.1.6.5). Also as a result of the elimination of the belting system, the noise level would be considerably reduced. Driving electronics would be developed to run the custom BLDC motor and electrically commutate the windings in an efficient manner. There is a wealth of knowledge available in the field of BLDC motor design to assist with the design this particular BLDC motor including rotor, winding and driving electronics design.

Additional improvements could be made to allow near-silent operation. Using the custom BLDC methods, it is entirely possible to completely seal all the moving parts in both the transmitter and receiver devices. This is because unlike a traditional motor or generator, there is no need for an exposed shaft. Therefore the all moving parts can be encased in sound insulating materials, reducing the noise considerably.

5.1.9.3 Long term development outlook

There are also a few improvements best suited for long term development. One of these improvements would be potentially using a less-conventional, inside-out, BLDC motor and
generator topology. This type of BLDC motor has a stator with windings on the inside, and a PM rotor on the outside. The on-axis cross-sectional view is illustrated in Figure 42(a).

![Figure 42: Inside-out BLDC motor and generator cross-sectional view](image)

This topology reverses the positions of the PM rotor and copper stator of common BLDC machines, so that the PM could be moved to the exterior, and the copper windings could be moved to the interior. This is advantageous because the copper windings do not assist with the coupling between the transmitter and receiver PM, and they currently occupy the space between the two PM rotors. The proposed new method moves the windings out of the way and as a result could potentially allow greater separation distances. In addition, the windings no longer surround the exterior of the PM. This allows the inner windings in the stator to use soft-magnetic core without shielding the external far-field. This could potentially increase the coiling efficiency with less volume of copper. However, it remains to be clarified whether the field produced by the annular PM rotor would indeed be improved, compared to the conventional inner-rotor BLDC machine design. Far-field estimates, near-field estimates, as well as numerical estimates would be possible to verify any advantage.

Another potential improvement could be an alternate manufacturing technique for the PM rotor. The current PM rotor design makes considerable effort to mount the cylindrical Neodymium magnet onto shafts in a structurally robust way, discussed in Section 5.1.3.5. A potentially better method involves embedding a shaft inside the cylindrical magnet at the time of the manufacturing, illustrated in Figure 43.
Figure 43: Manufacturing of Neodymium magnet with built-in shaft

Bonded permanent magnets, including bonded Neodymium magnets have the ability to be manufactured in this way. The shaft could be manufactured independently from high strength materials such as stainless steel, and simple operations such as stamping could make the shaft non-symmetric to allow stronger attachment between the shaft and PM. Finally the PM cylindrical magnet could be formed over the shaft. A sleeve, possibly made of stainless steel could be optionally fitted over the outer diameter of the magnet to serve as extra protection against centripetal forces.

This method of manufacturing of the PM rotors results in the central shaft composed of a single piece of stainless steel, rather than two-pieces, one on each side. It could potentially be stronger and more accurate than the current design. This method requires a more substantial initial investment, but this investment would be justified for even moderate manufacturing volumes.

5.2 60 Watt scale application

In addition to the kW size-scale system, a smaller sized transfer system capable of transferring few tens of watts was designed, constructed and tested. The main application of
the device at this power and size scale was aimed at powering of biomedical implants and household electronics.

5.2.1 Biomedical application

As mentioned in previous sections, in terms of potential risks to human health the new power transfer method has a distinct advantage over the method of high frequency induction. Therefore the currently proposed power transfer method would be especially suited for applications which operate in the direct vicinity of people. One such application, which requires substantial amounts of power transferred through the human body, would be the powering of an artificial heart\textsuperscript{46}.

The need for wireless transfer in this case would be obvious. Electrical wires which penetrate the skin easily lead to infection, and are extremely cumbersome. More so here, than in any other application, is would be important for the electro-magnetic fields to be safe to human tissue and organs. As explained in previous sections, the fields created by the proposed method were strictly magnetic, and were of much lower frequency than inductive transfer devices. Further analysis of the risk to human health is discusses in Chapter 6.

Other applications at this size scale may also include household appliances and personal electronics.

But, the biomedical application of wirelessly powering an artificial heart appears to have the most scientific value, and so this was the main target application.

5.2.2 Design procedure

The design process of this system was very similar to the design process of the kilo-watt system discussed in Section 5.1.3. Most of the design considerations were based on the application to power an artificial heart. The final design is also useful for most other applications.
5.2.2.1 Magnet type

For the same reasons as the kilo-watt model, bonded Neodymium magnets were used. This is because their magnetic strength, low cost and availability. Refer to Section 5.1.3.1 for complete justification.

5.2.2.2 Separation and power requirement

The power capacity of the transfer device is designed to match the power requirements of a modern artificial heart. A brief survey of some artificial hearts has found the power requirements to be in the order of 7 Watts\(^47\), although eventually 60 Watts was eventually achieved.

![Diagram of wireless power transfer system for an artificial heart](image)

**Figure 44:** Placement of transmitter and receiver components of wireless power transfer system for an artificial heart

Figure 44 assumes the transmitter device is mounted on the chest, directly opposite the receiver device, and the receiver was assumed to be implanted deep inside the human body. Using this orientation, the separation distance was one-half the thickness of an average human torso, which is determined to be of the order of 10 cm.
5.2.2.3  Size, shape and speed of PM rotors

Unlike the kilo-watt model, the transmitter and receiver magnets were designed to be unequal in size and shape. The main reason is because it is necessary to minimize the size of the receiver, since a real device would be implanted inside the human body. The size of the transmitter was less critical because it is outside the human body. Therefore, the volume of the receiver device was reduced, offset by an increase in volume of the transmitter device. Multiple transmitter and receiver devices with PM rotors of different sizes were constructed to obtain experimental data of various combinations.

The shape for the receiving PM used was spherical. The shape was not critical because it is the total volume of the magnet that determines the far-field. However, the spherical shape had benefits such as having a known analytic solution for the near and far-field.

The size of the receiver PM meant to be implanted, was designed to have a diameter of 2.5 cm. This was a rough estimate of the maximum space the receiver can take inside the human body.

The maximum rotation frequency of this size of PM was determined using Equation (23). Similar to the procedure carried out in Section 5.1.3.3, the maximum rotation speed was determined to be 300 Hz. While spinning at this speed, the stress induced by the centripetal forces does not exceed 1/20th of the stress it can withstand, considering the tensile strength of bonded Neodymium magnets.

The shape of the transmitter PM was cylindrical. This was because the transmitter PM was driven by a standard electric motor, in which case the cylindrical PM offers ease of mounting. The size of the transmitter PM was varied to observe different performance results. The diameter and length of the cylinder magnet range from 1.9 to 5 cm. Using 5 cm diameter PMs, the maximum operating speed was calculated to be 150 Hz, half the speed of 2.5 cm diameter PMs.
The shape, size and speed of the PM rotors was summarized in Table 4.

<table>
<thead>
<tr>
<th>PM size, shape and rotation speed</th>
<th>Receiver</th>
<th>Transmitter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shape</td>
<td>Spherical</td>
<td>Cylindrical</td>
</tr>
<tr>
<td>Size</td>
<td>2.5 cm diameter</td>
<td>1.9 – 5 cm diameter</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.9 – 5 cm length</td>
</tr>
<tr>
<td>Max speed</td>
<td>300 Hz</td>
<td>112 – 300 Hz</td>
</tr>
</tbody>
</table>

Table 4: The range of shape, size and speed of rotation of receiver and transmitter used

5.2.2.4 Method of motoring and generating

The method of motoring and generating are very similar to the methods of the kilo-watt model in Section 5.1.3.4.

In this case, the motoring method implemented a separate electric motor to drive the transmitter PM (discussed in Section 3.2.1), while the generation method implemented a custom BLDC generator using the 2.5 cm diameter spherical PM to serve both as a component in the magnetic gear, as well as a component in the generator (discussed in Section 3.3.2). The motivation for this design was very similar to the design of the kilo-watt model in Section 5.1.3.4.

5.2.2.5 Bearing selection

At this smaller size-scale, there is much greater variety in types of bearings that are commercially available. Two types of bearings were used to compare performance.

First, a miniature ball bearing with adequate speed and load ratings was used. Second, a jewel bearing was used, known for its exceptional low frictional losses, and are used in many
precision applications such as instrumentation. A jewel bearing essentially comprised of a shaft spinning inside an assembly which resembles a cup. The components of the cup which is in contact with the shaft were made of a hard material such as synthetic sapphire, which have a hardness rating comparable to diamond. The shaft was constructed of stainless steel. The contact area of sliding surfaces between sapphire cup and stainless steel shaft was minimized by rounding one or both of the contact surfaces. This design resulted in low frictional losses due to a small contact area, and resulted in high durability due to hardness of the materials. In comparison to ball bearings, jewel bearings have lower friction than ball bearings, but they are far more expensive and their behavior is not completely known for high speed operation, wear and life. In this study, both types of bearings were implemented to compare performance.

5.2.2.6 Coiling design

The model developed in Chapter 3 applies to this device as well. Unlike the kilo-watt device, a three phase winding was chosen. The main reason for this choice was ease of construction and mounting of the windings. Two different types of windings were built, where the first was easy to manufacture, and the second was optimized for winding factor.

The first type of windings were constructed with circular cross-sections. The windings were mounted around the PM as shown in Figure 45.
Windings 1, 2, and 3 are labeled in the diagram. Each winding occupies 120° in angular space, so that while the 2-pole PM magnet rotates, the EMF produced from each winding would be 120° out of phase from each other. Each winding was wound with 30 gauge magnet wire, to have the proper voltage and cross-sectional area. The main advantage of this type of winding was the simplicity of construction.

The second type of winding differs considerably in the cross-section. Instead of a round cross-section, the cross-sectional shape and size was determined by numerical modeling methods of Section 3.3.2.5.1. The analytic expression for the magnetic field of a uniformly magnetized sphere was used in the numerical model. This is one of a few cases where the magnetic field was easily expressed analytically both in the near and far-field.49

Furthermore, the area of each winding was constrained to 120° of the angular space. Using these criteria, the cross-section of the winding was allowed to “grow” to the most advantageous locations with maximum flux and minimum resistance. The result was graphed in Figure 46, where the winding factor is shown to increase as the total cross-sectional area increases.
Figure 46: Winding factor increases, and then decreases as a function of winding cross-sectional area

A compromise was made between maximizing the winding factor and minimizing the size. The final cross-sectional shape and size is illustrated in Figure 47.

Figure 47: Cross-section of optimized winding design
Since this type of winding had relatively complex curves in cross-section, the construction would be complex if all the curves were realized. A compromise was made, where complex curves were implemented only when it contributes greatly towards the final winding factor. The only such curve was the one located next to the spherical PM. The other curve, which was located away from the spherical PM, was not as crucial since the winding factor was not sensitive to it. Therefore it was linearly approximated as shown in Figure 48.

![Figure 48: Cross-section of linearly approximated optimized winding design](image)

The construction of this winding required the construction of a specially machined bobbin, discussed in more detail in Section 5.2.3.3.
5.2.3 Construction and assembly

A number of custom BLDC generator models were built. The PM used in each model was identical. The models differed in the type of bearings and windings used. Separate construction methods were employed depending on the type of bearings used.

5.2.3.1 Receiver permanent magnet rotor

The receiver PM rotates at a speed of 300Hz. A method was devised to attach the spherical PM to shafts.

Since the design described in Section 5.1.3.5 is only possible for cylindrical magnets, the technique used previously for the kilo-watt model was not possible in this case. Instead, careful machining of the spherical magnet is performed instead.

![Diagram of two methods of attaching shaft to spherical PM](image)

(a) (b)

**Figure 49:** Two methods of attaching shaft to spherical PM

Depending on the diameter of the shaft required, a hole would either be drilled all the way through the sphere as illustrated in Figure 49(a), or two blind holes would be drilled into opposite ends of the sphere as illustrated in Figure 49(b). Drilling all the way through was only possible for certain shaft diameters. The shaft diameter was dependent on the type of bearings used. Since two types of bearings were used, both types of machining techniques were used.

Most Neodymium PM manufacturers recommend against drilling into the magnet. Therefore special considerations were taken in order to drill through them. The drilling required
specialized carbide drill bits, and careful alignment to ensure the shafts are centered and co-
axial. After the shafts are inserted, it was bonded to the inner surface of the hole with special
adhesives.

5.2.3.2 Receiver bearings

Two types of bearings were used for the receiver PM rotor, jewel bearings and ball bearings.
Each required separate techniques for construction. The jewel bearing receiver employed two
shafts and two jewel assemblies, shown in Figure 50.

![Figure 50: Receiver PM rotor assembly using jewel bearings](image)

The shafts were mounted into the spherical PM according to the technique of Figure 49(b). A
hole could not be completely drilled through because the diameter of the shaft was too small.
With a drill bit diameter of only 1.2 mm, the drill bit had trouble drilling deep, accurate
holes. After the shaft was installed on the PM, the protruding end of each shaft was rounded
to a radius of 1.9 mm to minimize contact surface, illustrated in Figure 51.
A flat end-stone was fitted to the jewel assembly, made of synthetic sapphire. It makes contact with the rounded end of the stainless steel shaft. This handled axial loads. A ring-jewel made of sapphire was also fitted to the jewel assembly, around the shaft. Contact area was again minimized by rounding the inner surface of the ring, called an olive hole. This handled radial loads.

The outer surface of the jewel assembly had threads machined into it to allow installation. The jewel assemblies are installed on a one-piece acrylic frame shown in Figure 50. The holes and threads in the acrylic frame for the bearings, were machined in a single operation to ensure proper alignment. A small drop of oil was placed inside of the jewel assembly for lubrication prior to the mating of shaft and jewel assembly. The assembly of the ball bearing receiver is illustrated in Figure 52.
The ball bearing used for this receiver was of the miniature type. It has a shaft diameter of 2 mm, ABEC 7 rating, lubricated with oil. The shaft was installed into the PM rotor using the technique in Figure 49(a). The hole was drilled all the way through because the diameter was sufficiently large to allow a drill bit to do so. A sleeve was inserted on either side of the PM between the bearings and the PM to ensure that the PM did not slide along the shaft. The frame also was constructed from a one-piece acrylic block, with both holes drilled for the bearings in one machining operation, as it was the case for the jewel bearing assembly for accuracy.

5.2.3.3 Winding construction

Two types of windings were designed in Section 5.2.2.6, each with its own method of construction. The construction of the first type, with circular cross-section, was relatively straight-forward, illustrated in Figure 53. The magnet wire was wound around a cylindrical rod by hand. The diameter of the rod was designed to be large enough to allow a 2.5 cm sphere to fit inside 3 such windings that were subsequently attached to each other (see Figure 45). After winding the required number of turns, the winding was removed from the rod and secured by further applying wire in a torroidal fashion to secure the windings to each other, as show in Figure 53.
This naturally produced a circular cross-section, and produces a self-supporting winding using only conventional magnet wire.

Afterwards, each winding was briefly soaked in an epoxy adhesive which was allowed to cure. The windings were then installed directly around the assembled PM rotor, mounted in the acrylic frame with epoxy. Extreme care was taken to make sure that neither the windings nor epoxy touched the PM rotor. This was simplified by using a spacer made of Teflon film, which was wrapped around the spherical PM during assembly. After the windings were mounted, the spacer was removed.

The construction of the second type of winding was more complicated. Because the cross-section was no longer a simple circle, a specially made bobbin was used, illustrated in Figure 54.
Figure 54: Custom bobbin used to construct optimized winding

The bobbin was constructed from 3 parts, A, B and C illustrated in Figure 54. Part A was a cylinder with one end machined to be a section of a 2.8 cm diameter sphere. This allows the windings to be placed very close to the magnet, with sufficient spacing between the PM and windings. Part B served to constrain the windings in the linearly approximated sections. Part C served to constrain the winding within each winding’s 120° of angular space.

Part C required special consideration due to the 30° overhang illustrated in Figure 54(a). The presence of this overhang prevented the placement of wire in the space directly under the overhang. This had been solved by removing the 30° overhang from a portion of part C illustrated in Figure 54(b). For this to be useful, part C was also made to slide with respect to parts A and B allowing the “no overhang” portion of part C to remain stationary with respect to the incoming wire. This allowed the incoming wire to always approach through the section which was free from the 30° overhang.

Since the final winding had to be self-supporting, the wires were kept in a bath of epoxy during the entire winding process. An epoxy designed for potting electronics with a long cure time was used for this purpose. To prevent the epoxy from bonding to the bobbin, all parts of
the bobbin were made of PTFE. To allow the removal of the winding once cured, the 3 parts of the bobbin were made to be separable.

After three such windings were made, reusing the same bobbin each time, they were assembled onto the acrylic frame around the PM rotor in the same fashion as that of the windings with circular cross-sections.

5.2.3.4 Transmitter device construction

The transmitter device used a separate off-the-shelf motor to drive the PM, and a design that was of simpler construction than the receiver. Unlike the transmitter device for the kilowatt model, the PM is coupled directly to the motor. The electric motors are of the brushed-DC, and brushless-DC kind, and are designed to operate at speeds of 300 Hz, or 18,000 rpm.

The technique used for the coupling of cylindrical transmitter PMs onto the motor shaft is illustrated in Figure 55.

![Figure 55: Transmitter construction of 1.9 and 2.5 cm diameter PM](image)

The motor shaft and PM were directly coupled with a custom-made shaft coupler. One side of the shaft coupler had the inner diameter of the motor shaft, and the other side of the shaft coupler had an inner diameter of 1.9 or 2.5 cm, depending on the diameter of the PM magnets used. Each side of the shaft coupler was secured using set screws. The motor was then securely fastened to the table.
For transmitter PM of 5 cm diameter, a slightly more secure method of direct coupling was used, illustrated in Figure 56.

![Figure 56: Transmitter construction of 5 cm diameter PM](image)

Shafts were mounted onto the 5 cm diameter, 5 cm long PM using shaft couplers similar to ones used in Figure 55. In this case, these shaft couplers were installed on both sides of the PM. The shafts were secured onto bearings which are seated in a one-piece acrylic frame, and the motor was coupled to one end of shaft using an off-the-shelf direct-drive shaft-coupler. The motor was mounted and secured to the acrylic frame.

5.2.4 Experimental verification of power transfer

The completed transmitter and receiver devices were tested under various conditions to verify performance, similar to the procedure used in the kilowatt model. Since a variety of
receivers and transmitters were built for this size scale with varying winding factors and dimensions, the effects of these parameters were emphasized in this analysis.

The first experiments were intended to explore the limits of synchronization, as it relates to speed of rotation and load resistance connected to the receiver, plotted in Figure 57(a). These tests were conducted using a transmitting PM, 5 cm in diameter and 5 cm in length.

![Graph](a)

**Figure 57:** Power and speed results with respect to load resistance at two separation distances (12.05cm and 13.35cm). Uncertainties not visible: $R_L \pm 0.5 \Omega$, Power $\pm 0.4$ W, rotational frequency $\pm 0.1$ Hz, $d \pm 0.1$ cm
The results in Figure 57 are very similar to the results obtained in Figure 34 when dealing with the kilowatt-scale model. Figure 57(a) displays the maximum speed sustainable under synchronization, as a function of load resistance, at two separation distances. The reason why the slope of the best-fit line was greater for smaller separation distances, could be understood by Equation (19), where the slope at small $R_L$ was proportional to the maximum time-averaged torque, which was inversely related to the distance between the two PMs. The frequency of rotation was allowed to increase until the mechanical speed limit of 150 Hz was reached.

The power obtained across the load, at the frequencies of Figure 57(a), is plotted in Figure 57(b). This relationship agreed closely with the theoretical model developed in Equation (20). It is clear from this plot that when the receiver and transmitter are closer, the power obtained would be greater.

The relationship of frequency and power was explored in Figure 58.

![Figure 58: Power results with respect to rotational frequency at several distances, Uncertainties not visible: Power ±0.4 W, rotational frequency ±0.1 Hz, d ±0.1 cm](image-url)
The maximum power obtained at any given frequency was plotted, for four different separation distances in Figure 58. These results were also very similar to the kilowatt model results, and in agreement with the theoretical model obtained in Equation (8). The power varied linearly with the frequency of rotation due to the definition of mechanical, rotational power, $P = \omega T$. Since the time-averaged maximum torque was independent of frequency, at increased frequency, the power increased proportionally. The slope of this linear relationship was greater for smaller separation distances. This was due to the fact that the slope was the time-averaged maximum torque, which was inversely proportional to separation distance.

The explicit relationship between maximum power, previously called $P_{\text{max}}$, and the separation distance, is plotted in Figure 59.

![Figure 59: Power results with respect to separation distance](image)

**Figure 59:** Power results with respect to separation distance, Uncertainties not visible: Power ±0.4 W, $d$ ±0.1 cm

Again, it was apparent the power was decreasing as a function of separation distance, due to the decreasing magnetic field as a function of distance. This was very similar to the behavior seen for the kilowatt model in Figure 36, which followed the theoretical model for the magnetic gear in Equation (35).

The effect of winding factor was investigated using two receivers with different winding factors. The experimentally measured winding factor of the optimized winding was shown to be 4.55 times higher than the winding factor of the less optimized winding. The experiment was conducted at identical conditions, and the resulting power from two different types of windings was plotted in Figure 60 with respect to frequency.
The advantage of the optimized winding is apparent from the graph in Figure 60, where the same separation distance, transmitting magnet, receiving PM rotor and bearings were used in each case. The optimized winding was able to produce more power under the same conditions. This advantage can be explained when analyzing Equation (24) developed for the theoretical model for maximum power attainable, $P_{\text{max}}$, under mechanical speed limitation.

In the approximation where mechanical friction was neglected, that expression simplifies to (40).

$$P = T\omega - \frac{2}{(\Phi_0^2/R_c)}T^2$$

(40)

From the above expression, it was clear why the two plots in Figure 60 resembled linear relationships with differing y-intercepts. The first term on the right-hand-side of (40) described the linear relationship between mechanical power and angular velocity. The second term in (40) described the a negative y-intercept in Figure 60. The second term relates to the power loss due to mechanical to electrical conversion, related to the winding factor, $\Phi_0^2/R_c$. 

Figure 60: The effect of winding factor on performance

Uncertainties not visible: Power ±0.4 W, Frequency ±0.1 Hz
The measurement of Figure 60 was performed with identical transmitter and receiver PM rotors, and very similar separation distances. Therefore, the torque sustained by the magnetic gear is relatively constant for any frequency and coiling type. Therefore the slope between the two plots of Figure 60 was nearly identical. The two plots differed only by $\Phi_0^2/R_c$, which affects the relationship only in terms of the y-intercept, which is evident in Figure 60 and Equation (40).

5.2.5 Experimental verification of efficiency

Figure 61 illustrates the power flow of the transfer device.

**Figure 61:** Power flow diagram of wireless power transfer system

The efficiency of the system was divided into three portions and analyzed independently. The first portion, the receiver efficiency illustrated in Figure 61(c), was the efficiency of the generator portion, the ability of the receiver to transform mechanical power of the receiver PM into electrical power. This efficiency was found to be relatively high, discussed later in this section. The second portion was the efficiency of the wireless channel, or the magnetic gear illustrated in Figure 61(b). This efficiency was near 100% efficient, with losses too small to be measured with techniques used. The third portion was the transmitter efficiency illustrated in Figure 61(a). This was the efficiency of the motor. It took into account all the
losses internal to the motor, as well as any losses in the rotation of the transmitter PM. This efficiency was relatively low, as expected, mainly due to the low efficiency of the off-the-shelf motor. Each portion of the efficiency was analyzed in more detail in the following sections.

5.2.5.1 Receiver efficiency

5.2.5.1.1 Method

Of all three efficiency portions, the measurement of the receiver efficiency was the most complex. The receiver efficiency was defined by Figure 61(c), and the output power was simply the power dissipated across the load and easy to measure. In contrast, the input power was more difficult to measure. The input power was the mechanical power applied by the transmitting PM, to the receiving PM, through the wireless channel. The torque which was applied across the wireless channel (labeled as “air-gap torque” in Figure 61) was measured.

One assumption was made in this measurement, which was that the wireless channel efficiency is 100%, and this assumption was verified in the next section. Using this assumption, the air gap torque was measured by observing the speed of the PM rotors decelerating under frictional and load forces. First, the deceleration of the transmitter and receiver rotors were measured at the moment power to the motor was cut and left open-circuit. The PM rotors at that point were under synchronization, with a certain load resistance connected.

Using this data, similar to the case of the kilowatt model, the torque was calculated. This is the amount of torque to provide the power in the electrical load, as well as the losses associated with the receiver, transmitter and associated motor. Subsequently, the same measurement was conducted without the presence of the receiver device. The difference in torque between the two measurements was the portion of torque used to turn the receiver alone, and is therefore the “air-gap torque” in question.
5.2.5.1.2 Moment of Inertia

In order to calculate torque from a deceleration curve mentioned previously, the moment of inertia was determined for all the rotating components, including the motor shaft, the transmitter PM rotor, the receiver PM rotor, as well as all shaft coupling components. Using these values of moments of inertia ($I$) and deceleration ($\dot{\omega}$), the torque, $T(\omega)$, was calculated using Equation (41).

\[ I\dot{\omega} = T(\omega) \quad (41) \]

The moment of inertia of the receiver PM rotor was estimated to be that of a sphere, 2.5 cm in diameter, with uniformly distributed mass-density. The known expression for the moment of inertia of a sphere in Equation (42) was used, where $M$ is the total mass and $r$ is the radius of the sphere.

\[ I = \frac{2}{5} Mr^2 \quad (42) \]

This yields a moment of inertia of $4.169 \times 10^{-6}$ kgm$^2$ for the receiver PM rotor.

Calculating the moments of inertia of the motor shaft and the transmitter PM rotor was more involved, since the shape of those components could not be simplified, so an alternate method of measuring the moment of inertia was used. By imposing a known torque, $T(\omega)$, to the shaft and rotor at standstill, and observing the resultant acceleration, $\dot{\omega}$, the moment of inertia was calculated using (41).

To impose a torque, the stall torque of the motor was used. The stall torque was imposed on the motor shaft connected to the PM rotor, while both were stationary. As the rotors accelerated, the speed was recorded using linear Hall-Effect sensors near the PM. The motor was current-limited, so a small but constant amount of torque was applied each time. The stall torque was measured by measuring the force applied by a lever-arm attached to the motor shaft.
Using these methods, the total moment of inertia of the motor shaft coupled to a 5 cm diameter transmitter PM rotor, was measured to be \(3.04 \times 10^{-4}\) kgm\(^2\). These values were used to calculate torque and efficiency in the following sections.

5.2.5.1.3 Measuring the air-gap torque and receiver efficiency

The air-gap torque was measured using the previously determined moments of inertia. With a certain electrical load resistance connected, the transmitter and receiver rotor speeds reached a certain value in synchronization. Then, power to the motor was suddenly disconnected and the motor leads were left open-circuit. The receiver and transmitter PM rotors thus decelerated under the influence of the receiver, transmitter and motor forces. The magnitude of this acceleration was multiplied by the sum of the moments of inertia of the receiver, transmitter and motor to obtain total torque. This total torque was plotted against speed of rotation in Figure 62.

![Figure 62: Calculation of air-gap torque in order to calculate efficiency of the receiver](image)

This measurement was then repeated with only the motor and transmitter rotors, without the of the receiver. This time the magnitude of the acceleration was multiplied with the moment of inertia of the motor and transmitter rotor only. This was also plotted in Figure 62. The
difference between the two torques was the “air-gap torque” as a function of speed of rotation, plotted as the shaded area in Figure 62.

The receiver efficiency was determined by the ratio of the power dissipated across the load and the air-gap torque times the angular frequency in question. It took into account all the losses of the receiver including mechanical friction of the bearing, as well as eddy current losses in the bearings and the windings. This procedure was repeated for multiple load resistance values to obtain the receiver efficiency as a function of load resistance, illustrated in Figure 63.

Figure 63: Receiver efficiency as function of $R_L$ at different speeds
Uncertainties not visible: $R_L \pm 0.5 \ \Omega$, efficiency $\pm 0.02$

Multiple data points for each load resistance were taken at different frequencies ranging from 30 Hz to 100 Hz. The data points were compared against the theoretical efficiency, also plotted in Figure 63.
The data shows that the efficiency reaches 90% for the majority of load resistances. At low load resistances, the load resistance approached the winding resistance. Therefore in that region, the losses were mainly due to copper losses in the windings.

5.2.5.2 Channel efficiency

The channel efficiency was illustrated in Figure 61(b). The assumption that the magnetic gear is near 100% efficient was tested by observing the total system efficiency as a function of separation distance. This relied on the assumption that if there were relevant losses inherent to the magnetic gear, such as eddy current and hysteresis losses in the magnet themselves, these losses would vary as a function of distance between the two PM rotors.

For this purpose, the total efficiency was measured at various separation distances, ranging from 7.4 to 12.8 cm as a function of frequency. This was done for two different load resistances, illustrated in Figure 64.

![Figure 64: Efficiency independent of separation distance](image)

Uncertainties not visible: Frequency ±0.1 Hz, efficiency ±0.005
The results showed that the system’s total transfer efficiency varied as a function of frequency and load resistance, but never as a function of separation distance. At each load resistance, the efficiency fell on the same line no matter what separation distance was used. Therefore it could be reasonably concluded that the inherent losses of the magnetic gear, was sufficiently low compared to other losses that it cannot be measured, and were negligible.

As a side note, the efficiency of the new method of wireless power transfer device was independent of distance. This is in stark contrast to the resonant inductive wireless power transfer methods, for which efficiency is inversely proportional to separation. This is clarified in Appendix C, where it is shown that the efficiency is dependant on the mutual inductance between the two windings, which is heavily dependant on separation. As a second side note, the overall efficiencies in Figure 64 were quite low. This was due to low efficiency of the motor, measured in the next section.

5.2.5.3 Transmitter efficiency

The transmitter efficiency was illustrated in Figure 61(a). It included the switching losses in the motor driver, the electromagnetic losses in the motor, and the frictional losses in the motor and transmitter PM rotor. An off-the-shelf DC motor was used for these measurements.

The input power was the electrical power supplied to the motor driver, and was determined by measuring the voltage and current. The output power is the air-gap power previously calculated assuming magnetic gear efficiency of 100%. The efficiency of the transmitter was calculated as the ratio of output power against input power, plotted in Figure 65.

The efficiency of the receiver, at the speeds in question, was also plotted here, in Figure 65. The total system efficiency was the product of the transmitter and receiver efficiency, which is plotted here in Figure 65 as well.
The most noticeable feature was that the efficiency of the transmitter was considerably lower than that of the receiver. Since a separate motor was used to spin the transmitter PM instead of a custom BLDC solution, the lower efficiency was to be expected. In an ideal implementation, the transmitting and receiving stages would be implemented by custom BLDC machines. In that case, the efficiency of the transmitter would be comparable to that of the receiver, significantly improving total efficiency.

5.2.6 Misalignment tolerance

The power transfer capacity of the system was measured while the transmitter and receiver were placed in less than optimal positions and orientations relative to one another.

Relative displacements and orientations were achieved by moving and rotating the receiver device while the transmitter device remained stationary. Although the work in this thesis mainly used one orientation, there were in fact, two ideal orientations for power transfer, co-axial and parallel, illustrated in Figure 66(a) and 66(b) respectfully.

Figure 65: Receiver, transmitter and overall efficiency

Uncertainties not visible: $R_L \pm 0.5 \, \Omega$, efficiency $\pm 0.02$
Although most of the work of this thesis concentrated on parallel transfer orientation, co-axial orientation was also valid. Here the misalignment tolerance is described for both orientations.

5.2.6.1 **Co-axial transfer orientation**

The co-axial transfer orientation is illustrated in Figure 66(a). Translational and angular misalignments are introduced to the ideal orientation. Due to the symmetry of the overall transfer orientation around the mutual axis, translations of the receiver in the $z$ and $x$ directions are equivalent. A translation in the $y$ direction is the effect of range previously explored in Figure 59. Therefore, translation in the $z$ direction alone is illustrative. Likewise, rotations around the $x$ and $z$ axes were equivalent due to symmetry, and a rotation around the $y$ axis had no effect at all. Therefore, rotation around the $x$ axis was explored.

The effect of the translational and angular misalignments are plotted in Figure 67(a) and (b) respectfully, with a separation distance between the transmitter and receiver devices kept constant at 14.5 cm.
Figure 67: The effect of misalignment on power transfer in the co-axial transfer orientation

Figure 67(a) shows that the range of positions which achieve 90% of full power or more was approximately ±6 cm. Figure 67(b) shows that the range of angles which achieve 90% of full power or more was approximately ±35°. The range of acceptable misalignment was substantial, which is encouraging from an application perspective.

5.2.6.2 Parallel transfer orientation

Ideal parallel transfer orientation is illustrated in Figure 66(b). In this orientation, the translational misalignments must be considered separately, in the z and x directions, due to
the absence of the co-axial symmetry. Likewise, the angular misalignments must be considered around both the x and y axes.

The effect of translations in the x and z directions is plotted in Figure 68(a), and the effect of rotations in the x and y axes is plotted in Figure 68(b). For this experiment, the separation distance between the transmitter and receiver devices were kept constant at 14.8 cm.

![Figure 68](image)

**Figure 68:** The effect of misalignment on power transfer in the parallel transfer orientation

Figure 68(a) shows that the range of positions which achieve 90% of full power or more, was approximately ±3 cm in the z axis, and ±3.5 cm in the x axis. Figure 68(b) shows that the
range of angles which achieve 90% of full power or more, was approximately $\pm 35^\circ$ around the $x$ axis, and $\pm 40^\circ$ around the $y$ axis. Overall the parallel transfer orientation is similar in misalignment to that of the co-axial transfer orientation.

### 5.2.6.3 Misalignment tolerance conclusion

In either transfer orientation, for 90% full power tolerance, the range of misalignment is estimated at $\pm 35^\circ$ angular, and $\pm 3$ cm translational for a transfer distance of 14.5 cm. This was well within the misalignment tolerance requirements of most applications, including biomedical applications where transmitters and receivers were allowed to shift, affording patients greater mobility. Other applications where misalignment tolerance would be valuable would be the self-charging of autonomous robots, and the wireless charging of electric vehicles.

### 5.2.7 Control

There was a comprehensive set of controls that were implemented to manage the power transfer. These controls include proximity sensing, alignment sensing, and synchronization slip angle monitoring ($\theta$ in Figure 8). The implementation required a few well placed, linear Hall Effect sensors. Depending on the need of the application, all the sensing and control intelligence could be mounted on the transmitter side. The receiver side could remain sensor-less. There was no communication needed between the transmitter and receiver to implement these controls. The proximity sensor in the transmitter side detects the presence and distance of any incoming receiver. The alignment sensors detect the presence and magnitude of the misalignment. The synchronization slip angle sensor informs the control software when the limit of synchronization is being approached. Subsequently the controller can then take action to avoid synchronization loss.

All the controls mentioned were implemented and shown to perform reliably using linear Hall sensors on the transmitter side alone. A micro-controller was used with multiple analog to digital (A/D) converter inputs to accept Hall sensor signals. A pulse-width-modulated
(PWM) output was used to control the motor in the transmitter. The PWM output was amplified to serve as a voltage-controlled power supply for the DC motor.

These controls were implemented only on the 60 W model to date, but there is no reason to believe they would not be equally practical in the kilowatt scale model.

The code of the software program used to implement the control system was provided in Appendix A.

5.2.7.1 Proximity detection

In a practical application, such as a charging station, the transmitter would likely be configured to detect the presence of any suitable receivers and would automatically initiate the power transfer. There are many ways to sense proximity, one of which is RF communication. An alternative, which we decided to explore, is to sense the field of the PM in the receiver, by means of Hall Effect sensors.

For this to work, the larger field created by the nearby transmitter PM must not interfere too much with the detection of the field of more distant receiver PM. To achieve this, two Hall sensors were used, oppositely mounted on the stationary frame, \( u \) and \( v \), on the transmitter side as illustrated in Figure 69.
Figure 69: Hall sensor placement for control system

In the orientation of Figure 69, the Hall sensors, $H1$ and $H2$ were mounted on the stationary frame, on opposite sides of the transmitter magnet as shown, sensitive to the magnetic fields in directions shown. The axes of rotation of the PMs were into the page. The signal from each Hall sensor was linearly proportional to a magnetic field in the outward radial direction as shown. Considering only the transmitter side in this orientation, the sum of the signals does not vary substantially as the transmitter PM is rotated around the Hall sensors.

However, when a receiving PM is in the vicinity of the transmitting PM along the $u$-axis, the poles of the PMs would line up in the orientation as illustrated Figure 69. In this case, although the transmitter PM has a very small effect on the signal of $(H1+H2)$, the receiver PM induces a significant change in the signal $(H1+H2)$. Using this method, proximity was determined by calculating how far the $(H1+H2)$ signal deviated from a pre-calibrated level. This was implemented in micro-controller software. The $H1$ and $H2$ signals were fed separately to the micro-controller’s A/D converter inputs and the $(H1+H2)$ signal was internally calculated. The micro-controller was programmed to initiate the power transfer once the $(H1+H2)$ deviates sufficiently from a pre-programmed value. The pre-programmed value was determined when was no receiver present. This proximity detection approach was found to perform reliably.
5.2.7.2  Synchronization slip angle monitoring

We selected the following algorithm for initiating the power transfer. Once close proximity is detected, voltage to the DC motor is increased until the desired frequency is reached. As the rotation rate is ramping up, the synchronization slip angle (θ) is constantly monitored. If for whatever reason this angle approaches too close to 90°, the angular acceleration would decreased to avoid the loss of synchronization.

Thus an important piece of information at any time is the phase difference between rotating transmitter and receiver PMs. This can be determined from the Hall sensors already present. As the receiver PM rotates, the (H1+H2) signal oscillates at the frequency of the rotation, in phase with the rotation of the receiver PM. Further, the available difference signal, (H1-H2), primarily provides phase information for the transmitter magnet.

Therefore the phase difference in the rotating transmitter and receiver PMs is related to the phase difference between the signals (H1+H2) and (H1-H2)

In this regard, it was important to realize that in the orientation of Figure 69, when the two PMs are synchronized with zero slip angle, the phase difference between (H1+H2) and (H1-H2) is actually 90°. As the slip angle increases, the phase difference of the signals deviates in equal magnitude to the slip angle. The sign of the deviation depends on the direction of rotation of the PM rotors.

As with proximity sensing, both (H1+H2) and (H1-H2) signals were calculated internally in the micro-controller. The values were polled and stored internally for a minimum of one period to calculate the relative phase difference between (H1+H2) and (H1-H2) signals. As mentioned earlier, as the calculated phase difference approaches too closely to 90°, the loss of synchronization is imminent. As this limit is approached, the software was programmed to reduce the voltage to the DC motor, thus reducing its angular acceleration and, in some cases, its angular velocity. Conversely, if the measured synchronization slip angle was sufficiently small, the voltage to the motor was increased, increasing the slip angle. Therefore the frequency of operation was controlled in a closed-loop fashion to maximize power transfer without losing synchronization.
Using this implementation, synchronization was reliably maintained. Moving the receiver closer to the transmitter allowed the system to spin faster. This is because to the microcontroller sensed the increase torque transfer capability by sensing a decrease in slip angle. Frequency would then increase to yield greater power transfer. Conversely, moving the receiver farther from the transmitter would cause the system to spin slower. This control scheme served as a demonstration of ability to sense the slip angle, which might be helpful in some applications.

5.2.7.3 Alignment detection

In addition to proximity detection, alignment detection is also possible. Alignment detection would provide information about the magnitude and direction of misalignment before attempting to initiate power transfer. This could be especially useful for wireless EV charging, where the driver could be directed to compensate for the misalignment, if it were necessary.

Such detection of the misalignment could be implemented using 2 or 4 linear Hall sensors mounted oppositely on the transmitter side, in order to sense the position of the receiver. In the case of wireless charging of EVs, a visual or audio notification could be delivered to the driver to recommend adjustment of the position of the vehicle in order to improve power transfer.

5.2.7.4 Load sensing

Load sensing could be used in some circumstances to initiate a shut down of the transfer. The rate of transfer could be determined by monitoring the power being delivered to the motor, which, combined with motor frequency enables a fairly accurate assessment of the power transfer rate. When the transfer rate drops to a pre-determined level, which for example could indicate complete charging of an EV battery, the transmitter could be shut off to save energy and wear.
6 HUMAN EXPOSURE LIMITS TO MAGNETIC FIELDS

During the operation of the power transfer device, there is a time-varying magnetic field produced by the rotating magnets. It is important to consider whether these fields could pose any risk to human health. For this reason, the magnetic field magnitudes were measured and compared to exposure limits set by various governing bodies.

6.1 Standards

The two main organizations who set standards for human exposure to electromagnetic fields are the Institute of Electrical and Electronics Engineers (IEEE) and the International Commission on Non-Ionizing Radiation Protection (ICNIRP). Apparently, the exercise of setting exposure limits was a challenging one, as the limit depends heavily on the frequency of the time-changing fields, the magnitudes of the electric and magnetic fields, and whether short term or long term effects are considered. The standards are based on experiment results which are far from comprehensive and often not conclusive. Especially difficult is the determination of long term exposure limits, for which data was even more incomplete. Therefore most standard setting bodies, such as the IEEE, have not set specific long term exposure limits. For this reason, we began with the IEEE standard50, “IEEE Standard for Safety Levels with Respect to Human Exposure to Electromagnetic Fields, 0–3 kHz”, for short term exposure limits.

For long term exposure limits, the ICNIRP standard51, “Guidelines for Limiting Exposure to Time-Varying Electric, Magnetic, and Electromagnetic Fields (up to 300 GHz)”, was used. This seemed appropriate since the ICNIRP standard was set partly in consultation of data from long term exposure experiments.

According to ICNIRP, the exposure to time-varying magnetic fields at 150 Hz was limited to 33 μT_{rms} for the general public.

This exposure limit set by ICNIRP was based mainly on the induction of currents inside the human body. It was based on a variety of studies such as on epidemiological studies of
cancer risk in relation to exposure to power-frequency fields, occupational studies of
electrical workers as well as cellular and animals studies52.

According to IEEE, the exposure to time-varying magnetic fields at 150 Hz was limited to
904 μT_{rms} for the head and torso, and 25 mT_{rms} for the arms and legs.

These exposure limits set by IEEE were based on known biophysical interactions with
magnetic fields. It was based on experimental results gathered from human and animal data.
The limit values were set to avoid adverse biological effects such as painful stimulation,
muscle excitation, alteration of synaptic activity and cardiac excitation53.

### 6.2 Magnetic field measurements

For the current kilowatt scale version, the peripheral magnetic field was measured and
compared to the exposure limits. The 60 W model would likely produce weaker fields. The
field was measured with the orientation illustrated in Figure 70.

![Figure 70: Measurement of ambient magnetic fields created by device](image)

This was the orientation in which 1.6kW was transferred previously. The field was measured
along the \(x\)-axis. The positions along the \(x\)-axis experiences the greatest magnetic fields since
they are inline with both magnets. The magnets were held stationary in the orientation shown
during the measurement. In this orientation, the magnets are producing a field magnitude
corresponding to the peak value when the magnets are spinning. The measured peak values
were converted to rms values. Measurements were taken using a Gauss meter, starting \(x=0\)
corresponding to the surface of the receiver. The measurements were then continued by
moving the Gauss meter probe further down the x-axis away from the devices. The rms magnetic field values with respect to displacement along the x-axis are plotted in Figure 71.

![Graph showing magnetic field strength as a function of distance with limit points set by standards.](image)

**Figure 71:** Magnitude of ambient magnetic field created by kW system as function of distance with limit points set by standards

Not surprisingly, the magnetic field strength fell dramatically as a function of distance. This is because the far-field is inversely proportional to the 3\textsuperscript{rd} power of distance. At a distance of 75 cm, the field strength was sufficiently low that the Gauss meter was not able to separate signal from noise. The field shown beyond that point is extrapolated, assuming it continues to fall off like an ideal dipole field.

### 6.3 Human exposure evaluation

In Figure 71, the limit values in the IEEE and ICNIRP standards are also plotted at the appropriate distances.

For short-term exposure of the arms and legs, the fields met the IEEE standard at a distance beyond 6 cm. For short-term exposure of the head and torso, the fields met the IEEE standard at a distance beyond 30 cm.
For chronic exposure, the fields met the ICNIRP standards at a distance beyond approximately 1 meter. These restrictions seem reasonable for most applications, such as wireless vehicle charging. Even if the driver were to spend extended amounts of time in and around the vehicle while charging, the exposure would fall within the allowable chronic level for constant exposure. During periods when it was necessary to be closer than 1 meter, such as vehicle maintenance, the hands and head of the operator would typically not come closer than the limiting distances described above. Therefore it is generally safe to perform average vehicle maintenance while the power transfer device is still running, which is reassuring even though there would rarely be a need to do so. If in doubt, a simple interlock system could turn off power if a person were too close. Overall, this system is extremely safe, likely much safer than any electrical cord and plug.

6.4 Perceived RF health hazard

Often to the general public, the perception of danger is almost as important as real danger. One of the disadvantages that have plagued the inductive wireless transfer technologies has been the perceived danger of high frequency electro-magnetic field radiation\(^5\). Some inductive methods have conformed to the limit values, and some have exceeded the limit values. Even if the device conforms to the standard, the wide-spread adoption still required the public to accept the presence of an otherwise unnecessary high frequency and high power radiation source. The lack of understanding of electromagnetic radiation by the general public might lead unjustified, as well as justified, perceptions of danger.

However, the general public, in general, has a good understanding of the permanent magnet. It is usually perceived to be harmless. Therefore it could be argued that the public’s perception of the new transfer device will not have those automatic negative perceptions.
7 CONCLUSION

It has been supported through the findings of this thesis that the mechanical interaction of synchronized, rotating PMs is a viable coupling mechanism for electrical wireless power transfer. With the use of common and economically viable Neodymium permanent magnets spinning at mechanically reliable speeds, a useful amount of electrical power for many applications can be transferred. These applications include the powering of medical implants, consumer electronics, electrical vehicles, and other industrial devices.

The efficiency of the transfer was also demonstrated to be sufficiently high for most applications. The efficiency of the custom BLDC generator has been measured to be around 90%. The mechanisms of power loss for the BLDC generator have been identified, and methods for reducing this loss have been identified. The first of the two main methods involved increasing the winding factor by increasing the magnetic flux and decreasing resistance in the winding. This could be done by placing turns closer to the PM, increasing the packing factor of the turns, and increasing the magnetization strength of the PM. The second of the two main methods involved reducing eddy current losses in the windings. This could be done by constructing the windings from smaller diameter wire and greater number of turns. The winding factor should remain unchanged if the same amount of total cross-sectional area of wire is used.

The efficiency of the magnetic gear has also been indirectly measured to be very close to 100%. Although in the current implementation, the motoring portion did not have a high efficiency, it could be reasonably extrapolated that a custom BLDC motor implementation would have an efficiency very close to the BLDC generator, again around 90%. This was also a reasonable extrapolation judging from literature, in which BLDC motors of higher efficiencies have been built. This suggests that the total power transfer device should be able to achieve an overall efficiency of 81% if implemented with a custom BLDC generator and a custom BLDC motor.

This method of wireless power transfer is also highly scalable. It was experimentally verified that the power transfer capacity could be increased by increasing the physical size of the
PMs. A model which quantitatively characterizes the behavior of performance parameters as a result of physical size-scale was developed and experimentally verified. This suggests that future transfer systems with any reasonable power capacity can be constructed by scaling the physical dimensions.

Through the process of constructing the PM rotors, it had been shown that Neodymium PMs can be reliably mounted onto rotating shafts. Although neodymium PM material itself has relatively poor mechanical properties and is often regarded as fragile, compared to steel or other alloys, techniques developed and reported here, for machining a containment and mounting system for large Neodymium PMs, can be reliably executed to ensure safe and reliable rotation.

In many applications of wireless power, relative misalignment of the transmitter and receiver devices must be tolerated. The effect of misalignment on power transfer capacity has been measured. The results have shown that this type of mechanical PM coupling is fairly forgiving of misalignment, making it a viable technique for most applications.

Finally, most applications of wireless power require operation in an environment in close proximity to human beings. The magnetic fields created by the transfer system must not pose a risk to human health. The magnitude and frequency of the time-varying magnetic fields have been measured and evaluated against human exposure limits set by standards. It was concluded from that evaluation that the transfer device poses no risks to human health at reasonable distances. It is likely safer than any alternative method, including a power cord.
REFERENCES


3 Ibid 2


6 Ibid 2

7 Ibid 2


13 Ibid 9


19 Ibid 18


22 Ibid 15, p. 246

23 Ibid 15, p. 244

24 Ibid 15, p. 228


31 Ibid 26, p. 56

32 Ibid 29, p. 128


37 http://www.friendlyrobotics.com/

38 http://www.kjmagnetics.com/custom.asp

39 http://www.kjmagnetics.com/specs.asp

40 Ibid 26

41 G-10/FR 4, manufactured by American Epoxy & Metal Inc., Bronx, NY, 10461, USA


43 Ibid 26, p. 28

44 Ibid 26, p. 3


49 Ibid 20
50 IEEE Standards Coordinating Committee 28., *C95.6 IEEE standard for safety levels with respect to human exposure to electromagnetic fields, 0-3 kHz*, p. 10, IEEE, New York, NY, 2002


52 Ibid 51

53 Ibid 50


57 Ibid 15, p. 246

58 Ibid 15, p. 244


60 Ibid 55, p. 305

61 Ibid 55, p. 305

62 Ibid 15, p. 454


64 Ibid 55
APPENDIX A : CODE FOR CONTROL SOFTWARE

The software implementation of the controls of Section 5.2.7 was written in the programming language C included here. It was implemented in a microcontroller to control the motor of the transmitting device. The microcontroller receives feedback from the transmitting PM through two Hall Effect sensors directly fed into the analog-to-digital inputs, illustrated in Section 5.2.7.1. The microcontroller controls the motor through a pulse-width-modulated output which was amplified to directly drive a DC motor.

Upon activation, the program was designed to wait for the presence of a receiver using proximity detection methods described previously. Once proximity of a receiver was detected, speed of the DC motor was increased, while monitoring the lagging angle of the receiver magnetic moment with respect to the field produced by the transmitter PM, previously coined \( \theta \). The lagging angle, \( \theta \), was adjusted in a closed-loop fashion to an angle less than, but close to, 90\(^\circ\), by adjusting the speed of the motor.

It was also designed to revert back to the initial state once the proximity of the receiver was no longer detected. At that point, the DC motor was stopped, and the device waits until the receiver comes into sufficient proximity to start again.

```c
//pin assignments
int pwmPin = 3;      // pwm to D pin 3
int Hapin = 3;    // potentiometer connected to A pin 3
int Hbpin = 5;    // current sense connected to A pin 5
int ledPin = 13;     // LED to D pin 13
int timingPin = 12;

//proximity detection
int proxdet=0;
long sumiterProx=0;
long diffiterProx=0;
int sumaveProx=0;
int diffaveProx=0;

//startup
int startupDC = 40;
int maintainDC = 40;

//readsumdiff and average calc
int Ha=0;
int Hb=0;

//average summing
```
long sumiter=0;
long diffiter=0;
int sumave=0;
int diffave=0;

//edge detect
int diffnormA=1;
int diffnormB=1;
int edgedetected=0;
int halfperiods=0;

//phasemeasure
int t=0;
long SeeD=0;
long SeeDt=0;
boolean sign = false;

//readsumdiff
int diffnormread=1;
int sumnormread=1;

//driver
long preloopphase=0;
long inloopphase=0;

int motorspeed=0;
int speedchange=0;

boolean ledonoff=false;

//sumrms
int sumrms=0;
long sumrmsiter=0;

void setup()
{
  pinMode(ledPin, OUTPUT);      // sets the digital pin as output
  pinMode(timingPin, OUTPUT);      // sets the digital pin as output
  TCCR2A = 0x23;
  TCCR2B = 0x0C;  // select timer2 clock as 16 MHz I/O clock / 64 = 250 kHz
  OCR2A = 249;  // top/overflow value is 249 => produces a 1000 Hz PWM
  pinMode(pwmpin, OUTPUT);  // enable the PWM output (you now have a PWM signal on digital pin 3)
  OCR2B = 125;  // set the PWM to 50% duty cycle
}

void loop(){

digitalWrite(ledPin, LOW);
analogWrite(pwmpin, 0);

  //*** PROXIMITY DETECTION
  for(int i = 0;i<5000;i++){
    Ha=analogRead(Hapin);
Hb=analogRead(Hbpin);

sumiterProx = (Ha+Hb)+sumiterProx;
diffiterProx = (Ha-Hb)+diffiterProx;
}
sumaveProx=sumiterProx/5000;
diffaveProx=diffiterProx/5000;
digitalWrite(ledPin, LOW);

while(1){ //infinite loop until receiver in proximity
    proxdet=0;
    while(proxdet<10){
        Ha=analogRead(Hapin);
        Hb=analogRead(Hbpin);
        if(abs((Ha+Hb)-sumaveProx)>15){
            proxdet++;
            digitalWrite(ledPin, HIGH);
        }else{
            proxdet=0;
            digitalWrite(ledPin, LOW);
        }
    }
}

Ha=0;
Hb=0;
sumiter=0;
diffiter=0;
sumave=0;
diffave=0;
digitalWrite(ledPin, LOW);

//*** END PROXIMITY DET

/**///(1) open loop startup
//delay(1000);
analogWrite(pwmpin, startupDC);
delay(250);
analogWrite(pwmpin, maintainDC);
delay(1000);
/**/// end (1) open loop startup

/*** average calculation: calculate midpoints for phase calculations
for(int i = 0;i<5000;i++){
    Ha=analogRead(Hapin);
    Hb=analogRead(Hbpin);
    sumiter = (Ha+Hb)+sumiter;
    diffiter = (Ha-Hb)+diffiter;
}
sumave=sumiter/5000;
diffave=diffiter/5000;
/***/ END average calculation
/***pre-loop measure phase
preloopphase=0;
for(int i=0;i<10;i++){
    preloopphase=preloopphase+measurephase(1);
}
preloopphase=preloopphase/10;

/***starting loop
motorspeed=maintainDC;
speedchange=0;
digitalWrite(ledPin, HIGH);
while(sumrms>300){
    if(speedchange<0){
        motorspeed=motorspeed-10;
    }else if(speedchange>0){
        motorspeed++;
    }
analogWrite(pwmpin, motorspeed);
inloopphase=0;
for(int i=0;i<1;i++){
    inloopphase=inloopphase+measurephase(1);
}
inloopphase=inloopphase/1;
if(abs(inloopphase-preloopphase)<400){// closed loop control of speed acroding to phase
    speedchange=1;
    //digitalWrite(ledPin, LOW);
}else if(abs(inloopphase-preloopphase)<420){
    speedchange=0;
    //digitalWrite(ledPin, LOW);
}else {
    speedchange=-1;
    //digitalWrite(ledPin, HIGH);
}
}//END closed loop phase det
analogWrite(pwmpin, 0);

}/**BACK to proximity (no longer in proxmimy, transmitter stopped pending re-approach of receiver.

}

int measurephase(int param){ // measure phase difference between (H1+H2) and (H1-H2)
diffnormA=1;
```
diffnormB=1;
edgedetected=0;
halfperiods=0;

//phasemeasure
t=0;
SeeD=0;
SeeDt=0;

//sum rms
sumrmsiter=0;
sumrms=0;

//***edge detect
readsumdiff(1);
difffnormA=difffnormread;

while(halfperiods<1){
    delayMicroseconds(3);
    readsumdiff(1);
    difffnormB=difffnormread;
    if(difffnormB!=difffnormA){
        halfperiods++;
        //delayMicroseconds(10);
    }
    difffnormA=difffnormB;
}

//***integrating
halfperiods=0;
while(halfperiods<4){
    //summing here
    if(sumnormread==difffnormread){
        SeeD++;
        SeeDt=SeeDt+t;
    }
    t++;

    //sum rms
    sumrmsiter=sumrmsiter + (Ha+Hb-sumave)*(Ha+Hb-sumave);

    readsumdiff(1);
    difffnormB=difffnormread;
    if(difffnormB!=difffnormA){
        halfperiods++;
        //delayMicroseconds(10);
    }
    difffnormA=difffnormB;
}

// returning
sumrms=sumrmsiter*10/t;
```
if(SeeDt*10000/t/SeeD>5000){
    sign=true;
    return (1800-SeeD*1800/t)+1800;
}else{
    sign=false;
    return (SeeD*1800/t-1800)+1800;
}
}

void readsumdiff(int param){//helper function
    Ha=analogRead(Hapin);
    Hb=analogRead(Hbpin);
    if(Ha-Hb-diffave>0){
        diffnormread=1;
    }else{
        diffnormread=-1;
    }
    if(Ha+Hb-sumave>0){
        sumnormread=1;
    }else{
        sumnormread=-1;
    }
}
APPENDIX B : ALGORITHM FOR WINDING OPTIMIZATION

Matlab® version R2006b was used to implement an algorithm to determine the optimum winding size and shape. For the example of cylindrical PM rotor of the kW scale system, the code and description of the optimization is provided below.

The following code calculates the magnetic flux and resistance contribution of each turn of wire in a 2-D on-axis, cross sectional coordinate system \((x, y)\). The magnetic flux contribution for turns located inside the 1.1 inch inner radius (IR) was set to zero to ensure no turns will be placed there during the optimization. Likewise, the magnetic flux contribution for turns located outside 1.5 inch outer radius (OR) was also set to zero. This process is repeated for different OR to determine optimum OR.

```matlab
clear; clc
[x, y] = meshgrid(0.01:.01:1.6, 0.01:.01:1.6);
% [x, y] = meshgrid(1.1, 0.01:0.01:10);
x=x+realmin;
y=y+realmin;
Rin=x; % set dimensions of Rin
Rout=abs(y);

RR=2.*(4)+2.*pi.*Rout; % R(x, y)

% calculate S in and out
for i = 1:length(x(:,1))
    for j = 1:length(x(1,:))
        if (x(i,j)<1)
            Rin(i,j)=sqrt(1-x(i,j)^2);
            Sin(i,j)=3*Rin(i,j)*2;
            Sout(i,j)=3*2*(BoutInt(x(i,j),Rout(i,j))-BoutInt(x(i,j),Rin(i,j)));
        else
            Rin(i,j)=0;
            Sin(i,j)=0;
            Sout(i,j)=3*2*(BoutInt(x(i,j),Rout(i,j))-BoutInt(x(i,j),Rin(i,j)));
        end
    end
end

% set S=0 for inside and beyond 90deg locations
for i = 1:length(x(:,1))
    for j = 1:length(x(1,:))
        if ((x(i,j)^2+y(i,j)^2)<1.1^2)
            Sin(i,j)=0;
            Sout(i,j)=0;
        end
    end
end
```

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if ((x(i,j)^2+y(i,j)^2)>1.5^2)
    Sin(i,j)=0;
    Sout(i,j)=0;
end
end
end

Stot=Sin+Sout;
Stot=(0.64/(39.37^2))*Stot; %***** SET Bres
RR=0.013469*RR;

contour(x,y,Stot)
axis equal

The above code uses helper function, BoutInt(x,y), which calculates the magnetic field of an infinite cylinder according to literature. Due to the fact that the infinite cylinder approximation was only valid for turns near the center of the length of the cylinder, the magnetic flux at each end had been set to zero for 0.5 inches for conservative estimation.

This produces a contour plot of the magnetic flux at each location in cross-section, plotted in Figure 72.
Using the magnetic flux and resistance of each turn calculated in the last program, the following program ranks each location starting according to the best locations to place a turn.

```matlab
clc;

StotDy=Stot;
RRDy=RR;

% find first point
Q=Stot.^2./RR;
[val,i,j]=maxmatrix(Q);
output=[0 0 0 0; i j Stot(i,j) RR(i,j)] ;

% delete first point
StotDy(i,j)=0;
RRDy(i,j)=9999;

% iterate all other points
while (sum(sum(StotDy))>9.0968e-004)
    Ssofar=sum(output(:,3));
```
RRsofar=sum(output(:,4));

Qnext= (Ssofar+StotDy).^2./(RRsofar+RRDy);
[val2,i2,j2]=maxmatrix(Qnext);
output=[output; i2 j2 Stot(i2,j2) RR(i2,j2)];

StotDy(i2,j2)=0;
RRDy(i2,j2)=9999;
end

The program does this by first selecting the single best location by comparing \( \frac{\Phi_{0i}^2}{R_{Ci}} \) values of each location. The coordinates, magnetic flux and resistance were recorded for this location. Then the program sets the flux of that location to zero so it is not selected again. It then selects the next best location by calculating 
\[
\frac{\Phi_0^2}{R_C} = \frac{\left( \sum_i \Phi_{0i} \right)^2}{\sum_i R_{Ci}}
\] 
for each other location. It selects the next location based on the highest value of total winding factor based on locations already selected and the location of the additional turn. The flux of each selected location is set to zero subsequently to ensure it is not selected again. The program repeats until all the locations are selected. The result is the matrix “output”, which ranks locations in decreasing order of preference to achieve a high winding factor.

Using the following program, the total winding factor is plotted in Figure 73, against number of turns to determine where the point of diminished returns lies.

[rows collums]=size(output);
Q1D=zeros(1,rows-1);
pixelnum=zeros(1,rows-1);
for k=2:rows
    S1D(k-1)=sum(output(1:k,3));
    R1D(k-1)=sum(output(1:k,4));
    Q1D(k-1)=(sum(output(1:k,3))^2/sum(output(1:k,4)));
    pixelnum(k-1)=k-1;
end
plot(pixelnum,Q1D, 'k')
xlabel('number of turns')
ylabel('total winding factor')
[maxQ, maxN] = max(Q1D)
maxQ*2

Figure 73: The increase of total winding factor with increasing number of turns

It can be determined here the maximum winding factor achievable at 1.5 inch OR, and the optimum number of turns. The shape of the winding which corresponds to this optimal condition was plotted with the following program in Figure 74.

tempQQ=x-x;
[rows, collums] = size(output);
for k=2:6707  %********pick number of points+1
    tempx=output(k,1);
    tempy=output(k,2);
    tempQQ(tempx,tempy)=1;
end
contour(x, y, tempQQ);
axis([0 1.6 0 1.6 ])
xlabel('inches')
ylabel('inches')
Figure 74: Shape of optimal winding

This cross-sectional plot, shows the approximate $\theta_{gap}$ required to achieve the maximum winding factor for this particular winding factor.

This entire process was repeated to determine the relationship of the maximum winding factor as a function of OD. Using this process, an OR of 1.5 inches was chosen for the kilowatt scale transfer device.

The process is very similar for the spherical PM case, with slightly modified calculations.
APPENDIX C : RESONANT INDUCTIVE COMPARISON

In order to make a meaningful comparison between the PM enhanced wireless transfer system and resonant inductive wireless transfer systems, the systems illustrated in Figure 75 (a) and (b) were evaluated and compared using basic principles.

Figure 75:  Comparison between resonant inductive (b) and PM enhanced (a) transfer systems

The system illustrated in Figure 75 (a) is similar to the experimental model described in Section 5.1. The length of the cylindrical PM and the corresponding windings extend roughly
As discussed in Section 5.1, this device was capable of transferring approximately 1.6kW of power with 15cm separation, operating at 150 Hz, with 81% efficiency. The system illustrated in Figure 75 (b) is identical to that in Figure 75 (a), except the PMs have been removed. The receiver and transmitter windings were connected to mutually resonant circuits shown in Figure 76. The efficiency of this resonant inductive system was evaluated for a wide range of operating frequencies.

**Figure 76:** Receiver and transmitter windings \((L_1 \text{ and } L_2)\) connected to resonant circuits

\[ R_1 = R_2 = \frac{l_c}{ab\sigma}(1 + \frac{a^4 \omega^2 \mu_0^2 \sigma^2}{720}) \]  

\( l_c \) is the total length of the wire in the winding, \( a \) and \( b \) are the height and width of the cross-section of wire, and \( \sigma \) is the electrical conductivity. \( \omega \) is the angular frequency and \( \mu_0 \) is the permittivity of free space. \( l_c \) is approximated to be the circumference of a 10 cm by 10 cm rectangle multiplied by 128 turns, or 51 m. Both \( a \) and \( b \) were approximated to be the diameter of the wire, 3.1 mm. \( \sigma \) is the conductivity of copper is \( 59.6 \times 10^6 \text{ S/m} \).

The mutual inductance, \( M \), was approximated by using its definition\(^56\) (44). 

\[ M = \frac{N_2 \Phi_{21}}{i_1} \]  

\( \Phi_{21} \) is the magnetic flux linking \( L_2 \) and \( L_1 \).
The variable, \( N_2 \), is the number of turns in the receiver winding, \( \Phi_{21} \) is the magnetic flux through the receiver winding induced by current through the transmitter winding, and \( i_1 \) is the current through the transmitter winding. The flux through the receiver winding was approximated by the product of the area of the winding, \( A_2 \), and the field magnitude normal to the area, \( B_2 \), expressed in Equation (45).

\[
\Phi_{21} = B_2 A_2 \tag{45}
\]

\( B_2 \) is the field magnitude inside the receiver winding due to current in the transmitter winding. \( B_2 \) was approximated by making the assumption that the transmitter winding produces an idealized dipole field\(^57\), expressed in Equation (46).

\[
B_2(m_1, r, \lambda) = \frac{\mu_0 m_1}{4\pi r^2} \sqrt{1 + \frac{3}{7} \sin^2(\lambda)} \tag{46}
\]

The variable \( r \) is the distance between the centers of the windings, and \( \lambda \) is illustrated in Figure 6, equal to 90° in this case. The variable \( m_1 \) is the magnetic moment of the transmitter windings, approximated by current carrying wire loops\(^58\) (47).

\[
m_1 = i_1 A_1 N_1 \tag{47}
\]

Combining Equations (44) to (47), an optimistic estimate of the mutual inductance is expressed in Equation (48).

\[
M = N_1 N_2 A_1 A_2 \frac{\mu_0}{4\pi r^3} \frac{2}{r^2} \tag{48}
\]

In this case, \( N_1 \) and \( N_2 \) are both 128 turns, \( A_1 \) and \( A_2 \) are both the area of a 5 cm by 10 cm rectangle, and \( r \) is 15 cm. Using this method, the mutual inductance, \( M \), was calculated to be \( 2.42 \times 10^{-5} \) Henry.

The capacitors \( C_1 \) and \( C_2 \) were assumed to be ideal, with values which resonate with \( L_1 \) and \( L_2 \) at the desired operating frequency, \( \omega \), (49)
\[ C_1 = C_2 = \frac{1}{\omega^2 L_1} \]  

(49)

The voltage source in the circuit, \( V_{\text{in}} \), is assumed to have a voltage of 110 V\text{rms} and oscillating at the resonance frequency, \( \omega \).

An important feature of this doubly resonant circuit is that the series impedance of \( L_1 \) and \( C_1 \) (and equivalently \( L_2 \) and \( C_2 \)) reduce to zero when operating at resonance. This can be seen by Equations (50) to (53)

\[ Z_1 = Z_2 = j \omega L_1 + \frac{1}{j \omega C_1} \]  

(50)

\[ Z_1 = Z_2 = j \left( \omega L_1 - \frac{1}{\omega C_1} \right) \]  

(51)

\[ Z_1 = Z_2 = j \left( \omega L_1 - \frac{\omega^2 L_1}{\omega} \right) \]  

(52)

\[ Z_1 = Z_2 = 0 \]  

(53)

Equation (52) employed the capacitor value expressed in the Equation (49), and \( j \) is the imaginary number, \( \sqrt{-1} \).

Using the definition of mutual inductance\(^5^9\), \( V_2 \), the voltage across the receiver winding induced by the transmitter winding is expressed in Equation (54).

\[ V_{2,\text{rms}} = i_{1,\text{rms}} \omega M \]  

(54)

Since \( L_2 \) and \( C_2 \) combined produce an impedance of zero, \( V_2 \) is divided between \( R_2 \) and \( R_L \).

The power dissipated across \( R_L \), the load, is expressed in Equation (55), and the power dissipated across \( R_2 \), the winding resistance, is expressed in Equation (56).

\[ P_L = i_{1,\text{rms}}^2 \omega^2 M^2 \frac{R_L}{(R_L + R_2)^2} \]  

(55)
\[ P_2 = i_{1, rms}^2 \omega^2 M^2 \frac{R_2}{(R_L + R_2)^2} \]  

The power dissipated across \( R_1 \) is expressed in equation (57).

\[ P_1 = i_{1, rms}^2 R_1 \]  

Since the remaining circuit elements were ideal inductors and capacitors which do not dissipate power, \( P_L, P_1, P_2 \) were the only elements which dissipate power in this simplified model. \( P_L \) is the power delivered to the load, and \( P_1 \) and \( P_2 \) are losses.

The efficiency of the system is expressed in Equation (58).

\[ \eta = \frac{P_L}{P_L + P_1 + P_2} \]  

Substituting \( P_L, P_1, P_2 \) into (58), the expression for efficiency was simplified to expression in Equation (59).

\[ \eta = \frac{\omega^2 M^2 R_L}{\omega^2 M^2 (R_L + R_i) + R_1} \]  

The load resistance, \( R_L \), in this case, was set to a value that produces a \( P_L \) of 1.6kW, in order to operate at a condition comparable to the PM enhanced system. This was done by solving Equation (55) for \( R_L \). The current in the transmitter circuit was determined simply as 
\( i_{1, rms} = \frac{V_{in}}{R_1} \), were \( V_{in} \) has previously been assumed to be 110 V rms. The \( R_L \) solved in this manner, is expressed in (60).

\[ R_L = \frac{\frac{V_{in}^2}{R_i^2} \omega^2 M^2 - 2R_i + \left( 2R_i - \frac{V_{in}^2}{R_i^2} \omega^2 M^2 \right)^2 - 4P_L^2}{2P_L} \]  

For operating conditions which cannot provide \( P_L \) of 1.6kW, Equation (60) produces a complex number for \( R_L \). In such conditions, the efficiency was set to zero in this comparison.
Using Equation (59) with $R_L$ expressed in (60), $R_1$ expressed in (43) and $M$ expressed in (48), the efficiency is plotted as a function of operating angular frequency in Figure 77.

![Figure 77: Efficiency of PM enhanced system, and resonant inductive system, as a function of frequency](image)

It is shown in Figure 77 that the resonant inductive transfer system has a near-zero efficiency operating at low frequencies. The efficiency of the PM enhanced system, operating at 150Hz, was approximately 81%.

The efficiency of this idealized resonant inductive system is higher for high frequencies (approximately 1kHz) mainly due to the fact that $P_L$ is proportional to $\omega^2$. At higher frequencies (approximately 100kHz), winding resistances $R_1$ and $R_2$ are both proportional to $\omega^2$ due to the AC resistance of the windings. This causes the efficiency to decrease. The efficiency of this idealized resonant system is maximum at approximately 10kHz, roughly in agreement with the operating frequency of practical resonant inductive wireless power transfer systems.
The quality factor of the transmitting resonator was calculated by using the definition of an electrical resonator (61).

\[ Q = \frac{\omega L_1}{R_1} \]  

(61)

\( L_1 \) was estimated using an expression for the inductance of a winding with square cross-section60 (62).

\[ L_1 = \frac{\mu_0 N_1^2 \pi a^2}{b} \frac{1}{1 + 0.9(a/b) + 0.32(c/a) + 0.84(c/b)} \]  

(62)

The variables \( a, b \) and \( c \) are various dimensions of the winding, illustrated in page 305 of the reference61.

The resulting quality factor is plotted as a function of operating frequency in Figure 78.

![Figure 78: Quality factor of the resonator](image.png)
It is shown in Figure 78 that the maximum quality factor is approximately 130, at the frequency near 10kHz. The fact that the quality factor and efficiency reaches a maximum at the same frequency range supports the fact that a high quality factor is needed for high efficiency transfer.

Figures 77 and 78 show the inability for resonant inductive system to operate efficiently at low frequencies. This is in stark contrast to PM enhanced systems, which are quite efficient at low frequencies.

This idealized model of the resonant inductive system made a number of optimistic approximations, which in practice would produce a device with lower efficiency. These include the value of the mutual inductance, the use of ideal capacitors and inductors, and the lack of radiation losses. In practice, when operating at higher frequencies, litz wire would be used to lower AC resistance of the winding. This makes it advantageous to operate at even higher frequencies.

The technique of using litz wire and operating at higher frequency is effective until radiation losses become substantial. The radiation losses of the transmitter winding was approximated by the power emitted by an oscillating magnetic dipole62 (63).

\[ P = \frac{\mu_0 m_1^2 \omega^4}{12\pi^3} \]  

(63)

The variable \( m_1 \) is the amplitude of the magnetic moment of the receiving winding. The variable \( c \) is the speed of light.

\( m_1 \) here, was calculated in a similar fashion as in Equation (47). AC resistance of the winding in this approximation, was assumed to be completely eliminated due to the use of litz wire. The power radiated by the transmitting winding is plotted in Figure 79.
Figure 79: Radiated power of the receiver winding operating at 1MHz to 10MHz

It is shown in Figure 79 that operation beyond 1 MHz will produce substantial radiation losses. Unlike some other losses, these losses are unavoidable during high frequency operation.

The radiation loss of the PM enhanced transfer system was also approximated by that of an oscillating dipole in order to make the comparison between the two systems. The dipole moment of the PM is calculated using Equation (2), with the frequency of operation of 150Hz. The radiation loss of the PM enhanced system estimated to be $5.14 \times 10^{-17}$ W using this method. Clearly, the radiation losses of a PM enhanced system would be negligible compared to the radiation losses of an resonant inductive system operating at the required, high frequency.
The total efficiency of the resonant inductive system is further reduced by the losses incurred during the conversion from low frequency (60Hz) to high frequency (10kHz) and back. Advanced DC-chopping, high-frequency power sources have been reported to operate with 90%-97% efficiency\textsuperscript{63}. However, these devices produce square-wave output. The frequency components of a square-wave is distributed across a wide frequency range. Only a small portion of these components can be transmitted to the receiver since the circuits are resonant only in a narrow frequency range. Therefore, the efficiency to produce power at a single frequency is much lower than the efficiency reported. In comparison, the PM enhanced transfer system does not suffer from these losses since it does not need to operate at high frequencies.

Possible health concerns over time-varying magnetic fields emitted by the resonant inductive system were also evaluated and compared against the PM enhanced system. An operating frequency in the upper range of Figure 77 was considered for the resonant inductive system, at approximately 30kHz. The IEEE standard sets an upper limit on the rate of change of the magnetic field, $\frac{\partial B}{\partial t}$, at 1.45 T/s for brain synapse exposure\textsuperscript{64}. The rationale behind this limit is based on the magnetic induction model where currents are induced inside the body. According to this model, the induced current densities are proportional to the rate of change of the magnetic field.

$\frac{\partial B}{\partial t}$ of the resonant inductive system was calculated at locations along the $x$-axis indicated in Figure 80.
Figure 80: \( \frac{\partial B}{\partial t} \) calculated along the x-axis

The magnitude of \( \frac{\partial B}{\partial t} \) was calculated along the x-axis, which is a distance, \( y \), away from the center of the transmitting winding. Since the magnetic field is a sinusoidal time-varying function, the magnitude of \( \frac{\partial B}{\partial t} \) is simply \( \omega B \), where \( B \) is the magnitude of the field. The magnitude of the field was estimated assuming the field produced by the transmitter winding is an ideal dipole-field of the form in equation (46). The frequency of operation of the PM system was 150Hz, and the dipole moment of the PM is calculated using equation (2). The results are plotted in Figure 81.
Figure 81: Calculated $\partial B/\partial t$ at $y = 35$cm for resonant inductive system and PM enhanced system, compared to the IEEE limit

At $y = 35$ cm, the values of $\partial B/\partial t$ produced by the PM enhanced system is below the IEEE limit along the entire $x$-axis, whereas those produced by the resonant inductive system far exceeds the IEEE limit for 60 cm in each direction along the $x$-axis.

This was due to the higher operating frequency of the resonant inductive system. Although the magnetic dipole moment of the PM enhanced system was 10 times higher than that of the resonant inductive system, the frequency at which that oscillates was a 200 times higher for the resonant system than that of the PM system. Therefore, the values of $\partial B/\partial t$ for the resonant inductive system is roughly 20 times higher than those of PM enhanced system, for the same amount of power transferred. This comparison thus shows the advantage of using PM enhanced transfer systems over resonant inductive transfer systems from the perspective of potential human health risks.
APPENDIX D: PHOTOGRAPH OF KILOWATT SCALE DEVICE