SHIFTED FREQUENCY ANALYSIS FOR EMTP SIMULATION OF POWER SYSTEM DYNAMICS

by

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Abstract

Electromagnetic Transients Program (EMTP) simulators are being widely used in power system dynamics studies. However, their capability in real time simulation of power systems is compromised due to the small time step required resulting in slow simulation speeds.

This thesis proposes a Shifted Frequency Analysis (SFA) theory to accelerate EMTP solutions for simulation of power system operational dynamics. A main advantage of the SFA is that it allows the use of large time steps in the EMTP solution environment to accurately simulate dynamic frequencies within a band centered around the fundamental frequency.

The thesis presents a new synchronous machine model based on the SFA theory, which uses dynamic phasor variables rather than instantaneous time domain variables. Apart from using complex numbers, discrete-time SFA synchronous machine models have the same form as the standard EMTP models. Dynamic phasors provide envelopes of the time domain waveforms and can be accurately transformed back to instantaneous time values. When the frequency spectra of the signals are close to the fundamental power frequency, the SFA model allows the use of large time steps without sacrificing accuracy. Speedups of more than fifty times over the traditional EMTP synchronous machine model were obtained for a case of mechanical torque step changes.

This thesis also extends the SFA method to model induction machines in the EMTP. By analyzing the relationship between rotor and stator physical variables, a phase-coordinate model with lower number of equations is first derived. Based on this, a SFA model is proposed as a general purpose model capable of simulating both fast transients and slow dynamics in induction machines. Case study results show that the SFA model is in excess of seventy times faster than the phase-coordinate EMTP model when simulating the slow dynamics.

In order to realize the advantage of SFA models in the context of the simulation of the complete electrical network, a dynamic-phasor-based EMTP simulation tool has been developed.

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Chapter 1

Introduction

1.1 Background

Although a power system blackout is a low-probability event, whenever it happens, its impact on the power system and the society is catastrophic. A typical example is the Aug. 14, 2003 Blackout in the USA and Canada, which caused a total loss of 62,000 MW of loads and tripping of 531 generators [1]. Fifty million people were affected by this system collapse. During the next two years, twelve major blackouts happened in Europe, Asia and North America. A recent blackout happened in Los Angeles on September 12, 2005 affecting millions in California.

In today's deregulated electricity market environment, power system planning and operations are largely driven by economic motivations, without sufficient investments in new generation and transmission equipments. This might have contributed to the higher frequency of blackouts in recent years [2]. Because of aging infrastructures operating under stressed conditions, power system stability, including transient stability, voltage stability, and inter-area oscillations have become a major concern in the North American power grid.

Traditionally in power system studies a number of simplifying assumptions are made to analyze different types of stability problems with specific tools [3]-[5]. For instance, the static techniques for long-term voltage stability analysis [6] only solve the steady-state (algebraic) response of the system. Another example is quasi-steady-state (QSS) analysis [7], which neglects the fast network transients and only considers electromechanical dynamics. Because the time varying electrical quantities in QSS are represented with phasors, these tools can only capture snapshots of the system operation ignoring the dynamics between states.

Problems such as voltage stability depend strongly not only on the machine-network interaction, but also on the interaction between the electrical network and the loads. Moreover,

severe disturbances in the system often involve frequency excursions in addition to voltage excursions [8]. Especially when the power system is approaching critical collapse points, the QSS or steady-state assumptions can result in inaccurate predictions.

Tools such as the EMTP [9][10] can most closely simulate the real power system dynamics by continuously tracing the evolution of the system state in arbitrary multi-phase networks with lumped or distributed parameters. Therefore EMTP-type simulators [11]-[15] are very appealing to be used for power system stability assessment. These tools, however, require small discretization steps dictated by the need to trace the instantaneous values of all waveforms. This makes the EMTP unnecessarily slow to trace phenomena around the 60 Hz fundamental frequency. As a matter of fact, nowadays one can only simulate small-scale equivalent power systems on the real-time EMTP simulators [10] [16]. Therefore, efforts should be taken to bridge the gap between the EMTP-type simulators and a unified power system analysis tool.

Reference [18] aims at combining electromagnetic and electromechanical simulations [19] by considering both the network and machine differential equations without making QSS assumptions. However, no efficient solution algorithms are provided in this reference and the presented machine models are still reduced order models in phasor form. Reference [20] provides the first systematic method for a unified steady-state and transient power system analysis tool, by combining dynamic phasor concepts with EMTP simulation. Dynamic phasor models for transmission lines and linear elements are presented in this reference. However, the machine model is still based on the $dq0$ model, which can present numerical stability problems for large time steps and may not be the most appropriate for a general EMTP-based dynamic phasor unified theory. Important contributions to dynamic phasor modeling of electric machines have been presented in [21], [22], [23] based on the approach of the generalized averaging method [23] which, however, is also an approximate method with higher frequency components truncated.

In general, it is a challenging topic to correctly, efficiently obtain the time-domain simulation results in the neighborhood of the fundamental frequency without making QSS or other simplifying assumptions. Hence, the intention of this thesis is to explore an effective way to combine the EMTP solution and the dynamic phasor concept, to build dynamic phasor models for power system components, and to develop a general-purpose simulation tool to obtain dynamic phasor results from EMTP solutions.

1.2 Motivation

It is important to have some insight into the power system signals before we can further develop an efficient and accurate method for power system dynamic simulation.

A common observation is that, when a contingency happens, the frequencies in a power system are usually close to fundamental frequency ω_s (60 Hz). That is to say, in the dynamic situation, the electrical signals in power systems have a fundamental frequency component modulated by slower events. This fact is analogous to the situation in communication systems where the carrier frequency is very high and carries on its sidebands the much lower frequency (e.g., audio or video) information. In communications theory, the carrier frequency is modulated to send out the information and then demodulated to recover the information at the receiving end [24]. Typically the carrier in a communication system is a sinusoidally varying signal at some frequency which is much higher than the frequencies contained in the information or message. The process of modulation gives the signal a nonzero bandwidth that is usually much smaller than the carrier frequency. Thus the signal can be regarded as a narrowband process, and can be accurately modeled as the product of a bandlimited ‘message’ waveform and a sinusoidal carrier. By this analogy, one can directly draw the conclusion that usually a power system signal $u(t)$ can be modeled by a narrowband signal which has a small bandwidth around ω_s .

In steady-state AC circuit theory, one uses complex exponentials (i.e. phasors) to represent real sinusoidal signals with the understanding that the real part of the complex exponential gives the physical time-domain quantity of the AC signal.

Going beyond the steady-state analysis, the analytic signal of a real function plays the similar role for a dynamic waveform. In fact, the analytic signal $z(t)$ of a bandpass signal $u(t)$ can be derived as:

$$z(t) = U(t)e^{j\omega_s t} \quad (1.1)$$

where $U(t)$ is called the dynamic phasor of $u(t)$, $z(t) = u(t) + jH[u(t)]$, and $H[\bullet]$ denotes the Hilbert transform [25][26].

The dynamic phasor $U(t)$ is the complex envelope of $u(t)$, and will degenerate to the phasor U if $u(t)$ becomes a sinusoidal signal in steady state. In fact, the analytic signal of a steady-state sinusoidal function $A\cos(\omega_s t + \varphi)$ is $Be^{j\omega_s t}$ where $B = Ae^{j\varphi}$. Here the dynamic phasor

becomes $Ae^{j\varphi}$, which is the phasor of $A\cos(\omega_s t + \varphi)$.

Similar to the phasor, the dynamic phasors can be transformed back to time domain waveforms by taking the real part of the analytic signal in (1.1) (proof is shown in Chapter 2)

It can be seen in (1.1) that the spectrum of $U(t)$ is the spectrum of the analytic signal of $u(t)$ shifted by synchronous angular frequency $-\omega_s$. This can be called the “frequency shifting” property. If the power network is formulated by using dynamic phasors instead of the instantaneous time quantities, it is said to be modeled in the shifted frequency domain, as opposed to the model in the time domain. Similar idea is proposed in reference [27]. The way to build power system component models and solve the network equations in the shifted frequency domain is called the Shifted Frequency Analysis (SFA).

A major advantage of the Shifted Frequency Analysis is that it allows the use of large time steps in the EMTP solution environment to accurately simulate dynamic frequencies within a band centered around a fundamental frequency. The original system is transformed into a shifted frequency system where the frequencies around the power frequency (e.g. 60Hz) become frequencies around dc (0 Hz) [28][29]. Because the time step in EMTP simulations is limited by the Nyquist frequency, low frequencies in the shifted frequency system imply the feasibility of using large integration time steps.

By coupling SFA models into an EMTP-type simulator, the shifted system is then numerically integrated to obtain dynamic phasor solutions, which are more easily understood by power system operators and planners than instantaneous time domain results. At the same time, the dynamic phasor results can then be transformed back to time domain waveforms using the inverse transformation.

EMTP uses the trapezoidal integration rule that is A-stable, reasonably accurate and simple. The discrete-time nodal equations formed in EMTP are elegant and can preserve the sparsity of the power system structure. All these features makes EMTP a standard simulation tool in the power industry that can model the power system at the device level. The goal of this thesis is to implement the SFA method into EMTP, which is the first practical step for building a unified power system analysis tool based on the EMTP solution.

1.3 Contributions

A two-step strategy is employed in the research of Shifted Frequency Analysis. First, the SFA-based models for power network components are developed and tested. Second, a general-purpose SFA-based EMTP tool is built to simulate a power network. The main contributions are presented in Chapter 2 to Chapter 5 and are briefly summarized as follows.

I. The definition of dynamic phasor and the theoretical basis of the SFA method are introduced. The basic procedures for analyzing time functions (signals) in the SFA domain are discussed, and the numerical accuracy of the discrete-time SFA solution is analyzed. Chapter 2 summarizes these theoretical works.

II. A few types of power system components are modeled in the SFA domain. Chapter 2 describes the SFA-based models for the linear circuit components, transformer and some load models such as exponential load and steady state induction motor load.

III. A new efficient synchronous machine model for the simulation of slow system dynamics using EMTP modelling in the Shifted Frequency Analysis framework is presented in Chapter 3. Discrete-time SFA synchronous machine models are derived which have a similar form as the EMTP component models.

IV. In Chapter 4, the SFA method is extended to model the induction machines in EMTP. By analyzing the relationship between rotor and stator physical variables, a phase-coordinate model with lower number of equations is first derived. Based on this, a SFA model is proposed as a general purpose model capable of simulating both fast transients and slow dynamics. Case study results have confirmed that the SFA induction machine model is a valuable component for real-time EMTP simulations. It is observed that the SFA model is in excess of 70 times faster than the EMTP phase-coordinate model when simulating dynamics with frequency spectra close to the fundamental power frequency.

V. A final contribution in this thesis is a dynamic-phasor-based EMTP simulation tool. The structure of the tool and the test system results are reported in Chapter 5.

Chapter 2

Shifted Frequency Analysis

2.1 Analytic Signal and Hilbert Transform

The Fourier transform $F(\omega)$ of a real-valued function $f(t)$ is a Hermitian function, which means $F(-\omega) = F^*(\omega)$ in the frequency domain. Thus the F for negative frequencies can be expressed by F^* (complex conjugate of F) for positive ones. This means that the positive frequency spectra is adequate to represent a real signal, and the negative frequency components of the Fourier transform can be discarded without loss of information. Now one can construct a function $F_a(\omega)$ that contains only the non-negative frequency components of $F(\omega)$ by defining

$$F_a(\omega) = F(\omega) + \text{sgn}(\omega) \cdot F(\omega) \quad (2.1)$$

where

$$\text{sgn}(\omega) = \begin{cases} 1 & \omega > 0 \\ 0 & \omega = 0 \\ -1 & \omega < 0 \end{cases}$$

The analytic signal $f_a(t)$ is defined to be the inverse Fourier transform of $F_a(\omega)$

$$\begin{aligned} f_a(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} F_a(\omega) e^{j\omega t} d\omega \\ &= \frac{1}{\pi} \int_0^{\infty} F(\omega) e^{j\omega t} d\omega \end{aligned} \quad (2.2)$$

It is clearly seen that $f_a(t)$ is a complex function. If we define the Hilbert transform of $f(t)$ as the imaginary part of $f_a(t)$, and denote it by $H[f(t)] = \hat{f}(t)$, then $f_a(t)$ can be expressed as

$$f_a(t) = f(t) + j \hat{f}(t) \quad (2.3)$$

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From equation (2.1) and (2.3) we can get the following Fourier pair

$$f(t) + j\hat{f}(t) \xleftrightarrow{F} F(\omega) + \text{sgn}(\omega) \cdot F(\omega)$$

Because we have $f(t) \xleftrightarrow{F} F(\omega)$ by definition, we can derive that

$$\hat{f}(t) \xleftrightarrow{F} -j\text{sgn}(\omega) \cdot F(\omega)$$

Using the transform pair $\frac{1}{\pi t} \xleftrightarrow{F} -j\text{sgn}(\omega)$ and taking the inverse Fourier transform of $-j\text{sgn}(\omega) \cdot F(\omega)$, we can get

$$H[f(t)] = \hat{f}(t) = \frac{1}{\pi t} * f(t) = \frac{1}{\pi} PV \int_{-\infty}^{+\infty} \frac{f(\tau)}{t - \tau} d\tau \quad (2.4)$$

Note that the Hilbert transform is an improper integral, thus it is calculated using the Cauchy principal value (see the ‘PV’ in (2.4)), i.e. $\hat{f}(t) = \lim_{\substack{\varepsilon \rightarrow 0 \\ A \rightarrow \infty}} \frac{1}{\pi} \left[\int_{-A}^{-\varepsilon} \frac{f(\tau)}{t - \tau} d\tau + \int_{+\varepsilon}^A \frac{f(\tau)}{t - \tau} d\tau \right]$.

Obviously, $H[f(t)]$ is real.

The analytic signal has no negative frequency components; moreover, the original real signal can be converted back from it by simply dropping the imaginary part. The analytic signal is thus a generalization of the phasor concept. This important implication leads to the shifted frequency analysis method as is explained in the following section.

2.1.1 Shifted Frequency Analysis

If a power system signal $u(t)$ is a bandpass one, it can then be represented [20] as

$$u(t) = u_I(t) \cos \omega_s t - u_Q(t) \sin \omega_s t \quad (2.5)$$

where the lowpass signals $u_I(t)$ and $u_Q(t)$ are the in-phase and quadrature components of the bandpass signal, respectively.

The dynamic phasor $U(t)$ for the signal $u(t)$ is defined as

$$U(t) = u_I(t) + j u_Q(t)$$

which is the complex envelope of $u(t)$.

Assume that $z(t)$ is the analytic signal of $u(t)$, then

$$z(t) e^{-j\omega_s t} = \{u(t) + jH[u(t)]\} e^{-j\omega_s t}$$

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$$\begin{aligned}
 &= \{u_I(t) \cos \omega_s t - u_Q(t) \sin \omega_s t + j[u_I(t) \sin \omega_s t + u_Q(t) \cos \omega_s t]\} e^{-j\omega_s t} \\
 &= u_I(t) + j u_Q(t)
 \end{aligned} \tag{2.6}$$

where $H[\bullet]$ denotes the Hilbert transform.

Therefore,

$$U(t) = z(t) e^{-j\omega_s t} \tag{2.7}$$

Note that the dynamic phasor $U(t)$ will degenerate to the phasor U if $u(t)$ becomes a sinusoidal signal in steady state.

Equation (2.7) shows that the spectrum of $U(t)$ is the spectrum of the analytic signal of $u(t)$ shifted by synchronous angular frequency $-\omega_s$. This is called “frequency shifting”.

Shifted frequency analysis (SFA) [24] allows the exact simulation of frequencies within a band centered around a fundamental frequency using large time steps in a discrete-time EMTP-type of solution environment. The original system is transformed into a shifted frequency system where the frequencies around the power frequency (60Hz or 50Hz) become frequencies around dc (0 Hz). The shifted system is then solved using an EMTP solution.

The equivalent circuit for network components in the shifted frequency domain can be derived in three steps:

- (i) Create the phase-coordinate differential equations of the component in the normal unshifted domain;
- (ii) Transform phase quantities into dynamic phasor variables according to Equation (2.7);
- (iii) Discretize the dynamic phasor equations using an integration method, e.g. the trapezoidal rule, and build the equivalent circuit suitable for an EMTP solution.

For instance, the time domain voltage equations for M coupled inductances, shown in Figure 2.1(a), can be written as

$$\mathbf{v}(t) = \mathbf{L} \frac{d\mathbf{i}(t)}{dt} \tag{2.8}$$

As illustrated in Figure 2.1(b), the standard EMTP equivalent circuit derived with the trapezoidal rule can be written as

$$\mathbf{i}(t) = \mathbf{R}_L^{-1} \mathbf{v}(t) - \mathbf{h}_L(t) \tag{2.9}$$

where

$$\mathbf{R}_L = \frac{2\mathbf{L}}{\Delta t} \quad (2.10)$$

is an $M \times M$ matrix of resistances, and

$$\mathbf{h}_L(t) = \mathbf{L}^{-1} \mathbf{v}(t - \Delta t) \Delta t + \mathbf{h}_L(t - \Delta t) \quad (2.11)$$

is a vector of past histories.

To obtain the SFA equivalent circuit (Figure 2.1(c)), we transform (2.8) using (2.7) to get

$$\mathbf{V}(t) = \mathbf{L} \frac{d\mathbf{I}(t)}{dt} + j\omega_s \mathbf{L} \mathbf{I}(t) \quad (2.12)$$

where $\mathbf{V}(t)$ and $\mathbf{I}(t)$ are the dynamic phasor vectors corresponding to the physical time vectors $\mathbf{v}(t)$ and $\mathbf{i}(t)$, respectively.

We can now transform (2.12) to discrete time using, for example, the trapezoidal rule,

$$\mathbf{I}(t) = \mathbf{R}_L^{-1} \mathbf{V}(t) - \mathbf{H}_L(t) \quad (2.13)$$

where the equivalent resistances $\mathbf{R}_L$ and history terms $\mathbf{H}_L(t)$ are expressed as

$$\mathbf{R}_L = \frac{2\mathbf{L}}{\Delta t} + \mathbf{K}_L = \left(\frac{2}{\Delta t} + j\omega_s \right) \mathbf{L} \quad (2.14)$$

$$\mathbf{H}_L(t) = \frac{-\frac{4}{\Delta t}}{\left(\frac{2}{\Delta t} + j\omega_s \right)^2} \mathbf{L}^{-1} \mathbf{V}(t - \Delta t) - \frac{-\frac{2}{\Delta t} + j\omega_s}{\frac{2}{\Delta t} + j\omega_s} \mathbf{H}_L(t - \Delta t) \quad (2.15)$$

The corresponding SFA equivalent circuit is shown in Figure 2.1(c).

Figure 2.1 (a) M Coupled Inductances; (b) EMTP Equivalent Circuit; (c) SFA Equivalent Circuit

A major advantage of SFA modelling is that since the equivalent circuits are centered

around 60 Hz ($j\omega_s L$ for the inductances of Figure 2.1), deviations from 60 Hz correspond numerically to very low frequencies (like deviations from 0 Hz in the unshifted domain) and a large integration step can be used in the solution. For example, respecting the accuracy limits imposed by the Nyquist frequency and the distortion introduced by the discretization rule [30], allowing a 3% error with the trapezoidal rule, the integration step can go, for example, from about three 60 Hz cycles, 50ms, for frequency deviations of ± 2 Hz to as large as 1s for frequency deviations of ± 0.1 Hz. With these time steps, the SFA method provides an effective way to make the EMTP program a general purpose simulator for power system stability problems.

By applying SFA in an EMTP-type simulator, time varying phasor solutions are obtained, which can be easily understood by power system operators and planners. At the same time, detailed waveform results can also be traced back from the SFA results by using the inverse transformation

$$u(t) = \text{Re}[U(t)e^{j\omega_s t}] \quad (2.16)$$

Unless specifically noted, in this paper uppercase letters are used to represent dynamic phasors in the shifted frequency domain, while lowercase letters denote real variables in the time domain.

2.2 SFA-Based Network Component Models

The component models in the SFA domain are building blocks for a SFA-based network simulator. In this section, we implement the SFA modeling technique described in Section 2.1.1 and build component models for the linear *RLC* components, transformer, exponential load and steady-state induction machine. SFA equivalent circuits for other network components can also be derived following the procedures in Section 2.1.1. Particularly, two important dynamic components in the power system, i.e. synchronous machine and induction machine, are modeled in SFA domain in Chapter 3 and Chapter 4.

2.2.1 Equivalent Circuits for RLC in the Shifted Frequency Domain

The equivalent circuits for network components in the shifted frequency domain can be derived by first writing the component equations in the phase domain and then relating phase and dynamic phasor variables according to the frequency shifting transformation.

A. M-Phase Resistances

The time domain voltage equations for M-Phase resistances are expressed by

$$v(t) = R i(t) \quad (2.17)$$

By using (2.7), we can obtain the SFA form of equation (2.17)

$$V(t) = R I(t) \quad (2.18)$$

Series resistance matrix sometimes appears as a part of the π -circuit representation of the transmission line. If an M-phase transmission line is modeled as an M-phase π -circuit, the series resistance matrix will be a full $M \times M$ matrix. The off-diagonal elements come about because the earth return is eliminated as the $(M+1)$ th conductor [9].

B. M-Phase Coupled Inductances

Equations (2.12)-(2.15) give us the shifted frequency domain equations and the discrete-time equivalent circuit for L .

C. M-Phase Coupled Capacitances

The time domain voltage equations for the M-Phase capacitances are written as follows

$$i(t) = C \frac{dv(t)}{dt} \quad (2.19)$$

The SFA form of equation (2.8) is obtained by employing the shifted frequency transformation,

$$I(t) = C \frac{dV(t)}{dt} + K_C V(t) \quad (2.20)$$

where $K_C = j\omega C$.

By discretizing (2.20) by the trapezoidal integration rule, the difference equations are obtained

$$I(t) = G_C V(t) - h_C(t) \quad (2.21)$$

where

$$\mathbf{R}_C = (\mathbf{G}_C)^{-1} = \left(\frac{2}{\Delta t} \mathbf{C} + \mathbf{K}_c \right)^{-1}$$

$$\mathbf{h}_C(t) = \frac{4}{\Delta t} \mathbf{C} \mathbf{V}(t - \Delta t) - \mathbf{h}_C(t - \Delta t)$$

Equation (2.21) is the EMTP equivalent circuit for $\mathbf{C}$ in the shifted frequency domain.

D. M-Phase Coupled R-L Branches

The time domain voltage equations for a series connection of M-Phase resistances and coupled inductances, shown in Figure 2.2(a), can be written as

$$\mathbf{v}(t) = \mathbf{R} \mathbf{i}(t) + \mathbf{L} \frac{d\mathbf{i}(t)}{dt} \quad (2.22)$$

The standard EMTP equivalent circuit (Figure 2.2(b)) for the series connection of $\mathbf{R}$ and $\mathbf{L}$ are reproduced as follows

$$\mathbf{i}(t) = \mathbf{r}_{RL}^{-1} \mathbf{v}(t) - \mathbf{h}_{RL}(t)$$

where

$$\mathbf{r}_{RL} = \mathbf{R} + \frac{2}{\Delta t} \mathbf{L} = \mathbf{g}_{RL}^{-1}$$

is a matrix of resistances, and

$$\mathbf{h}_{RL}(t) = \mathbf{g}_{RL} \left[\left(\mathbf{R} - \frac{2}{\Delta t} \mathbf{L} \right) \mathbf{g}_{RL} - \mathbf{I} \right] \mathbf{v}(t - \Delta t) - \mathbf{g}_{RL} \left(\mathbf{R} - \frac{2}{\Delta t} \mathbf{L} \right) \mathbf{h}_{RL}(t - \Delta t)$$

is a vector of history terms.

We now transform (2.22) using (2.7) to obtain the dynamic phasor equations

$$\mathbf{V}(t) = \mathbf{R} \mathbf{I}(t) + \mathbf{L} \frac{d\mathbf{I}(t)}{dt} + \mathbf{K}_L \mathbf{I}(t) \quad (2.23)$$

where $\mathbf{K}_L = j\omega_s \mathbf{L}$, $\mathbf{V}(t)$ and $\mathbf{I}(t)$ are the dynamic phasor vectors corresponding to the time-domain real variables $\mathbf{v}(t)$ and $\mathbf{i}(t)$, respectively.

Equation (2.23) can be transformed to discrete time by using the trapezoidal rule,

$$\mathbf{I}(t) = \mathbf{R}_{RL}^{-1} \mathbf{V}(t) - \mathbf{H}_{RL}(t) \quad (2.24)$$

where the equivalent resistances $\mathbf{R}_{RL}$ and past histories $\mathbf{H}_{RL}(t)$ are expressed as

$$\mathbf{R}_{RL} = \left(\mathbf{R} + \frac{2}{\Delta t} \mathbf{L} + \mathbf{K}_L \right) = \mathbf{G}_{RL}^{-1}$$

$$\mathbf{H}_{RL}(t) = \mathbf{G}_{RL} \left[\left(\mathbf{R} - \frac{2}{\Delta t} \mathbf{L} + \mathbf{K}_L \right) \mathbf{G}_{RL} - \mathbf{I} \right] \mathbf{V}(t - \Delta t) - \mathbf{G}_{RL} \left(\mathbf{R} - \frac{2}{\Delta t} \mathbf{L} + \mathbf{K}_L \right) \mathbf{H}_{RL}(t - \Delta t)$$

The SFA equivalent circuit described in (2.24) is illustrated in Figure 2.2(c).

Figure 2.2 (a) Series Connection of M-phase R and L; (b) EMTP Equivalent Circuit; (c) SFA Equivalent Circuit

E. Case Study

Two test cases are simulated to explore the feasibility of the SFA modeling method. The first test case is to simulate the switching operations in a typical linear time invariant (LTI) circuit that is shown in Figure 2.3. A voltage source is switched into the circuit at $t = 0$ s. After 8 ms, the switch S_1 opens once the current flowing through the switch crosses zero. In this circuit, $R_1 = 3 \Omega$, $R_2 = 50 \Omega$, $L_1 = 300$ mH, $L_2 = 1000$ mH, $C_1 = 20 \mu\text{F}$, $C_2 = 6 \mu\text{F}$. The voltage source has a rms value of 230 kV and a frequency of 60 Hz.

Figure 2.3 Linear Test Case

The current following through the branch connecting node 1 and node 2 is illustrated in Figure 2.4. From the simulation result we can find

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- (1) During the transient state, the SFA solutions are the envelop of the time domain solutions;
- (2) During the steady state, the SFA solutions degenerate to the phasor, which is the concept being widely used in power industry;
- (3) Time domain simulation results can be accurately traced back from the SFA solutions. We can see from Figure 2.4 that the time domain curve is identical to that obtained from the EMTP time-domain simulation.

Figure 2.4 Current Flowing through Branch 1-2 ($\Delta t = 1\text{ms}$)

The second test case is a linear time varying (LTV) circuit. A sinusoidal current source $i(t) = A\cos(\omega t)$ is applied to a time varying inductance $L\cos(\omega_0 t)$, as is shown in Figure 2.5. The frequency of the current source is $\omega = 60\text{Hz}$, and $\omega_0 = 10\text{Hz}$. The current source has a magnitude of 1 A, and we choose an integration step $\Delta t = 500\mu\text{s}$.

The unknown variable $v(t)$ is the voltage across the inductance.

Figure 2.5 Linear Time Varying Test Case

The numerical solution to the dynamic phasor $V(t)$ can be readily obtained by using the trapezoidal rule, which is given by

$$V(t) = -V(t - \Delta t) + \left[j\omega L \cos(\omega_0 t) + \omega_0 L \sin(\omega_0 t) + \frac{2L}{\Delta t} \cos(\omega_0 t) \right] I(t) + \left[j\omega L \cos(\omega_0 (t - \Delta t)) + \omega_0 L \sin(\omega_0 (t - \Delta t)) - \frac{2L}{\Delta t} \cos(\omega_0 (t - \Delta t)) \right] I(t - \Delta t)$$

The time domain solution of voltage across the inductance can be transformed back from $V(t)$ by $v(t) = \text{Re}[V(t)e^{j\omega t}]$.

Figure 2.6 Voltage across the Time Varying Inductance ($\Delta t = 0.5$ ms)

The simulation result is shown in Figure 2.6. It is clearly seen that the dynamic phasor solution is the envelope of the time domain solution. This test case validates that the SFA method is suitable for the LTV circuit analysis.

2.2.2 Options between Complex Arithmetic and Real Arithmetic

As can be seen in Section 2.2.1, the discretized power systems equations based on SFA models will be a system of linear complex equations, which can be solved by using either the complex arithmetic or the real arithmetic. Some references [31][32] concluded that, for the complex matrices, the complex inversion of them might be up to twice as fast as the real inversion, and that the rounding error bound for complex inversion is tighter than that for real inversion in terms of Gauss elimination. Fortunately, this may no longer be a notable issue with the modern computer architectures. Therefore, the real arithmetic method can be used as an alternative to the complex-arithmetic-based system solver. This section gives some examples about building real valued equivalent circuit for the RLC elements.

A. M-Phase Resistances

The discretized equations (2.18) can be decomposed into two equation sets corresponding to the real and imaginary parts of the dynamic phasors for voltages and currents. This leads to the real valued equivalent circuit of $\mathbf{R}$.

$$\begin{bmatrix} \mathbf{I}_{re}(t) \\ \mathbf{I}_{im}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{R}^{-1} & \\ & \mathbf{R}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{re}(t) \\ \mathbf{V}_{im}(t) \end{bmatrix} \quad (2.25)$$

B. M-Phase Coupled Inductances

The decomposed difference equations for the M-Phase coupled inductances can be derived as

$$\begin{bmatrix} \mathbf{I}_{re}(t) \\ \mathbf{I}_{im}(t) \end{bmatrix} = \begin{bmatrix} a\mathbf{L}^{-1} & -b\mathbf{L}^{-1} \\ b\mathbf{L}^{-1} & a\mathbf{L}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{re}(t) \\ \mathbf{V}_{im}(t) \end{bmatrix} - \begin{bmatrix} \mathbf{h}_{L,re}(t) \\ \mathbf{h}_{L,im}(t) \end{bmatrix} \quad (2.26)$$

where

$$\begin{bmatrix} \mathbf{h}_{L,re}(t) \\ \mathbf{h}_{L,im}(t) \end{bmatrix} = \begin{bmatrix} c\mathbf{L}^{-1} & -d\mathbf{L}^{-1} \\ d\mathbf{L}^{-1} & c\mathbf{L}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{re}(t - \Delta t) \\ \mathbf{V}_{im}(t - \Delta t) \end{bmatrix} + \begin{bmatrix} e & -f \\ f & e \end{bmatrix} \begin{bmatrix} \mathbf{h}_{L,re}(t - \Delta t) \\ \mathbf{h}_{L,im}(t - \Delta t) \end{bmatrix}$$

$$\begin{aligned} a &= \frac{\frac{2}{\Delta t}}{\left(\frac{2}{\Delta t}\right)^2 + \omega_s^2} & b &= \frac{-\omega_s}{\left(\frac{2}{\Delta t}\right)^2 + \omega_s^2} \\ c &= \frac{-\frac{4}{\Delta t} \left[\left(\frac{2}{\Delta t}\right)^2 - \omega_s^2 \right]}{\left[\left(\frac{2}{\Delta t}\right)^2 + \omega_s^2 \right]^2} & d &= \frac{\omega_s \left(\frac{4}{\Delta t}\right)^2}{\left[\left(\frac{2}{\Delta t}\right)^2 + \omega_s^2 \right]^2} \\ e &= \frac{-\left(\frac{2}{\Delta t}\right)^2 + \omega_s^2}{\left(\frac{2}{\Delta t}\right)^2 + \omega_s^2} & f &= \frac{\omega_s \frac{4}{\Delta t}}{\left(\frac{2}{\Delta t}\right)^2 + \omega_s^2} \end{aligned}$$

C. M-Phase Coupled Capacitances

We can get the real valued equivalent circuit for $\mathbf{C}$ as the following

$$\begin{bmatrix} \mathbf{I}_{re}(t) \\ \mathbf{I}_{im}(t) \end{bmatrix} = \begin{bmatrix} \frac{2}{\Delta t} \mathbf{C} & -\omega_s \mathbf{C} \\ \omega_s \mathbf{C} & \frac{2}{\Delta t} \mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{re}(t) \\ \mathbf{V}_{im}(t) \end{bmatrix} - \begin{bmatrix} \mathbf{h}_{C,re}(t) \\ \mathbf{h}_{C,im}(t) \end{bmatrix} \quad (2.27)$$

where

$$\begin{bmatrix} \mathbf{h}_{C,re}(t) \\ \mathbf{h}_{C,im}(t) \end{bmatrix} = \begin{bmatrix} \frac{4}{\Delta t} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \frac{4}{\Delta t} \mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{re}(t - \Delta t) \\ \mathbf{V}_{im}(t - \Delta t) \end{bmatrix} - \begin{bmatrix} \mathbf{h}_{C,re}(t - \Delta t) \\ \mathbf{h}_{C,im}(t - \Delta t) \end{bmatrix}$$

D. M-Phase Coupled R-L Branch

By decomposing the equivalent circuit in (2.24), we can obtain the real valued equivalent for a series RL branch

$$\begin{bmatrix} \mathbf{I}_{re}(t) \\ \mathbf{I}_{im}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{G}_{RL,re} & -\mathbf{G}_{RL,im} \\ \mathbf{G}_{RL,im} & \mathbf{G}_{RL,re} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{re}(t) \\ \mathbf{V}_{im}(t) \end{bmatrix} - \begin{bmatrix} \mathbf{h}_{RL,re}(t) \\ \mathbf{h}_{RL,im}(t) \end{bmatrix} \quad (2.28)$$

where

$$\begin{bmatrix} \mathbf{h}_{RL,re}(t) \\ \mathbf{h}_{RL,im}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{C} & -\mathbf{D} \\ \mathbf{D} & \mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{re}(t - \Delta t) \\ \mathbf{V}_{im}(t - \Delta t) \end{bmatrix} - \begin{bmatrix} \mathbf{E} & -\mathbf{F} \\ \mathbf{F} & \mathbf{E} \end{bmatrix} \begin{bmatrix} \mathbf{h}_{C,re}(t - \Delta t) \\ \mathbf{h}_{C,im}(t - \Delta t) \end{bmatrix}$$

$$\mathbf{A} = \mathbf{R} - \frac{2}{\Delta t} \mathbf{L}$$

$$\mathbf{B} = \omega_s \mathbf{L}$$

$$\mathbf{C} = -\mathbf{G}_{RL,re} + \mathbf{G}_{RL,re} \mathbf{A} \mathbf{G}_{RL,re} - \mathbf{G}_{RL,im} \mathbf{B} \mathbf{G}_{RL,re} - \mathbf{G}_{RL,re} \mathbf{B} \mathbf{G}_{RL,im} - \mathbf{G}_{RL,im} \mathbf{A} \mathbf{G}_{RL,im}$$

$$\mathbf{D} = -\mathbf{G}_{RL,im} + \mathbf{G}_{RL,re} \mathbf{B} \mathbf{G}_{RL,re} + \mathbf{G}_{RL,im} \mathbf{A} \mathbf{G}_{RL,re} + \mathbf{G}_{RL,re} \mathbf{A} \mathbf{G}_{RL,im} - \mathbf{G}_{RL,im} \mathbf{B} \mathbf{G}_{RL,im}$$

$$\mathbf{E} = \mathbf{G}_{RL,re} \mathbf{A} - \mathbf{G}_{RL,im} \mathbf{B}$$

$$\mathbf{F} = \mathbf{G}_{RL,im} \mathbf{A} + \mathbf{G}_{RL,re} \mathbf{B}$$

2.2.3 Transformer Model in the Shifted Frequency Domain

Since the SFA modeling focuses on the low frequency dynamics as opposed to the standard EMTP modeling for simulating the fast transients, here only the low frequency transformer model is built in the SFA domain. For transients with highest frequency less than 2-3

kHz, the transformer can be modeled as a series connection of multi-phase coupled $\mathbf{R}$ and $\mathbf{L}$ branches, in which each RL branch can represent one transformer winding [9]. Therefore, the SFA domain model of a transformer follows directly the model described in equation (2.24).

Here a major issue is how to obtain the $\mathbf{G}_{RL}$ (thus the history terms) in (2.24). A traditional way for single-phase transformers is the $[\mathbf{Z}]$ matrix method, which first builds the coupled impedances from the transformer parameters. For example, the $[\mathbf{Z}]$ matrix of a two winding transformer illustrated in Figure 2.7 can be obtained as below

$$[\mathbf{Z}] = \begin{bmatrix} Z_1 + Z_m & \frac{N_2}{N_1} Z_m \\ \frac{N_2}{N_1} Z_m & Z_2 + \left(\frac{N_2}{N_1}\right)^2 Z_m \end{bmatrix}$$

Then, $[\mathbf{Z}]$ is inverted to get the admittance matrix $[\mathbf{Y}]$ that will be used to calculate $\mathbf{G}_{RL}$. This method, however, has limitations such as:

(i) Magnetizing impedance Z_m cannot be neglected or set to be ∞ in this model. Otherwise the model cannot work. (ii) Z_m usually dominates $[\mathbf{Z}]$ matrix because $Z_m \gg$ leakage impedances Z_1 and Z_2 . However, it is the leakage impedances that largely determine the simulation results. This means all data inputs are required to be very precise. Moreover, $[\mathbf{Z}]^{-1}$ is ill-conditioned because of the dominating Z_m , which may negatively affect the accuracy in the simulation results.

Figure 2.7 Single-Phase Two-Winding Transformer

In this section the $[\mathbf{L}]^{-1}$ model [9] adopted in MicroTran, rather than the $[\mathbf{Z}]$ matrix model, is used. This model can directly formulate the $[\mathbf{L}]^{-1}$ matrix without doing inversion on the $[\mathbf{Z}]$ matrix. It works for any number of windings, and for single-phase as well as three-phase transformers. For instance, the $[\mathbf{L}]^{-1}$ matrix of a two winding transformer in Figure 2.7 can be built by

$$[\mathbf{L}]^{-1} = \begin{bmatrix} \frac{1}{L_s} & -\frac{N_1}{N_2} \frac{1}{L_s} \\ -\frac{N_1}{N_2} \frac{1}{L_s} & \left(\frac{N_1}{N_2}\right)^2 \frac{1}{L_s} \end{bmatrix} \quad (2.29)$$

where

$$L_s = L_1 + \left(\frac{N_1}{N_2}\right)^2 L_2$$

An alternative to the $[\mathbf{L}]^{-1}$ model is to use an inductance matrix in series with an ideal transformer [14].

The theory behind the $[\mathbf{L}]^{-1}$ model and the $[\mathbf{L}]^{-1}$ matrices for other types of transformers can be found in [9]. The equivalent conductance and history terms matrices can be calculated by using (2.24). Note that some modifications in (2.24) are needed for the usage of $[\mathbf{L}]^{-1}$, as expressed below

$$\begin{aligned} \mathbf{G}_{RL} &= \left(\mathbf{R} + \frac{2}{\Delta t} \mathbf{L} + \mathbf{K}_L \right)^{-1} = \left[\mathbf{L}^{-1} \mathbf{R} + \left(\frac{2}{\Delta t} + j\omega_s \right) \mathbf{I} \right]^{-1} \mathbf{L}^{-1} \\ \mathbf{H}_{RL}(t) &= \left\{ \left[\mathbf{L}^{-1} \mathbf{R} + \left(\frac{2}{\Delta t} + j\omega_s \right) \mathbf{I} \right]^{-1} \left[\mathbf{L}^{-1} \mathbf{R} + \left(-\frac{2}{\Delta t} + j\omega_s \right) \mathbf{I} \right] \mathbf{G}_{RL} - \mathbf{G}_{RL} \right\} \mathbf{V}(t - \Delta t) - \\ &\quad \left[\mathbf{L}^{-1} \mathbf{R} + \left(\frac{2}{\Delta t} + j\omega_s \right) \mathbf{I} \right]^{-1} \left[\mathbf{L}^{-1} \mathbf{R} + \left(-\frac{2}{\Delta t} + j\omega_s \right) \mathbf{I} \right] \mathbf{H}_{RL}(t - \Delta t) \end{aligned}$$

The magnetizing branch is not required in this model. It can be added at the terminals when it is needed. This will allow one to model the saturation effect of the core by adding the nonlinear L_m at the terminals.

As a test case, a two winding transformer with the on-load tap changer (OLTC) is simulated using the SFA model. The rated line-to-line voltage of the primary winding is 110kV. The winding ratio is $N_1 : N_2 = 110 : 28.4$. $R = 10 \, \Omega$, $X_s = 33.2174 \, \Omega$. The transformer is first connected to the rated voltages and operating until the primary winding voltage suddenly drops to 0.95 p.u. at $t = 10s$. Five seconds later, the secondary side OLTC changes the tap ratios to 1+0.0125, attempting to bring the voltage back. The total simulation time is 20s. Figure 2.8 illustrates the dynamic phasor result together with the time domain result transformed back from SFA solution. In this simulation, a very large time step $\Delta t = 50 \, ms$ is used. On the other hand, if

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we simulate this case using the EMTP algorithm, the time step for an accurate solution should at least satisfy $\Delta t < 1/(5 \times 60) = 3.3 \text{ ms}$ to respect the Nyquist frequency limit.

Figure 2.8 Secondary Winding Voltage ($\Delta t = 50 \text{ ms}$)

2.2.4 Load Models

A. Exponential Load Model

An exponential load means the power consumed by the load depends exponentially on the load voltage, which can be expressed as

$$P(t) = P_0 \left(\frac{V}{V_0} \right)^{np} \quad (2.30)$$

$$Q(t) = Q_0 \left(\frac{V}{V_0} \right)^{nq} \quad (2.31)$$

where P_0 , Q_0 , and V_0 are the pre-disturbance conditions of the load. For special cases such that load parameters np or nq becomes 0, 1, or 2, the load model will represent the constant power, constant current, or constant impedance load, respectively.

The following equations express the discrete-time equivalent circuit (see Figure 2.9) for the exponential load. They can be directly derived from (2.30) and (2.31) by using the backward Euler rule. The backward Euler rule avoids predictions in the simulation with load models, though it may introduce some damping into the equivalent circuit. The trapezoidal rule can also

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be used but iterations have to be applied at each time step due to the nonlinearity of the exponential load model.

$$R_{load}(t) = \frac{|V(t - \Delta t)|^2}{P(t)} \quad (2.32)$$

$$L_{load}(t) = \frac{|V(t - \Delta t)|^2}{\omega_s Q(t)} \quad (2.33)$$

$$R_{Lload}(t) = \left(\frac{2}{\Delta t} + j\omega_s \right) L_{load}(t) \quad (2.34)$$

$$h_{Lload}(t) = \frac{\frac{-4}{\Delta t}}{\left(\frac{2}{\Delta t} + j\omega_s \right)^2} L_{load}^{-1}(t) V(t - \Delta t) + \frac{\frac{2}{\Delta t} - j\omega_s}{\frac{2}{\Delta t} + j\omega_s} h_{Lload}(t - \Delta t) \quad (2.35)$$

$$P(t) = P_0 \left(\frac{|V(t - \Delta t)|}{V_0} \right)^{np} \quad (2.36)$$

$$Q(t) = Q_0 \left(\frac{|V(t - \Delta t)|}{V_0} \right)^{nq} \quad (2.37)$$

where $|V(t)|$ is the magnitude of dynamic phasor $V(t)$.

Figure 2.9 Equivalent Circuit for the Exponential Load

In this section, a two node test case shown in Figure 2.10 is simulated to demonstrate the load modeling by the SFA method. This test case for voltage stability studies is taken from [33].

Figure 2.10 Two Node Test Case

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First, the system is operating with the load $P=0.8\text{p.u.}$, and $Q=0.4\text{p.u.}$ The load voltage at node B is illustrated in Figure 2.11. It can be seen that the SFA curve is the complex envelope of the time domain curve, to which the SFA result can be accurately transformed.

Figure 2.11 Simulation Results of the Node Voltage (a) SFA Solution ($\Delta t = 10\text{ ms}$) (b) EMTP Solution ($\Delta t = 0.5\text{ ms}$)

Second, the system is operating with a heavier load of $P=1.85\text{p.u.}$, $Q=0.15\text{p.u.}$ From the nodal voltage shown in Figure 2.12, it can be seen that the system collapses in this case. These results in Figure 2.12 are almost the same as the results obtained in [33], where the system is found to collapse when $P=1.843\text{p.u.}$ and $Q=0.15\text{p.u.}$

Figure 2.12 Simulation Results of Voltage Collapse (a) SFA Solution ($\Delta t = 10\text{ ms}$) (b) EMTP Solution ($\Delta t = 0.5\text{ ms}$)

Now the system performance is simulated when the system load keeps increasing with time. The system load is assumed to change in the way shown in Table 2.1. The SFA results of

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the load node voltage are illustrated in Figure 2.13. It can be seen that the SFA model can accurately trace the voltage stability behavior of the system when the exponential dynamics of the load are taken into consideration.

Table 2.1 Load Increased with the Time

Time interval (s)	[0, 2]	[3, 4)	[4, 5]	[5, 6]	[6, 7]	[7, 8]
Active Load (p.u.)	0.7	0.8	0.8	0.9	1.0	1.85
Reactive Load (p.u.)	0.3	0.4	0.5	0.5	0.6	0.15

Figure 2.13 SFA Simulation Results of the Voltage Collapse Process ($\Delta t = 10$ ms)

B. Steady State Induction Machine Load

The steady state induction motor load is modeled in this section. The machine may be first initialized at the beginning of the simulation. Usually, the induction machine parameters such as R_s , X_s , X_M , X_R , R_R are known quantities as input data. In addition, the real power P and the terminal voltage are also known, where V can be determined by the steady state power flow calculations. Hence the unknowns for initialization are the slip s , and reactive power Q .

With the core loss neglected, the equivalent circuit of the induction motor is shown in Figure 2.14.

Figure 2.14 Steady State Equivalent Circuit

The formula to calculate the initial slip s and the reactive power Q can be derived [34] based on the equivalent circuit in Figure 2.14, as expressed below.

$$s = \frac{2A}{-B + \sqrt{B^2 - 4AC}} R_R \quad (2.38)$$

$$Q = \text{Im}(V^2 \bar{Y}) \quad (2.39)$$

where

$$A = A' \frac{P}{V^2} - R_s \quad B = B' \frac{P}{V^2} - X_M^2 \quad C = C' \frac{P}{V^2} - R_s (X_s + X_M)$$

$$A' = R_s^2 + (X_s + X_M)^2 \quad B' = 2R_s X_M^2 \quad C' = (X_s X_M + X_s X_R + X_R X_M)^2 + R_s^2 (X_R + X_M)^2$$

Then the initial slip can be validated by checking whether the power factor falls into a reasonable range, for instance,

$$0.7 < \cos \varphi = \cos \left(\arctan \frac{Q}{P} \right) < 0.9$$

If the inequality is not satisfied, an error message will be printed.

The discrete time equivalent circuit in the shifted frequency domain is shown in Figure 2.15. The phasor dynamics are introduced to the steady-state equivalent circuit to capture the dynamics in the electrical part. The parameters in the SFA model can be obtained as follows.

Figure 2.15 Discrete Time Equivalent Circuit

$$G_{23} = \frac{1}{R_s + \frac{2L_s}{\Delta t} + K_{Ls}} \quad (2.40)$$

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$$G_m = \frac{1}{\frac{2L_m}{\Delta t} + K_{Lm}} \quad (2.41)$$

$$G_{s0} = \frac{1}{\frac{R_r}{s(t-\Delta t)} + \frac{2L_r}{\Delta t} + K_{Lr}} \quad (2.42)$$

$$h_{stator}(t) = -[V_2(t-\Delta t) - V_3(t-\Delta t)] \frac{\frac{4L_s}{\Delta t}}{\left(R_s + \frac{2L_s}{\Delta t} + K_{Ls}\right)^2} + \frac{-R_s + \frac{2L_s}{\Delta t} - K_{Ls}}{R_s + \frac{2L_s}{\Delta t} + K_{Ls}} h_{stator}(t-\Delta t) \quad (2.43)$$

$$h_m(t) = -V_3(t-\Delta t) \frac{\frac{4L_m}{\Delta t}}{\left(\frac{2L_m}{\Delta t} + K_{Lm}\right)^2} + \frac{\frac{2L_m}{\Delta t} - K_{Lm}}{\frac{2L_m}{\Delta t} + K_{Lm}} h_m(t-\Delta t) \quad (2.44)$$

$$h_{rotor}(t) = -V_3(t-\Delta t) \frac{\frac{4L_r}{\Delta t}}{\left(\frac{R_r}{s(t-\Delta t)} + \frac{2L_r}{\Delta t} + K_{Lr}\right)^2} + \frac{-\frac{R_r}{s(t-\Delta t)} + \frac{2L_r}{\Delta t} - K_{Lr}}{\frac{R_r}{s(t-\Delta t)} + \frac{2L_r}{\Delta t} + K_{Lr}} h_{rotor}(t-\Delta t) \quad (2.45)$$

When the system voltages change, there is a mismatch between the electromagnetic torque and mechanical torque since mechanical torque usually cannot change instantaneously. Then the acceleration equation of the rotor mass is represented as

$$T_e - T_m = J \frac{d\omega_m}{dt} = -J \frac{2\omega_0}{p_f} \frac{ds}{dt} \quad (2.46)$$

where p_f is the number of poles.

Therefore the slip can be calculated using the backward Euler rule,

$$s(t) = s(t-\Delta t) - \frac{p_f}{2\omega_0 J} [T_e(t) - T_m(t)] \Delta t \quad (2.47)$$

where

$$T_e(t) = 3 \frac{p_f}{2} \frac{V_{TH}^2(t) \frac{R_r}{s\omega_0}}{\left[\left(R_{TH} + \frac{R_r}{s(t-\Delta t)} \right)^2 + (X_{TH} + X_r)^2 \right]} \quad (2.48)$$

If the mechanical torque is assumed to be a quadratic function of ω ,

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$$\begin{aligned} T_m(t) &= a + b\omega + c\omega^2 \\ &= \alpha + \beta s(t - \Delta t) + \gamma s^2(t - \Delta t) \end{aligned} \quad (2.49)$$

where

$$\begin{aligned} \alpha &= a + b\omega_s + c\omega_s^2 & \beta &= -b\omega_s - 2c\omega_s^2 & \gamma &= c\omega_s^2 \\ V_{TH} &= V_\phi \frac{X_m}{\sqrt{R_s^2 + (X_s + X_m)^2}} & R_{TH} &= \frac{R_s X_m^2}{R_s^2 + (X_s + X_m)^2} \\ X_{TH} &= \frac{R_s^2 X_m + X_s X_m (X_s + X_m)}{R_s^2 + (X_s + X_m)^2} \end{aligned}$$

Equation (2.49) represents constant torque if $a = 0$ and $b = 0$, and a torque linearly dependent on ω if $c = 0$.

In this section, a test system is simulated to illustrate the capability of the SFA method for modeling induction machines in steady state. As shown in Figure 2.16, an induction motor is supplied by an infinite voltage source through a double-circuit line. One of the parallel lines is tripped off at $t = 2$ s. The total simulation time is 10 s. The time step for integration is 5 ms. The parameters are $R_s = 0.0092$ pu, $X_s = 0.0717$ pu, $X_m = 4.1375$ pu, $R_r = .00698$ pu, $X_r = .0717$ pu, inertia constant $H = 0.6$ s, line-to-line voltage = 6.9 kV, $p_f = 2$, $a = b = 0$, $c = 0.00308$, $V_0 = 1.0064$ p.u., $P_0 = 0.4$ p.u., $R_{line} = 0.161$ Ω , $L_{line} = 0.0042707$ H, $C = 0.209$ mF. The per unit values are based on $V_{base} = 6.9$ kV and $S_{base} = 10$ MVA.

Figure 2.16 Motor Test Case

The following are the SFA solutions of the test case, including the load bus voltage, induction motor slip and real power absorbed by the motor. These results are illustrated in Figure 2.17, Figure 2.18 and Figure 2.19. As shown in Figure 2.17, the voltage has dropped from steady state to collapsing state with the motor stalled in about 6 seconds. Artificial numerical oscillations appear in the simulation results because of imperfect initialization of the test system. In order to capture detailed dynamics between the steady state and the collapse, one needs to apply the SFA method to a detailed motor model considering the rotor and stator transients. This

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is further investigated in Chapter 4, where a general-purpose induction machine model in the shifted frequency domain is proposed.

Figure 2.17 Induction Motor Terminal Voltage ($\Delta t = 5$ ms)

Figure 2.18 Real Power Absorbed by the Induction Motor ($\Delta t = 5$ ms)

Figure 2.19 Slip of Induction Motor ($\Delta t = 5$ ms)

2.3 Numerical Accuracy Analysis

As mentioned in Chapter 1, the theory of Shifted Frequency Analysis (SFA) needs to be implemented in a circuit analysis program such as the EMTP to achieve its advantage of using large solution step and getting correct simulation results in the neighborhood of the 60 Hz frequency. In the EMTP, a numerical discretization rule (integration rule) is used to convert the

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equivalent circuit of each network component into an equivalent discrete time model consisting of an equivalent resistance and a history term. These equivalent circuits of components are used to build the nodal equations of the whole system, which can be solved by using the numerical linear algebra. The trapezoidal and backward Euler rules are the most common choices to perform the discretization from continuous time models into discrete time models.

An illustrative and effective way to analyze the numerical accuracy of an integrator (integration rule) is to examine the behavior of an inductance L with $v(t)$ as input and $i(t)$ as output (or a capacitance C with $i(t)$ as input and $v(t)$ as output) [30]. The magnitude and phase distortion introduced by an integrator can be quantitatively analyzed by calculating the frequency domain equivalent circuit of L which is discretized by the given integration rule. This approach is adopted in this section to analyze the numerical accuracy of an integrator in the SFA domain.

To analyze frequency responses of an inductance L , first take an input that has only one frequency ω (close to ω_s) and a unity magnitude, i.e. $v(t) = \cos \omega t$.

The dynamic phasor of $v(t)$ in the SFA domain is

$$V(t) = [v(t) + jH(v(t))]e^{-j\omega_s t} = e^{j(\omega - \omega_s)t} = e^{j\Delta\omega t} \quad (2.50)$$

Now suppose the output is

$$I(t) = Y_e(\Delta\omega)e^{j\Delta\omega t}. \quad (2.51)$$

where $Y_e(\Delta\omega)$ is the admittance of an L in the discrete-time SFA domain.

A. Frequency Domain Equivalent Circuits for L

The SFA domain equivalent circuit of L is obtained with the trapezoidal and backward Euler rule. The equivalent circuit parameters can be plotted as functions of $\Delta f (= \frac{\Delta\omega}{2\pi})$, as is shown in Figure 2.20.

Substitute (2.50), (2.51) into the difference equation it is easy to find $Y_e(\Delta\omega)$ for each discretization rule.

(1) Trapezoidal rule

With trapezoidal rule,

$$V(t) = \left(\frac{2L}{\Delta t} + j\omega_s L \right) I(t) - V(t - \Delta t) + \left(-\frac{2L}{\Delta t} + j\omega_s L \right) I(t - \Delta t) \quad (2.52)$$

Substitute (2.50), (2.51) into (2.52),

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$$e^{j\Delta\omega t} + e^{j\Delta\omega(t-\Delta t)} = \left(\frac{2L}{\Delta t} + j\omega_s L\right) Y_e e^{j\Delta\omega t} + \left(-\frac{2L}{\Delta t} + j\omega_s L\right) Y_e e^{j\Delta\omega(t-\Delta t)}$$

Then

$$Y_e(\Delta\omega) = \frac{1}{j\omega_s L + \frac{2L}{\Delta t} \cdot \frac{e^{j\Delta\omega\Delta t} - 1}{e^{j\Delta\omega\Delta t} + 1}} = \frac{1}{j\omega_s L + j\Delta\omega L \cdot \frac{\tan \frac{\Delta\omega\Delta t}{2}}{\frac{\Delta\omega\Delta t}{2}}}$$

where the equivalent inductance is

$$L_e = \left(1 + \frac{\Delta\omega}{\omega_s} \frac{\tan \frac{\Delta\omega\Delta t}{2}}{\frac{\Delta\omega\Delta t}{2}}\right) L \quad (2.53)$$

(2) Backward Euler rule

The difference equation is

$$I(t) - I(t - \Delta t) = \frac{\Delta t}{L} (V(t) - j\omega_s L I(t)) \quad (2.54)$$

Then

$$Y_e e^{j\Delta\omega t} - Y_e e^{j\Delta\omega(t-\Delta t)} = \frac{\Delta t}{L} (e^{j\Delta\omega t} - j\omega_s L Y_e e^{j\Delta\omega t})$$

Therefore the admittance can be derived

$$1/Y_e(\Delta\omega) = \frac{L}{\Delta t} (j\omega_s \Delta t + 1 - e^{-j\Delta\omega\Delta t}) = j\omega_s L + \frac{1}{\frac{\Delta t}{L} \cdot \frac{e^{j\Delta\omega\Delta t/2} - e^{-j\Delta\omega\Delta t/2}}{e^{j\Delta\omega\Delta t/2} + e^{-j\Delta\omega\Delta t/2}}}$$

$$\text{Let } Y_e(\Delta\omega) = \frac{1}{R_e} + \frac{1}{j\omega_s L_e}, \text{ then}$$

$$\begin{cases} R_e = 1/\text{Re}(Y_e(\Delta\omega)) \\ L_e = -1/[\omega_s \text{Im}(Y_e(\Delta\omega))] \end{cases} \quad (2.55)$$

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Figure 2.20 Equivalent Circuit L_e for Backward Euler and Forward Euler ($L=1$; $\Delta t=1$ cycle, 3 cycles, 5 cycles)

B. Accuracy of Discretization Rules

The accuracy of the discrete-time integration rules can be expressed by the following ratio

$$\frac{H_e(\Delta\omega)}{H(\Delta\omega)} = \frac{Y_e(\Delta\omega)}{Y(\Delta\omega)} \Big|_{L=1} \quad (2.56)$$

(1) Trapezoidal rule

$$\begin{aligned} \frac{H_e(\omega)}{H(\omega)} &= \frac{j\omega_s}{j\omega_s + \frac{2}{\Delta t} \cdot \frac{e^{j\Delta\omega\Delta t} - 1}{e^{j\Delta\omega\Delta t} + 1}} \\ &= \frac{j\omega_s}{j\omega_s + \frac{2}{\Delta t} \cdot \frac{e^{j2\pi\Delta f\Delta t} - 1}{e^{j2\pi\Delta f\Delta t} + 1}} \end{aligned}$$

(2) Backward Euler

$$\begin{aligned} \frac{H_e(\omega)}{H(\omega)} &= \frac{j\omega_s\Delta t}{j\omega_s\Delta t + 1 - e^{-j\Delta\omega\Delta t}} \\ &= \frac{j\omega_s\Delta t}{j\omega_s\Delta t + 1 - e^{-j2\pi\Delta f\Delta t}} \end{aligned}$$

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Frequency domain accuracy analysis results are shown in Figure 2.21. The frequency deviation ranges from -3 to 3 Hz.

(a)

(b)

Figure 2.21 Accuracy of integration rules (a) Magnitude (b) Phase

From Figure 2.20 we can find that the distortion on L is very small for f close to 60 Hz. When $\Delta f \rightarrow 0$, $L_e \rightarrow L$ and there is no distortion. The error grows as Δf increases. The distortion in the equivalent inductance also increases with the time step.

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Figure 2.21(a) shows that the trapezoidal rule is slightly less accurate than the backward Euler rule in the shifted frequency domain. On the other hand, Figure 2.21(b) shows that the trapezoidal rule has no phase distortion while the backward Euler rule can cause larger phase distortions especially when Δf becomes larger.

It is clearly shown in equation (2.55) that the backward Euler rule adds a fictitious resistance to the circuit, which introduces numerical damping. Therefore, it may be advantageous when the trapezoidal rule has the risk of numerical oscillations when it is used as a differentiator. Here the backward Euler rule may be used over a few integration steps to damp numerical oscillations whenever some discontinuities occur, which is the major idea of CDA [30].

Unless specially noted, in this thesis the trapezoidal rule is used for the component modelling and the system solver because SFA equivalent circuits similar to those in the EMTP can facilitate the implementation of SFA concepts in the EMTP algorithm.

Chapter 3

Synchronous Machine Modelling Based on Shifted Frequency Analysis

3.1 Introduction

This chapter describes a new synchronous machine model based on the Shifted Frequency Analysis (SFA) method, which uses dynamic phasor variables rather than instantaneous time domain variables. Discrete-time SFA component models have a form which is similar to the EMTP component models. Dynamic phasors provide envelopes of the time domain waveforms and can be accurately transformed back to instantaneous time values. When the frequency spectra of the signals are close to the fundamental power frequency, the SFA model allows the use of large time steps without sacrificing accuracy. This makes the SFA method particularly efficient for power system dynamics.

3.2 Voltage-behind-Reactance Synchronous Machine Model

The SFA method can be applied to a number of synchronous machine models, such as the phase domain synchronous machine model of [35], the voltage-behind-reactance (VBR) model of [36], and others. As reported in [37], the VBR model is more efficient and stable than other traditional machine models used in the EMTP and it was chosen for the SFA implementation in this chapter.

Figure 3.1 Salient-Pole Synchronous Machine and Its Windings

The cross-section of a salient pole synchronous generator is shown in Figure 3.1. The generator has a 3-phase stator winding, a rotor field winding (fd winding), M rotor damper windings in the q -axis ($kq1, \dots, kqM$), and N rotor damper windings in the d -axis ($kd1, \dots, kdN$). The equivalent circuit in Figure 3.1 can be used for modelling different types of synchronous machines without losing generality since straightforward changes can be made without much difficulty to adapt this model for other types of synchronous machines. In this chapter, some assumptions [38] are made in developing the mathematical model of the synchronous machine:

The 3-phase stator winding is assumed to be symmetrical;

The capacitances of all the windings are neglected;

Each of the distributed winding can be represented by a concentrated winding;

The change in the inductance of the stator windings due to rotor position is sinusoidal and does not contain higher harmonics;

Hysteresis loss is negligible while the effect of eddy current is included in the damper winding models.

In Figure 3.1, the motor convention is adopted for the machine modelling, which means the stator currents are positive when flowing into the machine terminals. The damper windings are shown to represent the paths for induced rotor currents. In a salient-pole machine, the rotor is laminated and therefore the damper winding currents are largely confined to the cage windings embedded in the rotor surface. Usually the dynamic behaviour of the salient-pole machine can be predicted accurately enough by using one equivalent damper winding $kq2$ in the q -axis and one damper winding kd in the d -axis. On the other hand, a cylindrical-rotor machine has a solid iron rotor with a cage-type winding embedded in the rotor surface, and the damper winding currents can flow either in the cage winding or in the solid iron. In order to accurately simulate the transient process in the cylindrical-rotor machine, it is necessary to put two equivalent damper windings $kq1$, $kq2$ in the q -axis and one damper winding kd in the d -axis.

The $dq0$ synchronous machine model is most widely used in power system electromagnetic transient simulations. However, to interface the $dq0$ model to the network in the phases a , b , c coordinates, some electrical variables have to be predicted, which may theoretically cause numerical instability. The $dq0$ model needs prediction because the $dq0$ electrical variables are not solved simultaneously with abc external network variables. A phase-domain model avoids predicting the electrical quantities and therefore is more robust than the $dq0$ model. The SFA model implemented in this Chapter is based on phase coordinates and therefore does not need predictions of the electrical variables. A detailed discussion of the prediction in EMTP machine models can be found in Chapter 8 of [9].

The original phase-coordinate synchronous machine model consists of three stator voltage equations, $M + N + 1$ rotor voltage equations, $M + N + 4$ flux linkage equations, and mechanical part equations for the torque and speed. To improve the numerical efficiency and stability, a new phase-coordinate model called Voltage-behind-Reactance (VBR) model has been derived in [36]. The rotor voltage equations and flux linkages equations are manipulated so that the derivatives of the rotor flux linkages are algebraically incorporated into the stator voltage equations, resulting in an efficient voltage-behind-reactance form model. The VBR model for the

synchronous machine in Figure 3.1 includes the stator voltage equations, flux linkages equations and rotor mechanical part equations, which can be stated as follows.

A. Stator Voltage Equations

$$\mathbf{v}_{abcs}(t) = \mathbf{r}_s \mathbf{i}_{abcs}(t) + p[\mathbf{L}_{abcs}''(\theta_r) \mathbf{i}_{abcs}(t)] + \mathbf{v}_{abcs}''(t) \quad (3.1)$$

where $\mathbf{r}_s$ is the stator winding resistance matrix, and

$$\mathbf{L}_{abcs}''(\theta_r) = \mathbf{L}_0'' + (-L_b'')\mathbf{A} \quad (3.2)$$

$$\mathbf{L}_0'' = \begin{bmatrix} L_{ls} + L_a'' & -\frac{L_a''}{2} & -\frac{L_a''}{2} \\ -\frac{L_a''}{2} & L_{ls} + L_a'' & -\frac{L_a''}{2} \\ -\frac{L_a''}{2} & -\frac{L_a''}{2} & L_{ls} + L_a'' \end{bmatrix},$$

$$\mathbf{A} = \begin{bmatrix} \cos(2\theta_r) & \cos(2\theta_r - 2\pi/3) & \cos(2\theta_r + 2\pi/3) \\ \cos(2\theta_r - 2\pi/3) & \cos(2\theta_r - 4\pi/3) & \cos(2\theta_r) \\ \cos(2\theta_r + 2\pi/3) & \cos(2\theta_r) & \cos(2\theta_r + 4\pi/3) \end{bmatrix},$$

$$L_a'' = \frac{L_{mq}'' + L_{md}''}{3},$$

$$L_b'' = \frac{L_d'' - L_q''}{3},$$

$$L_{mq}'' = \left(1/L_{mq} + \sum_{j=1}^M 1/L_{lkqj} \right)^{-1},$$

$$L_{md}'' = \left(1/L_{md} + \sum_{j=1}^N 1/L_{lkdj} + 1/L_{lfd} \right)^{-1},$$

(As an example, for a salient-pole machine, L_{mq}'' and L_{md}'' may be expressed as

$$L_{mq}'' = (1/L_{mq} + 1/L_{lkq2})^{-1}, L_{md}'' = (1/L_{md} + 1/L_{lkd} + 1/L_{lfd})^{-1}. \text{ Otherwise, } L_{mq}'' = (1/L_{mq} + 1/L_{lkq1} + 1/L_{lkq2})^{-1},$$

$L_{md}'' = (1/L_{md} + 1/L_{lkd} + 1/L_{lfd})^{-1}$ may be used in modelling a cylindrical-rotor machine.)

The subtransient voltages in equation (3.1) can be expanded

$$\mathbf{v}_{abcs}''(t) = [\mathbf{K}_s'(\theta_r(t))]^{-1} \begin{bmatrix} \mathbf{v}_q'' \\ \mathbf{v}_d'' \\ 0 \end{bmatrix} \quad (3.3)$$

where

$$v_q = \omega_r L_{mq} \left(\frac{\lambda_{fd}}{L_{lfd}} + \sum_{j=1}^N \frac{\lambda_{kdj}}{L_{lkdj}} \right) + \sum_{j=1}^M \left(\frac{L_{mq} r_{kqj}}{L_{lkqj}^2} (L_{mq} \sum_{j=1}^M \frac{\lambda_{kqj}}{L_{lkqj}} - \lambda_{kqj}) + \frac{L_{mq}^2 r_{kqj}}{L_{lkqj}^2} i_{qs}^r \right) \quad (3.4)$$

$$\begin{aligned} v_d = & -\omega_r L_{mq} \sum_{j=1}^M \frac{\lambda_{kqj}}{L_{lkqj}} + \sum_{j=1}^N \left(\frac{L_{md} r_{kdj}}{L_{lkdj}^2} \left[L_{md} \left(\frac{\lambda_{fd}}{L_{lfd}} + \sum_{j=1}^N \frac{\lambda_{kdj}}{L_{lkdj}} \right) - \lambda_{kdj} \right] + \frac{r_{kdj}}{L_{lkdj}^2} L_{md}^2 i_{ds}^r \right) \\ & + \frac{L_{md} r_{fd}}{L_{lfd}^2} \left[L_{md} \left(\frac{\lambda_{fd}}{L_{lfd}} + \sum_{j=1}^N \frac{\lambda_{kdj}}{L_{lkdj}} \right) - \lambda_{fd} \right] + \frac{r_{fd}}{L_{lfd}^2} L_{md}^2 i_{ds}^r + \frac{L_{md}}{L_{lfd}} v_{fd} \end{aligned} \quad (3.5)$$

B. Flux Linkages Equations

$$p\lambda_{kqj} = -\frac{r_{kqj}}{L_{lkqj}} (\lambda_{kqj} - \lambda_{mq}), j = 1, 2, \dots, M \quad (3.6)$$

$$p\lambda_{kdj} = -\frac{r_{kdj}}{L_{lkdj}} (\lambda_{kdj} - \lambda_{md}), j = 1, 2, \dots, N \quad (3.7)$$

$$p\lambda_{fd} = -\frac{r_{fd}}{L_{lfd}} (\lambda_{fd} - \lambda_{md}) + v_{fd} \quad (3.8)$$

where

$$\lambda_{mq} = L_{mq} \left(\sum_{j=1}^M \frac{\lambda_{kqj}}{L_{lkqj}} + i_{qs} \right) \quad (3.9)$$

$$\lambda_{md} = L_{md} \left(\frac{\lambda_{fd}}{L_{lfd}} + \sum_{j=1}^N \frac{\lambda_{kdj}}{L_{lkdj}} + i_{ds} \right) \quad (3.10)$$

C. Mechanical Part Equations

$$p\theta_r = \omega_r \quad (3.11)$$

$$p\omega_r = \frac{P}{2J} (T_e - T_m) \quad (3.12)$$

$$T_e = \frac{3P}{4} (\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}) \quad (3.13)$$

where

$$\lambda_{qs} = L_{mq} i_{qs} + L_{mq} \sum_{j=1}^M \frac{\lambda_{kqj}}{L_{lkqj}} \quad (3.14)$$

$$\lambda_{ds} = L''_d i_{ds} + L''_{md} \left(\frac{\lambda_{fd}}{L_{ffd}} + \sum_{j=1}^N \frac{\lambda_{kdj}}{L_{lkdj}} \right) \quad (3.15)$$

Discretizing the above VBR model with the trapezoidal integration results in a more flexible and robust EMTP-type model than the $qd0$ model [37]. The $qd0$ model is numerically efficient and is widely used in EMTP simulators. However, the interfacing of $qd0$ quantities with the electrical network, which is modeled in abc coordinates with the EMTP, is not direct and requires some prediction. The SFA model proposed in this chapter is based on the VBR model. Therefore, the SFA model is still in abc coordinates and the interfacing in the EMTP or in a hybrid environment like the OVNI [39] simulator does not require prediction but interpolation (in a similar way to what is discussed in reference [40] for the coupling of different size time steps). Section 3.3 details the development of the SFA-based synchronous machine model.

3.3 Synchronous Machine Modelling with SFA

3.3.1 Synchronous Machine Model Based on SFA

The following equations are based on the salient-pole generator with one field winding fd , one damper winding kd in the d -axis, and one $kq2$ damper winding in the q -axis. These formulas can be modified with minor efforts for synchronous machines with arbitrary numbers of windings in the d - and q -axis, following the procedures in Section 2.1.1.

To construct the SFA-based stator voltage equations, we first rewrite the electrical variables using equation (2.5)

$$\mathbf{i}_{abcs}(t) = \mathbf{i}_{abcsI}(t) \cos \omega_s t - \mathbf{i}_{abcsQ}(t) \sin \omega_s t \quad (3.16)$$

$$\mathbf{v}_{abcs}(t) = \mathbf{v}_{abcsI}(t) \cos \omega_s t - \mathbf{v}_{abcsQ}(t) \sin \omega_s t \quad (3.17)$$

Substituting equation (3.16) and equation (3.17) into equation (3.1) results in

$$\begin{aligned} & \mathbf{v}_{abcsI}(t) \cos \omega_s t - \mathbf{v}_{abcsQ}(t) \sin \omega_s t \\ &= \mathbf{r}_s [\mathbf{i}_{abcsI}(t) \cos \omega_s t - \mathbf{i}_{abcsQ}(t) \sin \omega_s t] + \mathbf{L}_0'' [p \mathbf{i}_{abcsI}(t) \cos \omega_s t - \mathbf{i}_{abcsI}(t) \omega_s \sin \omega_s t \\ & - p \mathbf{i}_{abcsQ}(t) \sin \omega_s t - \mathbf{i}_{abcsQ}(t) \omega_s \cos \omega_s t] - L_b'' \{ p [\mathbf{A} \cos \omega_s t] \mathbf{i}_{abcsI}(t) + \mathbf{A} \cos \omega_s t \cdot p \mathbf{i}_{abcsI}(t) \\ & - p [\mathbf{A} \sin \omega_s t] \mathbf{i}_{abcsQ}(t) - \mathbf{A} \sin \omega_s t \cdot p \mathbf{i}_{abcsQ}(t) \} + \mathbf{v}_{abcsI}''(t) \cos \omega_s t - \mathbf{v}_{abcsQ}''(t) \sin \omega_s t \end{aligned} \quad (3.18)$$

Applying the Hilbert Transform to the left hand side (*LHS*) and right hand side (*RHS*) of (3.18), respectively, we can construct the analytic signal (3.19), as follows

$$\begin{aligned} \mathbf{V}_{abcs}(t)e^{j\omega_s t} &= \mathbf{r}_s \mathbf{I}_{abcs}(t)e^{j\omega_s t} + \mathbf{L}_0'' [p\mathbf{I}_{abcs}(t) + j\omega_s \mathbf{I}_{abcs}(t)]e^{j\omega_s t} \\ &\quad - \frac{L_b''}{2} \{j(2\omega_r + \omega_s)\mathbf{K}_a \mathbf{I}_{abcs}(t) + j(2\omega_r - \omega_s)\mathbf{K}_b \mathbf{I}_{abcs}^*(t) \\ &\quad + \mathbf{K}_a p\mathbf{I}_{abcs}(t) + \mathbf{K}_b p\mathbf{I}_{abcs}^*(t)\} + \mathbf{V}_{abcs}''(t)e^{j\omega_s t} \end{aligned} \quad (3.19)$$

Here $\mathbf{I}_{abcs}(t)$, $\mathbf{V}_{abcs}(t)$ are the dynamic phasors of stator currents and voltages, respectively; $\mathbf{I}_{abcs}^*(t)$ is the conjugate of $\mathbf{I}_{abcs}(t)$. $\mathbf{K}_a$ and $\mathbf{K}_b$ are expressed as

$$\begin{aligned} \mathbf{K}_a &= \begin{bmatrix} e^{j(2\theta_r + \omega_s t)} & e^{j(2\theta_r + \omega_s t - 2\pi/3)} & e^{j(2\theta_r + \omega_s t + 2\pi/3)} \\ e^{j(2\theta_r + \omega_s t - 2\pi/3)} & e^{j(2\theta_r + \omega_s t - 4\pi/3)} & e^{j(2\theta_r + \omega_s t)} \\ e^{j(2\theta_r + \omega_s t + 2\pi/3)} & e^{j(2\theta_r + \omega_s t)} & e^{j(2\theta_r + \omega_s t + 4\pi/3)} \end{bmatrix}, \\ \mathbf{K}_b &= \begin{bmatrix} e^{j(2\theta_r - \omega_s t)} & e^{j(2\theta_r - \omega_s t - 2\pi/3)} & e^{j(2\theta_r - \omega_s t + 2\pi/3)} \\ e^{j(2\theta_r - \omega_s t - 2\pi/3)} & e^{j(2\theta_r - \omega_s t - 4\pi/3)} & e^{j(2\theta_r - \omega_s t)} \\ e^{j(2\theta_r - \omega_s t + 2\pi/3)} & e^{j(2\theta_r - \omega_s t)} & e^{j(2\theta_r - \omega_s t + 4\pi/3)} \end{bmatrix} \end{aligned}$$

Performing the frequency shifting as explained in equation (2.7), the SFA based formulas for the stator voltage equations can be expressed as

$$\begin{aligned} \mathbf{V}_{abcs}(t) &= \mathbf{r}_s \mathbf{I}_{abcs}(t) + \mathbf{L}_0'' [p\mathbf{I}_{abcs}(t) + j\omega_s \mathbf{I}_{abcs}(t)] + \\ &\quad + j(2\omega_r + \omega_s)\mathbf{L}'' e^{j2\theta_r} \mathbf{I}_{abcs}(t) + j(2\omega_r - \omega_s)\mathbf{L}'' e^{j(2\theta_r - 2\omega_s t)} \mathbf{I}_{abcs}^*(t) \\ &\quad + \mathbf{L}'' e^{j2\theta_r} p\mathbf{I}_{abcs}(t) + \mathbf{L}'' e^{j(2\theta_r - 2\omega_s t)} p\mathbf{I}_{abcs}^*(t) + \mathbf{V}_{abcs}''(t) \end{aligned} \quad (3.20)$$

where

$$\mathbf{L}'' = -\frac{L_b''}{2} \begin{bmatrix} 1 & a^2 & a \\ a^2 & a & 1 \\ a & 1 & a^2 \end{bmatrix}, \quad a = e^{j2\pi/3}.$$

The SFA formulas for the subtransient voltages $\mathbf{V}_{abcs}''(t)$ can be derived using the same approach described above.

3.3.2 Discrete Time Model

Discretizing (3.20) with the trapezoidal rule of integration and rearranging,

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$$\begin{aligned} \mathbf{V}_{abcs}(t) = & \mathbf{A}_1 \mathbf{I}_{abcs}(t) + \mathbf{A}_2 \mathbf{I}_{abcs}^*(t) + \mathbf{A}_3 \mathbf{I}_{abcs}(t - \Delta t) + \mathbf{A}_4 \mathbf{I}_{abcs}^*(t - \Delta t) + \mathbf{V}_{abcs}''(t) \\ & + \mathbf{V}_{abcs}''(t - \Delta t) - \mathbf{V}_{abcs}(t - \Delta t) \end{aligned} \quad (3.21)$$

where

$$\begin{aligned} \mathbf{A}_1 &= \mathbf{r}_s + \left(\frac{2}{\Delta t} + j\omega_s \right) \mathbf{L}_0'' + \left(\frac{2}{\Delta t} + j\omega_s \right) \mathbf{L}'' e^{j2\theta_r(t)} \\ \mathbf{A}_2 &= \left(\frac{2}{\Delta t} + j\omega_s \right) \mathbf{L}'' e^{j(2\theta_r(t) - 2\omega_s t)} \\ \mathbf{A}_3 &= \mathbf{r}_s + \left(-\frac{2}{\Delta t} + j\omega_s \right) \mathbf{L}_0'' + \left(-\frac{2}{\Delta t} + j\omega_s \right) \mathbf{L}'' e^{j2\theta_r(t - \Delta t)} \\ \mathbf{A}_4 &= \left(-\frac{2}{\Delta t} + j\omega_s \right) \mathbf{L}'' e^{j(2\theta_r(t - \Delta t) - 2\omega_s(t - \Delta t))} \end{aligned}$$

Equation (3.21) can be further decomposed into real and imaginary parts

$$\begin{aligned} \begin{bmatrix} \mathbf{V}_{abcs,r}(t) \\ \mathbf{V}_{abcs,i}(t) \end{bmatrix} &= \begin{bmatrix} \mathbf{A}_{1,r} + \mathbf{A}_{2,r} & -\mathbf{A}_{1,i} + \mathbf{A}_{2,i} \\ \mathbf{A}_{1,i} + \mathbf{A}_{2,i} & \mathbf{A}_{1,r} - \mathbf{A}_{2,r} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{abcs,r}(t) \\ \mathbf{I}_{abcs,i}(t) \end{bmatrix} \\ &+ \begin{bmatrix} \mathbf{V}_{abcs,r}''(t) \\ \mathbf{V}_{abcs,i}''(t) \end{bmatrix} + \begin{bmatrix} \mathbf{A}_{3,r} + \mathbf{A}_{4,r} & -\mathbf{A}_{3,i} + \mathbf{A}_{4,i} \\ \mathbf{A}_{3,i} + \mathbf{A}_{4,i} & \mathbf{A}_{3,r} - \mathbf{A}_{4,r} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{abcs,r}(t - \Delta t) \\ \mathbf{I}_{abcs,i}(t - \Delta t) \end{bmatrix} \\ &+ \begin{bmatrix} \mathbf{V}_{abcs,r}''(t - \Delta t) \\ \mathbf{V}_{abcs,i}''(t - \Delta t) \end{bmatrix} - \begin{bmatrix} \mathbf{V}_{abcs,r}(t - \Delta t) \\ \mathbf{V}_{abcs,i}(t - \Delta t) \end{bmatrix} \end{aligned} \quad (3.22)$$

The discrete time formulas for the subtransient voltage can be similarly derived, giving

$$\mathbf{V}_{abcs}''(t) = \mathbf{K}(t) \mathbf{I}_{abcs}(t) + \mathbf{e}_h''(t) \quad (3.23)$$

The time-varying factors $\mathbf{K}(t)$ and $\mathbf{e}_h''(t)$ are expanded as follows

$$\begin{aligned} \mathbf{e}_h''(t) &= \begin{bmatrix} e^{j(\theta_r - \omega_s t)} & e^{j(\theta_r - \omega_s t - \pi/2)} & 1 \\ e^{j(\theta_r - \omega_s t - 2\pi/3)} & e^{j(\theta_r - \omega_s t - 2\pi/3 - \pi/2)} & 1 \\ e^{j(\theta_r - \omega_s t + 2\pi/3)} & e^{j(\theta_r - \omega_s t + 2\pi/3 - \pi/2)} & 1 \end{bmatrix} \\ &\cdot \begin{bmatrix} \mathbf{K}_1 \lambda_{kq2}(t - \Delta t) + \mathbf{K}_2 \begin{bmatrix} \lambda_{jd}(t - \Delta t) \\ \lambda_{kd}(t - \Delta t) \\ 0 \end{bmatrix} + \mathbf{K}_3 \mathbf{v}_f(t) + \mathbf{K}_4 \mathbf{v}_f(t - \Delta t) \\ + [\mathbf{K}'_s(\theta_r(t))]^{-1} \end{bmatrix} \\ &\cdot \begin{bmatrix} [\mathbf{K}_s \quad \mathbf{K}_6] \frac{2}{3} \begin{bmatrix} \cos(\theta_r(t - \Delta t)) & \cos(\theta_r(t - \Delta t) - 2\pi/3) & \cos(\theta_r(t - \Delta t) + 2\pi/3) \\ \sin(\theta_r(t - \Delta t)) & \sin(\theta_r(t - \Delta t) - 2\pi/3) & \sin(\theta_r(t - \Delta t) + 2\pi/3) \\ 0 \end{bmatrix} \mathbf{I}_{abcs}(t - \Delta t) \end{bmatrix} \end{aligned} \quad (3.24)$$

$$\mathbf{K}(t) = [\mathbf{K}'_s(\theta_r(t))]^{-1} \begin{bmatrix} \mathbf{K}_9 & \mathbf{K}_{10} & \mathbf{0} \\ 0 & 0 & 0 \end{bmatrix} [\mathbf{K}'_s(\theta_r(t))] \quad (3.25)$$

where

$$\mathbf{K}'_s(\theta_r) = \frac{2}{3} \begin{bmatrix} \cos(\theta_r) & \cos(\theta_r - 2\pi/3) & \cos(\theta_r + 2\pi/3) \\ \sin(\theta_r) & \sin(\theta_r - 2\pi/3) & \sin(\theta_r + 2\pi/3) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

is the Park's transformation matrix, and

$$\begin{aligned} \mathbf{K}_1 &= \begin{bmatrix} \frac{L''_{mq} r_{kq2}}{L_{lkq2}^2} \left(\frac{L''_{mq}}{L_{lkq2}} - 1 \right) & 2 - \Delta t \frac{r_{kq2}}{L_{lkq2}} \left(1 - \frac{L''_{mq}}{L_{lkq2}} \right) \\ -\omega_r(t) \frac{L''_{mq}}{L_{lkq2}} & 2 + \Delta t \frac{r_{kq2}}{L_{lkq2}} \left(1 - \frac{L''_{mq}}{L_{lkq2}} \right) \end{bmatrix} \\ \mathbf{K}_2 &= \mathbf{K}_7 \cdot \mathbf{K}_8 \cdot \begin{bmatrix} 2 - \Delta t \frac{r_{fd}}{L_{lfd}} \left(1 - \frac{L''_{md}}{L_{lfd}} \right) & \Delta t \frac{r_{fd}}{L_{lfd}} \frac{L''_{md}}{L_{lfd}} \\ \Delta t \frac{r_{kd}}{L_{lkd}} \frac{L''_{md}}{L_{lfd}} & 2 - \Delta t \frac{r_{kd}}{L_{lkd}} \left(1 - \frac{L''_{md}}{L_{lkd}} \right) \end{bmatrix} \\ \mathbf{K}_3 &= \mathbf{K}_4 + \begin{bmatrix} 0 \\ \frac{L''_{md}}{L_{lfd}} \end{bmatrix} \\ \mathbf{K}_4 &= \mathbf{K}_7 \cdot \mathbf{K}_8 \cdot \begin{bmatrix} \Delta t \\ 0 \end{bmatrix} \\ \mathbf{K}_5 &= \begin{bmatrix} \frac{L''_{mq} r_{kq2}}{L_{lkq2}^2} \left(\frac{L''_{mq}}{L_{lkq2}} - 1 \right) & \Delta t \frac{r_{kq2} L''_{mq}}{L_{lkq2}} \\ -\omega_r(t) \frac{L''_{mq}}{L_{lkq2}} & 2 + \Delta t \frac{r_{kq2}}{L_{lkq2}} \left(1 - \frac{L''_{mq}}{L_{lkq2}} \right) \end{bmatrix} \\ \mathbf{K}_6 &= \mathbf{K}_7 \cdot \mathbf{K}_8 \cdot \begin{bmatrix} \Delta t \frac{r_{fd} L''_{md}}{L_{lfd}} \\ \Delta t \frac{r_{kd} L''_{md}}{L_{lkd}} \end{bmatrix} \\ \mathbf{K}_7 &= \begin{bmatrix} \omega_r(t) \frac{L''_{md}}{L_{lfd}} & \omega_r(t) \frac{L''_{md}}{L_{lkd}} \\ \frac{L''_{md} r_{kd}}{L_{lkd} L_{lfd}} + \frac{L''_{md} r_{fd}}{L_{lfd}} \left(\frac{L''_{md}}{L_{lfd}} - 1 \right) & \frac{L''_{md} r_{fd}}{L_{lfd} L_{lkd}} + \frac{L''_{md} r_{kd}}{L_{lkd}} \left(\frac{L''_{md}}{L_{lkd}} - 1 \right) \end{bmatrix} \end{aligned}$$

$$\mathbf{K}_8 = \begin{bmatrix} 2 + \Delta t \frac{r_{fd}}{L_{lfd}} \left(1 - \frac{L_{md}''}{L_{lfd}} \right) & -\Delta t \frac{r_{fd}}{L_{lfd}} \frac{L_{md}''}{L_{lkd}} \\ -\Delta t \frac{r_{kd}}{L_{lkd}} \frac{L_{md}''}{L_{lfd}} & 2 + \Delta t \frac{r_{kd}}{L_{lkd}} \left(1 - \frac{L_{md}''}{L_{lkd}} \right) \end{bmatrix}^{-1}$$

$$\mathbf{K}_9 = \mathbf{K}_5 + \begin{bmatrix} \frac{r_{kq2} (L_{mq}'')^2}{L_{lkq2}^2} \\ 0 \end{bmatrix}$$

$$\mathbf{K}_{10} = \mathbf{K}_6 + \begin{bmatrix} 0 \\ \left(\frac{r_{kd}}{L_{lkd}^2} + \frac{r_{fd}}{L_{lfd}^2} \right) (L_{md}'')^2 \end{bmatrix}$$

Equation (3.23) is then decomposed into real and imaginary parts

$$\begin{bmatrix} \mathbf{V}_{abcs,r}''(t) \\ \mathbf{V}_{abcs,i}''(t) \end{bmatrix} = \begin{bmatrix} \mathbf{K}_r & -\mathbf{K}_i \\ \mathbf{K}_i & \mathbf{K}_r \end{bmatrix} \begin{bmatrix} \mathbf{I}_{abcs,r}(t) \\ \mathbf{I}_{abcs,i}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{e}_{h,r}''(t) \\ \mathbf{e}_{h,i}''(t) \end{bmatrix} \quad (3.26)$$

Substituting equation (3.24) into equation (3.22) produces

$$\begin{bmatrix} \mathbf{V}_{abcs,r}(t) \\ \mathbf{V}_{abcs,i}(t) \end{bmatrix} = \mathbf{R}_{eq} \begin{bmatrix} \mathbf{I}_{abcs,r}(t) \\ \mathbf{I}_{abcs,i}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{e}_{h,r}(t) \\ \mathbf{e}_{h,i}(t) \end{bmatrix} \quad (3.27)$$

where

$$\mathbf{R}_{eq} = \begin{bmatrix} \mathbf{A}_{1,r} + \mathbf{A}_{2,r} + \mathbf{K} & -\mathbf{A}_{1,i} + \mathbf{A}_{2,i} \\ \mathbf{A}_{1,i} + \mathbf{A}_{2,i} & \mathbf{A}_{1,r} - \mathbf{A}_{2,r} + \mathbf{K} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{e}_{h,r}(t) \\ \mathbf{e}_{h,i}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{3,r} + \mathbf{A}_{4,r} & -\mathbf{A}_{3,i} + \mathbf{A}_{4,i} \\ \mathbf{A}_{3,i} + \mathbf{A}_{4,i} & \mathbf{A}_{3,r} - \mathbf{A}_{4,r} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{abcs,r}(t - \Delta t) \\ \mathbf{I}_{abcs,i}(t - \Delta t) \end{bmatrix} + \begin{bmatrix} \mathbf{V}_{abcs,r}''(t - \Delta t) \\ \mathbf{V}_{abcs,i}''(t - \Delta t) \end{bmatrix} - \begin{bmatrix} \mathbf{V}_{abcs,r}(t - \Delta t) \\ \mathbf{V}_{abcs,i}(t - \Delta t) \end{bmatrix} +$$

$$\begin{bmatrix} \mathbf{e}_{h,r}''(t) \\ \mathbf{e}_{h,i}''(t) \end{bmatrix}$$

Equation (3.27) is the SFA-based formula, which has a similar form to the phase coordinates EMTP synchronous machine model of [41]. The SFA equivalent circuit is shown in Figure 3.2.

Figure 3.2 SFA Equivalent Circuit of a Synchronous Generator

In each time step, the flux linkages can be updated as follows

$$\lambda_{kq2}(t) = \frac{\Delta t \frac{r_{kq2} L''_{mq}}{L_{lkq2}}}{2 + \Delta t \frac{r_{kq2}}{L_{lkq2}} \left(1 - \frac{L''_{mq}}{L_{lkq2}}\right)} \cdot \frac{2}{3} [\cos(\theta_r(t)) \quad \cos(\theta_r(t) - 2\pi/3) \quad \cos(\theta_r(t) + 2\pi/3)] \text{Re}[\mathbf{I}_{abcs}(t) e^{j\omega_e t}]$$

$$+ \frac{2 - \Delta t \frac{r_{kq2}}{L_{lkq2}} \left(1 - \frac{L''_{mq}}{L_{lkq2}}\right)}{2 + \Delta t \frac{r_{kq2}}{L_{lkq2}} \left(1 - \frac{L''_{mq}}{L_{lkq2}}\right)} \lambda_{kq2}(t - \Delta t) + \frac{\Delta t \frac{r_{kq2} L''_{mq}}{L_{lkq2}}}{2 + \Delta t \frac{r_{kq2}}{L_{lkq2}} \left(1 - \frac{L''_{mq}}{L_{lkq2}}\right)} \cdot \frac{2}{3} [\cos(\theta_r(t - \Delta t)) \quad \cos(\theta_r(t - \Delta t) - 2\pi/3) \quad \cos(\theta_r(t - \Delta t) + 2\pi/3)] \text{Re}[\mathbf{I}_{abcs}(t - \Delta t) e^{j\omega_e t}] \quad (3.28)$$

$$\begin{bmatrix} \lambda_{fd}(t) \\ \lambda_{kd}(t) \end{bmatrix} = \begin{bmatrix} 2 + \Delta t \frac{r_{fd}}{L_{lfd}} \left(1 - \frac{L''_{md}}{L_{lfd}}\right) & -\Delta t \frac{r_{fd}}{L_{lfd}} \frac{L''_{md}}{L_{lfd}} \\ -\Delta t \frac{r_{kd}}{L_{lkd}} \frac{L''_{md}}{L_{lfd}} & 2 + \Delta t \frac{r_{kd}}{L_{lkd}} \left(1 - \frac{L''_{md}}{L_{lkd}}\right) \end{bmatrix}^{-1} \begin{bmatrix} \Delta t \frac{r_{fd} L''_{md}}{L_{lfd}} \\ \Delta t \frac{r_{kd} L''_{md}}{L_{lkd}} \end{bmatrix}$$

$$\cdot \frac{2}{3} [\sin(\theta_r(t)) \quad \sin(\theta_r(t) - 2\pi/3) \quad \sin(\theta_r(t) + 2\pi/3)] \text{Re}[\mathbf{I}_{abcs}(t) e^{j\omega_e t}]$$

$$+ \begin{bmatrix} 2 + \Delta t \frac{r_{fd}}{L_{lfd}} \left(1 - \frac{L''_{md}}{L_{lfd}}\right) & -\Delta t \frac{r_{fd}}{L_{lfd}} \frac{L''_{md}}{L_{lfd}} \\ -\Delta t \frac{r_{kd}}{L_{lkd}} \frac{L''_{md}}{L_{lfd}} & 2 + \Delta t \frac{r_{kd}}{L_{lkd}} \left(1 - \frac{L''_{md}}{L_{lkd}}\right) \end{bmatrix}^{-1} \begin{bmatrix} 2 - \Delta t \frac{r_{fd}}{L_{lfd}} \left(1 - \frac{L''_{md}}{L_{lfd}}\right) & \Delta t \frac{r_{fd}}{L_{lfd}} \frac{L''_{md}}{L_{lfd}} \\ \Delta t \frac{r_{kd}}{L_{lkd}} \frac{L''_{md}}{L_{lfd}} & 2 - \Delta t \frac{r_{kd}}{L_{lkd}} \left(1 - \frac{L''_{md}}{L_{lkd}}\right) \end{bmatrix} \begin{bmatrix} \lambda_{fd}(t - \Delta t) \\ \lambda_{kd}(t - \Delta t) \end{bmatrix}$$

$$\begin{aligned}
 & + \begin{bmatrix} 2 + \Delta t \frac{r_{fd}}{L_{ffd}} \left(1 - \frac{L''_{md}}{L_{ffd}}\right) & -\Delta t \frac{r_{fd}}{L_{ffd}} \frac{L''_{md}}{L_{ffd}} \\ -\Delta t \frac{r_{kd}}{L_{lkd}} \frac{L''_{md}}{L_{ffd}} & 2 + \Delta t \frac{r_{kd}}{L_{lkd}} \left(1 - \frac{L''_{md}}{L_{lkd}}\right) \end{bmatrix}^{-1} \begin{bmatrix} \Delta t \frac{r_{fd} L''_{md}}{L_{ffd}} \\ \Delta t \frac{r_{kd} L''_{md}}{L_{lkd}} \end{bmatrix} \\
 & \cdot \frac{2}{3} [\sin(\theta_r(t - \Delta t)) \quad \sin(\theta_r(t - \Delta t) - 2\pi/3) \quad \sin(\theta_r(t - \Delta t) + 2\pi/3)] \text{Re}[\mathbf{I}_{abcs}(t - \Delta t) e^{j\omega_e t}] \\
 & + \begin{bmatrix} 2 + \Delta t \frac{r_{fd}}{L_{ffd}} \left(1 - \frac{L''_{md}}{L_{ffd}}\right) & -\Delta t \frac{r_{fd}}{L_{ffd}} \frac{L''_{md}}{L_{ffd}} \\ -\Delta t \frac{r_{kd}}{L_{lkd}} \frac{L''_{md}}{L_{ffd}} & 2 + \Delta t \frac{r_{kd}}{L_{lkd}} \left(1 - \frac{L''_{md}}{L_{lkd}}\right) \end{bmatrix}^{-1} \begin{bmatrix} \Delta t \\ 0 \end{bmatrix} [\mathbf{v}_{fd}(t) + \mathbf{v}_{fd}(t - \Delta t)] \quad (3.29)
 \end{aligned}$$

$$\lambda_{qs}(t) = L_q'' \frac{2}{3} [\cos(\theta_r(t)) \quad \cos(\theta_r(t) - 2\pi/3) \quad \cos(\theta_r(t) + 2\pi/3)] \text{Re}[\mathbf{I}_{abcs}(t) e^{j\omega_e t}] + L_{mq}'' \frac{\lambda_{kq2}(t)}{L_{lkq2}} \quad (3.30)$$

$$\lambda_{ds}(t) = L_d'' \frac{2}{3} [\sin(\theta_r(t)) \quad \sin(\theta_r(t) - 2\pi/3) \quad \sin(\theta_r(t) + 2\pi/3)] \text{Re}[\mathbf{I}_{abcs}(t) e^{j\omega_e t}] + L_{md}'' \left[\frac{\lambda_{fd}(t)}{L_{ffd}} + \frac{\lambda_{kd}(t)}{L_{lkd}} \right] \quad (3.31)$$

The differential equations of the generator's mechanical part also have to be discretized and can be solved together with the electrical equations. The discrete time equations of the mechanical part are expressed

$$\omega_r(t) = \omega_r(t - \Delta t) + \frac{\Delta t p}{4J} [T_m(t) + T_m(t - \Delta t) - T_e(t) + T_e(t - \Delta t)] \quad (3.32)$$

$$T_e(t) = -\frac{3p}{4} [\lambda_{ds}(t) i_{qs}(t) + \lambda_{qs}(t) i_{ds}(t)] \quad (3.33)$$

$$\theta_r(t) = \theta_r(t - \Delta t) + \frac{\Delta t}{2} [\omega_r(t) + \omega_r(t - \Delta t)] \quad (3.34)$$

$$\delta_r(t) = \delta_r(t - \Delta t) + \Delta t \left[\frac{\omega_r(t) + \omega_r(t - \Delta t)}{2} - \omega_s \right] \quad (3.35)$$

where

p is the number of poles and

$$\mathbf{i}_{qd0s} = \mathbf{K}_s'(\theta_r(t)) \text{Re}[\mathbf{I}_{abcs}(t) e^{j\omega_s t}]$$

Refer to [9] for more detailed multi-mass mechanical part models.

3.3.3 Note on the Cylindrical-Rotor Machine Model

The shifted-frequency-domain equivalent circuit of a cylindrical-rotor synchronous machine is similar to that of the salient-pole machine. As mentioned in Section 3.2, however, the cylindrical-rotor machine has different rotor windings from the salient-pole machine. Therefore, particular equations and factors in Sections 3.3.1 and 3.3.2 have to be re-written., which can be expressed as follows.

$$\mathbf{e}_h''(t) = \begin{bmatrix} e^{j(\theta_r - \omega_s t)} & e^{j(\theta_r - \omega_s t - \pi/2)} & 1 \\ e^{j(\theta_r - \omega_s t - 2\pi/3)} & e^{j(\theta_r - \omega_s t - 2\pi/3 - \pi/2)} & 1 \\ e^{j(\theta_r - \omega_s t + 2\pi/3)} & e^{j(\theta_r - \omega_s t + 2\pi/3 - \pi/2)} & 1 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{K}_1 \begin{bmatrix} \lambda_{kq1}(t - \Delta t) \\ \lambda_{kq2}(t - \Delta t) \end{bmatrix} \lambda_{kq2}(t - \Delta t) + \mathbf{K}_2 \begin{bmatrix} \lambda_{fd}(t - \Delta t) \\ \lambda_{kd}(t - \Delta t) \end{bmatrix} + \mathbf{K}_3 v_f(t) + \mathbf{K}_4 v_f(t - \Delta t) \\ 0 \end{bmatrix} + [\mathbf{K}_s'(\theta_r(t))]^{-1} \cdot \begin{bmatrix} [\mathbf{K}_5 \quad \mathbf{K}_6] \frac{2}{3} \begin{bmatrix} \cos(\theta_r(t - \Delta t)) & \cos(\theta_r(t - \Delta t) - 2\pi/3) & \cos(\theta_r(t - \Delta t) + 2\pi/3) \\ \sin(\theta_r(t - \Delta t)) & \sin(\theta_r(t - \Delta t) - 2\pi/3) & \sin(\theta_r(t - \Delta t) + 2\pi/3) \end{bmatrix} \mathbf{I}_{abc}(t - \Delta t) \\ 0 \end{bmatrix} \quad (3.36)$$

$$\mathbf{K}_1 = \begin{bmatrix} \frac{L_{mq}'' r_{kq1}}{L_{lkq1}^2} \left(\frac{L_{mq}''}{L_{lkq1}} - 1 \right) + \frac{L_{mq}''^2 r_{kq2}}{L_{lkq2}^2 L_{lkq1}} & \frac{L_{mq}'' r_{kq2}}{L_{lkq2}^2} \left(\frac{L_{mq}''}{L_{lkq2}} - 1 \right) + \frac{L_{mq}''^2 r_{kq1}}{L_{lkq1}^2 L_{lkq2}} \\ -\omega_r(t) \frac{L_{mq}''}{L_{lkq1}} & -\omega_r(t) \frac{L_{mq}''}{L_{lkq2}} \end{bmatrix}$$

$$\begin{bmatrix} 2 - \Delta t \frac{r_{kq1}}{L_{lkq1}} \left(\frac{L_{mq}''}{L_{lkq1}} - 1 \right) & -\Delta t \frac{r_{kq1} L_{mq}''}{L_{lkq1} L_{lkq2}} \\ -\Delta t \frac{r_{kq2} L_{mq}''}{L_{lkq1} L_{lkq2}} & 2 - \Delta t \frac{r_{kq2}}{L_{lkq2}} \left(\frac{L_{mq}''}{L_{lkq2}} - 1 \right) \end{bmatrix}^{-1} \begin{bmatrix} 2 + \Delta t \frac{r_{kq1}}{L_{lkq1}} \left(\frac{L_{mq}''}{L_{lkq1}} - 1 \right) & \Delta t \frac{r_{kq1} L_{mq}''}{L_{lkq1} L_{lkq2}} \\ \Delta t \frac{r_{kq2} L_{mq}''}{L_{lkq1} L_{lkq2}} & 2 + \Delta t \frac{r_{kq2}}{L_{lkq2}} \left(\frac{L_{mq}''}{L_{lkq2}} - 1 \right) \end{bmatrix}$$

$$\mathbf{K}_5 = \begin{bmatrix} \frac{L_{mq}'' r_{kq1}}{L_{lkq1}^2} \left(\frac{L_{mq}''}{L_{lkq1}} - 1 \right) + \frac{L_{mq}''^2 r_{kq2}}{L_{lkq2}^2 L_{lkq1}} & \frac{L_{mq}'' r_{kq2}}{L_{lkq2}^2} \left(\frac{L_{mq}''}{L_{lkq2}} - 1 \right) + \frac{L_{mq}''^2 r_{kq1}}{L_{lkq1}^2 L_{lkq2}} \\ -\omega_r(t) \frac{L_{mq}''}{L_{lkq1}} & -\omega_r(t) \frac{L_{mq}''}{L_{lkq2}} \end{bmatrix}$$

$$\begin{aligned}
 & \begin{bmatrix} 2 - \Delta t \frac{r_{kq1}}{L_{lkq1}} \left(\frac{L_{mq}''}{L_{lkq1}} - 1 \right) & -\Delta t \frac{r_{kq1} L_{mq}''}{L_{lkq1} L_{lkq2}} \\ -\Delta t \frac{r_{kq2} L_{mq}''}{L_{lkq1} L_{lkq2}} & 2 - \Delta t \frac{r_{kq2}}{L_{lkq2}} \left(\frac{L_{mq}''}{L_{lkq2}} - 1 \right) \end{bmatrix}^{-1} \begin{bmatrix} \Delta t \frac{r_{kq1} L_{mq}''}{L_{lkq1}} \\ \Delta t \frac{r_{kq2} L_{mq}''}{L_{lkq2}} \end{bmatrix} \\
 \mathbf{K}_9 = \mathbf{K}_5 & + \begin{bmatrix} \left(\frac{r_{kq1}}{L_{lkq1}^2} + \frac{r_{kq2}}{L_{lkq2}^2} \right) (L_{mq}'')^2 \\ 0 \end{bmatrix} \\
 \begin{bmatrix} \lambda_{kq1}(t) \\ \lambda_{kq2}(t) \end{bmatrix} & = \begin{bmatrix} 2 - \Delta t \frac{r_{kq1}}{L_{lkq1}} \left(\frac{L_{mq}''}{L_{lkq1}} - 1 \right) & -\Delta t \frac{r_{kq1} L_{mq}''}{L_{lkq1} L_{lkq2}} \\ -\Delta t \frac{r_{kq2} L_{mq}''}{L_{lkq1} L_{lkq2}} & 2 - \Delta t \frac{r_{kq2}}{L_{lkq2}} \left(\frac{L_{mq}''}{L_{lkq2}} - 1 \right) \end{bmatrix}^{-1} \begin{bmatrix} \Delta t \frac{r_{kq1} L_{mq}''}{L_{lkq1}} \\ \Delta t \frac{r_{kq2} L_{mq}''}{L_{lkq2}} \end{bmatrix} \\
 & \frac{2}{3} [\cos(\theta_r(t)) \quad \cos(\theta_r(t) - 2\pi/3) \quad \cos(\theta_r(t) + 2\pi/3)] \text{Re}[\mathbf{I}_{abcs}(t) e^{j\omega_e t}] + \\
 & \begin{bmatrix} 2 - \Delta t \frac{r_{kq1}}{L_{lkq1}} \left(\frac{L_{mq}''}{L_{lkq1}} - 1 \right) & -\Delta t \frac{r_{kq1} L_{mq}''}{L_{lkq1} L_{lkq2}} \\ -\Delta t \frac{r_{kq2} L_{mq}''}{L_{lkq1} L_{lkq2}} & 2 - \Delta t \frac{r_{kq2}}{L_{lkq2}} \left(\frac{L_{mq}''}{L_{lkq2}} - 1 \right) \end{bmatrix}^{-1} \begin{bmatrix} 2 + \Delta t \frac{r_{kq1}}{L_{lkq1}} \left(\frac{L_{mq}''}{L_{lkq1}} - 1 \right) & \Delta t \frac{r_{kq1} L_{mq}''}{L_{lkq1} L_{lkq2}} \\ \Delta t \frac{r_{kq2} L_{mq}''}{L_{lkq1} L_{lkq2}} & 2 + \Delta t \frac{r_{kq2}}{L_{lkq2}} \left(\frac{L_{mq}''}{L_{lkq2}} - 1 \right) \end{bmatrix} \\
 & \begin{bmatrix} \lambda_{kq1}(t - \Delta t) \\ \lambda_{kq2}(t - \Delta t) \end{bmatrix} + \begin{bmatrix} 2 - \Delta t \frac{r_{kq1}}{L_{lkq1}} \left(\frac{L_{mq}''}{L_{lkq1}} - 1 \right) & -\Delta t \frac{r_{kq1} L_{mq}''}{L_{lkq1} L_{lkq2}} \\ -\Delta t \frac{r_{kq2} L_{mq}''}{L_{lkq1} L_{lkq2}} & 2 - \Delta t \frac{r_{kq2}}{L_{lkq2}} \left(\frac{L_{mq}''}{L_{lkq2}} - 1 \right) \end{bmatrix}^{-1} \begin{bmatrix} \Delta t \frac{r_{kq1} L_{mq}''}{L_{lkq1}} \\ \Delta t \frac{r_{kq2} L_{mq}''}{L_{lkq2}} \end{bmatrix} \\
 & \frac{2}{3} [\cos(\theta_r(t - \Delta t)) \quad \cos(\theta_r(t - \Delta t) - 2\pi/3) \quad \cos(\theta_r(t - \Delta t) + 2\pi/3)] \text{Re}[\mathbf{I}_{abcs}(t - \Delta t) e^{j\omega_e t}] \quad (3.37) \\
 \lambda_{qs}(t) & = L_q'' \frac{2}{3} [\cos(\theta_r(t)) \quad \cos(\theta_r(t) - 2\pi/3) \quad \cos(\theta_r(t) + 2\pi/3)] \text{Re}[\mathbf{I}_{abcs}(t) e^{j\omega_e t}] + \\
 & L_{mq}'' \left(\frac{\lambda_{kq1}(t)}{L_{lkq1}} + \frac{\lambda_{kq2}(t)}{L_{lkq2}} \right) \quad (3.38)
 \end{aligned}$$

Thus, it can be seen that separated subroutines have to be coded for salient-pole and cylindrical-rotor machines when implementing them in a computer program. If both the salient-pole and the cylindrical-rotor machine could use a single model with the same number of damper windings, this would be more convenient for the code maintenance and extension.

Before we solve the above problem, let us first examine the subtransient reactance matrix in the stator equation (3.2), where $\mathbf{L}_{abs}'' = \mathbf{L}_0'' + (-L_b'')\mathbf{A}$. If the dynamic saliency effect, which means $L_d'' \neq L_q''$, can be neglected from the stator equations, we get $L_b'' = \frac{L_d'' - L_q''}{3} = 0$ and $\mathbf{L}_{abs}''$ becomes a constant matrix. This will effectively improve the numerical efficiency in simulations. References [42] has proposed a method to neglect the dynamic saliency effect. First, the operational impedances are calculated based on the original parameters of the machine. Second, an artificial q -axis winding is added in the machine model such that $X_q'' = X_d''$, at the same time, the frequency response curve of the new operational impedance accurately matches that of the original $X_q(s)$ over a frequency range $[-\infty, f_r]$, where f_r is a user-defined frequency. By fitting the frequency response curve of the new operational impedance, a new set of machine parameters can be obtained.

Therefore, we can always build a three damper windings model to represent both salient-pole machine and the cylindrical-rotor machine with acceptable frequency response. Because the dynamic saliency is eliminated, the refitted model is numerically more efficient than that with the dynamic saliency. We then need to develop only one uniform subroutine for different types of synchronous machines. Thus the problem is solved.

3.4 Simulation Results

Three test cases have been simulated to illustrate the efficiency and accuracy of the SFA model. The first two cases illustrate that the SFA model is a general purpose model, which can simulate both slow and fast dynamics. The third case shows that the SFA model is numerically more efficient and stable than the phase-domain EMTP model. Throughout this chapter, both the EMTP and SFA programs for simulating the single machine dynamics are implemented and tested in the same MATLAB environment.

The synchronous machine parameters for the three cases are shown in Appendix A taken from [43].

A. Simulation of Slow Dynamics

The first test case consists of a salient-pole hydro turbine generator connected to an

infinite bus. It was assumed that the system is initially operating in steady state with a mechanical torque $T_m = 0$ and a field voltage $V_f = 1.0$ p.u.

At $t = 1.0$ s, the field voltage is stepped down to 0.8 p.u. Later, at $t = 7.0$ s, the field voltage is stepped up to 1.2 p.u. Finally, at $t = 15.0$ s the mechanical torque is stepped up to $T_m = 0.85$ p.u. The total simulation time is 20 s.

The simulation results with the SFA model are shown in Figure 3.3. The variables plotted in this figure are i_{as} (phase A stator current), i_{fd} (field current), T_e (electromagnetic torque), δ_r (rotor angle), ω_r (rotor speed), P (real power) and Q (reactive power); all values except for δ_r are in per unit of the machine ratings. In this simulation, a large time step $\Delta t = 7$ ms is used. Note that the SFA model can produce both dynamic phasor and time domain results (refer to equation (2.7)). The uppermost subplot in Figure 3.3 shows the magnitude of dynamic phasor for phase A stator current. The dynamic phasor solution gives the envelope of the time domain results. The reconstructed instantaneous time domain values were identical to those obtained with the EMTP simulation with the synchronous machine represented with our own VBR model implementation. The EMTP and SFA programs for simulating the single machine dynamics are both developed in MATLAB script. The total CPU time for the SFA responses on a 1.83G Hz PC was 7.0 s.

B. Simulation of Fast Transients

The second simulation is a three phase fault for a single machine infinite bus system. The parameters of the hydro turbine generator used for this case are also those in Appendix A. The system is initially operating with $T_m = 0.85$ p.u. and $V_f = 1.0$ p.u. At $t = 1.0$ s, a three-phase fault is applied to the generator terminals. Then, at $t = 1.4$ s, the fault is removed with the simulation continuing to $t = 8.0$ s. The simulation is then repeated with the fault cleared at $t = 1.5$ s.

A Δt of 0.5ms was used in order to capture the fast transients caused by the fault. When the fault is cleared in 0.4 s the system remains stable. However, when the fault is removed in 0.5 s the system loses synchronism. By repeating the simulation, we found that the critical fault clearing time to prevent instability is close to 0.462 s. The SFA-based model gave identical results to those of the instantaneous time domain simulation. The time domain results are from the same MATLAB program mentioned in Section A. The CPU time needed to calculate the SFA responses for the stable case and the unstable case were 6.13 s and 6.14 s, respectively. The simulation results of the three-phase fault case are illustrated in Figure 3.4-Figure 3.6. Zoomed-

in views of the unstable case with fault clearing time of 0.5 s are shown in Figure 3.6, which has clearly captured the exponentially decaying of dc offsets in three-phase stator currents.

Simulation results for a single-phase-to-ground fault are presented in Figure 3.7. This figure shows the stator currents i_{as} , i_{bs} and i_{cs} with a fault occurring at $t = 1.1$ s on phase A. Two zoomed-in segments for i_{as} are embedded in the top subplot to depict the relationship between dynamic phasor and time domain results.

Case studies (a) and (b) show that the SFA model is adequate as a general purpose machine model. It allows us to use large time steps to simulate slow transients (case (a)), while it can also accurately capture the fast transients if an appropriate small time step is used (cases (b)).

C. Comparison between SFA and VBR Models

In order to further illustrate the advantages of the SFA solution domain and of the proposed SFA phase-coordinates salient-pole synchronous generator model, a third case is simulated comparing the SFA solution with a conventional EMTP solution that uses the improved VBR phase-coordinates salient-pole synchronous generator model. In this example, a steam turbine generator is connected to an infinite bus that supplies rated three-phase voltages. The system is running in no-load steady state with $T_m = 0$ and $V_f = 1.0$ p.u. The torque T_m is stepped up to 1.1 p.u. at $t = 0.1$ s.

The SFA model was used with different time steps $\Delta t = 0.1$ ms, $\Delta t = 1$ ms and $\Delta t = 10$ ms. The study was repeated using the VBR model with $\Delta t = 0.1$ ms. The VBR model was carefully verified in [37] against the standard EMTP model and its results are assumed to be the reference accurate time domain results. The results in Figure 3.8-Figure 3.12 show that the proposed SFA model can accurately simulate the system dynamics with a very large time step of 10 ms. In fact, the simulations show that if the time step is made as large as 15 ms, we still get accurate and stable results. Table 3.1 indicates the CPU time needed for the VBR and SFA simulations. For the same accuracy, the VBR model needed a $\Delta t = 0.1$ ms and a total CPU time of 13.266 s, while the SFA model needed a $\Delta t = 10$ ms and a total CPU time of 0.26 s. A zoomed in view of phase A stator current is shown in Figure 3.9. The SFA model was in this case 50 times more efficient than the VBR model. As reported in [37], for the same accuracy the EMTP VBR model can use a 50 times larger time step than the conventional $dq0$ model [41] (e.g. 0.5 ms vs 10 μ s) at the expense of about 5 times the computational time per solution step. For this rotor dynamics

example, the VBR model in the EMTP solution frame was, therefore, about 10 times more efficient than the $dq0$ EMTP model, while the new proposed SFA model in the SFA solution frame was about 500 times faster than the $dq0$ EMTP solution. This example illustrates the very high advantage of the SFA model for slow system dynamics. Table II also shows that SFA simulation is computationally more intensive than EMTP simulation for the same Δt step. This is due to the higher computational cost of operating with complex numbers (SFA) as compared to real numbers (EMTP) and also to the overhead involved in transferring between shifted frequency domain and normal time domain. These higher costs, however, are more than compensated for by the gains achieved by using a much larger integration step Δt .

It is also interesting to compare the numerical stability of the SFA machine model in the SFA frame with that of the VBR model in the EMTP standard frame. Figure 3.13 shows that if we run an EMTP simulation with the VBR model using the time step of 10 ms that we use for the SFA model, the VBR model results are no longer numerically stable. It is known that the integration rule can only give reasonably correct results when the maximum frequency in the simulation is at least five times less than the Nyquist frequency $f_{\text{Nyquist}} = 1/(2f_{\text{max}})$ [30]. In other words, the time step Δt should be less than $1/(5 \times 2 \times 60) = 1.67 \text{ ms}$ for an EMTP simulation of fundamental frequency dynamics. Table 3.2 shows the VBR model with a 5 ms time step has 11% numerical error, which makes the result no longer usable. When the time step used in the simulation is too large, i.e. $\Delta t = 10 \text{ ms}$, the simulation in the unshifted time domain will become numerically unstable. On the other hand, as can be seen in Table 3.2, the SFA model with 10 ms time step provides much more accurate result than the VBR model with 1 ms time step, and at the same time is five times faster than the latter.

Table 3.1 CPU Times for 4s Simulation in Case C

		VBR Model	SFA Model
Time Steps	0.1 ms *	13.266 s *	27.012 s
	1 ms	1.328 s	2.668 s
	10 ms *	- **	0.260 s *

*values for same accuracy ** numerically unstable

Figure 3.3 Simulation Results with the SFA Model for Field Voltage and Mechanical Torque Changes ($\Delta t = 7$ ms)

Figure 3.4 Simulation Results with the SFA Model for a Three Phase Fault ($\Delta t = 0.5$ ms): A Stable Case (fault removed at 1.4s)

Figure 3.5 Simulation Results with the SFA Model for a Three Phase Fault ($\Delta t = 0.5$ ms): An Unstable Case (fault removed at 1.5s)

Figure 3.6 Simulation Results with the SFA Model for a Three Phase Fault ($\Delta t = 0.5$ ms): Zoomed-in View of a Portion of Results in Figure 3.5

Figure 3.7 Simulation Results with the SFA Model for a Single-Phase-to-Ground Fault ($\Delta t = 0.5$ ms)

Figure 3.8 Simulation Results by SFA: Phase A Stator Current

Figure 3.9 Simulation Results by SFA: Zoomed-in View of Phase A Stator Current

Figure 3.10 Simulation Results by SFA: Field Current

Figure 3.11 Simulation Results by SFA: Electromagnetic Torque

Figure 3.12 Simulation Results by SFA: Rotor Speed

Table 3.2 Accuracy Comparison between SFA Model and VBR EMTP Model in Case C

Model	Time Step	CPU Time	Numerical Error in Stator Current
VBR	1 ms	1.328 s	1.02 %
VBR	5 ms	0.271 s	11.0 %
SFA	10 ms *	0.260 s	0.21 %

Figure 3.13 Time Domain Results Using the VBR Model ($\Delta t = 10$ ms)

3.4 Summary

This chapter has presented a very efficient synchronous machine model for the simulation of slow system dynamics using EMTP modelling in the Shifted Frequency Analysis framework. Instead of instantaneous time domain variables, the model uses time varying complex variables (dynamic phasors). The SFA model results in an EMTP-type equivalent circuit that retains the numerical properties of the EMTP solution at much larger integration steps. The synchronous machine model implemented in this chapter for the SFA framework is based on the VBR synchronous machine model, which is a very efficient implementation of a phase-coordinates synchronous machine model. Working in phase coordinates provides a more stable machine model than the traditional $dq0$ model by avoiding predictions at the interface with the external phase-coordinates power system network. The SFA framework reduces the size of the equation set that describes the synchronous machine behaviour for large efficiency gains. During system dynamics around the fundamental 60 Hz operating state of the system, a very large time step can be used to capture time domain results without loss of accuracy. Speedups of fifty times over traditional EMTP simulation were obtained for a case of mechanical torque step changes. This proves the feasibility of using EMTP solutions in the Shifted Frequency Domain for on-line assessment of large-scale power system dynamics. The SFA model is also capable of simulating fast transients caused by topological changes in the electrical network by using smaller time steps, but the extra overhead makes it slower than traditional EMTP for these situations.

Chapter 4

Induction Machine Modelling Based on Shifted Frequency Analysis

4.1 Introduction

The intent of this chapter is to extend the SFA method to induction machine modelling. Induction machines form a large portion of power system loads and serve as some Independent Power Producer-owned generators. An accurate simulation of their dynamic behavior is therefore required for voltage stability and transient stability, etc [7][8]. The efficient and stable SFA model for induction machines is well suited for such simulations in the SFA domain.

This chapter proposes a new efficient SFA model for induction machines, by combining the SFA method with the phase-domain EMTP induction machine model. An equivalent-reduction technique, which reduces the number of equations of stator, rotor, and flux linkages into only those of stator variables without sacrificing numerical accuracy, is used in deriving the SFA induction machine model. It allows the use of large time steps in simulating slow dynamics, but the SFA model can also accurately simulate the fast transients in the induction machine, which makes it a general purpose model.

Section 4.2 first proposes a new phase-domain induction machine model used in EMTP. The philosophy inside the proposed equivalent-reduction (ER) approach will be described, and case study results will also be discussed in this section. Section 4.3 extends the Shifted Frequency Analysis method to the ER-based induction machine model, resulting in a very

efficient, numerically stable model that is a valuable component for real-time EMTP simulations. Section 4.4 summarizes the new contributions made in this chapter.

4.2 Equivalent-Reduction Approach to Induction Machine Modelling in EMTP

Induction machines form a large portion of power system loads. Accurate EMTP simulation of their transient behavior may be required for voltage stability, transient stability, power quality analysis, and motor starting calculations, etc [8][44]. An efficient and stable EMTP model for induction machines is therefore of importance.

Induction machine models being used in EMTP-type simulators are mainly $dq0$ -domain models [41], [45]-[48]. In terms of the ways they interface with EMTP, the $dq0$ -domain models [49] can be further classified into several groups. One group of $dq0$ models, e.g. the universal machine model [50], utilizes the compensation technique where the external system is modeled as a three-phase Thevenin equivalent circuit and solved simultaneously with the machine equations to obtain machine variables. Then the system node voltages are updated by using linear superposition to take into consideration the machine effects. In order to use the Thevenin equivalent and linear superposition, the external system seen from the machine terminals has to be linear. Therefore, a distributed-parameter stub line has to be inserted to separate the machine model from other machines or nonlinear elements, which usually requires very small integration time steps [45][46]. The synchronous machine model in [41] can be directly converted to the induction machine model by feeding it with modified data entries [46]. However, this model requires predictions of flux linkages and mechanical variables. With this approach, the $dq0$ model is transformed to a phase quantities model, after application of the trapezoidal rule and equations reduction to stator variables only. To keep the equivalent resistances constant and not rotor-angle dependent, the d - and q -resistances are averaged, and a correction term is added to compensate for this error. This correction term requires prediction of the stator currents. All of these predictions require a smaller time step than actually needed, to avoid numerical instabilities. Other $dq0$ models include the voltage-behind-subtransient-reactance (and armature resistance) model and Norton-equivalent-based model [47], etc, which have similar

disadvantages as described above. A detailed review of $dq0$ machine models can be found in [47].

To overcome the numerical instability and the limitation of smaller time steps in $dq0$ models, the phase domain model has been proposed [35][51]. Based on the coupled-circuit induction machine equations [43], a phase domain model can be directly coupled into system equations; thus a simultaneous machine-network solution is achieved.

This chapter proposes an efficient phase-domain EMTP model for induction machines based on the equivalent-reduction (ER) technique. By using ER, the equations of stator, rotor, and flux linkages are reduced to only those of stator variables, without sacrificing numerical accuracy. The ER model is more efficient than and as accurate as other phase domain models in the EMTP.

4.2.1 Equivalent-Reduction (ER) to Stator Quantities

As seen from the terminal nodes, a power system component can be described by the terminal variables.

A power system component can often be modeled as a set of linear (time-varying) differential equations (DEs). When the differential equations are discretized, the resulting circuit can be equivalenced to a simple equivalent conductance matrix and a current source vector that retain only the terminal variables.

The ER method will significantly improve the simulator efficiency when the reduced nodal conductance matrix of the component is used quite often, for example, in a step-by-step EMTP simulation.

The following subsection 4.2.2 derives the so-called ER induction machine model for the EMTP simulation.

4.2.2 Induction Machine Modelling Based on ER Technique

A. Induction Machine Model

The original voltage and flux linkage equations of the induction machine can be expressed in the arbitrary reference frame as

$$\mathbf{v}_{qd0s} = \mathbf{r}_s \mathbf{i}_{qd0s} + \omega \lambda_{dqs} + p \lambda_{qd0s} \quad (4.1)$$

$$0 = \mathbf{r}_r \mathbf{i}_{qd0r} + (\omega - \omega_r) \lambda_{dqr} + p \lambda_{qd0r} \quad (4.2)$$

$$\begin{bmatrix} \lambda_{qd0s} \\ \lambda_{qd0r} \end{bmatrix} = \begin{bmatrix} \mathbf{K}_s \mathbf{L}_s \mathbf{K}_s^{-1} & \mathbf{K}_s \mathbf{L}_{sr} \mathbf{K}_r^{-1} \\ \mathbf{K}_r \mathbf{L}_{sr} \mathbf{K}_s^{-1} & \mathbf{K}_r \mathbf{L}_r \mathbf{K}_r^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{qd0s} \\ \mathbf{i}_{qd0r} \end{bmatrix} \quad (4.3)$$

where ω is the speed of the reference frame. $\mathbf{K}_s$ and $\mathbf{K}_r$ are the transformation matrices that map the stator variables and rotor variables to the reference frame [43], respectively.

B. Machine Modelling via ER

Rewriting equations (4.1)-(4.3) into a new matrix form, excluding the 0 equations because it is already decoupled from the d and q equations, we get

$$\begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{qds} \\ \mathbf{i}_{qdr} \end{bmatrix} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{qds} \\ \mathbf{i}_{qdr} \end{bmatrix} + \begin{bmatrix} \mathbf{v}_{qds} \\ \mathbf{0} \end{bmatrix} \quad (4.4)$$

where

$$\mathbf{A}_{11} = \begin{bmatrix} L_{ls} + L_M & & \\ & L_{ls} + L_M & \\ & & L_{ls} \end{bmatrix}$$

$$\mathbf{A}_{12} = \mathbf{A}_{21} = \begin{bmatrix} L_M & & \\ & L_M & \\ & & 0 \end{bmatrix}$$

$$\mathbf{A}_{22} = \begin{bmatrix} L_{lr} + L_M & & \\ & L_{lr} + L_M & \\ & & L_{lr} \end{bmatrix}$$

$$\mathbf{B}_{11} = \begin{bmatrix} -r_s & -\omega(L_{ls} + L_M) & 0 \\ \omega(L_{ls} + L_M) & -r_s & \\ & & -r_s \end{bmatrix}$$

$$\mathbf{B}_{12} = \begin{bmatrix} 0 & -\omega L_M \\ \omega L_M & 0 \\ & & 0 \end{bmatrix}$$

$$\mathbf{B}_{21} = \begin{bmatrix} 0 & -(\omega - \omega_r)L_M \\ (\omega - \omega_r)L_M & 0 \\ & & 0 \end{bmatrix}$$

$$\mathbf{B}_{22} = \begin{bmatrix} -r_r & -(\omega - \omega_r)(L_{lr} + L_M) \\ (\omega - \omega_r)(L_{lr} + L_M) & -r_r \\ & & -r_r \end{bmatrix}$$

The number of equations in (4.4) can be reduced by eliminating rotor equations via ER, as follows

$$\tilde{\mathbf{A}}_{11} \dot{\mathbf{i}}_{qds} = \tilde{\mathbf{B}}_{11} \mathbf{i}_{qds} + \tilde{\mathbf{B}}_{12} \mathbf{i}_{qdr} + \mathbf{v}_{qds} \quad (4.5)$$

where

$$\begin{aligned} \tilde{\mathbf{A}}_{11} &= \begin{bmatrix} L'' & \\ & L'' \end{bmatrix} \\ \tilde{\mathbf{B}}_{11} &= \begin{bmatrix} -r_s & -\omega L'' - \omega_r \frac{L_M^2}{L_{lr} + L_M} \\ \omega L'' + \omega_r \frac{L_M^2}{L_{lr} + L_M} & -r_s \end{bmatrix} \\ \tilde{\mathbf{B}}_{12} &= \begin{bmatrix} \frac{L_M}{L_{lr} + L_M} r_r & -\omega_r L_M \\ \omega_r L_M & \frac{L_M}{L_{lr} + L_M} r_r \end{bmatrix} \\ L'' &= L_{ls} + L_M'' \\ L_M'' &= \frac{L_{lr} L_M}{L_{lr} + L_M} \end{aligned}$$

Re-arranging (4.5), including the 0 equations, and applying Park's inverse transformation to it, we get

$$\mathbf{v}_{abcs} = \mathbf{r}_s \mathbf{i}_{abcs} + \tilde{\mathbf{L}}_{abcs} \dot{\mathbf{i}}_{abcs} + \mathbf{v}_{res} \quad (4.6)$$

where

$$\tilde{\mathbf{L}}_{abcs} = \begin{bmatrix} L_{ls} + \frac{2}{3} L_M'' & -\frac{L_M''}{3} & -\frac{L_M''}{3} \\ -\frac{L_M''}{3} & L_{ls} + \frac{2}{3} L_M'' & -\frac{L_M''}{3} \\ -\frac{L_M''}{3} & -\frac{L_M''}{3} & L_{ls} + \frac{2}{3} L_M'' \end{bmatrix},$$

$$\mathbf{v}_{res} = [\mathbf{K}_s(\theta)]^{-1} \cdot \begin{bmatrix} \left[\begin{array}{cc} \frac{\omega_r L_M^2}{L_{lr} + L_M} & \\ -\frac{\omega_r L_M^2}{L_{lr} + L_M} & \end{array} \right] \mathbf{i}_{qds} - \left[\begin{array}{cc} \frac{L_M r_r}{L_{lr} + L_M} & -\omega_r L_M \\ \omega_r L_M & \frac{L_M r_r}{L_{lr} + L_M} \end{array} \right] \mathbf{i}_{qdr} \\ 0 \end{bmatrix} \quad (4.7)$$

In equation (4.6), no information for rotor electrical part is lost. In fact, the rotor information is included in $\tilde{\mathbf{L}}_{abcs}$ and $\mathbf{v}_{res}$, which is an equivalent seen from the stator. Rotor currents $\mathbf{i}_{qdr}$ can be represented using the following differential equations derived from (4.4)

$$\mathbf{A}_{22} \dot{\mathbf{i}}_{qdr} = -\mathbf{A}_{21} \dot{\mathbf{i}}_{qds} + \mathbf{B}_{21} \mathbf{i}_{qds} + \mathbf{B}_{22} \mathbf{i}_{qdr}$$

Discretizing the above equation for $\mathbf{i}_{qdr}$, we can obtain the discrete-time equations (see equation (4.9) in subsection C) that can be used to update $\mathbf{i}_{qdr}$ at each time step.

The rotor circuits have to be retained explicitly when power electronics devices are connected to them.

C. Discrete-Time ER Model

Discretizing (4.6) with the trapezoidal rule of integration gives us

$$\mathbf{v}_{abcs}(t) = \left(\mathbf{r}_s + \frac{2}{\Delta t} \tilde{\mathbf{L}}_{abcs} \right) \mathbf{i}_{abcs}(t) + \mathbf{v}_{res}(t) + \mathbf{e}_{h1}(t) \quad (4.8)$$

where

$$\mathbf{e}_{h1}(t) = \left(\mathbf{r}_s - \frac{2}{\Delta t} \tilde{\mathbf{L}}_{abcs} \right) \mathbf{i}_{abcs}(t - \Delta t) + \mathbf{v}_{res}(t - \Delta t) - \mathbf{v}_{abcs}(t - \Delta t)$$

After applying Kron's reduction formula [52] to the discrete-time form of equation (4.4), we can get

$$\mathbf{i}_{qdr}(t) = \mathbf{k}_1(t) \mathbf{i}_{qds}(t) + \mathbf{k}_2(t) \quad (4.9)$$

Substituting (4.9) into (4.7) results in

$$\mathbf{v}_{res} = \mathbf{k}(t) \mathbf{i}_{abcs}(t) + \mathbf{e}_{h2}(t) \quad (4.10)$$

The equivalent circuit of the induction machine therefore becomes

$$\mathbf{v}_{abcs}(t) = \mathbf{R}_{eq} \mathbf{i}_{abcs}(t) + \mathbf{e}_h(t) \quad (4.11)$$

where

$$\mathbf{R}_{eq} = \mathbf{r}_s + \frac{2}{\Delta t} \tilde{\mathbf{L}}_{abc} + \mathbf{k}(t) \quad (\text{Equivalent resistances})$$

$$\mathbf{e}_h(t) = \mathbf{e}_{h1}(t) + \mathbf{e}_{h2}(t) \quad (\text{History term})$$

The factor matrices and history terms in (4.9) and (4.10) can be expanded as follows

$$\begin{aligned} \mathbf{k}_1(t) &= \left(\mathbf{A}_{22} - \mathbf{B}_{22}(t) \frac{\Delta t}{2} \right)^{-1} \left(-\mathbf{A}_{21} + \mathbf{B}_{21}(t) \frac{\Delta t}{2} \right) \\ &= - \left[\begin{array}{cc} \frac{\Delta t r_r}{2} + L_{lr} + L_M & \frac{\Delta t [\omega(t) - \omega_r(t)] (L_{lr} + L_M)}{2} \\ -\frac{\Delta t [\omega(t) - \omega_r(t)] (L_{lr} + L_M)}{2} & \frac{\Delta t r_r}{2} + L_{lr} + L_M \end{array} \right]^{-1} \\ &\quad \left[\begin{array}{cc} L_M & \frac{\Delta t}{2} [\omega(t) - \omega_r(t)] L_M \\ -\frac{\Delta t}{2} [\omega(t) - \omega_r(t)] L_M & L_M \end{array} \right] \\ \mathbf{k}_2(t) &= \left(\mathbf{A}_{22} - \mathbf{B}_{22}(t) \frac{\Delta t}{2} \right)^{-1} \left[\left(\mathbf{A}_{21} + \mathbf{B}_{21}(t - \Delta t) \frac{\Delta t}{2} \right) \mathbf{i}_{qds}(t - \Delta t) \right. \\ &\quad \left. + \left(\mathbf{A}_{22} + \mathbf{B}_{22}(t - \Delta t) \frac{\Delta t}{2} \right) \mathbf{i}_{qdr}(t - \Delta t) \right] \\ &= - \left[\begin{array}{cc} \frac{\Delta t r_r}{2} + L_{lr} + L_M & \frac{\Delta t [\omega(t) - \omega_r(t)] (L_{lr} + L_M)}{2} \\ -\frac{\Delta t [\omega(t) - \omega_r(t)] (L_{lr} + L_M)}{2} & \frac{\Delta t r_r}{2} + L_{lr} + L_M \end{array} \right]^{-1} \\ &\quad \left\{ \left[\begin{array}{cc} L_M & -\frac{\Delta t}{2} [\omega(t - \Delta t) - \omega_r(t - \Delta t)] L_M \\ \frac{\Delta t}{2} [\omega(t - \Delta t) - \omega_r(t - \Delta t)] L_M & L_M \end{array} \right] \mathbf{i}_{qds}(t - \Delta t) \right. \\ &\quad \left. + \left[\begin{array}{cc} -\frac{\Delta t r_r}{2} + L_{lr} + L_M & \frac{-\Delta t [\omega(t - \Delta t) - \omega_r(t - \Delta t)] (L_{lr} + L_M)}{2} \\ \frac{\Delta t [\omega(t - \Delta t) - \omega_r(t - \Delta t)] (L_{lr} + L_M)}{2} & -\frac{\Delta t r_r}{2} + L_{lr} + L_M \end{array} \right] \mathbf{i}_{qdr}(t - \Delta t) \right\} \end{aligned}$$

The time-varying factors $\mathbf{k}(t)$ and $\mathbf{e}_{h2}(t)$ in (4.10) are expressed as

$$\mathbf{k}(t) = [\mathbf{K}_s(\theta)]^{-1} \begin{bmatrix} \mathbf{k}_3(t) \\ 0 \end{bmatrix} [\mathbf{K}_s(\theta)]$$

$$\mathbf{e}_{h2}(t) = [\mathbf{K}_s(\theta)]^{-1} \begin{bmatrix} \mathbf{k}_4(t) \\ 0 \end{bmatrix}$$

$$\mathbf{k}_3(t) = \begin{bmatrix} \frac{\omega_r(t)L_M^2}{L_r + L_M} & -\frac{L_M r_r}{L_r + L_M} \\ -\frac{\omega_r(t)L_M^2}{L_r + L_M} & \frac{L_M r_r}{L_r + L_M} \end{bmatrix} - \begin{bmatrix} \frac{L_M r_r}{L_r + L_M} & -\omega_r(t)L_M \\ \omega_r(t)L_M & \frac{L_M r_r}{L_r + L_M} \end{bmatrix} \mathbf{k}_1(t)$$

$$\mathbf{k}_4(t) = - \begin{bmatrix} \frac{L_M r_r}{L_r + L_M} & -\omega_r(t)L_M \\ \omega_r(t)L_M & \frac{L_M r_r}{L_r + L_M} \end{bmatrix} \mathbf{k}_2(t)$$

It should be noted that, although the equations for the rotor electrical part are eliminated, the rotor currents could be updated at each simulation step via (4.9).

Refer to [9] and [51] for the discrete-time equations of the mechanical part.

The ER induction machine model is actually a full-order accurate phase-domain model because the information for the rotor electrical part has already been integrated into the stator part equations. Also, because Park's transformation is implicitly used in the ER model, the elegance and simplicity in the discrete-time equivalent circuit are achieved. With a lower number of equations in the model, it is simpler and faster than traditional phase-domain models in the EMTP, without compromise in accuracy.

4.2.3 Simulation Results

Three test cases are simulated to illustrate the efficiency and accuracy of the ER model. The first case illustrates free acceleration characteristics of a 3-hp induction machine and a 2250-hp induction machine. The second case is a load torque change test which shows the slow dynamics of the 3-hp machine. The third case simulates the fast transients caused by a three phase fault. The induction machines parameters for the three cases are shown in Table A-II taken from [43]. Rotor parameters have been converted to the stator side.

As reported in [37], the voltage-behind-reactance (VBR) model [36] is more efficient than traditional phase-domain EMTP models and more stable than $dq0$ -domain models; hence a discrete-time VBR induction machine model used in EMTP [53] was chosen to compare with the

ER model. To facilitate the comparison of CPU times, both machine models are deliberately implemented in the language of MATLAB[®] script.

A. Simulation of Free Acceleration

The start-up transients of the 3-hp and 2250-hp induction machines during free acceleration from stall are simulated. The total simulation times are 1 s for the 3-hp induction machine and 3 s for the 2250-hp induction machine, respectively. In this simulation, a time step $\Delta t = 500 \mu s$ is used.

Figure 4.1 illustrates the torque-speed characteristics of the 2250-hp induction machine during free acceleration. The simulation results for the 3-hp induction machine are shown in Figure 4.2. The variables plotted in this figure are i_{as} (phase A stator current), ψ_m (magnetizing flux linkage), T_e (electromagnetic torque) and mechanical rotor speed. These machine variables are observed in the rotor reference frame; results viewed in other reference frames are omitted here.

It can be seen that the time domain values obtained from the EMTP simulation with the ER induction machine model are practically identical to those with the VBR model. In Figure 4.1 and Figure 4.2, the results from the ER model and the VBR model are so close that they become indistinguishable from each other. Numerical tests also indicate that both ER and VBR model, when a large time step of 1 ms is used, can still get accurate, stable, and identical solutions.

Figure 4.1 Torque-Speed Characteristics during Free Acceleration of a 2250-hp Induction Machine ($\Delta t = 500 \mu s$)

Figure 4.2 Dynamic Performance of a 3-hp Induction Machine during Free Acceleration ($\Delta t = 500 \mu s$)

B. Load Torque Change Test

Slow dynamics during step changes in load torque are simulated in this case. The 3-hp machine originally operates in steady state with no load. A mechanical torque T_m of 12 N·m is applied at $t = 2.05 s$. Then T_m is reversed to $-12 N \cdot m$ at $t = 2.5 s$. The simulation time is 3 s with a time step of $500 \mu s$. Figure 4.3 shows the phase A stator current, slip, real power, and reactive power during load torque changes. The machine variables are observed in the synchronous reference frame.

Comparisons of the EMTP simulation results obtained from the ER model with those from the VBR model show again that the results are identical.

Figure 4.3 Dynamic Performance of a 3-hp Induction Machine during Step Changes in Load Torque ($\Delta t = 500 \mu s$)

C. Three Phase Fault Test

In this simulation of fast transients, the 2250-hp induction machine is first operating under rated conditions with a load torque equal to T_B . Here T_B is the base torque defined by $T_B = \frac{P_B}{(2/p)\omega_b}$, where P_B is the rated power output of the machine and ω_b is the rated speed of the machine. A 3-phase fault is then applied at the terminals at $t = 6.1 s$. After 6 cycles, the fault is cleared. The simulation time is 10 s with a time step of $100 \mu s$. Figure 4.4 illustrates that the proposed ER model can simulate fast dynamics of the machine as accurately as the VBR model.

Figure 4.4 Simulation Results for a 3-Phase Fault at the Terminals of a 2250-hp Induction Machine ($\Delta t = 100 \mu s$)

Cases A to C show that the ER model is as accurate as the VBR model for the same Δt step. It is also interesting to explore the numerical accuracy and robustness of the ER model by using different Δt steps in the fast transients simulation of Case C. Results obtained from the VBR simulation [53] with a very small $\Delta t = 10 \mu s$ are assumed to be the reference accurate time domain results. A zoomed-in view of phase A stator current is shown in Figure 4.5. Figure 4.5 indicates that, if we run an EMTF simulation with the ER model using a time step of $1 ms$, relatively accurate results are still obtained. Even with a time step of $1.5 ms$, the numerical solution remains stable.

Figure 4.5 Simulation Results for a 3-Phase Fault with Different Time Steps

Table 4.1 summarizes the CPU times needed for the VBR and ER simulations in cases A to C. The timing in Table 4.1 was obtained on a 1.83G Hz PC. It can be calculated that the ER is usually 33%-36% faster than the VBR model for the same accuracy. The efficiency of the ER model is due to its concise form and the low computer burden in updating electrical variables and flux linkages.

The ER model has a further advantage in addition to the faster speed. Instead of being specifically designed for the machines to achieve efficient phase domain models, the ER technique used in the induction machine modeling is a general-purpose approach. ER can also be used for building more efficient models for other types of machines, transformers and power system elements, as long as they can be mathematically represented by differential equations with mutually coupled (or decoupled) state variables.

Since the ER-based model proposed in this chapter is actually a full-order model, it is different from the Model-Order-Reduction-based models. Model Order Reduction (MOR) approaches have been proposed, either for constructing a reduced-order model of the machine itself by neglecting stator flux derivatives, or for constructing a reduced-order model of the external system by using techniques such as Krylov-subspace or Fourier methods [54]-[57]. MOR methods improve the simulation speed; however, undesirable inaccurate step-by-step simulation results will be produced, especially from the latter one.

Table 4.1 CPU Times for Simulations

Test Case	VBR Model	ER Model
Case A (3-hp induction machine, $T = 1 \text{ s}$, $\Delta t = 500 \mu\text{s}$)*	0.875 s	0.556 s
Case A (2250-hp induction machine, $T = 3 \text{ s}$, $\Delta t = 500 \mu\text{s}$)	2.601 s	1.698 s
Case B (3-hp induction machine, $T = 3 \text{ s}$, $\Delta t = 500 \mu\text{s}$)	2.606 s	1.696 s
Case C (2250-hp induction machine, $T = 10 \text{ s}$, $\Delta t = 500 \mu\text{s}$)	8.391 s	5.562 s
Case C (2250-hp induction machine, $T = 10 \text{ s}$, $\Delta t = 50 \mu\text{s}$)	82.796 s	55.031 s

* T stands for the total simulation time.

4.3 Induction Machine Modelling with SFA

4.3.1 Induction Machine Modelling Based on SFA

To build the dynamic-phasor-based stator equations, the stator variables are first expressed as

$$\mathbf{i}_{abcs}(t) = \mathbf{i}_{abcsI}(t) \cos \omega_s t - \mathbf{i}_{abcsQ}(t) \sin \omega_s t \quad (4.12)$$

$$\mathbf{v}_{abcs}(t) = \mathbf{v}_{abcsI}(t) \cos \omega_s t - \mathbf{v}_{abcsQ}(t) \sin \omega_s t \quad (4.13)$$

Substituting (4.12) and (4.13) into (4.6), we get

$$\begin{aligned} & \mathbf{v}_{abcsI}(t) \cos \omega_s t - \mathbf{v}_{abcsQ}(t) \sin \omega_s t \\ &= \mathbf{r}_s [\mathbf{i}_{abcsI}(t) \cos \omega_s t - \mathbf{i}_{abcsQ}(t) \sin \omega_s t] + \tilde{\mathbf{L}}_{abcs} [p\mathbf{i}_{abcsI}(t) \cos \omega_s t - \mathbf{i}_{abcsI}(t) \omega_s \sin \omega_s t \\ & \quad - p\mathbf{i}_{abcsQ}(t) \sin \omega_s t - \mathbf{i}_{abcsQ}(t) \omega_s \cos \omega_s t] \end{aligned} \quad (4.14)$$

Then we apply the Hilbert transform to both sides of (4.14) and construct the analytic signal, which leads to

$$\mathbf{V}_{abcsI}(t)e^{j\omega_s t} = \mathbf{r}_s \mathbf{I}_{abcs}(t)e^{j\omega_s t} + \tilde{\mathbf{L}}_{abcs} [p\mathbf{I}_{abcs}(t) + j\omega_s \mathbf{I}_{abcs}(t)]e^{j\omega_s t} + \mathbf{V}_{res}(t)e^{j\omega_s t} \quad (4.15)$$

where $\mathbf{V}_{abcs}(t)$ and $\mathbf{I}_{abcs}(t)$ are dynamic phasors of stator voltages $\mathbf{v}_{abcs}(t)$ and currents $\mathbf{i}_{abcs}(t)$, respectively, and $\mathbf{V}_{res}(t)$ is the dynamic phasor of $\mathbf{v}_{res}(t)$.

Applying frequency shifting as described in (2.7), we can obtain the dynamic-phasor equations for the stator part as

$$\mathbf{V}_{abc}(t) = \mathbf{r}_s \mathbf{I}_{abc}(t) + \tilde{\mathbf{L}}_{abc} [p \mathbf{I}_{abc}(t) + j\omega_s \mathbf{I}_{abc}(t)] + \mathbf{V}_{res}(t) \quad (4.16)$$

4.3.2 Discrete Time Model

Discretizing (4.16) with the trapezoidal rule of integration gives us

$$\mathbf{V}_{abc}(t) = \left[\mathbf{r}_s + \left(\frac{2}{\Delta t} + j\omega_s \right) \tilde{\mathbf{L}}_{abc} \right] \mathbf{I}_{abc}(t) + \mathbf{V}_{res}(t) + \mathbf{E}_{h1}(t) \quad (4.17)$$

where

$$\mathbf{E}_{h1}(t) = \left[\mathbf{r}_s + \left(-\frac{2}{\Delta t} + j\omega_s \right) \tilde{\mathbf{L}}_{abc} \right] \mathbf{I}_{abc}(t - \Delta t) - \mathbf{V}_{abc}(t - \Delta t) + \mathbf{V}_{res}(t - \Delta t)$$

From the discrete-time equation of $\mathbf{v}_{res}(t)$ (see Section 4.2), the corresponding discrete-time equation of $\mathbf{V}_{res}(t)$ can be derived as

$$\begin{aligned} \mathbf{V}_{res} &= \mathbf{k}(t) \mathbf{I}_{abc}(t) + \mathbf{E}_{res,h}(t) \\ &= \mathbf{k}(t) \mathbf{I}_{abc}(t) + \begin{bmatrix} e^{j(\theta - \omega_s t)} & e^{j(\theta - \omega_s t - \pi/2)} & 1 \\ e^{j(\theta - \omega_s t - 2\pi/3)} & e^{j(\theta - \omega_s t - 2\pi/3 - \pi/2)} & 1 \\ e^{j(\theta - \omega_s t + 2\pi/3)} & e^{j(\theta - \omega_s t + 2\pi/3 - \pi/2)} & 1 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{k}_4(t) \\ 0 \end{bmatrix} \end{aligned} \quad (4.18)$$

The factors $\mathbf{k}(t)$ and $\mathbf{k}_4(t)$ are explained in Section 4.2.

By substituting (4.18) into (4.17), we get the equivalent circuit of the induction machine (4.11) in the SFA domain as follows

$$\mathbf{V}_{abc}(t) = \mathbf{R}_{eq} \mathbf{I}_{abc}(t) + \mathbf{E}_h(t) \quad (4.19)$$

where

$$\begin{aligned} \mathbf{R}_{eq} &= \mathbf{r}_s + \left(\frac{2}{\Delta t} + j\omega_s \right) \tilde{\mathbf{L}}_{abc} + \mathbf{k}(t) && \text{(Equivalent resistances)} \\ \mathbf{E}_h(t) &= \mathbf{E}_{h1}(t) + \mathbf{E}_{res,h}(t) && \text{(History term)} \end{aligned}$$

The SFA equivalent circuit for the induction machine is illustrated in Figure 4.6.

Figure 4.6 SFA Equivalent Circuit of an Induction Machine

Discretizing the rotor mechanical part equations [9], we can get the discrete time equations of the mechanical part

$$\omega_r(t) = \omega_r(t - \Delta t) + \frac{\Delta t p}{4J} [T_e(t) + T_e(t - \Delta t) - T_m(t) - T_m(t - \Delta t)] \quad (4.20)$$

$$T_e(t) = \frac{3p}{4} [\lambda_{ds}(t)i_{qs}(t) + \lambda_{qs}(t)i_{ds}(t)] \quad (4.21)$$

$$\theta_r(t) = \theta_r(t - \Delta t) + \frac{\Delta t}{2} [\omega_r(t) + \omega_r(t - \Delta t)] \quad (4.22)$$

where

p is the number of poles,

$$\mathbf{i}_{qd0s} = \mathbf{K}_s^r(\theta_r(t)) \text{Re}[\mathbf{I}_{abcs}(t)e^{j\omega_s t}]$$

$$\lambda_{qs} = L_{ls}i_{qs} + \lambda_{mq} = L_{ls}i_{qs} + L_M(i_{qs} + i_{qr})$$

$$\lambda_{ds} = L_{ls}i_{ds} + \lambda_{md} = L_{ls}i_{ds} + L_M(i_{ds} + i_{dr})$$

and $\mathbf{i}_{qdr}(t)$ can be updated by using (4.9).

The more complex mechanical part equations based on the multi-mass model can be found in [9].

4.3.3 Simulation Results

The same three test cases described in Section 4.2.3 are simulated on a 1.83 GHz PC to illustrate the efficiency and accuracy of the SFA model. The first case illustrates free acceleration characteristics of a 3-hp induction machine and a 2250-hp induction machine. The second case is a load torque change test which shows the slow dynamics of the 3-hp machine. The third case simulates the fast transients caused by a three phase fault. The following subsections A, B, and C explain that the SFA-based model is an accurate and general-purpose one. Subsection D explores the very high efficiency of the SFA model. The author has developed

a MATLAB program based on the EMTF algorithm and the ER machine model. All SFA results in the following subsections have been compared with the results from this EMTF program in the same MATLAB environment. The induction machine parameters for the three cases can be found in Appendix A. Rotor parameters have been converted to the stator side.

A. Simulation of Free Acceleration

The start-up transients of the 3-hp and 2250-hp induction machines during free acceleration from stall are simulated. The total simulation times are 1 s for the 3-hp induction machine and 3 s for the 2250-hp induction machine, respectively. In this simulation, a time step of $\Delta t = 500 \mu s$ is used.

Figure 4.7 illustrates the stator current of the 2250-hp induction machine during free acceleration. Note that the SFA model can produce both dynamic phasor and time domain results.

The simulation results for the 3-hp induction machine are shown in Figure 4.8. The variables plotted in this figure are i_{as} (phase A stator current), ψ_m (magnetizing flux linkage), T_e (electromagnetic torque) and mechanical rotor speed. These machine variables are observed in the rotor reference frame; results viewed in other reference frames are omitted here. As is shown in Figure 4.8, the time domain values reconstructed from the dynamic phasor results are identical to those obtained from the EMTF algorithm using a ER model. The CPU times used for simulations are listed in Table 4.2.

Figure 4.7 Stator Current during Free Acceleration of a 2250-hp Induction Machine ($\Delta t = 500 \mu s$)

Figure 4.8 Dynamic Performance of a 3-hp Induction Machine during Free Acceleration ($\Delta t = 500 \mu s$)

Table 4.2 CPU Times for Simulations

Free Acceleration Case	SFA Model	ER Model
3-hp induction machine, $T = 1 s$, $\Delta t = 500 \mu s$	0.728 s	0.556 s
2250-hp induction machine, $T = 3 s$, $\Delta t = 500 \mu s$	2.439 s	1.698 s

* T stands for the total simulation time.

B. Load Torque Change Test

Slow dynamics during step changes in load torque are simulated in this case. The 3-hp machine originally operates in steady state with no load. A mechanical torque T_m of 12 N·m is applied at $t = 2.05 s$. Then T_m is reversed to $-12 \text{ N}\cdot\text{m}$ at $t = 2.5 s$. The simulation time is 3 s with a

time step of 0.5 ms for the ER model, and a time step of 5 ms for the SFA model. Figure 4.9 shows the phase A stator current and the slip during load torque changes.

Figure 4.9 shows that simulation results from the SFA model with $\Delta t = 5\text{ ms}$ are as accurate as those from an EMTP solution with $\Delta t = 0.5\text{ ms}$. The CPU time for a 3 s simulation with the SFA model is 0.266 s , while the simulation with the ER model uses 1.696 s . Therefore, to achieve the same numerical accuracy, the SFA induction machine model can be more than 5 times faster than its corresponding ER model in this case.

Figure 4.9 Dynamic Performance of a 3-hp Induction Machine during Step Changes in Load Torque (RE $\Delta t = 0.5\text{ ms}$, SFA $\Delta t = 5\text{ ms}$)

C. Three Phase Fault Test

In this simulation of fast transients, the 2250-hp induction machine is first operating

under rated conditions with a load torque equal to T_B . A 3-phase fault is then applied at the terminals at $t = 6.1$ s. After 6 cycles, the fault is cleared. The simulation time is 10 s with a time step of $500 \mu\text{s}$. Figure 4.9 illustrates that the proposed SFA model can simulate fast dynamics of the machine as accurately as the ER model. The CPU times are 7.258s and 5.562 s for SFA simulation and ER simulation, respectively.

Figure 4.10 Simulation Results for a 3-Phase Fault at the Terminals of a 2250-hp Induction Machine ($\Delta t = 500 \mu\text{s}$)

D. Comparison between SFA and an EMTP Phase-Domain Model

Cases A to C show that the SFA model is as accurate as EMTP phase-domain models for the same time step Δt . Now we will carefully explore the numerical accuracy and robustness of

the SFA model, by using different time steps Δt in the slow dynamics simulation. The 2250-hp machine originally operates in steady state with rated torque of 8900 N·m. The machine dynamics are recorded when a sudden reverse of mechanical torque to -8900 N·m is applied at $t = 6$ s. The total simulation time is 10 s.

The SFA model was run with different time steps $\Delta t = 0.1$ ms, $\Delta t = 1$ ms, $\Delta t = 5$ ms, and $\Delta t = 10$ ms. The results obtained from an original phase-domain model (refer to p. 142-147 in [43]) with a small $\Delta t = 0.1$ ms are assumed to be the reference accurate time domain results. A zoomed-in view of phase A stator current is shown in Figure 4.11, which shows that the SFA model can accurately simulate the system dynamics with a large time step of 10 ms. In fact, even with a time step of 15 ms, the numerical solution with SFA remains stable. On the contrary, an EMTP simulation in the time domain with a time step of more than 4 ms became numerically unstable, as is shown in Figure 4.12.

Figure 4.11 Simulation Results by SFA

Figure 4.12 Time Domain Results from an EMTP Algorithm Implemented in MATLAB

Table 4.3 summarizes the CPU times needed for the SFA and ER simulations in this case. It can be seen that the SFA model was about 10 times faster (0.370 s vs 2.801 s) than the EMTP phase-domain model (ER model) to achieve a comparable accuracy. In certain situations, for instance, when simulating a system with power electronics components, an EMTP-type simulator may have to use very small time steps, for example, less than 0.1 ms. As can be seen from Table 4.3 that a SFA model with $\Delta t = 10\text{ms}$ can be about 75 times faster than an EMTP model with $\Delta t = 0.1\text{ms}$, for reasonable accuracy. Therefore, it is very promising to apply the SFA model in hybrid simulators, where the power electronics components as well as the network can be simulated with a small time step, while the SFA can be used for machine models with a larger time step. Further, the new SFA model has the capability to be extended to model the saturation and deep bar effects in induction machines. The SFA modeling framework is shown to be successful for machine modeling as a new method, which opens the door for more future research such as applying the SFA to wind turbine generator modeling, and developing SFA-based EMTP simulators or hybrid simulators, and so on.

Table 4.3 CPU Times for Simulations

2250-hp induction machine Torque Change Case	SFA Model	ER Model
$T = 10\text{ s}, \Delta t = 0.1\text{ ms}$	36.108 s	27.328 s
$T = 10\text{ s}, \Delta t = 1\text{ ms}$	3.725 s	2.801 s
$T = 10\text{ s}, \Delta t = 5\text{ ms}$	0.738 s	--*
$T = 10\text{ s}, \Delta t = 10\text{ ms}$	0.370 s	--*

* Numerically unstable.

4.4 Summary

(1) An Equivalent-Reduction-based induction machine model is proposed. With a lower number of model equations, it is actually a full-order phase-domain model but is simpler and faster. Park's transformation is implicitly used in the ER model in order to maintain the elegance and simplicity in the discrete-time equivalent circuit. As a non-ad hoc approach, the ER can also be used for building more efficient models for other power system elements.

(2) Based on the ER model, a SFA model is proposed as a general-purpose model capable of simulating both fast transients and slow dynamics. Case study results have confirmed the SFA induction machine model is a valuable component for real-time EMTP simulations. It is observed that the SFA model is in excess of 70 times faster than the EMTP solution with the RE model when simulating dynamics with frequency spectra close to the fundamental power frequency.

Chapter 5

EMTP Implementation

5.1 Introduction

Since a number of power system component models in the shifted frequency domain have been built in the previous chapters, it is now time to ask how they can be used in a general purpose simulation tool based on the Shifted Frequency Analysis.

EMTP, which was originally developed for calculating the transient overvoltages in transmission systems, has been significantly expanded to tracing the evolution of the system states in arbitrary multi-phase power networks consisting of all types of components. With improved functionality, accuracy and numerical stability, EMTP has become a standard tool being widely used in the power industry for system planning and designing purposes. Now EMTP is seeing broader applications in power system steady state [58] and dynamics studies and will remain one of the mainstreams in power system research. The intention of this thesis is to expand EMTP to efficiently simulate the slow dynamics in power systems, and to bridge the gap between the EMTP and a unified power system analysis tool. This means that the EMTP algorithm will be adopted in the SFA-based simulation tool. What makes a difference here is that the electrical variables are described by dynamic phasors instead of instantaneous time values.

5.2 Program Structure

The implicit trapezoidal rule of integration, which has attractive characteristics in terms of accuracy and numerical stability, is used in EMTP and is followed in the SFA-based modelling of system components. By using the trapezoidal rule, the differential equations representing all network components are converted into algebraic relationships which can be interpreted as equivalent resistances or admittances, voltages, currents, and known history terms. Then the nodal equations of the system can be written directly as follows.

$$[\mathbf{G}][\mathbf{V}(t)] = [\mathbf{I}(t)] + [\mathbf{H}(t)] \quad (5.1)$$

where

$[\mathbf{G}]$ is the matrix of the nodal equivalent admittances

$[\mathbf{V}(t)]$ is the vector of the nodal voltages, which are dynamic phasors

$[\mathbf{I}(t)]$ is the vector of the nodal currents, which are dynamic phasors

$[\mathbf{H}(t)]$ is the vector of the nodal history terms

The voltages of the nodes connected to voltage sources are known quantities, therefore the corresponding equations can be eliminated. Suppose A is the index set denoting the nodes with unknown voltages, and B is the index set for nodes with known voltages, then the nodal equations can be written in a block matrix form

$$\begin{bmatrix} \mathbf{G}_{AA} & \mathbf{G}_{AB} \\ \mathbf{G}_{BA} & \mathbf{G}_{BB} \end{bmatrix} \begin{bmatrix} \mathbf{V}_A(t) \\ \mathbf{V}_B(t) \end{bmatrix} = \begin{bmatrix} \mathbf{I}_A(t) + \mathbf{H}_A(t) \\ \mathbf{I}_B(t) + \mathbf{H}_B(t) \end{bmatrix} \quad (5.2)$$

Thus the unknown voltages can be obtained at time t by solving the following equations

$$\mathbf{G}_{AA} \mathbf{V}_A(t) = [\mathbf{I}_A(t) + \mathbf{H}_A(t)] - \mathbf{G}_{AB} \mathbf{V}_B(t) \quad (5.3)$$

In this thesis, a toolbox for simulating power system dynamics in the shifted frequency domain has been developed with MATLAB. This dynamic phasor tool consists of several files, which are depicted in the schematic structure in Figure 5.1.

A. Input Data File

Input data file provides the data needed for the simulation of a multi-phase power network. These include transmission line data, load data, machine data, switching operations

Chapter 5 EMTP Implementation

data, voltage sources data, etc. All these data are documented in a MATLAB script file and most of the data are provided as the MATLAB arrays. Refer to Appendix C for the input data format.

Figure 5.1 Schematic Structure for the EMTP Dynamic Phasor Simulation Tool

B. Data Processing Files

When the input data are read by the main program, the data processing functions are called to establish the equivalent circuit for different power system components, and initialize their history terms for the simulation at the first time step. The equivalent circuit for different components can be found in previous chapters.

The initialization in the SFA simulations is based on the snapshot method. First, let the dynamic phasor program run with zero initial conditions and reaches the steady state. Then, a snapshot of the system is taken by saving system variables and history terms at a particular time step. These system variables and history terms from the snapshot file are fed to the SFA program as the initial conditions. With system variables and history terms initialized, the SFA simulation

will run with a 'flat' start, and the advantage of using large time step in simulating 60 HZ dynamics is achieved.

C. File for Building G Matrix

The file 'Gsys.m' builds the system $[G]$ matrix in equation (5.1) and returns the $[G_{AA}]$ and $[G_{AB}]$ to the system solver for calculating the unknown voltages. The following explains how this simulation tool inserts power system components into equivalent admittance matrix $[G]$ by taking the π -circuit model of a transmission line as an example.

Suppose there is a three-phase π -circuit for a transmission line connecting node set J ($=[j_1, j_2, j_3]$) and node set K ($=[k_1, k_2, k_3]$) as depicted in Figure 5.2. As can be seen in Figure 5.2, the π -circuit model consists of a coupled RL branch and two coupled capacitances. When discretized by the trapezoidal rule, the coupled RL branch will contribute a 3×3 equivalent admittance matrix G_{RL} to $[G]$, and contributes a 3×1 history term h_{RL} to $[H]$. Similarly, the coupled capacitances will also contribute a 3×3 equivalent admittance matrix G_C and a 3×1 history term h_C . From equations (2.21) and (2.24), it can be found that G_{RL} will be entered into $[G]_{J,J}$ and $[G]_{K,K}$ in the system admittance matrix $[G]$, and $-G_{RL}$ into $[G]_{J,K}$ and $[G]_{K,J}$. For instance, $G_{RL(1,1)}$ will be added to the element $[G]_{j_1,j_1}$, $G_{RL(1,2)}$ will be added to $[G]_{j_1,j_2}$, $G_{RL(1,2)}$ to $[G]_{j_1,j_3}$, $G_{RL(2,1)}$ to $[G]_{j_2,j_1}$, and so on. This procedure is illustrated in Figure 5.3. The MATLAB implementation of this process is shown in Figure 5.4.

This routine also moves the nodes connecting to the voltage sources to the bottom of the system equation set, and then extract the sub-blocks $[G_{AA}]$ and $[G_{AB}]$ from $[G]$.

Figure 5.2 The Π Transmission Line Model

D. File for Updating $[G]$ and $[H]$

The MATLAB files UpdateG.m will update the G matrix when changes happen in the network configuration, e.g. line tripping or faults. The file UpdateHist.m will calculate the history terms at each time step and return them to the solver for calculating unknown variables.

E. System Solver

The system solver Main.m is the core function in the simulation tool. In each time step, this routine assembles the right hand side of (5.3), performs downward operations on it, and does the back-substitution to obtain the unknown nodal voltages. After the unknowns are found, the routines for updating history terms and/or admittance matrix are called, in preparation for the calculations at the next step. The simulation run will continue until the total simulation time is reached. A schematic flow chart of the system solver is presented in Figure 5.5.

Figure 5.3 Contributions of the II-Circuit Transmission Line Model to the Nodal Admittance Matrix


```

% Pi Lines Contribution to G Matrix

for kk = 1 : nLine
    ii = (LineFromBus(kk)-1)*3 + 1;
    jj = (LineToBus(kk)-1)*3 + 1;
    Gsystem( ii: ii + 2, ii: ii + 2) = Gsystem( ii: ii + 2, ii: ii + 2) + ...
        GRL(:, :, kk) + GC(:, :, kk);
    Gsystem( ii: ii + 2, jj: jj + 2) = Gsystem( ii: ii + 2, jj: jj + 2) - ...
        GRL(:, :, kk);
    Gsystem( jj: jj + 2, ii: ii + 2) = Gsystem( jj: jj + 2, ii: ii + 2) - ...
        GRL(:, :, kk);
    Gsystem( jj: jj + 2, jj: jj + 2) = Gsystem( jj: jj + 2, jj: jj + 2) + ...
        GRL(:, :, kk) + GC(:, :, kk);
end

```

Figure 5.4 The MATLAB Code for Inserting Π -Circuit Transmission Line Model into G Matrix

5.3 Test Cases

Four test cases are tested in the SFA-based EMTP simulator developed in this chapter.

A. Radial Transmission Line Case

The first case is a radial network consisting of a voltage source, a three phase load and a double-circuit overhead line between the source and the load. The one-line diagram is shown in Figure 5.6. The transmission line is energized at $t = 0$. At $t = 3s$, one of the parallel circuits is tripped. The total simulation time is 5 seconds. A time step of 5 ms is used in this simulation. Detailed data for this case are included in Appendix C. Figure 5.12-Figure 5.12 illustrates the three phase voltages at the load node, including the zoomed-in view for these voltages in the chosen time interval from 2.9s to 3.1s. A MATLAB program based on the EMTP pi-circuit model is used to generate the time domain results for comparison. It can be seen that the EMTP solutions in the shifted frequency domain and those in the time domain are almost identical, and the dynamic phasor result is the envelop of the time domain curve.

Figure 5.5 Flow Chart for Dynamic-Phasor-Based EMTP Simulator

Figure 5.6 One-line Diagram of a Radial Test System

Figure 5.7 Phase A Voltage at the Load Node ($\Delta t = 5$ ms)

Figure 5.8 Zoomed-in View of Phase A Voltage at the Load Node ($\Delta t = 5$ ms)

Figure 5.9 Phase B Voltage at the Load Node ($\Delta t = 5$ ms)

Figure 5.10 Zoomed-in View of Phase B Voltage at the Load Node ($\Delta t = 5$ ms)

Figure 5.11 Phase C Voltage at the Load Node ($\Delta t = 5$ ms)

Figure 5.12 Zoomed-in View of Phase C Voltage at the Load Node ($\Delta t = 5$ ms)

B. Distribution Network Case

The second case is a 13 bus network with the configuration adapted from the IEEE test feeder [59]. The network configuration is shown in Figure 5.13. The detailed network data are documented in Appendix C. The network is energized at $t = 0$. The induction machine load connected to node 11 is initially operating at no-load. At $t = 3s$, a rated mechanical torque is applied on the induction machine. Then one circuit of the double-circuit lines connecting node 0 and node 1 is tripped at $t = 6s$. The total simulation time is 8 s. A time step of 1 ms is used in this simulation. Figure 5.14-Figure 5.18 illustrate part of the three phase voltages at three different nodes. A MATLAB program using the EMTP algorithm is used to produce the time domain results. The time domain simulation results and the dynamic phasor results are both shown in each figure to illustrate the correctness of the dynamic phasor results. From the figures, it can be seen that the increase of the induction motor load causes drops in the system voltage. Loss of one of the double circuit lines further weakens the systems and causes voltage dip problems.

Figure 5.13 One Line Diagram for the Test Feeder

Figure 5.14 Phase A Voltage at the Induction Machine Node ($\Delta t = 1$ ms)

Figure 5.15 Zoomed-in View of Phase A Voltage at the Induction Machine Node ($\Delta t = 1$ ms)

Figure 5.16 Phase B Voltage at Load Node 6 ($\Delta t = 1$ ms)

Figure 5.17 Zoomed-in View of Phase B Voltage at Load Node 6 ($\Delta t = 1$ ms)

Figure 5.18 Phase C Voltage at Node 1 ($\Delta t = 1$ ms)

C. First Benchmark System for the Subsynchronous Resonance Studies

This test system was prepared by an IEEE Subsynchronous Resonance Task Force [60] as a standard test case for computer programs to simulate subsynchronous resonance phenomena. The test system consists of an 892.4 MVA turbine-generator connected through a step-up transformer to a 500 kV transmission line with series capacitor compensation. The power system at the receiving end is represented by a Thevenin equivalent circuit (infinite bus behind reactance) [61]. Figure 5.19 shows the one-line diagram for the first benchmark system. In

Figure 5.19, all data are represented in p.u. based on 892.4 MVA and 500 kV. The generator parameters of the electrical part are listed as below

$X_d = 1.79$ p.u.	$X_q = 1.71$ p.u.	$R_a = 0$
$X'_d = 0.169$ p.u.	$X'_q = 0.228$ p.u.	$X_l = 0.13$ p.u.
$X''_d = 0.135$ p.u.	$X''_q = 0.2$ p.u.	$f = 60$ Hz
$T'_{do} = 4.3$ s	$T'_{qo} = 0.85$ s	$i_f(0) = 1$ p.u.
$T''_{do} = 0.032$ s	$T''_{qo} = 0.05$ s	

The original purpose of this case was to simulate the interaction between the mechanical torque placed on the generator turbines and the electrical torque related to the power network, and the resulting shaft torsional oscillations. Accordingly, a detailed multi-mass model of the mechanical shaft is adopted in [60], [61], where the generator shaft system is modeled by 6 masses including 4 turbine sections HP, IP, LPA and LPB, 1 generator and 1 exciter. This thesis focuses on the feasibility of applying the SFA method, and thus uses a simpler single-mass representation of the mechanical part. The total inertia constant of the turbine and generator is 2.894 seconds. The SFA synchronous machine model in Chapter 3 is used in this test case. It should be noted that the SFA synchronous machine model of Chapter 3 can be extended to model the multi-mass shaft system by adding differential equations for individual spring-masses to the differential equations of the shaft system. The self and mutual damping effect can also be easily included in the shaft system equations.

The detailed network parameters can be found in [60] and [61].

Figure 5.19 The First Benchmark Network for Subsynchronous Resonance Studies

In this test, a three-phase fault occurs at bus B at $t = 3 \text{ s}$. After $4\frac{1}{2}$ cycles, the fault is cleared. The total simulation time is 3.5 s . A time step of 1 ms is used in this simulation. The whole system is represented using the SFA models proposed in the previous chapters. The generator terminal voltages, the voltages at bus A and bus B, and the voltages across the series capacitors are monitored during the SFA simulation. Figure 5.20 to Figure 5.23 illustrate the dynamic phasor results for the system voltages at different locations as well as the time domain results transformed back from the corresponding dynamic phasors. The dynamic phasor results show that, after the fault is applied and cleared, there are low frequency oscillations happening at the generator terminals, on the transformer sides, and across the series capacitor banks.

Figure 5.20 Generator Terminal Voltage: Phase A ($\Delta t = 1 \text{ ms}$)

Figure 5.21 Transformer High Side (Bus A) Voltage: Phase A ($\Delta t = 1$ ms)

Figure 5.22 Voltage across Series Capacitor: Phase A ($\Delta t = 1$ ms)

These oscillations are an electrical phenomenon because the shaft system is modeled as a single mass. The slow oscillations are caused by resonances with the series capacitor, which are excited by the fault and the switching operations. The resonance mode(s) is determined by the inherent characteristics of the power network. In fact, the modes or natural frequencies can be quantitatively found by either performing a frequency scan (steady-state solutions over a frequency range) or by calculating the eigenvalues of the admittance matrix of the electrical

network. Similar to the network resonance analysis, if the shaft system is represented by the detailed multi-mass model, we can also determine the torsional natural frequencies (eigenvalues) and mode shapes (eigenvectors) by applying the modal analysis to the shaft differential equations. If the complement of the natural frequency of the network is close to one of the torsional frequencies of the shaft system, torsional oscillations will be excited [33]. This is the mechanism of the subsynchronous resonance. The dynamic phasor results obtained from the SFA simulations show that the oscillations with frequencies around 30 Hz have been excited in this benchmark system. On the other hand, the natural frequencies of the 6-mass shaft system were found to be 15.71 Hz, 20.21 Hz, 25.55 Hz, and 32.28 Hz [60]. That means the complement of the natural frequency of the series-compensated network is close to the torsional natural frequencies, which would result in a subsynchronous resonance in this system and would build up torsional oscillations on the shaft. This has been verified by EMTP simulations [61].

In summary, the SFA simulations are able to capture the slow oscillations in the benchmark system and can be used for subsynchronous resonance studies once the detailed shaft model is incorporated into the SFA synchronous machine models.

Figure 5.23 Infinite Bus (Bus B) Voltage: Phase A ($\Delta t = 1$ ms)

D. Second Benchmark System for the Subsynchronous Resonance Studies

Eight years after the first subsynchronous resonance benchmark was published, the IEEE Subsynchronous Resonance Working Group proposed a second benchmark [62]. The second

benchmark is a case consisting of a 600 MVA generator connected through a step-up transformer to two parallel lines, one of which is series compensated. The compensation rate of the line is 55%. The power system at the receiving end is represented by a Thevenin equivalent circuit (infinite bus behind impedance). Figure 5.24 shows the one-line diagram for the second benchmark system. All data in Figure 5.24 are represented in p.u. on a 100 MVA, 500kV base. The mechanical part of the round rotor turbine generator is again represented as a single-mass model with a total inertia of 2.683 seconds in the SFA simulation. In [62], a more detailed multi-mass representation is used with 4 masses. The parameters of the electrical part are based on the machine ratings, which are given below.

$X_d = 1.65$ p.u.	$X_q = 1.59$ p.u.	$R_a = 0.0045$ p.u.
$X'_d = 0.25$ p.u.	$X'_q = 0.46$ p.u.	$X_l = 0.14$ p.u.
$X''_d = 0.20$ p.u.	$X''_q = 0.20$ p.u.	$f = 60$ Hz
$T'_{do} = 4.5$ s	$T'_{qo} = 0.55$ s	$i_f(0) = 1$ p.u.
$T''_{do} = 0.04$ s	$T''_{qo} = 0.09$ s	number of poles = 2;

The detailed benchmark network parameters can be found in [62] and [61].

In this test, a three-phase fault is applied at the high voltage side of the transformer connecting to bus 2 at $t = 3$ s. The fault clearing time is 1 cycle, and the total simulation time is 3.5 s. The time step used in this simulation is 1 ms. Figure 5.25 to Figure 5.28 show the dynamic phasor results for the system voltages at different locations together with the time domain results transformed back from the corresponding dynamic phasors. Obviously, oscillations slower than the fundamental frequency are excited in the system after the fault occurs, which can be seen from the dynamic phasors of the generator terminal voltage, the transformer high side voltage and the voltage across the series capacitor. Note that bus 1 does not see large deviations in the voltage because it is electrically close to the infinite external system, and also because the series capacitor is acting as a highpass filter that blocks the slow oscillations in the system. The test results on the second benchmark system further verify that the SFA method can capture the so called ‘parallel resonance’ phenomenon [62] in the meshed power system.

Note that relatively small time-step is used for the simulation of the subsynchronous resonance because large frequency deviation occurs after the fault is applied. The reason why a small time step has to be used is that large frequency deviation defines a wide bandwidth in the shifted frequency domain, which in turn requires smaller time steps to respect the Nyquist

frequency limit in the shifted frequency domain. This indicates that a variable time step scheme is a future research direction to realize full potential of the SFA method. The time step can be reduced when system states are changing rapidly in order to achieve better accuracy in the SFA simulation. On the other hand, when system dynamics slowdown, large time steps can be used to avoid unnecessarily long computational time while still achieving reasonable accuracy.

Figure 5.24 The Second Benchmark Network for Subsynchronous Resonance Studies

Figure 5.25 Generator Terminal Voltages: Phase A ($\Delta t = 1$ ms)

Figure 5.26 Voltage across Series Capacitor: Phase A ($\Delta t = 1$ ms)

Figure 5.27 Transformer High Side (Bus 2) Voltage: Phase A ($\Delta t = 1$ ms)

Figure 5.28 Infinite Bus (Bus 1) Voltage: Phase A ($\Delta t = 1$ ms)

The test results from all the above cases indicate that the dynamic phasor is a generalization of the phasor concept, which can represent the dynamic waveform in power systems, without loss of important information. The SFA method with its implementation in the EMTP environment can integrate the differential equations of the power system in the SFA domain and can produce dynamic phasors for electrical variables, which are visually clear and

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easy to follow for power engineers. With the SFA method, power engineers may gain better insight into the EMTP simulation results, and we would expect broader applications of the EMTP in power system steady state and dynamics studies, beyond the fast transient simulations.

Chapter 6

Conclusions

6.1 Summary of Contributions

The goal of this thesis is to extend EMTP functionality for power system dynamic simulation, especially for simulating dynamics with frequency spectra close to the fundamental power frequency. This has been accomplished by developing the Shifted Frequency Analysis (SFA) method, modeling system components with SFA, and accelerating the EMTP simulations for dynamics around 60Hz. A series of contributions made in this thesis are the following.

- I. The theory of Shifted Frequency Analysis is proposed with the help of Hilbert transform and analytic signal concept. Numerical accuracy analysis is performed for the discrete-time SFA simulation.
- II. Linear circuit components, transformer, exponential load and steady-state induction motor are modeled in the shifted frequency domain.
- III. An efficient SFA-based synchronous machine model for the simulation of slow system dynamics is developed. This model is a general-purpose model that can be used for evaluating the dynamic performance of both the salient-pole and the cylindrical-rotor machines.
- IV. The SFA method is extended to model the induction machines. This thesis proposes a new phase-domain induction machine model based on the equivalent-reduction (ER) approach. The ER model has a concise discrete-time equivalent circuit that can be directly incorporated into EMTP-type simulators. Based on the ER model, a SFA induction machine model is proposed as a general purpose model capable of simulating both fast transients and slow dynamics.
- V. An EMTP simulation tool based on the SFA is developed. Simulation results

validate that the SFA method is capable to efficiently simulate power system fundamental frequency dynamics. This is the first practical accomplishment to build a unified power system analysis tool based on the EMTP solution.

6.2 Future Research

The accomplishment achieved in this thesis will inspire researchers to apply SFA and associated techniques to power system analysis and other potential areas. The success of the applications of SFA may be achieved in different aspects such as:

- I. Apply the SFA to the modeling and analysis of renewable energy resources, dispersed generation plants, and industrial power systems. In the foreseeable future, more and more independent-power-producer-owned (IPP-owned) generation units will be connected into the power transmission and distribution system due to a deregulated electricity market. Many of these plants will be cleaner, and many of them will be renewable energy sources such as biomass, solar, wind, geothermal, small hydro, ocean energy, and so on. The IPP interconnection impact studies, which identify the system constraints brought about by the integration of IPPs, and determine network upgrades and remedial action schemes, are therefore critical for the reliable and safe operation of the IPPs and associated power systems. By applying the SFA technique in the IPP modeling, it may lead to accurate and efficient solutions in the IPP impact study, which is hardly achievable with the current phasor tools.
- II. Develop new component models and controller models, and improve the SFA-based EMTP solver. In addition, the future SFA simulator will adopt more object-oriented design such that each module for new components can simply 'plug into' the simulation engine without modifying the core codes. The robustness and efficiency of the core codes will also be improved by using up-to-date sparsity techniques.
- III. The equivalent-reduction idea can be used for building more efficient models for various types of machines, transformers and power system equipments, as long as they can be mathematically represented by differential equations with mutually coupled state variables. These new models can then be further explored in the shifted frequency domain.

- IV. Transform the SFA-based simulation tool into parallel programs. The UBC Object Virtual Network Integrator (OVNI) [63] is a real time parallel simulator, which uses PC clusters [64] as hardware and Muti-Area Thevenin Equivalent (MATE) algorithm as solution engine [65]-[69]. MATE partitions the power system into subsystems and solves them in parallel. By changing the real-valued component models to the dynamic phasor models, a new OVNI simulator based on SFA can be implemented. In the future, one may expect this new simulator to serve as a distributed simulation tool for the supervision and control of self-healing power infrastructures.
- V. Further investigate the basic theory of the SFA method. This will still be an interesting field in future research. There are two limitations in the current SFA method. First, SFA simulation is computationally more expensive than EMTP simulation for the same integration step Δt . This is due to the higher computational cost of operating with complex numbers in SFA as compared to real numbers in EMTP and also to the computational cost involved in transferring between shifted frequency domain and time domain. Second, the aliasing effect may occur when simulating very fast transients in a system [24]. This may introduce error or distortion in SFA simulation result. Some new theories such as discrete-time analytic signal, Hilbert-Huang transform, and new antialiasing techniques are likely to bring theoretical breakthroughs and may lead to the next generation SFA method, which may be more flexible and accurate than the current SFA.
- VI. It may be worthwhile to look into transformations to d,q,0-quantities as an alternative to dynamic phasors [70], particularly for cases where the three-phase impedances are balanced, and where the faults are symmetrical three-phase faults.
- VII. Last but not least, the applications of the SFA method is not limited to the EMTP solution. The Shifted Frequency Analysis can also be introduced into any other circuit simulators, for instances, some state-space-based circuit and/or electric machinery solvers, SPICE, and so on. It is equally suitable to be implemented in the variable time step simulators such as Simulink®. It can also be used in hybrid simulations, and hardware-in-loop simulators.

All in all, the SFA is believed to have opened new doors of opportunity for research relating to dynamic simulations, and further theoretical work and applications are justified.

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Appendix A

Machine Parameters

A.1 Synchronous Machine Parameters

TABLE A-I
SYNCHRONOUS MACHINE PARAMETERS

Parameters	Hydro Turbine Generator	Steam Turbine Generator
r_s	0.00234 Ω	0.00243 Ω
X_{ls}	0.1478 Ω	0.1538 Ω
X_q	0.5911 Ω	1.457 Ω
X_d	1.0467 Ω	1.457 Ω
r_{fd}	0.0005 Ω	0.00075 Ω
X_{lfd}	0.2523 Ω	0.1145 Ω
r_{kq1}	-	0.00144 Ω
X_{lkq1}	-	0.6578 Ω
r_{kq2}	0.01675 Ω	0.00681 Ω
X_{lkq2}	0.1267 Ω	0.07602 Ω
r_{kd}	0.01736 Ω	0.0108 Ω
X_{lkd}	0.1970 Ω	0.06577 Ω
J	$3.51 \times 10^7 \text{ J} \cdot \text{s}^2$	$6.58 \times 10^4 \text{ J} \cdot \text{s}^2$
Rating	325 MVA	835 MVA
Line-to-line voltage	20 KV	26 KV
Poles	64	2

A.2 Induction Machine Parameters

TABLE A-II
INDUCTION MACHINE PARAMETERS

Parameters	3-hp Induction Machine	2250-hp Induction Machine
r_s	0.435 Ω	0.029 Ω
X_{ls}	0.754 Ω	0.226 Ω
X_M	26.13 Ω	13.04 Ω
X_{lr}	0.754 Ω	0.226 Ω
r_r	0.816 Ω	0.022 Ω
J	0.089 kg $\cdot$ m ²	63.87 kg $\cdot$ m ²
T_B	11.9 N $\cdot$ m	8.9 $\times 10^3$ N $\cdot$ m
Line-to-line voltage	220 V	2.3 kV
Poles	4	4

Appendix B

Voltage behind Reactance Induction Machine Model

This appendix faithfully reproduces reference [53], a project report for UBC course EECE 549. The only difference here is that all equation numbers and figure numbers are added with a 'B.'. This report implemented the Voltage behind Reactance (VBR) model for the symmetrical induction machine simulations. The discrete-time VBR model has the similar form as the EMTP-type equivalent circuits. The simulation results validate the accuracy and efficiency of this new induction machine model.

B. 1 Voltage Behind Reactance Model of Induction Machine

The equations of the induction machine can be expressed in the arbitrary reference frame as

$$v_{qs} = r_s i_{qs} + \omega \lambda_{ds} + p \lambda_{qs} \quad (\text{B.1})$$

$$v_{ds} = r_s i_{ds} - \omega \lambda_{qs} + p \lambda_{ds} \quad (\text{B.2})$$

$$v_{0s} = r_s i_{0s} + p \lambda_{0s} \quad (\text{B.3})$$

$$0 = r_r i_{qr} + (\omega - \omega_r) \lambda_{dr} + p \lambda_{qr} \quad (\text{B.4})$$

$$0 = r_r i_{dr} - (\omega - \omega_r) \lambda_{qr} + p \lambda_{dr} \quad (\text{B.5})$$

$$0 = r_r i_{0r} + p \lambda_{0r} \quad (\text{B.6})$$

where each variable and parameter have been converted to stator side, and

$\omega = 0$ for stationary reference frame, $\omega = \omega_r$ for rotor reference frame and $\omega = \omega_e$ for

Appendix B Voltage behind Reactance Induction Machine Model

synchronous reference frame.

Flux linkages equations are expressed as

$$\lambda_{qs} = L_{ls} i_{qs} + \lambda_{mq} = L_{ls} i_{qs} + L_M (i_{qs} + i_{qr}) \quad (\text{B.7})$$

$$\lambda_{ds} = L_{ls} i_{ds} + \lambda_{md} = L_{ls} i_{ds} + L_M (i_{ds} + i_{dr}) \quad (\text{B.8})$$

$$\lambda_{0s} = L_{ls} i_{0s} \quad (\text{B.9})$$

$$\lambda_{qr} = L_{lr} i_{qr} + \lambda_{mq} \quad (\text{B.10})$$

$$\lambda_{dr} = L_{lr} i_{dr} + \lambda_{md} \quad (\text{B.11})$$

$$\lambda_{0r} = L_{lr} i_{0r} \quad (\text{B.12})$$

Rearrange and manipulate (B.7) - (B.12), we can obtain

$$\lambda_{mq} = L_M'' \left(i_{qs} + \frac{\lambda_{qr}}{L_{lr}} \right) \quad (\text{B.13})$$

$$\lambda_{md} = L_M'' \left(i_{ds} + \frac{\lambda_{dr}}{L_{lr}} \right) \quad (\text{B.14})$$

where

$$L_M'' = \left(\frac{1}{L_M} + \frac{1}{L_{lr}} \right)^{-1}$$

Substitute (B.13), (B.14) into (B.7), (B.8), the q -axis and d -axis flux linkages can be rewritten as

$$\lambda_{qs} = L'' i_{qs} + \lambda_q'' \quad (\text{B.15})$$

$$\lambda_{ds} = L'' i_{ds} + \lambda_d'' \quad (\text{B.16})$$

where

$$L'' = L_{ls} + L_M''$$

$$\lambda_q'' = L_M'' \frac{\lambda_{qr}}{L_{lr}} \quad (\text{B.17})$$

$$\lambda_d'' = L_M'' \frac{\lambda_{dr}}{L_{lr}} \quad (\text{B.18})$$

Therefore (B.1) and (B.2) can be written as

$$v_{qs} = r_s i_{qs} + \omega (L'' i_{ds} + \lambda_d'') + p (L'' i_{qs} + \lambda_q'') \quad (\text{B.19})$$

Appendix B Voltage behind Reactance Induction Machine Model

$$v_{ds} = r_s i_{ds} - \omega(L'' i_{qs} + \lambda_q'') + p(L'' i_{ds} + \lambda_d'') \quad (\text{B.20})$$

By manipulating (B.1)-(B.18), we can obtain

$$p\lambda_q'' = \frac{L_M''}{L_r} \left[\frac{r_r}{L_r} (\lambda_q'' + L_M'' i_{qs} - \lambda_{qr}) - (\omega - \omega_r) \lambda_{dr} \right] \quad (\text{B.21})$$

$$p\lambda_d'' = \frac{L_M''}{L_r} \left[\frac{r_r}{L_r} (\lambda_d'' + L_M'' i_{ds} - \lambda_{dr}) + (\omega - \omega_r) \lambda_{qr} \right] \quad (\text{B.22})$$

Substitute (B.21), (B.22) into (B.19), (B.20), the voltage behind reactance form of the voltage equations can be expressed

$$v_{qs} = r_s i_{qs} + \omega L'' i_{ds} + p(L'' i_{qs}) + v_q'' \quad (\text{B.23})$$

$$v_{ds} = r_s i_{ds} - \omega L'' i_{qs} + p(L'' i_{ds}) + v_d'' \quad (\text{B.24})$$

where

$$v_q'' = \omega \lambda_d'' + \frac{L_M'' r_r}{L_r^2} (\lambda_q'' - \lambda_{qr}) + \frac{L_M''^2 r_r}{L_r^2} i_{qs} - \frac{L_M''}{L_r} (\omega - \omega_r) \lambda_{dr} \quad (\text{B.25})$$

$$v_d'' = -\omega \lambda_q'' + \frac{L_M'' r_r}{L_r^2} (\lambda_d'' - \lambda_{dr}) + \frac{L_M''^2 r_r}{L_r^2} i_{ds} + \frac{L_M''}{L_r} (\omega - \omega_r) \lambda_{qr} \quad (\text{B.26})$$

Applying the inverse transformation to (B.23), (B.24), we can get the phase domain equations

$$\mathbf{v}_{abcs}(t) = \mathbf{r}_s \mathbf{i}_{abcs}(t) + p[\mathbf{L}_{abcs}'' \mathbf{i}_{abcs}(t)] + \mathbf{v}_{abcs}''(t) \quad (\text{B.27})$$

where

$$\mathbf{v}_{abcs}''(t) = [\mathbf{K}_s(\theta)]^{-1} \begin{bmatrix} v_q'' \\ v_d'' \\ 0 \end{bmatrix}$$

$$\mathbf{L}_{abcs}'' = \begin{bmatrix} L_{ls} + \frac{2}{3} L_M'' & -\frac{L_M''}{3} & -\frac{L_M''}{3} \\ -\frac{L_M''}{3} & L_{ls} + \frac{2}{3} L_M'' & -\frac{L_M''}{3} \\ -\frac{L_M''}{3} & -\frac{L_M''}{3} & L_{ls} + \frac{2}{3} L_M'' \end{bmatrix}.$$

B. 2 Discrete Time VBR Model

Based on the trapezoidal rule, equation (B.27) can be discretized and rearranged as

$$\mathbf{v}_{abc}(t) = \left(\mathbf{r}_s + \frac{2}{\Delta t} \mathbf{L}_{abc}'' \right) \mathbf{i}_{abc}(t) + \mathbf{v}_{abc}''(t) + \mathbf{e}_h(t) \quad (\text{B.28})$$

where

$$\mathbf{e}_h(t) = \left(\mathbf{r}_s - \frac{2}{\Delta t} \mathbf{L}_{abc}'' \right) \mathbf{i}_{abc}(t - \Delta t) + \mathbf{v}_{abc}''(t - \Delta t) - \mathbf{v}_{abc}(t - \Delta t) \quad (\text{B.29})$$

Rewrite (B.25) and (B.26), we can get the matrix form of the subtransient voltages

$$\begin{bmatrix} \mathbf{v}_q''(t) \\ \mathbf{v}_d''(t) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \lambda_{qr}(t) \\ \lambda_{dr}(t) \end{bmatrix} + \begin{bmatrix} a_{13} & 0 \\ 0 & a_{23} \end{bmatrix} \begin{bmatrix} i_{qs}(t) \\ i_{ds}(t) \end{bmatrix} \quad (\text{B.30})$$

where

$$\begin{aligned} a_{11} &= \frac{L_M'' r_r}{L_r^2} \left(\frac{L_M''}{L_r} - 1 \right) & a_{12} &= \omega_r(t) \frac{L_M''}{L_r} \\ a_{21} &= -a_{12} & a_{22} &= a_{11} \\ a_{13} &= \frac{L_M''^2 r_r}{L_r^2} & a_{23} &= a_{13} \end{aligned}$$

By rearranging and manipulating (B.4), (B.5), (B.10) and (B.11), the rotor flux linkage equations can be obtained

$$p \begin{bmatrix} \lambda_{qr}(t) \\ \lambda_{dr}(t) \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \lambda_{qr}(t) \\ \lambda_{dr}(t) \end{bmatrix} + \begin{bmatrix} b_{13} & 0 \\ 0 & b_{23} \end{bmatrix} \begin{bmatrix} i_{qs}(t) \\ i_{ds}(t) \end{bmatrix} \quad (\text{B.31})$$

where

$$\begin{aligned} b_{11} &= \frac{r_r}{L_r} \left(\frac{L_M''}{L_r} - 1 \right) & b_{12} &= -[\omega(t) - \omega_r(t)] & b_{13} &= \frac{r_r}{L_r} L_M'' \\ b_{21} &= -b_{12} & b_{22} &= b_{11} & b_{23} &= b_{13} \end{aligned}$$

Discretizing (B.31) by using the Trapezoidal rule, then

$$\begin{aligned} \begin{bmatrix} \lambda_{qr}(t) \\ \lambda_{dr}(t) \end{bmatrix} &= \begin{bmatrix} 2 - b_{11}\Delta t & -b_{12}(t)\Delta t \\ -b_{21}(t)\Delta t & 2 - b_{22}\Delta t \end{bmatrix}^{-1} \begin{bmatrix} 2 + b_{11}\Delta t & b_{12}(t - \Delta t)\Delta t \\ b_{21}(t - \Delta t)\Delta t & 2 + b_{22}\Delta t \end{bmatrix} \begin{bmatrix} \lambda_{qr}(t - \Delta t) \\ \lambda_{dr}(t - \Delta t) \end{bmatrix} \\ &+ \begin{bmatrix} 2 - b_{11}\Delta t & -b_{12}(t)\Delta t \\ -b_{21}(t)\Delta t & 2 - b_{22}\Delta t \end{bmatrix}^{-1} \begin{bmatrix} b_{13}\Delta t & 0 \\ 0 & b_{23}\Delta t \end{bmatrix} \left\{ \begin{bmatrix} i_{qs}(t) \\ i_{ds}(t) \end{bmatrix} + \begin{bmatrix} i_{qs}(t - \Delta t) \\ i_{ds}(t - \Delta t) \end{bmatrix} \right\} \end{aligned} \quad (\text{B.32})$$

Appendix B Voltage behind Reactance Induction Machine Model

Substitute (B.32) into (B.30), we can get the discrete time equations

$$\begin{bmatrix} v_q''(t) \\ v_d''(t) \end{bmatrix} = \mathbf{K}_1(t) \begin{bmatrix} i_{qs}(t) \\ i_{ds}(t) \end{bmatrix} + \mathbf{K}_2(t) \quad (\text{B.33})$$

where

$$\begin{aligned} \mathbf{K}_1(t) &= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} 2 - b_{11}\Delta t & -b_{12}(t)\Delta t \\ -b_{21}(t)\Delta t & 2 - b_{22}\Delta t \end{bmatrix}^{-1} \begin{bmatrix} b_{13}\Delta t & 0 \\ 0 & b_{23}\Delta t \end{bmatrix} + \begin{bmatrix} a_{13} & 0 \\ 0 & a_{23} \end{bmatrix} \\ \mathbf{K}_2(t) &= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} 2 - b_{11}\Delta t & -b_{12}(t)\Delta t \\ -b_{21}(t)\Delta t & 2 - b_{22}\Delta t \end{bmatrix} \begin{bmatrix} 2 + b_{11}\Delta t & b_{12}(t - \Delta t)\Delta t \\ b_{21}(t - \Delta t)\Delta t & 2 + b_{22}\Delta t \end{bmatrix} \begin{bmatrix} \lambda_{qr}(t - \Delta t) \\ \lambda_{dr}(t - \Delta t) \end{bmatrix} \\ &\quad + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} 2 - b_{11}\Delta t & -b_{12}(t)\Delta t \\ -b_{21}(t)\Delta t & 2 - b_{22}\Delta t \end{bmatrix}^{-1} \begin{bmatrix} b_{13}\Delta t & 0 \\ 0 & b_{23}\Delta t \end{bmatrix} \begin{bmatrix} i_{qs}(t - \Delta t) \\ i_{ds}(t - \Delta t) \end{bmatrix} \end{aligned}$$

Applying the inverse transformation to (B.33), we can transfer the $qd0$ variables to the phase domain

$$\mathbf{v}_{abs}''(t) = \mathbf{K}(t) \mathbf{i}_{abs}(t) + \mathbf{e}_{hs}(t) \quad (\text{B.34})$$

where

$$\begin{aligned} \mathbf{K}(t) &= [\mathbf{K}_s(\theta(t))]^{-1} \begin{bmatrix} \mathbf{K}_1(t) & 0 \\ 0 & 0 \end{bmatrix} [\mathbf{K}_s(\theta(t))] \\ \mathbf{e}_{hs}(t) &= [\mathbf{K}_s(\theta(t))]^{-1} \begin{bmatrix} \mathbf{K}_2(t) \\ 0 \end{bmatrix} \end{aligned}$$

Finally, by substituting (B.34) into (B.28), the equivalent circuit for the stator voltage equations can be written as

$$\mathbf{i}_{abs}(t) = \mathbf{G}_{eq} \mathbf{v}_{abs}(t) - \mathbf{h}(t) \quad (\text{B.35})$$

where

$$\begin{aligned} \mathbf{G}_{eq} &= \left[\mathbf{r}_s + \frac{2}{\Delta t} \mathbf{L}_{abs}'' + \mathbf{K}(t) \right]^{-1} \\ \mathbf{h}(t) &= \mathbf{G}_{eq} [\mathbf{e}_h(t) + \mathbf{e}_{hs}(t)] \end{aligned}$$

B. 3 Free Acceleration Simulation of a 3-hp Induction Machine

The first test case is to simulate the start-up transients of a 3-hp induction machine. The parameters of the machine are

$$P=4 \quad V_{LL}=220V \quad r_s = 0.435\Omega \quad X_{ls} = 0.754 \Omega \quad X_M = 26.13 \Omega \quad X_{lr} = 0.754\Omega \\ r_r = 0.816\Omega \quad J = 0.089 \text{ kg.m}^2.$$

The simulation results viewed from different reference frames are shown in Fig. B.1- B.3.

(1) Figure B.1: Stationary Reference Frame

Because the initially position of the stationary $qd0$ reference frame is zero, the $f_{as} = f_{qs}$. Therefore the i_{qs} and v_{qs} are the same as i_{as} and v_{as} , respectively. The rotor variables are varying at 60 HZ after referred to the stationary reference frame.

(2) Figure B.2: Rotor Reference Frame

At the very beginning, when the speed of the machine is slow, the variables in the rotor reference frame are similar to those in the stationary reference frame. Then the stator variables will vary at slip frequency when the rotor speed is increasing. After the speed reach the synchronous speed, all variables in rotor reference frames becomes constant.

(3) Figure B.3: Synchronous Reference Frame

At stall the variables in the synchronous reference are varying at 60 HZ. The stator and rotor variables will become constant when the rotor speed reaches synchronous speed. Note that the zero position of synchronous reference frame that we chose is zero, therefore the magnitude of v_{qs} is identical to this of v_{as} and v_{ds} is zero.

B. 4 Dynamic Performance of Induction Machine during Mechanical Torque Changes

In this case, the machine originally operates in the steady state. At $t = 0.05s$, a constant mechanical torque of 12 N.m^2 is applied. Then the mechanical torque is reversed to -12 N.m^2 at $t = 0.5s$. The simulation time is $1s$.

Figure B.4 illustrates the simulation results with the VBR model.

Appendix B Voltage behind Reactance Induction Machine Model

Figure B.1 Free Acceleration Characteristics in Stationary Reference Frame

Appendix B Voltage behind Reactance Induction Machine Model

Figure B.2 Free Acceleration Characteristics in Rotor Reference Frame

Appendix B Voltage behind Reactance Induction Machine Model

Figure B.3 Free Acceleration Characteristics in Synchronous Reference Frame

Appendix B Voltage behind Reactance Induction Machine Model

Figure B.4 Dynamic Performance of a 3-hp Induction Machine during Step Changes in Load Torque

Appendix C

Test Case Data

```

%*****
% Test Case 1
% An Electromagnetic Transient Program with Dynamic Phasor Solution
%
% =====
%      =      Version 1.0      =
%      =      Last revised: 24 Feb.2007      =
%      =      Peng Zhang      =
%      =      Copyright 2008 © Peng Zhang      =
%      =====

nBus = 2;
% PI Line data
nLine=1; % Three phase line
LineFromBus(1)=1;
LineToBus(1)=2;

%      |      R1 ohm      |      X1 ohm      |      C1 muF      | R2 | X2 | C2 |....
LineParameters =[0.0868455 0.0298305 0.0288883 0.2025449 0.0847210 0.0719161 0.00274 -0.0007 -0.00034 ;
                0.0298305 0.0887966 0.0298305 0.0847210 0.1961452 0.0847210 -0.0007 0.00296 -0.00071 ;
                0.0288883 0.0298305 0.0868455 0.0719161 0.0847210 0.2025449 -0.00034 -0.00071 0.00274 ]*10.56

% Load data
nLoad=1;% Three phase load
LoadBus(1)=1;
rLoad(1,1)=(12.47*1000/sqrt(3))^2/(2000*1000); % phase A resistance
xLoad(1,1)=(12.47*1000/sqrt(3))^2/(0.32868410517886306346562595373367*2000*1000); % phase A reactance
rLoad(1,2)=(12.47*1000/sqrt(3))^2/(2000*1000); % phase B resistance
xLoad(1,2)=(12.47*1000/sqrt(3))^2/(0.32868410517886306346562595373367*2000*1000); % phase B reactance
rLoad(1,3)=(12.47*1000/sqrt(3))^2/(2000*1000); % phase C resistance
xLoad(1,3)=(12.47*1000/sqrt(3))^2/(0.32868410517886306346562595373367*2000*1000); % phase C reactance

% Voltage source data
nVSource=1;
VSourceBus(1)=2;
Vabcs(1,1) = 10182 ; % Vamp
Vabcs(1,2) = 10182 * exp(-j*2 * pi / 3);

```

Appendix C Test Case Data

```
Vabcs(1,3) = 10182 * exp( jay * 2 * pi / 3);
```

```
T10= 3;
```

```
LineTripped = 1;
```

```
return;
```

```
%*****
```

```
% Test Case 2
```

```
% An Electromagnetic Transient Program with Dynamic Phasor Solution
```

```
%  
%=====
```

%	=	Version 1.0	=
%	=	Last revised: 24 Feb.2007	=
%	=	Peng Zhang	=
%	=	Copyright 2008 © Peng Zhang	=

```
%=====
```

```
simT = 8;      % total simulation time
```

```
jay=sqrt(-1);
```

```
ws = 2 * pi * 60;
```

```
nBus = 12;
```

```
% PI Line data
```

```
nLine=11; % Three phase line
```

```
LineFromBus(1)=12;
```

```
LineToBus(1)=1;
```

```
LineFromBus(2)=1;
```

```
LineToBus(2)=4;
```

```
LineFromBus(3)=4;
```

```
LineToBus(3)=5;
```

```
LineFromBus(4)=1;
```

```
LineToBus(4)=2;
```

```
LineFromBus(5)=2;
```

```
LineToBus(5)=3;
```

```
LineFromBus(6)=1;
```

```
LineToBus(6)=6;
```

```
LineFromBus(7)=6;
```

```
LineToBus(7)=8;
```

```
LineFromBus(8)=8;
```

```
LineToBus(8)=9;
```

```
LineFromBus(9)=8;
```

```
LineToBus(9)=10;
```

```
LineFromBus(10)=6;
```

```
LineToBus(10)=11;
```

```
LineFromBus(11)=6;
```

Appendix C Test Case Data

LineToBus(11)=7;

%	R1			X1			C1 muF			R2 X2 C2 ...		
LineParameters(1:3,1:9)=[	0.3465	0.0	0.0	1.0179	0.5017	0.4236	6.2998/(2*pi*60)	-1.2958/(2*pi*60)	-1.2595/(2*pi*60)			
	0.0	0.3375	0.0	0.5017	1.0478	0.3849	-1.2958/(2*pi*60)	5.9597/(2*pi*60)	-1.2595/(2*pi*60)			
	0.0	0.0	0.3414	0.4236	0.3849	1.0348	-1.2595/(2*pi*60)	-1.2595/(2*pi*60)	5.6386/(2*pi*60)	]		
LineParameters(1:3,10:18)=[	1.3238	0.0	0.0	1.3569	0.4591	0.4591	4.6658/(2*pi*60)	-0.8999/(2*pi*60)	-0.8999/(2*pi*60)			
	0.0	1.3294	0.0	0.4591	1.3471	0.4591	-0.8999/(2*pi*60)	4.7097/(2*pi*60)	-0.8999/(2*pi*60)			
	0.0	0.0	1.3238	0.4591	0.4591	1.3569	-0.8999/(2*pi*60)	-0.8999/(2*pi*60)	4.6658/(2*pi*60)	]		
LineParameters(1:3,19:27)=[	1.3238	0.0	0.0	1.3569	0.4591	0.4591	4.6658/(2*pi*60)	-0.8999/(2*pi*60)	-0.8999/(2*pi*60)			
	0.0	1.3294	0.0	0.4591	1.3471	0.4591	-0.8999/(2*pi*60)	4.7097/(2*pi*60)	-0.8999/(2*pi*60)			
	0.0	0.0	1.3238	0.4591	0.4591	1.3569	-0.8999/(2*pi*60)	-0.8999/(2*pi*60)	4.6658/(2*pi*60)	]		
LineParameters(1:3,28:36)=[	0.7526	0.0	0.0	1.1814	0.4236	0.5017	5.6990/(2*pi*60)	-1.0817/(2*pi*60)	-1.6905/(2*pi*60)			
	0.0	0.7475	0.0	0.4236	1.1983	0.3849	-1.0817/(2*pi*60)	5.1795/(2*pi*60)	-0.6588/(2*pi*60)			
	0.0	0.0	0.7436	0.5017	0.3849	1.2112	-1.6905/(2*pi*60)	-0.6588/(2*pi*60)	5.4246/(2*pi*60)	]		
LineParameters(1:3,37:45)=[	0.3807	0.0000	0.0000	0.6922	0.0000	0.0000	0.0001	0.0000	0.0000			
	0.0000	0.3807	0.0000	0.0000	0.6922	0.0000	0.0000	0.0001	0.0000			
	0.0000	0.0000	0.3807	0.0000	0.0000	0.6922	0.0000	0.0000	0.0001	]		
LineParameters(1:3,46:54)=[	0.3465	0.0	0.0	1.0179	0.5017	0.4236	6.2998/(2*pi*60)	-1.2595/(2*pi*60)	-1.2595/(2*pi*60)			
	0.0	0.3375	0.0	0.5017	1.0478	0.3849	-1.2595/(2*pi*60)	5.9597/(2*pi*60)	-1.2595/(2*pi*60)			
	0.0	0.0	0.3414	0.4236	0.3849	1.0348	-1.2595/(2*pi*60)	-1.2595/(2*pi*60)	5.6386/(2*pi*60)	]		
LineParameters(1:3,55:63)=[	1.3238	0.0	0.0	1.3569	0.4591	0.4591	4.6658/(2*pi*60)	-0.8999/(2*pi*60)	-0.8999/(2*pi*60)			
	0.0	1.3238	0.0	0.4591	0.4591	0.4591	-0.8999/(2*pi*60)	4.7097/(2*pi*60)	-0.8999/(2*pi*60)			
	0.0	0.0	1.3294	0.4591	0.4591	1.3471	-0.8999/(2*pi*60)	-0.8999/(2*pi*60)	4.7097/(2*pi*60)	]		
LineParameters(1:3,64:72)=[	1.3292	0.0000	0.0000	1.3475	0.0000	0.0000	4.5193/(2*pi*60)	0.0000	0.0000			
	0.0000	1.3292	0.0000	0.0000	1.3475	0.0000	0.0000	4.5193/(2*pi*60)	0.0000			
	0.0000	0.0000	1.3292	0.0000	0.0000	1.3475	0.0000	0.0000	4.5193/(2*pi*60)	]		
LineParameters(1:3,73:81)=[	1.3425	0.0000	0.0000	0.5124	0.0000	0.0000	88.9912/(2*pi*60)	0.0000	0.0000			
	0.0000	1.3425	0.0000	0.0000	0.5124	0.0000	0.0000	88.9912/(2*pi*60)	0.0000			
	0.0000	0.0000	1.3425	0.0000	0.0000	0.5124	0.0000	0.0000	88.9912/(2*pi*60)	]		
LineParameters(1:3,82:90)=[	0.3465	0.0	0.0	1.0179	0.5017	0.4236	6.2998/(2*pi*60)	-1.9958/(2*pi*60)	-1.2595/(2*pi*60)			
	0.0	0.3375	0.0	0.5017	1.0478	0.3849	-1.9958/(2*pi*60)	5.9597/(2*pi*60)	-0.7417/(2*pi*60)			
	0.0	0.0	0.3414	0.4236	0.3849	1.0348	-1.2595/(2*pi*60)	-0.7417/(2*pi*60)	5.6386/(2*pi*60)	]		
LineParameters(1:3,91:99)=[	0.7982	0.0	0.0	0.4463	0.0328	-0.0143	96.8897/(2*pi*60)	0.0000	0.0000			
	0.0	0.7891	0.0	0.0328	0.4041	0.0328	0.0000	96.8897/(2*pi*60)	0.0000			
	0.0	0.0	0.7982	-0.0143	0.0328	0.4463	0.0000	0.0000	96.8897/(2*pi*60)	]		

% Load data

nLoad=8;% Three phase load

LoadBus(1)=3;

LoadBus(2)=4;

LoadBus(3)=5;

LoadBus(4)=10;

LoadBus(5)=6;

LoadBus(6)=7;

LoadBus(7)=11;

LoadBus(8)=9;

rLoad=zeros(8,3);

Appendix C Test Case Data

```
rLoad(:,:)=1e6;
xLoad=zeros(8,3);
xLoad(:,:)=1e6;

rLoad(5,1)=(4.16*1000/sqrt(3))^2/(385*1000); % phase A resistance
xLoad(5,1)=(4.16*1000/sqrt(3))^2/(220*1000); % phase A reactance
rLoad(5,2)=(4.16*1000/sqrt(3))^2/(385*1000); % phase B resistance
xLoad(5,2)=(4.16*1000/sqrt(3))^2/(220*1000); % phase B reactance
rLoad(5,3)=(4.16*1000/sqrt(3))^2/(385*1000); % phase C resistance
xLoad(5,3)=(4.16*1000/sqrt(3))^2/(220*1000); % phase C reactance

% Induction machine data
IndMacBus=11;
Ttorq=3; % torque applied
%          |   S   | VI | P | J   | rs   | Lls          | LM   | rr   | Llr   |
IndMacParameters = [500/(1.341e-3) 4600 4 11.06 0.262*4 1.206*4/ws 54.02*4/ws 0.187*4 1.206*4/ws];

% Voltage source data
nVSource=1;
VSourceBus(1)=12;
Vabcs(1,1) = (4.16*1000/sqrt(3))*sqrt(2) ; % Vamp
Vabcs(1,2) = (4.16*1000/sqrt(3))*sqrt(2) * exp(-j*2 * pi / 3);
Vabcs(1,3) = (4.16*1000/sqrt(3))*sqrt(2) * exp(j*2 * pi / 3);

return;
```