MAGIC CARPET:

DIGITAL INTERPRETATION OF TRADITIONAL TESSELLATION PATTERNS

by

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Abstract

Contemporary architecture has failed to engage the rich culture of planar and spatial transformations of historical Muslim architecture, often relegating it to a form of naive pastiche or, at best, to the realm of historical reconstruction. In this project we make use of current digital technologies in an attempt to revisit and reinterpret, in modern terms, the geometric structure of patterns embedded in the historic Islamic architecture of Iran. The original contribution of this project lies in extending traditional two dimensional tiling patterns into a dynamic three dimensional state with the help of computational tools. The analogy to the classical Persian carpet as well as mobile character of design can also be seen as original. The notion of 'transparency' and 'dynamism' are interpreted using Autodesk's Maya and Bentley's Generative Components software. This report illustrates initial explorations and outlines future possibilities.

In the past architects of the country were responsible for making the enclosure heaven-like while it was carpet weavers' job to make the floor heaven-like. In this project as a symbolic approach, carpet and weaving becomes both the enclosure and the floor to define both floor and roof and symbolize the new approach through which we as architects use other disciplines and new tools such as new software to learn and shape the space and discover new vocabulary for a contemporary and local architecture for Iran.

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Note: Figures are all original work of the author unless acknowledged otherwise.

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Dedications

I dedicate this work to

The best that I have got

In this world;

My entire family

CHAPTER 1: OVERVIEW AND THEMES

Overview

In Iran, during the last century especially, the traditional processes of architecture were mostly abandoned due to the absence of a conversation between modern and traditional approaches powerful enough to create a new direction for local architecture. Both the 'modern' and the 'traditional' discourses have been judged by and received attributes from each other which have in turn made each of them more and more distant from the other. If we personify the 'modern' and 'traditional' discourses, we could observe that they have not been listening to each other, but rather listening to their own agenda about the other and as a result the 'traditional' has not continued its natural gradual development in the architectural realm. Now, the 'modern', for the most part, is present, as a naive replacement across Iran. Nevertheless, the 'traditional' has still remained powerfully alive and has evolved in other aspects of Iranian life, such as music, Persian-carpets and Kilims, poetry, calligraphy etc. and in contrast with the architectural realm it has found ways to have a more constructive dialogue with the 'modern'. Within the areas in which the 'traditional' has been able to continue generating a powerful discourse, the one area that we find that might be the closest to architecture, being related in materiality, geometry, color and also in being a craft, is the art of the Persian-carpet and Kilims.

This project is intended to symbolize the possibility of a new path for investigation in Iranian architecture. The path itself brings the 'contemporary traditional' from other disciplines or realms in which the 'traditional' continued generating the discourse, as with Persian Kilims, into the architectural realm materializing those secondary traditional interpretations in architectural form or process thereby creating a local foundation for a future development.

This way, even if we could not always develop and create a piece of architecture which is local and at the same time inspiring from within the architectural discourse, we can still adjust our approaches with the new developments in the source disciplines, like Persian Carpet or Calligraphy until we again have a powerful local architectural vocabulary in which tradition is alive and real. In other words we use those other disciplines where traditional discourse has been able to evolve, generate and inspire as therapy. In life an injured person whose legs have disabilities and can't walk properly would continue going to physiotherapy till he again gets back his competency in walking. Here the therapy clinic is other disciplines like Persian Carpet and Calligraphy from which we can learn how possibly traditional conversation can evolve, generate results and inspire. We as architects continue looking till we again learn walking with our own (local) contemporary architectural feet rather than a couple modern, western feet which never

quite took us to where we wanted with the architecture of Iran, something like: 'a local contemporary Iranian sense of place'.

Traditional conversation extends itself from the past, not to so much bring forward a literal tradition but to shape a locally contextual future, inside which people can be proud of who they are as a nation and be empowered in their own Iranian physical space which they and their body relates to and has more workability with.

Figure 1: *Flying Carpet by Victor Vasnetsov, 1880.*

The 'Magic carpet' is a vehicle from the folklore of the past. We do not initially want it for its own sake; rather it is a vehicle which allows for a distinct experience during the trip. It does not need fossil fuel; it is quick and intentional and has a kind of intelligence to get guidance from the driver. It can get to the destination in a split second. It is a beautiful piece of cloth, it is a surface that we can stand or sit on just as with normal carpets. (See fig.1)

The work which will be described in this report uses the symbolic character of a magic carpet. The final presented design is a colorful, retractable, woven, carpet-like sunshade flying above a pedestrian access into a significant piece of Iran's heritage - Maydan Naghshe Jahan, in Isfahan, and defines the entrance to Research Center for Islamic Arts.

Persian Carpets and also Persian Kilims, well-known worldwide, are among the few traditional crafts of the country that have continued to evolve and have become vibrant today in a way that grand Iranian architecture never has. The intent is to use live traditional crafts like Persian

carpets as a means or vehicle to inspire and enliven again a grand local yet contemporary Iranian architecture. In other words they become the vehicles which symbolize this act of extending the tradition to create and define the future.

As we mentioned before, some parts of Iranian identity are not lost, like Persian Carpets, and we can again weave the rest of our cultural identity, such as our local architecture, in a way that once again has integrity and workability and that, breathes magic. Placing the design on the entrance of the Research Center for Islamic Arts pronounces the idea and will inspire those who are involved in the same kind of research, investigating Islamic Arts, to create something equally new and magical. In this inquiry we first take a brief look at some aspects of traditional Iranian Architecture that we can bring into our work and demonstrate the possibility of revisiting the embedded characteristics of Islamic architecture in Iran with the use of new digital technology. We believe that those embedded characteristics did not evolve through practice or at least were not re-interpreted in contemporary architectural practice in Iran in an inspiring way and thus were therefore lost from the architectural scene.

Digital 3-D Software with programming capabilities helped us to look at the underlying principles of traditional architecture in a different and contemporary way, offering new methods and tools to create new avenues for local design solutions. The aspects of traditional Iranian Architecture which we chose to work with, in our explorations are:

The issue of Color in Iranian Architecture and cityscape

In the traditional architecture of the cities in Iran, as part of the Muslim world, there are some elaborate and colorful tiled surfaces set as jewels against the silent adobe background of the urban fabric. In the past, whenever people had enough wealth they would add to those surfaces, following precise geometric rules trying to make the city even more heaven-like (See fig.2).

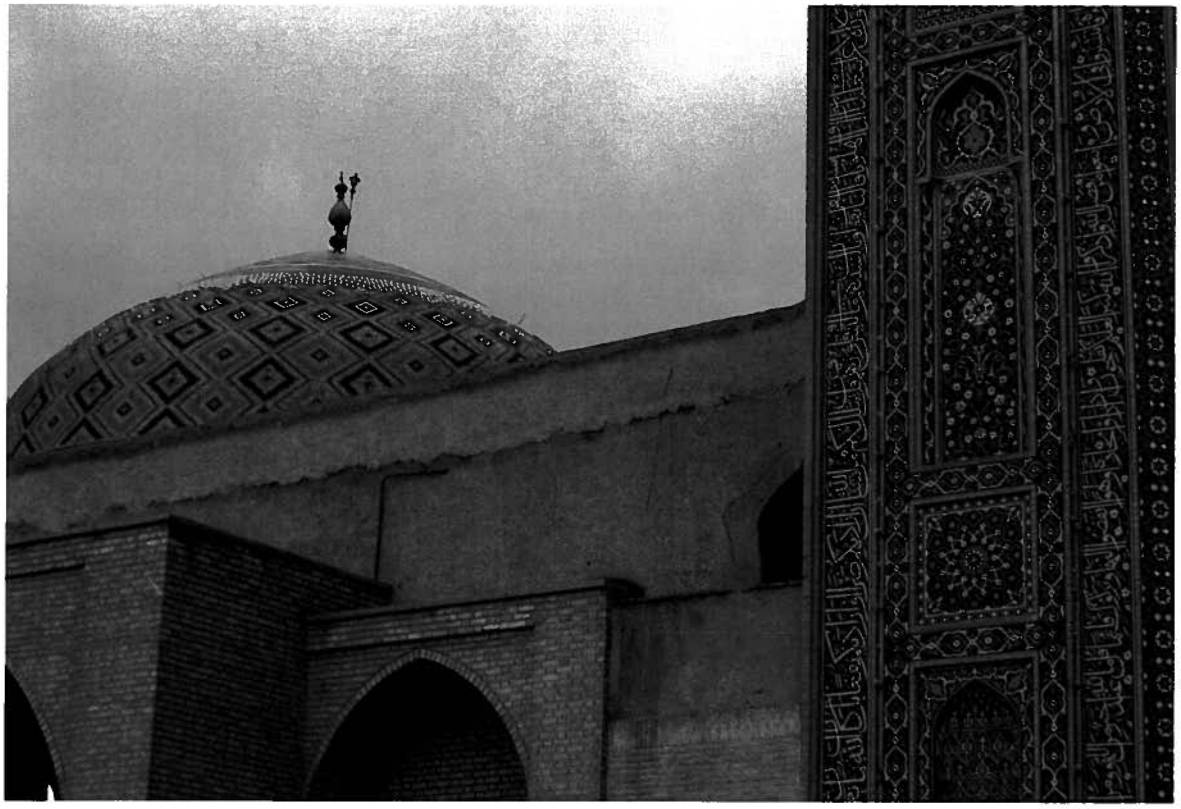

Figure 2: *Tiled Surfaces against the adobe background; Masjid Jame, Yazd, Iran; © Mehdi Hashemi, adaptation is by permission of copyright holder: Mehdi Hashemi.*

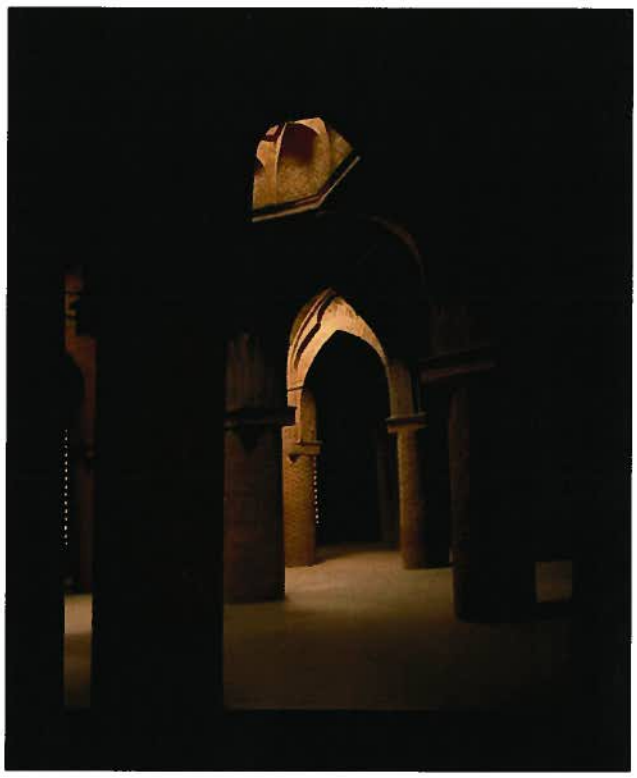

Figure 3: *Natural lighting condition in Masjid Jāmé, Isfahan.*

The issue of Light in Iranian Architecture

In places like old mosques, palaces or even old houses, depending on their age and in what era they were built one will find special controlled lighting qualities that are essential parts of the vocabulary of the space. These lighting features are not just added on top, they might even be seen as the most important inseparable part of the vocabulary of the space in certain spaces and certain buildings. The way the roof and wall openings are shaped to direct and spread the daylight inwards and the way in which the light is further controlled by colorful translucent stone sheets or glass in some cases is an essential part of Iranian Architecture especially during the Islamic period. Relating to the issue of light in traditional Architecture of Iran, in this project we will examine transparency and translucency, light, shadow and color and explore the ways in which these can be incorporated in our design (See fig.3).

Architecture and Carpets in the traditional architecture of Iran

The Persian carpet is one of the most alive of the traditional crafts of Iran and is a craft that is well known throughout the world. Many people in villages and cities are still involved in its production. Persian Carpet Design is so prevalent as an art that one can study and acquire a university degree in it.

Traditionally, Architects were responsible for the design of the enclosure and the ceilings of rooms (where they deployed all of those three dimensional Muqarnas geometries to make them heaven-like), while the carpet weavers were responsible for making the floors heaven-like and similar to the way heaven was described in Qur`an, the holy book of Muslims. Architects were used to designing rooms based on the standard sizes of carpets. In this project the carpet is 'flying', representing a living tradition by inversely defining the ceiling and taking care of the whole space (See fig.4).

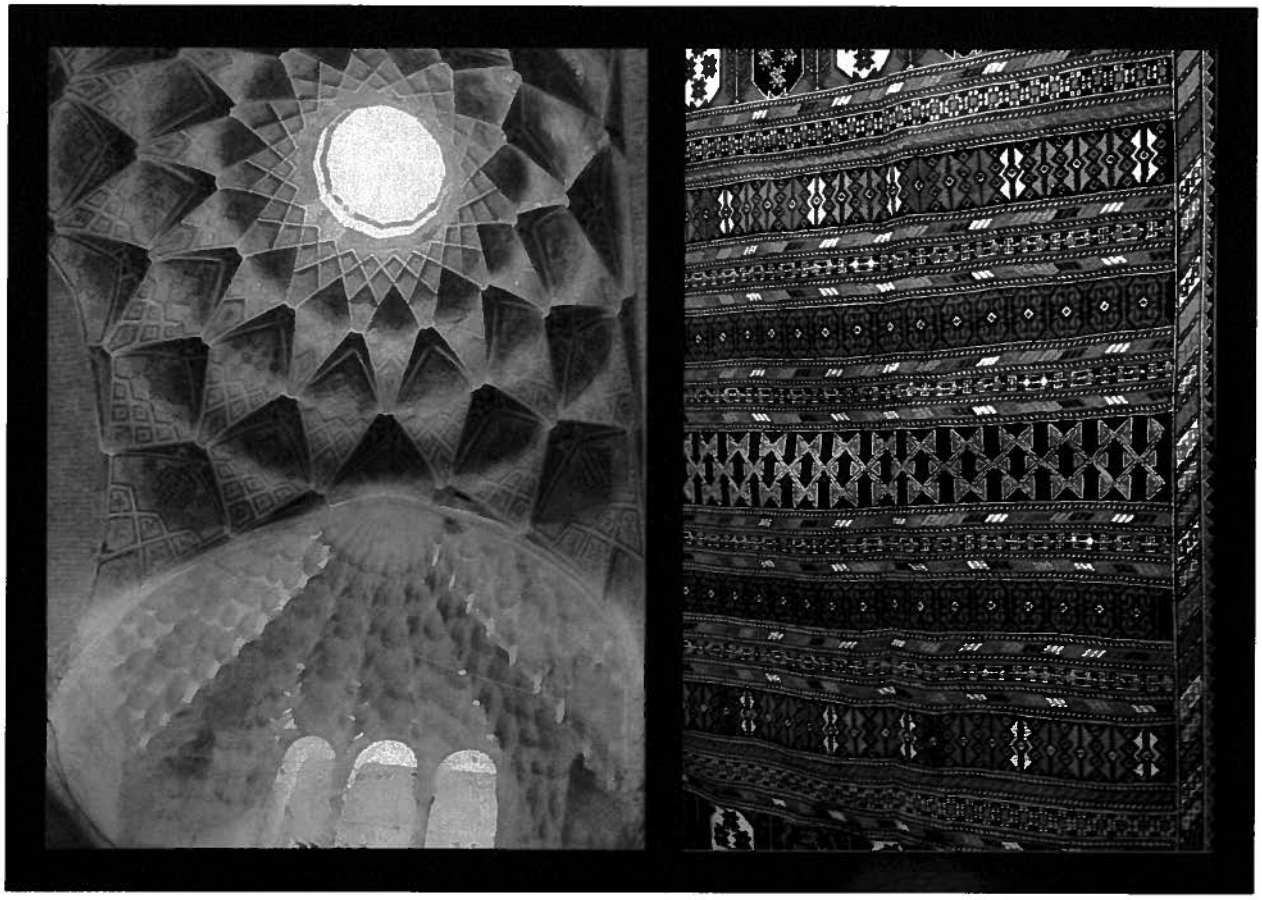

Figure 4: *Left: Iranian Architect making the enclosure heaven-like. Right: Persian Carpet weavers responsible to make the floor heaven-like; © Leila Hosseini, adaptation is by permission of copyright holder: Leila Hosseini.*

On the roofs of Persian Bāzārs, dyed wool in different colors is hung to dry before being used in the weaving of carpets. Our retractable porous and colorful roof, as a flying carpet in the texture of the city becomes readable and well-grounded, positioned beside the dyed wools on the roof of the Bāzār of Isfahan and the colorful domes and minarets of the mosques nearby [Masjid Imam and Masjid Shaykh Lotfollāh).

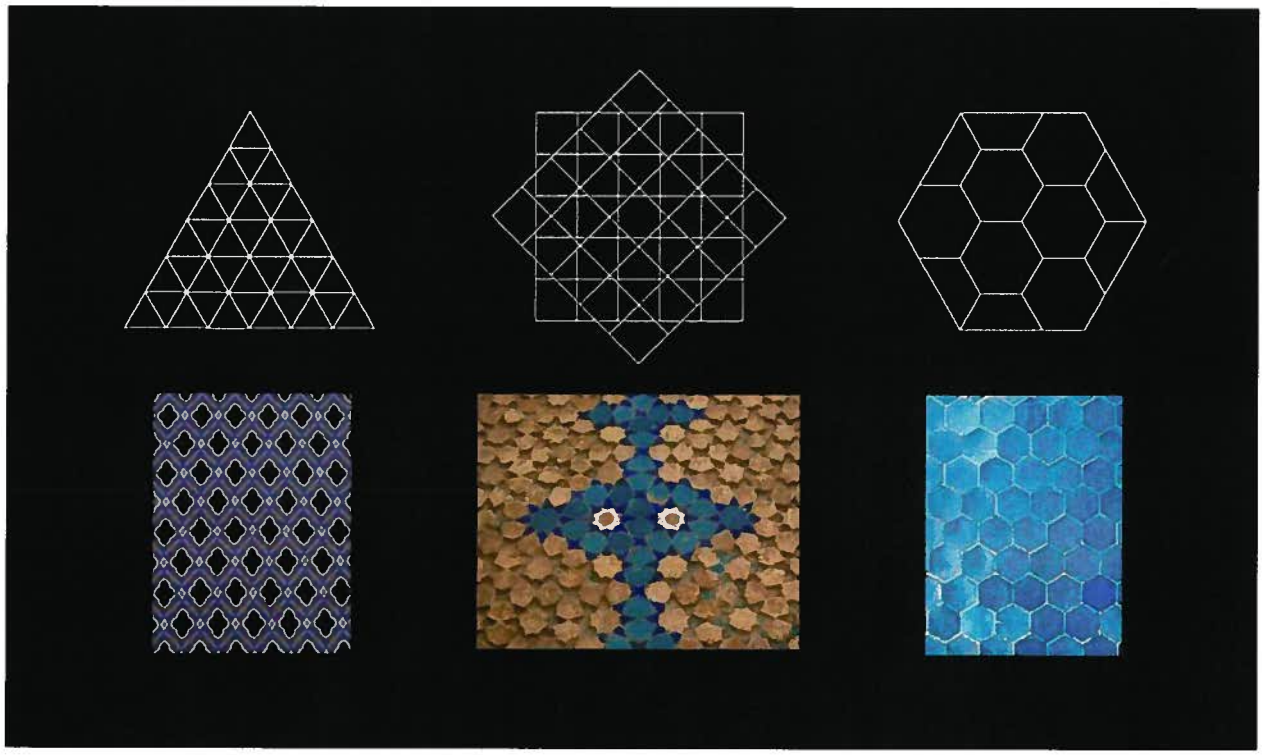

***Figure 5**: The three essential geometric shapes - equilateral triangle, square and hexagon -That can cover a flat surface are the primary grids used for many of the tiled surfaces in Iranian Architecture.*

Traditional use of simple geometry in two and three dimensional context

Traditional geometric tiling patterns were used throughout the Muslim world. When we walk around the lands of the Muslim world we see many different expressions of geometric tile-works and patterns, they might seem very different, but behind the complex aesthetic, there is just a simple geometric grid of squares or equilateral triangles (See figs.5,6).The main geometrical shapes used in the majority of these patterns are equilateral triangles, squares and hexagons.

Figure 6 has been removed due to copyright restrictions. The information removed is: Left- the Hexagonal grid behind the curvilinear aesthetic. Right- Real ornamentation based on equilateral triangle grid. The source was: El-Said, I., Geometric Concepts in Islamic Art, World of Islam Festival Publishing Company, Ltd. London, 1976.

***Figure 6:** Left- the Hexagonal grid behind the curvilinear aesthetic. Right- Real ornamentation based on equilateral triangle grid.*

By a simple application of center-based rotation on each of the lines of these basic grids (i.e. the edges of the triangles or squares) a new base or grid can be generated on the basis of which various new patterns can get created (See fig.7).

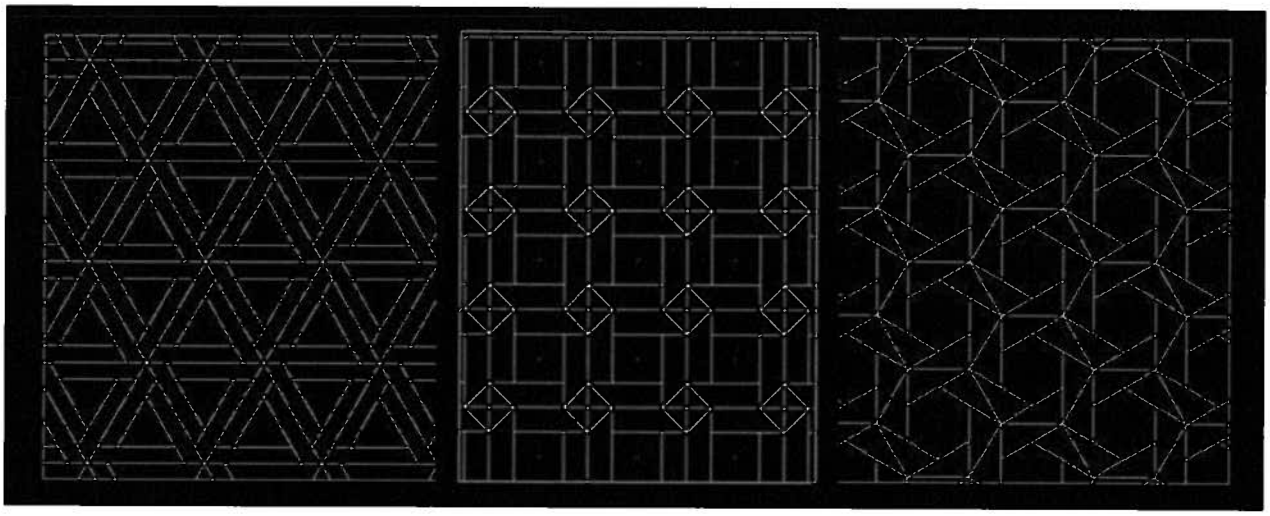

Figure 7: *There exists only three regular tessellations, that is with equilateral triangles, squares and hexagons.*

These three planar shapes (equilateral triangle, square and hexagon) are the only ones that independently can cover a flat surface without any gaps.

Dr. Keith Critchlow, a leading expert in sacred architecture and the cofounder of the journal Temenos, as well as the author of numerous books on sacred geometry, including Order in Space and Time Stands Still, Professor Emeritus at The Prince's School of Traditional Arts in London, and a former professor of Islamic Art at the Royal College of Art says: "Symbols can exhaust verbal explanation but verbal explanation can in no way exhaust symbols – symbols are directed toward undifferentiated unity, while verbal explanations involve never less than two- the donor and the recipient; even if they become the same there still exists the separation of reference: the subject itself and the explanation. Pattern like number, is one of the fundamental conditions of existence and is likewise a vehicle of archetypes. As arrangements both emerge from simplicity and unity and return towards it, they exhibit some fundamental relationships which become hierarchical."[1]

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[1] Keith Critchlow, Islamic Patterns, an analytical and cosmological approach, (New York: Schocken Books, 1976) 24.

CHAPTER 2: GEOMETRY AND COMPUTER SOFTWARE

Geometry and Originality

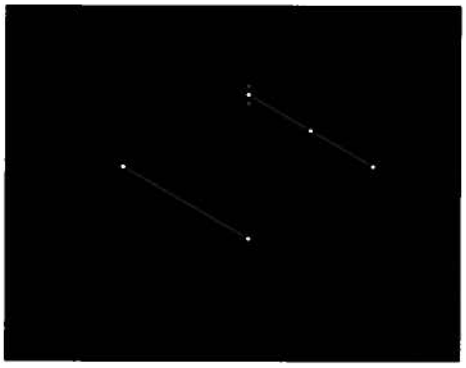

Figure 8: *The line-path can be taken as representing the point 'externalizing' itself. Prepared by Alex Liu Cheng; © Alex Liu Cheng, adaptation is by permission of copyright holder: Alex Liu Cheng.*

Doctor Critchlow says: "The manifestation of an action, object or thought (if it can be defined) necessitates a point of origin or departure, in relation both to the manifestation itself and to the person who is conscious of its emergence. The point of emergence does not necessarily reveal its causation either in the field of its emergence or in the mind of the viewer. In the mind the point represents a unitary focus of conscious awareness; in the physical world it represents a focal event in a field which was previously uninterrupted."[2] (See fig.8) and he continues:

"If the manifestation of the point is indicative of a departure from its source, the direction is implied. Direction in space is qualitative, and hence the first departure or line path from the point is qualitative."[3]

"The line-path can be taken as representing the point 'externalizing' itself. A line, i.e. when a point has moved outside and away from its original position, symbolizes the polarity of existence, although it consists essentially of three elements- two ends and a relationship between them. Having a limited departure from the point of origin, polarity expresses itself in the relationship of the central (essentially passive) 'original' point and the outer projected (active) point. This expression forms an arc with the line representing our original departure as radius."[4]

As the Arc closes, it defines a domain. The origin, the center of the circle, is hidden.

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[2] Critchlow 9.

[3] Critchlow 9.

[4] Critchlow 9.

Dr. Critchlow says: "Once the enclosing circle is completed, a unity is obtained; this reflects the unity of the original point."[5] And he continues:

"Having established the existence of a unity between the point and the circle, we can express the process of departure or externalization again in terms of the whole circle."[6] (See fig.9)

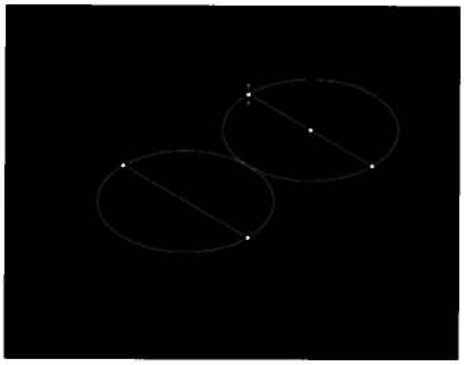

Figure 9: *The new circle originates from the original circle and moves out of the domain of the first circle till the connection between the two circles is reduced to a point; Prepared by Alex Liu Cheng; © Alex Liu Cheng, adaptation is by permission of copyright holder: Alex Liu Cheng.*

Now a new circle originates from the original circle. As it departs there is two significant moments, one is when it is halfway out of the original circle, and the other is just before it is totally out, where the connection between the two has been reduced to a point. In the first the distance between the centers of the two circles is equal to the radius of the circles and in the second case it is equal to twice the radius of the circles. Now a third circle can originate from the second one, like we created the arc out of the original point. The third circle's center continues on the arc till the third circle touches the second circle (and first circle) in just one point. Now each circle touches each of the other two circles in just one point. The three centers of the circles make the first polygon, the equilateral triangle.

The fourth circle originates from the third one in the same way and we can continue this till we have six circles around the originating circle, and the centers of these 6 circles define a hexagon.

Now if we repeat all of the process from the beginning and each circle just departs half-way from its originating circle, that is equal to the length of the radius, we come to the primary paths on which we will be constructing the project which is discussed in this report.

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[5] Critchlow 9.

[6] Critchlow 14.

Explorations with the software

On the basis of the fact that the simple action of 'central rotation' applied to the edges of a shape can result in various distinct patterns, we began our exploration with the computer software Maya. Starting with a square grid, all the lines were set to rotate on their own centers. In this way, by selecting the geometries inside the software and 'center pivoting', in Maya terminology, we could create an animation where all the grid lines rotate on their own center in the same direction and with the same degree of rotation.

In Maya we can define surfaces by selecting two lines, and the software will create a surface between those two lines. In Maya terminology, we 'loft' between the two lines.

In order to create such a surface there are different settings that we can set subjectively. After choosing certain settings and selecting our line couples the 'loft' surfaces get created. If we now rotate the edges, the surface created with these settings would deform and follow the edges in a smooth manner.

During our experiments with a square grid we experienced a problem which later led us to opt for an equilateral triangle grid. Comparing the square grid with the equilateral triangle grid, the squares have two distinct characteristics. One is that they have parallel edges and second is that they have right angles. Our approach towards square was using the legs of the right angles to define surfaces. The problem with our approach towards the square was that we actually started making use of the 'right angle' aspect of the square's character instead of the 'parallel sides' aspect. By rotating the side by side edges after 45 degrees of rotation the tangent of the surface attached to them would abruptly jump from plus to minus. With our settings in Maya the direction of the defined surface would change suddenly and that did not seem possible to achieve in the real material world. So we chose to continue our explorations with equilateral triangle grid. Regarding the square grid there is the possibility of using the parallel edges of the square rather than the right angle legs which is out of the scope of this paper. In short, using the parallel edges in square grid would help us to have a two dimensional surface with no stretch in the surfaces attached to these edges, because by rotating the parallel edges the surface attached to these two edges would just change from square to rhombus without any stretch (See fig.10).

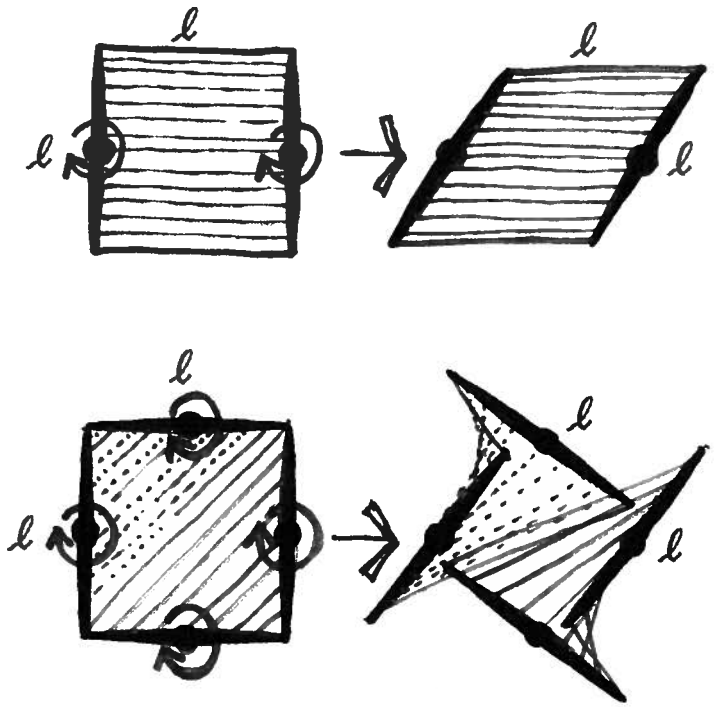

Figure 10: *Top: The two parallel edges of the square define the surface, and rotation.*
No stretch.
Bottom: The right angle legs define the surfaces. Stretch involved.

Continuing on the previous talk if we again move the circles outward there is different stages. There is a time where alternate circles among the 6 circles are in contact at their outer points; at this stage a central domain opens up which is clear of all the departing circles. The second stage is when the arcs which were cutting the perimeter of the center circle in 12 places are exactly equidistant. If we connect the points where the circles cut each other we will see the emergence of square.

Doctor Critchlow says: "We can now see that the square emerges as a product of our basic pattern. Hexagon and triangle are dual and complementary, the one co-existing within the other. The square is self-reflecting and self-dueling as squares emerge from the center of a square matrix. These three basic shapes are used to symbolize the square of earth or materiality, the triangle of human consciousness, and the hexagon (or circle) of Heaven."[7]

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[7] Critchlow 24.

Below (See fig.11) you can see a real traditional example of a square grid based tile work that expresses the whole idea of rotation. These examples are actually located beside the Maydan (city plaza) Naghshe Jahan near which the experimental design part of this project is situated.

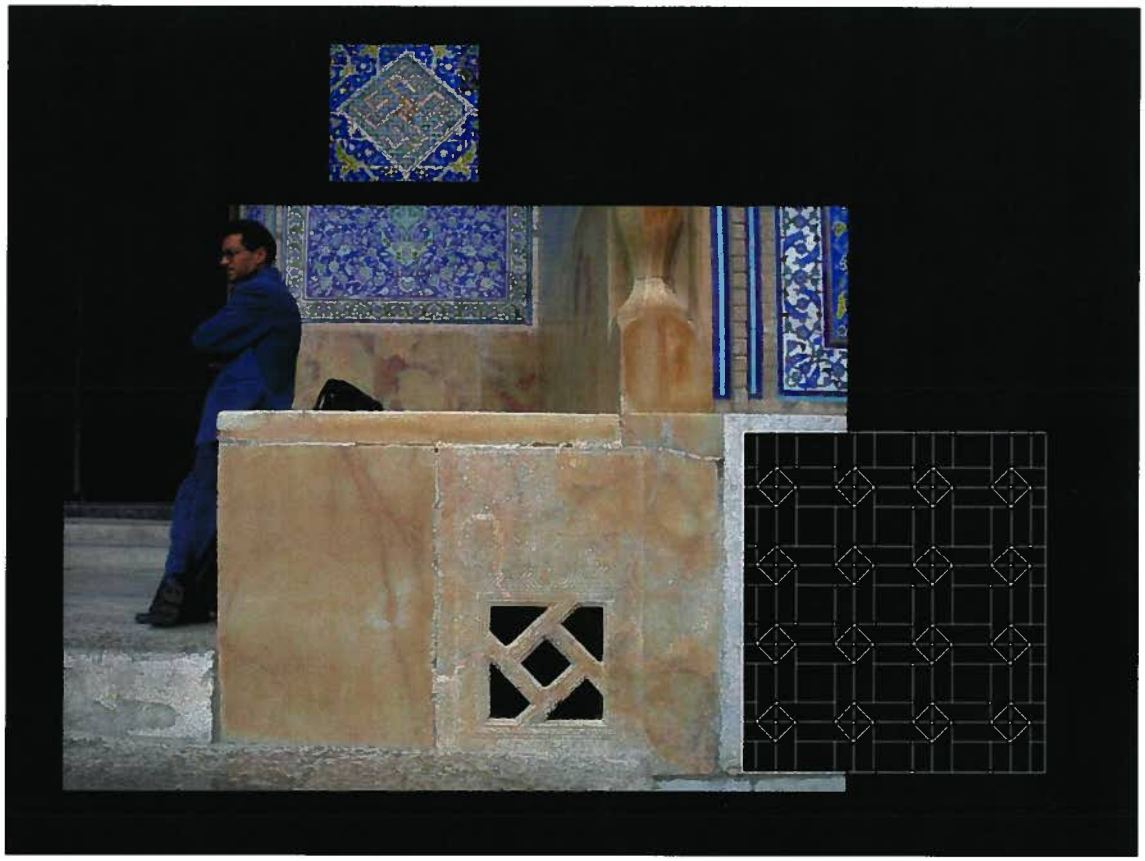


Figure 11: *Rotation in square-based tile work and stone-work from Masjid Shaykh Lotfollāh, Isfahan, Iran.*

An animation got created in Maya for each grid, in each of which the edges would rotate and cause the surfaces attached to them to rotate along with them. On the next two pages we present some of the frames of this animation. In the square animation, the final amount of rotation was 180 degrees. For the triangle grid it went up to 60 degrees and then returned to zero degrees of rotation. The reason for this was that in our final design with the triangle grid our range of rotation would be limited between -60 and +60 degrees. The reason for that restriction is described later in this report (See figs.12, 13).

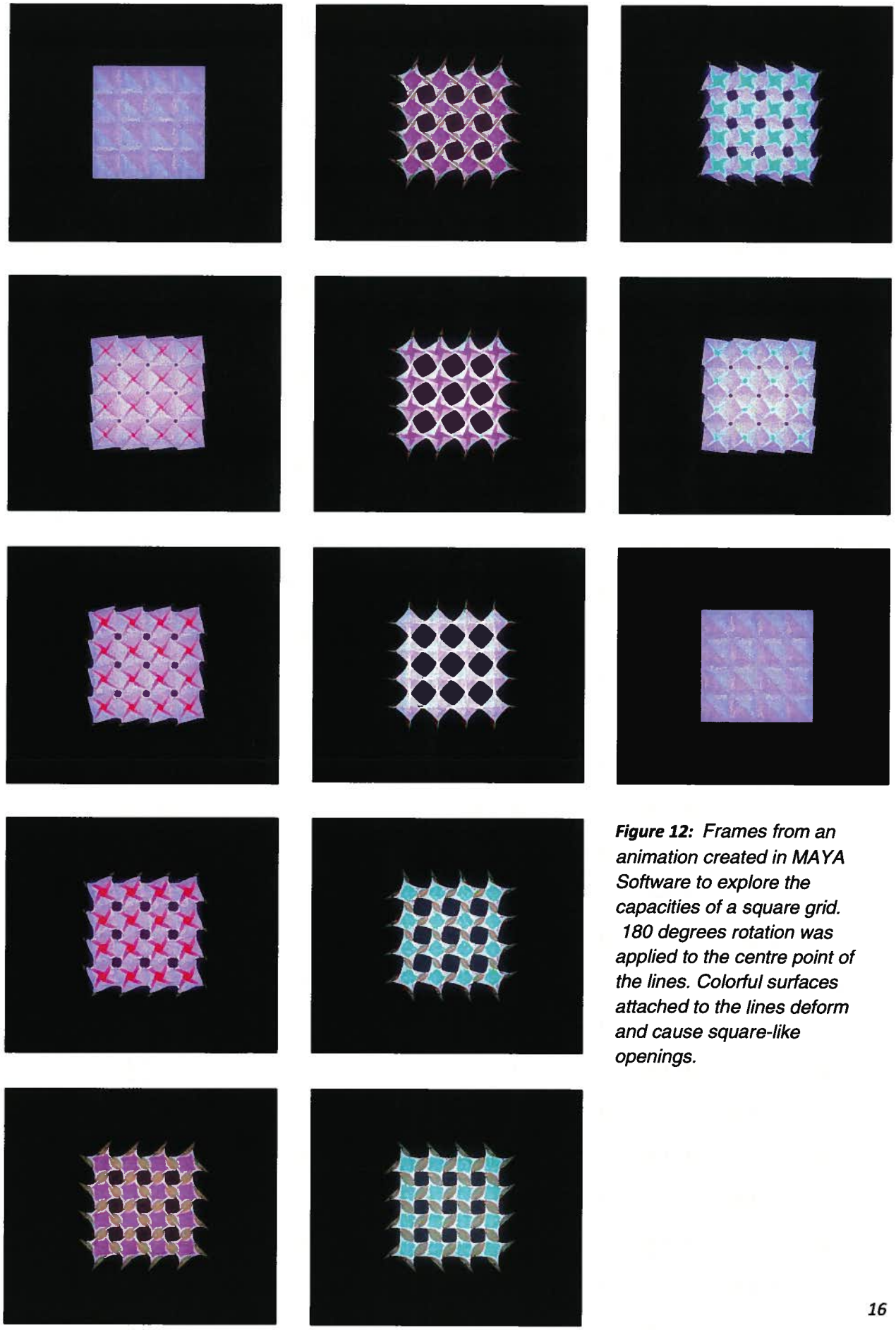


Figure 12: *Frames from an animation created in MAYA Software to explore the capacities of a square grid. 180 degrees rotation was applied to the centre point of the lines. Colorful surfaces attached to the lines deform and cause square-like openings.*

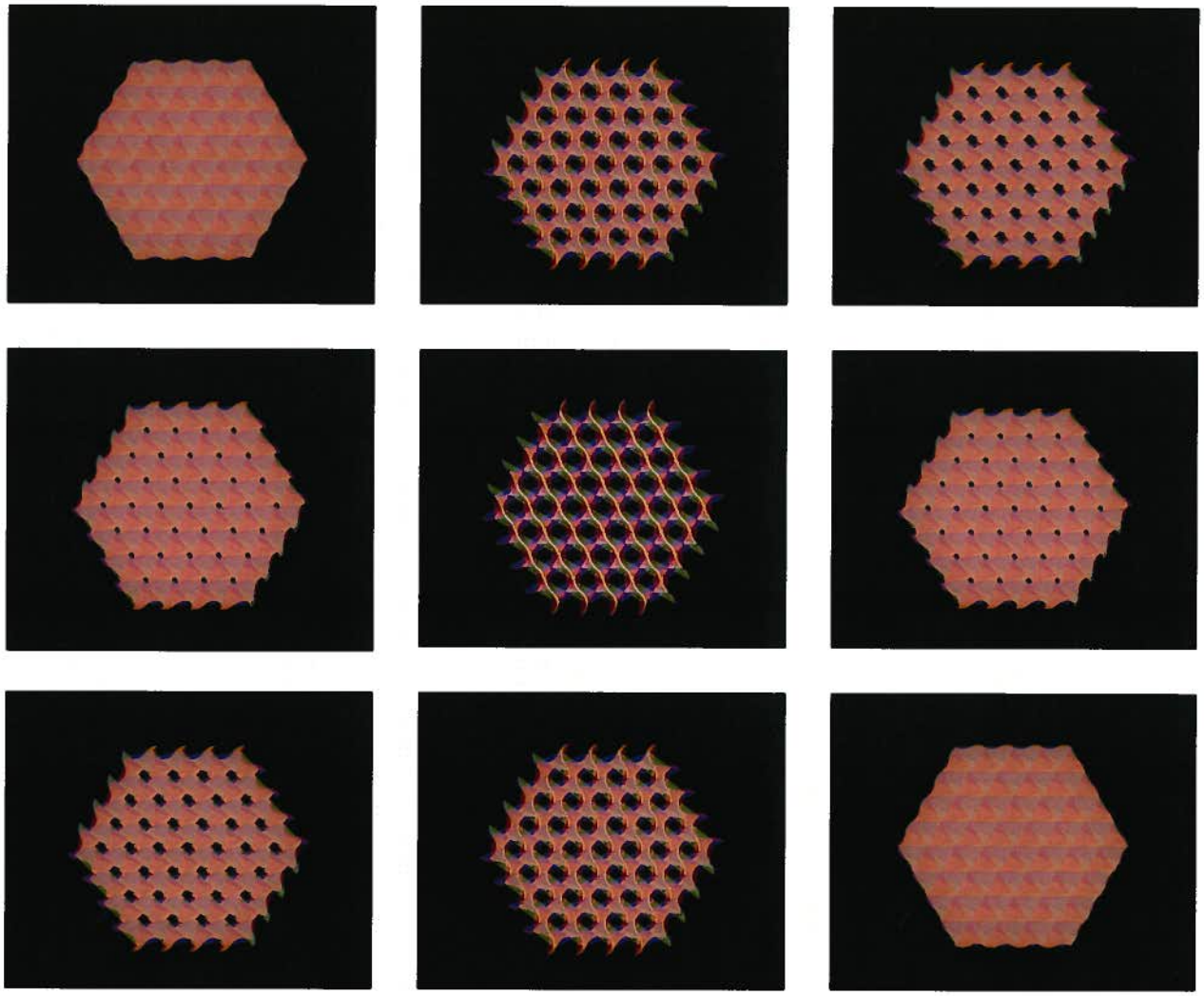


Figure 13: *Frames from an animation for triangle grid, created in MAYA Software; to explore the capacities of an equilateral triangle grid.*
180 degrees rotation was applied to the centre point of the lines. Colorful surfaces attached to the lines deform and cause hexagon-like openings.

In this project, in each unit of the design, what happens is a hierarchical trip from an originating point to a line, then to a circle, and then to the second circle originating from the first circle. The first points are the final pivot points, the lines are the final rotating arms and the circles are the paths that the ends of the rotating arms define. Various circles originate from the first circle and we start to have a grid of our design unit.

The Design Unit

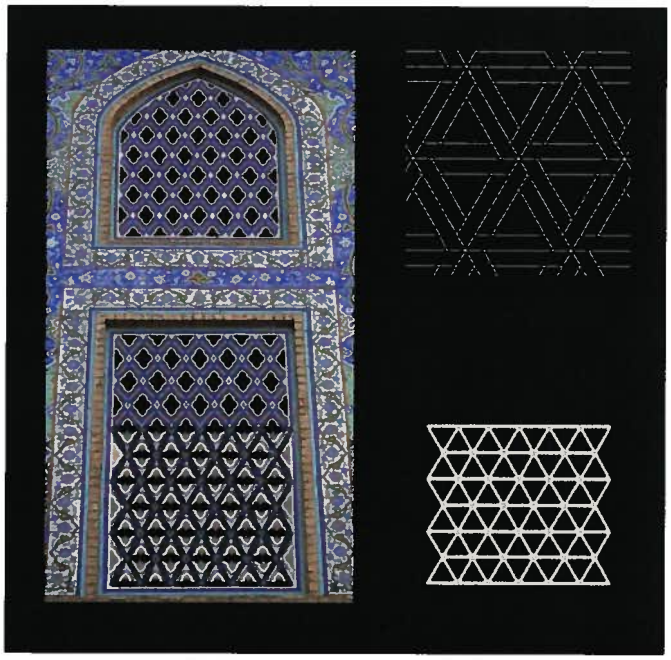


***Figure 14**: The equilateral triangle grid got chosen to work with - The example from a triangle-based light screen, Masjid Shaykh Lotfollah. Isfahan.*

Finally we chose the triangular grid to work with (See fig.14). In the diagram below (See fig.15) we present a single design unit of the grid - one triangle. It is defined by two edges of the triangle with a surface attached between them. The next page's illustrations (See fig.16) show a row of the units and how they would transform by applying 30 CW and 60 CW rotations to the edges or arms. Isoparms are the lines on the surface that can represent the surface.

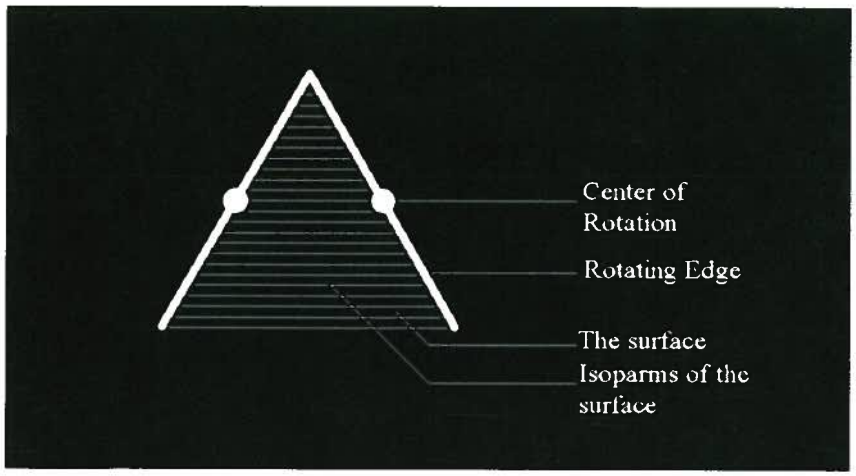


***Figure 15:** The Triangular design unit.*

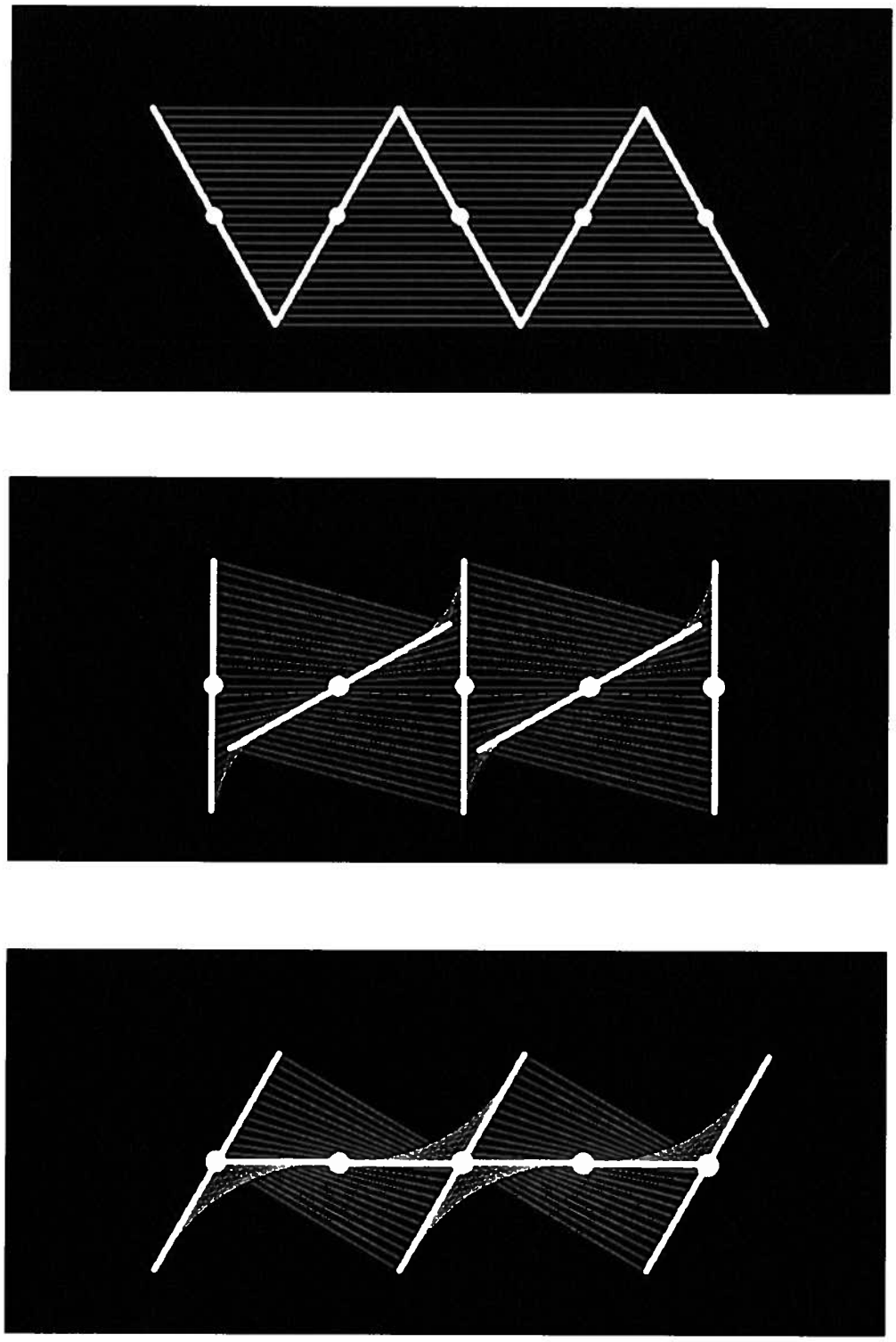


***Figure 16**: Top: One row of design units without rotation, Middle: One row of design Units with 30 degree CW rotation, Bottom: One row of design units with 60 degrees CW of rotation.*

CHAPTER 3: DEVELOPING GEOMETRICAL THEME INTO MATERIAL REALITY

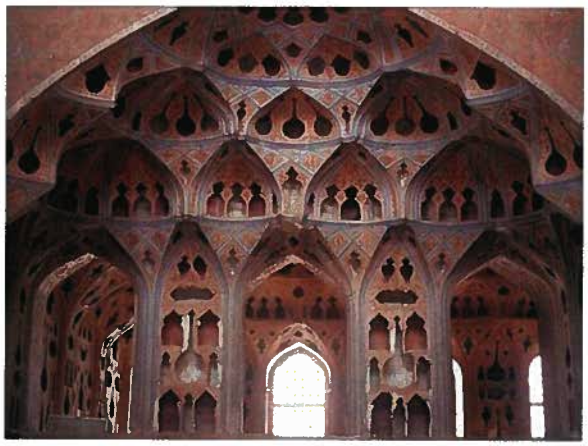


***Figure 17**: Example of three dimensional geometries in Iranian architecture- Auli Qapu building-Isfahan.*

In our journey towards creating something new out of a simple geometric grid by attaching surfaces to the rotating lines and expecting a dynamic nature from the final result, we came across some models that we needed to address in order for the idea to be expressed materially. We took on different approaches for the development of the geometrical theme into three dimensional material reality (See fig.17).

One stumbling block was the question over the existence of any kind of inexpensive material that could be used as a surface that would tolerate the amount of stretch required as a result of the rotation of the arms. If at all such a material existed, the problem would need to work for repeated long term use, considering the different amount of stretches that it would have to withstand. In certain circumstances the material would even have to stretch from nothing to something, in a planar geometry. This would occur, for example, in the triangle that would occur at the corners where the two planar arms defining the surface would touch each other's end points and then after some rotation would start to become distant from each other. As a result the surface material attached to these edges of the geometry at these critical end-points would have to stretch from theoretically nothing (edges touching each other) to some defined length (edges separated from each other) (See fig.18).

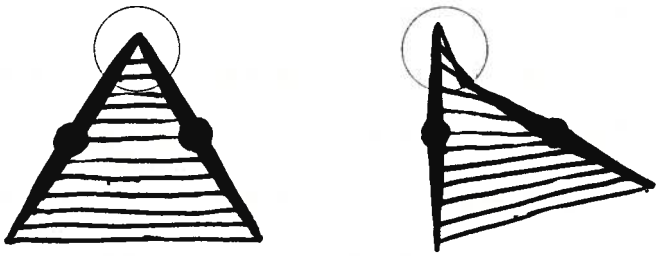


***Figure 18:** Extending from nothing to something is not possible in the material world.*

The first thing that came to mind was that we needed to propose some ideas in terms of geometry or materials to avoid the stretch or to minimize the range of the stretch in order to make the roof design feasible.

Here are two approaches we considered:

1. Keep the two rotating arms out of the same plane to avoid the critical condition of our material stretching from nothing to something. In other words we avoid the problem which was caused by the 2 Dimensional constraints by taking the design into the 3-Dimensional realm. This required a calculation of the optimum distance or height (in the Z direction) between the two parallel planes on which each of the arms rotate in order to minimize the range of deformation or stretch of the surface attached to the arms. Also we began to look at solutions that could simply provide the extra length needed without undergoing any stretch forces, because the stretch forces would probably require some sort of external force to neutralize them and that would create unnecessary complexity from our point of view.

2. Keep the arms in the same plane but use a length of material for the surface that would be sufficient for the maximum extra-length needed and would drop away (let's say by gravity) if less planar length was needed.

For a better understanding of both approaches we began simulating the surface with a series of stretchable lines going from one arm to the other. In more technical words we replaced the surface with its own isoparms.

For Approach 1 (moving the arms into separate planes), the solutions that were found include the following:

a. Select a few isoparms of the whole surface and turn them into telescopic elements attached to the arms by universal joints. This telescopic approach would allow for a neat change of length. Like a telescope with different pieces whenever that special isoparm needed more or less length as a result of the rotation of the arms by sliding in or out the telescopic pieces would be able to adjust to that and still keep their linear look from one arm to the other without any drop, or curvature (See fig.19).

b. Select a few isoparms of the whole surface and turn them into strings and provide reels on the arms that would collect and/or provide extra length of string when needed (See fig.19).

c. Select a few isoparms of the whole surface and thicken them (make them thick as if they were straws). Have them all be the same length to the maximum length needed for the maximum stretch. Provide holes in one of the rotating arms to accommodate them passing through the arm whenever the extra length was not needed and they would be attached to the other arm by universal joints to allow for free spatial rotation needed (See fig.19).

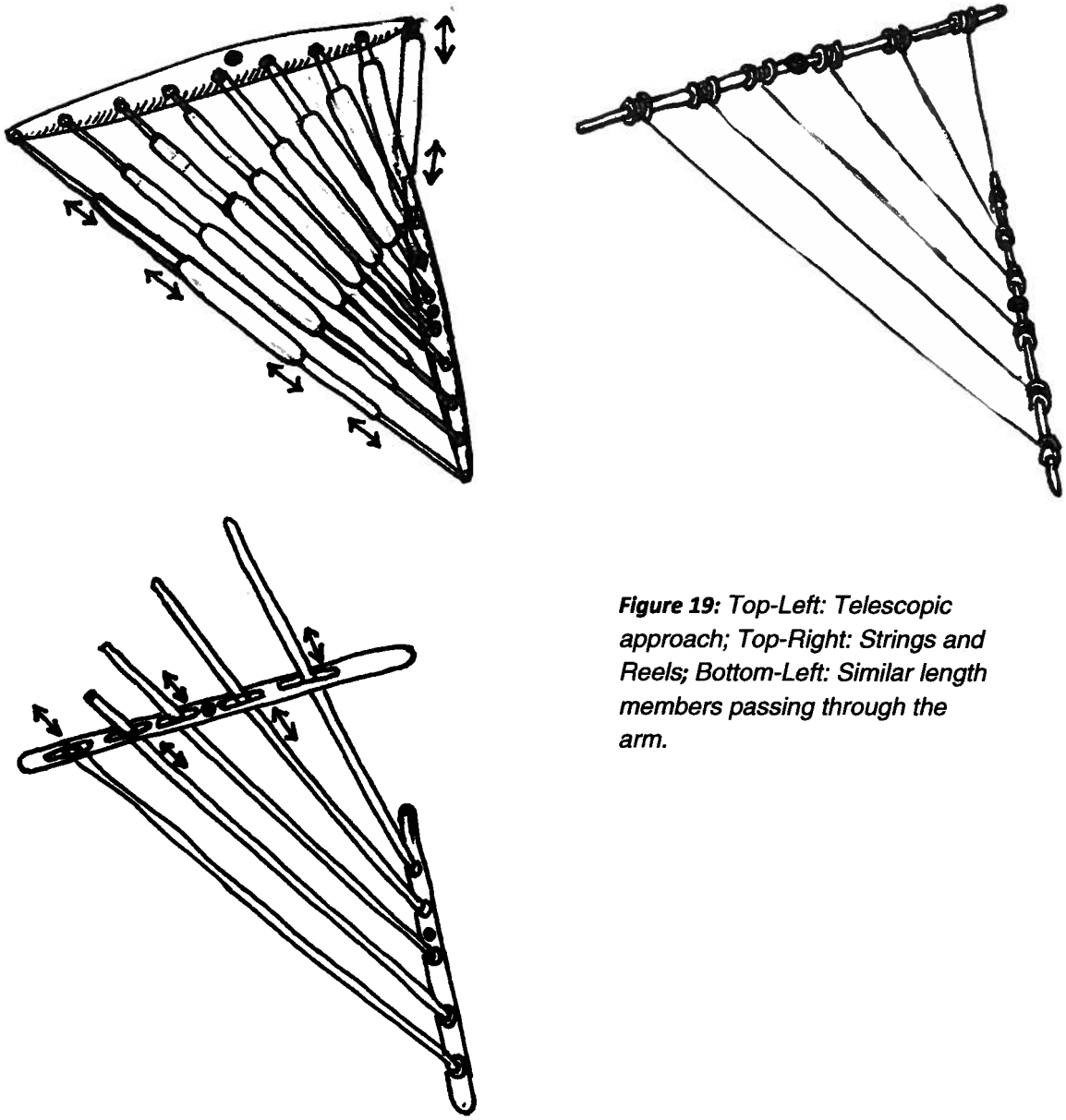


Figure 19: *Top-Left: Telescopic approach; Top-Right: Strings and Reels; Bottom-Left: Similar length members passing through the arm.*

For Approach 2 (keeping the arms in the same surface), the solution that we found was:

a. Select a few isoparms of the whole surface and have all the lengths be equal to the maximum needed. In some way define one or more joints (hinges) along each of them. This would allow for a natural, less detailed change of length when needed. When the isoparms are at maximum length there is no bend and when less length is needed they bend to accommodate that. In this approach the entire thing can be replaced with a normal surface without any stretch qualities to it, and we would have a smoother curved shape where the surface needs to drop because there is less planar length needed. The original shape of our surface when laid on a flat surface would be a simple rectangle or a square if all sides of it are equal to the length of the rotating arms. Interestingly enough this takes us back to our approach with square grids, and shows that if we selected two parallel edges in a square rather than two perpendicular ones we could possibly work the design out using a square grid and in just a 2D surface again. It is the same 2D surface between the two parallel edges of the square that if we bring the two ends of our edges towards each other it will drop at one end to become and shape a 3D surface as you see in the figure below (See fig.20).

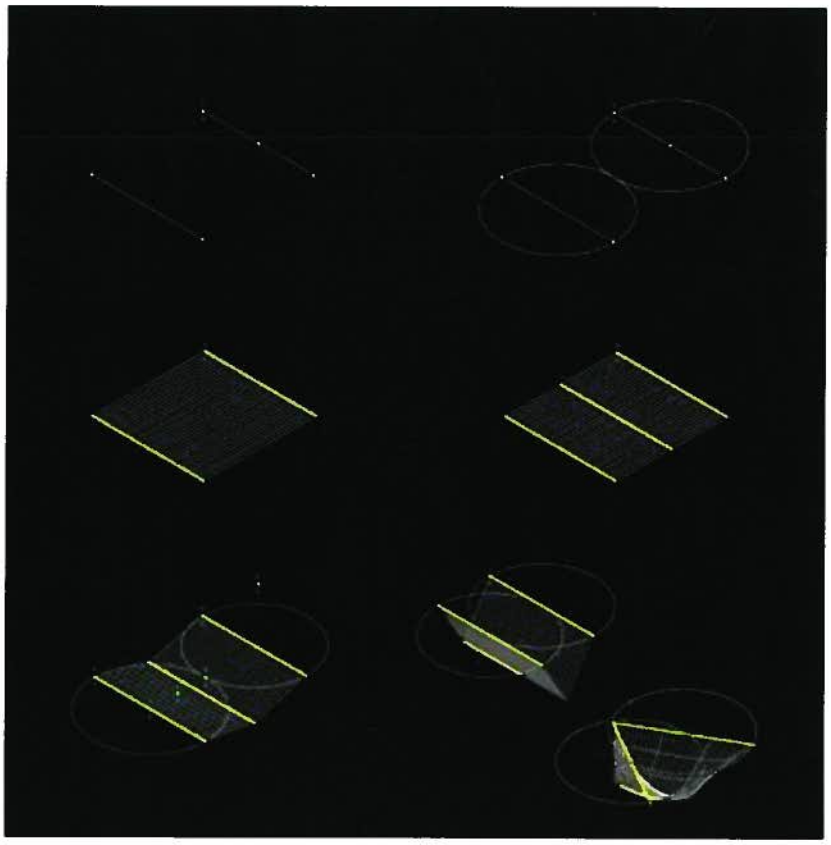


***Figure 20**: Process of creating the three dimensional design unit; The unit model used in this figures was prepared by Alex Liu Cheng in Bentley's Generative Components software. © Alex Liu Cheng, adaptation is by permission of copyright holder: Alex Liu Cheng.*

Imagine that half way between one arm and the other on our series of surface isoparms we attach small weights - these will act as a hinge keeping our surfaces straight. In this way the final 3 dimensional shape that is derived can remind us and relates to the historical 3 dimensional (muqarnas) shapes that we see in Islamic architecture (See fig.21).

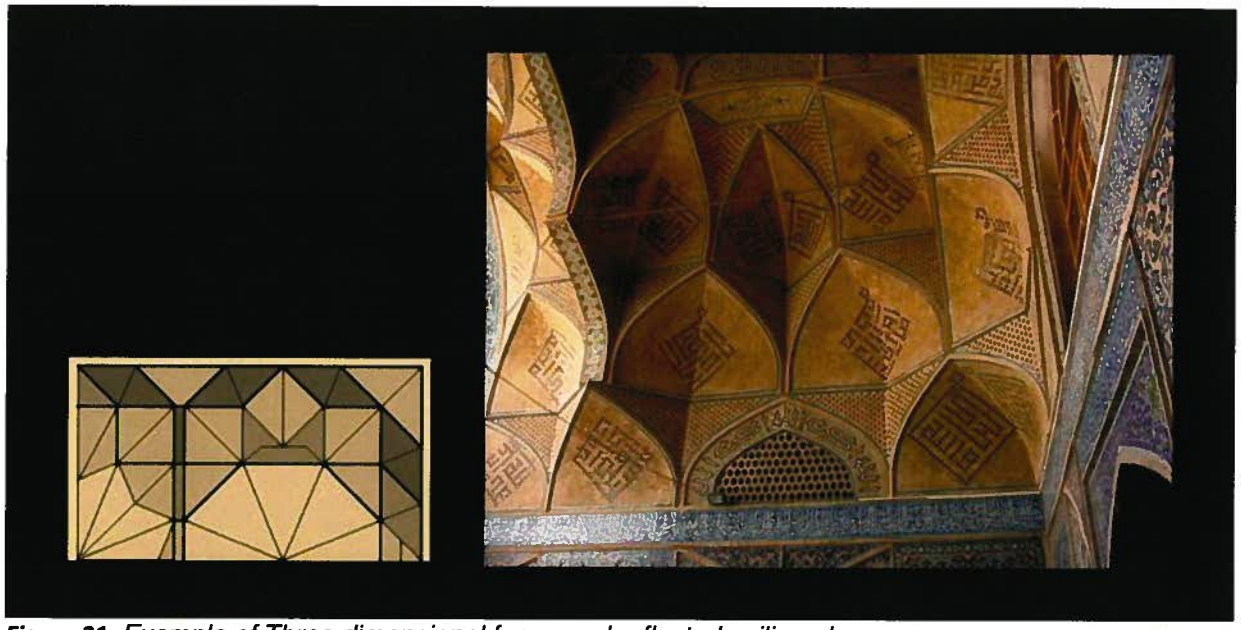


***Figure 21**: Example of Three dimensional forms and reflected ceiling plan which our ancestors used to draw on the ground underneath the ceiling and from which they transferred the points to their appropriate spatial height, Masjid Jamé, Isfahan, Iran.*

The different approaches discussed above can be developed or even combined with each other to enrich the various possible shapes. They bring new possibilities to the design that can be customized for various uses with different techniques.

Figure below (See fig.22) shows the 3D unit that we just distinguished and how it would look like by applying rotation to the arms, clockwise and counter-clockwise. The unit model used in this table was prepared by Alex Liu Cheng in Bentley's GC software by means of generative algorithmic approach available inside the software. Different variables and relationships were defined inside the software to have the dropped surface follow the rotating arms in a real fashion. Different variables were defined by which we could control the amount of the drop of the surface, the number of isoparms etc.

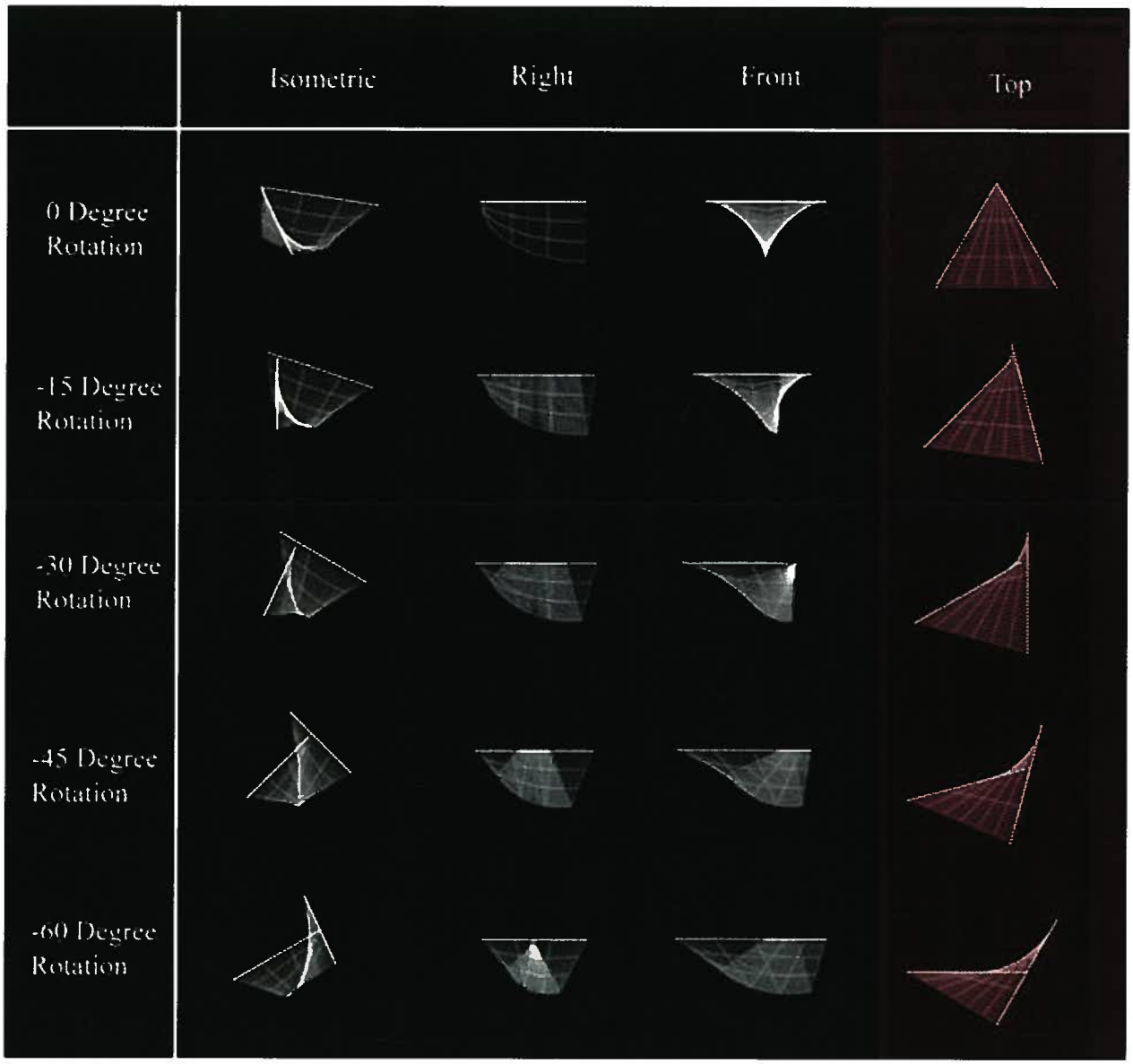


***Figure 22**: Three Dimensional design unit with rotation applied, different views; Drawings of this figure and the unit model is prepared by Alex Liu Cheng in Bentley's Generative Components software. © Alex Liu Cheng, adaptation is by permission of copyright holder : Alex Liu Cheng.*

Below is how the shadow of the unit would look like on the ground. We will refer to the role of these shadows in the design, later on in the pages to come (See fig.23).

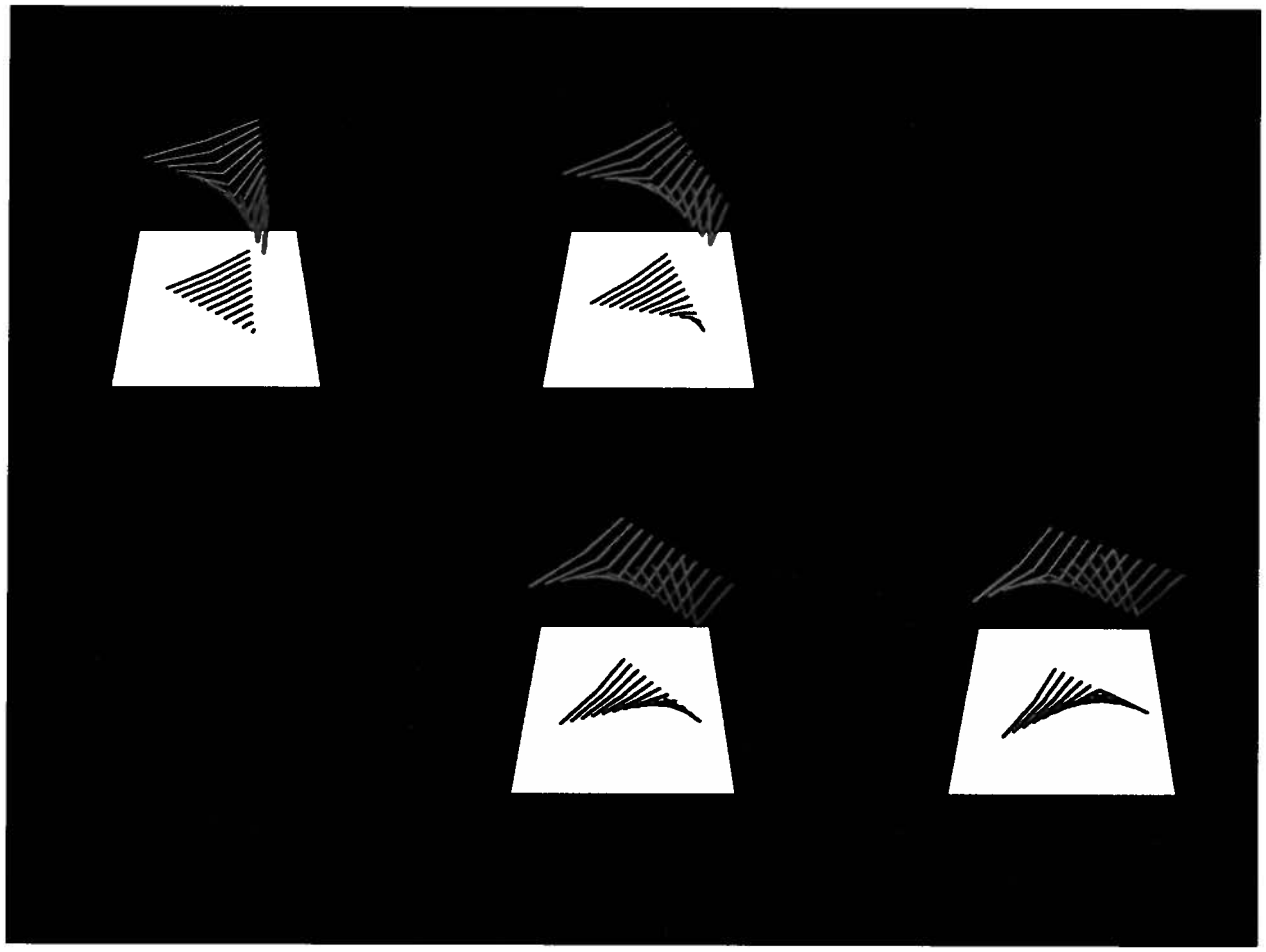


***Figure 23**: Three Dimensional unit, rotation and shadows.*

Three layers

One issue during the design process that always showed up was that due to the extreme stretch or at least extreme complexity of defining the materiality, we tended to define the surface between just two edges of a triangle or square, and not all three or all four edges. However this choice of dealing with just two edges of the geometry later on became a powerful part of the design and part of the flexibility of it.

We found that we could define layers and in each layer have different sets of edge couples of the triangle to define the surface. Now, if we put the resulting three layers beneath each other, view the system from above and then apply the same rotation to all layers, the three layers would appear as if we had defined the surface between all edges.

Consequently we would be able to attach different colors to these 3 separate layers of detached surfaces, and have the surfaces made of colorful translucent materials. The final product would then have something to say about light and the mixing of color as light passes through one layer and reaches the others.

It also makes it possible to have the final design be in one layer, two layers or three layers depending on what light and color effects we want to achieve.

Further, we can open and close the separate layers (by rotating the arms) in different amounts for any purpose, from sustainable envelopes that control heat penetration to defining new patterns and colors and controlling vision through the openings (See figs.24-34).

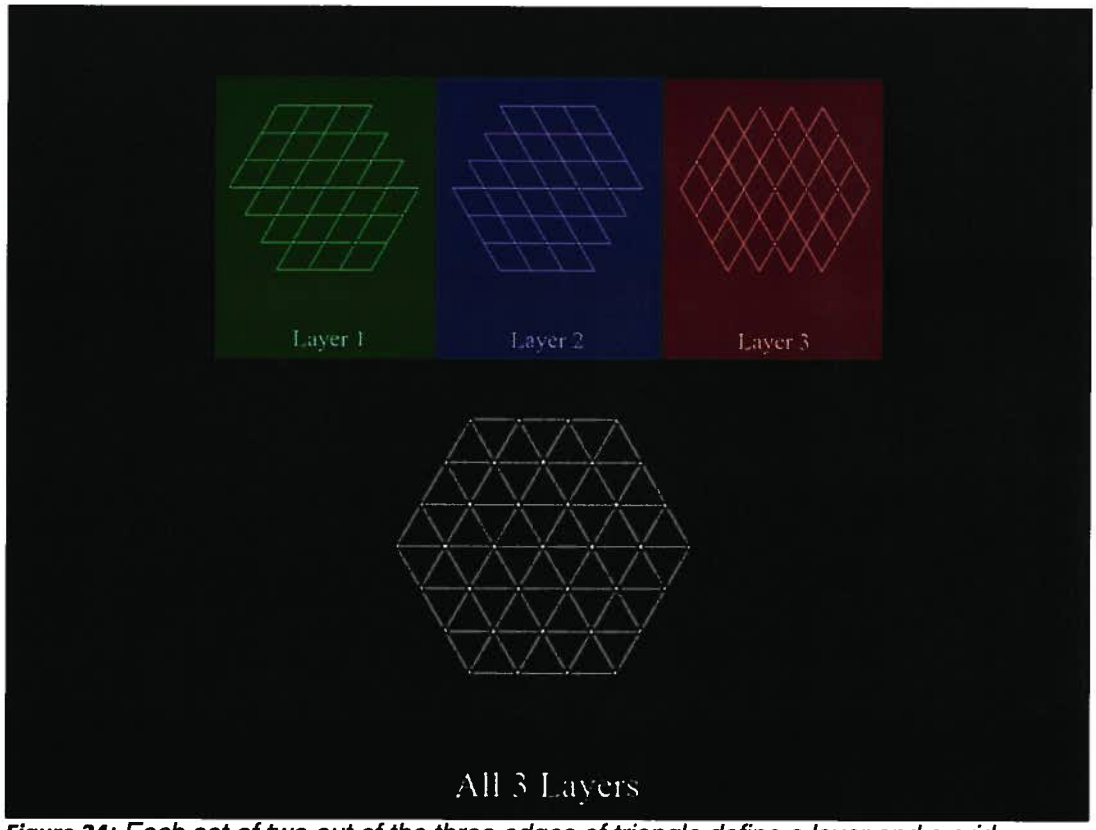
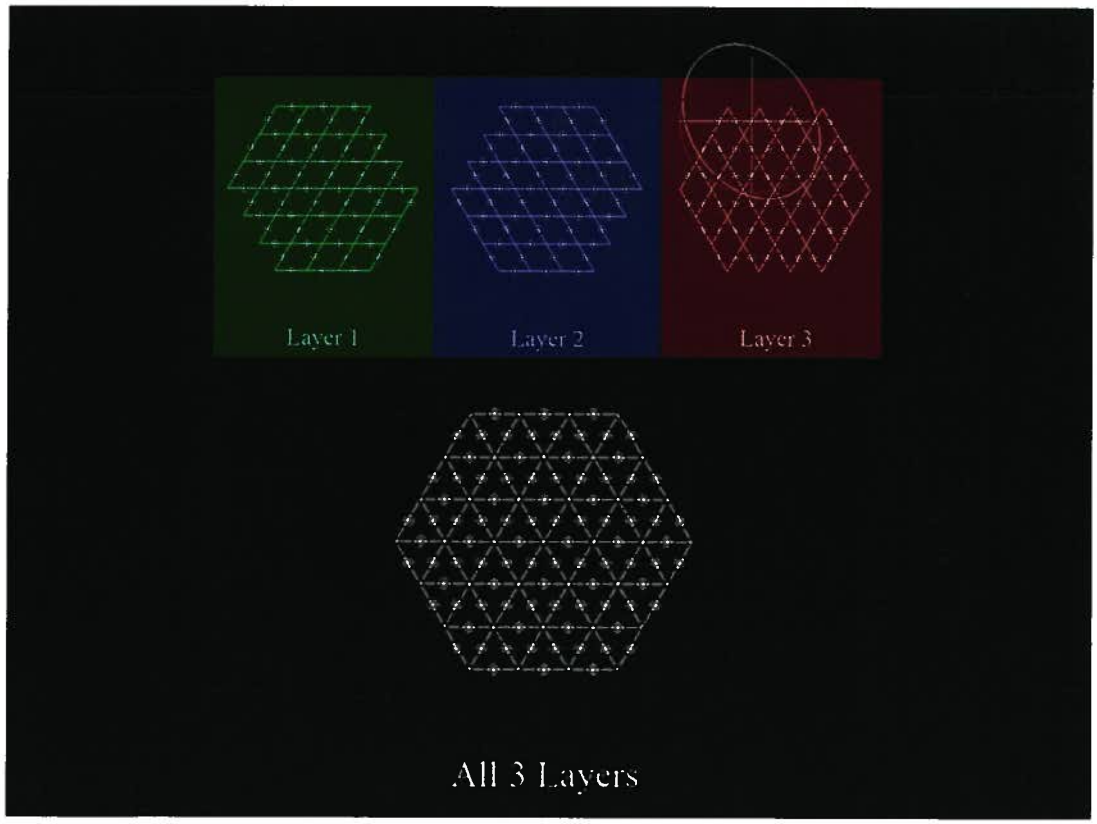


***Figure 24**: Each set of two out of the three edges of triangle define a layer and a grid.*

***Figure 25**: All the triangle edges Center Pivoted in Maya software.*



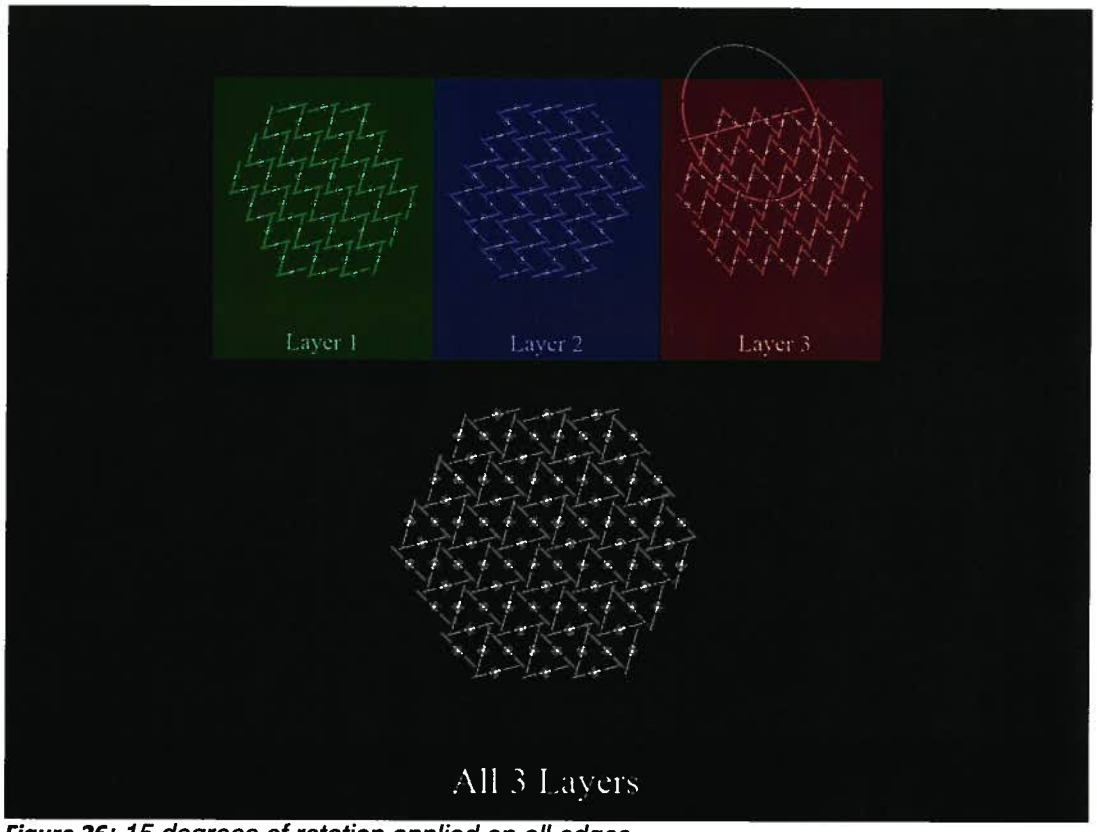
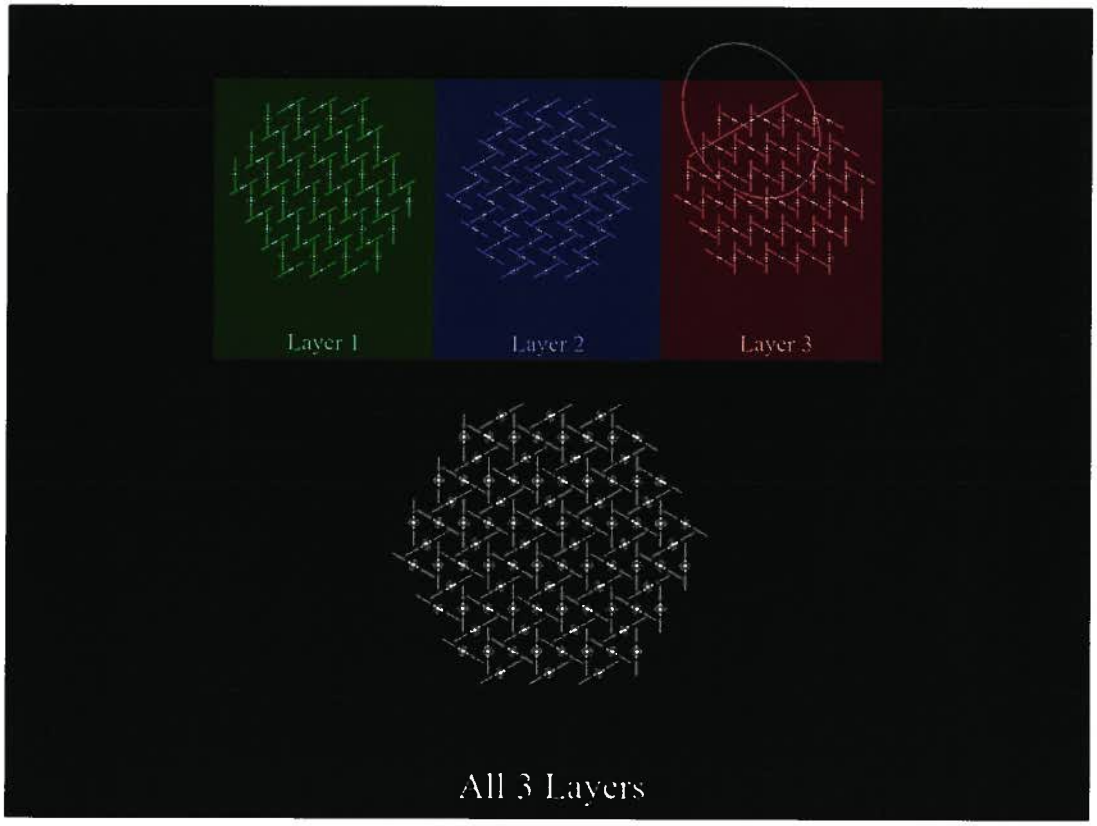


Figure 26: *15 degrees of rotation applied on all edges.*

Figure 27: *30 degrees of rotation applied on all edges.*



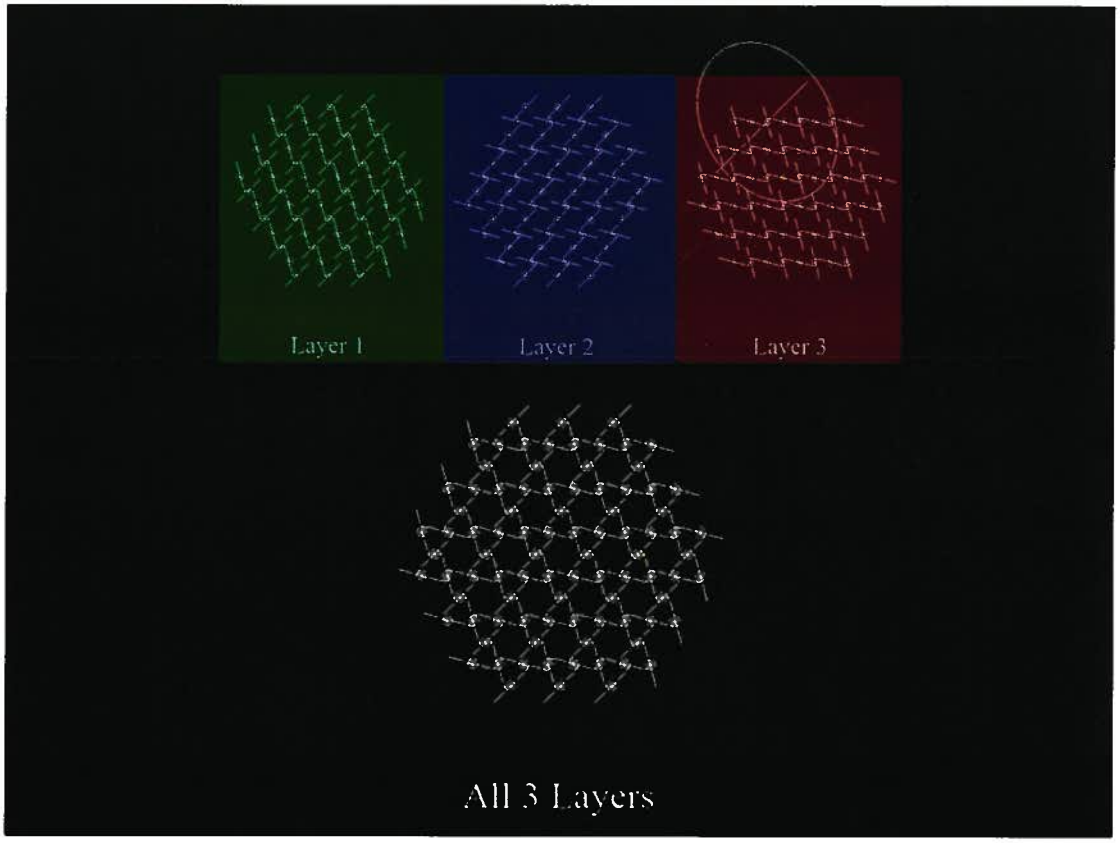
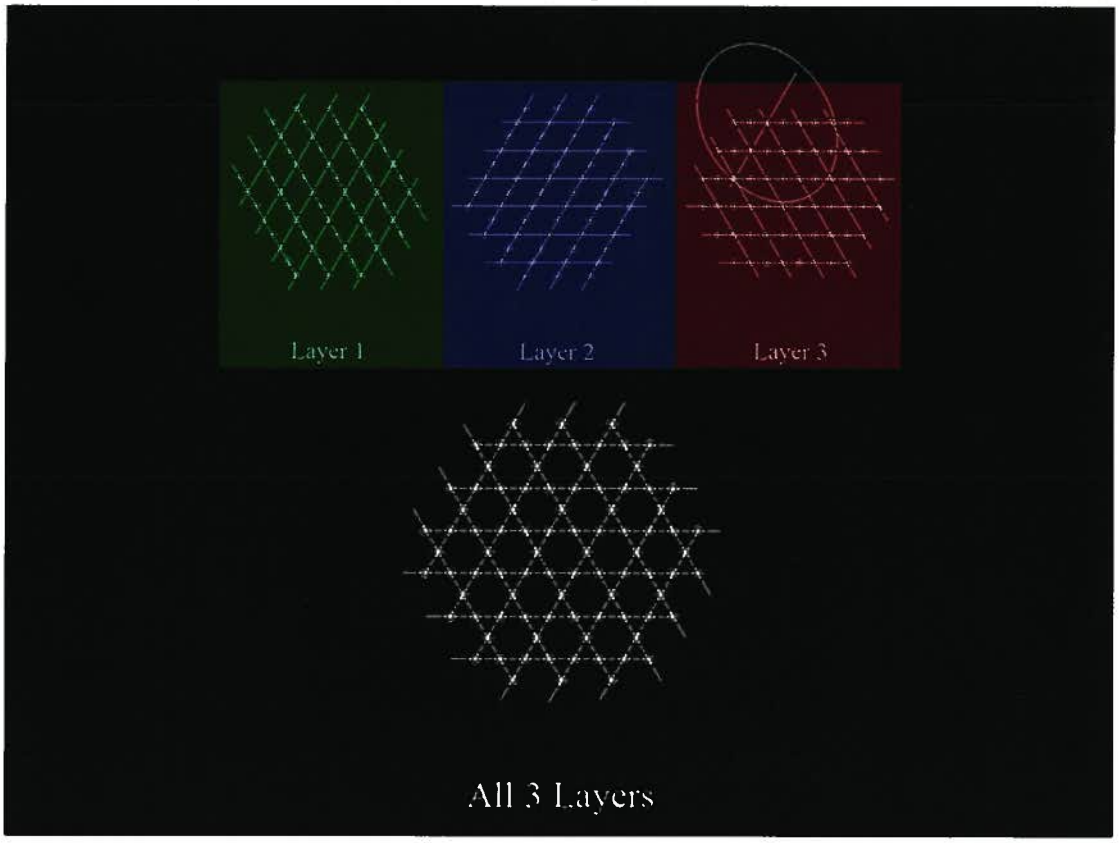


***Figure 28**: 45 degrees of rotation applied on all edges.*

***Figure 29**: 60 degrees of rotation applied on all edges.*



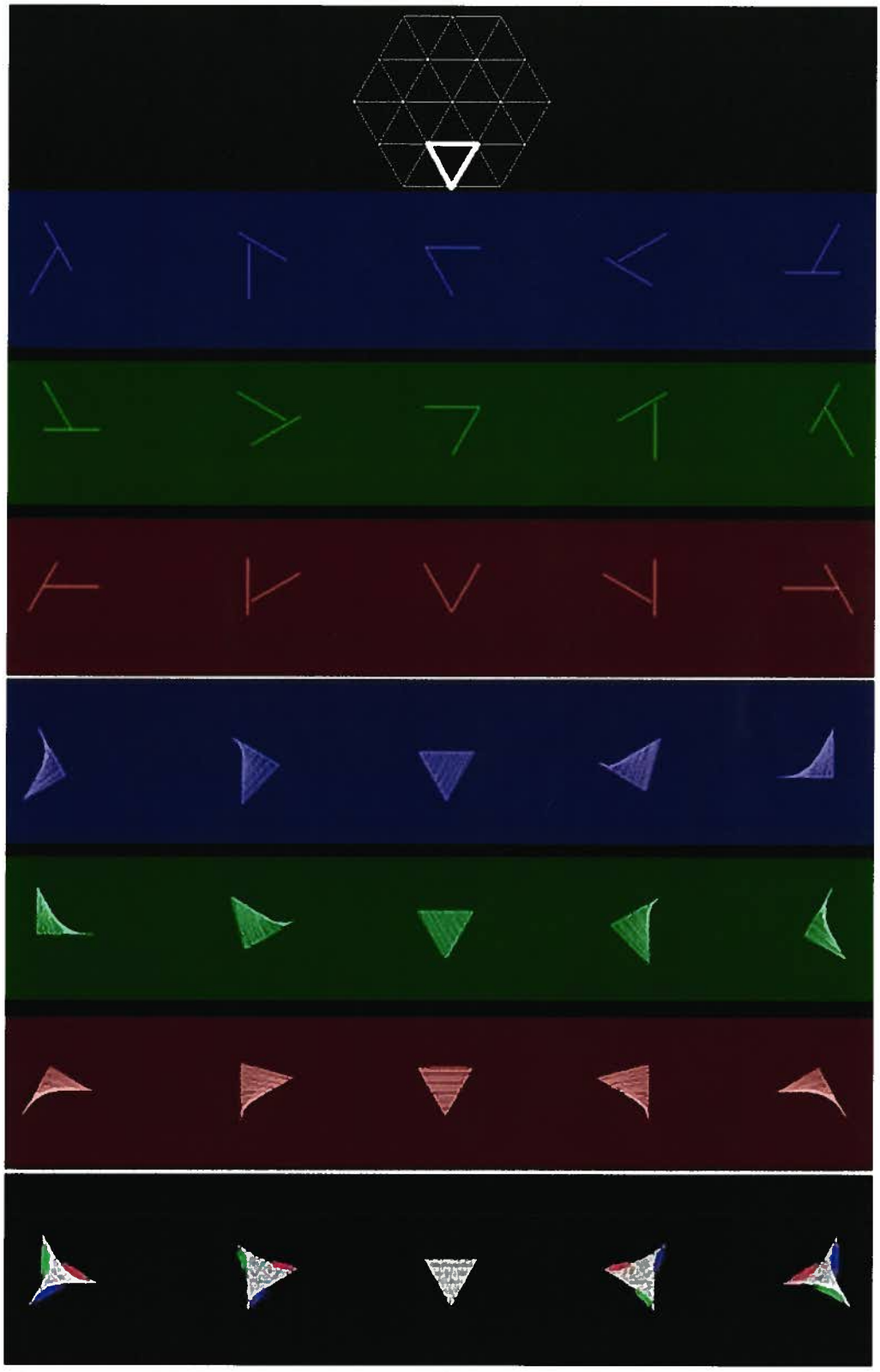


Figure 30: *Bottom: One triangle of the grid picked; Middle: Triangle made into three layers of two edges, surfaces attached to the two edges, ±30 and ±60 degree rotations applied; Top: Three layers on top of each other.*

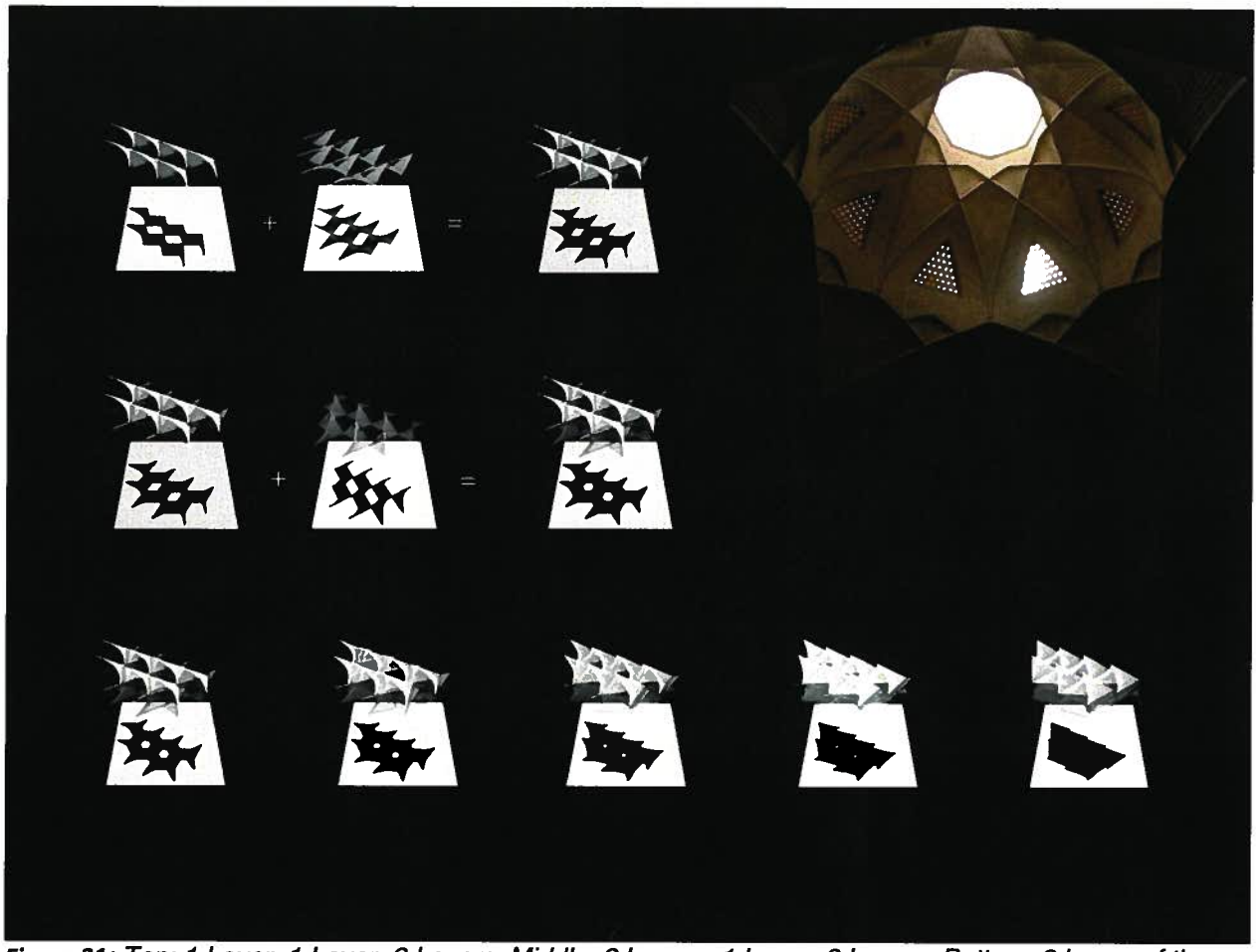


***Figure 31**: Top: 1 Layer+1 Layer=2 Layers; Middle: 2 Layers+1 Layer=3 Layers; Bottom: 3 layers of the design gradually close and the shadow.*

***Figure 32**: RGB (Red, Green, and Blue) color wheel*

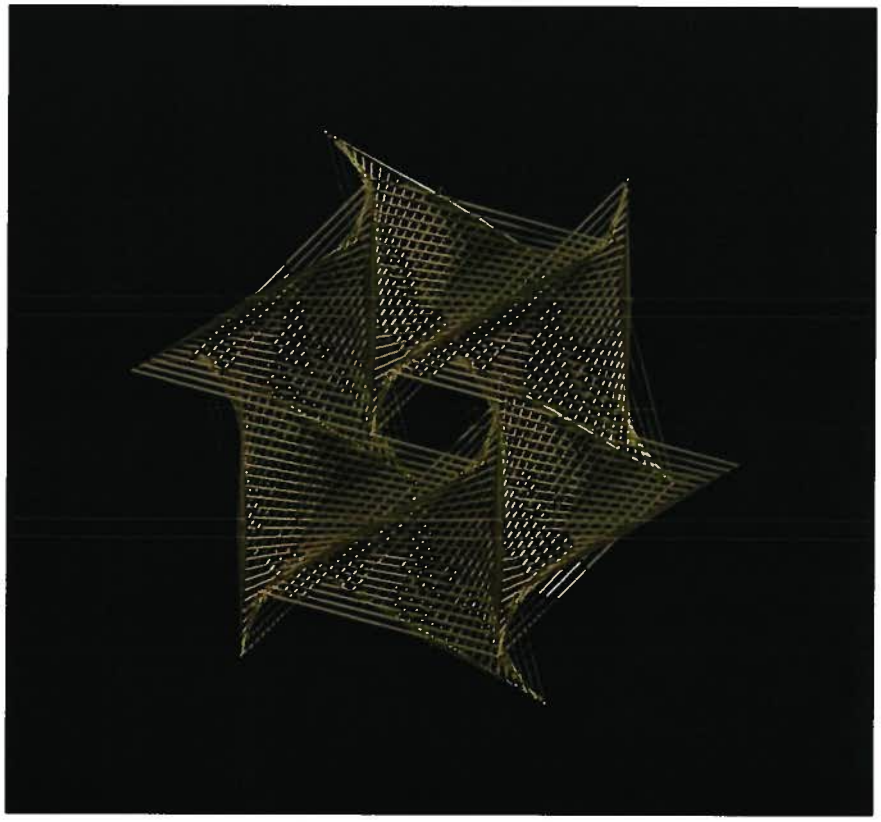
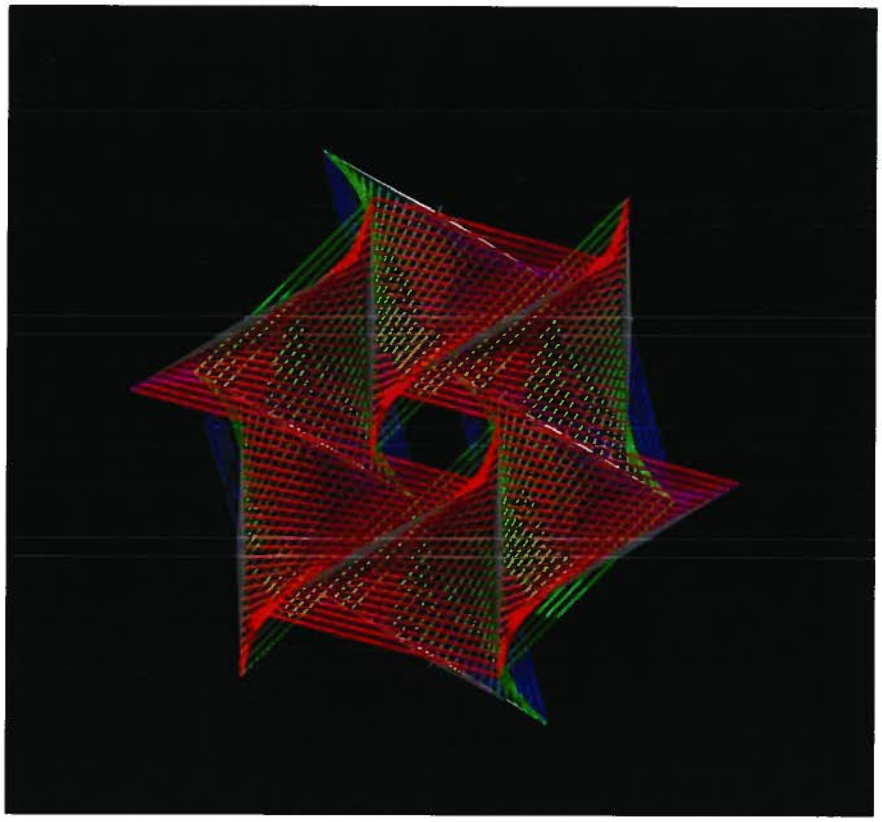


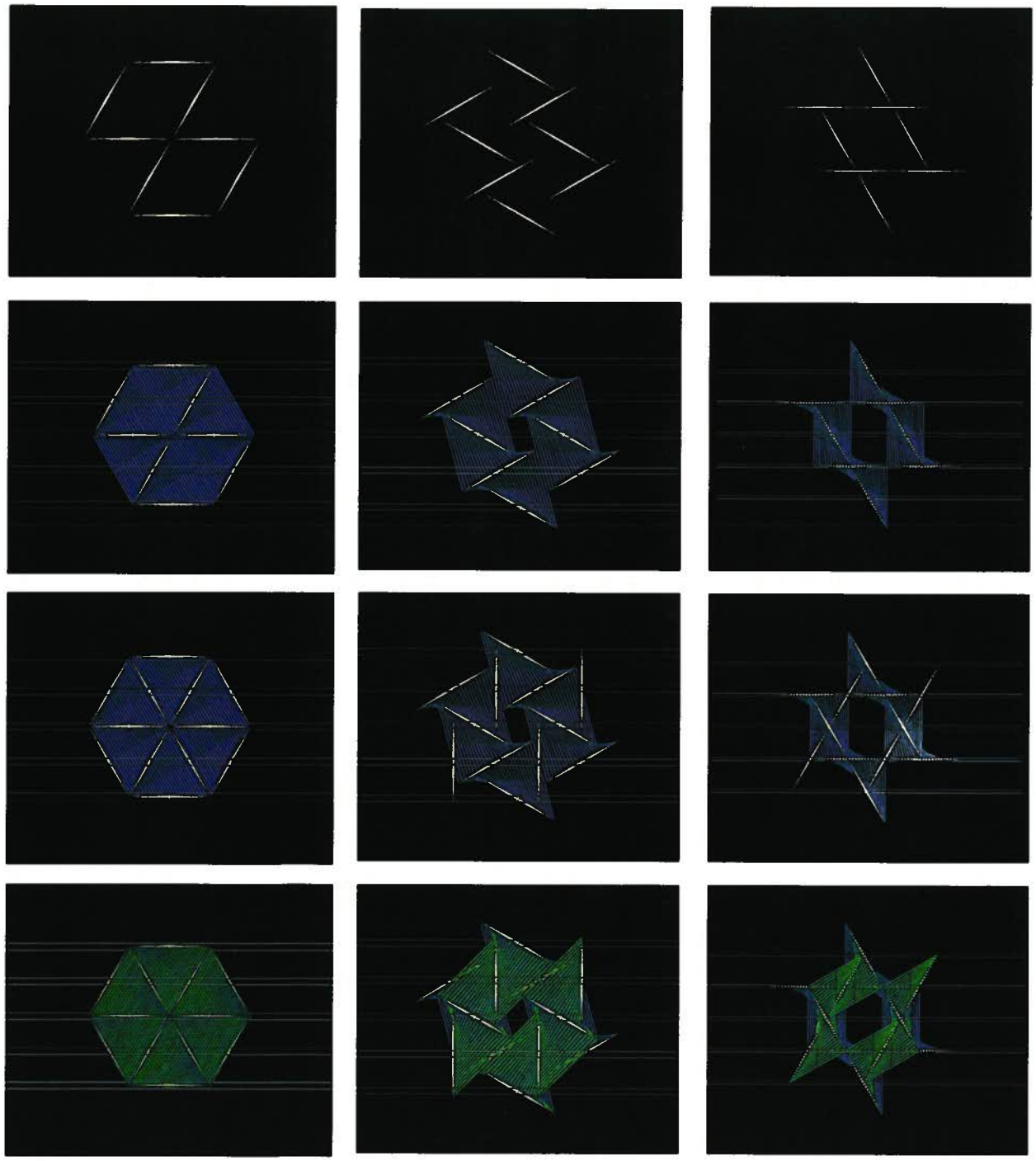
***Figure 33**: Hexagonal unit in three layers with moving cables.*

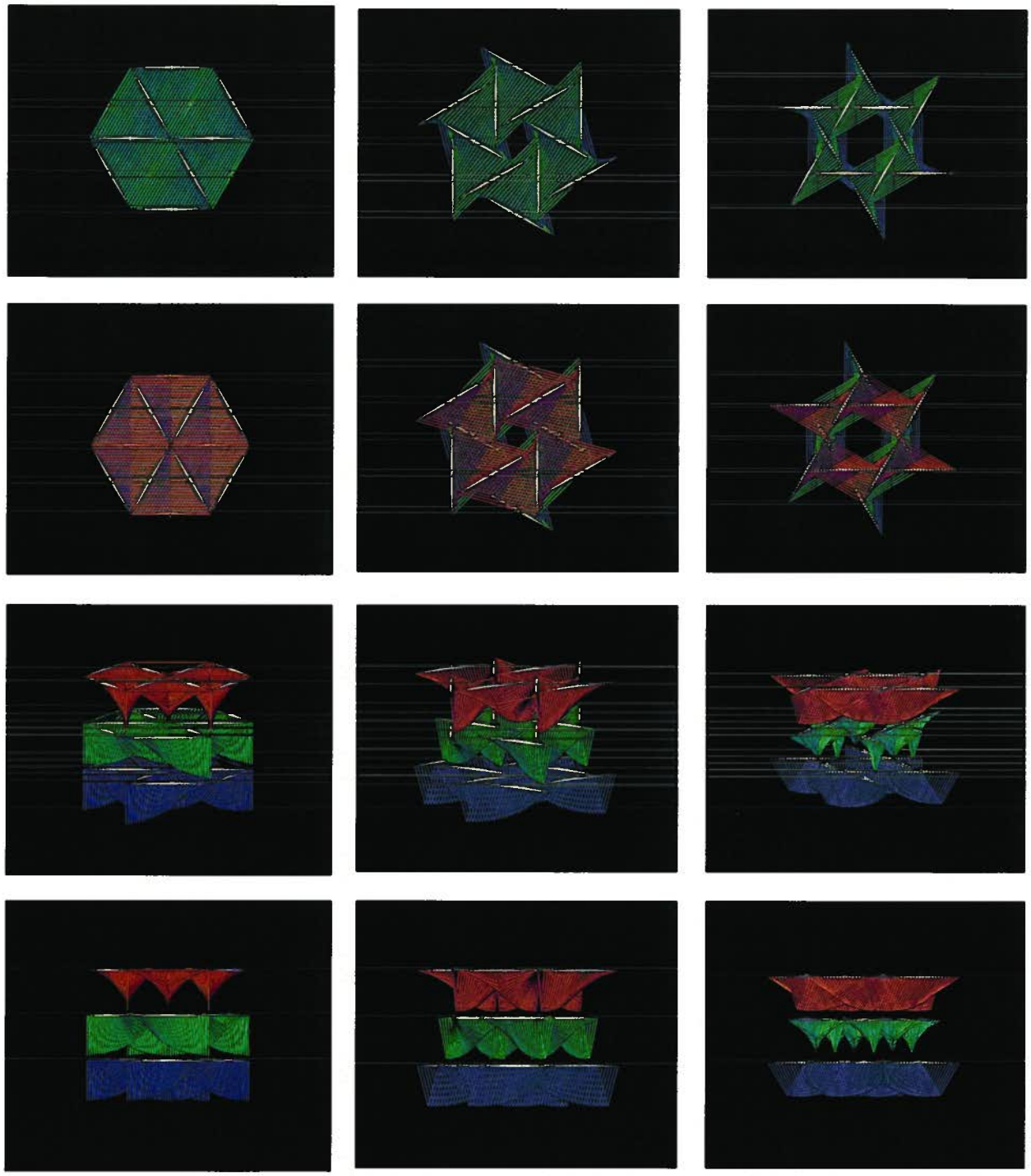
***Figure 34**: Hexagonal unit in three layers with moving cables and color.*



Below in three columns we can see six adjacent units of the design making a bigger hexagonal unit and the effects of the addition of different layers on top of each other with 0, 30 and 60 degrees of rotation applied to the arms (See fig.35).

***Figure 35**: Left column-The Hexagonal unit of the design in three layers; Middle column- 30 degrees rotation; Right column- 60 degrees rotation [This page and next page together].*





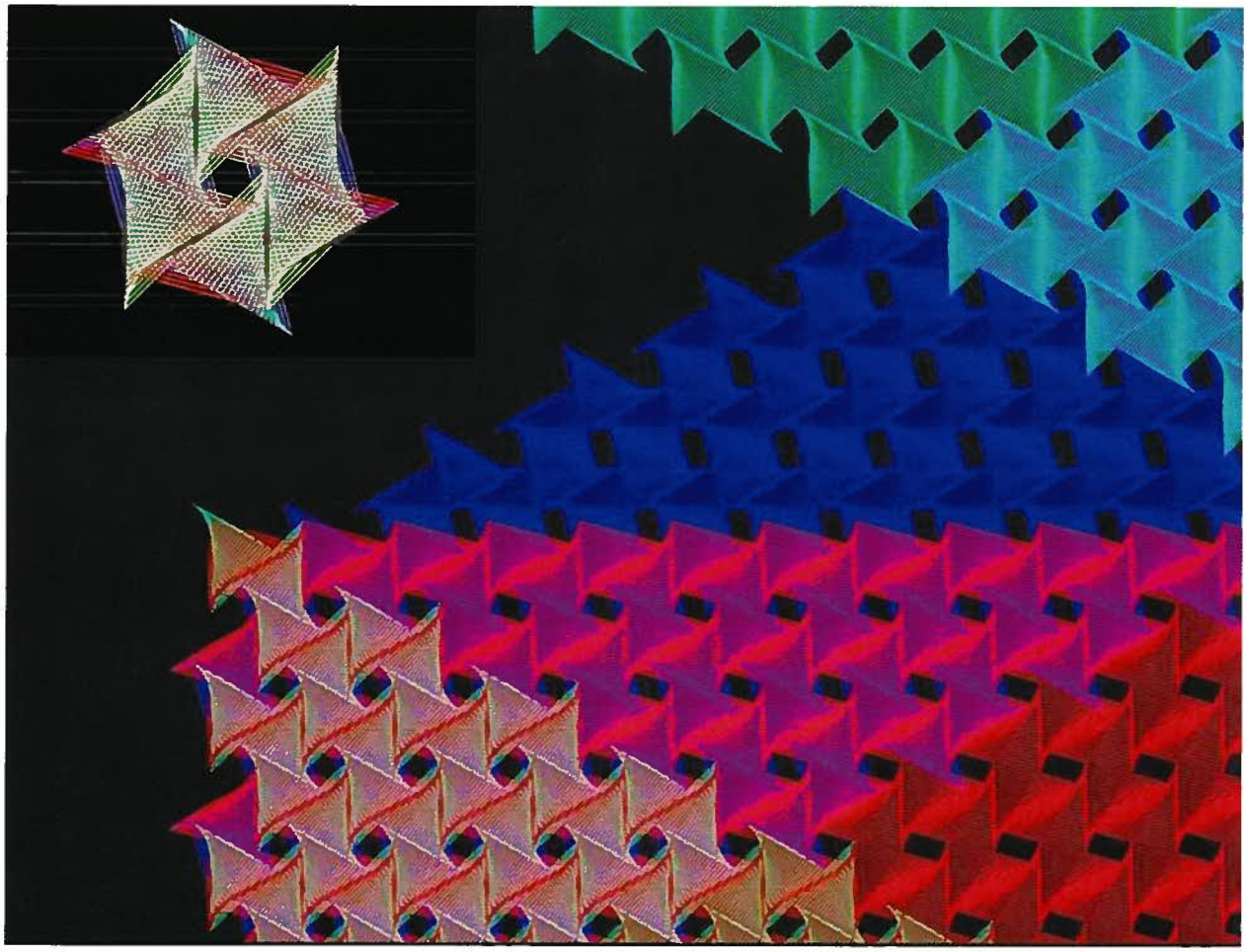


***Figure 36**: Design could be three layers on top of each other, two layers or even one layer, giving the possibility of different color mixtures.*

In the final design we chose to orient the pattern in such a way that we could use all the possible variations of having just one of the layers, or different combinations of two of them or all three layers. If the colors selected for the three layers are Red, Green and Blue (RGB system for colors), then red and blue mixing would make Magenta; Green and Blue mixing would make Cyan and Red and Green would make Yellow. The three layers together would make white. However when layers cross over top of each other only at the points where the isoparm members of two different layers cross each other, would that point's color be a mix of both layers' colors and in elsewhere the color of each layer would reveal itself causing a gradual change of color. This can be seen in a close-up view of the hexagonal unit in the illustration above. For a different result in color and light patterns, we could choose any three colors including the choice of Cyan, Magenta and Yellow (CMYK system for colors). We could also choose colors based on the colors of the context or place in which the design is going to be installed (See fig.36).

Design for the Movement and Persian carpet analogy

The dynamic rotational movement of the arms is created through the linear back and forth movements of a series of parallel cables that are attached to the arms (See fig.37).

For each arm there are two parallel cables, attached to the arm in two points exactly the same distance from the center of the arm but in different sides of the center. Now, if we pull the cables in opposite directions because the rotating arm is attached to them a rotational movement in the arm is created. A certain amount of linear movement of the cables would cause a certain amount of rotation in the arm to which they are attached, e.g. if we pull one cable 5 cm to the right and the other one 5 cm to the left then the arm to which the cables are attached would rotate for example 30 degrees clockwise and that would be the same in all the cables and arms throughout the grid.

For our purposes we wanted to be able to control all the pivoting arms at the same time with the same amount of pull and push. Because the amount of linear movement needed for a certain amount of rotation is related to the cosine of the angle of rotation rather the angle itself and the cosine curve is not a linear curve, and because of the fact that the arms that we wanted to control were set in 3 different directions (3 sides of a triangle); we decided to create a series of secondary small pieces attached to each pivoting arm to which the two parallel cables get attached. These secondary pieces unlike the arms (edges of the triangle) are parallel to each other on all the triangle edges and initially perpendicular to the direction of the moving cables; this is regardless of the direction of the arm itself. In this way a certain amount of linear movement in the cables would cause a certain amount of rotation in the secondary parallel pieces attached to the arms causing the arms to rotate exactly the same amount and also in the same direction, clockwise or counter-clockwise.

In shaping the final look of the primary arms and secondary pieces we shaped and thickened the middle of the primary and secondary arms leading to a point at each end of the arm, this was to create a better base for attaching the secondary small arms and also hiding them behind the thickness of the primary arms to reduce their visual impact on the design, avoiding an unnecessary layer of information that might have been too much for the audience to pick out.

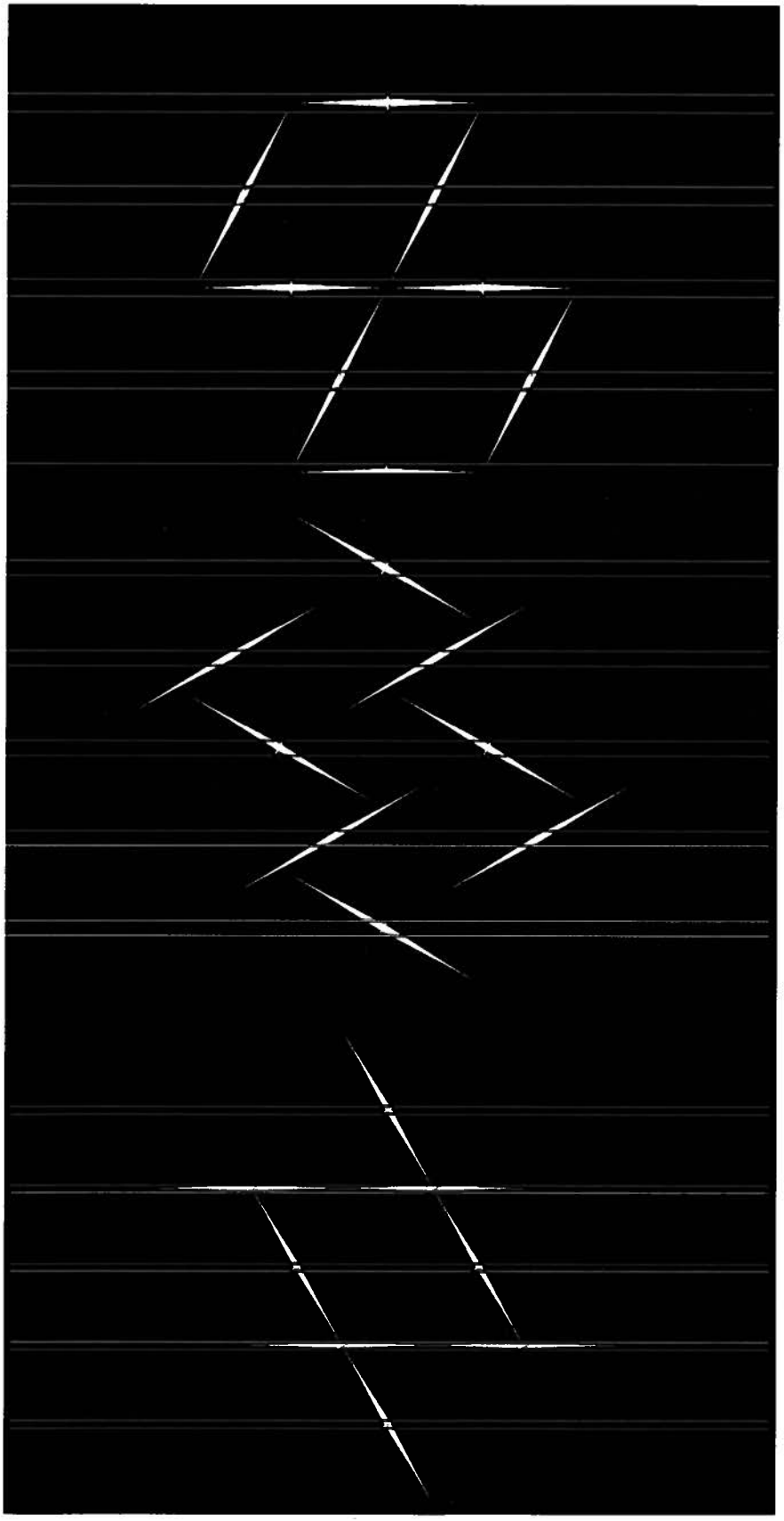


***Figure 37**: Moving structure. Consisting of Big arms, secondary parallel arms and parallel moving cables; Top: 0 degree rotation; Middle: 30 degree rotation; Bottom: 60 degree rotation.*

In this project the two moving cables passing through and attached to each rotating arm (by means of the small secondary pieces) are the same distance from the pivoting center of each arm and not located on the point of rotation, which would have made the act of rotation impossible.

"Even the most elementary particles of the atomic nucleus surround an un-manifest center."[8]

In the Architecture of the Muslim world the idea of the dome and its central peak as the point of origin versus the whole diversity of geometries under the dome resulting from the point, and the abstract nature of the point versus the reality of the geometries resulted from it, is an idea that we wanted to embed in the design. It first showed up in the rotational movement around a point and now it manifested itself in a different way: in order to effect the rotation and all the diversity that would result from it, we have to avoid the centers-the pivot points. They are like Zeros and, interestingly enough, the shorter the distance between the moving cables and the pivot centers, the bigger the angle the rotating arms will sweep with a smaller linear movement of the cable. However on the pivot point everything reduces zero. In our design, depending on what result we want to get, we can choose an appropriate distance between the cables and the centers.

While arms rotate the gap between the two parallel cables rotating each arm changes , which in its turn has impact on the final look of the design and also the shadows on the ground part of which shadows gets created by these cables as well as other dynamic elements. The bigger the openings of the design become the closer to each other the two parallel cables become.

In its final assembly the cables could be understood as the parallel warps in a carpet, upon which the beautiful carpet is going to get woven. The Warps in a carpet or a Kilim are the structure of it that holds it together. In this design we also will have static structural cables distinct from the moving cables that will carry the weight of the assembly and keep the pivot points in place. However, these static cables are not the warps of the flying carpet we are designing, the static character of the warps is changed here in our design, and the moving cables are the dynamic warps of our dynamic carpet. Their movement causes a constantly changing 'flying' carpet (See fig.38).

---

[8] Critchlow 9.

Part of Figure 38 has been removed due to copyright restrictions. The information removed is the picture of a Turkish woman weaving Kilim and the intention was to compare sarpet waeving and the elements of the carpet, like the parallel warps and colorful weaving with our project. The source was:
Wood, Rob, clobber photography,Weaver: Turkish village woman hard at work weaving a rug. Goreme, Turkey, 2005; 29 Sep 2008 <http://topendsports.com/photography/gallery/people/turkish_weaver?full=1>

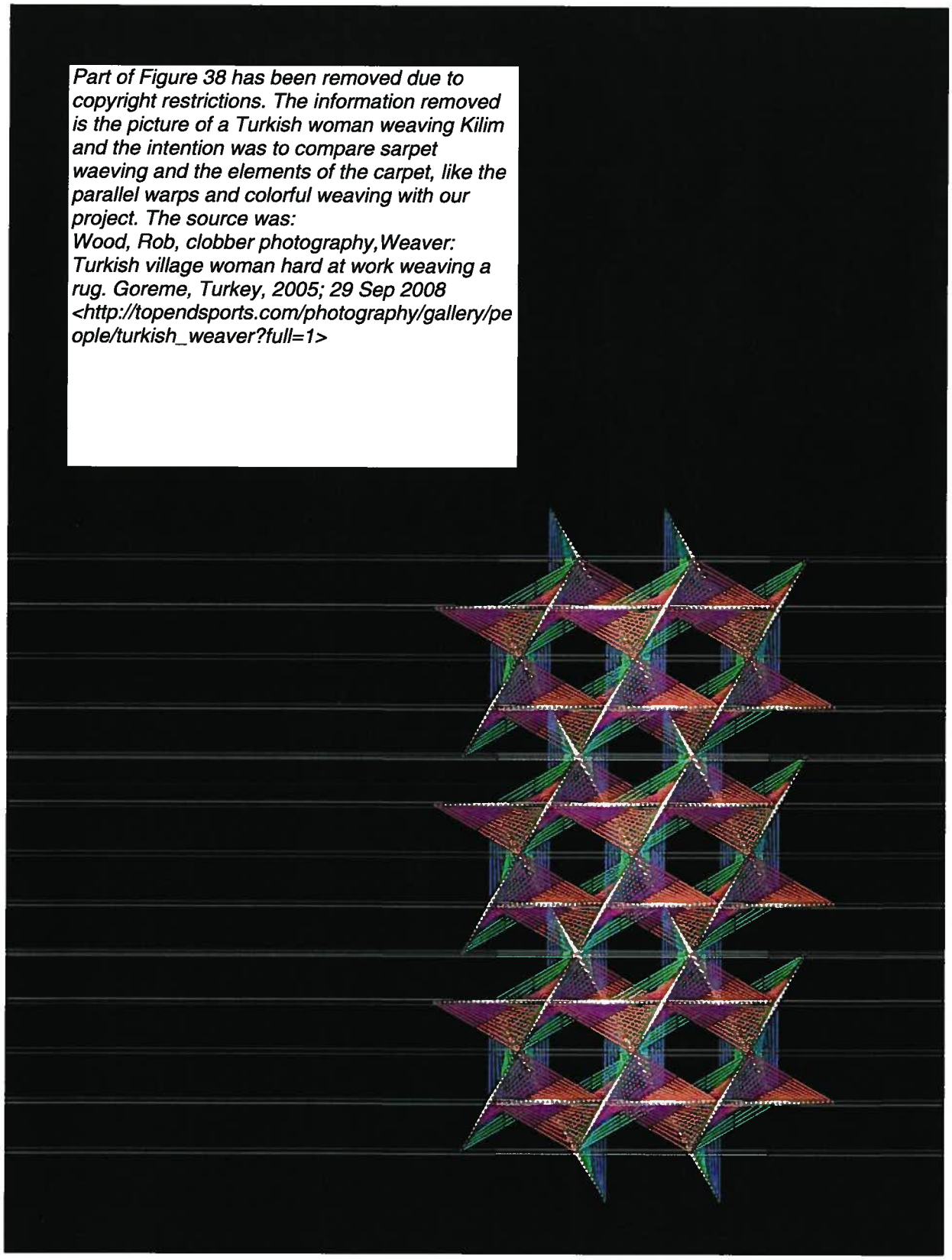


***Figure 38**: The moving cables become like the warps in a carpet or Kilim. On top of which the colorful translucent material get woven.*

CHAPTER 4 CONTEXT AND PLACE; CONSTRUCTION AND DETAIL

Place

With all the ideas explored in the design process we chose to solidify the design in the form of a roof, a horizontal colorful three dimensional sunshade, over an open pedestrian access to the historical Maydan Naghshe' Jahan in Isfahan, Iran (See figs. 39,40,41).

Isfahan is sometimes referred to as the cultural capital of the Muslim world and Maydan Naghshe Jahan is one the largest historical city squares in the world designated as world heritage. The Maydan has a rectangular shape and is surrounded by small shops on all four sides which usually create or sell traditional goods, from carpets and metal dishes to traditional sweets and confectionery. There are four landmark buildings beside the Maydan, two of which are mosques, namely Masjid Imam and Masjid Shaykh Lotfollāh. Opposite to the Masjid Shaykh Lotfollāh on the other long edge of the Maydan is the entrance to the Safavid era palaces, The entrance building is called Ali Qapu. Opposite to the Masjid Imam mosque on one the shorter sides of the Maydan is the entrance to the Bāzār, the large traditional market of Isfahan. Many of the photos used in this entire report are actually of these buildings close to which our roof design will be located.

Pedestrian access to the Maydan is located between Ali Qapu and Masjid Imam and is the only uncovered pedestrian access into the Maydan. Despite a recent renovation of the walls of this pedestrian route, the route is not often used by the people of the city. Furthermore, tourists, who would typically get out of their cabs inside the Maydan through the vehicular accesses to the Maydan, seldom experience the symphonic hierarchy of spaces that can be experienced when entering the Maydan (See figs.42,43,44).

The objective of this design, therefore, is to achieve several goals by defining this colorful modern sunshade on a symbolic part of the pedestrian route. Firstly, it would give some interest and rouse people's natural curiosity to rediscover the use of this pedestrian route. Secondly, it would mark the entrance to the Research center for Islamic Arts located beside and accessed from the route. This could potentially inspire, sparkle and open up new possibilities for many students and researchers who would enter and exit the institute as well as those who would walk beneath it: The possibility of new possible ways we can look at creating local Architecture or think through the tradition to get an answer for contemporary. By having the design be located near the great historical buildings it would be more contextually grounded and further motivate the new possible ways that the conversation between 'Modern' and 'Traditional' could take place (See figs. 45-52).

Figure 40 has been removed due to copyright restrictions. The information removed is the Aerial photo of Maydān Naghshé Jahān, comparing it to a carpet in the texture of Isfahan. The source was: Google Maps, 30 Sep 2008 <http://maps.google.ca/maps?client=firefox-a&rls=org.mozilla:en-US:official&hl=en&tab=wl>

Part of Figure 42 has been removed due to copyright restrictions. The information removed is the plan of Maydan and Bazaar, specifying the site of the project. The source was: Ardalan, N., Bakhtiar L., The Sense of Unity, The Sufi Tradition in Persian Architecture, The University of Chicago, 1975, page 98.

Figure 43 has been removed due to copyright restrictions. The information removed is the Aerial photo of pedestrian access between Masjid Imam and Auli Qapu (the palace). It showed the placing of the project in site if viewed from above. The source was: Google Maps, 30 Sep 2008 <http://maps.google.ca/maps?client=firefox-a&rls=org.mozilla:en-US:official&hl=en&tab=wl>

***Figure 39**: Top Left: Maydān Naghshé Jahān from Bāzār entrance. © Arad Mojtahedi, adaptation is by permission of copyright holder: Arad Mojtahedi.*
***Figure 40**: Top Right: Aerial photo of Maydān Naghshé Jahān, it looks like a carpet itself.*
***Figure 41**: Middle Left: Masjid Imam beside Maydān Naghshé Jahān.*
***Figure 42**: Middle Right: right: Masjid Imam Entrance; left: Maydan's plan, gray frame is project site.*
***Figure 43**: Bottom Left: Project site + design, pedestrian access between Masjid Imam and Auli Qapu.*
***Figure 44**: Bottom Right: Pedestrian access view from Maydān, the project site.*

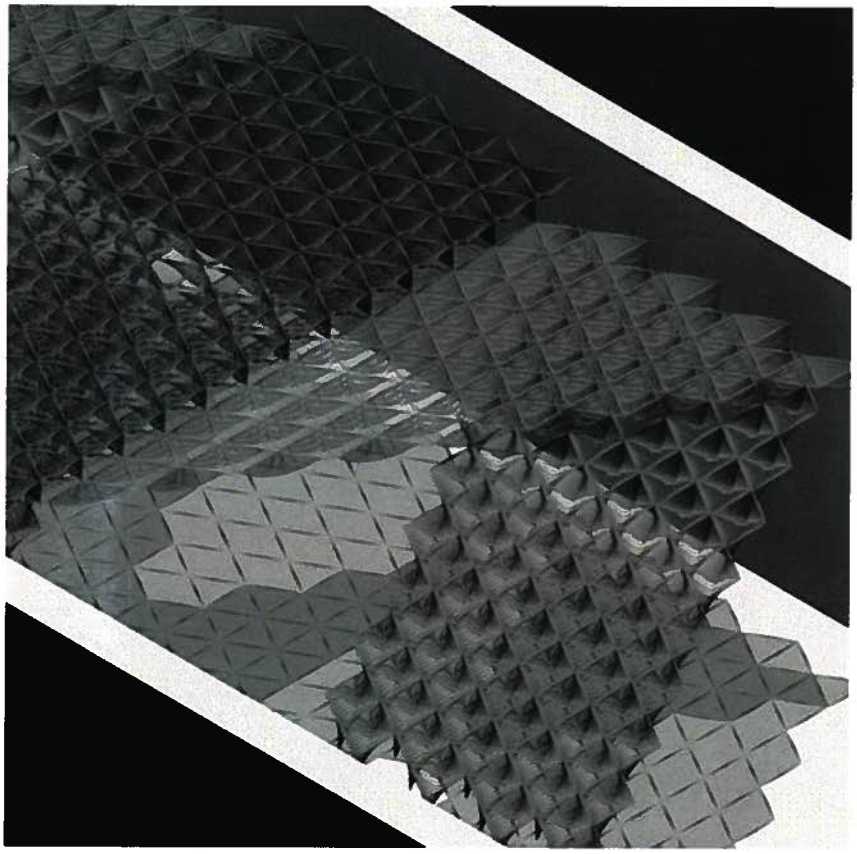


***Figure 45**: Primary look of the design with shadow and 0 degree rotation.*

***Figure 46:** Two strips of Green, Two strips of Blue, Red defines entrance.*

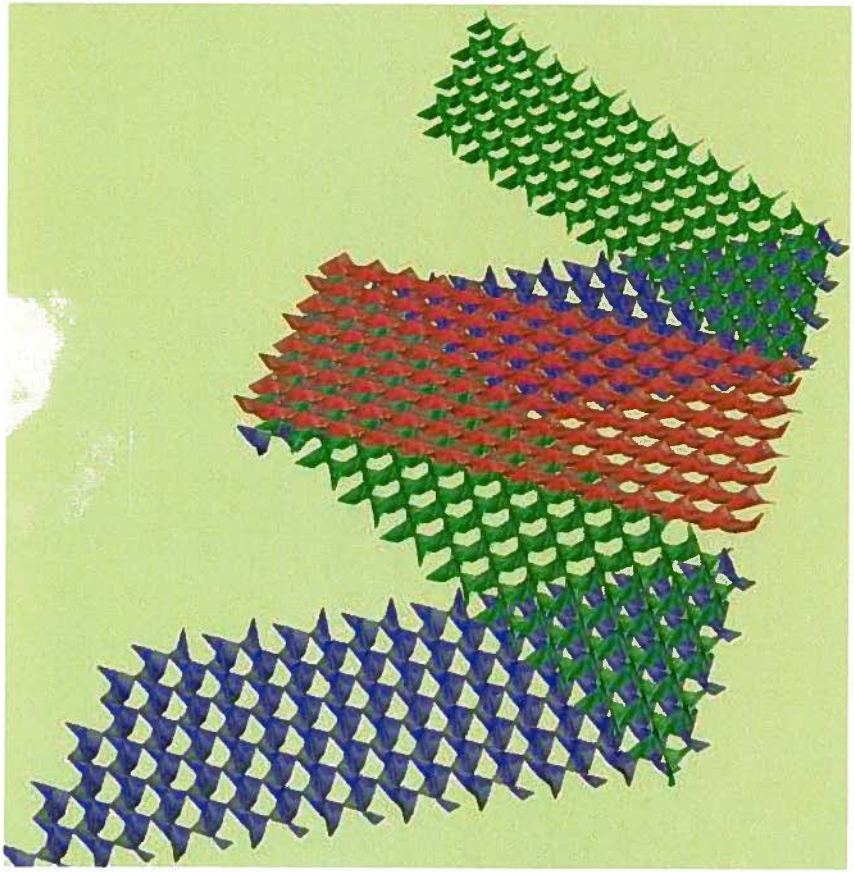
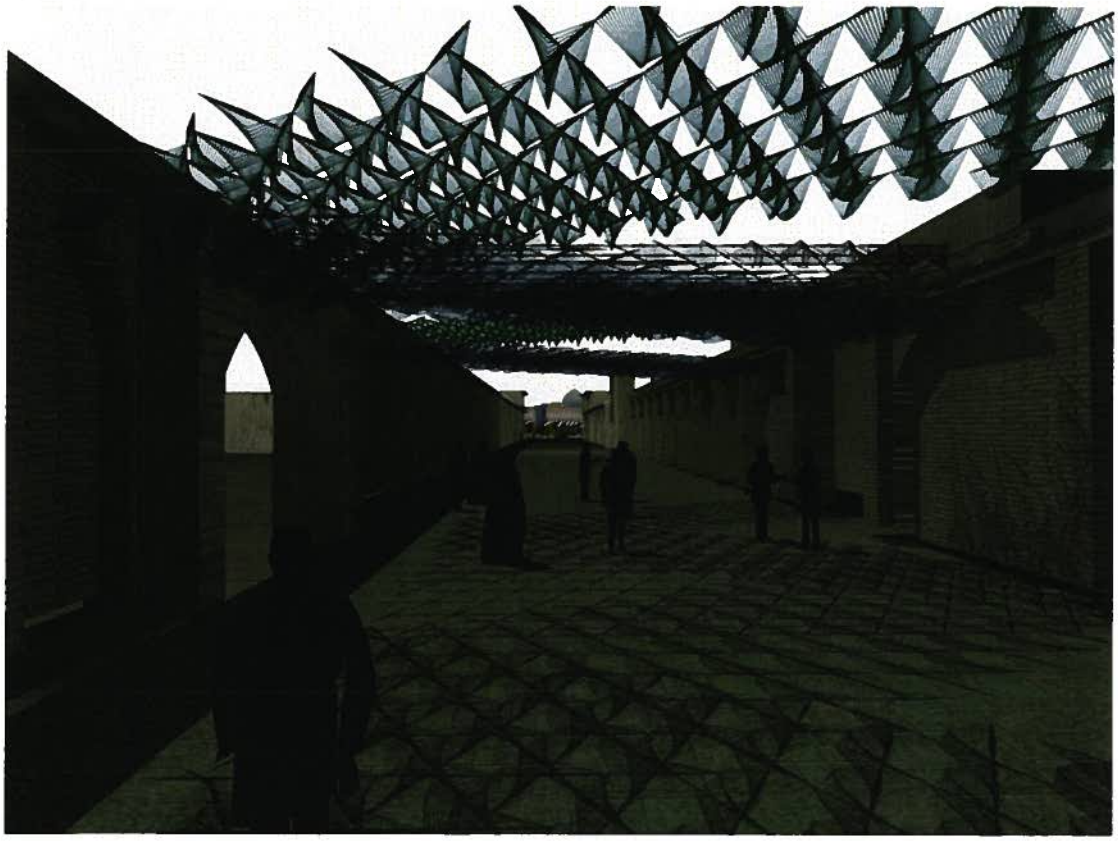


***Figure 47**: Towards Shaykh Lotfollāh.*

***Figure 48**: Under the roof towards Shaykh Lotfollāh. Notice the shadows on the ground.*



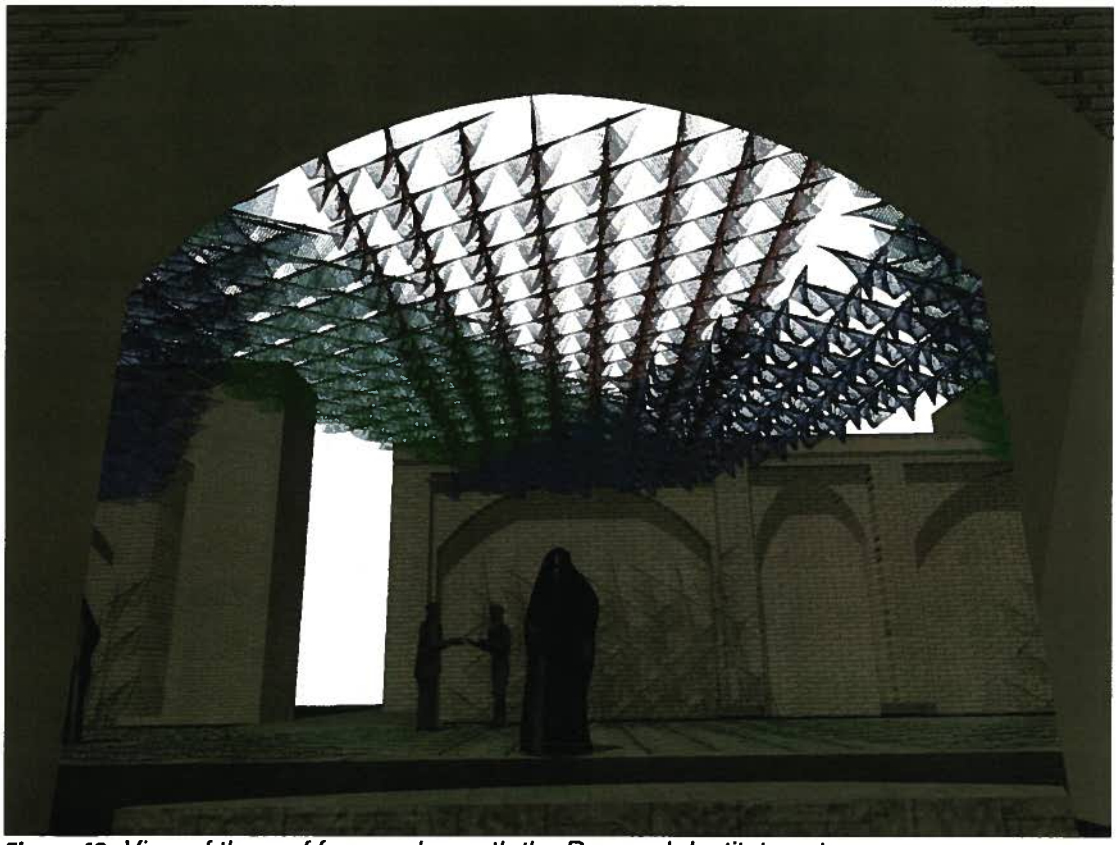


***Figure 49**: View of the roof from underneath the Research Institute entrance.*

***Figure 50**: View through the Research Institute entrance.*

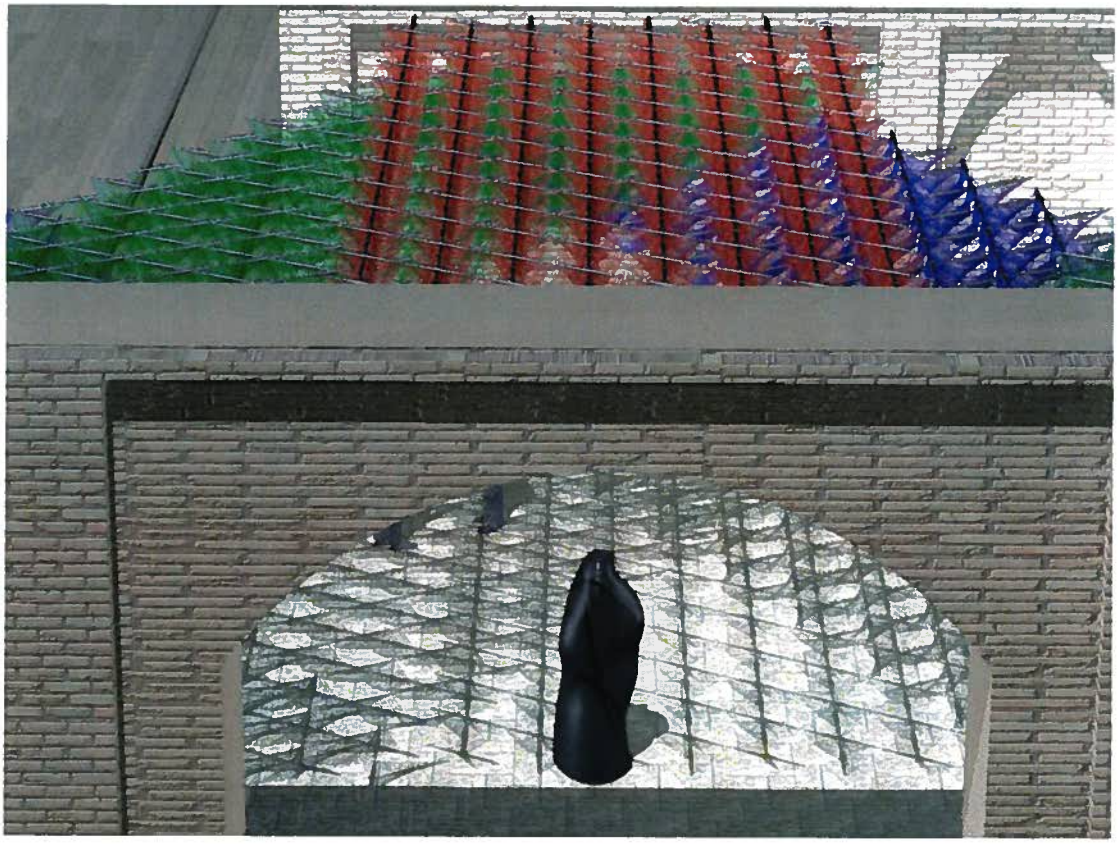
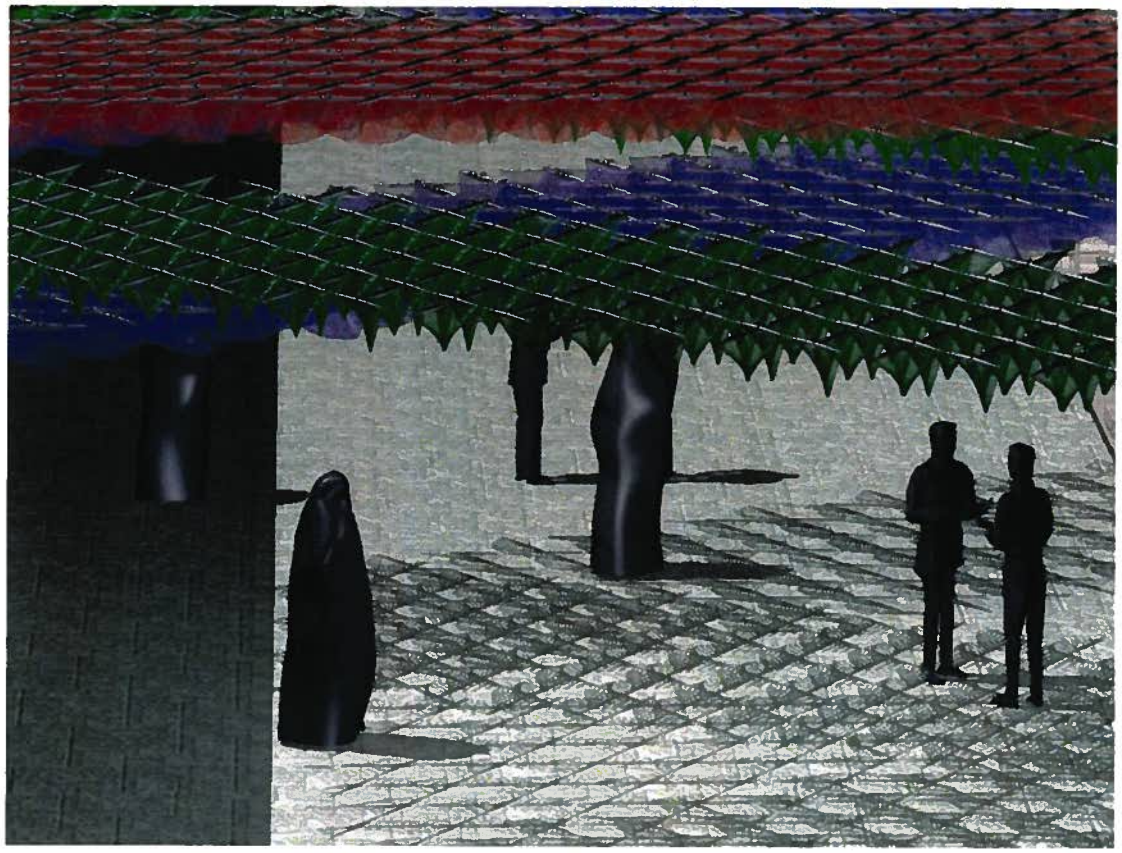


***Figure 51**: The shadows and the roof.*

***Figure 52**: The shadows and the roof.*



Detail

In designing the roof one of the things that became evident is that whatever we do in the design of one triangular unit would be multiplied by the number of triangle units in the entire roof. Therefore, for construction purposes, we have to keep in mind that every small detail that we might add, its weight is going to multiply by the number of the units. Our intention is to have a light weight roof and that has an impact on the design of the roof. We want to use hollow shaped materials for the moving parts so that we can still shape them without unnecessary extra mass.

At the same time we want to make sure that our roof as a whole has a quality that saves it from possible damage by wind forces. Hollow materials would assist here by allowing wind to pass through them. That is also one of the reasons for having the translucent surfaces as parallel translucent strips with hollow parts in between them to let the wind pass through the gaps.

We need to have universal joints at the end of the translucent strips where they get attached to the rotating arms (See fig.53). A universal joint at the end of the strips connecting to the arm would allow for the strips to be able to freely rotate in the 3-D space regardless of the arms direction. For the universal joints the most inexpensive solution would be creating knots at the end of the strips. The end part of each strip with is shaped thinner and this part after passing through a narrow slot in the arm makes a big knot inside the moving arm in a way that the knot can't come out of the slot. The person who is weaving these strips to the arms will pull the narrow end of the strip through the slot up to the wide part of the strip. Then he makes a knot out of the thin part of the strip and tightens it up right behind the slot and if necessary cuts the extra ends of the strip; then he goes to the next strip and does the same. Depending on the design of the whole roof, the color of each strip can be different. This whole process of weaving with different colors of strip and passing the strip through the slot, making a knot and then cutting the extras is very closely similar to the process that Persian-carpet weavers use. After we are done with one end of each strip we do the same thing with the other end of the strip and connect it to the second rotating arm (See fig.53).

Then we start to add the small centre weights that will stretch down and shape our strips. The nature of these weights is important. They can be very light weight, like key rings, or safety pins or even more shaped ones that could be made in the bazaar of Isfahan by metal-shops as part of using the local sources for the project and getting the local people involved and interested in its final production (See fig.53).

There are vertical structural hangers hanging from a net of structural cables to hold the centers of the rotating arms in place and relative to each other. When necessary hangers pass through the pivoting center of one arm to reach the arms in lower layers meanwhile holding up the upper layer arms with ball-bearing connections to maintain the rotation of the arm and still hold it in place. (See fig.53)

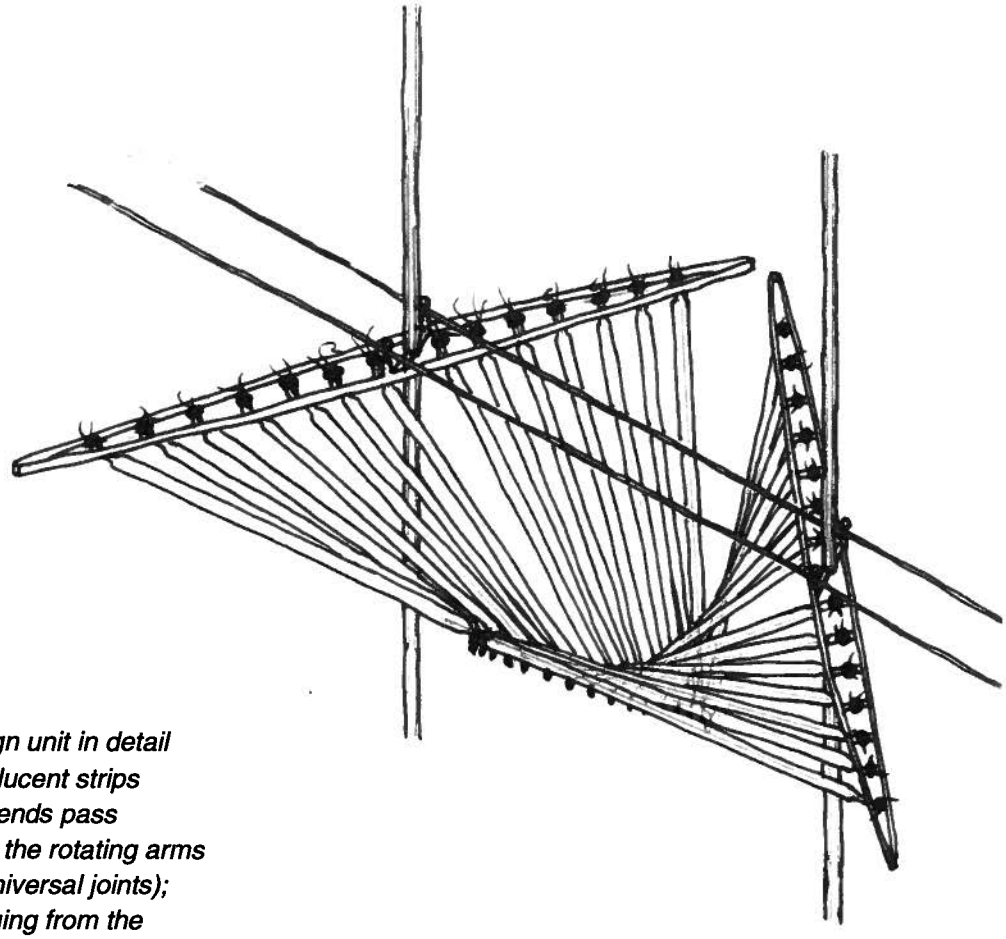


***Figure 53**: The design unit in detail shows: Wide translucent strips which their narrow ends pass through the slots in the rotating arms and make knots (universal joints); Small weights hanging from the middle could be bought from the bazaar of Isfahan; The moving cables; the small secondary arms and the hangers.*

The movements for each layer of the roof are controlled by two moving bars parallel to the walls of the alley moving towards or away from each other (See fig.54). When the bars move towards each other (that is towards the center of the alley) they push one of the wings of the small rotating pin. The center of this pin is fixed, and as a result of that push towards one of the wings it would rotate in its place like a compass. Each of the two wings is attached to a cable and this rotation will cause a pull or push in the cables. These cables as we saw before pass through

and are attached to the small secondary arms and the secondary arms are attached to the primary rotating arms. The end rotating pins as discussed now are parallel to the secondary arms. When these end pins of each row close to each wall rotate one end of the pin pulls one of the cables towards the wall and the other cable gets pulled from the pin on the other end of the row on the other side of the alley and because the length of cable between where it is attached to secondary arms is always the same and all the moving secondary arms and the end pins are parallel to each other all the small secondary arms that cause the rotation of the primary arms start to rotate the same amount and still remain parallel to each other. This means that the primary rotating arms with different initial directions rotate with the same amount and angle. As we saw we made the linear movement of the parallel bars into a rotational movement of the end pins, then to again the linear movement of the parallel cables and from there again to the rotational movement of the secondary arms (See fig.54).

When the two long bars situated next to the walls for each layer of the roof move far from each other the pins with the kind of spring-like character that they have and also because of the magnets that are attached to the end of the pins would rotate backwards and cause the roof holes to open. The release point is where the roof is totally closed and there is no tension (See fig.54).

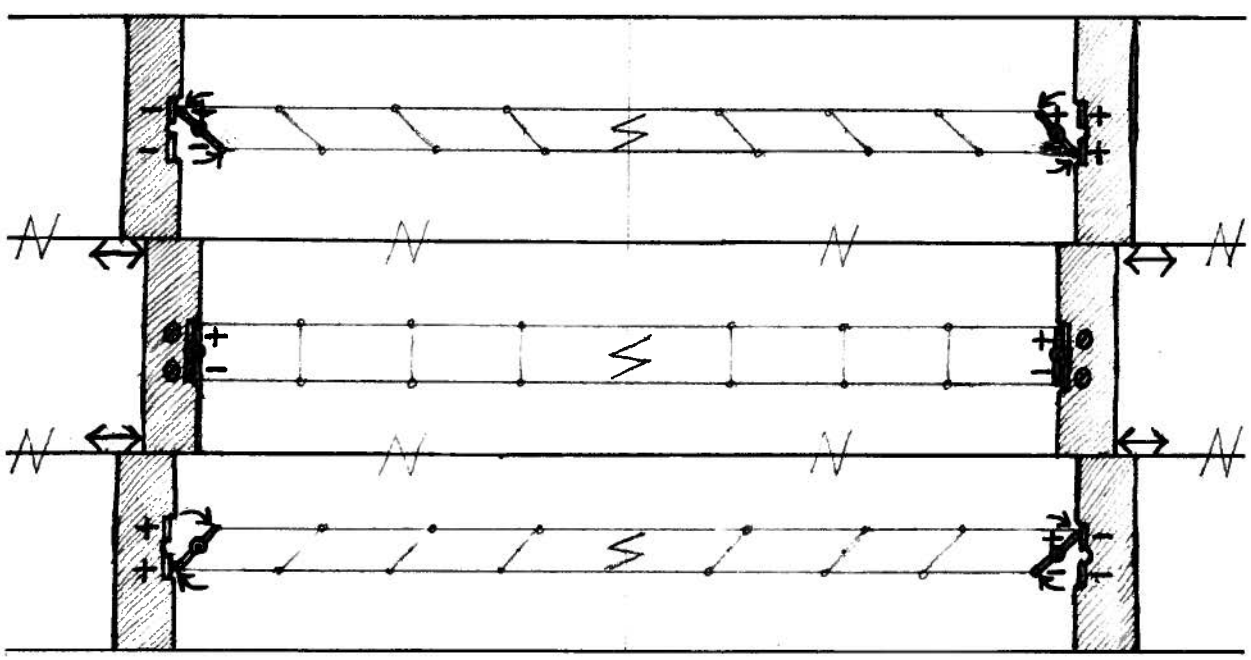


***Figure 54**: The mechanism of movement in the end pins from above. When the bars move away the roof opens. The pins have magnets at both ends; the moving bars also as shown can be magnetized for further control.*

The structural support and the mechanism for the roof opening and closing are located at the two sides of the alley. The structural net of cables that the roof is hanging form is anchored beyond the two sides of the alley to a secondary structure behind the walls. Also the dynamic cables causing the rotation for each layer of the roof would come from the sides of the alley. The sides of the alley and also the structure behind both play the role of the frame that holds the warps in weaving a carpet. Moving cables in turn, could be operated by the small electric engines installed out of sight that are ultimately controlled with computer program. The dynamic performance of the piece could be further extended if we deploy sensors and actuators responding dynamically to diverse environmental or cultural conditions on the site (See fig.55).

Because of the 3 dimensional shapes of the surfaces the drop shadows become quite complex. However the interesting thing is that the shadows are still repetitive and create different complex patterns on the ground and the walls before they meet the roof. These patterns would also be different at different times of the day and on different days of the year giving more transformational and dynamic character to the design (See figs.56-63).

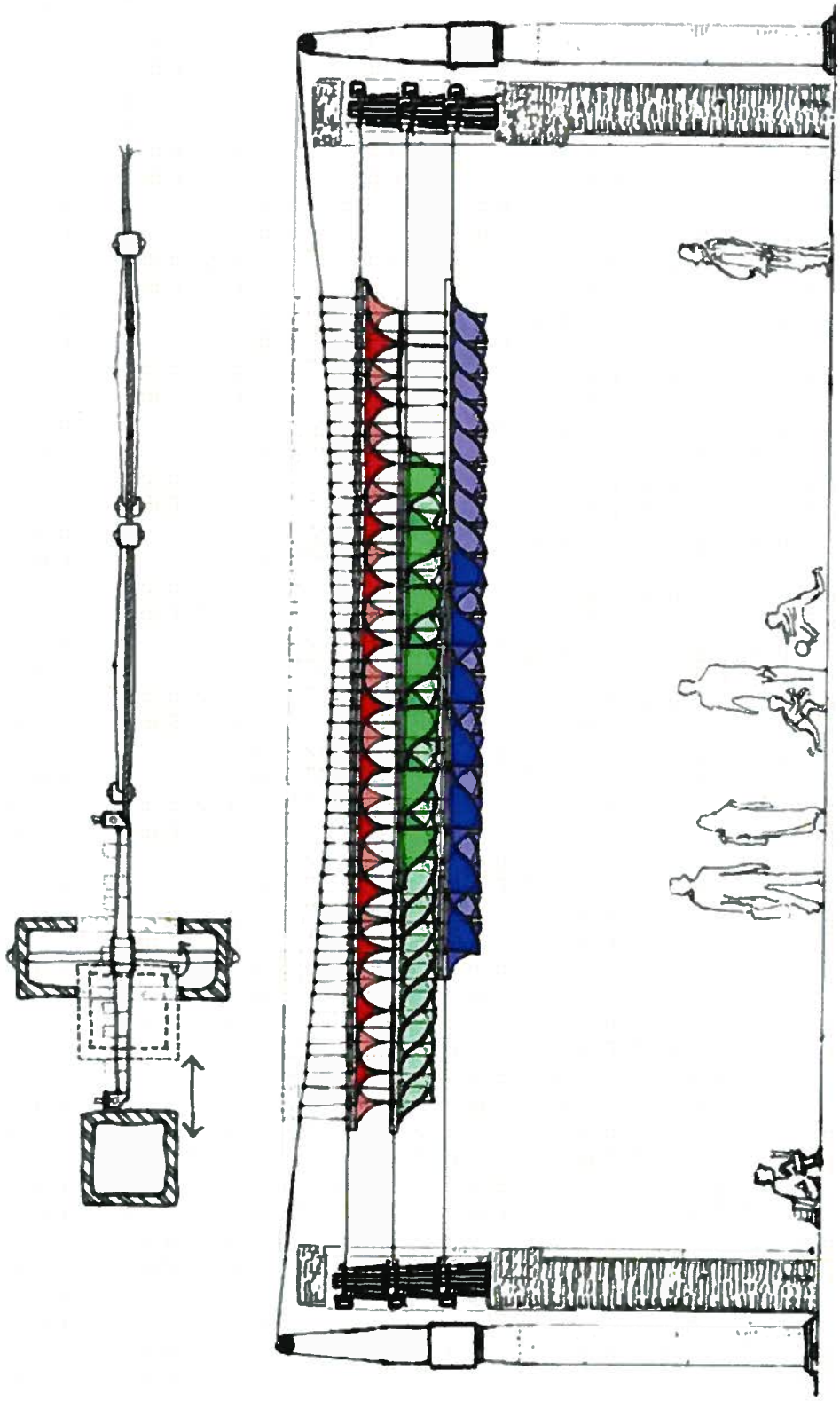


Figure 55: *Section: Roof at its closed status. Detail: How the moving parts work.*

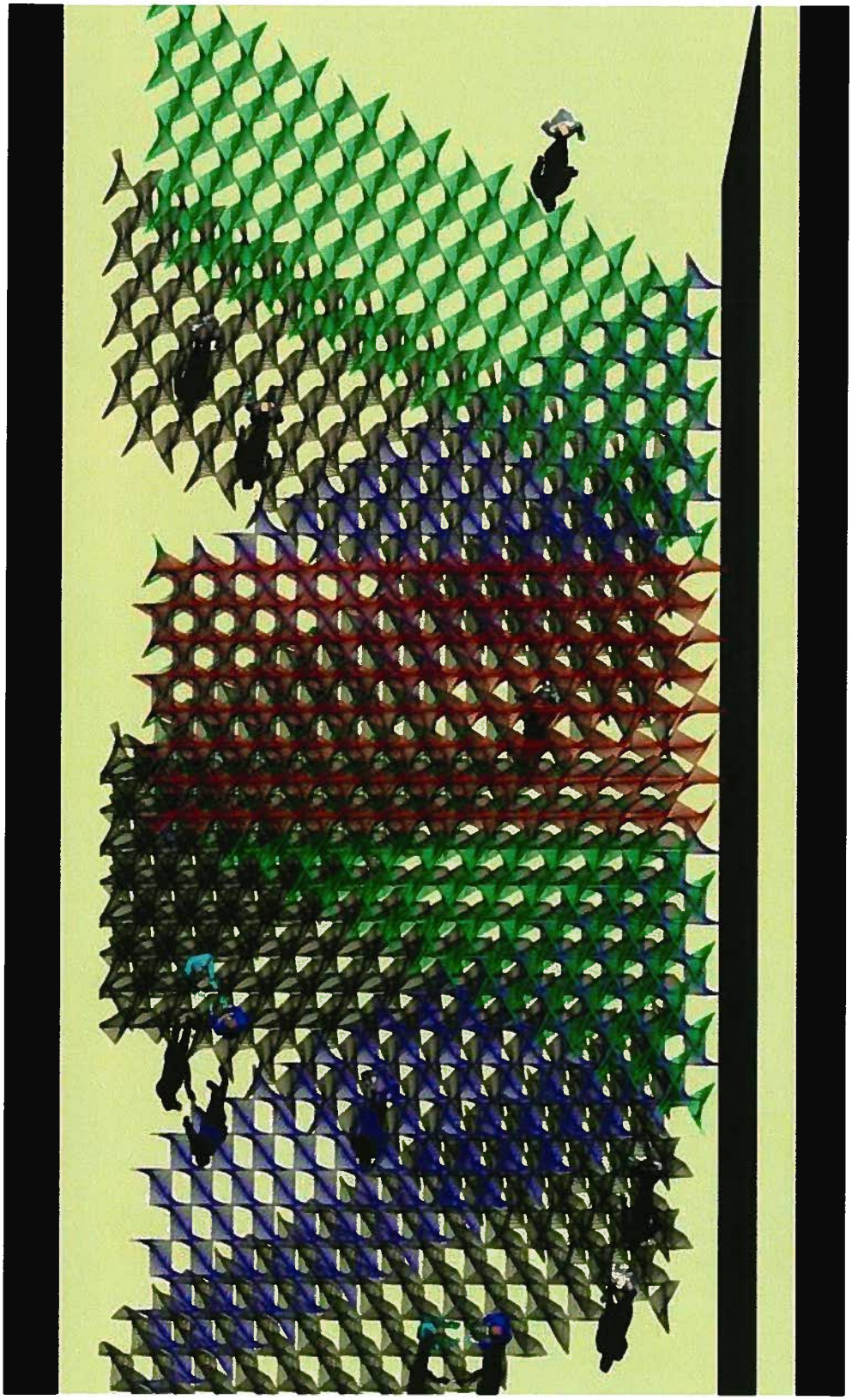


Figure 56: *Roof, Shadows and People; Roof is 60 degrees open at its maximum openness.*

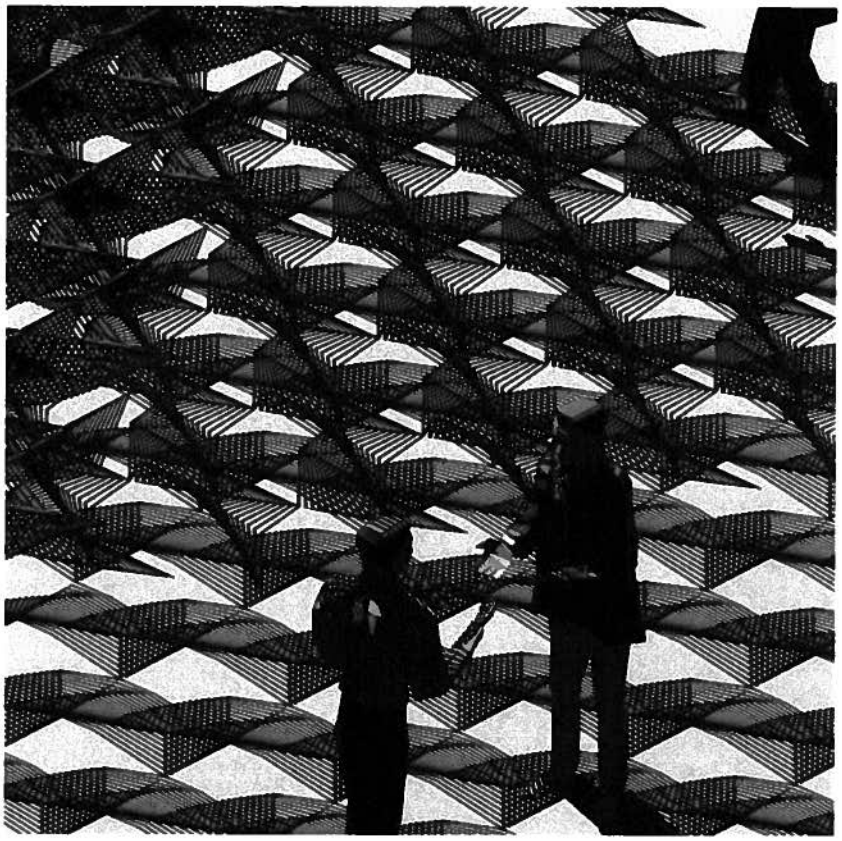
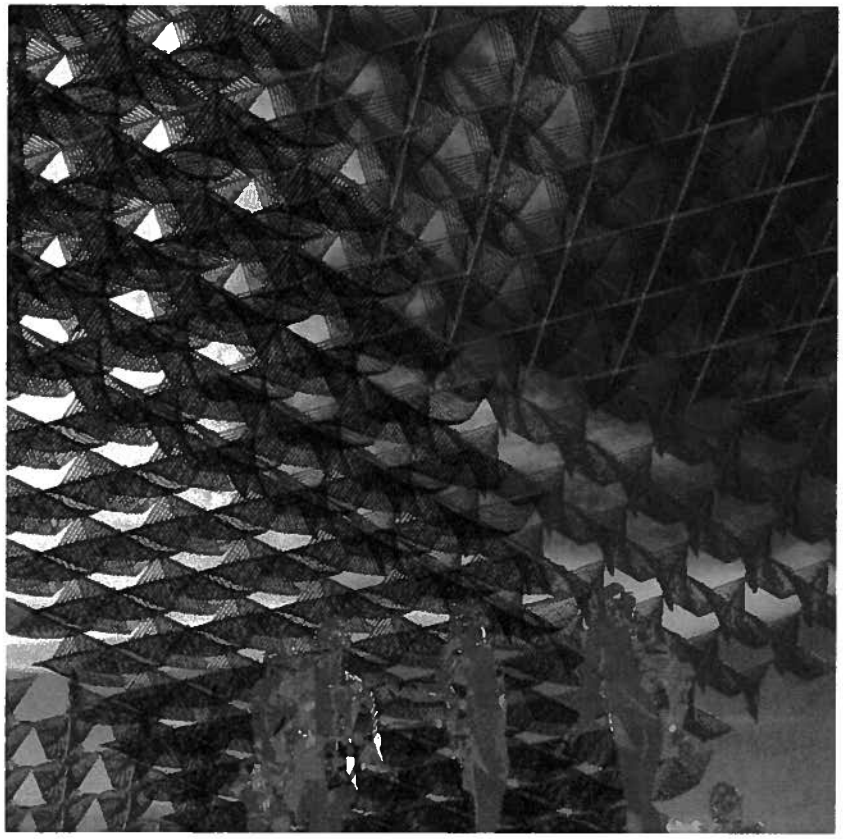


Figure 57: *Shadows- Bottom: 1 layer roof shadow, Middle: 2 layer and Top Left: 3 Layer shadow.*
Figure 58: *Under the roof towards sky- Roof 60 degrees open.*



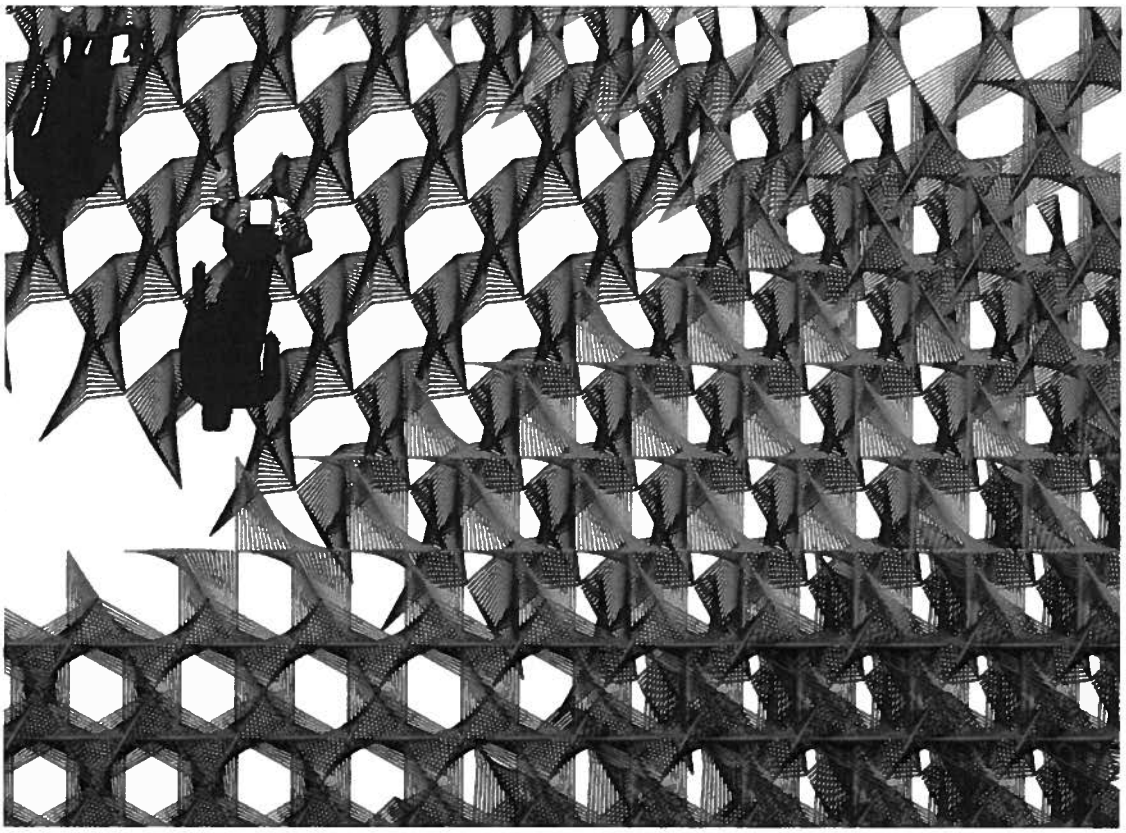
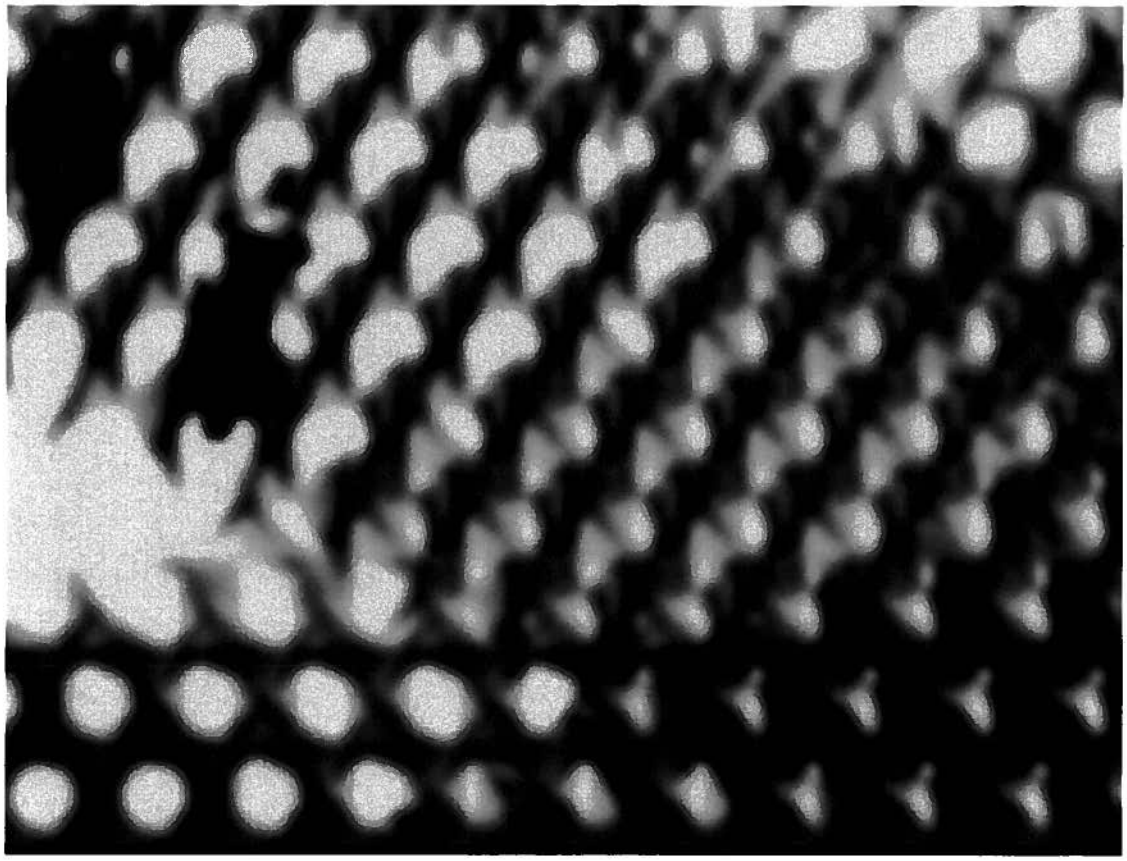


Figure 59: *Roof and its shadows- 60 degrees open.*

Figure 60: *Roof and its shadows in reality- 60 degrees open.*



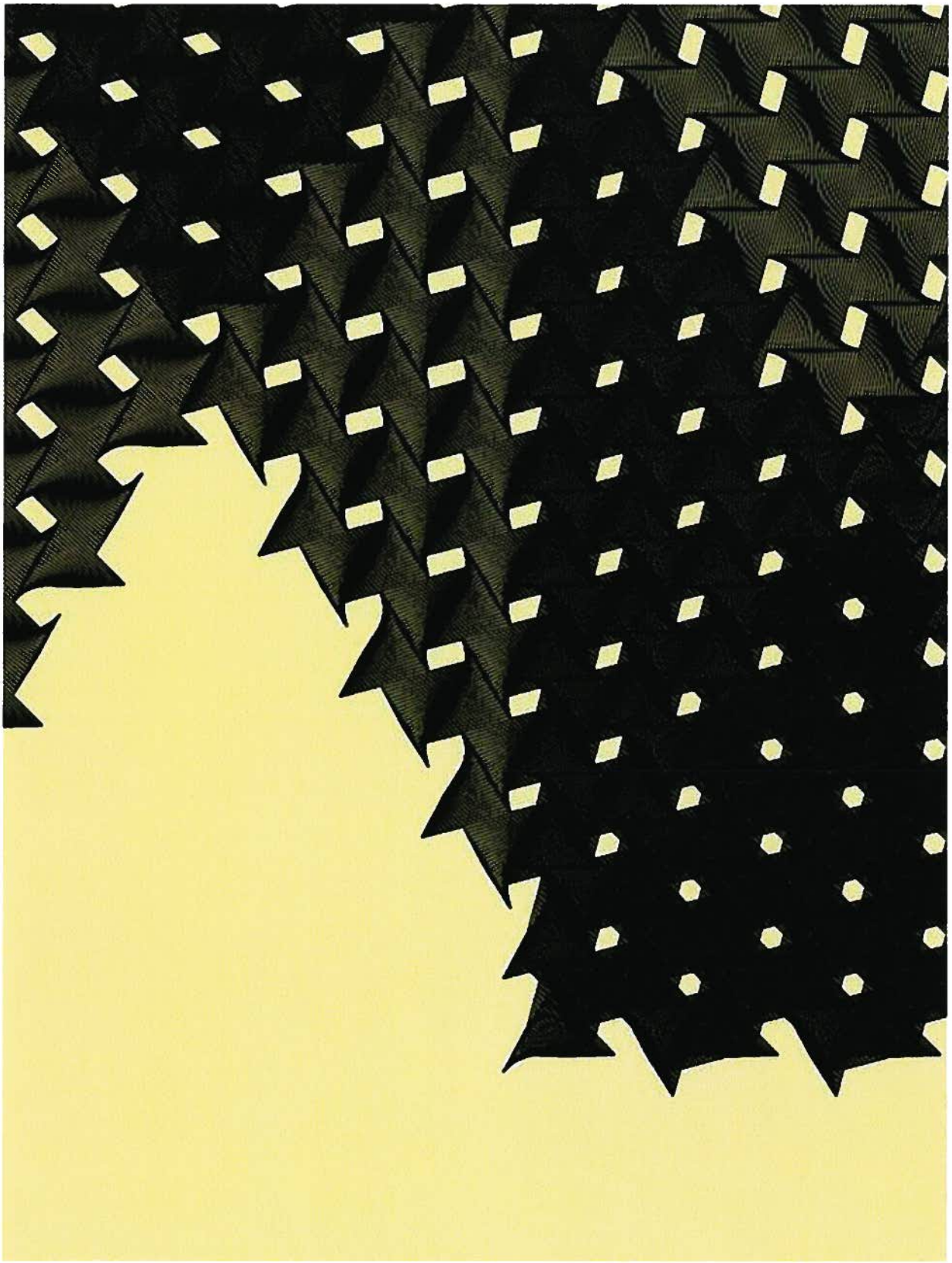


Figure 61: *The gradual change in the planar shape of the openings from rectangle to rhombus and finally to hexagon.*

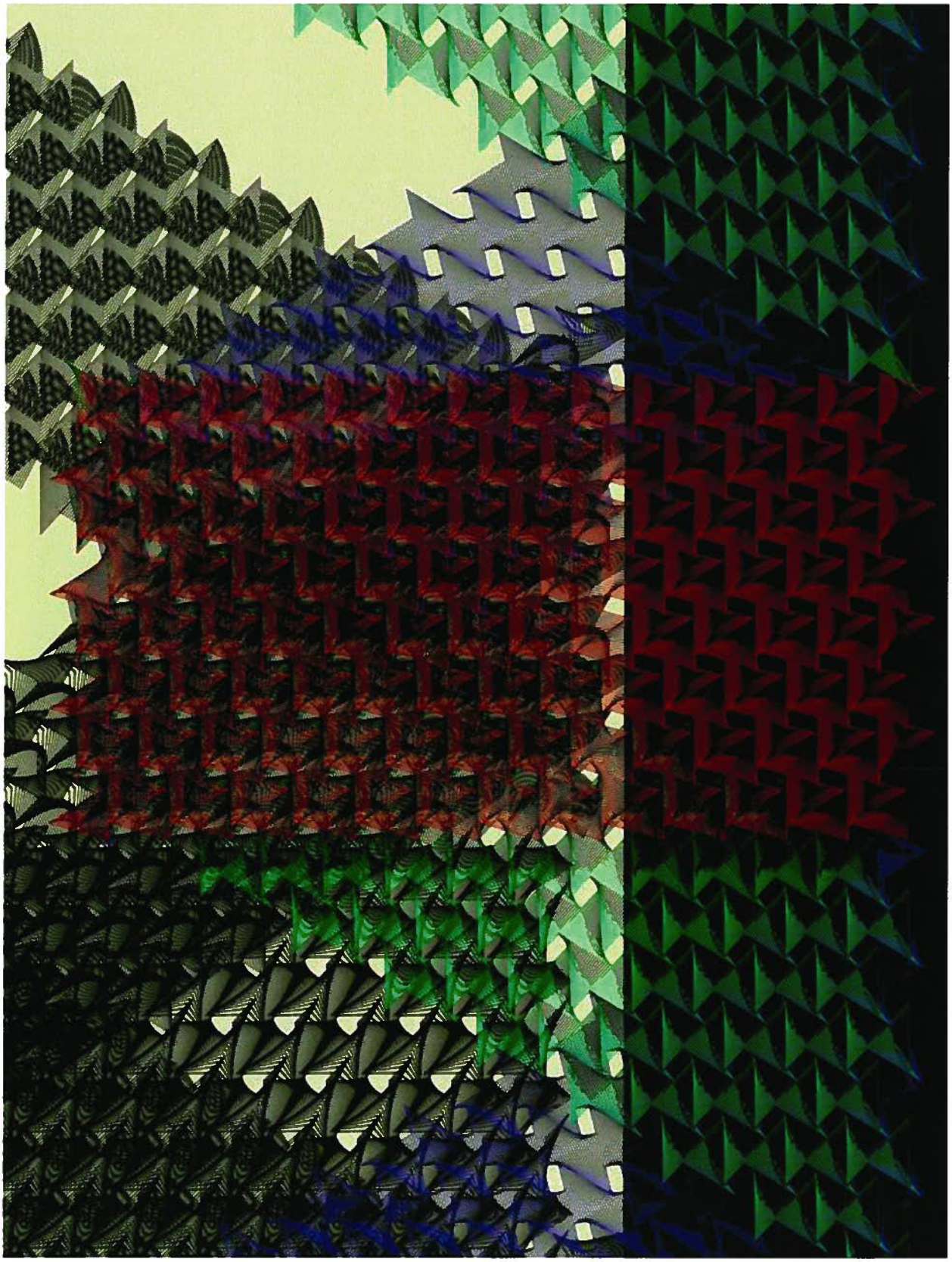


***Figure 62**: Roof and its shadows- 30 degrees open.*

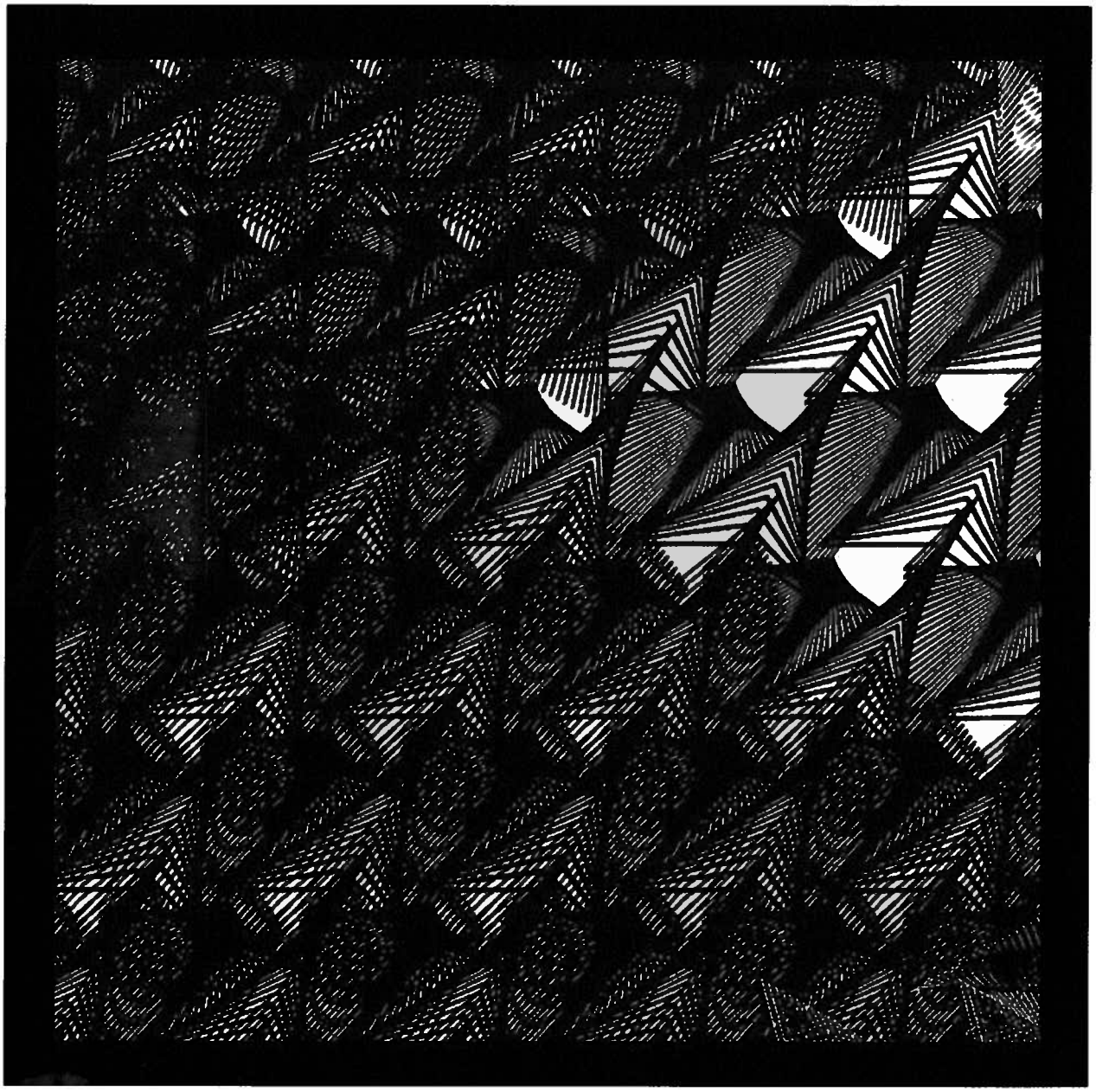


Figure 63: *Shadows.*
Right: 1 Layer shadows; Top and Bottom: 2 Layers shadows; Left: 3 layers shadows- Roof open 30 degrees.

CHAPTER 5: RETRIEVE AND OPENING; CONCLUSION AND FUTURE

Retrieve

The magic carpet: It was a vehicle. It remained a vehicle without any other claim. The magic carpet is a piece of conversation; it has always been, even in the folk tales of the past. It symbolizes that which is possible. In our case, the possibility of a contribution from other disciplines into architecture in a way that causes a transformation in how we perceive what is possible.

This work had a premise to address tradition in a contemporary way, controlling and creating spectacular lighting conditions (See fig.65), colorful surfaces, geometry and patterns in traditional tile-works and Persian carpets and Kilims. It even went further starting with 2D tile patterns and addressing the 3 dimensional qualities present in historical Iranian architecture (See fig.64).

The carpet wrapped the space taking care of it like a caring mother that holds the baby in her arms creating shadows on the ground and walls under a colorful roof. Nobody could walk under the colorful roof and not look above to gaze through the source of the shadows (See fig.66).

Opening

It can be detailed as a very finely woven Persian carpet, by means of choosing different colors for each and every isoparm of the hanging surfaces or like a rough Kilim with big patterns when each layer of the roof takes just one color or any place between these two extremes is possible. Perhaps we are weaving a new kind of carpet. This is where we could contribute to the Carpet and Kilim discipline. Maybe the carpet designers could advise us as to how many isoparms or surface of the same colors should be used and where exactly in the roof. That could be a further aspect of the design in which to be involved and to pursue (See fig.67).

Conclusion and future

In this project we introduced 'dynamics' and 'transparency' as new vocabulary to the existing traditional framework and extended it into a 3 Dimensional retractable surface. Future work can include investigation of vocabulary from more contemporary discourse. In addition further examination of remaining traditional geometrical patterns, like the combination of triangles, squares and hexagons are to be explored. Future work could examine issues of mass-customization and diversity of the size and shape of the elements of the roof and new variables in software like Generative Components could be defined resulting in new geometrical patterns and more complex forms. The precise materiality needs to be further examined and the investigation of these geometries for the design of performative sun screen walls in a contemporary sustainable enclosure is a possible avenue to explore. This project started with three dimensional digital interpretations of tessellation patterns of flat surfaces. The historical architecture of Iran is full of geometries that cover minarets, and domes, also the spatial Muqarnas forms defining the entrances to the mosques and Bāzār. They all await a revisit and reinterpretation with the new, digital tools (See figs.68, 69).

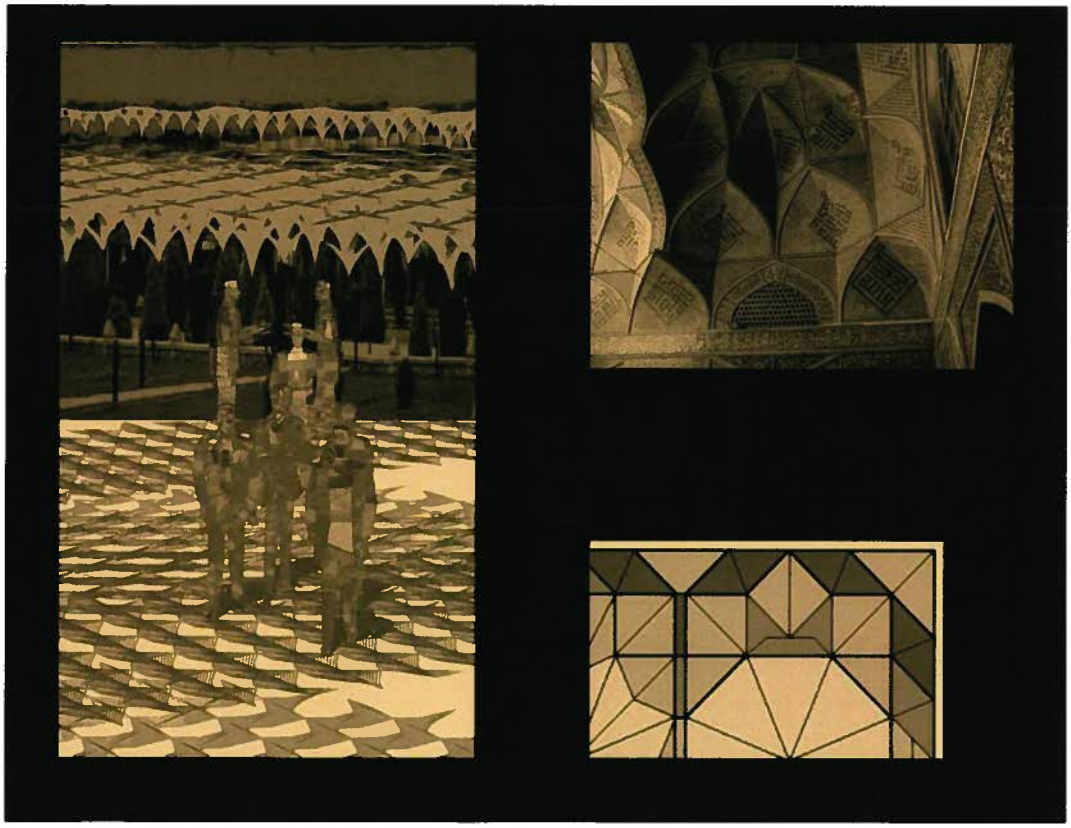


***Figure 64**: Roof shape resembles the Right tradition- Shadows resemble the Right tradition.*

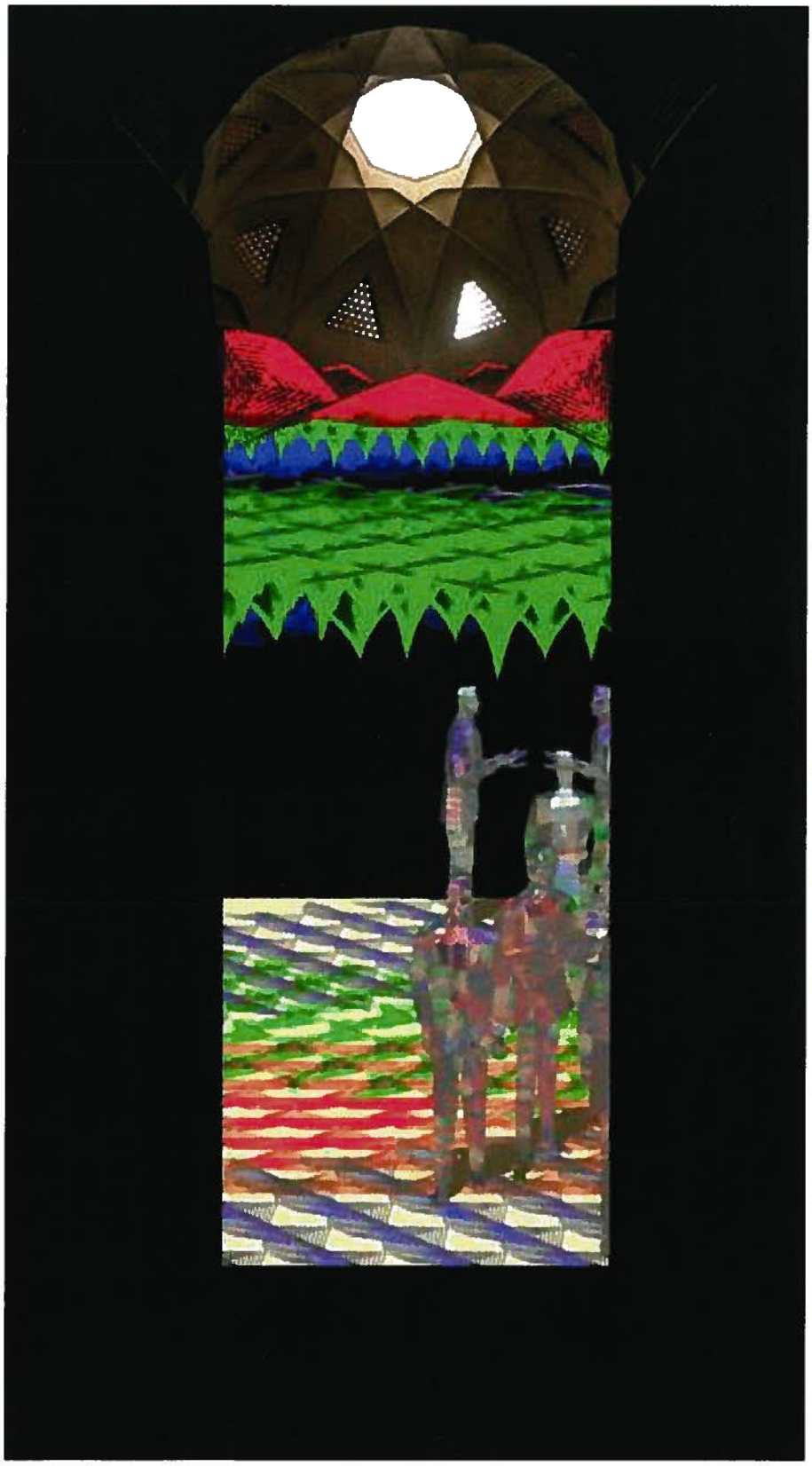


***Figure 65**: Light design and control- Tradition and our ever changing and maybe even colorful shadows.*

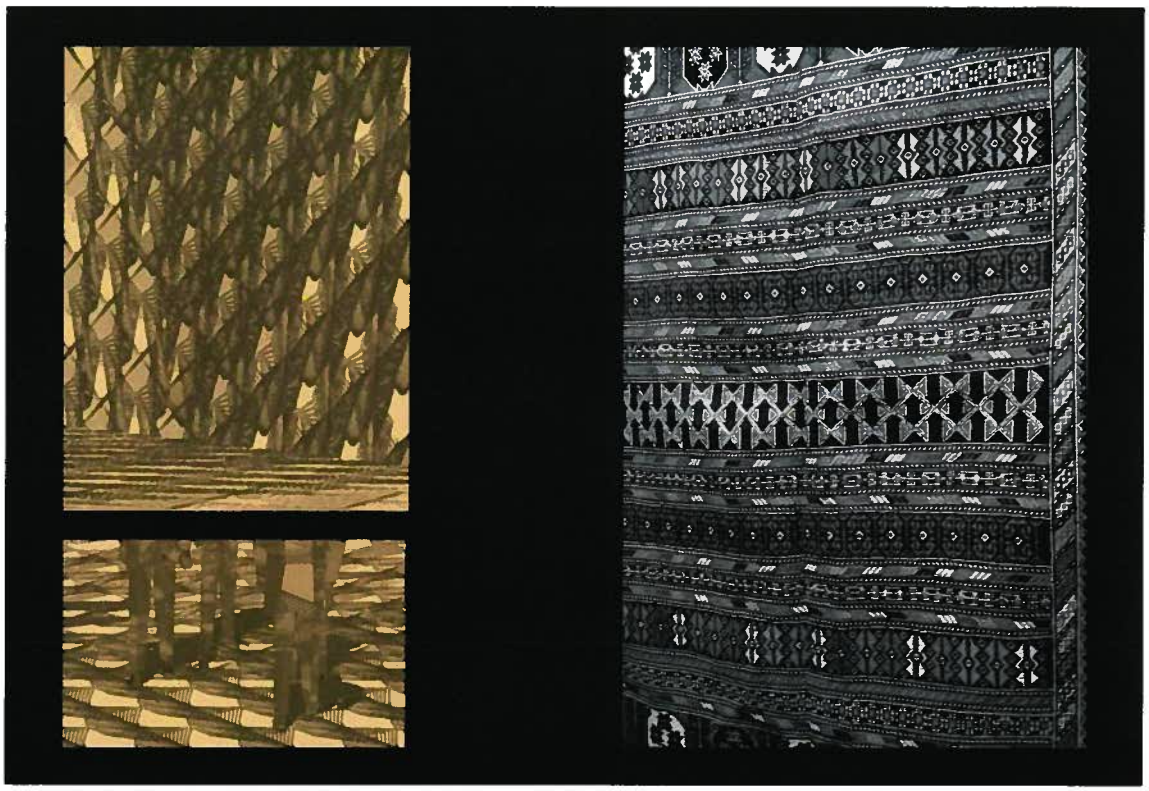
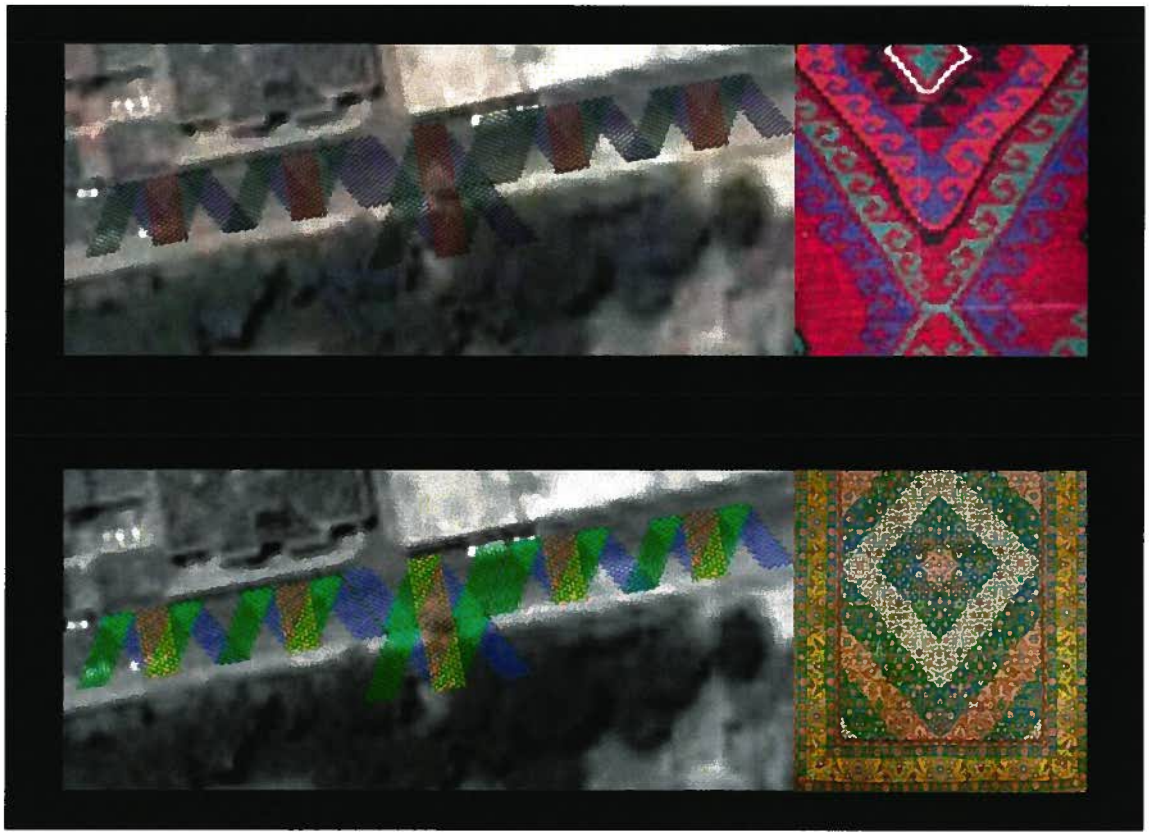


Figure 66: *In our design Shadow patterns define the floor like a carpet and through the walls reach and touch the roof itself. The magic carpet and the carpet weaver take responsibility for both ground and enclosure, till the architect stands up again for making a local and contemporary architecture in Iran. Carpet picture: © Leila Hosseini, adaptation is by permission of copyright holder: Leila Hosseini.*

Figure 67*: Different colors, Different Carpets- How long do you want to weave? How big a pattern? Each Layer has a different color or each triangle or each isoparm of each triangle?*



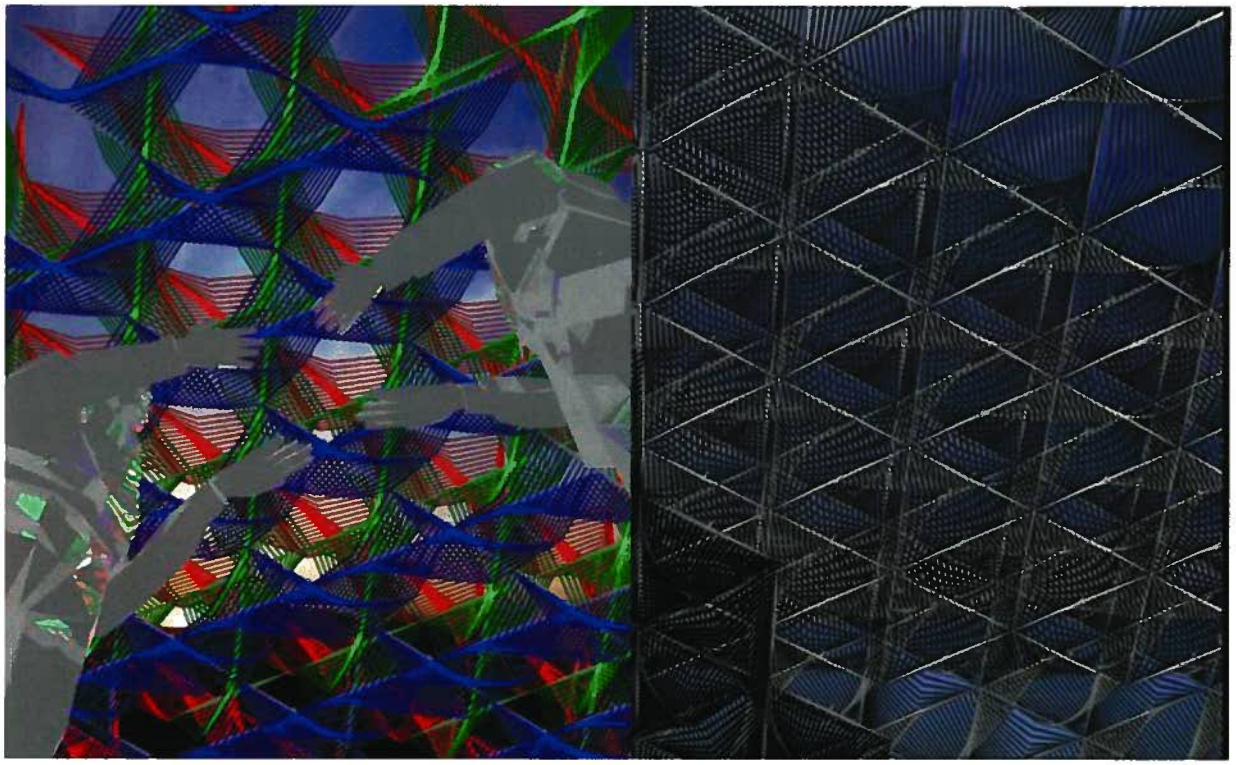
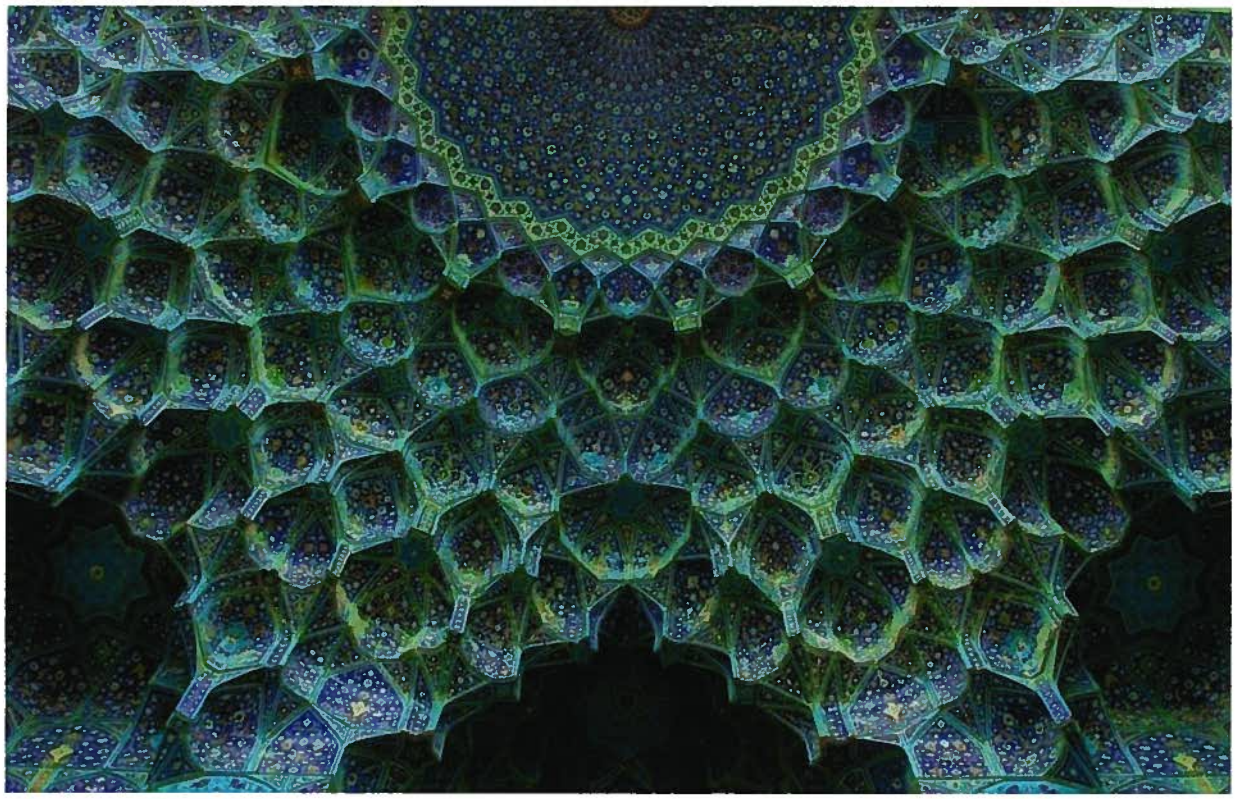


Figure 68: *Roof Closed- Roof Open.*

Figure 69: *Masjid Imam Entrance Muqarnas, there is still too much work to be done.*



Bibliography

1. Albarn, K., Smith, J.M., Steele, S., Walker, D., The Language of Pattern, an enquiry inspired by Islamic decoration, Harper & Row, Publishers, Inc., New York, 1974.

2. Alexander, Ch., Ishikawa, S., Silverstein, M., A Pattern Language: Towns, Buildings, Construction, Oxford University Press, USA, 1977.

3. Alexander, Ch., The timeless way of building, Harvard University Press, USA, 1979.

4. Alexander, Ch., The Oregon experiment, Oxford University Press, USA, 1975.

5. Ardalan, N., Bakhtiar L., The Sense of Unity, The Sufi Tradition in Persian Architecture, The University of Chicago, 1975.

6. Autodesk Maya, 17 Feb 2008 <http://usa.autodesk.com/adsk/servlet/index?siteID=123112&id=7635018>.

7. Bentley's Generative Components, 17 Feb 2008 <http://www.bentley.com/en-US/Markets/Building/generativecomponents/ >.

8. The Bridges Organization, Mathematical Connections in Art, Music, and Science, 4 Mar 2008 <http://www.bridgesmathart.org/>.

9. Critchlow, Keith, Islamic Patterns, an analytical and cosmological approach, Schocken Books, New York, 1976.

10. Eilouti, B.H., Al-Jokhadar, A.M.I., A Generative System for Mamluk Madrasa Form-Making, Nexus Network Journal, Architecture and Mathematics, 2007, 9(1), 7-59.

11. El-Said, I., Geometric Concepts in Islamic Art, World of Islam Festival Publishing Company, Ltd. London, 1976.

12. Hejazi, M., Geometry in nature and Persian architecture, Building and Environment, 2005, 40, 1413-1427.

13. Kaplan, Craig S. Computer Generated Islamic Star Patterns, In Bridges 2000: Mathematical Connections in Art, Music and Science, 2000.

14. Kaplan, Craig S. Computer Graphics and Geometric Ornamental Design, University of Washington, 2002, 4 Mar 2008 <http://www.cgl.uwaterloo.ca/~csk/phd/>.

15. Kaplan, Craig S. and Salesin, David H. , Islamic Star Patterns in Absolute Geometry, ACM Transactions on Graphics 23(2):97-119, 2004.

16. Mathematica software, 4 March 2008 <http://demonstrations.wolfram.com/>.

17. Muqarnas Geometries, 28 Sept 2008 <http://www.tamabi.ac.jp/idd/shiro/muqarnas/data/isfahan.htm>.

18. Pottmann, H., Asperl, A., Hofer, M., and Kilian , A., Architectural Geometry, Bentley Institute Press, Exton, 2007.

19. Rogmann, sascha, Wachstumsfunktionen von Pflasterungen, Heinrich-Heine-Universitat, Dusseldorf, 1997, 4 March 2008 <http://www.rogmann.org/math/diplom/>.

20. Scharein, Robert G. Interactive Topological Drawing, The University of British Columbia, 1998, 4 March 2008 <http://www.knotplot.com/thesis/>.