CHARACTERIZATION OF DISCRETE FRACTURE NETWORKS AND THEIR INFLUENCE ON CAVEABILITY AND FRAGMENTATION

by

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ABSTRACT

This thesis focuses on the use of Discrete Fracture Network (DFN) modeling to simulate rock masses with different characteristics by varying fracture spacing, persistence and dispersion, and assessing block instability without failure due to brittle fracture. The DFN method was used together with block theory to assess block volumes, characterize block shapes, evaluate block failure modes and estimate block size distributions in simulated ore bodies. A model was built to simulate block caving and run tests of specific rock mass parameters to evaluate their impact on caveability and fragmentation.

The potential of the Block Shape Characterization Method (BSCM) for evaluating the block shape distribution within a rock mass was further confirmed, especially when used with the DFN method. The stability of the generated blocks was evaluated based on the factors of safety obtained from the FracMan stability analysis. The information gathered during modeling suggested that of the variables analyzed, fracture persistence has the largest influence on the generation of drawbell blocking block sizes. Qualitative similarities between the apparent block volume and the blockiness character were observed and confirmed previous studies. The results indicate that caveability in this model is most sensitive to changes in fracture spacing.

This research indicates that DFN modeling has great potential for fragmentation evaluation and determination, caveability assessment, and investigating the factors influencing the caving process.
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1.0 INTRODUCTION

1.1 Problem Statement

In recent years, the mining industry has been faced with a fresh array of old and new challenges. These include aging mines (most of which are surface mines), deeper deposits, lower grades and an increase in demand for mineral resources. The block caving mining method has emerged as an answer to many of these problems. It allows mining of massive, low grade deposits at depth, and has the lowest production costs and the highest production rates of any underground mining method used today. It also provides high levels of safety for the personnel and a good platform for automation. However, recent experiences in some block cave operations around the world, such as Northparkes in Australia and Palabora in South Africa, have highlighted the lack of understanding of the geotechnical processes involved in caving. Among the most important factors in block cave mines are fragmentation and caveability. Poor estimation of both of these variables can lead to production and processing problems, or in the worst scenario, failure of the project.

In order to gain more understanding of the caving process, research was undertaken in this thesis to study fragmentation and caveability. The investigation focussed on the use of Discrete Fracture Network (DFN) modeling to simulate rock masses with different characteristics by varying fracture spacing, persistence and dispersion, and assessing block instability without failure due to brittle fracture. DFN modeling is a technique of fracture simulation which allows the generation of three dimensional, synthetic fractures. The method first saw use in the characterization of permeability of fractured rock masses and generic studies on fracture influences. More recently it has been used as a tool in mining for rock mass characterization, either by itself or together with other methods (e.g. synthetic rock
masses). In this thesis, the DFN method was used together with block theory (Goodman and Shi, 1985) to assess block volumes, characterize block shapes, evaluate block failure modes and estimate block size distributions in the simulated ore bodies.

1.2 Research Objectives

The key objectives of this thesis were as follows:

1. Characterize the block shapes generated with DFN models by varying fracture intensity, persistence and dispersion;
2. Evaluate the block failure modes with different fracture spacing, persistence and dispersion;
3. Investigate the effects on block size distribution of fracture intensity, persistence and dispersion with DFN models and block theory;
4. Relate block volumes obtained from the models with the apparent block volume;
5. Study trace block areas (on the undercut) and block volumes for the generated models.

1.3 Thesis Organization

Chapter 1 describes the problem and outlines the research objectives.

Chapter 2 presents a literature review on block caving as a mining method, the methods currently employed for evaluating caveability and fragmentation, as well as describing the Discrete Fracture Network method and Block Theory.

Chapter 3 explains the methodology used to complete this research, describing the variables, model and procedure followed to accomplish the objectives.
Chapter 4 presents the results and analysis of the modeling performed.

Chapter 5 summarizes the conclusions of this research and gives recommendations for future work.
2.0 LITERATURE REVIEW

2.1 Block Caving

2.1.1 Description

Block Caving is an underground mining method normally used to mine massive ore bodies that have a consistent and generally low grade throughout. The method relies on the gradual and controlled collapse of the ore rock under its own weight. Caving is typically initiated by drilling and blasting a zone below the ore body to be mined, called the undercut. The undercut is blasted in sequence, allowing the broken ore to be drawn off with the aim of creating a void into which initial caving of the ore can take place (Figure 2.1). Caving continues as more ore is drawn off, propagating the cave upward until it reaches the surface, generating subsidence (Brown, 2003).

Figure 2.1: Cut away view of a block cave (Duplancic, 2001).
Duplancic and Brady (1999) developed a conceptual caving model based on studies of microseismic monitoring. They described 5 distinctive zones (Figure 2.2):

1. **Caved Zone** – region that comprises the material that has already fallen from the cave back. The broken rock provides support for the cave walls.

2. **Air Gap** – zone that develops between the cave back and the caved zone during continuous extraction. The size of the air gap is a function of the extraction and caving rates.

3. **Zone of Discontinuous Deformation (Zone of Loosening)** – region of the cave back where there are large rock displacements. This is where the disintegration of the rock mass occurs, therefore, this zone does not provide any support for the overlying rock. It is important to mention that no seismicity is recorded from this section.

4. **Seismogenic Zone** – area of the cave with changing stress conditions caused by the advancing undercut and the progression of the cave. Seismic activity occurs due to the brittle failure of the rock and slip on joints.

5. **Surrounding Rock Mass (Pseudo Continuous Domain)** – only elastic deformation occurs in the rock mass ahead of the Seismic Zone and away from the cave walls.
2.1.2 Brief History

The precursor to modern block caving was developed in the late 19th century in the iron ore mines of northern Michigan. During the early 20th century the method was further developed and began to be used in a wide variety of mines with large, weak ore bodies. In the late 1950’s, block caving was introduced into the southern African diamond and chrysotile asbestos mines. During the 1960’s, developments in mechanized equipment, particularly Load-Haul-Dump (LHD) loaders, allowed for the introduction of mechanized and trackless cave mining (Brown, 2003). Today it is a method that is used all around the world (Figure 2.3), having been extended to stronger ore bodies and larger block heights (higher than 200m), such as Palabora and Northparkes respectively.

Figure 2.2: Conceptual caving model developed by Duplancic and Brady (1999).
2.1.3 Types of Caving and Advantages of the Method

The block caving method is generally divided into two categories:

- **Block Caving**: method in which an ore block, usually encompassing the entire ore body, is undercut and caved.

- **Panel Caving**: caving method where the ore body is undercut in stages, and caved in a series of panels.

There are many variations of these two categories which resulted from a need to adapt block caving for local conditions.

Block caving has the lowest operating costs of any underground mining method. This is because of the high rates of extraction achievable with a small workforce, and because no additional primary blasting is required after the undercut is opened. The method is also
considered one of the safest, since most of the operations are conducted in the production level with little personnel (Duplancic, 2001), i.e. there are no workers in stopes and they are away from the caving zone.

2.2 Caveability Assessment

Caveability assessment methods are generally classified into two groups: empirical and numerical.

2.2.1 Empirical Methods

Empirical methods have been developed from large amounts of data (based on previous caving experience) and assumptions about its significance. This leads to the disadvantage, though, that these methods have difficulty combining different variables in a simple robust tool (Brown, 2003). The empirical methods used for block caving prediction include:

a) Laubscher's Caving Chart (Laubscher, 1990) – it is generally recognized as the industry standard for assessing caveability, although it is not used in all caving mines. The chart consists of a graph plotting MRMR (Mining Rock Mass Rating) values against Hydraulic Radius (term derived by dividing the area of the opening by its perimeter of an opening) populated by a database of other values from cave operations around the world. The graph is then divided into 3 zones, based on these (Figure 2.4): stable, transitional and caving. The MRMR is a development of Bieniawski's (1976) RMR (Rock Mass Rating) system that incorporates adjustments for weathering, joint orientation, blast damage and mining induced stresses (Laubscher, 1990). The method is quite successful in predicting caveability in weak
and large ore bodies. However, it has shown limitations when applied to stronger rocks and smaller footprints.

![Graph showing different geometries and hydraulic radius](image)

**Figure 2.4:** Laubscher’s (2000) caving chart incorporating the shape factor for caves with different geometries.
b) *Mathews Stability Graph* (Mathews et al., 1980) – this is very similar to Laubscher’s chart, but predates it by several years. It was developed initially for open stope design. The method consists of a stability number, $N'$, which is calculated based on the multiplication of $Q'$, a rock stress factor A, a joint orientation adjustment factor B and a gravity adjustment factor C (Figure 2.5). $Q'$ is a modification of Barton et al. (1974) NGI Q system with the Stress Reduction Factor (SRF) and the Joint Water Reduction Factor ($J_w$) set equal to one, as they are accounted for separately within the analysis (Factor A).

![Factor A: Rock stress factor](image1)

![Factor B: Joint orientation adjustment](image2)

![Factor C: Design surface orientation factor](image3)

*Figure 2.5: Adjustment factors in the Mathews stability method (Mathews et al., 1980).*
The $N'$ factor is plotted against the hydraulic radius. The graph was originally divided into 3 zones: stable, potentially unstable and potentially caving (Figure 2.6).

![Mathews stability graph](image)

**Figure 2.6:** Mathews stability graph (Mathews et al., 1980; Brown, 2003).

The stability graph has been updated several times, adding more data and redefining the stability zones. It is worth mentioning the extended Mathew’s Stability Graph developed by Mawdesley et al. (2001), where the zones of stability were defined as isoprobability contours; and Capes and Milne (2008) who compiled additional dilution graph data for open stope hangingwall design.

Matthew’s method has its limitations when used for predicting caveability. The data used for the stability graph does not include block cave mines. Mawdesley (2002) developed an extended version of the graph including caving cases and defining a caving line within the
graph (Figure 2.7). However, more caving data needs to be incorporated in order to increase the confidence in this method for caving prediction (Brown, 2003).

Figure 2.7: Extended Mathews stability graph based on logistic regression showing the stable and caving lines (Mawdesley 2002).

2.2.2 Numerical Methods

Numerical methods use mathematical functions and constitutive relationships to model the behaviour of rock, or other materials. They have the advantage of being able to account for a wide range of geotechnical behaviour and for inhomogeneous properties within the problem domain, as well as incorporating different failure criteria. Some of the methods currently in use are:

a) Continuum Models – these models treat the rock mass as a continuum, based on the assumption that the rock is so variable that its behaviour will not be kinematically
controlled by specific discontinuity sets. The rock mass properties are defined as equivalent parameters (i.e. incorporating both intact rock and joints). They assume the material response may be described by the theories of elasticity or plasticity (Brown, 2003), represented as flexural deformation and plastic yield. Some examples of continuous models are: the finite difference codes FLAC (Itasca, 2005) and FLAC3D (Itasca, 2004c); the finite element programs Phase2 (Rocsience, 2004) and Abaqus (Simulia, 2007); and the boundary element code Map3D (Mine Modeling, 2006). The most commonly used continuum model codes for caveability and subsidence assessment are FLAC, FLAC3D and Abaqus. Work using FLAC and FLAC3D has been performed by Singh et al. (1993) for Rajpura Dariba and Kiruna mines, Karzulovic et al. (1999) for El Teniente, and Flores and Karzulovic (2003) for the International Caving Study (conceptual models). Studies with Abaqus have been carried out for BHP's Nickel West and Diamond divisions by Beck et al. (2006a, 2006b). FLAC3D has been used for the pre-feasibility and feasibility caveability assessments at Northparkes in Australia (Lift 2 and E48) and Palabora in South Africa. Abaqus was employed for caveability assessments at Argyle Diamonds in Australia.

b) **Discontinuum Models** – rock is inherently discontinuous, therefore methods that explicitly include the presence of discontinuities present an attractive option for caveability assessment. Discrete element methods (DEM) are more complex and computationally intensive which makes them currently unsuitable for large scale 3D modeling (Brown, 2003). Nevertheless, they provide an aid in understanding the caving process. DEM codes represent rock masses in two ways: as an assembly of
deformable or rigid blocks subject to block movement and/or block deformation (e.g. UDEC and 3DEC, Itasca, 2004a, b); or as an assembly of rigid bonded particles under the influence of bond breakages and particle movements (e.g. PFC and PFC3D, Itasca, 2008, and Rebop, Itasca, 2004d). UDEC and 3DEC have been used for caveability assessment (Palabora), but are more commonly employed for surface subsidence and pillar stability investigations, such as the work performed by Li and Brummer (2005) at Palabora. Caveability studies using DEM codes are generally carried out using PFC or PFC3D, for example, the research performed by Pierce et al. (2007) and Reyes-Montes et al. (2007) back analyzing Northparkes' Lift 2 cave, and Mas Ivars et al. (2008) looking at the strain-strain response curve from synthetic rock mass UCS tests on carbonatite material from Palabora. PFC3D was also used by Gilbride et al. (2005) to assess subsidence at the Questa mine.

c) Hybrid Models – these models incorporate combined continuum-discontinuum techniques providing a platform for the analysis of complex engineering problems. One such code is ELFEN (Rockfield, 2006), which is a program developed originally for dynamic modeling of impact loading on brittle materials, incorporating finite and discrete element analysis techniques (Elmo et al., 2007a). The code has two constitutive fracture models implemented: the Rankine rotating crack model and the Mohr-Coulomb model with a Rankine cut off (Vyazmensky et al., 2007). ELFEN has been applied to block caving by Escl and Dutko (2003) and Pine et al. (2006) showing promising results. More recently, Elmo at al. (2007a) and Vyazmensky et al. (2007) have been using ELFEN to characterize surface subsidence and open-pit/block
cave interaction, and Rance et al. (2007) has been applying it to fragmentation estimations.

Recent developments in numerical modeling have involved new methods of simulating caving and surface subsidence to incorporate the pre-caving natural fracture system. Two examples are the combinations of Synthetic Rock Mass (SRM)-DEM (Mas Ivars et al., 2008), Finite Difference/SRM (Cundall et al., 2008; Sainsbury et al., 2008) and FEM/DEM-Discrete Fracture Network (DFN) simulations.

The SRM-DEM method consists of simulating a rock mass as an assembly of bonded spheres with an embedded discrete network of disc-shaped joints. The three main properties necessary for the construction of a SRM model are the intact rock properties, a discrete fracture network and the joint properties (Pierce et al., 2007). All samples are generated in PFC3D, where the analysis is conducted simulating the stress conditions typical of deep block caved ore bodies (Reyes-Montes et al., 2007). The back analysis of Northparkes’ Lift 2 mine using microseismicity records (Reyes-Montes et al., 2007) and rock mass behaviour simulation of the same mine (Pierce et al., 2007) have been able to accurately represent what was observed during the mine’s operation.

The FEM/DEM-DFN method also uses a synthetic rock mass, but this is developed from DFN models in conjunction with FEM/DEM simulations to derive the rock mass properties. This provides a link between mapped fracture systems and rock mass strength as opposed to using empirical rock mass classifications alone (Elmo et al., 2008a). The synthetic rock mass is used for modeling the effects of caving on the surface or in surface-underground interactions. Elmo et al. (2008a) have reported that this method has realistically captured the effects of block cave mining on surface subsidence. Further work is underway to incorporate
other variables, including in-situ rock stress, different surface geometries and undercut drawing sequences.

Numerical models show great promise for the future as ongoing advances in computing power and refinement of algorithms and models will allow more realistic simulation of caving processes and caveability assessment. Nevertheless, a great deal of work needs to be done to develop realistic geotechnical models.

2.3 Fragmentation

Fragmentation has an important impact on the overall success and profitability of block cave operations. There are many design and operating parameters influenced by fragmentation, including (Laubscher, 2000; Brown, 2003):

- Drawpoint size and spacing;
- Equipment selection;
- Draw control;
- Production rates;
- Dilution entry;
- Hang-ups;
- Secondary breakage;
- Staffing levels; and
- Processing plant design and costs.

Table 2.1 describes the potential effects of different fragmentation sizes on caving operations.

Fragmentation is generally classified into three components (Laubscher, 2000; Brown, 2003):
a) *In situ Fragmentation* – this is the inherent fragmentation of the rock mass generated by natural discontinuities;

b) *Primary Fragmentation* – these are the blocks that separate from the cave back once the caving process has started. The blocks are located in the vicinity of the cave back; and

c) *Secondary Fragmentation* – the fragmentation that occurs as the rock moves through the draw column from the cave back to the drawpoint.

Both, primary and secondary fragmentations include brittle fracturing.

**Table 2.1:** Rock fragmentation sizes and their potential effects on caving operations (Laubscher, 2000).

<table>
<thead>
<tr>
<th>Rock Fragmentation Size</th>
<th>Potential Effects</th>
<th>Length Range (m)</th>
<th>Mean Length L (m)</th>
<th>Mean Volume L x L/2 x L/2 (m³)</th>
<th>Maximum Volume (m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>100% through 1.5m x 0.3m grizzly</td>
<td>&lt;0.5</td>
<td>0.25</td>
<td>0.004</td>
<td>0.031</td>
</tr>
<tr>
<td>B</td>
<td>Grizzly</td>
<td>0.5 to 1.0</td>
<td>0.75</td>
<td>0.11</td>
<td>0.25</td>
</tr>
<tr>
<td>C</td>
<td>100% into LHD bucket</td>
<td>1.0 to 2.0</td>
<td>1.5</td>
<td>0.8</td>
<td>2</td>
</tr>
<tr>
<td>D</td>
<td>Hang-up in drawpoint throat</td>
<td>2.0 to 4.0</td>
<td>3</td>
<td>7</td>
<td>16</td>
</tr>
<tr>
<td>F</td>
<td>High hang-up</td>
<td>4.0 to 8.0</td>
<td>6</td>
<td>54</td>
<td>128</td>
</tr>
<tr>
<td>F</td>
<td>Drawbell blocker</td>
<td>8.0 to 16</td>
<td>12</td>
<td>432</td>
<td>1024</td>
</tr>
<tr>
<td>G</td>
<td>Double drawbell blocker</td>
<td>&gt;16</td>
<td>24</td>
<td>3456</td>
<td>Infinite</td>
</tr>
</tbody>
</table>
The common measurements for block size determination are (Stille and Palmström, 2008):

- Rock quality designation (RQD) in drill core;
- Joint spacing in mapped surfaces, drill core or scan lines;
- Density of joints in mapped surfaces, drill core or scan lines; and
- Block volume in mapped surfaces.

RQD was introduced by Deere (1963) as a way of providing a quantitative estimate of rock mass quality from drill core by measuring the percentage of core pieces longer than 10cm across the drilled core length. Grenon and Hadjigeorgiou (2003) indicated that RQD can result in sampling bias, due to the preferential orientation of certain discontinuities. RQD also does not account for the length of the discontinuities, and it is insensitive when the total frequency is greater than 3m⁻¹ (moderately fractured). Moreover, RQD gives no information about core pieces shorter than 0.1m (Palmström and Broch, 2006).

Palmström (1974) proposed the volumetric joint count (Jᵥ) as a way of assessing the amount of joints in a determined volume of rock. Volumetric joint count is defined as the number of joints intersecting a volume of 1m³. It was modified by Palmström (1982) to add an extra parameter to account for random joints:

\[
Jᵥ = \frac{1}{S₁} + \frac{1}{S₂} + \frac{1}{S₃} + \ldots + \frac{1}{Sₙ} + \frac{Nr}{5\sqrt{A}}
\]

where \(S₁, S₂\) and \(S₃\) are average spacing for the joint sets, \(Nr\) being the number of random joints in the actual location, and \(A\) is the area in m². \(Jᵥ\) is related to RQD using the following equation (Palmström, 1974):

\[
RQD = 115 - 3.3Jᵥ
\]
However, it was shown by Palmstrom (2005) that RQD is very difficult to relate to other joint measurements (such as $J_v$ or block volume).

Fragmentation or block size (i.e. volume) is an important parameter in rock mechanics and it is usually represented in the rock mass rating systems:

- In the NGI rock tunnel quality index Q (Barton et al., 1974), block size is represented by the ratio between RQD and the joint number factor ($J_n$);
- The RMR system (Bieniawski, 1976) incorporates fragmentation with the RQD and joint spacing factors;
- The Rock Mass Index (RMi) system (Palmstrom, 1996) uses the block volume ($V_b$) and the number of joint sets ($n_j$).

However, the representations of block volume in the Q and RMR systems is directly affected by inherent problems with RQD which forms one component of the classification parameters used in both rock mass rating systems (as discussed before). Other rock mass rating systems like MRMR (Laubscher, 1990), Geological Strength Index (GSI) (Hoek et al., 2002), or the modified basic RMR (MBR) (Kendorski et al, 1982) are indirectly affected by the RQD, since most of them are based on or use RMR as an input factor.

The RMi system directly estimates the mean block volume by using the following equation (Palmstrom, 1996):

$$V_0 = \frac{S_1 S_2 S_3}{\sin \gamma_1 \sin \gamma_2 \sin \gamma_3}$$

where $S_1$, $S_2$, $S_3$ are the mean spacing between joint sets and $\gamma_1$, $\gamma_2$, $\gamma_3$ are the mean acute angles between the joint sets.
Cai et al. (2004) modified Palmstrøm's mean volume equation to account for impersistent joints by adding a joint persistence factor to each fracture set, to calculate an equivalent block volume:

\[ V_b = \frac{s_1 s_2 s_3}{\sin \gamma_1 \sin \gamma_2 \sin \gamma_3 \sqrt[3]{p_1 p_2 p_3}} \]

where \( p_i \) is the joint persistence factor estimated by using the following criteria:

\[ p_i = \begin{cases} \frac{l_i}{L} & l_i < L; \\ 1 & l_i \geq L; \end{cases} \]

where \( l_i \) is the accumulated joint length of set \( i \) in the sampling plane, and \( L \) is the characteristic length of the rock mass under consideration. Cai et al. (2004) used the equivalent block volume and their joint condition factor as a supplementary quantitative approach to the GSI system.

Kim et al. (2007) performed a correlation analysis relating block sizes generated with UDEC and 3DEC against those calculated using the equivalent block volume equation, validating the method proposed by Cai et al. (2004).

Kalenchuk et al. (2006) developed the Block Shape Characterization Method (BSCM) for identifying the shape characteristics of individual blocks, as well as the shape distribution of an entire rock mass. Two parameters were derived:

- The \( \alpha \) parameter describing the shortening of the minor principal axis of the block;
- The \( \beta \) parameter describing the elongation of the major axis of the block.
These parameters are combined with the block volume distribution in a Block Shape Diagram, making it possible to determine the shape characteristics for a single block or an entire rock mass (Figure 2.8).

![Figure 2.8: Modified block shape diagram (Kalenchuk et al., 2007a) illustrating how BSCM classifies various shapes.](image)

Kalenchuk et al. (2006) also analyzed the block volume distributions and the total volume distribution based on the block shapes for different types of rock masses. The method was formulated and calibrated using rock masses simulated with 3DEC. The BSCM has also been validated using field data, e.g. Ekati Mine rock mass characterization (Kalenchuk et al., 2007a, b).

There have been several programs and codes specifically developed for the estimation of in-situ fragmentation with different degrees of success. Among them are Joints (Villaescusa, 1991), Blocks (Maerz and Germain, 1995), Stereoblock (Hadjigeorgiou et al., 1995), BCF (Esterhuizen, 1994), JKFrag (Eadie, 2002), MAKEBLOCK (Wang et al., 2003) and Split-Desktop (Split-Engineering, 2008).

More recently, the use of DFN (Discrete Fracture Network) models has shown promising results as a new tool for in situ fragmentation assessment. Rogers et al. (2007) have
demonstrated that DFN modeling clearly has application in the estimation of fragmentation. Elmo et al. (2008b) have characterized fragmentation through DFN models and have related them to Cai et al.'s (2004) work with GSI. The DFN has also been used for defining fracture networks in synthetic rock masses (Pierce et al., 2007), together with hybrid codes (ELFEN) (Rance et al., 2007) to estimate fragmentation.

2.4 The Discrete Fracture Network (DFN)

The DFN is a stochastic method of fracture simulation which allows the generation of 3D, synthetic, probabilistically simulated fractures. DFN's can produce realistic, stochastically similar discontinuity models based on limited field data (see Table 2.2). DFN models try to describe the heterogeneous nature of rock masses by representing characteristics such as joint shape, size, orientation of fracture sets and termination explicitly using probability distribution functions (Dershowitz and Einstein, 1988). The fracture network models developed up to the late 1980's are reviewed in detail in Dershowitz and Einstein (1988).

Table 2.2: Fracture data and derived input data for a DFN model (Staub et al, 2002). See section 2.4.2 for definitions of $P_{10}$, $P_{21}$ and $P_{32}$.

<table>
<thead>
<tr>
<th>&quot;Raw&quot; Fracture Data</th>
<th>Source</th>
<th>DFN Input Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fracture orientation (strike, dip)</td>
<td>Boreholes, outcrops, tunnels</td>
<td>Fracture sets, orientation of fractures in each set</td>
</tr>
<tr>
<td>Trace Length</td>
<td>Tunnels, outcrops, lineaments</td>
<td>Size distribution</td>
</tr>
<tr>
<td>Termination</td>
<td>Tunnels, outcrops, lineaments</td>
<td>Choice of the model, hierarchy of the sets</td>
</tr>
<tr>
<td>Fracture intensity</td>
<td>Boreholes, scanlines ($P_{10}$), outcrops ($P_{21}$)</td>
<td>Fracture intensity ($P_{10}$ or $P_{32}$)</td>
</tr>
</tbody>
</table>
Since its conception, the DFN method has been continuously developed, with many applications in civil, environmental and reservoir engineering (Jing, 2003). The method first saw use in characterization of the permeability of fractured rock masses and generic studies on fracture properties. Some examples are the work of Layton et al. (1992) and Watanabe and Takahashi (1995) on hot-dry-rock reservoirs; Dershowitz (1992) on characterization of the permeability of fractured rocks; and Rouleau and Gale (1987) on water effects on underground excavations in rock. DFN modeling has been used in the oil and gas industry for the simulation of hydrocarbon reservoirs (Dershowitz et al., 1998a) and in the nuclear industry for the modeling of nuclear waste repositories. It has also been identified as a useful tool for dealing with geomechanical problems in rock. A few examples are: Starzec and Tsang (2002) looking at the stability of tunnels; Grenon and Hadjigeorgiou (2003) studying open stope stability; Rogers et al. (2007) and Elmo et al. (2008b) analyzing fragmentation of fractured rock; Vyazmensky et al. (2008) using DFNs as input for hybrid brittle fracture models in the analysis of progressive rock slope failure in response to underground block cave mining; and Hadjigeorgiou et al. (2008) analysing the stability of vertical excavations in hard rock by integrating a DFN system into a PFC model.

2.4.1 Fracture Size

Fracture size or persistence is one of the critical factors establishing the formation and size of 3D blocks or incomplete blocks in a rock mass (Rogers et al., 2007), as well as having a significant influence on the rock mass properties. Fracture length is often a critical input in DFN models and a key parameter for sensitivity studies (Rogers et al., 2006).
It is almost impossible to determine persistence without taking apart the rock volume being studied and measuring it directly. Therefore, the length of the rock discontinuities must be inferred from the data sampled at outcrops, rock cuts or tunnel faces. This information at the same time suffers from statistical biases generated at several levels. Zhang and Einstein (1998) attributed these sampling errors to four different type of bias:

1. Orientation bias: the probability of a joint appearing in an outcrop depends on the relative orientation between the outcrop and the joint;

2. Size bias: large joints are more likely to be sampled than small joints. This bias results in two ways: a) a larger joint is more likely to appear in an outcrop than a smaller one; and b) a longer trace is more likely to appear in a sampling area than a shorter one;

3. Truncation bias: very small trace lengths are difficult or sometimes impossible to measure. Therefore, trace lengths below some known cut-off length are not recorded; and

4. Censoring bias: long joint traces may extend beyond the visible exposure so that one end or both ends of the joint traces cannot be seen (Figure 2.9).
Figure 2.9: Discontinuities intersecting a circular sampling window in 3 ways; a) both ends censored, b) one end censored, and c) both ends observable (Zhang and Einstein, 1998).

For orientation bias, Terzaghi (1965) devised a simple correction procedure based on a trigonometric correction factor, assuming the discontinuity spacing of different sets is equal. Baecher (1983) suggested that the orientation bias can be easily corrected by weighting the data in inverse proportion to their probability of appearing in the sample population.

Size bias converts many common distributions into lognormal forms. When applying goodness-of-fit tests to linearly biased exponential and lognormal distributions, lognormals better satisfy the Kolmogorov-Smirnov (K-S) criteria at the 5% level (Baecher, 1983). Einstein and Baecher (1983) explain by visual inspection that lognormal and gamma distributions provide good fits for persistence data. But after running K-S tests on the data, it shows that only the lognormal distribution provides an acceptable fit (at 5% confidence intervals).

Truncation is not significant in the formation of blocks, since the truncation threshold can be decreased to reduce its influence on the formation of medium to large blocks (Jimenez-
Rodriguez and Sitar, 2006). According to Baecher (1983), truncation may be safely ignored in most cases if the truncation level is small compared to the problem scale.

Censoring bias is a very significant issue. This bias is more likely to adversely affect the analysis of the rock mass, since it occurs with proportionally higher probability for longer traces. This causes the samples to be biased towards shorter lengths (Baecher, 1983), potentially affecting the generation of large blocks in the model. Mauldon (1998) developed a method for overcoming this bias by using density and mean trace length estimators. Zhang and Einstein (2000) also proposed a method for obtaining the true trace length distribution for circular windows.

In the future, laser scanning (LiDAR) and digital photogrammetry technology may provide another source of information for fracture length assessment. These systems might even help overcome some of the sample biases (e.g. truncation bias). They also allow the measurement of large exposed faces that have difficult or limited access.

2.4.2 Fracture Density and Spacing

Fracture density is defined as the mean number of trace centers per unit area (Mauldon, 1998). Discontinuities in a rock mass can only be characterized in a finite volume of rock. This information is generally obtained through boreholes, or through outcrop mapping and scanlines.

Data gathered using boreholes and scanlines is considered to be one-dimensional and is usually denoted as fracture frequency. In DFN terminology, this parameter is defined as \( P_{10} \) \((m^{-1})\), which is the fracture frequency along a scanline or borehole.
The 2D equivalent to fracture frequency is collected from outcrop mapping and it is known as $P_{21}$ (m/m$^2$). This is the total length of fractures, divided by the area, intersecting an outcrop surface. $P_{10}$ and $P_{21}$ are both subject to sampling bias since both factors are ruled by the orientation and scale of the sampling domain.

There is a third 3D parameter described as the total fracture area per unit volume of rock, $P_{32}$ (m$^2$/m$^3$). $P_{32}$ cannot be measured directly from the rock mass, however it can be linearly correlated to $P_{10}$ and $P_{21}$ (after sampling biases have been removed) using Dershowitz and Herda's (1992) relation of proportionality correlating the fracture intensity parameters:

$$P_{32} = C_{21} \times P_{21};$$

$$P_{32} = C_{10} \times P_{10};$$

where $C_{10}$ and $C_{21}$ are constants of proportionality that depend on the orientation and radius size distribution of the fractures, and the orientation of outcrops ($P_{21}$), or scanlines or boreholes ($P_{10}$).

Fracture spacing is generally defined as the distance between a pair of discontinuities measured along a line of specified location and orientation. If the discontinuity occurrence across a scanline or borehole is considered to be random, then the location of one discontinuity intersection has no influence upon the location of any other (Figure 2.10). In this case, the intersections obey a one dimensional Poisson process (Priest, 1993). Following these assumptions, the resultant probability density distribution is a negative exponential function of the form:

$$F(x) = \lambda e^{-\lambda x},$$

with $\lambda$ being the total discontinuity frequency, $x$ being a randomly located interval and $1/\lambda$ being the mean discontinuity spacing.
The observed discontinuity spacing distributions tend to be negative exponential functions suggesting, but not confirming, that fracture occurrences are random (Priest, 1993).

Figure 2.10: Random intersections along a line produced by variable discontinuity orientations (Priest, 1993).

2.4.3 Fracture Orientation

This variable is also defined from scanline or trace mapping data, and it is represented by either dip and dip direction or strike and dip notation. The mean orientation of each fracture set is determined using stereonet analysis. As with the previous parameters, fracture orientation is also subject to bias due to the relative orientation of the borehole, scanline, or outcrop with respect to the joint.

Sometimes the information gathered can be well organized and defined, and can be easily fitted to known statistical distribution forms. The more adequate distributions for this purpose are the Fisher, Bivariate Fisher and Bingham distributions (Dershowitz and Einstein, 1988). The most commonly used is the Fisher distribution since it is the analog for the normal distribution in fracture data and because of the ease to derive parameters from field data (Staub et al., 2002). A Fisher Distribution models the distribution of 3D orientation
vectors, like the distribution of joint orientations (pole vectors) on a sphere (Fisher, 1953). The Fisher Distribution describes the angular distribution of orientations about a mean orientation vector, and is symmetric about the mean. The probability density function can be expressed as:

\[ f(\theta) = \frac{\kappa \sin(\theta)e^{\kappa \cos \theta}}{e^{\kappa} - e^{-\kappa}}, \]

where \( \theta \) is the angular deviation from the mean vector, in degrees, and \( \kappa \) is the dispersion factor. The dispersion factor describes the tightness or dispersion of an orientation cluster (Fisher, 1953). A larger \( \kappa \) value (e.g. 50) implies a smaller cluster, and a smaller \( \kappa \) value (e.g. 8) implies a more dispersed cluster (Figure 2.11).

![Figure 2.11](image.png)

**Figure 2.11**: Schmidt equal area, lower hemisphere stereonets representing three fracture sets displaying the effects of different Fisher distributions. (a) \( \kappa = 8 \), (b) \( \kappa = 50 \).

The \( \kappa \) value can be estimated from the following equation:

\[
\kappa \approx \frac{N - 1}{N - R},
\]

where \( N \) is the number of poles, and \( R \) is the magnitude of the resultant vector, i.e. the magnitude of the vector sum of all pole vectors in the set (Fisher, 1953).
2.4.4 Fracture Spatial Model

Several conceptual models to describe the spatial distribution of discontinuities have been developed. There are three different types of distributions employed to describe the spatial distribution of fractures: i) considering that the fractures are ubiquitous (i.e. random in space following a Poisson distribution); ii) clumped or clustered around a certain feature, e.g. a fault; iii) close to constant fracturing, like in layered systems such as sedimentary rocks, where spacing is strongly related to bed thickness (Staub et al., 2002; Rogers et al., 2007). Most of these share common characteristics, such as size, termination and shape of fractures.

Among the models used for ubiquitous fractures is the Enhanced Baecher model. As in the conventional Baecher model (Baecher et al., 1978), the fracture centers are located uniformly in space using a Poisson process. The Enhanced Baecher model however, depicts fractures as polygons with a given radius and location, and not as disks (Staub et al., 2002). It also allows for the fracture termination to be specified.

The Nearest Neighbour model is generally utilized to simulate fractures clustered around some major feature (for example a fault) by producing new discontinuities near earlier fractures (Dershowitz et al., 1998a). The model organizes fractures into primary, secondary and tertiary groups and it generates them in that sequence. The Nearest Neighbour model is identical to the Enhanced Baecher model except for its assumptions regarding the spatial distribution (Staub et al., 2002).

The Levy-Lee Fractal model is a commonly used model to represent layered systems. It accounts for the chronology of fracture formation, since centers are created sequentially by the Levy flight process in 3D. The size of the fracture is related to the distance from the previous fracture, and fracturing can be bounded with spacing controlled by bed thickness
(Staub et al., 2002). Figure 2.12 shows examples of DFN generated models for Enhanced Baecher, Nearest Neighbour and Levy-Lee spatial distributions.

![Example of DFN models generated using different fracture spatial models for equivalent fracture orientation and radius distributions.](image)

**Figure 2.12:** Example of DFN models generated using different fracture spatial models for equivalent fracture orientation and radius distributions. (a) Enhanced Baecher model, (b) Nearest-Neighbour model and (c) Fractal Levy-Lee model (Elmo et al., 2007b).

### 2.4.5 Fracture Termination

This property quantifies the connectivity between fractures within a rock mass identifying the resulting network. Joint termination is related to and can be expressed by the characteristic shape, planarity, size and to some extent by location and orientation (Dershowitz and Einstein, 1988). Fracture termination is critical to block formation because it determines the potential of the rock mass to fully form blocks. It can also be used to determine the sequence of fracture formation in a rock mass.

### 2.5 Block Theory

Block theory is a geometrically based analysis developed by Warburton (1981) and Goodman and Shi (1985), and is typically used for determining potentially unstable blocks within a jointed rock mass. The analysis is based on data describing variously oriented discontinuities in 2D or 3D space and the wedges they form relative to a free face in the form
of an underground excavation (Goodman, 1995). This makes block theory useful for the analysis of blocks that separate from the cave back once the caving process has started (primary fragmentation). The theory divides blocks into removable and non removable. Removable blocks are finite and kinematically free to fall or slide. The latter involves a stability check under the applied forces (generally block weight and friction). Blocks which are unstable (i.e. factor of safety less than one) and removable are the "key blocks" of the excavation. Blocks that are stable (factor of safety greater than 1; i.e. friction preventing sliding) and removable are called "potential key blocks". There is a third category called "safe removable blocks" which are blocks that are removable, but their face orientations prevent them from moving (Table 2.3).
Table 2.3: Types of finite blocks identified Goodman and Shi's (1985).

<table>
<thead>
<tr>
<th>Goodman and classification scheme</th>
<th>Shi's classification scheme</th>
<th>Description of Block Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>I: Key Block</td>
<td></td>
<td>Block moves in direction of resultant driving force or slides with factor of safety &lt; 1</td>
</tr>
<tr>
<td>II: Potential Key Block</td>
<td></td>
<td>Factor of safety &gt; 1; friction alone prevents block from moving</td>
</tr>
<tr>
<td>III: Safe Removable</td>
<td></td>
<td>Combination of fixed face orientations alone is sufficient to prevent block from moving if direction of resultant driving force is as given, but would not be sufficient for certain other directions of that force</td>
</tr>
<tr>
<td>IV: Tapered (non removable) Block</td>
<td></td>
<td>Kinematic infeasibility: block could not be removed without disturbing rest of rock (assumed fixed)</td>
</tr>
<tr>
<td>V: Infinite Block</td>
<td></td>
<td>Combination of fixed face orientations alone is sufficient to prevent block from moving if direction of resultant driving force is as given</td>
</tr>
<tr>
<td>VI: Joint Block</td>
<td></td>
<td>No faces on excavation perimeter</td>
</tr>
</tbody>
</table>

Non removable blocks cannot be key blocks unless there is fracture development and/or key blocks are removed. Figure 2.13 describes the definitions for the different types of blocks outlined in Table 2.3.
Figure 2.13: Description of the block types identified by Goodman and Shi (1985) as depicted in Table 2.3.

The basic assumptions of the original key block theory are the following:

- Rock blocks are undeformable;
- Rocks are separated by planar fractures with zero tensile strength; and
- Displacements are purely translational.

The method has been extended and refined to include rotational displacements and different types of block shapes (Mauldon and Goodman, 1990 and 1996; Mauldon, 1995; Tonon, 1998). In order to apply block theory, it is necessary to have persistent discontinuities, hard rock and low stresses. Discontinuities dominate rock mass deformations because stresses are too low to produce deformation and fracture of the rock blocks. This makes the underformable rock block assumption applicable. Conditions can also be approximated for excavations at greater depths if local stress relief has occurred (Warburton, 1981).
Block stability analysis with block theory is conducted either analytically by vector techniques, or graphically using stereographic projections. Block theory is based on the idea that a single plane divides the three dimensional space into upper and lower half space. Goodman and Shi (1985) use “0” to indicate the upper half space and “1” for the lower half space and a string to represent a block formed. For instance, the string 101 represents a block formed by three joints relative to an excavation face, which include: the lower side of joint set one, the upper side of joint set two and the lower side of joint set three. The theory translates each of the discontinuities and free faces so that they each pass through a common origin forming a series of pyramids:

- Block Pyramid – assemblage of planes forming a particular set of blocks;
- Joint Pyramid – group of discontinuity planes (rock to rock interfaces);
- Excavation Pyramid – group of excavation surfaces (rock to air interfaces).

Block theory uses full sphere stereographic projections with the reference plane plotting as a circle. Figure 2.14 shows a horizontal reference plane plotted as an upper hemisphere projection. In the case of Figure 2.14, the joint pyramid 100 lies outside the free face’s great circle, making it kinematically feasible. The joint pyramid 011 lies inside the great circle of the free face, making it kinematically feasible if the free face is the non overhanging floor of an excavation. The method can be extended to complex non concave polyhedra exposed at multiplanar convex or concave rock faces. But the surface area, the volume and the forces acting on each block have to be calculated using vector methods (Priest, 1993).
Figure 2.14: Application of block theory using a spherical projection (Priest, 1993).

Computer software implementing Goodman and Shi’s (1985) original block theory procedures allow the user to quickly perform a block stability analysis of a rock mass and to visualize of the three dimensional block geometries formed. Several numerical codes for key block prediction and rock support design for both underground facilities and rock slopes have been developed. Among them are Siromodel (Read and Ogden, 2006), KBTunnel (Pantechnica, 2000), Safex (Windsor and Thompson, 1991), Rock3D (Geo&Soft, 1999), SWEDGE (Rocscience, 2006), UNWEDGE (Rocscience, 2007), SATIRN (Priest and Samaniego, 1998), DRKBA (Stone, 1994) and MSB (Jakubowski, 1995). These codes are all based on similar principles but differ in terms of input data, model assumptions and the potential outcome from the analysis. It is also important to mention FracMan Geomechanics
(Golder Associates, 2007; Dershowitz et al., 1998b) which combines DFN simulation with block theory, in order to evaluate the stability of underground openings.

The procedures used by UNWEDGE to calculate the stability of a rock block follows a similar algorithm as FracMan Geomechanics. UNWEDGE determines all the possible wedges which can be formed with at least three distinct joint planes and an excavation face (Figure 2.15). In general, most of the wedges formed with UNWEDGE are tetrahedral in nature, but prismatic wedges can also be formed.

![Figure 2.15: Example of tunnel stability analysis performed with UNWEDGE (Rocscience, 2007).](image)

Once the program determines the wedge coordinates, it calculates the geometrical properties of each wedge including: wedge volume, wedge face areas and normal vectors for each plane. The forces on the wedge are classified as active or passive. Active forces involve the driving forces in the factor of safety calculation (e.g. wedge weight) and passive forces involving the resisting forces (e.g. support resistance). The individual force vectors are computed for magnitude and then the resultant active and passive force vectors are determined by vector summation of the individual forces (Rocscience, 2003). Once the program computes the wedge geometry, it calculates the sliding direction based on Goodman
and Shi's (1985) method. After the sliding direction has been determined, the normal forces to the planes are calculated which is followed by the shear and tensile strength computation (using either the Mohr-Coulomb, the Barton-Bandis or the Power Curve criterion). When all the forces are computed, the resultant factor of safety is determined (Rocscience, 2003).

Another approach to block stability is taken with the use of implicit DEM modeling, for example discontinuous deformation analysis (DDA) and distinct element modeling (e.g. UDEC). DDA can represent motion and deformation of the individual bodies by using an implicit solution with finite element discretization of the body interior (Jing and Stephansson, 2007). This is likewise done in UDEC (Cundall and Hart, 1993) and 3DEC by discretizing all the blocks to overcome the condition of undeformable blocks.

2.6 Chapter Summary

Block caving is an underground mining method that has been gaining importance because of its low costs and high production rates.

In order to assess the caveability of an ore body, two different methods are generally employed: empirical and numerical. Empirical methods are based on experiences in a large number of mines and numerical methods use mathematical algorithms to simulate the behaviour of rock. In the last few years, numerical methods have shown significant algorithm improvements, which has been accompanied by an increase in computing power. Fragmentation also plays a major role in block caving, particularly when it comes to the design and logistics of a mine.

The DFN is a stochastic method of fracture simulation which allows the generation of simulated fractures. It can produce realistic, stochastically similar discontinuity models based
on limited field data, describing the heterogeneous nature of rock masses by representing characteristics such as joint shape, size, orientation of fracture sets and termination explicitly using probability distribution functions.

Block theory is a geometrically based analysis used for determining potentially unstable blocks within a jointed rock mass. The analysis is based on data describing variously oriented discontinuities in 2D or 3D space, and the wedges they form relative to a free face in the form of an underground excavation or a slope.

Computer codes have been developed that combine both DFN simulation and the principles of block theory to assess the stability of underground openings. FracMan Geomechanics is one such programs and will be extensively used in this thesis.
3.0 METHODOLOGY

The discrete fracture network models simulated in this research were generated using the proprietary code FracMan Geomechanics (Golder Associates, 2007; Dershowitz et al., 1998b).

3.1 Characteristics of the Model

FracMan allows the user to choose a range of values and/or different models for fracture spatial distribution, fracture orientation and orientation distribution, fracture termination percentage, fracture radius distribution, and fracture intensity in order to simulate the conditions present in a given rock mass. After generating a DFN stochastic model from the assumed parameters, FracMan identifies the 3D blocks that have a common face with the opening being analyzed. In order to do this, the code computes the fracture intersections with the opening, iteratively defining trace maps for all the intersections until all discontinuities involved have a trace map of their own. After it identifies valid blocks that have formed, the code computes their volume based on a 3D process that builds the blocks by putting together the defined trace maps with no overlaps and no gaps (3D tessellation process). FracMan then carries out a stability analysis checking each block for unconditional stability, free fall or sliding (on one or two planes). The factor of safety for each block is assigned based on limit equilibrium assumptions (Rogers et al, 2006). As mentioned in chapter 3.5, the block stability analysis in FracMan is very similar to the one performed by UNWEDGE, utilizing Goodman and Shi’s (1985) block theory. UNWEDGE inputs the various combinations of assigned joint sets, looks for potential wedges and their factor of safety. FracMan generates a fracture network using a stochastic approach and identifies all blocks, determining their
factor of safety. The factor of safety (FS) is determined depending on the failure mode of a block. Stable blocks have an infinite factor of safety and free falling blocks have a factor of safety of zero. Between these two extremes are the cases of translational sliding on one- or two-planes. The factor of safety for these two cases can be calculated using either the Mohr-Coulomb or the Barton–Bandis criterion (Rogers et al., 2006). The Mohr-Coulomb model is shown below. For sliding on a single plane, the Mohr-Coulomb criterion:

\[ FS = \frac{A \cdot c + |N'| \cdot \tan \phi}{S} \]

where \( A \) is the area of the face, \( c \) is the cohesion parameter, \( N' \) is the normal force to the face, \( \phi \) is the friction angle and \( S \) is the magnitude of the shear force. For sliding on two planes using the Mohr-Coulomb criterion:

\[ FS = \frac{A_1 \cdot c_1 + |N'_1| \cdot \tan \phi_1 + A_2 \cdot c_2 + |N'_2| \cdot \tan \phi_2}{S_{12}} \]

where \( N'_1 \) and \( N'_2 \) are the normal forces to faces 1 and 2 respectively, \( A_1 \) and \( A_2 \) are the areas of faces 1 and 2 respectively, \( S_{12} \) is the shear force along the edge created by faces 1 and 2, \( c_i \) is the cohesion parameter of face \( i \) and \( \phi_i \) is the friction angle of face \( i \).

The DFN provides the possibility of generating multiple statistically equivalent realizations that allow the understanding of the frequency of occurrence for blocks of a particular size or factor of safety (Rogers et al, 2006). Figure 3.1 compares the UNWEDGE and FracMan stability analysis for a tunnel section with the same dimensions and joint sets. It can be observed how FracMan’s probabilistic approach is able to better characterize the potential blocks that can form in the tunnel walls.
Figure 3.1: Comparison between UNWEDGE and FracMan stability analysis for a tunnel and three joint sets. (a) UNWEDGE model for tunnel and three joint sets, (b) FracMan model and stability analysis for tunnel and three joint sets.

The following assumptions were made in the development of this thesis:

- Joints can be modeled as planar four sided polygons;
- The rock mass can be represented by a small number of joint sets, three in this case;
- Each joint set is modeled using a Fisher distribution for orientation dispersion, lognormal distribution for persistence, and the enhanced Baecher model for spatial distribution;
- Fracture termination was not assigned in order not to condition the fracture generation to it;
- Networks were modeled using FracMan’s default values for angle of friction (26.5°), cohesion (zero) and shear strength criterion (Mohr-Coloumb); and
- Stress effects were ignored.

FracMan does not account for stresses. The no stress assumption is valid for this thesis, since the goal is to evaluate the effects of joint persistence, spacing and dispersion on block
stability without the influence of brittle failure, the increase of FS due to increased shear strength, and the clamping and "pop out" effect on wedges.

3.2 Model Geometry

The model used for the simulation consisted of three parts (Figure 3.2):

- An "outer" box of 100m length, 100m width and 75 m height, representing the rock mass;
- An "inner" box in the center of the "outer" box with a length and width of 50m, and a height of 25m, representing the ore body;
- A 1m thick slab, at the bottom of the "inner" box, representing the undercut.

Figure 3.2: Model layout.
To condition the model to a given $P_{10}$ value, three orthogonal boreholes passing through the center of each of the boxes were inserted.

Because the simulations are conceptual models, it was decided initially to use three orthogonal sets of fractures with Fisher distributions, representing the most basic case. The fracture sets had orientations of (dip/dip direction): $00^\circ/000^\circ$, $00^\circ/090^\circ$ and $90^\circ/000^\circ$ (Figure 3.3a). A second set of fractures was subsequently used to evaluate the effects of joint set orientation with dispersion and will be referred to herein as the “modified orthogonal model”, with orientations of (dip/dip direction): $00^\circ/000^\circ$, $45^\circ/090^\circ$ and $45^\circ/270^\circ$ (Figure 3.3b).

![Figure 3.3: Schmidt equal area, lower hemisphere stereonet representing the three orthogonal fracture sets used in the simulations. a) Original orthogonal model, b) Modified orthogonal model.](image)

The fractures were generated in the “outer” box, in order to limit potential boundary problems affecting the subsequent analysis. The fractures were then clipped (Figure 3.4), leaving only the joints within the “inner” block where the analysis was to be conducted. The “inner” block represents the ore body that will be caved during the mining process.
In order to simulate an undercut in a block caving mine scenario, a slab was inserted at the bottom of the "inner" block. This provides a reference free surface for the subsequent block analysis. After clipping the fractures, the stability analysis was conducted (Figure 3.5).
3.3 Model Parameters

A set of 25 conceptual DFN models was set up to estimate the sensitivity of fracture intensity, dispersion and persistence of data when used in conjunction with the block analysis. For each conceptual model, only one variable at a time was changed, leaving the others fixed. The same changes were applied to all discontinuity sets for every model, thus creating the same fracture conditions in each dimension. The 25 conceptual models were divided into five cases:

- Five models for the evaluation of spacing
- Twelve models for the evaluation of dispersion, divided into two groups: six using the orthogonal model and six using the modified orthogonal model.
Ten models for the evaluation of persistence, divided into two groups: five using constant spacing and five using constant fracture count.

The studies for spacing, dispersion using the orthogonal model and persistence share one common model that was used as the base case (M3 model).

Based on work done by Starzec and Tsang (2002) regarding the effects of the number of realizations on predictions made with FracMan, the simulation was set at 50 iterations per model. The results of the assessment were based on the stability of the blocks generated (i.e. factor of safety), the volume of the blocks displaced and the mode of failure.

3.3.1 Fracture Intensity (Spacing)

In order to evaluate the effects of joint spacing, five models with different fracture intensities were constructed. The variables used are shown in Table 3.1.

**Table 3.1**: Description of values used for the variables in the fracture intensity analysis.

<table>
<thead>
<tr>
<th>Model</th>
<th>Spacing (m)</th>
<th>Dispersion ((\kappa))</th>
<th>Persistence (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>M1</td>
<td>.75</td>
<td>20</td>
<td>7</td>
</tr>
<tr>
<td>M2</td>
<td>1.35</td>
<td>20</td>
<td>7</td>
</tr>
<tr>
<td>M3</td>
<td>2</td>
<td>20</td>
<td>7</td>
</tr>
<tr>
<td>M4</td>
<td>3</td>
<td>20</td>
<td>7</td>
</tr>
<tr>
<td>M5</td>
<td>4</td>
<td>20</td>
<td>7</td>
</tr>
</tbody>
</table>

The values for persistence and dispersion were kept constant, and the values assumed for spacing were based on Bieniawski’s (1976) RMR\(_{76}\). The following ranges of values were defined as high, medium and low spacing:
Table 3.2: Description of chosen values for intensity.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>High</th>
<th>Medium</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spacing of Joints (RMR76)</td>
<td>&gt;3m</td>
<td>1-3m</td>
<td>0.3-1m</td>
</tr>
<tr>
<td>Value Used</td>
<td>4m</td>
<td>3m</td>
<td>2m 1.35m</td>
</tr>
</tbody>
</table>

The medium spacing classification was divided in three values because during modeling, it was the interval that showed the most of the variation. The values for low and high spacing were selected approximately in the middle of each of the ranges proposed by Bieniawski (1976).

3.3.2 Fracture Dispersion

The dispersion analysis was conducted using different dispersion factors ($\kappa$) for the fracture sets in the orthogonal and modified orthogonal models. In order to obtain suitable $\kappa$ values for the dispersion models, several tests were performed running models with FracMan to evaluate the size of the cluster of joints in the resulting stereonets. The variables used for the orthogonal model are outlined in Table 3.3.

Table 3.3: Variables used for dispersion analysis with the orthogonal model.

<table>
<thead>
<tr>
<th>Model</th>
<th>Spacing (m)</th>
<th>Dispersion ($\kappa$)</th>
<th>Persistence (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>M6</td>
<td>2</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>M3</td>
<td>2</td>
<td>20</td>
<td>7</td>
</tr>
<tr>
<td>M7</td>
<td>2</td>
<td>50</td>
<td>7</td>
</tr>
<tr>
<td>M8</td>
<td>2</td>
<td>100</td>
<td>7</td>
</tr>
<tr>
<td>M9</td>
<td>2</td>
<td>20000</td>
<td>7</td>
</tr>
<tr>
<td>M10</td>
<td>2</td>
<td>Constant</td>
<td>7</td>
</tr>
</tbody>
</table>

Spacing and persistence were kept constant for all models. The variables used for the modified orthogonal model are outlined in Table 3.4. The stereonets for several selected dispersion values are shown in Figure 3.6.
Table 3.4: Variables used for dispersion analysis with the orthogonal model.

<table>
<thead>
<tr>
<th>Model</th>
<th>Spacing (m)</th>
<th>Dispersion (κ)</th>
<th>Persistence (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>M11</td>
<td>2</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>M12</td>
<td>2</td>
<td>20</td>
<td>7</td>
</tr>
<tr>
<td>M13</td>
<td>2</td>
<td>50</td>
<td>7</td>
</tr>
<tr>
<td>M14</td>
<td>2</td>
<td>100</td>
<td>7</td>
</tr>
<tr>
<td>M15</td>
<td>2</td>
<td>20000</td>
<td>7</td>
</tr>
<tr>
<td>M16</td>
<td>2</td>
<td>Constant</td>
<td>7</td>
</tr>
</tbody>
</table>

Figure 3.6: Selected Schmidt equal area, lower hemisphere stereonets representing three fracture sets with varying dispersion for the orthogonal model. a) κ = 8, b) κ = 20, c) κ = 50, d) κ = constant.

Spacing and persistence were kept constant for all the modified orthogonal dispersion models, using the same values as in the orthogonal dispersion models (Table 3.4). The
stereonets for selected dispersion values for the modified dispersion models are shown in Figure 3.7.

![Stereonets](image)

**Figure 3.7:** Selected Schmidt equal area, lower hemisphere stereonets representing three fracture sets with varying dispersion for the modified orthogonal model. a) $\kappa = 8$, b) $\kappa = 20$, c) $\kappa = 50$, d) $\kappa = \text{constant}$.

### 3.3.3 Fracture Persistence

Two sets of simulations were carried out to assess persistence. The first set was defined using a constant spacing of 2m. Because the spacing was conditioned using $P_{10}$, FracMan produced fractures until the previously defined spacing condition was met. However, short joints have a lower probability of intersecting the borehole than long joints (i.e. size bias; Zhang and Einstein, 1998). Hence, a model using a low persistence would generate a substantially higher number of joints than the one with high persistence. This would not
affect the total fracture area of the models, due to the fact that the simulations were conditioned to $P_{10}$. To overcome this size bias, the second set of models was defined using different assumptions regarding spacing and fracture count. Instead of using constant spacing, it was decided to use the constant fracture count for all simulations instead. The value for the fracture count was taken from "M3", which is a common model for all analyses carried out with the orthogonal model. Once the simulations were run, the joint spacing was obtained by querying the boreholes.

The parameters for the first and second set of simulations are outlined in Table 3.5 and Table 3.6 respectively.

**Table 3.5**: Parameters used for persistence models with constant spacing.

<table>
<thead>
<tr>
<th>Model</th>
<th>Spacing (m)</th>
<th>Dispersion ($\kappa$)</th>
<th>Persistence (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>M17</td>
<td>2</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>M3</td>
<td>2</td>
<td>20</td>
<td>4</td>
</tr>
<tr>
<td>M18</td>
<td>2</td>
<td>20</td>
<td>7</td>
</tr>
<tr>
<td>M19</td>
<td>2</td>
<td>20</td>
<td>11</td>
</tr>
<tr>
<td>M20</td>
<td>2</td>
<td>20</td>
<td>15</td>
</tr>
</tbody>
</table>

**Table 3.6**: Set of parameters for persistence models with constant fracture count.

<table>
<thead>
<tr>
<th>Model</th>
<th>Fracture Count</th>
<th>Dispersion ($\kappa$)</th>
<th>Persistence (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>M21</td>
<td>48407</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>M22</td>
<td>48407</td>
<td>20</td>
<td>3</td>
</tr>
<tr>
<td>M23</td>
<td>48407</td>
<td>20</td>
<td>4</td>
</tr>
<tr>
<td>M3</td>
<td>48407</td>
<td>20</td>
<td>7</td>
</tr>
<tr>
<td>M24</td>
<td>48407</td>
<td>20</td>
<td>11</td>
</tr>
<tr>
<td>M25</td>
<td>48407</td>
<td>20</td>
<td>15</td>
</tr>
</tbody>
</table>

Joint dispersion was kept constant for all models. The mean values for persistence were obtained from ISRM standards for joint length (Table 3.7). Based on observations by
Einstein and Baecher (1983), Villaescusa and Brown (1992) and Munier (2004), a lognormal function was utilized to describe the probability distribution function of fracture lengths.

**Table 3.7:** Description of assumed persistence values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>High</th>
<th>Medium</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joint Length (ISRM)</td>
<td>10-20m</td>
<td>3-10m</td>
<td>1-3m</td>
</tr>
<tr>
<td>ISRM Classification</td>
<td>High Persistence</td>
<td>Medium Persistence</td>
<td>Low Persistence</td>
</tr>
</tbody>
</table>

### 3.4 Simulated Rock Mass Characterization

To characterize the rock masses that were being simulated, the rock masses were evaluated using Cai et al.’s (2004) quantified adaptation of the Geological Strength Index (GSI) chart (Hoek, 1994). Figure 3.8 describes the values and criteria used for determining the GSI. Based on the chosen parameters, the spacing used for the simulations does not allow the definition of any correlations except for massive to blocky rock masses. Within that range, a GSI can be estimated for different joint conditions. The only joint characteristic used was the angle of friction (26.5°) which based on Wyllie and Mah (2004) makes the rock mass fall into the low friction rock class (like schist, shale or marl). The approximate range that covers the simulated rock masses would potentially comprise a zone between good and fair joint surface conditions, and a massive to blocky rock mass (Figure 3.8).
The GSI values for the models were generally high, since most of the simulations have spacings equal or higher than 2m, and the GSI quantification chart is especially
sensitive to spacing. Under these conditions, the fragmentation for the model is expected to be coarse, i.e. with most of the blocks generated being between 0.1 and 10m³.

3.5 Chapter Summary

To assess the effects of joint persistence, spacing and dispersion on block stability without the influence of brittle failure (which would enable further degrees of freedom for block movement), 25 conceptual models were carried out using the DFN code FracMan Geomechanics. A summary of the variables used in the conceptual models is outlined in Table 3.8.

Table 3.8: Summary of the conceptual models generated in this thesis.

<table>
<thead>
<tr>
<th>Model</th>
<th>Spacing (m)</th>
<th>Dispersion (ε)</th>
<th>Persistence (m) Mean</th>
<th>Fracture Count</th>
<th>Simulation Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>0.75</td>
<td>20</td>
<td>7</td>
<td>7</td>
<td>Varied</td>
</tr>
<tr>
<td>M2</td>
<td>1.35</td>
<td>20</td>
<td>7</td>
<td>7</td>
<td>Varied</td>
</tr>
<tr>
<td>M3</td>
<td>2</td>
<td>20</td>
<td>7</td>
<td>7</td>
<td>Varied</td>
</tr>
<tr>
<td>M4</td>
<td>3</td>
<td>20</td>
<td>7</td>
<td>7</td>
<td>Varied</td>
</tr>
<tr>
<td>M5</td>
<td>4</td>
<td>20</td>
<td>7</td>
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<td>Varied</td>
</tr>
<tr>
<td>M6</td>
<td>2</td>
<td>8</td>
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<td>7</td>
<td>Varied</td>
</tr>
<tr>
<td>M7</td>
<td>2</td>
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</tr>
<tr>
<td>M8</td>
<td>2</td>
<td>100</td>
<td>7</td>
<td>7</td>
<td>Varied</td>
</tr>
<tr>
<td>M9</td>
<td>2</td>
<td>20000</td>
<td>7</td>
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<td>Varied</td>
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<td>Constant</td>
<td>7</td>
<td>7</td>
<td>Varied</td>
</tr>
<tr>
<td>M11</td>
<td>2</td>
<td>8</td>
<td>7</td>
<td>7</td>
<td>Varied</td>
</tr>
<tr>
<td>M12</td>
<td>2</td>
<td>20</td>
<td>7</td>
<td>7</td>
<td>Varied</td>
</tr>
<tr>
<td>M13</td>
<td>2</td>
<td>50</td>
<td>7</td>
<td>7</td>
<td>Varied</td>
</tr>
<tr>
<td>M14</td>
<td>2</td>
<td>100</td>
<td>7</td>
<td>7</td>
<td>Varied</td>
</tr>
<tr>
<td>M15</td>
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<td>Varied</td>
</tr>
<tr>
<td>M16</td>
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</tr>
<tr>
<td>M17</td>
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<td>Varied</td>
</tr>
<tr>
<td>M18</td>
<td>2</td>
<td>20</td>
<td>4</td>
<td>4</td>
<td>Varied Count</td>
</tr>
<tr>
<td>M19</td>
<td>2</td>
<td>20</td>
<td>11</td>
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<td>15</td>
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<tr>
<td>M21</td>
<td>Varied</td>
<td>20</td>
<td>2</td>
<td>2</td>
<td>48407</td>
</tr>
<tr>
<td>M22</td>
<td>Varied</td>
<td>20</td>
<td>3</td>
<td>3</td>
<td>48407</td>
</tr>
<tr>
<td>M23</td>
<td>Varied</td>
<td>20</td>
<td>4</td>
<td>4</td>
<td>48407</td>
</tr>
<tr>
<td>M24</td>
<td>Varied</td>
<td>20</td>
<td>11</td>
<td>11</td>
<td>48407</td>
</tr>
<tr>
<td>M25</td>
<td>Varied</td>
<td>20</td>
<td>15</td>
<td>15</td>
<td>48407</td>
</tr>
</tbody>
</table>
The assembled model consisted of three parts: an outer box representing the rock mass, an inner box representing the ore body and a slab under the inner box representing the undercut. FracMan Geomechanics conducted a block stability analysis based on Goodman and Shi’s (1985) block theory and calculated the factor of safety based on the Mohr-Coulomb criterion for free falling blocks, blocks sliding on one and two faces, and stable blocks.
4.0 RESULTS AND ANALYSIS

4.1 Block Shape Characterization

The average shape distribution for the blocks in each model was determined using Kalenchuk's et al. (2006) block shape characterization method (BSCM). It should be noted that the block shape characterization applies to primary fragmentation alone here, since this thesis only focuses on the first stage of the caving process (undercut opening). To evaluate the block shapes, a Java (Sun Microsystems, 2008) application was developed to extract the information from the files generated by FracMan Geomechanics and calculate the parameters for the BSCM analysis (Appendix I). The code was verified against the values presented by Kalenchuk et al. (2006). The program determines both the $\alpha$ and $\beta$ factors. The $\alpha$ factor is a dimensionless parameter relating the surface area and volume of an arbitrary object, and is defined as:

$$\alpha = \frac{A_s l_{avg}}{7.7V}$$

where $V$ is the block volume, $A_s$ is the surface area of the block, $l_{avg}$ is the average chord length and 7.7 is a numerical factor used to normalize $\alpha$ to a value of one for a cube (Kalenchuk et al., 2006). The $\beta$ parameter describes the elongation of an object and is estimated by first calculating all the inter-vertex dimensions of a rock block (including all edges, face diagonals and internal diagonals). Once all the chord lengths are calculated, those smaller than the median chord length are disregarded and the remainder is used to generate the $\beta$ factor as follows:

$$\beta = 10 \frac{\sum (a \cdot b)^2}{\sum \|a\|^2 \|b\|^2}$$
where a and b are the combination of chords equal or larger than the median chord length (Kalenchuk et al., 2006). As described in section 2.2, blocks are classified according to their shape based on three main groups:

- Cubic (C);
- Elongated (E);
- Platy (P);

and three transitional shape groups:

- Cubic-Elongated (CE);
- Elongated-Platy (EP); and
- Platy-Cubic (PC).

The average values of the simulations were utilized for the evaluation of the block shapes. The analysis was also carried out only on single blocks and only blocks larger than 0.001m³ were examined.

The shape distribution for the FracMan spacing simulations is described in Figure 4.1. In the smallest spacing (model M1), most of the blocks are cubic, cubic-elongated and elongated, with less platy-cubic and elongated-platy. However, as spacing increases the blocks distribution changes. The number of cubic blocks shows a progressive decrease and a clustering of the points starts to develop in the elongated block region of the diagram. This is especially noticeable in models M2 and M3 and is confirmed by observing the shape distribution plots. As the spacing further increases, most of the blocks become elongated. Still, for some of the spacing simulations it can be observed that all six shape types are generated. It would be expected that the generated rock masses would be primarily composed of equidimensional blocks, since the properties of all three joint sets are the same. This does
not occur because of the variances in joint spacing, length and orientation, which are responsible for the generation of non-equidimensional and transitional shapes.

<table>
<thead>
<tr>
<th>ID</th>
<th>Block Shape Diagram</th>
<th>Shape Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td><img src="image" alt="Block Shape Diagram" /></td>
<td><img src="image" alt="Shape Distribution" /></td>
</tr>
<tr>
<td>M2</td>
<td><img src="image" alt="Block Shape Diagram" /></td>
<td><img src="image" alt="Shape Distribution" /></td>
</tr>
<tr>
<td>M3</td>
<td><img src="image" alt="Block Shape Diagram" /></td>
<td><img src="image" alt="Shape Distribution" /></td>
</tr>
<tr>
<td>M4</td>
<td><img src="image" alt="Block Shape Diagram" /></td>
<td><img src="image" alt="Shape Distribution" /></td>
</tr>
<tr>
<td>M5</td>
<td><img src="image" alt="Block Shape Diagram" /></td>
<td><img src="image" alt="Shape Distribution" /></td>
</tr>
</tbody>
</table>

**Figure 4.1:** Block shape diagrams and block shape distribution plots for spacing simulations.
For the original dispersion models (Figure 4.2), the low $\kappa$ values (model M6) result in a clustering of the points in the elongated and cubic-elongated areas of the block shape diagram. As expected for this orthogonal model with equally spaced joints, the blocks become more equidimensional as the $\kappa$ value increases. When $\kappa$ is constant, most of the blocks are either cubic or cubic-elongated. However, there are still blocks in the elongated and platy-cubic areas, due to the variations in discontinuity spacing and persistence.

The modified dispersion models show similar trends as the original dispersion models (Figure 4.3). For low $\kappa$ values ($\kappa$ equal to 8 and 20) there is a slightly higher occurrence of cubic block than cubic-elongated and elongated blocks. This differs from the original dispersion models in Figure 4.2 which show a tendency of forming more cubic-elongated blocks. The difference between the original dispersion and modified dispersion models is credited to the orientation difference of two of the joint sets. For intermediate $\kappa$ values ($\kappa$ equal to 50 and 100) there is a small increase in the amount of elongated blocks. This changes as $\kappa$ is further increased. As expected (and similar to the original dispersion models), when dispersion is constant most blocks are equidimensional (cubic). Nevertheless, there are a large percentage of elongated cubic and elongated blocks, as well as a smaller quantity of platy-cubic blocks. The presence of other block shapes is again attributed to the variations in discontinuity spacing and persistence in the models.
<table>
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<tr>
<th>ID</th>
<th>Block Shape Diagram</th>
<th>Shape Distribution</th>
</tr>
</thead>
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<td><strong>C</strong> 60</td>
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<tr>
<td>N=353</td>
<td><strong>Alpha</strong></td>
<td><strong>C-E</strong> 30</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>E</strong> 10</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>E-P</strong> 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>P</strong> 0</td>
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<tr>
<td></td>
<td></td>
<td><strong>P-C</strong> 0</td>
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<tr>
<td>M3</td>
<td><img src="image3" alt="Block Shape Diagram" /></td>
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</tr>
<tr>
<td>k=20</td>
<td><strong>Beta</strong></td>
<td><strong>C</strong> 60</td>
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<td>N=244</td>
<td><strong>Alpha</strong></td>
<td><strong>C-E</strong> 30</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>E</strong> 10</td>
</tr>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td><strong>P-C</strong> 0</td>
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<td><img src="image6" alt="Shape Distribution" /></td>
</tr>
<tr>
<td>k=50</td>
<td><strong>Beta</strong></td>
<td><strong>C</strong> 60</td>
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<td>N=154</td>
<td><strong>Alpha</strong></td>
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<td></td>
<td></td>
<td><strong>E</strong> 10</td>
</tr>
<tr>
<td></td>
<td></td>
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<td><strong>P-C</strong> 0</td>
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<tr>
<td>k=20000</td>
<td><strong>Beta</strong></td>
<td><strong>C</strong> 60</td>
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<tr>
<td>N=81</td>
<td><strong>Alpha</strong></td>
<td><strong>C-E</strong> 30</td>
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<td></td>
<td></td>
<td><strong>E</strong> 10</td>
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<tr>
<td></td>
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<td></td>
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<td><strong>P</strong> 0</td>
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<tr>
<td></td>
<td></td>
<td><strong>P-C</strong> 0</td>
</tr>
<tr>
<td>M10</td>
<td><img src="image11" alt="Block Shape Diagram" /></td>
<td><img src="image12" alt="Shape Distribution" /></td>
</tr>
<tr>
<td>k=const.</td>
<td><strong>Beta</strong></td>
<td><strong>C</strong> 60</td>
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<td>N=76</td>
<td><strong>Alpha</strong></td>
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<td></td>
<td></td>
<td><strong>E</strong> 10</td>
</tr>
<tr>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td><strong>P</strong> 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>P-C</strong> 0</td>
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**Figure 4.2:** Block shape diagrams and block shape distribution plots for dispersion simulations.
<table>
<thead>
<tr>
<th>ID</th>
<th>Block Shape Diagram</th>
<th>Shape Distribution</th>
</tr>
</thead>
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<tr>
<td>M11</td>
<td><img src="attachment" alt="Image" /></td>
<td><img src="attachment" alt="Bar Chart" /></td>
</tr>
<tr>
<td>( \kappa=8 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( N=583 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| M12  | ![Image](attachment) | ![Bar Chart](attachment) |
| \( \kappa=20 \) |  |  |
| \( N=368 \) |  |  |

| M13  | ![Image](attachment) | ![Bar Chart](attachment) |
| \( \kappa=50 \) |  |  |
| \( N=479 \) |  |  |

| M14  | ![Image](attachment) | ![Bar Chart](attachment) |
| \( \kappa=100 \) |  |  |
| \( N=447 \) |  |  |

| M15  | ![Image](attachment) | ![Bar Chart](attachment) |
| \( \kappa=20000 \) |  |  |
| \( N=337 \) |  |  |

| M16  | ![Image](attachment) | ![Bar Chart](attachment) |
| \( \kappa=\text{const.} \) |  |  |
| \( N=209 \) |  |  |

**Figure 4.3:** Block shape diagrams and block shape distribution plots for modified dispersion simulations.
In the persistence models with constant spacing, most of the rock masses generated are dominated by elongated blocks (Figure 4.4). This is visible in the BSCM diagrams and the shape distribution plots. As with the other simulations (spacing and dispersion), different block shapes were produced, most of them being cubic and cubic-elongated. As the mean persistence is increased, blocks become more elongated and then cubic for the largest persistence value. This occurs because longer fractures have more probability of intersecting each other at regular intervals, therefore increasing the potential for generating more equidimensional shapes. Some platy-cubic, platy and platy elongated blocks were also produced, but most of the block shapes were again concentrated on the left side of the BSCM diagram.

Persistence models with constant fracture count produce similar block shape distribution patterns to the persistence models with constant spacing (Figure 4.5). In these simulations however, the blocks formed by short joint lengths (3 and 4m) are concentrated mostly in the elongated categories with no cubic and platy blocks being generated. As persistence is increased, more cubic blocks and less elongated blocks are produced.

Almost all models have most of the block shapes concentrated on the left side of the block shape diagram, i.e. cubic, cubic-elongated and elongated shapes. This can be attributed to the assumptions utilized for the modeling. The properties of all three joint sets were the same and only one variable was changed at a time. Since all the properties were the same, it is expected that the blocks generated will be approximately equidimensional. The generation of other shapes is a consequence of the range of values used for the variables (spacing, dispersion: and persistence). Kalenchuk et al. (2006) emphasized that in order to generate different ranges of shapes in a three joint set model, the values of one or two joint sets need
to be different to the remaining joint set or sets. This was not the case for the conditions adopted in this thesis.

<table>
<thead>
<tr>
<th>ID</th>
<th>Block Shape Diagram</th>
<th>Shape Distribution</th>
</tr>
</thead>
<tbody>
<tr>
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<td>![image2]</td>
</tr>
<tr>
<td>2m</td>
<td>![image3]</td>
<td>![image4]</td>
</tr>
<tr>
<td>N=47</td>
<td>![image5]</td>
<td>![image6]</td>
</tr>
<tr>
<td>M18</td>
<td><img src="image7" alt="" /></td>
<td>![image8]</td>
</tr>
<tr>
<td>4m</td>
<td>![image9]</td>
<td>![image10]</td>
</tr>
<tr>
<td>N=118</td>
<td>![image11]</td>
<td>![image12]</td>
</tr>
<tr>
<td>M3</td>
<td><img src="image13" alt="" /></td>
<td>![image14]</td>
</tr>
<tr>
<td>7m</td>
<td>![image15]</td>
<td>![image16]</td>
</tr>
<tr>
<td>N=244</td>
<td>![image17]</td>
<td>![image18]</td>
</tr>
<tr>
<td>M19</td>
<td><img src="image19" alt="" /></td>
<td>![image20]</td>
</tr>
<tr>
<td>11m</td>
<td>![image21]</td>
<td>![image22]</td>
</tr>
<tr>
<td>N=200</td>
<td>![image23]</td>
<td>![image24]</td>
</tr>
<tr>
<td>M20</td>
<td><img src="image25" alt="" /></td>
<td>![image26]</td>
</tr>
<tr>
<td>15m</td>
<td>![image27]</td>
<td>![image28]</td>
</tr>
<tr>
<td>N=457</td>
<td>![image29]</td>
<td>![image30]</td>
</tr>
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</table>

**Figure 4.4:** Block shape diagrams and block shape distribution plots for persistence simulations with constant spacing.
<table>
<thead>
<tr>
<th>ID</th>
<th>Block Shape Diagram</th>
<th>Shape Distribution</th>
</tr>
</thead>
<tbody>
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<td>M21</td>
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<td><img src="image2" alt="Shape Distribution" /></td>
</tr>
<tr>
<td>2m</td>
<td>Beta</td>
<td>% of Total Amount of Blocks</td>
</tr>
<tr>
<td>N=0</td>
<td>Alpha</td>
<td>C</td>
</tr>
<tr>
<td>M22</td>
<td><img src="image3" alt="Block Shape Diagram" /></td>
<td><img src="image4" alt="Shape Distribution" /></td>
</tr>
<tr>
<td>3m</td>
<td>Beta</td>
<td>% of Total Amount of Blocks</td>
</tr>
<tr>
<td>N=3</td>
<td>Alpha</td>
<td>C</td>
</tr>
<tr>
<td>M23</td>
<td><img src="image5" alt="Block Shape Diagram" /></td>
<td><img src="image6" alt="Shape Distribution" /></td>
</tr>
<tr>
<td>4m</td>
<td>Beta</td>
<td>% of Total Amount of Blocks</td>
</tr>
<tr>
<td>N=9</td>
<td>Alpha</td>
<td>C</td>
</tr>
<tr>
<td>M3</td>
<td><img src="image7" alt="Block Shape Diagram" /></td>
<td><img src="image8" alt="Shape Distribution" /></td>
</tr>
<tr>
<td>7m</td>
<td>Beta</td>
<td>% of Total Amount of Blocks</td>
</tr>
<tr>
<td>N=244</td>
<td>Alpha</td>
<td>C</td>
</tr>
<tr>
<td>M24</td>
<td><img src="image9" alt="Block Shape Diagram" /></td>
<td><img src="image10" alt="Shape Distribution" /></td>
</tr>
<tr>
<td>11m</td>
<td>Beta</td>
<td>% of Total Amount of Blocks</td>
</tr>
<tr>
<td>N=783</td>
<td>Alpha</td>
<td>C</td>
</tr>
<tr>
<td>M25</td>
<td><img src="image11" alt="Block Shape Diagram" /></td>
<td><img src="image12" alt="Shape Distribution" /></td>
</tr>
<tr>
<td>15m</td>
<td>Beta</td>
<td>% of Total Amount of Blocks</td>
</tr>
<tr>
<td>N=1930</td>
<td>Alpha</td>
<td>C</td>
</tr>
</tbody>
</table>

**Figure 4.5:** Block shape diagrams and block shape distribution plots for persistence simulations with constant fracture count.
4.2 Block Failure Mode

FracMan can provide information about the mode of failure of each block as part of its output. The failure types are divided into four categories: free falling, sliding on one plane, sliding on two planes and stable. The factor of safety of blocks sliding on one or two faces is estimated using the Mohr-Coulomb failure criterion (chapter 3.1). FracMan classifies the block failures in four types: the free falling, sliding on one plane blocks, sliding on two planes and stable blocks. Free falling blocks have a factor of safety less than one. Blocks sliding on one or two planes can have factors of safety greater or less than one. Stable blocks have a factor of safety greater than one. Under this definition, blocks free falling and sliding (on one or two planes) with a factor of safety less than one are considered key blocks (unstable block volume). The rest of the blocks are potential key blocks or tapered blocks (stable block volume). The results for the analysis of each model are described as a percentage of the total block volume generated and as a percentage of the total number of blocks. Only blocks larger than 0.001 m³ were examined.

Table 4.1 and Figure 4.6a and 4.6b show the failure type occurrence for the spacing simulations. Most of the block volume generated is stable with very little volume corresponding to blocks sliding on two planes Figure 4.6a. The unstable block volume is mostly composed of blocks sliding on one face with the free falling block volume representing approximately 20% of the total unstable block volume. By comparing the amount of blocks corresponding to each failure type (Figure 4.6b) to the volumes in Figure 4.6a, it can be inferred that the stable blocks have the largest volumes and the blocks sliding on two faces have the smallest. It can also be observed that most the blocks for model M5 are sliding on one face, however the largest volume is still represented by the stable blocks. The
small occurrence of blocks sliding on two faces is due mainly to three factors: the orthogonal nature of the model, the orientation and the dispersion of the joint sets.

**Table 4.1**: Values for failure modes for blocks generated during the spacing simulations.

<table>
<thead>
<tr>
<th></th>
<th>M1 (0.75m)</th>
<th>M2 (1.35m)</th>
<th>M3 (2m)</th>
<th>M4 (3m)</th>
<th>M5 (4m)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>As % of Total Block Volume</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Free Falling</td>
<td>0.9</td>
<td>1.0</td>
<td>1.8</td>
<td>3.4</td>
<td>7.7</td>
</tr>
<tr>
<td>Sliding on one Face</td>
<td>4.1</td>
<td>5.4</td>
<td>7.9</td>
<td>15.1</td>
<td>30.2</td>
</tr>
<tr>
<td>Sliding on two Faces</td>
<td>0.0</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>9.6</td>
</tr>
<tr>
<td>Stable</td>
<td>95.0</td>
<td>93.5</td>
<td>90.2</td>
<td>81.4</td>
<td>52.5</td>
</tr>
<tr>
<td><strong>As % of Total Number of Blocks</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Free Falling</td>
<td>6.2</td>
<td>8.3</td>
<td>10.7</td>
<td>13.7</td>
<td>18.7</td>
</tr>
<tr>
<td>Sliding on one Face</td>
<td>23.2</td>
<td>29.3</td>
<td>34.5</td>
<td>38.7</td>
<td>33.4</td>
</tr>
<tr>
<td>Sliding on two Faces</td>
<td>0.8</td>
<td>1.1</td>
<td>1.6</td>
<td>1.9</td>
<td>27.2</td>
</tr>
<tr>
<td>Stable</td>
<td>69.8</td>
<td>61.2</td>
<td>53.2</td>
<td>45.7</td>
<td>20.6</td>
</tr>
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</table>
Figure 4.6: Failure modes for blocks generated during the spacing simulations. (a) Failure mode as percentage of total block volume. (b) Failure mode as percentage of total number of blocks.

The results for the original dispersion models are shown in Table 4.2. For most of the models (models M6 to M9) the majority of the blocks and the block volume were stable.
(Figure 4.7a, b). Approximately 75% of the unstable blocks are sliding on one face, representing close to 80% of the potentially unstable block volume for models M6 to M9. In model M10 a shift in the block stability can be observed. Most of the blocks and the block volume generated become unstable, with a large percentage of the unstable blocks and unstable block volume being made up of free falling blocks. This can be explained by relating the block failure type to the block characterization carried out in chapter 4.1. The model becomes perfectly orthogonal for model M10, with two joint sets perpendicular between each other and perpendicular to the horizontal plane described by the undercut, and a third horizontal joint set parallel to the undercut. Therefore, the great majority of the blocks have cubic or near cubic shape (as observed in chapter 4.1) being then able to free fall. It must be emphasized that this is a kinematic gravitational analysis alone and no in-situ stresses are considered which might provide a “clamping” effect on the blocks. Conversely, this shows the importance of destressing on stability. It is also important to notice that as the $\kappa$ value increases, the occurrence of blocks sliding on two faces decreases, thus confirming the role of the dispersion and orientation of the joint sets in the generation of that type of block failure.

**Table 4.2:** Values for failure modes for blocks generated during the dispersion simulations.

<table>
<thead>
<tr>
<th></th>
<th>M6 $(k=8)$</th>
<th>M3 $(k=20)$</th>
<th>M7 $(k=50)$</th>
<th>M8 $(k=100)$</th>
<th>M9 $(k=20000)$</th>
<th>M10 $(k=const.)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>As % of Total Block Volume</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Free Falling</td>
<td>1.6</td>
<td>1.8</td>
<td>2.3</td>
<td>2.6</td>
<td>3.7</td>
<td>70.9</td>
</tr>
<tr>
<td>Sliding on one Face</td>
<td>7.6</td>
<td>7.9</td>
<td>8.0</td>
<td>9.8</td>
<td>11.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Sliding on two Faces</td>
<td>0.1</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Stable</td>
<td>90.7</td>
<td>90.2</td>
<td>89.7</td>
<td>87.6</td>
<td>85.2</td>
<td>29.0</td>
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<td><strong>As % of Total Number of Blocks</strong></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Free Falling</td>
<td>10.5</td>
<td>10.7</td>
<td>9.8</td>
<td>9.5</td>
<td>6.3</td>
<td>94.8</td>
</tr>
<tr>
<td>Sliding on one Face</td>
<td>33.5</td>
<td>34.5</td>
<td>31.4</td>
<td>29.4</td>
<td>17.9</td>
<td>0.1</td>
</tr>
<tr>
<td>Sliding on two Faces</td>
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<td>1.6</td>
<td>1.1</td>
<td>0.6</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Stable</td>
<td>54.3</td>
<td>53.2</td>
<td>57.8</td>
<td>60.5</td>
<td>75.8</td>
<td>5.1</td>
</tr>
</tbody>
</table>
Figure 4.7: Failure modes for blocks generated during the dispersion simulations. (a) Failure mode as percentage of total block volume. (b) Failure mode as percentage of total number of blocks.
The modified dispersion models (Table 4.3) show similarities with the original dispersion models. Predominantly stable blocks and block volume for most of the models (M11 to M15) is indicated. There is however a larger proportion of blocks that slide on one face than in the original dispersion simulations, due to the fact that the orientation of two joint sets is different (Figure 4.8a, b). The new joint orientations also have an effect on the blocks in the model with constant $k$. Even though the block shape distribution is very similar between the modified dispersion and the original dispersion models as observed in chapter 4.1 (Figure 4.3), approximately half of the blocks potentially unstable are blocks sliding on one face (in contrast to the original dispersion simulations where most of them were free falling). There are also more stable blocks when compared to the original dispersion models. As two of the perpendicular joint sets are now at 45 degrees to the horizontal plane described by the undercut, with the third set perpendicular to the undercut and the other two sets, more of the generated blocks become tapered and “potential key blocks”. In the modified dispersion models the blocks sliding on two faces also decrease as $k$ increases, further confirming the role of dispersion in the generation of that type of block failure.

**Table 4.3**: Values for failure modes for blocks generated during the modified dispersion simulations.

<table>
<thead>
<tr>
<th></th>
<th>M11 ($k=8$)</th>
<th>M12 ($k=20$)</th>
<th>M13 ($k=50$)</th>
<th>M14 ($k=100$)</th>
<th>M15 ($k=20000$)</th>
<th>M16 ($k=\text{const.}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>As % of Total Block Volume</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Free Falling</td>
<td>0.9</td>
<td>0.9</td>
<td>0.1</td>
<td>1.3</td>
<td>1.8</td>
<td>9.8</td>
</tr>
<tr>
<td>Sliding on one Face</td>
<td>4.9</td>
<td>6.4</td>
<td>1.8</td>
<td>7.6</td>
<td>8.5</td>
<td>27.4</td>
</tr>
<tr>
<td>Sliding on two Faces</td>
<td>0.1</td>
<td>0.1</td>
<td>7.9</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Stable</td>
<td>94.1</td>
<td>92.6</td>
<td>90.2</td>
<td>91.1</td>
<td>89.7</td>
<td>62.9</td>
</tr>
<tr>
<td>As % of Total Number of Blocks</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Free Falling</td>
<td>9.2</td>
<td>9.8</td>
<td>12.7</td>
<td>9.6</td>
<td>9.4</td>
<td>38.9</td>
</tr>
<tr>
<td>Sliding on one Face</td>
<td>31.1</td>
<td>32.6</td>
<td>37.7</td>
<td>32.1</td>
<td>30.9</td>
<td>43.0</td>
</tr>
<tr>
<td>Sliding on two Faces</td>
<td>1.7</td>
<td>1.5</td>
<td>2.2</td>
<td>0.8</td>
<td>0.1</td>
<td>0.0</td>
</tr>
<tr>
<td>Stable</td>
<td>58.0</td>
<td>56.1</td>
<td>47.5</td>
<td>57.4</td>
<td>59.7</td>
<td>18.1</td>
</tr>
</tbody>
</table>
Figure 4.8: Failure modes for blocks generated during the modified dispersion simulations. (a) Failure mode as percentage of total block volume. (b) Failure mode as percentage of total number of blocks.
Most of the block volume for all simulations in the persistence modeling with constant spacing comprises stable blocks (Table 4.4 and Figure 4.9a). Although for short to medium persistence (2, 4 and 7m) there are a greater percentage of potentially unstable blocks (Figure 4.9b). The ratio between sliding and free falling blocks decreases as persistence decreases, i.e. as fractures become shorter there are more free falling blocks, both in block quantity and block volume. By looking at the evolution of the block shapes (chapter 4.1) and relating it to the stability observed for these simulations, it can be speculated that the increase of stability with longer fractures is due to the presence of more equilateral (cubic) blocks. As observed in previous simulations (spacing, dispersion), there are blocks sliding on two faces, but their quantity is very small when compared to the other types of failure, and their presence is attributed to variations in the dispersion.

Table 4.4: Values for failure modes for blocks generated during the persistence with constant spacing simulations.

<table>
<thead>
<tr>
<th></th>
<th>M17 (2m)</th>
<th>M18 (4m)</th>
<th>M3 (7m)</th>
<th>M19 (11m)</th>
<th>M20 (15m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>As % of Total Block Volume</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Free Falling</td>
<td>10.4</td>
<td>5.4</td>
<td>1.4</td>
<td>0.1</td>
<td>1.4</td>
</tr>
<tr>
<td>Sliding on one Face</td>
<td>25.5</td>
<td>14.3</td>
<td>6.2</td>
<td>1.4</td>
<td>5.7</td>
</tr>
<tr>
<td>Sliding on two Faces</td>
<td>0.7</td>
<td>0.1</td>
<td>0.1</td>
<td>6.2</td>
<td>0.1</td>
</tr>
<tr>
<td>Stable</td>
<td>63.4</td>
<td>80.2</td>
<td>92.3</td>
<td>92.3</td>
<td>92.8</td>
</tr>
<tr>
<td>As % of Total Number of Blocks</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Free Falling</td>
<td>16.1</td>
<td>13.0</td>
<td>10.7</td>
<td>9.9</td>
<td>8.9</td>
</tr>
<tr>
<td>Sliding on one Face</td>
<td>45.1</td>
<td>39.1</td>
<td>34.5</td>
<td>30.8</td>
<td>29.9</td>
</tr>
<tr>
<td>Sliding on two Faces</td>
<td>2.4</td>
<td>2.1</td>
<td>1.6</td>
<td>1.3</td>
<td>1.4</td>
</tr>
<tr>
<td>Stable</td>
<td>36.5</td>
<td>45.7</td>
<td>53.2</td>
<td>57.9</td>
<td>59.9</td>
</tr>
</tbody>
</table>
Figure 4.9: Failure modes for blocks generated during the persistence with constant spacing simulations. (a) Failure mode as percentage of total block volume. (b) Failure mode as percentage of total number of blocks.
As in the persistence modeling with constant spacing, most of the block volume for the persistence modeling with constant fracture count is comprised of stable blocks (Table 4.5 and Figure 4.10a). Most of the potentially unstable blocks are sliding on one face, with a ratio between sliding and free falling blocks that remains approximately constant for all the models. Following the same pattern as in the persistence models with constant spacing, the quantity of stable blocks is less than that for the unstable blocks for short and medium persistence (2, 3, 4 and 7m). This changes as persistence increases (Figure 4.10b). Again, this is related to the presence of more cubic blocks in models with longer fractures.

Table 4.5: Values for failure modes for blocks generated during the persistence with constant fracture count simulations.

<table>
<thead>
<tr>
<th></th>
<th>M21 (2m)</th>
<th>M22 (3m)</th>
<th>M23 (4m)</th>
<th>M3 (7m)</th>
<th>M24 (11m)</th>
<th>M25 (15m)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>As % of Total Block Volume</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Free Falling</td>
<td>0.0</td>
<td>4.6</td>
<td>4.4</td>
<td>1.8</td>
<td>1.0</td>
<td>0.9</td>
</tr>
<tr>
<td>Sliding on one Face</td>
<td>0.0</td>
<td>16.6</td>
<td>12.7</td>
<td>7.9</td>
<td>4.5</td>
<td>4.2</td>
</tr>
<tr>
<td>Sliding on two Faces</td>
<td>0.0</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Stable</td>
<td>0.0</td>
<td>78.5</td>
<td>82.8</td>
<td>90.2</td>
<td>94.5</td>
<td>94.9</td>
</tr>
<tr>
<td><strong>As % of Total Number of Blocks</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Free Falling</td>
<td>0.0</td>
<td>22.4</td>
<td>19.4</td>
<td>10.7</td>
<td>8.2</td>
<td>6.4</td>
</tr>
<tr>
<td>Sliding on one Face</td>
<td>0.0</td>
<td>52.8</td>
<td>49.8</td>
<td>34.5</td>
<td>27.6</td>
<td>29.9</td>
</tr>
<tr>
<td>Sliding on two Faces</td>
<td>0.0</td>
<td>3.1</td>
<td>2.5</td>
<td>1.6</td>
<td>0.7</td>
<td>1.0</td>
</tr>
<tr>
<td>Stable</td>
<td>0.0</td>
<td>21.7</td>
<td>28.2</td>
<td>53.2</td>
<td>63.5</td>
<td>62.6</td>
</tr>
</tbody>
</table>
Figure 4.10: Failure modes for blocks generated during the persistence with constant fracture count simulations. (a) Failure mode as percentage of total block volume. (b) Failure mode as percentage of total number of blocks.
4.3 Block Size Distributions

The block size distribution analysis was based on a modified version of Laubscher's (2000) descriptions of the potential effects of the fragmentation size in block caving operations (Table 4.6). Category A' includes all blocks smaller than 0.25m$^3$ (100% of the blocks with pass through a 1.5 x 0.3m grizzly); category B' consists of all blocks larger than 0.25m$^3$ and smaller than 2m$^3$ (100% of the blocks fit into an LHD bucket); category C' comprises all blocks larger than 2m$^3$ and smaller than 128m$^3$ (drawpoint blocking blocks); and category D' is all the blocks bigger than 128m$^3$ (drawbell blocking blocks). Categories C' and D' directly affect production in block caving mines because they stop the extraction of ore from the drawpoints, requiring removal and causing production delays.

The block size distributions for each of the models were plotted as “percentage finer by volume” against block volume. This allows for an approximate estimation of the probable effects of the block sizes based on the average distributions obtained for each model. Since only the initial stages of caving are being studied in this thesis, only the block size distribution of the primary fragmentation is analyzed. The block size distribution evaluation includes all blocks, stable and unstable. FracMan generated blocks of volumes as small as 0.000001m$^3$. These volumes were considered unrealistic for primary fragmentation, therefore only blocks larger than 0.001 m$^3$ were examined. This limit was chosen based on work performed by Butcher and Thin (2007), and Laubscher (2000).

In order to assess the grading and curvature of the block size distributions, the coefficient of uniformity ($C_u$) and the coefficient of curvature ($C_c$) were calculated. The potential for the use of these factors as an indicator of fragmentation was assessed. The $C_u$ measures the variation in particle sizes. Steep curves, reflecting poorly graded mixtures have low $C_u$. 

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values, while flat curves reflecting well graded mixtures (or well sorted, in geological terms) have high values (Coduto, 1999). The Cc value is based on the following formula:

\[ C_u = \frac{D_{60}}{D_{10}} \]

where \( D_{60} \) corresponds to the 60 percent passing (i.e. 60 percent of the blocks finer than \( D_{60} \)) and \( D_{10} \) is the 10 percent passing. The Cc describes the shape of the gradation curve. For instance, materials with smooth curves have Cc values between 1 and 3, while most gap graded materials have values outside this range (Coduto, 1999). The Cc is defined as follows:

\[ C_c = \frac{(D_{30})^2}{D_{10}D_{60}} \]

where \( D_{30} \) is the 30 percent passing, \( D_{10} \) is the 10 percent passing and \( D_{60} \) is the 60 percent passing.

<table>
<thead>
<tr>
<th>Rock Fragmentation Size</th>
<th>Modified Rock Fragmentation Size Classification</th>
<th>Potential Effects</th>
<th>Length Range (m)</th>
<th>Mean Length L (m)</th>
<th>Mean Volume L x L/2 x L/2 (m³)</th>
<th>Maximum Volume (m³)</th>
<th>Drawpoint Images</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A'</td>
<td>100% through 1.5m x 0.3m grizzly</td>
<td>&lt;0.5</td>
<td>0.25</td>
<td>0.004</td>
<td>0.031</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>B'</td>
<td>100% into LHD bucket</td>
<td>0.5 to 1.0</td>
<td>0.75</td>
<td>0.11</td>
<td>0.25</td>
<td>(1)</td>
</tr>
<tr>
<td>C</td>
<td>C'</td>
<td>Hang-up in drawpoint throat</td>
<td>1.0 to 2.0</td>
<td>1.5</td>
<td>0.8</td>
<td>2</td>
<td>(2)</td>
</tr>
<tr>
<td>D</td>
<td>D'</td>
<td>High hang-up</td>
<td>2.0 to 4.0</td>
<td>3</td>
<td>7</td>
<td>16</td>
<td>(3)</td>
</tr>
<tr>
<td>E</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>D'</td>
<td>Drawbell blocker</td>
<td>8.0 to 16</td>
<td>12</td>
<td>432</td>
<td>1024</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td></td>
<td>Double drawbell blocker</td>
<td>&gt;16</td>
<td>24</td>
<td>3456</td>
<td>&gt;1024</td>
<td></td>
</tr>
</tbody>
</table>
In the spacing models (Table 4.7 and Figure 4.11), block volumes increase with increasing spacing, except for the last simulation. This is attributed to the boundary conditions relating spacing and the model’s scale. As expected, the likelihood of drawbell hang-ups increases with increase in block volume, reaching the largest drawbell hang-up potential in simulation M4. These percentages more than triple between models M1 and M4. Drawbell blocking potential is very low for all simulations. The coefficient of uniformity values are low for all models which reflects the relatively steep block size distribution curve. This is further confirmed by the coefficient of curvature values. However, no meaningful relationship can be observed between the coefficient of uniformity, the coefficient of curvature and the increase in spacing that might be related to the increase in block volume.

**Table 4.7:** Percentage of the total block volume generated in the spacing simulations for each of the classification groups.

<table>
<thead>
<tr>
<th>Model</th>
<th>Modified Rock Fragmentation Size Classification</th>
<th>Cu</th>
<th>Cc</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A' (% of Total)</td>
<td>B' (% of Total)</td>
<td>C' (% of Total)</td>
</tr>
<tr>
<td>M1 (0.75m)</td>
<td>75.0</td>
<td>19.1</td>
<td>5.8</td>
</tr>
<tr>
<td>M2 (1.35m)</td>
<td>61.0</td>
<td>24.3</td>
<td>14.6</td>
</tr>
<tr>
<td>M3 (2m)</td>
<td>58.3</td>
<td>24.6</td>
<td>16.9</td>
</tr>
<tr>
<td>M4 (3m)</td>
<td>57.7</td>
<td>24.1</td>
<td>18.1</td>
</tr>
<tr>
<td>M5 (4m)</td>
<td>61.1</td>
<td>25.1</td>
<td>13.7</td>
</tr>
<tr>
<td>Effect</td>
<td>No Production Problems</td>
<td>Hang Up</td>
<td>Drawbell Block</td>
</tr>
</tbody>
</table>
The average block volume distributions determined for dispersion (Table 4.8 and Figure 4.12) showed an increase in block size with increasing $\kappa$ values except for models M10. Referring to the observations in chapter 4.1, the cubic or equidimensional blocks in the model with constant dispersion seem to have smaller volumes than those that are slightly more irregular blocks (like the blocks observed in model M9). All models have the potential of having hang-ups, with the amount of drawpoint blocking blocks more than doubling between model M6 and M10. The data shows that hang-ups will be more problematic in ore bodies with higher $\kappa$ values. The amount of drawbell blocking blocks is very low, i.e. accounts for just 0.4% of the total volume of blocks in the worst case (model M9). As in the spacing modeling, the values for the coefficient of uniformity were low, being the values for
model M9 and M10 were lower than the rest of the models. The coefficient of curvature also remains low. This is a sign of well sorted block distribution curves.

**Table 4.8:** Percentage of the total block volume generated in the dispersion simulations for each of the classification groups.

<table>
<thead>
<tr>
<th>Model</th>
<th>Modified Rock Fragmentation Size Classification</th>
<th>Cu</th>
<th>Cc</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A' (% of Total)</td>
<td>B' (% of Total)</td>
<td>C' (% of Total)</td>
</tr>
<tr>
<td>M6 (k=8)</td>
<td>61.8</td>
<td>23.7</td>
<td>14.3</td>
</tr>
<tr>
<td>M3 (k=20)</td>
<td>58.3</td>
<td>24.6</td>
<td>16.9</td>
</tr>
<tr>
<td>M7 (k=50)</td>
<td>53.6</td>
<td>26.7</td>
<td>19.4</td>
</tr>
<tr>
<td>M8 (k=100)</td>
<td>52.6</td>
<td>26.3</td>
<td>20.8</td>
</tr>
<tr>
<td>M9 (k=20000)</td>
<td>37.9</td>
<td>32.4</td>
<td>29.4</td>
</tr>
<tr>
<td>M10 (k=const.)</td>
<td>39.1</td>
<td>32.3</td>
<td>28.4</td>
</tr>
</tbody>
</table>

**Figure 4.12:** Average block size distribution chart for dispersion simulations.

As with the original dispersion models, the modified dispersion simulations also show increasing block volume with increasing $k$ values except for model M16 (Table 4.9 and
Figure 4.13). The reason for this is the same as for the original dispersion models. The magnitude of the volume change between models for all size classifications is not as large as in the original dispersion simulations. Nevertheless, the amount of blocks with hang up and drawpoint blocking potential almost doubled between simulation M11 and M16. The amount of blocks with drawbell blocking potential remains very low (e.g. 0.3% of the total volume of blocks for simulation M15). Similar to what was observed in the original dispersion models, the coefficient of uniformity is low. The coefficient of curvature also remains low (most of the values below one). No correlation between the coefficient of uniformity, dispersion and fragmentation was observed. But the coefficient of curvature values increase with increasing \( k \).

Table 4.9: Percentage of the total block volume generated in the modified dispersion simulations for each of the classification groups.

<table>
<thead>
<tr>
<th>Model</th>
<th>Modified Rock Fragmentation Size Classification</th>
<th>Cu</th>
<th>Cc</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A' (% of Total)</td>
<td>B' (% of Total)</td>
<td>C' (% of Total)</td>
</tr>
<tr>
<td>M11 (k=8)</td>
<td>63.9</td>
<td>23.2</td>
<td>12.8</td>
</tr>
<tr>
<td>M12 (k=20)</td>
<td>61.2</td>
<td>23.9</td>
<td>14.8</td>
</tr>
<tr>
<td>M13 (k=50)</td>
<td>60.1</td>
<td>24.9</td>
<td>14.9</td>
</tr>
<tr>
<td>M14 (k=50)</td>
<td>57.5</td>
<td>26.0</td>
<td>16.3</td>
</tr>
<tr>
<td>M15 (k=20000)</td>
<td>50.0</td>
<td>29.3</td>
<td>20.3</td>
</tr>
<tr>
<td>M16 (k=const.)</td>
<td>48.8</td>
<td>30.7</td>
<td>20.3</td>
</tr>
<tr>
<td>Effect</td>
<td>No Production Problems</td>
<td>Hang Up</td>
<td>Drawbell Block</td>
</tr>
</tbody>
</table>

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Figure 4.13: Block size distribution chart for modified dispersion simulations.

The block volume distributions for the persistence models with constant spacing are described in Table 4.10 and Figure 4.14. As expected, the block volume is small for simulations with short joints and increases as fractures become longer. All of the simulations show hang up potential, but for models with short fractures (M17 and M18) this potential is very low. However, it increases by more than 20 fold between M17 and M20. The drawbell blocking potential was still low, but it reached the largest value for all the models tested in simulations M19 and M20 (0.5% of the total). The values for $C_u$ and $C_c$ indicate well sorted materials, with the $C_u$ and $C_c$ values being low. An increasing trend in the $C_c$ values was observed with increasing persistence, with large changes occurring between models M17 and M18, and models M18 and M3.
Table 4.10: Percentage of the total block volume generated in the persistence with constant spacing simulations for each of the classification groups.

<table>
<thead>
<tr>
<th>Model</th>
<th>Modified Rock Fragmentation Size Classification</th>
<th>Cu</th>
<th>Cc</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A' (% of Total)</td>
<td>B' (% of Total)</td>
<td>C' (% of Total)</td>
</tr>
<tr>
<td>M17 (2m)</td>
<td>88.8</td>
<td>9.6</td>
<td>1.5</td>
</tr>
<tr>
<td>M18 (4m)</td>
<td>79.0</td>
<td>15.4</td>
<td>5.6</td>
</tr>
<tr>
<td>M3 (7m)</td>
<td>58.3</td>
<td>24.6</td>
<td>16.9</td>
</tr>
<tr>
<td>M19 (11m)</td>
<td>52.1</td>
<td>27.0</td>
<td>20.4</td>
</tr>
<tr>
<td>M20 (15m)</td>
<td>50.4</td>
<td>26.7</td>
<td>22.4</td>
</tr>
</tbody>
</table>

Effect: No Production Problems, Hang Up, Drawbell Block

Figure 4.14: Block size distribution chart for persistence with constant spacing simulations.

By reviewing the block volume distributions for the persistence models with constant fracture count (Table 4.11 and Figure 4.15), it is evident that the effect of joint length on the block size is very similar to that in the persistence models with constant spacing. Block size becomes larger with increasing joint length, except for the last simulation. As in the spacing models, this is attributed to the boundary conditions relating spacing, persistence and the
model’s scale. The amount of blocks with drawpoint hang-up potential increases by more than 15 times between model M22 and model M24, again confirming the observations made for the persistence models with constant spacing regarding the impact of persistence on the block volume. Blocks with drawbell blocking potential are few (between 0 and 0.2% of the total number of blocks). No meaningful trend regarding the $C_u$ and $C_c$ values was observed although there is a slight tendency for the $C_u$ and $C_c$ values to become larger with increasing persistence.

Table 4.11: Percentage of the total block volume generated in the persistence with constant fracture count simulations for each of the classification groups.

<table>
<thead>
<tr>
<th>Model</th>
<th>Modified Rock Fragmentation Size Classification</th>
<th>$A'$ (% of Total)</th>
<th>$B'$ (% of Total)</th>
<th>$C'$ (% of Total)</th>
<th>$D'$ (% of Total)</th>
<th>$Cu$</th>
<th>$Cc$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M21 (2m)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td>M22 (3m)</td>
<td>84.5</td>
<td>14.3</td>
<td>1.2</td>
<td>0.0</td>
<td>0.0</td>
<td>23</td>
<td>0.61</td>
</tr>
<tr>
<td>M23 (4m)</td>
<td>75.0</td>
<td>18.6</td>
<td>6.4</td>
<td>0.0</td>
<td>0.0</td>
<td>39</td>
<td>0.77</td>
</tr>
<tr>
<td>M3 (7m)</td>
<td>58.3</td>
<td>24.6</td>
<td>16.9</td>
<td>0.2</td>
<td>0.0</td>
<td>87</td>
<td>0.72</td>
</tr>
<tr>
<td>M24 (11m)</td>
<td>57.3</td>
<td>25.4</td>
<td>17.1</td>
<td>0.2</td>
<td>0.0</td>
<td>79</td>
<td>0.79</td>
</tr>
<tr>
<td>M25 (15m)</td>
<td>74.9</td>
<td>25.3</td>
<td>13.7</td>
<td>0.1</td>
<td>0.0</td>
<td>71</td>
<td>0.71</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Effect</th>
<th>No Production Problems</th>
<th>Hang Up</th>
<th>Drawbell Block</th>
</tr>
</thead>
</table>

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Figure 4.15: Block size distribution chart for persistence with constant fracture count simulations.

The information gathered during the modeling suggests that of all the variables analyzed, fracture persistence has the largest influence on the generation of drawbell blocking block sizes (Table 4.12). This is followed by spacing, and then fracture dispersion. Results shown for spacing, dispersion and persistence where fragmentation size does not always increase as the value of the variables is increased, illustrates how block size is a function of the boundary conditions imposed by the spacing, persistence, dispersion and undercut size. No correlation was found between the coefficient of uniformity, the coefficient of curvature and the effects of block sizes in block cave mining production.
Table 4.12: Summary of the block size distributions for all simulations.

<table>
<thead>
<tr>
<th>Model</th>
<th>Modified Rock Fragmentation Size Classification</th>
<th>Cu</th>
<th>Cc</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A' (% of Total) B' (% of Total) C' (% of Total) D' (% of Total)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1 (0.75m)</td>
<td>75.0 19.1 5.8 0.0</td>
<td>38</td>
<td>0.78</td>
</tr>
<tr>
<td>M2 (1.35m)</td>
<td>61.0 24.3 14.6 0.1</td>
<td>86</td>
<td>0.64</td>
</tr>
<tr>
<td>M3 (2m)</td>
<td>58.3 24.6 16.9 0.2</td>
<td>87</td>
<td>0.72</td>
</tr>
<tr>
<td>M4 (3m)</td>
<td>57.7 24.1 18.1 0.1</td>
<td>79</td>
<td>0.64</td>
</tr>
<tr>
<td>M5 (4m)</td>
<td>61.1 25.1 13.7 0.1</td>
<td>64</td>
<td>0.70</td>
</tr>
<tr>
<td>M6 (k=8)</td>
<td>61.8 23.7 14.3 0.1</td>
<td>87</td>
<td>0.62</td>
</tr>
<tr>
<td>M8 (k=100)</td>
<td>52.6 26.3 20.8 0.3</td>
<td>81</td>
<td>0.78</td>
</tr>
<tr>
<td>M9 (k=20000)</td>
<td>37.9 32.4 29.4 0.4</td>
<td>52</td>
<td>1.05</td>
</tr>
<tr>
<td>M10 (k=const.</td>
<td>39.1 32.3 28.4 0.3</td>
<td>59</td>
<td>1.15</td>
</tr>
<tr>
<td>M11 (k=8)</td>
<td>63.9 23.2 12.8 0.1</td>
<td>67</td>
<td>0.67</td>
</tr>
<tr>
<td>M12 (k=20)</td>
<td>61.2 23.9 14.8 0.2</td>
<td>87</td>
<td>0.68</td>
</tr>
<tr>
<td>M13 (k=50)</td>
<td>60.1 24.9 14.9 0.2</td>
<td>88</td>
<td>0.77</td>
</tr>
<tr>
<td>M14 (k=50)</td>
<td>57.5 26.0 16.3 0.2</td>
<td>91</td>
<td>0.80</td>
</tr>
<tr>
<td>M15 (k=20000)</td>
<td>50.0 29.3 20.3 0.3</td>
<td>72</td>
<td>0.85</td>
</tr>
<tr>
<td>M16 (k=const.)</td>
<td>48.8 30.7 20.3 0.2</td>
<td>50</td>
<td>1.13</td>
</tr>
<tr>
<td>M17 (2m)</td>
<td>88.8 9.6 1.5 0.0</td>
<td>18</td>
<td>0.53</td>
</tr>
<tr>
<td>M18 (4m)</td>
<td>79.0 15.4 5.6 0.0</td>
<td>53</td>
<td>1.15</td>
</tr>
<tr>
<td>M19 (11m)</td>
<td>58.3 24.6 16.9 0.2</td>
<td>87</td>
<td>0.72</td>
</tr>
<tr>
<td>M20 (15m)</td>
<td>52.1 27.0 20.4 0.5</td>
<td>87</td>
<td>0.74</td>
</tr>
<tr>
<td>M21 (2m)</td>
<td>50.4 26.7 22.4 0.5</td>
<td>115</td>
<td>0.67</td>
</tr>
<tr>
<td>M22 (3m)</td>
<td>48.8 30.7 20.3 0.2</td>
<td>50</td>
<td>1.13</td>
</tr>
<tr>
<td>M23 (4m)</td>
<td>88.8 9.6 1.5 0.0</td>
<td>18</td>
<td>0.53</td>
</tr>
<tr>
<td>M24 (11m)</td>
<td>79.0 15.4 5.6 0.0</td>
<td>53</td>
<td>1.15</td>
</tr>
<tr>
<td>M25 (15m)</td>
<td>58.3 24.6 16.9 0.2</td>
<td>87</td>
<td>0.72</td>
</tr>
<tr>
<td>Effect</td>
<td>No Production Problems Hang Up Drawbell Block</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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4.4 Assessment of Apparent Block Volume

As mentioned in section 3.3, Cai et al. (2004) developed a method to account for the persistence of discontinuities to assist in the use of the GSI system for rock mass classification and introduced the concept of apparent block volume. This concept has since been verified by Kim et al. (2007). The apparent block volume is a way of calculating how massive or fragmented the rock mass is. It is based on the fracture persistence expressed as the joint persistence factor (average joint length divided by the characteristic length of the rock mass under consideration), the angle between the joint sets and the spacing (for more detailed information refer to chapter 2.3). According to Cai et al. (2004), the apparent volume should be larger for rock masses with non persistent fractures, i.e. the rock mass should be more massive. Based on the work carried out by Elmo et al. (2008b), the apparent block volume for the persistence modeling with constant spacing was calculated and plotted versus the persistence factor for the 50m box region employed in this thesis (Figure 4.16a). In order to compare the apparent block volume to the true volume obtained from the DFN analysis, Elmo et al. (2008b) developed a quantitative index of the character of blockiness of the rock mass, that uses the inverse of the number of blocks estimated in FracMan. This blockiness character was plotted against the persistence factor for the 50m box region (Figure 4.16b).
Figure 4.16: (a) Apparent block volume against persistence factor for persistence simulations with constant spacing of 2m, (b) Blockiness character against persistence factor for persistence simulations with constant spacing of 2m.

Confirming observations by Elmo et al. (2008b), there is a qualitative agreement between the trends estimated between both indices. The apparent block volume was also calculated for the spacing simulations. The information was plotted against spacing (Figure 4.17a) and
then compared with the blockiness character (inverse of the number of blocks) which was also plotted against spacing (Figure 4.17b). Again a qualitative agreement between the trends calculated with the two indices is observed.

Figure 4.17: (a) Apparent block volume against spacing for spacing simulations with constant persistence of 7m, (b) Blockiness character against spacing factor for spacing simulations with constant persistence of 7m.
There is a similar qualitative agreement observed for the persistence simulations with constant fracture count (Figure 4.18).

**Figure 4.18:** (a) Apparent block volume against persistence factor for persistence simulations with constant fracture count, (b) Blockiness character against spacing factor for persistence simulations with constant fracture count.
The dispersion simulations were again plotted against the persistence factor, however in this case no agreement between the apparent block volume and the blockiness character was observed (Figure 4.19 Figure 4.20) In Figures 4.19a and 4.20a, all values are concentrated at one point because the apparent volume does not take into consideration dispersion. As observed in Figure 4.19b and 4.20b, dispersion has an effect in the blockiness of the rockmass; the blockiness character increasing with increasing $\kappa$. 
**Figure 4.19:** (a) Apparent block volume against persistence factor for varying dispersion simulations with constant persistence of 7m and constant spacing of 2m, (b) Blockiness character against spacing factor for spacing simulations with the original orthogonal model, constant persistence of 7m and constant spacing of 2m.
Figure 4.20: (a) Apparent block volume against persistence factor for varying dispersion simulations with the modified orthogonal model, constant persistence of 7m and constant spacing of 2m, (b) Blockiness character against spacing factor for spacing simulations with constant persistence of 7m and constant spacing of 2m.

The qualitative agreement between the apparent block volume and the blockiness character shows the potential of relating the results obtained using FracMan to existing rock
mass classification systems. However, the apparent block volume needs to be modified to incorporate the effect of fracture dispersion.

4.5 Block Trace Areas and Block Volumes for Generated Models

Comparing the block trace areas on the undercut and block volumes generated can be used to assess the influence on the initial caveability potential of each of the tested variables. To compare the models, the block areas on the undercut were normalized against the total area of the undercut. The volume of blocks generated was also normalized, but to the volume of the ore body or “inner box” (defined in Section 3.1.2.) and expressed as a percentage (%) of total ore body volume. FracMan does not consider the propagation of the cave, so this investigation only relates to the initial stage of caving (i.e. the opening of the undercut). The following parameters were used in the analysis:

1. The total block area – this is the total area of the blocks generated as seen from below the undercut (Figure 4.21);
2. The total unstable block area – this is the area of all blocks with a factor of safety less than one seen in the undercut, i.e. the area on the undercut of the “key blocks” using Goodman and Shi’s (1985) classification scheme. Key blocks are important because their removal can trigger the mobilization of stable blocks;
Figure 4.21: View of the undercut for model M2. Red areas represent unstable blocks and green areas represent stable blocks.

3. The total block volume – this is the total volume of all the blocks (stable and unstable) generated in the model, and reflects the total initial caveability potential of the model (Figure 4.22);

4. The total unstable block volume – this is the volume of all blocks with factor of safety less than one, i.e. blocks considered as “key blocks”.
Figure 4.22: Three dimensional view of the blocks generated for model M2. Red blocks are unstable and green blocks are stable.

The spacing data showed an exponential increase in the total block area and total unstable block area with decreasing spacing (Figure 4.23a). The same was observed for the total block volume and total unstable block volume (Figure 4.23b). The exponential increase in block volume with decreasing spacing is in agreement with the functions obtained for spacing versus volume by Starzec and Tsang (2002), even though a different model geometry (tabular to represent a tunnel) was employed. They utilized different assumptions for their model, simulating a tunnel with four joint sets instead of three, using different orientations, length, shape and spacing of fractures. The fact that both studies share a common function fitting the total block volume data as compared to spacing (negative exponential), suggests that the type of function describing the relationship between spacing and caved volume might
be independent of the number of joint sets, the orientation of the sets, length, shape and spacing of fractures, and the geometry of the free face. In the case of a comparison between spacing and area, and spacing and volume the function describing the relationship will be a negative exponential curve, as long as the fractures are randomly distributed in space (Poisson process). In the case of these models (Starzec and Tsang’s and the models in this thesis) this applies, since they were simulated using the Enhanced Baecher model. As described in Chapter 4, the Enhanced Baecher model follows a 3D Poisson process, and the probability density function of a Poisson process is a negative exponential function.
Figure 4.23: a) Total block area and total unstable block area plotted against spacing as a percentage of total undercut area, b) Total block volume and total unstable block volume plotted against spacing as a percentage of total ore body volume.

For the dispersion analysis (original and modified), the value of $\kappa$ equal to 100000 was used in order to plot the results for the “constant” dispersion simulation. $\kappa$ equal to 100000
was chosen because this value is high enough to approximate constant dispersion. To facilitate the presentation of the data, the dispersion axis (x axis) in the graphs was displayed using a log scale.

For the original dispersion models, the block area and unstable block area do not follow any recognizable pattern (Figure 4.24a). Moreover, $\kappa$ values less than 100 show increases and decreases in block area, and an increase in unstable block area. This is in contrast to the total block volume and the total unstable block volume which steadily decrease in the same interval (Figure 4.24b). This highlights the limitations of looking at caveability potential in a two dimensional manner, particularly when low $\kappa$ values are involved. The total block volume remains stable for $\kappa$ values larger than 100. For constant dispersion, the total block area and total unstable areas are almost equivalent. The same is observed in the total block volume and the total unstable volume. This can be explained by referring back to chapters 4.1 and 4.2. Since most of the blocks become approximately equidimensional for constant dispersion, they are not affected by friction and are then free to fall as a consequence of the orientations of the joint sets and the orthogonal nature of the model. As mentioned in chapter 4.2, there is no in-situ stress clamping effect considered in the modeling that could increase the stability of the blocks. This does not occur with lower $\kappa$ values which produce more irregular shapes that have a higher probability of sliding on one or more faces.

For the modified dispersion, the total block area and total unstable block area show the same pattern as in the original dispersion simulations (Figure 4.24a). Again, there is no observable trend in the change in block area with increasing $\kappa$. The total block volume steadily decreases with increasing $\kappa$ for $\kappa$ values lower than 100. As observed in chapter 4.3, the percentage of blocks with large volumes is higher for the model with $\kappa$ equal to 20000.
than for the model with constant dispersion. This is reflected in the total block volume in which the same trend is seen. The unstable block volume follows a very similar trend to the original dispersion modeling, remaining approximately at the same value for all the simulations except for the model with constant dispersion. With constant dispersion the total unstable block volume does not become as large a percentage of the total block volume as in the dispersion simulations. This is attributed to the different orientation of two of the joint sets which allows for the formation of tapered and potential key blocks.
Figure 4.24: a) Total block area and total unstable block area plotted against dispersion (original and modified) as a percentage of total undercut area, b) Total block volume and total unstable block volume plotted against dispersion (original and modified) as a percentage of total ore body volume.

The total block area and total unstable area for the persistence models with constant spacing data increases linearly with increasing persistence (Figure 4.25a). The same trend is
observed for the total block volume and unstable block volume (Figure 4.25b). When comparing the generation of block areas and block volumes for a certain fracture length, significant block areas (greater than 5%) are generated for relatively short fractures (4m). But for a 4m fracture length there is very little total volume produced. This shows again the limitations of using a two dimensional analysis (the block area) to describe a three dimensional process (caveability).
Figure 4.25: a) Total block area and total unstable block area plotted against persistence as a percentage of total undercut area for persistence models with constant spacing, b) Total block volume and total unstable block volume plotted against persistence as a percentage of total ore body volume for persistence models with constant spacing.
In the persistence models with constant fracture count the total block and unstable block area increase linearly with increasing persistence (Figure 4.26a). The total block volume and total unstable block volume follow similar trends also increasing linearly with increasing persistence (Figure 4.26b). In these simulations fractures were generated until a specified number was reached. No spacing was assigned and persistence alone was changed. These conditions indirectly modified the spacing of each of the models. If the ore bodies were to be surveyed for spacing, it would be observed that the changes in persistence lead to a change in fracture spacing. A decrease in spacing related to an increase in joint persistence would be detected, due to the fact that longer fractures have a higher probability of intersecting a borehole or a wall than short fractures. The fracture spacing for all the persistence models with constant fracture count was calculated from measurements in the predefined boreholes in the model. For the models with joint persistence of 2, 3 and 4m the spacing is 9.4, 5.2 and 3.5m respectively. These values correlate with the results for the spacing modeling in Figure 4.23b. This may explain the differences between the persistence models with constant spacing and the persistence models with constant fracture count.
Figure 4.26: a) Total block area and total unstable block area plotted against persistence as a percentage of total undercut area for persistence models with constant fracture count, b) Total block volume and total unstable block volume plotted against persistence as a percentage of total ore body volume for persistence models with constant fracture count.

When comparing the models, it can be observed that the largest percentage of ore body volume which is potentially caveable was generated in the persistence simulations with
constant fracture count. However, as mentioned before, the persistence simulations with constant fracture count cannot be used for comparison because more than one variable was changed at the same time (persistence and indirectly the spacing). The total block volume is most sensitive to changes in spacing, followed by changes in persistence and then by changes in dispersion. This is in agreement with the findings of Chan and Goodman (1987), and Hoerger and Young (1990), who observed that in a three joint set network, the total volume generated is sensitive mainly to fracture spacing (Table 4.13).

Table 4.13: Summary of the impact on caveability potential of the different modeled variables.

<table>
<thead>
<tr>
<th>Simulation Type</th>
<th>Impact on Caveability Potential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spacing</td>
<td>High</td>
</tr>
<tr>
<td>Dispersion</td>
<td>Low</td>
</tr>
<tr>
<td>Modified Dispersion</td>
<td>Low</td>
</tr>
<tr>
<td>Persistence with Constant Spacing</td>
<td>Medium</td>
</tr>
<tr>
<td>Persistence with Constant Fracture Count</td>
<td>High</td>
</tr>
</tbody>
</table>

Another aspect worth considering is the usual practice in underground exploration. The data obtained for geotechnical evaluation and design of underground mines is predominantly obtained through boreholes, since the rock is usually covered by overburden, vegetation, snow restricting outcrop mapping. However, there are only two of the three variables employed in this thesis that can be determined from boreholes: spacing and dispersion. Spacing can be measured from the core or borehole surveying; dispersion can be obtained indirectly by discontinuity orientation techniques and stereographic analysis. Persistence cannot be measured from boreholes, however, fracture length will manifest itself in a borehole as fracture intensity (size bias). This has to be taken into consideration when using borehole data to generate any type of fracture network.
4.6 Brief Analysis of the Effects of Stress on Stability

It was decided to carry out a limited analysis on the effects of stress in the stability of blocks. UNWEDGE (Rocscience, 2007) was used to perform the analysis, since it utilizes the same algorithm as FracMan to calculate the stability of blocks. The analysis was simple because of the limitations that the program presented. Only one block at the time could be tested and only tetrahedral shapes could be modeled. It was decided to model symmetrical and asymmetrical block shapes. The symmetrical shapes consisted of three different types of blocks with faces at 10, 45 and 80 degrees from horizontal respectively. The asymmetrical blocks composed different shapes with two faces at 10, 45 and 80 degrees and one at 90 degrees from horizontal respectively. Several stress regimes were also tested. The ratios between principal stresses were obtained from observations performed by Martin et al. (2003) in Canada and Sweden. The stress ratios used for $\sigma_1:\sigma_2:\sigma_3$ were: 1:1:1, 2:1.5:1, 2:1:1 and 2:2:1. The following values for $\sigma_1$ were employed: 1, 5, 10, 20, 40, 60, 80 and 120 MPa. These values were chosen to represent the range of stress magnitudes that might be observed in shallow to deep excavations. All the simulations were carried out on a 5x5m tunnel (Figure 4.27). The analysis ignored stress distributions around the excavation and $\sigma_1$ was always kept horizontal, in order to generate a “clamping effect” on the blocks.
As expected, the stresses had the highest impacts on the blocks with faces at steeper angles from horizontal (Figures 4.28 - 4.31). This was equally valid for the symmetrical and asymmetrical blocks. It is apparent that the stability of blocks (measured as factor of safety) has larger changes for stresses lower than 20 MPa, due to lower clamping stress. Differences in stress ratios have very little influence in the factors of safety, except for symmetrical blocks with low stresses (below 20 MPa) in the 2:2:1 stress regime.

**Figure 4.27:** Basic tunnel model used for UNWEDGE simulations. In this case modeling an asymmetrical block.
Figure 4.28: Factors of safety for the different block shapes tested for a principal stress ratio of 1:1:1.

Figure 4.29: Factors of safety for the different block shapes tested for a principal stress ratio of 2:1.5:1.
Figure 4.30: Factors of safety for the different block shapes tested for a principal stress ratio of 2:1:1.

Figure 4.31: Factors of safety for the different block shapes tested for a principal stress ratio of 2:2:1.
5.0 CONCLUSION AND RECOMMENDATIONS

5.1 Conclusions

Discrete fracture network modeling has been used to simulate block caving and evaluate the impact of fracture spacing, dispersion and persistence on fragmentation and caveability. The block shape characterization was determined using Kalenchuk's et al. (2006) block shape characterization method (BSCM). There were clear tendencies observed by changing the values of the variables. For instance, blocks became more elongated with larger spacing, they became more cubic with constant dispersion, and they changed from elongated to cubic with higher fracture persistence. Almost all models had most of the block shapes concentrated in the left side of the block shape diagram, i.e. cubic, cubic-elongated and elongated shapes. This was attributed to the fact that the properties of all three joint sets were the same and only one variable was changed at a time. Therefore, the blocks generated were approximately equidimensional. There were also other shapes generated, but in less quantity, as a consequence of the range of values used for the variables (spacing, dispersion and persistence). However, the potential of the BSCM for evaluating the block shape distribution within a rock mass was further confirmed, especially when used with the DFN method.

The stability of the blocks generated was evaluated based on the factors of safety obtained from the stability analysis performed by FracMan. Only a kinematic gravitational analysis was carried out and no in-situ stresses were considered. As spacing increased, most of the failures shifted from being free falling to sliding on one face. For the persistence models, more blocks became stable with increasing persistence. For the dispersion simulations, as the κ value increased, blocks became more equidimensional increasing the occurrence of free falling blocks. This occurrence is due to the geometry used for the model
and the absence of the "clamping effect" of the stresses, conversely showing the importance of destressing on stability. It was also possible to relate block shapes to stability.

The block size distribution analysis was based on a modified version of Laubscher's (2000) descriptions of the potential effects of the fragmentation size in block caving operations. The information gathered during the modeling suggested that of all the variables analyzed, fracture persistence has the largest influence on the generation of drawbell blocking block sizes. This was followed by spacing and fracture dispersion. Results showed that fragmentation size did not always increase as the value of the variables was increased, illustrating how block size was a function of the boundary conditions imposed by the spacing, persistence, dispersion and undercut size. Correlations between the coefficient of uniformity, the coefficient of curvature and the effects of block sizes in block cave mining production were investigated, but none was found.

There was qualitative agreement between the apparent block volume (Cai et al., 2004) and the blockiness character (Elmo et al. 2008b). This showed the potential of relating the results obtained using FracMan to existing rock mass classification systems. Nevertheless, there were disagreements between the apparent block volume and the blockiness character when the volumes generated during the fracture dispersion modeling were compared. This occurred because the apparent block volume does not incorporate the effect of fracture dispersion.

This research indicates that when comparing all the models, the largest percentage of ore body volume which was potentially caveable was generated in the spacing simulations. The total block volume is most sensitive to changes in spacing, followed by changes in persistence and then by changes in dispersion. This was in agreement with the findings of
Chan and Goodman (1987), and Hoerger and Young (1990). They observed that in a three joint set network, the total volume generated was sensitive mainly to fracture spacing. There were limitations observed between the two dimensional (total block area on undercut) and three dimensional (total block volume) evaluation of potential caveability. The measurements of the block area on the undercut proved to be unreliable as means of assessing the blocks that were potentially caveable. It is recommended that the block volume is used for this purpose.

A brief analysis was carried out using UNWEDGE to determine the effect of stress on the stability of blocks. Increases in stress had the largest impact on the factor of safety for blocks with steep vertices. This was expected, since the stress acts more perpendicular to the block face.

Based on the work carried out in this thesis, the potential of DFN for primary fragmentation evaluation and determination was confirmed. DFN modeling shows great potential for caveability assessment, and the study of the factors influencing the caving process.

5.2 Recommendations for Further Work

Some of the recommendations for further study include extending the modeling performed in this thesis to inhomogeneous ore bodies. As observed during the block shape analysis, it is necessary to evaluate models with different values of spacing, dispersion and persistence in all three dimensions. To further extend this research is also necessary to incorporate rock mass properties into the DFN models including rock strength and joint properties, as well as stress fields.
It is also suggested that the apparent block volume is modified to incorporate the effects of dispersion.

No reliable data was found in order to verify the results. Case studies with reliable data need to be modeled and compared to the synthetic simulations generated. This research showed the potential of the DFN for evaluating fragmentation. It is important to verify these findings by relating information gathered during the exploration and development faces of a block cave mine with the fragmentation and hang up data from the production face.

It is also important to investigate the impact of secondary fragmentation on the blocks formed during the primary fragmentation process.
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APPENDIX A
Java Application Code
package rockblock;

import java.io.BufferedReader;
import java.io.IOException;
import java.io.InputStreamReader;
import java.util.ArrayList;
import java.util.HashSet;
import java.util.List;
import java.util.Set;
import java.util.regex.Matcher;
import java.util.regex.Pattern;

public class Main {

    /**
     * @param args
     */
    private static final int READ_AHEAD_LIMIT = 300;
    // This is used in calls to mark, in case we need to reset the BufferedReader.

    private static R3Vector vec(String coords) {
        // System.err.println("Parsing vector coordinates: "+ coords);
        String[] coordinates = coords.split(" ",3);
        double x = (Double.valueOf(coordinates[0])).doubleValue();
        double y = (Double.valueOf(coordinates[1])).doubleValue();
        double z = (Double.valueOf(coordinates[2])).doubleValue();
        return new R3Vector(x,y,z);
    }

    public static void main(String[] args) {
        // List<Block> blocks;
        // Appears of this list are commented out at present,
        // since it doesn't really seem necessary
        int blockCount = 0;

        List<Set<Face>> blockFaces = new ArrayList<Set<Face>>();
        // Each element of this list is a list of faces for one block
        List<Double> blockVolumes = null;
        BufferedReader input = new BufferedReader(new InputStreamReader(System.in));
String currentLine = new String("\\n");
int lineNumber = 0;
try {
    while (!currentLine.contains("End Of File")) {
        // During this first part, we're getting lists of faces for each
        // adding these lists to blockFaces. We can't construct the
        // until we have the volumes.
        ++lineNumber;
        currentLine = input.readLine();

        if (currentLine.contains("Block Number")) {
            ++blockCount;
            Set<Face> faces = new HashSet<Face>()
            currentLine = new String("\\n"); // this prevents a
        }

        while (!currentLine.contains("Block Number") &&
        !currentLine.contains("End Of File")) {
            // reading face data until next block
            input.mark(READ_AHEAD_LIMIT);
            ++lineNumber;
            currentLine = input.readLine();
            if (currentLine.startsWith("Face ")) {
                List<R3Vector> corners = new
                R3Vector normal = null;
                currentLine = new String("\\n");

            }
        }

        while (!currentLine.contains("Block Number") &&
        !currentLine.contains("End Of File")) {
            input.mark(READ_AHEAD_LIMIT);
            ++lineNumber;
            currentLine = input.readLine();
        }
    }
}
if (currentLine.contains("Outward normal")) {
    String coords =
currentLine.substring(currentLine.lastIndexOf(':')+2);
    normal = vec(coords);
}
Matcher m =
Pattern.compile("v\d:\s([-\d\.]+)\s([-\d\.]+)\s([-\d\.]+)").matcher(currentLine);
if (m.find()) {
    //
    System.err.println("Match: " + m.group(1) + " # " + m.group(2) + " # " + m.group(3));
    corners.add(new
R3Vector(Double.valueOf(m.group(1)), Double.valueOf(m.group(2)), Double.valueOf(m.group(3))));
}
input.reset();
Face face = new Face(corners, normal);
faces.add(face);
//
blockCount + ", Face " + faces.size();
//
face.area();

was " + currentLine);
at line " + String.valueOf(lineNumber));
initialized with null normal: ");

System.err.println(faces.toString());

System.exit(1);
if (currentLine == null) {
    System.err.println("Unexpected end of file while parsing blocks and faces.");
    System.exit(1);
}
while ((currentLine = input.readLine()) != null) {
    ++lineNumber;
    if (currentLine.startsWith("BlockNo")) {
        break;
    } // Volumes should begin on the second line after this
} if (currentLine == null) {
    System.err.println("Unexpected end of file while looking for block volumes.");
    System.exit(1);
}
input.readLine(); ++lineNumber; // Skipping a (blank) line

blockVolumes = new ArrayList<Double>(blockCount);
for (int i = 1; i <= blockCount; ++i) {
    // The block numbering starts at 1.
    currentLine = input.readLine(); ++lineNumber;
    String[] numbers = currentLine.split(" ");
    if (Integer.valueOf(numbers[0]).intValue() != i) {
        System.err.println("Invalid format online" + String.valueOf(lineNumber) + " Line expected to start with "+ String.valueOf(i) + ", but ");
        System.err.println(numbers[0] + " read instead.");
    } // Each line should begin with a block number, followed by the volume
    blockVolumes.add(Double.valueOf(numbers[1]));
} // Any unexpected null on readLine here is clearly an error.
} catch (IOException e) {
    System.err.println(e.toString());
    System.exit(2);
}
// Now, we are ready to set up the blocks and perform computations.
blocks = new ArrayList<Block>(blockCount);
System.err.println("Counted " + blockCount + " block" + (blockCount == 1 ? "" : "s") + ".");
```java
for (int i = 0; i < blockCount; ++i) {
    blocks.add(new Block(faces.get(i), blockVolumes.get(i))); //
    System.err.println(String.valueOf(i));
    System.err.println(block.to_string());
    System.out.print(String.valueOf(i + 1) + "t");
    System.out.print(String.valueOf(block.alpha()) + "t");
    System.out.println(String.valueOf(block.beta()));
}
}

package rockblock;

import java.util.ArrayList;
import java.util.Collections;
import java.util.Comparator;
import java.util.ListIterator;
import java.util.List;
import java.util.Set;

This comparator is inconsistent with equals(), as it only compares lengths of vectors.
class lengthComp implements Comparator<R3Vector> {
    public int compare(R3Vector u, R3Vector v) {
        return (int) Math.signum(u.length() - v.length());
    }
}

public class Block {
    private Set<Face> faces;
    private List<R3Vector> points;
    private double volume;
    private List<R3Vector> chords;
    // private double median_length; // This isn't currently used
    private int median_index; // This denotes the index in the sorted list chords where
                               // the first entry of at least
    private Double alpha = null;
```
private Double beta = null;
// These are used to cache the values of alpha and beta, so that if they are queried more
// than once on any particular Block, they need not be recomputed.
// (Double objects are used instead of doubles because null is a convenient initial value.)

public Block(Set<Face> faces, double volume) {
    this.volume = volume;
    this.faces = faces;
    points = new ArrayList<R3Vector>();
    // We take care here not to add the same point twice. I was going to use Set for this (with
    // HashSet implementing), but it's handy to use a List in the next part.
    for (Face face : faces) {
        for (R3Vector point : face.corners()) {
            if (!points.contains(point)) points.add(point);
        }
    }
    // Now we go on to set up our list of all chord lengths from one distinct point
to another
    // with a total of p*(p-1)/2 such lengths
    int p = points.size();
    chords = new ArrayList<R3Vector>(p*(p-1)/2);
    for (int i = 0; i < p; ++i) {
        for (int j = i + 1; j < p; ++j) {
            R3Vector chord = points.get(i).sub(points.get(j));
            // System.err.print("Chord " + chord.toString() + " computed from point "+ i);
            System.err.println(" minus point "+ j + ".");
            chords.add(chord);
        }
    }
    Collections.sort(chords, new lengthComp());
    int n = chords.size();

    // System.err.println("Found "+ n + " chords."); // DIAGNOSTIC
    if (n % 2 == 0) {
        medianindex = n/2;
        // e.g. with 14 chords (indices 0-13), index 7 is just above median
medianlength = (chords.get(medianindex - 1).length() +
chords.get(medianindex).length()) / 2;
} else {
    medianindex = (n - 1) / 2;               // e.g. with 15 chords, index 7 is
at the median
    //
    medianlength = chords.get(medianindex).length();
    //
}

public double alpha() {
    if (alpha != null) {
        return alpha.doubleValue();
    }
    double surfaceArea = 0;
    for (Face face : faces) {
        surfaceArea += face.area();
    }
    double meanLength = 0;
    double n = chords.size();
    for (R3Vector v : chords) {
        meanLength += v.length() / n;
    }
    // System.err.println("Total length: " + meanLength*n + " with " + chords.size()
    + " chords.");  // DIAGNOSTIC
    // System.err.println("Surface area: " + surfaceArea + "\t" + "Mean length: " +
    meanLength);
    // System.err.println("\tVolume: " + volume);  // DIAGNOSTIC

    return (alpha = surfaceArea * meanLength) / (7.7 * volume);  // return and store value
}

public double beta() {
    if (beta != null) {
        return beta.doubleValue();
    }
    // Here, we take only those chords of at least median length.
    List<R3Vector> longChords = chords.subList(medianindex, chords.size());
    // DIAGNOSTIC
System.err.print("Of " + chords.size() + " total chords, " + longChords.size() + " of at least");
System.err.println(" median length are used to compute beta.");

// In the numerator, we take a sum of squares of dot products over all pairs
// (a,b) of long enough chords s.t. a != b and only one of (a,b), (b,a) appears.
double topSum = 0;
// In the denominator, there is a sum of $|a|^2|b|^2$ over such pairs.
double bottomSum = 0;

String numeratorDiagnostics = "";
String denominatorDiagnostics = "";

for (ListIterator<R3Vector> i = longChords.listIterator(); i.hasNext(); ) {
    R3Vector a = i.next();
    System.err.println("Long chord " + a.toString() + " found. Its length is " + a.length());
    // DIAGNOSTIC
    for (ListIterator<R3Vector> j = longChords.listIterator(i.nextIndex()); j.hasNext(); ) {
        R3Vector b = j.next();
        double numeratorTerm = Math.pow(a.dot(b), 2);
        double denominatorTerm = a.dot(a) * b.dot(b);
        numeratorDiagnostics += "Adding \( (a \cdot b)^2 \) = 
" + numeratorTerm + " to numerator.\n";
        denominatorDiagnostics += "Adding \[ |a|^2 |b|^2 \] = 
" + denominatorTerm + " to denominator.\n";
        topSum += numeratorTerm;
        bottomSum += denominatorTerm; //a.dot(a) * b.dot(b);
    }
    System.err.println(numeratorDiagnostics);
    System.err.println(denominatorDiagnostics);
    return beta = 10 * Math.pow((topSum / bottomSum), 2); // return
}

public String toString() {
    return "Volume: " + volume + "\n" + Faces: " + faces.toString() + "\n" + "Points: " + points.toString();
}
public class Face {
    private List<R3Vector> corners;
    private R3Vector normal;

    public Face(List<R3Vector> corners, R3Vector normal) {
        this.corners = corners;
        this.normal = normal;
    }

    public List<R3Vector> corners() {
        return this.corners;
    }

    public R3Vector normal() {
        return normal;
    }

    public double area() {
        // As an unfortunate hack, I'm currently going to return zero area in situations
        // such as having no corners. I probably want to revisit this and maybe use
        // exceptions.
        if (corners.isEmpty()) return 0;
        R3Vector cumulative = R3Vector.origin;

        // Here we want to add up successive cross products, starting with the first
        // point cross the
        // second, etc. up to the last point cross the first.
        Iterator<R3Vector> i = corners.iterator();
        R3Vector firstPoint = i.next();
        R3Vector currentPoint;
        R3Vector nextPoint = firstPoint;
        while (i.hasNext()) {
            currentPoint = nextPoint;
            nextPoint = i.next();
            cumulative = cumulative.add(currentPoint.cross(nextPoint));
        }
        cumulative = cumulative.add(nextPoint.cross(firstPoint));
    }
}
// Hopefully taking the absolute value will make sure we get an actual area.
return Math.abs(normal.dot(cumulative))/2;
// Note that, if there is only one point in the set of corners, the while loop will
never activate
// and we will obtain zero == firstPoint.cross(firstPoint) in the current
implementation.

public String toString() {
    return "Normal " + normal.toString() + ", Corners: " + corners.toString();
}

package rockblock;

public class R3Vector {
    private double x, y, z;
    final public static R3Vector origin = new R3Vector(0,0,0);

    public R3Vector(double x, double y, double z) {
        this.x = x;
        this.y = y;
        this.z = z;
    }

    public double x() {
        return x;
    }
    public double y() {
        return y;
    }
    public double z() {
        return z;
    }

    public R3Vector scalarmult(double c) {
        return new R3Vector(c * this.x, c * this.y, c * this.z);
    }

    public R3Vector add(R3Vector v) {
        return new R3Vector(this.x + v.x, this.y + v.y, this.z + v.z);
    }

    public R3Vector sub(R3Vector v) {
return new R3Vector(this.x - v.x, this.y - v.y, this.z - v.z);
}
public double dot(R3Vector v) {
    return this.x * v.x + this.y * v.y + this.z * v.z;
}
public R3Vector cross(R3Vector v) {
    return new R3Vector(this.y * v.z - this.z * v.y,
                         this.z * v.x - this.x * v.z,
                         this.x * v.y - this.y * v.x);
}
public double distance(R3Vector v) {
    return java.lang.Math.sqrt((this.x - v.x) * (this.x - v.x) +
                               (this.y - v.y) * (this.y -
                               v.y) +
                               (this.z - v.z) * (this.z -
                               v.z));
}
public double length() {
    return distance(origin);
}

public boolean equals(Object o) {
    if (!(o instanceof R3Vector)) return false;
    R3Vector v = (R3Vector) o;
    if (this.x == v.x() && this.y == v.y() && this.z == v.z())
        return true;
    else return false;
}
public int hashCode() {
    return Double.valueOf(this.x).hashCode() ^
           Double.valueOf(this.y).hashCode()
                                                ^ Double.valueOf(this.z).hashCode();
}
public String toString() {
    return "<" + String.valueOf(this.x) + "," + String.valueOf(this.y) + "," +
            String.valueOf(this.z) + ">";
}