CONTOUR REDUCTION ALGORITHMS: A THEORY OF PITCH AND DURATION HIERARCHIES FOR POST-TONAL MUSIC

by

MUSTAFA BOR

B.A., Bilkent University, 2001
M.Mus., The University of Alberta, 2003

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

in

THE FACULTY OF GRADUATE STUDIES

(Music)

THE UNIVERSITY OF BRITISH COLUMBIA

(Vancouver)

April 2009

ABSTRACT

This dissertation takes work on contour by Robert Morris as a point of departure and develops a set of contour reduction algorithms, called *window algorithms*. These involve the notion of a hypothetical window or frame of a specific width (i.e. number of events) through which the contour succession in a given melody is experienced temporally (much like the way a landscape is experienced visually through the side window of a moving automobile or train). Certain normative principles relevant to windows of various widths are devised and represented with the help of symbolic logic and flowcharts. Reiterative application of the window algorithms on a melody to "prune" pitches at a series of successive levels, introduces notions of melodic contour hierarchy that are explored in various ways throughout the dissertation. The application of the algorithms is demonstrated on a variety of 20th century musical excerpts reflecting a wide range of melodic archetypes, thereby enabling observation of the behavior of the algorithms in different musical contexts. Phenomenological and cognitive implications of the algorithms are discussed from the perspective of a listener implementing the algorithm on the fly. An analysis focusing on the *Hauptstimmen* in the first movement of Schoenberg's Third String Quartet explores how intervallic features of the reduced contours can form the basis for a tonal-formal reading of the movement. The possibility of extending the theory to the duration domain is also introduced; following a preliminary analysis involving instrumental gestures from the opening of Webern's *Variations for Orchestra*, and a discussion of analytical, conceptual, and methodological inconsistencies arising from the application of the algorithms on duration contours, an alternative approach that attaches appropriate durations to (reduced) pitch contours is developed, examined, and advocated. The relationships and interactions between the pitch and duration contours in Berio's *Sequenza I* are examined in the light of the proposed theory.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ACKNOWLEDGEMENTS

This dissertation would not have been possible without the keen insight and perceptive guidance of my advisor, Richard Kurth. His enthusiasm, his trust in my abilities, and his generosity with both time and ideas have provided me with an invaluable experience throughout the research and writing process. I am also grateful to the other members of my committee, John Roeder and Alan Dodson, for their careful reading and thoughtful critiques, which have greatly improved and enriched this dissertation.

I would like to extend my thanks to William Benjamin for stimulating discussions during the course of my studies and to fellow students, Stephanie Lind, Rebecca Simpson-Litke, Scott Cook, Robin Attas, and Alfredo Santa Ana for their camaraderie. I am also indebted to Henry Klumpenhouwer for introducing me to the fascinating and multifaceted world of music theory some years ago.

Finally, I wish to thank my parents, Şakir and Şule, and my sister, Ayşe, who wholeheartedly supported me in every possible way, and most of all, Jana for her unconditional love and understanding during the completion of my doctoral work.

CHAPTER 1

INTRODUCTION

Definition and Context

In the most general sense, the term "contour" refers to an overall outline, profile, or gradient, which represents the shape of something. A representation of any shape requires at least two dimensions, and a generalized contour can be defined as an ordering of values in one dimension (or parameter) relative to the ordering of values in another dimension (or parameter). For example, in meteorological cartography, the parameters often include humidity, precipitation, or pressure, relative to geographic position. In music, the term contour is most often understood to be an ordering of relative pitch height (y-axis on a Cartesian coordinate system) in relation to sequential temporal ordering (x-axis on a Cartesian coordinate system). Nevertheless, parameters other than these (especially duration) have been examined increasingly in recent advancements to contour theory. In principle, one could define and study musical contours involving not just pitch height or duration relative to temporal ordering, but also any parameters such as pitch height, duration, loudness, accent, timbre (etc.) relative to one another.[1] The potential interest of these new sorts of contours notwithstanding, this dissertation focuses on contours involving pitch and duration, and then explores notions of hierarchy in those particular contexts. Potentially, contour relationships involving parameters other than duration and pitch could also be examined hierarchically, but such an investigation will not be pursued here.

[1] For a detailed formal definition of contour in music, see Robert D. Morris, *Composition with Pitch-Classes* (New Haven: Yale University Press, 1987), 26-27.

The concept of contour plays a prominent role in the perception of post-tonal music since, as psychologist Judy Edworthy, among others, has suggested that contour is arguably "more important, relative to interval information, under those circumstances that make the establishment of a key difficult."[2] Despite the significance of contour in a post-tonal musical context, most of the analytical tools developed for this repertoire are formally conditioned by the modularity of pitch-class space, and do not take registral characteristics into account. By contrast, the concept of contour is established upon non-modular space, and it provides a unique opportunity to analyze post-tonal music from a hierarchical perspective—one that is not explicitly conditioned by the concept of key.

The three primary goals of this dissertation are: (1) to develop a set of contour reduction algorithms, called *window algorithms*, in order to explore musical hierarchies based on contour features rather than tonality or other traditional criteria; (2) to extend the applicability of the algorithms to the domain of durations; and (3) to demonstrate the analytical possibilities provided by the theory.

Outline of Chapters

Chapter two lays out the theoretical foundation for this study by reviewing the literature on musical contour from the mid-twentieth century up to the present. Emphasis is placed on the more recent developments in contour theory, since the mid-1980s.[3]

---

[2] Edworthy, Judy. "Melodic Contour and Musical Structure." In *Musical Structure and Cognition*, edited by Peter Howell, Ian Cross and Robert West, 184. London: Academic Press, 1985.

[3] Note that the scope of this dissertation is limited to "contour theory" and although there is significant conceptual connection between the theories of gesture and contour, the present study does not adopt a semantic or semiotic approach. For a collection of essays in musical gesture, see, Anthony Gritten and Elaine C. King, eds., *Music and Gesture* (Aldershot: Ashgate Publishing, 2006). See also, Robert S. Hatten, *Interpreting*

Chapter three opens with an evaluation of Robert Morris's contour reduction algorithm and Rob Schultz's recent refinements to that algorithm.[4] This section is followed by an introduction to *window algorithms* for reducing contours. These are established on the notion of a hypothetical window of a specific width moving across a given musical segment.[5] In particular, two of the window algorithms, the 3-window algorithm and the 5-window algorithm, are given special emphasis. After a discussion of how repeating pitches can be incorporated into the algorithms, formal definitions of the algorithms are presented via the help of symbolic logic and flowcharts. Based on the successive applications of these algorithms in various combinations, the concept of *contour reduction functions* is then introduced. A single-staff representation of all hierarchical "depth levels" is proposed for notational convenience. The chapter concludes with a comparison of Morris's reduction algorithm, the 3-window algorithm, and the 5-window algorithm.

Chapter four explores the application of the algorithms on a variety of musical excerpts taken from twentieth-century works by a wide range of composers. The excerpts reflect a variety of melodic archetypes, some with global ascent/descent, some with zigzags, and others with successive or non-successive repeating pitches. This chapter aims to demonstrate the behavior of the algorithms on these distinct melodic archetypes, in order to form a clearer sense of contour reduction in a range of contexts.

Musical Gestures, Topics, and Tropes: Mozart, Beethoven, Schubert (Bloomington: Indiana University Press, 2004) and Michael Spitzer, *Metaphor and Musical Thought* (Chicago: University of Chicago Press, 2004).

[4] Robert D. Morris, "New Directions in the Theory and Analysis of Musical Contour," *Music Theory Spectrum* 15.2 (1993): 205-28; Rob Schultz, "Melodic Contour and Nonretrogradable Structure in the Birdsong of Olivier Messiaen," *Music Theory Spectrum* 30.1 (2008): 89-137.

[5] A similar idea of *cursor-window* is introduced by David Lewin from a transformational perspective in his article "Thoughts on Klumpenhouwer Networks and Perle-Lansky Cycles," *Music Theory Spectrum* 24.2 (2002): 196-230.

Chapter five is speculative in nature. It investigates the phenomenological and cognitive implications of the algorithms from a listener's point of view. In this chapter, a set of alternative window models is introduced, exploring the premise of shifting the "reference point" from the center of the window to the edges of the window. Including the original window model presented in Chapter two, a set of eight models (three for the 3-window algorithm and five for the 5-window algorithm) is developed. An evaluation and comparison of the models is presented from a computational standpoint that concludes with a certain preference in favor of the model proposed in Chapter two.

Chapter six presents an analytical application of the theory on the *Hauptstimmen* of the first movement of Schoenberg's Third String Quartet. The reduction process of each *Hauptstimme* is followed by an examination of the intervallic content of contours at various "depth levels" that are invoked by successive contour reduction functions. The observations arising from this second stage invoke the question of whether the algorithms maintain certain notes, while excluding others, and whether they reveal a tonal/modal framework for the movement. The latter possibility is discussed in detail in the light of both statistical significance and the tonal implications suggested by the traditional sonata form of the movement.

Chapter seven proposes a methodology to extend the algorithms to the duration domain. First, a brief analysis is presented in order to demonstrate the merits of approaching the duration domain from a contour perspective. Then, the possibility of applying the algorithms on duration contours is discussed. After a demonstration of the analytical, conceptual, and methodological inconsistencies arising from the application of the algorithms on duration contours, a solution is presented that adopts a "cumulative" approach.

Chapter eight presents an analysis of Berio's *Sequenza I* based on the reduced pitch and duration contours. The interaction between the reduced pitch and duration contour for each segment is examined in detail. After the observation that different types of relationships between the multi-parametric contours signify different formal sections of the work, a transformational network is constructed to represent the overall formal scheme delineated by the interaction types.

Chapter nine presents the conclusions drawn from the dissertation. This chapter also offers ideas for further theoretical and analytical research.

Theoretical Concepts and Terminology

The terminology used in the dissertation for the most part follows the conventions set by previous literature. A detailed explanation of common terms will be provided in Chapter two, as the contour literature is reviewed, and new terminology will be introduced in the following chapters, particularly in chapters three, five, and seven. Nonetheless, some of the key terms used in this dissertation are summarized below for the purposes of introduction and reference. The first subsection includes standard terminology and the second subsection includes new terminology introduced in the dissertation.

pc • Pitch class. An equivalence class of all pitches that are related by octave(s).

cp • Contour position.[6] Contour positions correspond to an increasing (or decreasing) ordering in any specified parameter, and are indicated by the integers 0 through n–1, given a

[6] In the current literature cp denotes "contour *pitch,*" which was initially coined by Robert Morris in his book *Composition with Pitch-Classes*. The term *pitch* is replaced by *position* in order to prevent potential confusion and to obtain a domain-neutral term.

context involving n distinct positions. Note that cp is a generic term and does not indicate the parameter in question. (See in particular the two cp types, pp and dp, indicated below.)

cseg • Contour segment.[7] An ordered set or segment of cps; e.g. <142320>. Note that cseg is a generic term and does not indicate the parameter in question. (See in particular the two cseg types, pseg and dseg, indicated below.)

csubseg • Contour subsegment. An (ordered) subset of cps, not necessarily continuous, from a cseg. For example, given the cseg <142320> its csubsegs include <1430>, <422>, <1232>, <423>, <320>, etc.

op • Ordinal (or order) position in a cseg (first, second, third, etc.) Similar to contour positions, order positions are indicated by integers 0 to n–1, given a context involving n distinct positions. For instance, given the cseg <142320>, cp 1 is in op 0; cp 4 is in op 1; cp 2 is in op 2; cp 3 is in op 3; cp 2 is in op 4; cp 0 is in op 5.[8]

Cardinality • Total number of ops in a given cseg. For example, cseg <142320> has a total of six ops (op 0-op 5), and thus, has a cardinality of six.

pp • Pitch position. A cp whose ordering is in the pitch domain.

dp • Duration position. A cp whose ordering is in the duration domain.

---

[7] In this study we use the term cseg following Marvin and Laprade (1987), however, the idea of ordering registral position of pitches in relation to sequential time was introduced earlier by Friedmann (1985), who refers to this ordering as *Contour Class*, or CC for short. See Michael Friedmann, "A Methodology for the Discussion of Contour: Its Application to Schoenberg's Music," *Journal of Music Theory* 29.2 (1985): 223-48; and Elizabeth West Marvin and Paul Laprade, "Relating Musical Contours: Extensions of a Theory for Contour," *Journal of Music Theory* 31.2 (1987): 228.

[8] Note that cp 2 is in both op 2 and op 4 in this cseg.

pseg • Pitch contour segment. A cseg—i.e. an ordered set of pps—in which the cps denote *relative* pitch height, with pp 0 corresponding to the lowest note and pp n-1 corresponding to the highest note. Thus, given the pseg <142320>, the lowest note (pp 0) corresponds to op 5 and the highest note (pp 4) corresponds to op 1.

dseg • Duration contour segment. A cseg—i.e. an ordered set of dps—in which the cps denote *relative* duration, with dp 0 corresponding to the longest note and dp n–1 corresponding to the shortest note.[9] Thus, given the dseg <142320>, the longest note (dp 0) corresponds op 5 and the shortest note (dp 4) corresponds to op 1.

CAS • Contour Adjacency Series. "An ordered series of +s and –s corresponding to moves upward and downward in a musical unit. For example, the theme of the finale of Mozart's 'Jupiter' Symphony has a CAS of <+,+,–>."[10]

Translation/Renumbering • A csubseg (or pruned cseg) may no longer include all cps between 0 and n–1. In such cases, the csubseg can be "translated" or "renumbered" using adjusted cps from 0 to m–1, where m is the number of distinct cps in the csubseg. For example, the cseg <2837311> is equivalent to cseg <1423200> under translation/renumbering.[11] The procedure is referred to as "translation" by Marvin and Laprade.[12] Henceforth, we will use "translation" and "renumbering" interchangeably.

---

[9] In the current literature, the values used in the duration domain are ordered from short to long and set in correspondence with the integers 0, 1, 2, etc., in the same manner with pitch contours. Thus, the shortest duration is denoted by cp "0," and longer durations are indicated by larger cps. However, the approach adopted in this study reverses this association. In Chapter 6, we will discuss this issue in detail.

[10] Friedmann, "A Methodology for the Discussion of Contour," 246.

[11] Note that the cardinality of the exemplary cseg is 7 but there are only 5 distinct cps, two of which are repeated.

[12] The concept of translation was introduced by Marvin and Laprade in "Relating Musical Contours," 228.

Maximum [cp] • A cp that is higher than (a specified number of) preceding and succeeding cps.

Minimum [cp] • A cp that is lower than (a specified number of) preceding and succeeding cps. [13, 14]

Contour Reduction Algorithm • An algorithm proposed by Robert Morris and refined by Rob Schultz that reduces a cseg by dividing it into two strata (called max-list and min-list) and subjecting each to a set of reduction criteria based on the concepts of maximum and minimum.[15]

* * *

3-Window Algorithm • An algorithm that reduces a cseg by applying a width-3 window successively on each cp. The reduction criteria for the window are based on an extended and more flexible notion of maximum and minimum.[16]

5-Window Algorithm • An algorithm that reduces a cseg by applying a width-5 window successively on each cp. The reduction criteria for the window are based on an extended and more flexible notion of maximum and minimum. [17]

[13] Note that the notion of maxima and minima as introduced by Robert Morris involves only the preceding and succeeding cps, whereas our approach extends the definition to include a specified number of preceding and succeeding cps. See Morris, "New Directions in the Theory and Analysis of Musical Contour."
[14] In Chapters 7 and 8 we will extend the concepts of maximum and minimum to the duration domain and redefine them as “shorter” and “longer” rather than “higher” and “lower.”
[15] Morris, "New Directions in the Theory and Analysis of Musical Contour" and Schultz, "Melodic Contour and Nonretrogradable Structure in the Birdsong of Olivier Messiaen."
[16] Maximum and minimum are not limited only to the central cp anymore, as we shall see in Chapter 5.

Contour Reduction Function • Any combination of successive applications of window algorithms on a cseg. Contour reduction functions will be indicated by the letter R followed by a string of numbers indicating the successive window algorithms employed by the reduction function. Thus, given a cseg x:

- **R3(x)** is the function that reduces x by applying the "3-window" algorithm.
- **R5(x)** is the function that reduces x by applying the "5-window" algorithm.
- **R35(x)** is the function that reduces x by applying the 3-window algorithm first, and reduces the output further by then applying the 5-window algorithm; i.e. R35(x) = R5(R3(x)).
- **R53(x)** is the function that reduces x by applying the 5-window algorithm first, and reduces the output further by then applying the 3-window algorithm; i.e. R53(x) = R3(R5(x)).
- **R55(x)** is the function that reduces x by applying the 5-window algorithm twice in succession; i.e. R55(x) = R5(R5(x)).
- **R355(x)** is the function that reduces x by applying the 3-window algorithm first, and reduces the output further by then applying the 5-window algorithm, twice in succession; i.e. R355(x) = R5(R5(R3(x))), and so forth.

Depth Level (of a Contour Reduction Function) • The depth level of a contour reduction function is equal to the number of successive window algorithms it involves. R3(x) and

[17] Maximum and minimum in 5-window are neither limited to the central cp, nor limited to three cps, as in Morris's algorithm.

R5(x) both have depth 1; R35(x) and R55(x) both have depth 2; R355(x) has depth 3, and so forth.

Contour Framework • The maximally-reduced form of a contour under the application of contour reduction functions (i.e. the depth level which cannot be reduced any further).

Depth Level Contour • A hyper contour (contour of a contour) in which the cps denote depth levels, with cp 0 corresponding to the shallowest depth level (no reduction) and cp n-1 corresponding to the furthest depth level (maximal reduction).[18]

Left-, Center-, and Right-Frame Windows • A set of window models whose "points of reference" are located at different order positions within a window. For example, a left-frame model indicates that the referential op falls on the left-of-center within the frame and similarly, a right-frame model indicates that the referential op falls on the right-of-center within the frame. A center-frame model indicates that the referential op is at the center of the frame and is adopted throughout the dissertation for the reasons explained in Chapter five. Given three order positions (op 0, op 1, op 2) for the 3-window, and five order positions (op 0, op 1, op 2, op 3, op 4, op 5) for the 5-window:

- **LF** (Left-Frame) is the model whose reference point is at op 0 for the 3-window and op 1 for the 5-window. (In both windows, the reference cp is left-adjacent to the center cp.)
- **CF** (Center-Frame) is the model whose reference point is at op 1 for the 3-window and op 2 for the 5-window. (In both windows, the reference point is the center cp.)

[18] An example of this concept requires a more detailed explanation of the window algorithms, and thus, is left for a later chapter.

- **RF** (Right-Frame) is the model whose reference point is at op 2 for the 3-window and op 3 for the 5-window. (In both windows, the reference cp is right-adjacent to the center cp.)
- **LLF** (Leftmost-Frame) is the 5-window model whose reference point is at op 0.
- **RRF** (Rightmost-Frame) is the 5-window model whose reference point is at op 5.

CDV • Contour Difference Vector. The ordered values of the respective pp and dp differences (i.e. pp–dp) in a segment. For example, the CDV of pseg(x) = <3021> and dseg(x) = <1032> is <+2, 0, –1, –1>. Note that it is also possible to notate the CDV by using absolute values (i.e. |2011|).

CHAPTER 2

THEORETICAL CONTEXT: SELECTED RECENT THEORIES OF CONTOUR IN MUSIC

Origins

Two precursors to contemporary contour theory, both from the end of 1940s, are Arnold Schoenberg's *Fundamentals of Musical Composition* and Ernst Toch's *The Shaping Forces in Music*.[19] In these sources, both composers approach melodic contour from a global perspective and discuss its general features (i.e. wave-like shapes, climaxes, etc.) with regard to tonal melodies. For example, Schoenberg's graphical (curvilinear) depiction of the opening of the Menuetto movement from Mozart's String Quartet in D major, K 575, spans more than sixteen measures and shows the two climaxes that respectively characterize the formal sections of the opening.[20] Whereas Schoenberg in similar manner illustrates the contour content of selected examples from Bach, Beethoven, and Mozart, among others, Toch focuses on general observations, asserting that there are three basic types of contour: a line that rises and falls around a fixed pitch; a gradually ascending line that reaches climax towards the end; and a series of curved, wave-like arches ascending gradually to build up a larger wave. In both Schoenberg's and Toch's discussions, the concept of contour is limited to the pitch domain.

---

[19] Arnold Schoenberg, *Fundamentals of Musical Composition*, eds. Gerald Strang and Leonard Stein (London: Faber & Faber, 1967); and Ernst Toch, *The Shaping Forces in Music: An Inquiry into the Nature of Harmony, Melody, Counterpoint, Form* (New York: Criterion Music Corp., 1948). Schoenberg worked on the notes published posthumously as *Fundamentals of Musical Composition* between 1937 and 1948.

[20] *Fundamentals of Musical Composition*, 114.

A more generalized notion of musical contour, which encompasses parameters other than pitch, was proposed in 1960 by the musicologist and composer Charles Seeger.[21] Although Seeger does not use the term contour, his description of pitches, dynamics, and tempi "in only one direction or its opposite,"[22] which he refers to as *variance of direction*, clearly implies a contour-based approach to these parameters. Here, an increase in one direction of variance in pitch, dynamics, or tempo is referred to as *tension* and indicated by a plus sign (+), while a decrease in any of these parameters is referred to as *detension* and indicated by a minus sign (–).[23] By combining binary and ternary chains of signs, he classifies *twelve basic moods*,[24] in which any sign chain that begins with + is classified as tense, and any chain that begins with – is classified as detense; Seeger thus classifies two binary tense moods (+ + and + –), two binary detense moods (– – and – +), four ternary tense moods (+ + +, + + –, + – –, and + – +), and four ternary detense moods (– – –, – – +, – + +, and – + –). Seeger's application of these twelve moods to pitch, rhythm, dynamics, and tempo is followed by an extension of moods from binary and ternary to quaternary, and several tonal passages are used to exemplify the concepts.

Another systematic categorization of contours, though limited to the pitch domain, was proposed by the ethnomusicologist Charles Adams in 1976.[25] Adams's method accounts for rise or fall (+ or –) between not only successive pitches but also non-successive pitches. Adams lists four "minimal boundaries" for any melodic segment: the initial pitch (I) and the

---

[21] Charles Seeger, "On the Moods of a Musical Logic," *Journal of the American Musicological Society* 13.1 (1960): 224-61.
[22] Ibid., 236.
[23] Ibid., 236. On the same page, Seeger also refers to invariance as tonicity (=), but does not incorporate it into his theory.
[24] Ibid., 241.
[25] Charles Adams, "Melodic Contour Typology," *Ethnomusicology* 20.2 (1976): 179-215.

final pitch (F), which are referred to as temporal boundaries, and the highest pitch (H) and the lowest pitch (L), which are referred to as tonal boundaries. Then he defines three relationships between any two pitches, successive or non-successive: higher than (>); same as (=); and lower than (<). Adams then defines two important relational concepts. The *slope* of a contour indicates one of the three possibilities for the relationship between the initial and final pitches: I>F, or I=F, or I<F.[26] The *deviation* of a contour corresponds to whether I or F is also the H or L of the contour; this consideration results in situations involving three different cardinalities: <I, H, L, F> and <I, L, H, F> contour types have cardinality 4; <I, H, F> and <I, L, F> contour types have cardinality 3; and the <I, F> contour type has cardinality 2.[27] Using these two concepts, Adams proposes a classification of 15 contour types.[28] He then applies the proposed typology to Native American songs, specifically Flathead Indian songs and Southern Paiute songs.

Another ethnomusicologist, Mieczyslaw Kolinski, approaches contour from a statistical perspective in his 1965 study “The General Direction of Melodic Movement.”[29] Kolinski investigates the statistical correlation of the “general direction of melodic movement” between Western and non-Western music by considering the initial, final, lowest, and highest tones in melodies from various musical repertoires.[30] For each repertoire, he calculates (a) the average interval between the initial and lowest pitches, (b) the average interval between the lowest and final pitches, and (c) the range between lowest and highest pitches for a body of work. Kolinski then examines the ratios a:c and b:c, which he refers to

---

[26] Ibid., 197.
[27] Ibid., 197.
[28] Ibid., 199.
[29] Mieczyslaw Kolinski, "The General Direction of Melodic Movement," *Ethnomusicology* 9.3 (1965): 240-64.
[30] Adams does not fail to acknowledge Kolinski’s contribution with regard to his minimal boundaries of I, F, L, and H.

as *initial level* and *final level* for the repertoire. These two levels form the *level formula*, which is further used to determine what he calls the lower and upper melody curves. Kolinski's work could be seen as the first statistical approach to contour, an approach that has been developed further, with increasing methodological sophistication, by theorists such as Ian Quinn and Zohar Eitan.

Contour Perception in Non-Tonal Context

Since contour is one of the most fundamental aspects of auditory perception, it is hardly surprising that it has drawn interest among psychologists since the 1970s.[31] Although most of the research in this area discusses contour within a tonal context, Dowling and Fujitani examined the significance of contour in perception and memorization of non-tonal melodies as early as 1971.[32]

Experiments by Sandra Trehub, Dale Bull, and Leigh Thorpe have shown that for infants, contour is more significant than is tonally-defined interval.[33] In a set of three experiments, they demonstrate that infants aged 8-11 months, who are unable to distinguish a melody from its contour-identical transformation, can nonetheless discriminate a melodic transformation that violates the contour of the original melody.

---

[31] See, for instance, W. Jay Dowling, "Recognition of Inversions of Melodies and Melodic Contours," *Perception and Psychophysics* 9.3B (1971): 348-49; John B. Davies and Anne Yelland, "Effects of Two Training Procedures on the Production of Melodic Contour, in Short-Term Memory for Tonal Sequences," *Psychology of Music* 5.2 (1977): 3-9; W. Jay Dowling, "Scale and Contour: Two Components of a Theory of Memory for Melodies," *Psychological Reports* 85.4 (1978): 341-54; and Gerald B. Olson, "Intersensory and Intrasensory Transfer of Melodic Contour Perception by Children," *Journal of Research in Music Education* 26.1 (1978): 41-47.

[32] W. Jay Dowling and Diane S. Fujitani, "Contour, Interval, and Pitch Recognition in Memory for Melodies," *The Journal of the Acoustical Society of America* 49.2 (1971): 524-31.

[33] Sandra E. Trehub, Dale Bull, and Leigh A. Thorpe, "Infants' Perception of Melodies: The Role of Melodic Contour," *Child Development* 55 (1984): 821-30.

Another experiment by Judy Edworthy, in which musician subjects were asked to detect interval and contour alterations in a set of transposed melodies ranging from 3 to 15 notes, indicates similar results.[34] Edworthy notes that the subjects were evidently more successful in detecting contour alterations for melodies up to 11 notes and more successful in detecting interval alterations in 15-note melodies. Melodies of 13 notes show no significant correlation for either. The superior interval identification for longer melodies likely results from the establishment of a tonal framework, which requires quite a few notes. (All of the melodies used in Edworthy's experiment were tonal, and it appears that on average, for the range of listeners and range of melodies used in the experiment, it took about 13 notes to establish tonal orientation.) In "atonal" contexts with little or no tonal orientation, one would expect the greater significance of contour identification over interval identification to extend to melodies longer than 13 notes. Edworthy notes that "contour can be encoded independently of key,"[35] while "it has been empirically demonstrated many times (e.g., Francès, 1958; Cuddy et al., 1979) that intervals are encoded and recognized more accurately when heard in the presence of an established tonal framework than when heard in isolation or in weak tonal framework."[36] Thus, contour identification is likely to play a more significant role than interval identification in atonal melodies that are even longer than 13 notes.

In a more recent study, Mark Schmuckler assesses similarity measures set forth by music theorists Michael Friedmann, Elizabeth West Marvin and Paul Laprade—which we will review shortly; he proposes an alternative theory for measuring contour similarity, and

---

[34] Judy Edworthy, "Interval and Contour in Melody Processing," *Music Perception* 2.3 (1985): 375-88.
[35] Ibid., 388.
[36] Ibid., 386.

subjects all three theories to empirical scrutiny.[37] Schmuckler argues that a combinatorial approach to contour in determining similarity does not preserve the “global contour shape,” which he believes to be significant for contour perception in general and perceived contour similarity in particular. He proposes to use Fourier analysis and contour oscillations (i.e. up and down motion), which quantify the global shape of the contour, and discusses the advantages and disadvantages of the methodology. Finally, he reports the results of two experiments—one based on tonal and the other based twelve-tone melodies—using the similarity measures discussed earlier in the study. His results suggest that the similarity measures based on Fourier analysis and contour oscillation are a better fit for assessing contour similarity than those proposed by Friedmann, and Marvin and Laprade, particularly for twelve-tone melodies.

Foundations of Contemporary Contour Theory

The studies evaluated by Schmuckler stand as the first full-fledged theoretical formalization of contour in North American music theoretical discourse, today referred to as “contour theory.” The first of a series of theoretical studies that emerged in the second half of 1980s, Michael Friedmann’s 1985 article “A Methodology for the Discussion of Contour: Its Application to Schoenberg's Music” introduces a number of ways to measure and discuss musical contour and applies the proposed methodology to analysis of Schoenberg’s music.[38] In this article, Friedmann first defines what he calls a Contour Adjacency Series (CAS), which simply denotes the ordered series of directional moves (+ or –) without the particular

[37] Mark A. Schmuckler, "Testing Models of Melodic Contour Similarity," *Music Perception* 16.3 (1999): 295-326.
[38] Michael Friedmann, "A Methodology for the Discussion of Contour: Its Application to Schoenberg's Music," *Journal of Music Theory* 29.2 (1985): 223-48.

distance (i.e. interval) between adjacent pitches. For example, the pitches <C4, F4, E4, D4, G4, A3> have a CAS of <+, –, –, +, –>. A bipartite summation of the pluses and minuses in a CAS is referred to as *Contour Adjacency Series Vector* (CASV), which could be understood as the relative degree of upward and downward motion within a contour. The example above has a CASV of <2,3> since the CAS involves two plus signs and three minus signs. Neither the CAS nor the CASV indicates the relative position (i.e. height) of pitches that are non-adjacent. For instance, in the CAS <+, –> the third pitch could be higher or lower than the first pitch. However, Friedmann proposes a measure that provides the relative positions between both adjacent and non-adjacent pitches. The term *Contour Class* (CC) refers to an ordering of pitches from low to high by assigning to each pitch an integer from 0 to n–1 (where n is the distinct number of pitches). The pitch segment <C4, F4, E4, D4, G4, A3>, given above, has a CC of <1, 4, 3, 2, 5, 0>. Note that, typically, in a CC the ordering is relative to the temporal domain (i.e. sequential time). For instance, the initial integer 1 takes place before all other integers; the second integer 4 takes place before integers 3, 2, 5, and 0, and after integer 1; the third integer 3 takes place before integers 2, 5, and 0, and after integers 1 and 4, etc. Based on the concept of Contour Class, Friedmann devises other measurements such as *Contour Interval* (CI) and *Contour Interval Succession* (CIS); the former measures the distance between the adjacent or non-adjacent contour integers within a CC and the latter lists the successive Contour Intervals in order. For example, CC <1, 4, 3, 2, 5, 0> has a CIS of <+3, –1, –1, +3, –5> and each integer within the CIS is referred to as a *Contour Interval* (CI). Furthermore, the listing of the entire CI content of a contour, as opposed to the successive CI content of CIS, is referred to as *Contour Interval Array* (CIA). This measurement essentially lists the total number for each CI separately: <CI+1, CI+2,

CI+3, etc. / CI–1, CI–2, CI–3, etc.> For example, CC <1, 4, 3, 2, 5, 0> has a CIA of <2,2,2,1,0 / 3,2,1,1,1>. Based on the CIA, Friedmann proposes two types of *Contour Class Vector* (CCV), namely CCV I and CCV II. The first type calculates the total distance spanned within the CC, between all integers (adjacent and non-adjacent). The calculation distinguishes the pluses and minuses, and thus, lists the total distances for each separately. For example, CIA <2,2,2,1,0 / 3,2,1,1,1> has a CCV I of <2(+1) + 2(+2) + 2(+3) + 1(+4) + 0(+5) / 3(–1) + 2(–2) + 1(–3) + 1(–4) + 1(–5)>, or <16/19>. The second type of CCV basically sums all of the positive and negative integers separately. For example, the CCV II of the CIA above is <2+2+2+1+0 / 3+2+1+1+1>, or <7/8>. Friedmann demonstrates his methodology on excerpts from Schoenberg's music, some of which include segmentations based on timbre and registral proximity. Furthermore, he discusses how some of these concepts behave when twelve-tone operations and the rotation operation are applied on specific contours. Lastly, he introduces the notion of contour subsets, which allows a contour to be examined based on the successive or non-successive sub-contours within the original contour.

Beyond some important similarities with Friedmann's approach to contour, Robert Morris's definitions of *contour space* (c-space) and *c-pitches* (cps) provide the necessary formal grounds for a generalized theory of contour. Morris defines these concepts as follows: "A c-space of order n, is a pitch-space of n elements, called c-pitches (cps). C-pitches are numbered in order from low to high, beginning with 0 up to n–1. The intervallic distance between the cps is ignored and left undefined."[39] Thus, a contour is defined as an ordered set

[39] Robert D. Morris, *Composition with Pitch-Classes*, 26. See also Larry Polansky, "Morphological Metrics: An Introduction to a Theory of Formal Distances," in Proceedings of the International Computer Music Conference 1987 (San Francisco: International Computer Music Association, 1987), 197-205.

of c-pitches. Furthermore, Morris introduces a comparison function, COM (a,b), in order to compare any pair of c-pitches, a and b. This function formalizes the pluses and minuses of Friedmann's CAS. If b is higher than a, COM(a,b) = +1; if b is the same as a, COM(a,b) = 0; and if b is lower than a, COM(a,b) = –1. Based on the comparison function, Morris constructs a *comparison matrix*, in which all c-pitches of a pair of contours (same or different) are compared. Figure 2.1 illustrates the comparison matrix for contour <1, 4, 3, 2, 5, 0>, compared to itself.

Figure 2.1. Comparison Matrix for <1, 4, 3, 2, 5, 0>

COM	1	4	3	2	5	0
1	0	+	+	+	+	–
4	–	0	–	–	+	–
3	–	+	0	–	+	–
2	–	+	+	0	+	–
5	–	–	–	–	0	–
0	+	+	+	+	+	0

Morris's concept of contour-class, which he calls "segment class," is based on the comparison matrices. According to his criterion, if two contours generate the same comparison matrix, then they are assumed to be equivalent, and thus, belong to the same contour class. For example, the contours <143> and <142>, which are embedded in the contour illustrated in Figure 2.1, are both members of the contour-class (segment class)

<021>, with which they share the same comparison matrix. Morris further investigates how certain operations on contours affect comparison matrices, and he uses the four operations P (identity), I (inversion), R (retrograde), and RI (retrograde inversion) to propose a classification of contours. According to this classification, "any two contours related by R, I, and/or contour-equivalence are members of the same c-space segment-class."[40] Later on, in his *Composition with Pitch-Classes*, Morris defines musical contour in a more generalized manner, proposing that "a contour is a set of points in one sequential dimension ordered by any other sequential dimension," [41] and this suggests the employment of musical dimensions other than pitch, such as loudness, timbre, envelope, and so forth. Furthermore, this definition introduces the possibility of excluding the temporal domain (i.e. sequential time) altogether, as discussed by Morris in an article published six years later.[42]

Following the formal groundwork laid out by Morris, Elizabeth West Marvin and Paul Laprade propose a methodology for measuring contour similarity based on comparison matrices (COM-matrices), and further formalize the classification system suggested by Morris (i.e. c-space segment-classes) with an extension of contour cardinality up to six.[43] While Marvin and Laprade retain Morris's terminology (i.e. contour-space and contour-pitches) for the most part, they simply replace the term contour with *c-segment* (cseg) and introduce the term *c-subsegment* (csubseg) as "any ordered subgrouping of a given cseg" that "may be comprised of either contiguous or non-contiguous c-pitches from the original

[40] Ibid., 32.
[41] Ibid., 283.
[42] Robert D. Morris, "New Directions in the Theory and Analysis of Musical Contour."
[43] Elizabeth West Marvin, and Paul Laprade, "Relating Musical Contours."

cseg."[44] Marvin and Laprade propose two methods for measuring similarity between csegs; the notion of csubseg is at the heart of their second method, as we shall see shortly. In the first of these methods, the *contour similarity function* (CSIM) compares the upper right-hand triangle of the respective COM-Matrices of the csegs and returns a ratio of match between the pluses and minuses between the matrices. Figure 2.2 provides an example of CSIM between csegs A: <143250> and B: <320451>. In this figure, the circled signs indicate a match between the COM-matrices and the ratios of match to total are placed under the matrices.

Figure 2.2. CSIM between csegs <143250> and <320451>

COM	1	4	3	2	5	0
1	0	+	+	+	+	–
4	–	0	–	–	+	–
3	–	+	0	–	+	–
2	–	+	+	0	+	–
5	–	–	–	–	0	–
0	+	+	+	+	+	0

COM	3	2	0	4	5	1
3	0	–	–	+	+	–
2	+	0	–	+	+	–
0	+	+	0	+	+	+
4	–	–	–	0	+	–
5	–	–	–	–	0	–
1	+	+	–	+	+	0

CSIM(A,B) = 10/15 = .67

---

[44] Ibid., 255. Note that these terms are essentially the same with what Friedmann refers to as contour class (CC) and contour subset.

While the first similarity function, CSIM, compares only contours of the same size (i.e. cardinality), the second similarity function, CEMB (contour embedding function), compares contours of different sizes by counting the number of times the smaller cseg is embedded in the larger cseg both successively and non-successively.[45] Figure 2.3 shows an example of CEMB between cseg C <120> and cseg D <32041> by first listing all successive and non-successive cardinality-3 csubsegs of cseg D <32041>, and then translating them in order to observe which ones are equivalent to <120>.[46] The csubsegs that are equivalent to <120> are denoted by asterisks. Note that, although visually more convoluted, it is possible to illustrate CEMB using COM-matrices as well.

[45] Note that Marvin and Laprade acknowledge David Lewin, Robert Morris, and John Rahn in adopting their notion of embedding functions. Departing from Allen Forte's "interval vector" and Eric Regener's "common-note function," David Lewin develops an embedding function for pcsets (although he extends the notion to chord types, etc.) in his 1977 article "Forte's Interval Vector, My Interval Function, and Regener's Common-Note Function." Marvin and Laprade's terminology is mainly drawn from John Rahn's "Relating Sets," in which the concepts laid out by Lewin and Morris are further developed and formalized. In "Transformational Techniques in Atonal and Other Music Theories," David Lewin discusses embedding of specific pc collections (which he refers to as X- and Y-forms, representing certain trichords and tetrachords, respectively), as well as embedding of graphs within other graphs. In "Combinatoriality without the Aggregate," published in the same volume with Lewin's article, Robert Morris discusses partition embedding by using subsets of combination matrices (CM) which form vertical aggregate subsets as a result of aligning horizontal aggregate subsets in certain ways. See David Lewin, "Forte's Interval Vector, My Interval Function, and Regener's Common-Note Function," *Journal of Music Theory* 21.2 (1977): 194-237; John Rahn, "Relating Sets," *Perspectives of New Music* 18.1-2 (1979-80): 483-98; Robert D. Morris, "A Similarity Index for Pc Sets," *Perspectives of New Music* 18 (1979-80): 445-60; David Lewin, "Transformational Techniques in Atonal and Other Music Theories," *Perspectives of New Music* 21.1-2 (1982): 312-71; Robert D. Morris, "Combinatoriality without the Aggregate," *Perspectives of New Music* 21.1-2 (1982): 432-86.

[46] Marvin and Laprade define translation as "an operation through which a csubseg is renumbered from 0 for the lowest c-pitch to (n–1) for the highest" (Ibid. 255). For example, cseg <130> is equivalent to cseg <120> under translation.

Figure 2.3. CEMB between csegs <120> and <32041>

csubsegs of <32041>		translation	
<320>	=	<210>	
<324>	=	<102>	
<321>	=	<210>	
<304>	=	<102>	
<301>	=	<201>	**CEMB(C,D) = 2/10 = .20**
<341>	=	<120>*	
<204>	=	<102>	
<201>	=	<201>	
<241>	=	<120>*	
<041>	=	<021>	

Further developing this idea of embedded csubsegs, Marvin and Laprade suggest another type of embedding function that compares csegs of equal or unequal size. First they introduce the notion of "mutually embedded" csubsegs and propose a function that lists and compares the mutually embedded csubsegs of some designated cardinality found within two larger csegs. They denote this function as $\text{CMEMB}_n(\text{X,A,B})$, in which n refers to the cardinality, X refers to the csubseg, and A and B refer to csegs. Based on this function, they propose another all-embracing function, which calculates all csubseg cardinalities to return a similarity ratio. They denote this function as ACMEMB(A,B). Figure 2.4 illustrates an example of ACMEMB between cseg A <1203> and cseg B <32041>.

Figure 2.4. ACMEMB between csegs <1203> and <32041>

csubsegs of <32041>		csubsegs of <1203>
<32> = <10>	<3204> = <2103>	<12> = <01>
<30> = <10>	<3201> = <3201>	<10> = <10>
<34> = <01>	<3241> = <2130>	<13> = <01>
<31> = <10>	<3201> = <3201>	<20> = <10>
<20> = <10>	<3041> = <2031>	<23> = <01>
<24> = <01>	<32041> = <32041>	<03> = <01>
<21> = <10>		<120> = <120>
<04> = <01>		<123> = <012>
<01> = <01>		<103> = <102>
<41> = <10>		<203> = <102>
<320> = <210>		<1203> = <1203>
<324> = <102>		
<321> = <210>		
<304> = <102>		
<301> = <201>		
<341> = <120>		
<204> = <102>		
<201> = <201>		
<241> = <120>		
<041> = <021>		

ACMEMB(A,B) = 24/37 = .65

Here, first each csubseg of cseg A (left-hand columns) is evaluated to determine whether it has a correspondence in the csubseg list of cseg B (right-hand column) and then each csubseg of cseg B (right-hand column) is evaluated to determine whether it has a correspondence in the csubseg list of cseg A (left-hand column). After the evaluation of all 37 csubsegs, 24 of them are found to have correspondences within the csubseg list of the other cseg.

Marvin and Laprade offer an analytical application of the methodology proposed, and an appendix that provides a computer-generated tabulation of the "prime forms" for csegs of cardinalities 2 through 6, based on the criterion of similarity to within inversion and retrograde.

Extensions to the Theory

Marvin's 1989 dissertation includes a great deal of the methodology proposed in Marvin and Laprade 1987, but extends the methodology to the temporal domain by treating durations as contour-positions, forming duration contours (dsegs).[47] A readjusted version of the dissertation chapter that discusses duration contours was also published in 1991.[48] In this article, Marvin proposes a "duration space, analogous to contour space, that models relative duration in the same way that contour space models relative pitch height."[49] Her application of duration contours is purposefully confined to non-metrical contexts, since with the lack of a consistent beat, "rhythmic contours of relative shorts and longs best model the listener's

[47] Elizabeth West Marvin, "A Generalized Theory of Musical Contour: Its Application to Melodic and Rhythmic Analysis of Non-Tonal Music and Its Perceptual and Pedagogical Implications," Ph.D. dissertation, University of Rochester, 1989.
[48] Elizabeth West Marvin, "The Perception of Rhythm in Non-Tonal Music: Rhythmic Contours in the Music of Edgard Varese," *Music Theory Spectrum* 13.1 (1991): 61-78.
[49] Ibid., 65.

perception."[50] In Marvin's duration space (d-space), the durations are assigned to numbers in order from short to long, 0 being the shortest duration and n–1 being the longest duration, where n refers as usual to the cardinality of the contour space.[51] It is important to note that in this ordering, inter-onset durations are taken into account. In other words, each duration is measured from the onset of one pitch to the onset of the following pitch, regardless of whether a rest intervenes between the pitches.

After applying to duration contours the methodology proposed in Marvin and Laprade, and her dissertation (i.e. the COM-matrix and equivalence classes based on inversion and retrograde), Marvin presents an analysis of Varèse's *Density 21.5*, in which she discusses pitch and duration contours—as well as their csubsegs—pointing out the relationships between segments in one or both of the domains.

Taking a different perspective on contour theory, Larry Polansky and Richard Bassein investigate the formal implications of combinatorial approaches presented in the earlier studies (i.e. Morris 1987, and Marvin and Laprade 1987), and introduce the concept of "impossible" contours.[52] After replacing the +, 0, and – used in COM-matrices with the numbers 0, 1, and 2, Polansky and Bassein propose to notate the combinatorial relationships between all c-pitch pairs as a string of numbers 0, 1, and 2. As an example, the cseg <102> is represented in their approach by 200, since 1>0 (resulting in the 2), 0<2 (resulting in the first 0), and 1<2 (resulting in the second 0). By using the *binomial coefficient*, Polansky and

[50] Ibid., 65.
[51] This ordering will be discussed and reversed in Chapter 6.
[52] Larry Polansky, and Richard Bassein, "Possible and Impossible Melody: Some Formal Aspects of Contour," *Journal of Music Theory* 36.2 (1992): 259-84.

Bassein calculate the string length for contours of higher cardinality.[53] In addition, they provide the formula to calculate the total number of "possible ternary descriptions" for a string of any length.[54] Taking cardinality-3 contours as an example, Polansky and Bassein calculate that there are 27 possible ternary descriptions for string length of 3. A listing of all 27 reveals that not all of the combinatorial contour descriptions are possible since some contain logical inconsistencies. For example, the string 010 ($a<b$, $b=c$, $a>c$) violates transitivity since if b is greater than a ($a<b$) and is equal to c ($b=c$), then c must be greater than a ($a<c$), which is contradicted by the last relationship of the string, $a>c$. Polansky and Bassein show that only 13 of the 27 cardinality-3 contours are possible and as the cardinality rises the ratio of possible contours decrease sharply. Polansky and Bassein are not, however, really describing contours that are "impossible"; they are merely demonstrating a flaw in their unconventional combinatorial approach, which produces strings that do not correspond to actual melodic shapes because they violate transitivity.

Using Morris's generalized definition of contour, which offers an extension to musical domains other than pitch and duration, Elizabeth West Marvin has analyzed works by Karlheinz Stockhausen and Luigi Dallapiccola by focusing on not only pitch and duration contours but also dynamic contours and pitch-span contours.[55] Comparisons of contour segments that belong to different musical domains are made using the methodologies discussed previously (i.e. COM-matrices, equivalence classes, csubsegs, etc.). Marvin places

---

[53] $SL = (C^2–C)/2$, where SL denotes string length and C denotes cardinality. For example, a contour of cardinality four has a string length of $(4^2–4)/2=6$. In other words, a cardinality four contour contains a total of six unordered distinct pairs (op 0-op 1, op 0-op 2, op 0-op 3, op 1-op 2, op 1-op 3, op 2-op 3).

[54] 3^{SL}, where SL denotes string length. For example, a string length of 6 with 3 variables (0, 1, and 2) can be ordered in 3^6, or 729 ways (000000, 000001, 000002, 000010, 000011, 000012, etc.)

[55] Elizabeth West Marvin, "Generalization of Contour Theory to Diverse Musical Spaces: Analytical Applications to the Music of Dallapiccola and Stockhausen," in *Concert Music, Rock, and Jazz since 1945: Essays and Analytical Studies,* eds. Elizabeth West Marvin and Richard Hermann (Rochester, N.Y.: University of Rochester Press, 1995), 135-71.

a special emphasis on how contour theory supplements set theory and formal design in her Dallapiccola analysis.

Robert Morris's Contour Reduction Algorithm

Robert Morris contributed another article on contour theory in 1993. After examining some contour relations in Schoenberg's Piano Piece, Op.19, No.4, Morris introduces a new theoretical and analytical tool in the form of an algorithm that reduces contour segments by pruning c-pitches based on a set of criteria, and thereby induces a degree of hierarchy among c-pitches in a segment.[56] This *Contour Reduction Algorithm* is founded upon the principle of retaining the local high and low c-pitches in a contour, while pruning the rest.[57] A local high c-pitch (cp), or *maximum*, can be understood to be the highest cp in comparison to the cps that are immediately adjacent to it (i.e. the preceding cp and the succeeding cp), and a local low cp, or *minimum*, can be understood to be the lowest cp in comparison to the cps that are immediately adjacent to it.[58] Since the pruning of some cps results in a new contour, some of whose cps were not adjacent previously, it is sometimes possible to apply another pass of the reduction algorithm on this newly formed (i.e. reduced) contour. The recursive aspect of the algorithm results in a hierarchization of the cps in the contour segment, and invokes additional concepts such as "depth" (i.e. how many times the algorithm is applied recursively) and "prime contour" (a contour that cannot be reduced any further). The

---

[56] Robert D. Morris, "New Directions in the Theory and Analysis of Musical Contour."
[57] In Morris's definition the first and last cps of a contour are always both maximum and minimum.
[58] If a maximum or minimum is repeated, both are retained in the first reduction (steps 1-4 in Figure 2.5). In the following reductions (steps 5-8 in Figure 2.5) either of the repetition is pruned given that none are first or last cps. Steps 6 and 7 in Figure 8 explain the procedure relating repeating cps.

algorithm in its entirety is reproduced in Figure 2.5 and the demonstrative example used by Morris is shown in Figure 2.6.

Figure 2.5. "Contour Reduction Algorithm" by Morris (1993, 212)

The algorithm prunes pitches from a contour until it is reduced to a "prime."

Definition: *Maximum pitch:* Given three adjacent pitches in a contour, if the second is higher than or equal to the others it is a *maximum*. A set of maximum pitches is called a *maxima*. The first and last pitches of a contour are maxima by definition.

Definition: *Minimum pitch:* Given three adjacent pitches in a contour, if the second is lower than or equal to the others it is a *minimum*. A set of minimum pitches is called a *minima*. The first and last pitches of a contour are minima by definition.

Algorithm: Given a contour C and a variable N:

step 0: Set N to 0.

step 1: Flag all maxima in C; call the resulting set the *max-list*.

step 2: Flag all minima in C; call the resulting set the *min-list*.

step 3: If all pitches in C are flagged, go to step 9.

step 4: Delete all non-flagged pitches in C.

step 5: N is incremented by 1 (i.e., N becomes N+1).

step 6: Flag all maxima in max-list. For any string of equal and adjacent maxima in max-list, either: (1) flag only one of them; or (2) if one pitch in the string is the first or last pitch of C, flag only it; or (3) if both the first and last pitch of C are in the string, flag (only) both the first and last pitch of C.

step 7: Flag all minima in min-list. For any string of equal and adjacent minima in min-list, either: (1) flag only one of them; or (2) if one pitch in the string is the first or last pitch of C, flag only it; or (3) if both the first and last pitch of C are in the string, flag (only) both the first and last pitch of C.

step 8: Go to step 3.

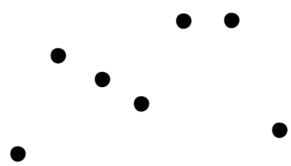
step 9: End. N is the "depth" of the original contour C.

The reduced contour is the prime of C; if N = 0, then the original C has not been reduced and is a prime itself.

Figure 2.6. Application of the “Contour Reduction Algorithm” on cseg <0432551>

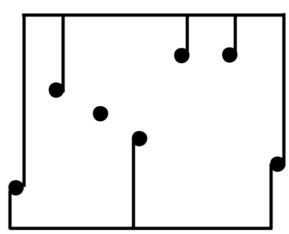
Let C = <0432551>

graph:

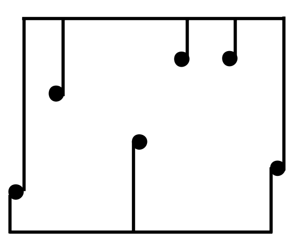


START:

steps 1 and 2: Upper beams show flagged max-list; lower beams show flagged min-list.

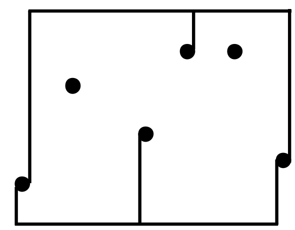


step 3: Not all pitches are flagged.
step 4: Delete 3rd (unflagged) pitch.

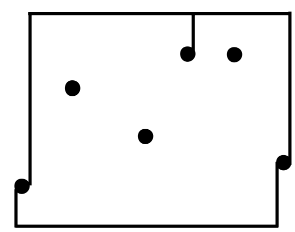


step 5: N = 1.

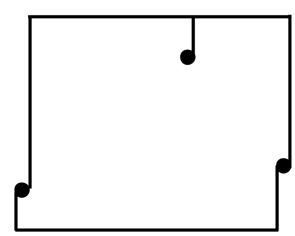
step 6: Flag maxima in max-list and flag either of the repetition in max-list.



step 7: Flag minima in min-list.



step 8: Go to step 3.
step 3: Not all pitches are flagged.
step 4: Delete unflagged pitches.



step 5: N = 2.
steps 6 and 7: All pitches are flagged.
step 8: Go to step 3.
step 9: END.

<0432551> has a depth of 2;
its prime is <021>.

A more detailed discussion of Morris's contour reduction algorithm will be presented in Chapter 3.

After parsing the Schoenberg piece into six phrases and applying the algorithm on each phrase (and then on the entire melody), Morris demonstrates how reduced contours complement the set theoretical approach to the work. Furthermore, Morris classifies primes—contours that are not reducible by the algorithm—into prime classes based on R, I, and RI invariance. The classification comprises 25 basic prime classes and 28 secondary prime classes, which include contours involving "simultaneous pitches." Note that out of 53 prime classes, only seven (<0>, <01>, <010>, <021>, <1021>, <1032>, and <1302>) do not involve simultaneous pitches. These prime classes are referred to as *linear prime classes*. Later in the article, by approaching contours as associations (i.e. mappings) between two sets, Morris demonstrates how contours with repetition and contours with simultaneity are both examples of what he calls "onto contours." In the former, one value in the pitch domain corresponds to two (or more) different values in the temporal domain, whereas in the latter, one value in the temporal domain corresponds to two (or more) different values in the pitch domain (i.e. same pitch onto different time-points and same time-point onto different pitches). Lastly, following the generalized notion of a contour as a mapping between two sets, Morris presents examples of contours that do not involve any temporal ordering, such as pitch-dynamic contours.

Recent Developments

A different type of study, which does not involve contour *per se* but investigates the relationship between melodic peaks and other musical domains (i.e. rhythm, meter, and harmony), is provided by Zohar Eitan in his recent book *Highpoints: A Study of Melodic Peaks*.[59] The primary hypothesis of the study is that the prominent points (i.e. peaks) in a melody have a tendency to correspond to specific configurations in these musical domains, which have "emphatic, intensifying, or tension-raising features."[60] In the temporal domain, Eitan suggests that melodic peaks tend to correlate with durational (agogic) and metric accent, syncopation, and temporal location (i.e. in the later portion of a segment or composition). From an interval perspective, Eitan hypothesizes that the melodic peaks are generally approached by large intervals, appear only once in a given segment but tend to be repeated in succession. The tonal implications of the hypothesis include certain harmonies and melodic scale degrees being associated with the peak. Finally, Eitan hypothesizes that pitch peaks tend to correspond to dynamic peaks. The hypotheses are tested in musical *corpora* that belong to three different styles: Classical, Romantic, and post-tonal. As the basis of his statistical tests, Eitan employs primarily the "chi-square test of homogeneity," which he complements by a "standard score test (*z*), as the statistical methodology in testing his hypotheses.[61]

In the conclusion to his statistical investigation based on works by Haydn, Chopin, and Berg, Eitan reports that although peaks in all three repertoires are "distinguished from the control group in some aspects … few features tend to be associated with peaks in all three repertoires, and some are peculiar to one repertory."[62] While in all three repertoires, peaks tend to be approached by large intervals and are presented only once in a segment, their associations with emphatic and intensifying features differ for each repertory: the weakest

[59] Zohar Eitan, *Highpoints: A Study of Melodic Peaks* (Philadelphia: University of Pennsylvania Press, 1997).
[60] Ibid., 6.
[61] Ibid., 19.
[62] Ibid., 145.

associations are found in the earliest repertory and the strongest associations are found in the latest repertory. Among the most important findings of the study, Eitan points out that surprisingly the statistical observations regarding treatment of melodic peaks in Chopin are more similar to those in Berg than in Haydn. Eitan discusses this finding in the light of Leonard B. Meyer's primary and secondary parameters, which include "discrete, proportional relationships" and "continuous processes," respectively. Tonality is an example of primary parameters whereas contour is an example of secondary parameters from this perspective. According to Eitan, the similarity between Chopin and Berg, instead of Chopin and Haydn, indicates that "syntactical" parameters (i.e. tonality) are not necessarily dependant upon "statistical" or "gestural" (i.e. contour) parameters, and thus, despite the fact that Chopin and Berg may not share a syntactical language such as tonality, the gestural dimension in their music is a commonality.

A new approach to contour similarity, which incorporates fuzzy set theory and fuzzy logic in order to provide certain flexibility to COM matrices, has recently been proposed by Ian Quinn.[63] As a point of departure, Quinn lists sixteen W-shaped melodies, each consisting of eleven notes, from Steve Reich's *The Desert Music*. Since all of the melodies share a similar overall shape, Quinn refers to them collectively as the **M** family (M and W being inversionally-related shapes) and then poses the question "given some other eleven-note contour *x*, is it possible to write a reasonably elegant algorithm capable of judging *x*'s fitness for membership in **M** based on similarity of contour?"[64] Quinn first lists the cseg-classes for all sixteen melodies in **M** and demonstrates that no two melodies belong to the same cseg-

[63] Ian Quinn, "Fuzzy Extensions to the Theory of Contour," *Music Theory Spectrum* 19.2 (1997): 232-63.
[64] Ibid., 236.

class. Then, he applies Morris's contour reduction algorithm on **M**. Although the results are more promising, the algorithm fails to reduce all melodies into the same prime contour. Finally, instead of relying on equivalence classes of the previous two approaches, Quinn adopts Marvin and Laprade's CSIM measure and shows that a minimum similarity threshold could be set in order to accept contours into one family, though he admits that CSIM returns significantly low matches for some of the melodies in **M**. Quinn's diagnosis of the inadequacies of these approaches is that all "assume, incorrectly, that it is possible to weigh a contour's compatibility with **M** simply by comparing it to **M**" and that they "seek to fix on some property possessed unilaterally."[65] He suggests replacing this approach with one that does not demand a melody to have any one common feature with any one melody in **M** in order to qualify for the membership, as long as it has a sufficient resemblance to the "average" member of **M**. It is at this point that Quinn introduces the concept of "fuzzy sets" and shows how they can be used to find the "average" member of a set of contours (e.g. all sixteen melodies of **M**), an "average contour" which he refers to as "fuzzy contour." First of all, Quinn constructs a C^+ matrix, which is based on the ascent relation between the contour-pitches, resulting in a binary relationship for any c-pitch pair (i.e. 0 for not ascending and 1 for ascending). Based on this matrix, he proposes to average any number of contours by simply summing the corresponding values for all contour matrices and dividing them by the total number of contours, and finally he converts a C^+ matrix into a COM matrix by employing a specific formula.[66] As Quinn demonstrates, this process returns not only integers but also ratios (i.e. fractions), which results in fuzzy relationships between c-pitches as

---

[65] Ibid., 240.

[66] $\mathrm{COM}(p,q) = \mu_{C+}(p,q) - \mu_{C+}(q,p)$. In this formula, $\mu_{C+}(p,q)$ receives a value of 1 if $p<q$ and a value of 0 if $p\geq q$. Similarly, $\mu_{C+}(q,p)$ receives a value of 1 if $q<p$ and a value of 0 if $q\geq p$.

opposed to the crisp relationships provided in the original COM matrix. After calculating the average contour for **M**, Quinn tests his methodology by comparing a very substantial number of randomly-generated eleven note melodic contours (32,768 of them) to the average contour for **M**, and reports promising results. In another experiment, Quinn tests the salience of non-successive contour-pitches (an important component of COM matrices) in perceiving melodic contour on musically-trained subjects.[67] The results indicate that although not as significant as successive c-pitches, non-successive c-pitches do play a role in the perception of contour similarity.

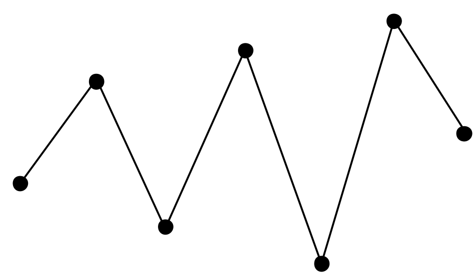
Rob Schultz's Refinements to the Contour Reduction Algorithm

As this dissertation was in the final stages of completion, an article by Rob Schultz has appeared which proposes further refinements to Morris's reduction algorithm.[68] The refinements proposed by Schultz are twofold. The first refinement includes some alterations and additions to Morris's algorithm in order to reduce an irreducible contour type when Morris's algorithm is applied. This contour type, which Schultz refers to as "wedge-shaped" contour, essentially includes any contour whose c-pitches are all either maxima or minima. An example of this contour type used by Schultz is illustrated in Figure 2.7.

---

[67] Ian Quinn, "The Combinatorial Model of Pitch Contour," *Music Perception* 16.4 (1999): 439-56.
[68] Rob Schultz, "Melodic Contour and Nonretrogradable Structure in the Birdsong of Olivier Messiaen."

Figure 2.7. Wedge-Shaped Contour



An application of Morris's algorithm to this contour would flag all c-pitches, following Steps 1 and 2 (see Figure 2.5), and the algorithm would conclude without any reduction since Step 3 instructs us to go to Step 9 ("End") as all pitches are flagged. Thus, the application of Morris's algorithm on this contour results in a cardinality-7 cseg, <2415063>, which cannot be reduced any further (i.e. a prime contour). Schultz points out that the application of the algorithm on this contour type produces a prime that does not belong to any of Morris's prime classes. In order to eliminate this inconsistency and to have the flexibility of reducing this specific type of contour, Schultz alters Step 3, in which he redirects us to Step 6, as opposed to Step 9. Similarly, Step 8 (which becomes Step 12 in the modified version) is altered to redirect us to Step 6. To accommodate the alteration, Schultz inserts four additional steps, namely steps 8-11, which are illustrated in Figure 2.8.[69]

[69] Also, note that there are some minor modifications, which are not necessarily related to the issue at hand. For example, the flag direction (i.e.: upward and downward) is included in steps 2, 3, 6, and 7, and the term "pitch" is replaced by "c-pitches" in steps 3, 4, 6, and 7.

Figure 2.8. Modified Version of Schultz's "Contour Reduction Algorithm" (2008, 98)

Algorithm: Given a contour C and a variable N:

step 0: Set N to 0.

step 1: Flag all maxima in C upwards; call the resulting set the *max-list*.

step 2: Flag all minima in C downwards; call the resulting set the *min-list*.

step 3: If all c-pitches are flagged, go to step 6.

step 4: Delete all non-flagged c-pitches in C.

step 5: N is incremented by 1 (i.e., N becomes N+1).

step 6: Flag all maxima in max-list upward. For any string of equal and adjacent maxima in the max-list, either: (1) flag only one of them; or (2) if one c-pitch in the string is the first or last c-pitch of C, flag only it; or (3) if both the first and last c-pitches of C are in the string, flag (only) both the first and last c-pitches of C.

step 7: Flag all minima in min-list downward. For any string of equal and adjacent minima in the min-list, either: (1) flag only one of them; or (2) if one c-pitch in the string is the first or last c-pitch of C, flag only it; or (3) if both the first and last c-pitches of C are in the string, flag (only) both the first and last c-pitches of C.

step 8: If all c-pitches are flagged, go to step 13.

step 9: Delete all non-flagged c-pitches in C.

step 10: If N≠0, N is incremented by 1 (i.e., N becomes N+1).

step 11: If N=0, N is incremented by 2 (i.e., N becomes N+2).

step 12: Go to step 6.

step 13: End. N is the "depth" of the original contour C.

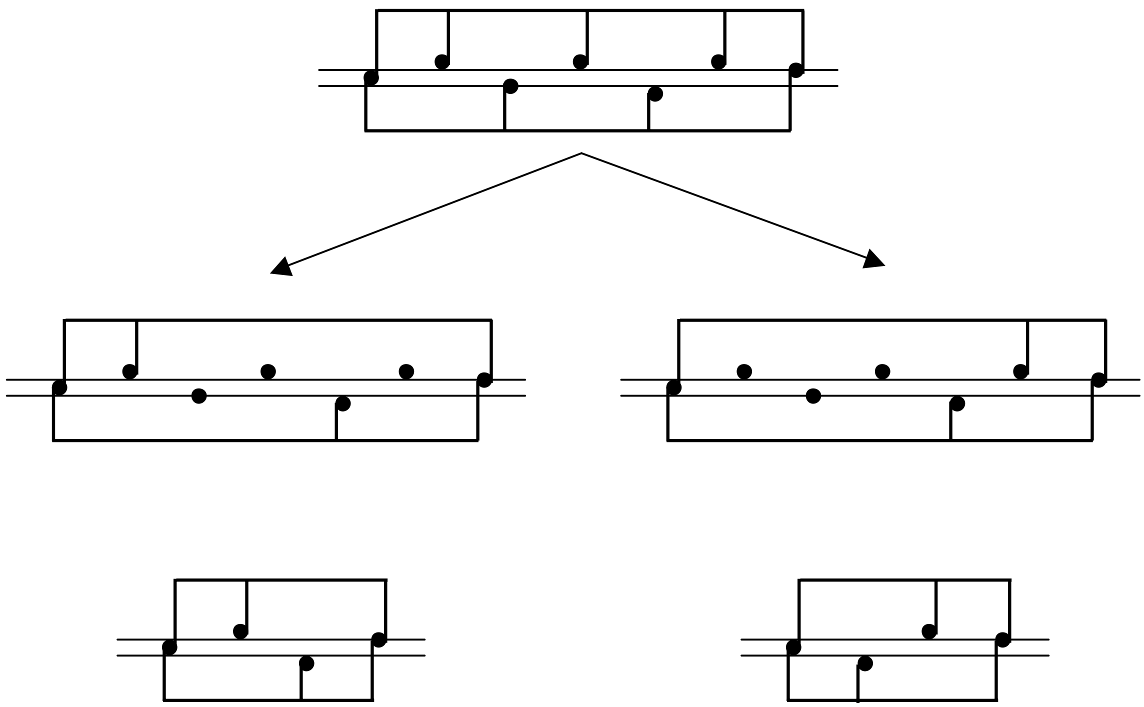
The reduced contour is the prime of C; if N=0, then the original C has not been reduced and is a prime itself.

The second refinement involves repeating c-pitches. Here, Schultz's concern about repeating c-pitches is strongly motivated by repetitions in Messiaen birdsong pieces that he is studying, although the concern does of course have general implications. Schultz demonstrates that arbitrary pruning of repeating maxima or minima, as instructed in Morris's algorithm, may result in different csegs in the deepest level. The reason for this difference is that if repeating maxima (or minima) are intervened by a minimum (or maximum), then retaining the first one would place it before the intervening minimum (or maximum), whereas retaining the second one would place it after. As a result, the first would have a different contour output than the second. Figure 2.9 illustrates this point by following the example provided by Schultz.[70] Here, two alternative ways to retain repeating c-pitches are shown following Step 6-1 ("flag only one of them"). The left-hand side solution flags (and retains) the first of the repeating c-pitches, whereas the right-hand side solution flags (and retains) the last. The retention of the cp before and after the intervening minimum (cp 0) results in different reductions, as is evident from the bottom portion of Figure 2.9. To avoid this inconsistency in the algorithm, Schultz proposes to retain both of the repeating maxima (or minima) if they are intervened by a minimum (or maximum). Note that at this point Schultz does not define which of the cps (i.e. the first or second cp 4) before the intervening minimum (cp 0) should be retained. The reduced cseg that results from Schultz's approach is illustrated in Figure 2.10.

[70] Note that for clarity purposes, Schultz employs staff lines in the representation of the reduction process, which we will adopt from now on.

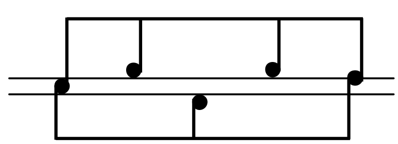
Figure 2.9. Two Possible Applications of Morris's "Contour Reduction Algorithm"

cseg <2414043>



cseg <1302> cseg <1032>

Figure 2.10. Schultz's Reduced cseg



cseg <13032>

Later on, Schultz presents a contour, <102021>, illustrated in Figure 2.11, in which both maxima and minima are repeated. The application of the criteria provided above to this contour retains one repeating c-pitch in max-list (or min-list) between the intervening minima (or maxima), which results in the retention of all repeating maxima and minima in the cseg. In order to reduce such a contour, which involves a chain of repeated maxima and minima, an additional criterion is required. Schultz suggests that repeating maxima or minima that are closer to the first or last c-pitches hold priority over the ones that are intermediary. Thus, he proposes to retain the repeating maxima or minima that are closest to the first and last c-pitches, and prune the others. The application of this criterion to cseg <102021> is shown in Figure 2.12, in which the intermediary c-pitches are pruned.

Figure 2.11. Cseg <102021>

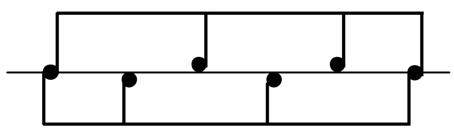
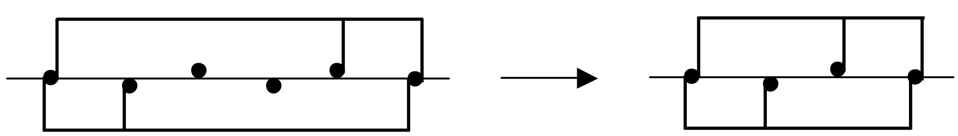


Figure 2.12. Application of Schultz's Algorithm on cseg <102021>



cseg <102021> cseg <1021>

The final version of Schultz's modified algorithm, which includes new rules regarding repeating c-pitches, is illustrated in Figure 2.13. Here, the alteration of Steps 6 and 7, and the insertion of Steps 8 and 9 solve the problem of intervening maxima/minima (previously illustrated in Figures 2.9 and 2.10), whereas the alteration of Step 10 (Step 8 in Figure 2.8), and the insertion of Steps 11 and 12, establish the retention of c-pitches based on their proximity to first and last c-pitches (previously illustrated in Figure 2.11 and 2.12).

The concerns which motivate the refinements proposed by Schultz are in fact addressed by the window algorithms that will be developed here, as we shall see in the following chapter. For now, suffice it to say that the disparity between the two approaches involves the criteria adopted in retaining repeating c-pitches.

Figure 2.13. Final Version of Schultz's "Contour Reduction Algorithm" (2008, 108)

Algorithm: Given a contour C and a variable N:

step 0: Set N to 0.

step 1: Flag all maxima in C upward; call the resulting set the *max-list*.

step 2: Flag all minima in C downward; call the resulting set the *min-list*.

step 3: If all c-pitches are flagged, go to step 6.

step 4: Delete all non-flagged c-pitches in C.

step 5: N is incremented by 1 (i.e., N becomes N+1).

step 6: Flag all maxima in max-list upward. For any string of equal and adjacent maxima in the max-list, flag all of them, unless: (1) one c-pitch in the string is the first or last c-pitch of C, then flag only it; or (2) both the first and last c-pitches of C are in the string, then flag (only) both the first and last c-pitches of C.

step 7: Flag all minima in min-list downward. For any string of equal and adjacent minima in the min-list, flag all of them, unless: (1) one c-pitch in the string is the first or last c-pitch of C, then flag only it; or (2) both the first and last c-pitches of C are in the string, then flag (only) both the first and last c-pitches of C.

step 8: For any string of equal and adjacent maxima in the max-list in which no minima intervene, remove the flag from all but (any) one c-pitch in the string.

step 9: For any string of equal and adjacent minima in the min-list in which no maxima intervene, remove the flag from all but (any) one c-pitch in the string.

step 10: If all c-pitches are flagged, and no more than one c-pitch repetition in the max-list and min-list (combined) exists, not including the first and last c-pitches of C, proceed directly to step 17.

step 11: If more than one c-pitch repetition in the max-list and/or min-list (combined) exists, not including the first and last c-pitches of C, remove the flags on all repeated c-pitches except those closest to the first and last c-pitches of C.

step 12: If both flagged c-pitches remaining from step 11 are members of the max-list, flag any one (and only one) former member of the min-list whose flag was removed in step 11; if both c-pitches are members of the min-list, flag any one (and only one) former member of the max-list whose flag was removed in step 11.

Figure 2.13 (cont.) Final Version of Schultz's *Contour Reduction Algorithm* (2008, 108)

step 13: Delete all non-flagged c-pitches in C.

step 14: If N≠0, N is incremented by 1 (i.e., N becomes N+1).

step 15: If N=0, N is incremented by 2 (i.e., N becomes N+2).

step 16: Go to step 6.

step 17: End. N is the "depth" of the original contour C.

The reduced contour is the prime of C; if N=0, then the original C has not been reduced and is a prime itself.

In addition to the sources reviewed in this chapter, the rising interest in contour theory over the past two decades has also resulted in a number of doctoral dissertations ranging from the application of complex mathematical and statistical modeling techniques such as *linear regression* and *trace analysis,*[71] to syntheses of contour theory with various other areas of research such as aural skills[72] and transformational theory.[73]

---

[71] Sean Hazard Carson, "Trace Analysis: Some Applications to Musical Contour and Voice Leading," Ph.D. dissertation, New York University, 2003; and R. Daniel Beard, "Contour Modeling by Multiple Linear Regression of the Nineteen Piano Sonatas by Mozart," Ph.D. dissertation, Florida State University, 2003. For a linguistic approach, see Suk Won Yi, "A Theory of Melodic Contour as Pitch-Time Interaction: The Linguistic Modeling and Statistical Analysis of Vocal Melodies in Selected Lied Collections of Schubert and Schumann," Ph.D. dissertation, University of California, Los Angeles, 1990. Note that the latter two focus on tonal repertoire. For analytical applications to atonal music, see Robert John Clifford, "Contour as a Structural Element in Selected Pre-Serial Works by Anton Webern," Ph.D. dissertation, University of Wisconsin, Madison, 1995.

[72] Laurdella Foulkes-Levy, "A Synthesis of Recent Theories of Tonal Melody, Contour, and the Diatonic Scale: Implications for Aural Perception and Cognition," Ph.D. dissertation, State University of New York, Buffalo, 1996. Note that in her dissertation, Foulkes-Levy engages tonal theories by a wide variety of theorists such as Heinrich Schenker, Fred Lerdahl, and John Clough.

[73] Noel Thomas Painter, "Exploring Contour Associations through Transformation Networks: Identification and Classification of Contour Relations in Modern Multiple Percussion Music," Ph.D. dissertation, University of Rochester, 2000.

Summary

As is evident from the review of the literature presented above, reduction of contours has not been central to previous research (with the exception of Morris 1993 and to a certain extent Quinn 1997). Despite the theoretical and analytical potential of Morris's contour reduction algorithm, the literature abounds with similarity relations from a combinatorial perspective (i.e. Marvin and Laprade 1987, Morris 1987, Quinn 1997, etc.), cognitive approaches (Dowling and Fujitani 1971, Trehub, Bull, and Thorpe 1984, Edworthy 1985), or both (i.e. Quinn 1999, Schmuckler 1999, etc.). Similarly, investigations into multi-parametric contour relations, which relate contours of different parameters (i.e. Marvin 1991, Morris 1993, Marvin 1995, etc.), do not take contour reduction into account and employ the same similarity measures provided by the combinatorial approach. One notable exception is the recent article by Rob Schultz (2008) which proposes some refinements to Morris's contour reduction algorithms. Although the doctoral dissertation by Laurdella Foulkes-Levy (1996) involves Morris's algorithm, along with theories by Schenker, Lerdahl, and Clough, the algorithm itself is not subject to alteration, but rather, the outcome of the algorithm is interpreted in a specific manner to accommodate tonal considerations such as neighbor tones, scale degrees, etc. The approach taken by Zohar Eitan (1997) implicitly engages with one specific aspect of Morris's algorithm, namely maxima (which Eitan refers to as *melodic peaks*) but does not take minima into account, nor involve the concept of reduction, and thus, hierarchy.

The aim of this dissertation is to contribute to this neglected area of research and provide a valuable analytical tool for a hierarchical interpretation of post-tonal compositions. The dissertation will propose a new set of algorithms based on Morris's contour reduction algorithm; extend the reductive process to the duration domain; and provide analyses to demonstrate the potential of the theory. The next chapter provides an essential step in this objective by introducing and discussing the concepts of *window algorithms* and *contour reduction functions*, which are vital to the theory proposed in this study.

CHAPTER 3

WINDOW ALGORITHMS AND CONTOUR REDUCTION FUNCTIONS

This chapter opens with a summary and critique of the contour reduction algorithm proposed by Robert Morris.[74] Based on this algorithm, an alternative set of algorithms, called "window algorithms," is introduced. This is followed by an investigation of the applicability of the algorithms on segments that contain successive and non-successive repeating pitches. Formalization of the algorithms through symbolic logic and flowcharts is followed by a discussion of "contour reduction functions" which arise from the successive applications of the window algorithms in various combinations. Finally, a comparison of Morris's reduction algorithm and the window algorithms is presented.

The Contour Reduction Algorithm by Morris

Essentially, the reduction algorithm Morris proposes (previously illustrated in Chapter 2, Figure 2.5) can be understood as having two distinct stages.

In the first stage of this algorithm (corresponding to steps 1-4), all of the cps that are neither maxima nor minima are first marked for pruning. A cp is a maximum if it is higher (and minimum if it is lower) than the cp that immediately precedes and also the cp that immediately follows it. If two successive cps have the same value, either one could be

[74] Robert D. Morris, "New Directions in the Theory and Analysis of Musical Contour."

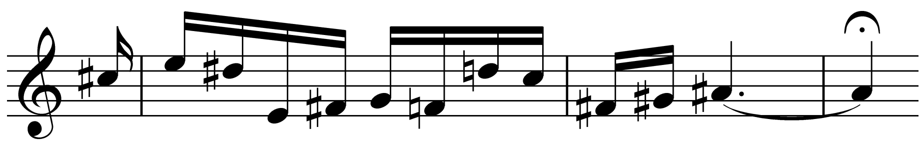
marked for pruning.[75] The first and last cps of the cseg are always considered as both maxima and minima, and thus should never be marked for pruning.[76] After the evaluation of all cps, the marked cps are pruned. In the resulting reduced cseg, every remaining cp will be either a (local) maximum or minimum, and the contour direction will change from + to –, or from – to +, with every cp.

In the second stage of Morris's algorithm (corresponding to steps 6-8), cps are given upward-stems if they are maxima, or are given downward-stems if they are minima. At this stage, we beam together all the upward-stemmed cps, and call this set of cps "Max-list"; similarly we also beam together all the downward-stemmed cps, and call this set of cps "Min-list." Now we consider Max-list, and identify those (upward-beamed) cps that are (also) *maxima* in the context of only Max-list, and we mark for pruning those that are not maxima in this context. Similarly, we consider Min-list, and identify those (downward-beamed) cps that are (also) *minima* in the context of only Min-list, and we mark for pruning those that are not minima in this context. If two successive cps have the same value, we flag either one for pruning. After examining every cp in the Max-list and Min-list, we prune those cps that have been marked for pruning. Note that, similar to the first stage, we consider the first and last cps to be maxima on the Max-list and minima on the Min-list.

Let us consider the second phrase of Schoenberg's op.19, no.4, which is also used by Morris in his analytical application of the contour reduction algorithm:

[75] This arbitrary pruning of cps is one of the main motivations behind Rob Schultz's refinements to Morris's algorithm. See, Rob Schultz, "Melodic Contour and Nonretrogradable Structure in the Birdsong of Olivier Messiaen."

[76] As we shall discuss later on in the chapter, this privilege granted to the first and last cps may or may not be eliminated in the "window" approach proposed in this study.

Figure 3.1. Schoenberg's op.19, no.4, Second Phrase

Arnold Schönberg, *6 kleine Klavierstücke*, op.19/4

Application of the first and second stages to this passage is demonstrated in Figures 3.2 and 3.3. In these figures, we adopt the notation devised by Schultz, which includes horizontal lines for visual clarity in representing the relative pitch height. Here, the pitches are placed on a 6-line staff according to their height; the highest pitch E5 above the top line, the second highest pitch D#5 on the top line, the third highest pitch D5 in between the top line and the line below it, and so forth.[77] It is important to note that if the cseg is long enough and sufficiently "zig-zaggy," it may be possible to reiterate stage 2 more than once. We can define the "depth" of the cseg to be the number of times the algorithm stages are applied. Since stage 1 can be applied only once, the depth of the cseg can be understood as "one plus the number of applications of stage 2". Figure 3.4 demonstrates the reiteration of the second stage for the same melody. Since stage 2 is applied twice on the Schoenberg phrase, the algorithm assigns a depth of 3 to the passage. All of the cps pruned by the first stage of the reduction algorithm (D#5, F#4, C5, and G#4) appear in metrically weak positions. An exploration of how contour relates to other forms of contextual salience will be presented in Chapter 6 but for now suffice it to say that in this excerpt, Schoenberg's sense of rhythm seems to coordinate the contour depth levels devised by this algorithm.

[77] In Schultz's notation it is not possible to identify pcs without comparing them to the original score. A notation which not only maintains the specific pcs but also demonstrates the entire reduction on a single staff will be introduced later in the chapter.

Figure 3.2. Stage 1 – Depth 1

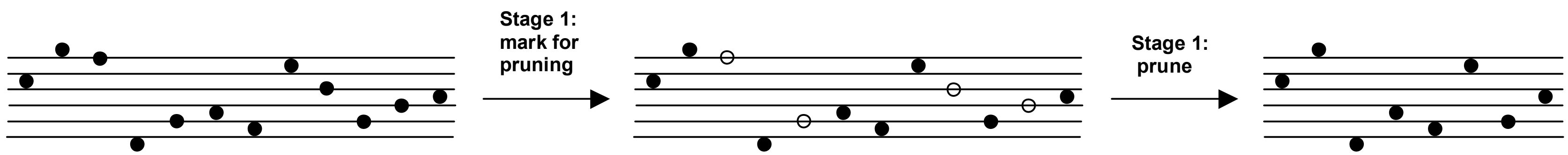


Figure 3.3. Stage 2 – Depth 2

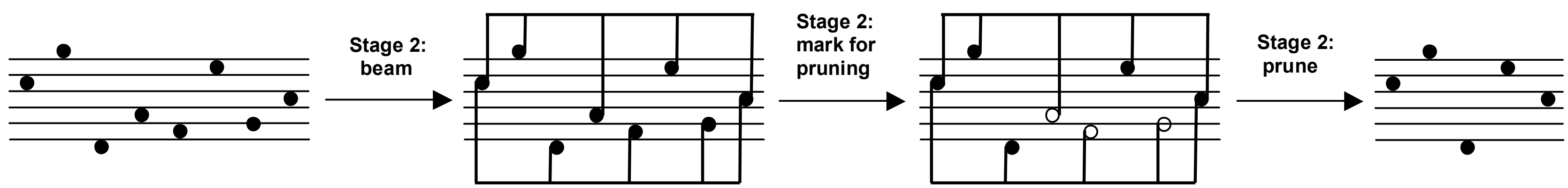
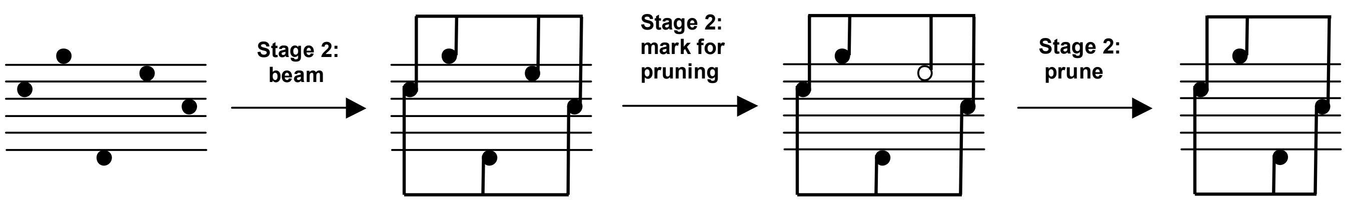


Figure 3.4. Stage 2 (reiteration) – Depth 3



Note that the first stage evaluates all of the cps in a single pass, while the second stage evaluates upper- and lower- beams (Max-list and Min-list) in separate passes. In particular, in the second stage, only maxima that are in the Max-list and minima that are in the Min-list are not pruned. Now, let us make an observation that we will not treat in depth in this chapter. In Figure 3.3, the lowest pitch in the maximum-stratum is closer (in pitch-space) to the pitches that immediately surround it but are beamed under the Min-list. Both visually and aurally, this pitch belongs more strongly to its immediate surrounding (due to the pitch proximity) rather than to the max stratification. In fact, in some cases it is possible to split a passage into upper and lower parts due to pitch proximity. In the following chapter, we will observe such an example by Anton Webern, which involves what is referred to as “auditory stream segregation.”[78]

It is also possible to apply the algorithm on the numerical cseg <7t9023186245> rather than on note-heads as exemplified in Figures 3.2-3.4. A demonstration on how to apply the algorithm to numbers alone is given below:

Cseg = <7t9023186245>

Stage 1: Mark for pruning (boldface/underline): <7t**9**0**2**318**6**2**4**5>

Stage 1: Prune 9, 2, 6, 4: <7t031825>

Renumber[79] pruned cseg: <57031624>

Stage 2: Max-list = <57364>. Mark for pruning (boldface/underline): <57**3**64>

Stage 2: Min-list = <50124>. Mark for pruning (boldface/underline): <50**12**4>

---

[78] See Albert S. Bregman, *Auditory Scene Analysis: The Perceptual Organization of Sound* (Cambridge: MIT Press, 1990).
[79] For a definition of “renumbering,” see Chapter 1, p. 7.

Stage 2: Prune 3, 1, and 2: <57064>

Renumber pruned cseg: <24031>

Stage 2: Max-list = <2431>. Mark for pruning (boldface/underline): <24**3**1>

Stage 2: Min-list = <201>. No cps to be marked for pruning.

Stage 2: Prune 3: <2401>

Renumber pruned cseg: <2301>

One could argue that using two distinct ways of evaluating cps for pruning (as in stage 1 and stage 2) weakens the unity and consistency of this algorithm. However, despite the dichotomy within the algorithm, the opportunity to reiterate the second stage multiple times offers some methodological flexibility since the reduction process can be stopped at interim depths should they be preferable in a given analytical situation.

It should be noted that Morris flags and beams the cps in stage one (steps 1-4), prior to any pruning. However, the process of flagging and beaming is redundant for the first-stage pruning: the pruning of the cps has nothing to do with the direction of the stems and with the beaming. In fact, as illustrated in Figure 3.5, Morris's algorithm could be reconstructed in a way to render the flagging and beaming in the first stage unnecessary.[80] Thus, the first stage and second stage pruning can be referred to as "pre-beaming" and "post-beaming" pruning, respectively.

---

[80] Note that this version of the algorithm also eliminates the wedge-contour problem pointed out by Schultz (see Chapter 2, Figure 2.7) without resorting to as many steps in Schultz's modification (Chapter 2, Figure 2.8).

Figure 3.5. Modified Version of Morris's Algorithm

step 0: Set N to 0.

step 1: Delete all pitches that are not maxima or minima.

step 2: If no pitch is pruned, go to step 4.

step 3: N is incremented by 1 (i.e., N becomes N+1).

step 4: Flag all maxima in C upwards; call the resulting set the *max-list*. For any string of equal and adjacent maxima in max-list, either: (1) flag only one of them; or (2) if one pitch in the string is the first or last pitch of C, flag only it; or (3) if both the first and last pitch of C are in the string, flag (only) both the first and last pitch of C.

step 5: Flag all minima in C downwards; call the resulting set the *min-list*. For any string of equal and adjacent minima in min-list, either: (1) flag only one of them; or (2) if one pitch in the string is the first or last pitch of C, flag only it; or (3) if both the first and last pitch of C are in the string, flag (only) both the first and last pitch of C.

step 6: Delete all pitches that are not maxima in the max-list and minima in the min-list.

step 7: If no pitch is pruned, go to step 9.

step 8: Go to step 3.

step 9: End. N is the "depth" of the original contour C.

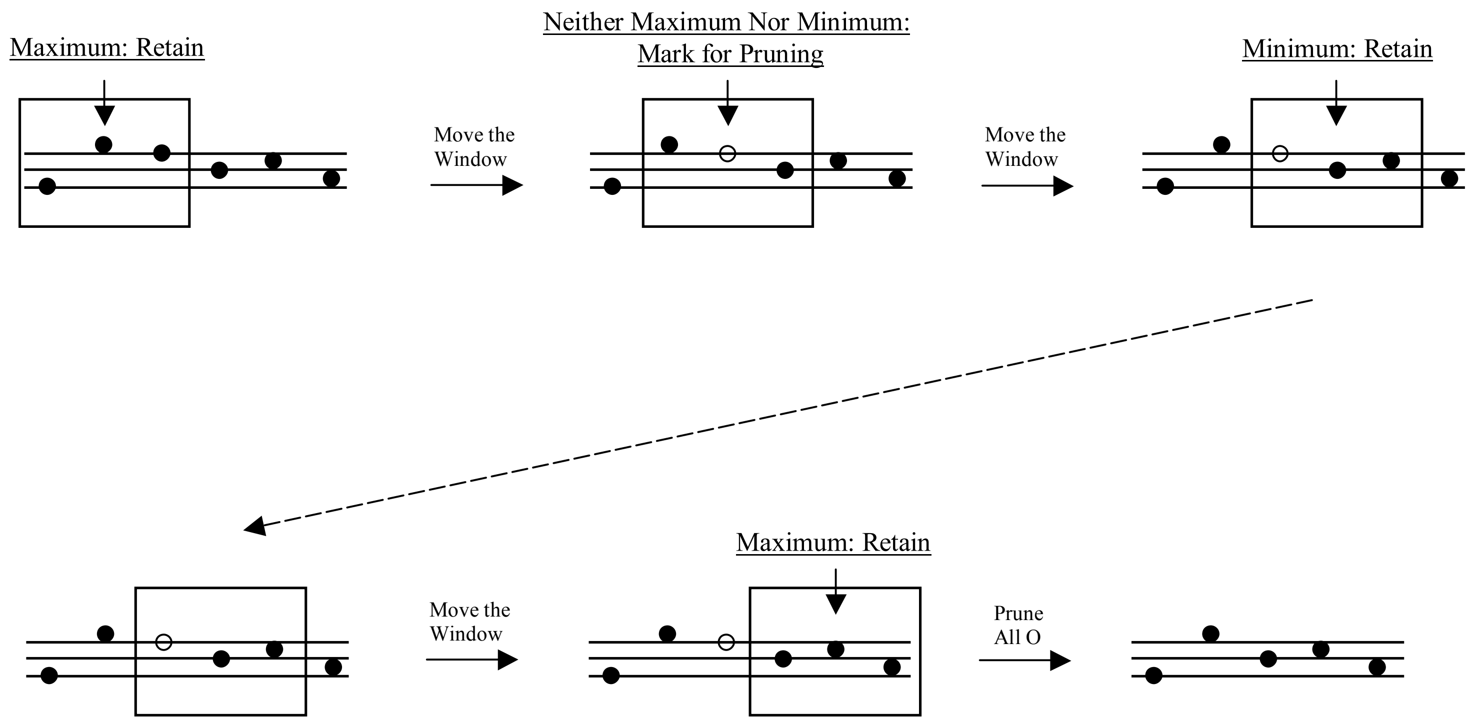
As explained above, on the one hand, Morris's algorithm provides a useful notion of "depth" due to the possibility for reiterating stage 2; on the other hand, the different types of pruning (one-pass pruning versus two-pass pruning) weaken the systematic consistency of the approach. The ideal reduction algorithm would provide the methodological benefits of reiteration, but would use a single type of pruning for all stages. In fact, such an algorithm is possible and will be introduced soon. First, we will develop the notion of "window algorithms," starting with the "3-window algorithm" as the most basic type.

The 3-Window Algorithm

Morris is particularly interested in the second stage and the end product of the algorithm, in which the resulting cseg cannot be reduced any further. Here, our focus will be restricted to the initial stage of the algorithm.

The first stage of Morris's algorithm can be understood essentially as a window consisting of 3 cps that moves through the cseg. The medial cp of the window is evaluated in comparison to the other two (preceding and succeeding) cps within the window. If the medial cp is a maximum or minimum (within the window), then it is retained. If not, it is marked for pruning. Here, as in Morris's algorithm, the first and last cps in the complete cseg are always retained. Figure 3.6 represents the 3-window algorithm on a shorter cseg <054231>.

Figure 3.6. 3-Window Algorithm Applied on cseg <054231>



The 3-window algorithm could also be applied on the strictly numerical cseg <054231>. If <xyz> represents the 3 cps in a window, we always evaluate the medial cp y; if y is a maximum or minimum in the window we mark it for retention and if y is neither a maximum nor a minimum in the window we mark it for pruning. An illustration of the numerical application on cseg <054231> is given below.

Cseg = <054231>

List successive 3-windows, with medial cp evaluated and set in boldface/underline if marked for pruning.

<054> evaluate 5: it is a maximum; mark it for retention

<542> evaluate 4: it is neither maximum nor minimum; mark it for pruning (<5**4**2>)

<423> evaluate 2: it is a minimum; mark it for retention

<231> evaluate 3: it is a maximum; mark it for retention

<054>, <5**4**2>, <423>, <231>

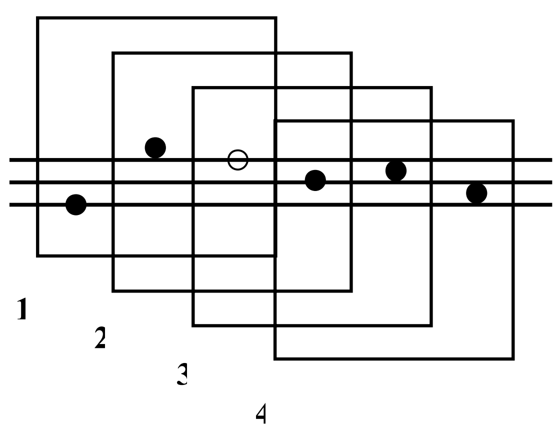
Prune cp 4: <05231>

Renumber pruned cseg: <04231>

It is important to note, from a phenomenological perspective, that in a listening context—as opposed to a score-based analytical context—one cannot evaluate the medial cp in the window until the last cp in the window is heard. Since such considerations deserve an in-depth investigation, they are left for Chapter 5 and the approach taken for the present is analytical and assumes perfect memory or a score-based interpretation.

Leaving the phenomenological considerations for later comment, it is important to point out that the notion of a "moving" window can be exchanged with the notion of a number of static windows, as demonstrated in Figure 3.7. The idea of using static windows by representing each window on the score distinctly provides a bird's eye view and does not necessarily correspond to the listener's experience of moving through the melody. The difference between the single moving window and multiple static windows could be understood as the difference between a listener implementing the algorithm on the fly in real-time, in contrast with an analyst working with a score—or some other non-temporal representation of the contour information, such as the diagrams above—or a listener who has a prior knowledge of the piece and "perfect" memory.

Figure 3.7. Immobile Windows



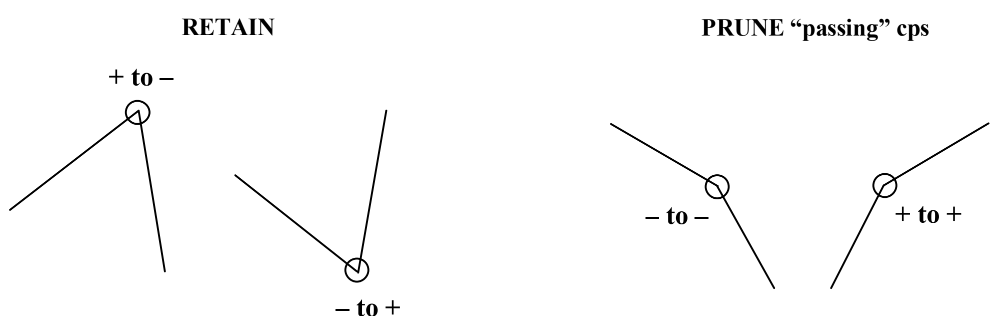
3-Window Algorithm as "Change in CAS"

In this section, we will discuss how the 3-window algorithm could be defined as change in Friedmann's *contour adjacency series* in order to gain an in-depth understanding of the true nature of the algorithm.

The term "contour adjacency series" (or CAS) coined by Michael Friedmann, refers to a series of pluses and minuses indicating contour changes between adjacent cps.[81] More formally, given a csubseg <xy>, the CAS of <xy> is <+> if cp x is less than cp y (i.e. x < y), and the CAS of <xy> is <–> if cp x is greater than the cp y (i.e. x > y). Consequently, csubsegs <01> and <10> are represented by "+" and "–," respectively. The cseg <054231> has a CAS of < +, –, –, +, – >. This particular CAS involves a change between all adjacent signs except the second and third signs, which are both minus.

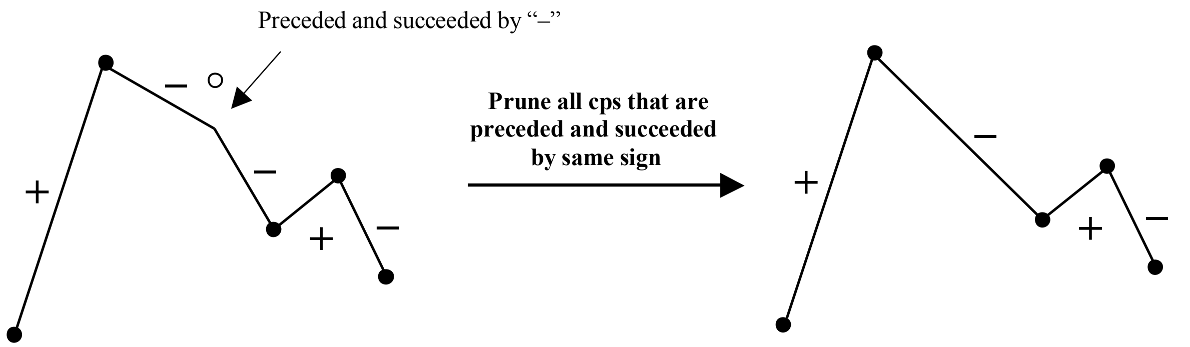
Let us define a "change in CAS algorithm" that reduces a cseg by pruning every cp that is preceded and succeeded by the same sign. Figure 3.8 demonstrates the two generalized situations in which cps are retained and pruned, and Figure3.9 illustrates the algorithm on cseg <054231>.

Figure 3.8. Retention and Pruning Based on CAS



[81] Michael Friedmann, "A Methodology for the Discussion of Contour."

Figure 3.9. Application of Change in CAS on cseg <054231>



Note that pruning cps that are preceded and succeeded by the same sign is just another way of pruning cps that are neither maxima nor minima; such cps must be "passing" as illustrated in Figure 3.8. Therefore, the change-in-CAS algorithm and the 3-window algorithm are two ways of applying the same principle.

Let us now demonstrate how the 3-window algorithm and the change-in-CAS-algorithm correspond to one another.

Let x, y and z be consecutive ordered cps in the 3-window <xyz>. Then the 3-window evaluate y as follows:

y is marked for retention if $x<y>z$

y is marked for retention if $x>y<z$

y is marked for pruning if $x>y>z$

y is marked for pruning if $x<y<z$

Now let us consider each of these four possibilities separately.

a) The condition $x<y>z$ indicates that:

y has the highest value of the three cps, and thus is a maximum in the 3-window algorithm;

y is preceded and succeeded by lower cp values, and thus preceded and succeeded by different signs, since the CAS is < +, – >. In both algorithms, y is marked for retention.

b) The condition $x>y<z$ indicates that:

y has the lowest value of the three cps, and thus is a minimum in the 3-window algorithm;

y is preceded and succeeded by higher cp values, and thus preceded and succeeded by different signs, since the CAS is < –, + >. In both algorithms, y is marked for retention.

c) The condition $x>y>z$ indicates that:

y has the medial cp value, and thus neither a maximum nor a minimum in the 3-window algorithm;

y is preceded and succeeded by higher and lower cp values, respectively. Thus it is preceded and succeeded by same signs, since the CAS is < –, – >. In both algorithms, y is marked for pruning.

d) The condition $x<y<z$ indicates that:

y has the medial cp value, and thus neither a maximum nor a minimum in the 3-window algorithm;

y is preceded and succeeded by lower and higher cp values, respectively. Thus it is preceded and succeeded by same signs, since the CAS is < +, + >. In both algorithms, y is marked for pruning.

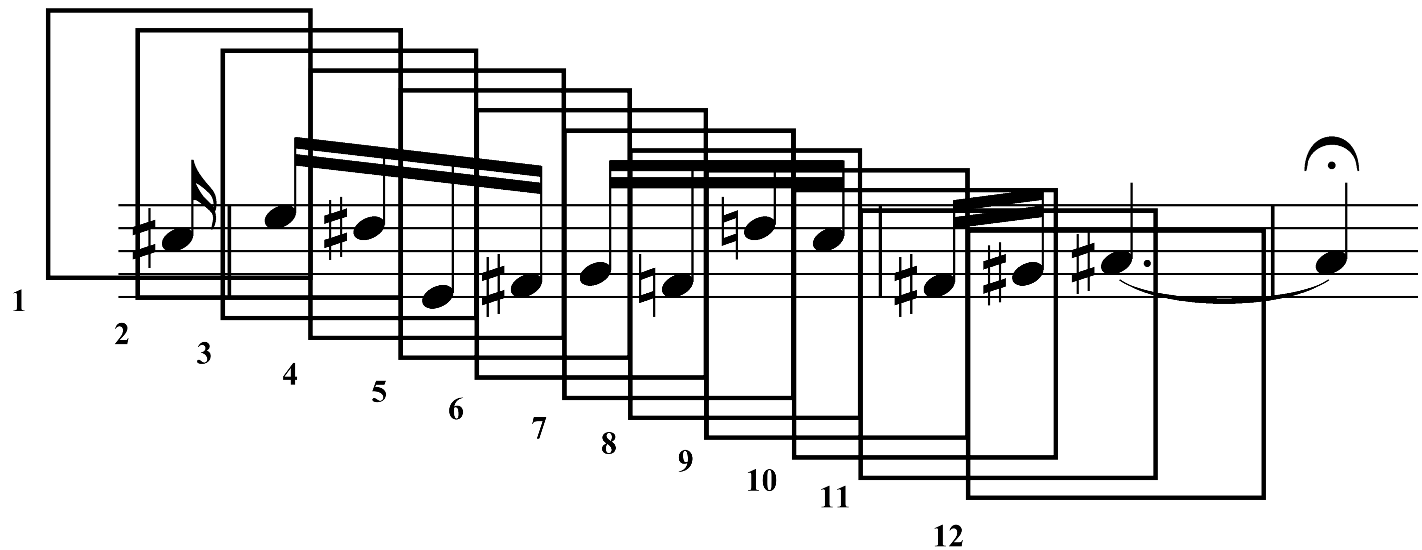
Since the overall shape of a contour can be defined by the points where a sign change occurs (i.e. where the cp is not passing), both (equivalent) algorithms reduce a given contour without changing its overall shape; the reduced contour always has strictly alternating signs such as < +, –, +, –, …> or < –, +, –, +, …>.

Note that Friedmann's definition of CAS omits the cases where two adjacent cps are identical, which constitute neither <+> nor <–>. Similarly, we have not yet included a discussion of repeated adjacent cps. The issue is an important one and will be addressed later in the chapter.

Application of the 3-Window Algorithm

An application of the 3-window algorithm on the Schoenberg phrase we discussed earlier is displayed in Figure 3.10.

Figure 3.10. Application of the 3-Window Algorithm on the Schoenberg Phrase



Despite the crowded representation of Figure 3.10, the principle behind the reduction is simple: for each note at the center of the window, determine whether it is a maximum or minimum in comparison to the preceding and succeeding notes (within the window). Now, let us walk through the application of the 3-window algorithm, discussing every note as such:

1. The C#5 is considered in the window <C#5, E5>; within this window, the C#5 is the minimum, and is therefore marked for retention.[82] There are various reasons to consider retaining the first and last cps regardless of its status, such as framing function, and primacy and recency effects, which will be discussed shortly.[83]
2. The E5 is considered in the window <C#5, E5, D#5>; within this window, the E5 is the maximum, and is therefore marked for retention.
3. The D#5 is considered in the window <E5, D#5, E4>; within this window, the D#5 is neither a maximum nor a minimum, and is marked for pruning.
4. The E4 is considered in the window <D#5, E4, F#4>; within this window, the E4 is the minimum, and is therefore marked for retention.
5. The F#4 is considered in the window <E4, F#4, G4>; within this window, the F#4 is neither a maximum nor a minimum, and is therefore marked for pruning.
6. The G4 is considered in the window <F#4, G4, F4>; within this window, the G4 is the maximum, and is therefore marked for retention.
7. The F4 is considered in the window <G4, F4, D5>; within this window, the F4 is the minimum, and is therefore marked for retention.

[82] Note that in a cardinality-2 window the evaluated pitch is always a maximum or minimum, and thus, the first and last pitches of a segment in the 3-window algorithm are always retained.
[83] The retention of the first and last cps is also the convention in Morris's algorithm which states that the first and last cps are always considered maximum and minimum.

8. The D5 is considered in the window <F4, D5, C5>; within this window, the D5 is the maximum, and is therefore marked for retention.
9. The C5 is considered in the window <D5, C5, F#4>; within this window, the C5 is neither a maximum nor a minimum, and is therefore marked for pruning.
10. The F#4 is considered in the window <C5, F#4, G#4>; within this window, the F#4 is the minimum, and is therefore marked for retention.
11. The G#4 is considered in the window <F#4, G#4, A#4>; within this window, the G#4 is neither a maximum nor a minimum, and is therefore marked for pruning.
12. The A#4 is considered in the window <G#4, A#4>; within this window, the A#4 is the maximum, and is therefore marked for retention.

The 3-window algorithm reduction could also be demonstrated numerically as exemplified below:

Cseg = <7t9023186245>

List successive 3-windows, with medial cp evaluated and set in boldface/underline if marked for pruning.

<Ø7t> evaluate 7: it is a minimum; mark it for retention

<7t9> evaluate t: it is a maximum; mark it for retention

<t90> evaluate 9: it is neither maximum nor minimum; mark it for pruning (<t**9**0>)

<902> evaluate 0: it is a minimum; mark it for retention

<023> evaluate 2: it is neither maximum nor minimum; mark it for pruning (<0**2**3>)

<231> evaluate 3: it is a maximum; mark it for retention

<318> evaluate 1: it is a minimum; mark it for retention

<186> evaluate 8: it is a maximum; mark it for retention

<862> evaluate 6: it is neither maximum nor minimum; mark it for pruning (<8**6**2>)

<624> evaluate 2: it is a minimum; mark it for retention

<245> evaluate 4: it is neither maximum nor minimum; mark it for pruning (<2**4**5>)

<45Ø> evaluate 5: it is a maximum; mark it for retention

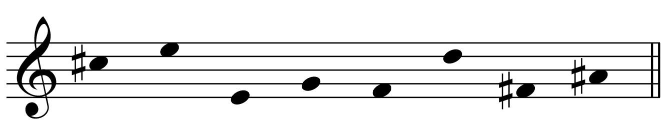
<Ø7t> <7t9> <t**9**0> <902> <0**2**3> <231> <318> <186> <8**6**2> <624> <2**4**5> <45Ø>

Prune marked cps (9, 2, 6, 4): <7t031825>

Renumber retained cps: <57031624>

The resulting pitch contour is illustrated in Figure 3.11.

Figure 3.11. Reduced cseg (3-Window)



<5 7 0 3 1 6 2 4>

In the next section, we will introduce and discuss the 5-window algorithm, which is also established upon the notion of maxima and minima, but involves a different window size.

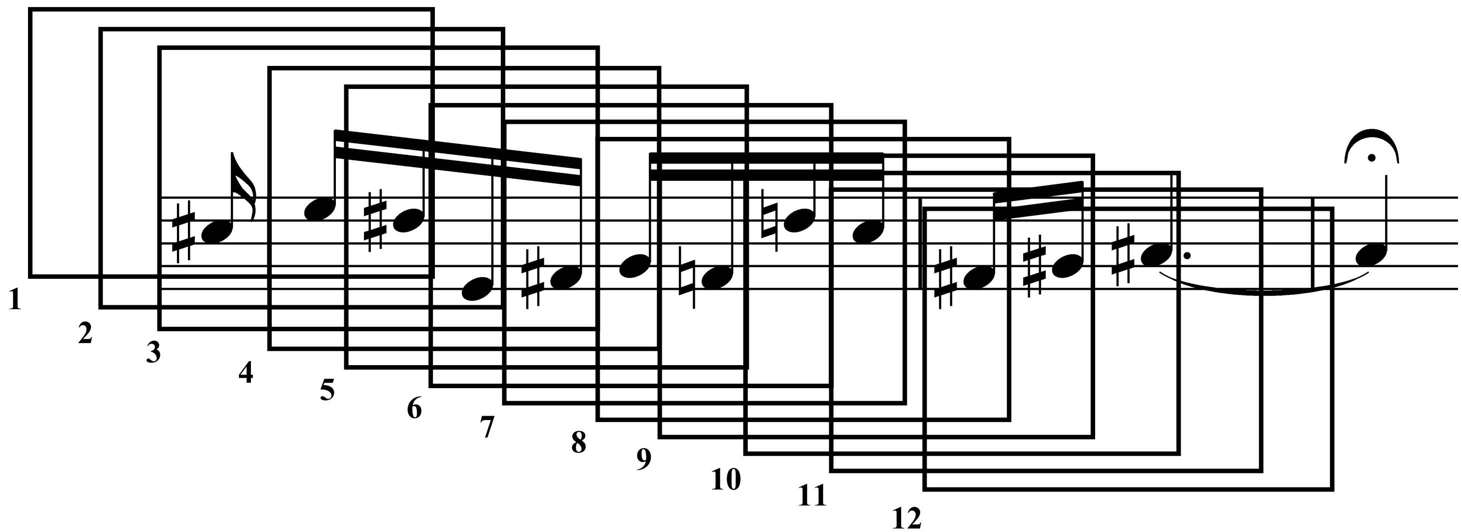
The 5-Window Algorithm

In essence, the 5-window algorithm can be understood as a version of the 3-window algorithm, in which the size of the window is enlarged to five cps, as opposed to three cps. This enlargement results in some significant conceptual and methodological differences: since the 3-window algorithm prunes cps that are preceded and succeeded by the same sign (i.e. <+, +> or <–, –>) and always result in a contour with alternating signs (i.e. < +, –> or < –, +>), it is not possible to apply the 3-window algorithm once more on the reduced contour. The 5-window algorithm, however, provides an opportunity for reiteration as a consequence of the enlargement of the window size. In addition, as indicated earlier, it promises only one type of evaluation for pruning, unlike Morris's algorithm, but nonetheless permits reiteration, and with it, the concept of depth, measured as the number of reiterations. Overall, the 5-window algorithm combines the useful properties of Morris's algorithm and the 3-window algorithm: a consistent methodology for pruning and a potential for multiple applications on a given contour. Some of the broader implications of the pruning processes in Morris's post-beaming stage and the 5-window algorithm will be discussed from a comparative point of view later in the chapter.

Application of the 5-Window Algorithm

Figure 3.12 demonstrates the 5-window algorithm on the Schoenberg phrase, showing twelve windows used to evaluate the twelve notes in the phrase. Note that successive applications of both the 3-window and the 5-window algorithms on a variety of melodies will be presented in Chapter 4.

Figure 3.12. Application of the 5-Window Algorithm on the Schoenberg Phrase



Here, similar to the 3-window algorithm, the center note within each window is evaluated to determine whether it is a maximum or minimum. However, now the evaluation encompasses the two preceding and two succeeding notes rather than just one. A step-by-step application of the algorithm is provided below.

1. The initial C#5 is marked for retention since it is the first cp of the segment (following Morris's convention).
2. The E5 is considered in the window <C#5, E5, D#5, E4>, which extends from the C#5 (only one event to the left) and to the E4 (two events to the right); within this window, the E5 is the maximum, and is therefore marked for retention.
3. The D#5 is considered in the window <C#5, E5, D#5, E4, F#4>, which extends two events to the left and right; within this window, the D#5 is neither a maximum nor a minimum, and is therefore not marked for retention.
4. The E4 is considered in the window <E5, D#5, E4, F#4, G4>; within this window, the E4 is the minimum, and is therefore marked for retention.
5. The F#4 is considered in the window <D#5, E4, F#4, G4, F4>; within this window, the F#4 is neither a maximum nor a minimum, and is therefore not marked for retention.
6. The G4 is considered in the window <E4, F#4, G4, F4, D5>; within this window, the G4 is neither a maximum nor a minimum, and is therefore note marked for retention.
7. The F4 is considered in the window <F#4, G4, F4, D5, C5>; within this window, the F4 is neither a maximum nor a minimum, and is therefore note marked for retention.
8. The D5 is considered in the window <G4, F4, D5, C5, F#4>; within this window, the D5 is the maximum, and is therefore marked for retention.

9. The C5 is considered in the window <F4, D5, C5, F#4, G#4>; within this window, the C5 is neither a maximum nor a minimum, and is therefore not marked for retention.
10. The F#4 is considered in the window <D5, C5, F#4, G#4, A#4>; within this window, the F#4 is the minimum, and is therefore marked for retention.
11. The G#4 is considered in the window <C5, F#4, G#4, A#4>, which extends only one event to the right; within this window, the G#4 is neither a maximum nor a minimum, and is therefore not marked for retention.
12. The A#4 is marked for retention since it is the last cp of the segment (following Morris's convention).

It is important to note that G4, which is retained in the 3-window algorithm as a local point of contour change, is pruned by the 5-window algorithm. This pitch is arguably aurally more significant than a passing cp such as the preceding F#4; however, both cps are pruned at the same stage. In order to differentiate between G4 and F#4, an application of the 3-window algorithm on this segment would be more suitable. We will discuss the principle in deciding the window size later on in the chapter.

Figure 3.13 illustrates the marking of maxima and minima within a 5-window centered on each respective cp, by circling them, where an upper-case **M** for "maximum" or a lower-case **m** for "minimum." All notes not marked with an **M** or **m** are pruned. The reduced cseg after the application of the 5-window algorithm is shown in Figure 3.14.[84]

---

[84] Note that the first and last cps are considered both maximum and minimum following Morris's definition.

Figure 3.13. 5-Window Maxima and Minima in the Schoenberg Phrase

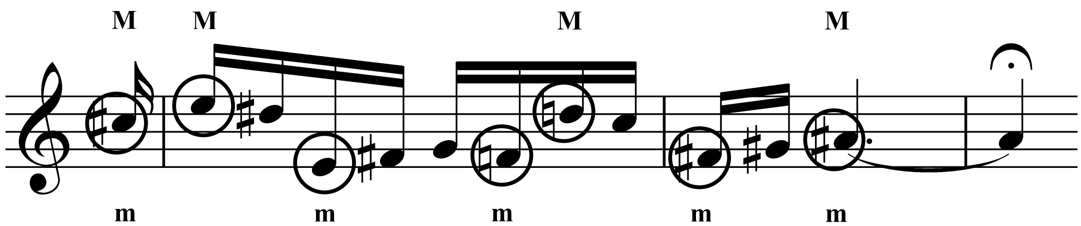
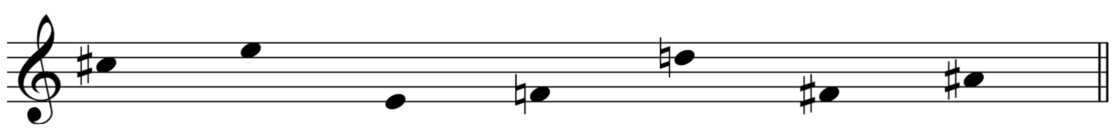


Figure 3.14. Reduced cseg (5-Window)

The 5-window algorithm could also be applied on the corresponding numerical cseg <7t9023186245>, as shown below:

Cseg = <7t9023186245>

List successive 5-windows, with medial cp evaluated and set in boldface/underline if marked for pruning:

<ØØ7t9> evaluate 7: it is the first cp; mark it for retention

<Ø7t90> evaluate t: it is a maximum; mark it for retention

<7t902> evaluate 9: it is neither maximum nor minimum; mark it for pruning (<7t**9**02>)

<t9023> evaluate 0: it is a minimum; mark it for retention

<90231> evaluate 2: it is neither maximum nor minimum; mark it for pruning (<90**2**31>)

<02318> evaluate 3: it is neither maximum nor minimum; mark it for pruning (<02**3**18>)

<23186> evaluate 1: it is a minimum; mark it for retention

<31862> evaluate 8: it is a maximum; mark it for retention

<18624> evaluate 3: it is neither maximum nor minimum; mark it for pruning (<18**6**24>)

<86245> evaluate 2: it is a minimum; mark it for retention

<6245Ø> evaluate 4: it is neither maximum nor minimum; mark it for pruning (<62**4**5Ø>)

<245ØØ> evaluate 5: it is the last cp; mark it for retention

<ØØ7t9> <Ø7t90> <7t**9**02> <t9023> <90**2**31> <02**3**18> <23186> <31862> <18**6**24> <86245> <62**4**5Ø> <245ØØ>

Prune marked cps (9, 2, 3, 6, 4): <7t01825>

Renumber retained cps: <4601523>

Note that in the 5-window algorithm, the first and the last cps are always retained; still, it is instructive to consider a different approach and treat the first and last cps not any different than rest of the cps in the segment. For example, if the last cp of the phrase happened to be a G natural rather than an A#, then it would neither be a maximum nor a minimum and would consequently be pruned. Technically, this is not possible in the 3-window algorithm since the first and last pitches are either maximum or minimum since there are only two cps to be evaluated (the succeeding cp for the first note and the preceding cp for the last note). This point is important as it offers a potential to remove the “retention privileges” of the initial and final cps. However, the methodological approach taken here favors the retention of the initial and final cps since they hold a framing function, which is significant to an understanding of melody in all musical repertoires. This is further supported by what psychologists call *primacy effect* and *recency effect* (collectively *serial position effect*), which refer to a "tendency for the items near the beginning and end of the series to be recalled best."[85] Nevertheless, it is possible to use the 5-window algorithm and allow pruning of the first or last notes if it can be convincingly supported within an analytical context, such as a phrase elision or a lack of accentuation in all musical parameters for the first or last cps.

The 5-window algorithm, in essence, addresses the problem of reducing the wedge-shaped contour, which Schultz points out. The inadequacy of Morris's algorithm in reducing this type of contour (previously illustrated in Chapter 2, Figure 2.7) and Schultz's remedial refinement to the algorithm were discussed in detail in the previous chapter, and thus, we will not reiterate the points made earlier. However, we will apply the 5-window algorithm on the wedge-shaped contour in order to demonstrate how this problem is addressed by the approach proposed in this study. Below is a numerical application of the 5-window algorithm on the contour <2415063>, which is employed by Schultz in his discussion of this issue.[86] The reduced cseg <1032> resulting from the application of the 5-window algorithm is the same as the reduced cseg that results from Schultz's refined algorithm.

[85] Andrew M. Colman, *A Dictionary of Psychology* (Oxford: Oxford University Press, 2006), 688. See also B.B. Murdock, "The Serial Position Effect of Free Recall," *Journal of Experimental Psychology* 64 (1962): 482-88; Alice F. Healy, David A. Havas, and James T. Parker, "Comparing Serial Position Effects in Semantic and Episodic Memory Using Reconstruction of Order Tasks," *Journal of Memory and Language* 42 (2000): 147-67; James V. Hinrichs, and Herman Buschke, "Running Missing Scan: Perception of Oldest Member in Serial Presentations," *Psychonomic Science* 19 (1970): 125-26; Herman Buschke, and James V. Hinrichs, "Controlled Rehearsal and Recall Order in Serial List Retention," *Journal of Experimental Psychology* 78 (1968): 502-09; Peter A. Frensch, "Composition During Serial Learning: A Serial Position Effect," *Journal of Experimental Psychology* 20, no. 2 (1994): 423-43.
[86] Schultz, "Melodic Contour and Nonretrogradable Structure," 96-101.

Cseg = <2415063>

List successive 5-windows, with medial cp evaluated and set in boldface/underline if marked for pruning:

<ØØ241> evaluate 2: it is the first cp; mark it for retention

<Ø2415> evaluate 4: it is neither maximum nor minimum; mark it for pruning (<Ø2**4**15>)

<24150> evaluate 1: it is neither maximum nor minimum; mark it for pruning (<24**1**50>)

<41506> evaluate 5: it is neither maximum nor minimum; mark it for pruning (<41**5**06>)

<15063> evaluate 0: it is a minimum; mark it for retention

<5063Ø> evaluate 8: it is a maximum; mark it for retention

<063ØØ> evaluate 3: it is the last cp; mark it for retention

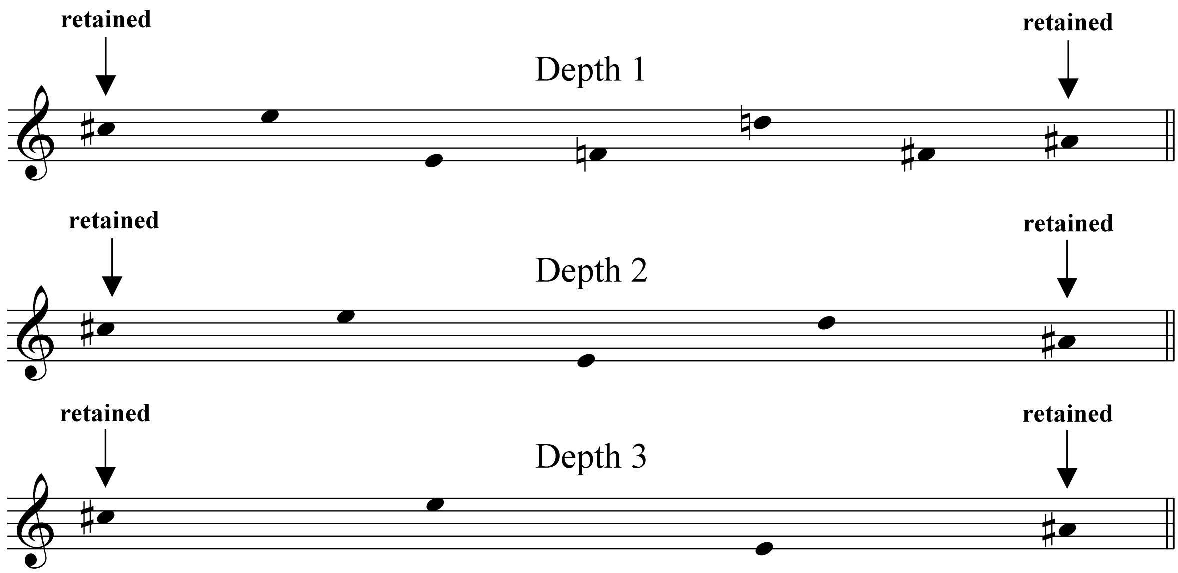
<ØØ241> <Ø2**4**15> <24**1**50> <41**5**06> <15063> <5063Ø> <063ØØ>

Prune marked cps (4, 1, 5): <2063>

Renumber retained cps: <1032>

As indicated earlier, the 5-window algorithm is reiterable. For example, the resulting cseg can be subject to the 5-window algorithm, and further iteration may produce smaller and smaller csegs, up to a certain point; every different cseg will reach a reduction limit with the 5-window algorithm, depending on the number of reiterations that continue to prune additional cps. Figure 3.15 illustrates the reiterative application of the 5-window algorithm on the Schoenberg phrase with the retention of the first and last cps. Each application of the algorithm gives a level of depth, indicated above the staves.

Figure 3.15. Recursive Application of the 5-Window Algorithm



It is also possible to indicate the pruned cps with small note heads, as shown in Figure 3.16. This is particularly useful as it provides an easy and immediate reference and a better sense of the context for the "deeper" notes.[87] Even better, it is also possible to present the entire reduction on a single staff by using different levels of beaming, as illustrated in Figure 3.17. Here, the total number of circles (i.e. intersection of stems and beams) indicates the depth level (i.e. D1, D2, D3, etc.) for each pitch. The numerals located at the leftmost portion of the figure indicate that all three depth levels involve a 5-window algorithm.

[87] Note that in this notation the retained pitches in one depth level are not vertically aligned with the corresponding pitches in the next depth level since the temporal distance between the cps is not essential to our present discussion. The issue of attack-points, durations, duration contours, and temporality in general, will be addressed in detail in Chapter 7.

Figure 3.16. Notation for Pruned cps

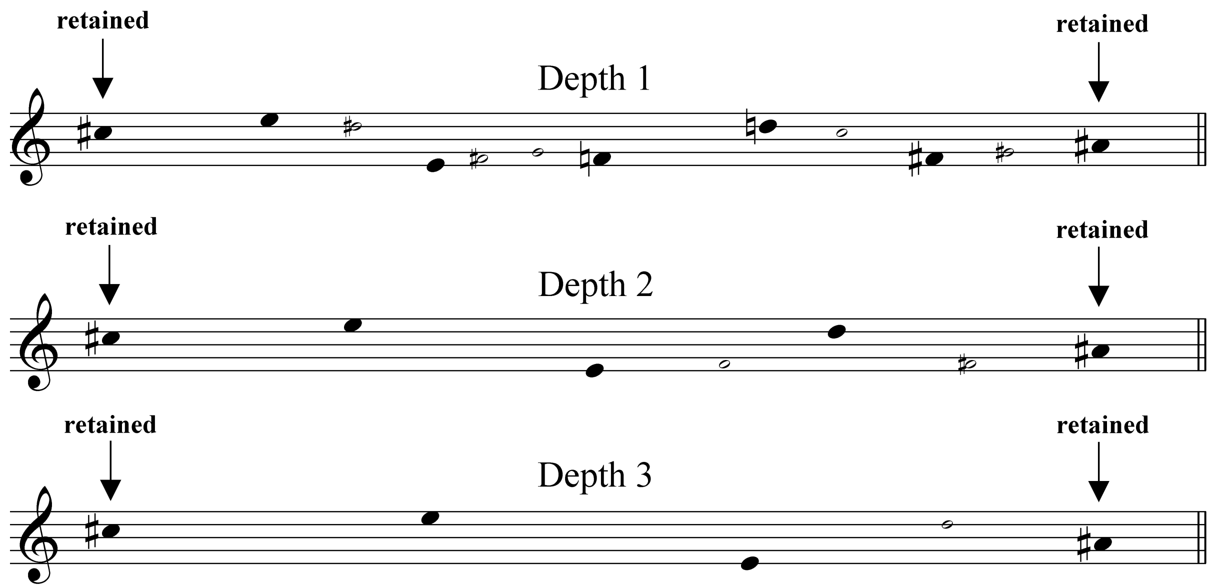
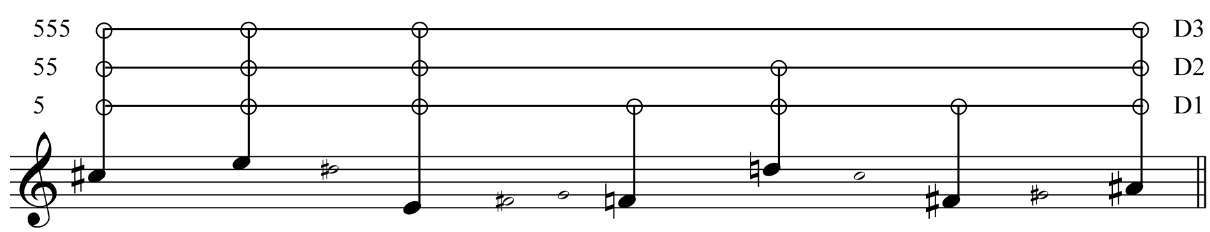


Figure 3.17. Single Staff Representation



It is important to note that an application of the 5-window algorithm could occasionally be followed by the 3-window algorithm, though this is not always possible. On Figure 3.17, for instance, Depth 1, which resulted from an application of the 5-window algorithm to the original segment, contains a "passing cp," namely F4, and at this point the analyst has two choices: to continue the reduction with either a 3-window or a 5-window algorithm. By contrast, depth 2, which resulted from two successive applications of the 5-window algorithm, cannot be reduced further by the 3-window algorithm since there are no passing cps. In other words, an application of the 3-window algorithm to this depth level would not prune any cps.

We noted that the first pass of the 5-window algorithm prunes both F#4 and G4, while the 3-window would prune only F#4. In principle, we will adopt the "smallest window-size" approach, which helps to differentiate the hierarchical status of pitches in as detailed a manner as possible. In other words, we will choose the smallest window-size possible for each depth level to continue further reduction. According to this principle, the 5-window algorithm should be used only if a segment (at any depth level) does not contain any passing cps, rendering the 3-window algorithm without effect. Rarely, we may want to use a larger window size in order to reduce a segment faster, to highlight an analytical observation, or to obtain a specific cseg cardinality. We will soon see that an application of 3-window, an application of 3-window followed by a 5-window, and an application of 5-window alone on the same segment may result in different cardinality csegs.

The following section will address the methodological complications arising from the issue of repeating cps. The discussion will be divided into two subsections, namely "successive" repeating cps in the 3-window and 5-window algorithms and "non-successive" repeating cps in the 5-window algorithm.

Segments with Repeating cps

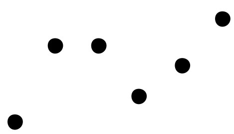
Successive Repeating cps

Let us consider the application of the 3-window algorithm on a hypothetical cseg <033124>, which includes successive identical cps. In such situations, Marvin and Laprade (1987), and Morris (1993) propose to prune one of the successive repeating cps without indicating which one of them should be pruned.[88] In the algorithm proposed by Morris, regarding the repeating pitches it is simply stated to “flag only one of them” (p.212, step 6). An application of Morris’s algorithm on this cseg prunes each of the repeating cps in a different depth level, as illustrated in Figure 3.18. Thus, one of the cp 3s is accepted to be hierarchically more salient than the other based solely on an arbitrary decision. This raises an important question regarding the methodology: if we can arbitrarily choose the cp we retain, which indicates that there is no apparent difference between the two, then why are we to assume one of the cps should be present in a deeper level than the other? Here the proposed approach for this specific context, in which both of the repeating cps are “passing” in the Max-List, is to prune both cps at the same time/same depth level. The formalism regarding at which stage the cps should be pruned will be presented shortly.

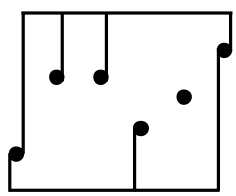
[88] At this point it is important to reiterate that at the first stage of his algorithm, Morris treats both of the repeating cps as maximum/minimum. His definition of maximum and minimum clearly articulates that “given three adjacent pitches in a contour, if the second is higher [or lower] than or equal to the others it is a *maximum* [or minimum].”

Figure 3.18. Application of Morris's algorithm on cseg <033124>

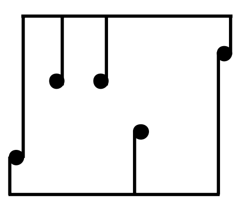
Let C = <033124>
graph:



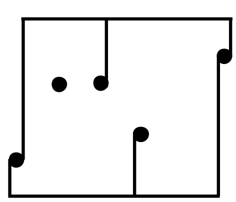
START:
steps 1 and 2: Upper beams show flagged max-list; lower beams show flagged min-list.



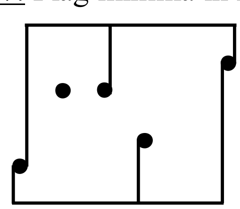
step 3: Not all pitches are flagged.
step 4: Delete 3rd (unflagged) pitch in min-list.



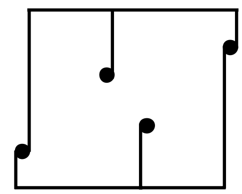
step 5: N = 1.
step 6: Flag maxima in max-list and flag either of the repeated cps in max-list.



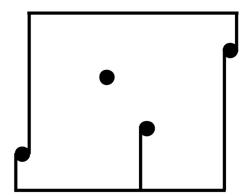
step 7: Flag minima in min-list.



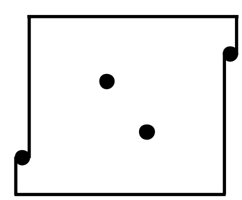
step 8: Go to step 3.
step 3: Not all pitches are flagged.
step 4: Delete unflagged pitches.



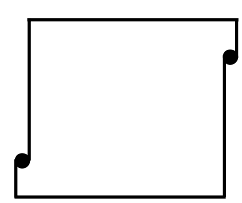
step 5: N = 2.
step 6: Flag maxima in max-list.



step 7: Flag minima in min-list.



step 8: Go to step 3.
step 3: Not all pitches are flagged.
step 4: Delete unflagged pitches.



step 5: N = 3.
steps 6 and 7: All pitches are flagged.
step 8: Go to step 3.
step 9: END.

Now let us look at another cseg, <03312>, which poses a certain type of challenge since cp 3 is the maximum in the segment. At this point, the method proposed above (i.e. pruning both cps simultaneously) does not work as we must retain one of them because it is a maximum. Arbitrary decision making could very well be adopted for these kinds of situations following Marvin and Laprade, and Morris. However, this approach results in complications once parameters other than pitch are included in the analysis, as we shall see in the following chapters. For instance in cseg <03312>, pruning the former cp instead of the latter would potentially result in a different duration contour of the segment. We will deal with duration contours in Chapter 7, but for now suffice it to say that the choice does indeed affect the result in non-pitch contours.

A criterion for retaining one of the two identical cps is difficult to assert. One way of setting such a criterion is to look for non-pitch domains. For example, one could argue that a change in texture or dynamic marking, for instance, could serve as a "tie-breaker." This is a completely valid approach; nevertheless, it would be ideal to solve the problem within the domain since there are various pitfalls in resorting to other domains: for example, both cps could have identical non-pitch parameters (i.e. both *mf*, both eighth-note, etc.), or each cp could be prominent in different parameters (i.e. one cp is assigned a higher dynamic marking, but the other is marked by a textural or timbral change, etc.).

From a purely temporal-pitch perspective, the only difference between the two cp 3s is their order-positions (i.e. one precedes and one succeeds the other). Thus, if we are to make a choice without resorting to other parameters, the criterion must be based on their temporal order, since it is the only difference between otherwise two identical cps. Considering the temporal order in which they appear, it is more viable to retain the initial cp

3 (op 1)[89] since at that time point, the listener's attention is drawn to a new pitch. On the contrary, the latter cp 3 (op 2) presents no change whatsoever. Needless to say, musical accent or a substantial change in any parameter is aurally more prominent (in terms of attention, and thus, memory) than the absence of change, which is referred to as "habituation" or "adaptation response" in psychology.[90] Phenomenological and cognitive issues concerning memory, forward- and backward-hearing, etc., which are pertinent to the issue at hand, will be further discussed in Chapter 5.

Henceforth, at any given stage of the reduction (whether 3-window or 5-window), when a segment contains two successive repeating cps, the second cp will be removed, if they are maxima/minima and both cps will be removed if they are not maxima/minima. Note that as a general rule, the first and last cps of a segment are always to be retained and if one of the repeating cps happens to be the first or last cp of the segment, the other cp should be pruned.

At this point, it is important to note that a contour may contain two successive repeating cps which collectively have a passing function. For example, cseg <01132> can be considered to contain two passing cps between cp 0 and cp 3. Following the fundamental premise of the 3-window algorithm—pruning the passing cps while maintaining the overall shape of the contour—both of the cp 1s can be pruned at the initial stage of the reduction

---

[89] Note that order positions are indicated by integers 0 to n–1. For example, the first entry in a cseg has an order position of 0, the second entry in a cseg has an order position 1, and so forth.

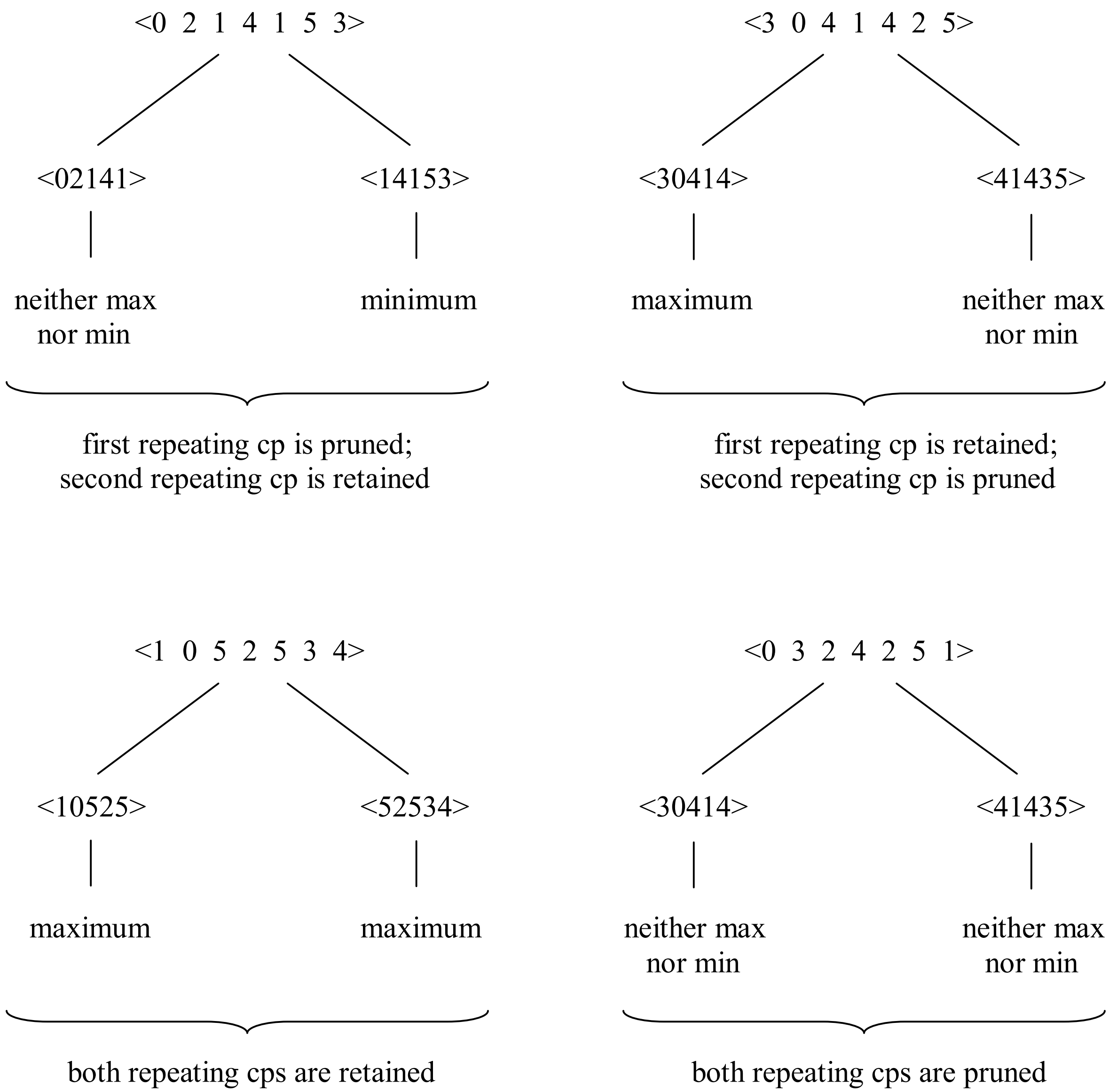
[90] See, for instance, Bernard Baars, *A Cognitive Theory of Consciousness* (New York: Oxford University Press, 1988); Fred Rieke, David Warland, Rob de Ruyter van Steveninck, and William Bialek, *Spikes: Exploring the Neural Code* (Cambridge: MIT Press, 1997); Michael Davis and M. David Egger, "Habituation and Sensitization in Vertebrates," in *Encyclopedia of Learning and Memory*, ed. Larry R. Squire (New York: Macmillan Publishing, 1992), 237-241.

since they can be understood as passing between cps 0 and 3. This approach, however, has some formal drawbacks as we will discuss later on.

Non-Successive Repeating cps

Non-successive repeating cps are of no concern in the 3-window algorithm since the middle cp is always adjacent to the other two cps within the window. However, it is possible to have non-successive repeating cps within a 5-window. In such cases, retaining the first of the non-successive repeating cps does not follow the rationale explained previously (with regard to adjacent repeating cps) since there is always a change in pitch between non-successive repeated cps. Thus, in principle we will retain non-successive repetitions unless they are neither maxima nor minima. For example, consider applying the 5-window algorithm on cseg <0214153>. Here, the 5-window <02141> prunes the former cp 1 since it is neither maximum (because of cps 2 and 4) nor minimum (because of cp 0). On the other hand, the 5-window <14153> retains the second cp 1 since it is a minimum. Thus, an application of the 5-window algorithm on cseg <0214153> prunes the first cp 1 and retains the second cp 1. Similarly, an application of the 5-window algorithm on cseg <3041425> prunes the second cp 4 and retains the first cp 4. Lastly, consider csegs <1052534> and <0324251>. Here, both of the repeating cps are retained in the former case and pruned in the latter case. These examples are illustrated in Figure 3.19.

Figure 3.19. Examples for Non-Successive Repeating cps



It is important to note that two non-successive cps, which are retained in one depth level, may become successive in the next level under the 5-window algorithm. For example, an application of the 5-window algorithm on cseg <0201020> would remove cp 1 and place the inner non-successive cp 0s next to one another in the following depth level since the resulting cseg would be <020020>=<010010>. This cseg could be further reduced to <01010> (an M-shaped contour) by pruning the successive repeating cps via either the 3- or the 5-window algorithm.

This approach to retaining and pruning non-successive cps suggests that <01010> cannot be reduced any further. This outcome is desirable since a reduction of an M-shaped contour to an inverted V-shaped contour (by pruning the interior cp 0) is counterintuitive given that M and W are the iconic forms of these contours, and those shapes should be included among the basic ones. Note that this approach is also in agreement with the refinement to Morris's algorithm proposed by Schultz. The only disparity between the two approaches is that Schultz does not allow, for example, cseg <0101010> to be a background (i.e. final depth level), or prime contour, whereas the approach taken in this study allows this contour to be a prime contour. In any case, situations such as these are uncommon in post-tonal music.

At this point, it is worth mentioning a conceptual distinction between the 3-window and the 5-window algorithms in the context of repeating cps. The 3-window algorithm is strictly ordered in the sense that swapping the outer ops within the 3-window would result in a different reduction. For example, the medial cp of a hypothetical window <332> would be pruned since it is the second of the repeating cps but a swap between the outer cps would result in a window of <233> and thus the retention of the medial cp. On the other hand, swapping the initial and the last ops within the 5-window would never affect the reduction. For example, the medial cp of the window <30321> would be retained even after the outer cps are swapped. Similarly, the medial cp of the window <20231> would be pruned regardless of the order of outer cps. Since the order in which the outer cps presented is of no consequence to the reduction, the 5-window can be understood to be a partially ordered set as opposed to the strictly ordered 3-window.

A formal definition of the algorithms including the incorporation of the successive and non-successive repeating cps discussed above will be presented in the next section.

Formal Definitions of the Window Algorithms

In most cases, applications of the algorithms are straightforward. Nonetheless, in certain situations (particularly, where repeating cps are involved) the applications may be less clear. In this section we will formalize the 3-window and the 5-window algorithms to account for all possible scenarios. Let us begin with a formal definition of the 3-window algorithm.

The 3-Window Algorithm

Let $<x_0, x_1, x_2, \ldots x_n>$ be a cseg

For each j from 0 to n, consider the cp x_j

Let $i = j-1$, let $k = j+1$, and let $l = j+2$[91]

If $j = n-1$, let $x_l = x_i$[92]

Let $x_j = [\max3]$ if $(x_j > x_i) \wedge (x_j \geq x_k)$

[91] The inclusion of xl, which is not adjacent to xj, enables us to prune the first of the two successive repeating cps which together have a passing function (i.e. cseg <0113>) as discussed earlier.

[92] Note that if j = n–1, then xk will be the last cp and therefore xl would be beyond the last cp. Here, we arbitrarily set xl to be equal to xi to nullify the effect of out-of-bounds, non-existing xl on the decision making process without altering the outcome.

Let x_j = [min3][93] if $(x_j < x_i) \wedge (x_j \leq x_k)$[94]

1. Let j = 0; mark x_j for retention and go to step 5.

2. If j = n, mark x_j for retention and go to step 6.

3. If $x_j = x_k$, then mark x_j for retention iff $(k \neq n) \wedge (((x_j > x_i) \wedge (x_j > x_l)) \vee ((x_j < x_i) \wedge (x_j < x_l))))$.[95]

4. If $x_j \neq x_k$, and if x_j = [max3] $\vee$ x_j = [min3], then mark x_j for retention.

5. Let j = j+1; go to step 2.

6. Delete every cp that has not been marked for retention.

7. "Translate" the remaining cps to obtain the reduced cseg.

Here, the first step starts the algorithm by assigning a value of 0 to j and retains x_j since it is the first cp.[96] The second step retains the last cp and directs us to the sixth step where the evaluation of cps has completed. The third step tells us to mark x_j for retention if it is equal to x_k—following the principle of retaining the first of the two successively repeating

---

[93] The terms "max3" and "min3" refer to a central cp being maximum and minimum within a width-3 window.
[94] Following our discussion above, the second of the successively repeating cps is always pruned. Thus, xj is not considered a maximum or minimum when it is equal to xi. However, given the other conditions are met (which will be presented in the discussion of the 5-window algorithm) xj can be a maximum or minimum even if it is equal to xh, xk, or xl.
[95] This entry prunes the repeated passing cps (i.e. first cp 1 in cseg <01132>) and also prevents the retention of xj when xk is the last cp. A detailed discussion of the repeated passing cps is to follow shortly. Note that this step can also be formulated as "If (xj = xk) ∧ ((k ≠ n) ∧ (((xj > xi) ∧ (xj > xl)) ∨ ((xj < xi) ∧ (xj < xl)))), then mark xj for retention."
[96] Let us reiterate that here we follow Morris's reduction principle in which the first and last cps are considered both maxima and minima, and thus, are retained.

cps—unless x_k is the last pitch or x_j is a repeated passing cp. Note that in the definition of [max3] and [min3], x_j is given as strictly greater than or less than x_i. This is because when x_j is equal to x_i, it should be marked for pruning (and not considered as a maximum or a minimum) since it is the second of the successive repeating cps.[97] The issue of repeated passing cps, which involves x_l in the formula, is discussed below.

The fourth step indicates that if x_j is not equal to x_k and is a maximum or minimum (which involves it not being equal to x_i, as discussed above), it should be marked for retention. The fifth step can be reached only if x_j is not the last cp. This step increments j by one and restarts the evaluation process for the incremented x_j from the second step. The sixth and seventh steps end the algorithm by deleting all cps that are not marked for retention and translating the retained cps to obtain the reduced cseg.

Note that if there are more than 2 identical cps in a row—a rarity in post-tonal music—that are passing, it is possible to revise step 3 by including x_m (for 3 identical cps), x_n (for 4 identical cps), etc. Although this solution would accommodate any number of repeating cps, a different type of problem, which is conceptual in essence, still persists: an evaluation of x_i and x_l imply a window size larger than 3.

One possible solution to this problem is to simply require contours to be already pruned of all immediate repetitions before applying the algorithms. However, this solution does not reflect the moving, phenomenological nature of the window. In addition, even after

[97] In effect, this means that we accept only the first of the two successive repeating cps as the maximum/minimum. Exceptions include the second cp being the last note of the segment and the first cp being a repeated passing cp.

pruning all successive repeating cps, it is possible to encounter other successive repeating cps at deeper levels, as we will see in the following chapters. In other words, pruning all successive repetitions at the beginning does not ensure that there will not be successive repetitions later on in the reduction process (if the 5-window algorithm is used).

Another potential solution is to use the 5-window algorithm when we come across "repeated passing cps." However, because the window will expand not only towards the succeeding repeated cps but also towards the preceding non-repeated cps, we would have to take the preceding cps into account only to be able to reduce the repeated passing cps. This, in effect, means that a cp that is not passing may be pruned; since the premise of the 3-window algorithm is to prune only passing cps, it is not viable to employ the 5-window algorithm in order to prune repeated passing cps.

Even though the implementation of repeated passing cps to the algorithm enlarges the window width temporarily, it provides the benefit of reducing such cps without resorting to two or more consecutive applications of the 3-window algorithm, and more importantly, it reflects the true nature of 3-window algorithms that is the deletion of passing cps and reducing a contour without changing its overall shape.

Based on the formal algorithm definition presented above, it is possible to construct a flowchart for the entire algorithm, as illustrated in Figure 3.20. The flowchart provides an easier way to apply the algorithm: each question requires a binary response (yes/no) and each answer leads to another question or to a conclusion of whether the cp in question is to be marked for retention. Each decision is followed by an increment of j by 1 ("let j = j + 1") until the last cp has been evaluated. In addition to the ease in applying the algorithm, it suggests a model for the cognitive process in evaluating the cps.

Following the conventional symbols used in flowcharts, ovals indicate the start and stop points, parallelograms indicate the input and output, diamonds indicate a decision point, rectangles indicate a process or action, and arrows indicate the flow between the symbols.

Note that applications of the algorithm on various musical passages will be presented in the following chapter.

Figure 3.20. Flowchart for the 3-Window Algorithm (Top Half)

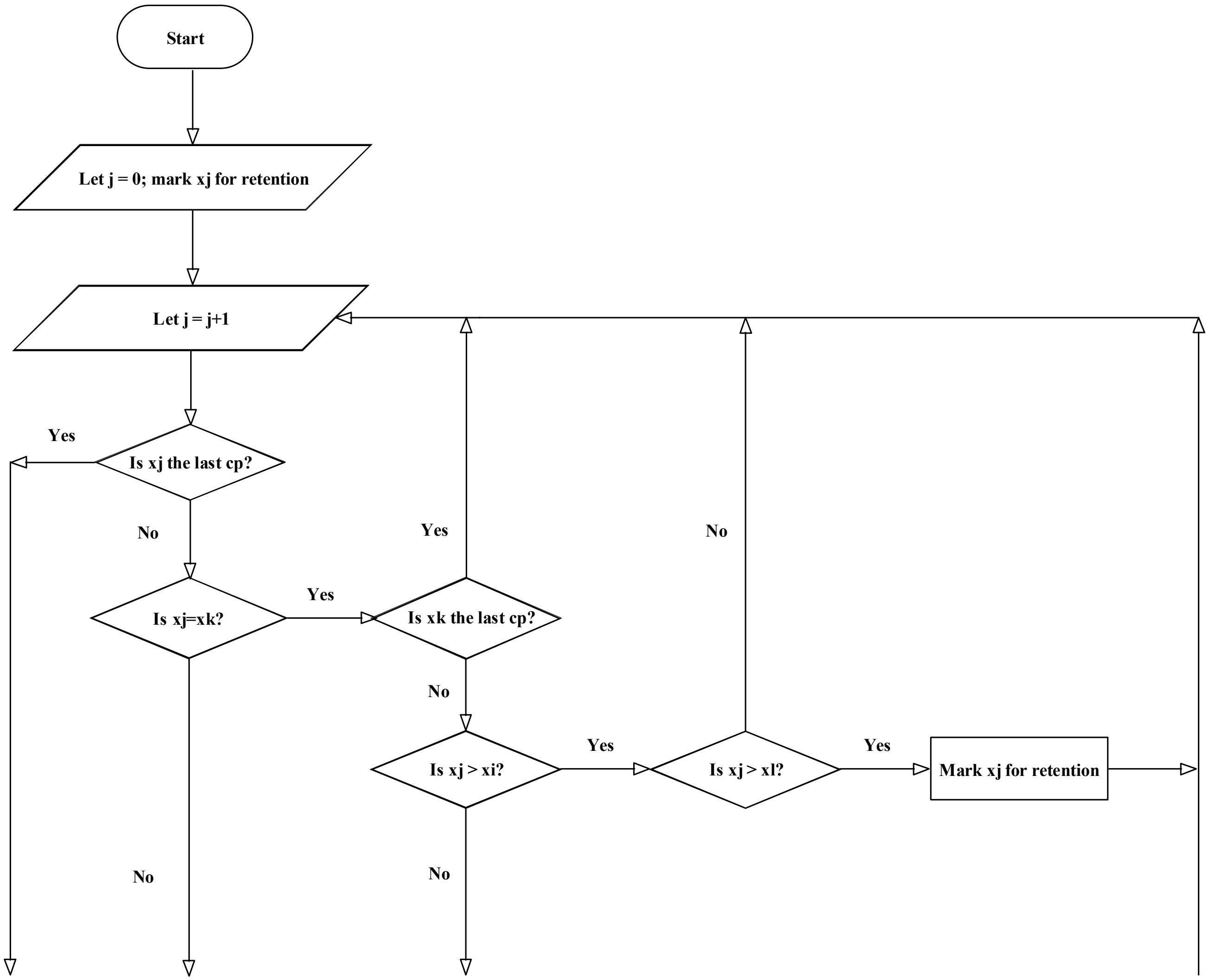
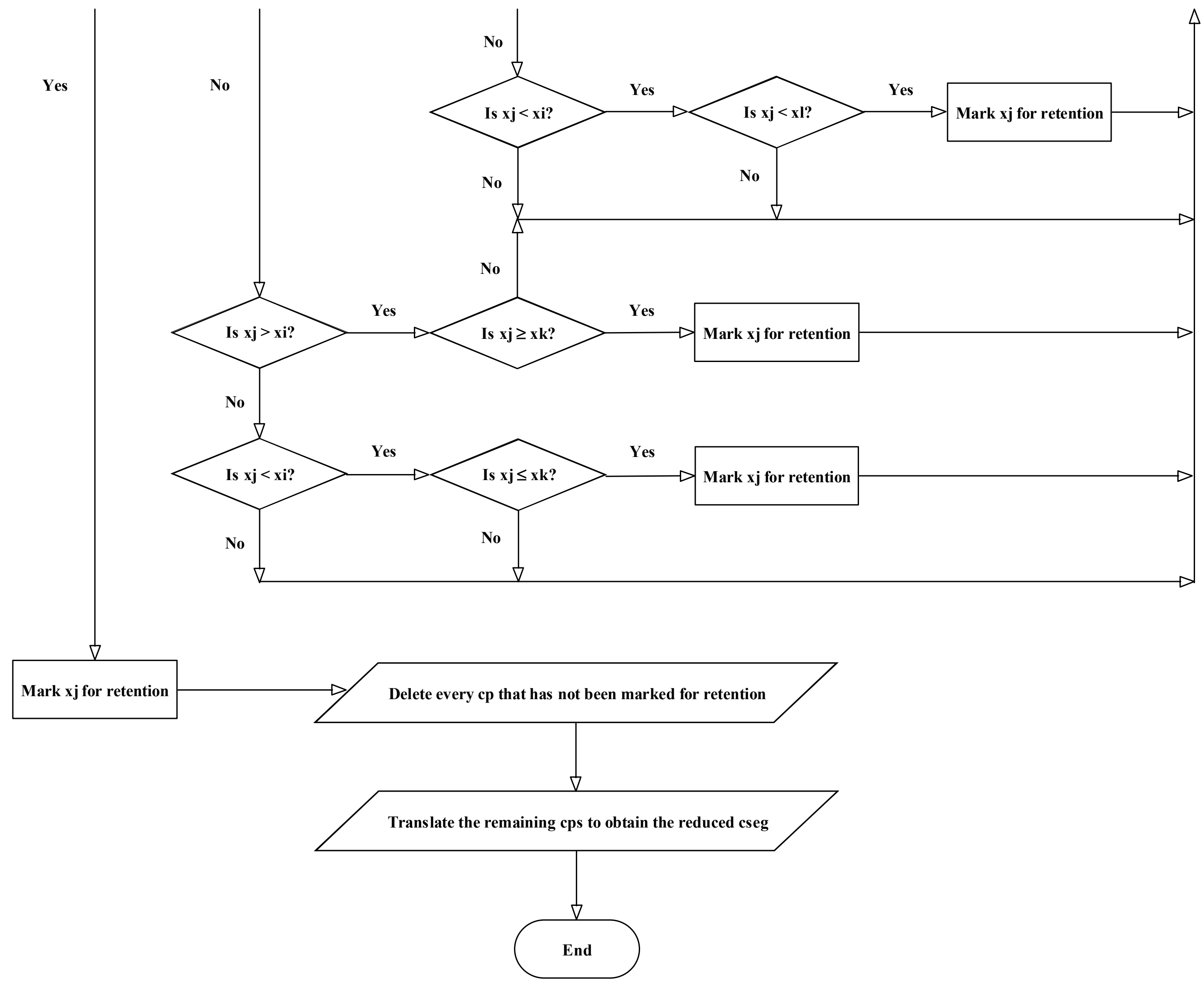


Figure 3.20 (cont.) Flowchart for the 3-Window Algorithm (Bottom Half)



The 5-Window Algorithm

A formal definition of the 5-window algorithm that is similar to the one presented in the discussion of the 3-window algorithm is given below.

Let $<x_0, x_1, x_2, \ldots x_n>$ be a cseg

For each j from 0 to n, consider the cp x_j

Let $i = j–1$, and let $k = j+1$

Let $h = j–2$, and let $l = j+2$

If $j = 1$, let $x_h = x_l$[98]

If $j = n–1$, let $x_l = x_h$

Let $x_j = [max5]$ if $(x_j > x_i) \wedge (x_j \geq x_h) \wedge (x_j \geq x_k) \wedge (x_j \geq x_l)$

Let $x_j = [min5]$ if $(x_j < x_i) \wedge (x_j \leq x_h) \wedge (x_j \leq x_k) \wedge (x_j \leq x_l)$

1. Let $j = 0$; mark x_j for retention and go to step 5.

2. If $j = n$, mark x_j for retention and go to step 6.

3. If $x_j = x_k$, then mark x_j for retention iff $(x_k \neq x_n) \wedge (x_j = [max5] \vee [min5])$.[99]

4. If $x_j \neq x_k$, and if $x_j = [max5] \vee x_j = [min5]$, then mark x_j for retention.

5. Let $j = j+1$; go to step 2.

[98] This statement and the following statement nullify the effect of out-of-bounds, non-existing xh and xl on the decision making process. The partially ordered 5-window algorithm, discussed in the previous section allows to swap xh and xl without altering the evaluation process.
[99] Note that this step can also be formulated as "If $(xj = xk) \wedge ((k \neq n) \wedge (xj = [max\ 5] \vee [min5]))$, then mark xj for retention.

6. Delete every cp that has not been marked for retention.

7. "Translate" the remaining cps to obtain the reduced cseg.

The principles behind the 5-window algorithm are very similar to those behind the 3-window algorithm. Here, just like the 3-window algorithm, the algorithm begins by assigning a value of 0 to j and retains x_j since it is the first cp, and the following step retains the last cp. The third step, however, is different. When x_j is equal to x_k, not only should x_k not be the last cp but also x_j should be greater (or smaller) than x_i and greater (or smaller) than or equal to both x_h and x_l. This point deserves a more detailed discussion. The rationale behind x_j being greater or smaller than x_i, and x_k not being the last cp is presented in the discussion of the 3-window algorithm. The greater or equal (≥) and smaller or equal (≤) cases for other cps (x_h, x_k, and x_l) ensure that the M-shaped and W-shaped contours, which we discussed earlier, are to be maintained by the algorithm. For example, let us take the cseg <13230>. An application of the 5-window algorithm on the first cp 3 marks it for retention, following "$x_j \geq x_l$." Similarly, the second cp 3 is retained following "$x_j \geq x_h$." In both cases, these highest cps would be pruned without the equality. Now let us take a cseg in which $x_j = x_h = x_k = x_l$. Here, according to the algorithm x_j still has the potential to be a maximum or minimum. Take the 5-window <Ø1333> from the cseg <13333240>, for example.[100] The application of the 5-window algorithm retains the first cp 3 as a maximum but prunes the rest of the cp 3s since

[100] Note that in this example the out-of-bounds xh equals xl (cp 3) following the statement "If j = 1, let xh = xl."

the algorithm states “$x_j > x_i$” rather than “$x_j \geq x_i$.” As a result, all of the successive repeating cps, with the exception of the very first one, are pruned. Similar to the 3-window algorithm, the fourth step indicates that if x_j is not equal to x_k and is [max5] or [min5], it should be marked for retention. The remaining steps in the 5-window algorithm are identical to the corresponding steps in the 3-window algorithm. A flowchart for the 5-window algorithm is illustrated in Figure 3.21.[101]

[101] Note that it would be useful to implement the algorithms in a programming language such as Pascal. However, this is beyond the scope of the present study and should await future projects.

Figure 3.21. Flowchart for the 5-Window Algorithm (Top Half)

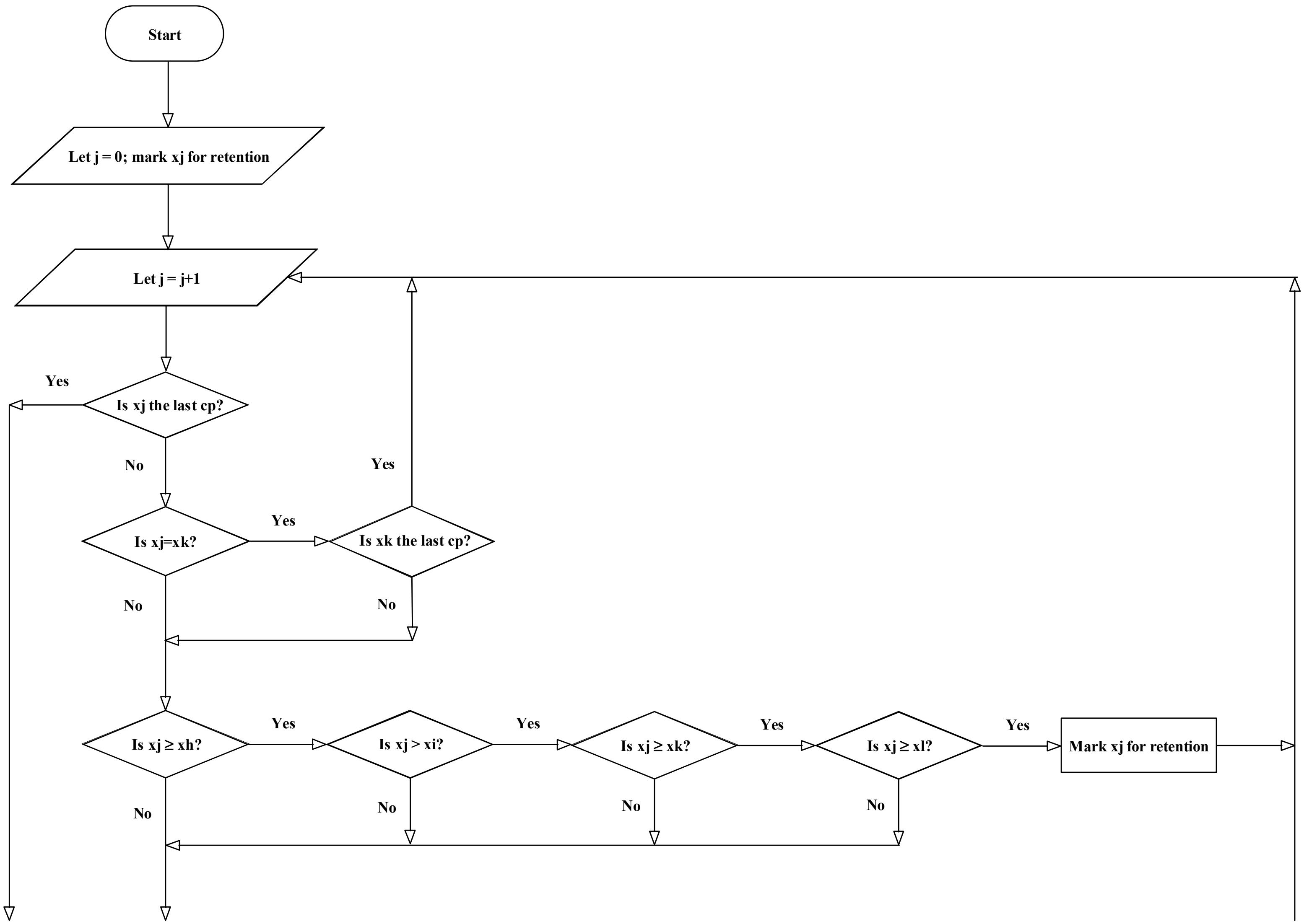
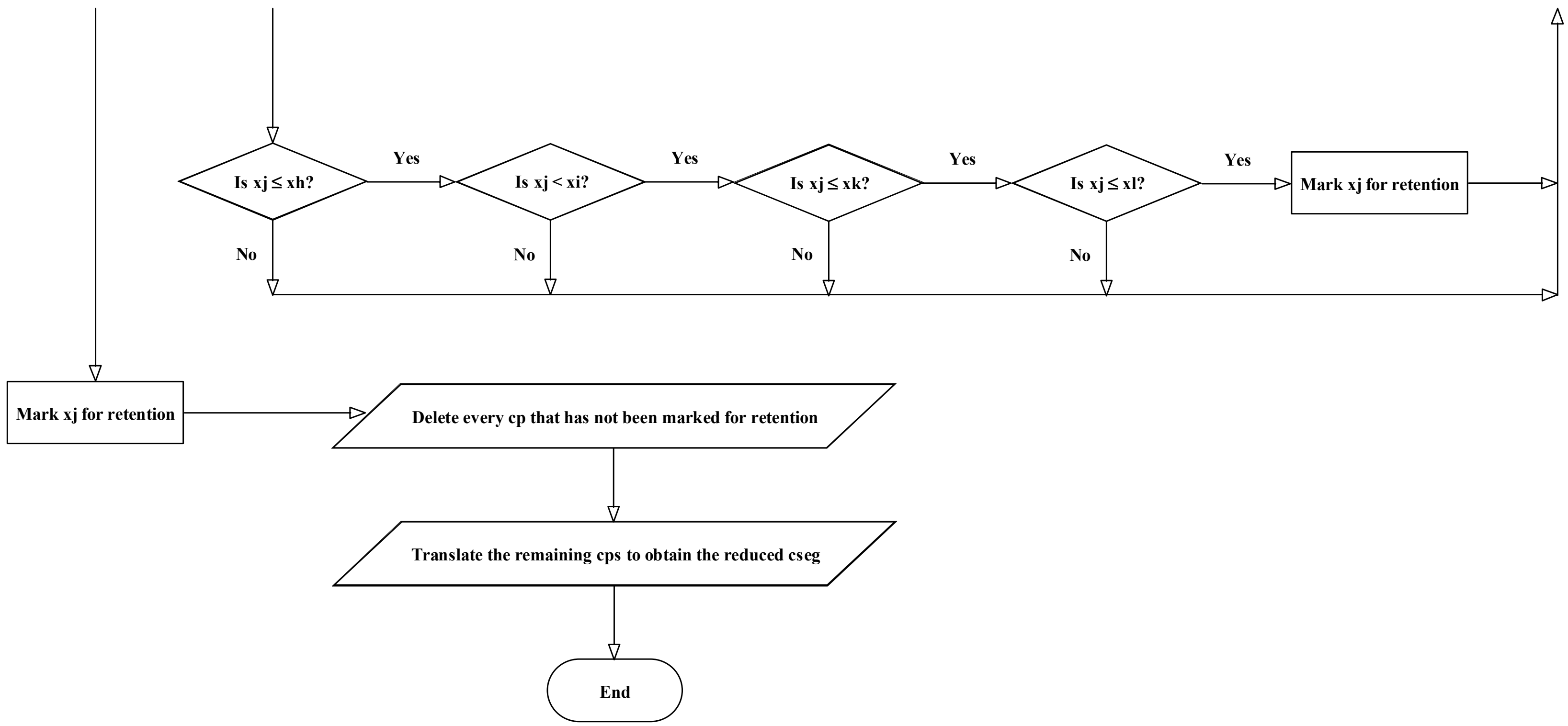


Figure 3.21 (cont.) Flowchart for the 5-Window Algorithm (Bottom Half)



In this flowchart, the flow following [max5] (sixth row) involves a point of potential confusion and thus, requires a detailed discussion. At first glance one could ask why all of the arrows following “No” go to “Is $x_j \leq x_h$?” and argue that it could be followed by “Is $x_j < x_i$?” rather than “Is $x_j \leq x_h$?” This seems reasonable: there is no point in asking whether “x_j is $\leq x_h$” since we already know the answer from the previous question, “Is $x_j \geq x_h$?” For example, if the answer to this question is “Yes” then we know that “$x_j \geq x_h$” and if the answer is “No” then we know that $x_i < x_h$.” However, skipping the question of “Is $x_j \leq x_h$?” prevents us from potentially answering this question as “No.” As an example, let us assume a cseg in which $x_j > x_h$ but $x_j < x_i$, $x_j < x_k$, and $x_j < x_l$ (i.e. <02143>). Now assuming that we follow this new flow, we will first respond to “Is $x_j \geq x_h$?” as “Yes” and then to “Is $x_j > x_i$?” as “No.” At this point if we continue with “Is $x_j < x_i$?” (skipping “Is $x_j \leq x_h$?”) we would continue all the way to “Mark for Retention” although x_j (cp 1) is not a minimum because it is higher than x_h (cp 0). However, if we continue with “Is $x_j \leq x_h$?” (which is the case in the flowchart), then we can answer the question as “No” and not mark it for retention (and eventually prune it). Thus, the negation of [max5] in the chart continues with the question of “Is $x_j \leq x_h$?” to avoid a non-minimum cp to be marked for retention, as exemplified above.

Contour Reduction Functions

As is evident from the discussion above, the window algorithms can be applied successively in various combinations (i.e. a 3-window algorithm followed by a 5-window algorithm, a 5-window algorithm followed by a 3-window algorithm, a 5-window algorithm followed by another 5-window algorithm, etc.). In essence, any algorithm combination applied on a cseg can be understood to be a "Contour Reduction Function," which can be denoted by an uppercase R followed by one or more integers. Let x be a cseg, then:

R3(x) denotes the function that reduces x by applying the 3-window algorithm.

R5(x) denotes the function that reduces x by applying the 5-window algorithm.

R35(x) denotes the function that reduces x by applying the 3-window algorithm first, and reduces the output further by then applying the 5-window algorithm; i.e. R35(x) = R5(R3(x)).

R355(x) denotes the function that reduces x by applying the 3-window algorithm first, and reduces the output further by then applying the 5-window algorithm, twice in succession; i.e. R355(x) = R5(R5(R3(x))).

Similarly, R535(x) = R5(R3(R5(x))), and so forth.

The "depth" of a contour reduction function is equal to the number of successive window algorithms it involves. Thus, R3(x) and R5(x) both have depth 1, R35(x) and R55(x) both have depth 2, and so forth. Note that in contour reduction functions with depth of 2 or more, the order of the algorithms corresponds to the order of integers from left-to-right in the R-label.

As a hypothetical example let us imagine a segment, denoted by S, which is first reduced by a 3-window algorithm (corresponding to first depth level), followed by a 7-

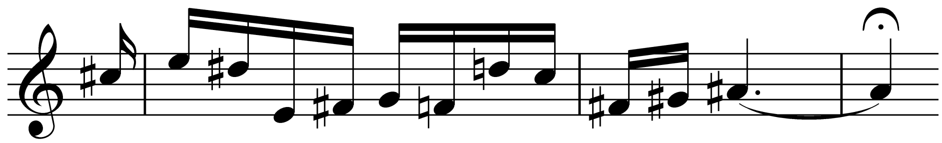
window algorithm[102] (corresponding to second depth level), a 5-window algorithm (corresponding to third depth level), and another 5-window algorithm (corresponding to fourth depth level). In this new notation, each depth level would be indicated by a different contour reduction function: R3(S), R37(S), R375(S), and R3755(S), respectively. Moreover, boldface could be used to indicate the final depth level. For example, assuming that it is the final depth level, the fourth depth level in this example would be notated in boldface: R375**5**(S). Henceforth, we will refer to the final depth level as "background" or "contour framework."

A Contour Representation of Depth Levels

Aside from the possibility of manipulating the cardinality of a given contour, the most important concept provided by the algorithms that are reiterable is the notion of "hierarchy." In this context, hierarchy could be defined in terms of the depth levels that result from the recursive nature of the algorithm: the more resistant a cp to pruning, the more hierarchically prominent it is. It is possible to associate a number with each note, indicating its depth with respect to a specific contour reduction function. Figure 3.22 shows the idea, on the Schoenberg phrase under the function R555. Note that the depth numbers (in this case, 0 through 3) are ordered and thus can be represented as a depth-level contour indicating the relative hierarchical prominence of each note. For this melody, the R555 depth-level contour is <330300120103>, as illustrated in Figure 3.22.

---

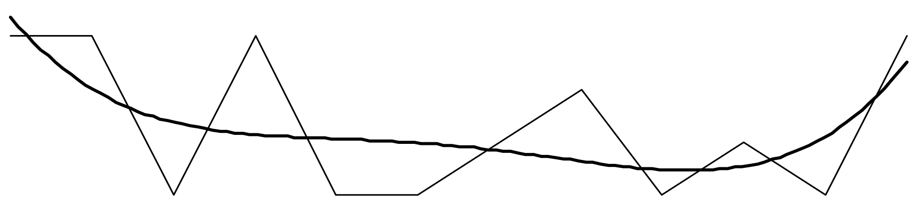
[102] Note that it is possible to employ algorithms that have larger window widths simply by enlarging the window-size and adapting the principles discussed in the 5-window algorithm.

Figure 3.22. Depth Level Contour of the Schoenberg Phrase under R555

Depth Level: < 3 3 0 3 0 0 1 2 0 1 0 3 >

The depth-level contour of the entire melody without any truncation offers an understanding of the overall cseg. As demonstrated in Figure 3.23, the depth-level contour of the melody clearly represents a general falling trend until the last cp. This trend is denoted by the trendline[103] (indicated by a thicker line) on the figure.

Figure 3.23. Visual Representation of the Depth Level Contour

Depth Level: <3 3 0 3 0 0 1 2 0 1 0 3>



The varying hierarchy contour is aurally most prominent at the boundary depth levels (i.e. depth level 0 and depth level 3). Depth level 0 indicates a cp that is not salient from a contour perspective since it is neither maximum nor minimum in its local surrounding. On

[103] Trendline can be described as a curve which provides the best fit to a series of data points. More formally, it is referred to as *best-fit linear regression line* in statistical analysis.

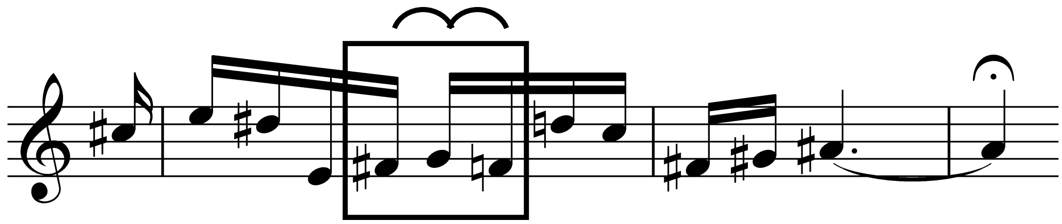
the contrary, the first and last cps are aurally striking as they mark the melodic frame. Similarly, the cps that are maximum and minimum with regard to the entire melody are significant in that they mark the highest and lowest pitches, forming a special type of accent. The intermediate levels of the contour hierarchy are aurally less immediate in comparison to the surface and background levels. Nonetheless, generally the distinction is clear enough to discern. For example, the D5 (depth 2) is aurally more salient than the following F#4 (depth 1). Since the preceding F4 is fairly proximate (in temporal terms) to F#4, it hampers the local minimum status of F#4 and attenuates its aural effect. D5, on the other hand, is temporally rather detached from the global maximum, E4, and thus, is likely to be heard as more significant in comparison to F#4.

It is important to note that it is possible to extend the "window-size" to 7 or even 9, and more. A wider window will filter more and more notes since the criteria for notes to qualify as maxima or minima become more selective. However, the larger the width of the window, the less the importance of local phenomena. Moreover, as the window width increases, the memory requirements increase for the listener. A discussion of memory and windows will be presented in Chapter 5.

Comparing the Window Algorithms and Morris's Reduction Algorithm

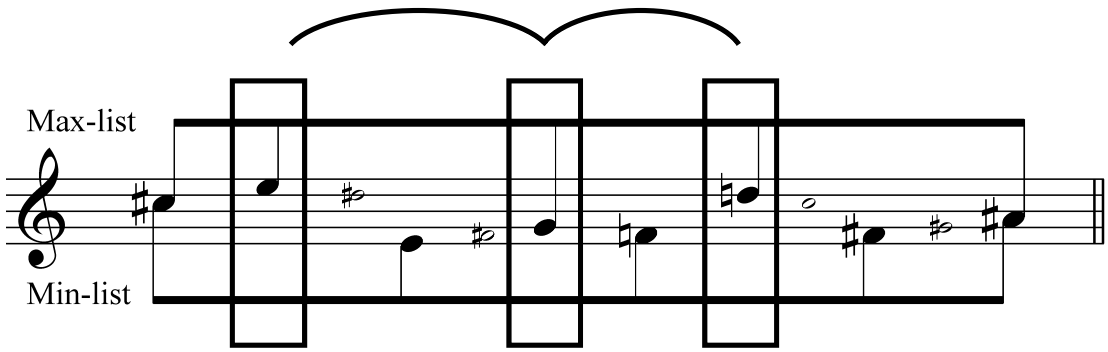
To demonstrate the similarities and differences between the reduction algorithms discussed above, let us take the 6th cp from the Schoenberg example, where G4 is the focal point. In the 3-window algorithm only <F#4-G4-F4> is taken into account in deciding whether G4 qualifies as a maximum or minimum, as illustrated in Figure 3.24.

Figure 3.24. Evaluation of G4 in 3-Window Algorithm



By contrast, Figure 3.25 shows that in stage 2 (i.e. post-beaming stage) of Morris's algorithm (corresponding to steps 6-8) not all cps that are evaluated are successive, due to separate Max-list and Min-list beaming. The surrounding F#4 and F4 do not belong to the Max-list and thus, to the same beam as does G4. In fact, F#4 is already pruned in the first stage (i.e. pre-beaming stage) of Morris's algorithm as indicated by the small note head, and as a result the pitches surrounding G4 in the second stage are E4 and F4. Nonetheless, since G4 is in the Max-list, the cps that belong to the same Max-list beam are not the immediate surrounding pitches, E4 and F4, but E5 and D5, as illustrated in Figure 3.25. In short, due to separate beaming, Morris's algorithm evaluates cps that are non-successive and temporally detached from G4 (i.e. E5 and D5) rather than the temporally immediate cps (i.e. E4 and F4) which are an integral part of our listening experience of G4. On the other hand, the 5-window algorithm takes all four surrounding pitches into account and provides a more localized representation of aural experience.

Figure 3.25. Morris's Contour Reduction Algorithm (Stage 2)



The two window algorithms offer distinct types of reduction. The 3-window algorithm retains the overall shape of a contour by pruning the passing cps; the 5-window algorithm reduces the local zigzags of a contour. These are important conceptual differences which are not explicitly apparent in Morris's algorithm.[104]

Moreover, the window algorithms provide a methodology to assist with repeated cps and to reduce the wedge-shaped contour type which is irreducible in Morris's algorithm. As indicated earlier, these two points are also discussed in a recent article by Schultz, which offers a different solution (by adding new steps to Morris's original algorithm).

Lastly, the contour reduction functions provide an opportunity to reduce a given contour by choosing different reduction paths (for example, R3**5**(S) as opposed to R5**5**(S)). Unlike Morris's reduction algorithm (and Schultz's refinement to it) this offers some analytical flexibility and an interpretative richness and plurality. Although we will be

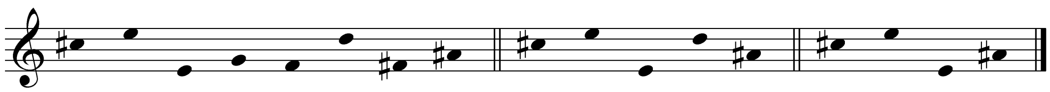
[104] Our partition of Morris's algorithm into two stages (i.e.: pre-beaming and post-beaming) at the beginning of the chapter is an attempt to make this distinction clearer.

following the "smallest window-size" approach in reducing contours for the most part, we will occasionally explore different reduction paths in order to highlight a particular analytical observation as we will see in the following chapters.

A comparison between the reduction algorithms operating on the Schoenberg phrase is illustrated in Figure 3.26.

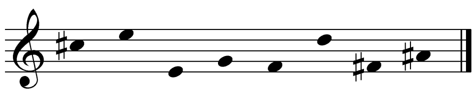
Figure 3.26. Comparison between the Algorithms (Schoenberg Phrase)

MORRIS



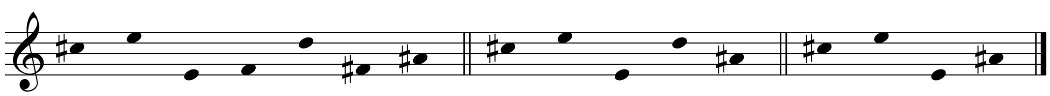
cseg	<5 7 0 3 1 6 2 4>	<3 4 0 2 1>	<2 3 0 1>
cardinality	8	5	4
depth	1	2	3

3-WINDOW (R3)



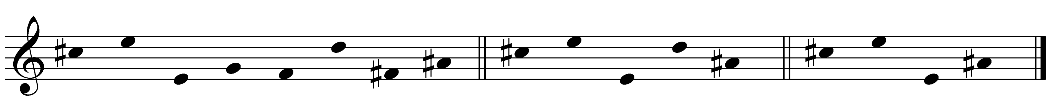
cseg	<5 7 0 3 1 6 2 4>
cardinality	8
depth	1

5-WINDOW (R555)



cseg	<4 6 0 1 5 2 3>	<3 4 0 2 1>	<2 3 0 1>
cardinality	7	5	4
depth	1	2	3

BOTH (R355)



cseg	<5 7 0 3 1 6 2 4>	<3 4 0 2 1>	<2 3 0 1>
cardinality	8	5	4
depth	1	2	3

In this figure, the first staff demonstrates the csegs from Morris's contour reduction algorithm for each depth level. The second and third staves exhibit the 3-window application and the 5-window application, respectively, on the segment, whereas the last staff illustrates a different contour reduction function in which the 3-window and 5-window are combined. The last two staves present two different reduction paths which eventually end with the same cseg in the contour framework.

It is interesting to observe that although the second and third applications of the 5-window algorithm (i.e. R55 and R55**5**) give the same output as Morris's algorithm, the results in each level of depth do not necessarily match: depth 1 of Morris's algorithm, which is identical to the 3-window algorithm (i.e. R3), is different than depth 1 of the 5-window algorithm (i.e. R5).

As indicated above, the results of Morris-Depth 1 and R3 are identical. Similarly, Morris-Depth 2, R55, and R35 are identical; so are Morris-Depth 3, R555, and R355; R5 is unique. This information can be formally represented by sets that contain the specific algorithms that give identical results:

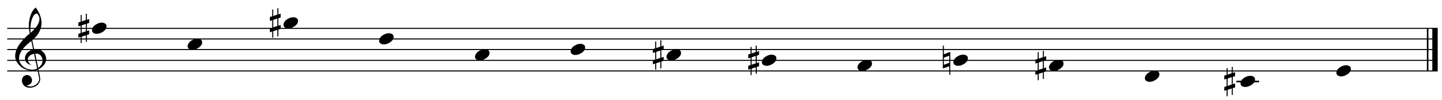
{R5} {M2, R55, R35} {M3, R555, R355}

Note that in this specific example, R55 and R35 result in the same reduced cseg, though this is not necessarily the case with all csegs. As is evident from R5, the window algorithms (or more specifically certain contour reduction functions) provide the analyst with a choice of depth, which is not possible with Morris's algorithm. In other words, in a special analytical context one could choose to employ a 5-window algorithm instead of a 3-window algorithm. This option provides numerous analytical advantages such as obtaining a specific contour cardinality, or highlighting a contextual observation.[105] Figure 3.27 illustrates this point with a hypothetical contour segment. Here, particular window algorithms, namely R5 and R53, reveal further depth levels which cannot be obtained by Morris's algorithm. These inner depth levels are analytically valuable since they provide the opportunity to differentiate between pitches that are hierarchically equivalent in Morris's algorithm. For example, in Morris's algorithm, C5, A4, and B4 are equally salient as they are pruned together at the same depth level. However, R5 and R35 reveal that C5 is more prominent than both A4 and B4, and A4 is more prominent than B4.

[105] We will see examples of this in the following chapters, where using the 5-window algorithm instead of the 3-window algorithm emphasizes a certain interpretation, which is prominent in the given analytical context.

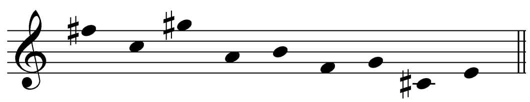
Figure 3.27. Comparison between the Algorithms (Hypothetical Segment)

Contour Segment



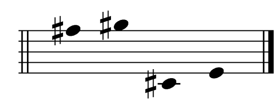
< 12 10 13 11 7 9 8 6 3 5 4 1 0 2 >

M1



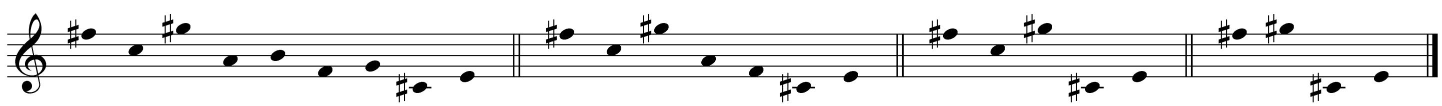
<7 6 8 4 5 2 3 0 1>

M2



<2 3 0 1>

R3 **R5** **R53** **R35**



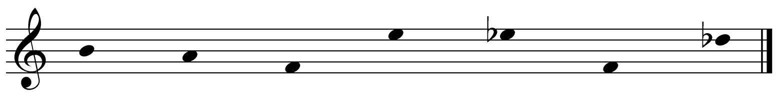
<7 6 8 4 5 2 3 0 1> <5 4 6 3 2 0 1> <3 2 4 0 1> <2 3 0 1>

cardinality **9** **7** **5** **4**

As indicated previously, it is also possible to extend the window width to 7 or even 9 cps, which could be referred to as 7-window and 9-window algorithms, respectively. The employment of such algorithms would result in an acceleration of the process and an increase in pruning per depth level. Thus, in addition to the analytical benefits mentioned above, they would also provide the opportunity to reduce a segment more quickly. However, these algorithms are more demanding on the listener's cognitive faculties, especially memory, and a flow-chart representation of the 7- or 9-window algorithm would show the cognitive overload on the listener's part. In any case, the analytical choice of using window widths larger than 5 cps is available to the analyst, especially if her focus is on the contour framework and not on the intermediate depth levels, as we shall see in Chapter 8.

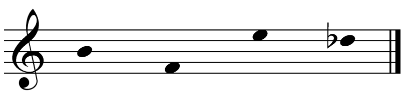
Lastly, let us reiterate that the window algorithms retain non-successive repeating maxima and minima, and preserve the M- and W-shaped contours at the framework level. In Morris's algorithm, such segments are ultimately reduced to an (inverted) N-shaped or an (inverted) V-shaped contour. A framework level comparison of the algorithms is illustrated in Figure 3.28, in which cseg <2105403> is reduced to an (inverted) N-shaped contour by Morris's algorithm and a W-shaped contour by the window algorithms.

Figure 3.28. Framework Comparison between the Algorithms



< 2 1 0 5 4 0 3 >

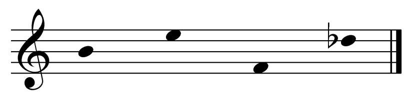
M2
(FRAMEWORK)



<1 0 3 2>

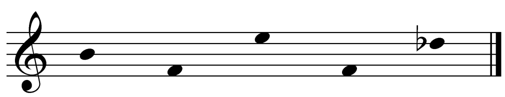
OR

M2
(FRAMEWORK)



<1 3 0 2>

R3
(FRAMEWORK)



<1 0 3 0 2>

A gallery of excerpts by a variety of 20th century composers will be subject to the employment of the newly introduced window algorithms in the following chapter and a more detailed analytical application of the algorithms will be presented in Chapter 6.

CHAPTER 4

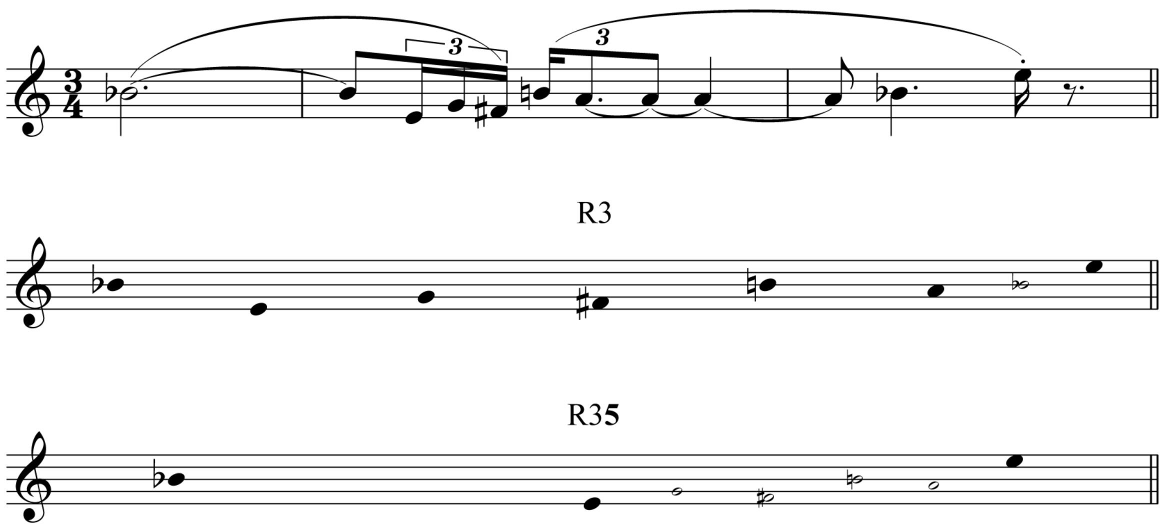
APPLYING THE ALGORITHMS: A GALLERY OF EXAMPLES

The present chapter demonstrates the application of the window algorithms to isolated passages that are extracted from compositions, from the 1920s to the 2000s, by Carter, Boulez, Xenakis, Hába, Babbitt, Webern, Birtwistle, and Stravinsky. The repertoire chosen for the chapter represents a range of musical styles which can be best described as lacking in tonal associations, at least in the conventional sense. Various melodic archetypes which involve zigzags, passing cps, and repeating cps (successively and non-successively) are specifically selected to observe the behavior of the algorithms on segments that are rather different in nature. This chapter also includes exemplary flowchart representations that demonstrate particular points in the reduction process. Although the "smallest possible window-size" approach is taken in reducing contours, the notion of different contour reduction paths, which involve windows that are larger than the smallest possible window-size, is explored via an example in which each reduction path results in a different reduced contour. It is important to note that the examples in this chapter serve for demonstration purposes in various contexts and are not in-depth analyses. A more detailed analysis, as well as phenomenological and cognitive considerations regarding the algorithms, will be presented in the next two chapters.

Elliott Carter's *A 6 Letter Letter* (1996)

Figure 4.1 illustrates the opening three measures of *A 6 Letter Letter* by Elliott Carter (1996) followed by a reduction of the passage.

Figure 4.1. Elliott Carter, *A 6 Letter Letter*, mm. 1–3



Here, the zigzag nature of the melody results in the pruning of only one cp in the first depth level (R3). As we can see from this example, the application of the 3-window algorithm to the passages that lack passing cps is limited but is nevertheless useful in highlighting the zigzag structure of the excerpt by pruning the passing cps. Figure 4.2 illustrates the flowchart for the evaluation of Bb4, in which the path followed is denoted by the circled answers, thicker arrows, and grey diamonds. As is evident from the figure, Bb4 is not marked for retention and as a result, is pruned at the last stage of the algorithm where cps that are not marked for retention are deleted.

It is interesting to point out that with the deletion of Bb4 in R3, both of the repeated adjacent interval-classes in the original segment (tritone and minor second) are removed. Thus, the emphasis placed on these interval-classes in the original segment is attenuated. Nevertheless, this removal could also be seen as a special type of emphasis: since the second Bb4 is the “odd one out,” the intervals preceding and succeeding it receive special attention.

The reduction following R3 employs the 5-window algorithm because an application of the 3-window algorithm does not reduce the segment any further (i.e. R33 = R3). Unlike R3, in which only one cp was pruned, the next depth level R35 prunes four cps and since it is not possible to apply another 5-window algorithm to the resulting segment, it is the deepest level with three retained pitches. Interestingly, the repeated pitch-class E is retained at the contour framework, whereas the only repeated pitch (not pitch-class) Bb4 is the only cp pruned in the first depth level. Although the main objective of this isolated passage is to illustrate an application of the algorithms on a “zigzaggy” melody and not to present a detailed analysis of the piece, it is interesting to note that the {E, Bb} tritone is emphasized throughout the piece. When Bb is paired with E, it is always treated as a passing cp up until m. 40 (almost the halfway point in the piece), whereas from then onwards it is presented as a local point of contour change. This shift in the role of Bb from a passing cp to a more structural cp is rather interesting and deserves further analytical exploration, which is beyond the scope of this short vignette.

Note that the cardinality-3 cseg framework results because the last pitch is the maximum of the segment. (If the melody stopped on the sixth note, the agogically accented A4, without repeating Bb and E, it would have a cardinality-4 framework with R35: <Bb4, Eb, B4, A4>).

Figure 4.2. Flowchart Path for Bb4 (Top Half)

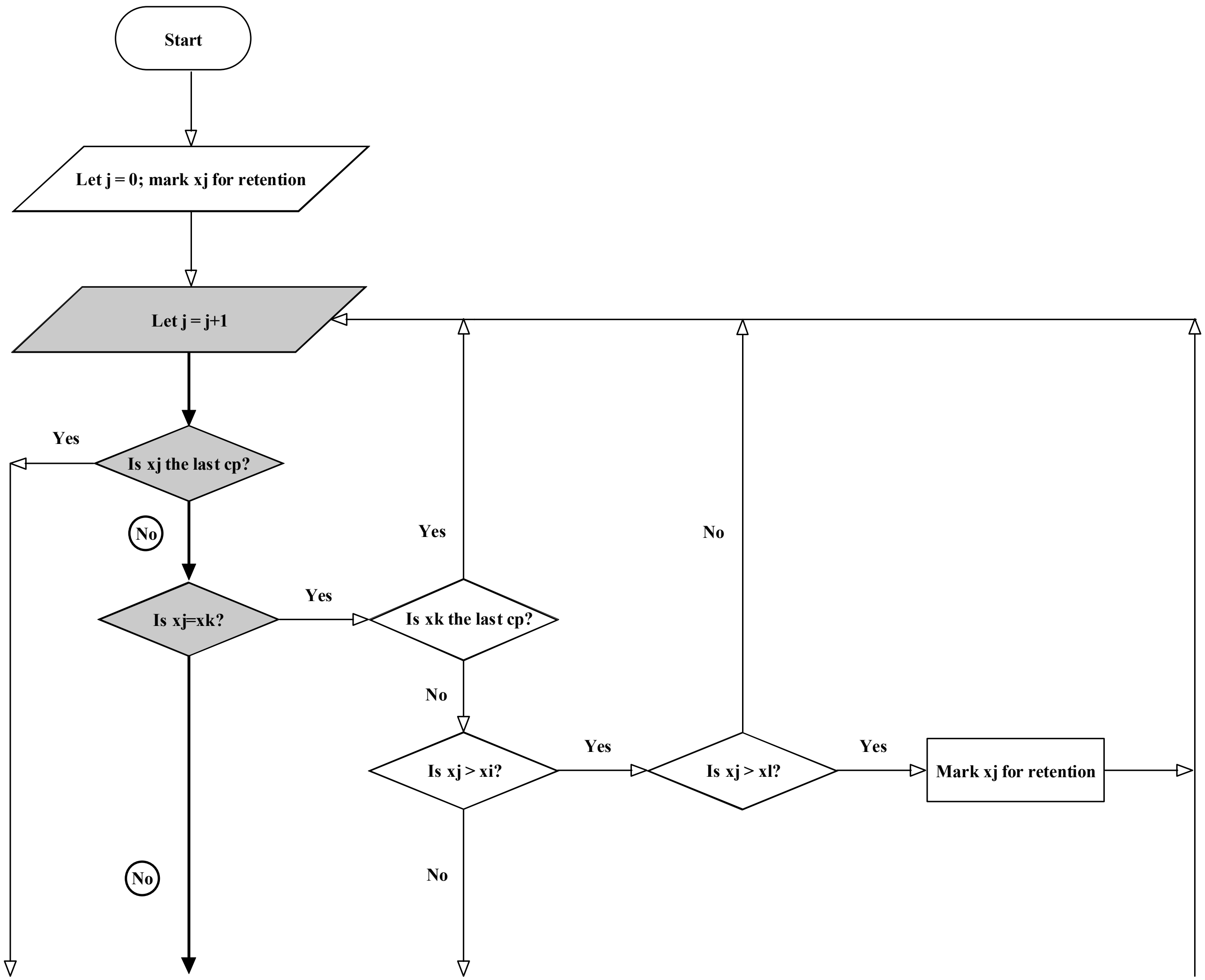
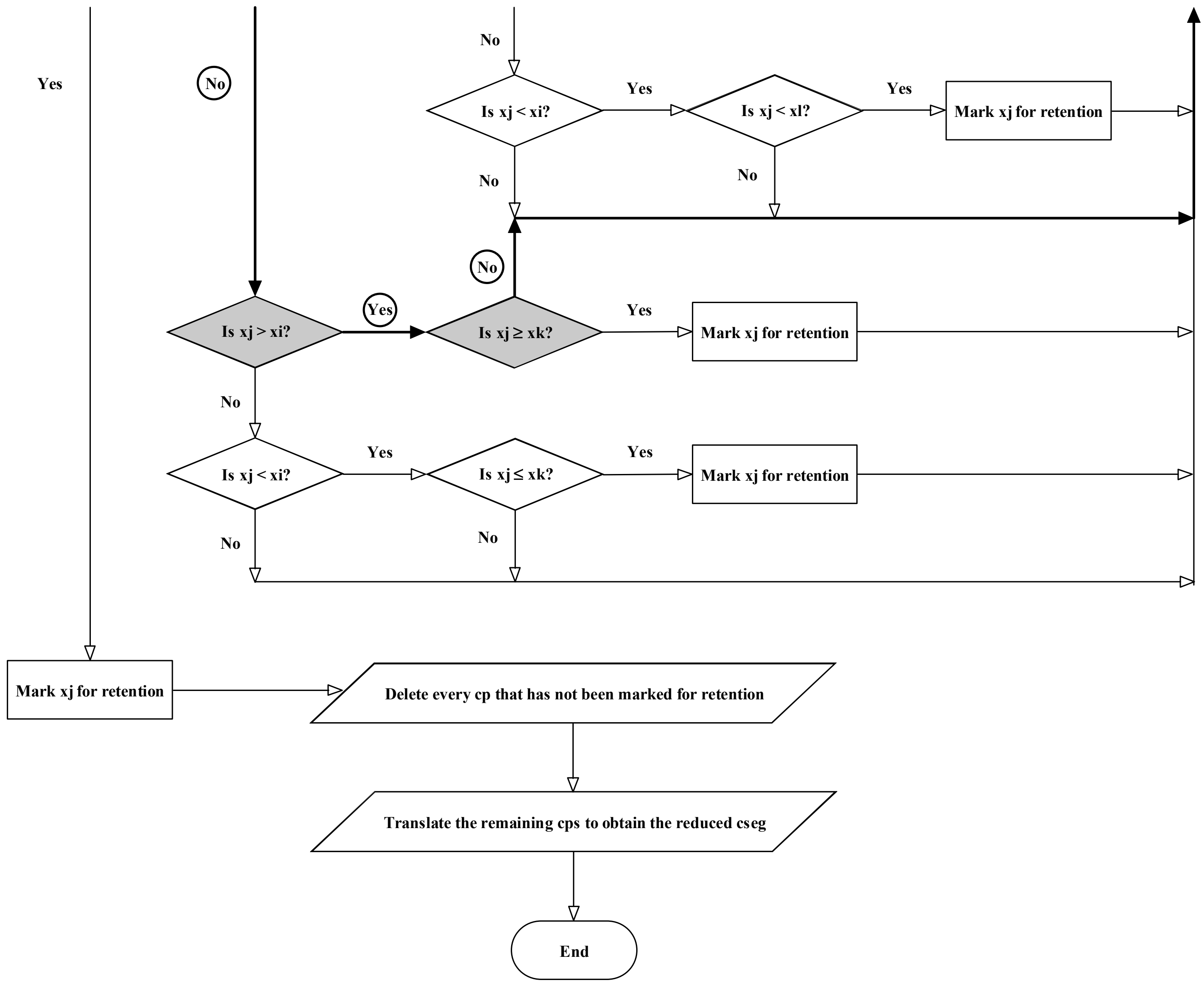


Figure 4.2 (cont.) Flowchart Path for Bb4 (Bottom Half)



Pierre Boulez's First Piano Sonata (1946)

An even more "zigzaggy" passage is illustrated in Figure 4.3. Here, the opening two measures of the second movement of Pierre Boulez's First Piano Sonata (1946) demonstrate a complete lack of passing cps and an emphasis on the direction change (or CAS change) due to the register extremities. Whereas the use of register in Carter example is limited to one octave, here the use of a span of over six octaves results in "sharp turns," drawing listener's attention to the change of direction.

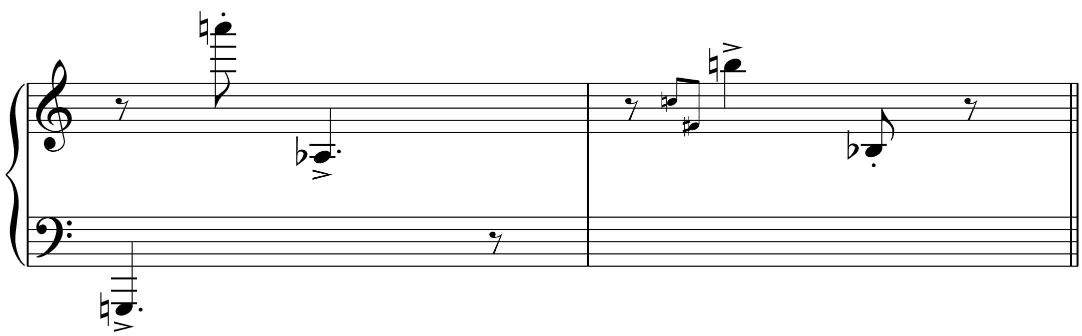
Note that in this example, the first pitch is sustained through the second pitch and beginning of the third one. This raises the question of whether these pitches should be treated as simultaneities following Robert Morris's prime classes that involve simultaneous cps.[106] Although it is possible to investigate the implementation of simultaneity cps into the algorithms, the excerpt given above does not qualify as a simultaneity contour according to Morris's definition because the first pitch has a different attack-point than both the second and third pitches. Thus, the segment constitutes a *linear* contour. Henceforth, we will adopt this approach and consider events in terms of their attack-points rather than how long they are sustained.

Due to the lack of passing cps as mentioned above, it is not possible to begin the reduction with the 3-window algorithm. Thus, the 5-window algorithm must be applied on the original segment, which in this case results in pruning the innermost pitches. From a contour perspective, the outermost pitches seem to have a certain prominence. At depth level 1 (R5), there is a passing cp. Following the guideline of employing the smallest possible window-size, a 3-window algorithm is applied on the resulting cseg. The order in which the

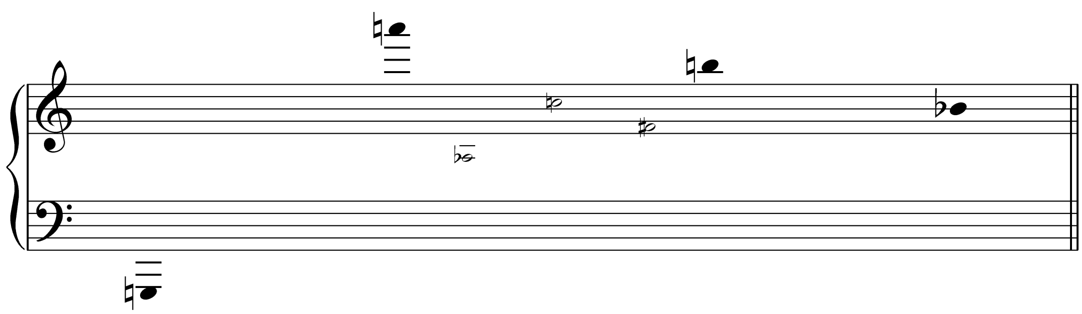
[106] Robert D. Morris, "New Directions in the Theory and Analysis of Musical Contour."

algorithms are applied is unusual (first a 5-window algorithm and then a 3-window algorithm). Most examples follow the exact opposite order for the first two reduction levels.

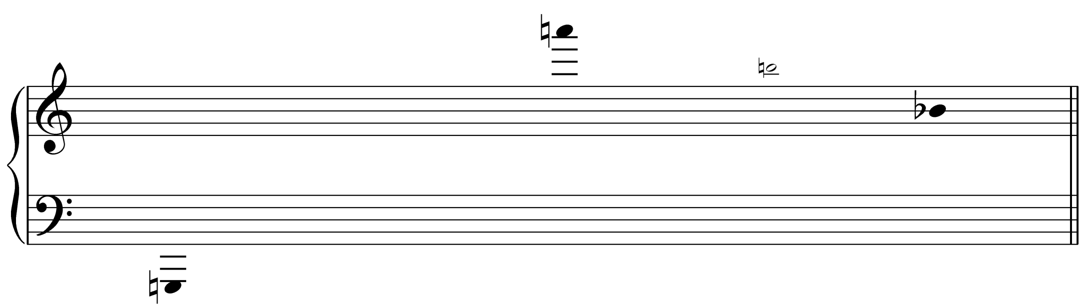
Figure 4.3. Pierre Boulez, First Piano Sonata (1946), mm. 1–2



R5

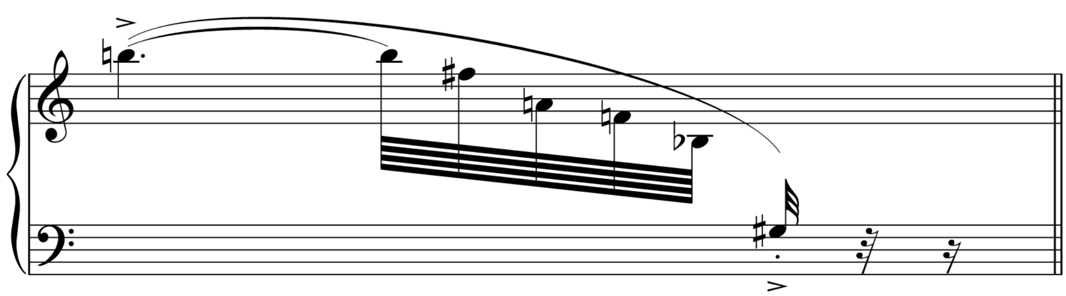


R53

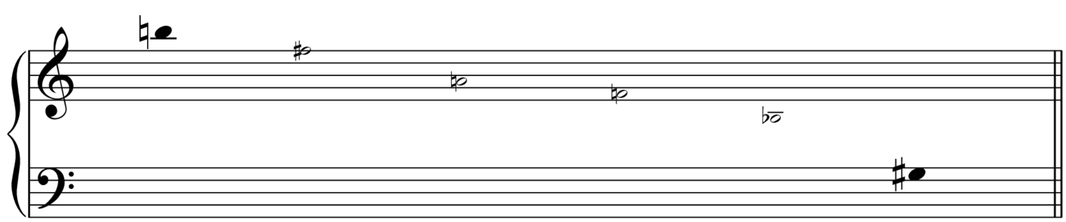


The preceding examples all presented one particular type of melodic archetype which involves successive zigzags.[107] The first movement of the same sonata by Boulez contains a passage in m. 14 which presents another type of melodic archetype with passing cps. The original segment and its R**3** reduction are shown in Figure 4.4.

Figure 4.4. Pierre Boulez, First Piano Sonata (1946), mm. 14



R**3**



[107] Note that the term melodic archetype used in this context could be loosely linked to some of Eugene Narmour's archetypes, such as duplication (D), process (P), and reversal (R), which we refer to as repeating cps, passing cps, and cps that form zigzags (i.e.: sign change in CAS), respectively. See Eugene Narmour, *The Analysis and Cognition of Basic Melodic Stuctures: The Implication-Realization Model* (University of Chicago Press, 1990).

Iannis Xenakis's *Herma* (1961)

This example also contains an employment of extreme registers, but the pitch successions could be understood as large-scale zigzags. In other words, this example combines the melodic archetypes of the previous examples as it involves some passing cps within the larger scale zigzags.

Figure 4.5 illustrates the opening three measures of Iannis Xenakis's *Herma* (1961) and its reductions.[108] Here the standard application of first the 3-window and then the 5-window algorithm is followed by a 3-window algorithm pruning D#7. This 15-note excerpt includes the aggregate plus the three repeated pitch-classes: E, B, and C#. Interestingly, the contour framework contains the repeated pcs E and B. In fact, the second half of the contour framework is a five-octave transposition of the first half of the framework, as illustrated in Figure 4.6.

[108] In this excerpt we adopt the attack-point approach discussed earlier in Figure 4.3. Also note that here it is possible to treat the segment as having more two, three, or even four separate parts due to its registral sparseness. However, even after the partition, the single parts would likely to involve pitches registrally too far apart for the listener to perceive them as belonging to a group. An example in which two distinct parts are audible will be presented at the end of the chapter.

Figure 4.5. Iannis Xenakis, *Herma* (1961), mm. 1–3

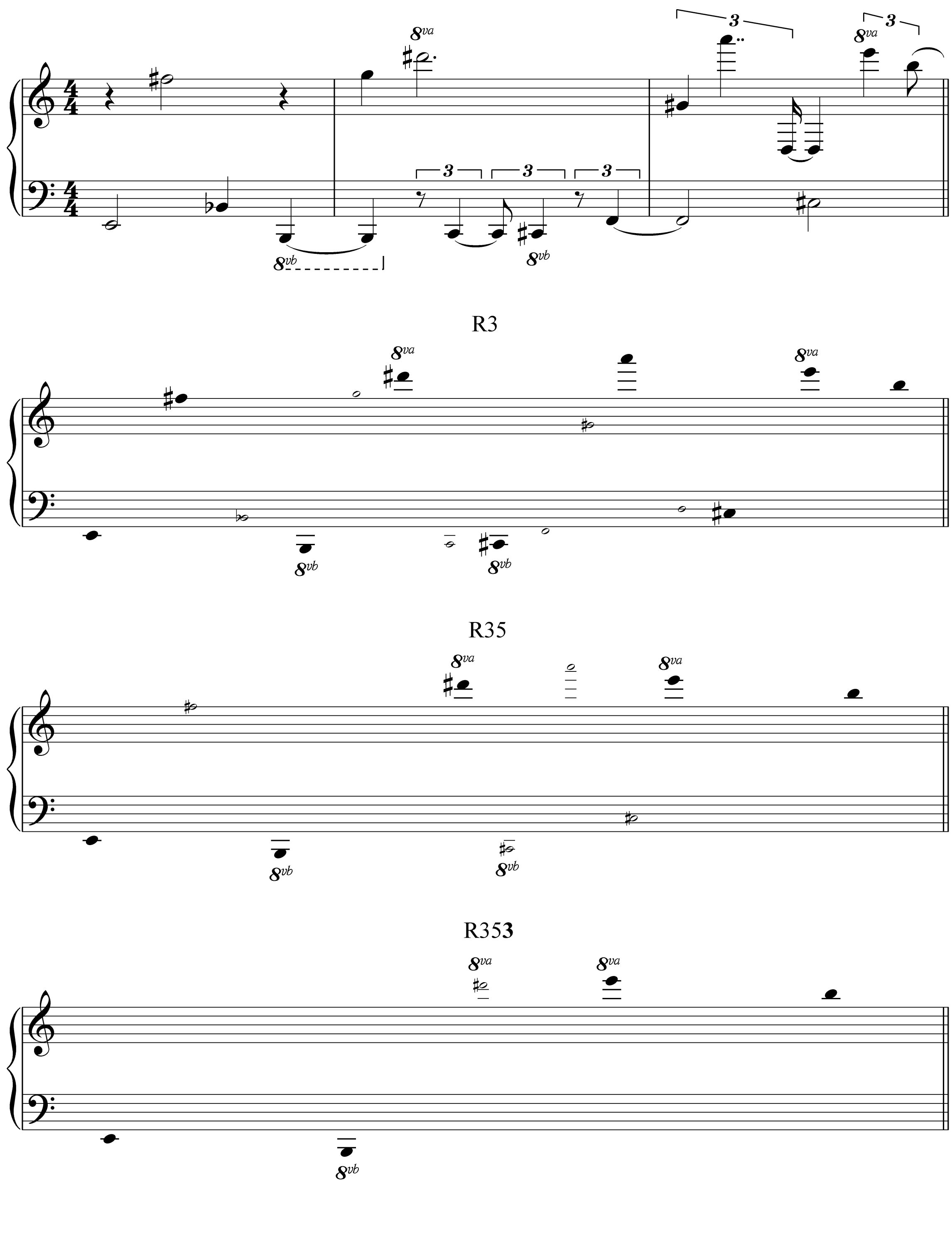
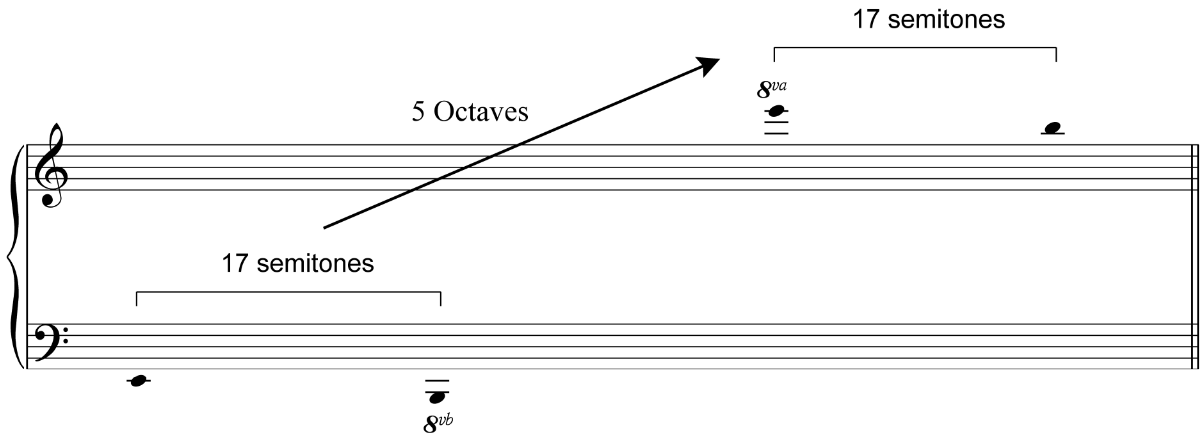



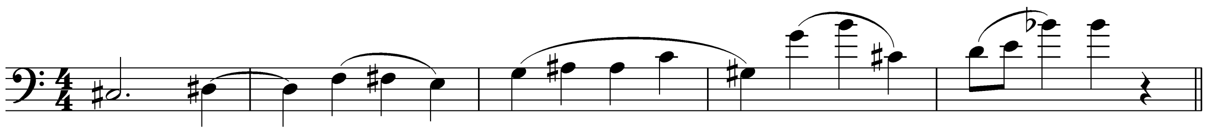
Figure 4.6. Intervallic Relationship in the Framework of *Herma*



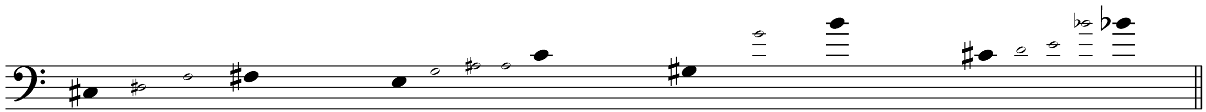
Alois Hába's *Nonet*, op. 40 (1931)

Our next example is from the single movement *Fantasy for Nonet* No.1, op. 40 (1931) by Alois Hába. Figure 4.7 illustrates the *Hauptstimme* from m. 293 to m. 297 and its reduction under R3 and R3**5**. This theme presents a clear overall ascent, temporarily interrupted three times by descents. The segment also includes two adjacent repeating cps, A#3 in the middle and Bb4 at the end.

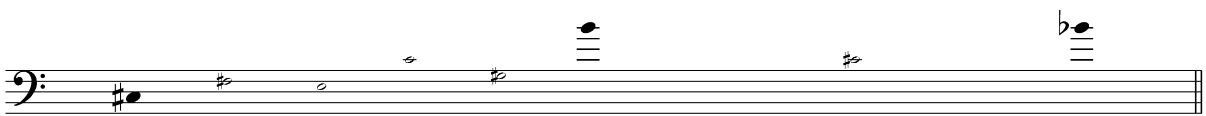
Figure 4.7. Alois Hába, *Fantasy for Nonet* No.1, op. 40, mm. 293–97



R3



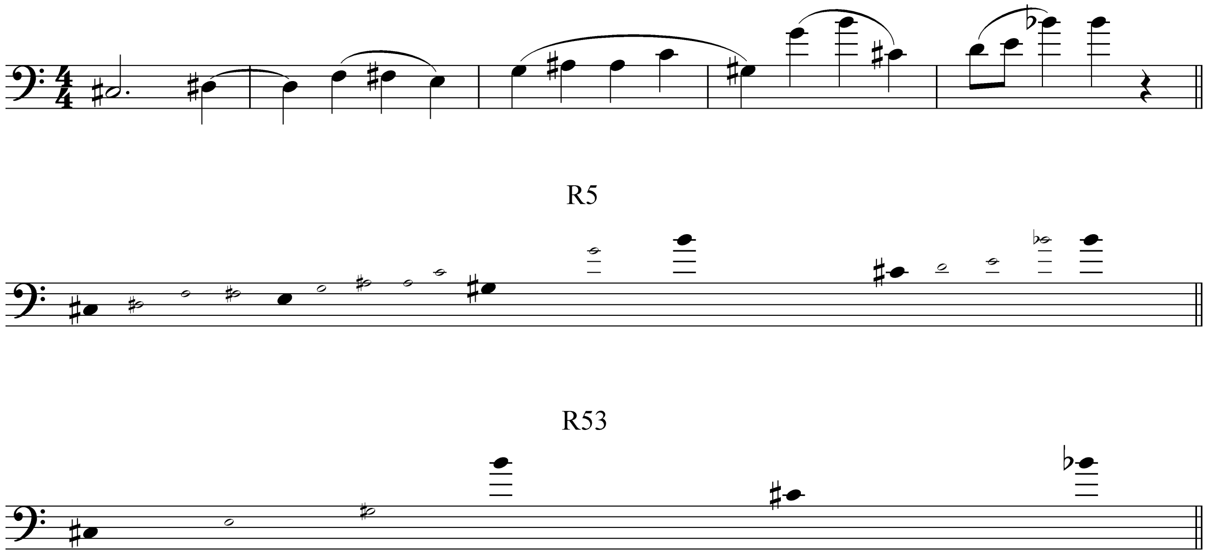
R3**5**



The second reduction level, R3**5**, prunes five cps because of the overall continuing ascent. This is the main reason why this 17-note cseg can be reduced to its framework in only two reductions. A generalization could be made based on this observation: an overall ascent or descent would generally result in fewer depth levels because the 5-window algorithm tends to even out local zigzagging.

Before we discuss the adjacent repeating cps, which are particularly interesting in this passage, let us consider an alternative reduction in order to reduce this segment by first employing a 5-window algorithm and only then a 3-window algorithm. This reduction, which follows the exact opposite order of the previous reduction, is illustrated in Figure 4.8.

Figure 4.8. Alternative Reduction for *Fantasy*



Interestingly, R5 essentially provides a depth level that is in between R3 and R3**5**: R3 is a cardinality-8 cseg and R3**5** is a cardinality-3 cseg, whereas R5 is a cardinality-6 cseg. If the analyst desires to obtain a depth level which contains 6 cps instead of 8 or 3, for example to compare contours of same cardinality, then she may choose to use R5. Note that for the most part in this study, we will not deal with comparing and contrasting contours of the same cardinality as some other theorists have done.[109] Rather, we will use contour reductions in order to prioritize pitches in music where priority is not particularly obvious or traditional. At a later stage, we will briefly consider comparing contours; however, our analytical focus will involve multi-parametric contours (i.e. pitch contour and duration contour) as we shall see in Chapter 8. Regardless, the possibility of obtaining a specific cardinality by altering the reduction path provides a potential to compare contours of different cardinalities.

---

[109] For example, Michael Friedmann adopts this approach in his analyses of Schoenberg's music in which nearly all of the comparisons take place between contours of equal cardinality. See Michael Friedmann, "A Methodology for the Discussion of Contour."

In the example given above, it is also possible to further "fine tune" the depth level between R3**5** and R5 by employing R53. R3**5** contains 3 cps and R5 contains 6 cps, whereas R53 contains 4 cps.

Note that R53 is not the last depth level and it is possible to apply another 5-window algorithm on this segment. As is evident from this illustration, one can manipulate cardinality and depth level by using different contour reduction functions. Table 4.1 illustrates how different contour reduction functions result in different cardinalities. Some of these options do not follow the protocol of smallest window-size possible at each stage.[110]

Table 4.1. Contour Reduction Functions with Different Cardinalities

	R3	R5	R53	R3**5** / R5**5** / 53**5**
cardinality	8	6	4	3

Now let us return to the reduction in Figure 4.7 and examine the adjacent repeating cps. The original segment involves two pairs of adjacent repeating cps: A#3 and Bb4. As is evident from R3, in the case of A#3, both cps are pruned since they are considered to be passing cps between G3 and C4. Such a situation provides the opportunity to observe the mechanics of the 3-window algorithm and also to examine the potentially confusing statement in the third step of the algorithm:

[110] Applying the 3-window algorithm first is generally very successful in modeling our listening experience, since we are very likely to hear passing cps as hierarchically less salient in most contexts.

3. If $(x_j = x_k) \wedge ((k \neq n) \wedge (((x_j > x_i) \wedge (x_j > x_l)) \vee ((x_j < x_i) \wedge (x_j < x_l))))$, then mark x_j for retention.

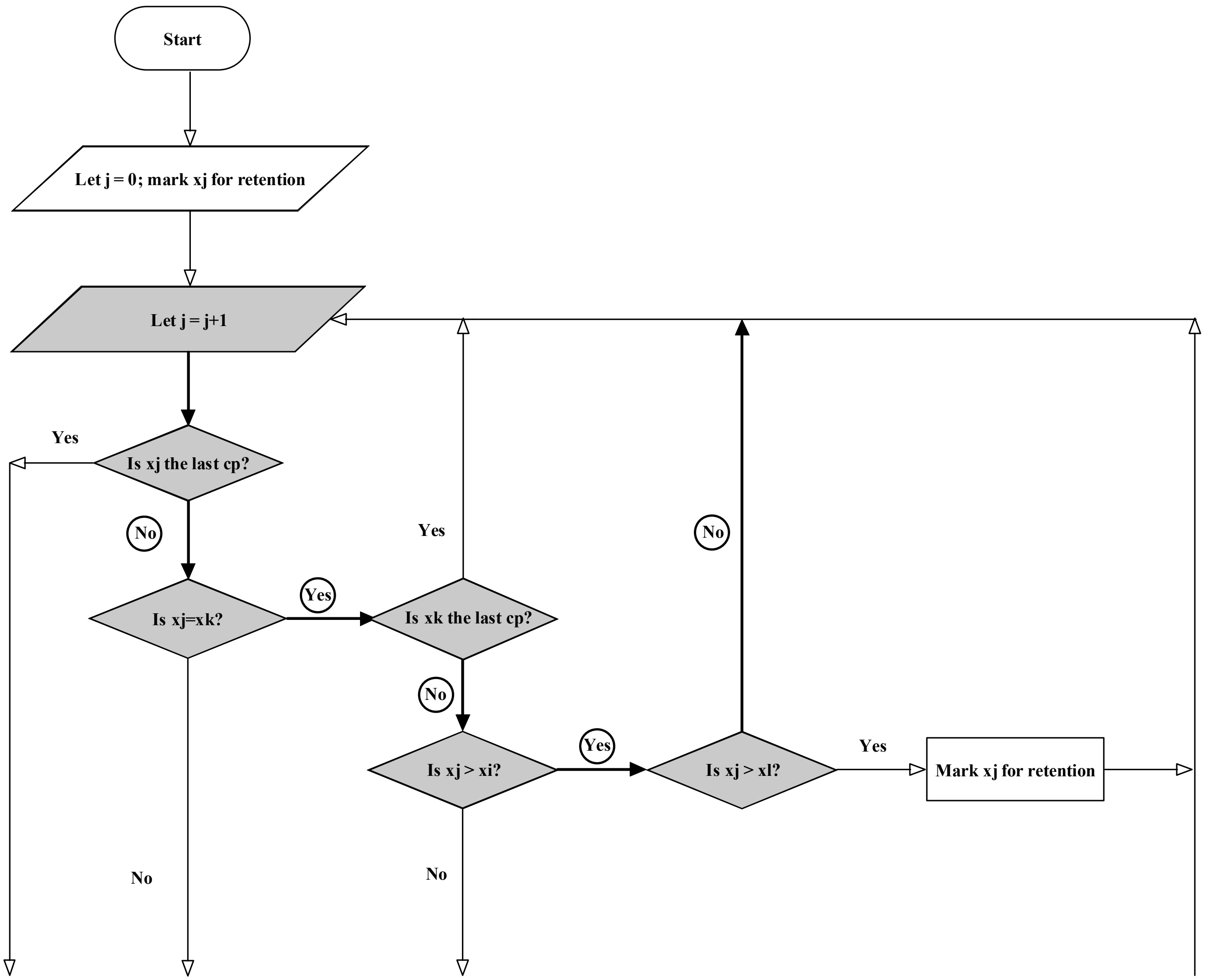
Here the involvement of x_l, which is outside the 3-window, serves the purpose of determining whether both of the adjacent repeating cps are passing. In this example, since x_j (the initial A#3) is neither maximum nor minimum with regards to x_i (G3) and x_l (C4), it is not to be retained. More formally:

$T \wedge (T \wedge ((T \wedge F) \vee (F \wedge T)) = T \wedge (T \wedge (F \vee F)) = T \wedge (T \wedge F) = T \wedge F = F$

Since the fourth step of the algorithm ($x_j \neq x_k \wedge (x_j = [max3] \vee x_j = [min3])$) is also false for the initial A#3, it is not marked for retention and eventually gets pruned. Note that the following A#3 is also pruned since none of the steps from two to four are true.

The path followed in the evaluation of the initial A#3 corresponds to the grey boxes of the flowchart shown in Figure 4.9.

Figure 4.9. Flowchart Path for the A#3



The second case of adjacent repeating cps (Bb4) is also interesting since the second cp is the last cp of the segment and thus is marked for retention. Only in this case the second repeating cp is retained and the first repeating cp is pruned. Let us first examine the initial Bb4. Once again, the third step in the algorithm states that:

3. If $(x_j = x_k) \wedge ((k \neq n) \wedge (((x_j > x_i) \wedge (x_j > x_l)) \vee ((x_j < x_i) \wedge (x_j < x_l))))$, then mark x_j for retention.

Here, the statement $k \neq n$ is false since $x_k = x_n$, in other words x_k is the last cp. Thus, the entire statement is false $(T \wedge F \wedge T) = F$ and we do not mark x_j for retention.

The latter Bb4 is marked for retention based on the second step in the algorithm:

2. If $j = n$, mark x_j for retention and go to step 6.

The flowcharts for the initial and final Bb4s are represented in Figures 4.10 and 4.11, respectively. Note that it is interesting to observe that both pairs of repeated adjacent cps in the segment belong to the same pitch-class A#/Bb.

Figure 4.10. Flowchart Path for the Initial Bb4

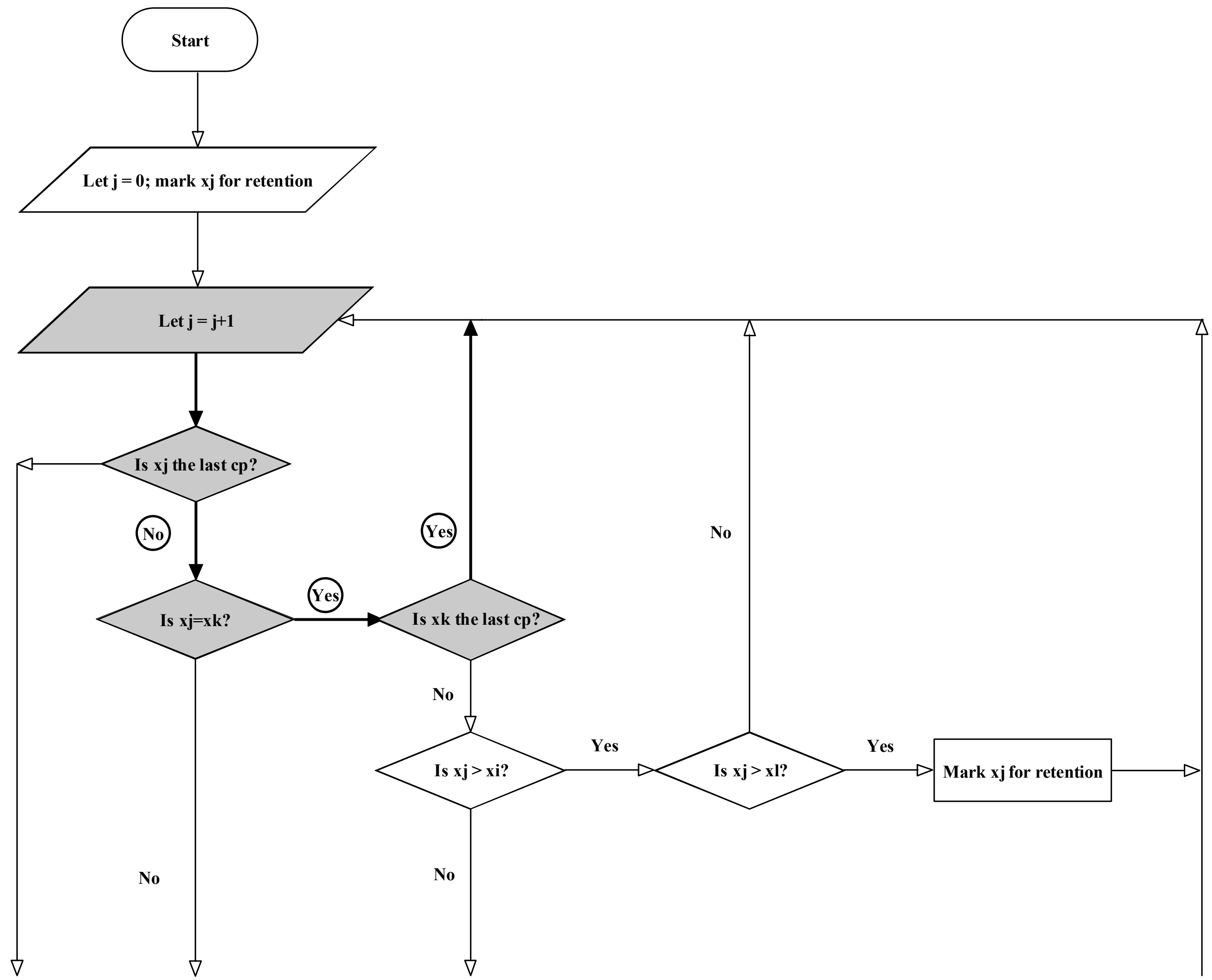


Figure 4.11. Flowchart Path for the Final Bb4 (Top Half)

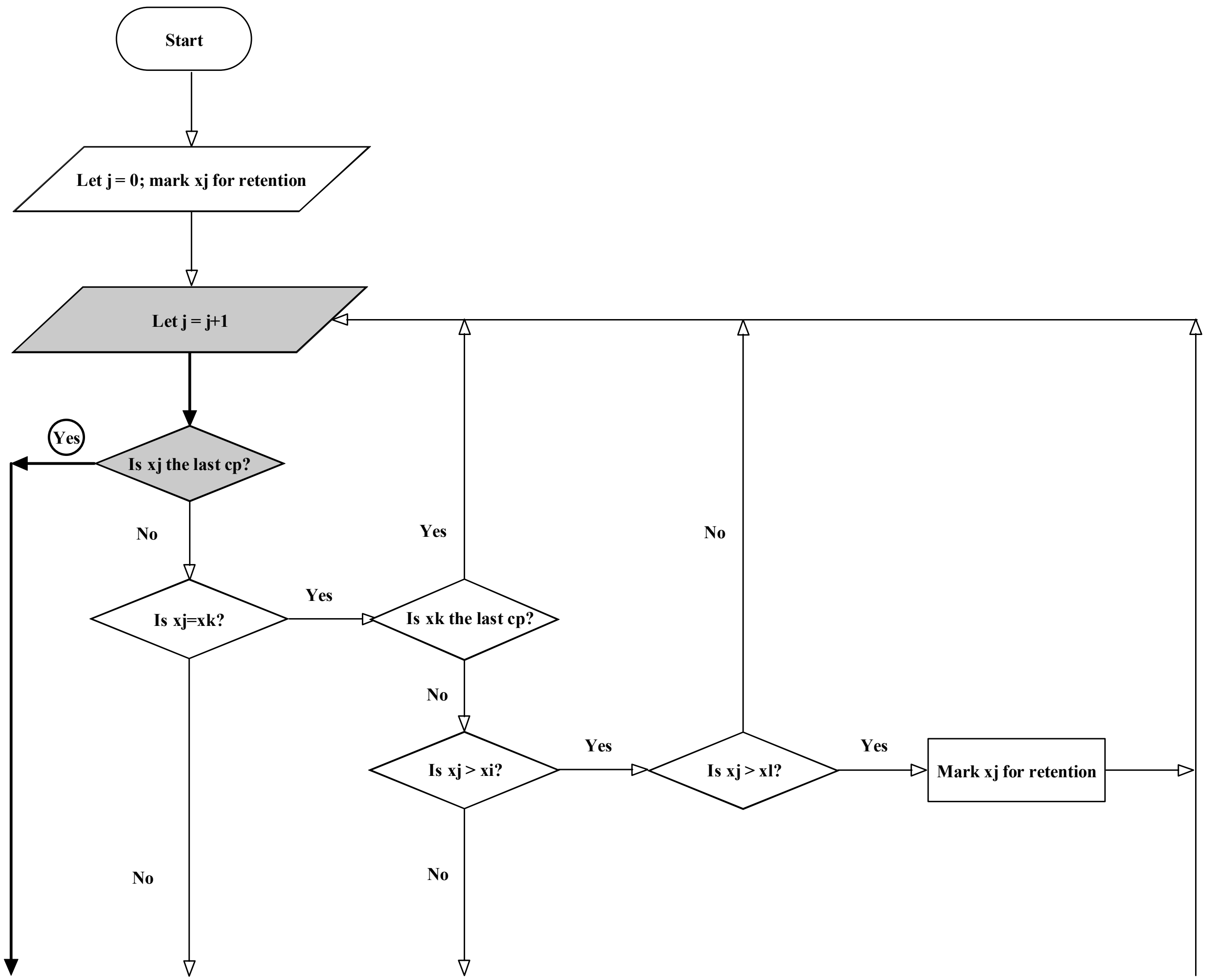
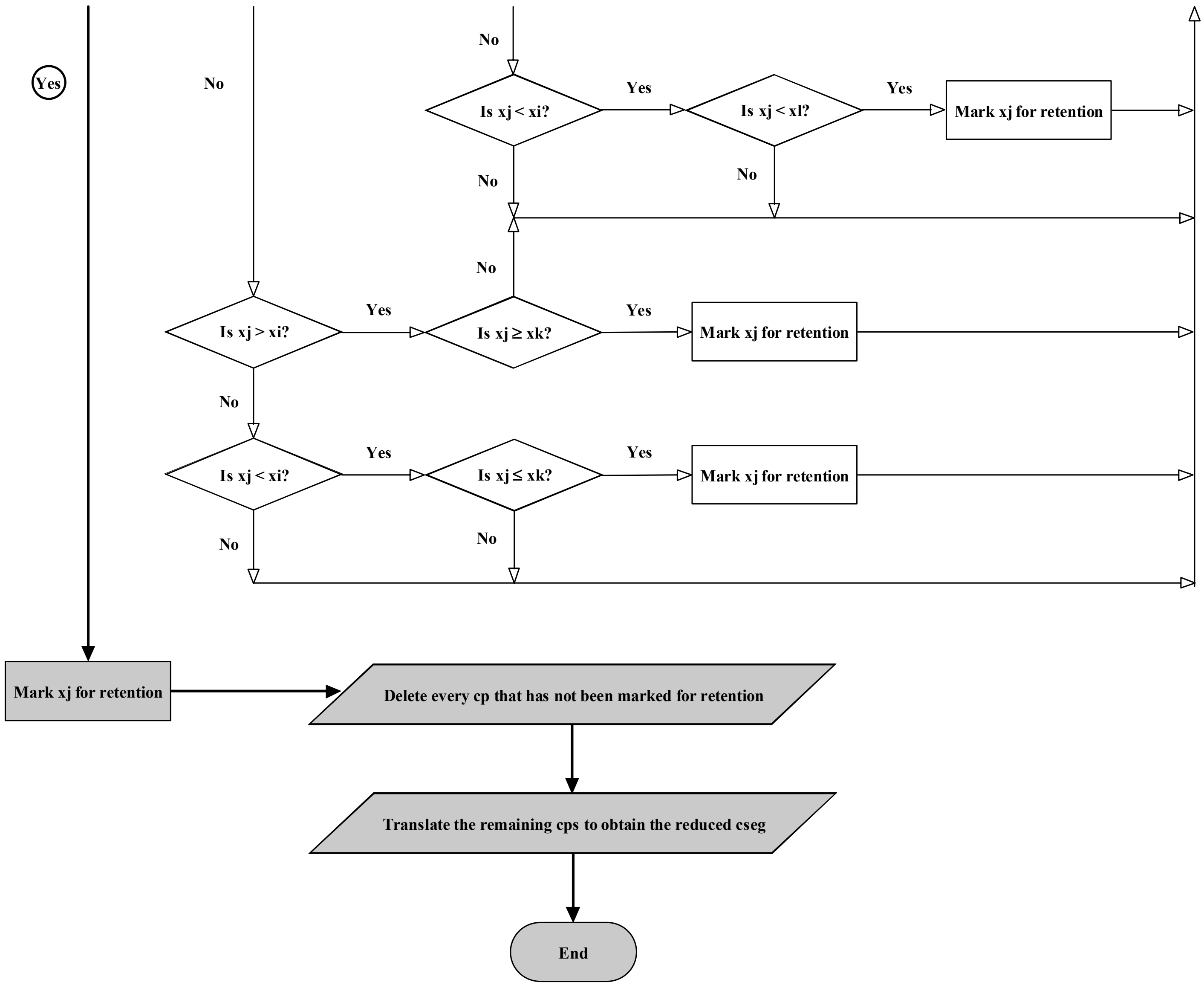


Figure 4.11 (cont.) Flowchart Path for the Final Bb4 (Bottom Half)

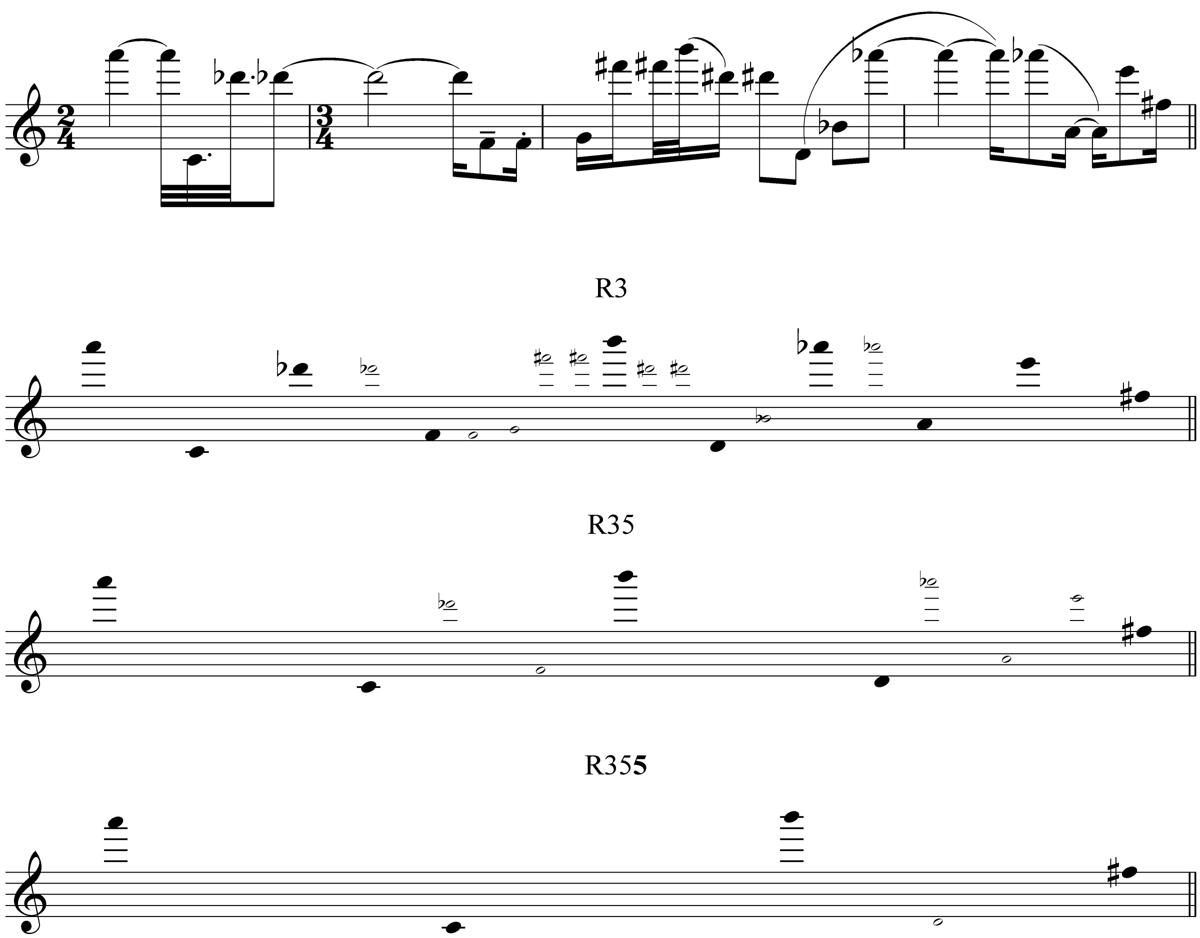


Milton Babbitt's *Soli e Duettini* for Flute and Guitar (1989)

Now let us look at another example of adjacent repeating cps, which involves neither retention of the last repeating cp nor passing adjacent repeating cps. Both of these cases are already examined in the previous Hába example. Figure 4.12 illustrates mm. 3–6 from *Soli e Duettini* for Flute and Guitar (1989) by Milton Babbitt. In this passage, the first and second pair of repeating adjacent pitches (Db6 and F4) are maximum and minimum, respectively. The following two pairs (F#6 and D#6) are passing and the last pair (Ab6) is a maximum. Our focus here will be on the pairs that are maximum or minimum.

This example is unique in that two adjacent repeated pairs are presented adjacently (i.e. <Db6, Db6, F4, F4>). As a result, the evaluation of the initial F4, which is repeated, involves the preceding Db6, which is also repeated.

Figure 4.12. Milton Babbitt, *Soli e Duettini* for Flute and Guitar, mm. 3–6



As is evident from R3, the first cp in the repeating pairs of Db6 and F4 are retained since they are maximum and minimum in their relatively global surrounding: two Db6s are surrounded by C4 and F4; two F4s are surrounded by Db6 and G4. Both of the retentions are due to the third step of the algorithm:

3. If $(x_j = x_k) \wedge ((k \neq n) \wedge (((x_j > x_i) \wedge (x_j > x_l)) \vee ((x_j < x_i) \wedge (x_j < x_l))))$, then mark x_j for retention.

for the initial Db6: $(T \wedge (T \wedge ((T \wedge T) \vee (F \wedge F)) = T \wedge (T \wedge (T \vee F)) = T \wedge (T \wedge T) = T \wedge T = T$; the initial Db6 is marked for retention.

for the initial F4: $(T \wedge (T \wedge ((F \wedge F) \vee (T \wedge T)) = T \wedge (T \wedge (F \vee T)) = T \wedge (T \wedge T) = T \wedge T = T$; the initial F4 is marked for retention.

Note that the latter Db6 and F4 are pruned since they do not meet the criteria of the second, third, or fourth steps:

2. If $j = n$, mark x_j for retention and go to step 6.

3. If $(x_j = x_k) \wedge ((k \neq n) \wedge (((x_j > x_i) \wedge (x_j > x_l)) \vee ((x_j < x_i) \wedge (x_j < x_l))))$, then mark x_j for retention.

4. If $x_j \neq x_k$, and if $x_j = [max3] \vee x_j = [min3]$, then mark x_j for retention.

2. $(F \wedge (T \wedge ((F \vee F)) = F \wedge (T \wedge F)) = F \wedge F = F$; the latter Db6 and F4 are <u>not</u> marked for retention.

3. $(T \wedge (F \vee F)) = T \wedge F = F$; the latter Db6 and F4 are <u>not</u> marked for retention.

4. F; the latter Db6 and F4 are <u>not</u> marked for retention.

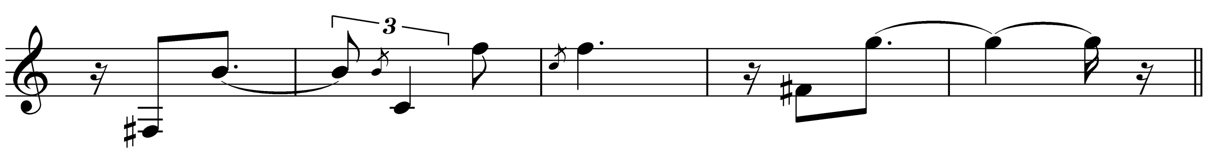
.

This example and the previous Hába example cover all possible scenarios involving adjacent repeating cps, namely: passing; not passing (maximum or minimum); and the first/second cp being the first/last cp of the segment.

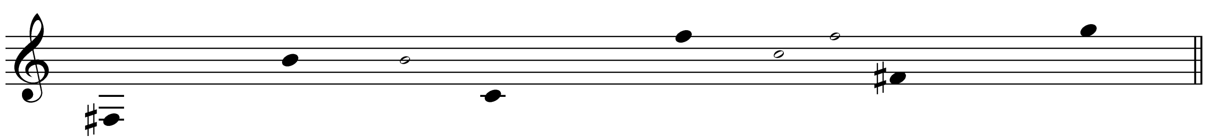
Harrison Birtwistle's *Verses* for Clarinet and Piano (1965)

Figure 4.13 illustrates the clarinet part from the *Verses* no.5 for Clarinet and Piano (1965) by Harrison Birtwistle. In this example, the minimum and maximum are at the same time the beginning and end cps of the passage, which results in a cardinality-2 contour framework. Note that this example is different from the Boulez example, which also involved a cardinality-two contour framework (see Figure 4.4). The Boulez example is formed by passing cps, whereas this passage does not involve any passing cps. This point is important since a series of consecutive passing cps must naturally result in a cardinality-2 contour framework whereas a passage with no passing cps rarely results in a cardinality-2 framework. In Figure 4.13, since there are no passing cps, the 5-window algorithm is applied on the original segment, which results in a zigzag ascent.

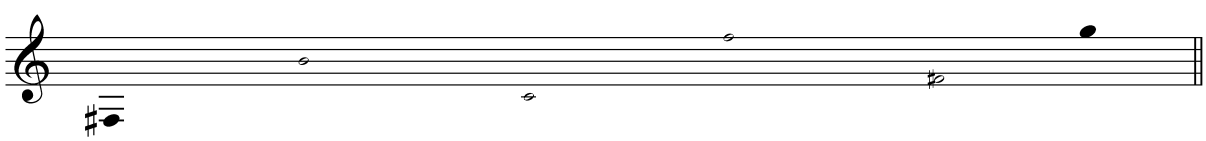
Figure 4.13. Harrison Birtwistle, *Verse* no.5 for Clarinet and Piano



R5



R5**5**



Here, for the first time, we also observe the application of the 5-window algorithm to repeating adjacent cps. In this example, the initial B4 is retained while the subsequent B4 is pruned. Since the principle for pruning adjacent repeating cps in the 3-window and the 5-window algorithms is the same, we do not need to demonstrate this point in detail. The issue of non-adjacent repeating cps, however, which are also found in this segment (two F5), needs further discussion.

Igor Stravinsky's *Septet* (1953)

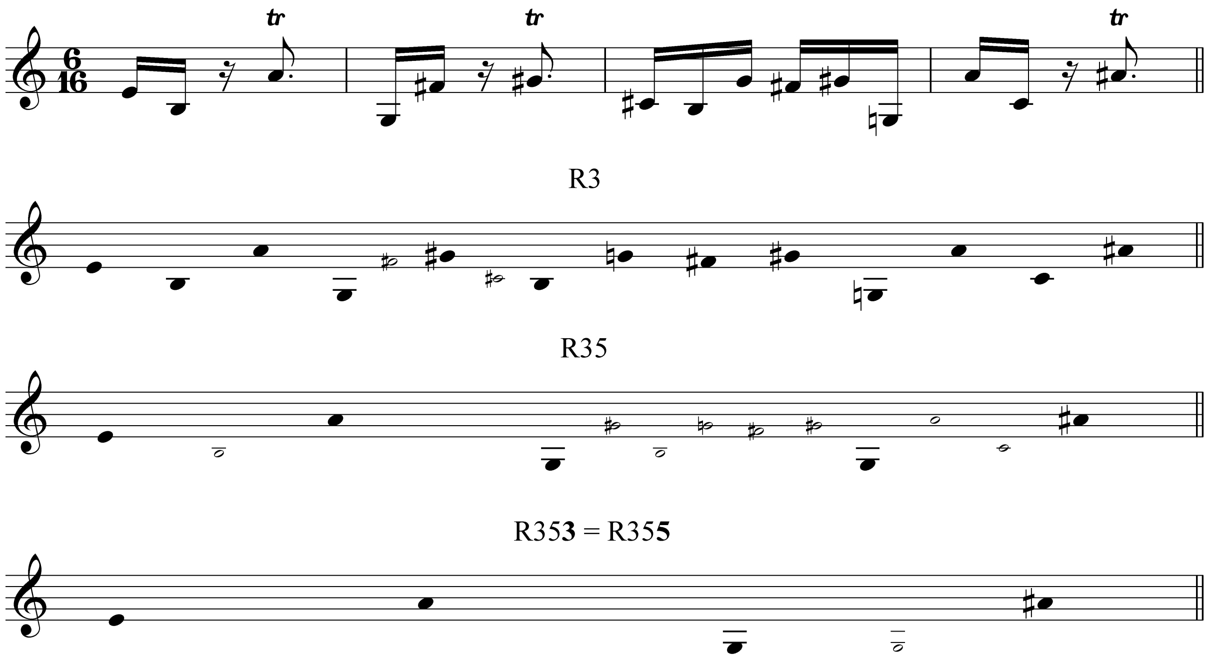
Figure 4.14 presents the subject (mm. 1–4) of the Gigue (the last movement) from Igor Stravinsky's *Septet*, a serial—though not dodecaphonic—work composed in 1953.

As is evident from the original segment (<418057316570829>), all repeated cps (i.e. cps 0, 1, 5, 7, 8) are non-adjacent and are repeated only once. Interestingly, since the repeating cps are too far apart from each other in the original segment and since no repeating pair is retained after the first reduction, except cp 0 (G3), no pair is assessed within the same 5-window.[111]

This passage demonstrates that an abundance of repeated cps does not necessarily secure the evaluation of repeated pairs within the same window. None of these cps, with the exception of the second cp 0, are pruned because they are repeated. (They are pruned for other reasons.)

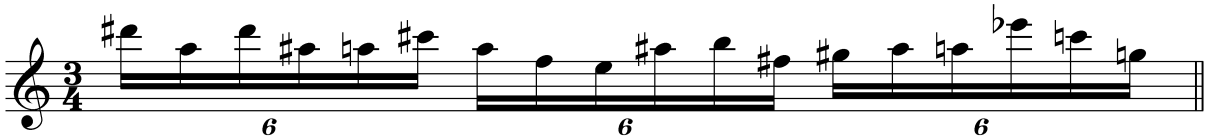
---

[111] Note that technically the third depth level employs the 3-window algorithm, following the principle of selecting the smallest possible window size. However, one could argue that it is more plausible to employ the 5-window algorithm instead of the 3-window algorithm since there are no passing cps in this depth level. In any case, this point is insignificant as both the 3- and the 5-window algorithms give the exact same result and both reductions are indicated on the score as R353 = R355. Note that this is also the case in the Birtwistle example (Figure 4.13), although here the application of the 5-window algorithm on the original segment gives a different result than the application of the 3-window algorithm. In that example the 5-window algorithm is chosen in order to "speed up" the reduction process.

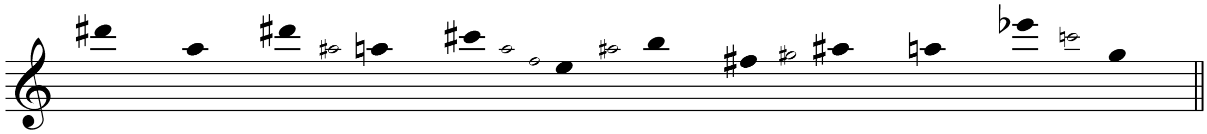
Figure 4.14. Igor Stravinsky, *Septet*, mm. 1–4

Elliott Carter's *Steep Steps* (2001)

An example which involves repeating cps that are not adjacent yet proximate enough to be evaluated in the same window is extracted from m. 46 of Elliott Carter's *Steep Steps* (2001), and is illustrated in Figure 4.15.

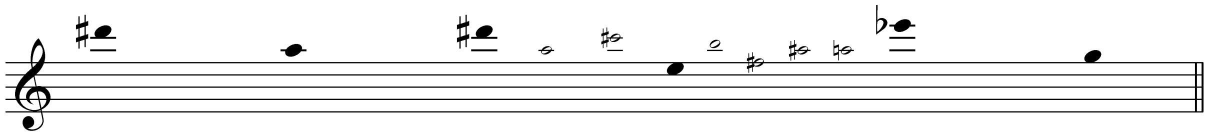
Figure 4.15. Elliott Carter, *Steep Steps*, m. 46



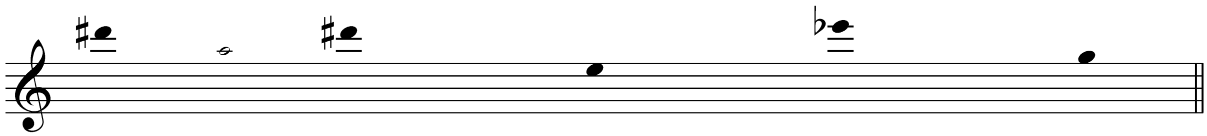
R3



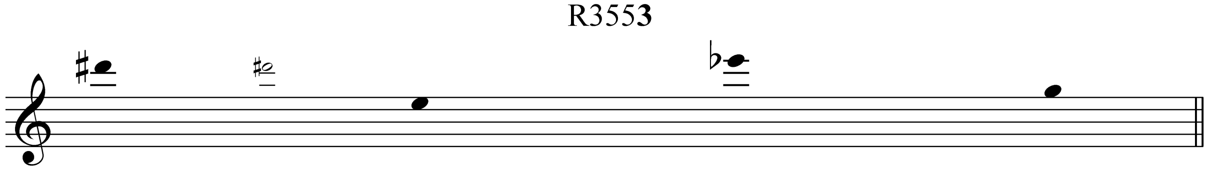
R35



R355



R355**3**



As always, R3 prunes all of the passing cps by applying a 3-window algorithm. In the following depth level R35, the 5-windows assessing the D#6 and the following A5 contain non-successive repeating cps unlike the Stravinsky example. At this depth, both of the D#6s are retained whereas the second A5 is pruned because it is neither maximum nor minimum within the 5-window.[112] The second D#6 is not pruned until the last depth level, in which it immediately succeeds the first D#6. It is interesting to observe that each level prunes a previously or currently repeating cp. It is also worth pointing out that although the second A5 is temporally slightly more detached from the first A5 than two D#6s are to each other, it is the first to be pruned due to the passing nature of its surrounding cps and also due to it not being a medial cp (i.e. neither maximum nor minimum) in its immediate surrounding after the deletion of passing cps. Note that in this example if the cps were assigned to relatively longer durations, then the durational detachment of A5 would play a more important role and would raise the question of whether the temporally detached medial (i.e. neither maximum nor minimum) A5 should hierarchically supersede the temporally more proximate maximum D#6. In any case, here the durational detachment of A5 is negligible because the inter-onset durations between the cps are fairly short. A discussion of temporal aspects, which involve durational contours, will be discussed in depth in Chapters 7 and 8.

Now let us observe the application of the algorithms on both of the A5s at R35. The fourth step of the 5-window algorithm, which marks the first A5 for retention, is provided below.

[112] Let us reiterate that here we follow Morris's inclusion of equality cases for maximum and minimum (i.e.: "higher/lower than or equal to"), with the exception of xi, for the reasons explained in Chapter 2.

4. If $x_j \neq x_k$, and if $x_j = [max5] \vee x_j = [min5]$, then mark x_j for retention.

Since [min5] is defined as $(x_j < x_i) \wedge (x_j \leq x_h) \wedge (x_j \leq x_k) \wedge (x_j \leq x_l)$, the first A5 is marked for retention. On the other hand, the second A5 is not marked for retention since it is neither maximum nor minimum according to the [max5] and [min5] definitions. Figure 4.16 illustrates the evaluation of the second A5 at R35.

The last depth level R355**3** returns using the 3-window algorithm in order to prune the successive repeating D#6. Note that the cardinality-4 cseg framework includes only three pitches as a result of the repeating maxima.

Figure 4.16. Flowchart Path for the second A5 at R35 (Top Half)

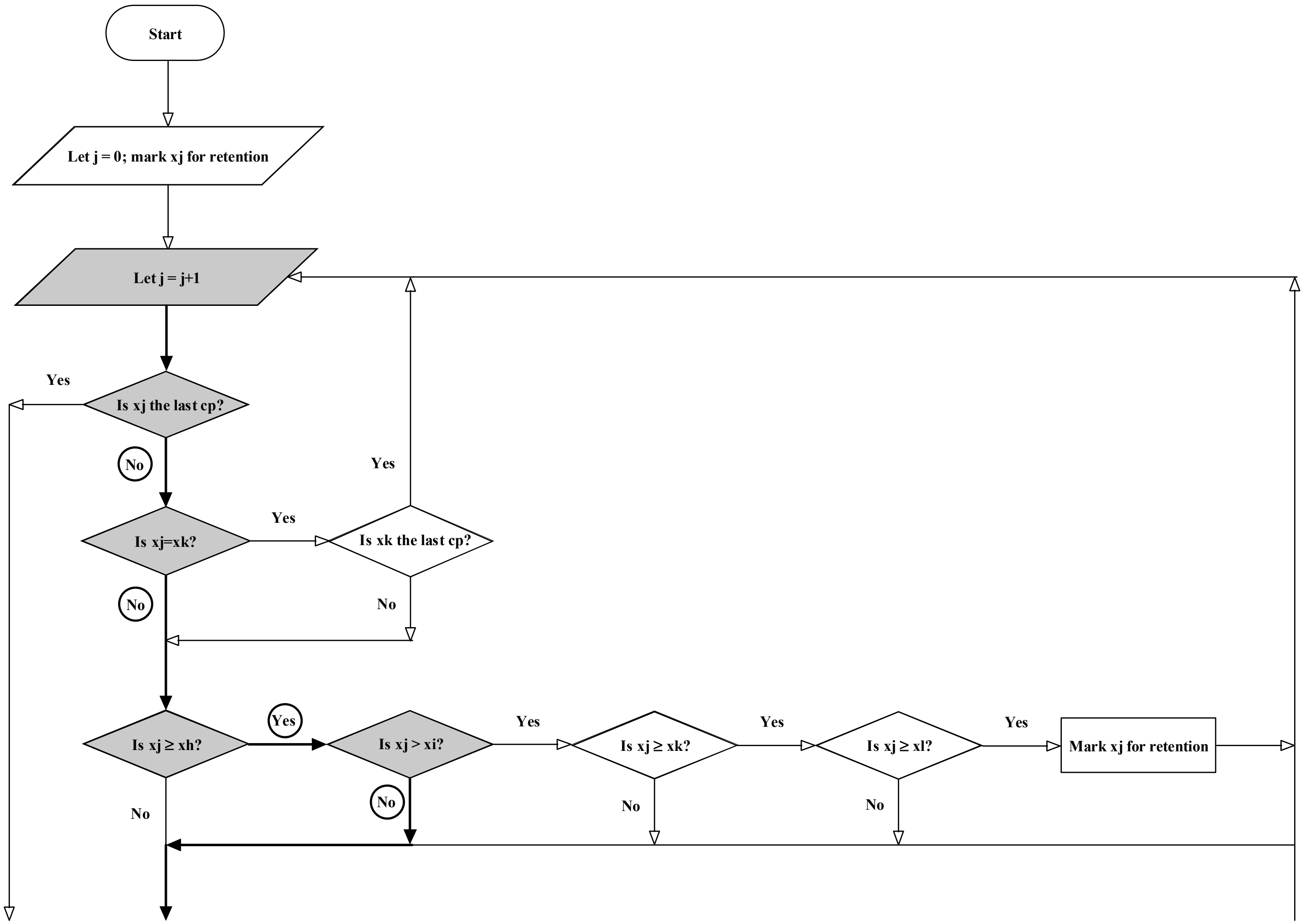
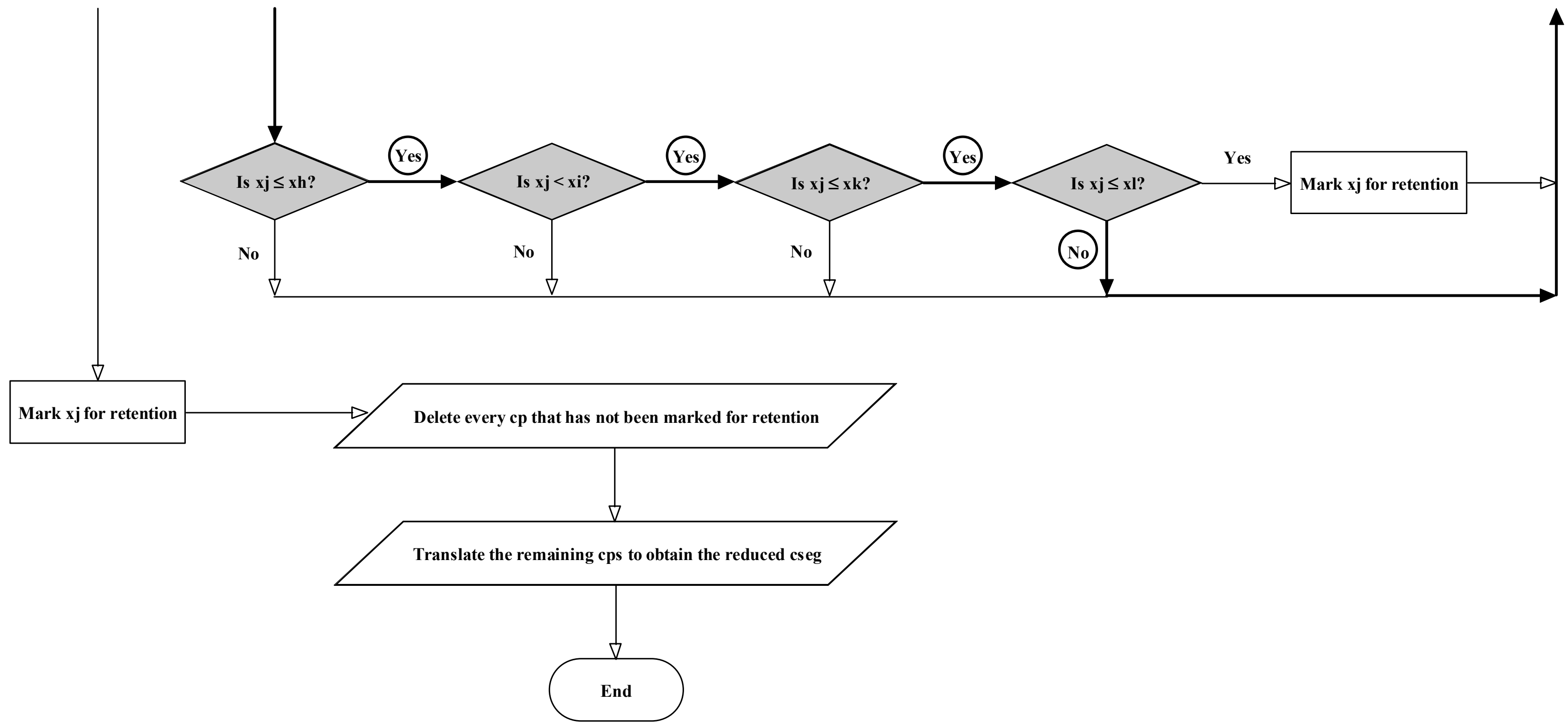


Figure 4.16 (cont.) Flowchart Path for the second A5 at R35 (Bottom Half)



Anton Webern's "Schatzerl klein," op.18, no.1 (1925)

The final example of this chapter can be seen as a transition to the following analytical chapter as it provides more analytical depth in comparison to the previous examples. The second stanza (mm. 4–8) from Anton Webern's "Schatzerl klein," op.18, no.1 (1925), which contains a global ascent and followed by a global descent, is illustrated in Figure 4.17. This passage displays an interesting jagged registral alternation, with low-high note pairs consistently ascending in the first half, and descending more irregularly in the second half. The uniformity of the opening pattern is partially disrupted in the fifth and sixth pairs. In the fifth pair the lower note is rising but the higher note is falling and in the sixth pair, it is just the opposite; the lower note is falling but the higher note is rising. However, the ensuing pairs continue the falling trend of the rising-falling wave formation.

The first depth level R3 reveals that the intervals of the pairs exhibit another interesting pattern: both the rise and fall are accompanied by increasing interval size, as illustrated in Figure 4.18. The only exception to this is the last interval.

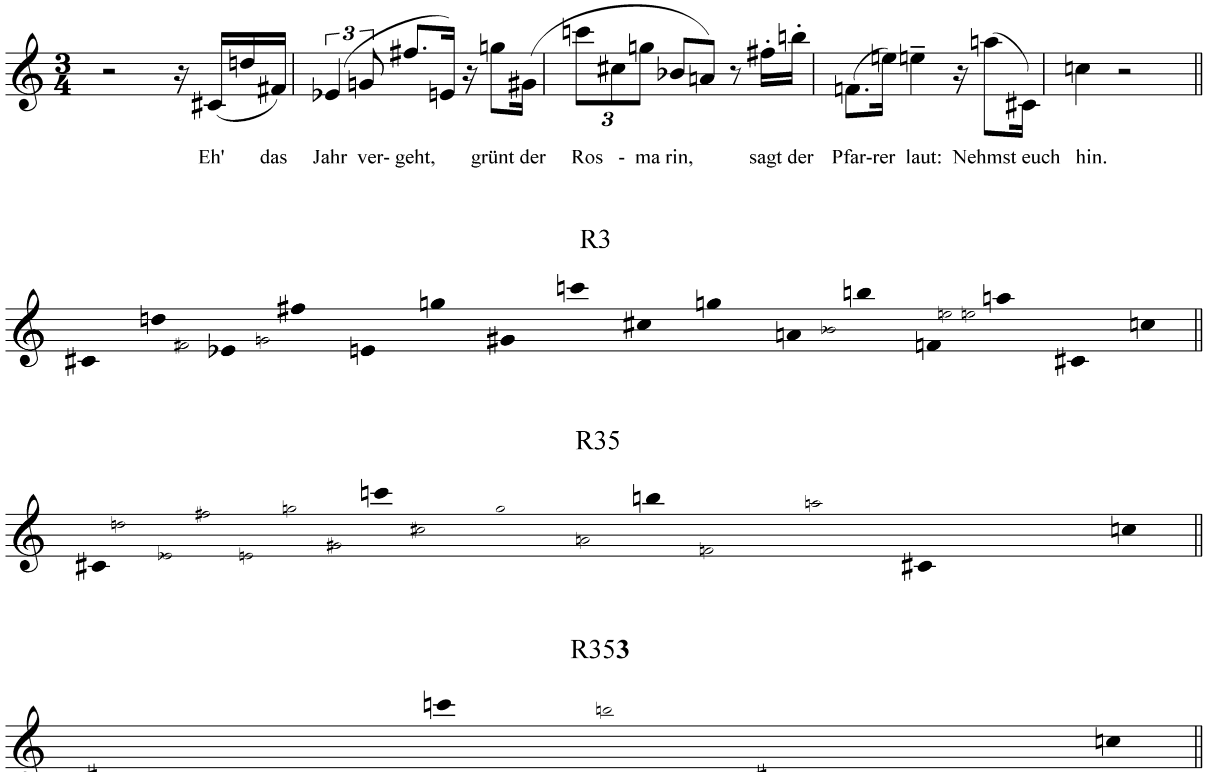
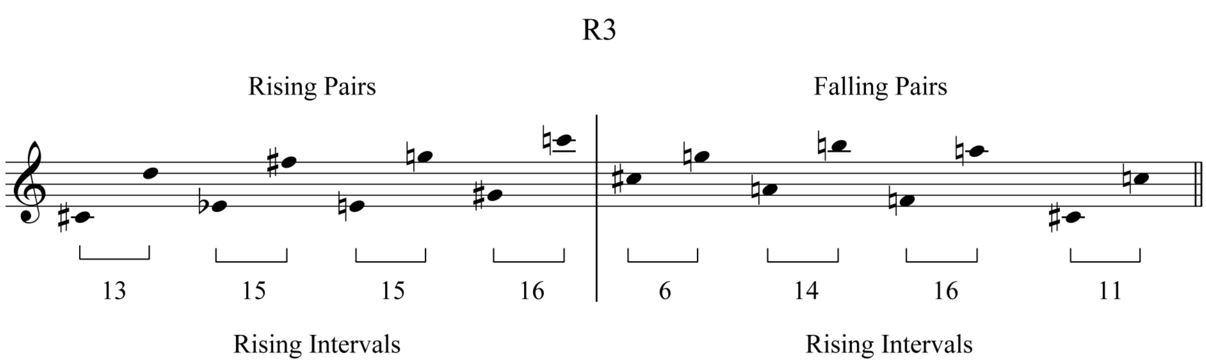
Figure 4.17. Anton Webern, "Schatzerl klein," op.18, no.1, mm. 4–8

Anton Webern, *3 Lieder für Gesang,* op.18/1

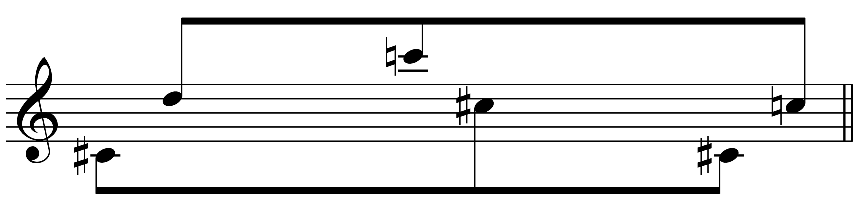
Figure 4.18. Rising and Falling Pairs at R3



Because almost all the intervals are larger than an octave, we can approach the reduced segment as two separate parts (upper and lower) due to the phenomenon known as auditory stream segregation.[113] Here, the upper part consists of <D5, F#5, G5, C6, G5, B5, A5, C5> and the lower part consists of <C#4, Eb4, E4, G#4, C#5, A4, F4, C#4>. Such an approach reveals interesting insights about the framework of the segment. As shown in Figure 4.19, the frameworks of the upper part and the lower part demonstrate a property similar to the Schenkerian concept of coupling, except at 11 or 13 semitones, rather than at the octave. Here, the lower part opens with C#4, reaches C#5 and descends back to C#4. In the upper part, D5 ascend up to C6 and this C6 is transferred an octave lower to C5. Arguably, the pitch-class stability of the lower stratum makes it the referential one, with the more variable upper stratum being coupled to it.

Figure 4.19. Framework Coupling of Two Parts

R35 (Framework of the upper part)



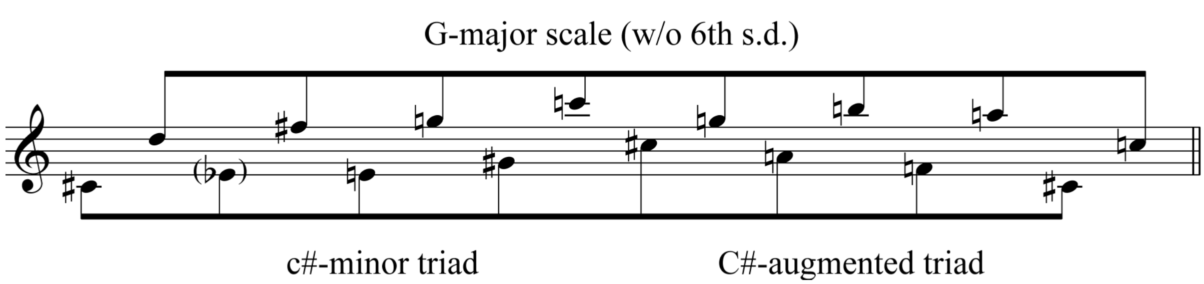
R3 (Framework of the lower part)

[113] For more information on this phenomenon, see Albert S. Bregman and Jeffrey Campbell, "Primary Auditory Stream Segregation and Perception of Order in Rapid Sequences of Tones." *Journal of Experimental Psychology* 89.2 (1971): 244–249; Albert S. Bregman, *Auditory Scene Analysis: The Perceptual Organization of Sound* (Cambridge: MIT Press, 1990); Roger Shepard, *Music, Cognition, and Computerized Sound: An Introduction to Psychoacoustics* (Cambridge: MIT Press, 1999).

Figure 4.19 gives the skeleton for the rising and falling wave, which could be derived from an application of the algorithm on both upper- and lower-parts separately. Here, the retained pcs form a (012) set-class and one notes in particular the cogent “central” line D5–C#5–C5. If the D5 is taken as a neighbour pitch class to the two upper-voice C’s, and reduced out on that count, the pcs exhibit an ic1 dyad. The deeper levels resulting from the reduction algorithm, which we will discuss below, would confirm or dispute such an interpretation.

The surface of the two-part approach is worth further investigation. For example, the pitch collection in each part reveals some interesting implications, shown in Figure 4.20. While the upper part exhibits a G-major scale without the 6th scale degree, the lower part strongly suggests c#-minor (with the parenthesized Eb conceived as D#) ending with a descending C# augmented triad. The G major and c# minor tonal residues are contrasted by modal opposition, at the tritone.

Figure 4.20. Tonal Implications of Two Parts at R3



As is evident from Figure 4.17, the (012) set-class in R35 is reduced to an ic1 dyad at R353, which also has been mentioned in Figure 4.19. Thus, R35 with its set-class (012), and

R353 with its ic1 dyad seem to support the "stream segregation" approach discussed previously. R353**5** is very similar to R353; the only difference is that the repeating pitch is pruned in the last depth level. This pruned pitch C#4 is the only metrically weak note in its immediate environment: both C6 and C5 are on the downbeat.

Note that an approach that segregates a segment into upper and lower parts could be taken in earlier examples, which contain register extremities such as the Boulez (Figure 4.3) and Xenakis (Figure 4.5) examples. However, these examples lack a clear pattern formed by separate parts, such as occurs in the Webern example. Also, note that a tonal interpretation resulting from the application of the window algorithms will be discussed in a more extensive analysis presented in Chapter 6.

CHAPTER 5

THE LISTENER'S WINDOW: REDUCTION ALGORITHMS, MEMORY, AND THE LISTENING EXPERIENCE

So far the discussion of the algorithms has been confined to a score-based analytical context or a context in which the listener has a prior knowledge of the piece. The phenomenological implications regarding the evaluation of the medial cp in an "on the fly" listening context have been raised in Chapter 3 without any further discussion. The present chapter addresses the phenomenological and cognitive aspects of the window algorithms from a listener's point of view.[114]

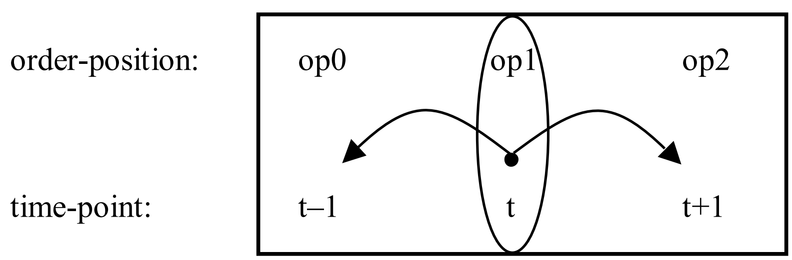
A Model for the 3-Window

Let us begin our discussion with modeling the experience of a listener who knows the piece and remembers accurately the temporally ordered collection of events. This listener's evaluation of the middle cp in a width-3 window is represented in Figure 5.1. Here, the dot represents the "time-point being evaluated," the arrowheads indicate the "remembered pitches," and the ellipse indicates the "reference point" (or the "evaluated cp"). For example, in Figure 5.1, the ellipse indicates that the medial pitch is the reference point, which means that op 1 is always the subject of the evaluation: op 1 is higher/lower than op 0; op 1 is higher/lower than op 2, etc. The single dot indicates that the evaluation (op 1 vs. op 0, and op 1 vs. op 2) takes place at time-point t. The arrowheads pointing to t-1 and t+1 indicate that op

[114] These two approaches are akin to what Jean-Jacques Nattiez refers to as "neutral level" and "esthesic" analyses in *Music and Discourse: Toward a Semiology of Music*, trans. Carolyn Abbate (Princeton: Princeton University Press, 1990).

0 and op 2 are remembered, respectively. Note that since the listener has prior knowledge and perfect memory of the piece, she relies on both her short-term and long-term memory while listening to the melody.[115] In other words, she evaluates the relative pitch height of op 1 at time-point t while hearing it (or imagining it), by relying on her short- and long-term memory of op 0 and on her long-term memory of op 2.[116]

Figure 5.1. 3-Window Model for a Listener with Prior Knowledge



Now let us imagine another listener who listens to the same window for the first time. This second listener's evaluation of the middle cp is shown in Figure 5.2. Here, the listener cannot fully evaluate op 1 upon reaching time-point t, since she does not know the succeeding event(s). For this listener op 1 can be evaluated, within a width-3 window, only after op 2 has been heard. This mode of listening can be referred to as "retrospective

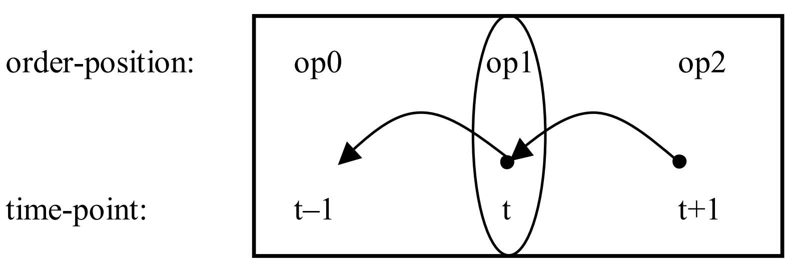
---

[115] Experiencing contour in real time but remembering it perfectly raises some interesting points. In one sense, it requires the listener to be encapsulated in the phenomenological present and at the same time to remember the events in the future which go beyond "now." The implications of the conceptual tension arising from a real-time experience and a perfect memory of what is to come are beyond the scope of this study. In any case, the remainder of the discussion in this chapter will involve a listener with no prior knowledge of the piece, modeling an ideal on the fly experience.

[116] Note that for the sake of clarity, the notion of expectation is omitted from the discussion. In the approach presented here, any given time-point involves a binary understanding of future events that is a lack of information (regardless of what is expected to follow) or a presence of information.

listening," because the listener can only consider the context around op 1 after reaching op 2, and thus must construct the context around op 1 in retrospect.[117] As a result, in this mode of listening there are two time-points of evaluation in comparing the middle cp with the preceding and succeeding cps: at time-point t, the listener compares the middle cp with the preceding cp; at the time-point t+1, the listener compares the middle cp with the succeeding cp, in retrospect. In both of the evaluations (taking place in two different time-points), the listener can only rely on her short-term memory depicted by the backward-pointing arrows. A backward-pointing arrow represents the short-term nature of the listener's memory and a forward-pointing arrow represents the long-term nature of the listener's memory. Since our focus here is on a listener without a score or a prior knowledge of the piece, the remainder of the figures will include only backward-pointing arrows, representing a listener on the fly who can only rely on her short-term memory in comparing cps.

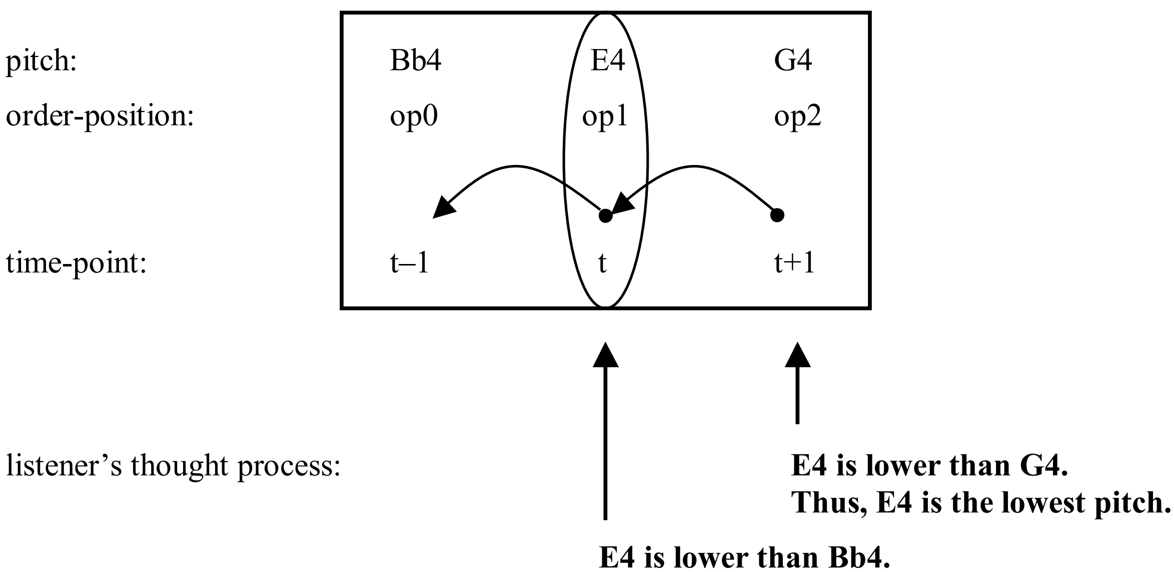
Figure 5.2. 3-Window Model for a Listener without Prior Knowledge



[117] Retrospective listening is essential to an understanding of a number of musical concepts such as modulation, harmonic function, etc. For instance, the pivoting function of a pivot chord can only be comprehended after the succeeding chord is heard. At the point of hearing the following chord, the listener reevaluates the pivot chord. Similarly, in this model, at the point of hearing the following cp, the listener reevaluates the medial cp.

Figure 5.3 demonstrates this mode of listening, by using the first full window (<402>) of *A 6 Letter Letter* by Elliot Carter (see also Figure 4.1).

Figure 5.3. 3-Window on csubseg <Bb4, E4, G4>



In this mode of listening, comparisons are made only between the contiguous order-positions and thus only the preceding event needs to be remembered. In Figure 5.3, when the listener hears E4 at time-point t, she takes this cp as her reference point and recognizes that it is lower than the previous cp (Bb4). This information is retained in her short-term memory. As she hears G4 at time-point t+1, she recognizes that her reference point, E4, is also lower than the present cp heard. Note once again that the memory requirement for each evaluation—at time-points t and t+1—is only one contiguous event (i.e. cp).[118]

[118] Note that the evaluation regarding Bb4 and E4 (i.e. "E4 is lower than Bb4") is made and stored at time-point t, and thus the memory requirement to recall it from t+1 is also one event.

The models put forward in Figures 5.1 and 5.2 correspond to the current application of the window algorithms, as discussed in the previous chapters, and since both models involve a reference point that is always in the center, they can be referred to as Center-Frame (CF) model. The distinction between the two CF models is the distinction between a listener who can resort to her long-term memory (or the score), and a listener who cannot do this and thus is bound to her short-term memory, which is entirely based on her immediate aural experience.

Short-Term Memory

Short-term memory, henceforth STM, is one of the three memory processes, along with echoic memory and long-term memory. Although the former functions on a smaller time scale and the latter functions on a larger time scale in comparison to STM, there has not been a clear consensus in the empirical literature regarding their precise boundaries. Arguably, the most widely acknowledged hypothesis regarding the boundaries of STM is known as the hypothesis of "magical number seven" by Miller (1956), which asserts an average of seven different elements plus or minus two (7±2) as the limit of STM.[119] In addition to "event" limits of STM, "duration" limits have also been subject of research. The duration span of short-term memory is believed to be an average of 3-5 seconds.[120] However, the boundaries may be as short as 2 seconds[121] and as long as 12 seconds.[122] At this point it is important to note that because of the potential variety in average rhythmic values and tempi

---

[119] George A. Miller, "The Magical Number Seven, Plus or Minus Two: Some Limits on Our Capacity for Processing Information," *Psychological Review* 63 (1956): 81-97.
[120] Bob Snyder, *Music and Memory: An Introduction* (Cambridge: MIT Press, 2000), 50.
[121] Richard L. Marsh, Marc M. Sebrechts, Jason L. Hicks, and Joshua D. Landau, "Processing Strategies and Secondary Memory in Very Rapid Forgetting," *Memory & Cognition* 25 (1997): 173-181.
[122] Bob Snyder, *Music and Memory*, 50.

between passages, our aural experience of two distinct 2-second-time-spans (i.e. the lower durational boundary for STM) or two distinct 12-second-time-spans (i.e. the higher boundary for STM) may be very different.

Generally speaking, the window and the segment seem to coincide with the opposite ends of this spectrum (specifically duration-wise). Note that although the lower boundary for duration (2-3 seconds) is reasonable for the lower boundary of 5 musical events to take place, the higher boundary for duration (12 seconds) is not necessarily compatible with the higher boundary for the number of events, which is 9. Effectively, one would expect more than 9 events to take place within the 12-second time-span, on average. In any case, the average window-size of 3 or 5 cps seems to lie on the lower boundary, while the average segment-size, which is rather flexible, seems to lie on the higher boundary.

Note that the 3-window alone is just below the event boundary and is also likely to be below the duration boundary. Interestingly, the concept of "focal awareness" supported by Mandler (1975), Klatzky (1984), Posner and Raichle (1994), but more importantly by Cowan (1995), indicates that it is possible for up to three elements to be present in the focus of consciousness without rehearsal.[123] If we are to assume this is the case with the 3-window, especially when the duration of the window is relatively short, the evaluation of the medial cp could be taking place almost simultaneously, without a genuine involvement of STM. Yet, the 5-window is within the lower boundary of STM and the 3-window is acceptably close to the boundary and thus can be regarded as taking place in the lower boundary of the STM.

---

[123] George Mandler, *Mind and Emotion* (New York: Wiley, 1975); Roberta L. Klatzky, *Memory and Awareness: An Information Processing Perspective* (New York: Freeman, 1984); Michael I. Posner and Marcus E. Raichle, *Images of Mind* (New York: Scientific American Library, 1994); Nelson Cowan, *Attention and Memory: An Integrated Framework.* Oxford Psychology Series Vol.26 (New York: Oxford University Press, 1995).

The respective locations of the 3- and 5-windows and an average-sized segment with regard to the STM boundaries are presented in Figure 5.4.

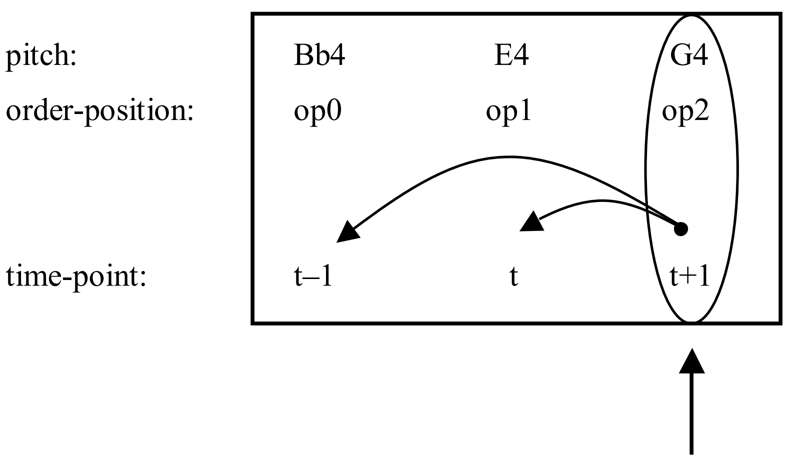
Figure 5.4. Placement of 3- and 5-Windows on the STM Scale

Interestingly, the concept of "focal awareness" implies a chunk-based evaluation rather than an event-based one as it asserts three elements to be present in our awareness simultaneously. Our current approach adopts an event-by-event evaluation. Nevertheless, the possibility of evaluating chunks rather than events is not entirely dismissed by the models proposed in this chapter. Since there is substantial overlap between successive windows (i.e. op 1 and op 2 in one window become op 0 and op 1 in the next one), it is possible to accommodate a chunk-based evaluation rather than an event-based evaluation. We will discuss the implications of overlapping windows later in the chapter; until then, our focus will be on individual windows and the evaluation of individual cps.

Alternative Models for the 3-Window

Now let us consider further possibilities for modeling the listener's window. In our previous model, the "point of reference" is located at the center of the window. In other words, the medial cp is the one that is evaluated with regards to other cps and is the subject of the evaluation statement. It is also possible to develop two additional modes of listening, in which the point of reference is not the medial cp. The first one, illustrated in Figure 5.5, takes the last cp as the reference point and compares it to the preceding cps, and the second one, illustrated in Figure 5.6, takes the initial cp as the reference point and compares it to the following cps.

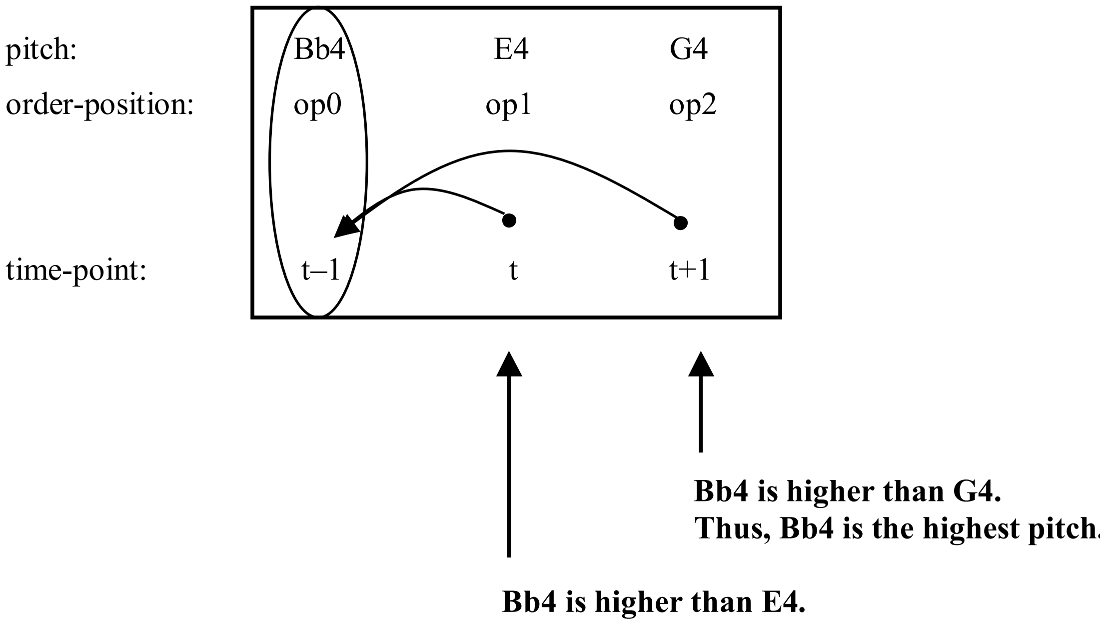
Figure 5.5. The First Alternative 3-Window Model



listener's thought process: **G4 is lower than Bb4.**

G4 is higher than E4.
Thus, G4 is neither the highest nor the lowest pitch.

Figure 5.6. The Second Alternative 3-Window Model



The model represented in Figure 5.5 is a Right-Frame (RF) model since the referential cp falls on the right-of-center within the frame. In this model, the listener evaluates op 2 (reference point) at the time-point of t+1. As she hears G4, she recognizes that this pitch is lower than Bb4 and higher than E4, making it neither the lowest nor the highest pitch. In this model, for an accurate evaluation she has to recall not only the preceding cp but also the one before it. Also, note that both of the comparisons (op 2 vs. op 1, and op 2 vs. op 0) take place on the same time-point, t+1. By contrast, in Figure 5.6, which represents a Left-Frame (LF) model, the listener evaluates op 0 (reference point) at two different time points: she compares op 0 and op 1 at time-point t and op 0 and op 2 at time-point t+1. At time-point t, the listener has to recall the preceding cp and similar to the model represented in Figure 5.5, at time-point t+1, she has to recall the cp before the preceding cp.

Note that in all of these approaches to listening, there is assumed to be one reference point per window. This is due to the fact that each window is assigned to evaluate the status of only one cp as outlined in Chapter 3. Having more than one reference point would simply mean that there is more than one window due to our (a priori) definition of the window.

All three models (CF, RF, LF) discussed so far seem to be plausible for modeling the listener's thought process involving STM. Which among the three modes of listening is a more accurate representation of our baseline ability to hear and recognize contours is a question worth investigating further. Here we will seek the model that requires the smallest memory span and the least amount of information to be processed at any given time-point. Note that this approach does not necessarily signify the normative mode of listening but computationally the most efficient mode of listening. At this point it is crucial to point out that although we will touch upon cognitive issues with regard to different models, the following discussion could perhaps be better understood as an exploration of the computationally most efficient model rather than an assertion of cognitively most suitable model. The objective of the chapter is by no means to provide conclusive cognitive constructs but to present a certain way of thinking about our listening experience (i.e. computationally most efficient) which could be tested experimentally by psychologists, if desired.

First, let us look at the number of evaluation points (represented by dots) in all three models. In the CF (Figure 5.3) and LF models (Figure 5.6), there are two time-points in which the evaluations take place, and in the RF model (Figure 5.5), there is only one time-point in which the evaluations take place. It is evident that the RF model is more demanding than the CF and LF model as it requires twice as much information to process at one time-point. The CF and LF models ease the amount of work on any time-point by distributing it to two time-points.

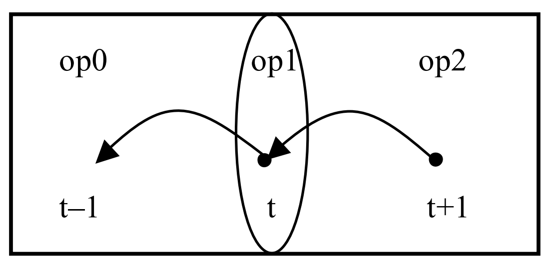
Secondly, let us look at the memory span that is required for each model. In the CF model, the total distance remembered, is less than the total distance remembered for both the RF and LF models. Also, in the CF model, a span of one event for any given time-point suffices, whereas the RF and LF models require a span of two events. In other words, the CF model is less demanding on memory and thus, is the computationally most efficient mode of listening. Figures 5.7, 5.8, and 5.9 summarize these observations regarding the memory span and maximum number of comparison per time-point.

Figure 5.7. CF Model Characteristics

CF Model

order-position:

time-point:



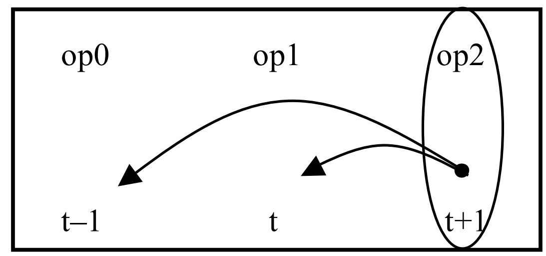
distance of the remembered event:	t–(t–1)=1	(t+1)–t=1
number of comparisons:	1	1
<u>total distance remembered</u>:	**2**	
<u>maximum distance recalled per time-point</u>:	**1**	
<u>maximum # of comparison per time-point</u>:	**1**	

Figure 5.8. RF Model Characteristics

RF Model

order-position:

time-point:



distance of the remembered event: (t+1)–t=1

(t+1)–(t–1)=2

number of comparisons: 2

<u>total distance remembered</u>: **3**

<u>maximum distance recalled per time-point</u>: **2**

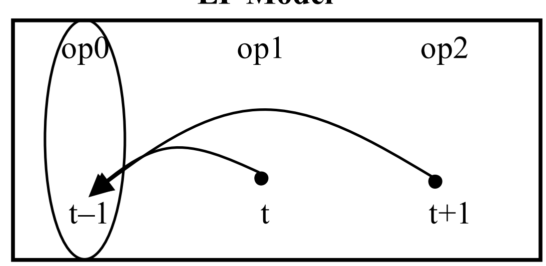
<u>maximum # of comparison per time-point</u>: **2**

Figure 5.9. LF Model Characteristics

LF Model

order-position:

time-point:



distance of the remembered event: t–(t–1)=1 (t+1)–(t–1)=2

number of comparisons: 1 1

<u>total distance remembered</u>: **3**

<u>maximum distance recalled per time-point</u>: **2**

<u>maximum # of comparison per time-point</u>: **1**

As is evident from Table 5.1, in terms of information processing, the CF model is the least and the RF model is the most demanding of the three models.

Table 5.1. 3-Window Model Comparison

Model	Total Distance Remembered	Max Distance Recalled per Time-Point	Max # of Comparison per Time-Point
CF	2	1	1
RF	3	2	2
LF	3	2	1

Models for the 5-Window

It is possible to readjust the width-3 window models for width-5 windows. Figure 5.10 illustrates all five models, namely RRF, RF, CF, LF, LLF, in order.

Figure 5.10. 5-Window Models

RRF Model

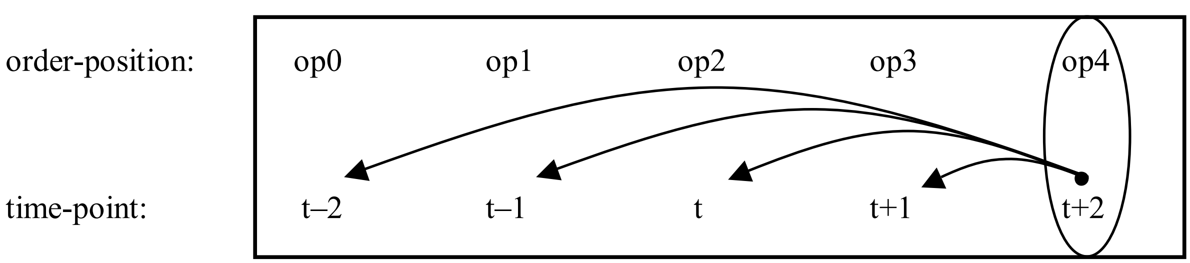
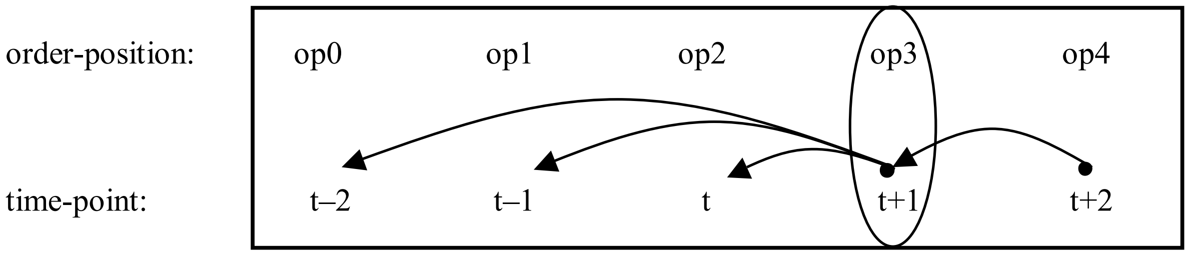
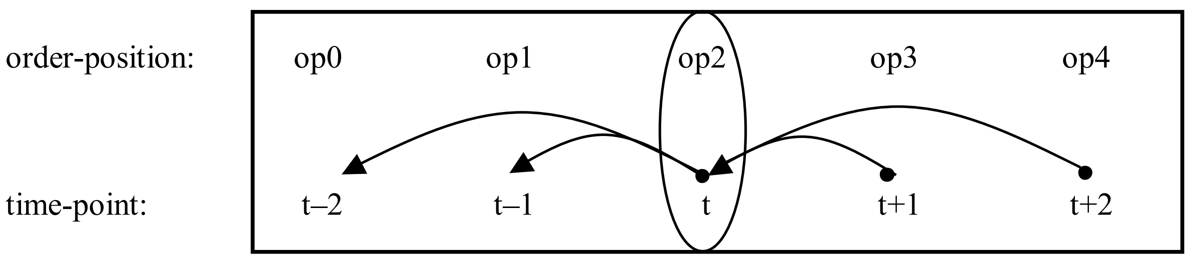


Figure 5.10 (cont.) 5-Window Models

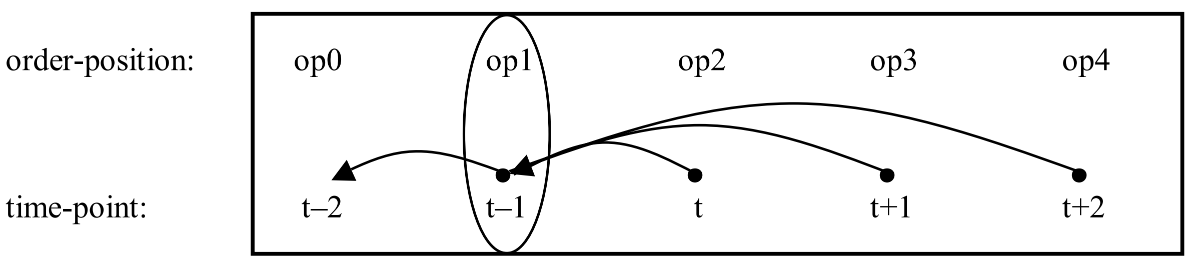
RF Model



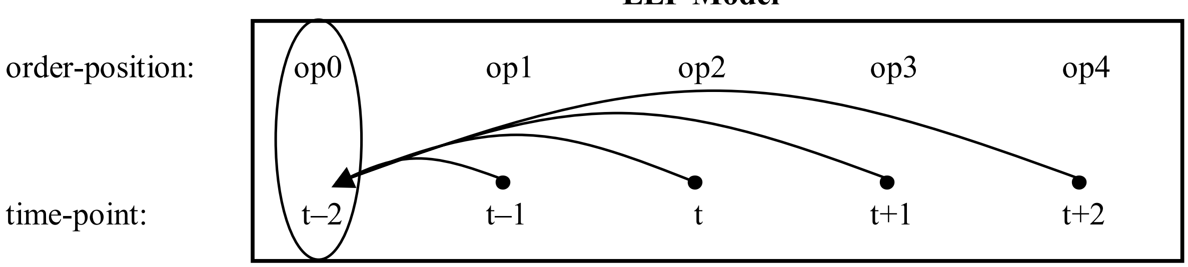
CF Model



LF Model



LLF Model



As the window size increases, the memory requirement and the amount of information to process at a given time-point also increases.[124] For example, the RRF model not only requires a memory of four events prior to t+2, but also requires the evaluation of all cps simultaneously. Although the RF model is less demanding than the RRF model, it still requires the listener to remember all three events preceding t+1. Yet, in this model the process load is less than in the RRF model. The CF model, which is the preferable model for the 3-window, is much less demanding on listener's cognitive faculties compared to the RRF and RF models: a maximum of two preceding events need to be recalled, and at time-point t two evaluations take place whereas at time-points t and t+1 only one evaluation for each takes place. Although the LF model requires a memory-span of three events at time-point t+2, which is more than the CF model, it does not demand the listener to evaluate two cps at any time-point. Similarly, LLF also has a maximum number of evaluation of one, but requires a memory of four events, just like RRF. It is important to note that our assessment of these models is based on single windows. A discussion of real-time processing of "successive" windows will be presented shortly.

---

[124] Let us point out once again that the computationally most efficient model does not necessarily imply the cognitively preferred model. The issue is too complex to draw such a simplistic conclusion. However, there seems to be agreement among psychologists and neuroscientists that temporal distance, attentional demand, and the complexity of the type of evaluation define the cognitive capacity of performing a short-term memory task. See, for example, Nelson Cowan, *Working Memory Capacity*. Essays in Cognitive Psychology (Psychology Press, 2005); L. Mark Carrier, and Harold Pashler, "Attentional Limits in Memory Retrieval," *Journal of Experimental Psychology: Learning, Memory, & Cognition* 21 (1995): 1339-48; Graeme S. Halford, William H. Wilson, and Steven Phillips, "Processing Capacity Defined by Relational Complexity: Implications for Comparative, Developmental, and Cognitive Psychology," *Behavioral and Brain Sciences* 21 (1998): 803-31; Graeme S. Halford, et al., "How Many Variables Can Humans Process?," *Psychological Science* 16.1 (2005): 70-76; Siegfried Lehrl, and Bernd Fischer, "The Basic Parameters of Human Information Processing: Their Role in the Determination of Intelligence," *Personality and individual Differences* 9 (1988): 883-96. Although the theory of "cognitive load" is geared towards educational psychology it employs the concepts of information processing and working memory load. See John Sweller, Jeroen Van Merriënboer, and Fred Paas, "Cognitive Architecture and Instructional Design," *Educational Psychology Review* 10 (1998): 251-96; Fred Paas, et al., "Cognitive Load Measurement as a Means to Advance Cognitive Load Theory," *Educational Psychologist* 38.1 (2003): 63-71.

Now, let us present the observations made above by calculating the total distance remembered, the maximum distance remembered per time-point, and the maximum number of comparison per time-point:

Table 5.2. 5-Window Model Comparison

Model	Total Distance Remembered	Max Distance Recalled per Time-Point	Max # of Comparison per Time-Point
RRF	10	4	4
RF	7	3	3
CF	6	2	2
LF	7	3	1
LLF	10	4	1

As is evident from Table 5.2 the least efficient model is the RRF model with the highest number of total distance, maximum distance recalled per time-point (both tying with LLF), and maximum number of evaluation per time-point. Although the LLF model has only one maximum number of evaluation per time-point in comparison to two for the CF, its total distance remembered and maximum distance remembered per time-point are considerably higher than CF. In fact, it holds the highest number for these criteria along with the RRF model. Similar to the 3-window models, once again, the CF model seems to be the best fit, along with the LF model. The LF model has the advantage of processing at most one evaluation per time-point. However, at no time-point does the CF model require a memory-span of three events, which is the case with the LF model at time-point t+2. In addition, the total distance remembered in the LF model is slightly higher than in the CF model. Thus, although either model could serve as a representation of the listener's cognitive processes, the CF model seems to hold a slight advantage over the LF model.[125]

An interesting observation that arises from the table is that the Left-Frame models, namely LF and LLF have an advantage over the Right-Frame models, namely RF and RRF. This is due to the fact that the Left-Frame models start evaluating earlier on, distributing the evaluation more or less evenly throughout the time-span, whereas the Right-Frame models leave the evaluations towards the end which results in heavier process load, as the same amount of evaluation has to be made in a shorter time-span.

Note that it is also possible to adopt an even-numbered window (i.e. 4-window, 6-window) approach, which we haven't discussed so far, but such an approach would be devoid of CF models. Thus, this approach does not seem to be particularly intuitive or fruitful, given that the CF model has proved to be the preferable model for the 3- and 5-window algorithms.

Successive Windows

Thus far our discussion has been limited to a single window; however, the reduction algorithms comprise multiple windows. Now let us compare the proposed models in a context of successive windows, as in a real-time reduction process. In the 3-window algorithm the medial and last cps of a window always overlap with the first and medial cps of the following window, regardless of the model. These shared cps are illustrated by dashed vertical lines in Figure 5.11.

[125] At this point, it is interesting to note that the RF 3-window and the LF 3-window models are, in fact, relevant to each side of the CF 5-window model. The leftmost three cps of the CF 5-window correspond to the RF 3-window and the rightmost three cps of the CF 5-window model correspond to the LF 3-window.

For a complete evaluation of the models, the shared cps should be taken into account, along with the different evaluations required in each case. Figure 5.12 illustrates all three 3-window models with successive windows. In the CF model, the successive windows share an arrow, denoted by dashed vertical lines. By contrast, in the RF and LF models, no arrow is shared between successive windows. The arrow overlap in the CF model indicates that only one new arrow is required in each new window, which eases the cognitive burden. Moreover, the new arrow points back only to the most recent event. Note that the evaluation of the relationship between op 1 and op 2 is changed in the second window since the subject (point of reference) changes. Here, the listener evaluates the relation between op 1 and op 2, and remembers this evaluation in the next time-point. Recalling the evaluation from the preceding time-point is cognitively more advantageous than recalling the pitch from the preceding time-point and evaluating its relationship with the current pitch. Even though in both cases the listener needs to remember data from the preceding time-point, only in the second case does she need to evaluate data (i.e. cps). Since the cognitive effort required in a model involving shared arrows is significantly lower than a model lacking shared arrows, once again, the CF model seems to be the best model to represent the listener's cognitive processes.

Figure 5.11. Shared cps in Successive 3-Windows

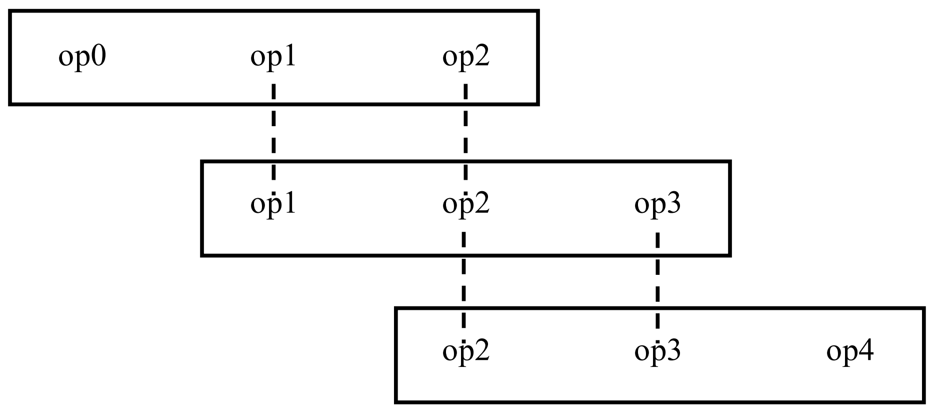
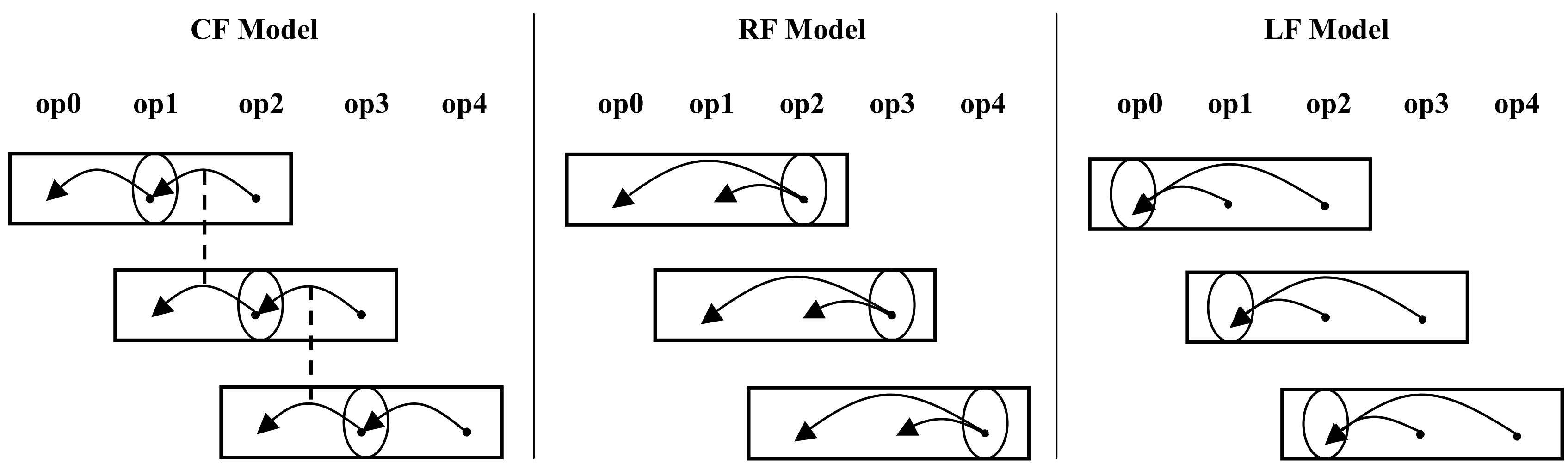


Figure 5.12. Shared Arrows in 3-Window Models



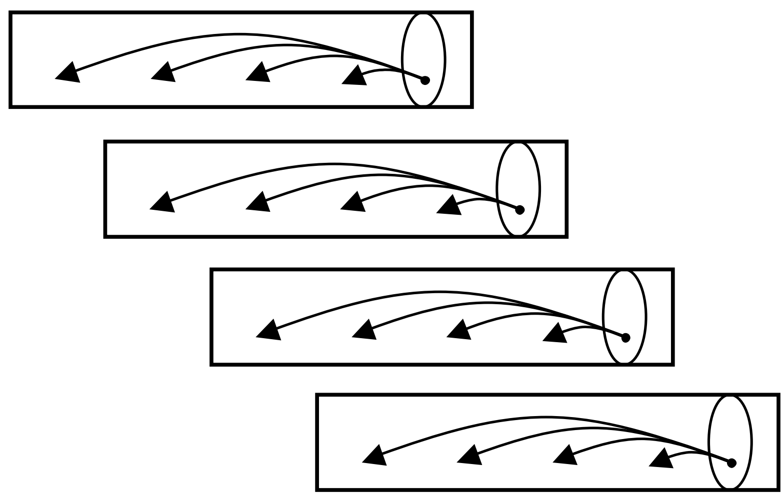
The same principle applies to the 5-window models, as illustrated in Figure 5.13. Here all shared arrows between the fourth window and the rest of the windows are marked by dashed lines; arrows that may be shared by other windows but do not involve the last window are omitted.[126] As is evident from the figure, the RRF and LLF models involve no shared arrows, while the RF and LF models each involve one. Once again, the CF model involves the highest number of shared arrows: an arrow of length one with the preceding window and an arrow of length two with the window two stages previous. Note that in the 5-window CF model, one of the arrows (involving op 3 and op 5) is shared by a non-adjacent window. In the CF model the relationship between op 3 and op 5 needs to be retained in memory for two time-points; similarly, in all other models the pitch corresponding to op 3 needs to be remembered. In other words, no matter how one counts the significance of memory, in all models data need to be retained for two time-points. However, in the CF model the listener does not need to compare op 3 and op 5 since this information (which was retained for two time-points) is immediately available, whereas in rest of the models op 3 and op 5 need to be evaluated. Therefore, the 5-window CF model appears to be the most suitable model for cognitive efficiency in the evaluation of successive, overlapping windows.

[126] Note that a 3-window cannot possibly share more than one arrow and a 5-window cannot possibly share more than four arrows, with any of the previous windows. Thus, four successive windows are adequate to evaluate the shared arrows of the last 5-window.

Figure 5.13. Shared Arrows in 5-Window Models

RRF Model

op0 op1 op2 op3 op4 op5 op6 op7



RF Model

op0 op1 op2 op3 op4 op5 op6 op7

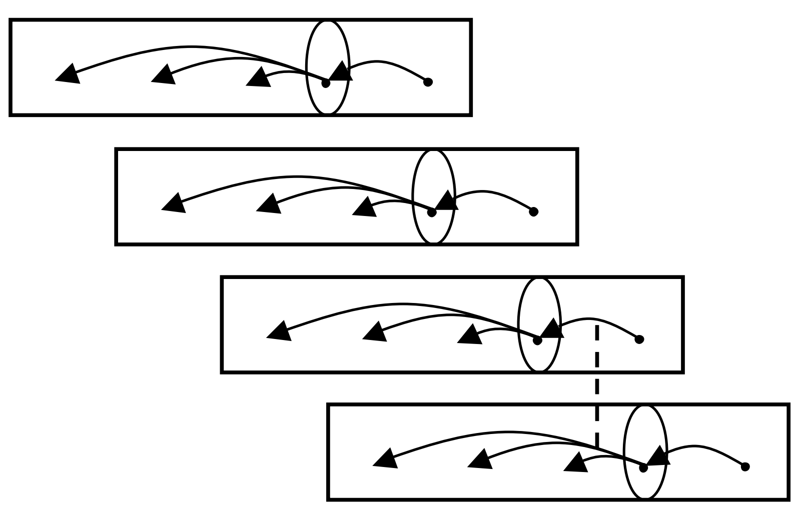
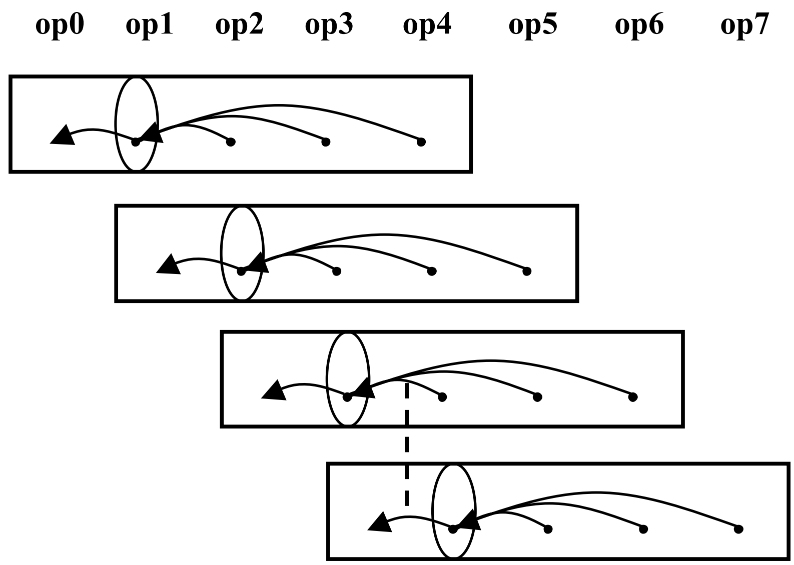


Figure 5.13 (cont.) Shared Arrows in 5-Window Models

LF Model



LLF Model

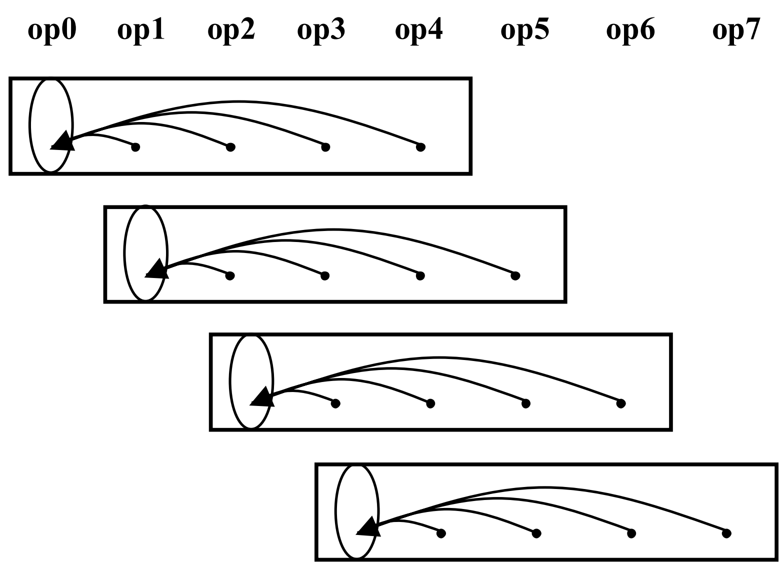
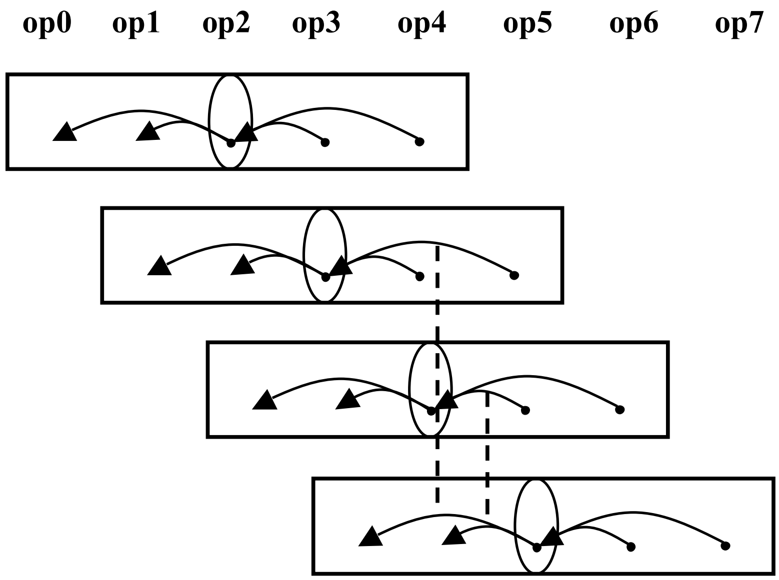


Figure 5.13 (cont.) Shared Arrows in 5-Window Models

CF Model



Hearing Depth Levels

One of the most interesting features of the window algorithms is their implication of hearing depth levels due to their recursive nature. First of all, it is important to point out that an "on the fly" reduction requires a substantially higher cognitive capacity compared to what we have been discussing so far and it is vital to keep in mind that a discussion of hearing depth levels is highly speculative.

Let us start our discussion of this issue by focusing on hearing the contour framework. According to our principle of "choosing the smallest window-size possible for each depth level," the listener is to hear various depth levels and their reductions simultaneously. This requires the listener to apply windows of various sizes to various depth level segments on the fly, which undoubtedly requires an extremely complex web of cognitive processes. For example, even a two-depth level reduction, such as the segment from Elliot Carter's *A 6 Letter Letter* (Chapter 4, Figure 4.1), requires the listener to assess each time-point from both the 3-window and the 5-window perspective. Furthermore, the 5-window evaluation (R3**5**(S)) should not be based on the original segment but the segment resulting from the 3-window reduction (R3(S)). This is due to the fact that an application of the 5-window on the original segment may be different than an application of the 5-window on the already reduced segment (i.e. R5(S) vs. R35(S)).[127] In short, according to this model, the listener should somehow evaluate each time-point based on the 3-window as well as on the 5-window, which is based on the 3-window segment. One could only imagine the complexity of this approach on a three-depth level reduction (for example, R355(S)), resulting in a 3-window evaluation, and a 5-window evaluation based on the 3-window evaluation, and a 5-window evaluation based on the 5-window evaluation based on the 3-window evaluation, all at the same time for every single time-point.

An alternative model, which is less cumbersome, is the listener hearing a larger window rather than multiple smaller windows. This model proposes that if a listener on the fly is hearing the contour framework, it is likely that she is hearing a larger window rather than hearing various depth levels and applying different smaller windows for each depth level. In effect, the listener is hearing the smallest possible window, which has the capability to reduce the entire segment at once. Such a listener applies R**7**(S) on the fly, rather than R3**5**(S), in order to hear the contour framework. Note that this does not necessarily mean that

---

[127] This issue has been discussed in Figures 4.7, 4.8, and Table 4.1 in Chapter 4.

the smaller-size windows are not implied in the bigger-size window and that the listener cannot hear the inner-depth levels.

As indicated previously, it is important to maintain a certain amount of skepticism in hearing deeper levels on the fly. We will keep using intermediate-depth levels in our analyses since the application of the algorithms is not limited to a listener on the fly and these intermediate-depth levels provide valuable insights into the contour segments, as we will see in the following chapter. Similarly, although the size of the segments in analytical contexts will commonly be in concordance with the higher boundary of STM, a score-based analysis, or an analysis which considers a listener who has prior knowledge of the piece and thus can compare events of longer spans due to her long-term memory, may refer to segments of higher cardinality.

Lastly, let us point out that it is possible to learn to hear certain contour reduction functions and reach beyond our natural, effortless way of hearing a segment. In other words, in our first hearing of a melody we might focus our attention on a certain window, for example R3. If we can hear the retained (and pruned) pitches in R3, next we might try to hear the application of a 5-window on the retained pitches (R35). This would require us to focus on the melody from a more global perspective as we are listening along. However, if we can fully recite the R3 melody, it would not be very difficult for us to hear the application of the 5-window. Although hearing the third depth level is considerably more difficult, if this level is the last (i.e. contour framework) or the second last depth level,[128] then we could choose to focus our attention primarily on the highest and lowest cps in the melody and try to hear their relation to other pitches. Assuming at this point we are familiar with the melody, when we

[128] Note that the majority of the examples in this study do not go beyond three depth levels.

hear a relatively very high (or very low) pitch, we could try to assess whether the pitch is higher (or lower) than a number of preceding cps (with the help of our short- and long- term memory) and succeeding cps (with the help of our long-term memory). Once we are comfortable with hearing a certain contour reduction path, we could try to hear a different reduction path (say, R53, instead of R55) by following the same strategy. At this point, we could compare the reduction paths and possibly realize that a certain path is easier to hear.[129] This ear-training strategy would help us learn to hear a melody in new ways.

It is important to emphasize that the technology proposed in this study is not merely a modeling of how we passively hear melodies but also a tool in enriching our musical understanding by actively focusing on certain aspects of the melodies.

[129] Other musical parameters would likely to play an important role in our preference.

CHAPTER 6

AN ANALYTICAL APPLICATION: THE *HAUPTSTIMMEN* IN SCHOENBERG'S STRING QUARTET NO.3, OP. 30, I

Introduction

A contour approach to Schoenberg's oeuvre is particularly suitable given his well-defined themes clearly indicated as *Hauptstimmen*.[130] This chapter demonstrates the analytical application of the window algorithms to the *Hauptstimmen* of the first movement of the Third String Quartet, op. 30.[131] The chapter is divided into two main sections, which present distinct analytical approaches to the movement. The first section involves the contour reduction process and the intervallic content of successive depth levels. A special emphasis is placed on the comparison of the reduced *Hauptstimmen* based on their interval prominence. The second section explores whether the reduced pitches suggest a tonal orientation that might support the traditional sonata design of the movement. A contour-based approach to

[130] Michael Friedmann analyzes excerpts from Schoenberg's other twelve-tone compositions, such as the Five Piano Pieces, op.23 (Waltz), the Phantasy, op. 47, the Suite, op. 25 (Mennuet; Trio), and the Klavierstueck, op. 33b. He compares and contrasts contours based on the certain properties of their content. See Friedmann, "A Methodology for the Discussion of Contour." The present study adopts a rather different approach, which involves reducing contours and investigating various implications of the intermediate reduction levels and the resultant contour frameworks.

[131] This movement has received considerable analytical attention over the last two decades. See, for instance, Carl Dahlhaus, "Arnold Schönberg: Drittes Streichquartett, Op. 30," *Melos* 50.1 (1988): 32-53; Ethan Haimo, *Schoenberg's Serial Odyssey: The Evolution of His Twelve-Tone Method, 1914-1928* (New York: Oxford University Press, 1990), 149-61; Reynold Simpson, "New Sketches, Old Fragments, and Schoenberg's Third String Quartet, Op. 30," *Theory and Practice* 17 (1992): 85-101; Jeff Nichols, "Metric Conflict as an Agent of Formal Design in the First Movement of Schoenberg's Quartet Opus 30," in *Music of My Future: The Schoenberg Quartets and Trio,* eds. Reinhold Brinkmann and Christoph Wolff (Cambridge: Harvard University Press, 2000), 95-116.

the movement can potentially contribute to the significant body of literature that addresses tonal implications in Schoenberg's music.[132]

In the first movement of the quartet, there are 12 fundamental *Hauptstimmen* melodies that are presented in the exposition and recapitulation. The rest of the *Hauptstimmen* function on a more local scale since they contain four notes or fewer. The formal layout of all the *Hauptstimmen* in the movement (27 in total) is demonstrated in Figure 6.1. Here the *Hauptstimmen* I am designating as "fundamental" are denoted by **H**, and the rest are denoted by **h**.[133] The cardinality and measure numbers of each *Hauptstimme* are shown underneath the formal scheme.

As Figure 6.1 shows, there are twelve **h** melodies that contain four notes or less: h2, h4-h12, and h13-h14. Ten of the twelve short *Haupstimmen* are in the development section, a fact that correlates with the generally unstable and less thematic nature of the development section. Since the window algorithms cannot be meaningfully applied to these short segments that comprise fewer than four or five notes, and since these short *Hauptstimmen* do not present strong thematic presence, the analysis here will focus on the 12 **H** melodies. Note that although the last *Hauptstimme* of the composition is substantially longer than the

---

[132] See, for instance, John MacKay, "On Tonality and Tonal Form in the Serial Music of Arnold Schoenberg," *Canadian University Music Review* 8 (1987): 62-77; Christopher Orlo Lewis, "Mirrors and Metaphors: Reflections on Schoenberg and Nineteenth-Century Tonality," *19th-Century Music* 11.1 (1987): 26-42; Michael Cherlin, "Schoenberg and Das Unheimliche: Spectres of Tonality," *Journal of Musicology* 11.3 (1993): 357-73; Michael Cherlin, "Memory and Rhetorical Trope in Schoenberg's String Trio," *Journal of the American Musicological Society* 51.3 (1998): 559-602; Richard B. Kurth, "Moments of Closure: Thoughts on the Suspension of Tonality in Schoenberg's Fourth Quartet and Trio," in *Music of My Future: The Schoenberg Quartets and Trio,* eds. Reinhold Brinkmann and Christoph Wolff (Cambridge: Harvard University Press, 2000), 139-60; William E. Benjamin, "Abstract Polyphonies: The Music of Schoenberg's Nietzschean Moment," in *Political and Religious Ideas in the Works of Arnold Schoenberg,* eds. Charlotte M. Cross and Russell A. Berman (New York: Garland, 2000), 1-39; Richard B. Kurth, "Suspended Tonalities in Schönberg's Twelve-Tone Compositions," *Journal of the Arnold Schönberg Center* 3 (2001): 239-65.

[133] In the score they are all marked **H**.

segments in the development section, it is not denoted as **H** since its contour lacks variety due to the arpeggiations and its double stops create challenges for a contour analysis.

Figure 6.1. ***Haupstimmen*** **in Schoenberg's Third String Quartet, op. 30, i**

	Exposition								**Development**				**Recapitulation**				**Coda**
	First Theme					Transition		Second Theme					First Theme	Transition	Second Theme		
	H1	**H2**	**H3**	**H4**	**H5**	**H6**	**H7**	**H8**	h1	h2	h3	h4-12	**H9**	**H10**	**H11**	**H12** h13-14	h15
Card-	10	10	9	5	10	5	31	48	5	4	6	2-3	48	14	10	10 3	29
mm.	5-12	13-18	20-25	25-27	27-32	40-42	41-61	62-94	139-69				174-206	235-44	245-50	252-58	297-308

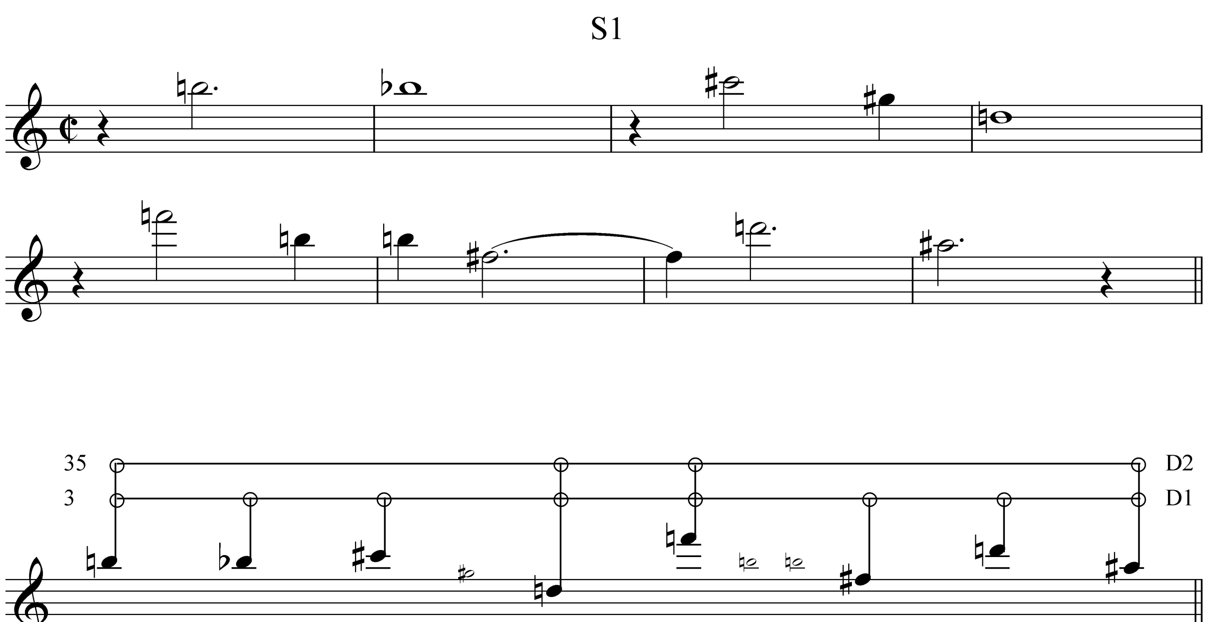
Now let us begin our analysis with an examination of each of the fundamental Ⱶ via the application of the window algorithms. We will be referring to these fundamental Ⱶ as "segments," henceforth S1, S2, S3, etc. Also, we will be adopting the notation presented in Chapter 3, in which all depth levels are presented on single staff.

Analytical Perspective 1: Intervallic Content at Successive Depth Levels

<u>*Segment 1*</u>

Figure 6.2 illustrates the initial segment and the reductions resulting from the application of the window algorithms.

Figure 6.2. Segment 1



A comparison between the original segment and R3(S1) shows that 2 notes, namely G#5 and B5 (repeated) are pruned with the initial 3-window reduction. It is interesting to observe that both notes are assigned to the shortest duration (i.e. quarter-note) of the segment and the onsets of both are in metrically weak positions. As illustrated in Figure 6.3, the pruning of G#5 and B5 results in consecutive pc dyads that contain the identical intervallic relationship of ic 1, i.e. <C#6, D5> and <F6, F#5>. Furthermore, the first six pcs alternate consecutive intervals of ic 1 and ic 3, often spanning ic 4 in consecutives pairs, i.e. <Bb5, C#6, D5>, <C#6, D5, F6>, <D5, F6, F#5>. Figure 6.4 illustrates the consecutive trichordal set-classes, in which 014 predominates (all trichords except the first and the last). In connection with the last 048, one also notes how the entire melody is structured by ic 1 perturbations with reference to the 048 pitch set {Bb5, D5, F#5, D6, A#5}.

Figure 6.3. Intervallic Content of R3(S1)

R3(S1)

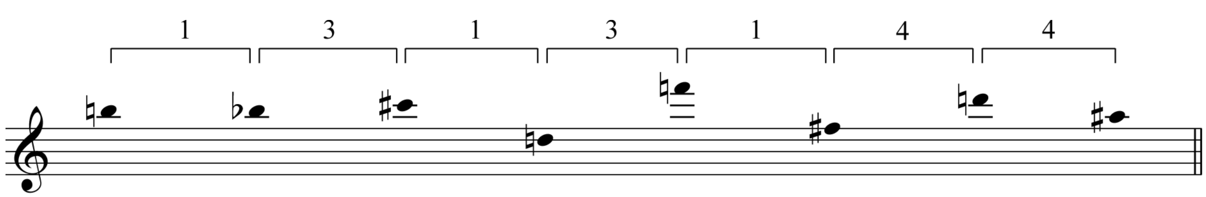
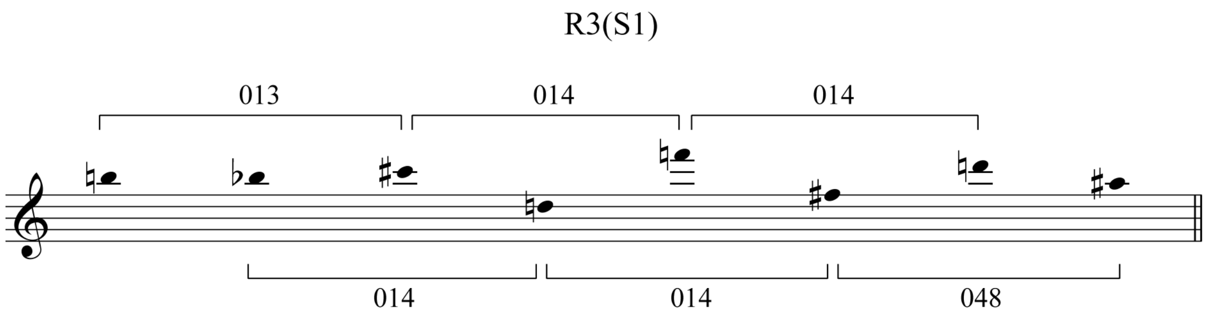
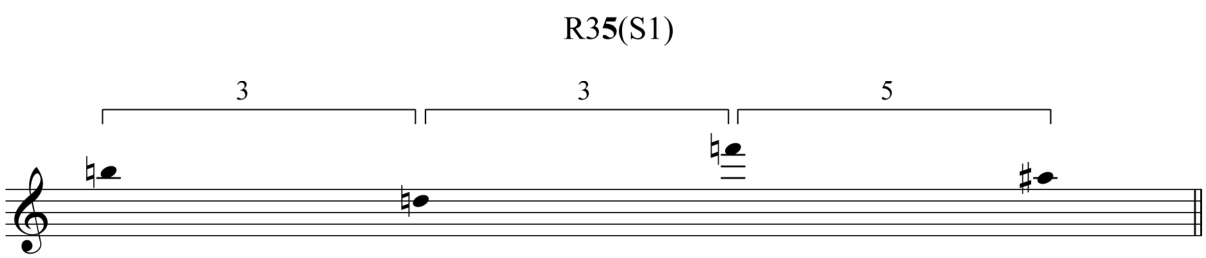



Figure 6.4. Trichords at R3(S1)



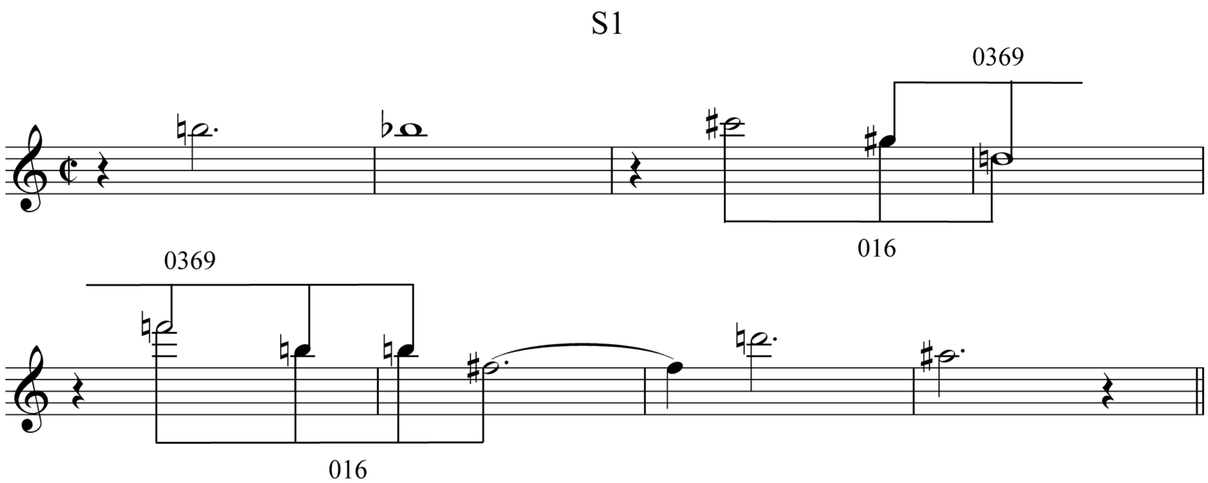
As is evident from the figures above, the "middle-ground" R3(S1) contour presents a consistent structure with repeating pc intervals and set-classes. Interestingly, the contour framework, i.e. R3**5**(S1), also maintains some intervallic uniformity, with two repeating ic 3s, as shown in Figure 6.5. Note however, how the 048 structuring is weakened.

Figure 6.5. Intervallic Content of R35(S1)



The 016 trichord (<B5, F6, A#5>) within R3**5**(S1) is rather interesting because the same trichord type presents itself twice in the middle part of S1, as shown in Figure 6.6. In addition, the consecutive ic3s that form an 0369 tetrachord in S1 (Figure 6.6) maintain their presence in R3**5**(S1) with the same pitches, but with the initial B5 rather than the later repeated ones. These observations demonstrate that the contour framework preserves some important characteristics of the surface. By contrast, R3(S1) emphasizes 014 trichords, which are not heard as adjacencies in S1, and the intermediate reduction therefore brings out something "hidden" that also continues as a presence in R3**5**(S1), in the 014 trichord <B5, D5, A#5>.

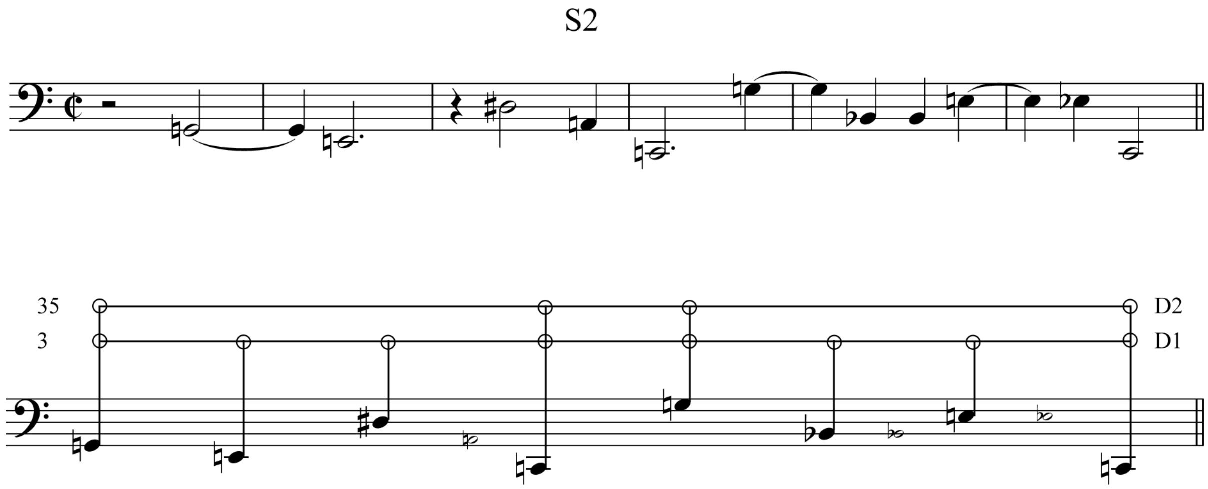
Figure 6.6. Surface Level Trichords and Tetrachords



Segment 2

The original segment and the three levels of reduction are shown in Figure 6.7.

Figure 6.7. Segment 2



Apart from the repeated Bb2 in m. 5, all the pitches pruned in R3(S2) and R3**5**(S2) are metrically weak, at least in the notated meter. (One notes also that the pitch onsets in the theme are generally assigned to weak metrical positions, in the notation.) However, the durational values of the pruned notes indicate a subtler correlation with the pruned pitches: the shortest durational values in the segment are quarter notes and there are four of them, three of which are pruned in the very first reduction. With the exception of E2 in m. 2, pitches with the second shortest durational value of half note are also pruned in the following reduction. In short, the first reduction decisively prunes the shortest notes and the second reduction selectively prunes the shortest notes remaining in the segment (G3 and the final C2 are retrieved by R3**5**(S2), but they are not longer than the pitches pruned at the second level). For this segment, the reduction technology demonstrates an interesting correlation between

Schoenberg's employment of register and duration: the weaker the pitch location in the contour domain (i.e. medial), the shorter the duration of the corresponding pitch.

The dyadic structuring of the retained pitches that was observed in S1 (see Figure 6.3 above) is found in this segment as well. Four dyads in R3(S2) involve similar structural relationships observed in R3(S1), as illustrated in Figure 6.8. R3(S2) deploys ic 3 in the same way that ic 1 was used in R3(S1), and both reduced segments end with ic 4. R3**5**(S1) ended with descending ic 5, the same interval featured twice, with repeating pcs, in R35(S2), as shown in Figure 6.9.

Figure 6.8. Intervallic Content of R3(S2)

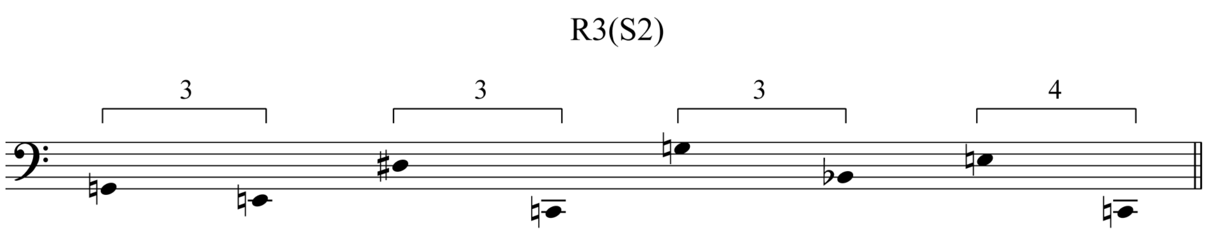
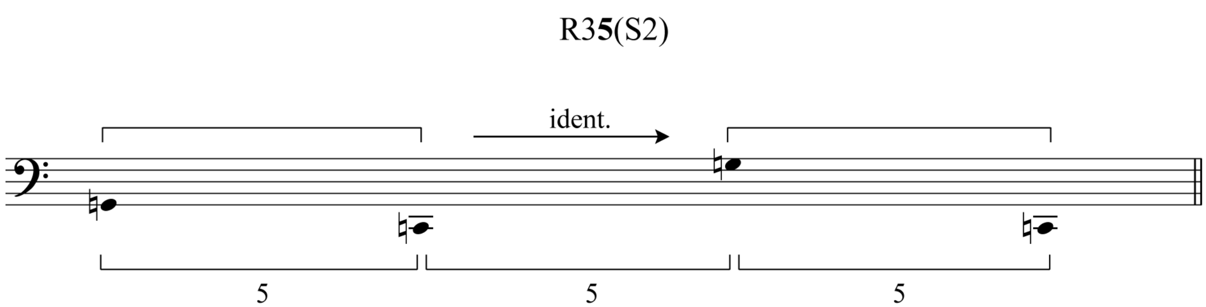


Figure 6.9. Intervallic Content of R35(S2)



The reiteration of the G-C descending fifth in this segment has potential tonal residues that will be considered later. For now, it is worth observing that S2 comprises two consecutive ordered pentachords <G, E, D#, A, C> and <G, Bb, E, Eb, C>. Ordering aside, they have 4 pcs in common (only A and Bb differ), and they are forms of set-class (01469) that are related under RI_7, which maps G and C to one another.[134] The facts that G and C begin and end each pentachord, are inversionally related, and are retained under R3**5**(S2) all reinforce one another.

Segment 3

The reductions of the third segment, illustrated in Figure 6.10, do not repeat intervals in the way just observed in reductions of S1 and S2. On the contrary, the emphasis seems to shift from intervallic uniformity to intervallic variety. This observation about contour is particularly interesting because S3 actually starts with the same five pcs as does S2, and the later part of S3 also has two pcs in common with the later part of S2, but the different contours of the two melodies bring out different intervals in their reductions. R3(S3) comprises six consecutive intervals and only one interval class is repeated (ic 3), as demonstrated in Figure 6.11. Note that the repeated ic 3, along with ic 1, has been the most prominent ic in reductions of the previous segments, which we have observed in Figures 6.3, 6.5, and 6.8 (R3(S1), R3**5**(S1), and R3(S2)).

[134] I7 also maps D# to E and A to Bb.

Figure 6.10. Segment 3

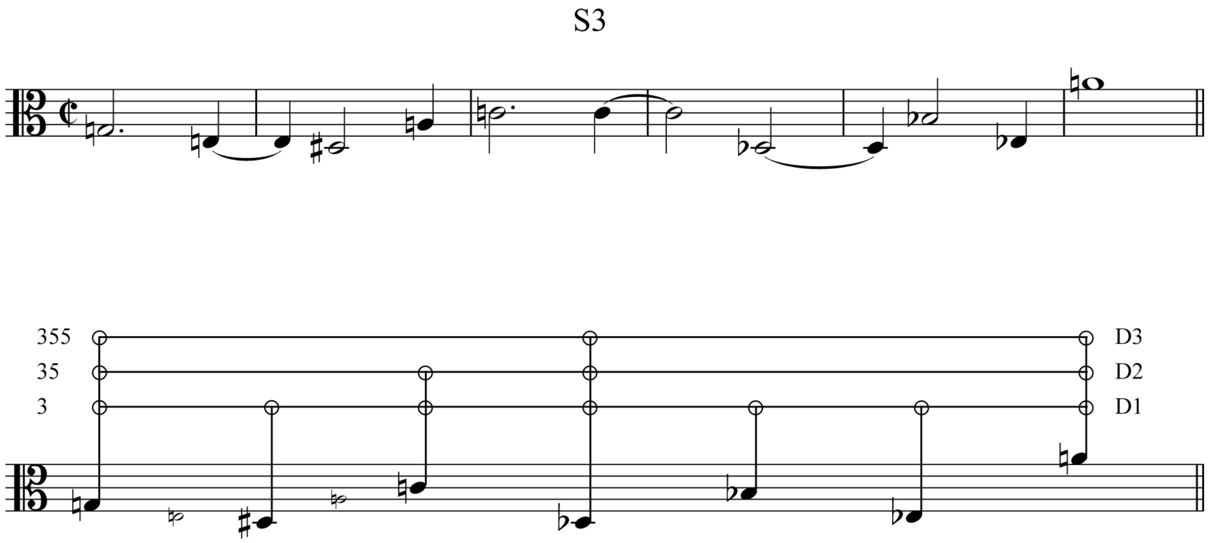
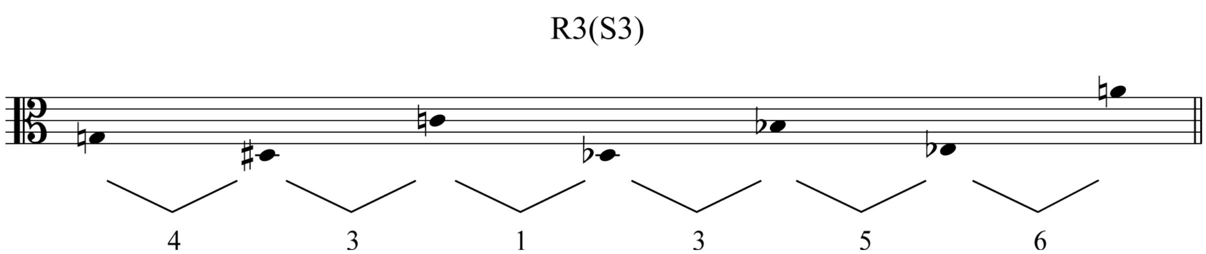


Figure 6.11. Interval-Classes at R3(S3)



Interestingly, the next reduction, R35(S3) excludes the ic 3s and the ic 6, and brings out ics <5,1,4>. Of these, only the ic 1 is a consecutive dyad retained from R3(S3). The last reduction, R35**5**(S), deletes C4 and thus introduces a new ic 6 and dismisses ics 1 and 5. The variety of ics, throughout different levels, constitutes a contrast with the first two segments.

Throughout the analysis, one should note that the intervals arising with each reduction are often (though not always) between pcs that were not adjacent at the preceding level. For example, in S3 the intervallic variety changes with each level as pitches are deleted and new consecutive intervals are formed at the next resulting level. The same is true for S1 and S2, in which intervallic uniformity dominates at each level, and also for the following segments.

Although the intervallic relationships are different between the first two segments and the third segment, the implicit principle of pruning of metrically and durationally weak pitches is instantiated in this segment as well.[135] The pitches pruned by R3(S3) and R35(S3) all begin on the second or fourth beat in the notated meter. It is also interesting to observe that the durational values of the pruned pitches in both levels are the shorter values within the segment, either quarter notes or half notes. All of the retained pitches in R35(S3) have longer durations than these. Note that since the durational values in this passage create a metric effect that is arguably different than the notated meter, here the concept of being "metrically weak" should be considered less decisive for pruning than being of shorter duration.

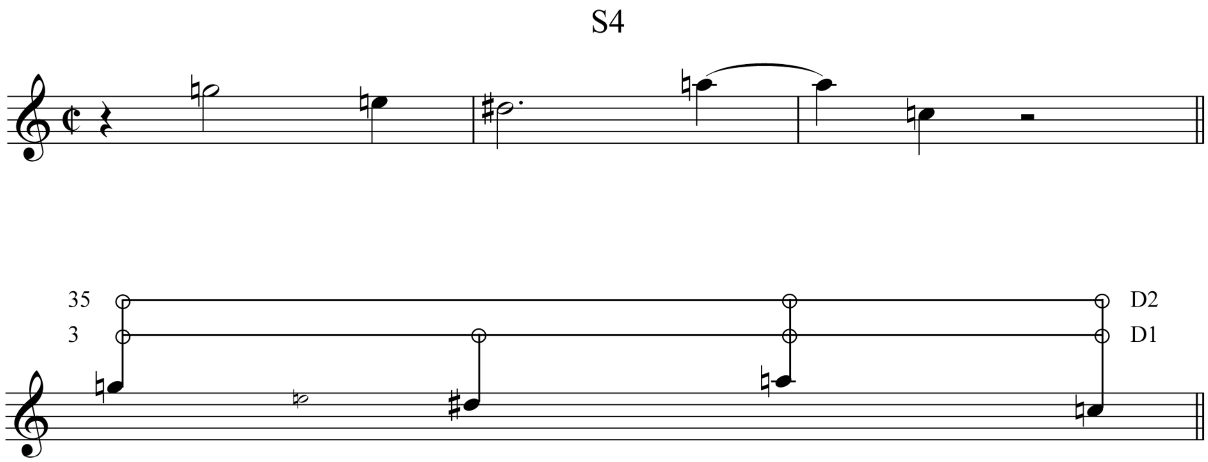
Segment 4

Along with the sixth segment, the fourth segment is the shortest fundamental 𝆦, with only five notes. Nevertheless, it is possible to apply the algorithm twice on the melody, as illustrated in Figure 6.12. S4 employs the same five pcs (in order) as the incipits of S2 and

[135] Note that this observation is based on the notated meter, but one could argue for different implicit meters invoked by the durations. For a detailed discussion of metrical implications within the movement, see Jeff Nichols, "Metric Conflict as an Agent of Formal Design in the First Movement of Schoenberg's Quartet Opus 30."

S3, but with a different contour yet again. The repetition of pc employment in S2, S3, and S4 is enriched by the use of a distinct contour for each segment.

Figure 6.12. Segment 4



In this segment, once again the medial cp with the weakest metric position and shortest duration is pruned in the first depth level. However, the second depth level prunes for the first time a pitch whose onset lands on a downbeat. Note that S3 and S4 share two of the notes in their contour framework, namely G and A, and the third note is lowered in S4 from D# to C. Moreover, the framework of S2, which comprises C and G, reveals itself within S4, as shown in Table 6.1.

Table 6.1. Contour Framework for S2, S3, and S4

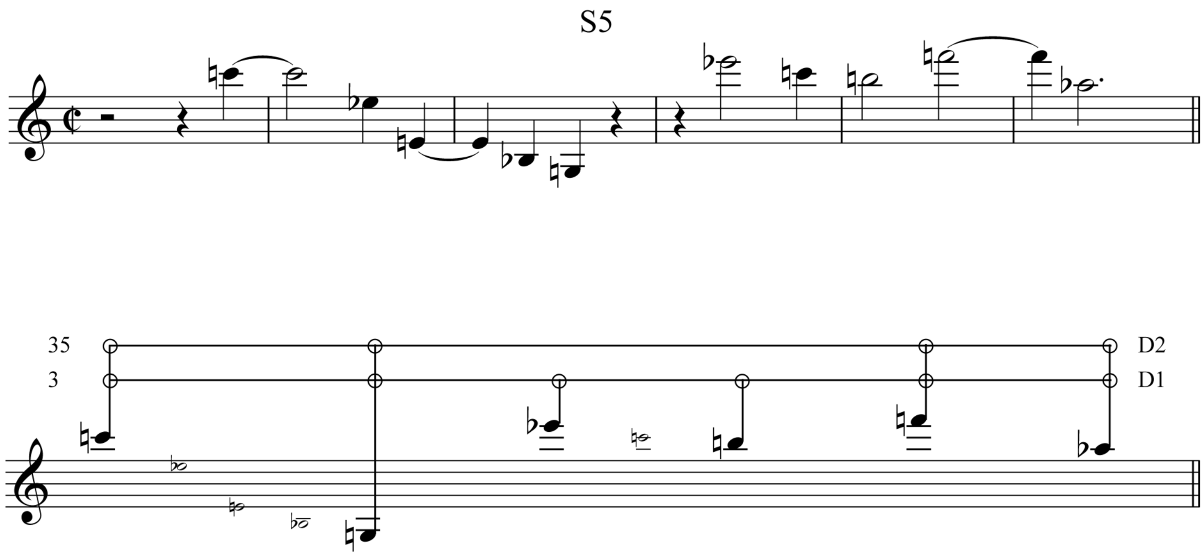
	Contour Framework		
	S2	S3	S4
Maximum	G	A	A
Medial	G (first cp)	G (first cp)	G (first cp)
Minimum	C	D#	C

As is evident from the table, G is the only pc that is retained in all three contour frameworks. Moreover, C and G are the only pcs that are retained at the second depth level in all three segments (R3**5**(S2), R35(S3), R3**5**(S4)). Note that G is retained in every case because it is first; in S4 it is also the maximum.

<u>*Segment 5*</u>

Although it contains twice as many notes as the fourth segment, the fifth segment also has only two depth levels, due to the abundance of passing cps. The first reduction prunes four notes and the second reduction prunes two notes, as illustrated in Figure 6.13. In the original segment two pcs occur twice: C and Eb. Although one of the Cs (the initial note) is retained in the contour framework, both of the Ebs are pruned in turn: Eb5 is pruned by R3(S5); Eb6 is pruned by R3**5**(S5). Note that S5, much like S2, presents two pentachords, <C, Eb, E, Bb, G> and <Eb, C, B, F, Ab>. They are both set-class (01469), which maps the repeated C and Eb to one another. The first of these pentachords is in fact the same pc-set as the second pentachord in S2.

Figure 6.13. Segment 5



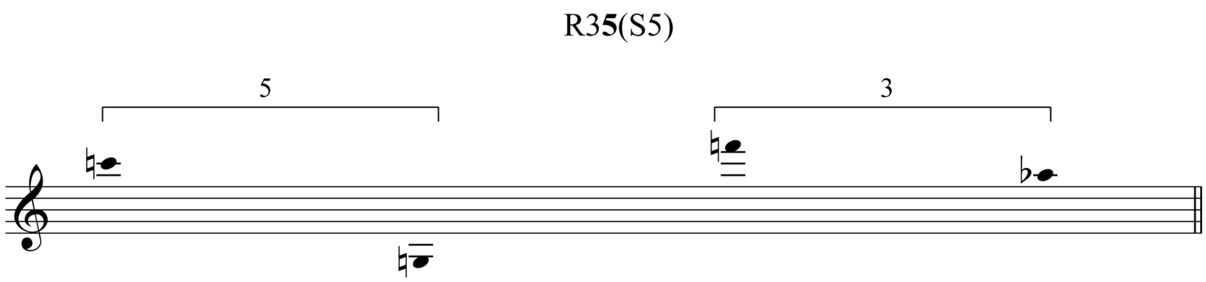
The notes retained in the first reduction, namely C6, G3, Eb6, B5, F6, and Ab5, collectively present strong tonal implications. The first three notes constitute a minor triad and the last three notes constitute a diminished triad, one that could be understood as leading-tone seventh chord (with its third missing) of the former tonic triad. The preceding low G3 enhances the sense of dominant function of the diminished triad; in fact, the last five notes can be heard as dominant minor ninth with augmented fifth (spelled as Eb instead of D#).

Note that C is retained in both levels of S5 because it is the first pitch (but neither highest nor lowest). By contrast, C was retained in the framework of S2 and S4 as the lowest and also the last note in both cases. In concordance with the preceding segments, all of the pruned notes in S5 are the shortest notes of the entire segment, with the exception of E4, which is assigned to the second shortest durational value.[136] The second depth level continues to prune the notes with the shortest remaining durations: Eb6 and B5, which are assigned to half note value. Interestingly, the remaining pitches in this level are the longest of the entire segment and all have the identical durational value of a dotted half note.

Although the correlation between the durational values and the retained notes resulting from the window-algorithm reductions seems straightforward, two of the pruned notes deserve further mention: E4 in R3(S5) and B5 in R3**5**(S5). With a half note value, E4 is longer than all of the other notes pruned in R3(S5), which have quarter-note values, and B5 is the only downbeat of the entire passage. Thus, the pruning of these pitches draws special attention. The pruning of E4 despite its relatively long durational value for R3(S5) prevents shadowing the chord quality in the first triad (c minor), whereas the pruning of B5, along with Eb6 in R3**5**(S5), seems to weaken a bit the C-centered perspective of R3(S5). R3**5**(S5), <C6, G3, F6, Ab5> contains only one triad, namely f minor. Thus, the second level of contour reduction seems to assert a shift from c minor to f minor, caused by the pruning of B5 and Eb6. Alternately, one could still hear G3-F6-Ab5 as a dominant ninth framework relative to the initial high C6, and the contour supports this hearing, in that it puts the G root in the bass and might encourage us to hear the F and Ab resolving characteristically down by step to notes of a c triad. Aside from these tonal implications, which we will discuss in the second section of the chapter, the intervallic content of the contour framework, namely adjacent ic 5, {C6, G3}, and ic 3, {F6, Ab5}, alludes to the previous segments, as shown in Figure 6.14. Interestingly, the ic 5 is again formed by pcs C and G, just like in S2 and S4.

[136] Note that throughout the analysis, we will adopt the approach taken by Marvin (1991), who treats rests as a part of the preceding note duration, focusing only on “inter-onset” durations. Here, the quarter note G3 has an inter-onset durational value of dotted half note.

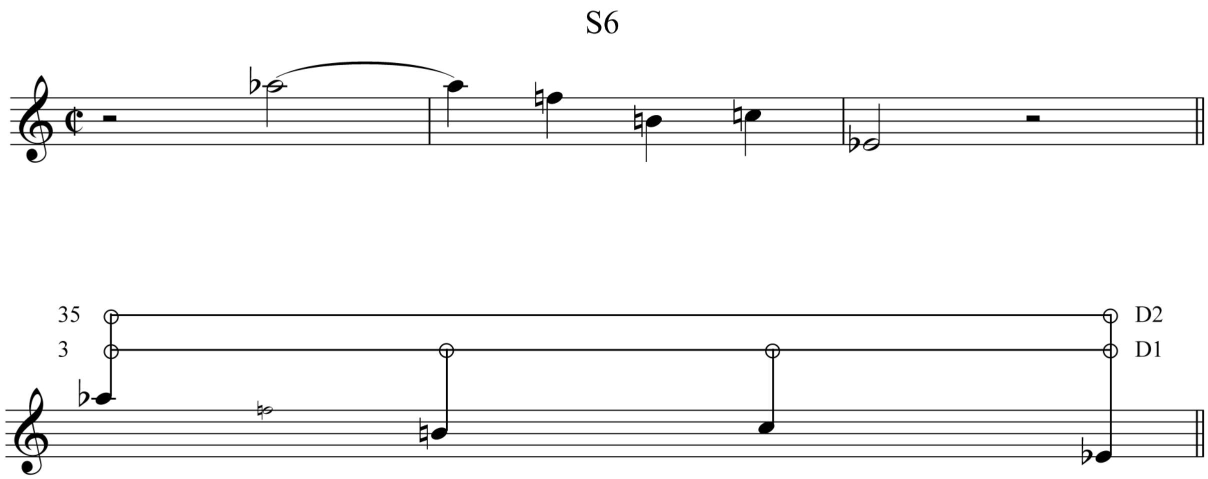
Figure 6.14. Intervallic Content of R35(S5)



<u>*Segment 6*</u>

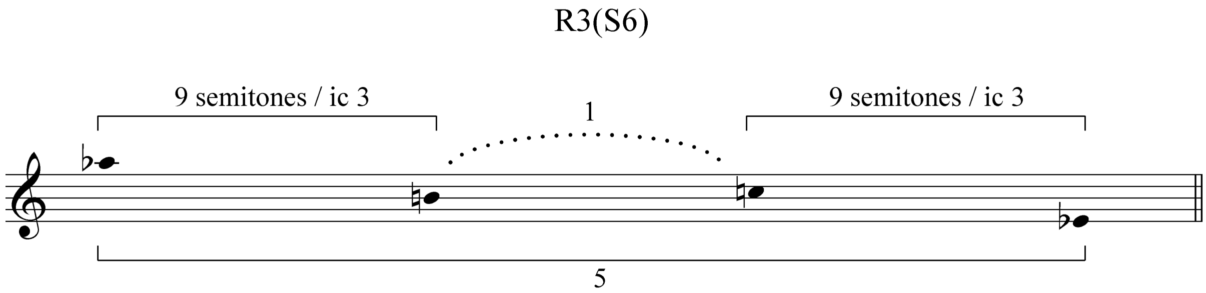
S6 has the same five pcs as the second pentachord in S5, but in retrograde order. S6 is also similar to S4 in that both segments contain only five notes and can be reduced twice by using the window algorithms, as illustrated in Figure 6.15. Once again, the pruned pitches in both levels have the shortest durations and also once again, this segment demonstrates a degree of intervallic repetition in its contour framework. It is intriguing to observe that the building block interval classes, ics 1, 3, and 5 are all manifested at the first depth level, which is also symmetrical in pitch space (see Figure 6.16). Ics 1 and 3 correspond to adjacent pitches; although ic 5 corresponds to non-adjacent pitches in R3(S6), it takes place between adjacent pitches in the contour framework, R3**5**(S6). The ic 5 relationship between the first/highest pitch and the last/lowest pitch is significant given that all previous segments, with the exception of S3, also contain this interval class at their framework level. Apart from the ic content, the outer pitches and the inner pitches are related by I_e in inversional pitch balance.[137] Note that the pruning of B4 and C5 in R3**5**(S6) does not effect the inversional balance since Ab5 and Eb4 are related by I_e.

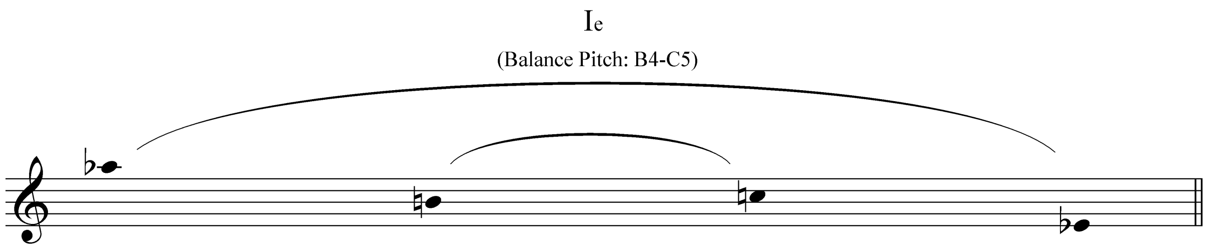
Figure 6.15. Segment 6



[137] Note that an "inversional pitch balance" contains a virtual pitch (not pitch-class) or pitch pair that the surrounding pitches are inverted around. In this case, both Ab5-Eb4 and B4-C5 are inverted around B4-C5. Had Ab5 or Eb4 been in another octave, the pitch balance point would have not been B4-C5 anymore.

Figure 6.16. Intervallic and Inversional Balance at R3(S6)



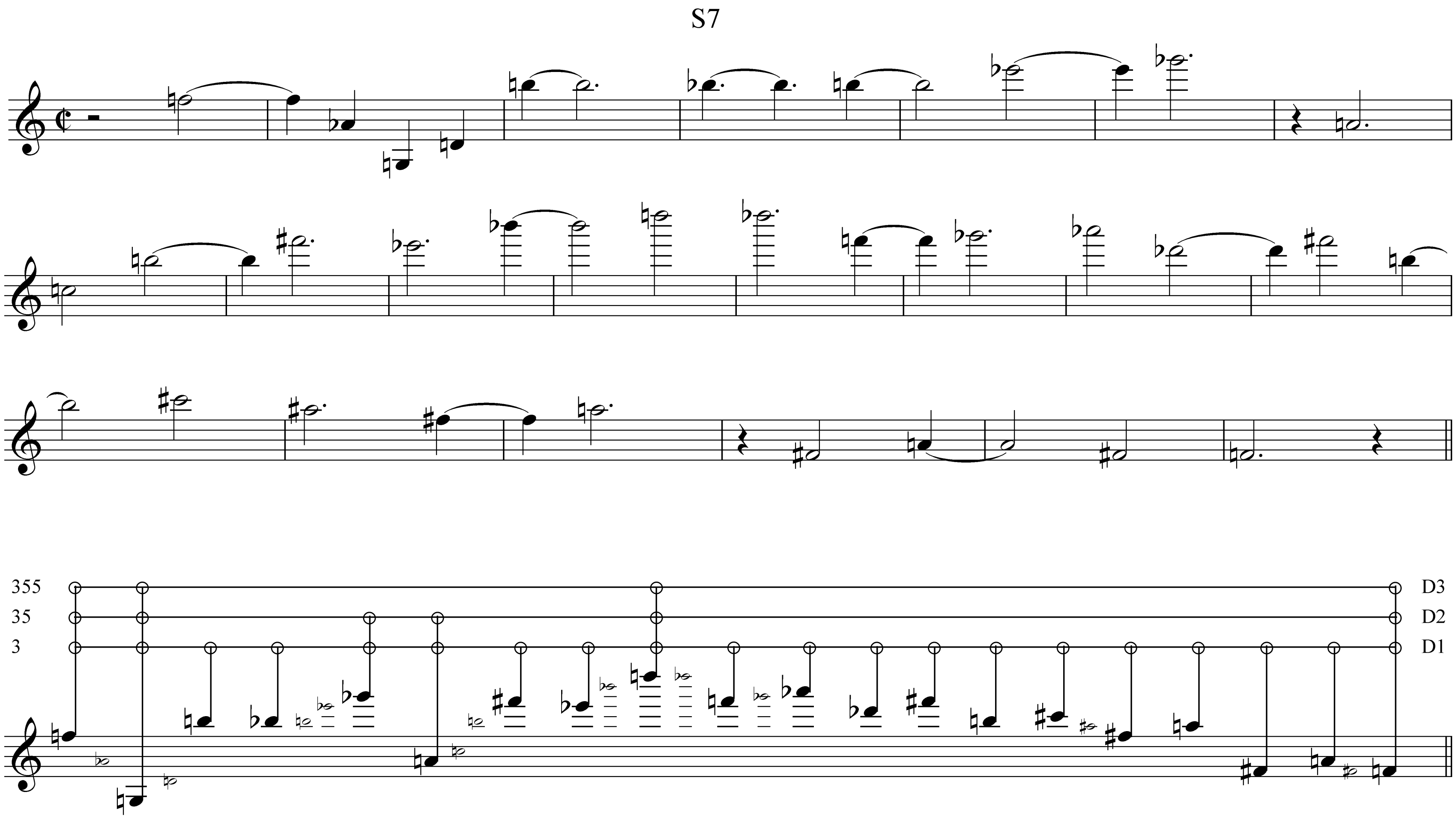


<u>*Segment 7*</u>

This segment, illustrated in Figure 6.17, has 31 notes and is the longest we have examined so far. However, despite its length, the window algorithms can be applied only three times on the original segment. The "depth level" of S7 is therefore the same as S3, which contains only nine pitches. All three levels of S7 are shown below the original segment.

The original segment contains seven F#s and four Bs, which collectively account for over a third of the segment. Interestingly, none of these repeated pcs are retained in the framework level. On the contrary, the retained notes in R35**5**(S7), G (lowest), D (highest), and two Fs (first and last), are among the scarcest notes presented in the original segment, which has only one G, two Ds, and three Fs. This point is remarkable in that it shows how the surface level is distinct from the background level: the most abundant notes are pruned and the underlying pitches of the segment are found scarcely on the surface level, but are crucially located with respect to the contour. The reduction algorithms help us observe that statistically insignificant pitches may nonetheless be prominent in terms of contour and can constitute a schematic "background" in that respect, while abundant notes, such as F# and B in this melody, may function as surface level ornamentations. Note that the retained notes of the contour framework form a seventh chord with a missing third, which is similar to another seventh chord with a missing third we encountered in R3(S5). Furthermore, we observe an ic 5 formed by the minimum G and maximums D at the framework level.

Figure 6.17. Segment 7



Note that with the exception of the last note, which repeats the initial pitch F5 an octave lower, all of the retained pitches in R35**5**(S7) fall on the second beat in the notated cut time. Although one could conceive (and perceive) the first note of the segment as the downbeat, which would shift all three pitches of R35**5**(S7) onto the downbeat, any assumption based on the notated meter requires caution.

Segment 8

Segment 8 is even longer than the previous segment and its 48 pitches span over 32 measures, as shown in Figure 6.18.

This *Hauptstimme* falls on a strategically important location in the movement since it is the first time Schoenberg presents the row horizontally, as a line in one voice, in its entirety. The four aggregates presented in the *Hauptstimme* naturally contain 48 pcs with each pc repeated four times.[138] Five applications of the window algorithms reduce this cardinality-48 segment to a cardinality-5 segment, as illustrated in Figure 6.19.

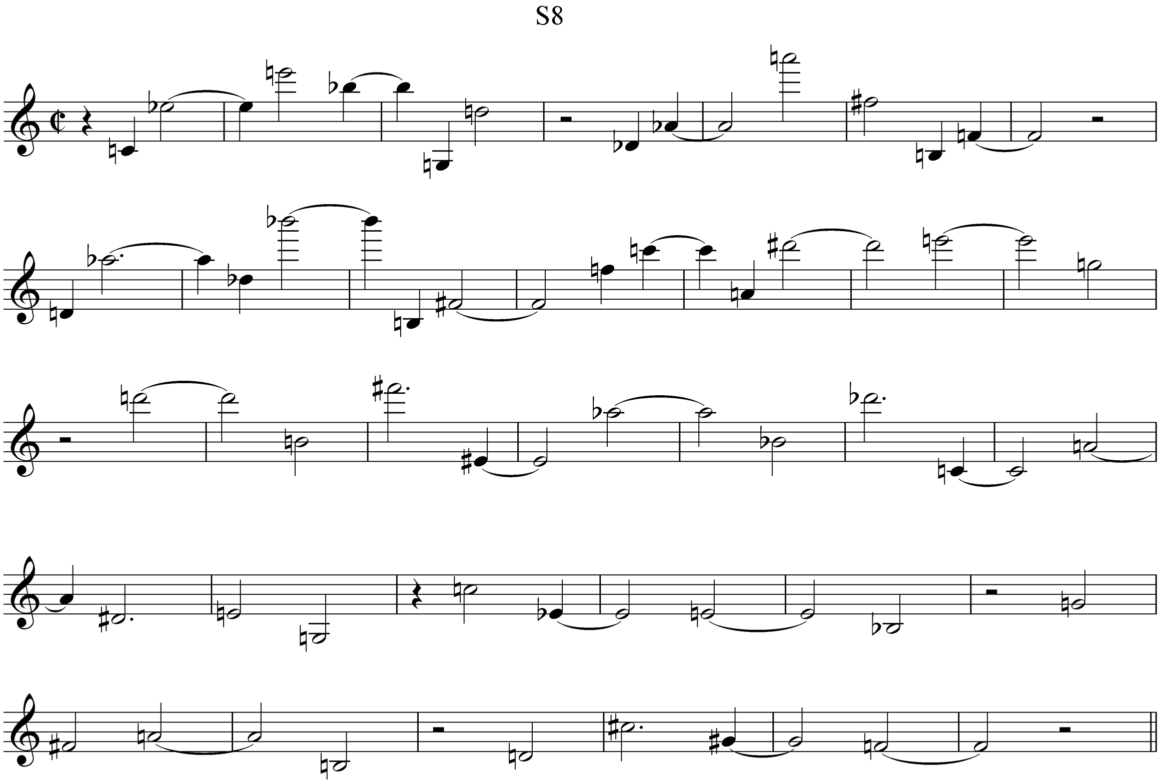
Unlike most of the retained cps of the previous segments, in this segment the first two retained pitches in the contour framework are not only metrically weak on the surface but also among the shortest. Here, almost half of the pruning takes place in the second depth level, with a total of 21 cps being pruned.

The intervallic content of the contour framework once again highlights ics 3 and 5, as shown in Figure 6.20. Interestingly, the framework levels of the first two segments, which open the "first theme" of the exposition, namely R3**5**(S1) (Figure 6.5) and R3**5**(S2) demonstrate a significant intervallic connection with the framework level of S8, which

[138] The first two of the aggregates present the row forms I5 and R0, whereas the following two do not include any of the 48 possibilities in isolation since the row is distributed among all four instruments.

open the "second theme" of the exposition. All of these segments contain ics 3 and 5 formed by adjacent pcs in their framework, as shown in Table 6.2.

Figure 6.18. Segment 8



Arnold Schönberg, 3. Streichquartett für 2 Violinen, Viola und Violoncello, op. 30

Figure 6.19. Reduction of Segment 8

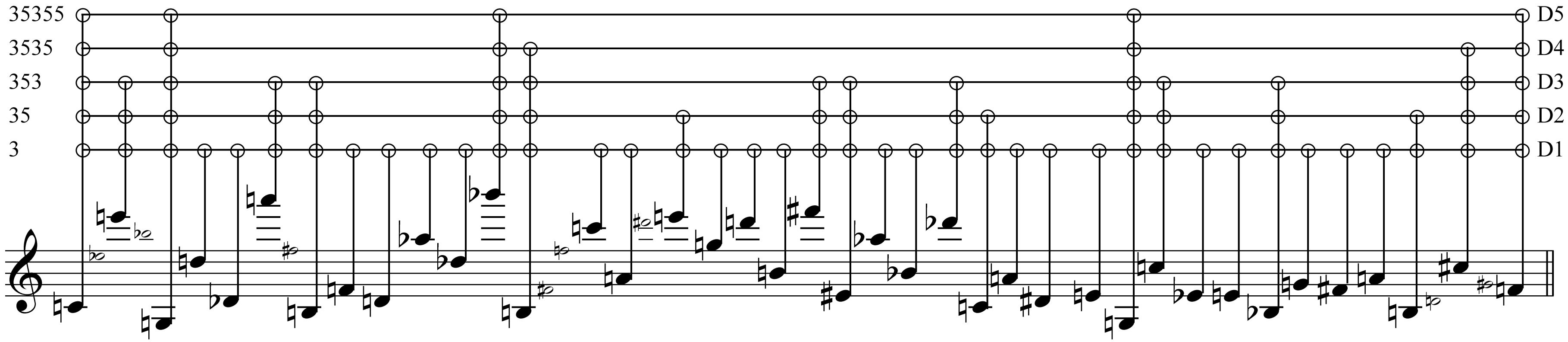


Figure 6.20. Intervallic Content of R35355(S8)

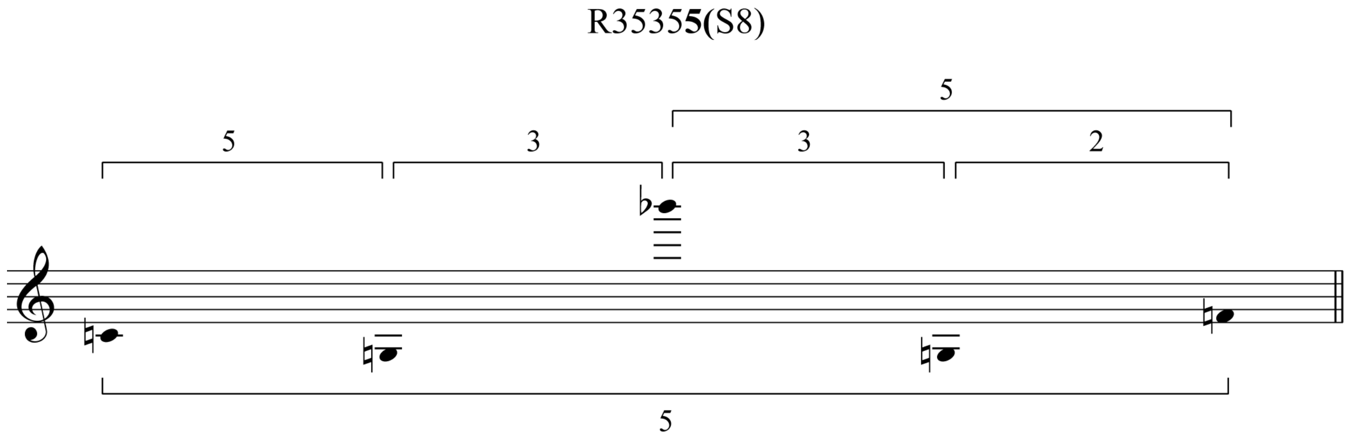


Table 6.2. Intervallic Content of Framework S1, S2, and S8

Segments	R3**5**(S1)	R3**5**(S2)	R3535**5**(S8)
Ordered pc Sets	<B,D,F,A#>	<G,C,G,C>	<C,G,Bb,G,F> (<C,G,Bb,F>)
Interval Classes	<3 3 5>	<5 5 5>	<5 3 3 2> (<5 3 5>)

Also note that the framework pcs of S8, which forms three ic 5 intervals (C-G, Bb-F, and C-F) can be understood to be derived from framework S1 (Bb-F) and S2 (C-G). Regardless of the differences between the surface levels of S1/S2 and S8, the contour reductions reveal a deep level of unity between the first and second theme segments, in terms of not only interval classes but also pitch classes.

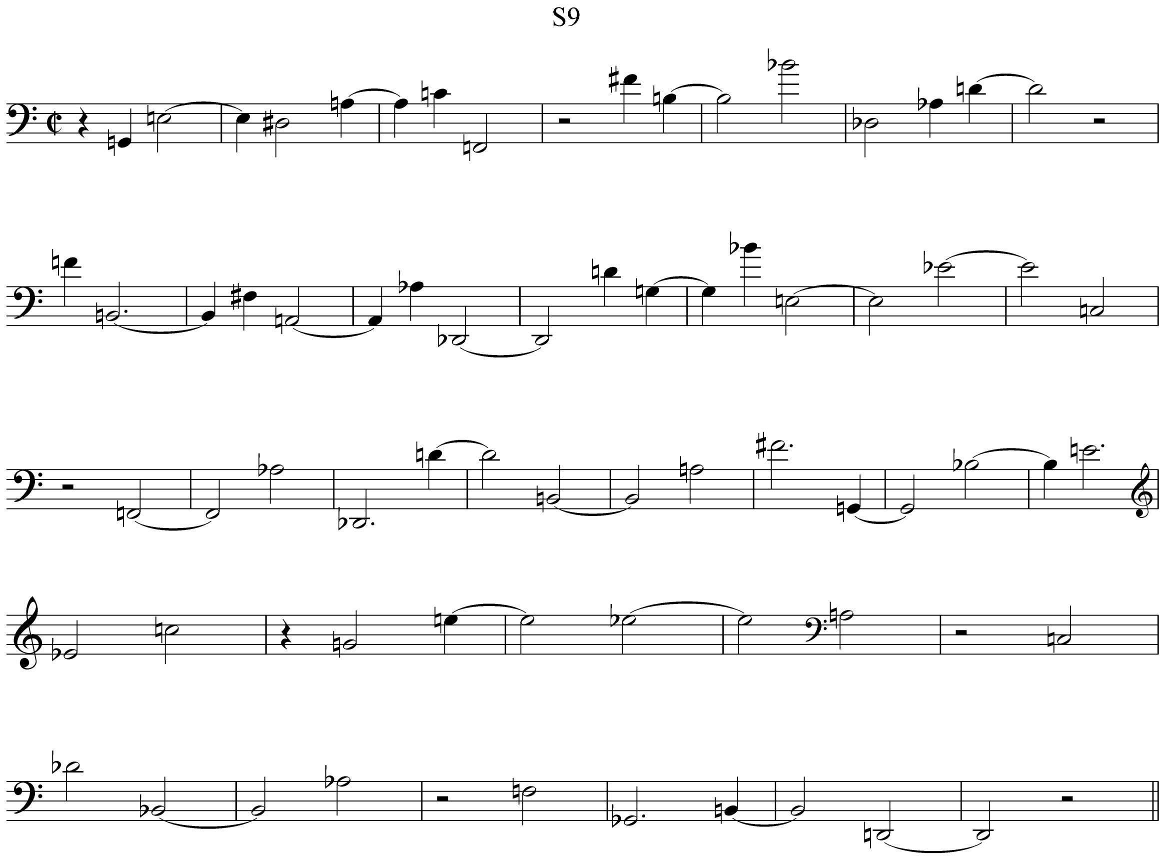
Segment 9

S9 begins the recapitulation section in the sonata form. Together with S8, S9 (illustrated in Figure 6.21) is the longest in the entire movement. Like S8, it also comprises four aggregates.[139]

Though S9 has the same number of notes as S8, it has one less level under the reduction algorithms. The four levels resulting from the application of the window algorithms are shown in Figure 6.22. Similar to the reductions of S8, the pruned notes of S9 are not necessarily metrically weak or short, as was the case for most of the segments in the exposition section. Interestingly, seven out of nine cps pruned in R3(S9) are on the second beat. Some of the pruned pitches are among the longest (i.e. inter-onset duration), such as D4), C3), Eb5, and A3.

[139] In this case, aggregates present the row forms P0 and RI5. Of course, the return to P0 at the opening of the recapitulation section (it also opened the exposition section, though not confined to the *Hauptstimme*), shows how Schoenberg continued to apply aspects of traditional musical forms in his twelve-tone pieces.

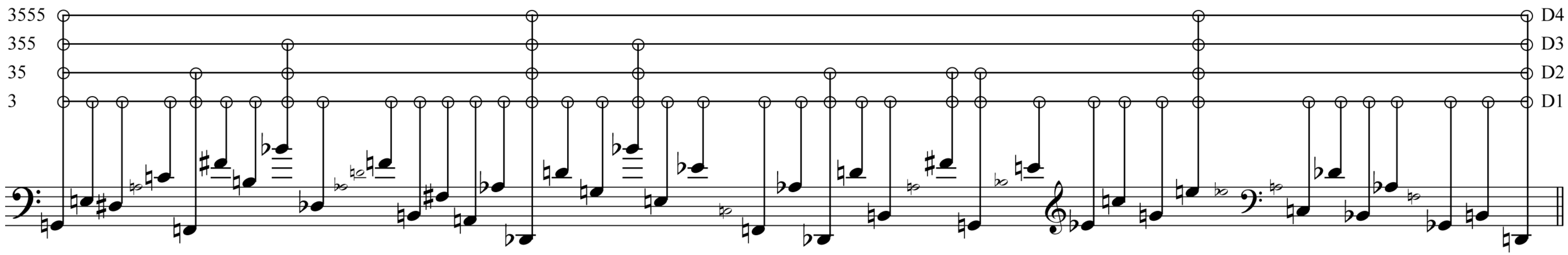
Figure 6.21. Segment 9



In S9, the second depth level holds an even more extreme pruning than the corresponding level of S8: 29 pitches are pruned in R35(S9), which amounts up to almost two-thirds of all pruned pitches in four depth levels.

The first three of the four pitches that are retained in the contour framework R355**5**(S9) form a diminished triad, just like S1. This point is intriguing since both S1 and S9 are the first segments of the first theme in the exposition and recapitulation, respectively. Furthermore, R355(S9) has only two more notes (both Bb4) than R355**5**(S9), and R355(S9) contains a complete diminished seventh chord <G, Bb, Db, E>, which contains ic 3 between each adjacent pc with the exclusion of the second (repeated) Bb. The diminished triad in R355**5**(S9) is a whole-tone above the diminished triad in R3**5**(S1). Interestingly, the last note D, which is not a part of the diminished triad in S9, is the only note shared between the two segments at the framework level. In fact, apart from the background of S7, this is the only framework level appearance of D since the initial segment.

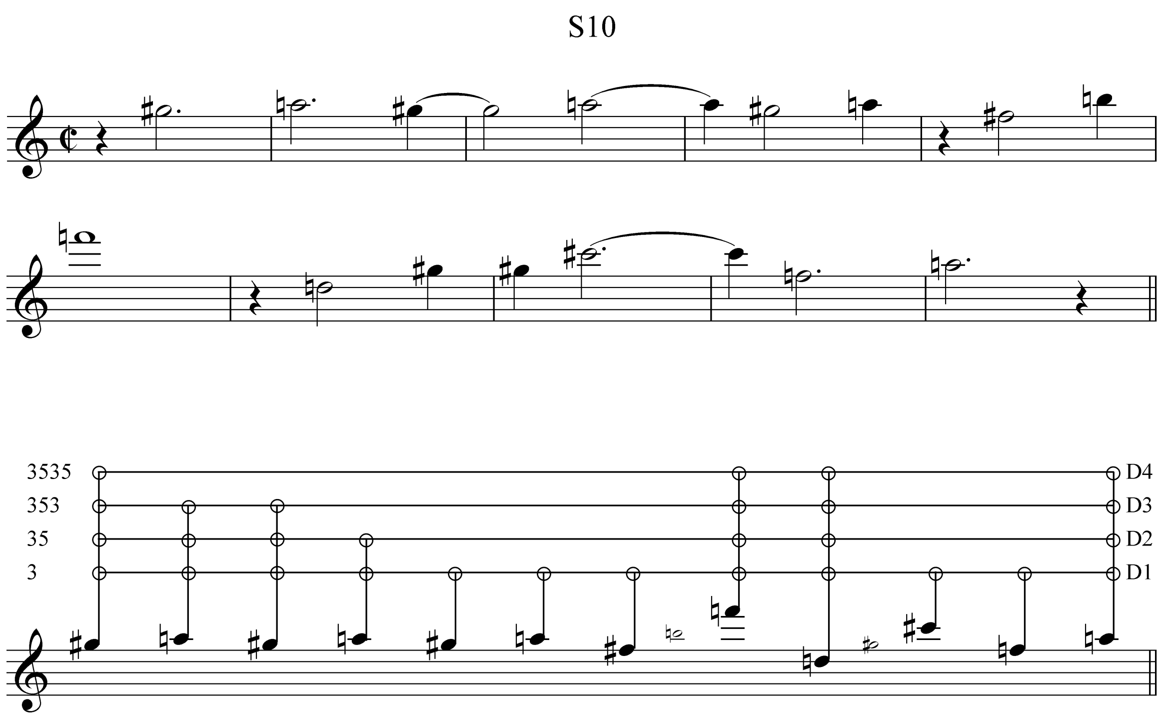
Figure 6.22. Reduction of Segment 9



Segment 10

In comparison with the previous segments, S10 is relatively short: it contains 14 pitches, many of which are repeated. This segment, illustrated in Figure 6.23, is in stark contrast with the previous ones, especially with regard to its repetitive nature and its limited pitch range of a minor 10th. Interestingly, because of the abundance of non-successive repetitions, this segment has four depth levels despite its size.

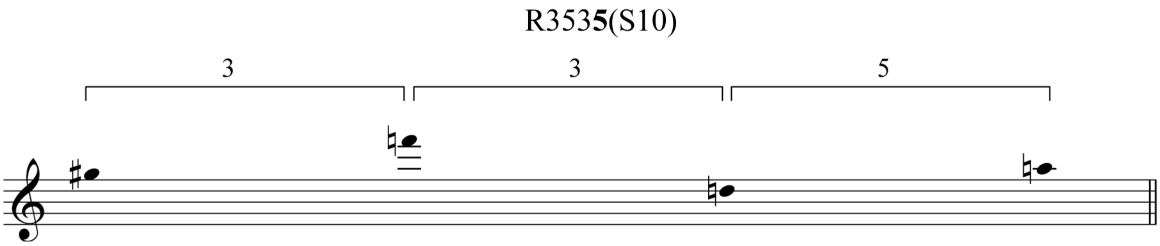
Figure 6.23. Segment 10



The first and the last two levels (R3, R353, and R353**5**) together prune only five cps, the same amount the second level (R35) prunes alone. The persistent repetition of two pitches, G#5 and A5, at the very beginning of the segment results in ups and downs in the contour, and thus are not pruned in the 3-window algorithm due to the change in CAS. They are pruned gradually through the reduction. This unusually repetitive excerpt (for a post-tonal work) provides an interesting example since the application of the 5-window algorithm demonstrates how repeated non-adjacent cps (in some cases three of them) are treated.

Interestingly, the first three notes of the contour framework form a diminished triad, <G#5, F6, D5>, just like the framework level of the previous segment. However, the diminished triad of R353**5**(S10) is a semitone above the diminished triad of R355**5**(S9). The pc following the diminished triad, A, forms yet another ic 5, as illustrated in Figure 6.24.[140] Note that the ordered ics in this figure are identical to the framework of S1 (Figure 6.5).

Figure 6.24. Intervallic Content of R3535(S10)



<u>*Segment 11*</u>

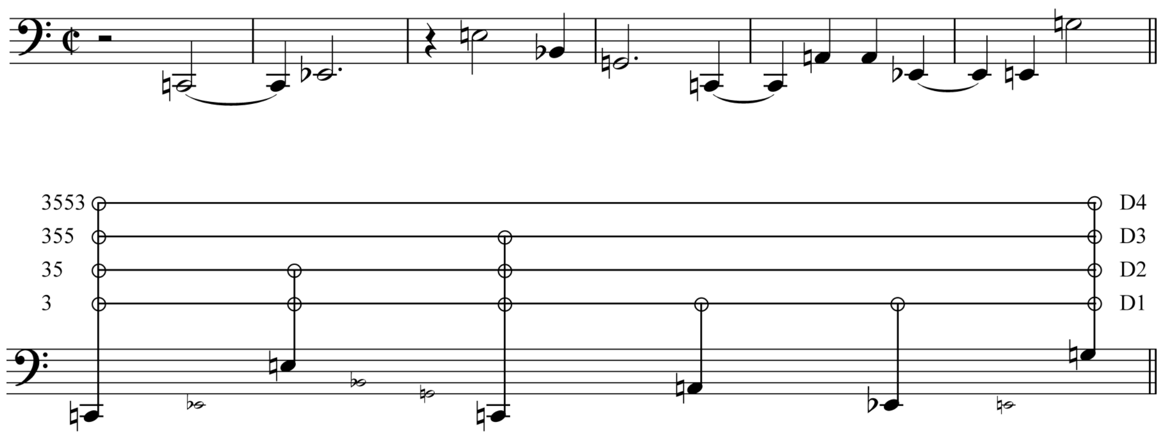
Last two segments contain exactly the same number of pitches as the first two segments. S11 marks the beginning of the recapitulation of the second theme; its reductions are illustrated in Figure 6.25. Along with S6, it is the only segment that reduces to two pitches. Interestingly, R35(S11) continues the triad formations we have

[140] It is also worth mentioning that the diminished triad could be interpreted as a G#^{o7} chord without a fifth, which resolves to an implied A triad.

observed previously: here the retained cps constitute a C major triad. The C-G ic 5 in R353**5**(S11) also bears a striking relation to the framework of S2, namely R3**5**(S2). Moreover, S11 involves the same two pentachords as S2. In fact, S11 and S2, in their entirety, are retrogrades of one another and are also related under I_7. Therefore, pentachords of S11, <C, Eb, E, Bb, G> and <C, A, Eb, E, G>, hold the same properties and relationships as pentachords of S2: they have 4 pcs in common (all pcs except A and Bb) and are related by RI_7. Nevertheless, contours of S11 and S2 do not present any of the relations observed above. S2 exhibits a more zigzaggy melody in comparison to S11, which contains an overall ascent and descent with passing cps in the first half of the segment. S11, with a cseg of <0165304127>, contains four passing cps (two of which are successive) and no adjacent repeating cps whereas S2, with a cseg of <21530744650>, involves two passing cps and one adjacent repeating cp. Despite their pc similarities, each segment involves a characteristic contour and thus a distinct reduction.

Figure 6.25. Segment 11

S11



<u>Segment 12</u>

S12 has three depth levels, as shown in Figure 6.26. Here, we observe that the original S12 and S3 are related by I_7, which also mapped S11 onto S2.[141] However, unlike S11 and S2, S12 and S3 are not related by retrograde. As we have observed in our comparison of S11 and S2, a pc inversion does not necessarily guarantee a contour inversion since pc inversion is indifferent to pitch direction. However, in this case both pcs and psegs of the segments present an inversional relationship, with the exception of the last pitch in S12. Since this last pitch is retained in the window algorithms, the reductions do not result in an exact inversion. Table 6.3 shows that with the exception of their last cp, the first two depth levels in both segments are inversionally related in both pc-space and pseg-space. However, as is evident from Table 6.3, the reductions result in considerably different csegs due to the added pitch at the end of S12. Interestingly, the framework csegs are identical (not inverted!) in both segments. In fact, there seems to be a transformation from inversion to identity taking place from the surface level (i.e. R3(S3) - R3(S12)) to background level (i.e. R35**5**(S3) - R35**3**(S12)). Nevertheless, this identity between the background level csegs is not reflected in pc-space: R35**5**(S3) consists of {Db, G, A} whereas R35**3**(S12) consists of {C, G, G}. The pcs of R35**3**(S12) are in fact identical to pcs of the framework levels of the previous segment S11, as well as S2.

[141] Note that there is an additional pitch at the end of S12, which prevents an exact mapping.

Figure 6.26. Segment 12

S12

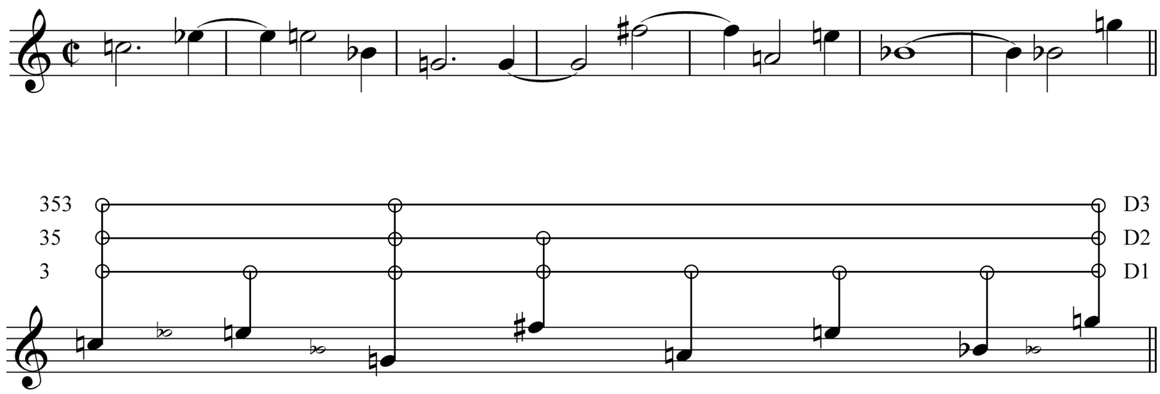


Table 6.3. Comparison of Ordered Pitch Intervals in Reduced S3 and S12

Segment	Ordered Pitch Intervals	Cseg
R3(S3)	<–4, +9, –11, +9, –7, +18>	<2140315>
R3(S12)	<+4, –9, +11, –9, +7, –6, +9>	<34051426>
R35(S3)	<+5, –11, +20>	<1203>
R35(S12)	<–5, +11, –6>	<1021>
R35**5**(S3)	<-6, +20>	<102>
R35**3**(S12)	<–5, +12>	<102>

As is evident from our previous observations, certain intervals seem to appear more frequently than others in the contour framework. With the exception of the third segment, the contour framework of all segments includes an ic 3 or an ic 5, or both. Four of the twelve segments (S2, S6, S11, and S12) contain only one interval class[142] and, remarkably, in each case it is ic 5. Tables 6.4 and 6.5 list the successive ics and their

[142] This is a result of either having only two cps or having a pitch doubling at the framework level.

frequencies for all segments at the framework level. Tables 6.6 and 6.7 list the successive as well as non-successive ics (by using an ic vector) and their frequencies for all segments at the framework level, similar to Tables 6.4 and 6.5.

Table 6.4. Successive IC Content of the Frameworks

Segment	ICs
S1	<335>
S2	<555>
S3	<46>
S4	<23>
S5	<523>
S6	<5>
S7	<253>
S8	<5332>
S9	<632>
S10	<335>
S11	<5>
S12	<5>

Table 6.5. Successive IC Frequencies of the Frameworks

IC	Frequency
1	0
2	5
3	10
4	1
5	11
6	2

Table 6.6. IC Vector of the Frameworks

Segment	IC Vector
S1	<102111>
S2	<000010>
S3	<010101>
S4	<011010>
S5	<111120>
S6	<000010>
S7	<011010>
S8	<021030>
S9	<112011>
S10	<102111>
S11	<000010>
S12	<000010>

Table 6.7. Successive and Non-Successive IC Frequencies of the Frameworks

IC	Frequency
1	4
2	7
3	10
4	4
5	14
6	4

The abundance of ics 3 and 5 (21/29), as well as the complete lack of ic 1 in Table 6.5 point toward a rather significant consonance and suggest further investigation of the framework pitches. This vein of thought exhibits interesting parallels with Richmond Browne's *position finding* technique, which is based on the scarcity of certain intervals in the diatonic collection to determine the tonic.[143] Although scholars have discussed tonality in Schoenberg's music from a variety of perspectives, including meter and

[143] Richmond Browne, "Tonal Implications of the Diatonic Set," *In Theory Only* 5.6-7 (1981): 3-21.

phrase-level structure,[144] rhythm and overlapping segments,[145] musical closure and interaction of conflicting keys,[146] memory and voice leading,[147] contour has not been explored to its fullest potential considering it is one of the key concepts to understand non-serial aspects of pitch organization and therefore holds the potential to reveal (and perhaps subconsciously implanted) tonal residues in Schoenberg's music.[148]

The following portion of the chapter will explore possible tonal associations that are implied by the intervallic content of the segments revealed by contour reductions.

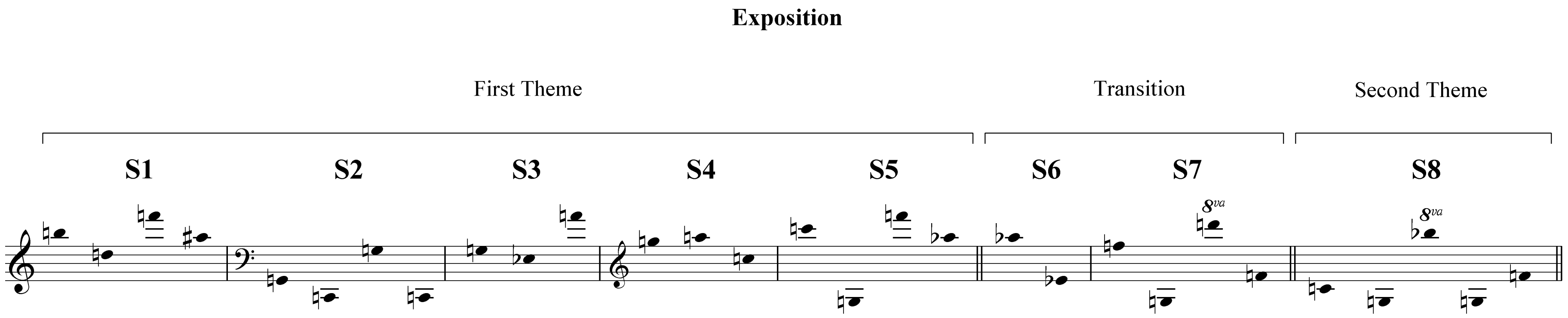
Analytical Perspective 2: Tonal Residues in the Contour Frameworks

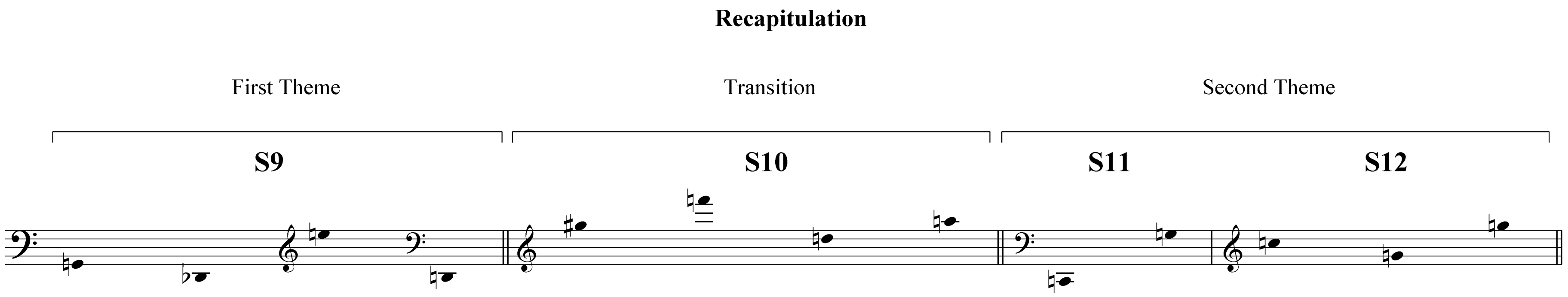
A closer examination of the reduced csegs and their relation to the formal outline of the movement provides interesting observations. For example, as noted previously, the frameworks for S12 and S2 share their pcs, which suggest some sense of overall formal unity since S2 is one of the opening segments and S12 is the closing fundamental segment of the movement. Moreover, the contour framework of S1 with pcs {D, F, A#, B} might be heard as having a dominant function relative to the {C, G} framework of S2. With the exception of A#, the three pitch classes {D, F, B} form the leading-tone triad of the following C-centered segment. The combined pc content of the S1 and S2 frameworks can elicit other tonal residues, e.g. F-G-Bb-B-C-D-F as an F-oriented scale or Bb-B-C-D-F-G-Bb as a Bb-oriented scale, or even G-Bb-B-C-D-F-G as a G-oriented scale. Hearing the combined content as C-oriented, C-D-F-G-A#-B-C responds, in particular, to the

144 William E. Benjamin, "Abstract Polyphonies."
145 Michael Cherlin, "Schoenberg and Das Unheimliche: Spectres of Tonality."
146 Richard B. Kurth, "Moments of Closure."
147 Michael Cherlin, "Memory and Rhetorical Trope in Schoenberg's String Trio."
148 Note that one particular aspect of the window algorithms, namely retention of initial and last pcs, is conditioned by serial techniques. Although the numerical (or pitch) outcome of a reduction is not affected by serial operations since any given pc could be placed as high or low as desired, the pc outcome of a reduction is influenced by serial manipulations. For example, a segment and its retrograde will always have the same pcs (but not necessarily pitches) retained as first/last.

prominence in S2 of the repeated low C2s, and of G2 and G3, as scale degrees 1 and 5 respectively. Other fairly consistent tonal references to C presented by the opening and closing background segments seem to be supported by many of the other segments. In order to explore such ideas, Figure 6.27 illustrates the concatenated reduced csegs in relation to the movement's sonata form design.

Figure 6.27. Concatenated Framework Contours





The rest of the first theme (S3, S4, S5) supports the C-centrality of the first two segments indicated above. Figure 6.28 shows the distribution of the reduced pcs in the first theme area.[149] The total pc content corresponds to the C melodic minor scale, and the first and fifth scale degrees are especially prominent.

Figure 6.28. Distribution of Framework pcs in the First Theme Area

C	**C#**	**D**	**D#**	**E**	**F**	**F#**	**G**	**G#**	**A**	**A#**	**B**
•		•	•		•		•	•	•	•	•
•					•		•		•		
•							•				
•							•				
							•				

Here, a G-centered interpretation also seems plausible, considering the dominance of G, but the missing leading-tone F# undermines such an interpretation. F- and Bb-centered alternatives also seem weaker than the C-weighting. As illustrated in Table 6.8, we can easily calculate the weighting of various (complete) triads, among the 18 pcs available.

Table 6.8. Weighting of Complete Triads in the Framework of the First Theme Area

Triad	Weighting
C minor	10
F major	8
F minor; G major; G minor; Eb major	7
Ab major	6
D minor	5
Bb major	4
Ab minor	3

[149] In this figure and other pc distribution figures, bullets (instead of numerals) are used for visual purposes.

The C minor triad has 10 (of 18) pcs; F major has 8 pcs; F minor has 7 pcs, as do G major, G minor, and Eb major triads; Ab major has 6 pcs; D minor has 5 pcs; Bb major has 4 pcs; Ab minor has 3 pcs. A discrete uniform distribution of any triad would be 25%, or 4.5 pcs out of 18.[150] The much higher distributions for the C minor, F major/minor, G major/minor, and Eb major triads collectively strengthen the notion of C minor weighting here, perhaps with some consideration also for its relative major, subdominant and dominant regions. In particular, the weighting of the C minor triad at over 50% (10/18=55.6%) is statistically very significant. Similarly, the uniform distribution of C harmonic minor scale is 58%, while the distribution outlined by the window algorithms is 15/18 (83.3%) for the C harmonic minor scale.[151] (The C melodic minor accounts for 100% of the pcs in Figure 6.28).

Figure 6.29 demonstrates the pc distribution of the 11 framework pitches in the transition and second theme areas. The focus on only seven pcs strongly refers to either the Eb major or C natural minor scales. The Bb major scale would lack seventh scale degree; the F major, D minor and G minor scales would lack not only the seventh scale degree but the third scale degree for F major and the second scale degree for D minor and G minor. Since

---

[150] Henceforth, “uniform distribution” is a distribution in which every possible value within a finite set is equally probable. In our case, the finite set comprises twelve notes and since all twelve notes are given equal importance in principle, a collection of notes is expected to have a uniform distribution.
[151] Statistical applications in musicological, theoretical, and music-psychological literature date back to 1950s and 1960s. See, for instance, Richard C. Pinkerton, "Information Theory and Melody," *Scientific American* 194.2 (1956): 77-86; Joel E. Cohen, "Information Theory and Music," *Behavioural Science* 7 (1962): 137-63; Albert Lyle Hanna, "A Statistical Analysis of Some Style Elements in the Solo Piano Sonatas of Franz Schubert," Ph.D. dissertation, Indiana University, 1965; Elinor Jane Perry Camp, "Temporal Proportion: A Study of Sonata Forms in the Piano Sonatas of Mozart," Ph.D. dissertation, Florida State University, 1968; Jens Brincker, "Statistical Analysis of Music: An Application of Information Theory," *Swedish Journal of Musicology* 52 (1970): 53-57. For more recent developments, see Carol L. Krumhansl, "Tonality Induction: A Statistical Approach Applied Cross-Culturally," *Music Perception* 17.4 (2000): 461-79; Jan Beran, *Statistics in Musicology* (Boca Raton, FL: Chapman & Hall/CRC, 2003); Andrea L. Snavely, "The Role of Statistical Cues in the Segmentation of Post-Tonal Music," Ph.D. dissertation, University of Wisconsin, Madison, 2004; Nico Schüler, "Towards a History and Evaluation of Statistical and Information-Theoretical Analysis of Melodic Incipits," *Theoria* 13 (2006): 113-26. For a historical summary of statistical approaches to music, see Nigel Nettheim, "A Bibliography of Statistical Applications in Musicology," *Musicology Australia* 20 (1997): 94-106.

the leading-tone, B, of C minor scale is missing, Eb major seems to be the clear choice, despite the stronger emphasis on its scale degrees 2 and 3, rather than 1.[152] Note that the Eb diatonic collection with an emphasis on F could also be interpreted as an F dorian, however, this interpretation does not address the tonal ambiguity presented earlier. The choice of Eb major, on the other hand, clarifies any modal ambiguity about the first theme area: a first theme in C major would conventionally imply a second theme in G major, whereas a first theme in C minor would imply a second theme in Eb major. With this idea in mind, the first theme might be understood to be in C minor, which is also supported by the role given to Eb in connection with Figures 6.28 and 6.29.

Figure 6.29. Distribution of Framework pcs in the Transition and Second Theme Areas

C	**C#**	**D**	**D#**	**E**	**F**	**F#**	**G**	**G#**	**A**	**A#**	**B**
•		•	•		•		•	•		•	
					•		•				
					•		•				

Nevertheless, this interpretation is conjectural, due to the weakness of the tonic Eb, which appears only once. In addition, the transition (S7) interestingly clings on to C minor residues since G-D-F dominant seventh chord (with an implied third) refers to C minor, undermining the Eb major tonality. Note that Eb occurs as a framework pitch only in S6, but not in S7 or S8. However, the combined S7 and S8 frameworks comprise a Bb pentatonic

---

[152] Schenkerian theory helps us to understand the potential background melodic significance of scale degrees 2 and 3.

collection, Bb-C-D-F-G, which could be construed as projecting dominant function relative to Eb.

It is also important to note that the unsegmented S7 and S8 are much longer than the previous *Haupstimmen*, and reduction therefore makes higher cognitive demands on the listener's memory. Thus, a re-evaluation of these segments could provide us with more accurate observations. Upon a closer examination, it is clear that S7 contains three subsegments, while S8 contains seven subsegments demarcated by rests. This new segmentation of the original S7 and S8 are shown in Figures 6.30 and 6.31.

Figure 6.30. Segmented S7

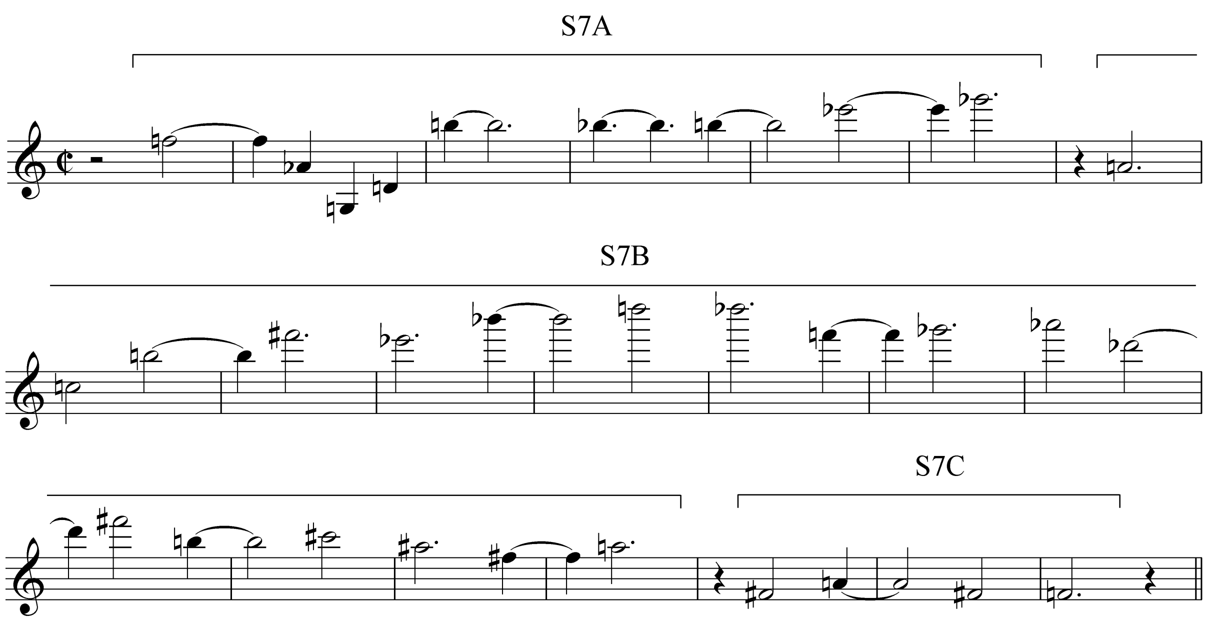
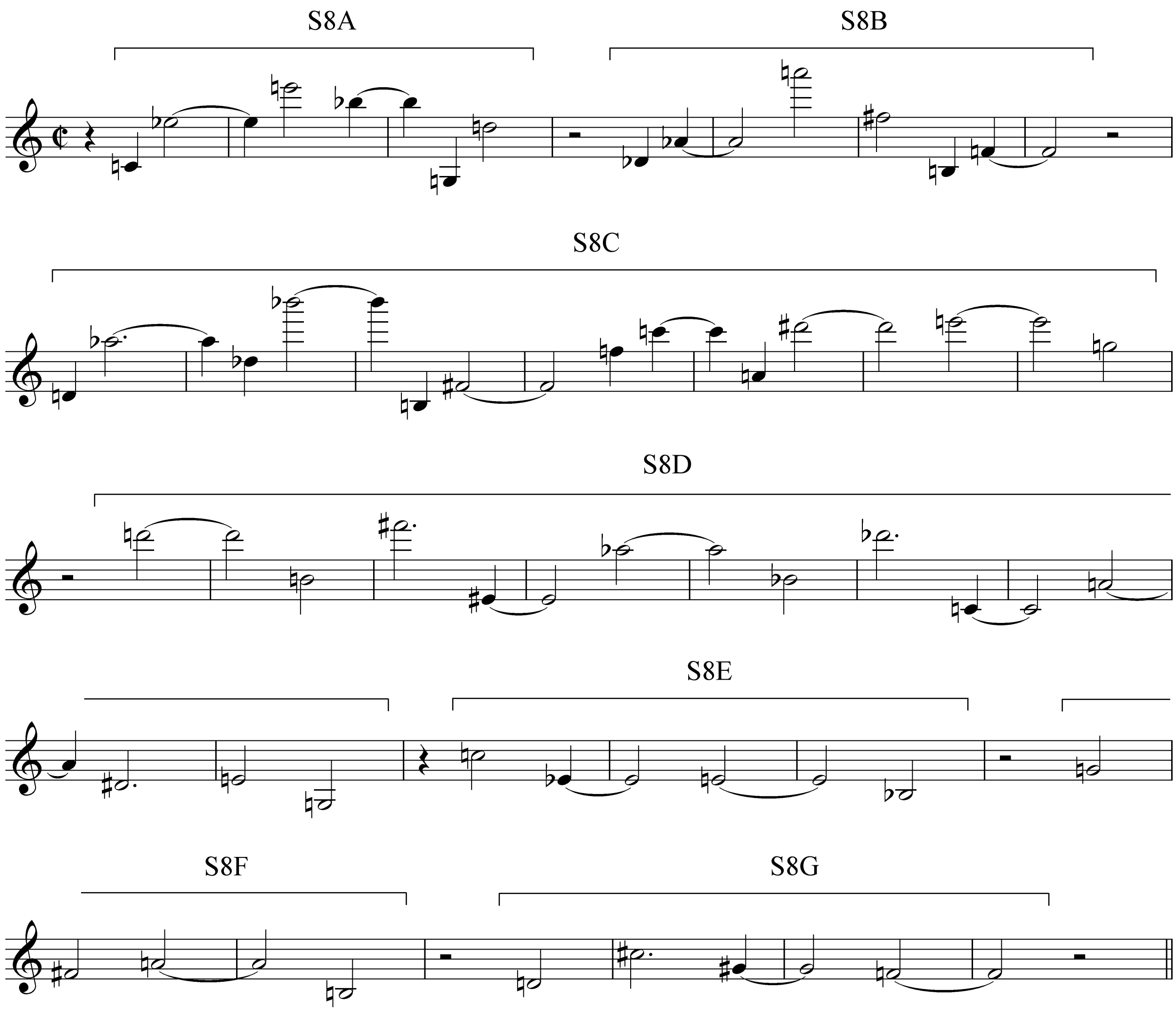


Figure 6.31. Segmented S8

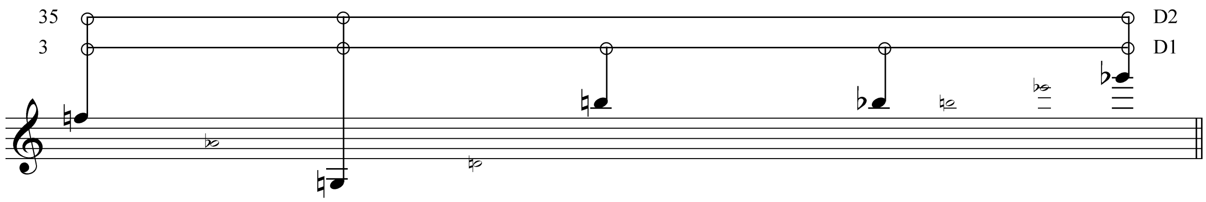


Arnold Schönberg, 3. Streichquartett für 2 Violinen, Viola und Violoncello, op. 30

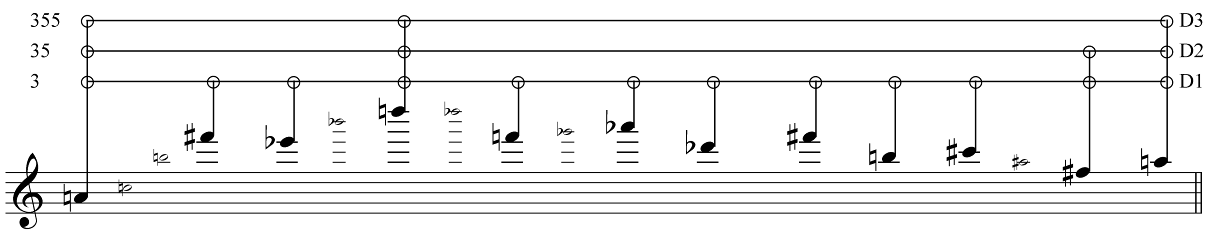
Figures 6.32 and 6.33 illustrate the reduction of these *Haupstimmen*, in which the segmentation is based on rests and each segment is denoted by an upper-case letter.

Figure 6.32. Reduction of Segmented S7

S7A



S7B



S7C

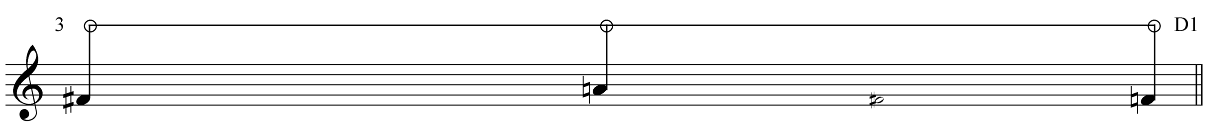


Figure 6.33. Reduction of Segmented S8

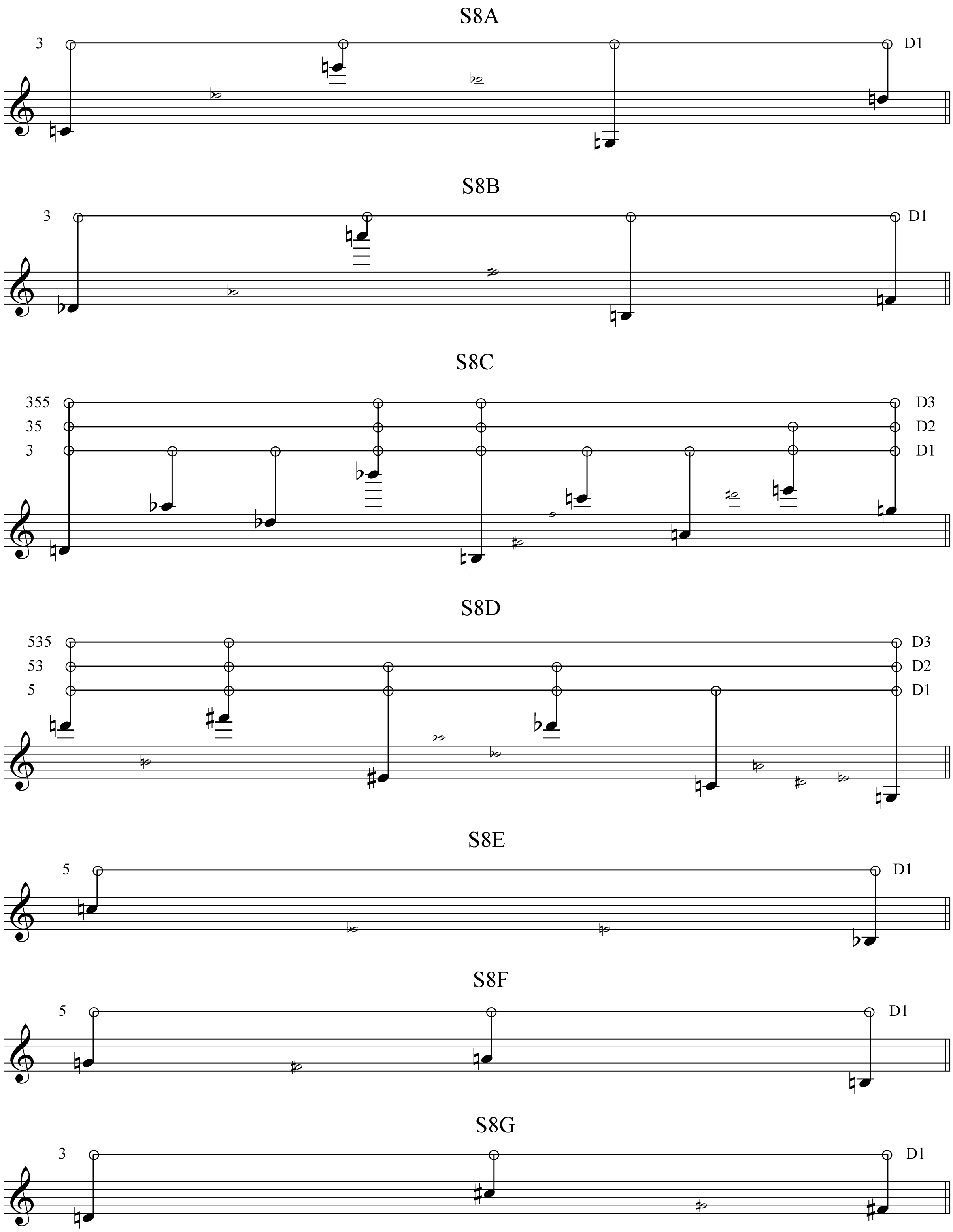
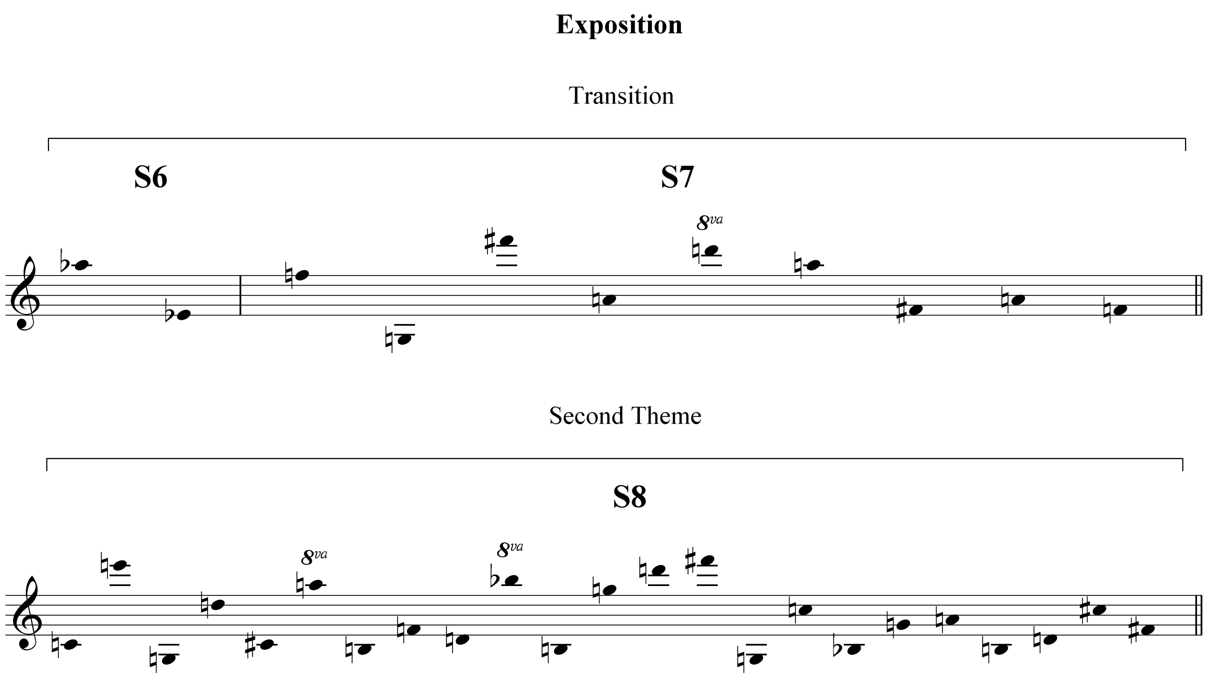


Figure 6.34 illustrates the framework pitches of S6 through S8, and a corresponding formal outline of this new interpretation.

Figure 6.34. Concatenated Framework Contours for S6, and Segmented S7 and S8



The concatenated background levels of S7, namely R3**5**(S7A), R35**5**(S7B), and R**3**(S7C) produce the string <F5, G3, Gb6, A4, D7, A5, F#4, A4, F4>. Eight notes out of nine belong to the D major or minor triad and six notes out of nine belong to the D major triad. This point is very significant since a uniform distribution for D major triad is 25%, while 67% of the framework pitches represent the D major triad. The root D is rather weak with only one appearance, but it is emphasized as the highest pitch of all three subsegments. If we follow our C-centered interpretation of the first theme (S1-S5), then this D major chord suggests a V of V in the conventional harmony and fulfills what one would expect from a “transition” in C major. Thus, this interpretation supports C major tonality rather than the C minor tonality put forward in our previous discussion. Naturally, this interpretation is heavily dependent upon the following second theme, S8. The melodic flow or harmonic progression of S8 is not as smooth or convincing as the previous *Hauptstimmen,* but its content points out remarkably clear tonal associations. Figure 6.35 demonstrates the pc distribution of the 23 framework pitches for S8A through S8G collectively.

Figure 6.35. Distribution of Framework pcs of Segmented S8

C	**C#**	**D**	**D#**	**E**	**F**	**F#**	**G**	**G#**	**A**	**A#**	**B**
•	•	•		•	•	•	•		•	•	•
•	•	•			•		•		•	•	•
		•					•				•
		•					•				

As is evident from the figure, the G-D ic 5 is heavily weighted, and the triads on those notes (especially on G) get substantial weighting. Counts for all complete triads are listed in Table 6.9.

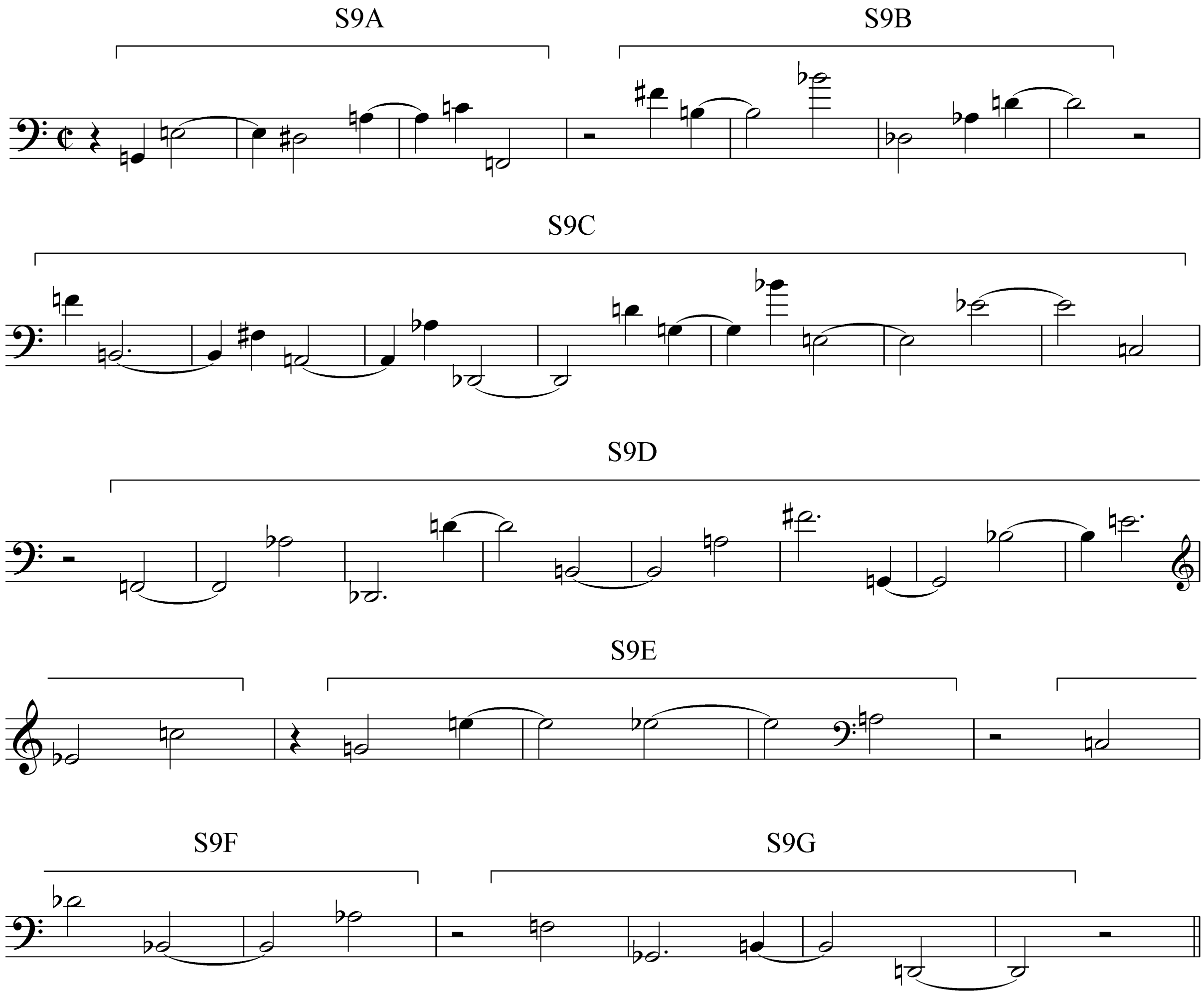
Table 6.9. Weighting of Complete Triads in the Framework of Segmented S8

Triad	Weighting
G major	11
G minor	10
D minor; E minor; Bb major; B minor	8
C major; D major	7
F major	6
F# major; F# minor; A major; A minor	5

Here, a D minor interpretation is certainly reasonable with a special emphasis on its subdominant. However, given the triadic prominence on G, which is marked by eleven notes out of twenty-three (48%), a G major interpretation seems to be convincing, despite the weak presence of its leading-tone. One should note that it is possible to hear S8 in either key but the G-oriented interpretation of S7 (as dominant) and S8 (as tonic) highlights the distinction between the formal functions of the transition and second theme, which was missing from the Eb major interpretation.

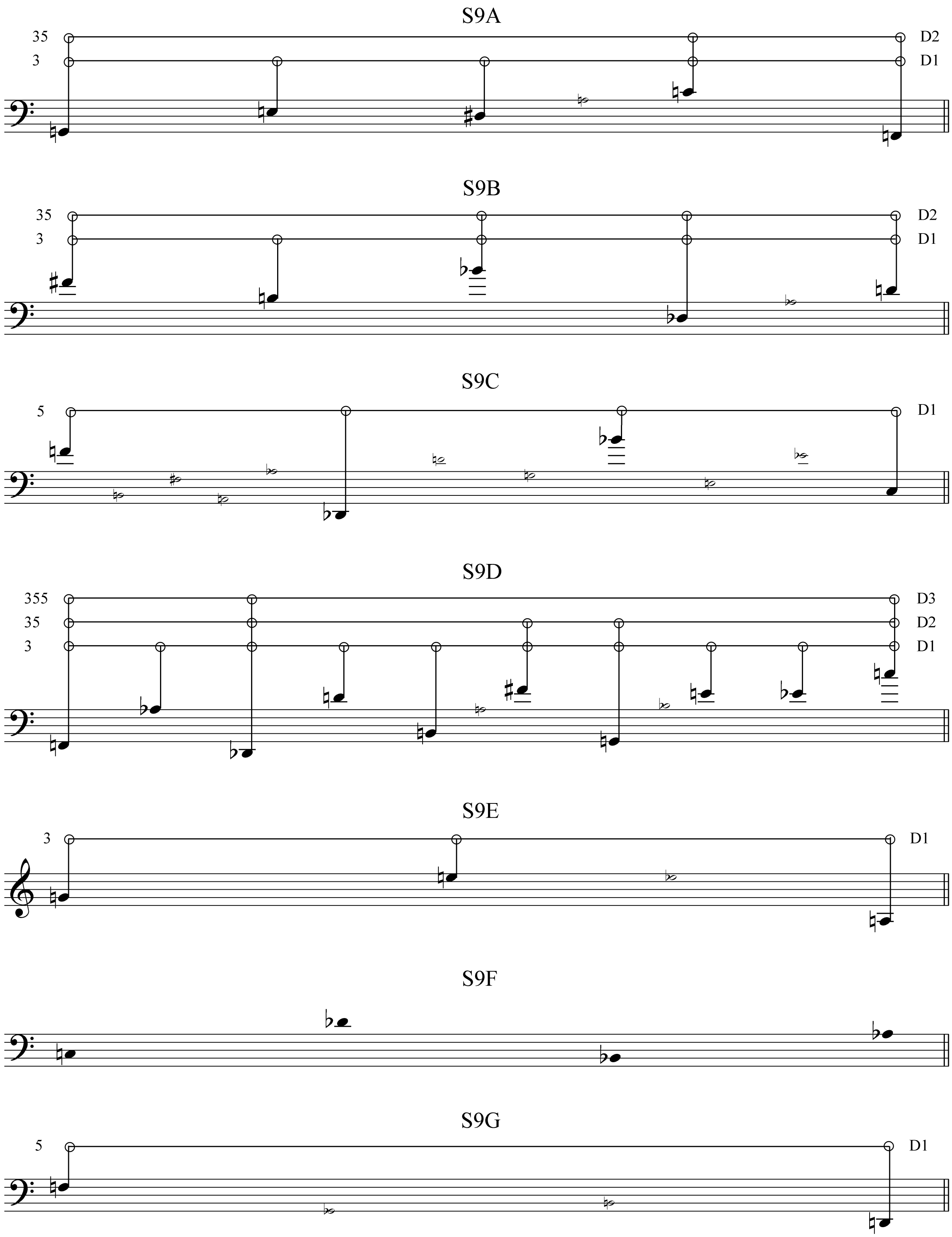
Now, let us continue our analysis with the recapitulation. The first theme and transition in the recapitulation section are the most challenging areas for a tonally-oriented interpretation is based on the contour reduction algorithms. S9 (Figure 6.27), seems to outline a C# diminished triad resolving to D, which could be interpreted as the local tonic. The previous level of reduction R355(S9) (Figure 6.22) supports this notion since this level includes Bb forming a C# diminished seventh chord rather than a triad. Since S9 is one of the three "long *Haupstimmen*," along with S7 and S8, it is possible to segment it further based on its interior rests, as illustrated in Figure 6.36.

Figure 6.36. Segmented S9



However, the reduction of this segmentation, which is shown in Figure 6.37, reveals that the contour framework does not endorse a C-centered interpretation either—as we might hope, on the conjecture that recapitulation of the first theme might project the same tonal residues as in the exposition.

Figure 6.37. Reduction of Segmented S9



The segmentation of S9 results in pitch collection <G2, C4, F2, F#4, Bb4, Db3, D4, F4, Db2, Bb4, C3, F2, Db2, C5, G4, E5, A3, C3, Db4, Bb2, Ab3, F3, D2>. Three pcs, namely C, C#, and F, occur four times each, while D# and B are entirely absent from the segmented framework. Figure 6.38 illustrates the pc distribution of the framework pitches for segmented S9.

Figure 6.38. Distribution of Framework pcs of Segmented S9

C	C#	D	D#	E	F	F#	G	G#	A	A#	B
•	•	•		•	•	•	•	•	•	•	
•	•	•			•		•			•	
•	•				•					•	
•	•				•						

In segmented S9, tonal residues are less apparent compared to many of the previous segments and the D minor interpretation of the unsegmented S9 is undermined to a certain degree. Table 6.10 lists the weighting of complete triads found in all 23 pcs.

Table 6.10. Weighting of Complete Triads in the Framework of Segmented S9

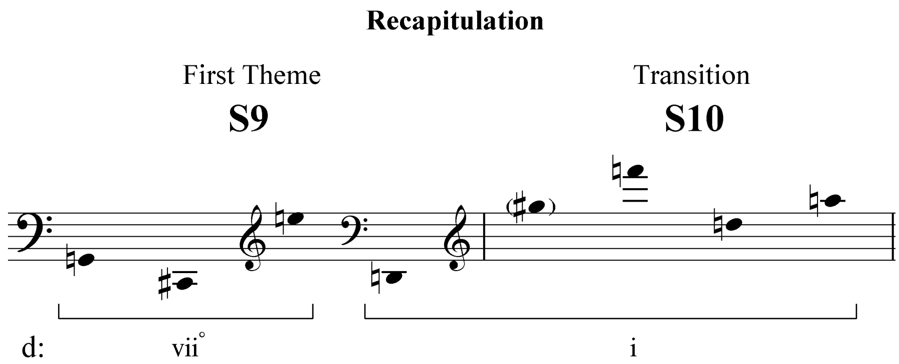
Triad	Weighting
Bb minor	11
Db major; F major; F minor; Bb major	9
Gb major	8
C major; D minor	7
F# minor; C# minor; A major; A minor	6
D major	4

As can be observed from the table, Bb minor is the most prominent triad with 11 pcs. This interpretation is also supported by the dominant F major triad and the relative Db major triad both with 9 pcs. The strong presence of Bb major, in this context, refers to modal mixture. Nevertheless, the complete lack of the subdominant triad, Eb minor, somewhat weakens this reading. Similarly, Db major and Bb major interpretations are dubious because the former lacks a dominant and the latter lacks a subdominant. By contrast, an F minor interpretation involves both the subdominant and dominant triads, Bb minor and C major with 11 and 7 pcs, respectively. An F major interpretation is also viable with a less emphasized subdominant, Bb major with 9 pcs. Unlike F minor, this interpretation is further supported by the relative triad, D minor with 7 pcs. The F interpretation is particularly interesting because subdominant is the second most commonly used recapitulation key area (after tonic) in common practice period.[153] A recapitulation in F, therefore, is hardly surprising in this context.

Although unsegmented S9 is tonally less conventional than segmented S9, it does provide a coherent harmonic progression together with S10. As shown in Figure 6.39, they present a D minor triad, plus one non-chord tone.

[153] For a detailed discussion of subdominant recapitulation, see, for instance, James Hepokoski, and Warren Darcy, *Elements of Sonata Theory: Norms, Types, and Deformations in the Late-Eighteenth-Century Sonata* (Oxford: Oxford University Press, 2006); and Charles Rosen, "Schubert's Inflections of Classical Form," in *The Cambridge Companion to Schubert,* ed. Christopher H. Gibbs (Cambridge: Cambridge University Press, 1997), 72-98.

Figure 6.39. Tonal Reinterpretation of S9 and S10



Considering S10 alone, one can also sense G#-F-D as viio7 of the final A. However, given the preceding segment, S9, the most plausible interpretation seems to be the D minor one. The C-centered interpretation of the exposition first theme raises the question of whether this D minor chord functions as a pivot chord and can in retrospect be heard as ii of C major.

The second theme in the recapitulation (Figure 6.27, S11-S12) with framework pitches <C2, G3, C5, G4, G5> can arguably be understood as an outline of the tonic C triad, supporting our earlier tonal-formal interpretation of the exposition. Interestingly, the mode ambiguity encountered in the first theme exposition is also prevalent at this crucial point in the movement. Note that this ambiguity is further supported by the possibility of the dual interpretation, which was evident from the two distinct segmentations of the transition (S6-S7) and second theme (S8) of the exposition—unsegmented second theme with residues of Eb major implying a C minor interpretation vs. segmented second theme with residues of G major implying a C major interpretation. In other words, if either of the two segmentations did point toward Eb major or C major tonality, then we could have safely asserted that it is analytically more satisfying and preferable. However in reality, each interpretation seems to hold analytical value, which results in a further veiling of the mode, as if Schoenberg decisively (yet perhaps subconsciously) aimed to obscure the mode in various formal locations by 1) avoiding a placement of the 3rd in the melodic maxima and minima (exposition: first theme; recapitulation: second theme) and 2) enabling a dual tonal interpretation based on the segmentations of the "long *Hauptstimmen*" (exposition: transition and second theme).

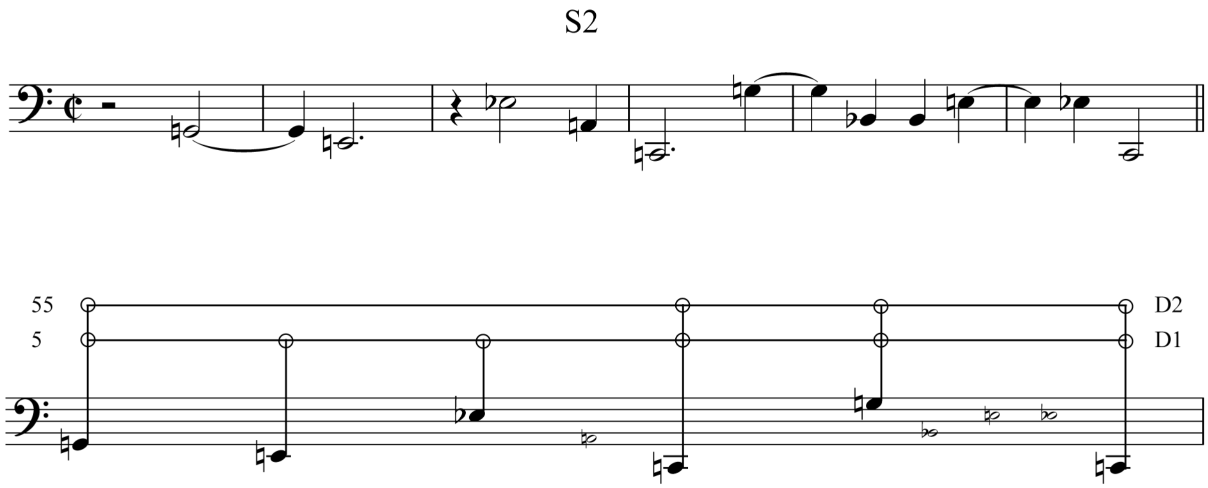
This dual tonality in the second theme area could be understood as a manifestation of what Christopher Lewis refers to as *tonal pairing*.[154] Our analytical investigation in determining an underlying tonality for the second theme area has decisively shown us that Eb major and G major tonalities coexist. As Lewis argues, "That there are paired tonics is more important than that there is progression from one to the other, for Schoenberg says that the tonic 'admits the rivalry of other tonics alongside it,' and either may emerge as the final tonic."[155]

It is also interesting to observe that the dual tonality is also present within S2. An alternative reduction routine, which does not necessarily follow the smallest window-size principle, has been discussed in previous chapters (Chapter 3 and Chapter 4, Figure 4.8). Figure 6.40 shows what happens if we begin the reduction of S2 with an application of the 5-window algorithm, rather than the 3-window algorithm. The first depth level, R5(S2), reveals a C major/minor triad with E and Eb both present, consecutively. The modal ambiguity that

[154] Christopher Lewis, "Mirrors and Metaphors," 29-33.
[155] Ibid., 29.

is to unfold is, in fact, present at the very beginning of the movement but is not as apparent in R3(S2) as it is in R5(S2).

Figure 6.40. Alternative Reduction (R55) of Segment 2



Arnold Schönberg, 3. Streichquartett für 2 Violinen, Viola und Violoncello, op. 30

Although the present analysis does not necessarily aim to base its claims on scientific grounds, a significant statistical correlation supports how the window algorithms reveal a potential or residual tonal background structure in this twelve-tone work, where one would expect a uniform distribution of pcs overall. Figure 6.41 demonstrates the pc distribution of framework pitches from the entire *Hauptstimmen* collectively (with separate tables, which account for the unsegmented and segmented versions of S7, S8, and S9).

Figure 6.41. Distribution of Framework pcs in the Movement

First Interpretation (with unsegmented S7, S8, S9)

C	C#	D	D#	E	F	F#	G	G#	A	A#	B
•	•	•	•	•	•		•	•	•	•	•
•	1	•	•	1	•		•	•	•	•	1
•		•	2		•		•	•	•	2	
•		•			•		•	3	3		
•		4			•		•				
•					•		•				
•					6		•				
7							•				
							•				
							•				
							•				
							•				
							12				

Second Interpretation (with segmented S7, S8, S9)

C	C#	D	D#	E	F	F#	G	G#	A	A#	B
•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	2	2	•	•	•	•	•	•	•
•	•	•			•	•	•	•	•	•	•
•	•	•			•	4	•	4	•	•	4
•	•	•			•		•		•	•	
•	6	•			•		•		•	6	
•		•			•		•		•		
•		•			•		•		•		
•		9			•		•		9		
•					•		•				
•					11		•				
12							•				
							•				
							•				
							15				

Both interpretations, and especially the second, give heavy weighting to the pentatonic set {C,D,F,G,A}, which indicates a general degree of consonance, and a certain proto-diatonic malleability that allows several specific tonics to emerge to various extents. For example, this pentatonic set supports F major and D minor as much as it supports C or G major. In fact, the D minor interpretation seems to fit the best since this pentatonic set includes both the tonic triad (1, 3, 5) and the harmonically most prominent scale degrees (1, 4, 5), as shown in Table 6.11. Nevertheless, the most prominent three notes in both the unsegmented and segmented interpretations (C, F, and G) point toward a C-centered interpretation. Moreover, the tonal-formal coherence (i.e. tonal residues and their connection with the sonata form) in both the unsegmented and segmented contour frameworks seems to favor the C-centered interpretation.

Table 6.11. Comparison of Possible Tonalities Based on Prominent Scale Degrees

	1	3	4	5
Dm	√	√	√	√
FM	√	√	x	√
CM/m	√	x	√	√
GM/m	√	x	√	√

As is evident from Figure 6.41, in both C major and minor interpretations the presence of pcs forming the tonic triad is above the uniform distribution (25%): the first interpretation results in 21/42 (50%) for the C minor triad and the second interpretation results in 29/84 (35%) for the C major triad. Furthermore, the tonic, subdominant, and dominant scale degrees outweigh the rest in both instances: 25/42 (60%) framework pitches in the first interpretation and 38/84 (45%) of the framework pitches in the second interpretation belong to one of these scale degrees. Considering the uniform distribution is only 25%, the contour framework asserted by the window algorithms points out the overwhelming abundance of these harmonically important scale degrees. Lastly, the pc distribution for both interpretations demonstrates a significant correlation with the C major and minor scales. The C minor interpretation contains 36/42 (86%) of the framework pitches that are members of C natural minor scale, and 35/42 (83%) that are members of C harmonic and melodic minor scales, whereas the uniform distribution is 58% for any of these scales. The second interpretation contains 62/84 (74%) of the framework pitches that are members of C major scale. Note that the segmented interpretation, with which we interpreted the second theme with residues of G major and its dominant in D major, unsurprisingly emphasizes pcs of G major triad and D major triad.

Despite the methodological and conceptual challenges of a statistical approach to scales or triadic representations, the contour frameworks do highlight a clear focus on certain pcs, in contrast with the more even distribution of the serial surface level. In this regard, the window algorithms prove to be analytically useful not only in revealing specific intervallic relationships that underlie each *Hauptstimme*, but also in providing a way to approach the sonata design of the movement from a tonal-formal perspective.

CHAPTER 7

DURATION CONTOURS: A CUMULATIVE APPROACH

Introduction

This chapter addresses the incorporation of durations into the analyses of pre-reduced, but more importantly post-reduced pitch contours. In the opening section of the chapter, a new numerical assignment for duration positions is proposed and an analytical tool, referred to as *contour difference vector*, is introduced. In the following section, the benefits of a contour approach to durations are presented through an analysis in which the pitch contours are not reduced. Conceptual and methodological inconsistencies arising from the application of the reduction algorithms to durations are examined next. Problems arising from reducing duration contours (dsegs) independently of reducing their respective pitch contours (psegs) are also discussed. Finally, a preferable alternative for reducing duration contours is presented.

Since this chapter will refer to cps and csegs in two parameters, we will use the abbreviations pp (pitch position) and dp (duration position) to refer to cps in the pitch and duration domains respectively. We will refer to pitch csegs as "psegs" and duration csegs as "dsegs."[156]

---

[156] Note that Robert Morris uses the term pseg to mean pitch segment, and not pitch-contour segment. See Robert D. Morris, *Composition with Pitch-Classes*, 45-46. As presented in Chapter 1, in this study cseg is used as a generic term without an indication of the parameter, whereas pseg and dseg are defined as a contour in pitch and duration domain, respectively. The term dseg is adopted from Marvin, "The Perception of Rhythm in Non-Tonal Music: Rhythmic Contours in the Music of Edgard Varese." It is important to reiterate that both psegs and dsegs list their respective contour positions relative to temporal ordering, rather than to some other parameter.

Duration Contours

Elizabeth West Marvin has already delineated some key aspects of approaching the duration domain from a contour perspective.[157] In her discussion, the ordering "lowest, low, high, highest" in relative pitch height, with pseg <0123>, is understood as having an analog in the rhythmic domain that will have the dseg <0123>, and for Marvin, this analog involves the duration sequence "shortest, short, long, longest." Formally, the values used in the duration domain have been ordered from short to long, so that the lower dps correspond with shorter durations. An example of a four-note segment with an identical pseg and dseg of <0123> is presented in Marvin's analysis of *Density 21.5* by Edgard Varèse.[158] An interest is thereby inherently expressed in close matching between the pseg and dseg contours of a melody. It is important to point out that although Marvin does not explicitly acknowledge such an analogy and claim that longer durations are "like" high pitches in any other sense than being assigned to larger cp values, the ensuing analysis in her article compares and contrasts the pitch and duration contours, thus forming a link between the two.

The assignment of contour-positions in duration segments (dsegs) deserves closer examination. It may at first appear that associating lower dps with shorter durations is logically consistent with the conventional association of lower pps with lower pitches. However, it is possible to generalize that in Western music longer durations take place in the lower voices, and shorter durations constituting melodies or ornaments take place in the higher voices. Consequently, it may be more appropriate to associate longer durations with

---

157 Marvin, "The Perception of Rhythm in Non-Tonal Music: Rhythmic Contours in the Music of Edgard Varèse."
158 Ibid., 78.

lower pitches.[159] A simple comparison between the violin and cello parts, or the trumpet and trombone parts, in most common practice period repertoire (and possibly also some 20th century art music) would generally confirm the claim made above, which can be simply formulated as: Longer Durations ~ Lower Pitches.[160] Throughout this chapter, we will therefore reverse the practice established by earlier theorists working with duration contours, and associate lower dps with longer durations.

This new assignment of dp values to relative durations requires inverting the dp values assigned by other theorists who have explored duration contours. For example, the ordering "shortest, short, long, longest," formerly represented as the dseg <0123>, will be revised by inverting each dp, resulting in the dseg <3210>. The ordering "lowest, low, high, highest" in relative pitch height (with pseg <0123>) will now correspond in the rhythmic domain with "longest, long, short, shortest" (with dseg <0123>), rather than with "shortest, short, long, longest" (which now has the dseg <3210>).

At this point, it is important to note that the correlation between the pitch and duration positions is less convincing when the pitches in a pseg are registrally very close to each other. For example, E1 receiving the longest duration is hardly surprising when other pitches of the segment are D1, C1, and D#1 (assuming that we expect the shortest duration for the highest pitch). This is especially true if the durational values are also proximate. However,

---

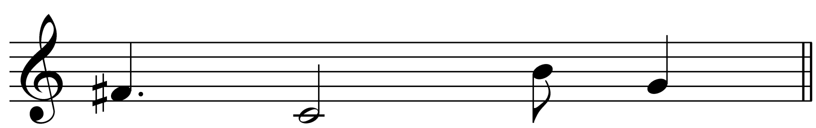
[159] The basis for this association (including aspects of human perception, the effects of instrument size and design on rhythmic agility, and other factors) is a research topic in its own.

[160] One should take this generalization with a grain of salt as there is a lack of empirical research regarding the correlation between pitch height and duration, and there are some instances that contradict this assumption (i.e.: any piano music with Alberti bass, Brandenburg concerti, etc.). A statistical investigation on a large number of musical corpora would help to solidify the claim; we do not expect that a statistical study would support the counterclaim (that longer durations are more often associated with higher pitches). A smaller scale inspection, which reveals that the longest notes (indicated by fermatas) in Berio's *Sequenza I* tend to coincide with lower pitches, is provided in the next chapter.

this particular issue is intrinsic to the notion of contour since it disregards particular interval sizes and other methodological tools are subject to similar problems.[161]

Figure 7.1 shows a hypothetical instance in which the independent parameters of pitch and duration all project the same cseg, relative to ordering in time. Figure 7.2 shows another hypothetical situation, in which each of these parameters presents a distinct cseg, again relative to ordering in time.

Figure 7.1. Identical Pitch and Duration Contours



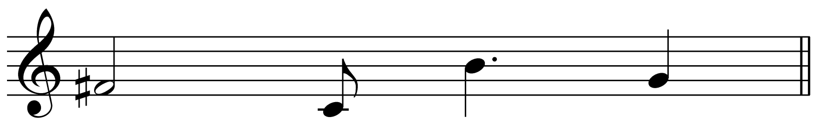
Pitch Position (pp)	1	0	3	2
Duration Position (dp)	1	0	3	2

Pitch Segment (pseg) <1032>

Duration Segment (dseg) <1032>

[161] Consider, for instance, Friedmann's CIS (Contour Interval Series), which indicates the cp distance between successive pitches. For example, cseg <0231> has a CIS of <+2, +1, –2>. Supposing the pitches of this cseg are <G3, A3, C5, Ab3>, the whole-tone pitch interval corresponding to the <+2> is smaller than the minor-third pitch interval corresponding to <+1> since contour does not reflect the actual pitch intervals. Likewise, the <–2> contour interval is also not commensurate in size, being 16 semitones. For a discussion of CIS, see Michael Friedmann, "A Methodology for the Discussion of Contour," 230.

Figure 7.2. Different Pitch and Duration Contours



Pitch Position (pp)	1	0	3	2
Duration Position (dp)	0	3	1	2

Pitch Segment (pseg) <1032>

Duration Segment (dseg) <0312>

In listening to both melodies, we hear contour accents on the lowest and highest notes. However, Figure 7.2 brings out a very crucial aspect of our realignment of dp renumbering. To the extent that we expect the shortest notes to be high, then the shortness of the lowest note C4, and the relative length of the highest note B4, are both surprising aspects of this melody (accents of distinct but related types); these are precisely the events in which the given notes' pp and dp values are most disparate. (The pp and dp values differ by 3 in the case of the C4, and by 2 in the case of the B4.) This sensation of tension between pitch and duration is particularly noticeable in comparison with the melody of Figure 7.1, in which the general comportment of each note's pitch and duration is much smoother, a fact that is appropriately linked to the identical pp and dp values for each note. (The pitch/agogic accents on C4 and B4 in Figure 7.1 are lighter than the corresponding accents in Figure 7.2.)

Figure 7.2 thus underscores how the new ordering of dp numbers (from long to short) draws special attention to moments of pitch/agogic mismatch. In Marvin's dp numbering, the dseg for Figure 7.2 would be <3021>, so the moment of greatest mismatch with the pseg <1032> would be the initial F#, even though it seems an unremarkable event and doesn't warrant the special attention it would thereby receive. The proposed new numbering, by contrast, offers us a clearer—and arguably more perceptually/phenomenally valid—way to treat and interpret the interaction of relative pitch height and relative duration.

The mismatch or disparity measure introduced above can be formalized as a difference vector between the pps and dps of a contour. This measurement, which we will refer to as *contour difference vector* (or CDV for short), lists the values for |pp–dp| for each order position. For example, the CDV of the contour illustrated in Figure 7.2 (i.e. pseg <1032> and dseg <0312>) is |1–0, 0–3, 3–1, 2–2|, or |1320|. The low numerical values such as 0 and 1 indicate events with well-matched pitch heights and durations. On the contrary, the higher numerical values such as 2 and 3 in a cardinality-4 CDV indicate notes with mismatched values, which signify notes of special interest that receive what might be called "multi-parametric contour accent." As we discussed in Chapter 2, in his book, *Highpoints: A Study of Melodic Peaks*, Zohar Eitan suggests that melodic peaks tend to correlate with various types of accent, including durational (agogic) accent and metric accent. This line of thought is in agreement with our dp assignment since the association of melodic peaks with long notes constitutes a certain type of accent (i.e.: multi-parametric contour accent). In other words, the association of melodic peaks with long notes, as pointed out by Eitan, should be seen as an accent, or an unusual event rather than a usual one.

Although a high numerical value such as 3 denotes the mismatch between the two domains according to the dp numbering proposed above, it does not inform us whether the note in question is relatively long for its height or too short for its lowness. By employing plus and minus signs (and removing the vertical bars, which indicate absolute value), however, it is possible to mark this distinction. For example, the CDV of the contour given above can be presented as <1–0, 0–3, 3–1, 2–2>, or <+1,–3,+2,0>. In this notation, the plus sign indicates that the note is too long for its height (or too high for its length), whereas the minus sign indicates that the note is relatively short for its lowness (or too low for its shortness). Thus, the CDV <+1,–3,+2,0> indicates that the first note is slightly long for its height, the second note is very short for its lowness, the third note is fairly long for its height, and the fourth note is relatively similar in height and duration.

In the present and following chapters, for the most part, we will use the CDV with plus and minus signs. However, when we are interested only in the mismatch value (for either analytical or theoretical reasons) we will employ the more simplistic CDV with the absolute values.

Webern, *Variations for Orchestra*: A Preliminary Analysis

A contour approach to the domain of durations is particularly suitable for analytical situations in which there is considerable variety among the durations that are employed. In fact, this approach provides a unique opportunity for comparing and integrating relationships between different musical parameters—in this case, the domains of pitch and duration.

For example, the pitch and duration contours in Webern's *Variations for Orchestra*, op. 30, exhibit some interesting relationships. Figure 7.3 illustrates mm. 1-20 (introduction), the first of the seven sections that are delineated by double bar lines in the score. In the figure, the 16 gestures are denoted by upper-case alphabetical letters from A to P. Table 7.1 collates the associated psegs and dsegs, and their contour difference vectors, along with the instrumentation of each gesture.

Figure 7.3. Anton Webern, *Variations for Orchestra*, op. 30, mm. 1-20

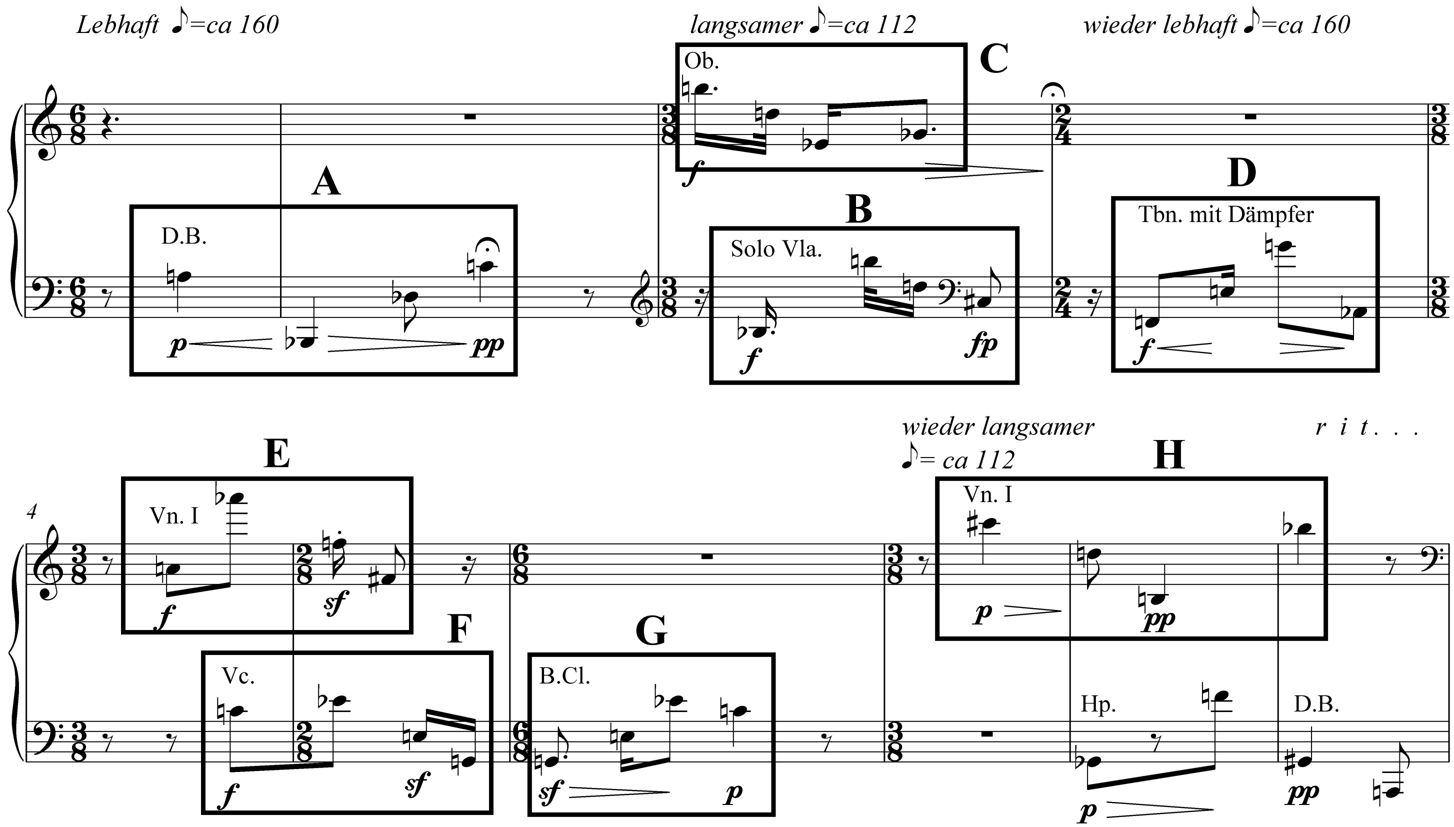


Figure 7.3 (cont.) Anton Webern, *Variations for Orchestra*, op. 30, mm. 1-20

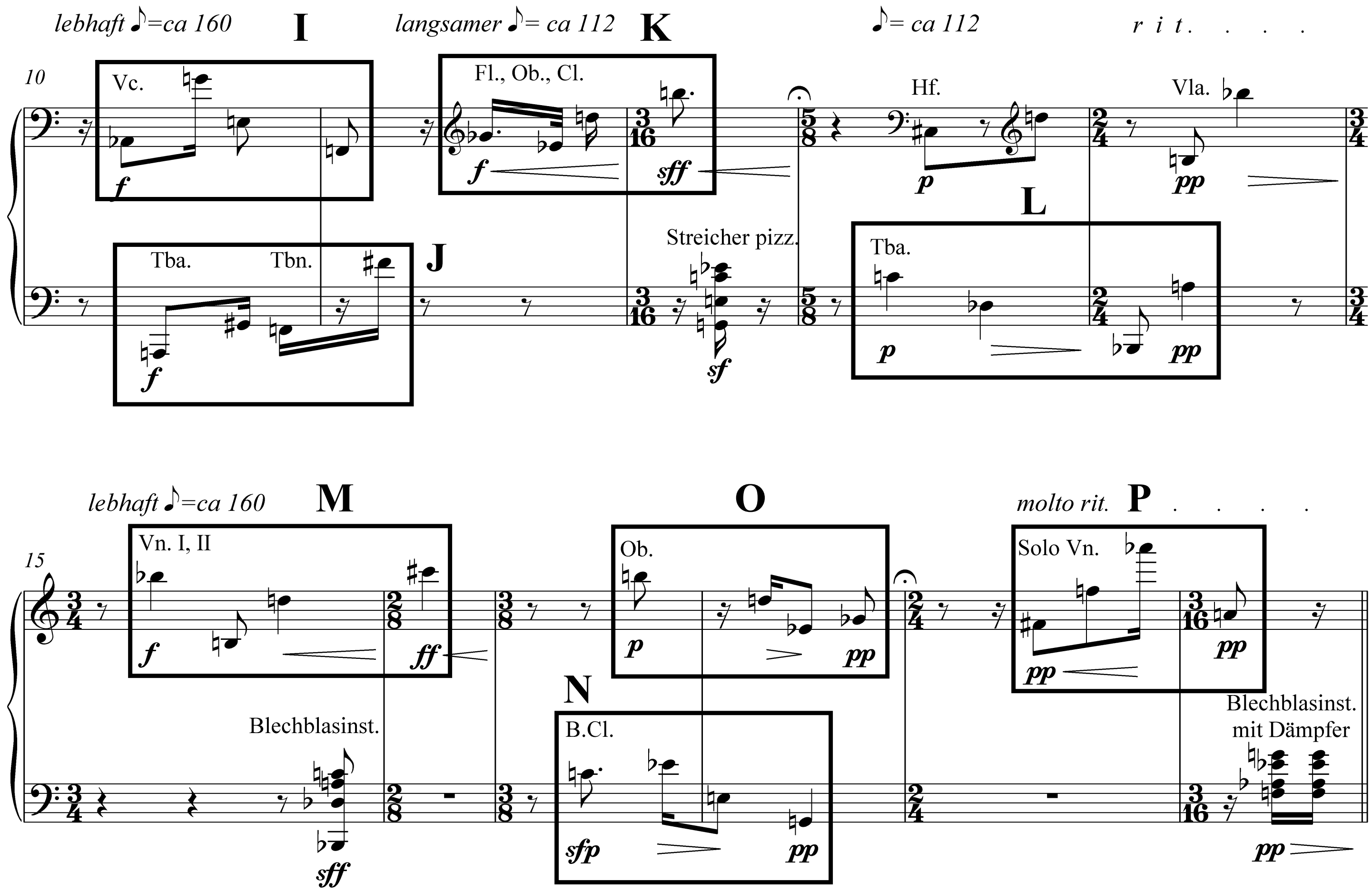


Table 7.1. List of psegs and dsegs, and their CDVs

Gesture	Instrument	mm.	pseg	dseg	CDV
A	Double Bass	1-2	<2013>	<1120>	<+1,–1,–1,+3>
B	Viola	3	<1320>	<1320>	<0,0,0,0>
C	Oboe	3	<3201>	<1320>	<+2,–1,–2,+1>
D	Trombone	4	<0231>	<0100>	<0,+1,+3,+1>
E	Violin 1	5-6	<1320>	<0010>	<+1,+3,+1,0>
F	Violoncello	5-6	<2310>	<0011>	<+2,+3,0,–1>
G	Bass Clarinet	6	<0132>	<1320>	<–1,–2,+1,+2>
H	Violin 1	7-9	<3102>	<0100>	<+3,0,0,+2>
I	Violoncello	10-11	<1320>	<0100>	<+1,+2,+2,0>
J	Tuba-Trombone	10-11	<0213>	<0111>	<0,+1,0,+2>
K	Flute-Oboe-Clarinet	11-12	<1023>	<1320>	<0,–3,0,+3>
L	Tuba	13-14	<3102>	<0010>	<+3,+1,–1,+2>
M	Violin 1-2	15-16	<2013>	<0100>	<+2,–1,+1,+3>
N	Bass Clarinet	17-18	<2310>	<1320>	<+1,0,–1,0>
O	Oboe	17-18	<3201>	<0100>	<+3,+1,0,+1>
P	Violin 1	19-20	<0231>	<0010>	<0,+2,+2,+1>

Since any fluctuation in the tempo would change the absolute durational value within a gesture, the issue of tempo changes in the score should be addressed immediately. Gestures L and P in mm. 14 and 19-20 respectively coincide with ritardandi. However, neither of these two ritardandi is likely to effect our perception of the dsegs because we hear a global slowing process rather than a change in proportions between the durational values preceding and succeeding the ritardandi. Thus, in Table 7.1, the dsegs of L and P do not incorporate any adjustment to reflect the ritardandi. The only alteration is made in the first gesture A, where a fermata is placed on the last dp.

Each gesture involves four distinct pitches, but many gestures do not involve four distinct durations. The dseg column in Table 7.1 reveals that only gestures B, C, G, K, and N give each distinct pitch a distinct duration. In fact, these five gestures all present dseg <1320>, as a result of Webern's employment of rhythmic augmentation. All but one of these gestures are presented by the woodwind instruments. (In fact, gesture O is the only woodwind gesture that does not involve four distinct durations.)

These five gestures, which I will henceforth call "main" gestures, mark formally important points in the section. Following the introductory gesture played by the double bass (ending with a fermata), the viola and oboe gestures B and C give the work its first true motion. The bass clarinet's gesture G in m. 6 closes the first phrase. Its dynamic level *p*, along with the change in tempo from 160 to 112 beats per minute, then signifies the start of the second phrase. Soon after, the first climax of the section indicated by *sff* and the unison in three different woodwind instruments (gesture K), is reached in mm. 11-12. Note that the second climax, which takes place in mm. 15-16, is not signified by different durational values in the corresponding cseg M. The bass clarinet in mm. 17-18 (gesture N), signals the closing of the introduction section by a dynamic descent from *sf* to *pp*. The isolated first violin after the fermata in m. 19 can be understood as a bridge to the following section.

These five gestures, which highlight the prominent points in the introduction section, reveal some interesting multi-parametric contour relationships. The viola's gesture B in m. 3 presents a certain type of unity between the domains: the ordered contour-position values in the pitch domain ("low, highest, high, lowest") match the contour-position values in the duration domain ("long, shortest, shorter, longest"), both resulting in cseg <1320>. In this gesture, the lowest pitch C#3 has the longest durational value of an eighth note, while the highest pitch B5 has the shortest durational value of thirty-second note. It is interesting to note that this gesture, the only non-woodwind instrument within the main gestures, is also the only one that presents a contour match between the domains.

Although the main gestures have identical duration contours, they all differ in their pitch contours (not to mention their specific durations). Nevertheless, the first woodwind gesture C can be referred to as the "source" gesture since the pitch contours of the following gestures, G, K, and N can be derived from it by standard serial transformations, as shown in Table 7.2.

Table 7.2. Derivation of Main Gestures from the Source Gesture

Gesture	C	G	K	N
dseg	<1320>	<1320>	<1320>	<1320>
pseg	<3201>	<0132>	<1023>	<2310>

C $\xrightarrow{\mathbf{I}}$ G, C $\xrightarrow{\mathbf{R}}$ K, C $\xrightarrow{\mathbf{RI}}$ N

Note that the timbrally-isolated viola is also differentiated in terms of its pitch contour as it does not relate to any of its woodwind counterparts in the ways presented in Table 7.2.

The application of the serial operators on the pseg of gesture C while retaining its dseg unaltered results in a maximum variety of cp relations: that is, each pp corresponds to a different dp until all of the possibilities are exhausted. The result, as can be seen on the lowest row of Table 7.3, is analogous to a *magic square*, one of Webern's favorite compositional paradigms.

Table 7.3. Pitch-Position and Duration-Position Correspondence in Main Gestures

cseg	**C**	**G**	**K**	**N**
pseg	<3 2 0 1>	<0 1 3 2>	<1 0 2 3>	<2 3 1 0>
dseg	<1 3 2 0>	<1 3 2 0>	<1 3 2 0>	<1 3 2 0>
pp0	dp2	dp1	dp3	dp0
pp1	dp0	dp3	dp1	dp2
pp2	dp3	dp0	dp2	dp1
pp3	dp1	dp2	dp0	dp3

This *maximum variety* of pp-dp relations is striking in the introduction of a non-traditional *variation* form. Here, the variety is provided by fixing one of the parameters and altering the other. It would have been equally plausible to alter each parameter separately, forming a variation “within” one parameter. However, Webern’s approach provides a variation between the pitch and duration domains, revealed by the multi-parametric contours. It is clear that the relationships just observed arise in close connection with the (twelve-tone) compositional technique. Even so, the methodology offers an effective way to observe, compare, and discuss such relationships in all sorts of other compositional contexts.

An examination of the CDVs associated with these gestures also reveals interesting analytic observations. The contour match between the domains of gesture B results in a minimal difference vector, <0,0,0,0>. The source gesture C, on the other hand, contains no matches between the pitch and duration domains and has a CDV of <+2,–1,–2,+1>. The first and last pitches of this gesture are relatively too long for their length and the medial pitches are relatively too short for their lowness. The first and third notes, also the highest and lowest respectively, constitute the highest difference with a numerical value of 2. Although the following main gesture G has a somewhat similar CDV of <–1,–2,+1,+2>, its greatest mismatch coincides with the second and fourth notes. The pitches of this gesture are heard initially as too short for their low (indicated by successive – signs) and then too long for their height (indicated by successive + signs). Interestingly, gesture K, which holds the climax among the main gestures, presents a very bumpy CDV of <0,–3,0,+3>, alternating between minimum and maximum differences. Here the second and fourth notes of the gesture are maximally disparate: the lowest pitch receives the shortest duration whereas the highest pitch receives the longest duration. This CDV involves the highest mismatch of all the main gestures and further supports the climax established by the dynamic level and the flute-oboe-clarinet unison. The last main gesture N has a contour difference vector of <+1,0,–1,0>, which contains as minimal disparity as possible without its multi-parametric contours being identical, (as with gesture B). The last note of gesture N presents a perfect match between the two domains, just like the entire CDV of the opening gesture B.

The CDV successions of the main gestures present a rather interesting overall organization. The perfect match between the multi-parametric contours of gesture B is gradually replaced by a sensation of tension through gestures C and G, which reaches a climax in gesture K. The last gesture N provides a resolution to the gradually raising disparity between the multi-parametric domains. This organization of an overall ABA form is also supported by *contour motion vectors* as we shall see shortly.

The content of the CDVs in the main gestures indicates that 0 is the most common and 3 is the least common CDV value. This distribution is in agreement with the *maximum variety* of pp-dp relations (in gestures C, G, K, and N) discussed previously. Table 7.4 shows the 16 possible pp-dp mappings (i.e. each pp value corresponding to a different dp value) and each corresponding CDV numerical difference. The bottom portion of the table lists the total number of occurrences for each numerical difference.

Table 7.4. Pitch-Position and Duration-Position Correspondence in Maximum Variety

pp0→dp0 = $\boxed{0}$	pp0→dp1 = $\boxed{-1}$	pp0→dp2 = $\boxed{-2}$	pp0→dp3 = $\boxed{-3}$
pp1→dp0 = $\boxed{+1}$	pp1→dp1 = $\boxed{0}$	pp1→dp2 = $\boxed{-1}$	pp1→dp3 = $\boxed{-2}$
pp2→dp0 = $\boxed{+2}$	pp2→dp1 = $\boxed{+1}$	pp2→dp2 = $\boxed{0}$	pp2→dp3 = $\boxed{-1}$
pp3→dp0 = $\boxed{+3}$	pp3→dp1 = $\boxed{+2}$	pp3→dp2 = $\boxed{+1}$	pp3→dp3 = $\boxed{0}$

<0>: 4 <+1>: 3 <–1>: 3 <+2>: 2 <–2>: 2 <+3>: 1 <–3>: 1

Note that Table 7.4 also presents all possible pp-dp mappings and therefore, we can deduce that a cseg is more likely to have a multi-parametric match than a multi-parametric accent; in the Webern example we observe that the uncommon CDV values of <+3> and <–3> are preserved for the climactic gesture K as discussed above.

Another type of variety that is related to, but different than the maximum variety of pp-dp relations, can be observed from the similar or different motions in each domain. For example, the contour motion of the last main gesture N involves a rise (+), then a fall (–), and then another fall (–) in both domains, and thus, can be represented as <sss>, indicating three similar motions. By contrast, gesture K involves a fall (–) in the pitch domain and a rise (+) in the duration domain, followed by two successive rises (+,+) in the pitch domain and two successive falls (–,–) in the duration domain, forming three contrary motions, or <ccc>. Henceforth, we will refer to these multi-parametric contour motions as *contour motion vector* (or CMV for short). Although motions involving different domains are somewhat abstract and speculative, they can be most easily understood as a measure of sign match/mismatch between the multi-parametric contours, where a plus sign indicates a rise in pitch or a decrease in duration and a minus sign indicates a fall in pitch or an increase in duration. For example, the CMV of gesture N involves only similar motions because a rise in pitch is always accompanied by a decrease in duration and a fall in pitch is always accompanied by an increase in duration. The CMV of gesture K, on the other hand, involves only contrary motions because a rise in pitch is always accompanied by an increase in duration and a fall in pitch is always accompanied by a decrease in duration. In essence, CMV is strongly related to the concept of "contour adjacency series:" a sign match in multi-parametric contours signifies similar motion and a sign mismatch in multi-parametric contours signifies contrary motion, as illustrated in Table 7.5.

Table 7.5. CAS and CMV correlation between Gestures N and K

	gesture N	**gesture K**
CAS (pseg)	<+, –, –>	<–, +, +>
CAS (dseg)	<+, –, –>	<+, –, –>
CMV	<s s s>	<c c c>

Table 7.6 shows the CMV of all five main gestures. Each motion is either similar or contrary, and abbreviated as s and c, respectively. There are no parallel or oblique motions in any of the gestures except for the case of gesture B, in which the cp values are identical; here e is employed in order to present the identity.[162] The consecutive motions between the cps are indicated in angle brackets below each gesture.

[162] Note that "e" will be used only when at two successive temporal positions, both dimensions (twice) have identical values. If only one pair is identical, the motion will be similar or contrary, as in N and K, respectively.

Table 7.6. CMV of Main Gestures

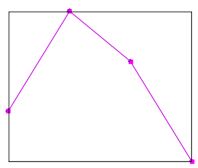
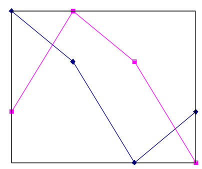
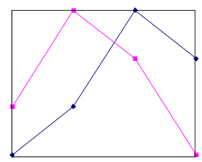
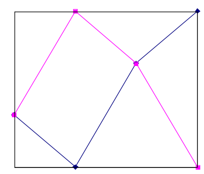
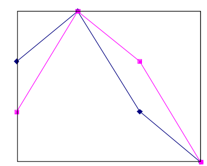
	gesture B	**gesture C**	**gesture G**	**gesture K**	**gesture N**
					
pseg	<1 3 2 0>	<3 2 0 1>	<0 1 3 2>	<1 0 2 3>	<2 3 1 0>
dseg	<1 3 2 0>	<1 3 2 0>	<1 3 2 0>	<1 3 2 0>	<1 3 2 0>
CMV	<eee>	<csc>	<scs>	<ccc>	<sss>

Table 7.6 demonstrates that none of the woodwind gestures (C, G, K, N) has identical CMV. Once again, a multi-parametric variation is established via motions between the pps and dps. Interestingly enough, the sense of closure in the last gesture N is further supported by a return to the CMV that was presented by the first gesture B. Both of the gestures have a CAS of <+, –, –> in their pitch and duration domains as shown in Table 7.7.

Table 7.7. CAS Comparison between Gestures B and N

	gesture B	**gesture N**
CAS (pseg)	<+, –, –>	<+, –, –>
CAS (dseg)	<+, –, –>	<+, –, –>

In contrast to the opening and closing gestures B and N, gesture K, where the climax takes place, has three consecutive contrary motions, adding to the tension already created by its timbral complexity, dynamic level, and maximal CDV. It is important to note that a maximal CDV (i.e. a CDV that involves two <3>s) does not always result in a CMV of <ccc>. For example, pseg <0123> and dseg <3120> have a CDV of <–3,0,0,+3> but a CMV of <csc>. The CMV of <ccc> (as opposed to <csc>, <scc>, or <ccs>) presented by gesture K further endorses the sense of tension.

Together with the climax, the opening and closing gestures B and N form a <sss>, <ccc>, <sss> "arch," similar to an ABA form, which we previously observed in our discussion of CDVs. We have already pointed out earlier in the analysis that gesture N provides a sectional closure by a dynamic descent. In addition, it is interesting to note that the last pp and dp values for gesture N are both "0." In other words, all three parameters support the formal function of closing: the lowest, longest and softest note take place at the last entry (i.e. op 3) of gesture N. Despite these observations, global closure is not fully achieved, as the ending does not provide a return to pseg <3201>. This is understandable, as a sense of finality should probably be avoided at the end of an introduction section. However, a local closure for the section is ensured by the means mentioned above, yet the expectation of a global closure is thwarted as the work continuous to the second section.

Let us also make some observations on the *subsidiary* gestures, which are not a part of the five main gestures discussed above. First, it is important to note that the notion of contour difference vector is somewhat moot when there is a significant amount of repetition in one of the domains. For example, the highest notes of gestures I and P correspond to the shortest notes of the segments respectively; however, these particular matches are not reflected in the CDV since there are only two distinct durational values. Similarly, the second lowest note of gesture H corresponds to the shortest note of the segment creating a sensation of tension, yet the CDV not only fails to demonstrate this but in fact returns a value of 0 for the note in question (see Table 7.1). Thus, the following discussion of subsidiary gestures will be restricted to pp and dp mappings and will not include CDV or CMV.

Table 7.8 shows that all subsidiary gestures—except A and F—have three repeating dps and in all but one (gesture J), the longest duration, dp 0, is the repeated dp.[163] A mapping between the pps and the corresponding dps of the subsidiary gestures reveals which pp is associated with the (single) non-repeating dp. The non-repeating (n-r) dp for each gesture is displayed at the bottom row of the table.

Table 7.8. Pitch-Position and Duration-Position Correspondence in Subsidiary Gestures

cseg	**D**	**E**	**H**	**I**	**J**	**L**	**M**	**O**
pseg	<0231>	<1320>	<3102>	<1320>	<0213>	<3102>	<2013>	<3201>
dseg	<0100>	<0010>	<0100>	<0100>	<0111>	<0010>	<0100>	<0100>
n-r dp	pp2	pp2	pp1	pp3	pp0	pp0	pp0	pp2

A similar degree of multi-parametric variety as we previously observed in Table 7.3, is also apparent in the subsidiary gestures: the non-repeating dps alternate between pp 2, pp 1, pp 3 and pp 0 in turn. In other words, the distinct durations correspond to all possibilities of pitch positions. In gestures D and E the distinct duration is heard in the second highest pitch; in gesture H the distinct duration is heard in the second lowest pitch; in gesture I the

[163] Note that gestures A and F are excluded because they do not have three repeating dps, and the very last gesture P is excluded because it functions as a bridge to the second section.

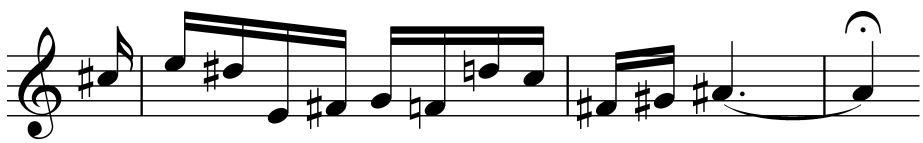
distinct duration is heard in the highest pitch; in gestures J, L and M the distinct duration is heard in the lowest pitch. Note that in gesture J the lowest pitch receives the longest (and non-repeating) duration while in gestures L and M the lowest pitches receive the shortest (and non-repeating) durations.

The analysis presented above, which reveals how variation form is articulated by multi-parametric contour relationships, demonstrates the analytical potential of a contour approach to the domain of durations, and of a multi-parametric approach to duration and pitch height.

Paucity in the Duration Domain as an Analytical Obstacle

An important observation regarding the work analyzed in the previous section is that there is a considerable amount of variety in the duration domain, as is evident from the main gestures. Furthermore, the analysis itself is based on rather short (cardinality-4) contours, which we referred to as gestures rather than segments due to their size. Although in some cases it is possible to meaningfully analyze dsegs with repeating values—as we did with the subsidiary gestures—the special circumstances mentioned above (i.e. the variety in the duration domain coupling with short contours) also resulted in duration contours of non-repeating dps, which turn out to be decisive for meaningful multi-parametric contour comparisons.

However, a significant number of compositions involve longer segments that do not present adequate duration variety. These present a challenge to contour analysis in the duration domain. (In fact, in many post-tonal works, pitch is the only domain that consistently possesses the considerable variety that is generally the subject of contour analysis.) The excerpt from Schoenberg's op.19, no.4, discussed in Chapter 3 and illustrated below in Figure 7.4, is one such example.

Figure 7.4. Schoenberg, op.19, no.4, second phrase

Here, the duration domain has only 2 duration positions, which results in a rather humdrum dseg: <111111111110>.[164] Such a representation of the duration domain does not provide many analytical benefits and thus, it is more appropriate to approach this domain from a contextual perspective rather than a mechanical one. For example, one can easily observe the emphasis on the last note due to the durational accent it receives, without consulting the contour representation of the ordered durations.

As is evident from the excerpt above, an application of contour theory to the domain of durations is highly dependant on the variety presented within the domain itself. Although a contour approach to durations provides valuable analytical insights into a certain corpus of

[164] Here, one possible approach would be to divide the segment into 4 units on the basis of register (separated by leaps larger than a 3rd), lasting 3, 4, 2, and 12 sixteenth notes—thus dseg <2130>. However, this methodology does not fully endorse contour based interpretation since pitch contour is indifferent to interval size. A different approach, which also incorporates the notion of contour reduction and depth levels will be presented below.

music (particularly those employing integral serialism), it is bound to lack a generalized applicability to a countless number of post-tonal compositions.

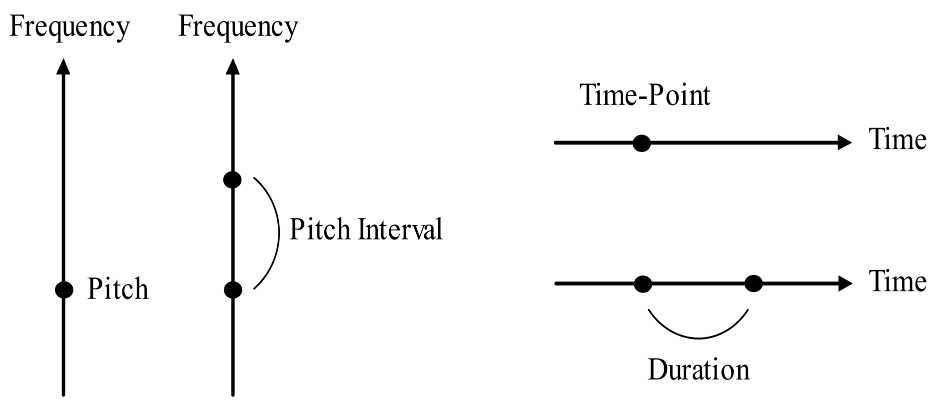
Conceptual and Methodological Inconsistencies Arising from the Application of the Window Algorithms to the Duration Domain

So far, we have been focusing on the contour approach to the duration domain without any reference to the reduction algorithms. Now, let us consider the application of the window algorithms to duration contours.

The application of the window algorithms to the duration domain poses a number of problems. The first of these problems is a conceptual one caused by the discrepancy between the concepts of duration and pitch, and by extension, duration contour and pitch contour. While a pitch corresponds to a single point or value on the frequency continuum, a duration is determined by a *pair* of time-points in the temporal continuum. In essence, a duration contour, considered relative to temporal ordering, would really be more analogous, in the domain of pitches, to "pitch interval contour" considered relative to pitch order (i.e. "pitch contour").[165] Figure 7.5 indicates how durations are analogous to pitch intervals.

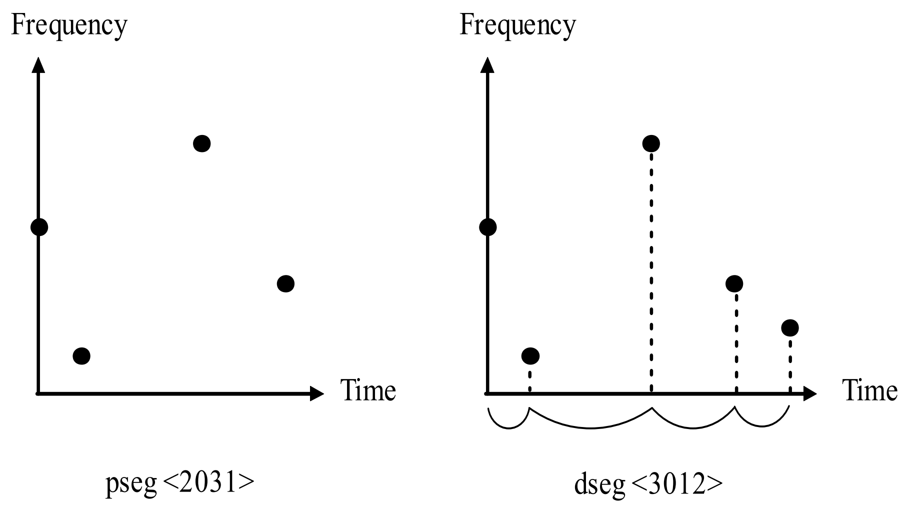
[165] Another closely-related factor that creates conceptual obstacles for duration contours occurs here because we are considering duration positions relative to temporal ordering, rather than relative to ordering in some other parameter. In this situation, we are comparing durations to the very parameter that defines them as values relative to one another, and this is the reason why duration/temporal-ordering contours correspond with intervals, rather than relative ordering in two different domains, as with pitch/temporal-ordering contours. We could mitigate this problem by restricting our purview to contours of durations relative to a different parameter but in this study we will continue to consider duration/temporal-ordering contours, since the concept of temporal ordering is so fundamental and ingrained in our conceptions of musical relationships and continuities.

Figure 7.5. Conceptual Comparison of Pitch and Duration



This discrepancy between pitch contour and duration contour is better understood by comparing two cardinality-4 contours (one pseg and one dseg), as illustrated in Figure 7.6. Here, the number of points (i.e. events) in the respective domain for each contour is different despite their identical cardinality: four points are adequate for a cardinality-4 pseg, whereas five points are required for a cardinality-4 dseg. This discrepancy arises because the duration of the fourth event cannot be identified until the onset of the next event, which is not the case in the pitch domain. Nevertheless, by convention, we shall agree that a four-note segment is considered to have pseg and dseg of cardinality-4 on the idea that the onset of the fifth event, which marks the end of the four-note segment, is nominally not taken into consideration.

Figure 7.6. Discrepancy between Pitch and Duration Contours

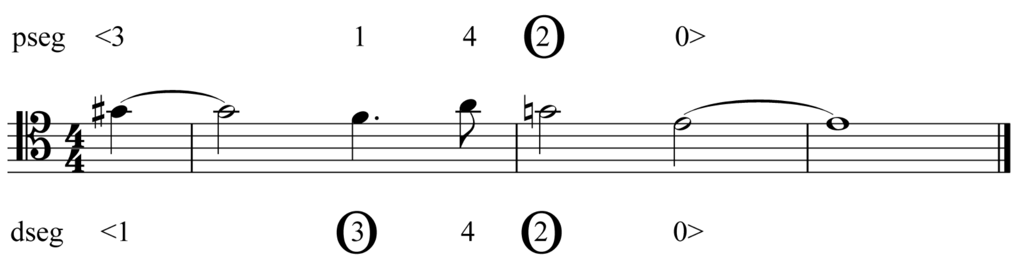


Another problem created by the misconception of relating the pitch domain with the duration domain is that there is no adequate durational analogy to the notion of a "passing pitch," a notion that plays an important role in the 3-window algorithm in Chapter 3. The proper analogy to the concept of "passing duration" would be a "passing pitch interval," and a reduction of an interval (whether pitch-interval or duration) based on the principles laid out by the 3-window algorithm is perceptually moot, unless it is a more extended series of progressive increases or decreases.[166] An alternative reduction method for duration contours will be introduced later on in this chapter. For the moment suffice it to say that an application of the window algorithms, which are developed to prune and retain single points in pitch-space, is not ideal for pruning and retaining intervals between two points in temporal-space.

[166] Note that this is not to say it is outright impossible to hear expanding intervals as one can easily hear them, for example, in traditional horn fifths (i.e.: E4-D4-C4 over C4-G3-E3). However, in this example, the aural recognition of expanding intervals does not warrant hearing the second interval as "passing," in the sense that it is hierarchically less salient on that account. Moreover, here, our recognition of expanding interval series is heavily influenced by our familiarity with the example. Such idiomatic examples are rare in post-tonal music.

Furthermore, other problems, which are methodological in nature, arise from the application of the algorithms on each domain separately. One of these is the potential—in fact, the likelihood—for conflicting deletions and retentions in the two domains. For example, a pruned pp might correspond to a retained dp, or vice versa. In this case, the reduction in one domain would be in conflict with the reduction in another domain. In addition, there is likely to be a cardinality mismatch after the parametric contours of a segment are reduced independently from one another, since a different number of cps may be pruned in each parameter. An example of this case can be observed in the first phrase of the opening *Hauptstimme* from the second movement of Schoenberg's *Five Pieces for Orchestra*, op. 16, which is illustrated in Figure 7.7.

Figure 7.7. Schoenberg, *Five Pieces for Orchestra*, op. 16, ii, mm. 1-3



The pseg and dseg of this passage, <31420> and <13420>, are indicated respectively above and below the staff. It is interesting to observe that the last three cps of both csegs match whereas the first two are merely swapped. But greater differences between the two contours come into view when we consider how the 3-window algorithm would prune cps in each domain. Circled cps indicate that in the pitch domain the fourth entry, pp 2, would be pruned while in the duration domain the second and fourth entries, dps 3 and 2, would be pruned. As is evident from the figure, the fourth entry, corresponding to half note G4, is pruned in both domains, and thus, does not cause methodological problems for the separate parametric application of the reduction algorithm.[167] However, the second entry, corresponding to the dotted-quarter F4, is problematic because it would be pruned in the duration domain but retained in the pitch domain. Thus, the applications of the algorithm separately on each domain would result in a methodological conflict: on the one hand the second note of the segment is required to be retained due to the pitch contour reduction, and on the other hand, it has to be pruned due to the duration contour reduction, and in any case, applying the 3-window algorithm to duration contours invokes the dubious notion of a "passing duration."

Moreover, applying the 3-window algorithm to the two domains independently results in different reduced cardinalities: the reduced pseg would be the cardinality-4 cseg <2130> while the reduced dseg would be the cardinality-3 cseg <120>. As a consequence of this cardinality mismatch, the correspondence between the pitch and duration positions is obscured. Assuming the first and last cps would be paired, the ambiguity concerns the middle cps. In this case, it is not clear whether dp 2 corresponds to pp 1 or pp 3. An answer to this methodological dilemma would leave one of the pitch positions, either pp 1 or pp 3, without a durational value.

[167] Note that here one could argue for pruning the eighth-note A4 before any other note since it could be interpreted as an appoggiatura embellishing the G#4-F4 and G4-E4, two paired minor thirds.

So far, we have identified the following problems with reduction algorithms involving duration contours:

1. The possible lack of variety in the duration domain.

2. The conceptual inconsistency between pitch (a point) and duration (an interval).

3. The implausibility of the notion of passing durations and the concomitant problems in applying the 3-window algorithm to duration contours.

4. The conflict between the pruned and retained cps in different domains when reduction algorithms are applied to each domain independently.

5. The potential cardinality inconsistency between the csegs in different domains when reduction algorithms are applied to each domain independently.

The third problem is a result of the second one, and the fourth and fifth problems are strongly related.[168] In fact, each of these five problems can be understood as being analytical, conceptual, or methodological in nature. The first problem is an analytical one because a paucity of different cps precludes opportunities for making convincing analytical distinctions. The second and third problems involve conceptual inconsistencies that render dubious the criteria on which the reduction algorithms prune out specific cps. The last two problems are methodological, since the application of the algorithms to separate sets of data is inconsistent and technically unreliable. In order to adopt a contour reduction approach to

[168] Note that the fourth problem is necessary but not sufficient for the existence of the fifth problem. For example, the conflicting deletion of one and only one cp in each domain would still result in identical cardinalities. The sufficiency would be met once one of the domains has more deletion than the other.

the duration domain, and to pitches and durations together, a solution that addresses each of these problems will need to be introduced.[169]

Two Solutions to the Inconsistencies

The first solution proposed here is to adopt an ad hoc approach to the duration domain, rather than a contour approach. Such an approach can point out some observations without taking duration contours into account. We observed this approach in Figure 7.4, where there was lack of variety in the duration domain. This approach not only eliminates the first problem (the lack of variety), but also eliminates the rest of the problems as it simply refuses to approach durations from a contour perspective. The major disadvantage in rejecting duration contours altogether is that they sometimes do provide valuable insights. For example, the analysis of Webern's op. 30 demonstrates that a contour approach to durations may sometimes yield powerful analytical observations. Thus, an alternative solution is desirable for working with multi-parametric contours. This solution should provide the benefit of comparing pitch and duration contours without being hampered by the problems listed above.

The second solution involves making the durational values—and durational contours—contingent upon the pitch contours. The reduction algorithm is not applied to the duration contours, but new durations are assigned after the reduction of the pitch contours. The assignment is executed by "adding" the duration of each pruned pp to the duration of the

[169] It is important to point out that there are other questions regarding duration contours which are related, yet distinct from the five listed in this study. For example, do we retain durations in memory in a similar way we retain pitches, or do we attribute duration relatively, in comparison to the duration last heard? How many durations can we remember and how similar can they be, for us still to be able to compare them? Do we perceive changes in duration trends the way we perceive changes in pitch trends? And so forth.

closest preceding non-pruned pp. This process of re-assigning durations, which results in new durational contours, is given below in four steps:

Step 1: Reduce "depth level n" pitch contour by employing one of the window algorithms to obtain "depth level n+1" pitch contour.

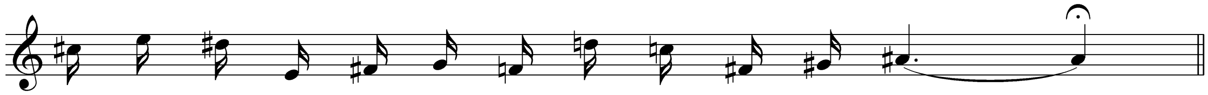
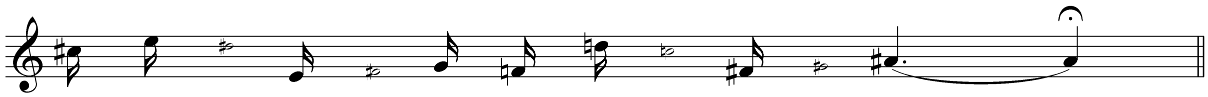
Step 2: Add the duration of each pruned pp to the duration of the closest preceding retained pp.

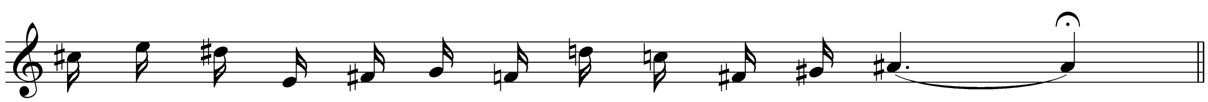
Step 3: Notate the ordered durations for each pp in "level n+1."

Step 4: Convert the ordered durations to a dseg in "level n+1."

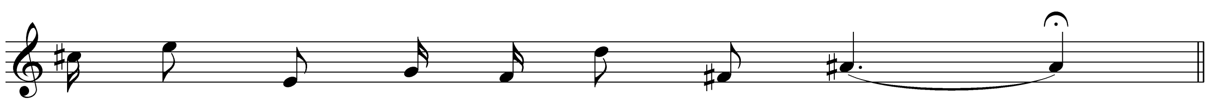
Figure 7.8 illustrates this process on the excerpt from Schoenberg's op. 19, no. 4.

Figure 7.8. Application of Four Steps to Schoenberg, op. 19, no.4

ORIGINAL SEGMENT S

STEP 1:
Pitch Contour Reduction R3(S)

STEP 2:
Durational Value Transfer from the Pruned pps to the Closest Preceding Retained pps



STEP 3:
Notation of the New Durational Values

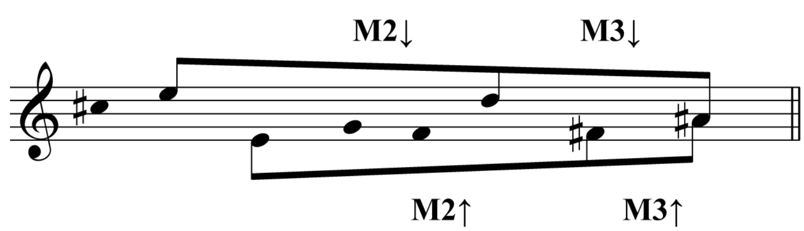


STEP 4:
Conversion of the New Durational Values to Durational Contour

dseg: <2 1 1 2 2 1 1 0>

Here, E5, E4, D5 and F#4 are each extended by the time spans of D#5, F#4, C5, and G #4, respectively. In other words, these retained pitches, whose duration is extended to an eighth note, can be considered as being "prolonged" (by the duration of any ensuing pruned pitch). Interestingly, the upper-register prolonged pitches (E5, D5, A#4) are related by "even" intervals (descending whole-step and descending major third), and so are the lower-register pitches (E4, F#4, A#4). The wedge formed by these prolonged pitches merges on the final A#4, which lies symmetrically between the other upper and lower prolonged pitches, as shown in Figure 7.9. Note that the non-prolonged (sixteenth-note) pitches are indicated without stems and belong to the complementary whole tone collection.

Figure 7.9. The Wedge-Shaped Contour at R3



It is important to note that the new durational values (and the contour) resulting from the process illustrated in Figure 7.8 belongs to the first depth level when the 3-window algorithm is applied on the segment. Already at this depth level, we can observe the increase in the durational variety from two dps (0 and 1) in the original contour to three dps (0, 1, and 2) in the reduced contour. Figures 7.10, 7.11, and 7.12 demonstrate the pitch contour reductions for R3, R35, and R35**5**, respectively, on the top staves, the durational values as well as the duration contours on the middle staves, and the combination of both domains on the bottom staves. Note that the bottom staff in each figure represents only one depth level unlike the single-staff notation adopted for pitch contour reductions. A single-staff notation involving both pitch and duration domains seems to be impractical, as it would likely harm the legibility of the reductions.

Figure 7.10. Pitch and Duration Contours for Depth Level 1

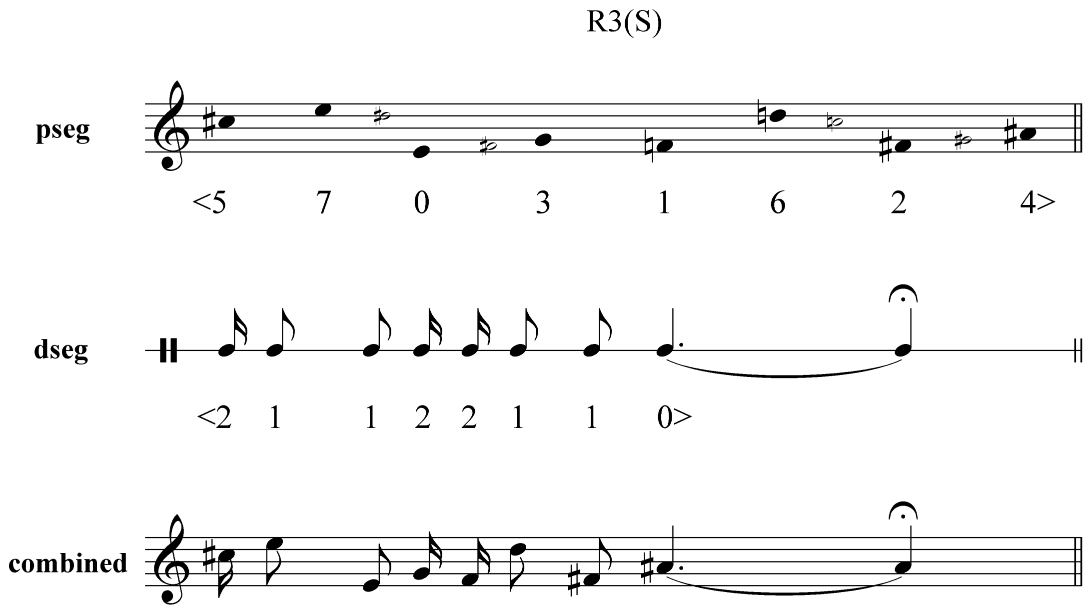


Figure 7.11. Pitch and Duration Contours for Depth Level 2

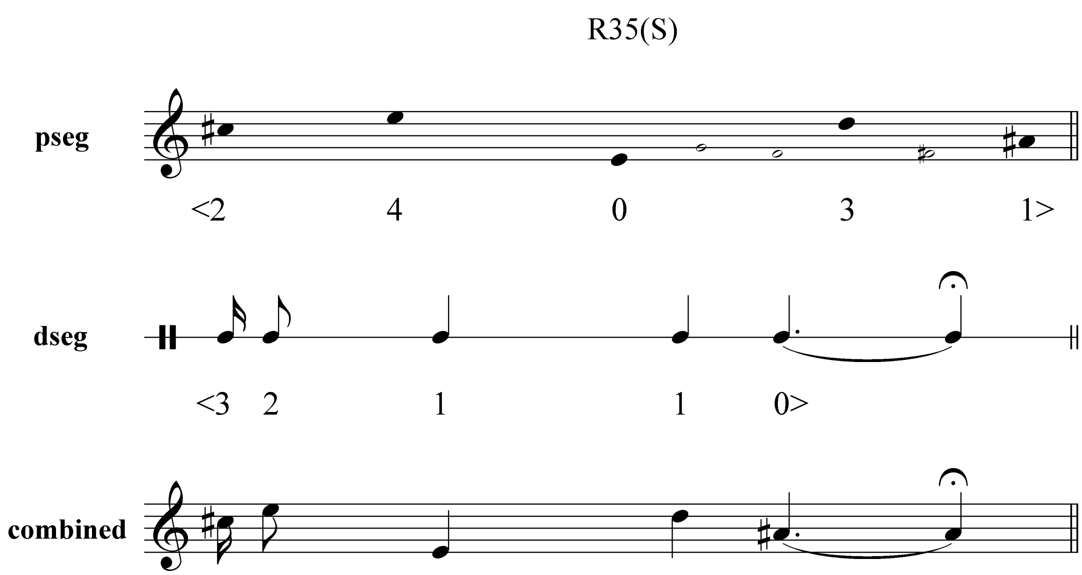
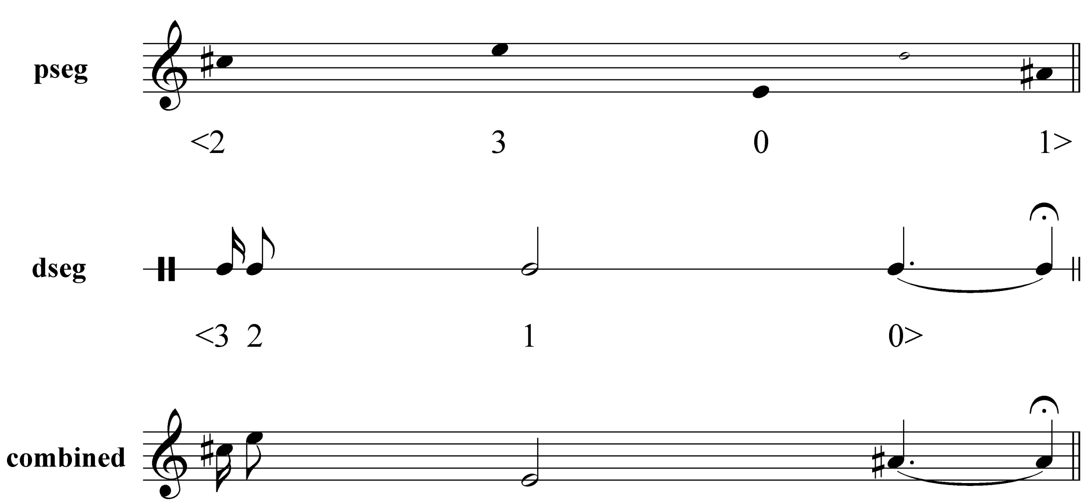


Figure 7.12. Pitch and Duration Contour Framework

R35**5**(S)

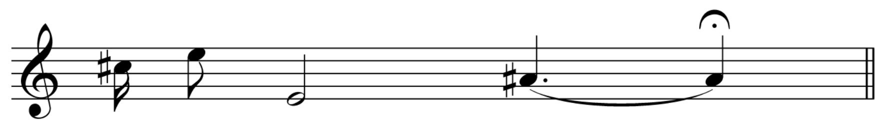


As is evident from the figures, starting with the second depth level, the durational variety increases gradually when this analytic methodology is employed. In this particular excerpt, the comparison of the duration contour with the pitch contour in the deepest level reveals that the maximum and minimum in the pitch domain always correspond to a medial cp in the duration domain, and vice versa. This can be understood as an equal distribution of all four maxima and minima—two pitch and duration maxima, and two pitch and duration minima. In other words, each event (i.e. note) receives either a maximum or a minimum in one (and only one) of the domains, as illustrated in Figure 7.13. Here, the upper-case M and lower-case m denote "maxima" and "minima," respectively. This equal distribution of all four maxima and minima results in a uniform CDV of <–1,+1,–1,+1> since the pseg <2301> and dseg <3210> have difference vectors of 1 for all notes. Note that the CDV of an equally distributed maxima and minima cannot involve "0" or "3:" a note whose CDV is <0> or <3>

receives maxima/minima in both domains and this means that one of the notes receives neither maximum nor minimum in either of the domains. In fact, the only possible CDVs for such cases are |1111|, |2222|, and the six combinations of two |2|s and two |1|s[170]. It is interesting to observe that the reduced multi-parametric contours in the Schoenberg example have the smoothest CDV possible for an equal distribution of maxima and minima. Also, we observe that the equal distribution in this example is not limited to maxima and minima: each note manifests the exact same level of multi-parametric contour accent—at least so far as the CDV can be taken as a measure of multi-parametric contour accent.

Figure 7.13. Distribution of Pitch and Duration Maxima and Minima

Pitch:		M	m	
Duration:	M			m



pseg	<2	3	0	1>
dseg	<3	2	1	0>
CDV	<-1	+1	-1	+1>

[170] These include |1122| (i.e.: <1203> and <0321>), |1212| (i.e.: <1023> and <0231>), |1221| (i.e.: <1032> and <0213>), |2112| (i.e.: <0123> and <2031>), |2121| (i.e.: <0132> and <2013>), and |2211| (i.e.: <0312> and <2103>). Note that the CDVs indicated in absolute value omit the plus and minus signs, and thereby only focus on the multi-parametric contour accent without paying attention to whether a note is too long for its height (+) or too short for its lowness (–). In other words, this type of CDV denotes the "distance" rather than the "difference" between the pps and dps.

This proposed method of adding durations in connection with reducing the pitch contour addresses all of the problems outlined above, with the exception of the second one. Let us examine how this method deals with each problem.

1. The possible lack of variety in the duration domain:

   When this method is applied, the lack of durational variety in the surface level will generally diminish at the deeper levels, as is evident from Figures 7.10, 7.11, and 7.12.

2. The conceptual inconsistency between pitch (a point) and duration (an interval):

   This problem is not addressed by any methodology that approaches duration from a contour perspective since it is embedded in the inherent nature of these two concepts. However, its direct consequence, which is posited in the third problem, is addressed with the proposed approach.

3. The implausibility of the notion of passing durations and the concomitant problems in applying the 3-window algorithm to duration contours:

   The application of the reduction algorithms on duration contours is avoided and the durations are "revised" following the pitch reduction. Durations are transferred instead of being pruned, and the total duration of the segment is not altered.

4-5. The conflict between the pruned and retained cps in different domains and the cardinality inconsistency between the csegs in different domains, when reduction algorithms are applied to each domain independently:

> Since the reduction algorithm is applied to only one domain, namely pitch, there can be no conflict or cardinality inconsistency which results from the separate applications of the algorithm to different domains.

The “cumulative” approach outlined above supersedes the ad hoc method since it resolves the problems and the resulting reduced/prolonged segments make stronger and more interesting analytic claims. An analytical application of the methodology, which focuses on the pitch and duration contour frameworks and the interactions that take place between them, is presented in the next chapter.

CHAPTER 8

ANALYSIS: INTERACTING PITCH AND DURATION CONTOURS IN BERIO'S *SEQUENZA I*

Introduction

Musical works, perhaps especially those belonging to the post-tonal idiom, can be viewed as multi-parametric interactions of contour-relations. According to this approach, as the music flows, different musical parameters and their contour-spaces interact in different ways, forming points of multi-parametric contour tension/relaxation, as well as a special type of contour polyphony, in which the multi-parametric contour relationships play an integral part. The present chapter demonstrates this approach on Berio's *Sequenza I* for solo flute. The analysis adopts the approach to durations and contour reductions developed in the preceding chapter: pitch contours are reduced, and the durations of the pruned pitches are added to the durations of the remaining pitches; the pseg and dseg of this reduced form can then be studied, to see how the (reduced) pitch and duration parameters interact.

The first of the fourteen Sequenzas, composed in 1958, was written in "proportional" or "spatial" notation, in which the duration between two consecutive pitches is defined by the horizontal distance between them. Interestingly, this notation does not represent the original design of the piece as intended by Berio. As indicated by Benedict Weisser, the spatial 1958 version was a "compromised version of an earlier attempt in metered notation that was extremely difficult to play."[171] Berio's former musical assistant Nicholas Hopkins, writing to Weisser, supports this claim: "He [Berio] originally wrote it in exceptionally fine detail (almost like Ferneyhough in the original form), but Gazzelloni could not handle it, so Berio decided to use proportional notation."[172] The fact that the piece was originally composed in standard notation is further substantiated by Paul Roberts, another assistant of Berio, as cited by Cynthia Folio and Alexander Brinkman.[173] In fact, in a 1981 interview, over twenty years after the initial publication in proportional notation, Berio expressed his dissatisfaction with the notation and his intention to rewrite the *Sequenza* in rhythmic notation.[174] In 1992, this new version of the *Sequenza* was published by Universal Edition. The present chapter will use the 1992 version, because it not only restores Berio's original (and preferred) conception of the *Sequenza*, but also includes precise durational values, which are indispensable for an analysis of duration contours.

The next section will deal with the segmentation of the *Sequenza*. The reader may skip ahead to the following section if not interested in the discussion of the segmentation and the rationale behind it.

---

[171] Benedict J. Weisser, "Notational Practice in Contemporary Music: A Critique of Three Compositional Models (Luciano Berio, John Cage, and Brian Ferneyhough)," Ph.D. dissertation, The City University of New York, 1998, 38.
[172] Ibid., 38.
[173] Cynthia Folio, and Alexander R. Brinkman, "Rhythm and Timing in the Two Versions of Berio's Sequenza I for Flute Solo: Psychological and Musical Differences in Performance," in *Berio'S Sequenzas: Essays on Performance, Composition and Analysis,* ed. Janet K. Halfyard (Ashgate Academic Publishers, 2007), 11-38.
[174] Balint Andras Varga, and Rossana Dalmonte, *Luciano Berio: Two Interviews*, trans. David Osmond-Smith, ed. David Osmond-Smith (New York: Marion Boyars, 1985).

Segmentation

The segmentation of the *Sequenza* is the first step towards a contour-based analysis. The two sources that segment the entire work, an essay by Folio and Brinkman and a dissertation by Lisa Cella,[175] contain a considerable amount of overlap: with the exception of the fourth section, Cella's segmentation is in agreement with Folio and Brinkman's. The main difference between the two is that Folio and Brinkman's segmentation is more detailed and closer to the musical surface than Cella's, which demarcates the formal components of the work, namely Section 1, Transitional Material, Section 2, Section 3 (quasi-development), Section 4 (return of pitch class string), and Coda. Folio and Brinkman's segmentation, on the other hand, contains 12 segments and 10 fermatas. In the present analysis we will adopt Folio and Brinkman's segmentation but further subdivide some of their segments, since they are fairly long and are not suitable for a contour-based analysis. Table 8.1 lists all three segmentations of the work, excluding the coda.[176] Since Folio and Brinkmann use the term "segment," we will denote to the present segmentation with a capital S followed by the segment number (i.e. "S1," "S2," etc.). Note that the "Page-Line" column in the table indicates the page and line numbers of the segment onsets in the 1992 edition, and the pitch indicates the first pitch of the segment with the corresponding dynamic markings.[177]

---

[175] Lisa M. Cella, "A Resource Manual for the Solo Flute Repertoire of the Twentieth Century," D.M.A. dissertation, University of California, San Diego, 2001.
[176] Fermatas of Folio and Brinkman's segmentation are omitted unless they coincide with the segmentation of Cella or the present segmentation.
[177] The grace notes are taken as surface level ornamentations and not as the structural framework, and are thus omitted from the analysis.

Table 8.1. Three Segmentations for *Sequenza I*

Page-Line	Initial Pitch	Cella (2001)	Folio et al. (2007)	Present
1-1	*ff* A4	Section 1	Segment 1	S1
1-2	*f* F#4			S2
1-3	*mf* F#4		Segment 2	S3
1-6	*ff* F5			S4
1-7	*ppp* D4	Transition	Fermata 2	S5
1-8	*p* E5	Section 2	Segment 3	S6
2-2	*pp* Bb4			S7
2-4	*mf* C#6		Segment 4	S8
2-5	*ff* A4	Section 3		
2-5	*p* G#5		Segment 5	S9
2-10	*p* F4		Segment 6	S10
3-2	*sffz* B6		Segment 7	
3-2	*f* C#5			S11
3-4	*pp* B5		Segment 8	S12
4-2	*p* Bb4		Segment 9	S13
4-3	*ppp* Ab5/Db6 <u>or</u> *p* Bb4			S14
4-3	*mf* C#5		Segment 10	
4-4	*ppp* A4	Section 4		

As is evident from the table, with the exception of their Segment 7 and Segment 10, all of Folio and Brinkman's segments start at points corresponding to the present segmentation. Folio and Brinkman's Segments 1, 2, and 3 are further segmented in the present version. The only drastic difference between all three interpretations is their last segments, none of which coincide. Cella's Section 3 also does not have a corresponding starting point in either of the other two interpretations.

The present segmentation chosen for the analysis is based on the rests and fermatas that demarcate the boundaries between the segments. In this segmentation, any rest that is longer than a dotted quarter note normally marks the ending of a segment. The exceptions to this are to be found in S5, S9, and S11, where a further segmentation would be counterintuitive.

In S5, a quarter note rest is accepted for segmentation, given the nature of following material (Figure 8.1). Here, a collection of fermatas following the two eighth note rests suggests that these fermatas belong together. The inclusion of these fermatas in the previous segment, S4, would have made the previous segment extremely long. Note that this segmentation is also supported by both Folio and Brinkman's, and Cella's segmentation. In S9, an eighth note rest followed by a quarter note rest takes place after E5, towards the very end of a long segment (Figure 8.2).[178] These rests are not marked for segmentation because the following portion of the segment attains closure by the longest fermata of the work. A segmentation at this point would clearly disrupt the musical process. In S11, if further segmented, the first sub-segment would have been too short (three pitches) to be called a segment (Figure 8.3).

Apart from these exceptions, in which it is not musically viable to segment, the method of segmentation based on rests is rigorously applied throughout the entire work. In addition, most of the fermatas in the composition indicate an ending of the current segment with a few indicating a beginning of a new segment. The only exception to this is S5,

[178] Note that this is only the last line of the segment, which is more than four-line long.

illustrated in Figure 8.1, in which a series of fermatas are presented consecutively making it impossible to further segment this section. Overall, the consistent application of the segmentation methodology aims to provide the rigor required for a contour analysis, which is heavily dependent upon the segmentations.

Figure 8.1. Segmentation of S5

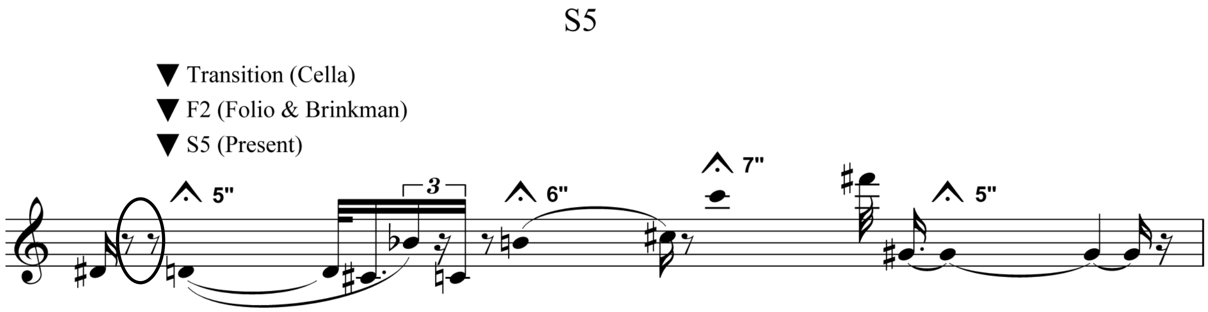


Figure 8.2. Segmentation of S9

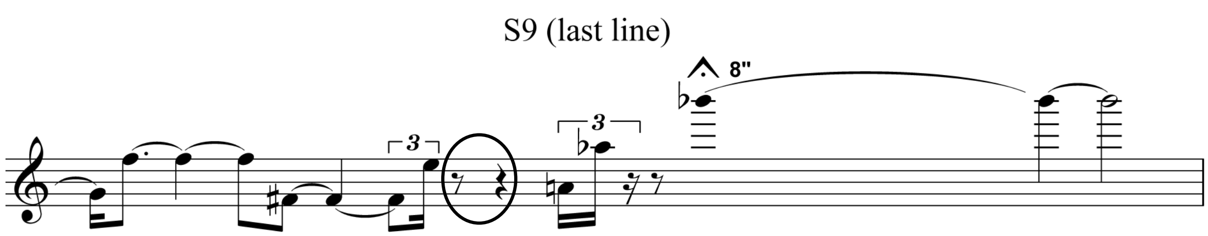


Figure 8.3. Segmentation of S11

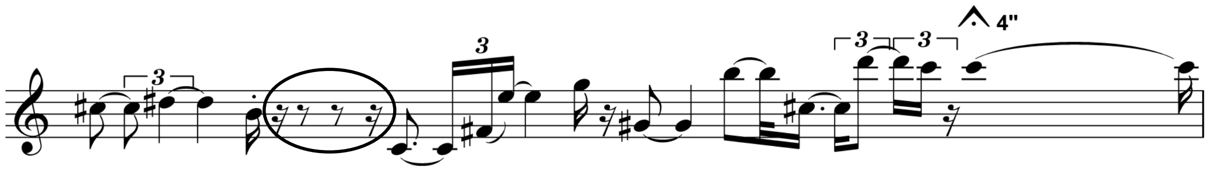
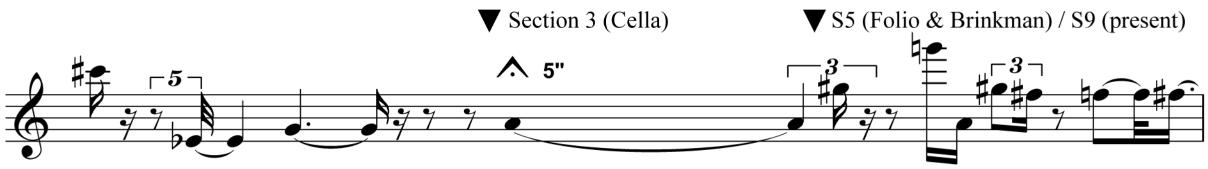
S11

Now let us briefly discuss the discrepancies between the three versions of segmentation (Cella, Folio and Brinkman, and present). First of all, Section 3 of Cella does not coincide with Segment 5 of Folio and Brinkman, as can be seen in Figure 8.4. The main reason for this is that Cella approaches the segment from a formal perspective noting the recurrence of the opening pitch classes. Thus, the new segment starts at the point of this recurrence. Although Folio and Brinkman acknowledge the "return," they mark the first recurring pitch class as Fermata 4 since this pitch is assigned to a five-seconds-long fermata. Accordingly, they start their segment on the following pitch, G#5. Since a fermata generally signals an ending of segmentation—and can be heard that way in this instance, because of the pacing and pitch proximity of the two preceding notes—we will adopt their interpretation. Therefore, Table 8.1 shows Segment 5 in Folio and Brinkman's segmentation corresponds to S9 in the present segmentation.

Figure 8.4. Segmentation of S8-S9

S8-S9 (first line)

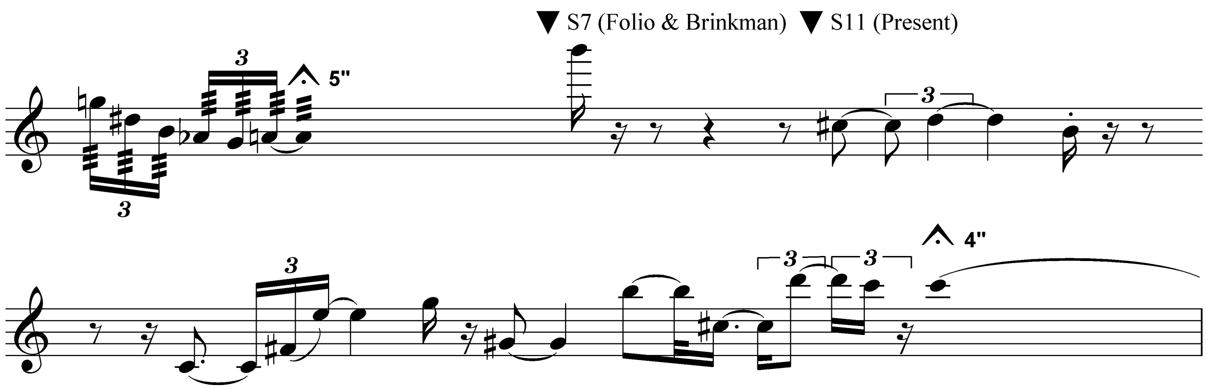


Secondly, the first note of Folio and Brinkman's Segment 7 is an isolated pitch, preceded by a five second-long fermata and succeeded by over a half-note-long rest, as can be seen in Figure 8.5.[179] Thus, the inclusion of this isolated pitch into a segment (either S10 or S11) is not considered by the present segmentation. Instead, the first pitch after the long rest marks the beginning of the new segment, S11, in the current segmentation. Note that the rests at the end of the first and the beginning of the second systems are addressed above in Figure 8.3.

[179] Compare this to Figure 8.4 in which the pitch following the fermata is interpreted as part of the next segment since it is followed by a much shorter rest and is not temporally isolated from the succeeding pitch unlike B6 in Figure 8.5. (Also, note that G#5 is considerably closer to G6 than B6 is to C#5.)

Figure 8.5. Segmentation of S10-S11

S10 (last line) - S11 (first two lines)

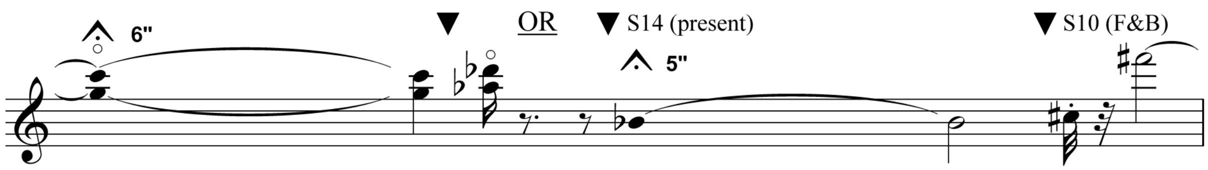


Luciano Berio, Sequenza I

Lastly, perhaps the most difficult of all segmentations, S14 should be addressed in relation to the segmentations of both Folio and Brinkman (Segment 10), and Cella (Section 4). At the beginning of the third line on page 4 (1992 edition), a fermata is followed by a sixteenth note dyad, which is followed by another fermata. This passage is illustrated in Figure 8.6. Since it is not preferable to include two fermatas that sum up to more than 11 seconds into one segment, it is clear that the segmentation must occur either before or after the sixteenth note dyad, leaving the fermatas disjunct. Folio and Brinkman's segmentation (Segment 10) takes place after the second fermata, leaving the previous two fermatas in the same segment, which they label as F9. Cella's segmentation does not take place until the middle of the fourth line and constitutes the same problem of including two long fermatas in line 3 into one segment, which is unlikely of how one hears the segment. Note that Cella's segmentation, which is not included in Figure 8.6, is based on the recurrence of the pitch class string that opened the work, and thus, marks the formal boundary, which does not always necessarily coincide with one's aural segmentation.

Figure 8.6. Segmentation of S13-S14

S13 (last line) -S14 (first line)



Luciano Berio, Sequenza I

Despite the minor differences between the segmentations, they support one another and could be understood as representing different musical levels. Cella's segmentation basically marks the formal components of the work, whereas Folio and Brinkman's segmentation could be understood as large-scale segmentation (mostly larger than what short term memory could retain). For the most part, the new segmentation adopted for this analysis is a further segmentation of Folio and Brinkman's segmentation. Their largest segments are subdivided into two segments, generally in correspondence with fermatas or long rests.

Pitch and Duration Contour Frameworks

Before starting the analysis of the contour frameworks for pitch and duration contours, let us examine the registral placement of the fermatas, the longest notes in the composition which are indicated in seconds. The registral distribution of the fermatas is particularly interesting as it provides us with information regarding the highest/lowest pitch correlation with the longest durations. There are a total of fifteen fermatas in *Sequenza I* with a registral range from C#4 to Bb6. Adopting the conventional tripartite registral division for the instrument that is low, middle, and high registers, we observe that nine pitches fall into the low register, four (or five) fall into the middle register, and one (or two) fall into the high register.[180] In other words, most of the longest notes take place in the lower registers, an observation which is in agreement with the hypothesis made in Chapter 7. Interestingly, most of the high register long notes are in the middle portion of the piece and seem to create a sense of tension whereas the pitches of the first three and last two fermatas are within the lowest octave of flute's registral range.

In the current chapter, the analytical emphasis will be on the interaction of post-reduced pitch and duration contours, and thus, only the deepest level of each segment will be provided (without an illustration of the reduction process, which has been exemplified amply in the preceding chapters). Since we are not particularly concerned with the intermediate depth levels in the present analysis, the background depth levels shown in each segment involve the 7-window algorithm. As we briefly mentioned at the end of Chapter 3, the employment of this algorithm results in an acceleration of the reduction process where more

[180] The registral divisions in Dave Black and Tom Gerou's "Essential Dictionary of Orchestration" are listed as C4-D5 for low register, D5-D6 for middle register, and D6-C7 for high register. See, Dave Black, and Tom Gerou, *Essential Dictionary of Orchestration* (Los Angeles: Alfred Publishing Co., 1998), 93.

cps are pruned in each depth level. Naturally, this reduction omits some of the intermediate depth levels and thus, passes over some of the hierarchical distinctions between cps which would be highlighted by 3- or 5-window reductions.

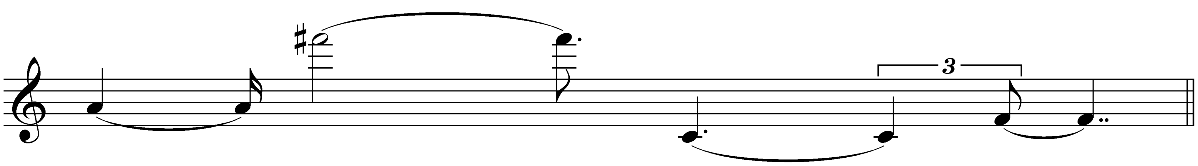
Figure 8.7 illustrates the retained pitches and their accumulative durations at the deepest level of S1. In this figure, the pitches and their corresponding durations in sixteenth notes are listed below the score.[181] The pitch contour of S1, which is given below, denotes the retained and repeating cps by underlined and boldface numerals, respectively. Henceforth, the pitch contour for each segment, on which the reduction is based, will be provided before to the relevant figure for the reader's reference.

S1<3,**2**,**7**,**2**,13,**7**,6,12,10,**5**,11,**5**,0,4,9,8,1>

[181] Note that *Sequenza I* contains triplets and quintuplets, as well as fermatas whose durations are indicated by Berio in seconds. In sixteenth notes, a quintuplet thirty-second counts as 0.4; a triplet sixteenth counts as 0.66; a quintuplet sixteenth counts as 0.8; a triplet eighth counts as 1.33; a quintuplet eighth counts as 1.6; a triplet quarter counts as 2.66; and so forth. Fermata lengths are converted into sixteenths using the designated tempo of 70MM for the quarter note, under which each second corresponds to 4.66 sixteenths.

Figure 8.7. Pitch and Duration Contour Framework of S1

R77(S1)



pitch:	A4	F#6	C4	F4
duration:	5	11	8.66	8.33
pseg:	< 2	3	0	1 >
dseg:	< 3	0	1	2 >
CDV:	< -1	+3	-1	-1 >

Here the ordered pitch collection of the background, <A4, F#6, C4, F4>, results in a pseg of <2301>, and the ordered cumulative durational values of these pitches, <5, 11, 8.66, 8.33>, result in a dseg of <3012>.[182] First, given that dseg = <3012>, CDV = <–1,+3,–1,–1>, the highest pitch receiving the longest duration is very striking. This is heard as a sort of "accent" or emphasis, in the sense that we argued in the preceding chapter that higher pitches would normally be associated with shorter durations. However, the dseg for this segment is questionable since the durational values for op 2 (8.66) and op 3 (8.33) are very close and the insignificant durational difference of 0.33 𝅘𝅥𝅯 between these cps require further investigation. An investigation into the performances of the segment by different performers could potentially serve as a "tie-breaker" between the cumulative durational values of C4 and F4.[183]

[182] Note that as is the case with all segments, the cumulative duration of F4 is the inter-onset duration. In other words, the duration of F4 is the distance between the onset of F4, which is the last pitch of S1, and the initial pitch of the following segment, S2.

[183] Although these values could be said to be essentially equal with respect to the other durations in the contour, an investigation of the performances holds the potential to differentiate between the two and is worth a close examination.

In their detailed examination of the performances of the old and new editions, Folio and Brinkman (2007) list eleven performers[184] with a specification of which edition each performer uses. According to their list, seven of the eleven performers play from the 1958 edition (Nicolet, Zöller, Sollberger P-L. Graf, Gazzelloni, Dick, E. Graf), three performers play from the 1958 edition but have studied the 1992 edition (Fabbriciani, Garzuly, Bezaly), and one performer plays from the 1992 edition (Cherrier). Since our current analysis is based on the 1992 edition, we will focus on the performances of Fabbriciani, Garzuly, Bezaly, and Cherrier.

Table 8.2 provides the cumulative durations based on the performances by these performers. The durational values of the cps in question (C4 and F4) are underlined in the table.

Table 8.2. Cumulative Durations for S1 in Four Performances

Performer	Fabbriciani	Bezaly	Garzuly	Cherrier
Cumulative Duration for A4	1.0"	1"	0.8"	0.9"
Cumulative Duration for F#6	2.6"	2.4"	2.3"	2.5"
Cumulative Duration for C4	<u>1.9"</u>	<u>3"</u>	<u>3"</u>	<u>2.3"</u>
Cumulative Duration for F4	<u>1.1"</u>	<u>1.4"</u>	<u>1.1"</u>	<u>1.7"</u>

As is evident from the table, in all four performances the cumulative duration of C4 is significantly longer than the cumulative duration of F4. Although this finding reveals the performers' misreading of the score, it also demonstrates that the original dp relationship (C4

[184] Aurèle Nicolet, Roberto Fabbriciani, Sophie Cherrier, Karlheinz Zöller, Harvey Sollberger, Anna Garzuly, Peter-Lukas Graf, Severino Gazzelloni, Robert Dick, Sharon Bezaly, and Erich Graf.

being longer than F4) is preserved regardless of the misreading. However, the degree of the durational difference between C4 and F4 presents the possibility of C4 being also longer than F#4, which is the longest pitch of the framework contour. This turns out to be the case in performances by Bezaly and Garzuly. The performances by Fabbriciani and Cherrier, however, maintain the original dseg of <3012> manifested by the score. Note that the interpretations by Bezaly and Garzuly could be understood as an inclination on the performers' parts to perform the lowest pitch with the longest duration. This tendency to associate lower pitches with longer durations, which is the premise of reassigning dp values, results in dseg <3102>. This interpretation, however, neglects the multi-parametric contour accent of F#6, since the corresponding CDV of this note no longer holds the value of <+3>. The Fabbriciani and Cherrier performances, which present this accent in agreement with the score, present a low CDV value of <-1> for the rest of the cps. These notes are slightly short for their relative lowness but their pitch/duration mismatch is small. Interestingly, the multi-parametric accent of F#6 is further supported by the pitch collection: an F major triad, which is formed by rest of the pitches, is disrupted by F#6, thereby establishing a dissonance accent.[185] In addition, the largest leap of the segment constitutes a tritone interval.

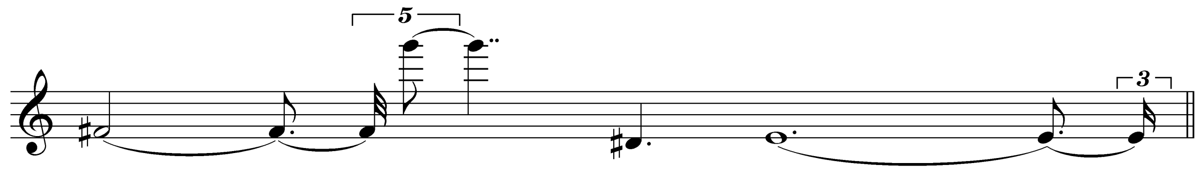
Now let us continue with the reduced background for segment S2 of the *Sequenza*, which is illustrated in Figure 8.8. The cumulative duration of the last pitch E4 results from a combination of simple durations, fermatas, and tuplets; more specifically, as the sum of a sixteenth note (1), a five-second-long fermata (23.33), an eighth note (2), and a triplet eighth note (1.33), or 27.66 sixteenths in total.

S2<2,8,10,4,9,12,**6**,13,7,11,3,5,0,**6**,1>

[185] Also, note that the pitches forming the triad take place within the same octave.

Figure 8.8. Pitch and Duration Contour Framework of S2

R77(S2)



pitch:	F#4	G6	D#4	E4
duration:	11.4	8.6	6	27.66
pseg:	< 2	3	0	1 >
dseg:	< 1	2	3	0 >
CDV:	<+1	+1	-3	+1 >

The pseg of this segment, <2301>, is identical to the pseg for S1. Furthermore, just like S1, it presents registral proximity of the lowest three pitches and registral isolation of the maximum which is a semitone above the isolated maximum of S1. Also, similar to the CDV of the previous segment, it contains a single multi-parametric contour accent and three notes with low pitch/duration disparity (i.e. <+1,+1,–3,+1>). However, this time the accent is shifted to the lowest note. This can be observed by a sign change from plus to minus in the CDV, which indicates that the lowest pitch, D#4 is too short relative to its lowness.[186]

Whereas the opening two segments of the *Sequenza* present multi-parametric contour accents on first the highest (and longest) and then the lowest (and the shortest) notes, the third segment, illustrated in Figure 8.9, provides a smoother interaction of pitch and duration contours.

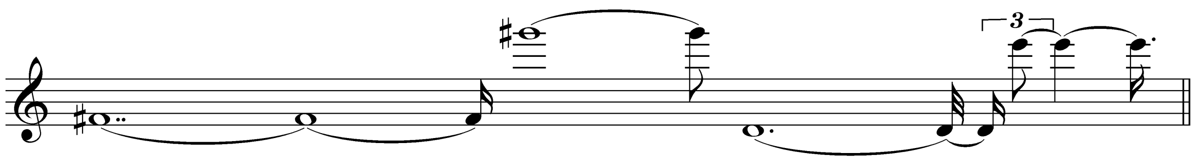
---

[186] For an analysis of this segment that discusses the musical accentuation in the passage by calculating the rate of change in different musical parameters, see John Roeder, "A Calculus of Accent," *Journal of Music Theory* 39.1 (1995), 24-25.

S3:<**3**,**3**,**16**,**18**,**21**,11,**14**,26,**13**,**22**,**23**,**21**,**6**,12,**14**,**20**,25,**10**,**9**,**19**,**9**,27,**19**,15,**20**,**6**,17,4,**19**,**22**,**8**,2,
0,1,7,5,3,**16**,**13**,**10**,**19**,**8**,**18**,**23**,**20**,24>

Figure 8.9. Pitch and Duration Contour Framework of S3

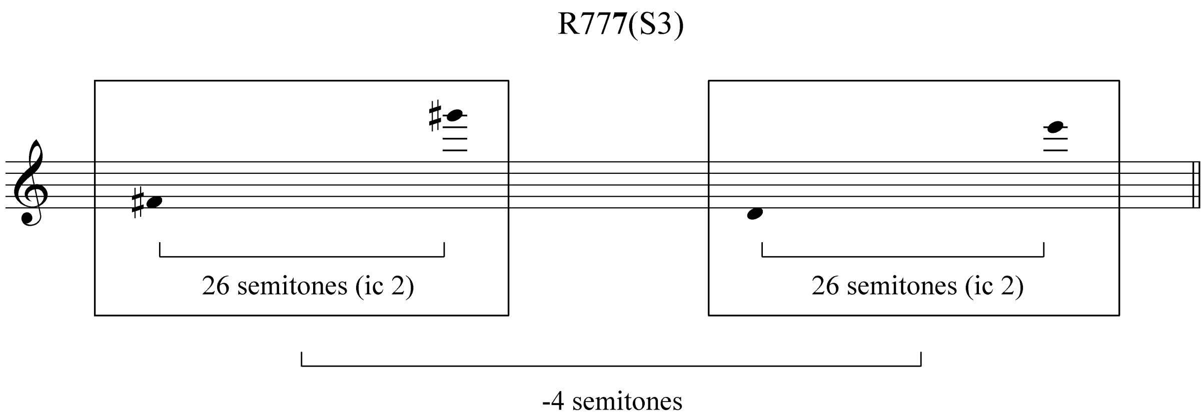
R777(S3)



pitch:	F#4	G#6	D4	E6
duration:	45	18	25.16	6.83
pseg:	< 1	3	0	2 >
dseg:	< 0	2	1	3 >
CDV:	<+1	+1	-1	-1 >

The CDV of S3 demonstrates a consistent slight discrepancy (value of |1|) between the pitch and duration cps, distributed evenly throughout the contour. Although the first two notes are long for their height and the following two notes are short for their lowness, this segments lacks the multi-parametric accent we observed in the previous two segments and provides a sense of relaxation, especially given that the two highest notes have the two shortest durations and the two lowest notes have the two longest durations. The intervallic content of the segment presents symmetrical properties, as illustrated in Figure 8.10.

Figure 8.10. Intervallic Content of Pitch Contour Framework for S3



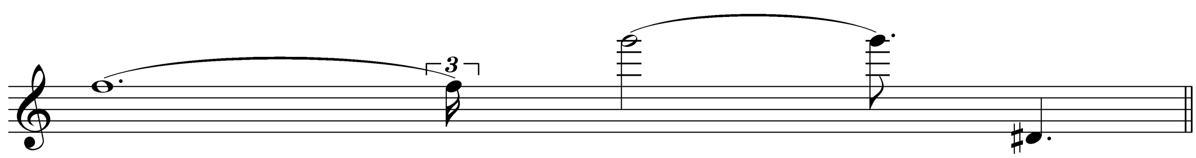
Here, the notes within the boxes are related by 26 semitones (major second) and the pairs are related by a descending major third. It is interesting to point out that the grouping of pcs asserted by the intervallic content is also in agreement with the CDV, in which the first two notes are too long and the last two notes are too short for their height and lowness, respectively. Furthermore, the whole-tone tetrachord, which contains identical adjacent intervals, supports the identical contour difference values of |1|. The symmetry of the whole-tone tetrachord and the evenly distributed CDV values (i.e. |1|) prevent any note from standing out from the others in the segment. The distribution of maxima and minima in both domains are shared equally as well: the first note has a minimum dp, the second note has a maximum pp, the third note has a minimum pp, and the fourth note has a maximum dp. Note that the CMV of the segment contains three similar motions: a pitch rise is always accompanied by a shortening of the corresponding duration and a pitch fall is always accompanied by a lengthening of the corresponding duration. The observations we have made so far all point toward a smooth flow of the multi-parametric contour.

The fourth segment presents a change to cardinality three in its reduced framework. Figure 8.11 illustrates the framework of S4.

S4<**10**,**14**,**11**,18,**6**,9,**15**,7,**11**,16,4,**13**,**10**,**14**,**15**,**12**,**14**,**5**,**13**,**5**,**13**,**5**,**13**,**17**,**10**,19,21,3,8,**17**,**5**,**6**,**14**, 2, **12**,20,**11**,1,0>

Figure 8.11. Pitch and Duration Contour Framework of S4

R777(S4)



pitch:	F5	G6	D#4
duration:	24.66	11	5
pseg:	< 1	2	0 >
dseg:	< 0	1	2 >
CDV:	<+1	+1	-2 >

Similar to the previous segment S3, the pc collection of this segment contains ics 2 and 4, forming a whole-tone trichord. Again similar to S3, the last note is the shortest one. However, here since the last note is also the lowest note of the segment, a multi-parametric contour accent is established.

Note that the framework of S4 contains only 3 cps unlike the previous segments which contain 4 cps. The reason for this difference is that the minimum of S4 is also the last cp of the segment and thus, the retained cps are <first, max, min/last> (three in total). On the other hand, in the previous segments the maximum, minimum, first cp, and last cp, are all distinct pitches. For example, the retained cps of S3 are <first, max, min, last> (four in total). Thus, whenever the maximum and/or minimum of a cseg is also the first and/or last cp of the segment, we obtain a cardinality-3 or cardinality-2 contour (see, for instance, Figures 4.3 and 4.4, Chapter 4).[187]

The ending of the fourth segment and the beginning of the fifth marks the end of "Section 1" and the beginning of the "transition" according to Cella's formal layout. Here, the cardinality-3 cseg seems to mark the ending of one formal section and the beginning of another. Interestingly, as we shall see later on, other cardinality-3 contours in the composition present a similar function. In this case, D#4, which is the minimum and the last note of the segment seems to mark a sectional boundary. Also, note that the multi-parametric accent on this note further emphasizes a prominent formal point in the composition.

At the surface level (unreduced), S5 contains consecutive fermatas with occasional notes inserted in between. Although the fermatas are long, this section is taken as one segment due to the impossibility of further segmentation. The background of S5 is illustrated in Figure 8.12.

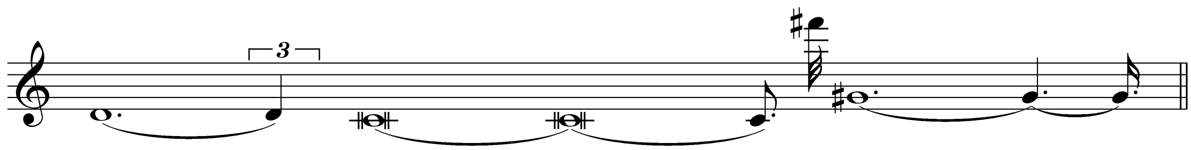
S5<2,1,4,0,5,6,7,8,3>

---

[187] Note that this generalization assumes that no cps are repeated in the framework.

Figure 8.12. Pitch and Duration Contour Framework of S5

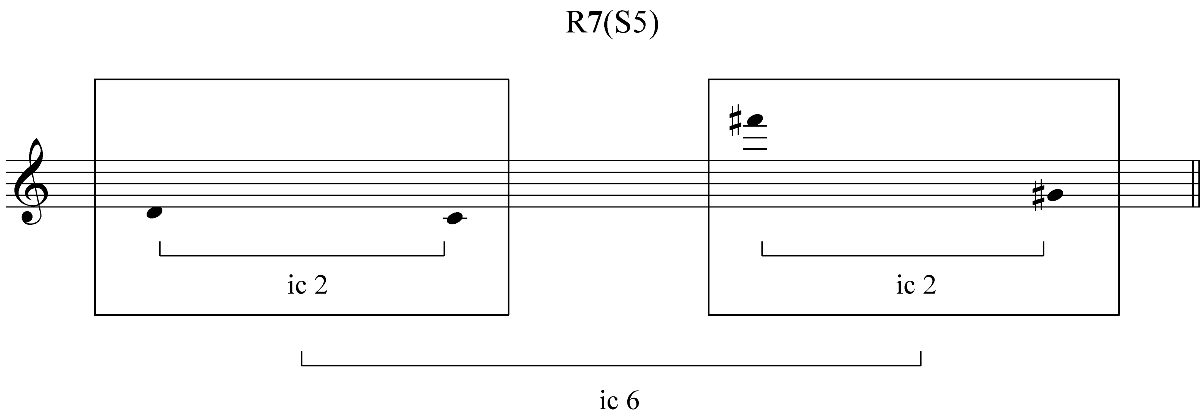
R7(S5)



pitch:	D4	C4	F#6	G#4
duration:	26.66	67	0.5	31.5
pseg:	< 1	0	3	2 >
dseg:	< 2	0	3	1 >
CDV:	< -1	0	0	+1 >

The pc interval symmetries in this segment, which are illustrated in Figure 8.13, are similar to S3. In addition, just like S1 and S3, the leap between the highest and lowest pitches constitutes a tritone. In fact, the leap in S5 takes place between the exact same pitches as it did in S1. Although this implies a return to the opening segment, establishing a sense of uniformity between the first and second sections of the composition, the leaps are distinguished by a reversal of direction: in S1 the leap motion is downward (i.e. pp3 to pp0), whereas in S5 the leap motion is upward (i.e. pp0 to pp3).

Figure 8.13. Intervallic Content of Pitch Contour Framework for S5



Another point of similarity between the two segments is the proximity of the lowest pitches and the isolation of the highest pitch. Nevertheless, unlike S1, the isolated pitch in this case does not hold a multi-parametric contour accent. On the contrary, for the first time we observe a perfect match of pp and dp values, which takes place not once but twice in the segment, as indicated by the CDV <–1,0,0,+1>. Here, the pitch contour <1032> and the duration contour <2031> share their maxima and minima (i.e. pp0 = dp0 and pp3 = dp3). In other words, the highest pitch is assigned to the shortest duration and the lowest pitch is assigned to the longest duration. We also observe that cp1 and cp2 are swapped between the contours and thus, the first (second lowest) note is slightly short for its relative lowness and the last (second highest) note is slightly long for its relative height. Neither of the minimal contour difference values of |0| take place at the end of the contour (op3), and thus, avoid a sense of resolution (or closure).

At this point, we should interject with a discussion of the notion of closure provided by multi-parametric contour match. A CDV value of |0| may indicate closure in the sense that our expectation of pitch height to duration is fulfilled, but not every note whose CDV is |0|

can be said to have a closure effect since other musical parameters may dictate a strong sense of continuation. Even when strictly speaking in contour terms, not all CDV |0| values exhibit closure to the same extent. For example, our sense of expectation (and fulfillment) for medial cps (i.e. cps 1 and 2) could be understood to be milder than our expectation (and fulfillment) for maxima and minima (i.e. cps 0 and 3) as the potential disparity for the former is smaller (a maximum of |2|) than the potential disparity for the latter (a maximum of |3|). Thus, a CDV of |0| on maxima and minima could be said to arouse a stronger sense of expectation (and tension) than the medial cps. Moreover, a short high note is not an idiomatic index of closure; thus, the term "closure" could perhaps be better understood as a "fulfillment of expectation," rather than a signal of conclusion in this context.

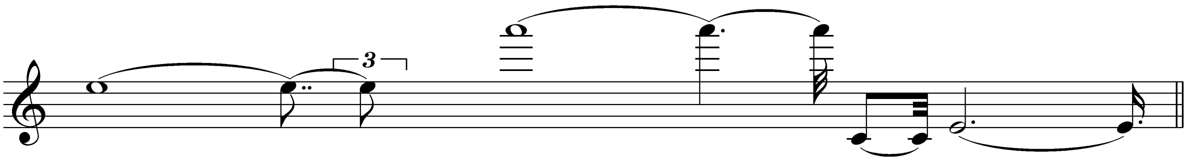
The CMV of S5 consists of three similar motions: the initial rise in pitch is accompanied by a decrease in duration; the following fall in pitch is accompanied by an increase in duration; and the final rise in pitch is accompanied by an increase in duration. The multi-parametric contour interaction of this segment presents a minimal amount of tension (with the exception of two identical contours).

Interestingly, the next segment, which marks the beginning of "Section 2" in Cella's formal scheme, brings a dramatic change to the interaction between the pitch and duration contours. Figure 8.14 illustrates the framework of S6.

S6: <11,**6**,13,8,**5**,1,15,14,**9**,10,**7**,16,**6**,3,**2**,4,12,**5**,**7**,**9**,0,**2**>

Figure 8.14. Pitch and Duration Contour Framework of S6

R777(S6)



pitch:	E5	A6	C4	E4
duration:	20.83	22.5	2.5	13.5
pseg:	< 2	3	0	1 >
dseg:	< 1	0	3	2 >
CDV:	<+1	+3	-3	-1 >

The pitch contour of this segment, <2301>, which opens the second section, is identical to the pitch contour of S1 and S2, from the beginning of the first section. However, the multi-parametric contour accent of this segment surpasses that of the opening segments with a CDV of <+1,+3,–3,–1>. In fact, collectively, this segment presents the most disparate pp and dp values among the segments we have discussed so far. The two maximal values in the CDV result from the highest pitch A6 receiving the longest duration and lowest pitch C4 receiving the shortest duration. The mismatch between the pitch and duration contours further stand out as it is preceded by the smoothest multi-parametric contours (with a CDV of <–1,0,0,+1>) presented by S5. The contrast between the two segments marks the formal boundary, which is marked in Cella's formal layout.

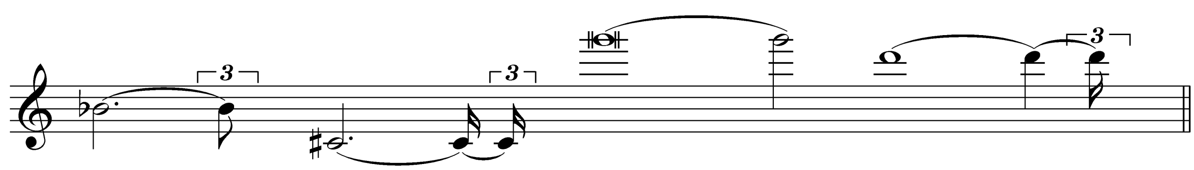
A different type of contrast between S5 and S6 can be observed by comparing their pc collections. In contrast to the tonally indefinite whole-tone tetrachord of S5, the pcs of S6 form an A minor triad. Here, for the first time a cardinality-4 contour forms a trichord rather than a tetrachord due to the reiteration of E. Interestingly, here the medial duration values are assigned to both Es, which are the first and last notes. It is also important to point out that the maximum pitch of this segment is, in fact, the highest pitch of the composition until now and the leap from the maximum to minimum is the largest of all the segments we have discussed so far. The highest maximum (or the "maximum of all maxima") and the largest leap together intensify the tension already created by the pitch/duration mismatch.

The following segment, S7, shown in Figure 8.15, contains two nearly-equal durations: 13.33 and 13.66. A performance-based analysis, similar to the one we have discussed in S1, would help us to determine the dp assignment for the pitches. Table 8.3 provides the cumulative durations of the segment based on the four performances.

S7<**9**,**5**,7,**17**,**19**,**9**,0,**11**,**12**,**22**,**21**,**4**,14,**17**,**24**,**16**,**18**,**21**,10,**11**,**1**,3,**5**,**2**,6,**16**,8,**22**,**19**,**12**,20,**2**,23,**4**, **17**,**15**,13,**24**,**18**,**22**>

Figure 8.15. Pitch and Duration Contour Framework of S7

R7777(S7)



pitch:	Bb4	C#4	G6	D6
duration:	13.33	13.66	40	20.66
pseg:	< 1	0	3	2 >
dseg:	< 3	2	0	1 >
CDV:	< -2	-2	+3	+1 >

Table 8.3. Cumulative Durations for S7 in Four Performances

Performer	Fabbriciani	Bezaly	Garzuly	Cherrier
Cumulative Duration for Bb4	<u>3"</u>	<u>3.4"</u>	<u>2.7"</u>	<u>2.8"</u>
Cumulative Duration for C#4	<u>2.7"</u>	<u>3.2"</u>	<u>2.8"</u>	<u>2.6"</u>
Cumulative Duration for G6	9.0"	10.7"	10.7"	9.5"
Cumulative Duration for D6	3.9"	3.9"	2.4"	3.4"

Although the durational difference between the first two cps (Bb4 and C#4) are minor, it can be seen from the table that all performances, with the exception of Garzuly's, result in a longer cumulative duration for Bb4 in comparison to C#4, forming a <2301> duration contour. Here, unlike the first segment (see Table 8.2), the durational difference in the performances are small enough to be taken as "interpretation" rather than "misreading." Interestingly, the dseg resulting from the performances is identical to the pseg of the previous segment, S6. Furthermore, the pseg <1032> of S7 is identical to the dseg of S6, resulting in a cross domain contour match between the two segments (i.e. pseg(S6) = dseg(S7) and dseg(S6) = pseg(S7)). Consequently, the CDVs of both segments are identical in absolute values (i.e. |1331|), indicating that the multi-parametric contour accent is placed in the inner ops. However, in S7, the first two notes are too short for their lowness and the last two notes are too long for their height (the order is reverse of S6). Yet, both contours present a similar interaction of their pitch and duration contours.

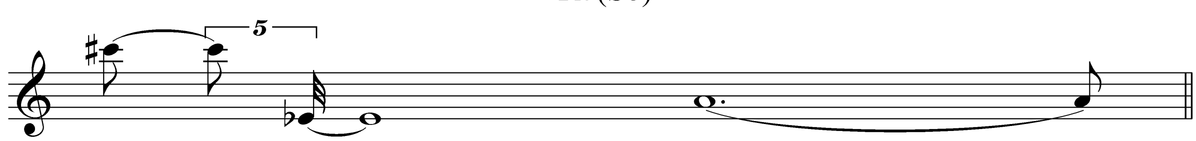
The pc content of S7 presents a G minor triad and a C# which disrupts the triad. Note that here, the disruption of the triad is not realized until the very end of the segment (if the listener is not familiar with the piece). The intervallic tension provided by C # (i.e. C#-G and

C#-D) is further endorsed by a multi-parametric accent. A triad with a disruptive note that is also emphasized by its pitch and duration contour difference is exactly what we have observed in S1. In fact, in both segments it is the second note (which is a maximum in S1 and a minimum in S7) that disrupts the triad. Furthermore, the leap from the second note to the third note (maximum to minimum in S1 and minimum to maximum in S7) constitutes yet another tritone, adding to the tension which is already established by the means discussed above. The overall pc content of the segments reveals that both segments share the same tetrachord type (0147). This is the only instance in the work where two framework segments share a set-class. Moreover, the ordered pc sets of S1, <A, F#, C, F> and S2, <Bb, C#, G, D>, are related by I_7.

The following segment, S8, which is represented in Figure 8.16, contains three cps. Interestingly, this cardinality-3 segment marks the ending of Section 2, similar to another cardinality-3 segment S4, which marked the ending of Section 1.

S8<3,0,1,2>

Figure 8.16. Pitch and Duration Contour Framework of S8

R7(S8)

pitch:	C#6	Eb4	A4
duration:	3.6	16.4	26
pseg:	< 2	0	1 >
dseg:	< 2	1	0 >
CDV:	< 0	-1	+1 >

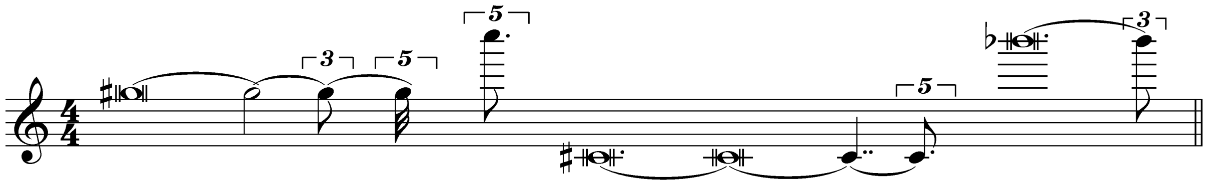
In this segment, the first note is also the highest pitch of the segment, thus holding a certain type of significance in comparison to other cps. Nevertheless, whereas the second and third notes are accented due to their multi-parametric contour disparity, the first note receives a minimal contour difference. Once again, we observe a tritone leap following the examples set by S1, S3, S5, and S7. The gradual slowing down of the segment, as is evident from the duration contour, could be understood to function as a closure.

The following section (Section 3) opens with S9, which is illustrated in Figure 8.17. This segment's reduced pitch contour <1302> is identical to S3, the third segment of Section 1. This point is an interesting one as the opening segment of Section 2 was identical to the first segment of Section 1, implying that the opening pitch contours of Sections 2 and 3 are derived from Section 1, as we will discuss later on. However, unlike S3, the highest pitch of the segment receives the shortest duration and the lowest pitch receives the longest duration, resulting in minimal CDV values of 0.

S9<**17,27,6,17,15,14,15,13,12,10,23,21,7,20,18**,2,**13,15,9,28,11,12**,25,**14,20,9**,7,**22**,24,**26,16**,
14,**23**,**13**,**22**,31,**21**,29,**28,30,27,26**,32,**26**,1,**0,20,5,9,18,19,5,16,18,14,10,20,17,21,13,11,12**,8,**1**
9,6,15,17,14,13,23,7,23,20,14,16,3,22,16,12,23,0,13,7,11,13,5,6,4,**14,3,13,6,17,30**>

Figure 8.17. Pitch and Duration Contour Framework of S9

R7777(S9)



pitch:	G#5	C7	C#4	Bb6
duration:	41.73	2.4	89.4	49.33
pseg:	< 1	3	0	2 >
dseg:	< 2	3	0	1 >
CDV:	< -1	0	0	+1 >

This segment is remarkably longer than the previous segments and the reduction results in an extreme duration of 89.4 sixteenth notes (or 19 seconds) for C#4. The durations assigned to G#5 and Bb6 are also considerably long and they last about 9 and 10.5 seconds, respectively. Since the durational difference between C#4 and G#5, as well as C#4 and Bb6 is easily discernable, especially considering that there are no intervening durations between them except the half-second-long C7, the length of C#4 does not constitute an analytical obstacle. However, the main difficulty with the segment lies in the comparison of the outer cps, namely G#5 and Bb6, which have relatively close durational values and are temporally far apart. Thus, the analysis of this segment could perhaps best be understood as a score-based one. In addition, the CDV values |0| and |+1| for C7 and Bb6 do not capture the huge durational distance between the two similar pitches.

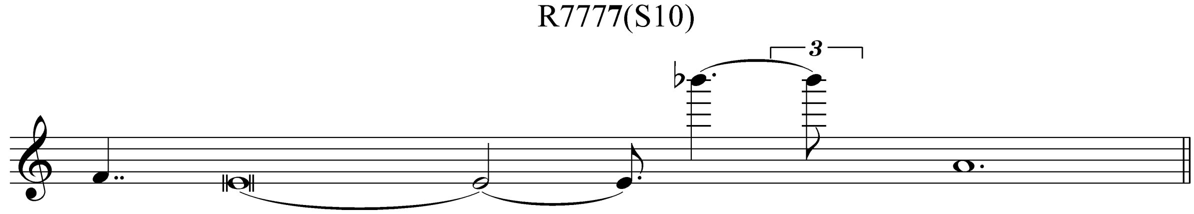
The maximum pitch of the segment, C7, is also the highest pitch of the entire composition; yet, its durational value is one of the shortest. The CDV value of this note could be said to attenuate the emphasis provided by its unusual pitch height. A long durational value assigned to such a high pitch (resulting in a higher CDV value) would certainly have a more prominent effect on the listener. The leap from this note to the minimum pitch constitutes the largest interval among the segments we have discussed so far. Similar to S5—the only segment whose highest and lowest pitches received the shortest and longest durations respectively—the minimal disparity between the pps and dps take place on the inner (i.e. second and third) notes. Interestingly, a multi-parametric contour approach to this passage does not give any emphasis to the very long duration assigned to Bb6 as there is little CDV discrepancy.

S10, illustrated in Figure 8.18, contains another pair of proximate cumulative durations which require further examination, as did S1 and S7. Table 8.4 lists and compares the cumulative durations of four performances.

S10<**1,0,10,8,9,10,8**,7,**9,11,9,10,8,9,11,15**,7,**13**,7,**11,1,6,15**,5,**11,13,18,13,3,0,7**,16,**18**,23,22,2
1, 20,19,17,**15**,12,14,**10,6,3**,2,4>

Figure 8.18. Pitch and Duration Contour Framework of S10

R7777(S10)



pitch:	F4	E4	Bb6	A4
duration:	7	43	7.33	24
pseg:	< 1	0	3	2 >
dseg:	< 3	0	2	1 >
CDV:	< -2	0	+1	+1 >

Table 8.4. Cumulative Durations for S10 in Four Performances

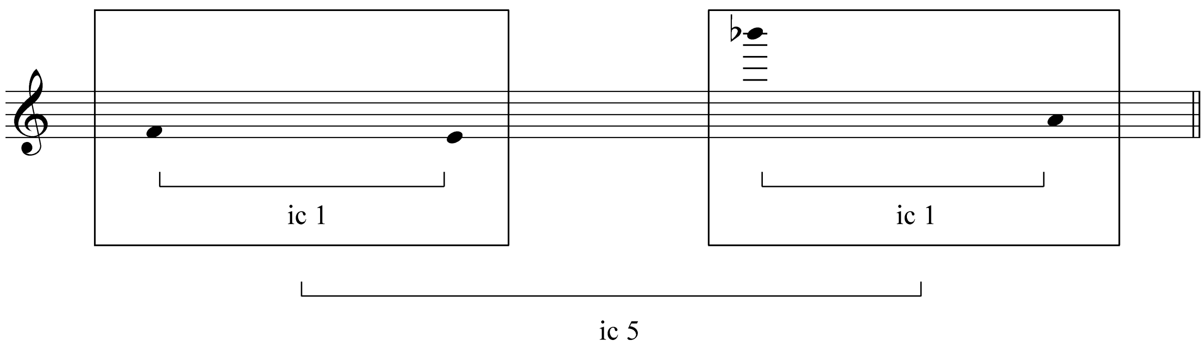
Performer	Fabbriciani	Bezaly	Garzuly	Cherrier
Cumulative Duration for F4	1.7"	2.4"	2.9"	2.5"
Cumulative Duration for E4	10.3"	14.5"	12.0"	10.4"
Cumulative Duration for Bb6	1.8"	1.5"	2.7"	1.4"
Cumulative Duration for A4	5.1"	3.2"	3.1"	3.7"

With the exception of Fabbriciani, all of the performances indicate a dseg of <2031>, rather than <3021>, which result in a CDV of <–1,0,0,+1>. This point is interesting because although this segment has distinct pitch and duration contours, it maintains the exact CDV of the previous segment, S9. The leap tritone between maximum and minimum, which has recurred throughout the analysis, is also present in this segment (E4-Bb6). It is worth pointing out the registral accent on Bb6 as a result of its isolation from F4, E4, and A4.

However, much like the previous segment, the emphasis on the maximum pitch, which is established by the tritone leap and the registral isolation, is somewhat weakened by the CDV value of 0. As illustrated in Figure 8.19, this segment exhibits intervallic symmetries that are similar to some of the earlier segments.

Figure 8.19. Intervallic Content of Pitch Contour Framework for S10

R777**7**(S10)

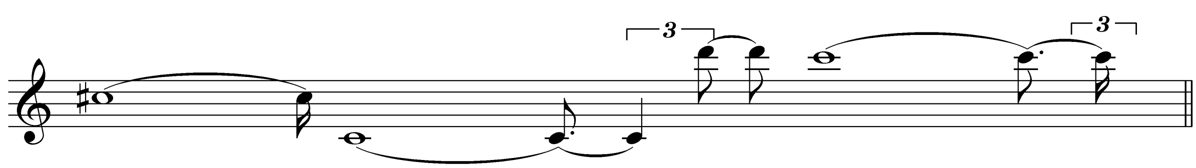


Interestingly, the pitch and duration contours of the following segment, S11, illustrated in Figure 8.20, are identical to S10 with a pseg of <1032> and a dseg of <2031>.[188]

S11: <**4**,5,3,0,1,6,7,2,**8**,**4**,10,**8**,9>

Figure 8.20. Pitch and Duration Contour Framework of S11

R7**7**(S11)



pitch:	C#5	C4	D6	C6
duration:	17	21.66	3.33	19.66
pseg:	< 1	0	3	2 >
dseg:	< 2	0	3	1 >
CDV:	< -1	0	0	+1 >

[188] Note that this figure does not include the repetition of the four-second-long fermata (page4-line4). Here, the second C6 is understood as an echo rather than an integral part of the segment.

It is intriguing to observe that the last three segments (S9, S10, S11) have the same CDV of <–1,0,0,+1> and their CMV consists of only similar motions (i.e. <sss>). On the other hand, the CMV of S6 and S7, whose CDVs involve two |3|s, consists of only contrary motion (i.e. <ccc>). These observations, summarized in Table 8.5, brings up the broader question of whether it is possible to generalize that when two multi-parametric contours have relatively low CDV values their CMV is likely to involve similar motion and when they have relatively high CDV values their CMV is likely to involve contrary motion. Although this seems to be the case in general, it would be erroneous to assume that when a CDV contains two |0|s and two |1|s, the corresponding CMV will always be <sss>. For example, pseg <0123> and dseg <0132> have a CDV of <0,0,–1,+1> but a CMV of <ssc>. Similarly, not all CDVs that involve two |3|s consist only of contrary motions. For example, pseg <3012> and dseg <0312> have a CDV of <+3,–3,0,0> but a CMV of <ccs>. Nevertheless, it is possible to say that no CDV that includes two |3|s has more than one <s>.[189] A detailed investigation of the relationships between CDV and CMV is beyond the scope of this work; however, in the following chapter, we will discuss and offer certain approaches for future research. For the present analysis, suffice it to say that the CMVs of S6 and S7 consist of only contrary motions and the CMVs of S9, S10, and S11 consist of only similar motions.

[189] Informally, this can be proven as follows: a CDV has two |3|s if and only if pp0 corresponds to dp3 and dp0 corresponds to pp 3. The motion preceding (and/or succeeding) pp0-dp3 match has to be contrary because pp0 has to be preceded (and/or succeeded) by a higher pp value whereas dp3 has to be preceded (and/or succeeded) by a lower dp value, resulting in a contrary motion. By applying the same logic to dp3-pp0 match, we can deduce that a CDV with two |3|s has to contain at least two contrary motions since both pp0-dp3 and pp3-dp0 should be at least preceded or succeeded by other cp(s), and thus, will have contrary motion with the preceding and/or succeeding cp.

Table 8.5. CDV and CDM Correlation in S6-S11

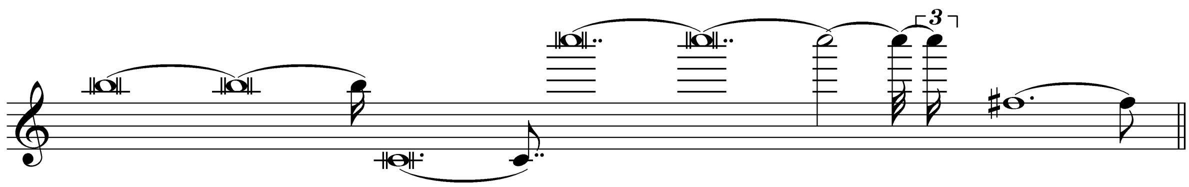
	S6	S7	S9	S10	S11
pseg	<2301>	<1032>	<1302>	<1032>	<1032>
dseg	<1032>	<2301>	<2301>	<2031>	<2031>
CDV	<+1,+3,–3,–1>	<–1,–3,+3,+1>	<–1,0,0,+1>	<–1,0,0,+1>	<–1,0,0,+1>
CMV	<ccc>	<ccc>	<sss>	<sss>	<sss>

Segment 12, which is by far the longest segment of the analysis, is illustrated in Figure 8.21. According to Folio and Brinkman, the "ideal" time of this segment is 41.36 seconds. Thus, it is not feasible to analyze this segment from an "on the fly" perspective. However, this segment can be taken into account from a score-based perspective.

S12<**23,24,22,7,23,25,14,6,17,21,20,22,19,23,27,13,26,22**,33,32,**25,22,26,23,15,17,16,18,24,18,20,16,17,14,3,12,23,14,10,14,11,21,20,25,0,6,4,5,4,6,3,14,0,7,10,22,12,14,8,7,25,15,16,6,8,10,23,12,10,21,20,6,16,17**,31,**21,22,12,11,24,2,16,17,7**,30,**15**,29,**9,19,22**,34,**26,15,25,17,19,18**,28,**14,24,25,23,24,23,22,26,17,11,15,4,2,27,26,13,15,21,17,18,16,7,20,12,1,11,14,11,12,11,12,11,12,10,7,11,1,2,6,12,10,9,5,8,10,7,11,3,1,2,10,8,9,5,7,3,6,0,10,2,11,3,5,4,6,0,6,8,4,5,2,3,6,4,2,5,4,6,2,3,2,4,6,7,5,6,18**>

Figure 8.21. Pitch and Duration Contour Framework of S12

R7777(S12)



pitch:	B5	C4	C7	F#5
duration:	65	51.5	117.16	26
pseg:	< 2	0	3	1 >
dseg:	< 1	2	0	3 >
CDV:	< +1	-2	+3	-2 >

The CDV of this segment includes three levels of multi-parametric contour mismatch (|1|, |2|, and |3|); a property we have not observed in any of the earlier segments. Furthermore, these levels indicate a gradual rise-then-fall in multi-parametric tension: first we have a note that is slightly long for its height, then another note that is fairly short for its lowness, a third note which is extremely long for its height, and a fourth note, fairly short for its relatively low position. The climax that has been thwarted in the last three segments is now very firmly established by the long C7 (long in accrued duration, not actual duration). Note that this climax is further supported by other aspects: C7 is the highest pitch in the entire composition (i.e. maximum of maxima)[190] and the leap leading to C7 constitutes the largest interval span among all of the segments. Furthermore, it is preceded by another C, drawing special attention to the second, repeated note.[191]

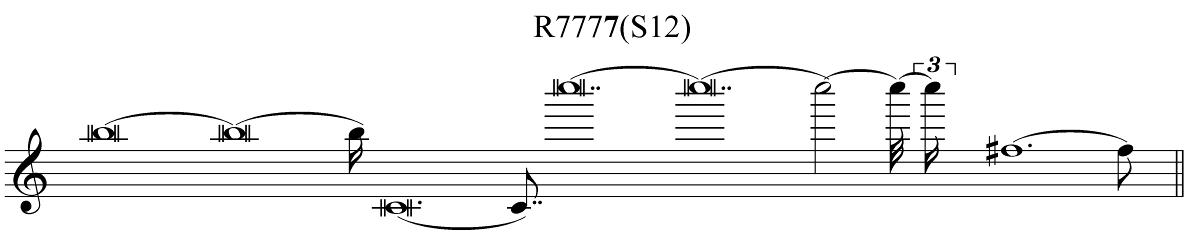
[190] Although S9 contains another C7, it has not been established as a climax in the way C7 in this segment has (i.e.: CDV and the leap interval size).

[191] The repetition of pc C reminds S6, in which pc E was repeated. However, in S6 repeating E corresponded to medial pps (i.e.: pp1 and pp2), whereas in S12, repeating C corresponds to minimum and maximum pps (i.e.: pp 0 and pp 3).

Segment 13 stands in contrast with the previous segment as it is a relatively short segment. Since the fermata at the very end of the segment employs multiphonics, the last cp of the background contains a vertical dyad of G5 and C6, as illustrated in Figure 8.22. Since both of the pitches would be cp 1 individually, it is suitable to accept them as cp 1 collectively. Note that if one of them were lower than Bb4 or higher than D6, we would face a dilemma in assigning a cp value to both pitches.[192]

S13<0,1,**5**,4,**5**,**3**,**5**,**3**,**5**,2>

Figure 8.22. Pitch and Duration Contour Framework of S13



pitch:	B5	C4	C7	F#5
duration:	65	51.5	117.16	26
pseg:	< 2	0	3	1 >
dseg:	< 1	2	0	3 >
CDV:	< +1	-2	+3	-2 >

The first note of the segment, which has a multi-parametric accent, is also the minimum pitch with a very short duration for its lowness. It is also relatively isolated from the other two members of the segment in terms of register. The pcs of the segment form a G minor triad similar to S7; however, here the fourth note is a C, instead of C#. It is interesting

[192] One possible approach to such an analytical problem would be to take the average of the pitches, which we do not need to recourse in this case.

to point out that although the cardinality of the segment is three, it involves four pitches as a result of the dyad and thus, form a tetrachord.

As discussed earlier in the chapter, S14 poses a difficult segmentation. One of the questions regarding this segmentation, which we have not addressed, is whether the segment should start with the Bb4 fermata or Ab5-Db6 multiphonics (both were included in Figure 8.6). The first option leaves the Ab5-Db6 multiphonic isolated, similar to B7 in S10-S11 segmentation (Figure 8.7). Regardless of the choice, both of the interpretations result in identical pitch and duration contours, as shown in Table 8.6.

Table 8.6. Two interpretations for S14 Segmentation

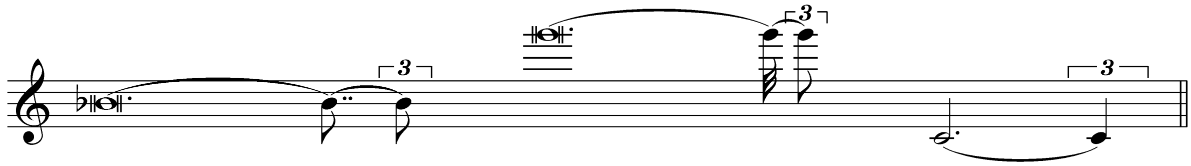
	First Interpretation	Second Interpretation
pseg	<Bb4, G6, C4> = <120>	<Ab5/Db6, G6, C4> = <120>
dseg	<52.83, 49.83, 14.66> = <012>	<58.8, 49.83, 14.66> = <012>

It is also worth mentioning that the last duration is, in fact, slightly longer than 14.66 sixteenth notes since the rests at the very end of the segment are in a slower tempo (MM60 instead of MM72). However, this tempo change does not effect the current dp assignments. Figure 8.23 illustrates the background of the first interpretation.

S14<6,**8**,18,**1**,**4**,**13**,**7**,15,17,**12**,19,**5**,**4**,**5**,**13**,**4**,11,**12**,10,16,**8**,9,**8**,**14**,**6**,**7**,**6**,**14**,**6**,**5**,2,**1**,3,0>

Figure 8.23. Pitch and Duration Contour Framework of S14

R777(S14)



pitch:	Bb4	G6	C4
duration:	52.83	49.83	14.66
pseg:	< 1	2	0 >
dseg:	< 0	1	2 >
CDV:	<+1	+1	-2 >

Here, once again the framework of the segment consists of three notes because the last op (as opposed to first op in S13) is also the lowest pitch. Interestingly, similar to S13, this note is too short (relatively) for its lowness, and thus, has a maximum pitch/duration disparity of <–2>.

It is important to point out that the last two segments, S13 and S14, seem to mark the end of the section, similar to S4 and S8. The coupling of two cardinality-3 segments emphasizes the closure since it is the formal ending of the entire piece. With S13 and S14, this effect is established twice successively at the end of the last section that precedes the Coda.

A Transformational Approach

The fourteen framework segments and their multi-parametric contours we have discussed so far reveal some interesting features. For example, S5, S9, S10, and S11 all present perfect multi-parametric matches in their maxima and minima. In other words, their highest pitches correspond to the shortest durations while their lowest pitches correspond to the longest durations in a consistent manner. Their medial pps and dps, however, present a slight disparity. In contrast to these four segments, S6 and S7 involve maxima and minima with greatest multi-parametric mismatch: their highest pitches receive the longest durations and their lowest pitches receive the shortest durations, resulting in multi-parametric contour accents. These observations suggest that it may be possible to approach the multi-parametric contour interaction from a serial perspective in which the mappings between pps and dps reveal certain types of operations. For example, the perfect multi-parametric match of maxima and minima, and the slight disparity between the medial pps and dps could be understood to present a "swap" relationship between the pitch and duration contours, in which cp 0 and cp 3 are retained while cp 1 and cp 2 are swapped between the multi-parametric contours.[193] The pitch and duration contours of S5, for instance, hold a "sw_{1-2}" relationship, as illustrated in Figure 8.24.

Figure 8.24. Swap Relationship between the Pitch and Duration Contours of S5

pseg(S5) < 1 0 3 2 >

sw_{1-2}

dseg(S5) < 2 0 3 1 >

[193] Henceforth, we will use index numerals to denote the cps that are being swapped. For a brief discussion of this operation, see Marvin, "The Perception of Rhythm in Non-Tonal Music: Rhythmic Contours in the Music of Edgard Varese," 70-71.

Similarly, the multi-parametric accents of S6 and S7 could be understood to display an inversion operation between the pps and dps. As demonstrated in Figure 8.25, the pitch and duration contours of S6 display an inversional relationship.

Figure 8.25. Inversional Relationship between the Pitch and Duration Contours of S6

pseg(S6)	< 2 3 0 1 >
	| | | |
dseg(S6)	< 1 0 3 2 >

Other segments, such as S1, S2, and S3, could be said to involve cyclic permutation, or rotation, which is an operator which cyclically permutes the members of an ordered set (i.e. segment). It is notated as r_y and could be formally defined as op(x–y) in mod z, where x is the order-position, y is the rotational index, and z is the cardinality of the segment. For example, an application of r_1 to a cardinality-4 cseg would simply map op 0 to op 3 (op 0–1 in mod 4), op 1 to op 0 (op 1–1 in mod 4), op 2 to op 1 (op 2–1 in mod 4), and op 3 to op 2 (op 3–2 in mod 4).[194] Thus, an application of r_1 on <2301> would result in <3012>, as is the case with the multi-parametric contours of S1. A visual representation of this rotation is provided in Figure 8.26.

[194] Note that here the notation used by Friedmann (1985) is adopted, which is different than the notation used by Morris (1987). Morris's rotation index is the inverse of Friedmann's rotation index and thus, r_1 of Friedmann is equivalent to r3 of Morris. The notation used by Morris can be obtained by replacing "x–y" with "x+y."

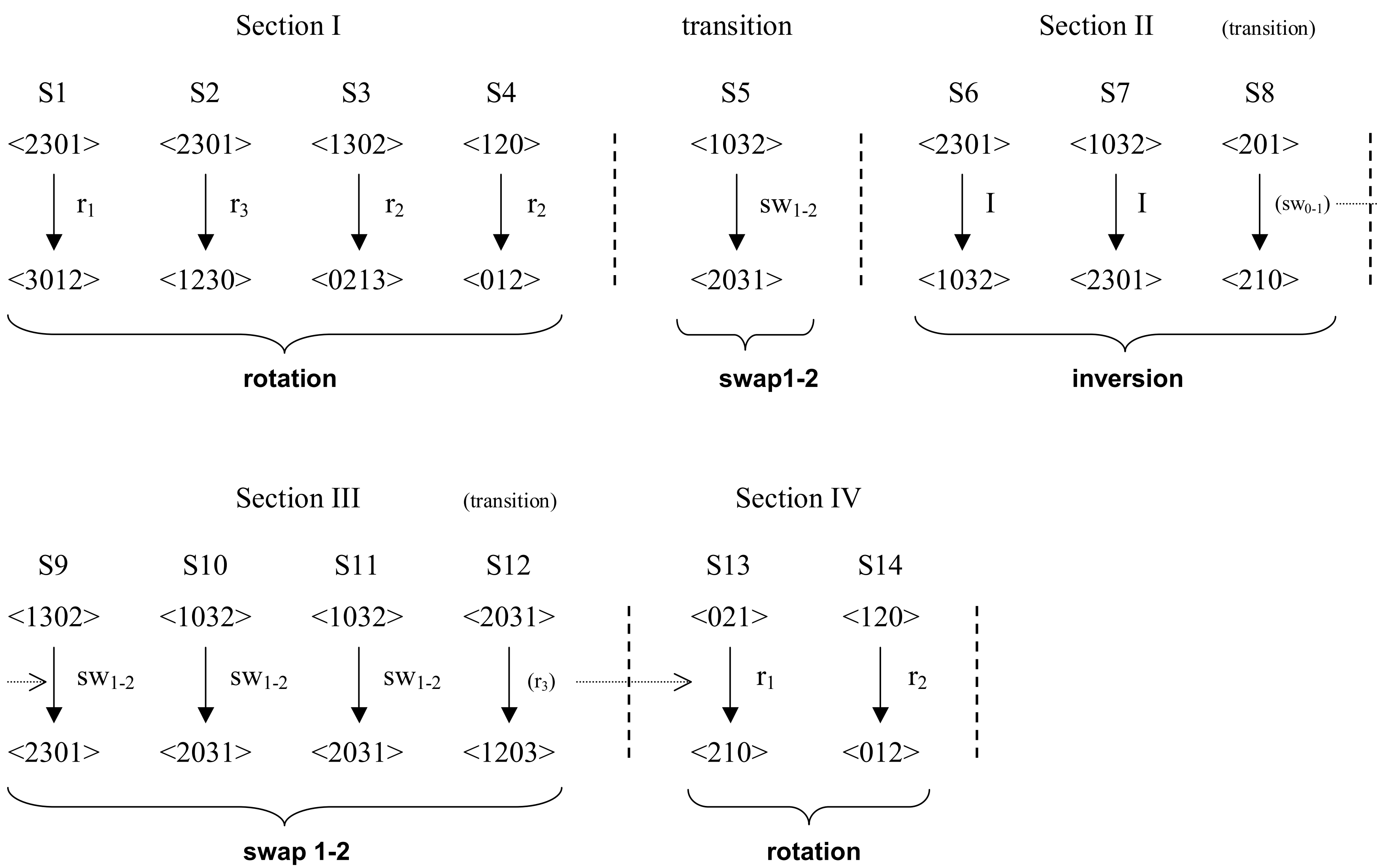
Figure 8.26. Rotation Relationship between the Pitch and Duration Contours of S1

pseg(S1)	operation r_1	dseg(S1)
< 2 3 0 1 >	< 2 3 0 1 2 >	< 3 0 1 2 >
	r_1	

Although a serial approach to multi-parametric contours is open to debate from an aural perspective, it raises some interesting analytical insights. Specifically, the interactions between pitch and duration contours seem to manifest the overall form, as illustrated in Figure 8.27. Here, each section is characterized by a specific type of interaction; Section 1 by rotation, Section 2 by inversion, Section 3 by swap, and Section 4 by rotation. Notice that the last segments of Sections 2 and 3 signal the upcoming interaction type. In other words, the swap relation in Section 2 and the rotation relation in Section 3 are precursors of the relations that are to come in the following sections. Thus, these segments could be understood to function as transitions from one section to the other. Also, as mentioned above, cardinality-3 csegs at the end of Sections 1, 2, and 4 signal an upcoming sectional boundary: whenever the maximum or minimum is the first or last cp of the segment, a new section, and thus a new interaction type, follows.

It is rather surprising to see how each section neatly outlines a multi-parametric interaction type. This is particularly intriguing since it is not conceivable that the composer has consciously designed the first, highest, lowest, and last pitches and their inter-onset durations to interact with each other the way they do. An examination of what it would mean to hear these interactions will be presented shortly but now let us continue with our discussion of the sections and their corresponding interaction types.

Figure 8.27. Pitch and Duration Contour Interaction as a Determinant of Overall Form



As is evident from the figure, the contrast between the sections is provided by different types of interactions between pitch and duration contours. The swap 1-2 operation affects only the medial values, so it maps the lowest pitch to the longest duration and the highest pitch to the shortest duration (pp0=dp0 and pp3=dp3), whereas the inversion operation maps the lowest pitch to the shortest duration and the highest pitch to the longest duration (pp0=dp3 and pp3=dp0), providing a certain type of contrast between the inner sections (sections 2 and 3).

Despite the contrast between the sections, in Sections III and IV there is a sense of unity in terms of the pp and dp mappings. In contrast to the Webern example of the previous chapter in which the "maximum variety" predominates (see Chapter 7, Table 7.3), here we observe that within Sections III and IV, each pp value maps onto only one dp value, forming a unique type of unity. This is illustrated in Table 8.7.

From this table it is apparent that all segments within each section (with the exception of the transition segment) have identical pp-to-dp mappings: pp0→dp0; pp1→dp2; pp2→dp1; pp3→dp3 for Section III and pp0→dp2; pp1→dp0; pp2→dp1 Section IV. This manifestation of unity is particularly interesting considering there are two different operations that characterize each section.

It is also worth mentioning that the overall layout of the sectional interactions presents a global retrograde, as illustrated in Figure 8.28.

Table 8.7. Pitch-Position and Duration-Position Correspondence in S9-S14

Section	**III**			**tr.**	**IV**	
Segment	**9**	**10**	**11**	**12**	**13**	**14**
pseg	< 1 3 0 2 >	< 1 0 3 2 >	< 1 0 3 2 >	< 2 0 3 1 >	< 0 2 1 >	< 1 2 0 >
dseg	< 2 3 0 1 >	< 2 0 3 1 >	< 2 0 3 1 >	< 1 2 0 3 >	< 2 1 0 >	< 0 1 2 >
pp0	dp0	dp0	dp0	dp2	dp2	dp2
pp1	dp2	dp2	dp2	dp3	dp0	dp0
pp2	dp1	dp1	dp1	dp1	dp1	dp1
pp3	dp3	dp3	dp3	dp0		
Cyclic notation	(12)	(12)	(12)	(0213)	(021)	(021)

Figure 8.28. Large-Scale Formal Retrograde

$\longrightarrow$

Rotation Swap1-2 I Swap1-2 Rotation

$\longleftarrow$

Now let us set the multi-parametric contour interactions aside and focus on the pitch contours. It is interesting to observe that the opening pitch segments of the opening section determine the entire flow of the piece. The pitch contours of S1 and S2 are identical to the pitch contour of S6, which opens the second section, whereas the pitch contour of S3 is identical to the pitch contour of S9, which opens the third section. In other words, the two cardinality-4 contours in Section I, namely pseg <2301> and pseg <1302>, open Section II and Section III, respectively. Thus, regardless of their difference in interaction types (I and swap), the opening segments in both Section II and Section III are derived from Section I, as demonstrated in Figure 8.29.

Figure 8.29. Derivation of Section II and Section III from Section I

Section I

S1 & S2 — S3

Section II S6 — Section III S9

This observation can be demonstrated in a transformational network, which also includes the rest of the cardinality-4 psegs, as illustrated in Figure 8.30. (Psegs 4, 8, 13, and 14 are omitted, as cardinality-3 psegs.) In this figure, "p" denotes the pseg, the numeral next to it denotes the segment number, the dashed line denotes the identity (e) operation, and the line with arrows indicates a swap or inversion operation. The vertical dashed lines demonstrate how pitch contours of S6 and S9 are derived from S1/S2 and S3, a point discussed above in Figure 8.29. Note that the path from p1 to p3 (opening) is identical to the path from p10 to p12 (closing).

The overall layout of the network, which involves only pitch contours, endorses the sections defined by pitch and duration interactions we have discussed in Figure 8.27. The top portion of the network presents the first section, which is characterized by multi-parametric contour rotation. The bottom part of the network presents the second and third sections. The opening pseg of each section is vertically aligned with the source pseg which they are derived from (i.e. p1-p6 and p3-p9). Figure 8.31 illustrates the same network with rectangular boxes to indicate the sections and their characteristic interaction types. Note that the fourth section is not present in the network since it contains only cardinality-3 csegs. Although this figure indicates the multi-parametric contour type for each section in parenthesis (i.e. rotation, inversion, and swap that take place between pitch and duration contours), these interactions are not included in the network as it displays only the pitch contours. Figure 8.32, on the other hand, illustrates both intra-parametric (one parameter-multiple segments; i.e. pseg(S1) and pseg(S2)) and inter-parametric (multiple parameters-one segment; i.e. pseg(S1) and dseg(S1)) contour relationships. Observe that within each box there is only one type of inter-parametric contour relationship, with the exception of the last segment (S12), which involves rotation rather than swap and functions as a transition to the following section. It is important to reiterate that cardinality-3 csegs are omitted from this network.

Figure 8.30. Transformational Network for Cardinality-4 psegs

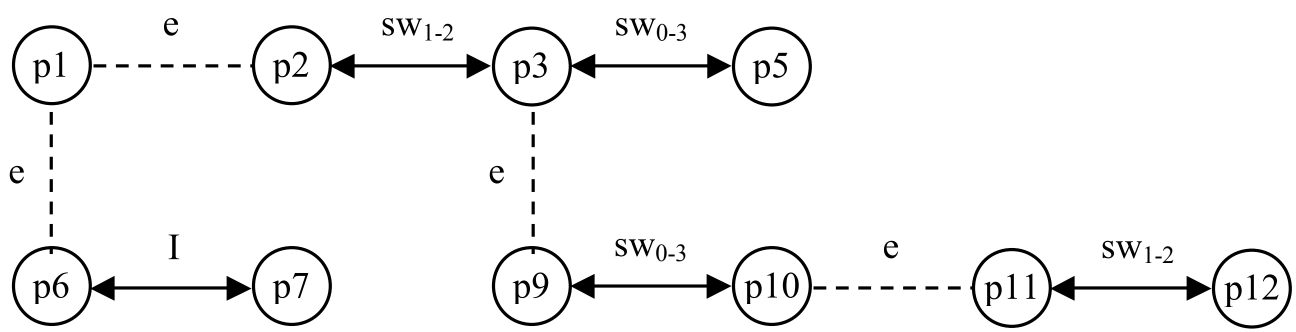


Figure 8.31. Transformational Network for Cardinality-4 psegs with Section Indications

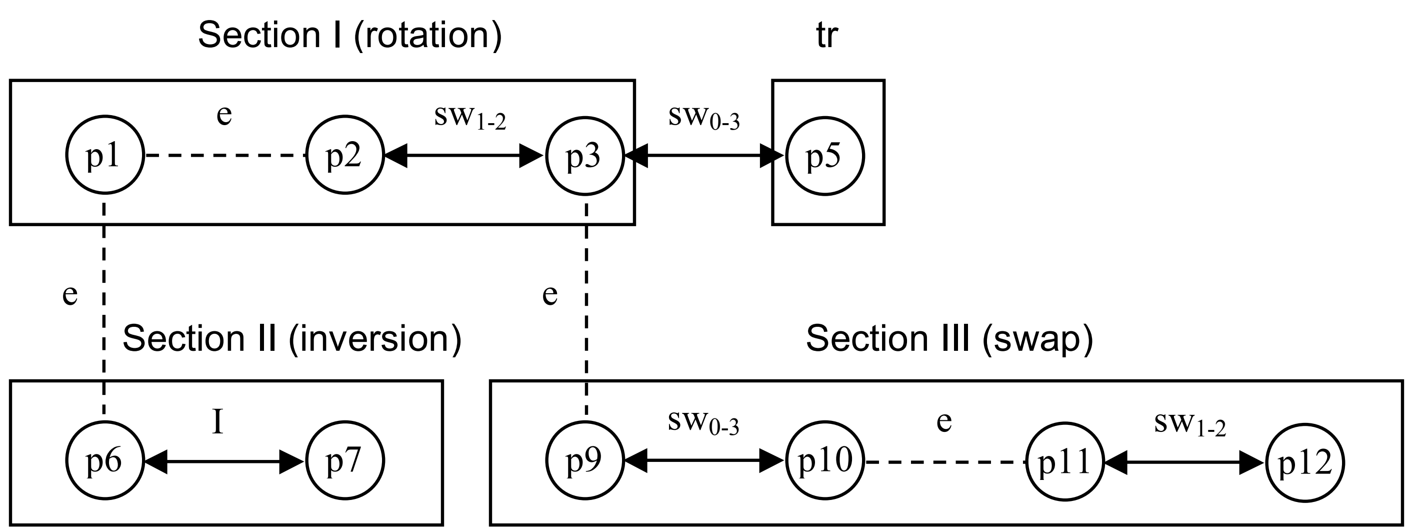
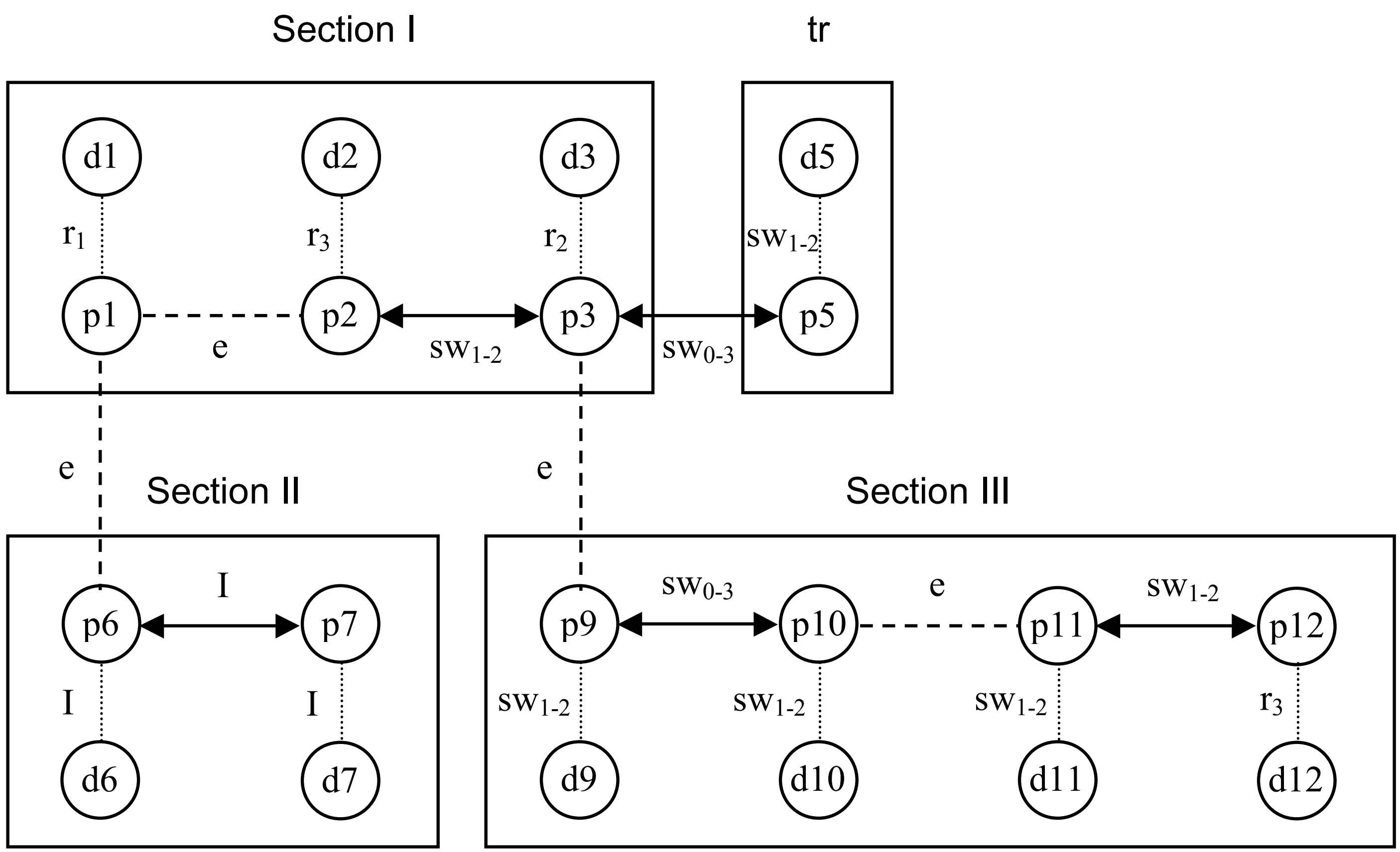


Figure 8.32. Transformational Network for Cardinality-4 psegs with Intra- and Inter-Parametric Contour Relationships



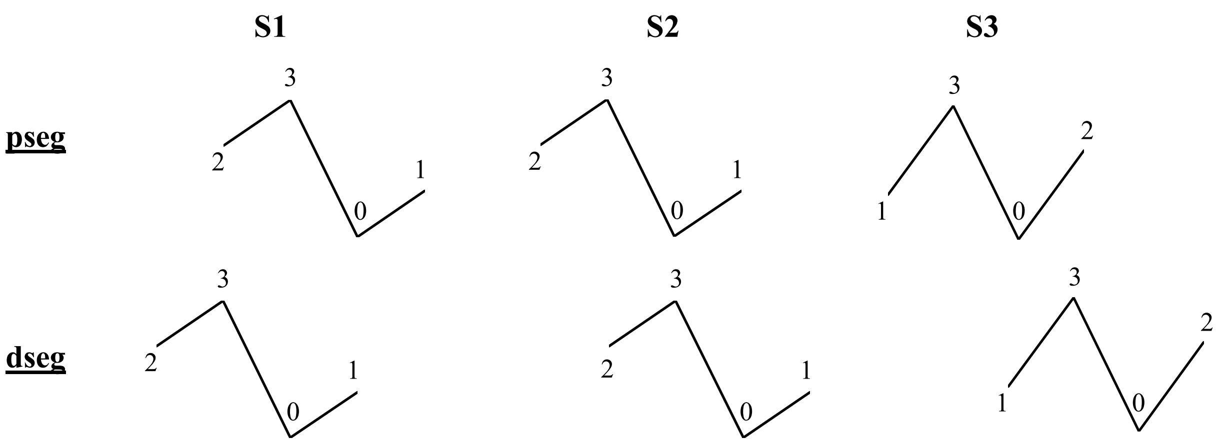
The analytical discussion presented in this section is arguably more abstract in comparison to the Schoenberg analysis. The main reason for this abstraction is the interaction between the multi-parametric contours in the background level. In the Schoenberg analysis our focus was not only on how the background level pitches are a manifestation of the sonata form but also on the intermediate reduction levels. By contrast, in this chapter we have focused solely on the background level contours. In addition, we have discussed the specific relationships between the reduced pitch and duration contours and how these relationships support the formal sections of the composition. The fundamental distinction between our approaches to the background level contours in both analyses is that the former takes a collection of objects (i.e. pitches) as the basis of the analytical methodology whereas the latter takes transformations (i.e: relationships between pitch and duration contours). The relationships between the background "pitches" in the Schoenberg analysis are aurally more accessible than the relationships between the background "pitch and duration relationships." Despite the fact that it is rather difficult to assert aural implications of background level multi-parametric contour relationships, we shall try to address briefly what it would mean to hear these relationships.[195]

Let us first start with the rotation operation, which is perhaps the most difficult to explain in aural terms. The aural effect of rotation can be understood as one of the two identical contours entering later than the other, forming a type of "contour imitation," as illustrated in Figure 8.33. In S1, the pseg enters one cp after the dseg; in S2, the dseg enters one cp after the pseg; and in S3, the dseg enters two cps after the pseg. It is conceivable to

[195] Even though we will address the aural implications as a real-time experience, it is important to remember that a realization of the relationships require retrospective listening, which was extensively discussed in Chapter 5. In fact, the notion of contour is heavily dependent on retrospective listening since for example the highest and lowest pitches of a segment (or window) are not recognized as such until the end of the segment (or window).

hear the delayed contour entries in pitch domain but it is considerably difficult to hear the delayed contour entries in two different domains, especially in the background level.

Figure 8.33. Rotation as Contour Imitation



S6 and S7, however, present a relationship, namely inversion, that is aurally more accessible.[196] In multi-parametric contour inversion, the highest pitch receives the longest duration while the lowest pitch receives the shortest duration. Similarly, the second highest pitch is assigned to the second longest duration and the second lowest pitch is assigned to the second shortest duration. This would in effect mean that when we hear the highest pitch, relatively long time passes until we hear the following pitch that is retained and when we hear the lowest pitch, the following retained cp is relatively imminent.

The swap1-2 relationship indicates the opposite in terms of duration assignments to the highest and lowest pitches. Here, the highest pitch receives the shortest duration and the

[196] Note that it is possible to interpret the multi-parametric relationships in S6 and S7 as retrogression rather than inversion; however, inversion seems to be a more suitable option given that it is more difficult to hear retrograde in long-range processes.

lowest pitch receives the longest duration, while the inner pps are assigned to the same inner dps in inversion. Thus, an attentive listener would recognize that when she hears the highest pitch, the next retained pitch is imminent, and when she hears the lowest pitch, relatively long time passes until she hears the following retained pitch. In both inversion and swap1-2 relationships, the highest and lowest pitches are very much present aurally while the inner cp relationships (i.e. pp vs. dp) are more difficult to hear. Table 8.8 shows the highest/lowest pitch correspondence with shortest/longest duration for both relationships.

Table 8.8. Inversion and Swap 1-2 Correspondence with Maxima and Minima

	Swap 1-2	**Swap 1-2**
Inversion	Highest Pitch	Longest Duration
Inversion	Shortest Duration	Lowest Pitch

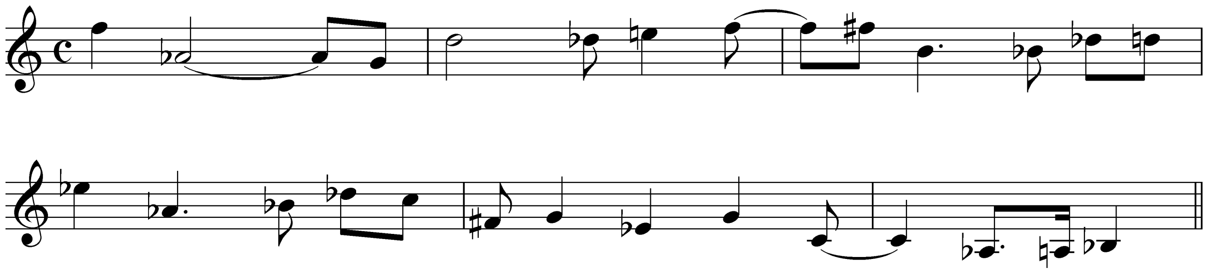
Although hearing all of these relations with equal ease may not be possible, the consecutive repetition of each relationship (as is the case within the sections) certainly raises an aural awareness of the relations and aids the listener in recognizing different sections of the work. As is evident from the analysis presented above, a multi-parametric contour approach to *Sequenza I* reveals not only the special interactions between the pitch and duration domains throughout the work but also the manifestation of the overall form through these interactions.

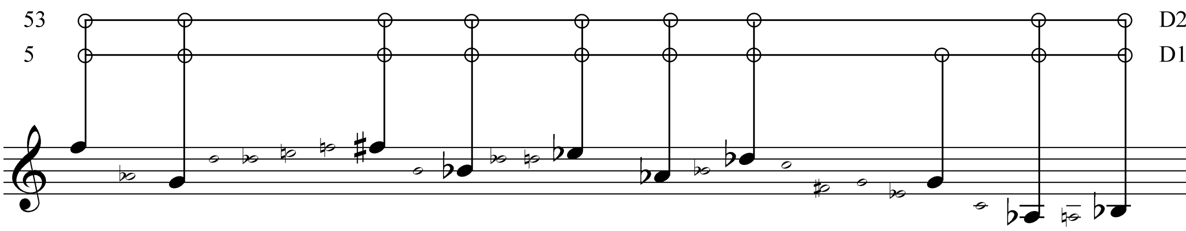
CHAPTER 9

CONCLUSIONS AND FUTURE DIRECTIONS

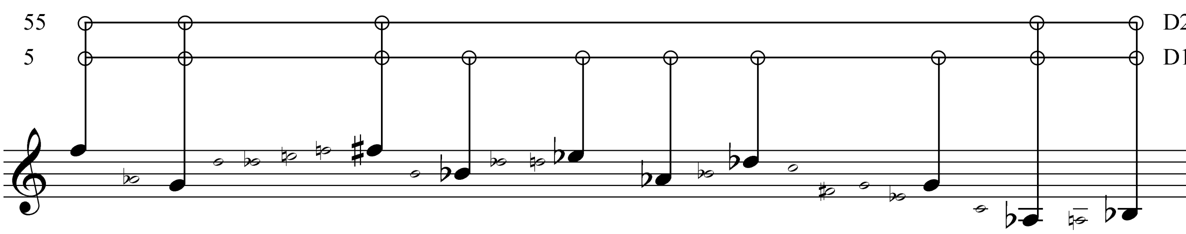
In this concluding chapter, I review the theoretical and analytical findings of this work, and suggest various avenues for further research.

After surveying the literature on contour theory and establishing the research framework, I proposed a theoretical approach to reducing musical contours based on the notion of cursor-windows of varying sizes (Chapter 3). I formalized this notion into a set of algorithms called 3-window and 5-window algorithms and then defined contour reduction functions to explore recursive applications of the algorithms on contour segments, and to formalize a specific concept of contour hierarchy and depth level. A single-staff notation was developed to represent the entire hierarchy of levels within a contour segment. One asset of the algorithms and contour reduction functions that has been only partially investigated in this study is the opportunity to apply a variety of different window sizes at any given depth level, and thus reduce a contour in a variety of different ways (and at varying speeds), as opposed to the *fixed reduction path* of Robert Morris's contour reduction algorithm, which is performed in one and only one way. Figure 9.1 shows, for example, that the contour reduction functions R53, R55, and R73 each reduce the same segment in different ways, producing (partially) reduced contours with different cardinalities. This excerpt is taken from Béla Bartók's String Quartet no.1, op. 7, mm. 1–5 (first violin).

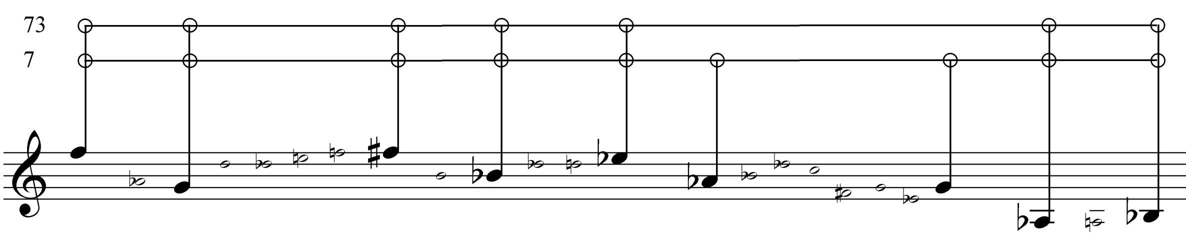
Figure 9.1. Béla Bartók, String Quartet no. 1, op. 7, mm. 1–5, first violin

R53



R55



R73



From Figure 9.1, we observe that R5 has cardinality 10; R7 has cardinality 9; R53 has cardinality 9, but not the same as R7 (Db5 vs. G4); R55 has cardinality 5, while R73 has cardinality 7. Interestingly, an application of the 5-window algorithm on segments resulting from R53 and R73 reveal that they are identical to R55 (i.e. R535 = R735 = R55).

Although earlier chapters adopted the principle of using the smallest applicable window size for each reduction, the application of different contour reduction functions on the same segment deserves further study.

The possibility of *multiple reduction paths* raises some interesting questions. For instance:

- Under what conditions do independent applications of R35(S) and R53(S) produce the same (or different) reduced contours? (In other words, do R3 and R5 ever commute?)

- Are there any conditions (or specific contours) for which independent applications of a *composite* contour reduction function—such as R35(S)—and a *simple* contour reduction function—such as R7(S)—give the same reduced contour?

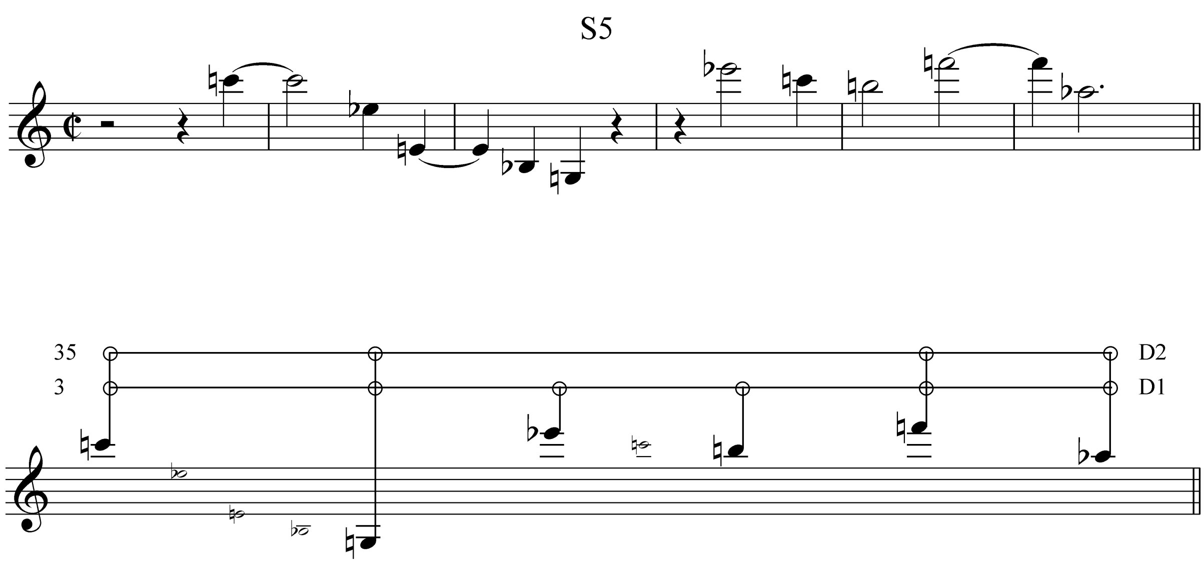
- Under what conditions would different reduction paths yield different depths? To what degree are they contingent upon the cardinality and curvature of the specific contour segment?

- Is there an easy way to tell how many possible reduction paths a given contour has, given a maximum and minimum window width (for example, minimum 5 and maximum 7)?

Underlying these considerations is the question whether it is possible to develop a set of rules that would easily evaluate any given contour without going through the reduction process.

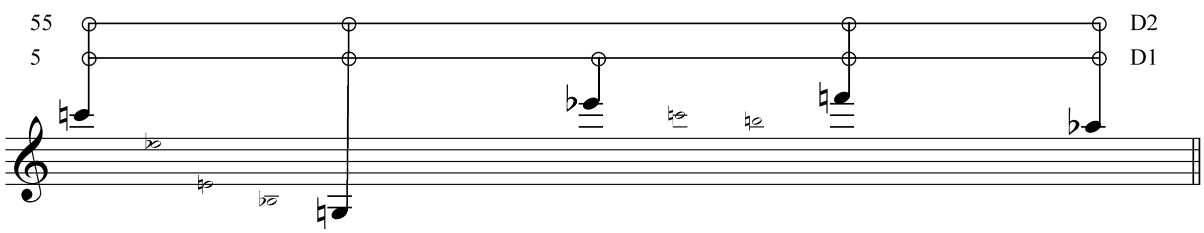
Another promising avenue for further research is the enumeration of all possible reduction paths as a means of achieving a full combinatorial understanding of the contour segment at hand. An exhaustive network of all possible reduction paths would reveal all the hierarchical contour relationships between the segment's cps, at least in the specific sense that contour hierarchy is defined by the window algorithms. As an example, let us look at the fifth *Hauptstimme* from the Schoenberg quartet analysis in Chapter 6. Figure 9.2 illustrates the original segment and the conventional application of the algorithm—R3**5**(S5)—on the segment. According to this figure, Eb6 and B5 are hierarchically equivalent to one another, because both are removed by the first pass of R3. Figure 9.3 illustrates a different reduction path, R55(S), which starts with an application of the 5-window algorithm. Although R55(S) produces overall the same depth-2 background as R35(S), this path reveals that Eb6 is, in fact, hierarchically more salient than B5, because Eb6 is retained while B5 is pruned by R5 at the first depth level. It is also interesting to observe that R5 reduces this contour more slowly than does R3. Note that similar observations can be made regarding S11 and S12 in the quartet, as well as the exemplary segment by Hába that was discussed in Chapter 4. A combinatorial approach to contour reduction functions has the potential to delineate all the hierarchical relationships between the pitches, including those that are overlooked by certain reduction paths. A computer program would be particularly useful to explore the full range of paths and test certain aspects of the algorithms by brute force. For example, reducing one of the longer segments in the Schoenberg quartet would likely include tens of contour reduction paths, some of which are redundant depending on the contour itself.[197]

Figure 9.2. Schoenberg, String Quartet, op. 30, i, R35(S5)



[197] For example, the contour reduction function R35**5**(S) may be equivalent to R5**5**(S) if the original contour segment does not involve any passing cps.

Figure 9.3. Schoenberg, String Quartet, op. 30, i, R55(S5)

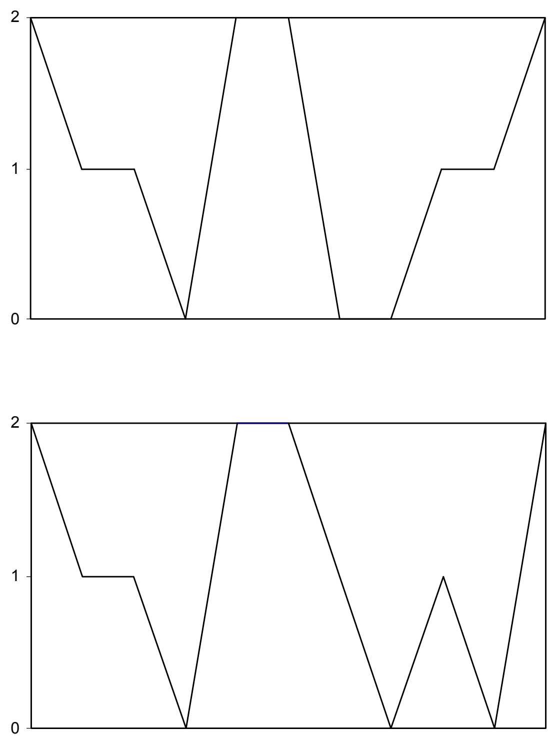


The notion of a contour representation of depth-level, which was briefly introduced in Chapter 3, also presents a promising area of research. The idea of representing depth-level as a contour itself may seem abstract at first. However, it provides a valuable tool for representing and comparing melodic segments. As an example, a numerical representation of the depth-level contours of the first two segments from the Schoenberg quartet is shown in Table 9.1. Figure 9.4 visually demonstrates the considerable similarity of the depth-levels of the segments, despite the fact that the first segment has a framework of <2031> whereas the second segment has a framework of <1020>. In both segments the contour salience (i.e. depth level of each successive cp) falls at the beginning. The middle sections of both segments illustrate a salience climax and interestingly, this climax is sustained in both segments as a result of the minimum-maximum succession. The climax is followed by a sharp salience drop in the first segment, and a more gradual drop in the second segment. Both segments eventually end with a rise (due to the retention privilege of the last cp) but the second segment involves a temporary descent (from depth level 1 to depth level 0) at the point in which the first segment maintains its depth level of 1.

Table 9.1. Depth Level Contours of R35(S1) and R35(S2)

	Contour Framework	
	R3**5**(S1)	R3**5**(S2)
Cseg	43520744163	21530744650
Depth Level	21102200112	21102210102

Table 9.2. Visual Representation of Depth Level Contours for R35(S1) and R35(S2)



Depth Level Contour of R3**5**(S1)

Depth Level Contour of R3**5**(S2)

In this study, the gallery of examples and the analytical chapters demonstrated the broad applicability of the theory to post-tonal repertoire. Particularly, the examples in the gallery (Chapter 4), included works from different time frames (ranging from 1925 to 2001) and by composers of different nationalities (American, French, Greek, Czech, Austrian, British, and Russian) who employ a variety of musical techniques (twelve-tone, integral serialism, stochastic process, etc.). Still, all of the works included in the dissertation share a commonality, namely a lack of tonal center, and an application of the theory to more tonally oriented music (i.e. common practice period repertoire, popular music, jazz music, or world music) could also offer some interesting analytical insights. For instance, contour and its properties as highlighted by the window algorithms could be used to compare and contrast music of different cultures, which do not have scales and tuning systems in common. Although tonal (or modal) associations found in common practice period, popular music, and jazz music, tend to override contour relationships, the algorithms could also reveal how certain aspects of tonality are manifested through the employment of melodic contour (i.e. certain scale degrees being highlighted by maxima and minima, modulation locations being marked by a change in CAS, etc.).

In Chapter 5, we explored the possibility of relocating the point of reference within the width-3 and width-5 windows. As we discussed in that chapter, different reference points imply considerably different modes of listening. Overall, the discussion of the distinct window-frame models involved phenomenological, cognitive, and computational concepts, resulting in an interdisciplinary and speculative discourse. Needless to say, an investigation by specialists working in these fields is indispensable to confirm or dispute the claims made, as well as to point towards areas of improvement in the models. Particularly, the computational approach to cognition adopted in this chapter is subject to empirical scrutiny, which is essential to substantiate the rather speculative claims made in this chapter.

The analysis of the Schoenberg quartet demonstrated how contour hierarchy, as revealed by the algorithms, exposed certain repeated intervallic relationships and implied certain tonal attributes, which are supported by the formal design of the movement. Following this train of thought, it would be worthwhile to examine Schoenberg's other twelve-tone compositions, which likewise typically lack obvious tonal associations on the surface. The analysis presented in this study should be seen as a first step towards understanding Schoenberg's employment of register, and more specifically, contour as a delineator of tonal residues, especially in relation to form.

Although we focused predominantly on applications of the contour reduction functions in the pitch domain, in Chapter 7 we reexamined the algorithms and their applicability to the duration domain. The reversal of the conventional duration-position associations proposed in Chapter 7 is subject to experimental inspection, which could reveal whether there is a significant aural correlation with the pitch boundaries and duration boundaries (i.e. highest/lowest pitch corresponding shortest/longest duration). Such an experiment, which could involve musician as well as non-musician subjects, requires a vigilant design set by psychologists.

In the same chapter, we also introduced the *contour difference vector* (CDV) in order to assess the pp and dp differences in a segment. Although we used this tool in our analyses, we did not study the classification of CDV types, which holds the potential to expose some interesting traits of the proposed tool. For example, we can immediately say that it is not possible to have more than two entries of |3| in a cardinality-4 CDV. On the other hand, it is interesting to note that it is possible to have a cardinality-4 CDV that consists solely of any other repeated single value, i.e. |0000|, |1111|, or |2222|.[198] A more generalized approach to

[198] Two entries of |3| occur when cps 0 and 3 are swapped between the pseg and dseg (i.e.: pseg <2013> and dseg <2310>). |0000| occurs when the pseg and dseg are identical (i.e.: <2310> and <2310>); |1111| occurs

CDVs would involve a comparison of all 24 cardinality-4 csegs[199] with each other, generating 288 CDVs (not all of which are distinct).[200] This would provide an opportunity to observe the CDVs that are relatively scarce or abundant. These particular CDVs, which correspond to particular multi-parametric contours, could eventually offer some significant analytical benefits. It is also interesting to point out that there are 256 hypothetical cardinality-4 CDVs,[201] but many of these cannot occur, in a somewhat similar fashion to Larry Polansky and Richard Bassein's "impossible" contours.[202]

In Chapter 8, we presented an analytical application of the cumulative approach to pitch/duration contours, which was proposed in the preceding chapter as a solution to the analytical, conceptual, and methodological inconsistencies arising from the application of the algorithms to the duration domain. In this analysis, we focused on the framework pitch and duration contours in Berio's *Sequenza I* and demonstrated the multi-parametric contour interaction from a transformational perspective. However, our analysis neglected the intermediate depth levels, and therefore, further analyses focusing on this particular aspect would be valuable. In addition, although the approach adopted for reducing duration contours avoids the analytical and methodological pitfalls discussed in Chapter 7, it occasionally results in pruning durationally prominent pitches. The theory would benefit from a methodological refinement addressing this limitation.

---

when cps 0 and 1, as well as cps 2 and 3, are swapped (i.e.: pseg <2310> and dseg <3201>); and |2222| occurs when cps 0 and 2, as well as cps 1 and 3, are swapped (i.e.: pseg <2310> and dseg <3201>).

[199] The number of distinct csegs without cp repetition is n!, where n is the cardinality of the cseg. For example, there are 3! distinct cardinality-3 csegs without cp repetition: <012>, <021>, <102>, <120>, <201>, and <210>. Similarly, there are 4! = 24 distinct cardinality-4 csegs; 5! = 120 distinct cardinality-5 csegs; 6! = 720 distinct cardinality-6 csegs, and so forth.

[200] (24*24)/2 = 288. For the CDV version with plus and minus signs, the result would be 24*24 = 576. For both calculations it is possible to subtract 24 from each result if one prefers to ignore the |0000| (or <0000>) cases.

[201] Each of the four positions can be filled by 0, 1, 2, or 3; hence, 4*4*4*4=256.

[202] Larry Polansky, and Richard Bassein, "Possible and Impossible Melody."

Lastly, it is important to note that both of the large analyses presented in this dissertation involve instrumental music. A particular area of interest for further research is the incorporation of the algorithms into text-based vocal music, which would shed light on the relationship between pitch retention and textual aspects (such as word and syllable accent).[203] Analyses of this sort could involve investigations of whether pitch retentions coincide with textually significant words or certain lexical categories (i.e. nouns, verbs, adjectives, etc.) and whether antonyms tend to take place at (local and global) maxima and minima.

[203] Although a 1990 doctoral dissertation by Suk Won Yi develops linguistic and statistical models to analyze vocal melodic compositions by Schubert and Schumann, it does not address the text-contour relationships beyond the surface level (i.e. from a hierarchical perspective). See Yi, Suk Won. "A Theory of Melodic Contour as Pitch-Time Interaction: The Linguistic Modeling and Statistical Analysis of Vocal Melodies in Selected Lied Collections of Schubert and Schumann." Ph.D. dissertation, University of California, Los Angeles, 1990.

BIBLIOGRAPHY

Adams, Charles. "Melodic Contour Typology." *Ethnomusicology* 20, no. 2 (1976): 179-215.

Agmon, Eytan. "Musical Durations as Mathematical Intervals: Some Implications for the Theory and Analysis of Rhythm." *Music Analysis* 16, no. 1 (1997): 45-75.

Babbitt, Milton. *Soli E Duettini, Flute and Guitar*. New York: C.F. Peters, 1991.

Bartók, Béla. *String Quartet, No. 1, Op. 7*. Budapest: Editio Musica, 1956.

Beard, R. Daniel. "Contour Modeling by Multiple Linear Regression of the Nineteen Piano Sonatas by Mozart." Ph.D. dissertation, Florida State University, 2003.

Benjamin, William. "A Theory of Musical Meter." *Music Perception* 1, no. 4 (1984): 355-413.

Benjamin, William E. "Abstract Polyphonies: The Music of Schoenberg's Nietzschean Moment." In *Political and Religious Ideas in the Works of Arnold Schoenberg*, ed. Charlotte M. Cross and Russell A. Berman, 1-39. New York: Garland, 2000.

Beran, Jan. *Statistics in Musicology*. Boca Raton, FL: Chapman & Hall/CRC, 2003.

Berio, Luciano. *Sequenza I: Per Flauto Solo*. Wien: Universal Edition, 1992.

Berry, Wallace. *Structural Functions in Music*. New York: Dover Publications, 1987.

Birtwistle, Harrison. *Verses: Clarinet and Piano*. London: Universal Edition, 1966.

Black, Dave, and Tom Gerou. *Essential Dictionary of Orchestration*. Los Angeles: Alfred Publishing Co., 1998.

Boulez, Pierre. *Première Sonate Pour Piano*. Paris: Amphion, 1951.

Brincker, Jens. "Statistical Analysis of Music: An Application of Information Theory." *Swedish Journal of Musicology* 52 (1970): 53-57.

Browne, Richmond. "Tonal Implications of the Diatonic Set." *In Theory Only* 5, no. 6-7 (1981): 3-21.

Buschke, Herman, and James V. Hinrichs. "Controlled Rehearsal and Recall Order in Serial List Retention." *Journal of Experimental Psychology* 78 (1968): 502-09.

Buteau, Chantal, and Guerino Mazzola. "From Contour Similarity to Motivic Topologies." *Musicae Scientiae* 4, no. 2 (2000): 125-49.

Camp, Elinor Jane Perry. "Temporal Proportion: A Study of Sonata Forms in the Piano Sonatas of Mozart." Ph.D. dissertation, Florida State University, 1968.

Carrier, L. Mark, and Harold Pashler. "Attentional Limits in Memory Retrieval." *Journal of Experimental Psychology: Learning, Memory, & Cognition* 21 (1995): 1339-1348.

Carson, Sean Hazard. "Trace Analysis: Some Applications to Musical Contour and Voice Leading." Ph.D. dissertation, New York University, 2003.

Carter, Elliott. *A 6 Letter Letter for English Horn*. New York: Boosey & Hawkes, 1996.

Carter, Elliott. *Steep Steps for Solo Bass Clarinet*. New York: Boosey & Hawkes, 2005.

Cella, Lisa M. "A Resource Manual for the Solo Flute Repertoire of the Twentieth Century." D.M.A. dissertation, University of California, San Diego, 2001.

Cherlin, Michael. "Schoenberg and Das Unheimliche: Spectres of Tonality." *Journal of Musicology* 11, no. 3 (1993): 357-73.

Cherlin, Michael. "Memory and Rhetorical Trope in Schoenberg's String Trio." *Journal of the American Musicological Society* 51, no. 3 (1998): 559-602.

Clifford, Robert John. "Contour as a Structural Element in Selected Pre-Serial Works by Anton Webern." Ph.D. dissertation, University of Wisconsin, Madison, 1995.

Cohen, Joel E. "Information Theory and Music." *Behavioural Science* 7 (1962): 137-163.

Colman, Andrew M. *A Dictionary of Psychology*. Oxford: Oxford University Press, 2006.

Cowan, Nelson. *Attention and Memory: An Integrated Framework*. Vol. 26 Oxford Psychology Series. New York: Oxford University Press, 1995.

________. *Working Memory Capacity* Essays in Cognitive Psychology: Psychology Press, 2005.

Dahlhaus, Carl. "Arnold Schönberg: Drittes Streichquartett, Op. 30." *Melos* 50, no. 1 (1988): 32-53.

Davies, John B., and Anne Yelland. "Effects of Two Training Procedures on the Production of Melodic Contour, in Short-Term Memory for Tonal Sequences." *Psychology of Music* 5, no. 2 (1977): 3-9.

Dowling, W. Jay. "Recognition of Inversions of Melodies and Melodic Contours." *Perception and Psychophysics* 9, no. 3B (1971): 348-49.

________. "Scale and Contour: Two Components of a Theory of Memory for Melodies." *Psychological Reports* 85, no. 4 (1978): 341-54.

________. "Melodic Contour in Hearing and Remembering Melodies." In *Musical Perceptions*, ed. Rita Aiello and John A. Sloboda, 173-90: New York: Oxford University Press, 1994.

Dowling, W. Jay , and Diane S. Fujitani. "Contour, Interval, and Pitch Recognition in Memory for Melodies." *The Journal of the Acoustical Society of America* 49, no. 2 (1971): 524-531.

Dowling, W. Jay, Aron Barbey, and Laura Adams. "Melodic and Rhythmic Contour in Perception and Memory." In *Music, Mind, and Science*, ed. Suk Won Yi, 166-88: Seoul, Korea: Seoul National University Press, 1999.

Edworthy, Judy. "Interval and Contour in Melody Processing." *Music Perception* 2, no. 3 (1985): 375-88.

Edworthy, Judy. "Melodic Contour and Musical Structure." In *Musical Structure and Cognition*, edited by Peter Howell, Ian Cross and Robert West, 169-88. London: Academic Press, 1985.

Eitan, Zohar. "Melodic Contour and Musical Style: A Quantitative Study." *Musikometrika* 5 (1993): 1-68.

________. *Highpoints: A Study of Melodic Peaks*. Philadelphia: University of Pennsylvania Press, 1997.

Folio, Cynthia, and Alexander R. Brinkman. "Rhythm and Timing in the Two Versions of Berio's Sequenza I for Flute Solo: Psychological and Musical Differences in Performance." In *Berio'S Sequenzas: Essays on Performance, Composition and Analysis*, ed. Janet K. Halfyard, 11-38: Ashgate Academic Publishers, 2007.

Foulkes-Levy, Laurdella. "A Synthesis of Recent Theories of Tonal Melody, Contour, and the Diatonic Scale: Implications for Aural Perception and Cognition." Ph.D. dissertation, State University of New York, Buffalo, 1996.

Frensch, Peter A. "Composition During Serial Learning: A Serial Position Effect." *Journal of Experimental Psychology* 20, no. 2 (1994): 423-43.

Friedmann, Michael. "A Methodology for the Discussion of Contour: Its Application to Schoenberg's Music." *Journal of Music Theory* 29, no. 2 (1985): 223-48.

________. "A Response: My Contour, Their Contour." *Journal of Music Theory* 31, no. 2 (1987): 268-74.

Gritten, Anthony, and Elaine C. King, eds. *Music and Gesture*. Aldershot: Ashgate Publishing, 2006.

Hába, Alois. *I. Nonet Pro Flétnu, Hoboj, Klarinet, Fagot, Lesní Roh, Housle, Violu, Violoncello a Kontrabas, Op. 40*. Praha: Státní hudební vydavatelství, 1967.

Haimo, Ethan. *Schoenberg's Serial Odyssey: The Evolution of His Twelve-Tone Method, 1914-1928*. New York: Oxford University Press, 1990.

Halford, Graeme S., Rosemary Baker, Julie E. McCredden, and John D. Bain. "How Many Variables Can Humans Process?" *Psychological Science* 16, no. 1 (2005): 70-76.

Halford, Graeme S., William H. Wilson, and Steven Phillips. "Processing Capacity Defined by Relational Complexity: Implications for Comparative, Developmental, and Cognitive Psychology." *Behavioral and Brain Sciences* 21 (1998): 803-831.

Hanna, Albert Lyle. "A Statistical Analysis of Some Style Elements in the Solo Piano Sonatas of Franz Schubert." Ph.D. dissertation, Indiana University, 1965.

Harrison, Daniel. *Harmonic Function in Chromatic Music: A Renewed Dualist Theory and an Account of Its Precedents*. Chicago: University of Chicago Press, 1994.

Hasty, Christopher. "Rhythm in Post-Tonal Music: Preliminary Questions of Duration and Motion." *Journal of Music Theory* 25, no. 2 (1981): 183-216.

________. "Segmentation and Process in Post-Tonal Music." *Music Theory Spectrum* 3 (1981): 54-73.

________. "Phrase Formation in Post-Tonal Music." *Journal of Music Theory* 28, no. 2 (1984): 167-90.

________. "On the Problem of Succession and Continuity in Twentieth-Century Music." *Music Theory Spectrum* 8 (1986): 58-74.

________. *Meter as Rhythm*: New York, U.S.A.: Oxford University Press, 1997.

Hatten, Robert S. *Interpreting Musical Gestures, Topics, and Tropes: Mozart, Beethoven, Schubert*. Bloomington: Indiana University Press, 2004.

Healy, Alice F., David A. Havas, and James T. Parker. "Comparing Serial Position Effects in Semantic and Episodic Memory Using Reconstruction of Order Tasks." *Journal of Memory and Language* 42 (2000): 147-67.

Hepokoski, James, and Warren Darcy. *Elements of Sonata Theory: Norms, Types, and Deformations in the Late-Eighteenth-Century Sonata*. Oxford: Oxford University Press, 2006.

Hinrichs, James V., and Herman Buschke. "Running Missing Scan: Perception of Oldest Member in Serial Presentations." *Psychonomic Science* 19 (1970): 125-26.

Imberty, Michel. *Signification and Meaning in Music*: Université de Montréal, 1976.

Klatzky, Roberta L. *Memory and Awareness: An Information Processing Perspective*. New York: Freeman, 1984.

Kolinski, Mieczyslaw. "The General Direction of Melodic Movement." *Ethnomusicology* 9, no. 3 (1965): 240-264.

Krumhansl, Carol L. "Tonality Induction: A Statistical Approach Applied Cross-Culturally." *Music Perception* 17, no. 4 (2000): 461-79.

Kurth, Richard B. "Moments of Closure: Thoughts on the Suspension of Tonality in Schoenberg's Fourth Quartet and Trio." In *Music of My Future: The Schoenberg Quartets and Trio*, ed. Reinhold Brinkmann and Christoph Wolff, 139-160. Cambridge: Harvard University Press, 2000.

________. "Suspended Tonalities in Schönberg's Twelve-Tone Compositions." *Journal of the Arnold Schönberg Center* 3 (2001): 239-265.

Lefkowitz, David S., and Kristin Taavola. "Segmentation in Music: Generalizing a Piece-Sensitive Approach." *Journal of Music Theory* 44, no. 1 (2000): 171-229.

Lehrl, Siegfried, and Bernd Fischer. "The Basic Parameters of Human Information Processing: Their Role in the Determination of Intelligence." *Personality and individual Differences* 9 (1988): 883-896.

Lewin, David. "Forte's Interval Vector, My Interval Function, and Regener's Common-Note Function." *Journal of Music Theory* 21, no. 2 (1977): 194-237.

________. "Some Investigations into Foreground Rhythmic and Metric Patterning." In *Music Theory: Special Topics*, ed. Richmond Browne, 101-37: Orlando, Fla.: Academic Press, 1981.

________. "Transformational Techniques in Atonal and Other Music Theories." *Perspectives of New Music* 21, no. 1-2 (1982): 312-71.

________. "Vocal Meter in Schoenberg's Atonal Music, with a Note on a Serial Hauptstimme." *In Theory Only* 6, no. 4 (1982): 12-36.

________. "Music Theory, Phenomenology, and Modes of Perception." *Music Perception* 3, no. 4 (1986): 327-92.

________. "Thoughts on Klumpenhouwer Networks and Perle-Lansky Cycles." *Music Theory Spectrum* 24, no. 2 (2002): 196-230.

________. *Generalized Musical Intervals and Transformations*. New York: Oxford University Press, 2007.

________. *Musical Form and Transformation: Four Analytic Essays*. New York: Oxford University Press, 2007.

Lewis, Christopher Orlo. "Mirrors and Metaphors: Reflections on Schoenberg and Nineteenth-Century Tonality." *19th-Century Music* 11, no. 1 (1987): 26-42.

London, Justin M. "Some Examples of Complex Meters and Their Implications for Models of Metric Perception." *Music Perception* 13, no. 1 (1995): 59-77.

________. "Cognitive Constraints on Metric Systems: Some Observations and Hypotheses." *Music Perception* 19, no. 4 (2002): 529-50.

________. "Rhythm in Twentieth-Century Theory." In *The Cambridge History of Western Music Theory*, ed. Thomas Christensen, 695-725: Cambridge, England: Cambridge University Press, 2002.

MacKay, John. "On Tonality and Tonal Form in the Serial Music of Arnold Schoenberg." *Canadian University Music Review* 8 (1987): 62-77.

Mandler, George. *Mind and Emotion*. New York: Wiley, 1975.

Marsh, Richard L., Marc M. Sebrechts, Jason L. Hicks, and Joshua D. Landau. "Processing Strategies and Secondary Memory in Very Rapid Forgetting." *Memory & Cognition* 25 (1997): 173-181.

Marvin, Elizabeth West. "A Generalized Theory of Musical Contour: Its Application to Melodic and Rhythmic Analysis of Non-Tonal Music and Its Perceptual and Pedagogical Implications." Ph.D. dissertation, University of Rochester, 1989.

________. "The Perception of Rhythm in Non-Tonal Music: Rhythmic Contours in the Music of Edgard Varèse." *Music Theory Spectrum* 13, no. 1 (1991): 61-78.

________. "Generalization of Contour Theory to Diverse Musical Spaces: Analytical Applications to the Music of Dallapiccola and Stockhausen." In *Concert Music, Rock, and Jazz since 1945: Essays and Analytical Studies*, ed. Elizabeth West Marvin and Richard Hermann, 135-71: Rochester, N.Y.: University of Rochester Press, 1995.

Marvin, Elizabeth West, and Paul Laprade. "Relating Musical Contours: Extensions of a Theory for Contour." *Journal of Music Theory* 31, no. 2 (1987): 225-67.

Meyer, Leonard B. *Emotion and Meaning in Music*: University of Chicago Press, 1956.

________. *Explaining Music: Essays and Explorations*: Berkeley: University of California Press, 1973.

Miller, George A. "The Magical Number Seven, Plus or Minus Two: Some Limits on Our Capacity for Processing Information." *Psychological Review* 63 (1956): 81-97.

Monelle, Raymond. *Linguistics and Semiotics in Music*. Chur, Switzerland: Harwood Academic Publishers, 1992.

Morris, Robert D. "A Similarity Index for Pc Sets." *Perspectives of New Music* 18 (1979-80): 445-460.

________. "Combinatoriality without the Aggregate." *Perspectives of New Music* 21, no. 1-2 (1982): 432-86.

________. *Composition with Pitch-Classes*: New Haven: Yale University Press, 1987.

________. "New Directions in the Theory and Analysis of Musical Contour." *Music Theory Spectrum* 15, no. 2 (1993): 205-28.

Murdock, B.B., Jr. "The Serial Position Effect of Free Recall." *Journal of Experimental Psychology* 64 (1962): 482-88.

Narmour, Eugene. *The Analysis and Cognition of Basic Melodic Stuctures: The Implication-Realization Model*: University of Chicago Press, 1990.

________. "Music Expectation by Cognitive Rule-Mapping." *Music Perception* 17, no. 3 (2000): 329-98.

Nattiez, Jean-Jacques. *Music and Discourse: Toward a Semiology of Music*. Translated by Carolyn Abbate. Princeton: Princeton University Press, 1990.

Nettheim, Nigel. "A Bibliography of Statistical Applications in Musicology." *Musicology Australia* 20 (1997): 94-106.

Nichols, Jeff. "Metric Conflict as an Agent of Formal Design in the First Movement of Schoenberg's Quartet Opus 30." In *Music of My Future: The Schoenberg Quartets and Trio*, ed. Reinhold Brinkmann and Christoph Wolff, 95-116. Cambridge: Harvard University Press, 2000.

Olson, Gerald B. "Intersensory and Intrasensory Transfer of Melodic Contour Perception by Children." *Journal of Research in Music Education* 26, no. 1 (1978): 41-47.

Paas, Fred, Juhani E. Tuovinen, Huib Tabbers, and Pascal W. M. Van Gerven. "Cognitive Load Measurement as a Means to Advance Cognitive Load Theory." *Educational Psychologist* 38, no. 1 (2003): 63-71.

Painter, Noel Thomas. "Exploring Contour Associations through Transformation Networks: Identification and Classification of Contour Relations in Modern Multiple Percussion Music." Ph.D. dissertation, University of Rochester, 2000.

Pinkerton, Richard C. "Information Theory and Melody." *Scientific American* 194, no. 2 (1956): 77-86.

Polansky, Larry. "Morphological Metrics: An Introduction to a Theory of Formal Distances." In *Proceedings of the International Computer Music Conference*, 197-205. San Francisco: International Computer Music Association, 1987.

Polansky, Larry, and Richard Bassein. "Possible and Impossible Melody: Some Formal Aspects of Contour." *Journal of Music Theory* 36, no. 2 (1992): 259-84.

Posner, Michael I., and Marcus E. Raichle. *Images of Mind*. New York: Scientific American Library, 1994.

Quinn, Ian. "Fuzzy Extensions to the Theory of Contour." *Music Theory Spectrum* 19, no. 2 (1997): 232-63.

________. "The Combinatorial Model of Pitch Contour." *Music Perception* 16, no. 4 (1999): 439-56.

Rahn, John. "Relating Sets." *Perspectives of New Music* 18, no. 1-2 (1979-80): 483-98.

Roeder, John. "Interacting Pulse Streams in Schoenberg's Atonal Polyphony." *Music Theory Spectrum* 16, no. 2 (1994): 231-49.

________. "A Calculus of Accent." *Journal of Music Theory* 39, no. 1 (1995): 1-46.

________. "Pulse Streams and Problems of Grouping and Metrical Dissonance in Bartok's with Drums and Pipes." *Music Theory Online* 7, no. 1 (2001).

Rosen, Charles. "Schubert's Inflections of Classical Form." In *The Cambridge Companion to Schubert*, edited by Christopher H. Gibbs, 72-98. Cambridge: Cambridge University Press, 1997.

Rosner, Burton S., and Leonard B. Meyer. "The Perceptual Roles of Melodic Process, Contour and Form." *Music Perception* 4, no. 1 (1986): 1-39.

Schmuckler, Mark A. "Testing Models of Melodic Contour Similarity." *Music Perception* 16, no. 3 (1999): 295-326.

Schoenberg, Arnold. *6 Kleine Klavierstücke, Op. 19*. Wien: Universal Edition, 1940.

________. *Five Pieces for Orchestra, Op. 16*. New York: C.F. Peters, 1952.

________. *Fundamentals of Musical Composition*, ed. Gerald Strang and Leonard Stein. London: Faber & Faber, 1967.

________. *String Quartet, No. 3, Op. 30*. Wien: Universal Edition, 1969.

Schüler, Nico. "Towards a History and Evaluation of Statistical and Information-Theoretical Analysis of Melodic Incipits." *Theoria* 13 (2006): 113-26.

Schultz, Rob. "Melodic Contour and Nonretrogradable Structure in the Birdsong of Olivier Messiaen." *Music Theory Spectrum* 30, no. 1 (2008): 89-137.

Seeger, Charles. "On the Moods of a Musical Logic." *Journal of the American Musicological Society* 13, no. 1 (1960): 224-261.

Shmulevich, Ilya. "A Note on the Pitch Contour Similarity Index." *Journal of New Music Research* 33, no. 1 (2004): 17-18.

Simpson, Reynold. "New Sketches, Old Fragments, and Schoenberg's Third String Quartet, Op. 30." *Theory and Practice* 17 (1992): 85-101.

Snavely, Andrea L. "The Role of Statistical Cues in the Segmentation of Post-Tonal Music." Ph.D. dissertation, University of Wisconsin, Madison, 2004.

Snyder, Bob. *Music and Memory: An Introduction*. Cambridge: MIT Press, 2000.

Spitzer, Michael. *Metaphor and Musical Thought*. Chicago: University of Chicago Press, 2004.

Stockhausen, Karlheinz. "...How Time Passes..." *Die Reihe (English Edition)* 3 (1959): 10-40.

Stravinsky, Igor. *Septet for Clarinet, Horn, Bassoon, Piano, Violin, Viola, and Violoncello*. New York: Boosey & Hawkes, 1953.

Sweller, John, Jeroen Van Merriënboer, and Fred Paas. "Cognitive Architecture and Instructional Design." *Educational Psychology Review* 10 (1998): 251-296.

Tarasti, Eero. *A Theory of Musical Semiotics*. Bloomington: Indiana University Press, 1994.

Tenney, James , and Larry Polansky. "Temporal Gestalt Perception in Music." *Journal of Music Theory* 24, no. 2 (1980): 205-41.

Toch, Ernst. *The Shaping Forces in Music: An Inquiry into the Nature of Harmony, Melody, Counterpoint, Form*. New York: Criterion Music Corp., 1948.

Trehub, Sandra E., Dale Bull, and Leigh A. Thorpe. "Infants' Perception of Melodies: The Role of Melodic Contour." *Child Development* 55 (1984): 821-830.

Varga, Balint Andras, and Rossana Dalmonte. *Luciano Berio: Two Interviews*. Translated by David Osmond-Smith, ed. David Osmond-Smith. New York: Marion Boyars, 1985.

Webern, Anton. *Drei Lieder für Gesang, Es-Klarinette und Gitarre, Op. 18*. Wien: Universal Edition, 1927.

________. *Variationen für Orchester, Op. 30*. Wien: Universal Edition, 1985.

Weisser, Benedict Jacob. "Notational Practice in Contemporary Music: A Critique of Three Compositional Models (Luciano Berio, John Cage, and Brian Ferneyhough)." Ph.D. dissertation, The City University of New York, 1998.

Xenakis, Iannis. *Herma*. New York: Boosey & Hawkes, 1967.

Yi, Suk Won. "A Theory of Melodic Contour as Pitch-Time Interaction: The Linguistic Modeling and Statistical Analysis of Vocal Melodies in Selected Lied Collections of Schubert and Schumann." Ph.D. dissertation, University of California, Los Angeles, 1990.