Non Tradeable Human Capital and Household Asset Allocation

by

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Abstract

This thesis is comprised of two essays that investigate household consumption and portfolio choices in dynamic life cycle frameworks.

In the first essay, I explain that stock market participation and stockholding are increasing in the level of education and financial wealth without relying on commonly used assumptions about differences in the cost of processing financial information among households. The key aspects of the model are recursive preferences, education attainment and stock market participation. Households with low risk aversion and high elasticity of intertemporal substitution (EIS) are more likely to exercise their education option, accumulate large wealth, invest in stock markets and invest heavily in stocks. These findings are consistent with three separate, but related, strands of the literature on i) household asset holding, ii) utility preferences based on household level financial data, and iii) utility preferences and education attainment. I find that, consistent with these studies, better educated households accumulate more financial wealth, hold a larger fraction of wealth in equity, have a higher EIS and are less risk averse than their less educated counterparts.

In the second essay, I investigate the fact that the fraction of financial wealth invested in equity is increasing in financial wealth in the cross section of households, a known fact that contradicts existing theories in the literature. I show that the contemporaneous positive correlation between human capital and financial wealth values is increasing in the persistence of labor income shocks. While human capital and financial wealth independently have opposing direct effects on equity shares, human capital effects dominate if labor income shocks are highly persistent, generating increasing equity shares in financial wealth. Both a simple model and a realistically calibrated life cycle model of consumption and portfolio choice are shown to generate the results. The predictions are supported empirically using data from Panel Study of Income Dynamics. The essay shows that rising equity shares in financial wealth is a consequence of persistence in labor income shocks and its effects on the joint distribution of human capital, financial wealth accumulation and wealth composition in the cross section, and not a consequence of financial wealth effects alone, as commonly assumed.

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Chapter 1

Introduction

One of the most important financial decisions agents face in their life times is the portfolio choice decision to save for their retirement. Optimal portfolio choice decisions depend on the financial assets that are available for investment, their risk and return profiles, the agent's utility preferences, background risks, and her current and future expectations of labor income. Therefore, an asset that is more important than any other is the investor's ability to earn income, or her human capital.

This thesis comprises two essays that examine issues in household finances. The over-aching, unifying theme of the two chapters is the agent's human capital and how it impacts investment decisions in a way to explain known regularities observed in the cross section of household finances that seem to be related to certain household characteristics such as wealth, labor income risk, utility preference and education attainment, but that have not yet been explained in theoretical models.

In the first essay, differences in stock market participation and equity holdings, as functions of the level of education, are results of the agents' endogenous investment decisions in both financial markets and their own human capital through the attainment of a college education. In conjunction with their consumption, participation and portfolio choice decisions, the agents can change the characteristics of their future labor income growth by obtaining a college education after incurring monetary and utility costs. A distinguishing feature of this essay is that it makes no suggestions about differences in the cost of assimilating market information across households to explain the heterogeneity in household finances.

The second essay investigates the positive correlation between equity shares and financial wealth evidenced in the cross section. The chapter investigates how the persistence in shocks to labor income impacts the joint distributions of equity shares, human capital and financial wealth in the cross section. Similar to the Permanent Income Hypothesis proposed by Friedman (1957), the essay proposes that the choices made by the agent regarding her wealth allocation patterns are determined by the agent's longer-term income expectations. The key conclusion of this theory is that weakly persistent, short-term changes in income have little effect on the agent's investment behavior. Permanent changes in income, however, have a pronounced effect on the current value of human capital, resulting in considerable heterogeneity in household finances, that on the surface, seem to be related to financial wealth.

The remainder of this chapter reviews the background, the motivations and the contributions of this thesis.

1.1 Heterogeneous Preferences, Education, Limited Stock Market Participation and Asset Allocation

Empirical studies of household finances have shown the relevance of education to the decision to participate in stock markets and how much to invest in stocks.[1] A robust finding is that more educated households appear to be better informed and aware of the types of assets and their return/risk characteristics, appear to be better able to take advantage of investment opportunities, and therefore are more likely to diversify their portfolios and participate in stock markets than their less educated counterparts. Also, as shown in the empirical literature, more educated households hold a larger fraction of their financial assets in risky securities, and accumulate more financial wealth than their less educated counterparts.

The first essay of the thesis generates consistent results without relying on heterogeneity in assimilating financial information across households. Instead, it models agents' decision to invest in their own human capital by attaining an education, in conjunction with the decision to pay a fixed participation cost to start investing in stock markets. The model in this paper is adapted from Gomes and Michaelides (2005) and the calibration follows the original source

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[1] Curcuru, Heaton, Lucas, and Moore (2006) is a comprehensive source for information on household portfolio choices as evidenced in household level finance data.

closely. The chapter shows that utility preferences can have an impact on the agent's motive to attain a college education, participate in stock markets and hold stocks consistent with qualitative aspects of the empirical findings. As in Gomes and Michaelides (2005), the results revolve around recursive preferences, and heterogeneity in risk aversion and elasticity of intertemporal substitution (EIS). However, the chapter shows that in a model of education attainment, the preference characteristics of stockholders are quite different from the results suggested by Gomes and Michaelides (2005) if one would like to capture the notion that the largest stockholders are well educated.

Gomes and Michaelides (2005) conclude that the marginal investor inherently has high risk aversion and a high EIS. This thesis, however, shows that utility preference parameters play a significant role in stock market participation among the agents who are liquidity constrained and not likely to play a significant role among college educated households. The conclusion is in stark contrast to Gomes and Michaelides (2005). To be able to qualitatively match education and wealth characteristics of stock market participants and their stock holdings, the largest stockholders must have high EIS and low risk aversion and the lowest stockholders must have a low EIS and high risk aversion, where the former comprises the college educated investors and the latter comprises the high school educated investors. Furthermore, the non-participants are likely to have a low risk aversion and low EIS, be young and more liquidity constrained and forego a college education.

The results attained are consistent with several other disparate facts evidenced empirically as well. They are consistent with the fact that stock holding and stock market participation are increasing in education and wealth, that both equity holding and education are increasing in the elasticity of intertemporal substitution and decreasing in risk aversion, and the anecdotally observed large group of poor households who gamble but do not invest in stocks. This thesis shows that it is possible to reconcile these disparate, but related, findings in a realistically calibrated life cycle model of limited stock market participation, education attainment and asset holdings.

1.2 Persistence in Labor Income Shocks and Rising Equity Shares in Financial Wealth

When comparing portfolio shares across groups of stockholding households, the portfolio share of risky assets are larger for the rich than for the poor, a known fact that potentially contradicts traditional theories on portfolio choice with non-tradeable labor income and constant relative risk aversion. While in principle reverse causality implies that the propensity to invest in risky assets that offer high rates of return causes successful risk takers to become wealthier, detailed studies presented by Luigi Guiso and Jappeli (2002) strongly suggest the converse. An alternative plausible explanation may be related to the notion that households have utility preferences whose relative risk is decreasing in the level of financial wealth. Indeed, the recent work by Wachter and Yogo (2007) has attempted to explain the aforementioned fact with decreasing relative risk aversion utility preferences. Again, while in principle this is plausible, a recent study dispels this possibility. Chiappori and Paiella (2006) show that agents' risk preferences are best described by constant relative risk aversion utility.

The second essay in the thesis shows how agents' investment decisions can diverge from one another if agents have highly persistent shocks to their labor income. The essay does not primarily rely on decreasing relative risk aversion utility or reverse causality to generate rising equity shares in wealth. It shows that rising equity shares in financial wealth is merely an artifact of the heterogeneity in human capital values in the cross section and their effects on equity shares, which is generated by highly persistent labor income shocks. The essay shows that persistence in labor income shocks leads human capital, financial wealth and equity shares to correlate in the cross section. Human capital and financial wealth independently have opposing direct effects on the equity share, a control in the optimization problem, but the effect of human capital dominates if labor income shocks are highly persistent, generating increasing equity shares in financial wealth. These results revolve around the time series property of the permanent component of labor income and how it impacts the contemporaneous joint distributions of human capital and financial wealth and their effects on equity holdings in the cross section. The model builds on existing theories of non-tradeable human capital and portfolio choice to reconcile an empirical fact that has been at odds with existing models of agents with constant relative risk aversion.

An attractive feature of the paper is how the aforementioned fact can be explained by drawing a parallel with the Permanent Income Hypothesis (PIH) of Friedman (1957). Measured income contains a permanent (anticipated and planned) element and a transitory (windfall gain/unexpected) element. Friedman's hypothesis point to the conclusion that individuals consume and save in accordance to their permanent income, not necessarily measured income. In Friedman's permanent income hypothesis model, agents make an estimation of their *anticipated* lifetime income and make appropriate consumption and savings decisions. I show that highly persistent shocks generate a wide cross sectional heterogeneity in anticipated lifetime incomes, resulting in a dispersed distribution of financial wealth and equity shares consistent with the empirical facts. However, weakly persistent shocks are akin to transitory shocks and do not significantly affect anticipations of future labor income. Consequently, the effect of human wealth in the cross section is weak, counterfactually generating equity shares that are decreasing in financial wealth.

Both a simple model and a realistically calibrated life cycle model of consumption and portfolio choice are shown to generate the results. To validate the predictions as a potential explanation, predictions are tested using data on household wages and portfolio choices. The essay shows that rising equity shares in financial wealth can be a consequence of persistence in labor income shocks and its effects on the joint distribution of human capital, financial wealth accumulation and wealth composition in the cross section, and not a consequence of financial wealth effects alone, as commonly assumed.

1.3 Literature on Human Capital and Portfolio Choice

Traditionally, the earlier theories on portfolio choice ignored the agent's ability to generate labor income. One of the most cited research in this area, which was based on the author's doctoral thesis and remains the canonical mathematical formulation of asset allocation problems both in theory and practice, is the single period Efficient Portfolio Frontier/Two Fund Separation Model by Markowitz

(1952). Markowitz showed how investors should choose investments if they only care about the mean and variance of returns. The continuous time stochastic model by Merton (1969) and Merton (1971), which were also based on the author's doctoral thesis, also proved to be seminal in the area of consumption and portfolio choice. It provided the mathematical formulation for other papers in consumption and portfolio choice, and in asset pricing. More recently, Bodie, Merton, and Samuelson (1992) incorporate periodic labor income in a finite horizon dynamic lifecycle setting to capture the effects that human capital has on portfolio choice. Bodie, Merton, and Samuelson (1992) treat human wealth as a non-tradable asset whose dividends are the stream of periodic labor income that the agent receives over her life. Since the investor has an implicit holding of a relatively low risk asset through her human capital, the agent's financial portfolio is tilted towards or away from the risky equity depending on the present value of her human capital.

A number of papers in the literature recently have extended the model of Bodie, Merton, and Samuelson (1992) to capture realism and consistency with empirical facts. Among many others, Cocco, Gomes, and Maenhout (2005) explain the hump shape in equity shares over the life cycle, among a number of other regularities found in empirical studies. Gomes and Michaelides (2005) explain that limited stock market participation and realistic holdings of stock and wealth accumulation can be a consequence of households with heterogeneous preferences. Benzoni, Collin-Dufresne, and Goldstein (2007) examine labor income that is cointegrated with stock dividends to explain limited participation in stock markets and low equity holdings among young adults. Lastly, Lynch and Tan (2006) show that predictability of labor income at business cycle frequency can generate negative hedging demand and realistic levels of stock holdings.

A major factor in these papers is the agent's labor income, or human capital. Likewise, this thesis builds from this area of research with human capital being the major focus or force driving the results. This thesis finds a way in the same literature and takes a step to contribute to our understanding of portfolio choice behaviors. A particular emphasis of this thesis, as opposed to what has traditionally been the focus in this literature, is its focus on explaining some facts observed in the broader cross section of household finances rather than in the aggregate or at particular stages in an agent's lifecycle, a feature of household finances that have been relatively unexplored in theoretical models.

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Chapter 2

Heterogeneous Preferences, Education, Limited Stock Market Participation and Asset Allocation[2]

> "Education is the best provision for old age."[3]
> - Aristotle, 384 BC – 322 BC -

2.1 Introduction

This paper presents a life cycle model of consumption and portfolio choice with stochastic uninsurable labor income that provides an explanation of some empirically observed facts about household finances; limited stock market participation and household investment behaviors related to education and preference parameters. The results of this paper revolve around heterogeneity in utility preferences and the determination of the agents' willingness to obtain a college education and to participate in stock markets, as well consumption and portfolio choices over the life cycle. The key aspects of the model in this paper are recursive preferences, education attainment, and stock market entry in a finite horizon life cycle model of consumption and portfolio choice. The model in this paper is adapted from Gomes and Michaelides (2005) and the calibration follows the original source closely. While the standard in the literature has been to specify background risk from labor income as an exogenous factor, this paper makes the determination of labor income growth an endogenous decision by the

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[2] A version of this chapter will be submitted for publication. Shim, K., Heterogeneous Preferences, Education, Limited Stock Market Participation and Asset Allocation.

[3] From Laertius (1938).

agent. In conjunction to her consumption and portfolio choices, the agent in this paper can change her future labor income growth profile by obtaining a college education after incurring monetary and utility costs. Contrary to most papers in the literature, in this paper differences in stock market participation and equity holdings, as functions of the level of education, are results of the agents' endogenous investment decisions in both financial markets and their own human capital through a college education. The model in this paper can potentially address some of the stylized facts on the heterogeneity of household finances that seem to be related to household characteristics such as preferences, education and wealth.

The results of the paper are as follows. The incentive to attain a college education is increasing in the elasticity of intertemporal substitution (EIS). Individuals with a higher EIS are more willing to accept lower current consumption as a trade off for higher future consumption by means of increasing current savings if return on savings is higher than the agent's subjective discount factor. Akin to savings in the form of investments in financial markets, education in and of itself is an investment in one's human capital and it is more likely the higher the EIS. On the other hand, the motive to attain a college education is decreasing in the coefficient of relative risk aversion. Even though human capital substitutes for a relatively low risk asset, trading one labor income stream for another by incurring significant monetary and opportunity cost is risky, and it is less likely to occur the higher the risk aversion.

Also, conditional on non-participation, the participation in stock markets is increasing in both the EIS and risk aversion. A higher EIS increases the motive to save for retirement while a higher risk aversion increases prudence, resulting in a larger buffer stock of assets and helping push up the wealth level to reach the participation boundary. However, utility preferences only play an important role in participation among high school educated households. The high school educated are more liquidity constrained and the effects of utility preferences are strong. The college educated households, however, easily accumulate the wealth threshold required to participate in stock markets and participation among this group is weakly sensitive to changing values of preference parameters, at least among the set of parameter values used in the calibrations.

Consequently, college educated households are more likely to have a high

EIS and low risk aversion than the average agent in the economy and easily accumulate the wealth threshold required to participate in stock markets. While participation among this group is weakly sensitive to their utility preferences, equity holding as a proportion of financial wealth is not. Therefore, college educated households become stock market participants early in life reaching full participation rates, and invest more aggressively in risky stocks than the average agent in the economy.

On the other hand, high school educated households tend to have a low EIS and a high coefficient of relative risk aversion. Therefore, they have a weak motive to attain a college education and do not save aggressively for retirement. They are also more likely to be liquidity constrained and some, particularly the young adults, do not pay the fixed cost to become a stock market participant. Due to their high risk aversion, these agents have an incentive to accumulate a buffer stock of assets and full participation rate eventually takes place much later in life. Conditioned on stock market participation, high school educated investors are inherently more risk averse and have a lower EIS than their college educated counterparts, accumulating lower wealth and investing less aggressively into stocks during their life time.

Lastly, agents who do not participate in stock markets are the liquidity constrained who forego a college education and accumulate little financial wealth. Non-market participants inherently are young, have very low risk aversion and low EIS. Being liquidity constrained with low incentive to accumulate a buffer stock of assets to hedge against labor income risk and with a low motive to save for retirement, these agents forego college education and do not accumulate enough wealth to reach the stock market participation threshold. This group is akin to the anecdotally observed large group of poor households who do not invest in stocks, but frequently take part in high risk gambles that offer negative expected returns, such as lotteries.

The aforementioned results are consistent with several facts evidenced empirically: i) that stock holding and stock market participation are increasing in education and wealth, ii) that equity holding, and iii) education are both increasing in the elasticity of intertemporal substitution and decreasing in risk aversion, and iv) the anecdotally observed large group of poor households who gamble but do not invest in stocks. I am able to reconcile these disparate, but related, findings in a realistically calibrated life cycle model of limited stock market participation, education attainment and asset holding. This paper makes no suggestions about differences in the cost of assimilating market information across households to explain the heterogeneity in household finances evidenced empirically.

This paper shows that college educated agents are more likely to participate in stock markets, invest more heavily in stocks, and accumulate more wealth than their high school educated counterparts. Curcuru, Heaton, Lucas, and Moore (2006) put these results into context. Using data on US household finances from the Survey of Consumer Finances (SCF) from year 2001, Curcuru, Heaton, Lucas, and Moore (2006) show that stock holding is increasing in household wealth and education. Using probit regression analysis, they also show that stock market participation is increasing in wealth and education. These evidences on market participation and stock holdings lend support to the claim that risky markets provide a habitat for wealthier and more educated households.

Another result of this paper is that college education, and subsequently stock market participation, are consequences of agents with a high EIS and low risk aversion. These agents invest the most in stocks and accumulate the most wealth, consistent with another strand of the literature. Using consumption data of stockholders from 1982 to 1996 available from the Consumer Expenditure Survey (CEX), Vissing-Jorgensen and Attanasio (2003) show that the top one third wealthiest have a significantly higher EIS than the rest of the sample. This result is robust to different instrumental variables and returns on assets used in the estimation of preference parameters. To lend further support to the claim, Vissing-Jorgensen (2002) uses the same data to show that estimates of stockholders' EIS ranges in values, but the top one third highest stock holding households have a significantly higher EIS than the entire sample while the bottom one third lowest stockholding households have a significantly lower EIS. On the evidence about risk aversion and stock holdings, Curcuru, Heaton, Lucas, and Moore (2006) show that risk tolerance plays a significant role as well.

Lastly, the results of this paper are also consistent with the literature on education attainment and utility preferences. Vissing-Jorgensen and Attanasio (2003) provide evidence that risk aversion is lower for the top one third wealthiest stockholders than the rest of the sample. Coupled with the known fact that educated households accumulate the most wealth and therefore are the wealthiest segment of the population, one can conjecture that educated households are less risk averse. Furthermore, using data from the Panel Study of Income Dynamics (PSID), Brown and Taylor (2006) find that the degree of risk aversion is inversely associated with the level of education among households in the U.S. Their finding is not unique to U.S. data. Using Italian panel data, Belzil and Leonardi (2005) find that schooling decreases with risk aversion. Lastly, Cagetti (2001) shows that college educated households are less risk averse and have a higher interest elasticity of wealth. Interest elasticity of wealth is a concept that is closely related to the EIS. Cagetti shows in a calibrated life cycle model that college educated households save a larger fraction of their wealth from the same increase in interest rate than do high school educated households. Since in his model an increase in future consumption can only be financed through an increase in current savings, college educated households are more willing to substitute consumption intertemporally. Therefore Cagetti's result lends support to the claim that college educated households have a higher EIS than high school educated households. Further support is given in Skinner (1985) who shows that the EIS is increasing in education both in OLS and SUR regressions using CEX data.

A number of papers have focused on explaining empirical facts about household finances and investment behaviors. To cite but a few, Cocco, Gomes, and Maenhout (2005) explain the hump shape in equity shares over the life cycle, among a number of other regularities found in empirical studies. Gomes and Michaelides (2005) explain that the lack of stock market participation among the young age groups and participation by their older counterparts with realistic levels of stock holdings can be a consequence of households that have preferences with low risk aversion paired with low elasticity of intertemporal substitution. Benzoni, Collin-Dufresne, and Goldstein (2007) examine labor income that is cointegrated with stock dividends to explain limited participation in stock markets and low equity holdings among young adults. Lynch and Tan (2006) show that predictability of labor income at business cycle frequency can generate negative hedging demand and realistic levels of stock holdings. And lastly, Wachter and Yogo (2007) explain rising equity shares in financial wealth within a model with preferences over consumption of basic and luxury goods.[4] While this paper broadly touches on the same issues as the other papers in the literature, most do not consider education as both an endogenous choice and as a characteristic related to stock market participation and portfolio choices, which is a key feature of this paper.

This paper builds on the model and findings from Gomes and Michaelides (2005). Gomes and Michaelides (2005) show that limited stock market participation can be a consequence of households that have preferences with low risk aversion paired with low elasticity of intertemporal substitution. These agents accumulate a low buffer stock of assets and never participate in stock markets. Therefore, the authors conclude that the marginal investor inherently has high risk aversion and a high EIS. Since this paper build on their model, the inner workings of preferences on the policies from Gomes and Michaelides (2005) also hold in this paper. However, Gomes and Michaelides (2005) do not consider education attainment as an endogenous decision in their model. In this paper I show that utility preference parameters play a significant role in stock market participation among the agents who are liquidity constrained and not likely to play a significant role among college educated households. Therefore, the conclusions of this paper differ from Gomes and Michaelides (2005). To be able to qualitatively match education and wealth characteristics of stock market participants and their stock holdings, the largest stockholders must have high EIS and low risk aversion and the lowest stockholders must have a low EIS and high risk aversion, where the former comprises the college educated investors and the latter comprises the high school educated investors. Moreover, the non-participants are likely to have a low risk aversion and low EIS, be young and more liquidity constrained and forego a college education.

Lastly, a recent paper by Roussanov (2004) also models education attainment within a life cycle model of consumption and portfolio choice with non-tradeable idiosyncratic labor income shocks. However, Roussanov (2004) and this paper have some fundamental differences. The focus of this paper is on the relationship between preference parameters and their effect on education attainment, stock market participation decision and subsequent wealth accumulation

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[4] The third chapter of this thesis also explains rising equity shares in financial wealth as evidenced in the cross section, however the main force driving the results is persistent shocks to labor income.

and financial wealth composition in light of the existing facts on household finances. Roussanov (2004), however, focus his paper around the effects of liquidity constraints on portfolio risk before and after the education choice is made, and does not investigate the effects of utility preferences on education choice. Moreover, Roussanov (2004) does not make the decision to participate in stock markets an endogenous decision, consequently can not potentially investigate the characteristics of the stock market participants related to education and utility preferences.

The remainder of the paper is presented as follows. Section 2 describes the life cycle model. Section 3 describes the dynamic programming problem related to the life cycle model. Section 4 describes the set of parameter values used to solve the model. Section 5 describes the properties of the optimal policies and controls associated with the model. Section 6 discusses simulation results. Section 7 summarizes the prior results and puts them into context in a heterogeneous agent framework in line with some empirical facts. Lastly, Section 8 concludes.

2.2 Life Cycle Model

This section describes the life cycle model. The model follows Gomes and Michaelides (2005) very closely with the augmentation to include an option to attain an education.

Consider an optimizing model of household consumption and portfolio choice. The agent's, or household's, life cycle consists of two phases; adult life employment and retirement. The agent enters adult life at the effective age of t_0 with financial wealth amounting to W_{t_0} and retires at age T with retirement savings in the amount of W_T. Labor income Y_t during her working years $t_0 \leq t \leq T$ depends on the agent's highest education attained. In her first year of adulthood, the agent is high school educated and her labor income in subsequent periods evolves according to the following processes

$$Y_t = P_t U_{Z_t,t} \tag{2.1}$$

$$P_t = \exp(f_{Z_t,t}) P_{t-1} N_{Z_t,t} \tag{2.2}$$

where $f_{Z_t,t}$ is a deterministic function reflecting the agent's income growth at age t characteristic of a typical household with education status Z_t, P_t is the permanent component, $U_{Z_t,t}$ is a transitory component and $N_{Z_t,t}$ is the shock to the permanent component of income. Z_t is a categorical variable that captures the agent's highest education attained. Following standard procedure in the literature, I assume that $\ln U_{Z_t,t} = u_{Z_t,t}$ and $\ln N_{Z_t,t} = n_{Z_t,t}$ are each independent and identically distributed with means $-\frac{1}{2}\sigma^2_{Z_t,u}$ and $-\frac{1}{2}\sigma^2_{Z_t,n}$, variances $\sigma^2_{Z_t,u}$ and $\sigma^2_{Z_t,n}$, and correlations with the stock market returns $\sigma_{\varepsilon^S,Z_t,u}$ and $\sigma_{\varepsilon^S,Z_t,n}$ respectively. All of the variance and covariance parameters are dependent on the education status Z_t.

It can be shown that the growth rate of an individual's labor income evolves as a random process

$$\Delta \ln Y_t = f_{t,Z_t} + \ln(N_{Z_t,t}) + \Delta \ln(U_{Z_t,t})$$

with an unconditional variance of $2\sigma^2_{Z_t,u} + \sigma^2_{Z_t,n}$.

In every period, the agent faces housing expenditure reflective of expenditures on rent for housing or housing consumption. Following Gomes and Michaelides (2005), housing expenditure is an exogenous percentage h_t of labor income and it does not affect consumption utility.[5] Therefore, at any time, the agent's disposal income available for consumption and savings is $(1 - h_t)Y_t$.

At any time during her adult life, the agent has the option to invest in her own human capital by obtaining a college education, at which point the labor income growth profile of the agent switches from one characteristic of a high school educated household to one characteristic of a college educated household. Education is attained in a single period and her education status changes immediately with the exercise of her education option. The cost of attaining a college education is E, a constant irreversible cost akin to tuition and student fees, and textbook costs. In conjunction to monetary costs, the agent suffers substantial financial hardship similar to foregone labor income students suffer during the time they spend attaining an education. To capture the notion of opportunity costs related to foregone labor income, when the option to educate

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[5] Modeling housing consumption significantly complicates an already complicated model. See Yao and Zhang (2005) for an example of life cycle consumption and portfolio choice model with housing consumption.

is exercised the agent suffers a utility penalty, or loss, dependant on a constant O proportional to the permanent component of labor income.

The agent also starts her adult life as a non-participant of stock markets. Therefore, prior to participation, the agent can save by investing only in a risk free security yielding a constant gross risk free rate of R_f. At any time prior to retirement, the agent can choose to pay a fixed cost F proportional to the permanent component of her labor income and become a stock market participant. This entry cost can be viewed as a transaction cost associated with opening a brokerage account.[6] Conditional on stock market participation, the agent can trade two financial assets. She can buy equity with a stochastic gross return of R_t^S, and save in the risk free security. The agent can not borrow or take short positions in equity. The equity premium above the risk free rate is

$$R_{t+1}^S - R^f = \mu^S + \varepsilon_{t+1}^S \tag{2.3}$$

where ε_t^S is a normally distributed shock with mean zero and variance $\sigma_{\varepsilon^S}^2$ and μ^S is the mean equity premium. Since $R^f, \sigma_{\varepsilon^S}^2$ and μ^S are fixed constants, the agent faces a constant investment opportunity set.

Central to the analysis in this paper are the agent's elasticity of intertemporal substitution (EIS), the coefficient of relative risk aversion and their effects on the policies. The conventionally used time separable power utility has the property that a single parameter controls both risk aversion and willingness to substitute consumption intertemporally. In the power utility function, the EIS ψ is equal to the reciprocal of the coefficient of risk aversion (i.e. $\gamma = 1/\psi$). ψ is related to an agent's willingness to move consumption between time periods. Such motive is well defined even in a riskless environment. In contrast, risk aversion is concerned with the agent's willingness to move consumption across states of the world in a risky environment. In a multi period and risky environment, such as the one modeled in this paper, such strong link between the EIS and risk aversion may not be appropriate.[7] Therefore, I consider a recursive utility function similar to Epstein and Zin (1989). The agent's utility over the

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[6] Alan (2006) provides some evidence on the plausibility of the participation cost assumption.

[7] See Bansal and Yaron (2004) for support to loosen the link between risk aversion and intertemporal substitution to obtain an equity premium and a risk free rate that are consistent with what have been observed in the recent past.

consumption good is given by

$$U_t(C_t, \mathrm{E}_t[U_{t+1}]) = \Big[(1-\beta)C_t^{\frac{1-\gamma}{\theta}} + \beta \mathrm{E}_t[U_{t+1}^{1-\gamma}]^{\frac{1}{\theta}}\Big]^{\frac{\theta}{(1-\gamma)}} \tag{2.4}$$

where $\theta = \frac{(1-\gamma)}{(1-\frac{1}{\psi})}$, ψ is the EIS, γ is the coefficient of relative risk aversion, β is the agent's subjective discount factor and C_t is consumption of the good at age t. At time of retirement T and thereafter, the agent's utility is given by $U_{t\geq T} = b\frac{\left(\frac{W_T+Y_T(1-h_t)}{b}\right)^{1-\gamma}}{1-\gamma}$. Each year during retirement the agent derives utility by consuming a fraction $\frac{1}{b}$ of pre-retirement final period cash on hand $W_T + Y_T(1-h_T)$. This is equivalent to modeling post-retirement consumption and investment decisions where the agent receives a constant annual income flow akin to a retirement annuity. The strength of the motive to save for retirement is captured by the parameter b.

2.3 The Dynamic Programming Problem

It is instructive to state the dynamic programming problem related to the life cycle model discussed in the previous section.

The law of motion of the agent's financial wealth W_t evolves according to the following process

$$W_{t+1} = [W_t + Y_t(1-h_t) - I_{Edu,t}E - I_{Part,t}FP_t - C_t] \times [I_{Stock,t}\alpha_t(R^S_{t+1} - R_f) + R_f] \tag{2.5}$$

where $I_{Edu,t}$ is a binary variable equal to one if the agent decides to pay the education cost and attain a college education or equal to zero otherwise, $I_{Part,t}$ is a binary variable equal to one if the agent decides to pay the entry cost and become a stock market participant or equal to zero otherwise, $I_{Stock,t}$ is equal to one if the agent is already a stock market participant, and α_t is the fraction of wealth invested in the stock market. $I_{Stock,t}$ is automatically equal to 1 when $I_{Part,t}$ is equal to 1. The agent receives labor income according to equations (2.1) and (2.2). Following Deaton (1991) in his aggregate savings model, denote cash on hand X_t the amount of liquid resources available for consumption, human capital investment and portfolio savings (ie. $X_t = W_t + Y_t(1-h_t)$). Then the new budget constraint equation can be rewritten as

$$\begin{aligned} X_{t+1} = {} & (X_t - I_{Edu,t}E - I_{Part,t}FP_t)(1-\kappa_t) \\ & \times [I_{Stock,t}\alpha_t(\mu^S + \varepsilon^S_{t+1}) + R_f] + [I_{Col,t+1}\exp(f_{1,t+1})P_tN_{1,t+1}U_{1,t+1} \\ & + (1 - I_{Col,t+1})\exp(f_{2,t+1})P_tN_{2,t+1}U_{2,t+1}](1-h_{t+1}) \end{aligned} \tag{2.6}$$

where Z_t has been substituted with 1 and 2 in $f_{Z_t,t}$, $U_{Z_t,t}$ and $N_{Z_t,t}$ if the agent's education status is college educated or high school educated respectively. $I_{Col,t+1}$ is a binary variable and it is equal to 1 if the agent is already college educated and zero otherwise. $I_{Col,t+1}$ is automatically equal to 1 if $I_{Edu,t}$ is equal to 1. Post housing expenditure cash on hand X_t is optimally allocated between education attainment and stock market participation costs, if applicable, and consumption and savings. κ_t is the fraction of post cost cash on hand that is consumed. The remainder $(1-\kappa_t)$ is saved and invested. A fraction α_t of the agent's savings is optimally allocated in equity, if she is a stock market participant, and the remainder $(1-\alpha_t)$ is invested in the riskless asset. Since the agent can not borrow, lend or short sell stocks, and along with the Inada condition that $C_t^{-\gamma} \to \infty$ as $C_t \to 0$, the controls κ_t and α_t must satisfy the following constraints

$$0 \le \alpha_t \le 1 \qquad \text{and} \qquad 0 < \kappa_t \le 1 \tag{2.7}$$

The agent's optimization problem is to choose optimal consumption control κ_t, portfolio holding control α_t, and education and stock market participation choices $I_{Edu,t}$ and $I_{Part,t}$ by maximizing her expected discounted life time utility contingent on her age, current values of X_t and P_t, and her education and stock market participation statuses $I_{Col,t+1}$ and $I_{Stock,t+1}$.

Following Bellman's Principle of Optimality, at ages $t_0 \le t \le T-1$ the Bellman equation for an agent who is both college educated and a stock market participant is given by

$$V^{E,P}(X_t,P_t) = \underset{\{\kappa_t,\alpha_t\}}{\mathrm{Max}} \left[(1-\beta)\left[X_t\kappa_t\right]^{\frac{(1-\gamma)}{\theta}} + \beta \mathrm{E}_t\left[V^{E,P}(X_{t+1},P_{t+1})^{1-\gamma}\right]^{\frac{1}{\theta}}\right]^{\frac{\theta}{(1-\gamma)}} \tag{2.8}$$

subject to equation (2.6) and constraints (2.7) with $I_{Edu,t}$ and $I_{Part,t}$ equal to zero, and $I_{Col,t+1}$ and $I_{Stock,t}$ equal to one.

The optimal value function for an agent who is already a stock market participant, but not yet college educated is given by

$$V^{NE,P}(X_t, P_t) = \tag{2.9}$$
$$\underset{\{\kappa_t,\alpha_t,I_{Edu,t}\}}{\text{Max}} \left[\begin{array}{c} (1-\beta)\left[(X_t - I_{Edu,t}E)\kappa_t\right]^{\frac{(1-\gamma)}{\theta}} \\ +\beta \mathrm{E}_t \left[\begin{array}{c} I_{Edu,t}V^{E,P}(X_{t+1} - I_{Edu,t}OP_t, P_{t+1})^{1-\gamma} \\ +(1-I_{Edu,t})V^{NE,P}(X_{t+1}, P_{t+1})^{1-\gamma} \end{array} \right]^{\frac{1}{\theta}} \end{array} \right]^{\frac{\theta}{(1-\gamma)}}$$

subject to equation (2.6) and constraints (2.7) with $I_{Part,t} = 0$ and $I_{Stock,t}$ equal to one.

The value function (2.9) above shows that if the agent finds it optimal to exercise her education option, she incurs a utility loss by receiving the next period expected value function corresponding to a lower cash on hand value of $X_{t+1} - OP_t$. In other words, the next period value function is $\mathrm{E}_t[V^{E,P}(X_{t+1} - OP_t, P_{t+1})]$ instead of $\mathrm{E}_t[V^{E,P}(X_{t+1}, P_{t+1})]$.

The optimal value function for an agent who is college educated, but not yet a stock market participant is given by

$$V^{E,NP}(X_t, P_t) = \tag{2.10}$$
$$\underset{\{\kappa_t,\alpha_t,I_{Part,t}\}}{\text{Max}} \left[\begin{array}{c} (1-\beta)\left[(X_t - I_{Part,t}FP_t)\kappa_t\right]^{\frac{(1-\gamma)}{\theta}} \\ +\beta \mathrm{E}_t \left[\begin{array}{c} I_{Part,t}V^{E,P}(X_{t+1}, P_{t+1})^{1-\gamma} \\ +(1-I_{Part,t})V^{E,NP}(X_{t+1}, P_{t+1})^{1-\gamma} \end{array} \right]^{\frac{1}{\theta}} \end{array} \right]^{\frac{\theta}{(1-\gamma)}}$$

subject to equation (2.6) and constraints (2.7) with I_{Edu} equal to zero.

Lastly, if the agent is not yet college educated and not yet a market participant, the Bellman equation is given by

$$V^{NE,NP}(X_t, P_t) = \underset{\{\kappa_t, \alpha_t, I_{Edu,t}, I_{Part,t}\}}{\text{Max}} \tag{2.11}$$

$$\left[\begin{array}{c} (1-\beta)\left[(X_t - I_{Edu,t}E - I_{Part,t}FP_t)\kappa_t\right]^{\frac{(1-\gamma)}{\theta}} \\ +\beta \mathrm{E}_t \left[\begin{array}{c} I_{Edu,t}I_{Part,t}V^{E,P}(X_{t+1} - I_{Edu,t}OP_t, P_{t+1})^{1-\gamma} \\ I_{Edu,t}(1-I_{Part,t})V^{E,NP}(X_{t+1} - I_{Edu,t}OP_t, P_{t+1})^{1-\gamma} \\ +I_{Part,t}(1-I_{Edu,t})V^{NE,P}(X_{t+1}, P_{t+1})^{1-\gamma} \\ +(1-I_{Part,t})(1-I_{Edu,t})V^{NE,NP}(X_{t+1}, P_{t+1})^{1-\gamma} \end{array}\right]^{\frac{1}{\theta}} \end{array}\right]^{\frac{\theta}{(1-\gamma)}}$$

subject to equation (2.6) and constraints (2.7).

The value function at time of retirement T for all two way classifications of education and participation is given by $V(X_T, P_T) = \left(\frac{b^\gamma}{1-\gamma}\right)^{\frac{1}{1-\gamma}} X_T$.

The agent's set of Bellman equations comprising equations (2.8) to (2.11) can be solved given a set of parameter values. The set of parameter values used to solve the model is described in the next section. The dynamic programming problem is solved using backward recursion in five state variables. The state variables are age t, permanent component of labor income P_t, available liquid wealth X_t, education status Z_t and stock market participation status I_{Stock}. A more detailed discussion of the dynamic program and how it is solved numerically is provided in one of the Appendices.

2.4 Calibration

This section describes the set of parameter values used in the life cycle model to attain the results that follow.

I assume that the agent enters adult working life at the age of $t_0 = 19$ and retires at age $T = 65$. I choose preference parameters which I find reasonable and on the same order of magnitude as the parameters found in Vissing-Jorgensen and Attanasio (2003) and Vissing-Jorgensen (2002). Vissing-Jorgensen and Attanasio (2003) find that the coefficient of risk aversion ranges widely depending on certain assumptions and the estimation method employed. They find that γ can be as large as 10, while a γ equal to unity is not too uncommon in calibrations of macroeconomics models. To account for this wide variation of possible values used in the literature, the coefficient of relative risk aversion that I use are 1.5, 3, 5, 7 and 9 where the baseline value is set to 7. I use ψ values of 0.3, 0.5, 0.7, 0.9 and 1.2 where 0.5 is my baseline value. These values are well within the range of estimates from Table 3 of Vissing-Jorgensen (2002). The subjective discount factor β is set at 0.96, which is commonly used in calibrations of macro and life cycle models. As for the strength of retirement savings motive b, I use a value of 10. A b value of 10 is equivalent to a motive to save for retirement where the agent receives a 20 year annuity equal to one half of her pre-retirement consumption.

Labor income growth profiles over the life cycle for typical high school and college educated households are required. To this end, I rely on the parametrization found in Gomes and Michaelides (2005) and Cocco, Gomes, and Maenhout (2005). The deterministic component of labor income growth f_{t,Z_t} in equation (2.2) is extracted from a 3rd order polynomial in age for each college and high school educated households. The standard deviation of the permanent component of labor income and the transitory shock for college educated households are set to be $\sigma_{1,n} = 0.13$ and $\sigma_{1,u} = 0.2417$ respectively. The standard deviations for the high school educated are given by $\sigma_{2,n} = 0.1030$ and $\sigma_{2,u} = 0.2717$. The coefficients of the polynomial in age for the deterministic component of labor income process can be found in Cocco, Gomes, and Maenhout (2005).

Prior empirical evidences using household level data show that there is weak correlation between labor income shocks and stock market returns. Baxter and Jermann (1997), however, state that estimations using longer horizon macro data yield more significant correlations. Also, a recent article by Benzoni, Collin-Dufresne, and Goldstein (2007) suggests that labor income that is cointegrated with stock dividends may be needed to explain stockholding among young adults. Instead of following Baxter and Jermann (1997) and Benzoni, Collin-Dufresne, and Goldstein (2007), I follow what is normal convention in the literature and specify a weaker correlation between labor income and stock market returns. More specifically, I use estimates given in Cocco, Gomes, and Maenhout (2005) and Gomes and Michaelides (2005). The correlation between the permanent component of labor income and stock returns are given by $\sigma_{\varepsilon^S,1,n} = -0.0175$ and $\sigma_{\varepsilon^S,2,n} = 0.0058$ for college and high school educated households respectively. I assume that there is no correlation between the transitory shocks of labor income and the stock market returns.

Housing expenditure shares h_t are from estimates found in Gomes and Michaelides (2005) where they fit a 3rd order polynomial in age using housing expenditure data from the Panel Study of Income Dynamics from 1976 to 1993.

Following the literature on asset pricing, I set $R_f - 1$ to 2%, which is commonly used as the real rate of return on short term government bonds. For the stock return process, I consider a mean equity premium μ^S of 4% and a standard deviation σ_{ε^S} of 18%. I choose μ^S of 4% instead of the historical average of 6%. This seems reasonable considering that investors are commonly subject to brokerage account costs, transaction costs, mutual fund management expenses, taxes on investment income and other costs when investing in financial securities. All of these factors lead to a lower effective return on financial investments.

I set the monetary cost of education E equal to 1.7 (scaled by \$10,000). According to the College Board website[8], the 2008-2009 annual tuition cost at four year private universities average about \$25,143, while at public universities the tuition averages \$4,500 for in state students. Moreover, roughly $\frac{2}{3}$ of the students receive grants. Grants at private institutions average about \$10,200 and \$2,300 at public schools. About 56% of the students attend public four year universities. I assume that 5% of students attend private universities. Based on this information, the expected annual tuition cost for a student is

$$\frac{5\% \times (\$25,143 - \frac{2}{3} \times \$10,200) + 56\% \times (\$4,500 - \frac{2}{3} \times \$2,300)}{5\% + 56\%} = \$4,227.02$$

which corresponds to about \$17,000 for four years.

As for the opportunity cost of education attainment, I set O equal to 2.5. One academic year, excluding the summer session, in a normal four year college degree program is about eight months of full time studies. This corresponds to a total of 32 months or 2.667 years of one's lifetime devoted towards the attainment of a college degree. An opportunity cost of 2.5 times the annual foregone labor income is reasonable provided that some students work during summer months to partially pay for education costs and supplement consumption.

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[8] http://www.collegeboard.com/student/pay/add-it-up/4494.html

Lastly, the fixed cost of stock market entry F is set to 2.5%, which is the value used in Gomes and Michaelides (2005) and close to the estimate provided in Alan (2006). Alan (2006) shows that the cost of market entry is 2.15% of the permanent component of labor income by estimating a structural model and matching important features of household portfolio data from Panel Study of Income Dynamics.

Table 2.1 conveniently summarizes the set of parameter values used in the calibration of the life cycle model. Once the life cycle model is solved and the optimal policies computed, numerical simulations can be carried out in order to assess the quantitative implications of the model. I simulate a cross section of 10,000 households. Each household enters adult life with an initial draw of financial wealth and permanent component of labor income. To make the simulations as realistic as possible, I use distributions estimated from real data. The initial wealth is drawn from a lognormal distribution based on estimates from the Consumer Expenditure Survey (CEX) which can be found in Gourinchas and Parker (2002). The mean of W_1/P_1 is set to 0.3 and its log standard deviation is set to 1.784. As for the labor income, I assume that initial P_{t_0-1} immediately before entering adult life is set to 1 (scaled by \$10,000) with a log standard deviation of 0.562, which are based on estimates from the CEX given in Wachter and Yogo (2007). A sample path of stock returns and labor income based on equations (2.3) and (2.1) are simulated and policies computed according to the solution of the optimization problem. The process is repeated 10,000 to generate a panel of households' consumption, investment, education and participation decisions at each time period. In order to investigate the implications from the preference parameters, I repeat the simulations for various values of γ and ψ once their respective optimization problems are solved. All simulation jobs are run from the same set of random numbers to avoid confounding effects that may arise from the computer system's random number generating algorithm.[9]

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[9] This can be done by using the same seed value in the computer codes to generate the random numbers for all jobs.

2.5 Results: Optimal Controls

This section discusses the optimal policies related to the dynamic programming problem.

2.5.1 Consumption Policy

Panel (a) of Figure 2.1 shows the optimal consumption share κ as a function of cash on hand X_t and permanent component of labor income P_t at age 45. The consumption share is qualitatively similar at different stages of the life cycle and the discussion follows similarly. As the figure shows, the consumption share is *decreasing* along the X_t dimension as cash on hand increases. This is consistent with the agent's motive to *smooth* consumption across states of nature. At very low levels of cash on hand, the agent is liquidity constrained and consumes all of her available liquid wealth. When X_t is very low, κ decreases rapidly, but past a certain level κ flatten along the X_t dimension as cash on hand increases. Thus, at low levels of cash on hand the rate of savings $(1 - \kappa)$ increases at a very fast pace as cash on hand increases. At substantially high levels of cash on hand, the consumption share is more or less insensitive to changes in liquid cash and most of it is saved.

Panel (a) also shows that κ is *increasing* along the P_t dimension. All else equal, a higher value of P_t implies a higher value of income in the future, which consequently, translates to a higher current value of human capital. One way to interpret this result is to perceive human capital as a factor that hinders precautionary saving motives. Precautionary savings leads one to reduce consumption and save more for the future as the agent becomes more risk averse. Since the presence of human capital reduces effective risk aversion, a higher value of P_t, other things equal, effectively lowers risk aversion and leads the agent to consume a larger fraction of available cash and reduce the fraction that is saved. This result is in line with the notion that human capital substitutes for a relatively low risk asset.

2.5.2 Portfolio Choice Policies

Panel (b) of Figure 2.1 shows the optimal equity share α as a function of cash on hand X_t and permanent component of labor income P_t at age 45. The equity share of wealth looks qualitatively similar over the state space at different stages of the life cycle, and for brevity they are not reproduced in the paper. As shown in the figure, α is *decreasing* in X_t. Eventually α asymptotes towards a constant as X_t grows unbounded. These findings are consistent with the standard literature on non-tradeable human capital and portfolio choice.

The figure also shows that α is *increasing* in P_t. It is interesting to note that even though labor income is very risky, as evident from the variances of labor income shocks used in the calibration, the existence of labor income can reduce overall consumption risk significantly. Larger values of P_t, other things equal, indicates that the agent has a larger value of human capital, leading to more aggressive holding of equity as proportion of financial wealth. The agent compensates for her larger value of human capital by holding more risky stocks because human capital reduces effective risk aversion.

2.5.3 Education Attainment

Investment in human capital has the potential to increase the value of human wealth of the agent by means of attaining higher future labor income growth. On the one hand, investment in human capital entails costs such as tuition expenses and foregone labor income. On the other hand, it allows for the possibility of higher labor income realizations once education is attained. From a technical point of view, the problem at hand is well known in the field of finance and economics. Such problems are common in the real investment literature and the solution entails an irreversible investment in a real asset.[10] With respect to the education decision, the agent faces a trade off analysis between costs and benefits and she exercises when it is optimal for her to do so. If the agent exercises the option, she must give up an asset whose payoffs are her existing future stream of labor income characteristic of the income of a high school educated household, pay a fixed cost and incur a utility loss, and in return

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[10] See Dixit and Pindyck (1994) for a review of the literature on investment under uncertainty from the view point of a firm.

she receives another asset whose payoffs are a future stream of labor income characteristic of the income of a college educated household. The finite horizon and the life-cycle aspects of the model imply that there is an optimal time and location along the state space, or a free boundary, where it is optimal for the agent to exercise the option.

Exercise Boundary

Panel (a) of Figure 2.2 shows the education free boundary at age 19 for the baseline set of parameter values. For any given level of P_t, if X_t rises to meet the curve, the education option is exercised. The figure shows that the free boundary is upward-sloping in the (P_t, X_t) diagram. This boundary is a function of both P_t and t and it reflects the minimum amount of liquid wealth required for the agent to have a motive to pursue a college education. The figure shows that the exercise region is to the left and above the boundary. Below or to the right of the curve, the education option is *out of the money* and it is not optimal to exercise it.

To understand the result, consider an agent with low current labor income. Since the opportunity cost of pursuing an education is low, given her level of permanent labor income P_t, she should have a strong incentive to pursue an education. Therefore, the minimum wealth required to attain an education should be low, insofar as the agent's wealth is sufficient to finance the cost of education and consumption during the year education is attained. On the other hand, consider an agent with high current labor income. Since the opportunity cost of a college education is high, given her current level of permanent labor income P_t, she should have a weak incentive to pursue an education. Therefore the minimum wealth required to pursue an education should be high. In light of this argument, Figure 2.2 depicts that a larger wealth requirement is equivalent to a weaker incentive to attain an education.

Education Expiry

Panel (b) of Figure 2.2 shows the education boundary at different stages of the young adult life. For any given level of P_t, the education boundary is higher the higher age is. In other words, there is weaker incentive to pursue a college education the older the agent gets.

To understand the results, it is instructive to look at the labor income profile for typical college and high school educated households. Panel (a) of Figure 2.3 shows the *levels* of labor income with stochastic shocks removed across different ages for both college and high school educated households. The figure shows that college educated households have much higher life time labor income than a typical high school educated household, justifying the value of a college education. However, education should be attained early in life to justify the costs. A younger agent has claims to more labor income than an older agent. Therefore, an agent would have a much stronger incentive to exercise her option to educate early in life, rather than later, to capitalize on higher future labor income growth characteristic of a college educated household.

To further understand the results, Panel (b) of Figure 2.3 shows the *growth rate* of labor income with stochastic shocks removed across different ages for both college and high school educated households. While both college and high school educated households see significant growth rates during young adulthood, the college educated households see a significantly higher growth until around the age of 45 at which point both income growth rates become negative. Around the age of 45 the high school only educated households have a higher, though negative, growth rate than the college educated. Therefore, past a certain age, the differences in labor income growth rates between college and high school educated households are not large enough to justify the significant costs of pursuing an education, and education is never attained past that age.

This result is in stark contrast to some of the existing results in the literature. Judd (2000) argues that there is a human capital underinvestment puzzle because education seems to increase life time utility greatly, and yet, many choose not to pursue it. The results in this paper, however, justify less than full college education rates across the population for those who missed the opportune time to go to college.

Preference Parameters and Education Attainment

The panels in Figure 2.4 show the education boundaries for various values of ψ and γ at age 19. As it can be seen, ψ and γ play important roles on the incentive to attain a college education.

Panel (a) of Figure 2.4 shows the education boundary for varying values of

ψ and baseline value of γ. It shows that the education exercise boundary shifts downwards as ψ increases. In other words, the incentive to exercise the option is strengthened as the EIS increases. The EIS, among other things, measures ones willingness to substitute consumption intertemporally. Individuals with a higher EIS are more willing to accept lower current consumption as a trade off for higher future consumption by means of increasing current savings if returns on savings are higher than the agent's subjective discount factor (i.e. $R > \frac{1}{\beta}$).[11] Akin to savings in the form of investments in financial markets, education in and of itself is an investment in one's human capital. Due to the large rate of return on a college education, particularly during the early stages of adult life, the education boundary is decreasing in ψ.

Panel (b) of Figure 2.4 shows the education boundary for varying values of γ and the baseline value of ψ. The figure shows that the education boundary shifts upwards as γ increases. In other words, the incentive to pursue a college education is weakened as risk aversion increases. It is apparent then that investment in human capital, like investments in financial assets, is dependant on the agent's measure of risk aversion. Even though human capital itself substitutes for a relatively low risk asset, as indicated by κ and α controls in Figure 2.1, trading one labor income stream for another by incurring significant monetary and utility costs is risky. The more risk averse the agent is, the weaker is her motive to attain an education because investing in her human capital is a risky endeavor.

2.5.4 Stock Market Participation

Panels (a) and (b) of Figure 2.5 show the stock market participation boundary for various values of ψ and γ holding other parameters fixed at baseline values. The first figure shows that a higher ψ leads to a stronger motive to become a stock market participant. This is because portfolios that invest in stocks can offer an expected rate of return that is high enough to offset the agent's preference for current consumption against future consumption ($\mu^S + R_f > \frac{1}{\beta}$).

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[11] This is the reason why the EIS is an important factor in the policy literature. For example, the EIS is an important factor in measuring the effects of changes in the interest rate, changes in expansionary fiscal policies, and changes in capital gains taxes, among many others, on household consumption and savings.

Panel (b) shows that, among the parameter values used, the motive to pay and participate in the stock market is strongest for a low γ value of 1.5, but it is a little less conclusive for the other values of γ. Lastly, the motive to become a stock market participant is decreasing in P_t since larger values of the latter entails larger costs of market participation.

2.6 Simulation Results

As discussed earlier, the choice to educate is similar to a real option, as is well known in the literature of investment under uncertainty. The choice to participate in the stock market can also be viewed as an irreversible investment since it entails a trade off between benefits and costs. However, the education and participation options still share one discrepancy with real options. While real options entail valuation and optimal exercise solutions in a risk neutral world, here the problem is preference dependent and the agent is not risk neutral. In conjunction to non risk neutrality, the agent also faces consumption and portfolio choice decisions, which consequently impact wealth accumulation and future utility. Also, the panels in Figures 2.4 and 2.5 provide a static view of the decisions to educate and participate in the stock market for different values of the state variables and preference parameters. They do not give any insights on how wealth accumulation is achieved towards the potential that education and participation will eventually happen. The same can be said for the consumption and investment policies depicted in Figure 2.1. As it turns out, preference heterogeneity has a significant impact on the differences in education, participation in the stock market, and wealth composition and accumulation among households. It is instructive then to turn to simulations to investigate the effects of preference heterogeneity in education and participating rates, along with consumption and portfolio choice decisions over the agent's life time.

2.6.1 Education Rates

Figure 2.6 shows the proportion of college educated households from a simulated cross section of 10,000 households. Panel (a) of the figure shows the education rate for varying values of ψ and the baseline value of γ. The percentage of college educated in the sample is increasing in the elasticity of intertemporal substitution. This result is consistent with the earlier discussion that a higher ψ leads to a stronger motive to attain a college education.

Panel (b) of the figure shows education rates for varying values of γ with ψ fixed at the baseline value. The percentage of college educated in the sample is decreasing in the coefficient of relative risk aversion. There are two opposing forces at play. On the one hand, as shown previously in Figure 2.4, the motive to attain a college education is decreasing in risk aversion. On the other hand, a higher risk aversion strengthens precautionary savings motive and the incentive to accumulate a higher buffer stock of assets. For example, Table 2.2 shows that the median consumption share is decreasing in γ, while Table 2.3 shows that the median wealth accumulation is increasing in γ. For any given value of P_t, a larger γ may 'push' agents to reach the education boundary sooner and lead to a larger proportion of college educated households. Figure 2.6, however, shows that the latter effects are weak and that education rates are decreasing in risk aversion.

Lastly, the proportion of college educated households ceases to increase around the age of 30. A college education is most valuable during early adulthood, consequently, a college education is never attain past a certain age. This result is due to the smaller difference in labor income growth between college and high school educated households after a certain age, as depicted in Figure 2.3

To summarize, both ψ and γ play meaningful roles in determining college education rates in the simulated sample, with γ playing a more dominating role. Education rates are increasing in ψ and decreasing in γ.

2.6.2 Stock Market Participation Rates

Figure 2.7 shows stock market participation rates for both college and high school educated households and various parameter values of ψ and γ.

Panel (a) of the figure shows that among high school educated households, lower values of ψ lead to lower participation rates, particularly among the young households. The participation rate increases in age and eventually reaches full participation between the ages of 40 and 45. However, Panel (b) shows that among college educated households, full participation is achieved very early in life for all parameter values of ψ, with slightly lower participation rates for ψ value of 3 among the very young. College educated households accumulate considerably higher levels of wealth than their high school educated counterparts due to their larger labor income, even during the early stages of adult life when education costs are incurred. Therefore, college educated households easily reach the participation boundary irrespective of the value of ψ. High school educated households however, are more liquidity constrained and the effects of the EIS are stronger. There are two reinforcing factors at play. First, a lower ψ results in a lower incentive to pursue a college education, as depicted in Figure 2.4. Second, a lower ψ results in lower savings for retirement consumption. All else equal, high school educated agents with a lower elasticity have a weaker incentive to save for retirement, accumulate a lower level of wealth, and many do not reach the participation boundary during early stages of their life when savings are likely to be low.

Panel (c) of Figure 2.7 shows that stock market participation rates is increasing in γ among high school educated households, however, panel (d) shows that γ plays a much weaker role among college educated households. A higher γ, other things equal, increases precautionary savings motive and results in a larger buffer stock of assets to insure against labor income risk, pushing wealth levels to reach the participation boundary. However, such effects are weaker for college educated households because they tend to accumulate wealth levels that are sufficiently high reaching the participation boundary, irrespective of their risk aversion. It is interesting to note that a γ of 1.5 results in very limited stock market participation rates early in life for the high school educated households, a result that has been difficult to achieve previously in the literature. At first, this result may seem surprising since one would expect that low risk aversion would lead one to invest in risky stocks. However, the aforementioned result may explain the anecdotal evidence that a large number of poor households who do not invest in stocks, but constantly take part in high risk gambles, such as lotteries.

To summarize, the effects of preference parameters on stock market participation rates seem to be strongest among the households that are liquidity constrained, which happens to occur among the young and high school educated households. Households that are not liquidity constrained, such as the college educated households, accumulate sufficient wealth and start investing in the stock market early in life and preference seems to play a much weaker role, if any. For the liquidity constrained, limited stock market participation is highest among the low γ and low ψ households.

2.7 Heterogeneous Preferences, Education, Limited Stock Market Participation, Wealth and Portfolio Composition

It is instructive to summarize the prior results discussed thus far and put them into context within a heterogeneous agents framework in line with some known empirical facts. This section describes the innate preference characteristics among the college and high school educated, the stock market participants and non-participants, along with their wealth accumulation and portfolio choices implied by a life cycle model with heterogeneous preferences.

College education attainment is more likely among the agents with high elasticity of intertemporal substitution and low risk aversion. Conditioned on the attainment of a college education, these agents easily accumulate the wealth required to participate in stock markets. Participation among this group is weakly related to their utility preference. Therefore, college educated stock market participants inherently have a higher elasticity of intertemporal substitution, and lower risk aversion than the average agent. Consequently they accumulate larger wealth for retirement and invest more aggressively in risky stocks than the average agent in the economy.

The high school educated households with a low elasticity of intertemporal substitution and high risk aversion have a weak motive to attain a college education and do not save aggressively for retirement. They are also more likely to be liquidity constrained and some do not pay the fixed cost to become a stock market participant early in their adult working lives. Due to their high risk aversion, these agents have an incentive to accumulate a large buffer stock of assets and full participation eventually takes place later in life. Therefore, high school educated stock market participants inherently have a lower EIS and a higher risk aversion than the average agent. Consequently, they accumulate lower wealth over their life time and do not invest as aggressively in stocks compared to their college educated participants.

To make the discussion thus far clear, Figure 2.8 shows stock market participation rates, the median fraction of equity holdings as a proportion of financial wealth, and the median wealth accumulation for $\psi = 1.2$, $\gamma = 3$ and college educated group compared to the high school educated group with $\psi = 0.3$ and $\gamma = 7$ in a simulated model of heterogeneous agents. Panel (a) shows that the college educated households exhibit a larger rate of stock market participation than the high school educated group, particularly during the first half of the working adult life. Contingent on participation, the college educated households invest a larger fraction of their wealth in risky stocks than the high school educated households. They also accumulate larger life time wealth and consequently enjoy larger retirement consumption.

Lastly, agents who do not participate in stock markets are the liquidity constrained who forego a college education and accumulate little financial wealth. Non-market participants inherently are young, have low risk aversion and low elasticity of intertemporal substitution. This group is the one depicted in Panels (a) and (c) in Figure 2.7. Being liquidity constrained, with low incentive to accumulate a buffer stock of assets to hedge against labor income risk, and low motive to save for retirement, these agents forego college education and do not accumulate enough wealth to reach the stock market participation threshold. This may explain the anecdotal evidence that a large number of poor households who do not invest in stocks, but constantly take part in high risk gambles, such as lotteries.

2.8 Conclusion

In this paper, I extend the life cycle model of consumption and portfolio choice with limited stock market participation presented in the literature by Gomes and Michaelides (2005). In conjunction to the choice to participate in stock markets along with consumption and portfolio choices, the agent in this model has an option to obtain a college education. I show qualitatively that stock market participation, equity holdings and wealth accumulation related to education levels can be explained from preferences alone. No suggestion in differences in the cost of processing information was made in the paper to explain the heterogeneity in household finances evidenced in reality.

In the model, households with low risk aversion and high elasticity of intertemporal substitution are more likely to "exercise" their "education option", subsequently accumulate more financial wealth and tend to hold a larger fraction of their wealth in stocks. High school stockholders are inherently more risk averse and have a weaker incentive to save for retirement. Lastly, nonparticipants forego college education despite their low risk aversion because they are liquidity constrained and likely have a low elasticity of intertemporal substitution. Qualitatively, these results are consistent with a number of findings from separate strands of the literature: i) the literature on education attainment and utility preferences, ii) the literature on household finances based on household characteristics, and iii) the literature on limited stock market participation.

Table 2.1: Parameters in the calibration of the life cycle model. Parameter values superscripted by * represent baseline parameter values. Select parameter values are taken from Cocco, Gomes and Maenhout (2005). See Gomes and Michaelides (2005) and Cocco, Gomes and Maenhout (2005) for coefficients of the 3rd degree polynomial in age of the labor income processes and housing expenditures.

Variable Name	Symbol	Parameter Value
Preferences:		
Discount Factor	β	0.96
Elasticity of Intertemporal Substitution	ψ	0.3, 0.5*, 0.7, 0.9 and 1.2
Risk Aversion	γ	1.5, 3, 5, 7* and 9
Bequest Motive	b	10
Labor Income:		
Standard Deviation: Permanent Income (College)	$\sigma_{1,n}$	0.1300
Standard Deviation: Temporary Income (College)	$\sigma_{1,u}$	0.2417
Standard Deviation: Permanent Income (High School)	$\sigma_{2,n}$	0.1030
Standard Deviation: Temporary Income (High School)	$\sigma_{2,u}$	0.2717
Asset Market:		
Risk Free Rate	$R_f - 1$	0.02
Equity Premium	μ^S	0.04
Standard Deviation: Equity	σ_{ε^S}	0.18
Correlation between Permanent Income and Stock Return (College)	$\sigma_{\varepsilon^S,1,n}$	-0.0175
Correlation between Permanent Income and Stock Return (High School)	$\sigma_{\varepsilon^S,2,n}$	0.0058
Education:		
Monetary Cost of College Education	E	1.7
Opportunity Cost of College Education	O	2.5
Stock Market Participation:		
Fixed Cost of Market Entry	F	0.025

Table 2.2: Median consumption share κ for different values of γ and ψ. Separate panels for different age and education groups. κ is decreasing in ψ and γ which means that savings is increasing in ψ and γ.

γ	College					High School				
	ψ					ψ				
	0.3	0.5	0.7	0.9	1.2	0.3	0.5	0.7	0.9	1.2
Age Group 19–30										
3	0.27	0.17	0.19	0.22	0.24	0.39	0.31	0.32	0.35	0.38
5	0.20	0.17	0.18	0.18	0.17	0.35	0.33	0.34	0.34	0.32
7	0.14	0.14	0.13	0.13	0.13	0.28	0.28	0.27	0.26	0.25
9	0.11	0.11	0.10	0.10	0.10	0.23	0.22	0.21	0.20	0.19
Age Group 31–40										
3	0.14	0.08	0.09	0.11	0.15	0.20	0.10	0.12	0.15	0.22
5	0.11	0.10	0.10	0.10	0.10	0.16	0.15	0.15	0.15	0.15
7	0.10	0.10	0.09	0.09	0.09	0.14	0.13	0.13	0.12	0.12
9	0.09	0.09	0.09	0.09	0.08	0.12	0.12	0.11	0.11	0.10
Age Group 41–50										
3	0.09	0.05	0.06	0.08	0.11	0.10	0.05	0.06	0.08	0.13
5	0.09	0.08	0.08	0.08	0.08	0.10	0.09	0.09	0.09	0.09
7	0.09	0.09	0.08	0.08	0.07	0.10	0.09	0.09	0.09	0.08
9	0.09	0.09	0.08	0.08	0.07	0.09	0.09	0.09	0.08	0.07
Age Group 51–65										
3	0.06	0.03	0.03	0.05	0.09	0.06	0.03	0.03	0.04	0.09
5	0.07	0.06	0.06	0.06	0.05	0.07	0.06	0.06	0.06	0.05
7	0.08	0.07	0.06	0.06	0.05	0.07	0.07	0.06	0.06	0.05
9	0.08	0.07	0.06	0.06	0.05	0.08	0.07	0.06	0.06	0.05

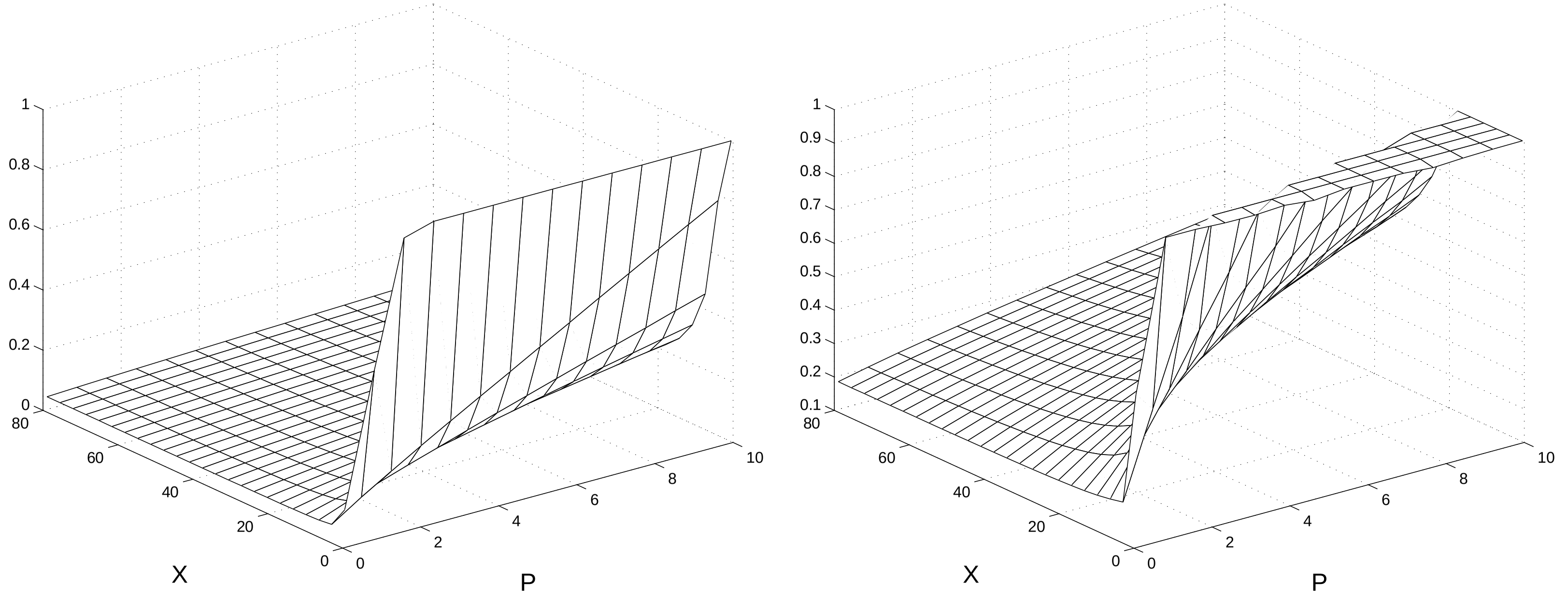
Table 2.3: Median \$ wealth accumulation amount (scaled by \$10,000) for different values of γ and ψ. Separate panels for different age and education groups. Wealth is increasing in ψ and decreasing in γ.

γ	College					High School				
	ψ					ψ				
	0.3	0.5	0.7	0.9	1.2	0.3	0.5	0.7	0.9	1.2
Age Group 19–30										
3	11.88	17.15	14.97	13.77	12.77	3.28	3.79	3.18	2.95	2.78
5	16.47	21.45	18.14	17.95	17.92	3.78	4.26	3.99	3.95	3.88
7	25.39	26.65	27.28	26.74	24.84	5.00	5.22	5.44	5.46	5.37
9	39.26	42.40	41.44	41.44	40.69	6.69	7.16	7.36	7.56	7.81
Age Group 31–40										
3	29.15	51.98	48.12	40.69	32.54	9.90	20.46	17.22	13.70	10.08
5	43.59	51.33	47.64	48.46	49.07	13.20	16.71	16.87	16.85	16.63
7	60.91	58.30	60.00	60.85	60.02	17.67	19.24	19.84	20.12	20.67
9	77.19	76.76	75.79	77.26	78.10	21.23	22.35	23.04	23.62	24.46
Age Group 41–50										
3	82.16	119.19	112.64	94.28	69.35	22.79	51.50	43.50	33.02	20.65
5	92.83	93.96	91.20	92.59	93.51	27.94	35.54	35.76	35.38	34.59
7	104.45	97.02	99.94	102.42	103.95	33.12	36.17	37.21	37.81	39.15
9	113.55	110.78	111.25	114.57	119.59	36.35	38.36	39.85	41.06	43.01
Age Group 51–65										
3	133.87	211.23	198.63	157.26	98.75	42.82	99.50	85.12	63.60	34.44
5	125.95	128.73	128.86	132.42	136.05	44.20	57.60	58.88	58.75	58.33
7	125.83	121.75	128.72	135.74	144.50	47.43	53.47	56.24	58.28	62.30
9	126.10	128.52	134.52	143.58	157.58	49.13	53.41	57.08	60.44	65.79

Table 2.4: Median share of wealth α invested in equity for different values of γ and ψ. Separate panels for different age and education groups.

γ	College					High School				
	ψ					ψ				
	0.3	0.5	0.7	0.9	1.2	0.3	0.5	0.7	0.9	1.2
					Age Group 19–30					
3	1.00	1.00	1.00	1.00	1.00	0	1.00	1.00	1.00	1.00
5	1.00	0.98	1.00	1.00	1.00	0.56	1.00	1.00	1.00	1.00
7	0.58	0.58	0.58	0.59	0.61	0.79	0.97	1.00	1.00	1.00
9	0.36	0.36	0.36	0.36	0.36	0.63	0.70	0.73	0.72	0.71
					Age Group 31–40					
3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
5	0.70	0.67	0.68	0.68	0.68	0.98	0.97	0.97	0.97	0.97
7	0.43	0.43	0.43	0.43	0.43	0.61	0.59	0.58	0.58	0.57
9	0.31	0.31	0.31	0.31	0.31	0.41	0.40	0.40	0.39	0.38
					Age Group 41–50					
3	0.96	0.91	0.92	0.94	0.97	1.00	0.91	0.93	0.99	1.00
5	0.54	0.53	0.53	0.53	0.53	0.59	0.58	0.58	0.58	0.58
7	0.36	0.36	0.36	0.36	0.36	0.40	0.40	0.39	0.39	0.39
9	0.27	0.27	0.27	0.27	0.27	0.29	0.29	0.29	0.29	0.28
					Age Group 51–65					
3	0.62	0.60	0.60	0.59	0.63	0.58	0.56	0.55	0.57	0.64
5	0.37	0.35	0.35	0.35	0.35	0.36	0.35	0.35	0.35	0.35
7	0.26	0.25	0.25	0.25	0.25	0.26	0.25	0.25	0.25	0.25
9	0.20	0.19	0.19	0.19	0.19	0.20	0.20	0.19	0.19	0.19

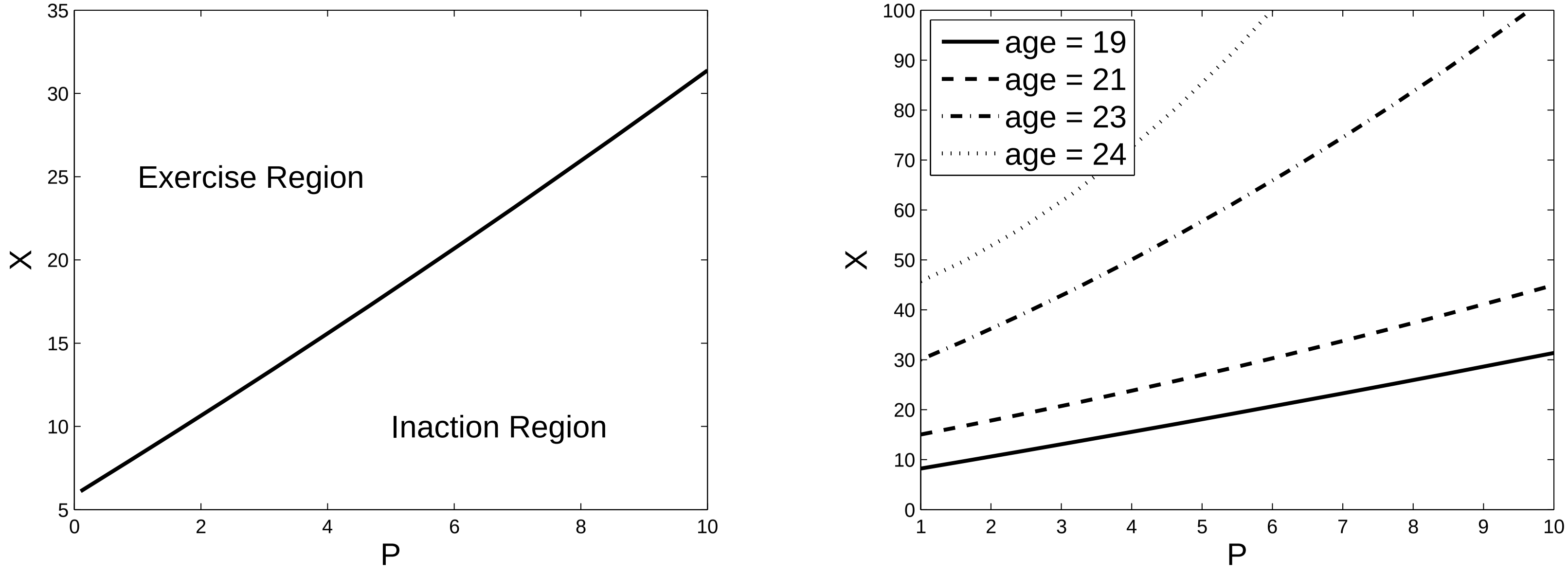
Figure 2.1: Life Cycle Model for baseline set of parameters: Panel (a) optimal consumption policy κ, and Panel (b) optimal investment policy in equity α at age $t = 45$ for college educated and stock market participant. The state variables are cash on hand X_t and permanent component of labor income P_t scaled by \$10,000.



(a) Optimal κ

(b) Optimal α

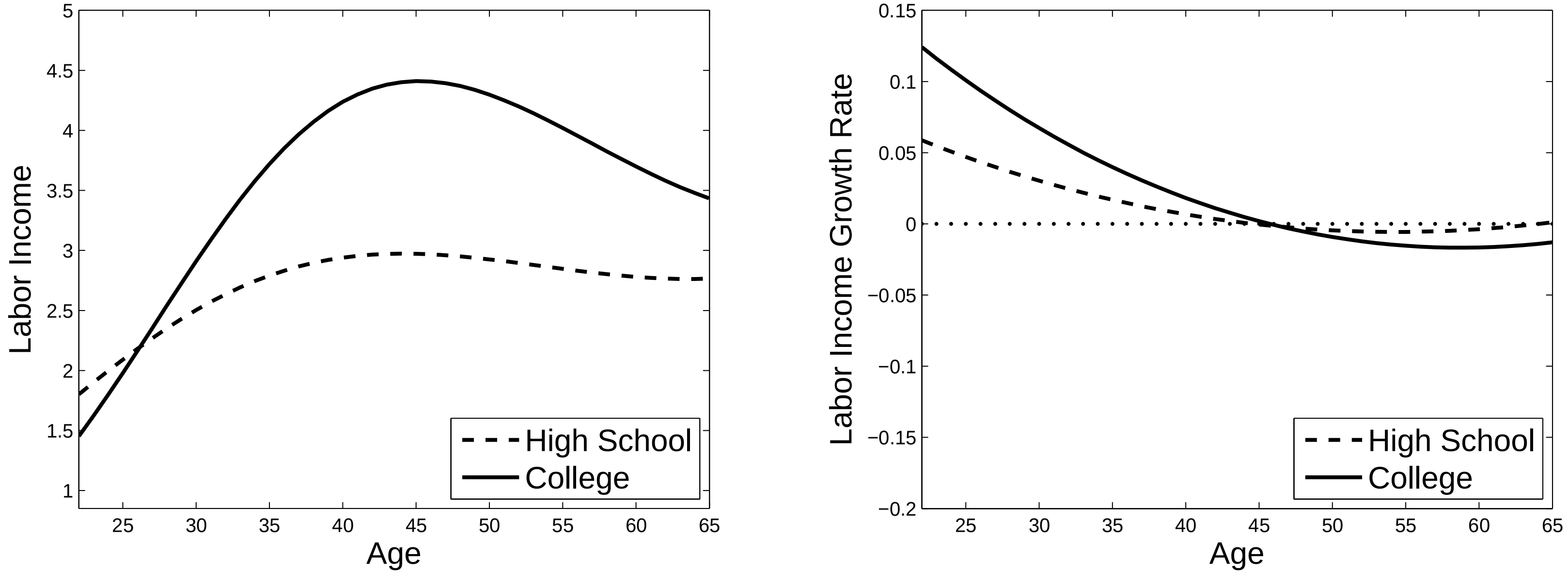
Figure 2.2: Education Attainment Free Boundary Along Values Of P And X. Panel (a) at age 19, and Panel (b) at different ages for $\gamma = 7$ and $\psi = 0.5$. Values on the axes are scaled by \$10,000. The incentive to educate is strongest when P is low. The education option loses value in age as it is most beneficial to educate as early as possible.



(a) Education Attainment Free Boundary at Age 19 and $\gamma = 7$ and $\psi = 0.5$

(b) Education Attainment Free Boundary at various ages and $\gamma = 7$ and $\psi = 0.5$

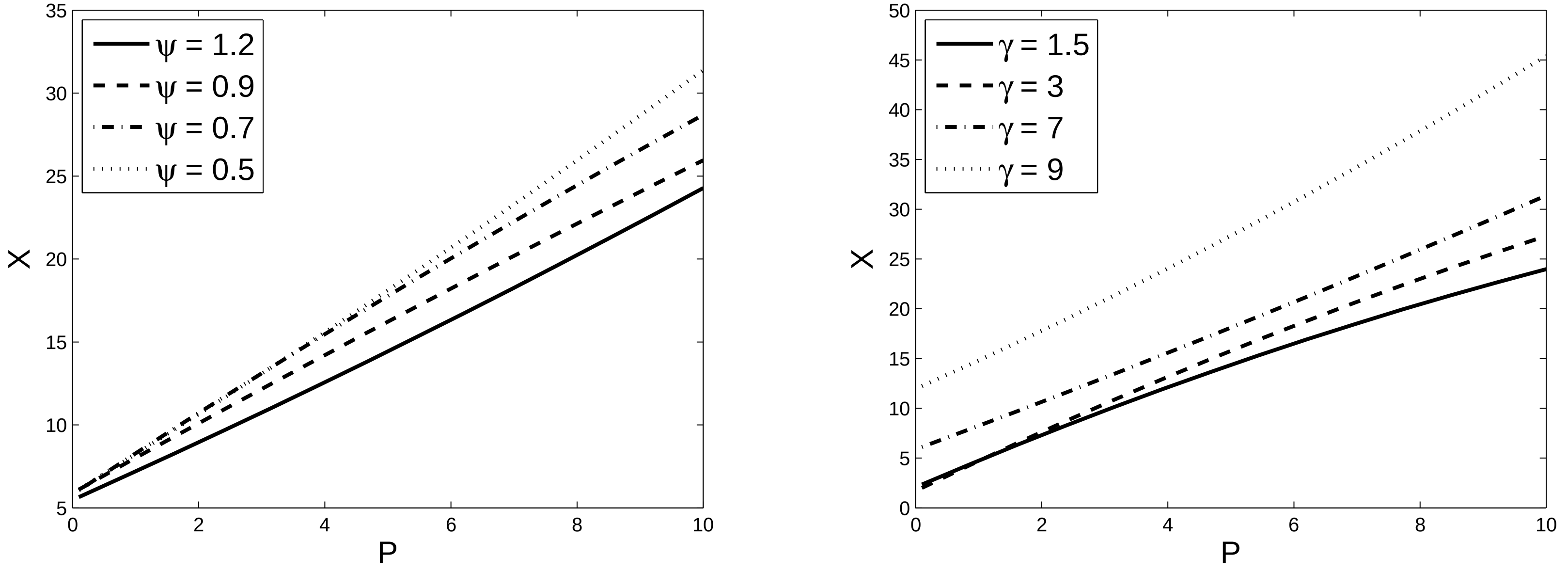
Figure 2.3: Third order polynomial fit in age. Panel (a) labor income, and Panel (b) labor income growth rate with stochastic shocks removed. Life cycle profiles given for both College and High School educated households. Values are scaled by $10,000.



(a) Deterministic Labor Income

(b) Growth Rate of Deterministic Labor Income

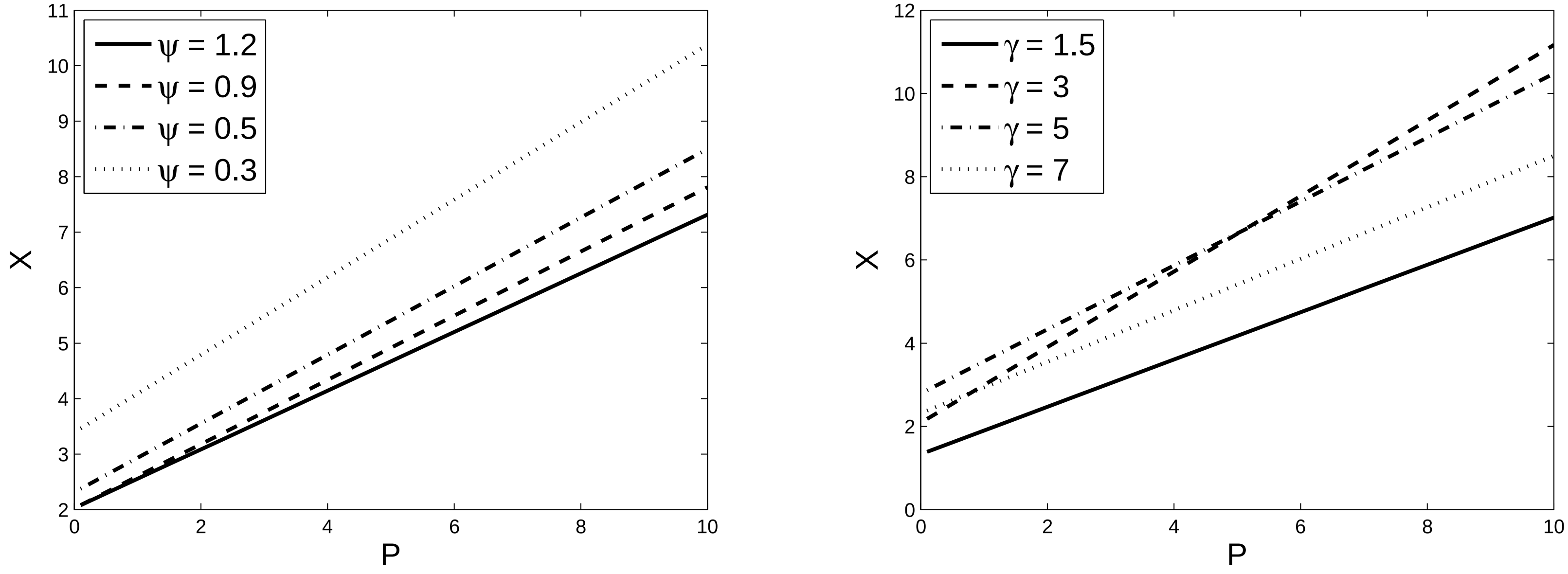
Figure 2.4: Education attainment free boundary along values of P and X. Panel (a) for various values of ψ and $\gamma = 7$, and Panel (b) for various values of γ and $\psi = 0.5$. Values on the axes are scaled by \$10,000. The motive to attain a college education is stronger for higher value of ψ and lower values of γ.



(a) Education Attainment Free Boundary at Age 19 for various values of ψ

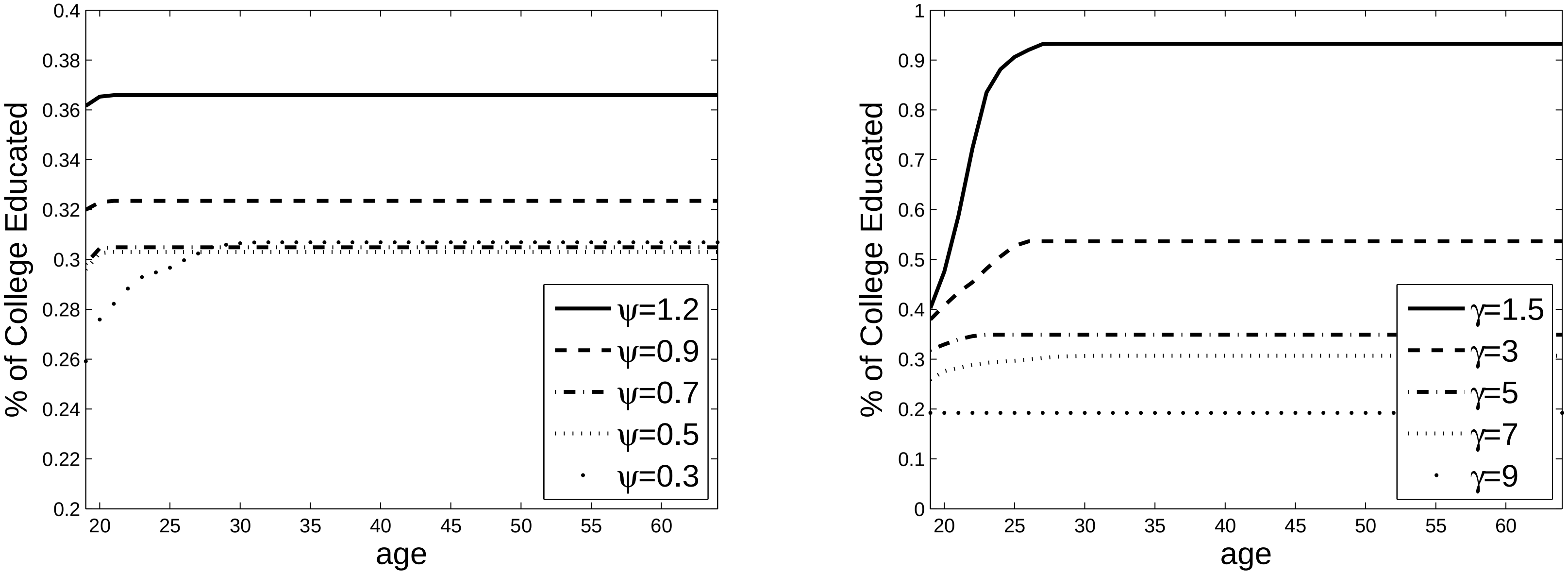
(b) Education Attainment Free Boundary at Age 19 for various values of γ

Figure 2.5: Stock market participation free boundary along values of P and X. Panel (a) for various values of ψ and $\gamma = 7$, and Panel (b) for various values of γ and $\psi = 0.5$. Values on the axes are scaled by \$10,000. The motive to participate is stronger for higher values of ψ. The motive to participate is strongest for the lowest value of γ.


(a) Participation Free Boundary at Age 19 for various values of ψ

(b) Participation Free Boundary at Age 19 for various values of γ

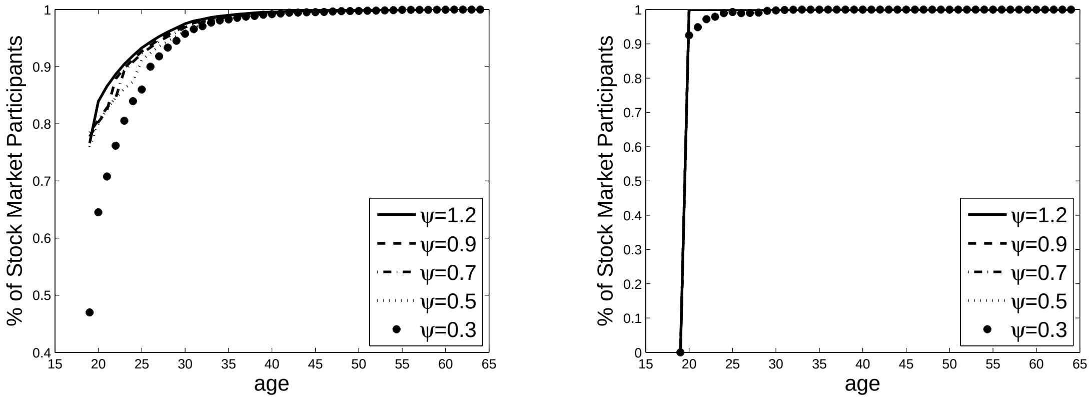
Figure 2.6: Simulations: Percentage of college educated households from a simulated cross section of 10,000. Panel (a) for various values of ψ and $\gamma = 7$, and Panel (b) for various values of γ and $\psi = 0.5$. Education rates are increasing in ψ and decreasing in γ.



(a) Percentage of College Educated
for various values of ψ

(b) Percentage of College Educated
for various values of γ

Figure 2.7: Simulations: Percentage of Stock Market Participants from a simulated cross section of 10,000. For various values of ψ Panel (a) among high school educated, Panel (b) among college educated, and various values of γ Panel (c) among high school educated, and Panel (d) among college educated.



(a) For various values of ψ among High School educated

(b) For various values of ψ among College School educated

(c) For various values of γ among High School educated

(d) For various values of γ among College School educated

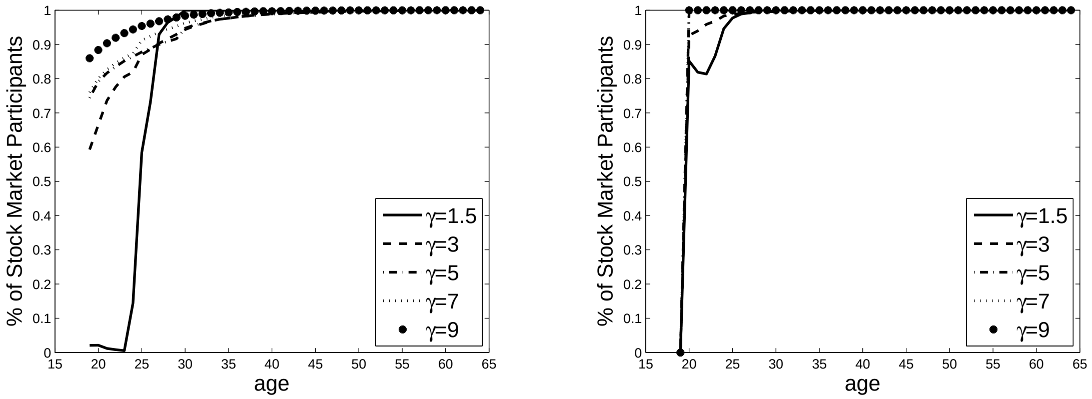
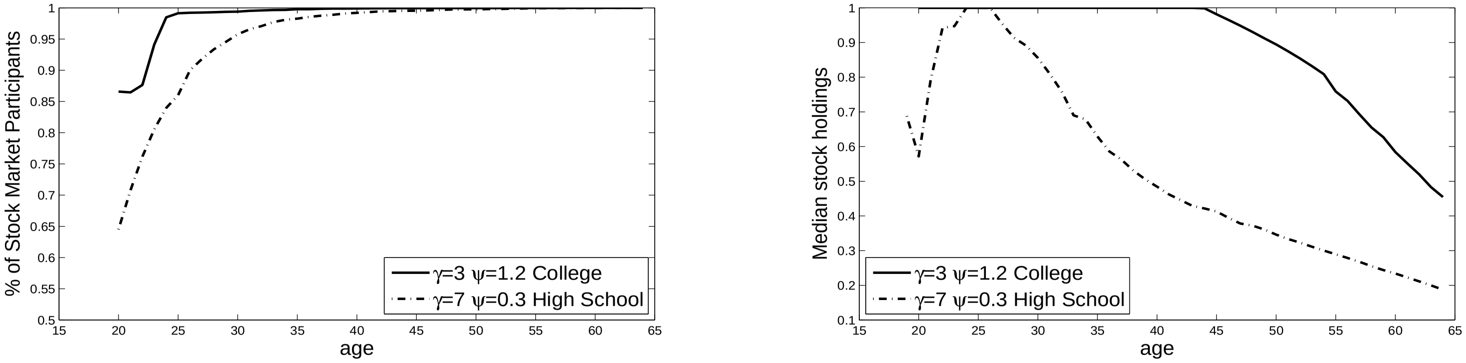


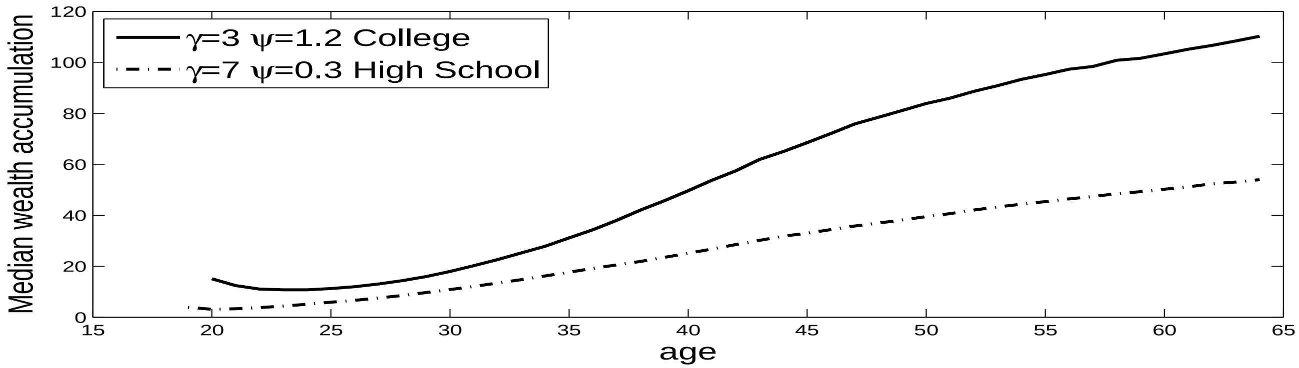
Figure 2.8: Simulations: Panel (a) stock market participation rates, Panel (b) median fraction of equity holdings as a proportion of financial wealth and (c) median wealth accumulation for high ψ, low γ and College educated subgroup vs. low ψ, high γ and high school educated subgroup in a model of heterogeneous agents.



(a) Stock Market Participation Rates

(b) Median α

(c) Median Wealth Accumulation



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Chapter 3

Household Portfolio Choice When Labor Income Shocks are Persistent and the Overallocation Problem[12]

> "The distribution of wealth - by which is meant here only nonhuman wealth - is typically very much more widely dispersed than the distribution of measured income ... If planned savings - by which we mean again only savings embodied in nonhuman wealth - are the same fraction of permanent income at all levels of permanent income, does this not imply a tendency for the distribution of wealth to become similar to the distribution of income?"[13]
>
> - Milton Friedman, 1912 – 2006 -

3.1 Introduction

U.S. household finance surveys show that in the cross section the proportion of financial wealth invested in equity (or the equity share) is increasing in financial wealth (hereafter, equity overallocation). This empirical fact is present within segments of different age groups, education levels, and even in different countries.[14] Traditional theories of consumption and portfolio choice with Constant Relative Risk Aversion and risky labor income, however, predict that the equity share observed in the cross section should be decreasing in the amount of financial wealth if the cross sectional distributions of financial wealth and hu-

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[12] A version of this chapter will be submitted for publication. Shim, K., Household Portfolio Choice When Labor Income Shocks are Persistent and the Overallocation Problem.

[13] From Friedman (1957).

[14] International evidence can be found in Luigi Guiso and Jappeli (2002).

man capital are independent. Bodie, Merton, and Samuelson (1992) show that in a model with labor income an investor's *total wealth* is composed of both *financial wealth* and *human capital.* Because human capital substitutes for a low risk bond, an agent with low financial wealth will hold a large fraction of it in stocks since her human capital composes a large portion of her total wealth. Increasing values of financial wealth implies that the agent's implicit holding of a low risk asset through her human capital makes up a decreasing portion of the agent's total wealth, giving rise to more conservative investing; hence the prediction that the equity share should be decreasing in financial wealth.

In this paper, I reconcile theory and empirical facts. This paper shows that a life cycle model of consumption and portfolio choice with highly persistent labor income shocks can explain the relationship between portfolio holdings and financial wealth observed empirically. I show that contemporaneously human capital and financial wealth become highly correlated if labor income shocks are highly persistent. While human capital and financial wealth independently have opposing effects on equity shares, if labor income shocks are highly persistent, the positive effects of human capital dominate in the cross section, resulting in increasing equity shares in financial wealth. I generate these results within the context of both a simple model and a realistically calibrated life cycle model of consumption and portfolio choice with non-tradeable human capital. And then, I test the predictions of the theoretical models using U.S household data on income and finances. The predictions are supported by the empirical results.

To understand the results of this paper, consider an *agent with highly persistent labor income shocks* in comparison with the *average agent* in the population. Since the *permanent* component of labor income is highly serially correlated for this agent, a *positive* shock today leads to higher expected labor income in the future as well. This leads to a substantial increase in the value of her human capital after an unexpectedly positive draw of labor income. Since labor income provides extra resources from which the agent derives utility beyond those from financial wealth, everything else equal, over time this agent will accumulate higher levels of financial wealth through savings of her earnings and have a higher value of human capital compared to the average agent. On the other hand, an agent who experiences a negative persistent shock to her labor income, everything else equal, will have a lower value of human capital and will accumulate a lower level of financial wealth than the average agent. Therefore, persistence in labor income shocks generates a strong contemporaneous positive correlation between human capital values and financial wealth in the cross section. Although human capital and financial wealth independently have opposing effects on equity shares (as predicted by the standard theories), if labor income shocks are highly persistent, the positive effects of human capital are stronger than financial wealth's in the cross section. Coupled with the result that financial wealth and human wealth are positively correlated in the cross section, equity overallocation is attained.

However, an agent with labor income shocks that are *weakly persistent* does not enjoy the same degree of future wealth accumulation from a current increase in labor income. A positive income shock realization today is not indicative of higher expected labor income in the future. The value of human capital is little changed after a shock to labor income and the agent's accumulation of wealth overtime becomes more reliant on returns from her financial portfolio, instead of increasing savings resulting from changes in her income earnings. Therefore, there is a weaker contemporaneous link between wealth accumulation and human capital in the cross section if labor income shocks are weakly persistent. Moreover, since the value of human capital is little changed from changes in labor income, human capital becomes a weaker factor when it comes to determining equity shares in the cross section. Consequently, financial wealth becomes the dominating factor, reestablishing the standard result from traditional theories that the equity share should be decreasing in financial wealth.

The main theoretical result of this paper is that the contemporaneous positive correlation between human capital, financial wealth and equity shares is increasing in the persistence in labor income shocks. Using U.S. household data on income and wealth composition from the Panel Study of Income Dynamics (PSID), I find that this prediction is supported empirically as well. After estimating the persistence parameter of labor income for each individual household in the data, I estimate a cross sectional model for equity shares with three-way interactions between wealth, human capital and the persistence parameter as predictors. I argue that incorporating three-way interactions is an appropriate way to account for the fact that human capital and financial wealth are not independently distributed in the cross section, a feature that is too commonly ignored in standard regression models, and that their joint distribution is crucially dependent on the persistence parameter. Unconditionally, equity shares are positively correlated with human capital and financial wealth. However, by estimating the three-way interaction model, I find that higher persistence strengthens human capital's positive effects on equity allocation in the cross section while financial wealth effects take a secondary role. I also find that equity overallocation is a consequence of persistence in income shocks and its effects on the joint distribution of human capital, financial wealth accumulation and wealth composition in the cross section, and not a consequence of financial wealth effects alone. From the estimations, I conclude that conditional on the existence of equity shares that are rising in financial wealth, overallocation is generated by households with highly persistent labor income shocks. Households with weaker persistence compose the middle of the distribution.

Understanding the nature of income risk has been a key factor in understanding a wide range of economic questions; such as the welfare cost of business cycles, as shown by Lucas (2003), the determination of asset prices, as shown by Constantinides and Duffie (1996), the significant rise in within-cohort consumption inequality, as shown by Deaton and Paxson (1994) and Storesletten, Telmer, and Yaron (2004), and consumption growth that parallels growth in income, as shown by Carroll (1992), among many others. While the economic literature has investigated the nature of labor income to study numerous problems in economics, this paper examines the role that the persistence of labor income shocks plays in the determination of the joint distribution between human capital, wealth accumulation and asset allocation at the micro household level, a subject that has been relatively unexplored.

Guvenen (2006) summarizes the current state of the empirical literature regarding estimates of the persistence parameter. The first view in the literature supports the notion that households have labor incomes that are subject to large and persistent shocks while facing similar life cycle profiles. A second alternative view supports the notion that households have labor incomes that are subject to mildly persistent shocks while facing large household specific life cycle profiles that vary widely in the cross section. In my empirical model, I accommodate both views by estimating a flexible model of labor income that depends on both, i) a household specific factor, by accounting for fixed effects, and ii) general life cycle factors such as age and education level, while at the same time, iii) allowing for cross sectional heterogeneity in the persistence of labor income shocks.

I place my paper in an active literature that studies consumption and portfolio choice in partial equilibrium and realistically calibrated life cycle models. More specifically, a number of papers have focused on explaining empirical facts that seem at odds with traditional theories or conventional wisdom on how to optimally invest financial wealth. To cite but a few, Cocco, Gomes, and Maenhout (2005) explain the hump shape in equity shares over the life cycle, among a number of other regularities found in empirical studies. Gomes and Michaelides (2005) explain that the lack of stock market participation among the young age groups and participation by their older counterparts can be a consequence of households that have preferences with low risk aversion paired with low elasticity of intertemporal substitution. Benzoni, Collin-Dufresne, and Goldstein (2007) examine labor income that is cointegrated with stock dividends to explain limited participation in stock markets and low equity holdings among young adults. Lynch and Tan (2006) show that predictability of labor income at business cycle frequency can generate negative hedging demand and realistic levels of stock holdings. And lastly, Wachter and Yogo (2007) explain rising equity shares in financial wealth within a model with preferences over consumption of basic and luxury goods. While most papers in this literature have provided theoretical predictions that seem consistent with empirical observations, most do not test their predictions using actual data. This paper, on the other hand, while providing a theoretical justification for the observed fact that is at odds with the current state of the literature, also tests the prediction of the proposed theory using actual real household level data. As it will be shown, the results from the empirical model supports the prediction of the theoretical models as well.

In conjunction with making a contribution to the broader literature on household consumption and portfolio choice, this paper also makes a contribution to the literature on household risk preferences. The literature on household risk preferences, as well as the broader literature on financial economics and asset pricing, have relied on assumptions that agents have decreasing relative risk aversion (DRRA) risk preferences perhaps mostly motivated by household level data.[15], [16] This paper suggests that a closer investigation of the persistence in labor income shocks, and its impact on the joint cross sectional distributions of human capital, wealth accumulation and wealth composition are imperative prior to making *a priori* assumption that households have DRRA risk preferences simply based on the fact that equity shares are rising in financial wealth in the cross section of households. In this paper, I am able to replicate investment behavior in the cross section that resembles households with DRRA preferences, however, I make no reliance on such preferences to generate these results; the agents in the theoretical model have *constant relative risk aversion* (CRRA). More recently, Chiappori and Paiella (2006) find compelling empirical support for CRRA risk preferences even though equity shares are rising in financial wealth in the cross section of households. This latter research supports and motivates the current paper.

The current paper is related to Wachter and Yogo (2007) since they also focus on explaining the equity overallocation phenomenon. However, this paper differs from theirs along a fundamental dimension. The results of their paper revolve around a *non-homothetic decreasing relative risk aversion* utility function motivated by consumption of luxury goods. They generate equity overallocation by explicitly modeling risk aversion that decreases in financial wealth (or in the consumption of luxury goods). An agent that is poor is very risk averse and invests very little in stocks while the consumption of basic goods composes a large fraction of her total consumption. In contrast, wealthy agent places a higher consumption priority in luxury goods. Since the agent is less risk averse over consumption of luxury goods, she invests more heavily in equity; hence attaining the result that equity share is increasing in financial wealth in the cross section. The results of my paper, however, revolve around the time series property of the permanent component of labor income and how it impacts the contemporaneous joint distributions of human capital and financial wealth

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[15] Rising equity shares in wealth is supportive of households having DRRA risk preferences. Support for DRRA risk preferences as evidenced in household level data have been around in the finance literature at least since the '70s. See Friend and Blume (1975) and Graves (1979).

[16] See Rubinstein (1976a) and Rubinstein (1976b) for examples with subsistence level of consumption, Sundaresan (1989) and Constantinides (1990) for examples with internal habit formation and Campbell and Cochrane (1999) for an example of external habit formation in asset pricing. All of these utility specifications generate portfolio patterns usually associated with DRRA. See Campbell and Viceira (2003) for an explanation.

and their effects on equity holdings in the cross section. Therefore, my model builds on existing theories of non-tradeable human capital and portfolio choice to reconcile an empirical fact that has been at odds with traditional models of agents with *homothetic* preferences. I make no reliance on preferences with DRRA to generate my results. I explain an investment behavior evidenced in reality that seems consistent, on the surface, with the notion that agents have risk preferences characteristic of DRRA, however my agents have CRRA utility, consistent with empirical findings as shown in Chiappori and Paiella (2006).

The remainder of the paper is presented as follows. Section 2 documents some relevant information about household asset holdings as evidenced in the Panel Study of Income Dynamics 1999 Wealth Supplemental Data File, and why the evidence may pose some challenges to traditional theories in portfolio choice. Section 3 develops a simplified model as a prelude to the more general life cycle model developed in Section 4. The simplified model will highlight the core factors driving the results in a simple and easy to understand manner. The more general life cycle model, additionally, will investigate other factors that may impact overallocation. Section 5 describes the chosen set of parameters used to solve the life cycle model and the simulations. Section 6 discusses the optimal policies followed by simulation results. Section 7 describes the data, estimation methods, and discusses the empirical results. Lastly, Section 8 concludes.

3.2 Equity Overallocation

This section documents some interesting facts about U.S. household portfolio choices. More specifically, it documents that the proportion of financial wealth invested in equity is increasing in financial wealth in the cross section of U.S. households. I will call this observed fact Equity *Overallocation.* This section also discusses the dominating standard theory on portfolio choice with non tradeable human capital and motivates the use of the term *overallocation* to describe the aforementioned facts on asset holdings.

3.2.1 Household Investment Behavior

Data on wealth composition are from U.S. households available from the Supplemental Wealth Data File for the year 1999 from the Panel Study of Income Dynamics (PSID). The empirical part of the current paper contains a more detailed description of the data.

Financial wealth W is defined to be the sum of all stated cash holdings, bonds and stocks either directly or indirectly through IRAs and pension, net of all amounts owed on those accounts. All cash and bond holdings, were treated as cash equivalents. The share of financial wealth invested in equity α, is defined to be the sum of direct and indirect holdings of stocks, net of all amounts owed on those accounts, divided by financial wealth, W. Further details of how the relevant variables were created are given in the empirical part of the paper. Lastly, following Wachter and Yogo (2007), I separate households into different age and education groups based on the age of the head of the household and his or her highest education attaintment. Since the data is from the same calendar year, separating the sample by age groups not only controls for life cycle effects, but it also controls for cohort effects. Within each age and education group, I sort households into quartiles based on their financial wealth and compute the median fraction of wealth invested in stocks for each quartile subsample.

Table 3.1 documents households' fraction of financial wealth invested in equity by age, education and wealth groups. The table shows an interesting fact about household asset holding. Conditional on stock market participation, α is increasing in total portfolio value. In other words, wealthier investors invest a larger fraction of their wealth in risky stocks. This is the case for both college and high school educated households, as evidenced in panels (a) and (b) of the table, but it seems more pronounced for the college group. Moreover, there seems to be a life cycle effect where overallocation seems strongest for the middle age groups. Overallocation seems mild for young adults (26-35 age group) in the college group, but negative for the same age group within the high school group. Overallocation strengthens for households in the high income growth group (36–45 age group) and top income earning group (46–55 age group), before declining again in the pre-retirement (56–65 age group) and post retirement (66–75 age group) groups. The high school group seems to exhibit a reversal of pattern in the post retirement age group with a higher overallocation compared to the other age groups. Idiosyncracies asides, one thing is clear; that generally α is increasing in W and this relationship differs among different age and education groups.

To lend further support to the claim, panels (c) and (d) in Table 3.1 also show select figures taken from Wachter and Yogo (2007), which are based on 2001 Survey of Consumer Finances (SCF). Wachter and Yogo's definitions of wealth and asset holdings are broader and more in line with the definition commonly used to define household net worth, which include illiquid assets such as businesses, public equity, real estate, business equity, risky bonds and homes. Regardless of how financial variables are defined, the table shows that the evidence from the SCF data is consistent with observations from PSID data, which indicates that equity overallocation is robust to different definitions of wealth and equity holdings.

3.2.2 Equity Overallocation and Portfolio Choice Theory

It is interesting to highlight why equity overallocation may pose some challenges to traditional theories in portfolio choice.

I start with a portfolio rule that is well know in theory; portfolio choice in complete markets faced by a power utility investor with a constant investment opportunity set. As shown by Merton (1969) and Merton (1971), the optimal proportion of wealth invested in stock is given by α in equation (3.1)

$$\alpha = \frac{\mu - r_f}{\gamma \sigma^2} \tag{3.1}$$

where $\mu - r_f$ is the mean equity premium above the risk free rate r_f, σ^2 is the variance of stock returns and γ is the coefficient of relative risk aversion of the agent. Merton's solution shows that the fraction of financial wealth invested in stock is constant and independent of financial wealth.

The dominating consensus in the literature views human capital as a substitute for a low risk asset. Curcuru, Heaton, Lucas, and Moore (2006), Heaton and Lucas (1997), Heaton and Lucas (2000), and Cocco, Gomes, and Maenhout (2005) highlight the results that periodic labor income flows serve as additional sources, beyond financial wealth, to finance consumption. Although one may expect risky labor income to be a significant source of background risk and reduce equity holding, this is not an implication of current theoretical models and not suggested by empirical findings. In fact, the literature suggests that investors should take leveraged positions in risky equity even in presence of substantial transaction costs.

Taking this line of reasoning, and following the insights given by Bodie, Merton, and Samuelson (1992) and Campbell and Viceira (2003), assume that the agent's financial wealth is given by FW_t and her human capital is HW_t. Then the investor's *total* wealth is the sum of her financial wealth FW_t and her human capital HW_t. If the investor was not constrained from trading her human wealth, she would sell her human wealth for HW_t dollars, invest $\alpha(FW_t + HW_t)$ in stock and invest the remainder $(1 - \alpha)(FW_t + HW_t)$ in the riskless asset, where α is given by Merton's solution in equation (3.1). It follows then that the optimal share of financial wealth invested in equity is now given by [17]

$$\alpha^* = \frac{\alpha(FW_t + HW_t)}{FW_t} = \frac{\mu - r_f}{\gamma\sigma^2}\left(1 + \frac{HW_t}{FW_t}\right) \tag{3.2}$$

Equation (3.2) shows that given a fixed value of human capital, α^* is decreasing in the value of financial wealth FW_t. The same conclusion should hold true in the data if the cross sectional distribution of financial wealth and human capital are independent. This is the central result implied by traditional theories that conflicts with the empirical evidence on household finances.

Figure 3.1 shows a graphical illustration of the results implied by Merton's original consumption and portfolio choice problem, a model with labor income, and what is evidenced in the cross section of U.S. households. While Merton's

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[17] If the agent is constrained from trading her human wealth, she will adjust her financial portfolio so that her total dollar asset holdings reflect, as close as possible, her optimal unconstrained asset holdings. The gross amount invested in equity is $\alpha(FW_t + HW_t)$ if it is less than the available amount of financial wealth FW_t, or her entire amount of financial wealth FW_t if $\alpha(FW_t + HW_t)$ is greater than FW_t. The former corresponds to the case of an interior solution where the agent has enough financial wealth to allocate into equity, and the latter corresponds to a corner solution where the agent does not have enough financial wealth. Therefore the optimal gross amount in stocks is given by $\mathrm{Min}(\alpha(FW_t + HW_t), FW_t)$. It follows, then, that the optimal share of financial wealth invested in equity is now given by

$$\alpha^* = \frac{\mathrm{Min}(\alpha(FW_t + HW_t), FW_t)}{FW_t} = \mathrm{Min}\left(\frac{\mu - r_f}{\gamma\sigma^2}\left(1 + \frac{HW_t}{FW_t}\right), 1\right)$$

and the optimal allocation into riskless asset is $1 - \alpha^* = \mathrm{Max}\left(1 - \frac{\mu - r_f}{\gamma\sigma^2}\left(1 + \frac{HW_t}{FW_t}\right), 0\right)$.

solution indicates a constant fraction of financial wealth invested in equity if risk preferences are homogeneous in the cross section, a model with human capital indicates that the equity share should be decreasing in financial wealth if the distribution of wealth and human capital are independent. Holding her human wealth constant, the agent with little wealth will hold a larger fraction of her financial wealth in stocks since her human capital, which substitutes for a riskless bond, composes a larger portion of her *total* wealth. Conversely, an agent with high financial wealth will hold a lower fraction of her financial wealth in stocks because her human capital composes a lower fraction of her total wealth, leading her to hold more riskless bonds in her financial portfolio. In the limit, as financial wealth grows unbounded, the optimal allocation in equity converges towards Merton's solution. A stark contrast is evident when it comes to asset holdings of U.S. households in the cross section. Figure 3.1, contrary to traditional theories, shows fitted values from PSID that households in the cross section have equity shares that are increasing in financial wealth.

It is interesting to note that equation (3.2) shows that α^* is increasing in the value of human capital. This result stems from the assumption that human capital is an implicit holding on a security that substitutes for a low risk security. Thus, human capital and financial wealth independently have opposing effects on α^*. Increases in financial wealth crowds out equity holding, while increases in human capital tilts portfolio weight towards equity. I exploit this result in this paper to generate contemporaneous joint distributions of financial wealth and human capital values consistent with equity overallocation as evidenced in the cross section. To generate equity overallocation, (i) contemporaneous wealth accumulation and human capital values must be positively correlated, (ii) and the positive effects of human capital on equity shares in the cross section must dominate the negative effects of financial wealth. I show that this is possible if labor income shocks are highly persistent.

3.3 A Simplified Model

Before proceeding to the more realistic life cycle model, it is useful to highlight the main forces that drive the core results of the paper within a simple and easy to understand model.

3.3.1 Model Specification

Assume an agent who is an infinite horizon power utility maximizer who invests in a market with a constant investment opportunity set. There are two assets available for investment: a riskless asset and a risky stock. The riskless asset has a constant log return r_f and the risky stock has a time stochastic log return r_t with constant mean μ and variance σ^2.

Moreover, consider a labor income process whose shocks to log levels follow a first order autoregressive (AR(1)) process. More specifically, assume that the log of labor income Y_t has the following dynamics

$$\log Y_t = y_t = gt + p_t \tag{3.3}$$

$$\log P_t = p_t = \rho p_{t-1} + \varepsilon_t \tag{3.4}$$

where g is the income growth rate, P_t is the permanent component of labor income and ε_t is the innovation to the $AR(1)$ process which has zero mean and variance σ_ε^2 and is independent of returns on financial assets. The parameter ρ governs how influential previous income shocks are to future income realizations through its size. The agent is subject to large and more persistent income shocks if ρ is closer to 1. Conversely, shocks are less persistent if ρ is closer to zero.

I make the optimal allocation solution given in equation (3.2) operational by developing a simple model to price human capital. To this end, I discount expected future labor income streams using a discount rate d.[18] The derivation of the human capital equation (3.5) is shown in the Appendix. The value of human capital at time t is

$$HW_t = \frac{Y_t}{P_t} \int_t^\infty e^{(g-d)(s-t)+e^{(t-s)\kappa}p_t+\frac{\left(1-e^{2(t-s)\kappa}\right)\sigma_p^2}{4\kappa}} \,\mathrm{d}s \tag{3.5}$$

where $\kappa = -log(\rho)$. Assume furthermore that financial wealth accumulation evolves according to the following process

$$FW_{t+1} = (FW_t + Y_t)\mathrm{E}_t[\alpha^*(exp(r_t) - exp(r_f)) + exp(r_f)] \tag{3.6}$$

To keep the focus on portfolio allocation, I make the simplifying assumption

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[18] Pricing of human capital must be done in an incomplete market framework if labor income shocks are correlated with stock markets, significantly complicating the simple model.

to preclude consumption in equation (3.6). I refrain from modeling optimal consumption to keep the model as simple as possible. Merton (1969) and Merton (1971) show that the consumption and portfolio choice problem can be separated into two separate optimization problems, one corresponding to the consumption choice problem and the other corresponding to the portfolio choice problem, whose solutions are independent of one another. Therefore precluding consumption in the model has no impact on the functional form of the optimal fraction of wealth invested in equities. While precluding consumption does not seem realistic in this setup, the section that follows develops a more general and realistic life cycle model which incorporates endogenous consumption along with endogenous portfolio allocation choices. As it turns out, the qualitative results of the simplified model are consistent with the results of the life cycle model.

Results from the simplified model can be attained through computer simulations. Given an initial value of y_0, p_0 and FW_0, I can simulate over time the labor income processes (3.3) and (3.4) for a given set of input parameter values γ, ρ, σ, σ_ε, μ, g, r_f and d, and compute relevant variables of interest such as the value of human capital, given in equation (3.5), the value of financial wealth accumulation, given in equation (3.6), and the optimal portfolio allocation in equity, given in equation (3.2). I now turn to some simulation results.

3.3.2 Model Results

To investigate the role that the persistence parameter ρ plays in asset allocation, I use different parameter values of ρ (1, 0.95, 0.7 and 0.4) and simulate a cross section of 10,000 households for each set of parameter values. The values for the remaining parameters are $\gamma = 10$, $g = 1.5\%$, $r_f = 2\%$, $\mu = 6\%$, $\sigma = 18\%$, $\varepsilon \sim N(0, .30^2)$ and $d = 6.5\%$. For each set of parameterizations, I separate the simulated data at different time periods, and rank and divide the sample according to wealth quartile values. Then I compute the median fraction of wealth invested in equity for each wealth quartile.

Table 3.2 shows the results from the simulations. The rows labeled nth Q represent the median α for the nth wealth ranked quartile sample. To aid in analyzing how α^* varies in financial wealth, the rows labeled Top Q - Bottom Q represent the difference between the median α^* of the Top and Bottom wealth quartiles. Results are shown for different values of ρ and time periods. The results in Table 3.2 show that equity overallocation consistent with empirical facts can be attained if households have labor income shocks that are highly persistent. Conversely, a negative relationship between equity shares and financial wealth is attained consistent with traditional theories of portfolio choice if labor income shocks are weakly persistent. As it can be seen from panel (a) of the table, α^* is greater for higher wealth quartile samples if shocks to labor income are highly persistent ($\rho = 1$). The difference between the median α^* for the top and bottom wealth ranked quartiles are 0.0511, 0.1710, 0.1877 and 0.1902 at time periods $t = 10$, 20, 40 and 60 respectively. There is a much different result if labor income shocks are not highly persistent ($\rho < 1$). Panel (d), for example, shows that the difference between the α^* of the top and bottom wealth ranked quartile samples are -0.043, -0.0275, -0.0144 and 0.0092 at time periods $t = 10$, 20, 40 and 60 respectively if ρ is 0.4.

While Table 3.2 shows that equity overallocation can be attained in a model with persistent labor income shocks, it is not clear how persistence generates the results. Table 3.3 provides the answer. It shows that contemporaneous positive correlation between human capital and wealth accumulation is increasing in the persistence parameter. The table shows 2–Way Frequency distributions when the simulated data is sorted in increasing order of Human Wealth HW_t and Financial Wealth FW_t values and then divided by median and quartile values into subsamples. For example, values under the column Top Quantile FW_t and Bottom Quantile HW_t gives the frequency of households that have the highest amount of financial wealth accumulated and the lowest value of Human Wealth. Values under other columns can be interpreted similarly. Frequencies are shown when the sample is divided by median values and quartile values at time periods 40 and 60. The results are qualitatively similar at other time periods and they are not reproduced in order to save space.

The table shows that if labor income shocks are highly persistent, households with large values of human capital tend to accumulate the most financial wealth over time. Conversely, high persistence also results in households with low values of human capital to accumulate the least financial wealth over time. For example, panel (b) of the table shows that at time $t = 40$, 16.60% out of a possible 25.00% of the sample concurrently have the highest amount of financial wealth accumulated and the highest Human Capital value. Moreover, the fraction of the sample that belongs concurrently in the poorest quartile as well as in the lowest quartile in human capital value is 13.70%. However, only a very small fraction of the sample belongs to either the wealthiest and lowest human capital value quartiles, or the poorest and highest human capital value quartiles (1.00% and 0.80% of the sample respectively). A stark contrast in frequency distribution emerges for lower values of parameter ρ. The frequencies are fairly uniform across all 2–way classifications of financial wealth and human capital. The conclusion is the same whether the samples are divided by median or quartile values of FW_t and HW_t, as shown in the other panels of the table.

Therefore, equity overallocation is a consequence of the persistence parameter and its impact on the joint distribution of human capital, financial wealth accumulation and equity shares in the cross section, and not a consequence of financial wealth effects alone. High persistence in income shocks implies that households with large human capital also accumulate the most wealth over their life time. Since financial wealth and human capital independently have opposing effects on equity shares, rising equity shares in financial wealth infers that the positive effects of human capital dominate in the cross section. However, the opposite conclusion is attained if labor income shocks are weakly persistent. If persistence is weak, there is a weaker link between human capital and financial wealth in the cross section. Moreover, equity shares are decreasing in levels of financial wealth as predicted by traditional theories. This implies that the effects of financial wealth on equity shares dominate in the cross section if persistence is weak.

To understand the results, consider an *agent with highly persistent labor income shocks* in comparison with the *average agent* in the population. Since the permanent component of labor income is highly serially correlated for this agent, a large and *positive* shock today leads her to raise her expectation of large and positive realizations of labor income in the future as well. This leads to a substantial increase in the value of her human capital after an unexpectedly high positive draw of labor income. Since labor income provides extra resources from which the agent derives utility beyond those from financial wealth, over time this agent will accumulate higher levels of financial wealth through savings of her earnings and have a higher value of human capital compared to the average agent. Therefore, persistence in labor income shocks generates a strong contemporaneous positive correlation between human capital values and financial wealth in the cross section. Although human capital and financial wealth independently have opposing effects on equity shares (as predicted by the standard theories), if labor income shocks are highly persistent, the positive effects of human capital are stronger than financial wealth's in the cross section. Coupled with the result that financial wealth and human wealth are positively correlated in the cross section, equity overallocation is attained.

However, an agent with labor income shocks that are *weakly persistent* does not enjoy the same degree of future wealth accumulation from a current increase in labor income. A positive and large income realization today is not indicative of large realizations of labor income in the future. The value of human capital is little changed and the agent's accumulation of wealth overtime becomes more reliant on returns from her financial portfolio, instead of increasing savings resulting from changes in her income earnings. Therefore, there is a weaker contemporaneous link between wealth accumulation and human capital in the cross section if labor income shocks are weakly persistent. Moreover, since the value of human capital is little changed from changes in labor income, human capital becomes a weaker factor when it comes to determining equity shares in the cross section. Consequently, financial wealth becomes the dominating factor, reestablishing the standard result from traditional theories that the equity share should be decreasing in financial wealth.

To make the point clear, Figure 3.2 shows illustrations of the joint distributions of financial wealth, human capital and equity shares in the cross section based on the simple model and its simulated data. Panels (a) and (b) of the figure show graphical illustrations profiling equity shares α^* (equation (3.2)) across joint values of human capital HW and financial wealth FW in the cross section if $\rho = 1$ and $\rho = 0.4$ respectively. Each joint values of human capital and financial wealth correspond to the joint median values of human capital and financial wealth for each quartile sample when the simulated data is sorted in increasing values of financial wealth and dividend into quartiles. Panel (a) shows that when labor income shocks are highly persistent ($\rho = 1$), there is a wide cross sectional heterogeneity in human capital values. The downward sloping curves corresponding to equity holdings for high and low values of human capital are distinct from each other. Since human capital and financial wealth are positively correlated, households with large values of human capital correspond to households with large amounts of financial wealth as well. Similarly, households with low values of human capital correspond to households with low amounts of financial wealth in the cross section. The square marks correspond to the values of equity shares for each joint values of human capital and financial wealth along the cross section. As it can be seen, equity shares is rising in financial wealth if the persistence parameter is unity. On the other hand, panel (b) shows that when labor income shocks are weakly persistent ($\rho = 0.4$), there is a very weak heterogeneity in human capital values in the cross section compared to financial wealth. The cross sectional heterogeneity in equity shares is mostly driven by differences in wealth levels, rather than human capital values. Consequently, financial wealth effects become the dominating factor in describing equity shares in the cross section, reestablishing the results from traditional theories that equity shares should be decreasing in financial wealth. To lend further support to the claim, panel (c) of the figure shows predicted values of α^* from a regression of equity shares on log of financial wealth using the simulated data. Equity share is decreasing in financial wealth if $\rho = 0.4$, and increasing in financial wealth if $\rho = 1$.

3.4 Life Cycle Model

The previous section highlighted the importance of persistence in labor income shocks to explain equity overallocation in the cross section in a simple model of portfolio choice. In this section I incorporate persistence in a more realistic life cycle model which includes both endogenous consumption and portfolio choices. The results from the more general model will highlight additional insights not possible from the simplified model.

I consider an optimizing model of household consumption and portfolio choice. The household life cycle consists of two phases; adult life employment and retirement. The agent enters adult life at the effective age of t_0 with financial wealth amounting to W_{t_0}. She retires at age T with retirement savings in the amount of W_T. She receives labor income Y_t during her working years $t_0 \leq t \leq T$. The logarithm of labor income evolves according to equations (3.7) and (3.8)

$$y_t = \log\, Y_t = f_t + p_t \tag{3.7}$$

$$p_t = \log\, P_t = \rho p_{t-1} + u_t + e_t \tag{3.8}$$

where f_t is a deterministic component that depends on certain household characteristics such as age and education, p_t is a permanent component, and u_t and e_t are shocks with means zero and variances σ_u^2 and σ_e^2, where u_t is correlated with stock market returns and e_t is not.[19,20] ρ is a parameter of persistence of labor income shocks. It tracks how influential previous period labor income shocks are to current and future period labor income.

Households can trade two financial assets. They can buy equity with a stochastic gross return of R_t^S, and save at a constant gross risk free rate of R_f. Households can not borrow or take short positions in equity. The equity premium above the risk free rate is

$$R_{t+1}^S - R^f = \mu^S + \varepsilon_{t+1}^S \tag{3.9}$$

where ε_t^S is a normally distributed shock with mean zero and variance $\sigma_{\varepsilon^S}^2$ and μ^S is the mean equity premium. Since $R^f, \sigma_{\varepsilon^S}^2$ and μ^S are fixed constants, households face a constant investment opportunity set.

I assume that the agent's preference is described by a standard constant relative risk aversion utility function.[21] The agent chooses a consumption and asset holding plan to maximize her utility. Her objective function is given by

$$\text{Max } \mathrm{E}_{t_0}\left[\sum_{t=t_0}^{T-1} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma} + \beta^T b \frac{\left(\frac{W_T+Y_T}{b}\right)^{1-\gamma}}{1-\gamma}\right] \tag{3.10}$$

subject to a budget constraint, where γ is the coefficient of relative risk aversion, β is the agent's subjective discount factor, C_t is consumption at age t, and W_T is financial wealth available to finance consumption during post-retirement years.

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[19] The labor income process follows a slightly different specification from some of the existing models in life cycle portfolio choice with human capital. The specification here is motivated by the estimation method employed in the empirical part of the paper.

[20] The actual structure of f_t is more specifically defined in the empirical part of paper.

[21] I also solve the problem using Epstein-Zin recursive utility. The formulation is described below.

At time of retirement T and thereafter, the agent derives utility by consuming a fraction $\frac{1}{b}$ of pre-retirement final period cash on hand $W_T + Y_T$. This is equivalent to modeling post-retirement consumption and investment decisions where the agent receives a constant annual income flow akin to a retirement annuity. The strength of motive to save for retirement is captured by the parameter b.

The budget constraint follows a process that is dependent on the agent's available financial wealth, income and chosen policies. The budget constraint is given by:

$$C_t + S_{t+1} + B_{t+1} \leq S_t R_t^S + B_t R_f + Y_t \tag{3.11}$$

where S_t and B_t are beginning of period amounts invested in stocks and riskless savings, which together multiplied with gross returns, compose the agent's financial wealth,

$$W_t = S_t R_t^S + B_t R_f \tag{3.12}$$

Interior solutions for the optimal policies C_t, S_{t+1} and B_{t+1} must satisfy a set of Euler equations from the optimization problem. During ages $t_0 \leq t < T - 1$, the Euler equations from the original optimization problem are

$$1 = \beta \mathrm{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{t+1}^S \right] \tag{3.13}$$

$$1 = \beta \mathrm{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} R_f \right] \tag{3.14}$$

and at age $t = T - 1$, the Euler equations are

$$1 = \beta \mathrm{E}_t \left[\left(\frac{W_T + Y_T}{b C_{T-1}} \right)^{-\gamma} R_{t+1}^S \right] \tag{3.15}$$

$$1 = \beta \mathrm{E}_t \left[\left(\frac{W_T + Y_T}{b C_{T-1}} \right)^{-\gamma} R_f \right] \tag{3.16}$$

Following Deaton (1991), denote cash on hand X_t as the amount of liquid resources available for consumption and portfolio savings at age t. Moreover, denote κ_t the fraction of available cash on hand chosen for consumption, α_t the fraction of financial wealth invested in equity and $(1 - \alpha_t)$ the remaining balance of financial wealth invested in riskless savings. Along with the agent's insatiability condition, equation (3.11) can be rewritten as a function of state variables X_t, P_t and t, and policies κ_t and α_t. The new budget equation is

$$X_{t+1} = W_{t+1} + Y_{t+1} = X_t(1-\kappa_t) \times [\alpha_t(R^S_{t+1} - R_f) + R_f] + \exp(f_{t+1}) P_t^\rho U_{t+1} E_{t+1} \tag{3.17}$$

where uppercase letters P_t, U_t and E_t stand for exponents of their respective lowercase variables. Since households are not allowed to borrow or short sell, and along with the Inada condition that $C_t^{-\gamma} \to \infty$ as $C_t \to 0$, the policies must satisfy the following constraints

$$0 \leq \alpha_t \leq 1 \qquad \text{and} \qquad 0 < \kappa_t \leq 1 \tag{3.18}$$

The set of Euler equations from the original optimization problem given in equations (3.13) to (3.16) can now be rewritten as functions of the new state variables and policies. At ages $t_0 \leq t < T-1$, interior solutions of the optimal policies κ_t and α_t must satisfy the new set of Euler equations

$$1 = \beta \mathrm{E}_t \left[\left(\frac{X_{t+1}\kappa_{t+1}}{X_t \kappa_t} \right)^{-\gamma} R^S_{t+1} \right] \tag{3.19}$$

$$1 = \beta \mathrm{E}_t \left[\left(\frac{X_{t+1}\kappa_{t+1}}{X_t \kappa_t} \right)^{-\gamma} R_f \right] \tag{3.20}$$

where the state variables P_t and X_t are subject to functional transitions given by equations (3.8) and (3.17) respectively, the policies are subject to constraints (3.18) and financial market returns are as given in equation (3.9). At age $t = T-1$, the optimal policies must satisfy the same set of Euler equations with the exception that κ_{t+1} is substituted by $\frac{1}{b}$.

Given a set of realistic parameter values, and any values of the state variables P_t and X_t, the optimal policies can be solved for at each age during working adult life starting from the pre-retirement period $t = T-1$ and working backwards in time until age $t = t_0$. An appendix contains further details of how the problem is solved numerically.

3.4.1 Epstein-Zin Utility

The time separable utility in equation (3.10) has the property that a single parameter controls both risk aversion and willingness to substitute consumption intertemporally.[22] The elasticity of intertemporal substitution (EIS) is related to an agent's willingness to move consumption between time periods. Such motive is well defined even in a riskless environment. In contrast, risk aversion is concerned with the agent's willingness to move consumption across states of the world in a risky environment. In a multi period and risky environment, such as the one modeled in this paper, such strong link between the EIS and risk aversion may not be appropriate. For instance, Bansal and Yaron (2004) provide support to loosen the link between risk aversion and intertemporal substitution to obtain an equity premium and a risk free rate that are consistent with what have been observed in the recent past. Therefore, in conjunction with the power utility function, it is instructive to solve the optimization problem if the agent has a time inseparable recursive utility as defined by Epstein and Zin (1989):

$$U_t(C_t, \mathrm{E}_t[U_{t+1}]) = \left[(1-\beta)C_t^{\frac{1-\gamma}{\theta}} + \beta \mathrm{E}_t[U_{t+1}^{1-\gamma}]^{\frac{1}{\theta}}\right]^{\frac{\theta}{(1-\gamma)}} \tag{3.21}$$

where $\theta = \frac{(1-\gamma)}{(1-\frac{1}{\psi})}$, ψ is the parameter of EIS, and the other parameters are as defined previously.

Given her preferences in (3.21), and state variables and policies as previously defined in the power utility model, the agent's optimization problem is characterized by the Bellman Equation:

$$V_t(X_t, P_t) = \underset{\{\kappa_t, \alpha_t\}}{\mathrm{Max}} \left[(1-\beta)(X_t\kappa_t)^{\frac{1-\gamma}{\theta}} + \beta \mathrm{E}_t\left[V_{t+1}(X_{t+1}, P_{t+1})^{1-\gamma}\right]^{\frac{1}{\theta}}\right]^{\frac{\theta}{(1-\gamma)}} \tag{3.22}$$

where the state variables P_t and X_t are subject to functional transitions given by equations (3.8) and (3.17) respectively, the policies are subject to constraints (3.18) and financial market returns are as given in equation (3.9). Appendix A contains further details of the numerical methods used to solve the dynamic programming problem.

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[22] In the power utility function, the elasticity of intertemporal substitution (EIS) ψ is equal to the reciprocal of the coefficient of risk aversion ($\gamma = 1/\psi$).

3.5 Calibration of Life Cycle Model

This section describes the set of parameter values used in the life cycle model to attain the results that follow.

I assume that the agent enters adult working life at the age of $t_0 = 19$ and retires at age $T = 65$. I choose preference parameters which I find reasonable and on the same order of magnitude as the parameters found in Vissing-Jorgensen and Attanasio (2003) and Vissing-Jorgensen (2002). Vissing-Jorgensen and Attanasio (2003) find that the coefficient of risk aversion ranges widely depending on certain assumptions and the estimation method employed. They find that γ can be as large as 10, while a γ equal to unity is not too uncommon in calibrations of macroeconomics models. To account for this wide variation of possible values used in the literature, the coefficient of relative risk aversion that I use are 3, 5 and 7, where the baseline value is set to 5. For the Epstein-Zin preferences, I use ψ values of 0.15, 0.3, 0.5 and 0.7, where 0.5 is my baseline value. These values are well within the range of estimates from Table 3 of Vissing-Jorgensen (2002). The subjective discount factor β is set at 0.96, which is commonly used in calibrations of macro and life cycle models. As for the strength of retirement savings motive b, I use values 1, 5 and 10, where 1 is the baseline value. A b value of 10, for example, is equivalent to a motive to save for retirement where the agent receives a 20 year annuity equal to one half of her pre-retirement consumption.

The central objective of the current paper is to investigate the persistence of labor income shocks and its effect on portfolio choice. As in the simplified model, I consider three parameter values for ρ, 1, 0.7 and 0.4, where 1 is the baseline value. The structure of the labor income process is estimated from PSID data. The deterministic component of the labor income process (3.7) is estimated using as regressors a quadratic polynomial in age for each college and high school educated households and a dummy variable to account for the marital status of the head of the household. The coefficients are estimated using pooled regression and the results are given in Table 3.10. Further details of the structure of the labor income model are given in the empirical portion of the paper. For the calibrations, I assume that all heads of households are married. The college group is the baseline group.

The variance of the error shocks in the labor income process are estimated from residuals after fitting a dynamic panel data model used in the empirical portion of the paper. I select the median values for the variances of u and e for each college and high school samples. The values for σ_u^2 and σ_e^2 are 0.0012 and 0.1477, and 0.0002 and 0.1455 for the college and high school groups respectively. Further details of the error structure of the labor income model are given in the empirical portion of the paper.

Prior empirical evidences using household level data show that there is weak correlation between labor income shocks and stock market returns. Baxter and Jermann (1997), however, state that estimations using longer horizon macro data yield more significant correlations. My estimates of covariances between u and R^S are weak consistent with prior findings and of the same order of magnitude as covariances found in Cocco, Gomes, and Maenhout (2005) and Gomes and Michaelides (2005). I find a covariance of -0.00435 and -0.0016 for college and high school groups respectively.

Following the literature on asset pricing, I set $R_f - 1$ to 2%, which is commonly used as the real rate of return on short term government bonds. For the stock return process, I consider a mean equity premium μ^S of 4% and a standard deviation σ_{ε^S} of 18%. I choose μ^S of 4% instead of the historical average of 6%. This seems reasonable considering that investors are commonly subject to brokerage account costs, transaction costs, mutual fund management expenses, taxes on investment income and other costs when investing in financial securities. All of these factors lead to a lower effective return on financial investments.

In the simulations, each household enters adult life with some initial financial wealth and a draw of the permanent component of labor income. The initial wealth is \$31,000 and \$27,000 for college and high school households respectively, which are median estimates attained from the 1999 PSID Wealth Supplemental data file for households whose heads were between the ages of 19 and 25. I assume that P_{t_0-1} immediately before entering adult life is equal to 1 with future random realizations based on the variances, covariances and persistence parameters values as discussed above. A sample path of stock returns and labor income is simulated according to equations (3.9) and (3.7) and optimal policies computed according to the solution of the optimization problem. The process is repeated 10,000 to generate a panel of households consumption and investment decisions at each time period. In order to investigate how different parameter values impact equity overallocation, I repeat the simulations for various parameters of ρ, γ, ψ (in the Epstein-Zin utility case) and b for each college and high school educated households once their respective optimization problems are solved. All simulation jobs are run from the same set of random numbers to avoid confounding effects that may arise from the computer system's random number generating algorithm.[23]

Table 3.4 conveniently summarizes the set of parameter values used in the calibration of the life cycle model and simulations.

3.6 Model Results: Optimal Policies and Simulations

Before looking at equity overallocation resulting from the simulations, this section discusses the optimal policy functions (or controls) and the underlying forces driving the main results. The discussion here will follow a parallel from the discussion of the simple model. When otherwise indicated, the results are for the baseline set of parameters, the power utility model and college educated group.

3.6.1 Consumption Policy

Panel (a) of Figure 3.3 shows the optimal share κ of cash on hand used for consumption at age 35 as a function of cash on hand X_t and permanent component of labor income P_t. The consumption share is qualitatively similar at different stages of the life cycle and the discussion follows similarly. As the figure shows, the consumption share is *decreasing* along the X_t dimension as cash on hand increases. This is consistent with the agent's motive to *smooth* consumption across states of nature. At very low levels of cash on hand, the agent is liquidity constrained and consumes all of her available liquid wealth. When X_t is very low, κ decreases rapidly, but past a certain level κ flatten along the X_t dimension as cash on hand increases. Thus, at low levels of cash on hand the rate

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[23] This can be done by using the same seed value in the computer codes to generate the random numbers for all jobs.

of savings $(1 - \kappa)$ increases at a very fast pace as cash on hand increases. At substantially high levels of cash on hand, the consumption share is more or less insensitive to changes in liquid cash and most of it is saved.

Panel (a) also shows that κ is *increasing* along the P_t dimension. Holding other variables constant, a higher value of P_t implies higher values of income in the future, which consequently, translates to a higher current value of human capital. A higher value of P_t results in an agent choosing to consume a higher fraction of cash on hand. One way to interpret this result is to perceive human capital as a factor that hinders precautionary saving motives. Precautionary savings leads one to reduce consumption and save more for the future as the agent becomes more risk averse. Since the presence of human capital reduces effective risk aversion, a higher value of P_t, other things equal, effectively lowers risk aversion and leads the agent to consume a larger fraction of available cash and reduce the fraction that is saved. This result is in line with the notion that human capital substitutes for a relatively low risk asset. The higher the value of the agent's *human capital*, the lower the risks of her *total wealth*, which leads her to reduce precautionary savings motive by increasing her consumption share.

3.6.2 Portfolio Choice Policies

Panel (b) of Figure 3.3 shows the optimal share α of financial wealth invested in risky stocks at age 35 as a function of cash on hand X_t and permanent component of labor income P_t. The equity share of wealth looks qualitatively similar over the state space at different stages of the life cycle, and for brevity they are not reproduced in the paper. Holding P_t constant, a larger value of cash on hand implies that financial wealth composes a larger fraction of the agent's *total wealth*. Therefore, α is *decreasing* in X_t. Eventually α asymptotes towards a constant as X_t grows unbounded, consistent with the standard literature on non-tradeable human capital and portfolio choice.

The figure also shows that α is *increasing* in P_t. While the simplified model assumed that human capital substitutes for a low risk asset, the latter is a result in the life cycle model. It is interesting to note that even though labor income is very risky, as evident from the variances of labor income shocks used in the calibration, the existence of labor income can reduce overall consumption risk significantly. Even though human capital is far from 'riskless' in every sense of the word, larger values of P_t, other things equal, indicates that a larger fraction of the agent's *total wealth* is composed of a relatively low risk asset. As in the simplified model, α is increasing in the value of human capital.

3.6.3 Persistence in Income Shocks and Human Capital's Effect on Equity Shares

It is useful to draw the similarities between the life cycle model and the simplified model. Holding all other factors constant, X_t and P_t proxy for financial wealth and human capital in the simplified model. Just like financial wealth and human capital, X_t and P_t have opposing effects on α; larger values of P_t leads the investor to tilt portfolio holdings towards equity, and larger values of X_t leads the agent to crowd out equity. This section adds to the insights attained from the simple model on the persistence parameter and its effect on the relative strengths of human capital and financial wealth to determine equity shares in the cross section.

Consider the effects that P_t has on α. Panel (a) of Figure 3.4 shows optimal α as a function of P_t and X_t at age 35 if ρ is equal to one and panel (b) shows the same variable if ρ is equal to 0.7. Comparing the two figures, it is easy to see that the α's surface has a steeper slope along the P_t dimension if ρ is equal to one, but it is relatively flat if ρ is equal to 0.7. I conclude from this observation that human capital's role as a determining factor of α is increasing in the persistence parameter. To lend further support to this claim, panel (c) of Figure 3.4 shows α along the P_t dimension when X_t and age are held fixed at $\$331,621$ and 35 for three different parameters of persistence ρ. α becomes more responsive to values of P_t at higher values of ρ. Conversely, α becomes fairly homogeneous to different values of P_t if ρ is low.

To understand this result, consider two agents; one subject to highly persistent labor income shocks and one subject to weakly persistent labor income shocks. The first agent will see the value of human capital change drastically with large unexpected changes to her labor income, whether the changes are positive or negative. Consequently, the agent whose labor income shocks are highly persistent places a strong emphasis on the current value of her human capital when it comes to making portfolio choice decisions. On the other hand, an agent who is exposed to weakly persistent labor income shocks will see little changes to the current value of her human capital even though her labor income could be very variable. Consequently, the agent whose labor income shocks are weakly persistent places a low emphasis on the current value of her human capital when it comes to making portfolio choice decisions.

3.6.4 Simulation Results

The previous section provides a static view of the optimal policies across different values of the state variables. I rely on numerical simulations in order to investigate the role that the persistence parameter plays in determining the joint distributions of human capital, financial wealth, equity shares, and consequently, equity overallocation in the cross section. This section also investigates the role of education, the coefficient of relative risk aversion, the elasticity of intertemporal substitution, the motive for retirement savings and life cycle effects on equity overallocation.

Persistence in labor income and implications for the joint distributions of W, Y and α

Since the optimal equity allocation is dependent on financial wealth and human capital, it is interesting to investigate how the persistence parameter affects the joint distributions of W, Y and α in the cross section. Table 3.5 shows 2–Way frequency distributions when the simulated data is sorted according to values of labor income Y_t and Financial Wealth W_t and then divided by median and quartile values into subsamples. 2–Way frequency distributions are shown at ages 45 and 55. The results are qualitatively similar at different stages of the life cycle and they are not reported.

As was the case in the simple model, the table shows that financial wealth is more strongly correlated with human wealth in the cross section if ρ is equal to one. Panel (a) of the table shows that at age 45 when ρ is equal to one, 42.45% of the top one half wealthiest were also top one half earners. The same frequency goes for the bottom one half poorest and bottom one half earners. However, if ρ is equal to 0.4, the frequency is almost evenly distributed across all 2–way classifications of financial wealth and labor income. The latter result indicates that the two variables are not strongly correlated if persistence in labor income shocks is weak. The same conclusion is reached at age 55 and when the simulated data is divided according to quartile and median values of W_t and Y_t.

This result follows naturally from the way investors accumulate financial wealth over time. Consider the case of highly persistent labor income shocks. At a given moment, agents receive labor income, which are subject to persistent shocks, some of which is consumed immediately while the remainder is added to current period financial wealth and invested in liquid financial assets. A mild unexpected positive shock to labor income at a given time leads to a pronounced increase in an agent's human capital. Due to higher future labor income realizations, in conjunction to returns from investments, the agent will tend to accumulate more financial wealth as a result of the positive shock. Even though the time series relationship between financial wealth and human capital is negative,[24] the contemporaneous cross sectional correlation is positive and it is increasing in the persistence parameter.

However, if labor income shocks are weakly persistent, agents do not enjoy the same degree of future wealth accumulation from a current positive shock to labor income. A positive and large labor income shock today is not indicative of large realizations of labor income in the future. The value of human capital is little changed and the agent's accumulation of wealth overtime becomes more reliant on returns from her financial portfolio. Therefore there is a weaker contemporaneous link between financial wealth accumulation and human capital values in the cross section if labor income shocks are weakly persistent.

Persistence in labor income and equity overallocation

Table 3.6 shows that equity overallocation, consistent with empirical facts, can be attained if households have labor income shocks that are highly persistent. Panel (a) shows that the differences between the median equity shares for the top and bottom wealth ranked quartiles are 0.11, 0.12, 0.11 and -0.10 for age groups 26–35, 36–45, 46–55 and 56–65 respectively for the college educated group if ρ is equal to one. There is equity overallocation in all age groups except for the

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[24] Generally, over time, human capital is a depreciating asset and financial wealth is an accumulating asset.

pre-retirement group.[25]

Overallocation is also present among the high school group as shown in panel (a) of the table, however, it seems weaker than in the college group. Human capital effects in the cross section is stronger for college educated households than for high school educated households due to higher variance in labor income shocks. It is interesting to note that equity overallocation evidenced in the U.S. household data is also more pronounced among the college group than the high school group. The results in Table 3.6 seem qualitatively consistent with empirical findings regarding education and equity overallocation.

An inverse relationship between equity shares and financial wealth is attained consistent with traditional theories of portfolio choice if labor income shocks are weakly persistent. Panels (c) and (d) show the median αs when ρ is equal to 0.7 and 0.4 respectively. α is inversely related to financial wealth for most age groups. If ρ is equal to 0.7, the differences between the median α for the top and bottom wealth ranked quartiles are 0.02, -0.32, -0.24 and -0.14 for age groups 26–35, 36–45, 46–55 and 56–65 respectively for the college educated group. Similar results hold for the high school group as well.

There are three forces driving the main effects that generate equity overallocation in the cross section. First, (i) persistence in labor income shocks leads the agents to place a higher weight on their human capital as a determining factor of portfolio weights. This follows because human capital not only has a strong influence on portfolio choices directly, but it also influences financial wealth accumulation over time. Second, (ii) high income persistence leads to a stronger positive contemporaneous correlation between financial wealth and human capital. Although financial wealth and human capital independently have opposing effects on equity allocation, human wealth plays a more dominating role than financial wealth in the cross section as shown in the simulation results. This leads α to be rising in financial wealth. Third, (iii) a higher persistence in income shocks generates a wider cross sectional dispersion in equity shares, human capital values and financial wealth accumulation, which strengthens equity overallocation due to larger dispersions of α values in the cross section when the sample is ranked according to financial wealth.

To summarize, this section showed that even though the optimal allocation

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[25] An explanation for this results is provided in the discussion of life cycle effects.

into equity is decreasing in financial wealth, equity overallocation is a consequence of persistence in labor income shocks and its effects on the joint distribution of human capital, financial wealth accumulation and wealth composition in the cross section, and not a consequence of financial wealth effects alone.

Risk aversion and equity overallocation

Consider the effects that P_t has on α for different values of γ. Panel (a) of Figure 3.5 shows α along the state space P_t and X_t at age 35 if γ is equal to 3 and panel (b) shows the same variable if γ is equal to 7. Comparing the two figures, it is apparent that α is higher in all nodes of the state space for a lower value of γ. Moreover, α has a much steeper slope along the P_t dimension if γ is lower. I conclude from this observation that the effects of human capital on equity shares is stronger if the coefficient of relative risk aversion is lower. To lend further support to this claim, the lower panel of Figure 3.5 shows α along the P_t dimension when X_t and age are held fixed at values $\$524,482$ and 35 for three different values of γ. α, not only is uniformly higher, but it also projects a much steeper slope for lower values of γ.

Now consider the effects that γ has on equity overallocation as shown in Table 3.7. Taking a quick glance through the three different panels in the table, all values of α are higher for lower values of γ for each age group, education group and wealth quartile samples. Focusing on the rows labeled Top Q - Bottom Q, the table shows that equity overallocation is more pronounced for lower values of γ. There are two forces at play; human wealth and financial wealth effects on equity shares. First, (i) a lower coefficient of relative risk aversion, every thing else equal, leads to uniformly higher αs across all quartile samples of wealth, however such effects are strongest for the top quartile wealth groups since they have the strongest human wealth effects. Second, (ii) a lower γ generally leads to higher financial wealth accumulation across all quartile samples of wealth due to the positive equity premium and greater allocation in equity, however the negative financial wealth effects on equity shares are strongest for the bottom quartile wealth groups. These two forces collectively give rise to an inverse relationship between equity overallocation in the cross section and the coefficient of risk aversion.

Elasticity of intertemporal substitution and equity overallocation

The results in Tables 3.6 and 3.7 show that the persistence in labor income shocks does not generate rising equity shares in financial wealth among the oldest age groups. These results seem inconsistent with empirical facts. This section considers the effects of the elasticity of intertemporal substitution in generating equity overallocation in the cross section.

Table 3.8 shows simulation results from the Epstein-Zin utility problem. It shows the median α for each wealth group for different values of ψ when ρ and γ are held fixed at their baseline values. Given the baseline γ value of 5, the power utility case corresponds to a low value of ψ equal to 1/5 in the Epstein-Zin utility problem. Consistent with results from Table 3.7, low values of ψ leads to negative overallocation for the oldest age groups. However, the table shows that overallocation among the oldest age groups is increasing in ψ and becomes positive if the elasticity is large enough.

An explanation is in order. It is generally the case that agents with higher elasticity of intertemporal substitution, coupled with positive expected returns on savings, have a stronger incentive to save for retirement than their low elasticity counterparts. Therefore, consumption shares are lower and agents accumulate more financial wealth in the latter stages of their lives if ψ is high. Consequently, equity shares are generally lower for higher values of ψ, however these effects are stronger among the lowest wealth quartile group than the top wealth quartile group. Therefore, equity overallocation in the cross section is increasing in ψ. Interestingly, as shown by Vissing-Jorgensen (2002) and Bansal and Yaron (2004), a higher value of ψ than the one that is implied by the power utility is also required to match historical values of the equity premium and the risk free rate. Therefore, this section's explanation of the equity overallocation phenomenon is consistent with explanations used to match certain macro-financial variables of interest in asset pricing.

Motive for Retirement Savings

Table 3.9 shows simulation results for different values of savings motive b. Panel (a) of the table shows that a stronger motive for retirement savings strengthens equity overallocation for all age groups, including the pre-retirement age group.

Panels (b) and (c) of the table show that a stronger savings motive lowers consumption shares and increases financial wealth for all age and wealth quartile subsamples. There are two opposing factors that play a role in the cross section. On the one hand, a stronger retirement savings motive may drive equity shares higher because equity offers a higher expected rate of return than the risk free asset. On the other hand, a stronger retirement savings motive may drive equity shares lower because the equity share control is decreasing in financial wealth. Table 3.9 shows that the former effect is most pronounced among the top wealth quartile sample while the latter effect is most pronounced among the bottom wealth quartile. Therefore, equity overallocation is increasing in the retirement savings motive.

Life cycle effects on equity overallocation

The simulation results so far show that equity overallocation is not constant over time and that if follows a lifecycle pattern. Equity overallocation generally seems to be present during early adult life, increases and peaks among high income growth and wealth accumulation age groups, and then decreases among the older age groups. This seems consistent with empirical facts, as shown in table 3.1. This section provides an explanation for this life cycle pattern.

Recall that equity overallocation is a consequence of the contemporaneous positive correlation between human capital and financial wealth values, which is strengthened by a high persistence in labor income shocks. Therefore, overallocation is strongest among the segment of the population that exhibits the highest contemporaneous positive correlation between human capital and financial wealth values in the cross section. Conventional wisdom suggests that the correlation is strongest during the middle age years (age groups 36–45 and 46–55). Since human capital is a depreciating asset over time, young households are likely to exhibit large heterogeneity in human capital values early in life. Heterogeneity in human capital values should be lowest for the oldest age group since this segment of the population is likely to have low values of human capital. On the other hand, the cross sectional heterogeneity in financial wealth is likely to be lowest for the young adults since agents in this age group have low accumulation of financial wealth. The greatest cross section dispersion in financial wealth is likely to happen later in the agents' lives. Consequently, conditional on human capital and financial wealth values being positively correlated, the correlation should be highest among the age groups whose human capital and financial wealth values jointly exhibit the highest cross sectional dispersion, which happens at the mid point of their adult lives.

3.7 Household Portfolio Choices: An Empirical Analysis

3.7.1 Data Source and Variables

This section empirically investigates the effects of the persistence of labor income shocks on household's portfolio choices using data from the Panel Study of Income Dynamics (PSID). PSID provides annual detailed information on the structure of respondents' labor income, among other economic, social and demographic variables, from years 1968 to 1997. From year 1997 and onwards PSID surveyed household units biennially. A relevant feature of the labor income estimation approach discussed below is that it requires a balanced panel data with contiguous observations for each household without any missing observations during the evaluated time period. Although PSID makes a strong effort to keep track of the same households overtime, attrition rates still exist in the data. A quick analysis of the data shows that the sample period beginning in year 1982 and ending in year 1999 provides the largest number of household units with non-missing data on labor income. I choose to use data from this sample period to estimate the labor income process described below.

I define labor income in a given year to be the sum of the stated labor income of the head of the household and the head's spouse. If the respondent is single, then labor income is defined to be the head's stated income only. All dollar values are stated in 1982 US dollars by deflating the stated amounts based on a price deflator available on the US Department of Labor's website. I exclude SEO[26] households from the sample and only select units that were not retired or classified as disabled during the sample period. Lastly, I select households

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[26] The Survey of Economic Opportunity sample oversamples low-income households. The remaining Survey Research Center (SRC) sample is more representative of the US household population. See PSID's website for further details.

whose age of the head was between 19 and 65, inclusive. The final sample size used to estimate the labor income process is 1196. Moreover, the covariances of labor income shocks with stock returns are computed for the calibration of the life cycle model. Annual stock return data on the value weighted equity index was collected from CRSP.

In conjunction to the main PSID data file, PSID keeps details of households' wealth and asset holdings stored in separate Wealth Supplemental data files. Data is available for years 1984, 1989, 1994, 1999 and 2001. The persistence parameter is estimated for each household and merged with the wealth data before I conduct cross sectional regressions. I use the wealth file from year 1999 to match the last year of the labor income data used to estimate the structure of the labor income process. To avoid possible spurious results and incidental holdings of stocks, I select households with net worth greater than $\$10,000$ and total stock holdings greater that \$500. The resulting final sample size for the cross sectional analysis is 570. The reduction in sample size from 1196 to 570 can be attributed to the lack of stock market participation among households in the sample.

Financial wealth W is defined to be the sum of all stated cash holdings, bonds and stocks either directly or indirectly through IRAs and pension, net of all amounts owed on those accounts. All cash and bond holdings were treated as cash equivalents. The share of financial wealth invested in equity α is defined to be the sum of direct and indirect holdings of stocks, net of all amounts owing on those accounts, divided by financial wealth. Furthermore, while there is no ambiguity in the stated dollar amount of direct holdings of stocks, the description available in PSID for indirect holdings of stocks through IRA and pension accounts is less precise. If respondents answered 'Mostly in stocks', 'Mostly in bonds' or 'Equally in stocks and bonds' to how IRA and pension funds are invested, then I treated the stated amounts to be fully invested in stocks, fully invested in bonds or one half in stocks and one half in bonds respectively according to their responses. I thought that this was an appropriate way of dealing with the lack of information and I feel that this will not bias the results.[27]

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[27] This type of measurement error is likely to result in larger standard errors, however it would not bias the results. Any attained significance would be more convincing due to large standard errors.

3.7.2 Estimation of Labor Income Process

I consider a labor income process $Y_{i,t}$ driven by:

$$Y_{i,t} = P_{i,t} F_{i,t} N_i \tag{3.23}$$

$$P_{i,t} = P_{i,t-1}^{\rho_i} U_{i,t} E_{i,t} \tag{3.24}$$

where $F_{i,t}$ is a deterministic function of household characteristics (such as the level of education, marital status and age of the Head), $P_{i,t}$ is a permanent component that tracks how persistent previous year shocks are to current labor income through a persistence parameter ρ_i, N_i is a time invariant, household specific constant, and $U_{i,t}$ and $E_{i,t}$ are shocks. The components of the income process can be written in logarithms:

$$y_{i,t} = \log Y_{i,t} = f_{i,t} + p_{i,t} + n_i \tag{3.25}$$

$$p_{i,t} = \log P_{i,t} = \rho_i p_{i,t-1} + u_{i,t} + e_{i,t} \tag{3.26}$$

where the lower case letters represent the logs of their respective factors represented by upper case letters. To correctly estimate the model parameters one must correctly control for fixed effects. To this end, I estimate the structure of the labor income process for each cross section i after removing the fixed effect n_i by conventional panel data estimation methods. I first fit a one way fixed effect panel data model to the logarithm of labor income and then fit Parks (1967) dynamic panel data model on the residuals from the first step. More specifically, the Parks model is used to fit $y^*_{i,t}$, where $y^*_{i,t}$ is given by

$$y^*_{i,t} = y_{i,t} - \widehat{n_i} \tag{3.27}$$

and $\widehat{n_i}$ is the estimate of the cross section specific, time invariant component from a one way fixed effect model. The specification for the Parks model is given by

$$y^*_{i,t} = \widehat{f_{i,t}} + \epsilon_{i,t} \tag{3.28}$$

$$\epsilon_{i,t} = \widehat{\rho_i} \epsilon_{i,t-1} + u_{i,t} + e_{i,t} \tag{3.29}$$

to estimate each component of the labor income process. I find it necessary to follow this two step approach to remove any fixed effects prior to estimating the dynamic model because the original Parks model does not control for fixed effects.

Parks (1967) model assumes a first-order autoregressive error structure with cross-sectional heteroscedasticity. Further details are available in the original reference and also in Greene (2003). One main advantage of the Parks estimation approach over other dynamic panel data methods for the purpose of this study is that the former approach treats the error components for each cross section separately, giving rise to separate estimates of the ρ_i parameter for each household unit i. This feature allows for further cross sectional analysis using the PSID Wealth data files. Other dynamic panel data estimation approaches provide a single estimate of the ρ parameter representative of all cross sections in the sample, which does not serve the purpose of the current paper.

I follow standard specification for the deterministic component of labor income, $f_{i,t}$. Since the deterministic function must capture conventionally assumed effects of age, education and marital status on labor income, I assume that $f_{i,t}$ is given by

$$\begin{aligned} f_{i,t} &= \beta_0 age_i \times I_{highschool,i} + \beta_1 age_i^2/10 \times I_{highschool,i} \\ &\quad + \beta_2 age_i \times I_{college,i} + \beta_3 age_i^2/10 \times I_{college,i} + \beta_4 \times I_{married,i} \end{aligned} \tag{3.30}$$

where $I_{highschool}$ is a dummy variable for households whose head has at most a high school degree, $I_{college}$ is a dummy for households whose head has at most a college degree, and $I_{married}$ is a dummy variable for households whose head is married.

Table 3.10 shows the estimates of the deterministic component of labor income using three different panel data estimation methods. All three estimation approaches give similar estimates for the deterministic function $f_{i,t}$. The coefficients in age and its quadratic term are positive and negative, respectively, reflecting the expected result that income is generally increasing in age but concave, consistent with a hump share. The pooled regression results show a larger age effect on labor income for college educated households than their high school educated counterparts, meaning that college educated households experience faster income growth and higher income levels. These results are not present from the fixed effects and Parks regressions because the cross section fixed effects $n_{i,t}$ captures most of the effects from education status. Lastly, as expected, married individuals collectively as a household earn more than their single counterparts throughout their lifetime as evidenced by the positive estimate for the dummy variable $I_{married}$.

The calibration of the life cycle model requires estimates of labor income shock variances and their covariances with stock market returns. To model the structure of the error terms, note that from equations (3.28) and (3.29) it is easy to see that the following holds

$$\begin{aligned} r_{i,t} &= y^*_{i,t} - \widehat{f_{i,t}} - \rho_i(y^*_{i,t-1} - \widehat{f_{i,t-1}}) && (3.31) \\ &= u_{i,t} + e_{i,t} && (3.32) \end{aligned}$$

I assume that $u_{i,t}$ is a shock to the permanent component of labor income that is perfectly correlated with the returns on the stock market and $e_{i,t}$ is independent. To estimate the error structure, it is convenient to express the shocks $r_{i,t}$ as a linear function of demeaned stock market returns and iid shocks.

$$r_{i,t} = u_{i,t} + e_{i,t} = \beta_i(R^S_t - R_f - \mu) + e_{i,t} \tag{3.33}$$

It is straight forward then to see that

$$\sigma^2_{r_i} = \beta^2_i \sigma^2_{R^S} + \sigma^2_{e_i} = \sigma^2_{u_i} + \sigma^2_{e_i} \qquad \text{where} \qquad \beta_i = \frac{Cov(r_i, R^S)}{\sigma^2_{R^S}} \tag{3.34}$$

I make use of these estimates as input parameters when solving and simulating the life cycle model.

3.7.3 Cross Sectional Analysis

Once the persistence parameters are estimated for each cross section, the estimates are merged with the variables in the wealth data file and cross sectional analysis is conducted. A hypothesis inherited from the theoretical portion of the paper is that the persistence parameter influences the effects that human capital and financial wealth have in determining portfolio allocation. However, the problem at hand does not simply call for the inclusion of the persistence parameter in an additive fashion as an explanatory variable in the regression for equity share, along with wealth and human capital. As predicted by the theory, persistence influences human capital and financial wealth in a rather complicated way. Therefore, I briefly discuss how the empirical study is structured, tested, and then interpreted before the results are discussed. Aiken and West (1991) contains a more thorough discussion of the methods employed in this section.

Modeling approach of cross sectional study

The standard practice in regression analysis is to enter the response variable as a linear function of the predictor variables. The coefficients inform the researcher whether the predictor variables have a nonzero linear relationship with the outcome variable in the population. However, sometimes the researcher may be interested in *how* a certain variable influences a predictor variable's effect on the response variable. For example, one may ask the question: does an explanatory variable's effect on the response variable depend on the levels of another variable? A natural way to proceed would be to first establish a hypothesis about a relationship between the predictor and response variables. Then an attempt is made to specify conditions under which such relationship would be 'strengthened' or 'weakened'. The latter step calls for interactions. For example, one may structure the following regression to investigate how the persistence parameter may influence the effects of human capital on equity allocation:

$$\log \alpha = \beta_0 + \beta_1 \rho \log Y \tag{3.35}$$

where labor income Y is a proxy for the value of human capital. The regression of $\log \alpha$ on $\log Y$ depends on specific values of ρ at which the slope of $\log \alpha$ on $\log Y$ is measured. In other words, there is a different line for the regression of $\log \alpha$ on $\log Y$ at each and every value of ρ. Because the regression of $\log \alpha$ on $\log Y$ depends on specific values of ρ, the effect of $\log Y$ is termed *conditional effect.*

If ρ is a categorical variable, equation (3.35) implies a separate regression for each category of the variable ρ. Therefore, one alternative approach for the study would be to conduct separate regressions for households who fall in different Quantile intervals of ρ. Unfortunately, this approach is associated with substantial costs. Quantile 'splits' of continuous variables throw away valuable information, reduce the power of statistical tests, and make it more difficult to detect significant effects when in fact they exist. Moreover, these costs tend to grow with the number of variables in interactions, which is of significant concern in this paper which uses three way interactions. The suggested approach to not 'split' the sample makes use of all the available information in the predictor variables without sacrificing power.

I follow the aforementioned procedure to structure my study on how the persistence parameter influences the effects that human capital and financial wealth have on equity allocation in the cross section. The full model to be looked at below calls for three way interaction between human capital, financial wealth and the persistence parameter, as well as two way interactions between the persistence parameter and each human capital and financial wealth. Such interactions can be embedded in a standard multiple regression framework simply by including product terms between the relevant variables as additional explanatory factors. Standard specification tests can be done to drop terms that are not significant before arriving at a suitable model.

In addition to interactions, a more general specification of complex relationships not captured by linear models calls for curvilinear relationships between the predictor and response variables. In such regressions, the regression equation may be structured to contain higher order terms of the individual explanatory variables in conjunction with interaction terms. Such specification is usually useful when the exact functional form of the response variable is unknown. For example, in the life cycle model, a closed form solution for the equity allocation α is not available. I merely know that at a particular time it must depend on wealth and the value of human capital. Since no functional solution is available, an approach commonly used in numerical analysis is to *approximate* unknown functions by expressing them as polynomials of relevant variables.[28] Therefore, in my study, I could extend the postulated functional form for α in equation (3.35) to contain a second order term in $\log Y$ interacting with ρ:

$$\log \alpha = \beta_0 + \beta_1 \rho \log Y + \beta_2 \rho \log^2 Y \tag{3.36}$$

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[28] See Judd (1998) for example.

where log α now can exhibit curvature in log Y that also depends on specific values of ρ.

There is no exact rule on the order of the polynomials to choose, except that most functions are fairly well approximated by low order polynomials. As for model specification, again, I can follow standard tests of significance to arrive at a refined and final model starting from a more general unrestricted specification. As it turns out, the final model with interactions between wealth, human capital and the persistence parameter contains first order and interaction terms.[29]

If an interaction exists, there are two approaches to *probe* the interaction to sharpen understanding of its meaning. The first approach is to *plot* the interactions and a second possible approach is to do *post hoc* statistical tests. Asides from the fact that the second approach can be subjected to some biases, I find that the first approach is more informative and an easier way to sharpen my understanding of interactions. The discussion of the results that follow will rely on graphical displays as a way to probe the interactions.

α as function of W and ρ

Consider first the effects of financial wealth W on equity holding α. The predictions of the current paper point to a positive relationship between α and financial wealth consistent with equity overallocation. Equity overallocation is a consequence of the positive contemporaneous correlation between human capital and financial wealth, which is strengthened by persistence in labor income shocks.

I investigate if such predictions hold in the data. To this end, I do cross sectional regression of α on a 2nd degree polynomial in the log of wealth W and their cross product with the persistence parameter ρ. The general form of the regression is:

$$\alpha = \beta_0 + \beta_1 \log W + \beta_2 \log^2 W + \beta_3 \rho \log W + \beta_4 \rho \log^2 W \qquad (3.37)$$

subject to further refinements based on regression specification error tests to determine insignificant terms.

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[29] The interaction terms are considered to be second order terms when a function is approximated using a Taylor series expansion.

Table 3.11 shows the estimates of regression (3.37) along with estimates for alternative specifications. The column under the full model (Model A) shows that the intercept estimate has a high standard error and none of the parameter estimates are statistically significant at the 10% significance level, even though the model itself is statistically significant as given by the F-Stat. This is an indication that the unrestricted model may have collinearity problems. Model B excludes the interaction terms and I arrive at the same conclusion as the unrestricted model. Although the model is significant, the intercept has a large standard error and the estimates are not significant. There seems to be some collinearity between the first and second order terms.

However, the alternative restricted model C, whose explanatory variables are composed of the cross product terms with the persistence parameter, is significant with all of its parameters significant at the 1% significance level. Moreover, the regression specification error test given by ${}_{A}F_{C}$ Stat of 1.08 indicates that one can not reject the null hypothesis that the coefficients of the factors independent of the persistent parameter are not statistically different from zero. This is an indication that the persistence parameter ρ is an important factor in determining equity holdings and it provides valuable information in the analysis.

Taking model C as the correct model, I probe the interaction based on its estimates. Figure 3.6 contains two panels with graphs of the function $\alpha(W, \rho)$ based on the estimates given in Table 3.11. Panel (a) shows $\alpha(W, \rho)$ in three dimensions along W and ρ. The graph shows that α is increasing in financial wealth and that this relationship becomes more pronounced the higher the persistence parameter is. To lend further support to the claim, panel (b) displays $\alpha(W)$ along different values of W if ρ is held fixed at values 1, 0.7 and 0.4. The conditional effect of W on α is positive and it becomes stronger the higher the value of ρ. The plots indicate that given a certain distribution of wealth in the sample, the households with more persistent labor income shocks will exhibit a wider cross sectional heterogeneity in equity allocation than households with weaker persistence. This conclusion is consistent with the theoretical predictions of the life cycle model that equity overallocation is more pronounced among households with more persistent labor income shocks.

α as function of Y and ρ

Consider now the effects of human capital on equity holding. The predictions of the current paper indicate that α should be increasing in human wealth and this relationship should be strongest for households who have labor income shocks that are highly persistent, while households with weakly persistent labor income shocks should have a more uniform holding of equity on the cross section. These are the predictions conveyed by Figure 3.4 from the theoretical part of the paper.

I investigate if such predictions also hold in the data. To this end, I do cross sectional regression of α on a 2nd degree polynomial in log of labor income Y, which is my proxy for the latent variable human capital, and their interaction terms with the persistence parameter ρ. The general form of the regression is:

$$\alpha = \beta_0 + \beta_1 \log Y + \beta_2 \log^2 Y + \beta_3 \rho \log Y + \beta_4 \rho \log^2 Y \tag{3.38}$$

subject to further refinements based on regression specification error tests to determine insignificant factors.

Table 3.12 shows the estimates of regression (3.38) along with estimates for alternative specifications. The column labeled Model A provides estimates for the unrestricted model given in equation (3.38). As it can be seen, while the intercept and the cross product terms with the persistence parameter ρ are statistically significant at the 5% significance level, the log of Y and its quadratic term are not. The model is refitted by dropping the terms that are independent of the persistence parameter and its results are given in the column under Model C. Model C is significant and has all of its coefficients significant at the 1% significance level and it has a better fit than the model without interactions (Model B). Again, this is an indication that the persistence parameter ρ is an important factor and it contains additional information beyond Y alone.

I probe the interaction in Model C. Figure 3.7 shows two panels with graphs of the function $\alpha(Y, \rho)$ based on the estimates given in Table 3.12. Panel (a) shows $\alpha(Y, \rho)$ in three dimensions along Y and ρ dimensions. The graph shows that α is increasing in the value of human capital and that this relationship becomes more pronounced the higher the persistence parameter. To lend further support to the claim, panel (b) displays $\alpha(Y)$ along different values of Y if ρ is held fixed at values 1, 0.7 and 0.4. The conditional effect of Y on α is positive and stronger the higher the value of ρ. As it can be seen, similarly to financial wealth, α is positively related to human wealth and the relation is strongest among households with most persistent income shocks. The plots indicate that given a certain distribution of labor income in the sample, the households with more persistent labor income shocks will exhibit a wider cross sectional heterogeneity in equity allocation than households with weaker persistence. This conclusion is consistent with the predictions depicted in Figure 3.4 and from the predictions of the theoretical part of the paper.

α as function of W, Y and ρ

A close look at the graphs in Figures 3.6 and 3.7 indicates that Human Wealth and Financial Wealth are closely related, at least when it comes to determining equity holdings in the cross section. While the previous analysis considered the effects of wealth and human capital on equity allocation separately (unconditional effects), it is useful to consider a model with joint effects since both variables share a certain commonality. This analysis provides additional insights on the interaction between human capital and financial wealth, and their dependence on the persistence parameter.

Recall that the predictions of the theoretical portion of the paper suggest that human wealth and financial wealth independently have opposing effects on α, however human wealth plays a more dominating role than financial wealth in the cross section if the persistence parameter is high. Additionally, if persistence is high financial wealth and human capital become positively correlated. Collectively, all of these effects help generate equity overallocation even though financial wealth alone reduces equity shares. These results are discussed in more detail in the theoretical portion of the paper and summarized in Tables 3.5 and 3.6.

I investigate if such predictions also hold in the data. To account for the possibility that equity overallocation is driven by human capital, rather than financial wealth, and to capture the effects that ρ has on the temporal relationship between the two variables, I introduce three-way interactions between these variables in my regression. I argue that incorporating three-way interactions is an appropriate way to capture the fact that human capital and financial wealth are not independently distributed in the cross section, a feature that is commonly ignored in standard regression models, and that their joint distribution is crucially dependent on the persistence parameter. I do cross sectional regression of log of α on log of labor income Y, log of financial wealth W, their cross product, and their interactions with the persistence parameter ρ. The general form of the regression is:

$$\begin{aligned}\log\alpha = \beta_0 + \beta_1 \log W + \beta_2 \log Y + \beta_3 \log W \log Y \\ + \beta_4\rho \log W + \beta_5\rho \log Y + \beta_6\rho \log W \log Y\end{aligned} \tag{3.39}$$

subject to further refinements based on regression specification error tests to determine insignificant factors.[30]

Panel (a) of Table 3.13 provides summary statistics for the explanatory variables in regression (3.39). The sample mean and median for ρ are 0.6506 and 0.7030 respectively, with a standard deviation of 0.2903, which are well within the range of values found in Guvenen (2006). The sample minimum and maximum are 0.0037 and 0.9991, which indicates that there is significant heterogeneity in labor income persistence in the cross section.

Before proceeding to the tables summarizing the results, a discussion about the covariances between predictor variables is in order. The theory predicts that the covariance between Y and W becomes more pronounced with persistence in labor income shocks. Panel (b) of Table 3.13 supports the claim. The table shows the correlations between the explanatory variables. It shows that while the sample correlation between $ln(W)$ and $ln(Y)$ is 0.27, the correlation between $ln(W) \times \rho$ and $ln(Y) \times \rho$ is 0.84, suggesting that persistence indeed strengthens the correlation between human capital and financial wealth.

On a different note, it is common in interaction studies to have the cross product between two 1st order terms become highly correlated with the variables it comprises. For example, Panel (b) of Table 3.13 shows that the correlation between $ln(W) \times ln(Y) \times \rho$ and $ln(W) \times \rho$ is 0.89. Such structural correlation in the sample may pose some challenges to regression models due to collinearity among the predictor variables. While there are no shortage of sophisticated methods proposed in the literature to handle *non-essential ill-conditioning*, I share with numerous other academics the opinion that collinearity that is not

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[30] 2nd order terms, except for interactions, turned out to be insignificant and are not included in the regression equation (3.39).

severe and that arises from *essential ill-conditioning*[31] is not harmful to the study. Collinearity does not reduce the predictive power or the reliability of the model as a whole; it only affects calculations regarding individual predictors. Furthermore, collinearity violates no regression assumptions, and estimates are still unbiased and consistent and their standard errors are still correctly estimated. The only perverse effect of collinearity is that it makes it difficult to obtain coefficient estimates with small standard errors. Collinearity produced by actual relationships between variables existing in the population does not harm the predictive power of the study, as such relationships are certain to continue to exist from sample to sample.

On the other hand, however, *severe* collinearity is still of concern in any type of study, whether the researcher is faced with an essentially or non-essentially ill-conditioned problem, *if* it results in estimates that are unreliable or inestimable. The most obvious sign of unreliable estimates due to severe collinearity is estimates that change drastically from minor changes in the data sample. I proceed with a vigil eye on signs of severe collinearity that may render my estimates unreliable. To this end, I re-estimate the final selected model using random sampling as a way to check the reliability of my original estimates.

Table 3.14 shows the estimates for various specifications of the regression model given by equation (3.39). The column representing model A provides coefficient estimates for the unrestricted model. The column labeled Model B provides estimates for the restricted model with factors without interactions with ρ only, and the column labeled model C provides estimates for the model with interaction terms only. The column under Model D provides estimates for a restricted version of Model C which excludes the first order effect of Y. The results for Model A seem to indicate that a more appropriate model is one that excludes terms independent of ρ, since their coefficient estimates are not statistically different from zero at the 5% significance level. This conclusion is consistent with the ${}_AF_B$-Stat which implies that Model B is misspecified (adding the interaction terms improve the fit of the model). Moreover, the ${}_AF_C$-Stat points to the conclusion that including the independent terms to Model C leads to over fitting. All of these results, again, reinforce the conclusion that

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[31] Essential ill-conditioning is the term coined by Marquardt (1980) to describe collinearity produced by actual relationships between variables that exist in the population, rather than in a sample.

the ρ parameter is a relevant factor in equity holdings and contain additional information beyond those contained in W and Y alone. Model D excludes the interaction term between $\log Y$ and ρ, which turned out to be insignificant at 5% significance level.

To alleviate the previously mentioned concerns about the possibility of unreliable estimates, I conduct robustness checks on the regression Model D by re-estimating its factor coefficients using random samples. For each of the ten samples, each household has the same probability of being selected and selection is done without replacement. Sample sizes are 500, 429 (3/4 of the original sample size of 570) and 400 for two, three and five of the samples respectively. Table 3.15 summarizes the results. As the table shows, the coefficient estimates and their standard errors are similar to estimates from the original sample summarized in Table 3.14 and they do not change drastically from sample to sample. Each sample gives estimates that are between -1 and + 1 standard error deviations from the original estimates. Therefore, Model D does not suffer from severe collinearity between its explanatory variables, and its coefficient estimates are reliable.

Taking model D to be the correct model, Figure 3.8 contains four panels with graphs of the function $\alpha(W, Y, \rho)$ based on estimates given in Table 3.14. Panels (a) and (b) show graphs of $\alpha(W, Y, \rho)$ when W is held fixed a two different values. As the figures show, α is weakly responsive to values of W in the cross section. Panels (c) and (d) show $\alpha(W, Y, \rho)$ when Y is held fixed a two different values. As the figures show, α is very responsive to values of Y in the cross section, and its sensitivity is increasing in the persistent parameter. This is a clear indication that in the sample human wealth has a stronger influence on equity holdings in the cross section than financial wealth and its influence is stronger for households with more persistent labor income shocks.

These results shed additional insights about the interaction between human capital and financial wealth, and its impact on asset holding. In the previous analysis when I considered α as functions of W and Y separately, I postulated that W's effect on α was a consequence of human wealth being positively correlated with financial wealth in the cross section, since both W and Y seemed to share some commonality regarding their effects on α. I concluded that even though human wealth and financial wealth independently have opposing effects on α, human wealth dominates when labor income shocks are highly persistent, resulting in rising α in W. When I consider α as a function of both W and Y and their interactions with ρ, I can see more clearly that rising equity shares in financial wealth is indeed mainly driven by human wealth and that financial wealth alone contributes weakly towards the observed phenomenon. Therefore, equity overallocation is a consequence of financial wealth being positively correlated with human capital, but it is human capital that drives it.

I attain a better understanding of the phenomenon by computing α's elasticity with respect to W based on estimates of Model D from Table 3.14.

$$\begin{aligned} \frac{\partial \log \alpha}{\partial \log W} &= -0.2726\rho + 0.0251\rho \log Y \\ &\geq 0 \quad \text{if} \quad Y \geq \$52,081.12 \end{aligned} \tag{3.40}$$

Equation (3.40) provides a measure of sensitivity of α to changes in W. It shows that the α's sensitivity to W depends on values of ρ and Y. Again, I turn to graphs to sharpen my intuition of the interactions. Figure 3.9 shows two panels. Panel (a) shows the elasticity in three dimensions along the ρ and $\log Y$ dimensions. The graph illustrates the elasticity for various levels of ρ and Y. For low values of Y, the elasticity is negative and more so for households with higher ρ. However, the elasticity is increasing in Y and more rapidly so for higher values of ρ. At some point, when Y is large enough (\$52,081.12), the elasticity becomes positive. Panel (b) shows a plot of the elasticity for confined values of ρ along different values of labor income. The elasticity is negative for low values of labor income, consistent with traditional theories that α is a decreasing function of financial wealth when human capital effects are weak or non-existent. However, when allowing for wealth interactions with human capital, the elasticity is increasing and turns positive for values of labor income greater than \$52,081.12. In other words, α's relationship with financial wealth is negative if human capital is low enough and it becomes more negative with decreasing values of human capital and larger ρ. On the other hand, α's relationship with financial wealth is positive if human capital is large enough and it gets stronger with larger values of ρ and human capital. A 1% increase (decrease) in financial wealth results in a larger percentage increase (decrease) in α for individuals with larger ρ than their low ρ counterparts. This is an indication that equity overallocation evidenced in the data is a consequence of contemporaneous effects that human capital has on financial wealth through the persistence of labor income shocks, rather than the effects of financial wealth alone. Equity overallocation is driven by households who have human capital effects that are strong enough. The strength of human capital is increasing in its value and in the persistence parameter.

3.8 Conclusions

In this article, I study the role that the persistence of labor income shocks plays in determining the joint contemporaneous distributions of human capital, wealth and wealth composition in the cross section of households. I show that the presence of highly persistent labor income shocks can generate equity shares that are rising in financial wealth in the cross section, an empirical observation that contradicts exiting theories of consumption and portfolio choice with CRRA utility. I show that equity overallocation is actually consistent with traditional theories of portfolio choice with non tradeable human capital among households with labor income shocks that are highly persistent. I show that contemporaneously human capital and financial wealth become highly correlated if labor income shocks are highly persistent. Moreover, while human capital and financial wealth independently have opposing effects on equity shares, if labor income shocks are highly persistent, human capital effects dominate in the cross section, resulting in increasing equity shares in financial wealth as evidenced in the cross section of U.S. households. These predictions are validated using PSID data after estimating a model with three way interactions between persistence parameters, financial wealth accumulations and human wealth and their joint effects on the equity shares in the cross section. Lastly, I caution researchers who base their motivation for DRRA utility on household level financial data. Empiricists may potentially encounter problems related to specification error when estimating risk parameters based on misspecified utility functions.

Table 3.1: Median fraction of wealth invested in stocks (α) sorted according to wealth quartiles based on 1999 PSID Wealth Supplemental Data File. The rows labeled nth Q represent the median α value for the nth quartile subsample when ranked by wealth. The rows labeled nth-mth represent the difference between the median αs of the nth and mth wealth ranked quartile subsamples. SCF 2001 figures are from Wachter and Yogo (2007). Overallocation is present in both PSID and SCF data and it seems stronger among the college educated group. Overallocation also exhibits a life cycle pattern.

	Age Group				
Wealth Quartile	26–35	36–45	46–55	56–65	66–75
	(a) 1999 PSID: College				
Top Q	0.49	0.75	0.75	0.72	0.79
2nd Q	0.70	0.68	0.57	0.52	0.74
3rd Q	0.59	0.62	0.63	0.65	0.68
Bottom Q	0.42	0.50	0.47	0.56	0.61
Top Q - Bottom Q	0.07	0.25	0.28	0.16	0.19
	(b) 1999 PSID: High School				
Top Q	0.49	0.52	0.59	0.55	0.72
2nd Q	0.54	0.47	0.50	0.64	0.59
3rd Q	0.47	0.48	0.47	0.49	0.57
Bottom Q	0.53	0.47	0.50	0.54	0.49
Top Q - Bottom Q	-0.04	0.05	0.09	0.02	0.23
	(c) 2001 SCF: College				
Top Q	0.68	0.71	0.70	0.64	0.64
2nd Q	0.53	0.69	0.60	0.66	0.80
3rd Q	0.41	0.55	0.46	0.53	0.64
Bottom Q	0.45	0.44	0.41	0.45	0.56
Top Q - Bottom Q	0.23	0.27	0.29	0.19	0.08
	(d) 2001 SCF: High School				
Top Q	0.31	0.58	0.60	0.46	0.62
2nd Q	0.49	0.57	0.59	0.68	0.37
3rd Q	0.47	0.49	0.60	0.39	0.09
Bottom Q	0.50	0.41	0.47	0.31	0.39
Top Q - Bottom Q	-0.19	0.17	0.13	0.15	0.23

Table 3.2: Simplified Model: Median fraction of wealth invested in stocks (α) sorted according to wealth quartiles from the simplified model of Section 3. Parameters used in the simulation are $\gamma = 10$, $g = 1.5\%$, $r_f = 2\%$, $\mu = 6\%$, $\sigma = 18\%$, $\varepsilon \sim N(0, .30^2)$, $d = 6.5\%$. The rows labeled nth Q represent the median α value for the nth wealth quartile subsample. The rows labeled nth-mth represent the difference between the median αs of the nth and mth wealth ranked quartile subsamples. Results are shown for various values of the persistence parameter (ρ) and time periods.

Wealth Quartile	α^*			
	t=10	t=20	t=40	t=60
	(a) $\rho = 1$			
Top Q	0.3713	0.3954	0.3455	0.3316
2nd Q	0.2856	0.2700	0.2480	0.2111
3rd Q	0.2819	0.2276	0.1769	0.1566
Bottom Q	0.3202	0.2244	0.1579	0.1414
Top Q - Bottom Q	0.0511	0.1710	0.1876	0.1902
	(b) $\rho = 0.95$			
Top Q	0.2484	0.2511	0.1981	0.1706
2nd Q	0.2342	0.2285	0.2021	0.1774
3rd Q	0.2351	0.2203	0.1966	0.1737
Bottom Q	0.2371	0.2177	0.1969	0.1808
Top Q - Bottom Q	0.0113	0.0334	0.0012	-0.0102
	(c) $\rho = 0.7$			
Top Q	0.2514	0.1923	0.1569	0.1445
2nd Q	0.2698	0.2038	0.1625	0.1475
3rd Q	0.2828	0.2141	0.1675	0.1506
Bottom Q	0.3062	0.2297	0.1756	0.1558
Top Q - Bottom Q	-0.0548	-0.0374	-0.0187	-0.0113
	(d) $\rho = 0.4$			
Top Q	0.2355	0.1873	0.1556	0.1436
2nd Q	0.2492	0.1956	0.1601	0.1465
3rd Q	0.2606	0.2030	0.1640	0.1491
Bottom Q	0.2786	0.2148	0.1700	0.1528
Top Q - Bottom Q	-0.0431	-0.0275	-0.0144	-0.0092

Table 3.3: Simplified Model: 2–way frequency tables in human wealth HW_t and financial wealth FW_t from the simplified model of Section 3. Values under column Top Quantile FW_t and Bottom Quantile HW_t gives the frequency within a sample of 10,000 simulated cross sections of top wealthiest investors who concurrently happened to have the lowest value of human wealth. Values under other columns are defined similarly. Frequencies are shown when the sample is divided by medians and quartiles at time periods 40 and 60.

ρ	Top Quantile HW_t		Bottom Quantile HW_t	
	Top Quantile FW_t	Bottom Quantile FW_t	Top Quantile FW_t	Bottom Quantile FW_t
		t = 40		
		(a) Median		
1	0.3820	0.1180	0.1180	0.3820
0.95	0.3820	0.1180	0.1180	0.3820
0.7	0.2720	0.2280	0.2280	0.2720
0.4	0.2600	0.2400	0.2400	0.2600
		(b) Quartile		
1	0.1660	0.0100	0.0080	0.1370
0.95	0.1610	0.0070	0.0060	0.1490
0.7	0.0610	0.0520	0.0460	0.0810
0.4	0.0510	0.0580	0.0670	0.0710
		t = 60		
		(c) Median		
1	0.3880	0.1120	0.1120	0.3880
0.95	0.3780	0.1220	0.1220	0.3780
0.7	0.2640	0.2360	0.2360	0.2640
0.4	0.2470	0.2530	0.2530	0.2470
		(d) Quartile		
1	0.1630	0.0070	0.0040	0.1350
0.95	0.1560	0.0080	0.0090	0.1370
0.7	0.0700	0.0680	0.0490	0.0720
0.4	0.0690	0.0540	0.0540	0.0760

Table 3.4: Parameters in the calibration of the life cycle model. Variances for labor income shocks and covariances with stock returns are estimated from PSID. See Table 3.10 for coefficients of the deterministic component of labor income f_t. Baseline parameter values are superscripted with *.

Variable Name	Symbol	Parameter Value
Adult Life:		
Beginning adult age	t_0	19
Retirement age	T	65
Preferences:		
Discount Factor	β	0.96
Risk Aversion	γ	$3, 5^*, 7$
Elasticity of Intertemporal Substitution	ψ	$0.15, 0.3, 0.5^*, 0.7$
Retirement Savings Motive	b	$1^*, 5, 10$
Labor Income:		
Persistence Parameter	ρ	$1^*, 0.7, 0.4$
Variance: (College)	$\sigma^2_{u_1}$	0.0012^*
Variance: (College)	$\sigma^2_{e_1}$	0.1477^*
Variance: (High School)	$\sigma^2_{u_2}$	0.0002
Variance: (High School)	$\sigma^2_{e_2}$	0.1455
Asset Market:		
Risk Free Rate	$R_f - 1$	0.02
Equity Premium	μ^S	0.04
Variance: Equity	$\sigma^2_{\varepsilon^S}$	0.0324
Covariance between u_1 and R^S(College)	$\sigma_{\varepsilon^S u_1}$	-0.00435^*
Covariance between u_2 and R^S (High School)	$\sigma_{\varepsilon^S u_2}$	-0.0016

Table 3.5: Life Cycle Model: 2–way frequency tables when data is divided based on quantile values of labor income Y_t and financial wealth W_t. Values under column Top Quantile W_t and Bottom Quantile Y_t gives the frequency within a sample of 10,000 simulated cross sections of top wealthiest investors who concurrently happened to have the lowest Y_t values. Values under other columns are defined similarly. Frequencies are shown when the sample is divided by medians and quartiles at ages 45 and 55.

ρ	Top Quantile Y_t		Bottom Quantile Y_t	
	Top Quantile W_t	Bottom Quantile W_t	Top Quantile W_t	Bottom Quantile W_t
		age = 45		
		(a) Median		
1	0.4245	0.0755	0.0755	0.4245
0.95	0.3829	0.1171	0.1171	0.3829
0.7	0.2934	0.2066	0.2066	0.2934
0.4	0.2733	0.2267	0.2267	0.2733
		(b) Quartile		
1	0.1952	0.0000	0.0004	0.1825
0.95	0.1609	0.0015	0.0058	0.1517
0.7	0.0918	0.0361	0.0388	0.0888
0.4	0.0758	0.0489	0.0517	0.0794
		age = 55		
		(c) Median		
1	0.4265	0.0735	0.0735	0.4265
0.95	0.3710	0.1290	0.1290	0.3710
0.7	0.2860	0.2140	0.2140	0.2860
0.4	0.2646	0.2354	0.2354	0.2646
		(d) Quartile		
1	0.2020	0.0000	0.0003	0.1799
0.95	0.1532	0.0039	0.0078	0.1440
0.7	0.0870	0.0363	0.0446	0.0855
0.4	0.0724	0.0515	0.0556	0.0716

Table 3.6: Life Cycle Model: Median fraction of wealth invested in stocks (α) sorted according to wealth quartiles from the life cycle model of Section 3. The rows labeled nth Q represent the median α value for the nth quartile subsample when ranked by financial wealth. The rows labeled nth-mth represent the difference between the median αs of the nth and mth wealth ranked quartiles. Results are shown for various values of the persistence parameter (ρ), age and education groups.

Wealth Quartile	College				High School			
	Age Group				Age Group			
	26–35	36–45	46–55	56–65	26–35	36–45	46–55	56–65
(a) $\rho = 1$								
Top Q	0.52	0.46	0.42	0.40	0.42	0.38	0.36	0.35
2nd	0.43	0.33	0.30	0.29	0.35	0.30	0.28	0.32
3rd	0.43	0.33	0.30	0.34	0.35	0.30	0.28	0.37
Bottom Q	0.41	0.33	0.31	0.50	0.34	0.30	0.30	0.41
Top Q - Bottom Q	0.11	0.12	0.11	-0.10	0.08	0.08	0.06	-0.06
(b) $\rho = 0.95$								
Top Q	0.54	0.46	0.42	0.40	0.43	0.38	0.36	0.36
2nd	0.54	0.43	0.38	0.39	0.41	0.36	0.34	0.36
3rd	0.54	0.43	0.38	0.40	0.41	0.36	0.34	0.35
Bottom Q	0.52	0.43	0.37	0.42	0.40	0.36	0.33	0.40
Top Q - Bottom Q	0.02	0.03	0.05	-0.03	0.03	0.02	0.03	-0.04
(c) $\rho = 0.7$								
Top Q	0.85	0.63	0.48	0.43	0.64	0.51	0.43	0.40
2nd	0.88	0.89	0.63	0.50	0.69	0.65	0.52	0.46
3rd	0.86	0.92	0.66	0.52	0.67	0.67	0.54	0.47
Bottom Q	0.83	0.96	0.73	0.57	0.66	0.69	0.56	0.52
Top Q - Bottom Q	0.02	-0.32	-0.24	-0.14	-0.02	-0.18	-0.13	-0.11
(d) $\rho = 0.4$								
Top Q	0.98	1.00	0.75	0.54	0.80	0.79	0.63	0.50
2nd	0.95	1.00	1.00	0.68	0.67	0.82	0.75	0.57
3rd	0.94	1.00	1.00	0.72	0.66	0.80	0.74	0.59
Bottom Q	0.92	1.00	1.00	0.82	0.74	0.76	0.72	0.64
Top Q - Bottom Q	0.06	-0.00	-0.25	-0.29	0.06	0.03	-0.09	-0.14

Table 3.7: Life Cycle Model: Median fraction of wealth invested in stocks (α) sorted according to wealth quartiles from the life cycle model of Section 4. The rows labeled nth Q represent the median α value for the nth quartile subsample when ranked according to wealth. The rows labeled nth-mth represent the difference between the median αs of the nth and mth wealth ranked quartile subsamples. Results are shown for ρ value of 1 and various parameters of relative risk aversion (γ), age groups and education groups.

Wealth Quartile	College Age Group				High School Age Group			
	26–35	36–45	46–55	56–65	26–35	36–45	46–55	56–65
				(a) $\gamma = 3$				
Top Q	0.96	0.83	0.75	0.70	0.76	0.69	0.65	0.61
2nd	0.74	0.57	0.51	0.49	0.66	0.53	0.49	0.59
3rd	0.73	0.56	0.50	0.61	0.66	0.53	0.48	0.66
Bottom Q	0.74	0.59	0.56	0.83	0.69	0.59	0.66	0.74
Top Q - Bottom Q	0.22	0.25	0.19	-0.13	0.08	0.10	-0.01	-0.13
				(b) $\gamma = 5$				
Top Q	0.52	0.46	0.42	0.40	0.42	0.38	0.36	0.35
2nd	0.43	0.33	0.30	0.29	0.35	0.30	0.28	0.32
3rd	0.43	0.33	0.30	0.34	0.35	0.30	0.28	0.37
Bottom Q	0.41	0.33	0.31	0.50	0.34	0.30	0.30	0.41
Top Q - Bottom Q	0.11	0.12	0.11	-0.10	0.08	0.08	0.06	-0.06
				(c) $\gamma = 7$				
Top Q	0.35	0.31	0.29	0.29	0.28	0.26	0.25	0.25
2nd	0.29	0.23	0.21	0.21	0.24	0.21	0.20	0.22
3rd	0.28	0.23	0.21	0.23	0.23	0.21	0.20	0.26
Bottom Q	0.27	0.23	0.22	0.35	0.23	0.21	0.21	0.29
Top Q - Bottom Q	0.08	0.08	0.08	-0.06	0.05	0.05	0.04	-0.05

Table 3.8: Life Cycle Model: Median fraction of wealth invested in stocks (α) sorted according to wealth quartiles from the life cycle model of Section 4. The rows labeled nth Q represent the median α value for the nth quartile subsample when ranked according to wealth. The rows labeled nth-mth represent the difference between the median αs of the nth and mth wealth ranked quartile subsamples. Results are shown for ρ and γ values of 1 and 5, respectively, and various parameters of EIS (ψ), age groups and education groups.

Wealth Quartile	College Age Group				High School Age Group			
	26–35	36–45	46–55	56–65	26–35	36–45	46–55	56–65
				(a) $\gamma = 5$ $\psi = 0.7$				
Top Q	0.52	0.45	0.41	0.34	0.41	0.37	0.35	0.31
2nd	0.42	0.33	0.29	0.27	0.34	0.30	0.28	0.26
3rd	0.42	0.33	0.29	0.27	0.34	0.29	0.28	0.26
Bottom Q	0.39	0.32	0.29	0.27	0.33	0.29	0.28	0.27
Top Q-Bottom Q	0.12	0.13	0.13	0.06	0.08	0.08	0.08	0.04
				(b) $\gamma = 5$ $\psi = 0.5$				
Top Q	0.52	0.46	0.42	0.35	0.42	0.38	0.36	0.32
2nd	0.43	0.33	0.30	0.28	0.34	0.30	0.28	0.27
3rd	0.42	0.33	0.29	0.28	0.34	0.29	0.28	0.27
Bottom Q	0.40	0.32	0.30	0.31	0.33	0.29	0.29	0.32
Top Q-Bottom Q	0.13	0.13	0.12	0.05	0.09	0.08	0.07	0.00
				(c) $\gamma = 5$ $\psi = 0.3$				
Top Q	0.53	0.46	0.43	0.38	0.42	0.38	0.36	0.34
2nd	0.43	0.33	0.30	0.28	0.35	0.30	0.28	0.28
3rd	0.43	0.33	0.30	0.29	0.34	0.30	0.28	0.33
Bottom Q	0.40	0.33	0.31	0.41	0.33	0.29	0.29	0.39
Top Q-Bottom Q	0.13	0.13	0.12	-0.03	0.09	0.09	0.07	-0.05
				(d) $\gamma = 5$ $\psi = 0.15$				
Top Q	0.53	0.46	0.43	0.40	0.42	0.38	0.36	0.35
2nd	0.43	0.34	0.30	0.29	0.35	0.30	0.28	0.32
3rd	0.43	0.33	0.30	0.33	0.34	0.30	0.28	0.37
Bottom Q	0.39	0.33	0.31	0.48	0.33	0.29	0.29	0.41
Top Q-Bottom Q	0.14	0.13	0.12	-0.08	0.09	0.09	0.07	-0.06

Table 3.9: Life Cycle Model: (a) Median fraction of wealth invested in stocks (α), (b) median consumption share (κ) and (c) median values of financial wealth (W) sorted according to wealth quartiles from the life cycle model of Section 4. The rows labeled nth Q represent the median value for the nth quartile subsample when ranked according to financial wealth. The rows labeled nth-mth represent the difference between the median of the nth and mth wealth ranked quartile subsamples.

Wealth Quartile	College Age Group				College Age Group			
	26–35	36–45	46–55	56–65	26–35	36–45	46–55	56–65
	$b = 5$				$b = 10$			
(a) α								
Top Q	0.52	0.45	0.42	0.34	0.57	0.49	0.50	0.42
2nd	0.43	0.33	0.30	0.27	0.40	0.25	0.31	0.28
3rd	0.43	0.33	0.29	0.27	0.42	0.23	0.30	0.28
Bottom Q	0.41	0.33	0.30	0.29	0.39	0.27	0.30	0.28
Top Q - Bottom Q	0.11	0.12	0.11	0.04	0.18	0.21	0.20	0.14
(b) κ								
Top Q	0.09	0.09	0.11	0.16	0.01	0.01	0.07	0.13
2nd	0.07	0.06	0.08	0.13	0.02	0.04	0.06	0.08
3rd	0.07	0.06	0.08	0.13	0.02	0.04	0.06	0.08
Bottom Q	0.07	0.06	0.08	0.16	0.03	0.05	0.06	0.08
Top Q - Bottom Q	0.02	0.03	0.03	0.01	-0.02	-0.04	0.01	0.04
(c) W in \$1,000,000's								
Top Q	0.31	1.15	2.33	3.33	0.56	2.65	4.50	5.42
2nd	0.06	0.08	0.09	0.06	0.09	0.15	0.16	0.15
3rd	0.05	0.07	0.07	0.05	0.08	0.12	0.12	0.11
Bottom Q	0.04	0.05	0.04	0.03	0.06	0.08	0.07	0.06
Top Q - Bottom Q	0.27	1.10	2.29	3.31	0.50	2.57	4.42	5.35

Table 3.10: Estimates for the deterministic component of labour income $ln(F_{i,t}) = f_{i,t}$. Three approaches are used. i) Pooled Regression, ii) One Way Fixed Effects and iii) Dynamic Panel (Parks) Method after controlling for cross section fixed effects. Standard errors are given in parenthesis.

Explanatory Variable	Estimates		
	Pooled Regression	One Way Fixed Effect	Dynamic Panel (Parks) Method
$age \times I_{highschool}$	0.3896^{***}	0.2440^{***}	0.2581^{***}
	(0.0056)	(0.0312)	(0.0030)
$age^2/10 \times I_{highschool}$	-0.0489^{***}	-0.0335^{***}	-0.0369^{***}
	(0.0011)	(0.0038)	(0.0051)
$age \times I_{college}$	0.4053^{***}	0.2349^{***}	0.2364^{***}
	(0.0106)	(0.0365)	(0.0170)
$age^2/10 \times I_{college}$	-0.0484^{***}	-0.0289^{***}	-0.0279^{***}
	(0.0024)	(0.0053)	(0.0029)
$I_{married}$	2.0969^{***}	1.8773^{***}	1.8926^{***}
	(0.0815)	(0.1256)	(0.0407)
R^2	0.8174	0.4310	0.7338
N	1196	1196	1196

$^{***}p < .01;^{**} p < .05;^{**} p < .1;$

Table 3.11: Estimates for $\alpha(W, \rho)$. Model A is the unrestricted model with explanatory factors composed of 1st and 2nd degree polynomials in $log(W)$ and their interactions with ρ. Models B and C are restricted models without interaction terms, and with interaction terms only, respectively.

Explanatory Variable	Estimates		
	Model A	Model B	Model C
$Intercept$	0.5468	0.6033	0.5770^{***}
	(0.5299)	(0.5272)	(0.0275)
$log(W)$	-0.0116	-0.0484	
	(0.0970)	(0.0925)	
$log^2(W)$	0.0012	0.0040	
	(0.0045)	(0.0040)	
$log(W) \times \rho$	-0.0419		-0.0644^{***}
	(0.0321)		(0.0144)
$log^2(W) \times \rho$	0.0037		0.0056^{***}
	(0.0027)		(0.0011)
R^2	0.0530	0.0494	0.0519
Adj-R^2	0.0463	0.0460	0.0486
F-Val	7.93^{***}	14.78^{***}	15.58^{***}
${}_AF_B$ Stat		0.32	
${}_AF_C$ Stat			1.08
N	570	570	570

$^{***}p < .01;^{**}p < .05;^{*}p < .1$; Ramsey's specification test is given by the ${}_{\text{Unrestricted Model}}F_{\text{Restricted Model}}$ Stat; Standard Errors in parenthesis

Table 3.12: Estimates for $\alpha(Y, \rho)$. Model A is the unrestricted model with explanatory factors composed of 1st and 2nd degree polynomials in $log(Y)$ and their interactions with ρ. Models B and C are restricted models without interaction terms, and with interaction terms only, respectively.

Explanatory Variable		Estimates	
	Model A	Model B	Model C
$Intercept$	1.7053^{**}	2.1205^{***}	0.5733^{***}
	(0.8103)	(0.7801)	(0.0279)
$log(Y)$	-0.1848	-0.3331^{**}	
	(0.1652)	(0.1470)	
$log^2(Y)$	0.0074	0.0175^{**}	
	(0.0086)	0.(0070)	
$log(Y) \times \rho$	-0.1278^{**}		-0.0959^{***}
	(0.0634)		(0.0309)
$log^2(Y) \times \rho$	0.0116^{**}		0.0087^{***}
	(0.0056)		(0.0027)
R^2	0.0269	0.0182	0.0215
Adj-R^2	0.0200	0.0147	0.0180
F-Val	3.90^{***}	5.25^{***}	6.22^{***}
${}_AF_B$ Stat		1.56	
${}_AF_C$ Stat			2.52^{*}
N	570	570	570

$^{***}p < .01;^{**} p < .05;^{*} p < .1$; Ramsey's specification test is given by the ${}_{\text{Unrestricted Model}}F_{\text{Restricted Model}}$ Stat; Standard Errors in parenthesis

Table 3.13: Summary statistics and correlations between the explanatory variables used in the cross sectional regression model. Summary statistics are sample size (N), mean, median, standard deviation (stdev), sum, minimum value (min) and maximum value (max).

(a) Summary Statistics

Explanatory Variable	N	mean	median	stdev	sum	min	max
ρ	570	0.6506	0.7030	0.2903	372.1665	0.0037	0.9991
$log(Y)$	570	11.1604	11.2152	0.7056	6361	5.0106	13.1422
$log(W)$	570	11.4658	11.4583	1.3691	6558	7.6009	16.7557
$log(W) \times log(Y)$	570	128.2577	127.7278	19.2900	73107	61.2100	202.2299
$log(Y) \times \rho$	570	7.2896	7.7974	3.3269	4155	0.0386	12.3563
$log(W) \times \rho$	570	7.5005	7.8820	3.5658	4290	0.0394	14.4692
$log(W) \times log(Y) \times \rho$	570	84.2304	87.5481	41.2442	48011	0.4113	169.7694

(b) Correlation Table

	$log(Y)$	$log(W)$	$log(W) \times log(Y)$	$log(Y) \times \rho$	$log(W) \times \rho$	$log(W) \times log(Y) \times \rho$
$log(Y)$	1	0.27	0.64	0.27	0.23	0.32
$log(W)$	0.27	1	0.71	0.14	0.33	0.36
$log(W) \times log(Y)$	0.64	0.71	1	0.22	0.36	0.42
$log(Y) \times \rho$	0.27	0.14	0.22	1	0.84	0.87
$log(W) \times \rho$	0.23	0.33	0.36	0.84	1	0.89
$log(W) \times log(Y) \times \rho$	0.32	0.36	0.42	0.87	0.89	1

Table 3.14: Estimates for $log(\alpha(W, Y, \rho))$. Model A is the unrestricted model with explanatory factors composed of $log(W)$, $log(Y)$, their cross product term and their interactions with ρ. Models B and C are restricted models without interaction terms and with interaction terms with ρ, respectively. Model D is a restricted version of Model C without interaction terms between $log(Y)$ and ρ.

Explanatory Variable	Estimates			
	Model A	Model B	Model C	Model D
$Intercept$	2.3025	1.4024	-0.7591^{***}	-0.7999^{***}
	(4.7313)	(4.7303)	(0.0825)	(0.0791)
$log(W)$	-0.1501	-0.2687		
	(0.4010)	(0.4000)		
$log(Y)$	-0.2555	-0.2411		
	(0.4304)	(0.4230)		
$log(W) \times log(Y)$	0.0116	0.0284		
	(0.0361)	(0.0356)		
$log(W) \times \rho$	-0.3995^{**}		-0.2354^{***}	-0.2726^{***}
	(0.1618)		(0.0850)	(0.0823)
$log(Y) \times \rho$	-0.0895		-0.0700^{*}	
	(0.0979)		(0.0410)	
$log(W) \times log(Y) \times \rho$	0.0439^{***}		0.0274^{***}	0.0251^{***}
	(0.0145)		(0.0072)	(0.0071)
R^2	0.0349	0.0185	0.0313	0.0263
Adj-R^2	0.0246	0.0133	0.0262	0.0229
F-Val	3.39^{***}	3.55^{**}	6.09^{***}	7.66^{***}
${}_AF_B$ Stat		3.19^{**}		
${}_AF_C$ Stat			0.69	
${}_CF_D$ Stat				2.91^{*}
N	570	570	570	570

$^{***}p < .01;^{**} p < .05;^{*} p < .1$; Ramsey's specification test is given by the ${}_{\text{Unrestricted Model}}F_{\text{Restricted Model}}$ Stat; Standard Errors in parenthesis

Table 3.15: Estimates for $log(\alpha(W, Y, \rho)) = \beta_0 + \beta_1 log(W)\rho + \beta_3 log(W) log(Y)\rho$ done with random sampling. Each unit has an equal probability of selection and sampling is done without replacement. Sample sizes are 500, 429 (3/4 of original sample) and 400 for two, three and five of the ten samples. Each sample gives estimates that are between -1 and + 1 standard error deviations from the original estimates given in Table 3.14 under Model D, indicating that the original estimates from Table 3.14 are reliable.

Explanatory Variable	Estimates				
	Sample A	Sample B	Sample C	Sample D	Sample E
$Intercept$	-0.8058^{***}	-0.8082^{***}	-0.765^{***}	-0.8360^{***}	-0.81189^{***}
	(0.0877)	(0.0889)	(0.0887)	(0.0957)	(0.0853)
$log(W) \times \rho$	-0.2873^{***}	-0.2701^{***}	-0.2465^{***}	-0.2794^{***}	-0.2235^{***}
	(0.0882)	(0.0897)	(0.0887)	(0.0988)	(0.0876)
$log(W) \times log(Y) \times \rho$	0.0265^{***}	0.0249^{***}	0.0225^{***}	0.0260^{***}	0.0213^{***}
	(0.0076)	(0.0078)	(0.0077)	(0.0085)	(0.0076)
R^2	0.0289	0.0251	0.0233	0.0287	0.0277
Adj-R^2	0.0211	0.0187	0.0242	0.0309	0.0231
F-Val	7.36^{***}	6.39^{***}	5.07^{***}	6.29^{***}	6.03^{***}
N	500	500	429	429	429
	Sample F	Sample G	Sample H	Sample I	Sample J
$Intercept$	-0.7909^{***}	-0.8726^{***}	-0.7826^{***}	-0.7598^{***}	-0.8309^{***}
	(0.0981)	(0.0994)	(0.0911)	(0.0878)	(0.0839)
$log(W) \times \rho$	-0.3271^{***}	-0.3059^{***}	-0.3174^{***}	-0.2618^{***}	-0.2509^{***}
	(0.1033)	(0.1023)	(0.0931)	(0.0888)	(0.0880)
$log(W) \times log(Y) \times \rho$	0.02985^{***}	0.0296^{***}	0.0290^{***}	0.0237^{***}	0.0237^{***}
	(0.0089)	(0.0088)	(0.0080)	(0.0077)	(0.0076)
R^2	0.0326	0.0361	0.0346	0.0237	0.0321
Adj-R^2	0.0313	0.0187	0.0301	0.0191	0.0276
F-Val	6.68^{***}	7.42^{***}	7.62^{***}	5.14^{***}	7.05^{***}
N	400	400	400	400	400

$^{***}p < .01; ^{**}p < .05; ^{*}p < .1$; Standard Errors in parenthesis

Figure 3.1: Fraction of financial wealth invested in equity according to Merton (1971), model with human capital, and fitted values on log of financial wealth using 1999 PSID wealth data. Parameters used for theoretical solutions are $\gamma = 2$, $r_f = 2\%$, $\mu = 6\%$, $\sigma = 18\%$ and $HW = 50,000$. PSID regression coefficients are estimates: $\alpha = 0.08409 + 0.04389 \times log(FW)$.

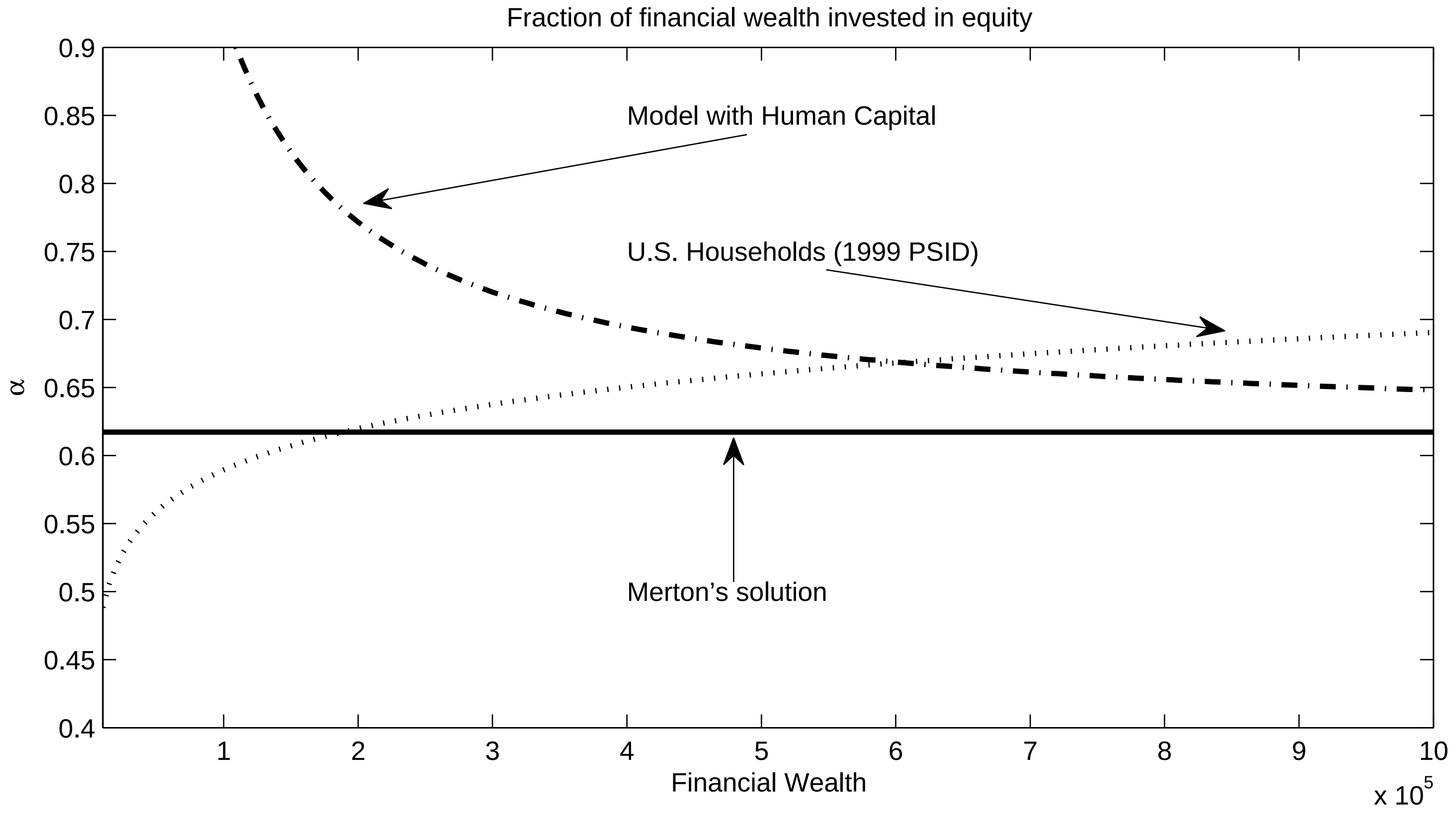
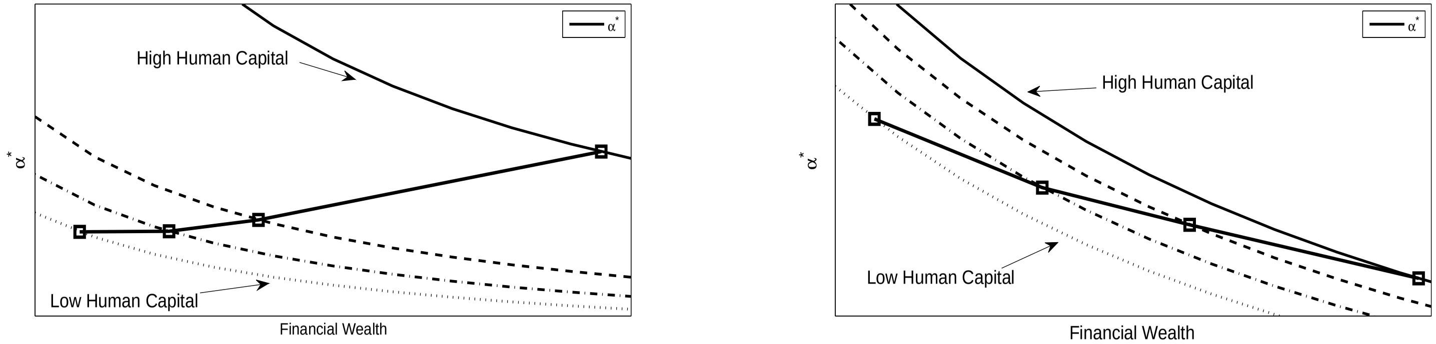


Figure 3.2: Simple Model: Illustration of joint distributions of FW, HW and α^* in the cross section. Panels (a) and (b) display α^*'s profile across likely joint values of FW and HW in the cross section if $\rho = 1$ and $\rho = 0.4$, respectively. Panel (c) shows predicted values of α^* from a regression of equity shares on $\log(FW)$ based on simulated data for ρ values of 1 and 0.4. $\alpha = 0.0849 + 0.0862 \times log(FW)$ if $\rho = 1$ and $\alpha = 0.6267 - .0632 \times log(FW)$ if $\rho = 0.4$



(a) FW, HC and α^* in the cross section if $\rho = 1$

(b) FW, HC and α^* in the cross section if $\rho = 0.4$

(c) Predicted values of α^* from simulated data

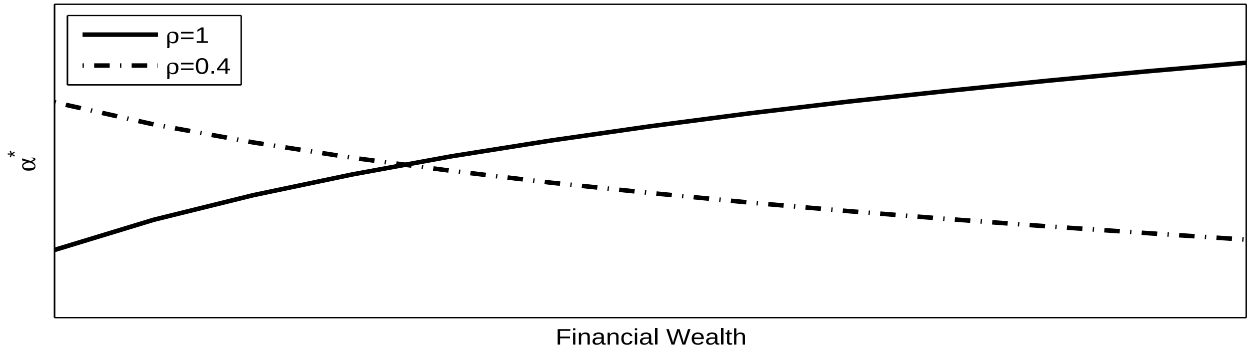
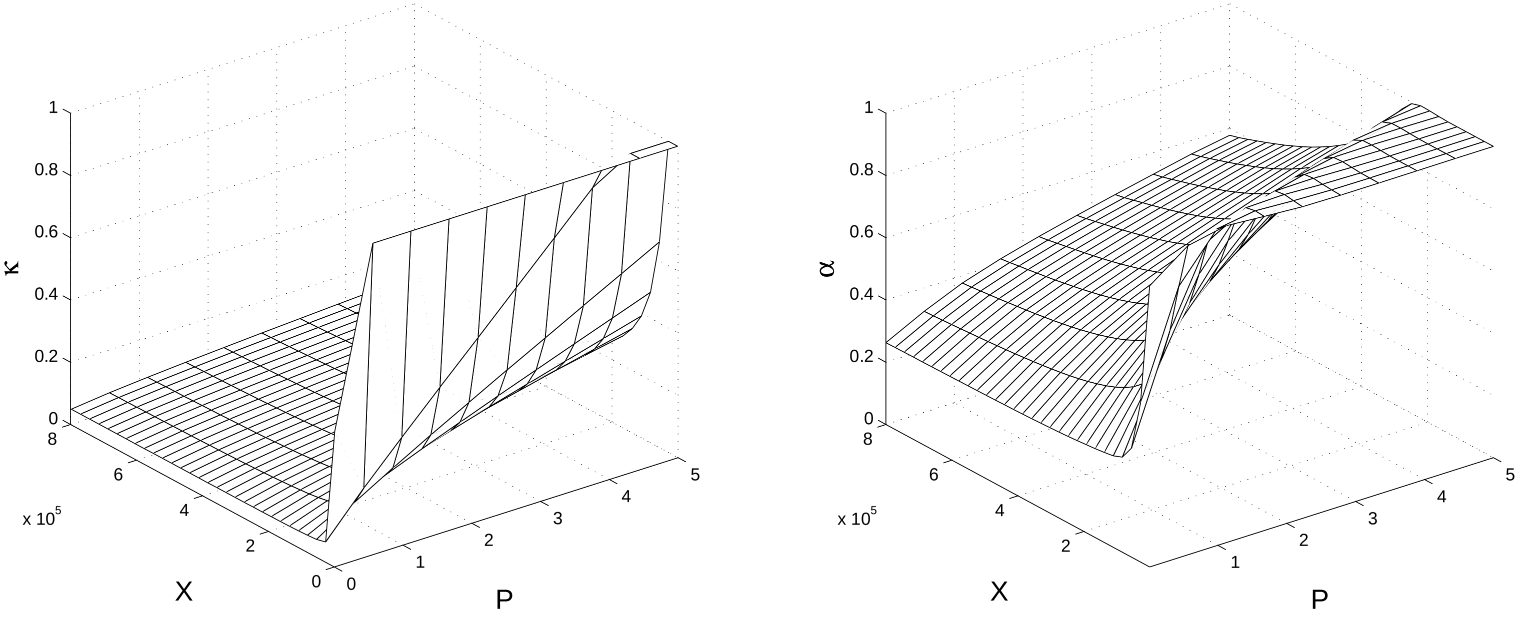


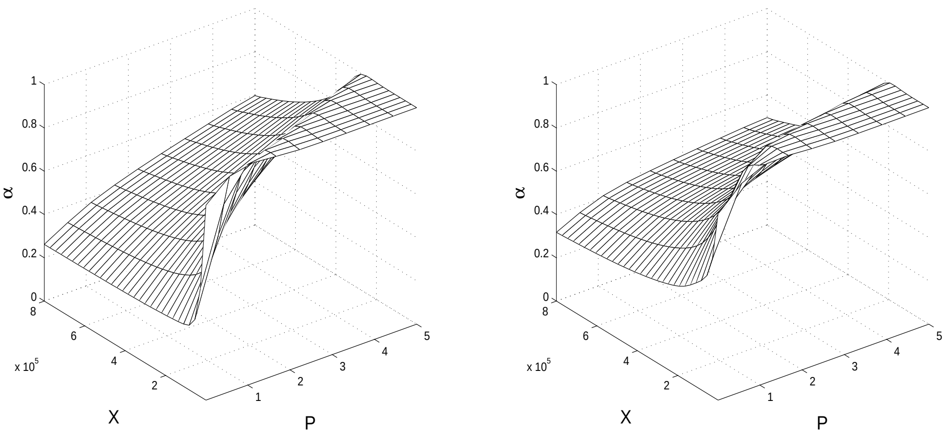
Figure 3.3: Life Cycle Model for baseline parameters: Panel (a) Optimal consumption policy κ at age 35. Panel (b) Optimal investment policy in equity α at age 35. The state variables are cash on hand X_t and permanent component of labor income P_t.



(a) Optimal κ at age 35 and $\rho = 1$

(b) Optimal α at age 35 and $\rho = 1$

Figure 3.4: Life Cycle Model: Optimal investment policy in equity α at age 35 for (a) $\rho = 1$, (b) $\rho = 0.7$ along the state variables X_t and P_t. Panel (c) shows optimal α along P_t values at age 35 for ρ values of 1, 0.95, 0.7 and 0.4. α's sensitive to values of P_t is increasing in the persistence in labor income shocks.



(a) $\alpha(P_t, X_t; t = 35, \rho = 1)$ (b) $\alpha(P_t, X_t; t = 35, \rho = 0.7)$

(c) $\alpha(P_t; X_t = 331621; t = 35, \rho = 1, 0.95, 0.7, 0.4)$

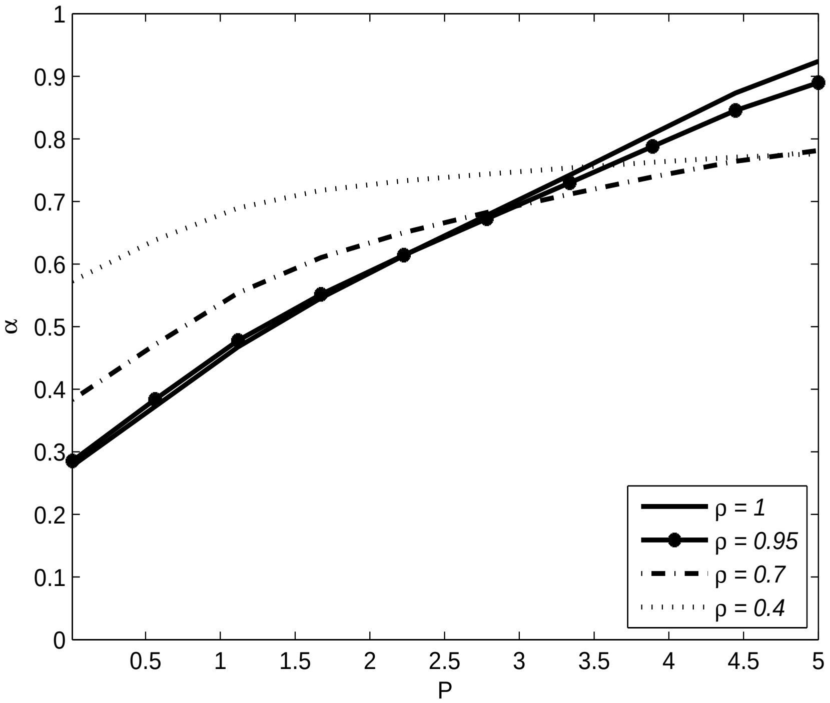
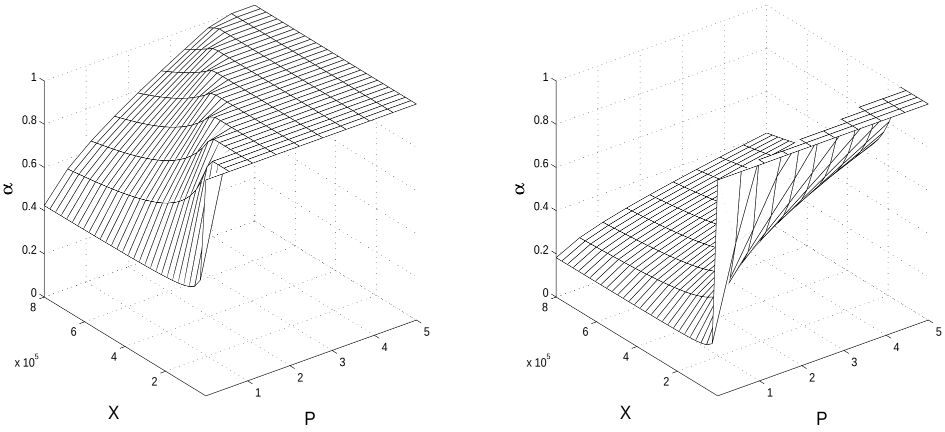


Figure 3.5: Life Cycle Model: Optimal investment policy in equity α at age 35 for (a) $\gamma = 3$, (b) $\gamma = 7$ along the state variables X_t and P_t. Panel (c) shows optimal α along P_t values at age 35 for γ values of 3, 5 and 7. α is uniformly higher and display greater sensitivity (slope) to P_t for lower values of γ.



(a) $\alpha(P_t, X_t; t = 35, \gamma = 3)$

(b) $\alpha(P_t, X_t; t = 35, \gamma = 7)$

(c) $\alpha(P_t; X_t = 524482; t = 35, \gamma = 3, 5, 7)$

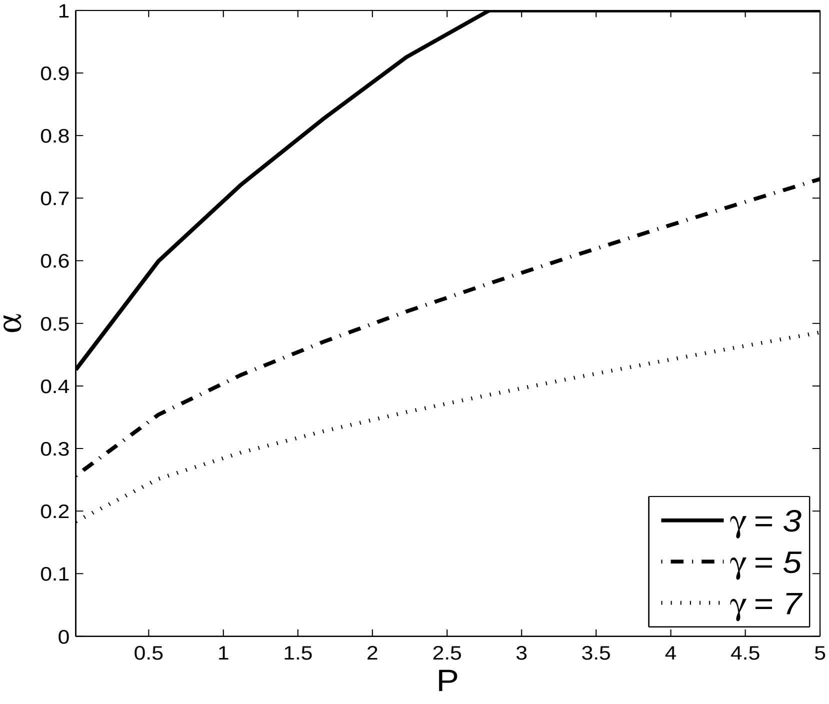
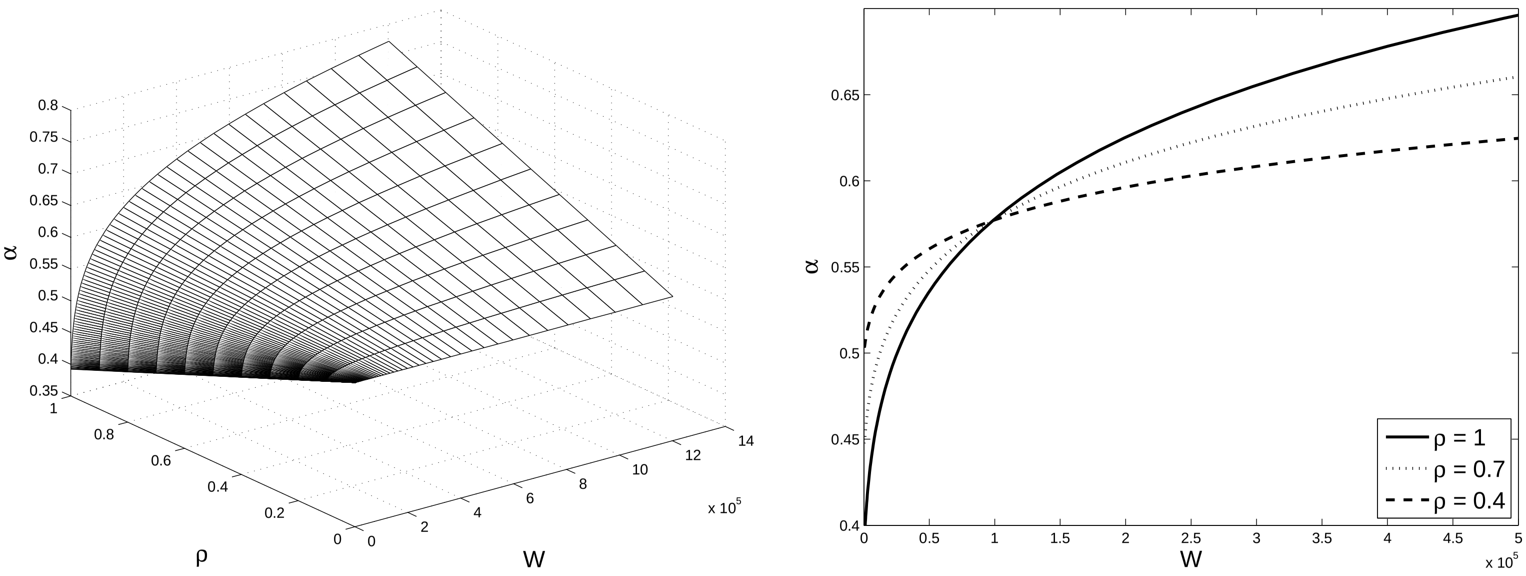


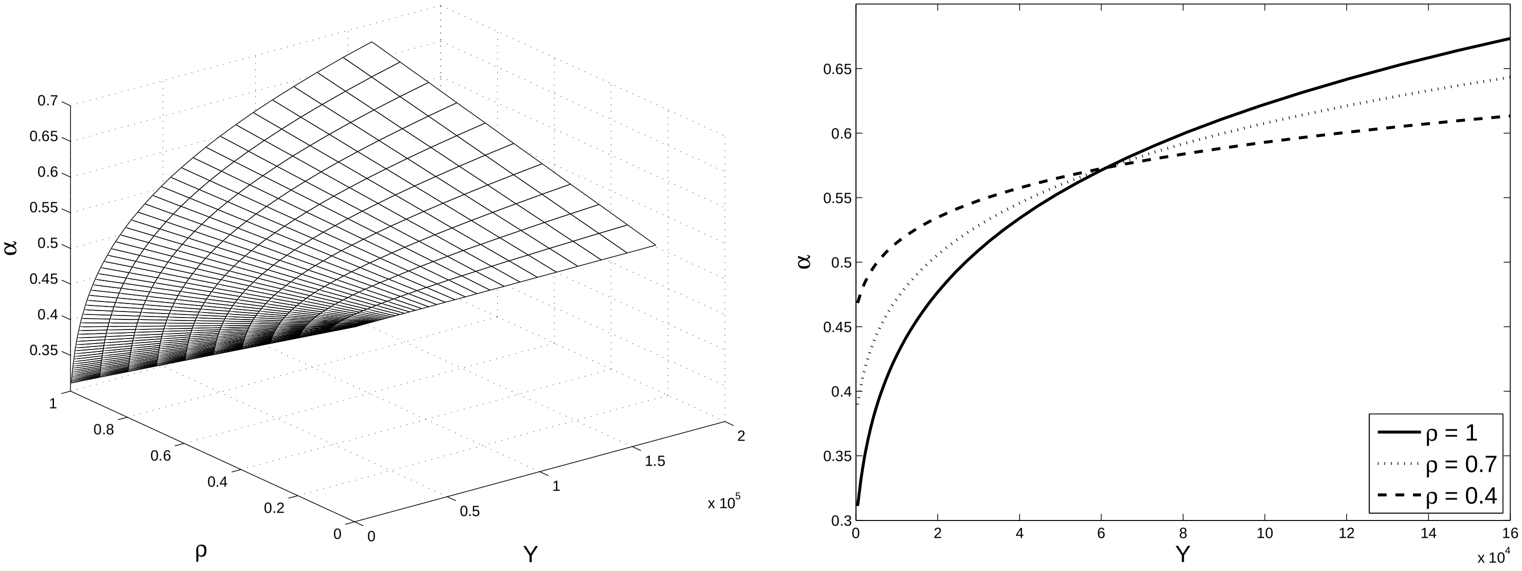
Figure 3.6: Empirical estimates: (a) $\alpha(W,\rho)$ and (b) $\alpha(W;\rho = 1, 0.7, 0.4)$ where $\alpha(W,\rho) = .5770^{***} - 0.0644^{***}log(W)\rho + 0.0056^{***}log(W)^2\rho$ is estimated from PSID 1999 wealth data. α is increasing in W reflective of the overallocation phenomenon present in the data. Overallocation in the cross section is increasing in ρ.



(a) $\alpha(W,\rho) \quad = 0.5770^{***} - 0.0644^{***}log(W)\rho$
$+0.0056^{***}log(W)^2\rho$

(b) $\alpha(W;\rho = 1, 0.7, 0.4)$

Figure 3.7: Empirical estimates: (a) $\alpha(Y,\rho)$ and (b) $\alpha(Y;\rho=1,0.7,0.4)$ where $\alpha(Y,\rho)=0.5733^{***}-0.0959^{***}log(Y)\rho+0.0087^{***}log(Y)^2\rho$ is estimated from PSID 1999 wealth data. α is increasing in Y. α's sensitive to Y is increasing in ρ. Persistent in labor income shocks strengthens human wealth effects on α in the cross section.

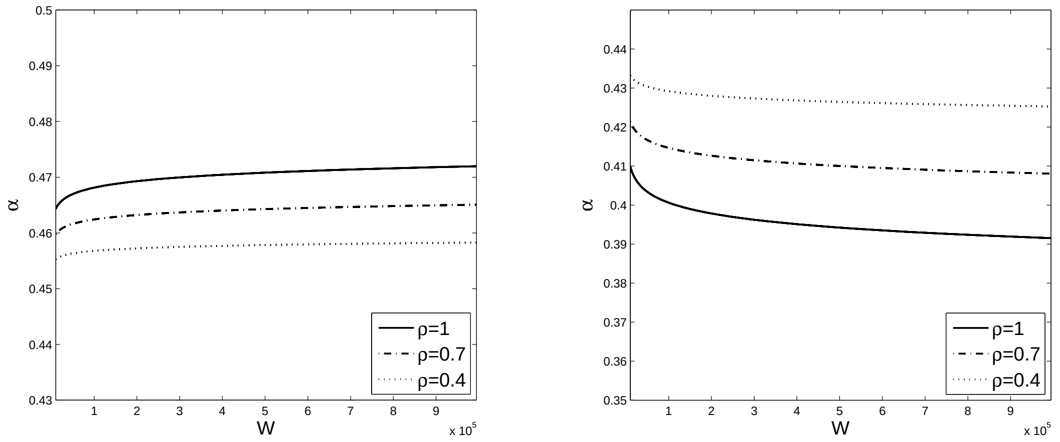


(a) $\alpha(Y,\rho) \quad = 0.5733^{***} - 0.0959^{***}log(Y)\rho$
$+0.0087^{***}log(Y)^2\rho$

(b) $\alpha(W;\rho=1,0.7,.04)$

Figure 3.8: Empirical estimates: (a) $\alpha(W;Y=35000,\rho=1,0.7,0.4)$, (b) $\alpha(W;Y=60000,\rho=1,0.7,0.4)$, (c) $\alpha(Y;W=250000,\rho=1,0.7,0.4)$ and (d) $\alpha(Y;W=500000,\rho=1,0.7,0.4)$ where $log(\alpha(W,Y,\rho)) = -0.7999^{***} - 0.2726^{***}log(W)\rho + 0.0251^{***}log(W)log(Y)\rho$ is estimated from PSID 1999 wealth data. Panels (a) and (b) show that α is weakly sensitive to values of W. Panels (c) and (d) show that α is sensitive to values of Y and its sensitivity is increasing in ρ. The 3-way interaction model shows that human wealth effects dominates financial wealth on α. Human wealth's relative importance is increasing in the persistence parameter.

(a) $\alpha(W;Y=35000,\rho=1,0.7,0.4)$ (b) $\alpha(W;Y=60000,\rho=1,0.7,0.4)$



(c) $\alpha(Y;W=250000,\rho=1,0.7,0.4)$ (d) $\alpha(Y;W=500000,\rho=1,0.7,0.4)$

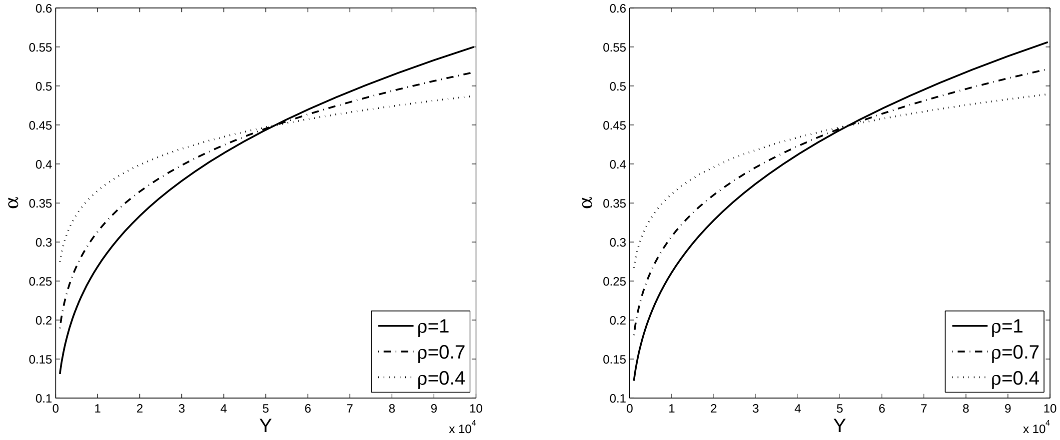
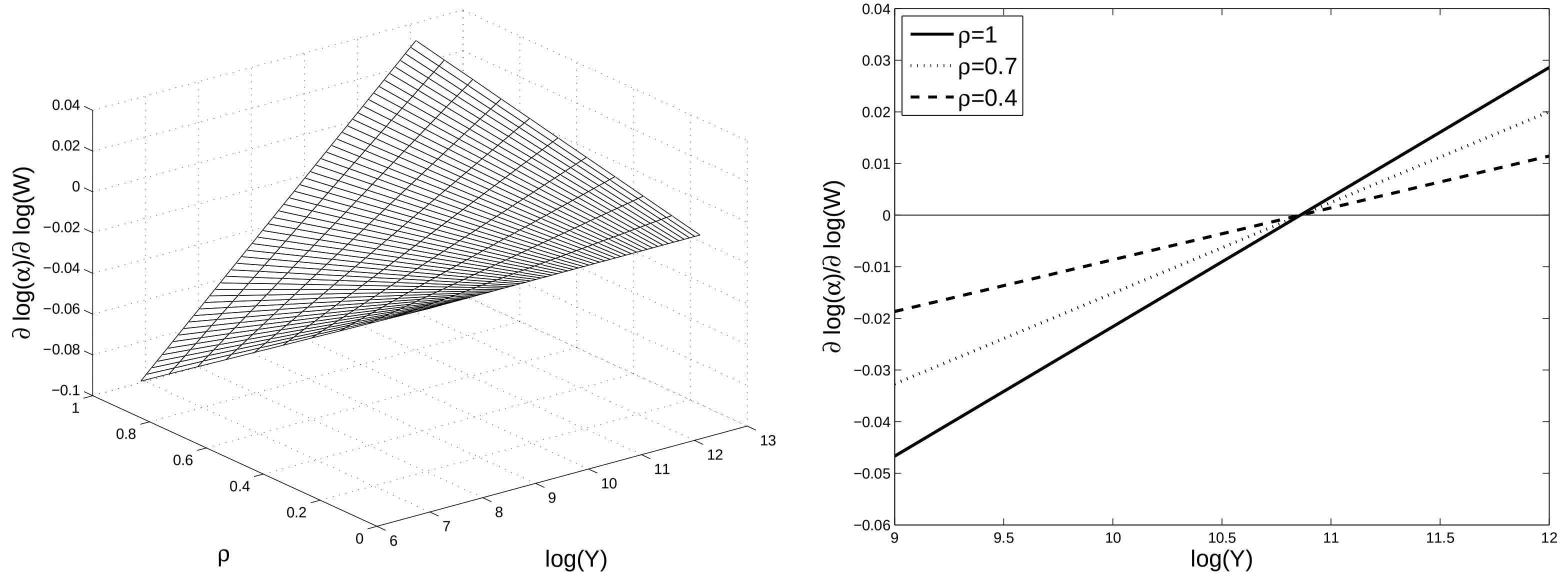


Figure 3.9: Empirical Estimates: (a)$\frac{\partial log(\alpha(W,Y,\rho))}{\partial log(W)}$ and (b)$\frac{\partial log(\alpha(W,Y;\rho=1,0.7,0.4))}{\partial log(W)}$ where $\frac{\partial log(\alpha(W,Y,\rho))}{\partial log(W)} = -0.2726^{***}\rho + 0.0251^{***}log(Y)\rho$ is estimated from PSID 1999 wealth data. Panel (a) shows the wealth elasticity of α along the Y and ρ dimensions. Panel (b) shows the elasticity in two dimension by holding the persistence parameter ρ fixed at values 1, 0.7 and 0.4. The elasticity is negative for low levels of Y and becomes positive if $Y \geq \$52,000.90$. The elasticity increases in the value of human capital and it increases at a faster pace for higher values of ρ.



(a) Wealth elasticity of α

(b) Wealth elasticity of $\alpha; \rho = 1, 0.7, 0.4$

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Chapter 4

Conclusions

In this thesis I discuss two essays that examine issues in household finances. The over-aching, unifying theme of the two chapters is the agent's human capital and how it impacts investment decisions in a way to explain known regularities observed in the cross section of household finances. In Chapter 2, I discuss that utility preferences can have an impact on the agent's motive to attain a college education, participate in stock markets and hold stocks consistent with qualitative aspects of the empirical findings. In Chapter 3, I explain that rising equity shares in financial wealth can be a consequence of persistence in labor income shocks and its effects on the joint distributions of human capital, financial wealth accumulation and wealth composition in the cross section, and not a consequence of financial wealth effects alone, as commonly assumed. In this chapter, I discuss my thesis in relation to recent research, describe some possible avenues for improvements and future research, and other potential applications from my findings.

A more exhaustive analysis of the models in this thesis reveals some results that are still counterfactual. For example, both chapters share results that, contingent on participation, the equity shares are declining in age. This result is common in the literature and not unique to this thesis. The dominating consensus in the literature views human capital as a substitute for a low risk asset, generating an equity share that is declining in age as the agent loses human capital value. Curcuru, Heaton, Lucas, and Moore (2006), Heaton and Lucas (1997), Heaton and Lucas (2000), and Cocco, Gomes, and Maenhout (2005) highlight the results that periodic labor income flows serves as additional sources, beyond financial wealth, to finance consumption, reducing overall risks. This thesis follows the same vein to describe human capital. However, a potential remedy for this is given by Benzoni, Collin-Dufresne, and Goldstein (2007) who examine labor income that is cointegrated with stock dividends to generate, among limited stock market participation, equity share that is declining in age. Elements of cointegration between the labor income and stock return processes, or other factors that make human capital 'stock like', can be included to improve on this thesis. This remains to be done in future revisions of the essays.

In the interest of keeping the models as simple as possible to explain the intended facts, this thesis did not consider a number of important factors that affect household finances. For example, personal debt and liabilities, student loans, mortgages, short selling, investments in real assets and housing, taxes, and alternative preferences based on habits and social status, among many others, are all substantial factors that play important roles on how agents consume, save, educate and invest. These factors are interesting extensions that have been, and are currently being considered by others in the literature. These will remain fruitful areas of research in the future and I fully intend to partake in them as well.

Furthermore, the thesis provides results that match the intended empirical facts on a *qualitative* level. Due to the intensive computational nature of the solution methods, it was not possible given the time constraint to solve the models with a more exhaustive set of parameter values in the calibrations of the theoretical models. My belief is that in addition to qualitative consistency, there is value in attempting to match facts quantitatively as well. Moreover, analysis with more set of parameters may possibly result in additional qualitative implications from the models. This also remains to be done in future revisions of the essays.

Some salient features of this thesis point to some important factors that affect investment decisions and how household units invest their wealth. However, I also believe that this thesis provides proposals for the broader field of asset pricing as well. One of the common and unifying results of both chapters is stock holding that is increasing in wealth in the cross section of households. The first essay generates wealth and equity holding that are rising in education, while the second essay generates equity share that is rising in financial wealth and human capital. These results are consistent with the fact that the wealthiest segment of the population disproportionately holds the largest share of risky assets and overall wealth of the economy. These results are not only consistent with empirical facts, as made clear in Curcuru, Heaton, Lucas, and Moore (2006) and in their citations, but they underline the importance of this wealthy subgroup in asset pricing models.

For example, most of the current literature on Consumption Based Asset Pricing prices risky assets based on aggregate macroeconomic factors, such as the aggregate level of consumption, that tend to be *too smooth* and weakly correlated with risky asset returns. Consequently, as pointed out by Campbell (2003), most of the Consumption Based Asset Pricing models are not able to capture a stochastic discount factor (SDF) that is variable enough to generate aggregate financial variables viewed in the recent past. To this end, researchers have searched and motivated alternative specifications for the SDF in search for a better asset pricing model.

If the wealthiest hold a disproportionate share of risky assets in the economy, hence owning the bulk of the stock of capital and technology that produces goods and services that employs large segments of the overall economy, then a closer investigation of the risk characteristics of their human capital is in order. This can potentially lead to better predictive models that can be of interest to academics and practitioner in capital markets. This thesis provides justification for the use of more volatile economic factors in pricing risky securities. Since stocks are held only by stock market participants, conventional wisdom would suggest that stock prices must be determined according to circumstances and preferences of a representative stockholder, not of a representative of all agents in the economy.

I also believe that the results from Chapter 2 of this thesis has applications in policy analysis. The essay tentatively describes risk aversion and intertemporal consumption preference characteristics for different segments of the population based on education, stock market participation, asset holdings and wealth accumulation. Knowledge of these characteristics of the population is of utmost importance in studying how effective competing policies are before they are implemented. Provided that the suggestions in this thesis are verifiable in a rigorous manner and validated, armed with the knowledge presented here policy makers would be better adept at engineering more effective policies targeted specifically to certain segments of the broader population.

In conclusion, this thesis takes a step to contribute to our understanding of portfolio choice behavior and household finances. The world in which we live is far more complicated than the economies that are modeled here, and as discussed in this short chapter, much still remains to be done. Although this thesis potentially answers some questions, viewed under a different light, it generates more questions as well. But in my opinion, so is the nature of research and how our understanding of this world evolves through time. Through research we answer some questions while generating some others in an attempt to better understand agents' behaviors. Despite its long history dating back at least to the seminal work of Markowitz, the literature on household consumption and portfolio choice is still evolving and its size is a testament of its importance. A great number of research in this area is still to be written and shared before puzzles and issues are resolved.

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Appendix A

Life Cycle Model - Numerical Solution for the Epstein-Zin Utility Model

This appendix describes the numerical solution method employed in the Epstein-Zin Utility model of Chapters 2 and 3. The problem is solved using numerical dynamic programming methods.

It can be shown that the Bellman equation is homogeneous of degree one in its arguments X_t and P_t. I take advantage of scale independence of the problem and normalize the state variables X_t and P_t by \$10,000. For example, a post normalized cash on hand value of 0.5 corresponds to a pre normalized cash on hand value of \$5,000. This scale lets me discretize the state space finely with fewer number of nodes.

I discretize the joint probability distributions of labor income shocks in equations (2.1), (2.2) and stock returns (2.3) using multidimensional gaussian quadrature

$$\{(u_{1,i}, w^u_{1,i})\}_{i=1}^{I} = \{(u_{1,1}, w^u_{1,1}), ..., (u_{1,I}, w^u_{1,I})\}, \tag{A.1}$$

$$\{(u_{2,i}, w^u_{2,i})\}_{i=1}^{I} = \{(u_{2,1}, w^u_{2,1}), ..., (u_{2,I}, w^u_{2,I})\}, \tag{A.2}$$

$$\{(n_{1,i}, w^n_{1,i})\}_{i=1}^{I} = \{(n_{1,1}, w^n_{1,1}), ..., (n_{1,I}, w^n_{1,I})\}, \tag{A.3}$$

$$\{(n_{2,i}, w^n_{2,i})\}_{i=1}^{I} = \{(n_{2,1}, w^n_{2,1}), ..., (n_{2,I}, w^n_{2,I})\}, \tag{A.4}$$

$$\{(\varepsilon_k, w^\varepsilon_k)\}_{k=1}^{K} = \{(\varepsilon_1, w^\varepsilon_1), ..., (\varepsilon_K, w^\varepsilon_K)\} \tag{A.5}$$

where u_i, n_j, ε_k are node values of u_{t+1}, n_{t+1}, ε_{t+1} respectively, and w^u_i, w^n_j and w^ε_k are their corresponding probability weights. Z_t has been substituted by 1 and 2 if the education status is college or high school educated respectively. I also discretize the state space as

$$\{(X_l)\}_{l=1}^{L} = \{(X_1, ..., X_L)\}, \tag{A.6}$$

$$\{(P_m)\}_{l=1}^{M} = \{(P_1, ..., P_M)\} \tag{A.7}$$

Consider first the simpler problem for an agent without any options to attain an education or to become a stock market participant. For simplicity, assume the agent is already college educated and a stock market participant. For each period $t < T-1$, and node values of X_l and P_m, I solve the following optimization problem for $\kappa_t(X_l, P_m)$ and $\alpha_t(X_l, P_m)$

$$V^{E,P}(X_l, P_m) = \underset{\{\kappa_t(X_l,P_m),\alpha_t(X_l,P_m)\}}{\text{Max}} \tag{A.8}$$

$$\left[\begin{array}{c} (1-\beta)\left[X_l \kappa_t(X_l, P_m)\right]^{\frac{(1-\gamma)}{\theta}} \\ +\beta \left[\begin{array}{c} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} w_{1,i}^{u} w_{1,j}^{n} w_{1,k}^{\varepsilon} \\ V^{E,P}(X_{t+1}(X_l, P_m; u_i, n_j, \varepsilon_k), P_{t+1}(P_m; n_j))^{1-\gamma} \end{array} \right]^{\frac{1}{\theta}} \end{array} \right]^{\frac{\theta}{(1-\gamma)}}$$

where $X_{t+1}(X_l, P_m; u_i, n_j, \varepsilon_k)$ is given by

$$\begin{aligned} X_{t+1}(X_l, P_m; u_i, n_j, \varepsilon_k) &= X_l(1-\kappa_t(X_l, P_m)) \times [\alpha_t(X_l, P_m)(\mu^S + \varepsilon_k - R_f) + R_f] \\ &\quad + \exp(f_{1,t+1}) P_m \exp(u_{1,i}) \exp(n_{1,j})(1-h_t) \end{aligned} \tag{A.9}$$

and $P_{t+1}(P_m; n_j)$ is given by

$$P_{t+1}(P_m; n_j) = \exp(f_{1,t+1}) P_m \exp(n_{1,j}) \tag{A.10}$$

$V^{E,P}(X_{t+1}(X_l, P_m; u_i, n_j, \varepsilon_k), P_{t+1}(P_m; n_j))$ values whose arguments $X_{t+1}(X_l, P_m; u_i, n_j, \varepsilon_k)$ and $P_{t+1}(P_m; n_j)$ do not lie on the node values in the $\{(X_l, P_m)\}_{l=1,m=1}^{L,M}$ grid are approximated using cubic spline interpolation.

At $t = T-1$, the problem is identical with the exception that $V^{E,P}(X_T(X_l, P_m; u_i, n_j, \varepsilon_k), P_T(X_l, P_m; , n_j)) = \left(\frac{b^{\gamma}}{1-\gamma}\right)^{\frac{1}{1-\gamma}} X_T(X_l, P_m; u_i, n_j, \varepsilon_k)$.

Now consider the more complicated problem when the agent is not yet college educated but a stock market participant. For each period $t < T-1$, and node values of X_l and P_m, I solve the following optimization problem for $\kappa_t(X_l, P_m)$, $\alpha_t(X_l, P_m)$ and $I_{Edu,t}(X_l, P_m)$

$$V^{NE,P}(X_l,P_m)=\underset{\{\kappa_t(X_l,P_m),\alpha_t(X_l,P_m),I_{Edu,t}(X_l,P_m)\}}{\text{Max}} \tag{A.11}$$

$$\left[\begin{array}{c} (1-\beta)\left[(X_l-I_{Edu,t}(X_l,P_m)E)\kappa_t(X_l,P_m)\right]^{\frac{(1-\gamma)}{\theta}} \\ +\beta\left[\begin{array}{c} I_{Edu,t}(X_l,P_m)\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}w^{u}_{1,i}w^{n}_{1,j}w^{\varepsilon}_{1,k} \\ V^{E,P}(X_{t+1}(X_l-I_{Edu,t}(X_l,P_m)OP_m,P_m;u_i,n_j,\varepsilon_k),P_{t+1}(P_m;n_j))^{1-\gamma} \\ +(1-I_{Edu,t}(X_l,P_m))\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}w^{u}_{2,i}w^{n}_{2,j}w^{\varepsilon}_{2,k} \\ V^{NE,P}(X_{t+1}(X_l,P_m;u_i,n_j,\varepsilon_k),P_{t+1}(P_m;n_j))^{1-\gamma} \end{array}\right]^{\frac{1}{\theta}} \end{array}\right]^{\frac{\theta}{(1-\gamma)}}$$

where $X_{t+1}(X_l,P_m;u_i,n_j,\varepsilon_k)$ is given by

$$\begin{aligned} X_{t+1}(X_l,P_m;u_i,n_j,\varepsilon_k) &= (X_l-I_{Edu,t}(X_l,P_m)E)(1-\kappa_t(X_l,P_m)) \qquad \text{(A.12)} \\ &\times[\alpha_t(X_l,P_m)(\mu^S+\varepsilon_k)+R_f]+(1-h_t) \\ &\times[I_{Col,t+1}(X_l,P_m)\exp(f_{1,t+1})P_m\exp(u_{1,i})\exp(n_{1,j}) \\ &+(1-I_{Col,t+1}(X_l,P_m))\exp(f_{2,t+1})P_m\exp(u_{2,i})\exp(n_{2,j})] \end{aligned}$$

and $P_{t+1}(P_m;n_j)$ is given by

$$P_{t+1}(P_m;n_j)=P_m[I_{Col,t+1}(X_l,P_m)\exp(f_{1,t+1})\exp(n_{1,j}) \tag{A.13}$$

$$+(1-I_{Col,t+1}(X_l,P_m))\exp(f_{2,t+1})\exp(n_{2,j})] \tag{A.14}$$

To solve the above problem, first set $I_{Edu,t}(X_l,P_m)$ and $I_{Col,t+1}(X_l,P_m)$ equal to 0 and solve the optimization problem. Store the results $\{\kappa_t(X_l,P_m;I_{Edu,t}(X_l,P_m)=0),\alpha_t(X_l,P_m;I_{Edu,t}(X_l,P_m)=0),V^{NE,P}(X_l,P_m;I_{Edu,t}(X_l,P_m)=0)\}$. Then set $I_{Edu,t}(X_l,P_m)$ and $I_{Col,t+1}(X_l,P_m)$ equal to 1 and solve the optimization problem again. Store the results $\{\kappa_t(X_l,P_m;I_{Edu,t}(X_l,P_m)=1),\alpha_t(X_l,P_m;I_{Edu,t}(X_l,P_m)=1),V^{NE,P}(X_l,P_m;I_{Edu,t}(X_l,P_m)=1)\}$. If $V^{NE,P}(X_l,P_m;I_{Edu,t}(X_l,P_m)=1)>V^{NE,P}(X_l,P_m;I_{Edu,t}(X_l,P_m)=0)$, then the optimal policies $\{\kappa_t(X_l,P_m),\alpha_t(X_l,P_m),I_{Edu,t}(X_l,P_m)\}$ are $\{\kappa_t(X_l,P_m;I_{Edu,t}(X_l,P_m)=1),\alpha_t(X_l,P_m;I_{Edu,t}(X_l,P_m)=1),1\}$ and the maximized value function $V^{NE,P}(X_l,P_m)$ is $V^{NE,P}(X_l,P_m;I_{Edu,t}(X_l,P_m)=1)$, otherwise the optimal policies are $\{\kappa_t(X_l,P_m;I_{Edu,t}(X_l,P_m)=0),\alpha_t(X_l,P_m;I_{Edu,t}(X_l,P_m)=0),0\}$ and the maximized value function is $V^{NE,P}(X_l,P_m;I_{Edu,t}(X_l,P_m)=0)$.

The process described above is similar when the agent has an option to become a stock market participant or when the agent has both options to attain an education and become a stock market participant. For brevity, the latter two cases are not shown.

All numerical optimizations are done using Nelder-Mead simplex algorithm.[32] I find that gradient-based optimization algorithms are not very reliable in the Epstein-Zin utility problem due to numerical instabilities.

Because the dynamic programming problem is a *constrained* optimization problem, and the general formulation of the Nelder-Mead simplex algorithm is an *unconstrained* one, a transformation of the original constrained problem is required to use the Nelder-Mead algorithm.[33] The transformed problem becomes an unconstrained optimization problem whose solution is the solution to the original constrained problem. To this end, the policies $\kappa(X_t, P_t)$ and $\alpha(X_t, P_t)$ in the Bellman equations and budget constraint are substituted by appropriate transformations. Since $\kappa(X_t, P_t)$ and $\alpha(X_t, P_t)$ are constrained to be between 0 and 1, the following trigonometric transformation works for both variables:

$$\kappa = 0 + (1 - 0)\frac{(cos(z_1) + 1)}{2} \tag{A.15}$$

$$\alpha = 0 + (1 - 0)\frac{(cos(z_2) + 1)}{2} \tag{A.16}$$

where z_1 and z_2 are the solutions to the unconstrained transformed optimization problem as given by the Nelder-Mead algorithm. As it can be seen from equations (A.15) and (A.16), both $\kappa(X_t, P_t)$ and $\alpha(X_t, P_t)$ are bounded between 0 and 1 for all real values of z_1 and z_2, as required.

Since the Bellman equations are recursive, the process described above starts at $t = T - 1$ and is iterated backwards in time until $t = t_0$, at which point the computations end.

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[32] See Judd (1998) for a thorough discussion of this and many other optimization algorithms and how they can be used in solving economic problems.

[33] An alternative to transforming the constrained variables is to use methods that apply a penalty function to the value function when the control variables fall outside the constrained set. These approaches tend to require algorithms that adapt the penalty function to values of the value function in each iteration. Consequently, penalty function methods are computationally more expensive and cumbersome to apply in this multi option model.

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Appendix B

Simplified Model - Present Value of Labor Income

I derive the value of human capital given in equation (3.5). To this end, it is useful to rewrite the labor income processes given in equations (3.3) and (3.4) in difference form

$$\Delta y_t = g\Delta t + \Delta p_t \tag{B.1}$$

$$\Delta p_t = (\rho - 1)p_{t-1} + \varepsilon_t \tag{B.2}$$

and transform them into continuous time differential equations by letting $\Delta t \rightarrow 0$. The continuous time analogs of the previous discrete time processes are

$$\mathrm{d}y_t = g\mathrm{d}t + \mathrm{d}p_t \tag{B.3}$$

$$\mathrm{d}p_t = -\kappa p_t\mathrm{d}t + \sigma_p\mathrm{d}z \tag{B.4}$$

$$\mathrm{d}f_t = g\mathrm{d}t \tag{B.5}$$

where $\kappa = -log(\rho)$ and f_t is the deterministic component of labor income. The solutions to the above equations are

$$y_s = f_s + p_s \tag{B.6}$$

$$= f_t + \int_t^s g\,\mathrm{d}u + p_t e^{(t-s)\kappa} + \int_t^s e^{(u-s)\kappa}\sigma_p\mathrm{d}z_u \tag{B.7}$$

$$= f_t + p_t - p_t + g(s-t) + p_t e^{(t-s)\kappa} + \int_t^s e^{(u-s)\kappa}\sigma_p\mathrm{d}z_u \tag{B.8}$$

$$= y_t - p_t + g(s-t) + p_t e^{(t-s)\kappa} + \int_t^s e^{(u-s)\kappa}\sigma_p\mathrm{d}z_u \tag{B.9}$$

and

$$p_s = p_t e^{(t-s)\kappa} + \int_t^s e^{(u-s)\kappa}\sigma_p\mathrm{d}z_u \tag{B.10}$$

for y_s and p_s respectively for any $s > t$. Y_s can be found by taking the exponent of y_s

$$Y_s = exp[y_s] = \frac{Y_t}{P_t} e^{g(s-t)+e^{(t-s)\kappa}p_t+\int_t^s e^{(u-s)\kappa}\sigma_p \mathrm{d}z_u} \tag{B.11}$$

and the value of human wealth can be found by computing the expected present value of Y_s over all $t \leqslant s \leqslant \infty$.

$$\begin{aligned} HC_t &= \mathrm{E}_t\left[\int_t^\infty e^{-\int_t^s d\mathrm{d}u} Y_s \mathrm{d}s\right] & \text{(B.12)}\\ &= \frac{Y_t}{P_t}\mathrm{E}_t\left[\int_t^\infty e^{(g-d)(s-t)+e^{(t-s)\kappa}p_t+\int_t^s e^{(u-s)\kappa}\sigma_p \mathrm{d}z_u}\mathrm{d}s\right] & \text{(B.13)}\\ &= \frac{Y_t}{P_t}\int_t^\infty \mathrm{E}_t[e^{\varphi_s}]\,\mathrm{d}s & \text{(B.14)}\\ &= \frac{Y_t}{P_t}\int_t^\infty e^{\mathrm{E}_t[\varphi_s]+\frac{1}{2}\mathrm{Var}[\varphi_s]}\,\mathrm{d}s & \text{(B.15)} \end{aligned}$$

where I have discounted labor income using a discount rate d,
$\varphi_s = (g-d)(s-t) + e^{(t-s)\kappa}p_t + \int_t^s e^{(u-s)\kappa}\sigma_p)\mathrm{d}z_u$ and
$\varphi_s \sim N\left[(g-d)(s-t) + e^{(t-s)\kappa}p_t, \frac{\left(1-e^{2(t-s)\kappa}\right)\sigma_p^2}{2\kappa}\right]$. Note that as $\kappa \to 0$,
$\varphi_s \sim N\left[(g-d)(s-t) + p_t, \sigma_p^2(s-t)\right]$.
After substituting for the mean and variance of φ_s the value of human capital (3.5) is attained. Moreover, if ρ is equal to unity then κ is equal to zero and the value of human capital is given by

$$HC_t = Y_t \int_t^\infty e^{(g-d+\frac{1}{2}\sigma_p^2)(s-t)}\,\mathrm{d}s = \frac{Y_t}{d-g-\frac{1}{2}\sigma_p^2} \tag{B.16}$$

provided the denominator is positive.

Appendix C

Life Cycle Model - Numerical Solution for the Power Utility Model

This appendix describes the numerical solution method employed in the model of Chapter 3.

I discretize the joint probability distributions of labor income shocks in equation (3.8) and stock returns (3.9) using multidimensional gaussian quadrature as

$$\{(u_i, w_i^u)\}_{i=1}^{I} = \{(u_1, w_1^u), ..., (u_I, w_I^u)\}, \tag{C.1}$$

$$\{(e_j, w_j^e)\}_{j=1}^{J} = \{(e_1, w_1^e), ..., (e_J, w_J^e)\}, \tag{C.2}$$

$$\{(\varepsilon_k, w_k^\varepsilon)\}_{k=1}^{K} = \{(\varepsilon_1, w_1^\varepsilon), ..., (\varepsilon_K, w_K^\varepsilon)\} \tag{C.3}$$

where u_i, e_j, ε_k are node values of u_{t+1}, e_{t+1}, ε_{t+1} respectively, and w_i^u, w_j^e and w_k^ε are their corresponding probability weights. I also discretize the state space as

$$\{(X_l)\}_{l=1}^{L} = \{(X_1, ..., X_L)\}, \tag{C.4}$$

$$\{(P_m)\}_{l=1}^{M} = \{(P_1, ..., P_M)\} \tag{C.5}$$

For each period $t < T - 1$, and node values of X_l and P_m, I solve the following system of equations for $\kappa_t(X_l, P_m)$ and $\alpha_t(X_l, P_m)$

$$1 = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} w_i^u w_j^e w_k^\varepsilon \beta \left[\left(\frac{X_{t+1}(X_l, P_m; u_i, e_j, \varepsilon_k)\kappa_{t+1}(X_l, P_m; u_i, e_j, \varepsilon_k)}{X_l \kappa_t(X_l, P_m)} \right)^{-\gamma} (\mu^S + \varepsilon_k) \right]$$

$$1 = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} w_i^u w_j^e w_k^\varepsilon \beta \left[\left(\frac{X_{t+1}(X_l, P_m; u_i, e_j, \varepsilon_k)\kappa_{t+1}(X_l, P_m; u_i, e_j, \varepsilon_k)}{X_l \kappa_t(X_l, P_m)} \right)^{-\gamma} R_f \right] \tag{C.6}$$

where $X_{t+1}(X_l, P_m; u_i, e_j, \varepsilon_k)$ is given by

$$\begin{aligned} X_l(1 - \kappa_t(X_l, P_m)) \times [\alpha_t(X_l, P_m)(\mu^S + \varepsilon_k - R_f) + R_f] \\ + \exp(f_{t+1}) P_m^\rho \exp(u_i) \exp(e_j) \end{aligned}$$

and $P_{t+1}(P_m; u_i, e_j)$ is given by

$$P_m^\rho \exp(u_i) \exp(e_j) \tag{C.7}$$

For $t = T - 1$, the system of equations to be solved are identical with the exception that $\kappa_{t+1}(X_l, P_m;\ u_i, e_j, \varepsilon_k) = \frac{1}{b}$. Note that the above system of equations is composed of two equations and two unknowns and the policies can be solved for exactly using any gradient-based constrained algorithm. Corner solutions imply inequalities ($>$) in the Euler equations. $\kappa_{t+1}(X_l, P_m; u_i, e_j, \varepsilon_k)$ values whose arguments P_{t+1} and X_{t+1} do not lie on the node values in the $\{(X_l, P_m)\}_{l=1,m=1}^{L,M}$ grid are approximated using cubic spline interpolation.

The problem is solved first at $t = T-1$ and iterated using backward recursion until $t = t_0$, at which time the algorithm is terminated.