Real Time Voltage Stability Monitoring by Thevenin Impedance Estimation with Local Measurement

by

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Abstract

As modern power systems operate closer to the limits due to load growth and financial imperatives, voltage stability becomes a more important issue and there have been more incidents caused by voltage collapse. For example, there have been 11 outages affecting more than 4000MW between 1984 and 2000 in North America [1]. In power systems, load voltages decrease as the supplied loads increase until the maximum power transfer point is reached. The voltage will collapse if the load is increased above this limit. Therefore, it is important to monitor the loadability of a system to avoid voltage collapse.

The loadability of a system can be calculated when the Thevenin impedance is available as the maximum power transfer occurs when the Thevenin impedance and the load impedance are the same in magnitude. This thesis suggests a method to estimate the Thevenin impedance of a system. ABB corporation suggests the Voltage Stability Predictor (VIP) method to estimate the Thevenin impedance, but there are problems with this method and it is not gaining popularity in industry. In this thesis, a method is suggested to estimate the Thevenin impedance by taking advantage of the existence of negative sequence components in the system.

The concept of this method has been proved mathematically. Simulations were performed on simple systems and on the modified IEEE 13 bus power flow test case to verify the feasibility of the method and the results are promising. Then, the method was verified with field measurements for a 25kV substation. The voltages and currents were analyzed to estimate the Thevenin equivalent impedance of the power system and the results were compared with the design Thevenin equivalent impedance. The result confirms the viability of the method as the estimated Thevenin impedance matched the design value.
# Table of Contents

ABSTRACT ................................................................................................................................. ii

TABLE OF CONTENTS ................................................................................................................. iii

LIST OF FIGURES ........................................................................................................................... v

ACKNOWLEDGMENTS ...................................................................................................................... vi

1. INTRODUCTION ......................................................................................................................... 1

2. THEORY ........................................................................................................................................ 3
   2.1. VOLTAGE STABILITY .............................................................................................................. 3
       2.1.1. Definition ............................................................................................................................ 5
       2.1.2. Time Scale .......................................................................................................................... 5
       2.1.3. Relationship between Power and Voltage .......................................................................... 6
       2.1.4. Stable Operating Point ...................................................................................................... 8
   2.2. VOLTAGE STABILITY MONITORING ................................................................................... 11
   2.3. SYMMETRICAL COMPONENTS AND SEQUENCE NETWORKS ............................................. 14
       2.3.1. Symmetrical Components .................................................................................................. 14
       2.3.2. Transformation .................................................................................................................. 15
   2.4. IMPEDANCE ESTIMATION ................................................................................................... 16
       2.4.1. Load Impedance ................................................................................................................ 16
       2.4.2. Thevenin Equivalent Impedance .................................................................................... 16
   2.5. DIFFERENCE BETWEEN POSITIVE AND NEGATIVE SEQUENCE IMPEDANCES IN SYNCHRONOUS MACHINES ........................................................................................................... 20
       2.5.1. Equivalent Model of Synchronous Machines .................................................................... 20
       2.5.2. Steady State ...................................................................................................................... 21
       2.5.3. Transient State .................................................................................................................. 21
       2.5.4. Subtransient State ............................................................................................................ 22
       2.5.5. Negative Sequence Steady State .................................................................................... 22
   2.6. INTERNAL AND EXTERNAL UNBALANCE .......................................................................... 23
   2.7. ACCURACY OF THE ESTIMATION ....................................................................................... 23

3. CASE STUDY .................................................................................................................................. 25
   3.1. SIMPLE CASE ......................................................................................................................... 25
   3.2. EFFECT ON ACCURACY OF THE INTERNAL UNBALANCE LEVEL ...................................... 26
   3.3. 14 BUS SYSTEM ....................................................................................................................... 27
       3.3.1. Equivalent Circuit .............................................................................................................. 27
       3.3.2. Current Injection ................................................................................................................ 29
       3.3.4 Results ................................................................................................................................ 29
   3.4. EFFECT ON ACCURACY OF THE EXTERNAL UNBALANCE ................................................ 31
       3.4.1. Calculation of the Thevenin Equivalent Impedance ............................................................ 31
       3.4.2. Error Caused by External Unbalances .............................................................................. 32
       3.4.3. Effect on Accuracy of the External Unbalance Level .......................................................... 32
       3.4.4. Balancing the External Unbalance .................................................................................... 33
   3.5. ERROR ANALYSIS OF THE EXTERNAL UNBALANCE EFFECTS ........................................ 34
       3.5.1. Error from External Unbalance in Phase a ........................................................................ 34
       3.5.2. Error from Unbalance in Phase b ..................................................................................... 35
       3.5.3. Error from Unbalance in Phase c ..................................................................................... 35
       3.5.4. Averaging the Error Components .................................................................................... 36
       3.5.5. Application ....................................................................................................................... 37
   3.6. BALANCING INTERNAL UNBALANCES TO COMPENSATE EXTERNAL UNBALANCES ERRORS .......................................................... 37
       3.6.1. Internal Unbalance in Phase a .......................................................................................... 37
       3.6.2. Internal Unbalance in Phase b .......................................................................................... 38
       3.6.3. Internal Unbalance in Phase c .......................................................................................... 38
3.6.4. Averaging Internal Unbalance Results ................................................................. 39
3.6.5. Application ............................................................................................................. 40
3.7. FIELD TEST WITH DATA FROM A 25kV SUBSTATION ........................................ 40
  3.7.1. Data Analysis with Steady-State ........................................................................... 40
  3.7.2. Average Thevenin Impedance .............................................................................. 44
  3.7.3. Analysis of Peak Load Data .................................................................................. 45
  3.7.4. Analysis with Time Domain Waveform Data ......................................................... 47
  3.7.5. Design System Impedance .................................................................................... 53

4. CONCLUSION ................................................................................................................... 55

REFERENCES ...................................................................................................................... 57

APPENDIX A VOLTAGE INSTABILITY PREDICTOR VIP .................................................. 59
APPENDIX B 14 BUSES POWER FLOW CASE DATA ..................................................... 62
APPENDIX C POWER FLOW DATA FORMAT ................................................................. 63
APPENDIX D SIMULATION CODE OF THE 14 BUS POWER FLOW CASE ....................... 66
List of Figures

Figure 1 Simplified transmission circuit................................................................. 4
Figure 2 Sample circuit.......................................................................................... 7
Figure 3 PV curve for the sample circuit................................................................. 7
Figure 4 PV curves for the sample circuit with different power factor............... 8
Figure 5 System with load-tap changer ................................................................. 9
Figure 6 PV curve of two-bus system with load-tap changer............................... 9
Figure 7 Misoperation of undervoltage relay......................................................... 12
Figure 8 Equivalent circuit.................................................................................... 13
Figure 9 Symmetrical components ...................................................................... 15
Figure 10 Equivalent circuit in negative sequence.............................................. 17
Figure 11 Sample circuit for equation derivation................................................. 18
Figure 12 Simplified equivalent model of a synchronous machine in positive sequence 20
Figure 13 Power system with parallel feeders..................................................... 23
Figure 14 Equivalent circuit in negative sequence with unbalance in parallel feeders 24
Figure 15 Circuit of case 3.1................................................................................ 25
Figure 16 14 buses power flow test case ............................................................ 27
Figure 17 Current injection.................................................................................... 29
Figure 18 Power system with parallel feeders...................................................... 31
Figure 19 Load impedance from measurements.................................................. 41
Figure 20 Thevenin impedance from measurements........................................... 41
Figure 21 Comparison of load impedance and Thevenin impedance............... 42
Figure 22 Unbalance levels of voltage and current.............................................. 43
Figure 23 Deviation of phases A, B, C load impedances with respect to their average value................................................................. 44
Figure 24 Thevenin impedance averaging in 3 hours.......................................... 45
Figure 25 Load impedance under peak load....................................................... 46
Figure 26 Thevenin impedance under peak load................................................ 46
Figure 27 Comparison of load impedance and Thevenin impedance under peak load .... 47
Figure 28 Time waveform of the voltages........................................................... 48
Figure 29 Time waveform of the currents ........................................................... 48
Figure 30 The 60Hz component of the positive sequence voltage time waveform 49
Figure 31 The 60Hz component of the negative sequence voltage time waveform 50
Figure 32 The 60Hz component of the positive sequence current time waveform 50
Figure 33 The 60Hz component of the negative sequence current time waveform 51
Figure 34 60Hz load impedance......................................................................... 51
Figure 35 60Hz Thevenin impedance................................................................. 52
Figure 36 Comparison of load impedance and Thevenin impedance............... 52
Figure 37 Equivalent circuit.............................................................................. 59
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1. Introduction

Voltage stability starts to gain more and more attention nowadays. Modern power systems are way more complex than in the older days. Moreover, they operate in much more stressed conditions compared to the past, due to the growing load consumptions and the economic considerations. For example, the peak load demand is forecasted to increase by almost 18% in the U.S. and 6.4% in Canada in the next 10 years. [2] However, the projected increase of transmission is only 8.8% in the U.S. and 4.8% in Canada. A more stressed system causes more power disruptions. In many cases, power disruptions are caused by the lack of reactive power or the drop of voltage level. They can cascade and lead to voltage collapse. For example, the massive blackout in Canada and the U.S. on August, 2003 was a result of voltage instability. The blackout affected approximately 50 million people in Canada and the U.S. and caused financial losses between 7 to 10 billion USD. Better ways to monitor voltage stability of power system are necessary to prevent this kind of incident. In this thesis, a real time voltage stability monitoring method is suggested by estimating the impedance of the power system.

Voltage collapse happens when the load is above the maximum amount the system can support. It can be prevented if the maximum loading of the system is well estimated. In the literature, several methods have been proposed to determine the maximum permissible loading; these methods include bifurcation theory, the energy method, the eigenvalue method and the multiple load flow solutions method. However, all of these methods have high computational burdens and cannot generate solutions fast enough for real time monitoring.

A simpler method can be used to monitor voltage stability in real time. This can be done by comparing the load impedance and the Thevenin equivalent impedance of the transmission network. It is known from circuit theory that maximum loading happens when the magnitude of the Thevenin impedance is the same as the magnitude of the load
impedance. Therefore, the maximum loading of a load bus can be determined if the
Thevenin equivalent of the network and the load impedance can be determined.

If the Thevenin equivalent can be estimated, the load can be monitored to ensure its value
is under the maximum permissible. Relay can sense the condition and instruct the
breakers to disconnect some loads before the maximum permissible loading is reached
and to reconnect the load when the system is able again to support the load. This process
can decrease the number of voltage collapses and blackouts. ABB corporation has done
research in this area and has suggested the method of Voltage Stability Predictor (VIP)
[3]. However, there are concerns about the limitations and the accuracy of this method.
The goal of this project is to find a better way to estimate the Thevenin equivalent. It is
believed that accurate estimations can be done by taking advantage of negative sequence
measurements.
2. Theory

Some background is introduced in this chapter to help the reader understand the method suggested for voltage stability monitoring. First, some basic concepts of voltage stability and stable operating point are defined. Also explained is how these concepts can be applied for power system stability monitoring. Then, a new method for voltage stability monitoring is suggested. The chapter ends with discussions of factors that affect the accuracy of the method suggested.

2.1. Voltage Stability

There are three main stability issues in power transmission: thermal, voltage and transient. Voltage stability issue is caused by electrical distance between generations and loads and depends on topology of the network. Voltage stability is the ability to maintain the voltage at an acceptable level under normal conditions. A power system affected by disturbances can enter a state of voltage instability after a progressive and uncontrollable drop in the voltage [4]. The main reason of voltage instability is usually the lack of reactive power since in power transmission voltage is closely coupled with reactive power [5]. This can be demonstrated with the simplified transmission circuit in Figure 1.

Voltage stability covers a wide range of phenomena. Induction motors, air conditioning or HVDC links can lead to fast acting system responses that within seconds can lead to system collapse. For on-load tap changers the process is slow, where a possible system collapse due to voltage instability may take several minutes to evolve. Thus, voltage stability might mean different things to different engineers.
In the circuit of figure 1, the relationship of power transferred and voltage can be easily calculated.

\[
\delta_r = P_r + jQ_r = E_r I^*
\]

\[
E_r \left[ \frac{E_s \cos \delta + jE_s \sin \delta - E_r}{jX} \right]^* (2.1)
\]

\[
\frac{E_sE_r \sin \delta + j}{X} \left[ \frac{E_sE_r \cos \delta - E_r^2}{X} \right] \right) (2.2)
\]

\[
P_r = \frac{E_sE_r}{X} \sin \delta = P_{\text{max}} \sin \delta (2.4)
\]

\[
Q_r = \frac{E_sE_r \cos \delta - E_r^2}{X} (2.5)
\]

In power systems, the angle across the transmission line is usually small. Then, the power transfer can be approximated with small angles approximation,

\[
P_r = P_{\text{max}} \delta (2.6)
\]

\[
Q_r = \frac{E_sE_r - E_r^2}{X} = \frac{E_r(E_s - E_r)}{X} (2.7)
\]

As shown in Equation 2.6, the real power is closely related to \( \delta \), which is the angle between the two buses. On the other hand, the reactive power transferred through the transmission line is given by Equation 2.7, which is closely coupled to the difference in voltage magnitude between the sending and receiving ends of the transmission line.
2.1.1. Definition

In this section, the definition of voltage stability, voltage collapse and voltage stability proposed by CIGRE are presented:

Voltage stability: A power system at a given operating state is small-disturbance voltage stable if, following any small disturbance, voltages near loads are identical or close to the pre-disturbance values. (Small-disturbance voltage stability corresponds to a related linearized dynamic model with eigenvalues having negative real parts. For analysis, discontinuous models for tap changers may have to be replaced with equivalent continuous models.)

Voltage collapse: A power system at a given operating state and subject to a given disturbance undergoes voltage collapse if post-disturbance equilibrium voltages are below acceptable limits. Voltage collapse may be total (blackout) or partial.

Voltage instability: voltage instability is the lack of voltage stability, and results in progressive voltage decrease (or increase).

2.1.2. Time Scale

Voltage collapse can be caused by a wide range of phenomena and there are different time ranges. They are classified as short-term (transient), mid-term or long-term time frame [6]:

Short-term time frame is in seconds. This phenomenon is often caused by fast acting power system equipments, or automatically controlled power system equipments, or both:

- Synchronous Condensers
- Automatic switched shunt capacitors
- Induction motor dynamics
- Static VAR Compensators
- Flexible AC Transmission System (FACTS) devices
- Excitation system dynamics
- Voltage-dependent loads
Mid-term time frame is from tens of seconds to several minutes and is caused by many different reasons such as load-tap changers, excitation limiters and thermostatic controls.

Long-term time frame is in tens of minutes and is caused by long-term dynamics such as governor action, load-voltage and load-frequency characteristic.

The voltage stability monitoring method suggested in this thesis takes steady-state measurements. Therefore, this thesis primarily focuses on phenomena of voltage stability from mid-term to long-term time frame.

2.1.3. Relationship between Power and Voltage

The relationship between power and voltage at the load is given by the PV curve. For a typical load, the load impedance drops in order to consume more power, up to the maximum power transfer point. For the following example in Figure 2, the relationship between the voltage and the power of the load is given by the PV curve in Figure 3. As indicated, the impedance of the load drops to consume more power. At the same time, the voltage decreases and the current increases as the impedance drops. However, when the voltage reaches the maximum power transfer point (nose point) at point C, the power transfer cannot increase any further. By theory, maximum power transfer happens whenever the magnitude of the thevenin impedance ($X_{\text{transmission}}$ in Figure 2) is the same as the load impedance. The shape of the PV curve depends on the load characteristic. A collection of curves with different load power factors is shown in Figure 4.
Figure 2 Sample circuit

Figure 3 PV curve for the sample circuit
2.1.4. Stable Operating Point

In power systems, there are usually two operating points for one given power output. However, only the value higher than the nose point is the stable one. Taking the circuit in Figure 2 as an example, there are two operating points for 15W of power output, A and B. Point A operates at a higher voltage and lower current than B. Point A is the stable operating point and Point B is the unstable one. The behavior of some system components, such as load tap changers, induction motors and heating thermostats unstabilize the system if the system works at the unstable operating point.

An example is given to show how the load tap changer unstabilizes the system at an unstable operating point. [7]
Figure 5 shows a simple power system with a load tap changer. The load tap changer regulates the voltage in the secondary side by adjusting the turns ratio. The ratio $r$ increases when the secondary voltage is too high. On the other hand, the ratio decreases when the secondary voltage is too low,

$$V_2 = rV$$  \hspace{1cm} (2.8)

The stability of the operating points can be checked by applying a disturbance consisting of a small decrease of $r$.

Figure 6 shows the PV curve and the load characteristic curves for the system of Figure 5. The transient load characteristic curve represents the load after a disturbance is applied. Figure 6 also shows how the transient characteristic curve changes after the ratio of the load tap changer changes. The popular exponential load model is applied to
characterize the load and is given by Equation 2.9. The operating point of the system is the intercept of the two curves.

\[ P = P_0 \left( \frac{V_s}{V_p^2} \right)^a \]  

(2.9)

At operating point S, which is above the nose point, the decrease of \( r \) will cause a lower power and a lower secondary voltage. As a result, \( r \) will increase to maintain the voltage and the operating point will go back to S. Hence, S is a stable operating point.

At operating point U, the decrease of \( r \) will cause a higher power and a higher secondary voltage. As a result, \( r \) will decrease to maintain the voltage and the operating point will go further away from U. Hence, U is an unstable operating point.

Motor load also unstabilizes power systems if the motor operates below the nose point. The relationship between rotor motion and mechanical and electrical torque is given by Equation 2.10

\[ 2H \dot{s} = T_m(s) - T_e(V, s) \]  

(2.10)

where \( H \) is the inertia constant, \( s \) is the motor slip, \( T_m \) is the mechanical torque and \( T_e \) is the electrical torque. The motor slip is defined as

\[ s = \frac{\omega_o - \dot{\theta}_r}{\omega_o} \]  

(2.11)

where \( \omega_o \) is the nominal angular frequency and \( \dot{\theta}_r \) is the rotor speed in electrical radians per second.

In steady state, the torque is at an equilibrium condition such that
\[ T_s(V, s) = T_m(s) \quad (2.12) \]

Assuming the mechanical load is constant, the stability of the operating points can be checked by applying a disturbance of a small increase of \( s \).

If the motor operates above the nose point, a slight increase in the slip increases the electrical torque. From Equation 2.10, the \( 2Hs \) will then be negative to reduce the slip and drive the motor back to the original operating point.

If the motor operates below the nose point, a slight increase in the slip decreases the electrical torque. From Equation 2.10, the \( 2Hs \) will become positive to reduce the slip and drive the motor further away from the original operating point.

Therefore, it is important to make sure that a power system operates above the nose point in the stable operating region.

### 2.2. Voltage Stability Monitoring

Nowadays, undervoltage relays are applied to the power system as a mitigation of voltage stability. They monitor the local voltage and order the breaker to shed load when the voltage stays lower than the defined voltage. It is a cost-effective and a simple method. However, the method is too simple that it can shed load prematurely or fail to detect voltage instability.
As shown in Figure 7, the blue circuit is the region of voltage instability. However, undervoltage operates in the red circuit region. The two circles do not totally overlap. Trajectory A and B are two examples of misoperation of undervoltage relay. The trajectory A will be treated as voltage instability prematurely by the undervoltage relay and the impedance trajectory B will not be detected by the undervoltage relay. As a result, a better method is needed to monitor voltage instability in real time.

As mentioned in Section 2.1.4, a power system operates at a stable operating point whenever it is above the nose point. Moreover, the nose point is reached when the magnitude of the Thevenin impedance is the same as the magnitude of the load impedance. Therefore, the stability of a power system can be defined if the equivalent circuit shown in Figure 8 can be found.
With the equivalent circuit of Figure 8, the stability of the system can be defined by comparing the magnitude of the Thevenin impedance and the load impedance. When the magnitude of the load at maximum loading is the same as the Thevenin impedance, the maximum loading power of the system can also be estimated from Equation 2.15.

\[
P_{\text{max}} = I_{\text{max}} Z_{\text{max}} \\
P_{\text{max}} = \frac{E_{\text{th}}}{Z_{\text{max}} + Z_{\text{thev}}} Z_{\text{max}} \\
P_{\text{max}} = \left| \frac{E_{\text{th}}}{Z_{\text{thev}} + Z_{\text{thev}} \angle \theta_{\text{load}} + Z_{\text{thev}}} \right| Z_{\text{thev}} \angle \theta_{\text{load}}
\]

However, the power factor of the load fluctuates as the load changes. Therefore, a good technique would be required to predict the power factor at the nose point.

By estimating the equivalent circuit, the minimum magnitude of the load impedance and the maximum power of a stable system can be defined. Therefore, two indices can be defined to monitor the condition of the power system.

\[
\text{Index}_{\text{impedance}} = \frac{|Z_{\text{load}}| - |Z_{\text{thev}}|}{|Z_{\text{thev}}|}
\]
\[
\text{Index}_{\text{power}} = \frac{P_{\text{max}} - P_{\text{load}}}{P_{\text{max}}}
\]
As the load impedance drops to draw more power, the indices decrease indicating that the system is approaching to the nose. The indices become zero when the system is operating at the nose point. As a result, it is important to make sure that the indices are positive and suitable action should be taken when the indices are close to zero.

### 2.3. Symmetrical Components and Sequence Networks

The method of symmetrical components is a useful tool to deal with unbalanced multiphase systems. In normal condition, the system is dominated by the positive sequence. However, significant unbalanced currents and voltages exist during unsymmetrical faults. Symmetrical components are commonly used to study unsymmetrical faults.

#### 2.3.1. Symmetrical Components

A set of unbalance voltages or currents can be broken down into three symmetrical components: positive, negative and zero sequence: [8]

Positive sequence consists of three phasors of the same magnitude that are 120° apart and have a counterclockwise phase rotation abc.

Negative sequence consists of three phasors of the same magnitude that are 120° apart and have a counterclockwise phase rotation acb.

Zero sequence consists of three phasors of the same magnitude that are in phase with each other.
2.3.2. Transformation

The original three phase components and the set of symmetrical components can be transformed to each other by the transformation matrices $A$ and $A^{-1}$. In the transformation matrices, $a$ is an operator that produces a $120^\circ$ phase shift.

\[ a = 1\angle 120^\circ \quad (2.18) \]

$A$ is the transformation matrix that converts the symmetrical components into the phase components.

\[
A = \begin{bmatrix}
1 & 1 & 1 \\
1 & a^2 & a \\
1 & a & a^2
\end{bmatrix}
\quad (2.19)
\]

\[
\begin{bmatrix}
V_a^+ \\
V_b^+ \\
V_c^+ 
\end{bmatrix} = \begin{bmatrix}
1 & 1 & 1 \\
1 & a^2 & a \\
1 & a & a^2
\end{bmatrix} \begin{bmatrix}
V_0^+ \\
V^+ \\
V^-
\end{bmatrix} = A \begin{bmatrix}
V_0^+ \\
V^+ \\
V^-
\end{bmatrix}
\quad (2.20)
\]

On the other hand, $A^{-1}$ is the transformation matrix to convert the phase components into the symmetrical components.
\[ A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \]  

(2.21)

\[
\begin{bmatrix} V^0 \\ V^+ \\ V^- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = A^{-1} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \]  

(2.22)

2.4. Impedance Estimation

As mentioned in Section 2.2, the voltage stability of a power system can be defined if the magnitude of the Thevenin impedance and the load impedance can be found. In this section, a method to estimate these impedances will be explained.

2.4.1. Load Impedance

The load impedance can be easily calculated by Ohm’s law. Power systems are three phase systems. Because loads are mostly balanced and motor torque is due to positive sequence currents, positive sequence quantities can be used to describe the load impedance,

\[
Z_{load}^+ = \frac{V^+}{I^+} \]  

(2.23)

From Equation 2.23, the load impedance can be calculated by dividing the positive sequence voltage by the positive sequence current.

2.4.2. Thevenin Equivalent Impedance

The Thenvein Impedance cannot be estimated as easily as the load impedance because there are voltage sources behind it. However, the negative sequence impedance can be estimated by taking advantage of the fact [9] that there are no negative sequence voltage sources in power systems.
2.4.2.a. Circuit Representation

Assuming that the system is all balanced except for the load where measurements are made, Figure 10 shows the equivalent circuit for the negative sequence components. By applying Ohm’s law, \( Z^- \) can be found,

\[
Z_{\text{thev}}^- = -\frac{V^-}{I^-}
\]  

(2.24)

With Equation 2.24, the Thevenin equivalent impedance can be calculated by dividing the negative sequence voltage by the negative sequence current.

![Figure 10 Equivalent circuit in negative sequence](image)

2.4.2.b. Equation Derivation

The following analysis shows that the method of finding the system Thevenin equivalent from the negative sequence quantities is valid when the load impedances are unbalanced. The circuit in Figure 11 is used for the derivation,
Figure 11 Sample circuit for equation derivation

In Figure 11, the Thevenin voltage source $E_{th}$ and the Thevenin impedances $Z_{th}$ are assumed to be balanced,

$$|E_a| = |E_b| = |E_c|$$  \hfill (2.25)
$$E_a = aE_b = a^2E_c$$  \hfill (2.26)
$$Z_{Ta} = Z_{Th} = Z_{Tc}$$  \hfill (2.27)

The magnitudes of the load impedance $Z_L$ are defined by the average impedance of the loads plus a deviation term,

$$Z_{La} = Z_L(1 + \alpha)$$  \hfill (2.28)
$$Z_{Lb} = Z_L(1 + \beta)$$  \hfill (2.29)
$$Z_{Lc} = Z_L(1 + \gamma)$$  \hfill (2.30)

The negative sequence impedance can be calculated as

$$\frac{V^-}{I^-} = \frac{\sqrt{3}(V_a + a^2V_b + aV_c)}{\sqrt{3}(I_a + a^2I_b + aI_c)} = \frac{(V_a + a^2V_b + aV_c)}{(I_a + a^2I_b + aI_c)}$$  \hfill (2.31)

$$V_{a,b,c} = \frac{E_{a,b,c}}{Z_T + Z_{La,b,c}}$$  \hfill (2.32)
$$I_{a,b,c} = \frac{E_{a,b,c}}{Z_T + Z_{La,b,c}}$$  \hfill (2.33)
\[
\frac{V^-}{I^-} = \frac{E_a(Z_1(1+\alpha)) + a^2 E_b(Z_1(1+\beta)) + a E_c(Z_1(1+\gamma))}{Z_T + (Z_L(1+\alpha))} + \frac{E_b(Z_1(1+\beta))}{Z_T + (Z_L(1+\beta))} + \frac{E_c(Z_1(1+\gamma))}{Z_T + (Z_L(1+\gamma))}
\]  
(2.34)

For simplification, the sum of the Thevenin impedance and the load impedances is combined as follows:

\(Z_T + Z_L = Z_{Ta}\)
\(Z_T + Z_L = Z_{Tb}\)
\(Z_T + Z_L = Z_{Tc}\)

\[
\frac{V^-}{I^-} = \frac{Z_a(Z_{Ta})(Z_{Tc}) + aZ_b(Z_{Ta})(Z_{Tc}) + a^2Z_c(Z_{Ta})(Z_{Tb})}{(Z_{Tb})(Z_{Tc}) + a(Z_{Ta})(Z_{Tc}) + a^2(Z_{Ta})(Z_{Tb})}
\]  
(2.38)

Incorporating the deviation terms in Equations 2.28-2.30, each of the terms in Equation 2.38 can be expanded as follows

\[
Z_a(Z_{Ta})(Z_{Tc}) = (1+\alpha)(1+\beta+Z)(1+\gamma+Z)
\]  
(2.39)

\[
aZ_b(Z_{Ta})(Z_{Tc}) = a(1+\beta)(1+\gamma+Z)(1+\alpha+Z)
\]  
(2.40)

\[
a^2Z_c(Z_{Ta})(Z_{Tb}) = a^2(1+\gamma)(1+\beta+Z)(1+\alpha+Z)
\]  
(2.41)

\[
(Z_{Tb})(Z_{Tc}) = (1+\beta+Z)(1+\gamma+Z)
\]  
(2.42)

\[
a(Z_{Ta})(Z_{Tc}) = a(1+\gamma+Z)(1+\alpha+Z)
\]  
(2.43)

\[
a^2(Z_{Ta})(Z_{Tb}) = a^2(1+\beta+Z)(1+\alpha+Z)
\]  
(2.44)

A new term \(Z\) is defined to simplify the derivation,

\[
Z = \frac{Z_T}{Z_L}
\]  
(2.45)

After expanding all the terms and substituting in Equation 2.45, the negative sequence impedance can be expressed as,

\[
\frac{V^-}{I^-} = \frac{Z_L[(\alpha Z^2 + \alpha Z - \beta \gamma Z) + a(\beta Z^2 + \beta Z - \gamma \alpha Z) + a^2(\gamma Z^2 + \gamma Z - \alpha \beta Z)]}{(-\alpha - \alpha Z + \beta \gamma) + a(-\beta - \beta Z + \gamma \alpha) + a^2(-\gamma - \gamma Z + \alpha \beta)}
\]  
(2.46)
\[
\frac{V^-}{I^-} = \frac{Z_L[(\alpha Z + \alpha - \beta \gamma) + a(\beta Z + \beta - \gamma \alpha) + a^2(\gamma Z + \gamma - \alpha \beta)]}{(-\alpha - \alpha Z + \beta \gamma) + a(-\beta - \beta Z + \gamma \alpha) + a^2(-\gamma - \gamma Z + \alpha \beta)}
\]  
(2.47)

\[
\frac{V^-}{I^-} = -Z_L
\]  
(2.48)

\[
\frac{V^-}{I^-} = -Z_r
\]  
(2.49)

From the derivation, it can be seen that the Thevenin impedance of the feeding system is equal to the negative value of the negative sequence voltage divided by the negative sequence current even when the load are imbalanced.

### 2.5. Difference between Positive and Negative Sequence Impedances in Synchronous Machines

The major component in a power system is the positive sequence. With the method suggested, however, \(Z_{\text{thev}^-}\) is estimated instead of \(Z_{\text{thev}^+}\). The difference between the positive and negative sequence is clarified in this section. The impedance of most components is the same in both negative and positive sequence. However, there are some differences in the impedance of machines between the positive and the negative sequences.

#### 2.5.1. Equivalent Model of Synchronous Machines

Figure 12 shows a simplified equivalent model of a synchronous machine. For the positive sequence, \(E_{\text{th}}\) is the Thevenin equivalent of the voltage source. \(X_d\) is the impedance of the machine which varies with the state of the machine. The notation of the machine impedances in different states are given in Table 1 [10].

![Figure 12 Simplified equivalent model of a synchronous machine in positive sequence](image-url)
<table>
<thead>
<tr>
<th>Equivalent Impedance of Machine</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_d$</td>
<td>Steady</td>
</tr>
<tr>
<td>$X_d'$</td>
<td>Transient</td>
</tr>
<tr>
<td>$X_d''$</td>
<td>Subtransient</td>
</tr>
</tbody>
</table>

Table 1 States of equivalent models

2.5.2. Steady State

$X_d$ is the simplified equivalent positive sequence impedance in the steady state. The flux takes the least reluctance path at the steady state. The flux takes the path of the magnetic core of the rotor. The relationship between the armature current and the flux linkage is given by Equation 2.50,

$$\Phi = LI = \frac{X_d I}{\omega}$$

(2.50)

Therefore, the flux wave has the greatest value for a given armature current with the least reluctance path.

2.5.3. Transient State

The transient impedance represents the relationship between the armature current and the voltage immediately after the field current is applied, instead of after a steady state voltage is reached as the case of steady-state. The sudden change in the armature current generates an m.m.f. that opposes the field winding. The field windings will then generate flux through the core to oppose the m.m.f. generated by the armature currents. As a result, the flux cannot get through the core and has to take the lower permanence leakage path largely in air. Therefore, with a certain amount of armature current, the flux linkages generated are smaller than in the steady state and the reactance $X_{d'}$ is smaller than $X_d$. 
2.5.4. Subtransient State

On top of the induced current of the field winding described in the last section, there is an additional transient current right after the armature current is applied. This current can either be generated by the damper windings in salient pole machines or eddy currents in round rotor machines. This current tries to keep the flux linkage of the rotor to be zero. As a result, the flux has to take an even higher permanence path than in the transient state. The flux linkages generated are the smallest in this state and the reactance $X_d''$ is the smallest.

As a conclusion, the reactance of machines changes as the state changes and the relationship is as follows:

\[ X_d'' < X_d' < X_d \]  \hspace{1cm} (2.51)

2.5.5. Negative Sequence Steady State

As suggested in Section 2.4.2, the negative sequence reactance at steady state can be estimated. While the negative sequence is turning at 60Hz in opposite direction to the positive sequence, it is running at 120Hz with respect to the rotor field and the reactance seen is similar to the subtransient state reactance. Running in 120Hz, the flux is not going through the magnetic path of d-axis only. The negative sequence magnetic path also rotates as the rotor in 120Hz. Therefore, the negative sequence steady state reactance is the average of the subtransient reactance of the q-axis and d-axis

\[ X_{neg} = \frac{X_q'' + X_d''}{2} \]  \hspace{1cm} (2.52)

There is a difference between the steady state reactance of the positive sequence and the negative sequence. However, the negative sequence reactance is a more appropriate representation of the system for stability monitoring because the reaction to voltage instability caused by sudden perturbations to the system and machines occur in the subtransient state.
2.6. Internal and External Unbalance

In power systems, there are usually many feeders connected to the same bus. Therefore, we can identify internal unbalance and external unbalance to clarify the unbalance between different feeders. Assuming the suggested negative sequence method is applied to estimate the Thevenin equivalent circuit in Figure 13 from $Z_{\text{load}}^1$, the unbalance of $Z_{\text{load}}^1$ is referred to as internal unbalance while the unbalance of $Z_{\text{load}}^2$ is referred to as external unbalance.

![Figure 13 Power system with parallel feeders](image)

2.7. Accuracy of the Estimation

Simulation results show that the suggested method works very well to estimate $Z_{\text{th}}$ if the external system contains only internal unbalance. The estimation also works better with a higher level of internal unbalance in the local load. Simulations are run to study this effect and the results are shown in Section 3.2 in the next chapter. The negative sequence equivalent circuit looks like Figure 10 when there is no external unbalance. However, there will be a negative voltage source as shown in Figure 14 in the equivalent circuit.
whenever there is external unbalance. As a result, the estimation is less accurate when the external unbalance level is high.

![Figure 14 Equivalent circuit in negative sequence with unbalance in parallel feeders](image)

Nonetheless, the combined unbalance from all external feeders should be smaller than the unbalance of the local feeder where measurements are made because the unbalance of all external feeders should tend to cancel out each other. In Section 3.4, simulations are run to show the effect of the accuracy of the estimation against different external unbalance levels.
3. Case Study

Simulations were run to test the proposed method and the results are shown in this chapter. First, a simple case will be provided to show how this method works and how the accuracy is affected by the unbalance level. Then, the method is applied to the IEEE standard 14 bus power flow case. Finally, a few more cases were simulated to demonstrate how the accuracy of the method is affected by the external unbalance.

3.1. Simple Case

A simple case (Figure 15) is used as an example to prove the concept and help the reader to understand the method. The transmission system is balanced while the load is slightly unbalanced.

Voltage and current can be calculated at the load and the results are given in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>Phase a</th>
<th>Phase b</th>
<th>Phase c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage (V)</td>
<td>0.9388∠-6.34°</td>
<td>0.99504∠-125.71°</td>
<td>0.99504∠114.29°</td>
</tr>
<tr>
<td>Current (A)</td>
<td>1.1043∠-6.34°</td>
<td>0.99504∠-125.71°</td>
<td>0.99504∠114.29°</td>
</tr>
</tbody>
</table>

Table 2 Phase voltage and current for case 3.1
Then, negative sequence voltage and current can be found as described in Section 2.3.

<table>
<thead>
<tr>
<th>$V_n$ (V)</th>
<th>0.0037∠-102.0805</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_n$ (A)</td>
<td>0.0366∠-12.0552</td>
</tr>
</tbody>
</table>

Table 3 Negative sequence voltage and current for case 1

The negative sequence impedance can be calculated and the result is almost identical to the Thevenin impedance.

<table>
<thead>
<tr>
<th>$Z_n$ Estimated</th>
<th>$0.1001e^{j89.9747}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_{Thevenin}$</td>
<td>$0.1000e^{j90}$</td>
</tr>
</tbody>
</table>

Table 4 Estimated Thevenin impedance

### 3.2. Effect on Accuracy of the Internal Unbalance Level

A higher internal unbalance gives more accurate results. A higher internal unbalance is equivalent to applying a stronger negative sequence voltage to the negative sequence external impedances. Using the circuit in Section 3.1, with varying loads in phase A while maintaining the same equal loads in phases B and C, different estimations of the equivalent impedance are made with different unbalance levels. These results are presented in Table 5.

<table>
<thead>
<tr>
<th>Load of phase a (Ω)</th>
<th>deviation of load magnitude</th>
<th>$Z_{th}$ estimated (Ω)</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.95</td>
<td>0.50%</td>
<td>0.1006</td>
<td>0.60%</td>
</tr>
<tr>
<td>0.96</td>
<td>0.40%</td>
<td>0.1015</td>
<td>1.50%</td>
</tr>
<tr>
<td>0.97</td>
<td>0.30%</td>
<td>0.1026</td>
<td>2.60%</td>
</tr>
<tr>
<td>0.98</td>
<td>0.20%</td>
<td>0.1038</td>
<td>3.80%</td>
</tr>
<tr>
<td>0.99</td>
<td>0.10%</td>
<td>0.1045</td>
<td>4.50%</td>
</tr>
</tbody>
</table>

Table 6 Error of estimation of Thevenin impedance in different internal unbalance level

As expected, the error of the estimation is smaller when the internal load unbalance is higher.
3.3. **14 Bus System**

For a more realistic study, the IEEE 14 bus power flow test case is taken to test the method. However, the IEEE 14 bus case is a power flow system, while in our case the currents and voltages, not the power flow, are the main focus. For our study, the system is interpreted as an equivalent circuit and it is simulated by the transient program MicroTran [11]. The parameters and data format of this case are given in Appendix B and Appendix C.

![14 bus test system bus code diagram](image)

**Figure 16 14 buses power flow test case**

### 3.3.1. Equivalent Circuit

The 14 bus case is translated into an equivalent circuit and the details of the translation are provided in this section.
3.3.1.a. Load

For power flow studies, loads are represented by their power consumptions. However, impedances are required for the simulation in MicroTran. Equivalent impedances are calculated with Equation 3.1 to replace the load,

\[ Z = \frac{|V|^2}{S^*} \]  

(3.1)

3.3.1.b. Generator and Synchronous Condenser

For power flow studies, generators are represented by their generated power. In MicroTran, electrical parameters can be defined to simulate the generators. Typical values of generator for the simulation are taken from Anderson [12].

For power flow studies, synchronous condensers are represented by the reactive power they generate. Synchronous condensers are modeled as generators and typical values are found from the literature [13].

3.3.1.c. System Unbalance

For power flow studies, the power system is simulated in single phase. However, the method presented in this thesis takes the advantage of the unbalance of the system and load unbalance is necessary to find the equivalent model. Following accepted practice, at 69kV to 500kV voltage unbalance shall not be greater than 1% and current unbalance shall not be greater than 5% under the requirements of British Columbia Transmission Corporation [14]. At 35kV and below, phase current unbalance of 10-20% and phase voltage unbalance up to 2-3% are the normal condition under the requirements of BC Hydro [15]. In an England and Wales transmission system, voltage unbalance level was measured in a 400kV system and the unbalance was 0.33% [16]. In a study of transposition from Australia, current unbalance was measured at the 220kV level and the
unbalance was 4.0% [17]. Unbalance of load is set to 5% such that the unbalance of voltage and current are close to the value measured in a real system.

3.3.2. Current Injection

Current injection is applied to find the $Z_{th}$ of the system. By definition, $Z_{th}$ is the equivalent impedance at the system terminals (Figure 17) when the independent sources are turned off. In calculating $Z_{th}$, all independent sources are first grounded. Then, the positive sequence voltage $V^+$ is measured when a positive sequence current $I^+$ is applied to the circuit. The equivalent impedance $Z_{th}$ can be found by dividing the voltage by the current,

$$Z_{th} = \frac{V^+}{I^+} \quad (3.2)$$

To make the calculation simpler, the current applied can be set to $1\angle 0$ such that the measured voltage is $Z_{th}$.

![Figure 17 Current injection](image)

3.3.4 Results

Assuming that the system does not have external unbalance, 5% of load unbalance is applied to the measured bus. The equivalent impedance is then estimated by dividing the negative sequence voltage by the negative sequence current. Alternatively, current injection is used to calculate the correct equivalent impedance. This is done in all buses.
and the results are shown in Table 7. The error of the magnitude of the impedance is calculated. The voltage and the current unbalance level of the system are also given in the Table 7.

\[
Error = \left( \frac{|Z_{th}^{5\% unbalance}| - |Z_{th}^{injection}|}{|Z_{th}^{injection}|} \right) \times 100\% \tag{3.3}
\]

The errors of the equivalent impedance estimated are less than 1% with 5% unbalance in the load.

<table>
<thead>
<tr>
<th>Injection</th>
<th>5% unbalance at measurement node only</th>
<th>Error</th>
<th>Current unbalance</th>
<th>Voltage unbalance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R X</td>
<td></td>
<td>Z</td>
<td>angle</td>
</tr>
<tr>
<td>Bus 14</td>
<td>0.14</td>
<td>0.31</td>
<td>0.34</td>
<td>65.24</td>
</tr>
<tr>
<td>Bus 13</td>
<td>0.11</td>
<td>0.24</td>
<td>0.26</td>
<td>64.34</td>
</tr>
<tr>
<td>Bus 12</td>
<td>0.16</td>
<td>0.30</td>
<td>0.34</td>
<td>61.41</td>
</tr>
<tr>
<td>Bus 11</td>
<td>0.11</td>
<td>0.25</td>
<td>0.27</td>
<td>65.71</td>
</tr>
<tr>
<td>Bus 10</td>
<td>0.09</td>
<td>0.23</td>
<td>0.25</td>
<td>67.77</td>
</tr>
<tr>
<td>Bus 9</td>
<td>0.07</td>
<td>0.20</td>
<td>0.21</td>
<td>70.55</td>
</tr>
<tr>
<td>Bus 6</td>
<td>0.07</td>
<td>0.17</td>
<td>0.19</td>
<td>66.91</td>
</tr>
<tr>
<td>Bus 5</td>
<td>0.05</td>
<td>0.12</td>
<td>0.13</td>
<td>67.87</td>
</tr>
<tr>
<td>Bus 4</td>
<td>0.05</td>
<td>0.12</td>
<td>0.13</td>
<td>68.70</td>
</tr>
<tr>
<td>Bus 3</td>
<td>0.06</td>
<td>0.15</td>
<td>0.16</td>
<td>68.80</td>
</tr>
<tr>
<td>Bus 2</td>
<td>0.04</td>
<td>0.12</td>
<td>0.13</td>
<td>70.24</td>
</tr>
</tbody>
</table>

Table 7 Results of the 14 buses power flow case
3.4. **Effect on Accuracy of the External Unbalance**

In the previous case studies, the estimate of the Thevenin impedance from negative sequence quantities was very accurate. However, the accuracy of the estimation is affected by the external unbalance, as mentioned in Section 2.7. A simple system is used next to study the accuracy of the method under these circumstances. The schematic of the circuit is given in Figure 18.

![Power system with parallel feeders](image)

Simulations are run to show the effect of the external unbalance on the accuracy of the estimation of the equivalent impedance. The equivalent impedance is estimated by measuring the current and voltage in $Z_{load}^1$ while loads 2 to 4 vary to have different amount of unbalance.

### 3.4.1. Calculation of the Thevenin Equivalent Impedance

First, the Thevenin equivalent impedance is calculated. By definition, the Thevenin equivalent impedance seen from load 1 can be calculated by taking the impedance of $Z_{transmission}$ in parallel with $Z_{load}^1$, $Z_{load}^2$, $Z_{load}^3$ and $Z_{load}^4$.

$$Z_{th} = Z_{transmission} \parallel Z_{load}^2 \parallel Z_{load}^3 \parallel Z_{load}^4 \quad (3.4)$$

This equivalent impedance is found to be $0.00187 + 0.0499i \cdot (0.0499e^{j87.85})$. 

---

31
3.4.2. Error Caused by External Unbalances

A case was run to show that error can be caused by external unbalance. In the previous cases, there was only internal unbalance from the load where measurements are made. In this case, there will be unbalance in all loads according to the circuit parameters given in Table 8. In this case, measurements and estimations are made at Z_{load}^1.

<table>
<thead>
<tr>
<th>Phase</th>
<th>E_{in}</th>
<th>Z_{transmission}</th>
<th>Z_{load}^1</th>
<th>Z_{load}^2</th>
<th>Z_{load}^3</th>
<th>Z_{load}^4</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1</td>
<td>0.05i</td>
<td>4.2</td>
<td>4.16</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>b</td>
<td>1</td>
<td>0.05i</td>
<td>4</td>
<td>4</td>
<td>4.2</td>
<td>4</td>
</tr>
<tr>
<td>c</td>
<td>1</td>
<td>0.05i</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4.2</td>
</tr>
</tbody>
</table>

Table 8 Parameters of the system of Figure 18 with external unbalances

<table>
<thead>
<tr>
<th>Phase</th>
<th>E_{in}</th>
<th>Z_{transmission}</th>
<th>Z_{load}^1</th>
<th>Z_{load}^2</th>
<th>Z_{load}^3</th>
<th>Z_{load}^4</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1</td>
<td>0.05i</td>
<td>4</td>
<td>4</td>
<td>4.2</td>
<td>4</td>
</tr>
<tr>
<td>b</td>
<td>1</td>
<td>0.05i</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4.2</td>
</tr>
<tr>
<td>c</td>
<td>1</td>
<td>0.05i</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4.2</td>
</tr>
</tbody>
</table>

Table 8 Parameters of the system of Figure 18 with external unbalances

\[ Z_{th} \text{ is estimated to be } 0.0016 + 0.0404i \left(0.0404e^{i \omega 87.7571}\right). \] Comparing to the result calculated from the last section, there is a 20% error in the estimation of the magnitude of the equivalent impedance.

3.4.3. Effect on Accuracy of the External Unbalance Level

The accuracy of the method is affected by the external unbalance level. A higher external unbalance is equivalent to applying a stronger negative voltage source against the signal. Next, different estimations of the equivalent impedance are made in the system of Figure 18 with different external unbalance levels at Z_{load}^2, as indicated in Table 11.

<table>
<thead>
<tr>
<th>Phase</th>
<th>E_{in}</th>
<th>Z_{transmission}</th>
<th>Z_{load}^1</th>
<th>Z_{load}^2</th>
<th>Z_{load}^3</th>
<th>Z_{load}^4</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1</td>
<td>0.05i</td>
<td>4</td>
<td>4*(1+d)</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>b</td>
<td>1</td>
<td>0.05i</td>
<td>4</td>
<td>4</td>
<td>4.2</td>
<td>4</td>
</tr>
<tr>
<td>c</td>
<td>1</td>
<td>0.05i</td>
<td>4.2</td>
<td>4</td>
<td>4</td>
<td>4.2</td>
</tr>
</tbody>
</table>

Table 12 Parameters of the system of Figure 18 with external unbalances
deviation of load magnitude | \( |Z_{th}| \) estimated | Error  
<table>
<thead>
<tr>
<th>(d)</th>
<th>(Ω)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4%</td>
<td>0.0554</td>
<td>11.09%</td>
</tr>
<tr>
<td>4.2%</td>
<td>0.0542</td>
<td>8.70%</td>
</tr>
<tr>
<td>4.4%</td>
<td>0.0531</td>
<td>6.39%</td>
</tr>
<tr>
<td>4.6%</td>
<td>0.0520</td>
<td>4.18%</td>
</tr>
<tr>
<td>4.8%</td>
<td>0.0509</td>
<td>2.08%</td>
</tr>
<tr>
<td>5.2%</td>
<td>0.0490</td>
<td>-1.83%</td>
</tr>
<tr>
<td>5.4%</td>
<td>0.0481</td>
<td>-3.62%</td>
</tr>
<tr>
<td>5.6%</td>
<td>0.0473</td>
<td>-5.27%</td>
</tr>
<tr>
<td>5.8%</td>
<td>0.0465</td>
<td>-6.80%</td>
</tr>
<tr>
<td>6%</td>
<td>0.0459</td>
<td>-8.08%</td>
</tr>
</tbody>
</table>

Table 13 Impedance estimated with external unbalance

The results are as expected: the error of the estimation of the equivalent impedance depends on the external unbalance level.

### 3.4.4. Balancing the External Unbalance

Even though the external unbalance generates error in the equivalent impedance estimations, the estimation is accurate when the external unbalances are the same in all three phases. A similar case to Section 3.4.2 is studied here, except that in this case the unbalances of the parallel feeders balance each other out. The parameters of this case are given in Table 14.

<table>
<thead>
<tr>
<th>Phase</th>
<th>( E_{\text{th}} )</th>
<th>( Z_{\text{transmission}} )</th>
<th>( Z_{\text{load}}^1 )</th>
<th>( Z_{\text{load}}^2 )</th>
<th>( Z_{\text{load}}^3 )</th>
<th>( Z_{\text{load}}^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1</td>
<td>0.05i</td>
<td>4.2</td>
<td>4.2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>b</td>
<td>1</td>
<td>0.05i</td>
<td>4</td>
<td>4</td>
<td>4.2</td>
<td>4</td>
</tr>
<tr>
<td>c</td>
<td>1</td>
<td>0.05i</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4.2</td>
</tr>
</tbody>
</table>

Table 14 Parameters of the system of Figure 18 with symmetrical external unbalance

<table>
<thead>
<tr>
<th>Voltage (V)</th>
<th>Phase a</th>
<th>Phase b</th>
<th>Phase c</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.9986601 ( \angle -2.7944^\circ )</td>
<td>0.9986310 ( \angle -122.8284^\circ )</td>
<td>0.9986310 ( \angle 117.1716^\circ )</td>
</tr>
<tr>
<td>Current (A)</td>
<td>(0.2377762 \angle -2.7944^\circ)</td>
<td>(0.2496577 \angle -122.8284^\circ)</td>
<td>(0.2496577 \angle 114.1716^\circ)</td>
</tr>
</tbody>
</table>

**Table 15 Phase voltage and current of the system with symmetrical external unbalance**

| \(V_n (V)\) | \(1.9777e^{-4} \angle 84.3774^\circ\) |
| \(I_n (A)\) | \(0.004 \angle 176.4912^\circ\) |

**Table 16 Negative Sequence voltage and current of the system with symmetrical external unbalance**

\(Z_{th}\) is estimated to be \(0.0018 + 0.0499i \ (0.0499e^{j\omega 87.8861})\) which is close to the calculated value. In this case, there are external unbalances in the parallel feeders. However, the external unbalances are equal to each other. Therefore, there is no external unbalance from the perspective of Load 1 and the estimation is accurate.

### 3.5. Error Analysis of the External Unbalance Effects

As shown in section 3.4.2, the external unbalance causes errors in the estimation of the equivalent impedance. However, it is found that the error from the three phases tends to cancel each other. For example, assume there is a certain amount of error due to the external unbalance in phase a. The errors from the same amount of external unbalance from phase b and phase c are 120° and -120° apart.

\[
\varepsilon_{\text{externalA}} + \varepsilon_{\text{externalB}} + \varepsilon_{\text{externalC}} = 0
\]  

(3.5)

The following example is done to show the cancellation of the error. Using the example in Figure 18, three cases are simulated with external unbalances generated from phase a, phase b and phase c.

#### 3.5.1. Error from External Unbalance in Phase a

The example in Section 3.4.2 is reused here. \(Z_{th}\) is estimated to be \(0.0016 + 0.0404i \ (0.0404e^{j\omega 87.7571})\). Comparing the result with the calculated one, the error is -0.0003 - 0.0096i \( (0.0096e^{j\omega -91.746})\).

\[
Z_{\text{th} A} = 0.0016 + 0.0404i
\]
\[ \varepsilon_{\text{external}B} = -0.0003 - 0.0096i \]

### 3.5.2. Error from Unbalance in Phase b

The same simulation is run with the unbalance in the parallel feeder in phase b.

<table>
<thead>
<tr>
<th>Phase</th>
<th>(E_{\text{th}})</th>
<th>(Z_{\text{transmission}})</th>
<th>(Z_{\text{load}}^1)</th>
<th>(Z_{\text{load}}^2)</th>
<th>(Z_{\text{load}}^3)</th>
<th>(Z_{\text{load}}^4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1</td>
<td>0.05i</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>b</td>
<td>1</td>
<td>0.05i</td>
<td>4</td>
<td>4</td>
<td>4.16</td>
<td>4</td>
</tr>
<tr>
<td>c</td>
<td>1</td>
<td>0.05i</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4.16</td>
</tr>
</tbody>
</table>

Table 17 Parameters of the system with external unbalance in phase b

<table>
<thead>
<tr>
<th>Phase a</th>
<th>Phase b</th>
<th>Phase c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage (V)</td>
<td>0.9986601 (\angle -2.7944^\circ)</td>
<td>0.9986253 (\angle -122.8349^\circ)</td>
</tr>
<tr>
<td>Current (A)</td>
<td>0.2377762 (\angle -2.7944^\circ)</td>
<td>0.2496563 (\angle -122.8349^\circ)</td>
</tr>
</tbody>
</table>

Table 18 Phase voltage and current of the system with external unbalance in phase b

| \(V_n\) (V) | 2.1909e\(^{-4}\) \(\angle 75.774^\circ\) |
| \(I_n\) (A) | 0.004 \(\angle 176.4272^\circ\) |

Table 19 Negative Sequence voltage and current of the system with external unbalance in phase b

\[ Z_{\text{th}} \text{ is estimated to be } 0.0102 + 0.0545i (0.0554e^{\angle 79.3468^\circ}) \]. Comparing the result with the calculated one, the error is \(0.0084 + 0.0045i (0.0095e^{\angle 28.5004^\circ})\).

\[ Z_{\text{th,B}} = 0.0102 + 0.0545i \]

\[ \varepsilon_{\text{external}B} = 0.0084 + 0.0045i \]

### 3.5.3. Error from Unbalance in Phase c

The same simulation is run with the unbalance in the parallel feeder in phase b.

<table>
<thead>
<tr>
<th>Phase</th>
<th>(E_{\text{th}})</th>
<th>(Z_{\text{transmission}})</th>
<th>(Z_{\text{load}}^1)</th>
<th>(Z_{\text{load}}^2)</th>
<th>(Z_{\text{load}}^3)</th>
<th>(Z_{\text{load}}^4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1</td>
<td>0.05i</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>b</td>
<td>1</td>
<td>0.05i</td>
<td>4</td>
<td>4</td>
<td>4.2</td>
<td>4</td>
</tr>
<tr>
<td>c</td>
<td>1</td>
<td>0.05i</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4.16</td>
</tr>
</tbody>
</table>

Table 20 Parameters of the system with external unbalance in phase c

<table>
<thead>
<tr>
<th>Phase a</th>
<th>Phase b</th>
<th>Phase c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage (V)</td>
<td>0.9986601 (\angle -2.7944^\circ)</td>
<td>0.9986310 (\angle -122.8284^\circ)</td>
</tr>
<tr>
<td>Current (A)</td>
<td>0.2377762 (\angle -2.7944^\circ)</td>
<td>0.2496577 (\angle -122.8284^\circ)</td>
</tr>
</tbody>
</table>
Table 21 Phase voltage and current of the system with external unbalance in phase c

<table>
<thead>
<tr>
<th>$V_n$ (V)</th>
<th>$I_n$ (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1919e-4</td>
<td>0.004</td>
</tr>
</tbody>
</table>

Table 22 Negative Sequence voltage and current of the system with external unbalance in phase c

$Z_{th}$ is estimated to be $-0.0063 + 0.0549i$ ($0.0552e^{|\omega|96.5429}$). Comparing the result with the calculated one, the error is $-0.0082 + 0.0049i$ ($0.0095e^{|\omega|148.8374}$).

$$Z_{th\text{evc}} = -0.0063 + 0.0549i$$

$$\varepsilon_{th\text{evc}} = -0.0082 + 0.049i$$

3.5.4. Averaging the Error Components

A summary of the estimations and the errors of the equivalent impedance are given in Table 23 and Table 24.

<table>
<thead>
<tr>
<th></th>
<th>$R_{th}$</th>
<th>$X_{th}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unbalance in phase a</td>
<td>0.0016</td>
<td>0.0404</td>
</tr>
<tr>
<td>Unbalance in phase b</td>
<td>0.0102</td>
<td>0.0545</td>
</tr>
<tr>
<td>Unbalance in phase c</td>
<td>-0.0063</td>
<td>0.0549</td>
</tr>
<tr>
<td>Average $Z_{th}$</td>
<td>0.001833</td>
<td>0.049933</td>
</tr>
</tbody>
</table>

Table 23 Estimated impedance

<table>
<thead>
<tr>
<th></th>
<th>Error $R_{th}$</th>
<th>Error $X_{th}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unbalance in phase a</td>
<td>-0.0003</td>
<td>-0.0096</td>
</tr>
<tr>
<td>Unbalance in phase b</td>
<td>0.0084</td>
<td>0.0045</td>
</tr>
<tr>
<td>Unbalance in phase c</td>
<td>-0.0082</td>
<td>0.0049</td>
</tr>
<tr>
<td>Sum of the error</td>
<td>-0.0001</td>
<td>-0.0002</td>
</tr>
</tbody>
</table>

Table 24 Errors of the estimated impedance

The average of the three estimations is $0.001833+0.049933i$ ($0.0499e^{|\omega|87.90}$) and it is close to the calculated value $0.00187+0.0499i$ ($0.0499e^{|\omega|87.85}$). The magnitudes of the error from the three phases are very close and they are 120° apart. As a result, the sum of them is close to zero and is negligible. From this result, it can be shown that the error from external unbalance from all three phases cancel each other.
3.5.5. Application

Error averaging of the external unbalance can be useful for improving the accuracy of the estimation. In power systems, external unbalance is unavoidable due to the unbalance of parallel feeders. However, external unbalances are caused by many feeders. Therefore, the time average of them should be quite balanced in all three phases. Therefore, the error could be balanced out if series of estimations is done continuously.

3.6. Balancing Internal Unbalances to Compensate External Unbalances Errors

Given a certain amount of external unbalance, the error of the internal unbalance also tends to cancel out as in the case of the external unbalance. Simulations were run with the circuit in Figure 18 and the results are shown in the following sections. In the following cases, there are external unbalances in the parallel feeders. Then, three estimations are made with internal unbalance added to one phase at a time.

3.6.1. Internal Unbalance in Phase a

First, internal unbalance is added to phase a. Parameters are given in Table 25 and results are given in Table 26 and Table 27.

<table>
<thead>
<tr>
<th>Phase</th>
<th>( E_{th} )</th>
<th>( Z_{transmission} )</th>
<th>( Z_{load}^1 )</th>
<th>( Z_{load}^2 )</th>
<th>( Z_{load}^3 )</th>
<th>( Z_{load}^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>0.05i</td>
<td>4.2</td>
<td>4.21</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>0.05i</td>
<td>4</td>
<td>4.15</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>0.05i</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4.22</td>
</tr>
</tbody>
</table>

Table 25 Parameters of the system with external unbalance and internal unbalance in phase a

<table>
<thead>
<tr>
<th>Phase</th>
<th>Phase a</th>
<th>Phase b</th>
<th>Phase c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage (V)</td>
<td>0.9986615∠2.7927°</td>
<td>0.9986239∠122.8366°</td>
<td>0.9986338∠117.1748°</td>
</tr>
<tr>
<td>Current (A)</td>
<td>0.2377765∠2.7927°</td>
<td>0.2496560∠122.8366°</td>
<td>0.2496584∠114.1748°</td>
</tr>
</tbody>
</table>

Table 26 Phase voltage and current of the system with external unbalance and internal unbalance in phase a
Table 27 Negative Sequence voltage and current of the system external unbalance and internal unbalance in phase a

\[ V_n (V) = 2.2944 \times 10^{-4} \quad \omega = 73.4733 \]
\[ I_n (A) = 0.004 \quad \omega = 176.4119 \]

\[ Z_{th} \text{ is estimated to be } 0.0165 + 0.0557i \quad (0.0581e^{i\omega_{73.4733}}). \]
Comparing the result with the calculated one, the error is \[ 0.0147 + 0.0058i \quad (0.0158e^{i\omega_{21.7085}}). \]

3.6.2. Internal Unbalance in Phase b

Internal unbalance is added to phase b. Parameters are given in Table 28 and results are given in Table 29 and Table 30.

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Phase} & E_{th} & Z_{transmission} & Z_{load}^1 & Z_{load}^2 & Z_{load}^3 \\
\hline
A & 1 & 0.05i & 4 & 4.21 & 4 \\
B & 1 & 0.05i & 4.2 & 4 & 4.15 \\
C & 1 & 0.05i & 4 & 4 & 4.22 \\
\hline
\end{array}
\]

Table 28 Parameters of the system with external unbalance and internal unbalance in phase b

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Phase} & \text{Voltage (V)} & \text{Current (A)} \\
\hline
\text{a} & 0.9986324 \angle -2.8268^\circ & 0.2496581 \angle -2.8268^\circ \\
\text{b} & 0.986531 \angle -122.8026^\circ & 0.2377746 \angle -122.8026^\circ \\
\text{c} & 0.9986338 \angle 117.1748^\circ & 0.2496584 \angle 114.1748^\circ \\
\hline
\end{array}
\]

Table 29 Phase voltage and current of the system with external unbalance and internal unbalance in phase b

\[ V_n (V) = 1.3636 \times 10^{-4} \quad \omega = 152.2330 \]
\[ I_n (A) = 0.004 \quad \omega = -63.2952 \]

Table 30 Negative Sequence voltage and current of the system with external unbalance and internal unbalance in phase b

\[ Z_{th} \text{ is estimated to be } -0.0006 + 0.0344i \quad (0.0344e^{i\omega_{152.2330}}). \]
Comparing the result with the calculated one, the error is \[ -0.0025 - 0.0155i \quad (0.0157e^{i\omega_{-99.1906}}). \]

3.6.3. Internal Unbalance in Phase c

Internal unbalance is added to phase c. Parameters are given in Table 31 and results are given in Table 32 and Table 33.

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Phase} & E_{th} & Z_{transmission} & Z_{load}^1 & Z_{load}^2 & Z_{load}^3 \\
\hline
A & 1 & 0.05i & 4 & 4.21 & 4 \\
\hline
\end{array}
\]
Table 31 Parameters of the system with external unbalance and internal unbalance in phase c

<table>
<thead>
<tr>
<th></th>
<th>Phase a</th>
<th>Phase b</th>
<th>Phase c</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>Voltage (V)</td>
<td>Current (A)</td>
<td>Voltage (V)</td>
</tr>
<tr>
<td></td>
<td>0.9986324∠-2.8268°</td>
<td>0.2496581∠-2.8268°</td>
<td>0.9986628∠117.2089°</td>
</tr>
</tbody>
</table>

Table 32 Phase voltage and current of the system with external unbalance and internal unbalance in phase c

<table>
<thead>
<tr>
<th></th>
<th>V_n (V)</th>
<th>I_n (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.4132e^-4∠-23.8103</td>
<td>0.004∠56.3495</td>
</tr>
</tbody>
</table>

Table 33 Negative Sequence voltage and current of the system with external unbalance and internal unbalance in phase C

Zth is estimated to be -0.0104 + 0.0599i (0.0607e^{i099.8401}). Comparing the result with the calculated one, the error is -0.0123 + 0.0100i (0.0158e^{i140.9163}).

3.6.4. Averaging Internal Unbalance Results

A summary of data is given in Table 34 and Table 35.

Table 34 Estimated Impedance

<table>
<thead>
<tr>
<th></th>
<th>Rth</th>
<th>Xth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unbalance in phase a</td>
<td>0.00165</td>
<td>0.0557</td>
</tr>
<tr>
<td>Unbalance in phase b</td>
<td>-0.0006</td>
<td>0.0344i</td>
</tr>
<tr>
<td>Unbalance in phase c</td>
<td>-0.0104</td>
<td>0.0599i</td>
</tr>
<tr>
<td>Average Z_th</td>
<td>0.0018</td>
<td>0.0500</td>
</tr>
</tbody>
</table>

Table 35 Error in estimated impedance

<table>
<thead>
<tr>
<th></th>
<th>Error Rth</th>
<th>Error Xth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unbalance in phase a</td>
<td>0.0147</td>
<td>0.0058</td>
</tr>
<tr>
<td>Unbalance in phase b</td>
<td>-0.0025</td>
<td>-0.0155</td>
</tr>
<tr>
<td>Unbalance in phase c</td>
<td>-0.0123</td>
<td>0.0100</td>
</tr>
<tr>
<td>Sum of the error</td>
<td>-8.9717e^{-003}</td>
<td>2.9067e^{-004}</td>
</tr>
</tbody>
</table>

The average of the three estimations is 0.0018+0.0500i (0.0500e^{i087.8922}) and it is close to the calculated value 0.00187+0.0499i (0.0499e^{i087.85}). The errors of all three of them are shown in Table 35. The errors have the same magnitude and are 120° apart. As a result, their sum is close to zero and is negligible. This is an important result to discover that the external unbalance error can be compensated by symmetrical internal unbalances.
3.6.5. Application

The external error compensation by symmetrical internal unbalance can be very useful for improving the accuracy of the estimation. Internal unbalance can be generated purposely in each phase before measurements are made. Then, the three results are averaged to cancel the external error. Some extra unbalances are needed to be injected to the system to apply this idea.

3.7. Field Test with Data from a 25kV Substation

This method in this work was applied to real system data from a 25kV substation to determine the feasibility of the application. Two sets of data were analyzed. First, steady-state measurements data for the voltages and currents are studied. Then, time waveforms of voltages and currents are studied.

3.7.1. Data Analysis with Steady-State

Steady-state voltage and current magnitudes for each phase were sequentially recorded. The data are taken every 15 minutes and a week of data was analyzed. The load impedance was found by dividing the positive sequence voltage by the positive sequence current and the equivalent Thevenin impedance was found by dividing the negative sequence voltage by the negative sequence current. The load impedance and the Thevenin impedance of one day are shown in Figure 19 and Figure 20. Figure 21 shows both functions on the same graph.
Figure 19 Load impedance from measurements

Figure 20 Thevenin impedance from measurements
From Figure 21, it can be observed that fluctuations of the magnitude of the load impedance are higher than those of the Thevenin impedance. This is expected since the load impedance changes continuously during the day. On the other hand, the Thevenin impedance depends on the transmission network impedance. The trend variations in the Thevenin impedance are probably caused by the operational changes in the transmission system, as for example, due to circuit breaker, tap changers and other voltage adjustment mechanisms. To a lesser degree they are also influenced by changes in the remote loads. The local fluctuations are probably caused by the errors in the method as discussed in Section 3.4. Another factor can be the noise in the measurements due to the finite accuracy of the current and voltage transformers.

3.7.1.a. Noise of Measurement

The estimation of the Thevenin impedance depends on the negative sequence. However, the magnitude of the negative sequence is small relative to the magnitude of the measurement. The unbalance level of the voltage and current can be found by Equation
3.6 and Equation 3.7. The unbalance of voltage and current of the data are calculated and shown in Figure 22. The unbalance of voltage and current is below 3% and 0.7%. However, the error of measurements of voltages and currents can be up to 5% from operation experience. Therefore, the negative sequence is sensitive to noise and the noise of measurements is one reason for the local oscillations.

\[
V_{unbalance} = \frac{V_{negative}}{V_{positive}} \tag{3.6}
\]

\[
I_{unbalance} = \frac{I_{negative}}{I_{positive}} \tag{3.7}
\]

![Figure 22 Unbalance levels of voltage and current](image)

**3.7.1.b. External Unbalance**

As mention in Section 3.4, the accuracy of the estimation is affected by the amount of the external unbalance. Also, the magnitude and phase of the error depends on the external unbalance. Figure 23 shows the difference between the load impedance of all phases and
their average value. Even though small, these deviations change relatively fast. As a result, the external unbalance change really fast and it can be the reason of the oscillation of the Thevenin impedance.

![Figure 23 Deviation of phases A, B, C load impedances with respect to their average value](image)

### 3.7.2. Average Thevenin Impedance

As shown in Figure 20, there are oscillations in the estimation of the Thevenin impedance. To smooth out the estimation, Figure 24 shows the average of the Thevenin impedance over 3 hour periods. There are two reasons that might cause the local oscillations, these are the noise of the measurement and the external unbalance. If the reason were the noise of the measurement, the oscillation caused by the noise tends to average out due to the nature of noise. If the reason were the external unbalance, the oscillations would also tend to average out as discussed in Section 3.5.
3.7.3. Analysis of Peak Load Data

To confirm the consistency of the method, an additional analysis was done at the same substation in another loading condition. The data in the peak load of the year was analyzed. The corresponding load impedance and Thevenin impedance for one peak day are shown in Figure 25 and Figure 26. It is observed that the load impedance is significantly lower for the peak load. The Thevenin system impedance has the same pattern during the day even though the value of the system impedance is somewhat lower in the peak load period than in the period of Figure 20. The lower Thevenin impedance is probably due to the action of voltage compensation devices, such as tap changers and the generators increased supply of reactive power.
Figure 25 Load impedance under peak load

Figure 26 Thevenin impedance under peak load
3.7.4. Analysis with Time Domain Waveform Data

The second set of data was a waveform recording of voltage and current sampled at 3840Hz so that there are 64 samples per cycle. Fourteen data cycles were analyzed. These data was measured at 9:50AM. The voltage and current waveforms are shown in Figure 28 and Figure 29.
Figure 28 Time waveform of the voltages

Figure 29 Time waveform of the currents
A DFT (Discrete Fourier Transform) algorithm was used to extract the 60Hz components of the voltages and currents. A 10 cycle window and 3846 point DFT were used to analyze the data. After the phase components were found, the transformation of Equation 2.22 was applied to find the sequential components. The positive sequence voltage and current are given in Figure 30 and Figure 32. The negative sequence voltage and current are given in Figure 31 and Figure 33. The load impedance was found by dividing the positive sequence voltage by the positive sequence current, and the equivalent Thevenin impedance was found by dividing the negative sequence voltage by the negative sequence current.

![Figure 30 The 60Hz component of the positive sequence voltage time waveform](image)
Figure 31 The 60Hz component of the negative sequence voltage time waveform

Figure 32 The 60Hz component of the positive sequence current time waveform
Figure 33 The 60Hz component of the negative sequence current time waveform

Figure 34 60Hz load impedance
Figure 35 60Hz Thevenin impedance

Figure 36 Comparison of load impedance and Thevenin impedance
As shown in Figure 34 and Figure 35, the load impedance is around 1.20 per unit and the equivalent impedance is around 0.24 per unit. The impedance values match well with the values from the phase measurements and are shown in Table 36.

<table>
<thead>
<tr>
<th></th>
<th>9:45 (Sequential Measurement)</th>
<th>9:50:03 (Waveform data)</th>
<th>10:00 (Sequential Measurement)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive Sequence</td>
<td>Positive Sequence Impedance (p.u.)</td>
<td>1.19</td>
<td>1.20</td>
</tr>
<tr>
<td>Negative Sequence</td>
<td>Negative Sequence Impedance (p.u.)</td>
<td>0.19</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Table 36 Impedance between 9:45 to 10:00

3.7.5. Design System Impedance

Using the design value of the system three phase short circuit capacity, the corresponding Thevenin impedance of the design system can be derived. The short circuit capacity (SCC) is the product of the open circuit voltage and the short circuit current.

\[
SCC = V_{oc} \times I_{sc}
\]  

(3.8)

If it is converted into per unit, the open circuit voltage is one and the short circuit capacity becomes the short circuit current.

\[
SCC(p.u) = I_{sc}(p.u)
\]

(3.9)

On the other hand, Thevenin impedance of the system in per unit is the inverse of the per unit short circuit current.

\[
Z_{th}(p.u) = V_{oc}(p.u)/I_{(SC)(p.u)} = 1/I_{(SC)(p.u)}
\]

(3.10)
The design system data is given as follows:

\[
V_{oc} = 25kV \quad (3.11)
\]
\[
I_{sc} = 15668.5 \angle -24.3A \quad (3.12)
\]
\[
SCC = \frac{V_{oc}}{\sqrt{3}} \times I_{sc} = 6.838MVA \quad (3.13)
\]
\[
SCC = \frac{V_{oc}}{\sqrt{3}} \times I_{sc} = 6.838MVA \quad (3.14)
\]
\[
I_{sc} (p.u.) = \frac{SCC (p.u.)}{SCC_{base}} = \frac{6.838}{SCC_{base}} \quad (3.15)
\]
\[
Z_{thev} (p.u.) = \frac{1}{I_{sc} (p.u.)} = 0.1462 \quad (3.16)
\]

Since the design value is taken for the worst short-circuit conditions, this number should be compared with the lowest value in the \( Z_{th} \) from the measured data. From Figure 26, the lowest observed value of \( Z_{th} \) is about 0.15 pu in the daily curve of the highest peak load in the year. This value of 0.15 pu matches exactly the \( Z_{thev} \) value calculated from the SCC in Equation 3.16.
4. Conclusion

As modern power systems operate under more stressed conditions, it is essential to monitor the voltage stability condition of the system. The condition of voltage stability can be monitored by comparing the Thevenin equivalent impedance and the load impedance. This thesis suggests a simple method to estimate the Thevenin equivalent impedance of a system in real time.

Taking advantage of the presence of negative sequence component, this method is a strong candidate to solve the voltage instability limit problem. It only needs one set of voltages and currents to estimate the Thevenin impedance. Therefore, it can monitor the system condition in real time and track the trajectory of the system condition. Moreover, the algorithm is really simple and it only needs the negative sequence voltage and current to evaluate the system Thevenin impedance. Also, this method only uses the magnitude components of the negative sequence voltage and current. As a result, it is very easy to implement and is a very cost effective solution. For example, the Schweitzer relay, which is widely used in the British Columbia power system, can be easily adapted to implement this method.

Even though there are some advantages, there are also drawbacks in the use of negative sequence components. Since the negative sequence components are relatively weak, the accuracy of the method is very sensitive to the unbalance of parallel loads and the noise of measurements. Fortunately, the errors are likely to be random and tend to cancel each other out. Also, if needed the accuracy of the measurement can be improved considerably by using optical transducers. Optical voltage transducers and current transducers can measure voltages and currents with error of 0.3% and 0.6% [18]. The new technology is more accurate, cheaper and easier to manage. Therefore, it is just a matter of time for the market to adapt the new technology.

In conclusion, this thesis presented a method to estimate the Thevenin equivalent impedance using negative sequence components. With this approach, the condition of voltage stability of a power system can be monitored by comparing the Thevenin
impedance and the load impedance. The method is derived mathematically. It is confirmed with simulations and field measurements.

Since the Thevenin equivalent impedance can be measured accurately, the next step of the research will focus on the operation scheme of the relay. For example, the critical distance between the Thevenin impedance and the load impedance has to be determined. Also, different types of load affect voltage stability in different ways depending on their dynamics. Therefore, the decisions of which type of load to shed and how much load to shed has to be studied to optimize the effectiveness of load shedding.
References


Appendix A Voltage Instability Predictor VIP

ABB suggested a method to measure the Thevenin equivalent of a power system. This method takes two sets of voltages and currents at different times to find the Thevenin equivalent. This method is illustrated in Figure 37,

\[ V_r, V_i, I_r \text{ and } I_i \text{ can be measured from the bus. Then, the unknowns can be found with measurements of two states,} \]

In this circuit,

\[
E_{th} = \overline{V} + Z_{th}I_L \quad (A.1)
\]

\[
E_r + jE_i = V_r + jV_i + (R_{th} + jZ_{th})(I_r + jI_i) \quad (A.2)
\]

Putting into matrix form of two linear equations and four unknowns,

\[
\begin{bmatrix}
V_r \\
V_i
\end{bmatrix}
= \begin{bmatrix}
E_r \\
E_i \\
R_{eq} \\
X_{eq}
\end{bmatrix}
\begin{bmatrix}
1 & 0 & -I_r & I_i \\
0 & 1 & -I_i & -I_r
\end{bmatrix}
\quad (A.3)
\]
By converting the matrix, the values of the Thevenin equivalent can be calculated, but there are two problems to calculate the Thevenin equivalent impedance with the Voltage Instability Predictor method. It assumes that the Thevenin equivalent voltage $E_{th}$ and the Thevenin equivalent impedance $Z_{th}$ are constant during the two sets of voltages and currents are measured. However, $E_{th}$ and $Z_{th}$ fluctuate; therefore, it is invalid to assume that $E_{th}$ and $Z_{th}$ are constant. The estimation of the Thevenin impedance is not accurate if the assumption of constant $E_{th}$ and $Z_{th}$ does not hold. This is illustrated by the following examples [19]. First, assuming two sets of load voltage and load current are measured when $E_{th}$ and $Z_{th}$ are constant.

\begin{align}
E_1 &= V_1 + Z_1 I_1 \\
E_2 &= V_2 + Z_2 I_2 \\
0 &= (V_1 - V_2) + Z_1 (I_1 - I_2) \\
Z_1 &= \frac{\Delta V}{\Delta I}
\end{align}

It shows that the Thevenin impedance can be estimated accurately if the $E_{th}$ and $Z_{th}$ are constant. In the next example, assuming there are changes in $E_{th}$ and $Z_{th}.$

\begin{align}
E_1 &= V_1 + Z_1 I_1
\end{align}
When \( E_{th} \) and \( Z_{th} \) are not constant, there is an error component in the estimated Thevenin impedance. As shown in Equation A.15, the error term is divided by the change of the load current; therefore, small changes in load current can give a large error in the estimated Thevenin impedance. As a result, the VIP method runs into a problem. It needs a time interval between the two measurements to be long enough for a change in the load. On the other hand, the time interval cannot be too long or else the \( Z_{th} \) and \( E_{th} \) will change.

\[
E_1 + \Delta E = V_2 + (Z_1 + \Delta Z)I_2
\]  
(A.11)

\[
\Delta E = \Delta V + Z_1 \Delta I + \Delta Z I_2
\]  
(A.12)

\[
Z_1 = -\frac{\Delta V}{\Delta I} + \frac{\Delta E}{\Delta I} - \frac{\Delta Z I_2}{\Delta I} = Z_{estimated} + \frac{\Delta E - \Delta Z I_2}{\Delta I}
\]  
(A.13)

\[
Z_{estimated} = Z_1 - \frac{\Delta E - \Delta Z I_2}{\Delta I}
\]  
(A.14)

\[
\varepsilon = -\frac{\Delta E - \Delta Z I_2}{\Delta I}
\]  
(A.15)
Appendix B 14 Buses Power Flow Case Data

08/19/93 UW ARCHIVE           100.0  1962 W IEEE 14 Bus Test Case

BUS DATA FOLLOWS                         14 ITEMS

1 Bus 1     HV  1  1  2 1.060  0.0      0.0  0.0  0.0  232.4 -16.9     0.0    0.0     0.0    0.0        0
2 Bus 2     HV  1  2 1.045 -4.98     21.7  12.7  40.0  42.4  0.0  1.045  50.0    0.0     0.0    0.0        0
3 Bus 3     HV  1  2 1.010 -12.72    94.2   19.0  0.0  23.4  0.0  1.010  40.0     0.0    0.0    0.0        0
4 Bus 4     HV  1  1 1.019 -10.33   12.4  16.4  0.0  0.0  0.0  0.0       0.0     0.0    0.0    0.0        0
5 Bus 5     HV  1  1 1.020  -8.78     9.0   1.6  0.0  0.0  0.0  0.0       0.0     0.0    0.0    0.0        0
6 Bus 6     LV  1  2 1.070 -14.22    1.2  11.2  0.0  0.0  0.0  0.0       0.0     0.0    0.0    0.0        0
7 Bus 7     ZV  1  1 1.062 -13.37    0.0   0.0  0.0  0.0  0.0  0.0       0.0     0.0    0.0    0.0        0
8 Bus 8     TV  1  2 1.090 -13.36    0.0   0.0  0.0  0.0  0.0  0.0       0.0     0.0    0.0    0.0        0
9 Bus 9     LV  1  1 1.056 -14.94    9.0   16.6  0.0  0.0  0.0  0.0       0.0     0.0    0.0    0.0        0
10 Bus 10    LV  1  1 1.051 -15.10    8.0   15.8  0.0  0.0  0.0  0.0       0.0     0.0    0.0    0.0        0
11 Bus 11    LV  1  3.0 1.057 -14.79  13.5   1.8  0.0  0.0  0.0  0.0       0.0     0.0    0.0    0.0        0
12 Bus 12    LV  1  1 1.055 -15.07    8.0   1.6  0.0  0.0  0.0  0.0       0.0     0.0    0.0    0.0        0
13 Bus 13    LV  1  1 1.050 -15.16    8.0   13.5  0.0  0.0  0.0  0.0       0.0     0.0    0.0    0.0        0
14 Bus 14    LV  1  1 1.036 -16.04    8.0   14.9  0.0  0.0  0.0  0.0       0.0     0.0    0.0    0.0        0

BRANCH DATA FOLLOWS                         20 ITEMS

INTERCHANGE DATA FOLLOWS                 1 ITEMS

TIE LINES FOLLOWS                     0 ITEMS

END OF DATA
Appendix C Power Flow Data Format

Partial Description of the IEEE Common Data Format for the Exchange of Solved Load Flow Data


The data file has lines of up to 128 characters. The lines are grouped into sections with section headers. Data items are entered in specific columns. No blank items are allowed, enter zeros instead. Floating point items should have explicit decimal point. No implicit decimal points are used.

Data type codes: A - Alphanumeric (no special characters)
I - Integer
F - Floating point
* - Mandatory item

Title Data
-----------

First card in file.

Columns 2-9 Date, in format DD/MM/YY with leading zeros. If no date provided, use 0b/0b/0b where b is blank.

Columns 11-30 Originator's name (A)
Columns 32-37 MVA Base (F*)
Columns 39-42 Year (I)
Column 44 Season (S - Summer, W - Winter)
Column 46-73 Case identification (A)

Bus Data *
-----------

Section start card *:
---------------------

Columns 1-16 BUS DATA FOLLOWS (not clear that any more than BUS in 1-3 is significant) *

Columns ?-? NNNNN ITEMS (column not clear, I would not count on this)

Bus data cards *:
-----------------

Columns 1-4 Bus number (I) *
Columns 7-17 Name (A) (left justify) *
Columns 19-20 Load flow area number (I) Don't use zero! *
Columns 21-23 Loss zone number (I)
Columns 25-26 Type (I) *
  0 - Unregulated (load, PQ)
  1 - Hold MVAR generation within voltage limits, (PQ)
  2 - Hold voltage within VAR limits (gen, PV)
  3 - Hold voltage and angle (swing, V-Theta) (must always have one)
Columns 28-33 Final voltage, p.u. (F) *
Columns 34-40 Final angle, degrees (F) *
Columns 41-49 Load MW (F) *
Columns 50-59 Load MVAR (F) *
| Columns 60-67 | Generation MW (F) * |
| Columns 68-75 | Generation MVAR (F) * |
| Columns 77-83 | Base KV (F) |
| Columns 85-90 | Desired volts (pu) (F) (This is desired remote voltage if this bus is controlling another bus.) |
| Columns 91-98 | Maximum MVAR or voltage limit (F) |
| Columns 99-106 | Minimum MVAR or voltage limit (F) |
| Columns 107-114 | Shunt conductance G (per unit) (F) * |
| Columns 115-122 | Shunt susceptance B (per unit) (F) * |
| Columns 124-127 | Remote controlled bus number |

Section end card:
-----------------

Columns 1- 4   -999

Branch Data *
-------------

Section start card *:
---------------------

Columns 1-16   BRANCH DATA FOLLOWS (not clear that any more than BRANCH is significant) *

Columns 40?- ? NNNNN ITEMS (column not clear, I would not count on this)

Branch data cards *:
---------------------

Columns 1- 4   Tap bus number (I) *
For transformers or phase shifters, the side of the model the non-unity tap is on

Columns 6- 9   Z bus number (I) *
For transformers and phase shifters, the side of the model the device impedance is on.

Columns 11-12  Load flow area (I)
Columns 13-14  Loss zone (I)
Column 17    Circuit (I) * (Use 1 for single lines)
Column 19    Type (I) *
  0 - Transmission line
  1 - Fixed tap
  2 - Variable tap for voltage control (TCUL, LTC)
  3 - Variable tap (turns ratio) for MVAR control
  4 - Variable phase angle for MW control (phase shifter)

Columns 20-29  Branch resistance R, per unit (F) *
Columns 30-40  Branch reactance X, per unit (F) * No zero impedance lines
Columns 41-50  Line charging B, per unit (F) * (total line charging, +B)
Columns 51-55  Line MVA rating No 1 (I) Left justify!
Columns 57-61  Line MVA rating No 2 (I) Left justify!
Columns 63-67  Line MVA rating No 3 (I) Left justify!
Columns 69-72  Control bus number
Column 74    Side (I)
  0 - Controlled bus is one of the terminals
  1 - Controlled bus is near the tap side
  2 - Controlled bus is near the impedance side (Z bus)

Columns 77-82  Transformer final turns ratio (F)
Columns 84-90  Transformer (phase shifter) final angle (F)
Columns 91-97  Minimum tap or phase shift (F)
Columns 98-104  Maximum tap or phase shift (F)
Columns 106-111  Step size (F)
Columns 113-119  Minimum voltage, MVAR or MW limit (F)
Columns 120-126  Maximum voltage, MVAR or MW limit (F)

Section end card:
-----------------

Columns 1- 4   -999

Loss Zone Data
--------------
Section start card
-------------------

Columns 1-16 LOSS ZONES FOLLOWS (not clear that any more than LOSS is significant)

Columns 40?-? NNNNNN ITEMS (column not clear, I would not count on this)

Loss Zone Cards:
----------------

Columns 1-3 Loss zone number (I)
Columns 5-16 Loss zone name (A)

Section end card:
-----------------

Columns 1-3 -99

Interchange Data *
-------------------

Section start card
-------------------

Columns 1-16 INTERCHANGE DATA FOLLOWS (not clear that any more than first word is significant).

Columns 40?-? NNNNNN ITEMS (column not clear, I would not count on this)

Interchange Data Cards *:
--------------------------

Columns 1-2 Area number (I) no zeros! *
Columns 4-7 Interchange slack bus number (I) *
Columns 9-20 Alternate swing bus name (A)
Columns 21-28 Area interchange export, MW (F) (+ = out) *
Columns 30-35 Area interchange tolerance, MW (F) *
Columns 38-43 Area code (abbreviated name) (A) *
Columns 46-75 Area name (A)

Section end card:
-----------------

Columns 1-2 -9

Tie Line Data
-------------

Section start card
-------------------

Columns 1-16 TIE LINES FOLLOW (not clear that any more than TIE is significant)

Columns 40?-? NNNNNN ITEMS (column not clear, I would not count on this)

Tie Line Cards:
---------------

Columns 1-4 Metered bus number (I)
Columns 7-8 Metered area number (I)
Columns 11-14 Non-metered bus number (I)
Columns 17-18 Non-metered area number (I)
Column 21 Circuit number

Section end card:
-----------------

Columns 1-3 -999
Appendix D Simulation Code of the 14 Bus Power Flow Case

* +--------------------------------------------------------------------+
* | Input file for Microtran |
* | Automatically Generated by Microtran Schematics |
* | Copyright (c) 1985-2000 Microtran Corporation |
* +--------------------------------------------------------------------+

* . . . . . . Case Identification Card *
* File information: *
* Data file name: C:\mt310\dat\untitled.dat *
* Schematic file name: c:\mt310\sch\untitled.sch *
* Date of creation of Data file: 01-29-2008; time: 15:33:47 *

Simulation 02 1 60 60 *
* . . . . . . . . . . Time card *
50.0E-6 -.1 5 100 0 *
* . . . . . . Lumped RLC branch *
* between 1 and 2 *
11a 2a .0194 .0591 .0528 0
11b 2b .0194 .0591 .0528 0
11c 2c .0194 .0591 .0528 0 *
* between 1 and 5 *
11a 5a .0540 .2230 .0492 0
11b 5b .0540 .2230 .0492 0
11c 5c .0540 .2230 .0492 0 *
* between 2 and 3 *
12a 3a .0470 .1980 .0438 0
12b 3b .0470 .1980 .0438 0
12c 3c .0470 .1980 .0438 0 *
* between 2 and 4 *
12a 4a .0581 .1763 .0340 0
12b 4b .0581 .1763 .0340 0
12c 4c .0581 .1763 .0340 0 *
* between 2 and 5 *
12a 5a .0570 .1739 .0346 0
12b 5b .0570 .1739 .0346 0
12c 5c .0570 .1739 .0346 0 *
* Load of 2 *
0 2a 3.748 2.194 0
0 2b 3.748 2.194 0
0 2c 3.748 2.194 0 *
* between 3 and 4 *
13a 4a .0670 .1710 .0128 0
13b 4b .0670 .1710 .0128 0
13c 4c .0670 .1710 .0128 0 *
* Load of 3 *
0 3a 1.041 .2098 0
0 3b 1.041 .2098 0
0 3c 1.041 .2098 0 *
* between 4 and 5 *
0 4a 5a .0134 .04211 0
0 4b 5b .0134 .04211 0
0 4c 5c .0134 .04211 0 *
* between 4 and 4T
0 4a 4T1a  0  .10456  0
0 4b 4T1b  0  .10456  0
0 4c 4T1c  0  .10456  0
* between 4T and 7T
184T1a  7Ta  .978  0
184T1b  7Tb  .978  0
184T1c  7Tc  .978  0
* between 4 and 4T2
0 4a 4T2a  0  .27809  0
0 4b 4T2b  0  .27809  0
0 4c 4T2c  0  .27809  0
* between 4T2 and 9T
184T2a  9Ta  .969  0
184T2b  9Tb  .969  0
184T2c  9Tc  .969  0
* Load of 4
0 4a  2.158  -.1761  0
0 4b  2.158  -.1761  0
0 4c  2.158  -.1761  0
* between 5 and 5T
0 5a 5Ta  0  .12601  0
0 5b 5Tb  0  .12601  0
0 5c 5Tc  0  .12601  0
* between 5T and 6T
185Ta  6Ta  .932  0
185Tb  6Tb  .932  0
185Tc  6Tc  .932  0
* Load of 5
0 5a  13.11  2.759  0
0 5b  13.11  2.759  0
0 5c  13.11  2.759  0
* between 6T and 6
0 6a 6Ta  0  .12601  0
0 6b 6Tb  0  .12601  0
0 6c 6Tc  0  .12601  0
* between 6 and 11
0 6a 11a  .0950  .19890  0
0 6b 11b  .0950  .19890  0
0 6c 11c  .0950  .19890  0
* between 6 and 12
0 6a 12a  .1229  .25581  0
0 6b 12b  .1229  .25581  0
0 6c 12c  .1229  .25581  0
* between 6 and 13
0 6a 13a  .0662  .13027  0
0 6b 13b  .0662  .13027  0
0 6c 13c  .0662  .13027  0
* Load of 6
0 6a  7.058  4.726  0
0 6b  7.058  4.726  0
0 6c  7.058  4.726  0
* between 7T and 7
0 7Ta  7a  0  .10456  0
0 7Tb  7b  0  .10456  0
0 7Tc  7c  0  .10456  0
* between 7 and 8
0 7a  8a  0  .17615  0
between 7 and 9
0 7a  9a  0 .11001  0
0 7b  9b  0 .11001  0
0 7c  9c  0 .11001  0
*

between 9T and 9
0 9Ta  9a  0 .27809  0
0 9Tb  9b  0 .27809  0
0 9Tc  9c  0 .27809  0
*

between 9 and 10
0 9a  10a  .0318 .08450  0
0 9b  10b  .0318 .08450  0
0 9c  10c  .0318 .08450  0
*

between 9 and 14
0 9a  14a  .1271 .27038  0
0 9b  14b  .1271 .27038  0
0 9c  14c  .1271 .27038  0
*

Load of 9
0 9a  2.871 1.6155  0
0 9b  2.871 1.6155  0
0 9c  2.871 1.6155  0
*

between 10 and 11
0 10a  11a  .0821 .19207  0
0 10b  11b  .0821 .19207  0
0 10c  11c  .0821 .19207  0
*

Load of 10
0 10a  8.672 5.589  0
0 10b  8.672 5.589  0
0 10c  8.672 5.589  0
*

Load of 11
0 11a  25.24 12.980  0
0 11b  25.24 12.980  0
0 11c  25.24 12.980  0
*

between 12 and 13
0 12a  13a  .2209 .19988  0
0 12b  13b  .2209 .19988  0
0 12c  13c  .2209 .19988  0
*

Load of 12
0 12a  17.07 4.478  0
0 12b  17.07 4.478  0
0 12c  17.07 4.478  0
*

between 13 and 14
0 13a  14a  .1709 .34802  0
0 13b  14b  .1709 .34802  0
0 13c  14c  .1709 .34802  0
*

Load of 13
0 13a  6.894 2.962  0
0 13b  6.894 2.962  0
0 13c  6.894 2.962  0
*

Load of 14
0 14a  6.474 2.173  0
0 14b  6.474 2.173  0
0 14c  6.474 2.173  0
*

$  =  End of level 1: Linear and nonlinear elements ...

* . . . . . . . Switches and piecewise linear elements

68
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$ ----- End of level 3: Sources$  
*   *   *   *   *   *   * Voltage-output nodes$  
1$  
$ ----- End of level 4: User-defined voltage output$  
$ ----- Level 5: End of data case$  

69