Abstract

Because the motion of a LEO satellite across the sky causes the Earth-space path to pass through any rain cells in the vicinity very quickly, the degree of rain fading on such paths changes more rapidly and leads to steeper fade slopes than in the geostationary case. A simulation model based on synthetic rain field technique is developed to obtain plausible estimates of the fade slope distributions for selected scenarios. Two methods of generating the rain field, Goldhirsh’s method based on the EXCELL model and Féral’s method based on the HYCELL model, are implemented. The simulation results for GEO satellites closely match the measurement data observed during the ACTS program. The results indicate that fade slopes, which could be between two and ten times greater than for GEO satellites at a given probability level, will steepen as: (1) the altitude of the satellite decreases, (2) the carrier frequency increases and (3) the average rain rate increases. The differences between the results yielded by the EXCELL and HYCELL model are very small and the slightly conservative estimates yielded by the EXCELL model are offset to some extent by the greater simplicity of the EXCELL model.

Scintillation on Earth-space paths increases greatly at low elevation angles and/or higher frequencies. On Earth-LEO links, because of the rapid change in elevation angle and motion of the satellite, both the length of the slant path to the turbulence layer and the velocity at which the slant path passes across the turbulence layer change rapidly. This affects both the intensity of the scintillation process, which generally reaches its maximum value at low elevation angles and/or periods of rain, and the corner frequency of the scintillation process, which generally reaches its maximum value at high elevation angles. Here a geometric model of propagation through the turbulence layer is developed in conjunction with Tatarskii’s theory of propagation through turbulent media to show that the effect becomes more pronounced as the orbital altitude decreases and as the height of the turbulence layer increases. These results have important implications for the design of power control algorithms and other fade mitigation techniques.
# Table of Contents

Abstract .......................................................................................................................... ii

Table of Contents .......................................................................................................... iii

List of Tables .................................................................................................................... vii

List of Figures ................................................................................................................... viii

List of Abbreviations ...................................................................................................... xi

Acknowledgments ........................................................................................................... xii

Co-Authorship Statement ............................................................................................. xiii

1 Introduction .................................................................................................................. 1

1.1 Implementation of Ka-band to satellite communications ........................................ 1
1.1.1 Ka-band Earth-GEO Links ................................................................................. 1
1.1.2 Ka-band Earth-LEO Links ................................................................................. 3

1.2 Major Propagation Impairments on Ka-band Earth-Space Links ......................... 5
1.2.1 Propagation Impairments at Ka-band .............................................................. 5
1.2.2 Difference between Earth-LEO and Earth-GEO Links .................................. 6
1.2.3 Fade Mitigation Techniques ............................................................................ 7

1.3 Motivation and Objectives ...................................................................................... 8

1.4 Thesis outline ........................................................................................................... 9

References ..................................................................................................................... 10

2 Fade Slope Analysis of Ka-band Earth-LEO Satellite Links Using a Synthetic Rain Field Model ........................................................................................................ 12
2.1 Introduction ................................................................................................................. 12

2.2 Formulation of the Simulation Scenario ........................................................................ 14

2.2.1 Satellites in Low Earth Station ............................................................................. 14

2.2.2 Fast and Slow Variations in Path Loss on Ka-band Earth-Space Links ............ 16

2.2.3 Fade Slope Analysis and Rain Field Modeling .................................................... 17

2.3 The Simulation Model ................................................................................................... 19

2.3.1 Approach and Simplifying Assumptions ................................................................. 19

2.3.2 Simulation Procedure ............................................................................................ 25

2.3.3 Typical Results ....................................................................................................... 27

2.4 Results ............................................................................................................................ 29

2.4.1 Simulation Database .............................................................................................. 29

2.4.2 Data Reduction Strategy ......................................................................................... 29

2.4.3 Model Validation ..................................................................................................... 31

2.4.4 Fade Slope Analysis ............................................................................................... 32

2.5 Conclusion ...................................................................................................................... 37

References ............................................................................................................................ 39

3 Effect of Rain Cell Shape on Fade Slope Statistics over Simulated Earth-LEO
Ka-band Links ......................................................................................................................... 42

3.1 Introduction ...................................................................................................................... 42

3.2 The Simulation Model .................................................................................................... 44

3.2.1 Synthesis of Two-dimensional Rain Fields Based upon HYCELL Model ........ 44

3.2.2 Simulation of Rain Attenuation Time Series ......................................................... 48

3.3 Results ............................................................................................................................. 49

3.3.1 Simulation Database and Data Processing ............................................................... 49

3.3.2 Model Validation on Earth-GEO Ka-band Links ................................................ 50
3.3.3 Comparison of EXCELL and HYCELL-based Predictions on Earth-LEO Ka-band Links ................................................................. 51

3.4 Conclusions ..................................................................................... 54

References............................................................................................... 55

4 Prediction of the Effect of Turbulence Layer Height and Satellite Altitude on Scintillation on Ka-band Earth-LEO Satellite Links............................... 57

4.1 Introduction ...................................................................................... 57

4.2 Characterization of Turbulence on Earth-Space Paths .................... 60
  4.2.1 Characteristics of the Turbulence Layer ..................................... 60
  4.2.2 Tatarskii's Theory of Scintillation ........................................... 61

4.3 Prediction of Scintillation Parameters on Earth-LEO Paths ............. 64
  4.3.1 Look Angle Determination for LEO Satellites ......................... 64
  4.3.2 Estimation of the Total and Transverse Velocities .................... 66
  4.3.3 Prediction of the Corner Frequency .......................................... 69
  4.3.4 Prediction of the Intensity of Scintillation ............................... 69

4.4 Evolution of Scintillation Parameters during Typical LEO Satellite Passes .... 71
  4.4.1 Low Earth Orbits of Particular Significance ............................. 71
  4.4.2 Evolution of Scintillation Parameters during Typical Passes ....... 72
  4.4.3 Statistics of Scintillation Parameters over Multiple Passes ........ 74
  4.4.4 Wind Effects on Corner Frequency ........................................ 74

4.5 Implications for Generation of Scintillation Time Series .................. 82

4.6 Conclusions .................................................................................... 86

References............................................................................................... 87

5 Conclusions and Recommendations.................................................... 90
List of Tables

Table 2.1 Processes that cause slow variation of signal strength on Earth-space links... 17
Table 2.2 Rain fall intensity statistics at selected locations........................................... 24
Table 2.3 Typical pass statistics for a satellite in low polar orbit.................................... 25

Table 3.1 The maximum fade slopes predicted using HYCELL and EXCELL-based synthetic rain fields over a 99% confidence interval............................................. 54
List of Figures

Figure 2.1 Distribution of the altitudes for 369 communications, earth observation and research satellites in low earth orbit. .............................................................................. 16

Figure 2.2 Percentage of the year the rain rate exceeds the abscissa (curve with stars), probability of exceeding abscissa given the rain rate exceeds 0.5 mm/h (curve with circles), and modeled conditional distribution (curve with diamond). (a) Tampa, FL. (b) White Sands, NM ................................................................. 23

Figure 2.3 CCDFs of pass durations for LEO satellites in 200, 800 and 1500-km polar orbits as seen by an earth station at latitude 30 degrees ............................................. 24

Figure 2.4 A simulated LEO satellite pass over Tampa, FL in former ITU-R rain zone N at an altitude of 800 km. (a) A simulated 150-km x 150-km rain-rate field + earth station location; —intersection of the path with the top of the rain layer. (b) Path gain observed at 20 GHz during the pass ............................................................... 28

Figure 2.5 Simulation model geometry, \( h \) is the rain height, \( L \) is the slant path length through the rain layer and is divided to small segments, \( L_{\text{horizon}} \) is the length of the horizontal projection of slant path ........................................................................ 29

Figure 2.6 Effect of fade slope interval on estimation of steep fade slopes .. 31

Figure 2.7 CCDFs of the fade slope observed on simulated Earth-space links to LEO satellites in 200, 800 and 1500-km polar orbits and ACTS in GEO from a site near Tampa, FL. (a) \( f = 20 \) GHz. (b) \( f = 27.5 \) GHz ................................................................. 34

Figure 2.8 CCDFs of the fade slope observed on simulated Earth-space links to LEO satellites in 200, 800 and 1500-km polar orbits and ACTS in GEO from a site near White Sands, NM. (a) \( f = 20 \) GHz. (b) \( f = 27.5 \) GHz ................................................................. 35

Figure 2.9 CCDFs of the fade slope observed on simulated 27.5 GHz Earth-space links to LEO satellites in 800-km polar orbits from sites corresponding to Tampa, FL and White Sands, NM ............................................................................. 36

Figure 2.10 Apparent angular rates of LEO satellites in 200, 800 and 1500-km polar orbits during passes that have a maximum elevation angle of 60 degrees ............ 36

Figure 2.11 Best fit of a lognormal distribution to CCDFs of the fade slope observed on simulated Earth-space links at 20 GHz to LEO satellites in 200, 800 and 1500-km polar orbits from a site near White Sands, NM .......................................................... 37
Figure 3.1 Comparison of long-term rain rate CCDFs for White Sands, NM and Tampa, FL and corresponding spatial rain-rate CCDFs from synthetic rain fields that were generated for each location.

Figure 3.2 The Earth-space link geometry where $h$ is the height of the rain layer, $L$ is the length of the slant path which is divided into short segments, $L_{\text{horizon}}$ is the length of the horizontal projection of the slant path and $\theta$ is the elevation angle.

Figure 3.3 CCDFs of the fade slopes observed on simulated Earth-space links to LEO satellites in 200, 800 and 1500-km polar orbits and ACTS in GEO from a site near White Sands, NM, and comparison to ACTS measurement data. (a) $f = 20$ GHz. (b) $f = 27.5$ GHz.

Figure 3.4 CCDFs of the fade slopes observed on simulated Earth-space links to LEO satellites in 200, 800 and 1500-km polar orbits and ACTS in GEO from a site near Tampa, FL, and comparison to ACTS measurement data. (a) $f = 20$ GHz. (b) $f = 27.5$ GHz.

Figure 4.1 Normalized power spectral density of the amplitude of scintillation.

Figure 4.2 The spherical triangle defined by the north pole $N$, the earth station $E$ and the sub-satellite point $S$. The sublayer point $P$ falls on the geodesic that links the Earth station and the subsatellite point.

Figure 4.3 The geometry of the Earth-space path with respect to the turbulence layer along the great circle that contains the earth station $E$, the sub-satellite point $S$, and the sub-layer point $P$. The remaining symbols are defined in the text.

Figure 4.4 The relationship between the direction to the satellite along the Earth-space path, the total velocity of the Earth-space path across the turbulence layer, and the component of the total velocity that is transverse to the Earth-space path. (a) Determination of total velocity $v_{\text{total}}$. (b) Determination of transverse velocity $v_r$.

Figure 4.5 Typical results for satellite in 200 km polar orbit, with 1 km layer height.

Figure 4.6 Typical results for satellite in 800 km polar orbit, with 1 km layer height.

Figure 4.7 Typical results for satellite in 1500 km polar orbit, with 1 km layer height.

Figure 4.8 Evolution of standard deviation and corner frequency during a pass. (a) Time series intensity, with a 1-km layer height. (b) Time series corner frequency, with 1-km layer height. (c) Time series corner frequencies for satellite in 200 km polar orbits, with turbulence layer at 1, 2, 3 and 4 km altitude.
Figure 4.9 CCDFs of the standard deviation and corner frequency. (a) Scintillation intensity for different satellite altitudes, with 1-km layer height. (b) Corner frequency for different satellite altitudes and a 1-km layer height, with and without wind. (c) Corner frequency for different layer heights for a satellite in a 200-km polar orbit.

Figure 4.10 Evolution of the corner frequency of scintillation during an overhead pass by a satellite in an 800-km orbit with an 8.3 m/s wind blowing at 30 degrees: (a) Evolution of the azimuth angle during the pass. (b) Contributions of wind and satellite motion to the transverse velocity during the pass. (c) Evolution of the total transverse velocity during the pass. (d) Evolution of the corner frequency during the pass.

Figure 4.11 Evolution of dry scintillation over time for typical links to satellites in both LEO and GEO. (a) Dry scintillation during a pass by a satellite in a 1500-km polar orbit. (b) Dry scintillation to a satellite in GEO at an elevation angle of 30 degrees over the same duration as in (a).

Figure 4.12 Accounting for wet scintillation during a pass by a satellite in a 1500-km polar orbit. (a) Evolution of path gain during the pass with account taken for range, atmospheric gases, cloud and fog, and rain fading only. (b) Evolution of wet scintillation during the pass. (c) Evolution of total path gain, including wet scintillation, during the pass.
### List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACA</td>
<td>Attenuation with respect to clear air</td>
</tr>
<tr>
<td>ACTS</td>
<td>Advanced communications technology satellite</td>
</tr>
<tr>
<td>AER</td>
<td>Azimuth, Elevation and Range</td>
</tr>
<tr>
<td>AFS</td>
<td>Attenuation with respect to free space</td>
</tr>
<tr>
<td>CCDF</td>
<td>Complementary cumulative distribution function</td>
</tr>
<tr>
<td>ECMWF</td>
<td>European Centre for Medium-Range Weather Forecast</td>
</tr>
<tr>
<td>ESA</td>
<td>European space agency</td>
</tr>
<tr>
<td>FMT</td>
<td>Fade mitigation technique</td>
</tr>
<tr>
<td>GEO</td>
<td>Geostationary earth orbit</td>
</tr>
<tr>
<td>ITU-R</td>
<td>International Telecommunications Union Radiocommunication Sector</td>
</tr>
<tr>
<td>LEO</td>
<td>Low earth orbit</td>
</tr>
<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
</tr>
<tr>
<td>STK</td>
<td>Satellite tool kit</td>
</tr>
<tr>
<td>TLE</td>
<td>Two-line element</td>
</tr>
</tbody>
</table>
Acknowledgments

I am most appreciative of the financial support that I received from Macdonald Dettwiler and Associates Ltd., the Natural Sciences and Engineering Research Council of Canada (NSERC), Western Economic Diversification Canada (WD), and Norsat International during the course of my work. I thank Dr. David V. Rogers and Dr. Cesar Amaya of the Communications Research Centre Canada (CRC) for the advice and suggestions that they offered at various times.

I thank my advisor, Prof. David G. Michelson, for his guidance, help and valuable suggestions over past two years.

I also thank my colleagues James Chuang, Sunny Xin, Simon Chiu and Anthony Liou for their help during my Masters program, and my friend Zilan Xiao for proof-reading my thesis.
Co-Authorship Statement

A version of Chapters 2, 3 and 4 in this thesis has been accepted or has been submitted to *IEEE Transactions on Vehicular Technology* for publication:


The project was initiated by Prof. Michelson. Prof. Michelson contributed greatly to: (1) the design of the project, (2) the research details, including some research methods, math derivations, explanation of the results, etc., and (3) organizing, drafting and editing the three manuscripts. My major contributions are: (1) conducting the research including the literature survey, and implementation and verification of the algorithms; (2) determining the details concerning the procedures of simulation, and conducting the simulations using STK and MATLAB; (3) Analysing the data including data reduction, comparison and organizing the results; (5) preparing the manuscripts including drafting and editing the papers and providing the results.


1 Introduction

1.1 Implementation of Ka-band to satellite communications

1.1.1 Ka-band Earth-GEO Links

In the last decades, satellite communications have been moving from lower frequency band, i.e., C-band (6/4 GHz) and Ku-band (14/12 GHz), to higher frequency band, i.e., Ka-band (30/20 GHz). Moving to higher frequencies will offer several advantages [1]:

1) less congested spectrum: C-band frequencies have been used for a long time for satellite communications and already been saturated. Afterwards, satellite communications migrated to Ku-band frequencies, but have been filling up rapidly. Higher frequency band as Ka-band is needed badly to solve the problem.

2) reduced interference potential: Because Ka-band has not yet been used widely, there is less chance that the performance of one link will be affected by another one operated at the same band.

3) higher data rates: When compared with C-band (downlink frequencies between 3.7 and 4.2GHz, and uplink frequencies between 5.925 and 6.425GHz) and Ku-band (uplink frequencies between 14 and 14.5GHz, and downlink frequencies between 11.7 and 12.7GHz), Ka-band offers wider bandwidths, and as a result, higher data rates.

4) smaller equipment size: The diameters of Ka-band antennas varies between 2 feet and 5 feet, while for C-band, large dish is required with diameter of about 6 feet, and for Ku-band, the diameter is between 2.6 and 5 feet.

Ka-band satellite service on Earth-GEO (geostationary earth orbit) links was first introduced as early as the 1970’s in Japan [2]. Afterwards, ESA and NASA both showed
interests in introducing higher frequency band, i.e., Ka-band, to satellite communications. Several significant measurement campaigns were conducted and contributed greatly to characterization of the propagation channel, investigating the propagation impairments, and developing/improving the prediction models and fade mitigation techniques for Ka-band Earth-GEO links.

(1) The OLYMPUS experiment was started in 1989 by ESA, and terminated in August 1993. During the satellite’s lifetime, the OLYMUPUS beacon signal for propagation research was received and analyzed at more than 50 locations covering all of Western Europe with different climatic regions, and a few in North America. The operating frequencies are Ku (12/14 GHz) and Ka-band (20/30 GHz) [3][4].

(2) The ITALSAT program was initiated by the Italian Space Agency, with the purpose of demonstrating advanced technologies in the field of Ka-band propagation. ITALSAT was launched in December 1991 into a geostationary orbit. The propagation measurements were performed at the frequencies of 20, 40 and 50 GHz [4].

(3) The Advanced Communications Technology Satellite (ACTS) was conceived at the National Aeronautics and Space Administration (NASA). It was designed to obtain slant-path attenuation statistics for locations within the United States and Canada for use in the design of low-margin Ka-band satellite communication systems. ACTS provided beacon signals at 20.2 and 27.5 GHz for use in making attenuation measurements [5],[6].

After 2000, there are still growing commercial interests in using Ka-band on Earth-GEO links to provide different kinds of service, especially low-price high-speed two-way broadband internet service [7],[8]. Past successful events and future launches include Anik-F2 (Canada, 2004), WildBlue-1 (U.S., 2006), Spaceway system (U.S., 2005 and 2006), KA-SAT (Europe, 2010) and ViaSat (U.S., 2011).
1.1.2 Ka-band Earth-LEO Links

The LEO satellite has some weaknesses, such as passing the earth station only for a short duration (around 20 minutes from horizon to horizon), and requiring more complicated antenna (multibeam antenna), etc. However, the use of LEO satellites presents some advantages over the use of GEO satellites [9][10]:

(1) Global coverage: LEO satellites are able to cover the entire surface of the earth, and make it possible to gather scientific or military data throughout the world, as well as offer the GPS services.

(2) Less propagation delay: The distance between LEO satellite and earth station is much shorter, and therefore the latency time is reduced;

(3) Less cost: The cost of launching and maintaining a LEO satellite in orbit is much lower than that of a GEO satellite.

Ka-band Earth-LEO links are far less common than Ka-band Earth-GEO links. Although several important events have been conducted, little measurement data is available in publications for characterizing the channel and establishing the prediction models for Earth-LEO links:

(1) Teledesic [11] was designed as a commercial broadband satellite constellation for Internet services. After original plan in 1995 of 840 satellites at an altitude of 700 km, the number of satellites decreased to 288 at 1400 km. The project was terminated before the system could be deployed due to lack of confidence in the underlying business model.

(2) Iridium satellite constellation came up with the concept of 77 LEO satellites at Ka-band, and then ended up with 66 active satellites. In 1998, it started to provide wireless global telephone access between any two locations on Earth. However, because of the huge expenses in launching and maintaining the satellite
constellation and that the service did not attract sufficient customers, IRIDIUM failed as a commercial venture [9][12]. As a commercial event, there is no published data available from IRIDIUM in open literature.

(3) ROCSAT-1 is a LEO satellite of Taiwan, PRC, for the purpose of conducting scientific experiments. The satellite was launched in January, 1999, into a circular orbit with an altitude of 600 km and an inclination of 35 degree. The carrier frequency of the uplink is 28.25 GHz, and downlink of 18.45 GHz. Although ROCSAT was a success in gathering significant amount of scientific data, there is little measurement data available for studying the propagation environment on Ka-band Earth-LEO links [13].

(4) FedSat is a Ka-band experiment on low-earth orbits (LEO). FedSat was launched into a near-polar, sun-synchronous circular orbit at an altitude of 800 km, resulting in a period of approximately 100 minutes. The earth terminal is located at Technical University of Sydney in Sydney, Australia. Since May 2003, over 60 experiments have been conducted for different satellite paths and under various weather conditions, ranging from clear and dry weather to heavy rain events. The experiment results, including the measurements in the clear days, the model of rain cells, cloudy conditions, can be found in [14].

Past interest in establishing Ka-band Earth-LEO links was deploying large satellites constellation, which requires huge cost for launching and maintaining the satellites. Based on the previous experience, current rising interest in Ka-band Earth-LEO links is a little more pragmatic. Two possible applications are: (1) retrieving gigabytes of data from Earth observation and scientific satellites during a single pass over an earth station [15], and (2) transferring terabytes of data between any two locations on Earth within twenty-four hours using a store-and-forward data transfer scheme based upon specialized LEO-based communications satellites [16][17]. In both cases, the objective is to accomplish such transfers of large amounts of data more efficiently and less expensively
than by relaying the data through a satellite in GEO/LEO.

1.2 Major Propagation Impairments on Ka-band Earth-Space Links

1.2.1 Propagation Impairments at Ka-band

One major problem of using Ka-band on Earth-space links is that the propagation impairments become more severe at such high frequencies [18][19].

Free space loss is the dominant component of the propagation attenuation. It can be calculated by using the equation

\[ L = 20 \log\left(\frac{4\pi d}{\lambda}\right) \text{ dB,} \]  

where \( L \) is the path loss, \( d \) is the distance between the satellite and the earth station, \( \lambda \) is the wavelength.

Gases attenuation is caused by gases molecules (oxygen and water vapour) absorbing energy from the radio waves passing through them. Gases attenuation increases with increasing frequency, and is dependent on temperature, pressure, and humidity. ITU-R P.676-6 includes and approximate model of calculating the gaseous attenuation.

Clouds and fog consist of water droplets (less than 0.1 mm in diameter), which absorb and scatter energy and causes reduction in signal amplitude. Although cloud attenuation is not severe, it usually presents for large percentage of the time. The method of obtaining cloud and fog attenuation is described in ITU-R 840.

Raindrops absorb and scatter radio wave energy, resulting in rain attenuation, which is the major impairments for frequency bands above 10 GHz. Because of the smaller wavelength, transmission at Ka-band is more susceptive to rain attenuation, which could
reach 40 dB at 30 GHz. Rain attenuation severely impairs the link performance, and therefore, fade mitigation techniques (FMT), such as power control and site diversity, are implemented to predict or compensate the rain fading [20].

Scintillation, here defined as tropospheric scintillation, is a rapid and random fluctuation in one or more of the characteristics (amplitude, phase, polarization, and direction of arrival) of a received signal, which is caused by refractive index fluctuations of turbulence due to turbulent mixing of air masses with different temperatures, pressure and water vapour content. At low elevation angles and higher frequencies, scintillation could reach the value comparable to rain fading and impairs low margin systems. Such fast fluctuation could interfere with power control algorithms used to mitigate rain fading [21].

1.2.2 Difference between Earth-LEO and Earth-GEO Links

Determining rain fading and scintillation are significant for estimating or designing the Earth-space links, as well as for the fade mitigation techniques. For Earth-GEO links, both rain fading and scintillation occurring along the path have been well studied, and many prediction models have been developed and tested by being compared with the measurement data.

For Earth-GEO links, the earth-space link is fixed. Therefore, the range between the satellite and earth station and the slant path through the atmosphere do not change, which make the prediction or calculation of the attenuation simple and straightforward.

Earth-LEO links are distinguished from Earth-GEO links by the limited time that the satellite is visible to a given earth station and the rapid rate at which the satellite passes across the sky. The rapid motion of the LEO satellite introduces several complications:

(1) During a satellite pass, the range from the satellite to the earth station is changing, which causes the free space loss varies.
(2) Gases, cloud and fog attenuation change during a satellite pass, because the slant path through the atmosphere changes rapidly.

(3) In the GEO case, fade slope, the rate at which rain fading changes, depends on how fast the rain rate varies, which due to both the advection wind passes the rain cell and the evolution inside the rain cell. In the LEO case, the Earth-space slant path passes through the rain cells in the vicinity very quickly, resulting in deeper fade slopes.

(4) Scintillation occurs as wind advects turbulence cells through the fixed Earth-space path for Earth-GEO links. In the LEO case, scintillation occurs as the motion of the satellite causes the Earth-space path to sweep through turbulence cells, which causes both the amplitude and the rate at which scintillation change during a satellite pass.

Both free space loss and attenuation due to gases, cloud and fog are easy to calculate by simply determining the range and elevation angle as seen by the earth station. The determination of the rain attenuation and scintillation require more efforts, because the motion of satellite, which does not exist in GEO cases, causes additional complications. The emergency issue is to determine the influence of satellite motion on how fast rain fading changes and scintillation, and develop similar well-documented models as for the GEO cases. However, until now, there is no such model describing the channel or predicting the rain fading/scintillation on Earth-LEO links. This is mainly because of lacking of the measurement data on Earth-LEO links useful for establishing the channel models.

1.2.3 Fade Mitigation Techniques

Fade mitigation techniques (FMT) are implemented in the satellite communications system to avoid or compensate for attenuations, mainly for countering rain fading. Two major techniques are power control and site diversity [18][22]. The objective of power
control is to make the received power stay constant, by varying the transmitted power in
direct proportion to the attenuation, mainly rain fading, on the link. Site diversity is a
technique implemented to overcome the effect of path attenuation during intense rain
events, by using two or more geographically separate earth stations to receive a common
signal from a satellite.

Effective power control and site diversity depend on the accurate prediction of fading
along the propagation paths, and the understanding of spatial behaviour of channels.
Therefore, channel models, which are able to characterise the propagation channels or
predict the fading along the propagation path, play a significant role in developing FMTs.

1.3 Motivation and Objectives

With little measurement data available, simulation-based study is therefore the best
option to characterise the rain fading/scintillation on Earth-LEO links. Simulation models
also offer a basis for evaluating and interpreting measurement data when they become
available. Therefore, in this work, simulation models are established to predict rain
fading and scintillation on Earth-space links, and the effect of satellite motion is
estimated.

The main objectives of this work are: (1) establishing simulation models both for
predicting rain attenuation, based on a synthetic rain field, and scintillation, based on a
geometry model, on Earth-LEO links, (2) estimating the rates at which rain fading
changes and scintillation happens, and (3) determining the factors, especially satellite
altitude, that affect the fade slope and scintillation. The algorithms and results described
in this work will give an initiative and good indication of the major impairments along
the Earth-LEO links, and help with the development of FMTs on such links.
This thesis is organized as follows. In Chapter 2, a simulation model based on a synthetic rain field technique is presented to predict the rain attenuation on Earth-LEO links. Fade slope statistics is calculated and analysed to show how fast rain fading changes along the Earth-LEO links. The effects of satellite altitude, local rain intensity and frequency are determined. In Chapter 3, a different rain cell model was implemented, and the fade slope statistics are compared with those in Chapter 2, to show how rain cell shape affects the fade slopes. In Chapter 4, a simulation procedure based on a geometry model of predicting scintillation on Earth-LEO links is presented. The effects of turbulence layer height and satellite altitude are determined. Finally, in Chapter 5, conclusions are drawn, advantages and limitations of the work are assessed, and some future work is recommended.
References


2 Fade Slope Analysis of Ka-band Earth-LEO Satellite Links Using a Synthetic Rain Field Model

2.1 Introduction

Although rain fading on Earth-space links is particularly troublesome on Ka-band, moving to higher frequencies offers many potential advantages including less congested spectrum, the possibility of supporting higher system bandwidths (and higher data rates), reduced interference potential, and smaller equipment size (especially smaller antennas) compared to lower frequencies [1]. At present, most Ka-band Earth-space links are used to increase the capacity of conventional communications satellites located in geostationary Earth orbit (GEO). In recent years, however, system designers have begun to show serious interest in using Ka-band links to provide high speed data communications with satellites in low Earth orbit (LEO) during the relatively short time that a LEO satellite passes within range of an Earth station.

In the late 1990’s, Iridium Satellite LLC pioneered the use of Ka-band Earth-space links to LEO with the establishment of several Ka-band terrestrial gateways to their constellation of 66 LEO communications satellites. (Following a major operational restructuring, only two gateways remain in operation.) Taiwan’s ROCSAT-1 [2] and Australia’s FedSat [3][4] have both recently carried experimental Ka-band transponders into LEO although only results from the latter have been reported in the literature to date. In the near future, MacDonald Dettwiler’s Cascade system will use high speed Ka-band Earth-space links to support a LEO-based store and forward data delivery system that

\[ \text{---} \]

\[ ^1 \text{A version of this chapter has been accepted for publication: W. Liu and D.G. Michelson, “Fade slope analysis of Ka-band Earth-LEO satellite links using a synthetic rain field model,” IEEE Trans. Veh. Technol., in press, 2009.} \]
will allow end users to transfer terabytes of data between any two locations on Earth within twenty-four hours [5][6]. Meanwhile, the European Space Agency is taking the first steps to relieve the congested high-rate downlink band at 8.2 GHz (X-band) that is currently used by LEO-based earth observation satellites by opening an alternative high-rate downlink band at 26 GHz (Ka-band) [7].

Because the rapid motion of the satellite across the sky causes the Earth-space path to pass through rain cells in the vicinity very quickly, the degree of rain fading on LEO satellite links changes more rapidly and, as we shall show, leads to steeper fade slopes, than in the well-studied case of links to geostationary satellites, e.g., [8]-[12]. This has important implications for the performance of the power control algorithms, forward error correction schemes, and other techniques used to mitigate such fading. However, any plans that NASA or ESA might have had following completion of the successful geostationary Ka-band measurement programs of the 1990’s to conduct programs aimed at characterizing fading on Ka-band LEO links were never realized. Although Iridium LLC may have collected propagation data at the handful of Ka-band gateways that they have operated since the late 1990’s, none has been published or referred to in the open literature [13]. Sample data from several FedSat passes over Australia were reported in 2005 [3][4], but little else has been released to date. The resulting dearth of statistical data concerning the rate of fading experienced on Ka-band links to LEO satellites places system designers at a severe disadvantage.

Until more extensive measurement programs are undertaken, simulation based upon reasonable models of the horizontal and vertical structure of rain is the likely best option for assessing the severity of fade slope due to rain fading on Ka-band links to LEO satellites. Several methods for generating realistic synthetic rain fields suitable for use in simulations of satellite communications systems have been proposed in recent years, e.g., [14]-[19]. In the past, such rain field models have generally been used to predict outage probability at a given location or to assess the performance of site diversity between terminals with wide geographic separation. To the best of our knowledge, we are among the first to use such models for fade slope prediction.
Here, we seek to obtain plausible estimates of the manner in which the fade slope distribution experienced on Ka-band links from a fixed station to a satellite in LEO is likely to be affected by: (1) the altitude of the satellite, (2) the carrier frequency and (3) the long-term rain statistics in the vicinity of the earth station. For simplicity, we use Goldhirsh's method [15] to obtain the key parameters of the well-known EXCELL model [14] of horizontal rain structure from long-term global rain statistics together with details concerning the rain layer that have been captured by the relevant ITU-R recommendations. We have accounted for advection of the rain cells by the wind by introducing a wind velocity model based upon the assumptions that average wind speed is lognormally distributed and average wind direction is uniformly distributed. Because we base our simulations on global rain statistics, our approach may be used to assess the performance of links between LEO satellites and earth stations located at essentially any location and in any climatic region on earth.

The remainder of this paper is organized as follows. In Section 2.2, we briefly describe the elements of our simulation scenario including issues peculiar to studies of earth-LEO satellite links and aspects of rain field modeling that should be considered when estimating fade slope. In Section 2.3, we describe our simulation procedure including our simplifying assumptions. In Section 2.4, we present our results. Finally, in Section 2.5, we summarize our findings and draw conclusions.

2.2 Formulation of the Simulation Scenario

2.2.1 Satellites in Low Earth Station

When formulating the link budget and predicting the reliability of a fixed link to a satellite in geostationary earth orbit, it is sufficient to specify the local rain statistics and the elevation angle of the satellite as seen by the earth station. When considering a link

2 The elevation angle of a GEO satellite as seen by the earth station determines the range to the satellite.
to a satellite in a low earth orbit, however, the situation is more complicated. The orbital altitude may range from 200 to 2000 km and the inclination angle may range from 0 degrees (equatorial) to 90 degrees (polar) to slightly beyond (sun-synchronous). The orbital altitude affects the minimum and maximum range to the satellite during each pass and the rate at which the satellite moves across the sky, as seen by the earth station. The inclination angle, combined with the latitude of the earth station, affects the probability distribution function of the satellite’s elevation angle as seen by the earth station [20]. Thus, before proceeding, we need to limit the number of cases that we consider by identifying low earth orbits of particular relevance or significance.

A histogram of the orbital altitudes of a representative set of 369 communications, earth observation and research satellites currently in low-earth orbit, based on two-line element (TLE) data sets obtained from Celestrack [21] is given in Figure 2.1. The lowest practical orbit has an altitude of 200 km; below that, atmospheric drag severely limits orbital lifetime. The majority of LEO satellites occupy orbits with altitudes near 800 and 1500 km with communications satellites having the smallest spread in orbital altitude and research satellites the greatest. Further examination of the TLE data sets also reveals that the vast majority of the LEO-based earth observation and research satellites, and perhaps half of the LEO-based communications satellites, are in near-polar circular orbits (including sun-synchronous orbits). In light of this, and for the sake of simplicity, we have focused our initial fade slope characterization efforts on LEO satellites in circular polar orbits with altitudes of 200, 800, and 1500 km. However, our approach can easily be applied to LEO satellites at other altitudes and inclinations, including those in elliptical orbits.
Figure 2.1 Distribution of the altitudes for 369 communications, earth observation and research satellites in low earth orbit.

2.2.2 Fast and Slow Variations in Path Loss on Ka-band Earth-Space Links

Over time, various processes will cause the received signal strength on Ka-band Earth-space links to vary over both fast and slow time scales. Fast fading is usually associated with processes such as scintillation effects due to turbulence in the atmosphere and multipath scattering from nearby obstacles and scatterers (buildings, trees, and foliage) in relative motion with respect to the earth station. Slow fading, which is our primary interest in this study, is associated with: (1) evolution of the individual rain cells that comprise the rain field, including changes in the physical extent of the cell(s), the rain rate, and the drop size distribution, (2) motion of the rain cells with respect to the Earth due to advection by wind, (3) movement of the ground terminal with respect to the Earth, (4) rapid changes in the range to the satellite (and the length of the slant path through the atmosphere and the rain layer) as the satellite passes across the sky, and (5) rapid changes
in the azimuth and elevation angle of the Earth-space path (and the intersection of the path with individual rain cells) as the satellite passes across the sky.

The significance of the five slow-fading processes for different user scenarios is summarized in Table 2.1: (1) For the well studied case of fixed links to satellites in geostationary orbit, only rain cell evolution and storm motion are significant. Synthetic storm techniques are an effort to distinguish between these two processes based upon measurements of received signal strength data or point rain rate and estimates of local wind speed [22][23]. (2) The case of links from mobile terminals to satellites in geostationary orbit has been considered by others in recent years, e.g., [23][24]. In such cases, the movement of the terminal with respect to both the storm and nearby structures that may temporarily block the path must also be accounted for. (3) In the case of fixed links to LEO, changes in the length of the slant path and rapid changes in the total amount of rain observed along the path as the satellite passes across the sky must be accounted for. (4) In the case of mobile links to LEO, all five of the processes identified above must be accounted for.

Table 2.1 Processes that cause slow variation of signal strength on Earth-space links.

<table>
<thead>
<tr>
<th></th>
<th>Rain event evolution</th>
<th>Storm motion</th>
<th>Terminal motion</th>
<th>Range variation</th>
<th>Az-el variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed to GEO</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mobile to GEO</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fixed to LEO</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Mobile to LEO</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

2.2.3 Fade Slope Analysis and Rain Field Modeling

Fade slope analysis is traditionally conducted using measured received signal strength data while synthetic rain fields, which depict the simulated two-dimensional distribution of rain rate over a certain area, are normally used in conjunction with wind velocity models to evaluate outage probabilities on single links and mutual correlation between
outages at geographically separated sites, *i.e.*, site diversity. The work we present here is apparently among the first to use synthetic rain field models to predict fade slope distributions.

Synthetic rain fields model the spatial structure of rain on three scales: (1) small scale (~20 x 20 km²) - the shape and intensity profiles of individual rain cells, (2) medium scale (~150 x 150 km²) - the random distribution of individual rain cells over small areas, (3) large scale (~1000 x 1000 km²) - the distribution of rain cell clusters due to the formation of weather fronts [18]. Because fade slope is determined by differences in rain intensity at points within the field that are typically less than one hundred metres apart, the spatial distribution of the rain cells is unlikely to play a significant role in determining the fade slope distribution. However, it seems likely that the fade slope distribution will be affected, to at least some degree, by the profile of the individual rain cells.

A rain cell is commonly defined as the region surrounding a local maximum within which the rain rate is larger than a certain threshold [16]. In recent years, three alternative rain cell models have been developed based upon radar observations of rain cells which show that rain rate in the outer regions of individual rain cells tends to decay exponentially. The EXCELL model, which is the simplest of the three models, assumes that the rain cell has an exponential profile with a prominent central peak [14], *i.e.*, the rain rate distribution in the horizontal plane is given by

\[ R(r) = R_M \exp \left( -\frac{r}{\rho_0} \right) \text{ mm/h}, \quad (2.1) \]

where \( R_M \) is the peak rain rate, \( r \) is the horizontal distance from the centre of the cell and \( \rho_0 \) is the distance scale factor. The Lowered EXCELL model is a recent variant of the EXCELL model that applies to stratiform rain [19]. The rain rate distribution in the horizontal plane is given by
\[ R(r) = \begin{cases} (R_M + R_{\text{low}}) \exp \left( -\frac{r}{\rho_0} \right) - R_{\text{low}} & r \leq r_{\text{max}} \\ 0 & r > r_{\text{max}} \end{cases} \text{ mm/h,} \tag{2.2} \]

where \( R_M \) is the peak rain rate, \( R_{\text{low}} \) is the so-called lowering factor, \( r \) is the horizontal distance from the centre of the cell and \( \rho_0 \) is the distance scale factor. The HYCELL model, the most complicated of the three, models the rain rate distribution in the horizontal plane as

\[ R(x,y) = \begin{cases} R_G \exp \left( -\frac{x^2 + y^2}{a_G^2 + b_G^2} \right) & r \leq r_1 \\ R_E \exp \left( -\left( \frac{x^2}{a_E^2} + \frac{y^2}{b_E^2} \right)^{1/2} \right) & r_2 \leq r < r_1 \end{cases} \text{ mm/h,} \tag{2.3} \]

where \( R_G, a_G \) and \( b_G \) define the Gaussian component and \( R_E, a_E \) and \( b_E \) define the exponential component [16]. This substantially reduces the amplitude of the central peak. The coordinates \( x \) and \( y \) and the radial distance \( r \) are measured with respect to the centre of the cell.

Our ultimate goal is to assess the sensitivity of the fade slope statistics to the degree of detail captured by these models. Here, we have started simply by using an approach based upon the EXCELL and the ITU-R one-layer rain models. As we shall note later, the next logical step will be to complete the assessment by conducting simulations using the more complicated rain cell models and rain layer models, e.g., [20][25].

2.3 The Simulation Model

2.3.1 Approach and Simplifying Assumptions

We used Goldhirsh’s method [15] to generate synthetic rain fields based upon long-term global rain statistics. The absolute probability of rain falling at a given rate is self-
The *conditional probability* refers to the probability over all instances when the rain rate exceeds a given threshold, e.g., 0.5mm/h. The major steps of Goldhirsh’s method for generating a two dimensional rain field are as follows:

1. For a specific location, determine the absolute complementary cumulative distribution function (CCDF) of rain rate as specified by ITU-R P. 837;

2. Set a rain rate threshold;

3. Determine the conditional CCDF by dividing the absolute CCDF by the absolute probability of the rain rate equaling the specified threshold (typical absolute and conditional CCDFs for Tampa and White Sands are shown in Figure 2.2);

4. Estimate the parameters $P_0$, $R^*$ and $\kappa$, by fitting the conditional CCDF to

$$P(R_q) = P_0 \left[ \ln \left( \frac{R^*}{R_q} \right) \right]^\kappa,$$  \hspace{1cm} (2.4)

where $R_q$ is an arbitrary threshold level of rain rate, $P(R_q)$ is the probability that the rain rate is greater than $R_q$;

5. Determine the peak rain rate $R_M$ values by setting the minimum peak rain rate, i.e., 2.5 mm/h, and the peak rain rate interval, i.e., 5 mm/h, and $R_M$ should not exceed $R^*$;

6. Calculate the number of rain cells belonging to certain peak rain rate interval, using

$$N(R_M) = \left( \frac{P_0}{2\pi \rho_0^2 R_M} \right)^{\kappa(\kappa-1)(\kappa-2)} \left[ \ln \left( \frac{R^*}{R_M} \right) \right]^{\kappa-3},$$  \hspace{1cm} (2.5)

$$\rho_0 = \frac{10^{-1.5 \log 10 R_M}}{\ln \left( \frac{R_M}{R_{\text{min}}} \right)},$$ \hspace{1cm} (2.6)

$$\text{NUM}(R_Q) = \left( \frac{N(R_M) + N(R_M + \delta)}{2} \right) \delta_A,$$ \hspace{1cm} (2.7)

20
where $p_0$ is the "characteristic distance" from the rain cell center to the bound that the rain rate reduces to $\exp(-1)$ of the peak value, $N(R_M)$ is the rain cell number density for a certain peak rain rate, $\delta$ is the peak rain rate interval, $A_0$ is the observation area, $Q$ is the index denoting the cells with peak rain rate within the interval, and $NUM(R_Q)$ is the number of rain cells with certain peak rain rate ranging from $R_M$ to $R_M + \delta$.

(7) Randomly distribute the rain cells (CELL1, CELL2, ..., CELL$_{NUM(R_Q)}$) throughout the rain field $A_0$, when $Q=1$;

(8) Specify the grid points within $A_0$ every 0.5 km x 0.5 km, starting with the grid location (0.25km, 0.25km), and for each grid, calculate the rain rates contributed from all rain cells based on EXCELL model, and sum them to yield the total rain rate,

$$R_{Q=1}(x, y) = \sum_{N=1}^{N_{\text{max}}} R_{Q=1,N}(x, y),$$

where $(x, y)$ is the location of the grid point, and $N_{\text{max}}$ equals to $NUM(R_Q)$;

(9) Repeat procedures (7) and (8) for $Q = 2, 3, ..., Q_{\text{max}}$. The total rain rate for each grid point is given by

$$R(x, y) = \sum_{Q=1}^{Q_{\text{max}}} R_Q(x, y).$$

We are then able to generate the rain rate values for all points in the grid in the form of a matrix that gives the rain rate throughout the rain field $A_0$.

Although Goldhirsh used the rain statistics from an early version of ITU-R P. 837-1 that divides the globe into 15 rain zones, ITU-R has since replaced this by global rain statistics obtained from the ECMWF (European Centre for Medium-Range Weather Forecast) ERA-40 re-analysis database, as described in the current version of ITU-R P.837-5. The rain rate intensity distributions for Tampa, FL (27.97° N, 82.53° W, in former ITU-R rain zone N) and White Sands, NM (32.38° N, 106.48° W, in former ITU-R rain zone E) obtained using the two approaches are compared in Table 2.2. The difference is small but noticeable. We generated absolute CCDFs based upon the current
ITU-R P.837-5 instead of the early version, and follow the remaining steps in Goldhirsh's method to generate the two-dimensional rain field.

In the absence of detailed information concerning temperature or drop size distribution, we use the method specified in ITU-R Rec. P.838, that the specific attenuation \( \gamma_R \) (dB/km) is obtained using the power-law relationship

\[
\gamma_R = kR^\alpha,
\]  

where \( R \) is the rain rate (mm/h), \( k \) and \( \alpha \) are coefficients depending on frequency and the calculation procedures can be found in ITU-R P.838 as well. We specify other details of the rain layer, such as its vertical extent, based upon the relevant ITU-R recommendations, e.g., ITU-R Rec. P.839. We account for advection of the rain cells due to wind by introducing a wind velocity model based upon the commonly held assumptions of lognormally distributed wind speed [22] and uniformly distributed wind direction. We define a pass as that portion of the orbit which carries the satellite to 10 degrees or greater above the horizon as seen by the earth station. Given that the rain layer is usually a few thousand metres in height [26][27], we can use a flat earth model without incurring significant errors in our estimate of the length of the slant path at low elevation angles.

Because the satellites are in low earth orbit, the duration of individual passes, when the elevation angle is greater than 10 degree above the horizon, is in the order of minutes or tens of minutes. We generate pass durations for satellites in 200, 800 and 1500-km polar orbits for an earth station located at latitude 30 degrees, using a commercial satellite orbit prediction tool, AGI's Satellite Tool Kit (STK). The CCDFs of pass duration are presented in Figure 2.3. A more detailed summary of pass statistics that includes earth stations at 0, 45 and 60 degrees latitude is presented in Table 2.3. Because pass durations are generally only minutes in length, we can reasonably assume that neither average wind speed and direction nor individual rain cells nor average rain rates change or evolve appreciably during each pass.
Figure 2.2 Percentage of the year the rain rate exceeds the abscissa (curve with stars), probability of exceeding abscissa given the rain rate exceeds 0.5 mm/h (curve with circles), and modeled conditional distribution (curve with diamond). (a) Tampa, FL. (b) White Sands, NM.
Figure 2.3 CCDFs of pass durations for LEO satellites in 200, 800 and 1500-km polar orbits as seen by an earth station at latitude 30 degrees.

Table 2.2 Rain fall intensity statistics at selected locations.

<table>
<thead>
<tr>
<th>Percentage of Time (%)</th>
<th>Rainfall intensity exceeded (mm/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>White Sands, NM</td>
</tr>
<tr>
<td></td>
<td>Rain zone E</td>
</tr>
<tr>
<td>1</td>
<td>0.6</td>
</tr>
<tr>
<td>0.3</td>
<td>2.4</td>
</tr>
<tr>
<td>0.1</td>
<td>6</td>
</tr>
<tr>
<td>0.03</td>
<td>12</td>
</tr>
<tr>
<td>0.01</td>
<td>22</td>
</tr>
<tr>
<td>0.003</td>
<td>41</td>
</tr>
<tr>
<td>0.001</td>
<td>70</td>
</tr>
</tbody>
</table>
Table 2.3 Typical pass statistics for a satellite in low polar orbit.

<table>
<thead>
<tr>
<th>Satellite altitude (km)</th>
<th>200 km</th>
<th>800 km</th>
<th>1500 km</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth station latitude (deg)</td>
<td>0</td>
<td>45</td>
<td>60</td>
</tr>
<tr>
<td>Average no. of passes per month (min)</td>
<td>40</td>
<td>59</td>
<td>85</td>
</tr>
<tr>
<td>Cumulative duration per month (min)</td>
<td>112</td>
<td>173</td>
<td>255</td>
</tr>
<tr>
<td>Average duration per pass (min)</td>
<td>2.8</td>
<td>2.9</td>
<td>3</td>
</tr>
<tr>
<td>Average velocity across the top of the rain layer (km/h)</td>
<td>1215</td>
<td>1173</td>
<td>1134</td>
</tr>
</tbody>
</table>

2.3.2 Simulation Procedure

We begin our simulation by specifying the earth station’s latitude and longitude then determining the rain rate CCDF from the ECMWF ERA-40 re-analysis database as specified in ITU-R Rec. P.837. We then use Goldhirsh’s method with the modifications described in the previous section to generate the two-dimensional rain field. As suggested by Goldhirsh, we truncate the horizontal extent of the exponential rain cell profile by setting a minimum rain rate. A representative rain field based upon parameters for Tampa, FL (27.97° N, 82.53° W) is shown in Figure 2.4(a). The corresponding rain rate statistics for this location is given in Table 2.2. Finally, we estimate the height of the rain field using the procedure outlined in ITU-R Recommendation P.839 so that we can convert the two-dimensional rain field into a three-dimensional one. The path length through the rain layer is a function of both the rain height and the elevation angle. As a result, the total rain attenuation along the path is given by

\[ A = \gamma_R \cdot \frac{h}{\sin \theta} \]  

where \( h \) is the rain height, and \( \theta \) is the elevation angle and \( \gamma_R \) is the mean value of specific attenuation along the path.
We assume that the earth station is fixed at a particular latitude and longitude and has a clear view of the satellite during the entire portion of the pass that is at least ten degrees above the horizon. We place the earth station at the centre of the rain field then select the altitude and inclination of the satellite orbit. We use STK to generate range, elevation and azimuth angle predictions at one-second intervals for many successive passes.

At the beginning of each pass, we randomly generate the speed and direction of the wind during the pass. Past work has suggested that, over the long term, the average wind speed in the mid-troposphere in Europe and North America is well modeled by the lognormal distribution with a median value of 30 km/h (8.33 m/s) and a standard deviation of \( \log_e 2 \) [22]. We use these values here but set a realistic upper limit on the wind speed of 100 km/h. During the several minutes of each pass, we simulate advection of the rain field by the wind by sliding the rain field past the earth station at a constant rate in the specified direction.

At any given instant, the total path loss in dB, \( L_p \), on Earth-space links is given by the product of several factors including path loss due to (1) propagation through free space, \( L_f \), (2) atmospheric gases, \( L_g \), (3) cloud and fog, \( L_c \), and (4) rain, \( L_r \), i.e.,

\[
L_p = L_f + L_g + L_c + L_r.
\]  

(2.12)

As the satellite sweeps across the sky, we determine the free space path loss using the Friis transmission formula and estimate attenuation due to atmospheric gases, cloud and fog using the methods outlined in ITU-R Recs. P.676, P.836, and P.840. We then determine the intersection of the Earth-space path with rain cells in the vicinity. We divide the slant path through the rain layer into short segments, and determine the average rain rate \( R_t \) for each of them. We predict the specific attenuation due to rain at each segment of the path using the method outlined in ITU-R P. 838, and then sum the contributions of each segment to yield the total rain attenuation in dB, as expressed by
\[ L_r = \sum_i \gamma_{R_i} \Delta l, \]  

(2.13)

where \( \gamma_{R_i} \) is the specific attenuation corresponding to rain rate \( R_i \) and \( \Delta l \) is the length of segment of the path. The simulation model geometry is show in Figure 2.5. In this manner, we determine the total path loss experienced along the path at successive instants during the satellite pass. Analysis of the time series over successive passes allows us to characterize slow fading due to rain, and the resulting fade slope, on Ka-band LEO satellite links.

### 2.3.3 Typical Results

A typical plot of path gain vs. time for a link between an earth station located near Tampa, FL (27.97° N, 82.53° W) and a satellite in an 800-km polar orbit is shown in Figure 2.4(b). It corresponds to the scenario depicted in Figure 2.4(a). During this pass, the elevation angle ranged from 10 to 48 degrees and the distance between the satellite and the earth station ranged from 1020 to 2400 km. We present the results in terms of path gain rather than path loss so that the curves will have the same form as those corresponding to received power. The simulated pass resembles, at least superficially, observations of received signal strength made during FedSat passes over Australia as reported in [3][4].
Figure 2.4 A simulated LEO satellite pass over Tampa, FL in former ITU-R rain zone N at an altitude of 800 km. (a) A simulated 150-km x 150-km rain-rate field + earth station location; —intersection of the path with the top of the rain layer. (b) Path gain observed at 20 GHz during the pass.
2.4 Results

2.4.1 Simulation Database

We used the path loss simulator described in the previous section to generate time series of path loss applicable to satellites in polar orbits at altitudes of 200, 800 and 1500 km as seen by fixed earth stations at locations that correspond to White Sands, NM (32.38° N, 106.48° W) and Tampa, FL (27.97° N, 82.53° W). We ran each simulation until the cumulative duration of the passes over each site exceeded 420 hours or 1.5 million seconds. For the 200, 800, and 1500-km orbits, this required almost 9000, 3000, and 1900 passes, respectively. From Table 2.3, it would take approximately 12 years, 2 years, and 1 year, respectively, to achieve such a cumulative duration over a particular site from a single satellite.

2.4.2 Data Reduction Strategy

ITU-R Rec. P.1623 offers recommendations for characterizing fade dynamics on Earth-space paths, including characterization of fade slope or the rate of change of attenuation
with time. A negative fade slope implies a rising signal while a positive fade slope implies a falling signal. Other parameters of interest include fade duration or the time interval between two crossings above the same attenuation threshold and interfade duration or the time interval between two crossings below the same attenuation threshold.

For Earth-space links to GEO, the range to the satellite and the length of the slant path through the atmosphere is fixed and the satellite is visible continuously making long observation times extremely practical. However, Earth-space links to LEO introduce two complications. First, the range to the satellite and the length of the slant path through the atmosphere is constantly changing during a pass. This makes it necessary to distinguish between AFS (attenuation with respect to free space) or ACA (attenuation with respect to clear air) when defining fade slope on such links. Second, the satellite pass will last for several minutes at most, as shown in Figure 2.3. The passes are generally so short compared to the duration of typical fading events that it is difficult to extract meaningful estimates of the fade and interfade durations, because of the possibility that the rain attenuation decreases or increases only during a pass. Accordingly, our focus here is on estimation of fade slope statistics based upon sample intervals of 2 seconds. Not only is knowledge of fade slope useful to those engaged in the analysis and design of fade mitigation techniques, it is easily estimated regardless of longer term variations in the fade depth. The fade slope is defined by,

$$\zeta(t) = \frac{A(t + \frac{1}{2} \Delta t) - A(t - \frac{1}{2} \Delta t)}{\Delta t}$$

(2.14)

where $A(t)$ is the attenuation level in dB at a given instant and $\Delta t$ is the time interval over which fade slope is calculated. We present our results in the form of CCDFs of the fade slope distributions. Those presented here are conditional, i.e., the vertical axis is the probability that the abscissa is exceeded given that it is raining.

When working with measured data, high frequency scintillation must be filtered from the received signal before one can estimate fade slope. Given the bandwidth of the
scintillation process, one cannot choose an arbitrarily small value of \( \Delta t \); ITU-R recommends a minimum value of 2 seconds. In Figure 2.6, we present CCDFs for a simulated Earth-space link where the earth station is at a location which experiences higher than average rainfall. The link is occasionally experiencing fade slopes of several dB/sec (many times higher than one would likely observe on a link to GEO) where fade slope has been calculated using time intervals of 1, 2, 6, 10 and 20 seconds. It is apparent that use of time intervals greater than 2 seconds in length impairs our ability to resolve the steeper fade slopes that are characteristic of links to LEO unless appropriate post-processing as specified in ITU-R P.1623 is applied. Those conducting measurement campaigns aimed at characterizing fade slope on actual Earth-space links to LEO should account for the possibility of encountering such steep fade slopes and plan accordingly.

![Figure 2.6 Effect of fade slope interval on estimation of steep fade slopes.](image)

2.4.3 Model Validation

Ideally, we would validate our simulation model by comparing its predictions with results obtained from hundreds of hours of measurement data from actual LEO Earth-space links at Ka-band. Because such data is not yet available, we took a different route.
and compared our simulator’s predictions of the long-term fade slope distributions likely
to be observed over geostationary links to earth stations located at Tampa, FL and White Sands, NM to those actually observed at these sites during the ACTS program [10]. Because the Earth-space path is fixed, we assume that fade slope is due solely to wind blowing the rain field past the earth station. As described in the previous section, we assumed a lognormally distributed wind speed with a median value of 30 km/h and a standard deviation of log_e2. In a manner similar to our LEO simulations, we ran 3000 simulations each of 20-minutes in length, yielding 1000 hours (or 3.6 million seconds) of time-series data. In processing our data, we used the same ACA threshold as the ACTS team used in processing theirs. The results, which are shown in Figure 2.7 and Figure 2.8 are a close match to the measurement data presented in [10]. In the absence of more complete validation data, this gives us confidence that the results of our simulations are likely useful representations of reality.

On Earth-space links to LEO, fade slope is mainly due to movement of the Earth-space path through the rain field. In Table 2.3, we present estimates of the average speed of the intersection of the Earth-space path with the top of the rain layer as the satellite sweeps across the sky. Because these speeds are so much higher than the median wind speed used in our simulations, it seems reasonable to conclude that simulations of Earth-space links to LEO conducted with a static rain field will yield essentially identical results to those obtained with a wind-blown rain field. Our simulation results confirm this. This also suggests that the fade slopes encountered by a mobile terminal that moves at conventional speeds (tens of km/h) and which has a clear view of the sky, will see fade slopes that are comparable to those encountered by fixed or transportable terminals.

2.4.4 Fade Slope Analysis

Conditional CCDFs of both rising and falling fade slopes observed on simulated 20 and 27.5 GHz links to satellites in 200, 800 and 1500-km polar orbits from earth stations located near White Sands, NM and Tampa, FL are presented in Figure 2.7, Figure 2.8 and Figure 2.9. The results suggest that links to LEO will encounter fade slopes that will
be twice to ten times greater than those reported previously for links to GEO. In particular, we note that fade slopes become much steeper: (1) as the altitude of the satellite decreases (and the angular velocity of the satellite across the sky increases, as suggested by Figure 2.10), (2) as the carrier frequency increases (the path geometry in our simulation implies that the frequency scaling factor for attenuation will also be the frequency scaling factor for fade slope), (3) as the average rain rate increases (and rain attenuation increases). Further, our results suggest that rising and falling fade slopes of a given value are equally likely.

Others have found that a lognormal distribution often fits the fade slope distribution on links to GEO, e.g., [9], so we attempted a similar fit here. The result is shown in Figure 2.11. In all cases, we found that the lognormal distribution fit well at probability levels > 0.01 but tends to over predict the probability of observing a given fade slope at probability levels < 0.01. The probability threshold at which fit is lost increases as rain rate increases and/or altitude decreases. Because the geometry of Earth-LEO links is considerably more complex than for GEO, we will conduct a detailed analysis of the manner in which the fade slope distribution jointly depends upon fade depth, elevation angle, and average rain rate during the next phase of our study.
Figure 2.7 CCDFs of the fade slope observed on simulated Earth-space links to LEO satellites in 200, 800 and 1500-km polar orbits and ACTS in GEO from a site near Tampa, FL. (a) f = 20 GHz. (b) f = 27.5 GHz.
Figure 2.8 CCDFs of the fade slope observed on simulated Earth-space links to LEO satellites in 200, 800 and 1500-km polar orbits and ACTS in GEO from a site near White Sands, NM. (a) f = 20 GHz. (b) f = 27.5 GHz.
Figure 2.9 CCDFs of the fade slope observed on simulated 27.5 GHz Earth-space links to LEO satellites in 800-km polar orbits from sites corresponding to Tampa, FL and White Sands, NM.

Figure 2.10 Apparent angular rates of LEO satellites in 200, 800 and 1500-km polar orbits during passes that have a maximum elevation angle of 60 degrees.
2.5 Conclusion

We have assessed fade slope statistics on Earth-space paths at Ka-band using rain field simulations based upon Goldhirsh’s method for estimating the key parameters of an EXCELL-based model of horizontal rain structure from ITU-R rain rate statistics and a simple wind velocity model that accounts for advection of the rain cells over time. Despite many simplifying assumptions, our predictions of the fade slope distributions on links to GEO closely match those observed at sites in White Sands, NM and Tampa, FL during the ACTS program. This gives us confidence that the fade slope distributions that we have predicted for links to LEO are likely a reasonable representation of reality.

Our results for LEO satellites show that fade slopes will become steeper as: (1) the altitude of the satellite decreases, (2) the frequency band of operation increases and (3) the average rain rate increases. Further, they suggest that at a given probability level, fade slopes could be between two and ten times greater than for geostationary satellites.
Furthermore, they also suggest that fade slopes encountered by a mobile terminal that moves at conventional speeds (tens of km/h) and which has a clear view of the sky, will see fade slopes that are comparable to those encountered by fixed or transportable terminals.

Although more complex rain field models may ultimately yield more accurate results at specific locations, the predicted fade slope distributions presented here represent a useful starting point for those engaged in the design of fade mitigation techniques, the development of rain field models for use in Earth-space link simulations, or the planning of propagation measurement campaigns involving Ka-band Earth-LEO links. The limitations of our work are largely those of the underlying rain field model. The next logical step is to gradually increase the complexity (and presumably the fidelity) of the underlying rain cell model and determine how the fade slope distribution predictions are affected. The ultimate step, of course, would be to obtain statistically significant amounts of measurement data from Ka-band Earth-LEO links to which the predictions presented here could be compared.
References


3 Effect of Rain Cell Shape on Fade Slope Statistics over Simulated Earth-LEO Ka-band Links

3.1 Introduction

An increasing number of Earth-space communications operators are moving from the X- and Ku-bands to Ka-band so that users can: (1) avoid growing congestion in lower bands, (2) achieve higher data rates and (3) use more compact antennas. Earth-GEO Ka-band links have attracted particular interest from companies that see commercial potential for massive deployment of satellite-based two-way broadband services [1]. Earth-LEO Ka-band links have attracted particular interest from operators of LEO-based scientific and Earth observation satellites who need to download large amounts of data during the short time that a satellite is within range of an earth station [2] and from companies that see commercial potential for using LEO-based store-and-forward data satellites to transfer terabytes of data from any place on Earth to any other within twenty-four hours [3]. However, Earth-space links at higher frequencies will experience more severe rain fading than will those at lower frequencies and fade mitigation techniques (FMT), such as uplink and downlink power control algorithms, are needed to improve the performance and reliability of such links. Knowledge of the rate at which rain fading varies over time, i.e., the fade slope statistics, is required so that the performance of power control algorithms can be assessed under realistic conditions [4]. While considerable measurement data for fading on Earth-GEO Ka-band links has been collected during the past twenty years, very little has been collected on Earth-LEO Ka-band links. Until

---

significant quantities of such data are available, designers of Earth-LEO Ka-band links will have to rely on simulations based upon realistic models of the atmosphere.

A rain field is comprised of a multiplicity of rain cells, each of which is defined as the region surrounding a local maximum within which the rain rate is higher than a certain threshold. In previous work [5], we simulated Earth-space Ka-band links in the presence of rain using synthetic rain fields based upon the EXCELL (exponential rain cell) model [6] with parameters derived from long-term rain rate statistics and showed that: (1) the slant path on an Earth-LEO link passes through rain cells in the vicinity much more quickly than the slant path on an Earth-GEO link does and, as a result, (2) fade slopes on Earth-LEO links are two to ten times greater than those observed on Earth-GEO links. Moreover, we showed how fade slopes increase as the orbital altitude decreases and as the local rain intensity increases.

The EXCELL rain cell model that we used in [5] was proposed over twenty years ago based upon observations of rain cells made using weather radars. Since then, other rain cell models have been proposed including the GAUCELL (Gaussian rain cell) [7], HYCELL (hybrid rain cell) [8],[9], lowered-EXCELL [10], and DEXCELL (double exponential rain cell) [11] models. All of them model the outer portions of the rain cell by a slow decay with increasing distance from the peak but differ from each other by modeling the central portion of the rain cell using either an exponential function to yield a sharp central peak or a Gaussian function to yield a rounded central peak. The EXCELL and GAUCELL models take the simplest forms but the EXCELL model tends to overestimate the peak rain rate while the GAUCELL systematically underestimates it [7]. The hybrid-models, HYCELL and DEXCELL, generally provide a better match to radar observations of typical rain cells, but at the expense of greatly increasing the complexity of the model and estimation of its parameters [9],[11].

An open question from our previous work concerns how changing the shape of the rain cell, e.g., using the HYCELL model, with its rounded peak, instead of the EXCELL model, with its sharper peak, will affect our predictions of long-term fade slope statistics.
Here, we show that simulations conducted using HYCELL-based rain fields yield slightly lower fade slopes than do EXCELL-based rain fields and fit measured data on Earth-GEO Ka-band links slightly better. However, the differences are very small and the slightly conservative estimates yielded by the EXCELL-based rain fields are offset to a large extent by the greater simplicity of the EXCELL model.

The remainder of this paper is organized as follows. In Section 3.2, we briefly review the procedure of generating a synthetic rain field based upon the HYCELL model and describe our simulation model for Earth-LEO links. In Section 3.3, we compare the distributions of fade slope statistics that we obtained using HYCELL and EXCELL-based rain fields. In Section 3.4, we draw conclusions and summarize our contributions.

3.2 The Simulation Model

3.2.1 Synthesis of Two-dimensional Rain Fields Based upon HYCELL Model

The HYCELL model was proposed in [8] as an improvement over the EXCELL model. It is a hybrid model that describes the spatial distribution of rain intensity within a rain cell by using a Gaussian function to model the convective-like high rain rate in the core of the rain cell and an exponential function to model the stratiform-like low rain rate spreading down to $R_2$, the minimum rain rate threshold, at the outer edge of the cell. The shape of a HYCELL rain cell is defined by seven parameters ($R_G, a_G, b_G, R_E, a_E, b_E, R_i$) and takes the form

$$R(x, y) = \begin{cases} 
R_G \exp \left[ -\frac{x^2}{a_G^2} - \frac{y^2}{b_G^2} \right] & \text{if} \quad R \geq R_1, \\
R_E \exp \left[ -\frac{x^2}{a_E^2} - \frac{y^2}{b_E^2} \right]^{1/2} & \text{if} \quad R_2 \leq R < R_1,
\end{cases}$$

where $R_G$ is the peak rain rate of the Gaussian component, $a_G$ and $b_G$ are the lengths along the major and minor axes along which the rain rate decreases by a factor 1/e with...
respect to $R_G$. The parameters $R_E$, $a_E$ and $b_E$ define the exponential component in a manner analogous to the Gaussian component. If the rain cells are assumed to be circular [9],[12], the number of parameters is reduced to just five ($R_G$, $a_G$, $R_E$, $a_E$, and $R_I$).

An iterative method for synthesizing a two-dimensional rain rate field comprised of HYCELL rain cells over an observation area $A_0$ based upon long-term rain rate statistics for a specified region and reasonable assumptions regarding the rain cell size distribution was described in detail in [9] and is summarized here: (1) For the specified location, determine the absolute complementary cumulative distribution function (CCDF) of the rain rate as specified in ITU-R P.837, which offers global rain rate statistics obtained from the ECMWF (European Centre for Medium-Range Weather Forecast) ERA-40 reanalysis database; (2) Determine the conditional CCDF by dividing the absolute CCDF by the absolute probability of the rain rate given a specified rain rate threshold $R_r$; (3) For a set of peak rain rates $R_G$ ranging from the minimum to the maximum rain rates observed on the CCDF, with an interval $\Delta R_G$ of 0.001 mm/h, and then estimate the rain cell density $n_p$ for individual peak rain rates $R_G$, using

$$n_p(R_G) = -\frac{4}{\pi} \left[ \frac{D_{\text{min}}^2}{\lambda} + \frac{2D_{\text{min}}}{\lambda^2} + \frac{2}{\lambda^3} \right]^{-1} \left[ \frac{d}{dR} P_r(R) \right]_{R = R_G} \Delta R_G A_r,$$

where $D_{\text{min}}$ is the minimum diameter of a circular rain cell, $\lambda$ is the climatic constant (approximately equal to 0.3), $P_r(R)$ is the conditional CCDF of rain rate, and $A_r$ is the area over which the rain rate exceeds the threshold. Usually, one will set $A_r = A_0$. Conduct a resampling process to adjust each interval in order yield one rain cell per instance $R_G$ and $N$ rain cells in total, i.e., adaptively increase and adjust $\Delta R_G$, until the conditions,

$$n_p(R_G) = 1,$$
$$N = \sum n_p(R_G),$$

(3.3)
are met and where the summation is taken over all values of $R_G$. (4) Set the value of the diameter $D$ at rain rate $R_2$ ranging from 2 to 30 km, with a bin size $\Delta D$ of 0.001 km, and then estimate the number of cells $n_D$ whose diameter $D$ lies between $D$ and $D+\Delta D$ using

$$n_D(R_2) = \frac{4}{\lambda \pi} \left[ \frac{D_{\text{min}}^2}{\lambda} + \frac{2D_{\text{min}}}{\lambda^2} + \frac{2}{\lambda^3} \right]^{-1} e^{\Delta D} P_r(R_2) \cdot \left[ e^{-\lambda D} - e^{-\lambda(D+\Delta D)} \right],$$

where $R_2$ is the rain rate threshold which defines the edge of a cell. Resample $D$ in order to yield one rain cell per instance $D$ and $N$ rain cell in total, i.e., adaptively increase and adjust $\Delta D$, until the conditions

$$n_D(R_2) = 1$$

$$N = \sum n_D(R_2)$$

Where the summation is taken over all values of $R_2$, and the spatial requirement

$$\sum_{i=1}^{N} \pi \frac{D_i^2(R_2)}{4} = A_P R \geq R_2 \mid R > R_r)$$

are all met, where $R_r$ is the rain rate threshold within the rain field. (5) For each rain cell with specified $R_G$ and $D$, apply fitting and continuity equations to determine the remaining parameters ($R_G, a_G, R_E, a_E, R_I$), and apply an iterative process to adjust the values of the parameters in order to guarantee that the distribution of rain rate within the rain field agrees with the local rain rate CCDF; (6) Randomly distribute the $N$ rain cells throughout the rain field $A_0$; (7) Specify the grid points within $A_0$ every 0.5 km x 0.5 km, starting with the grid point at (0.25 km, 0.25 km), and, for each grid point, determine the rain rates contributed by each of the $N$ rain cells that comprise the rain field, and sum them to obtain the total rain rate at that point, i.e.,

$$R_i(x, y) = \sum_{j=1}^{N} R_{i,j}(x, y),$$

where $(x_i, y_i)$ is the location of the $i$th grid point and $R_j(x_i, y_i)$ is the contribution of the $j$th rain cell at the point $(x_i, y_i)$. This yields the rain rate values for all grid points throughout the rain field $A_0$. 

46
We verified that the resulting synthetic rain fields are valid representations of the local climate by randomly picking rain fields that we generated for Tampa, FL and White Sands, NM and comparing the corresponding rain rate CCDFs over all grid points with the long-term rain rate CCDFs specified in ITU-R P.837’s rain rate statistics database. Comparison of the CCDFs shown in Figure 3.1 confirms that our rain fields are a good fit.

Figure 3.1 Comparison of long-term rain rate CCDFs for White Sands, NM and Tampa, FL and corresponding spatial rain-rate CCDFs from synthetic rain fields that were generated for each location.
3.2.2 Simulation of Rain Attenuation Time Series

We begin by selecting the altitude and inclination of the satellite orbit and the latitude and longitude of the earth station. We assume that the earth station has a clear view of the satellite during the entire portion of the pass that is at least ten degrees above the horizon. We use a commercial satellite orbit prediction tool, AGI's Satellite Tool Kit (STK), to predict the azimuth angle, elevation angle and range to the satellite at one-second intervals for many successive passes.

For each pass, we generated a two-dimensional 150-km × 150-km rain field based upon the procedures in Section 3.2.1, and placed the earth station at the centre of the rain field. We generated rain fields based upon the EXCELL model using the procedure described in [12] or the HYCELL model using the procedure described in [9], as appropriate. In either case, we estimated the height of the rain layer using the procedures in Rec. ITU-R P.839 in order to convert the two-dimensional rain field into a three-dimensional one. During a single satellite pass, we assume a fixed wind speed and wind direction. Over the long term, we assume that the wind speed is log-normally distributed with a median value of 30 km/h and a standard deviation of \( \log_2 2 \), as suggested in [13], and the wind direction is uniformly distributed. During the several minutes of each pass, we simulated advection of the rain field by the wind by sliding the rain field past the earth station at a constant rate in the specified direction.

As the simulated satellite swept across the sky, we determined the intersection of the Earth-space path with rain cells in the vicinity. We divided the portion of the slant path that passes through the rain layer into short segments, and determined the average rain rate \( R_i \) on each of them. We predicted the specific attenuation due to rain along each segment of the path using the method outlined in Rec. ITU-R P.838, and then summed the contributions of each segment to yield the total rain attenuation along the path, as given by
where $\gamma_{R_i}$ is the specific attenuation corresponding to rain rate $R_i$ and $\Delta l$ is the length of segment of the path. In this manner, we determined the rain fading experienced along the path at successive instants during the satellite pass. The model geometry is shown in Figure 3.2.

![Figure 3.2 The Earth-space link geometry where $h$ is the height of the rain layer, $L$ is the length of the slant path which is divided into short segments, $L_{\text{horizon}}$ is the length of the horizontal projection of the slant path and $\theta$ is the elevation angle.](image)

3.3 Results

3.3.1 Simulation Database and Data Processing

We used the procedure outlined in Section 3.2 to generate time series of attenuation with respect to clear air (ACA) that correspond to satellites in selected orbits as observed at specified locations in the presence of rain. In order to capture the rain attenuation dynamics, we focus on the fade slope, which is defined as

$$\zeta(t) = \frac{A(t + \frac{1}{2} \Delta t) - A(t - \frac{1}{2} \Delta t)}{\Delta t},$$

where $A(t)$ represents the attenuation at time $t$. This slope provides a measure of the rate of change of the attenuation over a small time interval, which is crucial for understanding the variability and intensity of the rain-induced fading.
where \( \zeta(t) \) is the fade slope at time \( t \), \( A(t) \) is the rain attenuation at \( t \), and \( \Delta t \) is the time interval over which the fade slope is calculated. Rec. ITU-R P.1623 suggests a minimum time interval of 2 seconds. In [5], we showed that longer time intervals would impair our ability to resolve the steeper fade slopes experienced on Earth-LEO links. Accordingly, we also use a time interval of 2 seconds here.

### 3.3.2 Model Validation on Earth-GEO Ka-band Links

The best way to validate our simulation model would be to compare our simulation results with substantial amounts of measurement data from actual Earth-LEO Ka-band links. Because such data is not yet available, we used our model to simulate the fade slope statistics on Earth-GEO Ka-band links for earth stations located near Tampa, FL and White Sands, NM, and then compared the predictions with long-term Ka-band measurement data observed during the ACTS project [14]. Over short time scales of tens of minutes in duration, variation in the rain rate over time on Earth-GEO links is mainly due to advection of the rain field past the earth station by the wind. We ran 3000 simulations each of which was 20 minutes in duration. When processing the data, we set the ACA threshold to the same values as in [14] and only accounted for the fade slopes when the ACA threshold was exceeded.

In Figure 3.3 and Figure 3.4, we show conditional CCDFs of fade slopes observed on 20 and 27.5 GHz links to satellites in geostationary orbit. The fade slope statistics based upon the HYCELL model are slightly lower than those based upon the EXCELL model, and show a slightly closer fit to measurement data. In particular, the average differences between simulated and measured fade slopes for each case fall between 0.023 and 0.119 dB/s for EXCELL-based simulations and between 0.033 and 0.063 dB/s for HYCELL-based simulations. In both cases, the close correspondence between the simulated results and the measured data gives us confidence that our simulation model is likely a useful representation of reality.
3.3.3 Comparison of EXCELL and HYCELL-based Predictions on Earth-LEO Ka-band Links

For satellites in 200, 800 and 1500-km polar orbits and earth stations located near Tampa, FL and White Sands, NM, we simulated almost 9000, 3000, and 1900 passes, respectively. The cumulative duration of the passes for each scenario exceeds 420 hours. In Figure 3.3 and Figure 3.4, we present conditional CCDFs of fade slopes observed on both 20 and 27.5 GHz links. At a specified probability level, simulations based upon the HYCELL model yield slightly lower fade slopes. This is almost certainly due to the smoother shape of the central core of the HYCELL-based rain cells. For 99% of the time, the fade slopes do not exceed the values given in Table 3.1. The difference between the EXCELL and HYCELL-based predictions increases with the magnitude of the fade slope, e.g., as the orbital altitude decreases, carrier frequency increases or average rain intensity increases, but never exceeds a few tenths of a dB/s. As in the Earth-GEO case considered in the previous paragraph, such differences are likely insignificant for the purposes of assessing the performance of fade mitigation techniques.
Figure 3.3 CCDFs of the fade slopes observed on simulated Earth-space links to LEO satellites in 200, 800 and 1500-km polar orbits and ACTS in GEO from a site near White Sands, NM, and comparison to ACTS measurement data. (a) $f = 20$ GHz. (b) $f = 27.5$ GHz.
Figure 3.4 CCDFs of the fade slopes observed on simulated Earth-space links to LEO satellites in 200, 800 and 1500-km polar orbits and ACTS in GEO from a site near Tampa, FL, and comparison to ACTS measurement data. (a) f = 20 GHz. (b) f = 27.5 GHz.
Table 3.1  The maximum fade slopes predicted using HYCELL and EXCELL-based synthetic rain fields over a 99% confidence interval.

<table>
<thead>
<tr>
<th>Satellite Altitude (km)</th>
<th>Fade slope difference (dB/sec)</th>
<th>White Sands, NM</th>
<th>Tampa, FL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20 GHz</td>
<td>27.5 GHz</td>
<td>20 GHz</td>
</tr>
<tr>
<td>1500</td>
<td>0.082</td>
<td>0.0126</td>
<td>0.081</td>
</tr>
<tr>
<td>800</td>
<td>0.091</td>
<td>0.154</td>
<td>0.111</td>
</tr>
<tr>
<td>200</td>
<td>0.337</td>
<td>0.528</td>
<td>0.405</td>
</tr>
</tbody>
</table>

3.4 Conclusions

We have simulated Earth-space Ka-band links at selected locations using: (1) synthetic rain fields that are comprised of standard rain cells and based upon long-term rain rate statistics and (2) a simple wind velocity model that accounts for advection of the rain cells over time. We have used the results to show how changing the shape of the rain cell, e.g., using the HYCELL model, with its rounded peak, instead of the EXCELL model, with its sharper peak, affects our predictions of long-term fade slope statistics on Earth-GEO and Earth-LEO links.

On simulated Earth-GEO Ka-band links to sites near White Sands, NM and Tampa, FL, we found that HYCELL-based fade slope predictions: (1) are a few-tenths of a dB/second lower than those predicted using EXCELL-based models at a given probability level and (2) provide a closer fit than EXCELL-based predictions to long-term fade slope statistics observed during the ACTS project. We observed similar trends on simulated Earth-LEO links to satellites in 200, 800 and 1500-km polar orbits where the difference between the EXCELL and HYCELL-based predictions increases with the magnitude of the fade slope. However, the differences are very small and are unlikely to have a significant impact on the assessment of alternative fade mitigation techniques. The slightly conservative fade slope estimates yielded by the EXCELL-based rain fields are offset to a large extent by the greater simplicity of the EXCELL model.
References


4 Prediction of the Effect of Turbulence Layer Height and Satellite Altitude on Scintillation on Ka-band Earth-LEO Satellite Links

4.1 Introduction

In recent years, Ka-band Earth-space links have attracted considerable attention from service providers wishing to take advantage of the broader spectrum allocations (and less congestion), higher system bandwidths (and higher data rates), reduced interference potential and smaller equipment size (especially smaller antennas) compared to lower frequencies [1]. Although most existing Ka-band Earth-space links are used to increase the capacity of conventional communications satellites located in geostationary Earth orbit (GEO) [2], much recent interest has focused on the use of Ka-band links to provide high speed data communications with satellites in low Earth orbit (LEO) during the relatively short time that a satellite passes within range of an Earth station.

A decade ago, interest in Ka-band links to LEO was driven largely by the efforts of Iridium, Teledesic and others to develop large constellations of LEO satellites capable of providing voice services and/or broadband wireless access across the globe and the need to provide such constellations with high speed Earth-space access and/or feeder links, e.g., [3]. Although commercial interest in the technology diminished for a time as confidence in the business models that supported the development of such networks evaporated, interest in Ka-band links to LEO has experienced resurgence in recent years. High speed links based upon Ka-band technology are now seen as an increasingly viable

---

method for reducing the cost of: (1) downloading gigabytes of data from scientific and Earth observation satellites during a single pass over an earth station [4] and/or (2) transferring terabytes of data between Earth stations at widely separated locations by using specialized LEO communications satellites or datasets to implement high capacity store-and-forward-based schemes [5].

Although rain fading and scintillation on Earth-space links are more pronounced at Ka-band than at lower frequencies and can greatly affect link performance, very little channel measurement data has been collected on Ka-band Earth-LEO links. For various reasons, any plans that NASA or ESA might have had to conduct measurement programs aimed at characterizing fading on Ka-band Earth-LEO links following completion of the successful geostationary Ka-band measurement programs of the 1990’s never materialized. Although Iridium LLC and Teledesic have both collected at least some Ka-band propagation data, none has been published or referred to in the open literature. Sample data from several FedSat passes over Australia were reported in 2005 [6], but little else has been released to date. As a result, measurement-based channel models for Ka-band LEO links do not yet exist.

Without a better understanding of the Ka-band Earth-LEO channel, it is difficult for designers to set link budgets and/or design and implement appropriate fade mitigation techniques. Until more extensive measurement programs are undertaken, simulation based upon reasonable models of the atmosphere is the likely best option for developing useful insights. Moreover, such an approach also provides a basis for evaluating and interpreting measurement data sets once they become more generally available. In [7], we used simulations based upon realistic synthetic rain field models to show how the rapid motion of a LEO satellite across the sky leads to steeper fade slopes than in the well-studied case of links to geostationary satellites, e.g., [8][9]. In this paper, we focus on the scintillation that is the result of the signal passing through layers of atmospheric turbulence and/or rain.
Scintillation increases greatly at low elevation angles and/or higher frequencies, may impair low margin systems and can interfere with power control algorithms used to mitigate rain fading. Our current understanding of the physics of scintillation is largely based upon the theoretical treatment developed by Tatarskii [10] in the early 1960's, based upon earlier work by Kolmogorov, that predicts both the lognormal distribution of signal amplitude over the short term and the characteristic form of the corresponding power spectrum. During the past twenty years, many researchers have conducted measurement-based studies of the amplitude and spectral characteristics of scintillation on Earth-GEO links, which are distinguished by their fixed elevation angles and path lengths. During the course of these studies, researchers have determined how the parameters of the scintillation process depend upon the height and thickness of the turbulence layer, the elevation angle of the slant path, tropospheric refractivity and water vapour content, wind speed, rain rate, carrier frequency and the size of the antenna aperture, and observed the manner in which the parameters of the scintillation process vary diurnally, seasonally and with geographic location [11]-[20].

With the exception of unpublished work such as [21], little has been reported concerning scintillation on Earth-LEO links. On such links, the satellite passes from horizon to horizon over several minutes and the motion of the satellite across the sky causes rapid changes in both the length of the slant path to the turbulence layer and the velocity at which the slant path passes across the turbulence layer. In this work, we use a geometric model of propagation through the turbulence layer during a LEO satellite pass in conjunction with Tatarskii's theory in order to obtain plausible estimates of: (1) the manner in which scintillation experienced on Ka-band Earth-LEO links is likely to evolve during the pass and (2) how scintillation will be affected by the altitude of the satellite and the height of the turbulence layer. The results will permit more effective assessment of the performance of scintillation suppression techniques and the effect of scintillation on fade mitigation techniques.

The remainder of this paper is organized as follows: In Section 4.2, we summarize current understanding of the turbulent layers that lead to scintillation. In Section 4.3, we
show how the parameters of the scintillation process can be estimated given knowledge of the look angle and angular velocity of the satellite across the sky and the height of the turbulence layer. In Section 4.4, we show how the motion and look angle of the satellite affects both: (1) the amplitude of the scintillation process, which generally increases as the elevation angle decreases and/or rain rate increases, and (2) the corner frequency of the scintillation process, which generally increases as the elevation angle increases and the height of the turbulence layer increases. In Section 4.5, we discuss both the implications of our results for simulation of scintillation. Finally, in Section 4.6, we summarize our findings and offer recommendations for future work.

4.2 Characterization of Turbulence on Earth-Space Paths

4.2.1 Characteristics of the Turbulence Layer

Scintillation is randomly occurring constructive and destructive interference caused by rapid, random fluctuations of the atmospheric refractive index caused by turbulent mixing of air masses with different temperatures and water vapour content. As an incident wave is randomly refracted by the turbulence, different versions of the incoming signal will arrive at the receiving antenna from different directions and with different amplitudes and phases. On Earth-GEO links, scintillation occurs as turbulence cells are advected past the propagation path by the wind. Analysis of scintillation events observed on such paths has long suggested that the turbulence that gives rise to the phenomenon resides in a relatively thin layer at the top of the planetary boundary layer, a relatively moist layer of air ranging from the surface of the earth up to a few kilometers [11].

Recent work has considerably refined our understanding of the turbulent layers that give rise to scintillation. In particular, it has been shown that the turbulent layers responsible for scintillation on slant paths are typically located at the topside of clouds, particularly fair weather cumulus clouds, where they are the result of an air entrainment mechanism. Measurement data collected over Belgium during a twelve-month period using a radiosonde suggests that in mid-latitude climates: (1) the layer height falls between 500
and 5000 metres and is well characterized by a Rician distribution and (2) the layer thickness is well characterized by a lognormal distribution with a median value of about 100 metres. Unlike the layer height and thickness, both the layer occurrence and the intensity of the turbulence were found to exhibit seasonal variability [12].

The refractive index structure constant $C_n^2$ is a widely used measure of atmospheric turbulence. It ranges in value from $10^{-10} \text{ m}^{-2/3}$ for a highly turbulent atmosphere to $10^{-20} \text{ m}^{-2/3}$ for weak turbulence [16]. Because the locus that defines the intersection of the slant path and the turbulence layer covers a broad extent during a typical LEO satellite pass, knowledge of the horizontal distribution of $C_n^2$ is essential for the simulation of scintillation on Earth-LEO links. Research concerning the horizontal distribution of atmospheric turbulence is still in its earliest stages, however, and much remains to be learned [22].

Previous work concerning scintillation on Earth-GEO links suggests that $C_n^2$ will be inside the turbulence layer for certain period of up to ten or fifteen minutes, within which the scintillation event is stationary with constant variance [16]. For GEO links, with wind velocity around 10 m/s, ten to fifteen minutes correspond to horizontal extent roughly within 10 km. However, for layer high from 1 km to 5 km and elevation angle above 10 degree, the horizontal extent for a satellite crossing the layer could vary from 11 km to 56 km. Therefore, it might not be reasonable to assume a constant structure parameter during a satellite pass for such large horizontal extension. Although we assume a uniform structure of turbulence in this work, further efforts should be put into the investigation of horizontal structure of turbulence layer.

### 4.2.2 Tatarskii's Theory of Scintillation

Most models of scintillation on Earth-space links, including ours, are based upon the theory proposed by Tatarskii [10] based upon earlier work by Kolmogorov. Here we briefly review the theory and consider its assumptions before applying it to scintillation on Earth-LEO links.
Then, if one can assume that the turbulence is locally homogeneous, isotropic and frozen, [22],[23] Kolmogorov’s theory of turbulence can be used to predict the variance of the scintillation amplitude. Kolmogorov’s theory characterizes turbulence eddies on two scales, the outer turbulence scale $L_0$ and the inner turbulence scale $l_0$ where, in the troposphere, $L_0$ is between 10 and 100 metres, corresponding to the instability of the medium, and $l_0$ is of the order of 1 mm, which is related to the fluid viscosity. When the eddy size $s$ falls within the inertial subrange between $L_0$ and $l_0$, the turbulence is isotropic, and the spectrum density is expressed by

$$
\Phi_n(\kappa) = 0.033C_n^2\kappa^{-11/3} \quad \text{for} \quad \frac{2\pi}{L_0} < \kappa < \frac{2\pi}{l_0},
$$

where $\kappa = 2\pi/s$ and $C_n^2$ is the structure constant of the refractive index.

Subject to the assumptions [16] that: (1) $l_0 << \sqrt{\lambda L} << L_0$, (2) the incident wave is a plane wave and (3) scintillation is weak, i.e., $\sigma_x^2 << 1$, Tatarskii [10] showed that the variance of the amplitude of a plane wave traveling through the turbulence layer is

$$
\sigma_x^2 = 0.307C_n^2k^{7/6}L^{1/6} \ Np^2 \\
= 23.17C_n^2k^{7/6}L^{1/6} \ dB^2,
$$

where $\lambda$ is the wavelength, $k$ is the wave number and $L$ is the distance through the turbulence layer. Tatarskii also showed that the scintillation spectrum can be expressed by

---

5 Some previous work has shown that (3) is valid for up to 0.2–0.5 dB², or 0.8 dB² of $\sigma_x^2$.

62
\[ W^g(f) = \frac{64.17}{f_0} C_n^2 k^{7/6} z^{11/6} \quad (dB^2 \text{ / Hz}) \quad f \ll f_c, \]
\[ W^e(f) = \frac{165.15}{f_0} C_n^2 k^{7/6} z^{11/6} \left( \frac{f_0}{f} \right)^{-8/3} \quad (dB^2 \text{ / Hz}) \quad f \gg f_c, \]

where \( z \) is the slant path distance to the turbulence layer, \( f_0 \) is the Fresnel frequency given by

\[ f_0 = \frac{v_t}{\sqrt{2\pi\lambda z}}, \]

where \( v_t \) is the transverse velocity. \( f_c \) is defined as the intersection between the two asymptotes; it is also known as the corner frequency and is equal to \( 1.43 f_0 \). The shape of spectrum of scintillation is shown in Figure 4.1.

For Ka-band (20/30 GHz), the wavelength \( \lambda \) falls between 0.01 to 0.015 metres. When the layer height \( h \) falls within the range 0.5 to 5 km, \( z \) falls between 5.8 and 28.8 km at the lowest elevation angle that we shall consider, \( \theta = 10^\circ \). Over this range, \( \sqrt{\lambda z} \) takes on values between 9 and 20 metres which satisfies the first of the three assumptions given earlier. The extreme ranges involved ensure that the incident wave is plane while experimental results collected over the years confirms that the amplitude of the scintillations observed on Earth-space links at Ka-band is generally low.
4.3 Prediction of Scintillation Parameters on Earth-LEO Paths

4.3.1 Look Angle Determination for LEO Satellites

The position of a satellite during a pass is defined by its azimuth, elevation and range relative to the earth station. The great circle angle $\gamma$ between the earth station and the sub-satellite point can be determined by constructing the spherical triangle shown in Figure 4.2 and applying the law of cosines to yield

$$\gamma = \arccos \left( \sin(L_e) \sin(L_s) + \cos(L_e) \cos(L_s) \cos(l_s - l_e) \right), \tag{4.5}$$

where $L_e$ and $l_e$ are the latitude and longitude of earth station, respectively and $L_s$ and $l_s$ are the latitude and longitude of sub-satellite point, respectively. The azimuth angle of the sub-satellite point relative to the earth station by applying the law of sines to yield the interior angle

$$\phi_{int} = \arcsin \left( \frac{\cos L_s \sin |l_s - l_e|}{\sin \gamma} \right), \tag{4.6}$$
then accounting for the quadrant in which the sub-satellite point is located relative to the earth station.

A cross-section of the great circle that contains the earth station $E$ and the sub-satellite point $S$ is shown in Figure 4.3. Applying the law of sines yields an expression for the elevation angle $\theta$

$$
\theta = \arcsin\left(\frac{R_E + H}{d}\sin \gamma\right) - \frac{\pi}{2}.
$$

(4.7)

where $R_E + H$ is the distance between the satellite and the centre of the earth and $d$, the length of the slant path to the satellite, is given by

$$
d = \sqrt{R_E^2 + (R_E + H)^2 - 2R_E H \cos \gamma}.
$$

(4.8)

In accordance with the discussion in Section 4.2.1, we assume that the turbulence layer is generally located between 0.5 and 5 kilometres above the earth’s surface.

Figure 4.2 The spherical triangle defined by the north pole $N$, the earth station $E$ and the sub-satellite point $S$. The sublayer point $P$ falls on the geodesic that links the Earth station and the subsatellite point.
4.3.2 Estimation of the Total and Transverse Velocities

We define the point $T_0$ as the intersection of the slant path with the middle of the turbulence layer of height $h$ at time $t_0$ and $v$ as the total velocity of the point $T_0$ across the layer at time $t_0$. In accordance with the discussion in Section 4.2, we assume that the turbulence layer is generally located between 0.5 and 5 kilometres above the earth’s surface.

For the case of a LEO satellite, one can determine $v$ using the geometric construction depicted in

Figure 4.3 The geometry of the Earth-space path with respect to the turbulence layer along the great circle that contains the earth station $E$, the sub-satellite point $S$, and the sub-layer point $P$. The remaining symbols are defined in the text.
Figure 4.4(a) and (b), where $T_1$ and $T_i$ are the locations of the point of intersection at one time interval $\Delta t$ after and before the current instant, respectively. The magnitude and direction of the total velocity are given by

$$\vec{v} = \frac{\vec{v}_0 + \vec{v}_1}{2} = \frac{T_1T_0 + T_0T_i}{2},$$

where $T_1T_0 = ET_0 - ET_i$, $T_0T_i = ET_i - ET_0$, and $E$ is the earth station. As shown in

Figure 4.4(c), the velocity $v$ of $T_0$ across the top of the turbulence layer at a given instant can be resolved into two components: the along path velocity $v_a$, which is parallel to the Earth-space path, and the transverse velocity $v_t$, which is perpendicular to the Earth-space path.

In the case of an overhead pass, the angle $\alpha$ between $v$ and $v_t$ depends only upon the elevation angle, and can easily be determined by constructing an appropriate plane triangle. In the general case, however, $\alpha$ is a function of both the elevation and azimuth angles to the satellite. As shown in

Figure 4.4(a), we could obtain the angle between vector $T_1T_i$ and $T_0E$, which is the same angle, $90-\alpha$, between $v$ and $v_a$. Therefore, the angle $\alpha$ is given by

$$\cos \left( \frac{\pi}{2} - \alpha \right) = \frac{T_1T_i \cdot T_0E}{|T_1T_i||T_0E|},$$

With $\alpha$ known, the transverse velocity can be determined from

$$v_t = v \cdot \cos \alpha.$$

67
From a simple geometric construction, one can show that the transverse velocity across a turbulence layer of height $h$ due to the motion of a satellite with given orbital altitude $H$ will peak at the zenith of an overhead pass. Given that the velocity of the satellite is given by

$$v_{sat} = \sqrt{\frac{GM_E}{R_E + H}}, \quad (4.12)$$

where $GM_E = 3.986 \times 10^5 \ km^3/s^2$ is the geocentric gravitational constant and $R_E = 6371 \ km$ is the mean radius of the Earth. It is then a simple matter to show that the peak transverse velocity due to satellite motion is given by

$$v_{t,\text{peak}} = \frac{h}{H} \sqrt{\frac{GM_E}{R_E + H}}. \quad (4.13)$$

In practical situations, and particularly in the case of fixed paths such as Earth-GEO links, advection of the turbulence cells by the wind must also be accounted for. We consider this in more detail in Section 4.4.4.
Figure 4.4 The relationship between the direction to the satellite along the Earth-space path, the total velocity of the Earth-space path across the turbulence layer, and the component of the total velocity that is transverse to the Earth-space path. (a) Determination of total velocity \( V_{total} \). (b) Determination of transverse velocity \( v_t \).

### 4.3.3 Prediction of the Corner Frequency

The corner frequency \( f_c \) of the scintillation power spectrum can be determined from Tatarskii’s theory using (4), which predicts that \( f_c \) increases as the transverse velocity \( v_t \) increases but decreases as the slant path length to the turbulence layer \( z \) increases. The slant path length \( z \) is directly proportional to the turbulence layer height \( h \), as shown in

\[
z = \frac{h}{\sin \theta}.
\]  

(4.14)

As discussed in Section 4.2.1, the layer height ranges from 0.5 to 5 km, and, as a result, \( z \) varies over a wide range. Because increasing \( h \) also leads to a higher total velocity \( v \) and, as a result, higher transverse velocity \( v_t \), the relationship is complex. In Section 4.4, we will predict and compare the manner in which corner frequencies evolve during a pass for layer heights of 1, 2, 3 and 4 km and satellite altitudes of 200, 800 and 1500 km.

For LEO links, the peak corner frequency that occurs at the zenith during an overhead pass, can be determined by substituting (14) into (4), yielding

\[
f_{c, \text{max}} = 1.43 \frac{h}{H} \frac{1}{\sqrt{2\pi \lambda h}} \sqrt{\frac{GM_E}{H + R_E}},
\]

\[
= 1.43 \frac{1}{\sqrt{2\pi \lambda}} \sqrt{\frac{GM_E}{R_E + H}} \sqrt{h}.
\]  

(4.15)
where $\lambda$ is the wavelength of the carrier.

### 4.3.4 Prediction of the Intensity of Scintillation

Over periods of several minutes, the instantaneous log-amplitude of scintillation on Earth-GEO links generally follows a zero-mean Gaussian distribution with an intensity $\sigma_x$ [16],[26].

$$P_1(X | \sigma_x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{X^2}{2\sigma_x^2}\right), \quad (4.16)$$

where $X$ is the amplitude of scintillation. Several models have been proposed to predict the mean standard deviation $\sigma_x$ experienced by Earth-GEO links with a specified elevation angle, frequency and antenna parameters [13]-[15]. Rec. ITU-R P. 618 recommends a model of the form

$$\sigma_x = \frac{\sigma_{ref} f^{7/2} g(x)}{(\sin \theta)^{1/2}}, \quad (4.17)$$

where $\sigma_{ref}$ is the reference (or normalized) standard deviation, $g(x)$ is the antenna aperture averaging factor, $f$ is the carrier frequency, and $\theta$ is the elevation angle.

For a period of one month or longer, the mean value of $\sigma_{ref}$ is given by

$$\bar{\sigma}_{ref} = 3.6 \times 10^{-3} + 1.03 \times 10^{-4} N_{\text{wet}}, \quad (4.18)$$

where $N_{\text{wet}}$ is the averaged wet term of the radio refractivity, and could be determined by the calculation procedure in Rec. ITU-R P. 453. Over the long term, $\sigma_x$ on Earth-GEO links has been found to follow a Gamma [11][13] or log-normal distribution [27][28]. Because $\sigma_{ref}$ is proportional to $\sigma_x$, we assume $\sigma_{ref}$ itself, which is independent of elevation, frequency and antenna effects, is also Gamma or log-normally distributed. If we assume that $\sigma_{ref}$ follows a Gamma distribution [13],

70
and where \( m_\sigma \) and \( \sigma_\sigma \) are the mean and standard deviation of \( \sigma_{\text{ref}} \), we can generate values for \( \sigma_{\text{ref}} \) that we can apply to successive passes.

For dry scintillation, we calculate time-series \( \sigma_x \) during a LEO satellite pass following the procedures: (1) determine the mean value \( m_\sigma \) of \( \sigma_{\text{ref}} \) from Rec. ITU-R P.618, and calculate \( \sigma_\sigma \); (2) calculate parameters \( \alpha \) and \( \beta \) to determine the Gamma distribution; (3) generate a random value from Gamma distribution as the \( \sigma_{\text{ref}} \) for the pass; and (4) calculate the time-series \( \sigma_x \) by incorporating the elevation, antenna and frequency effects using equation (4.18).

For wet scintillation, the model based upon the correlation between \( \sigma_x \) and rain attenuation is more appropriate. Matricciani [17] proposed a model originated from Tatarskii’s theory, and Van de Kamp [18] shows a statistical model based upon measurements. Based upon ITALSAT measurement, these two methods were rationalized [20] to yield

\[
\sigma_x = \sigma_0 \quad \text{dB if } A \leq 1 \text{ dB},
\]

\[
\sigma_x = \sigma_0 A^{3/2} \quad \text{dB if } A > 1 \text{ dB},
\]

(4.21)

for Matricciani model, and

\[
\sigma_x = \sigma_0 + 0.02A \quad \text{dB.}
\]

(4.22)

for the Van de Kamp model, where \( A \) is the rain attenuation. Combining such models with the simulated rain fading in [7], we are able to give a good estimation of time-series \( \sigma_x \) during a rain event for LEO satellite passes. Examples are presented in Section 4.5.
4.4 Evolution of Scintillation Parameters during Typical LEO Satellite Passes

4.4.1 Low Earth Orbits of Particular Significance

Specifying a link to a satellite in a low earth orbit involves many more degrees of freedom than in the geostationary case. The orbital altitude may range from 200 to 2000 km and the inclination angle may range from 0 degrees (equatorial) to 90 degrees (polar) to slightly beyond (sun-synchronous). The orbital altitude affects the minimum and maximum range to the satellite during each pass and the rate at which the satellite moves across the sky, as seen by the earth station. The inclination angle, combined with the latitude of the earth station, affects the probability distribution function of the satellite’s elevation angle as seen by the earth station [29]. Here, we have focused our initial efforts to characterize scintillation on Earth-LEO links on satellites in circular polar orbits with altitudes of 200, 800, and 1500 km. While such orbits are broadly representative of those occupied by many Earth observation and scientific satellites, the modeling approach described in the previous section can easily be extended to satellites in orbits with other altitudes, inclinations and ellipticities as required.

4.4.2 Evolution of Scintillation Parameters during Typical Passes

The motion of a LEO satellite across the sky causes rapid changes in both the length of the slant path to the turbulence layer and the velocity at which the slant path passes across the turbulence layer. As we showed in 4.3, this affects both the intensity of the scintillation process, which generally reaches its maximum value at low elevation angles and/or periods of rain, and the corner frequency of the scintillation process, which generally reaches its maximum value at high elevation angles.

We begin by selecting the altitude and inclination of the satellite orbit and the latitude and longitude of the earth station. We assume that the earth station has a clear view of the satellite during the entire portion of the pass that is at least ten degrees above the horizon.
We use a commercial satellite orbit prediction tool, AGI's Satellite Tool Kit (STK), to predict the azimuth angle, elevation angle and range to the satellite at one-second intervals for many successive passes. This allowed us to determine the corresponding values of the scintillation intensity $\sigma_x$ and corner frequency $f_c$ at each instant following the methods in Section 4.3.3 and 4.3.4. In our analysis, we focused on the portions of the satellite pass with an elevation angle greater than 10 degrees.

In Figure 4.5, Figure 4.6 and Figure 4.7, we show how the elevation angle $\theta$, total velocity $v$, transverse velocity $v_t$, the corner frequency $f_c$ and the standard deviation $\sigma_x$ evolve during overhead passes by LEO satellites in 200, 800 and 1500-km polar orbits with a thin layer of turbulence at an altitude of 1 km.

1. As the elevation angle increases, it is apparent that: (1) $\sigma_x$ decreases with increasing elevation angle, as given in eqn. (4.18); (2) the decreasing total velocity $v$ does not imply a corresponding reduction in the transverse velocity $v_t$, because $a$, the angle between $v$ and $v_t$, decreases, yielding that higher portion of $v$ contributes to $v_t$; (3) the corner frequency $f_c$ reaches a peak value at the highest elevation angle. The impact of the increased rate of scintillation at high elevation angles is considerably reduced by the sharp reduction in scintillation intensity in such directions.

2. As the orbital altitude increases, it is apparent that: (1) the rate at which the satellite passes through the zenith decreases markedly; (2) the minimum and maximum values of $\sigma_x$ are not affected by changes in the satellite's altitude, while the rate of change decreases because of the slower variation of the elevation angle; (3) the total velocity $v$ at which the point $T$ crosses the turbulence layer decreases less rapidly as the elevation angle increases, (4) the transverse velocity $v_t$ decreases, with the value between 15~38, 6~10 and 4~7 m/s for 200, 800 and 1500 km orbit, respectively; and (5) the maximum corner frequency $f_c$ experienced during a pass decreases, with the values of 5.6, 1.4 and 0.7 Hz for 200, 800 and 1500 km orbits, respectively. The manner in which $\sigma_x$ and $f_c$ evolve with orbital altitude are directly compared in Figure 4.8(a) and (b), respectively.
3. As the height of the turbulence layer increases, the corner frequency increases substantially, as shown in Figure 4.8(c). Although the increase in \( h \) causes both \( v_t \) and \( z \) to increase, which have contrary effects on corner frequency as discussed in Section 4.3.3, the result shows that the increase in \( v_t \) dominates. The peak corner frequencies for 200 km orbit, as shown in Figure 4.8(c), are roughly 5.6, 7.9, 9.7, 11.2 Hz for 1, 2, 3, and 4-km layer heights, respectively. For a given satellite altitude, the standard deviation of the scintillation amplitude is unaffected by the height \( h \) of the turbulence layer because the elevation angles are the same.

4.4.3 Statistics of Scintillation Parameters over Multiple Passes

In Figure 4.9(a), we present the CCDF of the standard deviation of the scintillation amplitude taken over hundreds of passes. We have included for the tendency for the intensity of the scintillation process to follow its own gamma distribution over the long term, as described in Section 4.3.4. As the satellite altitude increases the shape of the CCDF of \( \alpha_x \) evolves as shown in Figure 4.9(a) because the fraction of time that the satellite spends at higher elevation angles also increases.

In Figure 4.9(b) we present the CCDF of the corner frequency taken over hundreds of passes for the turbulence layer at a height of 1 km and satellite altitudes of 200, 800 and 1500 km, both for cases with and without wind. It is apparent that: (1) at higher satellite altitudes (800 and 1500 km), the CCDF of corner frequency drops off very suddenly which indicates that only a very narrow range of values of \( f_c \) are experienced by the link, (2) at lower altitudes (200 km), the range of corner frequencies experienced on the link increases but the probability of encountering a peak value is fairly rare. In Figure 4.9(c) we show the CCDFs of the corner frequency for the turbulence layer at heights of 1, 2, 3 and 4 km and the satellite at an altitude of 200 km. The result shows that higher corner
frequencies are reached at higher layer altitude, and wider range of $f_c$ are experienced by the link.

### 4.4.4 Wind Effects on Corner Frequency

Advection of turbulence cells past the Earth-space path by the wind contributes a random component to the transverse velocity that must be accounted for when predicting the corner frequency $f_c$ of the scintillation process. We introduce the effect of wind into our simulations as follows. First, we follow [25], and assume that the wind speed follows a log-normal distribution over the long term with a median value of 8.33 m/s and a standard deviation of $\log e 2$. Further, we assume that wind direction is uncorrelated with wind speed and follows a uniform distribution. Because a LEO satellite pass is only several minutes in length, we assume that the wind velocity is constant for the duration. As the direction of the azimuth angle changes during the pass, the wind’s contribution to the transverse velocity, $v_{L,\text{wind}}$, also changes.

A typical result for a satellite in an 800-km orbit is shown in Figure 4.10. The evolution of the azimuth angle during the pass is shown in Figure 4.10(a). The contributions of wind and satellite motion to the transverse velocity as the pass evolves are shown in Figure 4.10(b). In this case, the transverse velocity $v_{L,\text{wind}}$ decreases to zero when the azimuth angle points into the wind. The manner in which the total transverse velocity and corner frequency are distorted as the pass evolves is shown in Figure 4.10(c) and (d). In Figure 4.9(b), the CCDFs of the corner frequency over the long term for satellites in 200, 800 and 1500-km orbits can be compared directly with and without account taken for wind. It is apparent that advection of turbulence cells by the wind may cause relatively large changes in $f_c$, which cannot be ignored.
Figure 4.5 Typical results for satellite in 200 km polar orbit, with 1 km layer height.
Figure 4.6 Typical results for satellite in 800 km polar orbit, with 1 km layer height.
Figure 4.7 Typical results for satellite in 1500 km polar orbit, with 1 km layer height.
Figure 4.8 Evolution of standard deviation and corner frequency during a pass. (a) Time series intensity, with a 1-km layer height. (b) Time series corner frequency, with 1-km layer height. (c) Time series corner frequencies for satellite in 200 km polar orbits, with turbulence layer at 1, 2, 3 and 4 km altitude.
Figure 4.9 CCDFs of the standard deviation and corner frequency. (a) Scintillation intensity for different satellite altitudes, with 1-km layer height. (b) Corner frequency for different satellite altitudes and a 1-km layer height, with and without wind. (c) Corner frequency for different layer heights for a satellite in a 200-km polar orbit.
Figure 4.10 Evolution of the corner frequency of scintillation during an overhead pass by a satellite in an 800-km orbit with an 8.3 m/s wind blowing at 30 degrees: (a) Evolution of the azimuth angle during the pass. (b) Contributions of wind and satellite motion to the transverse velocity during the pass. (c) Evolution of the total transverse velocity during the pass. (d) Evolution of the corner frequency during the pass.
4.5 Implications for Generation of Scintillation Time Series

When simulating fade mitigation techniques or assessing scintillation suppression methods, it is usually necessary to provide time series data that mimic actual scintillation, *i.e.*, have the same first and second-order statistics. One method for generating scintillation time series that are representative of those observed on Earth-GEO links involves passing additive white Gaussian noise (AWGN) through a filter with a suitable low pass response then adjusting the scintillation intensity to the correct value [20]. For the case where precipitation is absent, *i.e.*, dry scintillation, the scintillation intensity is typically determined using the model contained in Rec. Rec. ITU-R P.618 while the distribution of the scintillation intensity over the long term is typically described by a Gamma distribution, as described in [13]. For the case where precipitation is present, *i.e.*, wet scintillation, the scintillation intensity depends upon the instantaneous depth of rain fading and can be calculated using either of the techniques described in [17] and [18], as described in Sec. 4.3.4. Another approach to simulating wet scintillation involves interpolating between the samples in a rain attenuation time series subject to the assumption that scintillation can be modeled as fractional Brownian motion [30].

Earth-LEO links introduce additional complications when simulating scintillation time series by low pass filtering white Gaussian because both the corner frequency and the scintillation intensity evolve rapidly during a pass. Filters with time-varying parameters are relatively simple to implement but care must be taken to avoid transient behavior when the filter parameters are updated [31][31]. Our time-varying scintillation time-series generator is based upon an AWGN generator followed by a fourth order low pass filter whose coefficients updated every sample. Following [20], we used the Yule-Walker equations to estimate the low pass filter coefficients required to realize a scintillation process with the normalized power spectral density shown in Figure 4.1. Although the scintillation power spectrum rolls off at -80/3 dB²/Hz, the corresponding spectral shaping filter rolls off at -40/3 dB/Hz [20].
Simulation of a scintillation time series proceeds as follows. We apply AWGN signal to the spectral shaping filter, and then, for every sample, we update the filter coefficient according to the instantaneous corner frequency that we predict using the geometric model described in Section 4.3.3. Finally, we scale the signal that appears at the filter output in obtain the desired scintillation intensity. Although our time series generator updates the spectral shaping filter every sample, it may be desirable in some cases to reduce the computational load by updating the filter once per frame. We leave this possibility and determination of the ideal frame length for future study.

In Figure 4.1 we show a typical result for dry scintillation during an overhead pass to a LEO satellite in a 1500-km polar orbit, and on an Earth-GEO link with an elevation angle of 30 degrees. It is apparent that: (1) at low elevation angles (at the start and the end of the pass), more intense scintillation is observed, and (2) at high elevation angles, the variation of the signal is relatively small. In Figure 4.12, we show a typical result for wet scintillation during an overhead pass by a LEO satellite in a 1500-km polar orbit, with a maximum elevation angle of 22 degrees. Substantial increases in scintillation intensity can be seen: (1) at the start and end of the pass, where they are a result of low elevation angle effects and (2) near \( t = 500 \) sec elapsed time, where they are the result of a sudden increase in rain attenuation.

Here, for lack of more detailed information concerning the spatial distribution of turbulence, we have assumed that the refractive index structure constant \( C_n^2 \), hence the value of \( \sigma_{\text{ref}} \), is constant along the path defined by the intersection of the slant path with the turbulence layer. In practice, it seems likely that the structure constant will change as the path passes from horizon and intersects different parts of the sky. Resolution of this issue will likely have to await general availability of actual measurement data from Earth-LEO Ka-band links.
Figure 4.11 Evolution of dry scintillation over time for typical links to satellites in both LEO and GEO. (a) Dry scintillation during a pass by a satellite in a 1500-km polar orbit. (b) Dry scintillation to a satellite in GEO at an elevation angle of 30 degree over the same duration as in (a).
Figure 4.12 Accounting for wet scintillation during a pass by a satellite in a 1500-km polar orbit. (a) Evolution of path gain during the pass with account taken for range, atmospheric gases, cloud and fog, and rain fading only. (b) Evolution of wet scintillation during the pass. (c) Evolution of total path gain, including wet scintillation, during the pass.
4.6 Conclusions

We have used Tatarskii's theory in conjunction with a geometric model of propagation through a uniform turbulence layer of broad extent during a LEO satellite pass to show how the amplitude and rate of the scintillation process will evolve as a function of the orbital altitude and height of the turbulence layer. While the amplitude of the scintillation process is only determined by the elevation angle to the satellite, the maximum corner frequency increases markedly as the orbital altitude decreases. The time-varying nature of the corresponding scintillation power spectrum makes it necessary to use time-varying filters when simulating scintillation on LEO links and introduces an extra level of complexity compared to simulation of scintillation on GEO links.

While our simulations have revealed the general manner in which the scintillation process will likely evolve during a LEO satellite pass, the accuracy of our results is limited by the simplifying assumptions that we have been forced to make concerning the horizontal structure of the turbulence layer and the manner in which it is affected over time by the wind. Research in this area is being pursued by various groups and the outcomes of that work will greatly benefit simulation studies of the sort presented here. Until more extensive measurement data is available, simulation studies of the type presented here are likely the best option for developing immediately useful insights concerning the nature of scintillation during LEO satellite passes. Moreover, once measurement data becomes more generally available, studies of this type will provide a benchmark against which measurement data can be assessed and interpreted.
References


5 Conclusions and Recommendations

5.1 Conclusions

Rain attenuation and scintillation are two major impairments on Ka-band Earth-space links. Although comprehensive research activities have been conducted to study these two impairments on Earth-GEO links, little work has been done concerning rain fading and scintillation on Earth-LEO links. The major contributions of this work were (1) development of two simulation models to predict rain attenuation and scintillation on Ka-band Earth-LEO links, (2) giving the first estimation of rate at which the rain attenuation changes and scintillation happens on Ka-band Earth-LEO links, and (3) showing the important effect of satellite altitude on fade slope statistics and scintillation process. Furthermore, the simulation results of rain fading and scintillation presented in this work could become a reference being compared to when the measurement data on Ka-band Earth-LEO links is available. This work and the results will help the people who (1) are planning the measurements on Ka-band Earth-LEO links, (2) need to design the power control algorithm on such links, (3) are interested in the channel modeling on such links, and (4) waiting for the simulation model or results to describe the existing measurement data.

The major contributions of this work were accomplished by three innovative concepts. (1) Traditionally for Earth-GEO links, the synthetic rain field technique is used to determine the site diversity of rain rate or rain attenuation. In this work, the spatial distribution of rain rate offered by the synthetic rain field technique plays an essential role in predicting the rain fading during a satellite pass, and is combined with the fade slope analysis on Earth-LEO links. (2) Traditional evaluation of different rain cell models is on the basis of comparing how accurate the model predicts the long term rain rate distribution, i.e., first order statistics. In this work, the shape of rain cell, i.e., how fast the rain rate increases/decreases, has been taken into account and linked to the second order statistics, fade slopes. (3) The traditional models of predicting intensity and
corner frequency of the scintillation on Ka-band Earth-GEO links have been combined with a geometry model of predicting the look angles and transverse velocity of the LEO satellite. The model captured the effect of satellite motion and time-variant elevation angles on intensity and corner frequency of the scintillation during a pass.

This work showed several advantages. (1) Both the statistical model of predicting rain fading and the geometrical model of simulating scintillation are straightforward in concept, (2) By adjusting the input parameters according to the specific location or weather condition, both models are adapted to variety situations. (3) Two models can be easily packaged to a complete simulator and put into practice to predict the time-series received signal on real Ka-band Earth-LEO links.

However, the weakness and limitation of this work still need to be paid attention to. (1) The temporal evolution of rain cells, which has been put aside, and the spatial distribution of turbulence layer, which has been assumed to be uniform, are two missing pieces of this work. (2) It is assumed in the simulations that the weather parameters, e.g., temperature, humidity and wind speed and velocity do not change during a satellite pass, which might not be true in reality. (3) The satellite orbit was limited to the polar orbit, which might exclude other factors, e.g., inclination angle, that affect the fade slope and scintillation process. (4) The performances of the models remain to be tested in the future by comparing the simulation results with the measurement data for more locations.

5.2 Recommendations for Further Work

In this work, first attempt of simulating the rain fading and scintillation on Earth-LEO links has been developed. Because of the limitation of this work and some further interests, several future research topics are suggested. (1) Study of temporal evolution of rain cells, the spatial distribution of turbulence [1] and the wind speed distribution as a function of location and altitude will greatly help to improve the simulation models. (2) Rain rate spatial distribution is essential when predicting the time-series rain fading. Radar observations [2] of rain rate can be considered as a reliable source to map the rain
rate distribution and be combined with the models. (3) Power spectral density (PSD) analysis [3] is a common technique having been implemented in GEO cases when dealing with the measurement data. This technique could also be introduced to analyse the simulation results on Earth-LEO links. (4) More satellite orbits with different inclination angles [5] need to be implemented to study if the satellite inclination has an effect on the fade slopes and scintillation. (5) When measurement data on Ka-band Earth-LEO links is available, the models might need some improvements or modifications. (6) Power control algorithms [6] of mitigating attenuation on Earth-LEO links can be designed based on the simulation models.
References


# APPENDICES

## Appendix A

Table A. 1 List of ITU-R Recommendations

<table>
<thead>
<tr>
<th>ITU-R Rec.</th>
<th>Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITU-R P.453-9</td>
<td>The radio refractive index: its formula and refractivity data</td>
</tr>
<tr>
<td>ITU-R P.618-9</td>
<td>Propagation data and prediction methods required for the design of Earth-space telecommunication systems</td>
</tr>
<tr>
<td>ITU-R P.676-6</td>
<td>Attenuation by atmospheric gases</td>
</tr>
<tr>
<td>ITU-R P.836-3</td>
<td>Water vapour: surface density and total columnar content</td>
</tr>
<tr>
<td>ITU-R P.837-5</td>
<td>Characteristics of precipitation for propagation modeling</td>
</tr>
<tr>
<td>ITU-R P.838-3</td>
<td>Specific attenuation model for rain for use in prediction methods</td>
</tr>
<tr>
<td>ITU-R P.839-3</td>
<td>Rain height model for prediction methods</td>
</tr>
<tr>
<td>ITU-R P.840-3</td>
<td>Attenuation due to clouds and fog</td>
</tr>
<tr>
<td>ITU-R P.1623-1</td>
<td>Prediction method of fade dynamics on earth-space paths</td>
</tr>
</tbody>
</table>
Appendix B

Additional figures for Chapter 2 are listed in this section.

Figure B.1 Histogram of satellite in different altitude.

Figure B.2 Satellite altitude versus inclination angle
Figure B.3 A 2D rain field based on EXCELL model, in the region of Tampa, FL.
Figure B.4 CCDFs of the fade slope observed on simulated Earth-space links to LEO satellites from sites corresponding to Tampa, FL and White Sands, NM. (a) satellite in 1500 km polar orbit. (b) satellite in 800 km polar orbit. (c) satellite in 200 km altitude polar orbit.
Appendix C

Additional figures and tables for Chapter 3 are listed in this section.

Figure C.1 Modeling of a two-dimensional rain field based upon HYCELL model, in the region of Tampa, FL.
Figure C.2 CCDF of the difference between the fade slopes predicted by HYCELL and EXCELL-based synthetic rain field models at specified probability levels as observed on simulate 27.5 GHz links over multiple passes to satellites in 200, 800 and 1500-km polar orbits from a site near White Sands, NM. (a) 20 GHz. (b) 27.5 GHz.
Figure C.3 CCDF of the difference between the fade slopes predicted by HYCELL and EXCELL-based synthetic rain field models at specified probability levels as observed on simulate 27.5 GHz links over multiple passes to satellites in 200, 800 and 1500-km polar orbits from a site near Tampa, FL. (a) 20 GHz. (b) 27.5 GHz.
Table C.1 Comparison of the Rain rate statistics Specified by ITU-R and Captured by a Randomly Selected Synthetic Rain Field

<table>
<thead>
<tr>
<th>Conditional probability level (rain rate &gt; 0.1 mm/h)</th>
<th>Rain rate exceeded (mm/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>White Sands, NM</td>
</tr>
<tr>
<td></td>
<td>Rain field</td>
</tr>
<tr>
<td>0.3</td>
<td>1.4</td>
</tr>
<tr>
<td>0.2</td>
<td>2.8</td>
</tr>
<tr>
<td>0.1</td>
<td>5.8</td>
</tr>
<tr>
<td>0.05</td>
<td>10.7</td>
</tr>
<tr>
<td>0.03</td>
<td>16.5</td>
</tr>
<tr>
<td>0.02</td>
<td>22.7</td>
</tr>
<tr>
<td>0.01</td>
<td>35.2</td>
</tr>
<tr>
<td>0.005</td>
<td>50.4</td>
</tr>
<tr>
<td>0.003</td>
<td>61.3</td>
</tr>
<tr>
<td>0.002</td>
<td>70.4</td>
</tr>
<tr>
<td>0.001</td>
<td>79.0</td>
</tr>
</tbody>
</table>
Appendix D

Here is a detailed calculation procedure for look angle determination. In order to determine the visibility of LEO satellite, azimuth, elevation angle and range (AER) are three important parameters.

Step 1: Determine the longitude and latitude of subsatellite point

We know that the period of Earth rotation is 24 hours. Therefore, the period of longitude changing 360 degrees is also 24 hours, and the rate of change, $\omega_{lon}$ is expressed as

$$\omega_{lon} = \frac{2\pi}{24 \times 3600} = \frac{\pi}{43200}\text{ rad/s.} \quad (C.1)$$

The period of latitude changing 360 degrees equals to the satellite orbital period.

$$T_{sat} = \frac{2\pi^{1.5}}{\mu^{0.5}} \text{ second,} \quad (C.2)$$

where $T_{sat}$ is the orbital period, $r_s$ is the distance between satellite and center of earth, $\mu$ is Kepler’s constant, and equals to $3.986004418 \times 10^5 \text{ km}^3/\text{s}^2$. Therefore, the rate of change of latitude is expressed as

$$\omega_{lat} = \frac{2\pi}{T_{sat}} = \frac{\mu^{0.5}}{r_s^{1.5}} \text{ rad/s.} \quad (C.3)$$

We assume the starting subsatellite point is located at zero latitude and zero longitude. Then the longitude of subsatellite point is

$$l_s = \omega_{lon} \cdot t - 2k_1\pi \text{ rad,} \quad (C.4)$$

where $t$ is time, $k_1$ is the number of full rounds that the earth has rotated and equals to the integer part of $\frac{\omega_{lon} \cdot t}{2\pi}$.

Assume that the satellite moves towards north first, the latitude of subsatellite point is
\[ L_s = \begin{cases} \omega_{lat} \cdot i - 2k\pi & i \leq \frac{\pi}{2\omega_{lat}} \\ \pi - \omega_{lat} \cdot i - 2k\pi & \frac{\pi}{2\omega_{lat}} < i \leq \frac{3\pi}{2\omega_{lat}} \\ \omega_{lat} \cdot i - 2\pi - 2k\pi & \frac{3\pi}{2\omega_{lat}} < i \leq \frac{2\pi}{\omega_{lat}} \end{cases} \] (C.5)

where \( k_2 \) is the number of full rounds that the satellite has traveled and equals to the integer part of \( \frac{\omega_{lat} \cdot i}{2\pi} \). Then the subsatellite location is \((L_o, l_o)\).

**Step 2: Determine the central angle \( \gamma \)**

The central angle \( \gamma \) is calculated using

\[ \cos \gamma = \cos(\frac{\pi}{2} - L_s) \cos(\frac{\pi}{2} - L_e) + \sin(\frac{\pi}{2} - L_s) \sin(\frac{\pi}{2} - L_e) \cos(l_s - l_e) \]

\[ = \sin(L_s) \sin(L_e) + \cos(L_s) \cos(L_e) \cos(l_s - l_e), \] (C.6)

where \( L_e \) and \( l_e \) are the latitude (north from equator) and longitude (west from Greenwich meridian) of earth station, respectively. \( L_s \) and \( l_s \) are the latitude and longitude of subsatellite point, respectively.

**Step 3: Calculate elevation angle**

The distance, \( d \), between satellite and earth is defined as the range of satellite, and is calculated geometrically as

\[ d = \sqrt{r_s^2 + r_e^2 - 2r_sr_e \cos \gamma} \text{ km}, \] (C.7)

where \( r_s \) is the distance from satellite to the earth center, \( r_e \) is the average radius of earth. By the law of sines, we have

\[ \frac{r_s}{\sin \psi} = \frac{d}{\sin \gamma}, \] (C.8)
where $\psi$ is the angle (within the triangle) measured from $r_e$ to $d$, and

$$\theta = \psi - \frac{\pi}{2} \text{ rad}, \quad \text{(C.9)}$$

where $\theta$ is the elevation angle. Combining (7), (8) and (9), we could get

$$\theta = \arcsin\left(\frac{r_e \sin \gamma}{d} \right) - \frac{\pi}{2} = \arcsin\left(\frac{r_e \sin \gamma}{\sqrt{r_s^2 + r_e^2 - 2r_s r_e \cos \gamma}}\right) - \frac{\pi}{2} \text{ rad.} \quad \text{(C.10)}$$

**Step 4: Calculate the azimuth angle**

The calculation of azimuth angle is more complicated because the geometry involves determining whether the subsatellite point is east or west of the earth station, and in which of the hemisphere the earth station and subsatellite point are located.

By the law of sines in a spherical triangle, we have

$$\frac{\sin \varphi_{int}}{\sin\left(\frac{\pi}{2} - L_s\right)} = \frac{\sin|L_s - L_e|}{\sin \gamma}, \quad \text{(C.11)}$$

where $\alpha$ is the intermediate angle for calculating the azimuth angle, and is between 0° and 90° degree. And then, we are able to get $\alpha$ in the form of

$$\varphi_{int} = \arcsin\left(\frac{\cos L_e}{\sin \gamma} \frac{\sin|L_s - L_e|}{\sin \gamma}\right) \text{ rad.} \quad \text{(C.12)}$$

Four cases need to be considered:

a. when the satellite is NE of the earth station, $\varphi = \varphi_{int}$;

b. when the satellite is NW of the earth station, $\varphi = 2\pi - \varphi_{int}$;

c. when the satellite is SE of the earth station, $\varphi = \pi - \varphi_{int}$;

d. when the satellite is SW of the earth station, $\varphi = \pi + \varphi_{int}$.