THREE ESSAYS IN REAL ESTATE MARKETS

by

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ABSTRACT

In this dissertation, I examine two research questions. In chapters 2 and 3, based on idea of reference value that was first proposed by Kahneman and Tversky, I look at a potential house seller’s pricing strategy when the reference value plays a role.

In chapter 2, I focus on the reference-dependence and its implications on loss aversion behavior, and I compare model predictions with documented empirical findings in the literature. In particular, I show that the stylized empirical evidence in the literature has relatively limited power on testing loss aversion, and I provide new specifications that aim to correctly test the loss aversion effect.

In chapter 3, I examine a reference-dependent seller’s pricing strategy in a less heterogeneous housing market such as the multi-unit residential market. Acknowledging the fact that units in the same building serve as close substitutes for each other, I show that the recent transaction price on a unit in the same building may generate two signaling effects. First, the average willingness to pay among buyers is positively correlated with the observed price, which generates a spatio-temporal autocorrelation effect; second, after observing the prior price, the heterogeneity of the potential buyer’s willingness to pay decreases, inducing house sellers to mark down their asking prices.

In chapter 4, I examine the power of monitoring and forcing contract on improving the managerial efficiency of REITs. I put particular emphasis on its implications regarding the choice of advisor type in REITs. I show that, for both internal and external advisors, increasing levels of monitoring power will increase their equilibrium effort under a stochastic forcing contract. Furthermore, I show that a crucial driving force regarding advisor choice is the heterogeneity of monitoring power between internal and external advisors and across REIT firms. Provided that the gap of monitoring power is large enough between internal and external advisors, shareholders could make use of the heterogeneity, and induce higher effort from external advisors. Hence, I am able to
provide a theoretical justification regarding the potential appeal of an external managerial structure, which is usually regarded as being inferior to an internal managerial structure.
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CHAPTER 1: INTRODUCTION

I examine two research questions in the field of real estate in this dissertation. The first question (addressed in chapters 2 and 3) regards reference-value and its impact on a house seller’s optimal pricing strategy. The second question (addressed in chapter 4) is on the power that monitoring and forcing contract has on improving the managerial efficiency of REITs and on the selection rule for REITs advisors.

The seminal work on prospect theory done by Kahneman and Tversky [1979] and Tversky and Kahneman [1991, 1992] proposes three components to help explain the decision-making process of individuals under uncertainty. The first component involves reference-dependence, in which people draw utility over gains and losses relative to a reference value such as a prior acquisition price or an initial endowment.\(^1\) In the second component, loss aversion effect, people treat losses and gains asymmetrically in their value functions. In particular, an equal-size loss looms larger than an equal-size gain.\(^2\) In the third component, diminishing sensitivity, the marginal value for both gains and losses declines with size. Although conceptually intuitive, most evidence that supports prospect theory comes from experimental studies (e.g., Kahneman, Knetsch and Thaler [1991], Tversky and Kahneman [1991], Knetsch, Tang and Thaler [2001], etc.). Not surprisingly, finding non-experimental evidence of loss aversion has become a popular topic in the empirical literature of the field. Genesove and Mayer [2001], as some of the most influential researchers in testing the loss aversion effect using real world data, have examined the behavior of sellers in the housing market. They found that: 1) compared to potential gainers, a seller subject to bigger potential loss sets a higher asking price and obtains a higher transaction price if the house is sold (finding one); and 2) the marginal mark up declines with the size of the seller’s potential loss exposure (finding two). Genesove and

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\(^1\) As pointed by Benartzi and Thaler [1995], this is in opposition to the classical expected utility theory in which the thing that matters is wealth itself.

\(^2\) That is, \(W(x) < -W(-x)\). See Loewenstein and Prelec [1992] for more discussion on this topic.
Mayer have interpreted finding one as a test of the loss aversion effect and finding two as a test of diminishing sensitivity in value function.

The purpose of chapter 2 is to examine the link between each component in prospect theory, and how its empirical implications relate to a seller’s pricing behavior in the housing market. We illustrate this by building a simple search model which incorporates Tversky and Kahneman’s prospect utility as a special case. In our model, we show that both findings provide evidence supporting a reference-dependent value function. Therefore, we find that they are valid tests of the first component in prospect theory. However, neither does finding one have a necessary relationship to loss aversion, nor does finding two have a necessary relationship to diminishing sensitivity. For example, we show that an increasing and concave relationship between a seller’s asking price and her potential loss exposure is fully consistent with a value function that has symmetric response between losses and gains and a marginal increasing sensitivity in both dimensions. As a result, there is a conceptual mismatch between the two empirical findings and their theoretical counterparts. Our results have important implications with regard to the literature on testing loss aversion since most studies interpret evidence of loss aversion in the same way as do Genesove and Mayer [2001]. For example, Neo, Ong and Somerville [2005] try to test loss aversion effect using housing auction data by finding a similar relationship between transaction prices and the loss exposures of sellers. In a slightly different manner, Chan [2001] and Engelhardt [2003] test loss aversion effect by examining the factors that influence household mobility. They both find that potential losses display a negative relationship to a households’ mobility. This is consistent with a seller’s behavior, since they are subject to a larger potential loss, and so will set a higher price when selling a house.

Since the testing power of findings one and two are very limited except with regard to reference-dependence, we show that by testing loss aversion and marginal diminishing sensitivity, a more important angle from which to examine this is to look at the behavior of sellers who have lower reference values. In particular, a value function that is fully consistent with prospect theory implies an S shaped asking price curve along reference
values, i.e., sellers’ asking prices increase convexly when reference values are small and increase concavely when they are big. In addition, there is a significant slope increase among sellers who are around break-even positions. Using a dataset of Vancouver single-family transactions, we find evidence that is consistent with the predictions made by prospect theory. Finally, our model also helps explain the positive price-volume relationship and why the price dispersion in a cold market is larger than in a hot one.

Chapter 3 holds a slightly different research focus. Instead of looking at reference-dependence effect per se, in this chapter I examine the source of the varying bidding heterogeneity from potential buyers and the impact of this varying heterogeneity on a reference-dependent seller’s optimal pricing strategy.

An intriguing finding in the housing market is that sellers may set very different prices for qualitatively similar units, even when facing the common housing market environment. To explain this phenomenon, two underlying ingredients regarding the market seem to be important: first, as suggested by prospect theory, instead of drawing utility directly from financial wealth, people draw utility over gains and losses relative to a reference value such as a prior acquisition price or an initial endowment. In other words, in this case, sellers do not maximize the expected selling proceeds. Second, the fact that it is actually possible for a seller to sell a house at very different prices implies that potential buyers should have heterogeneous valuations on a given house good. Then, an important research question here is what is the impact of this bidding heterogeneity on seller’s pricing strategy, in particular when the extent of this bidding heterogeneity changes.

The prior research that is most related with Chapter 3 is Zheng et al, [2008]. Using a search model, the authors show that a greater dispersion is bidding price leads to a higher asking price and they provide some empirical evidence on it. However, their research has some limitations. First, from the theoretical perspective, their modeling part doesn’t provide insight on the factors that may generate the varying bidding heterogeneity. The authors simply assume this variation exists when they examine the seller’s optimal pricing strategy. Second, their empirical measure on the varying bidding heterogeneity, the
standard deviation of the delisting prices of sold properties in different geographic submarkets, has some drawbacks. Following the model, each delisting price is determined by the extent of bidding heterogeneity. Therefore, using the second moment of a pool of the delisting prices as a measure of bidding heterogeneity causes the endogeneity problem when they regress the delisting price on this standard deviation measure, which is essentially the base of their empirical test. Meanwhile, since they use the whole sampling period when calculating this second moment, they inevitably face the problem of using future information to estimate the current outcome. Third, the reference-dependence effect is abstracted in their study. In another word, when facing the same bidding distribution, their model predicts a common optimal asking price across sellers, on which much counter evidence has been provided in the literature.

Despite a similar research focus as in Zheng et al. [2008], Chapter 4 aims to contribute to the literature in several aspects with the caution to avoid the above-mentioned limitations. Firstly and the most importantly, unlike Zheng et al. [2008], we explicitly identify one source of the varying bidding heterogeneity in the model, i.e., the signaling effect when market participants can observe the transaction prices of other units that are close comparables for the unit that a current seller wants to sell.

Acknowledging the fact that in the multi-unit residential market, units in the same building serve as close substitutes for each other, we show that the recent transaction price on a unit in the same building generates two signaling effects: first, it generates positive spatio-temporal autocorrelation with the subsequent price for the units in the same building; second, after observing the prior price, the heterogeneity of potential buyers’ willingness to pay decreases. The intuition is that, as the available housing goods become more comparable to each other, the transaction prices of other units may convey useful information on the valuation of a particular unit for sale on the market, which helps to reduce the disagreement on the willingness to pay across subsequent buyers and hence reduce the sellers’ bidding heterogeneity on that unit.
Adopting a simplified version of the search framework proposed in Sun [2007], our model predicts that sellers’ optimal asking prices would decrease when the potential buyers’ bidding heterogeneity decreases, which is consistent with the theoretical prediction of Zheng et al, [2008]. Using a geo-coded dataset on condominium transactions in Singapore from 1990 to 2001, we show that the spatio-autocorrelation on the quality-controlled price of one unit and the previously transacted units in the same building is significantly higher than with that of the units in neighboring buildings.

Meanwhile, using a two order spatio-temporal autoregressive model as proposed by Sun, Tu and Yu [2005], we find that, after controlling for the autocorrelation effect, sellers tend to mark down their asking prices by at least 1.6% if a recent transaction has occurred within the same building, which is consistent with the effect from the decreasing bidding heterogeneity among potential buyers. Since we use the incidence of a transaction in the same building within a very short time period (one month) as an indicator for a signal, our measure on the varying bidding heterogeneity is less likely to face the endogeneity problem as in Zheng et al, [2008].

Secondly, in the model we explicitly control for the impact of reference values on seller’s optimal pricing strategy. And our study re-affirms Sun [2007]’s finding that sellers’ optimal asking prices would increase with their reference values, which generates heterogeneous asking prices across sellers, and it is absent in the model of Zheng et al, [2008].

Another unique feature of our model is that, when decomposing potential buyers’ willingness to pay, we introduce a component which we call the substitution premium. We expect that in a heterogeneous housing market, potential buyers’ willingness to pay should also depend on the availability of finding a different house good which is a close substitute to a particular housing unit. Nevertheless, we don’t have any prior belief on to which direction would this substitution effect go. Our empirical finding suggests that potential buyers’ willingness to pay for any particular house unit is negatively related with the number of house substitutes in the market. This is consistent with a story of decreasing
opportunity cost for buyers to scarify a house good when outside substitutes are easier to find. Think of the case in which a buyer finds one unit that is attractive to him. If it is relatively easy for him to find another unit that is highly substitutable on the market, the potential cost of sacrificing the current shopping opportunity decreases and hence we should expect a lower willingness to pay. This can be contrasted to a case in which a potential buyer would have difficulty finding another substitute for this particular house unit. As a result, provided that the house goods on the market are equally accessible to all potential buyers, the mean level of the bidding distribution will decrease. As our theory predicts that the seller’s optimal asking price would decrease with the mean level of potential buyers’ bidding distribution, it implies that a seller’s optimal asking price should decrease with the empirical measure on this substitution premium.

Furthermore, our model predicts that the sellers’ optimal asking prices should also increase with the exogenous arrival rate of potential buyers, which is intuitive.

Our empirical findings are again consistent with the above mentioned model predictions. For example, we find significant evidence of the reference dependence effect. In addition, using a proxy measure on the exogenous arrival rate of potential buyers, we find sellers’ asking prices tend to decrease with this measure, as predicted by the model.

In chapter 4, I focus on REIT corporate governance. In particular, I look at the power of monitoring and forcing contract on the improvement in managerial efficiency of REITs. One puzzling question in real estate literature involves the justification of an external managerial structure in REIT firms. It is widely agreed that managing real estate investments through external advisors in REITs generates a larger conflict of interest between shareholders and agents than does managing real estate investments through internal advisors. For example, Howe and Shilling [1990] found that, after controlling for the firm’s characteristics and risk levels, externally advised REITs performed worse than did the general stock market until the late 1980s. Another convincing study by Capozza and Seguin [1998] also shows that in their sampling period, externally advised REITs underperformed internally advised REITs by over 7%. In addition to having an inferior
stock performance, Ambrose and Linneman [2001] found that externally advised REITs in general also incur higher financial expenses\(^3\). Because of the potentially high agency costs associated with the external advisor structure, the REIT industry has experienced a significant trend in converting to being internally advised since 1986, when private-letter rulings from the IRS first allowed REIT firms to convert to this structure. According to Chan, Erickson and Wang [2003], by the year 2000, more than 87\% of REITs chose to convert to an internal advisor structure. Despite a broad consensus among academics regarding the inferiority of external management, the fact that a non-trivial number of REITs keep to be externally managed seems controversial.

Chapter 4 further aims to provide a theoretical justification of the potential appeal of the external managerial structure. In particular, the agency problem is examined through focusing on the power of monitoring and forcing contract on the improvement of the advisor’s equilibrium effort supply and the selecting rule for REITs advisors. Our theory is closely linked to two branches of the literature. The first involves research on the effects of monitoring and forcing contract. Unlike the standard agency model, which assumes a lack of observability regarding an agent’s effort level, the theory on forcing contract assumes a partial observability of the agent’s efforts. For example, there may be a positive probability that the principal can detect an agent’s shirking behavior\(^4\) through monitoring. In such a situation, a new contract type called a ‘forcing contract’ is proposed. A forcing contract is characterized by a standard wage and a penalty wage. If no shirking behavior is detected, the agent gets a standard wage; instead, if the agent is detected to have engaged in shirking behavior, a penalty wage is applied, which is typically set to zero, and refers to the action of dismissal. The application of such measures is popular in the literature of labor economics. For example, Shapiro and Stiglitz [1984] apply this idea in an efficiency wage model. One conclusion that is relevant to this study is that with positive monitoring power, the principal can induce a higher effort level from the agent, thus increasing

\(^3\) See Chapter 4 of Chan, Erickson and Wang [2003] for a comprehensive review on the agency problems associated with REIT’s advisor types.

\(^4\) Shirking behavior refers to the fact that an agent spends less effort than the prescribed level set by the principal.
efficiency in the economy\textsuperscript{5}. The second branch involves research on optimal compensation schemes and the agency problem in the REIT industry. Despite a vast literature on empirical evidence regarding the sub-optimality of externally advised REITs, theoretical studies on the agency problem and the choice of advisor types in REITs are surprisingly scarce. The only research that sketches the connection between potential agency costs and compensation structures is that of Solt and Miller [1985], which points out the potential moral hazard resulting from the problem of the information asymmetry between REITs advisors and shareholders.

In this study, acknowledging the potential heterogeneity of monitoring power between internal and external advisors and across REIT firms, we show that in general, for both types of advisors, an increased monitoring power will increase their optimal effort. Furthermore, we argue that by choosing external structure, shareholders may enjoy a monitoring advantage, compared with internal management. We motive this argument from three aspects in the modeling part. The most important aspect is the dual role for an external advisory firm. On one hand, advisory firm serves as an agent for a REIT company and gets compensation from REIT shareholders; on the other hand, it is also the principal who in turn compensates external advisor it sends to the REIT company. We show how, by correctly specifying a compensation mechanism, REIT shareholder can free-ride on advisory firm’s superior monitoring ability; and it may induce higher effort levels from external advisors, despite a higher agency cost associated with this management structure. Acknowledging the potential heterogeneity of monitoring power between internal and external advisors and across REIT firms, the boundary of the optimal advisor type is then derived, and a numerical illustration on the advisor choice problem is presented in the modeling section.

Another contribution of this chapter involves a theoretical investigation of the optimal compensation scheme for REIT advisors. Our analytical results show clear implications regarding the power of retrospective monitoring and forcing contract in improving

\textsuperscript{5}The efficiency wage model also implies a trade-off between raising the equilibrium wage level and the saved monitoring cost. Since we abstract the monitoring cost in this study, we will not discuss implications regarding that dimension here.
production efficiency and shareholders’ welfare. In addition, we also compare the difference between a fixed and stochastic forcing contract\(^6\). Our findings show that with imperfect performance measures, the stochastic forcing contract always dominates the fixed one in increasing shareholders’ profits. Finally, we examine the impact of shareholder monitoring on the advisor’s welfare. One important finding is that monitoring power from shareholders may increase the advisor’s equilibrium utility when it gets below a certain threshold level, despite the fact that it also restricts the advisor’s flexibility regarding shirking. As a result, it could be in the advisor’s self-interest to help the shareholders monitor their effort levels. In other words, as an agent, the advisor may not necessarily be averse to a more efficient monitoring environment of his activities.

\(^6\) We will explain in detail the content of these two contracts in chapter 4. Simply speaking, a fixed forcing contract is a contract involving a fixed regular wage, a penalty wage, and a prescribed effort level. In general, agents receive regular wages unless they are captured for shirking, in which case a penalty wage is applied. In contrast, in the stochastic forcing contract, a regular wage is not fixed; rather it is based on some performance measure, which will be discussed thoroughly in the modeling section.
References


CHAPTER 2: LOSS AVERSION IN THE HOUSING MARKET: A REVISIT

2.1 Introduction

The seminal work on prospect theory by Kahneman and Tversky [1979] and Tversky and Kahneman [1991, 1992] propose three components to help explain the decision-making process of individuals under uncertainty. The first component is reference-dependence, in which people draw utility over gains and losses relative to a reference value such as prior acquisition price or an initial endowment. In the second component, loss aversion effect, people treat losses and gains asymmetrically in their value functions. In particular, an equal-size loss looms larger than an equal-size gain. In the third component, diminishing sensitivity, the marginal value for both gains and losses declines with size. Although conceptually intuitive, most evidence that supports prospect theory comes from experimental studies (e.g., Kahneman, Knetsch and Thaler [1991], Tversky and Kahneman [1991], Knetsch, Tang and Thaler [2001], etc.). Not surprisingly, finding non-experimental evidence of loss aversion becomes an active topic in the empirical literature of the field. As one of the influential efforts in testing loss aversion effect using real world data, Genesove and Mayer [2001] examine seller’s behavior in the housing market and find that: 1) compared to potential gainers, a seller subject to bigger potential loss sets a higher asking price and obtains a higher transaction price if the house is sold (finding one); and 2) the marginal mark up declines with the size of seller’s potential loss exposure (finding two. Genesove and Mayer interpret finding one as a test of loss aversion effect and finding two as a test of diminishing sensitivity in value function.

The purpose of this paper is to examine the link between each component in the prospect theory and how its empirical implications relate to a seller’s pricing behavior in housing

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7 A version of this chapter will be submitted for publication. Sun, H. Loss Aversion in the Housing Market, A Revisit.
8 As pointed by Benartzi and Thaler [1995], it is opposite to the classical expected utility theory in which the thing that matters is wealth itself.
9 That is, \( W(x) < -W(-x) \). See Loewenstein and Prelec [1992] for more discussion on it.
market. We illustrate this by building a simple search model, which incorporates Tversky and Kahneman’s prospect utility as a special case. In our model, we show that both findings are evidence supporting a reference-dependent value function. Therefore, they are valid tests of the first component in the prospect theory. However, neither does finding one have a necessary relationship with loss aversion, nor does finding two have a necessary relationship with diminishing sensitivity. For example, we show that an increasing and concave relationship between a seller’s asking price and her potential loss exposure is fully consistent with a value function that has symmetric response between losses and gains and a marginal increasing sensitivity in both dimensions. As a result, there is a conceptual mismatch between two empirical findings and their theoretical counterparts. Our result has important implication to literature on testing loss aversion since most studies interpret evidence of loss aversion in the same way as in Genesove and Mayer [2001]. For example, Neo, Ong and Somerville [2005] try to test loss aversion effect using housing auction data by finding a similar relationship between transaction prices and sellers’ loss exposures. Being slightly different, Chan [2001] and Engelhardt [2003] test loss aversion effect by examining the factors that influence household mobility. They both find that potential losses have a negative relationship with a households’ mobility, which is consistent with the behavior that a seller who is subject to a larger potential loss will set a higher price when selling a house.

Since the testing power of finding one and two are very limited except for reference-dependence, we show that by testing loss aversion and marginal diminishing sensitivity, a more important side is to look at the behavior of sellers who have lower reference values. In particular, a value function that is fully consistent with prospect theory implies an S shape asking price curve along reference values, i.e., sellers’ asking prices increase convexly when reference values are small and increase concavely when they are big. In addition, there is a significant slope increase among sellers who are around break-even positions. Using a dataset of Vancouver single-family transactions, we find evidence that is consistent with the predications made by prospect theory.
Finally, our model also helps to explain the positive price-volume relationship and why the price dispersion in a cold market is bigger than in a hot one.

The rest of the paper is structured as follows. Section 2.2 lays out the model for a seller’s decision problem which incorporates prospect utility as a special case and discusses its implications. The empirical predictions are tested in section 2.3. Section 2.4 points out the issues implied by prospect theory on price index construction. Section 2.5 concludes the paper.

2.2. Model Setup and Results

2.2.1 General Setup
Consider a large housing market with countable infinite number of potential sellers. Each of them has an ex-ante identical house for sale. To sell it, she must spend effort to search for a potential buyer, which arrives in independent Poisson process. For seller i, we define the proportional time spent on searching as $t_i$, which ranges from 0 to 1. During the time window of $\Delta t$, a buyer arrives with the probability of $B(t_i)\Delta t$. As a result, no buyer arrives with the probability of $1 - B(t_i)\Delta t$, and more than one buyer arrives with a probability that is smaller than the order of $\Delta t$. The time not spent on searching is consumed as leisure. During a time window of $\Delta t$, the working time incurs a cost of $H(t_i)\Delta t$. Later, we will study two cases: a costless search case and a costly search case. Although less realistic, we still want to study the first case since it will significantly simplify our model in order to get an analytical relationship between a seller’s asking price and her reference value, which parallels empirical finding one in Genesove and Mayer [2001]. In that case, we simply assume $B(t)=1$ and $H(t)=0$ for all of $t$. The underlying assumption is that in each period, one seller is guaranteed to meet one, and only one, potential buyer. In the model with costly search, we assume that arrival function $B$ has the following properties. First, it is twice continuously differentiable everywhere. Second, $B$ is increasing and strictly concave in $t_i$. This assumption implies a marginal decreasing productivity on a seller’s searching ability. To prevent corner solutions, we also assume $B(0) = 0, B'(0) = \infty$. As usual in literature, we also assume the search cost
function $H$ to be twice differentiable, increasing and strictly convex in $t_i$. Again, we assume $H'(0) = 0$ and $H'(1) = \infty$ to prevent any corner solutions.

When a potential buyer arrives due to a seller $i$’s search effort, the buyer inspects $i$’s house and finds out if the quality matches his own preference. Depending on the matching quality, the buyer decides the highest possible price $p_i$ that he is willing to pay. We assume $p_i$ is a random draw from distribution $G$. If there is no match at all, $p_i = 0$. If there is a perfect match, $p_i = 1$, which is only a normalization. Therefore, $G$ has a finite support $[0,1]$. Furthermore, we assume $p_i$ is independent across buyers, assets and time. The distribution $G$ could be very general in this regard. One typical assumption is that, in all cases the hazard function, $g/(1 - G)$ is non-decreasing. The seller chooses an asking price $r_i$ for her house. If $p_i \geq r_i$, the transaction is closed and the seller pays $r_i$. Therefore, we abstract any bargaining process here.\(^{10}\)

Collectively, the seller $i$’s objective is to solve the following maximization problem:

$$U^{v,c_i} = \max_{t_i,v_i} e^{-\beta t_i} \{ B(t_i)(1-G(r_i))\Delta t W(r_i,v_i) - H(t_i)\Delta t$$

$$+ (1-B(t_i)(1-G(r_i))\Delta t)[U^{v,c_i}-c_i\Delta t] \} + o(\Delta t)$$

subject to $0 \leq t_i \leq 1$ and $0 \leq r_i \leq 1$, for $i=1,2,\ldots$.

In equation 2.1, $\beta$ is the discount rate, and $c_i$ reflects the per-period net cost of staying in the current house, possibly due to the disutility of the mismatch between the seller’s preference and her house. A positive value of $c_i$ means the seller has an incentive to move sooner due to an attractive outside option. However, we acknowledge the possibility that for some sellers $c_i$ could be negative, which implies that remaining in the current house is more attractive than moving out. This idea generates the positive effect of spatial lock-in.

\(^{10}\)This is a simplification for the purposes of our paper] and dropping it will not change any qualitative implications in our model.
sellers, since they must ask potential buyers to compensate for giving up the benefit from superior matching. To rule out this trivial case, from now on we will only consider the case that $c_i$ is non-negative.

Conditional on a successful sale, seller $i$ receives a utility gain, as measured by value function $W(r_i, v_i)$. Particularly, we assume $W(r_i, v_i)$ has the following form:

$$W(r_i, v_i) = \begin{cases} (r_i - v_i)^\alpha & \text{if } r_i - v_i \geq 0 \\ -\lambda(v_i - r_i)^\alpha & \text{if } r_i - v_i < 0 \end{cases}$$

(2.2)

where $\lambda, \alpha > 0$. Our specification of $W(r_i, v_i)$ is very general. On one hand, when $v_i = 0$ for all $i$ and $\alpha = \lambda = 1$, equation 2.1 reduces to the traditional search model in which risk neutral sellers try to maximize the expected selling proceeds, as in Williams [1998]\textsuperscript{11}. On the other hand, $W(r_i, v_i)$ also incorporates Tversky and Kahneman’s prospect utility as a special case when $v_i \neq 0, \lambda > 1$ and $0 < \alpha < 1$. The component of reference-dependence corresponds to the existence of a non-zero $v_i$. Instead of drawing utility directly from selling proceeds, people will evaluate it relative to an inherited reference value. Therefore, $v_i$ is explicitly treated as an un-sunk cost opposite the classical model, in which all prior costs are sunk and hence irrelevant for the current decision. We also assume $v_i$ is drawn from distribution $V$. In our model, we don’t allow $v_i$ to change over time. It has been found in several studies, including that of Genesove and Mayer [2001], that people tend to have nominal reference values.

Since loss aversion refers to a behavior where an equal-size loss looms larger than an equal-size gain, it is clear that $\lambda > 1$ measures this asymmetric response. This is the reason why Kahneman and Tversky [1979, 1992] define $\lambda$ as the coefficient of loss aversion. Finally, the diminishing sensitivity is measured by $0 < \alpha < 1$. Tversky and Kahneman propose that $\lambda$ should be around 2.25 and $\alpha$ be around 0.88.

\textsuperscript{11} In Williams [1998], one difference is that households delegate this selling procedure to a broker.
We now discuss the implication as outlined by equation 2.1. Over the time interval $\Delta t$, the seller maximizes the discounted expected payoff from the following decision problem. First, it is possible that one buyer will arrive who is willing to pay $r_i$. The first term in the bracket of equation 2.1 measures this effect. Likewise, the second term measures the total search cost spent by seller $i$. In the third term, $1 - B(t_i)(1 - G(r_i))\Delta t$ refers to the probability of the remaining current state. In this case, the seller incurs a waiting cost, $c_i\Delta t$, and will repeat the current decision problem $U^{v_i, c_i}$. We should note that the terms in bracket correctly consider all possible events that could happen during period $\Delta t$. The other events can only occur with the probability of a smaller order than $\Delta t$. Those terms are collected as $o(\Delta t)$.

By Taylor expansion on $e^{-\beta\Delta t}$, equation 2.1 can be rewritten as:

$$
0 = \max_{t_i, r_i} \quad e^{-\beta\Delta t} \left\{ B(t_i)(1 - G(r_i))\Delta t[W(r_i, v_i) + c_i\Delta t - U^{v_i, c_i}] - H(t_i)\Delta t - c_i\Delta t \right\} - \beta U^{v_i, c_i}\Delta t
$$

(2.3)

Dividing the above equation by $\Delta t$, taking limit as $\Delta t \to 0$ and re-organizing the terms gives us

$$
U^{v_i, c_i} = \frac{B(t_i)(1 - G(r_i))W(r_i, v_i) - H(t_i) - c_i}{B(t_i)(1 - G(r_i)) + \beta}
$$

(2.4)

Now taking first order condition with respect to $t_i$ and $r_i$, we get

$$
B'(t_i^*)(1 - G(r_i^*))[W(r_i^*, v_i) - U^{v_i, c_i}] - H'(t_i^*) = 0
$$

(2.5)

$$
(1 - G(r_i^*))W_i(r_i^*, v_i) - g(r_i^*)[W(r_i^*, v_i) - U^{v_i, c_i}] = 0
$$

(2.6)

where the subscript means to take the partial derivative in respect to the corresponding variable. Equations 2.5 to 2.6 fully characterize equilibrium solutions, since we have two

12 A potential problem is that $W_i(r_i, v_i)$ may not exist at $r_i = v_i$ in some specifications of parameters, i.e., when $\alpha = 1$ and $\lambda \neq 1$. In this case, $W(r_i, v_i)$ is a linear asymmetric value function. When we numerically solve the model, we set $\alpha = 1.000001$ as an approximation of a linear function. Then, we can bypass the problem of a non-existed derivative.
equations to solve for three unknowns: \( t^*_i \) and \( r^*_i \). A proof of existence and a unique solution for this type of searching problem can be found in Williams [1998].

It is clear from equations 2.5 to 2.6 that equilibrium \( t^*_i, r^*_i \) are functions of \( v_i \) and \( c_i \). Supposing the C.D.F of its joint distribution is \( F(v, c) \). Conditional to it \( F(v, c) \), the expected market price is only the average of all transaction prices, weighted by the according probability of realizing such sales. As a result,

\[
\bar{P} = \int \int r^*(v, c)[B(t^*_i(r^*_i))(1-G(r^*_i))]f_{v,c}(v,c)d(v)d(c)
\]

\[
\int \int [B(t^*_i(r^*_i))(1-G(r^*_i))]f_{v,c}(v,c)d(v)d(c)
\]  

(2.7)

### 2.2.2 Results

Although the solving process is straightforward, equation 2.5 to 2.6 are too general to give us any clear implications on the relationship between a seller’s asking price and the reference value, in particular, the slope, \( \frac{dr^*_i}{dv_i} \). To get more concrete results, in the next subsection we will consider a case with a costless search. In doing this, we greatly simplify our model to get the analytical expression of \( \frac{dr^*_i}{dv_i} \).

#### 2.2.2.1 The Case of Costless Search

As mentioned before, in this case, we assume \( B(t)=1 \) and \( H(t)=0 \) for \( t \). As a result, equation 2.5 vanishes. Now we can rewrite equation 2.4 as

\[
U^{*v_i,c_i} = \frac{(1-G(r^*_i))W(r^*_i,v_i)-c_i}{(1-G(r^*_i)) + \beta}.
\]

(2.8)

Substituting equation 2.8 into equation 2.6 gives us

\[
(1-G(r^*_i))W(t^*_i(r^*_i),v_i) - g(r^*_i)[W(r^*_i,v_i) - \frac{(1-G(r^*_i))W(r^*_i,v_i)-c_i}{(1-G(r^*_i)) + \beta}] = 0
\]

(2.9)

Conditional on a common \( c_i \), we can total differentiate equation 2.9. After some algebraic manipulations, we get:
\[
\frac{dr^*}{dv_i} = \frac{\omega + \beta g(r_i^*)W_{v_i}(r_i^*,v_i)}{\omega - 2g(r_i^*)(G(r_i^*) - \beta - 1)W_{v_i}(r_i^*,v_i) - g(r_i^*)\beta W_{v_i}(r_i^*,v_i) + c_i}
\]  
(2.10)

where \( \omega = [\beta(G(r_i^*) - 1) - (G(r_i^*) - 1)^2]W_{v_i}(r_i^*,v_i) \).

To evaluate the validity of finding 1 in Genesove and Mayer [2001] as a test of loss aversion effect, we further assume that potential buyers bidding follows a uniform distribution and the value function \( W(r_i^*,v_i) \) is linear, i.e., \( \alpha = 1 \), which yields the following result:

**Lemma 1:** With costless search, linear value function and uniform bidding distribution, and conditional on a common \( c_i \),

\[
\frac{dr^*}{dv_i} = \frac{\beta}{-2(G(r_i^*) - \beta - 1)} > 0
\]

The proof comes directly from equation 2.10 due to the fact that \( \omega = 0 \) and \( g'(r_i^*) = 0 \).

One finding from Lemma 1 is that loss aversion plays no role in determining the sign of \( \frac{dr^*}{dv_i} \). As a result, a positive relationship between a seller’s asking price and the reference value doesn’t imply \( \lambda > 1 \), which is essentially the finding 1 in Genesove and Mayer [2001].\(^{13}\) Later, we will show that this finding still holds in a more general context with costly search and for other bidding distributions.

Nevertheless, finding 1 is a valid test for reference dependence, i.e., \( v_i \neq 0 \) for all i. If this was not the case, then from equation 2.4 we know \( U^{r_{v_i},c_i} \) is no longer a function of \( v_i \).

Therefore, it must be the case that \( \frac{dr^*}{dv_i} = 0 \) for all \( v_i \). The link between finding 1 and reference dependence is intuitive. If reference value plays no role in affecting seller’s

\(^{13}\) More accurately, Genesove and Mayer [2001] find a positive relationship between the asking price and a seller’s potential loss exposure, \( v_i - \bar{P} \). However, since \( \bar{P} \) is ex-post fixed, this finding is equivalent to a positive relationship between \( r_i^* \) and \( v_i \).
utility, it should not have any prediction power on the optimal asking price which a seller chooses.

To check the robustness of our argument, we now consider a more general case within the framework of costly search.

2.2.2.2 The Case of Costly Search

Now we need to further specify the functional form of search productivity $B(t)$, cost $C(t)$, the distribution of potential buyers valuation $G$, and the joint distribution of sellers’ reference values and net waiting costs $F(v, c)$. Hereafter, we will assume:

$$B(t) = \sqrt{2t - t^2}$$

$$H(t) = 1 - \sqrt{1 - t^2}.$$  \hspace{1cm} (2.11)  \hspace{1cm} (2.12)

Both $B(t)$ and $H(t)$ are quarter portions from unit circles. A handy feature of this specification is that they satisfy all the assumptions we want to make on the productivity and cost functions. They are both monotonically increasing and twice continuously differentiable. Also, we can check easily that they also satisfy the curvature assumptions we have. Furthermore, we assume $v_i$ and $c_i$ are independent variables. In other words, the reference value and net waiting cost for individual sellers are uncorrelated.

A more realistic assumption on potential buyer’s bidding distribution should have a bigger central tendency than the uniform distribution. Since $G$ has a finite support of $[0, 1]$, we cannot assume the usual normal distribution on $G$. Specifically, we use a quasi-triangular distribution:

$$G(r) = \begin{cases} 
3r^2 - 2r^3 & \text{for } 0 \leq r \leq 1. \\
0 & \text{otherwise}
\end{cases}$$

g(r) = 6r - 6r^2

The density of this distribution is shown in figure 2.1.
Similar to normal distribution, a quasi-triangular distribution is also symmetric and peaked at centre. The results presented below are based on this distribution. Since the result in Lemma 1 requires G to be a uniform distribution, it helps us to check the robustness of the finding in Lemma 1 to other distributions. Also, when doing parallel studies on uniform distribution we obtained very similar results.

Equipped with specifications on G, B, and H, we can substitute them into equation 2.5 to 2.6. For a given $v_i$, $c_i$, and $\beta$, we can solve the system numerically and get $t_i^*$ and $r_i^*$. In particular, we assume that per-period discount rate is 5% and waiting cost is 0.05\textsuperscript{14}. Hereafter, we will keep these parameter assumptions unless state otherwise. To show that finding 1 has no necessary relationship with loss aversion effect, we set $\lambda = 1$. We also assume $\alpha = 1$ to rule out any potential effect from diminishing sensitivity in value function. Hence $W(r_i, v_i) = r_i - v_i$.

\textsuperscript{14} We also tried other cases in which $c_i$ is random. We found very similar results. The advantage of using a constant $c_i$ is that, conditional to it, we can examine the relationship between a seller’s asking price, her reference value and the expected market price.
First, we examine the relationship among a seller’s asking price, searching effort and reference value. The results are presented in Figure 2.2.

Figure 2.2: Seller’s Asking Price, Searching Effort and Reference Value ( $\lambda = 1$)

In both cases, seller’s asking pricing is increasing with reference value, which is consistent with finding 1. Since we have set $\lambda = 1$, we can confirm the prior conclusion that finding 1 has a very limited power on testing loss aversion effect. The impact attained from reference dependence is intuitive. Since the reference value is regarded as an un-sunk cost by sellers, the seller will ask for more compensation the higher the reference value is, because the higher the reference value is, the more likely the seller will realize a loss if selling in the market. Another finding is that with a more heterogeneous pool of potential buyers (uniform case)$^{15}$, a seller tends to increase her asking price, holding other factors constant. This is true for all possible reference values. The intuition is that, since potential buyers are more heterogeneous in terms of their preferences, the probability of meeting a buyer who is willing to pay a higher matching premium becomes larger. As a result, it is more attractive for a seller to ask a higher price in the market. Finally, we find that the

$^{15}$ You could also think of it as a case with more asymmetric information in a housing market.
search effort decreases with the asking price. It implies that on one hand, a seller may want to fish on the market by asking for a higher price. On the other hand, she may also choose to spend less effort in searching for the potential buyers. One obvious force that helps to generate this finding is the role of the reference value. The higher the asking price is, the higher the reference value, because they are conditional to a given moving cost. Since the reference value is also a cost component in addition to searching, an increase in reference value may depress the seller’s searching incentive.

In Genesove and Mayer [2001], and many other studies, a seller’s asking price is examined with respect to her potential loss exposure, which is the difference between a seller’s reference value and the expected market price. To make an apple-to-apple comparison, we need to calculate the expected market price. To do so, we further assume the sellers’ reference values distribute uniformly in [0,1]. Each time we randomly draw 10000 sellers from this distribution and calculate the expected market price following equation 2.7. We then define $Loss_i = v_i - \bar{P}$, as consistent with the empirical studies. A negative potential loss means a positive potential gain.

In empirical studies, in order to control for the quality difference of houses, researchers look at the net asking price, which is the residual amount after subtracting the expected market price from the original asking price. To do it here, we define a new measure called fishing, which is the difference between seller’s asking price and the expected market price, i.e., $Fishing_i = r_i - \bar{P}$. With reference on potential loss/gain exposure, the measure of Fishing tells us about sellers within which range will systematically ask a higher or lower than the expected market price. One appeal of comparing this way is because there is a general impression in the empirical literature that fishing behavior is only relevant to potential losers. The intuition tells us that potential losers should ask for a higher price as they don’t want to realize a loss by selling at the expected market price. However, from our model, it is clear that the optimal asking price reflects a trade-off among waiting time, a reference-dependent utility and search cost. Since the expected market price is nothing but a weighted average of the realized transaction prices, there is no reason for us to believe that fishing behavior is only applicable to sellers whose reference values are below
the expected market price. Furthermore, as discussed in footnote 7, the slope and curvature relationship to \( \text{Loss}_i \) will be the same no matter we look at \( r_i^* \) directly or at \( \text{Fishing}_i \) instead. In another word, our judgment on the testing power of finding one and two still holds when we look at \( \text{Fishing}_i \) instead of \( r_i^* \) directly. The relationship between seller’s optimal asking price and potential loss exposure is presented in Figure 2.3.

**Figure 2.3: The Relationship between Fishing Amount and Loss Exposure \( (\lambda = 1) \)**

![Figure 2.3](image)

This figure sheds light on many testable behaviors in the housing market. First of all, since we maintain \( \lambda = 1 \), it again confirms our judgment on finding one. We can see that big potential losers will fish more in the market, even without loss aversion effect. One less obvious implication is regarding the range of fishing behavior. Figure 2.3 shows that the existence of fishing is not necessarily associated with a potential loss position. Actually, although the fishing amount decreases as the expected gain increases, the possibility may still exist for some sellers with small potential gains. Hereafter, we will call this phenomenon the small gainer fishing effect. The most surprising prediction is for the sellers whose expected gains are large. Our model predicts that those sellers actually want to sell their houses at a price lower than is expected on the market, and the higher the
expected gain, the higher the discount a seller is willing to offer. To our knowledge, this phenomenon has not been proposed or examined in the literature. Again, when a buyer’s bidding follows a uniform distribution, the seller will have a stronger incentive on fishing by consistently asking for a higher price. One implication is that an increasing in buyer’s preference heterogeneity may strengthen a seller’s fishing incentive, which is also empirically testable.

Another factor that influences a seller’s pricing behavior is the waiting cost. Clearly, a higher waiting cost makes fishing less attractive, since fishing will decrease the hazard rate of selling a house. Eventually, if the waiting cost is large enough, the stress of selling a house quickly will dominate the incentive of fishing and getting more proceeds. Hence, the fishing amount should decrease with the waiting cost. To check this intuition, we will try several waiting costs and plot the results in Figure 2.4. From now on, we only present the finding with a quasi-triangular bidding distribution, since the results from uniform distribution are very similar and available upon request.

**Figure 2.4: Fishing Behavior when Waiting Costs are Different (\( \lambda = 1 \))**
Consistent with our intuition, we find that the fishing amount tends to decrease when the waiting cost increases. The slope becomes flatter when the cost is higher. Eventually, if the waiting cost is extremely high (e.g., $c=0.8$), the fishing behavior becomes insignificant and all sellers tend to ask very similar prices no matter what their reference values are, since everybody wants to ask a price that facilitates a sale as soon as possible. Nevertheless, within the reasonable range of the waiting cost, the general findings in Figure 2.3 still hold. For example, we still find losers to fish on the market and big gainers to sell at discount. Therefore, our results turn out to be robust with a reasonable waiting cost.

Finally, we also want to compare the robustness of the above findings with various discount rates. We try a wide range of discount rates and plot the results in Figure 2.5.

![Figure 2.5: Fishing Behavior with Different Discount Rates ($\lambda = 1$)](image)

From Figure 2.5 we see that the general findings as shown in Figure 2.3 are quite robust in a wide range of discount rates. Therefore, we can safely conclude that a monotonically
increasing relationship between seller’s asking price and potential loss exposure has no necessary relationship with loss aversion behavior.

Now we turn to the second question. Does finding two necessarily imply diminishing sensitivity in value function? If not, do findings one and two jointly imply loss aversion and diminishing sensitivity? We will specify two kinds of value functions. For the one with increasing sensitivity, we set $\alpha = 1.2$. For the other one, with diminishing sensitivity, we set $\alpha = 0.88$, as proposed by Tversky and Kahneman. We also compare two cases in each specification, with and without loss aversion. The results are plotted in Figure 2.6.

![Figure 2.6: Asking Price with Different Value Functions](image)

Genesove and Mayer [2001] find that for sellers subject to a positive potential loss, the marginal markup declines as the size of the potential loss increases. This finding is only inconsistent with the case of diminishing sensitivity, and without loss aversion ($\alpha = 0.88$ and $\lambda = 1$). More importantly, it is consistent with increasing sensitivity, and this is true no matter we have loss aversion effect or not. As a result, neither does finding two necessarily imply diminishing sensitivity in value function, nor do findings one and two jointly imply loss aversion and diminishing sensitivity. This result is not as controversial as it was when first looked at. On one hand, for sellers with a big potential loss (hence, high reference values), asking a higher price could increase the prospective gain if sold successfully. On the other hand, a higher asking price could significantly reduce the
probability of realizing a sale and increase the duration time on the market, which is at cost to the seller. It is not surprising that the second effect could dominate the first one with non-trivial waiting cost. Remember buyers’ biddings are centered on the middle point, which is far below the optimal asking prices for sellers subject to a big potential loss. Because of the high asking price, the selling hazard rates for these sellers are quite low. As a result, the effective benefit from fishing more is relatively trivial since it is very unlikely to realize a sale. Being opposite, for sellers with small reference values, due to the lower asking price, their asking prices are actually close to the central point of bidding distribution. In this range, a small increase in the asking price will have a smaller impact on a decreasing selling hazard rate. Due to the increasing sensitivity from perspective gains, the first effect may tend to dominate the second. This is why we find that big gainers may increase their asking prices compared with small gainers in the left panel.

If finding one and two have limited power on testing their theoretical counterparts, what then is the appropriate test to look at? It should be obvious from Figure 6 that the correct approach is also to look at the pricing behavior for those sellers with small reference values, which are then subject to big potential gains. Loss aversion and diminishing sensitivity assumptions jointly predict that sellers with low reference values should increase their asking price in a convex way. This is in contrast to the case of increasing sensitivity, in which big gainers should ask higher prices than small gainers. As a result, a correct way to test prospect theory is to test a joint S shape of selling price curve, i.e., concave in loss area and convex in gain area. Base on this prediction, in the empirical part we are able to find evidence that is consistent with the predications made by prospect theory.

2.2.2.3 Implication on Price-volume Relationship and Dispersion Effect

One puzzling finding in housing market is the strong positive correlation between house price and transaction volume. For example, using national level data, Stein [1995] finds that a 10 percent drop in prices is associated with a reduction of transaction volume by over 1.6 million units in the United States. Another study by Ortalo-Magne and Rady [1998] show a similar relationship in U.K, too. In this section, we show that, with prospect
utility, the decision problem as outlined in equation 2.1 can help explain this phenomenon. As reflected by the right panel of Figure 2.6, with prospect utility, although house sellers tend to mark up their asking along with reference values, the upper slope of asking price curve is much flatter for sellers that have low reference values than for sellers that have high reference values. This is due to the fact of loss aversion effect. Compared to the expected market price, for sellers that have high reference values, it is more likely that they will encounter a loss, which yields a bigger disutility due to the asymmetric response in value function. As a result, they have bigger incentive to mark up the asking price so as to reduce this disutility. The heterogeneity of pricing behavior between sellers with low and high reference values naturally leads a positive price-volume relationship. The intuition is simple, conditional on a given distribution of reference values, when market becomes hot, the willingness to pay from potential sellers will increase. As a result, in hot market, the proportion of sellers that have low reference values relative to market price becomes bigger. Since these sellers are not subject to potential equity loss, the incentive for them to fish on the market is low. As more and more sellers in the market choose to sell at a moderate price, as reflected by the lower and flatter asking price curve, it is clear that the probability of a successful sale will increase, which in turn generates a higher transaction volume.

Furthermore, a flatter asking price curve for sellers with low reference values may shed light on the extent of price dispersion in different market conditions. As just mentioned, when market becomes hot, the proportion of sellers that have low reference values relative to market price becomes bigger. With high market price, as more and more sellers cluster to the range of low reference values, the difference among their asking prices becomes smaller. In aggregate, we should expect that, ceteris paribus, in hot market, the observed transaction prices will be less dispersed with respect to the expected market price than in a cold market.

To confirm the implication of price-volume relationship and dispersion effect from our model, we simulate different market conditions and compare the corresponding transaction volume and variance of asking prices in these markets. In particular, we define
market fundamental as the mean of the seller’s bidding distribution. Therefore, in the last section, the market fundamental for quasi-triangular distribution with support of [0,1] is 0.5. In the following simulation, we expand the quasi-triangular distribution by gradually increasing the market fundamental from 0.5 to 1.5\textsuperscript{16}. The simulation process remains the same as before. That is, we assume the sellers’ reference values distribute uniformly in [0,1]. Each time we randomly draw 10000 sellers from this distribution. Again, we set $\beta = 5\%$, $c_i = 0.05$, $\alpha = 0.88$ and $\lambda = 2.25$. We normalize the transaction volume per unit period as 100 when market fundamental equals 0.5. By doing this way, it is easier for us to infer the percentage change of transaction volume with expected market price. When calculating the variance of asking prices under each expected market price, we weight every seller’s asking price by the corresponding selling probability. The simulation results are presented in Figure 2.7.

\textbf{Figure 2.7: Price-volume Relationship and Dispersion Effect}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure27.png}
\caption{Price-volume Relationship and Dispersion Effect}
\end{figure}

\textsuperscript{16} I.e., the quasi-triangular distribution with market fundamental of 1.5 has the same shape as before but with a support of [0, 3], and peaks at 1.5.
As expected, the upper panel of Figure 2.7 reveals a positive price-volume relationship. Conditional on a given pool of potential sellers in the market, when the expected market price increases from about 0.6 to 1, the transaction volume also increases about 15%. A concave pattern of the increased transaction volume is not surprising. Since all sellers’ reference values ranges from 0 to 1, when the expected market get bigger and bigger, less sellers are subject to potential loss. As discussed before, the homogeneity of asking price among sellers also increases since the impact from reference values become weaker and weaker when buyers are willing to pay more. Eventually, when the expected market price is high enough, transaction volume tends to become stable. Consistent with this argument, we observe that the variance of asking price, a measurement of price dispersion on market, tends to decrease when expected market price increases. Being interestingly, Williams [1999] points out that, in hot market, sellers will ask prices that are not too different from the expected market prices whereas in cold market many sellers prefer to remain their properties unsold by asking much higher prices above the market ones. He refers it as a puzzling behavior in real estate market that needs to be addressed. Clearly, our model with prospect utility provides a theoretical justification on this phenomenon.

2.3. Empirical Tests

In section 2.2, we develop a simple theory to model a potential seller’s pricing strategy under different loss/gain exposures. The model implies important predictions that can be tested empirically. Before moving to the report of our empirical findings, we first list the three key predictions of the model: 1) loss aversion and marginal diminishing sensitivity jointly imply an S shape of asking price curve along the reference value; 2) asking price becomes lower when waiting cost increases; and 3) the heterogeneity of asking prices among sellers tends to decrease when market price is high. We test these three hypotheses using a comprehensive housing transaction dataset in Vancouver.

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17 Williams [1999] points out similar behavior in commercial real estate and rental market, too.
18 The price-volume relationship has been a well known finding in the literature. And in general, to test this relationship we need more aggregate level data in order to control for the location substitution effect in housing market. Hence we will not test price volume relationship in this paper.
2.3.1 Data

Our housing transaction data are based upon the complete record of single-family transactions in city of Vancouver for the period 1980(Q1)-2005(Q2), which is provided by Landcor from the British Columbia Assessment Authority (BCAA). Due to the strength of the data, which includes detailed records of housing characteristics in terms of structure and neighborhood information, we are able to obtain a relatively accurate estimate on the expected market price by using a hedonic model. We drop those records that have missing values in the used hedonic characteristics, leaving us with 37,224 transaction records during the sample period that are available with which to test our hypothesis. Among them, 27,273 were sold by the repeated sellers.

2.3.2 Methodology

Define unit i’s expected log market value at time t as:

\[ V_{it} = X_i \beta + \delta_i \, . \]  

(3.1)

where \( X_i \) is a vector of hedonic characteristics, \( \delta_i \) is a time dummy for period t. However, in reality, we cannot observe this expected market value. Instead, what we observe is the transaction price at time t, in log form, we express it as:

\[ P_{it} = X_i \beta + \delta_i + e_{it} = V_{it} + e_{it} \]  

(3.2)

where the additional component \( e_{it} \) is the amount that is over or under-paid by the buyer. In the theory part, we assume housing units are ex-ante identical in terms of its structure characteristics and quality. In other words, all housing units should have the same expected market price. To control the quality difference in real data, we perform a two-stage process. In stage 1, we run a hedonic regression, through which we can recover the expected market price for each unit. We then subtract the expected market prices of different housing units and only focus on the residual term, \( e_{it} \), which measures a sellers’ heterogeneous selling prices after controlling the quality difference. In stage 2, we perform tests on a different hypothesis, using \( e_{it} \) as the measure of the net asking price.
One key prediction from our theory is that \( e_{it} \) depends on the potential loss or gain exposure, which reflects seller’s heterogeneous reference value. We measure it by a variable called \( \text{Loss}_{ist} \), to be consistent with Genesove and Mayer [2001]. The relationship is specified as:

\[
e_{it} = f(\text{Loss}_{ist}) + \beta^* c + \epsilon_{it}
\]  

(3.3)

where \( \epsilon_{it} \) is the error term with the usual assumptions. \( c \) refers to the seller’s waiting cost upon the sale. As is typically done in the literature, we use the original purchase price \( P_{is} \) at time \( s \) as reference value and, hence, \( \text{Loss}_{ist} \) is defined as the difference between prior transaction price and the current expected value:

\[
\text{Loss}_{ist} = (P_{is} - V_{it}) = \delta_s - \delta_i + e_{is}.
\]  

(3.4)

Substituting equation 3.4 into equation 3.3 yields our ideal econometric specification:

\[
e_{it} = f(\delta_s - \delta_i + e_{is}) + \beta^* c + \epsilon_{it}.
\]  

(3.5)

The fishing behavior is measured by the positive part of \( e_{it} \), \( (e_{it})^+ \), which refers to the fact that a seller will eventually sell her house at a higher than the expected market price. Likewise, the amount of discount is measured as \( (V_{it} - P_{it})^+ = |(e_{it})^-| \). Similarly, the potential loss is captured by \( (\delta_s - \delta_i + e_{is})^+ \), and the potential gain is therefore \( |(\delta_s - \delta_i + e_{is})^-| \).

One difficulty in testing the waiting cost effect is that one is unable to observe an individual seller’s moving pressure. Nevertheless, holding other factors constant, a smaller waiting cost implies less stress with regard to moving and a longer duration time in the current home. Fortunately, for a seller of each repeat-sales house, we have information on the length of time between her initial purchase and her next move so we can track the length of staying in the current house before a sale. Ideally, if we can get the demographic information of house sellers, we can use it to estimate the moving pressure conditional to the demographic information. Unfortunately, such information is unavailable in our data.
However, since we do have detailed information on housing characteristics, we can use it as a proxy of household characteristics and still get some systematic relationships between housing attributes and a household’s expected duration time. Of course, the validity of this procedure depends upon the assumption that there is a systematic relationship between the household moving pressure and the housing characteristics. Therefore, the remaining question is: should we expect any statistical relationship between a seller’s moving pressure and house characteristics? The answer is probably yes. One possible factor is the age of house. One may suspect that the older the house is, the less likely for the seller to wait too long in the market, since the proximity to a house’s demolition is closer. As a result, seller of an older house may have a tighter horizon constraint and, therefore, a higher cost of waiting. On the other hand, family size could have positive effect on seller’s horizon constraint. The intuition is that a larger family size could imply that this family has a tighter social tie with local community and, therefore, the opportunity cost for moving could be higher. As a result, all factors being equal, the bigger the family is, the lower the ci is. Unfortunately, our dataset doesn’t have information about the family size for each house. However, we do have information on the floor area of the house and the number of bedrooms, which seems to be reasonable proxies for family size.

To confirm our intuition about the effect of the above factors on moving cost, we use the total time the household lives in each house\(^\text{19}\) as a dependent variable and regress the variable on the age of the house, total floor area of the house and the number of bedrooms in the house. We use the Heckman two-stage regression to control the possible selecting issue which arises here. The intuition is that ex-ante people who have the intention to sell their houses later may select some housing attributes intentionally. For example, they may tend to purchase a newer house, since it could be easier for them to sell in the future. However, it doesn’t mean that buying a newer house tends to shorten a seller’s horizon if she sells the house ex-post. To estimate the selecting issue, we restrict our data before the Q2 1985, and track whether a household who bought a house before this period sold it within the next twenty years or not. The occurrence of a selling is tracked as 1 and not selling is tracked as 0. In the first stage of the Heckman model we estimate a Probit model

\(^{19}\) We assume it is the duration time between initial purchase and next selling.
decision of whether the house is sold within next twenty years on a set of housing characteristics. In the second stage, we regress the realized duration time on the factors that we raised in prior discussions to see whether they have any influence on expected duration time, resulting in some idea of the reasonability of our speculation. The results of the Heckman model are presented in Table 2.1.

<table>
<thead>
<tr>
<th>Explanatory variables:</th>
<th>Panel A: First-stage regression (Probit)</th>
<th>Dependent variable: Dummy on Selling or Not</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>1.1833***</td>
<td>(0.0622)</td>
</tr>
<tr>
<td>Age of House when Purchase</td>
<td>-0.0011***</td>
<td>(0.0002)</td>
</tr>
<tr>
<td>Floor Area</td>
<td>0.0000</td>
<td>(0.0000)</td>
</tr>
<tr>
<td># Bedrooms</td>
<td>0.0028</td>
<td>(0.0071)</td>
</tr>
<tr>
<td>House Price Index</td>
<td>-0.0001</td>
<td>(0.0002)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Explanatory variables:</th>
<th>Panel B: Second-stage regression</th>
<th>Dependent variable: Duration (In Month)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>69.0520***</td>
<td>(2.7243)</td>
</tr>
<tr>
<td>Age of House when Purchased</td>
<td>-0.0265**</td>
<td>(0.0114)</td>
</tr>
<tr>
<td>Floor Area</td>
<td>0.0020***</td>
<td>(0.0006)</td>
</tr>
<tr>
<td># Bedrooms</td>
<td>0.7006</td>
<td>(0.5123)</td>
</tr>
<tr>
<td># observations</td>
<td>5870</td>
<td></td>
</tr>
</tbody>
</table>

Note: 1) * Significant at 0.10 level. ** Significant at 0.05 level. *** Significant at 0.01 level.

Consistent with our intuition, we find significant evidence that people who sell their houses ex-post tend to buy relatively new ones ex-ante in their initial purchase. Therefore, there are some selecting issues arising from this data. Furthermore, we find that after controlling for a sample selecting problem, our intuition about the factors that could
influence horizon constraint makes sense. For example, people living in older homes tend to stay for a shorter time and move faster, and people living in bigger houses tend to live there longer before their next move. The coefficient for the number of bedrooms is positive, although insignificant. The possible reason for the lack of significance could be due to the high correlation between the number of bedrooms and the floor area. Actually, when we drop floor area from the equation, the coefficient for the number of bedrooms becomes significant at 5% level.

The above finding suggests some explanation power of housing characteristics on a seller’s ex-post moving pressure. To get a concrete measure of the moving cost, we use the Cox proportional hazard model to estimate a seller’s hazard rate of moving, conditional on the given housing characteristics that are used in our hedonic regression. We use the hazard rate as a proxy for the underlying moving cost of each seller. Since a higher hazard rate implies a higher likelihood of moving and a higher waiting cost, we should expect a negative coefficient when we regress the realized transaction price on this rate. 20

2.3.3 Estimation Results

Table 2.2 presents our basic results on the relationship between the selling price and a seller’s potential loss/gain exposures.

---

20 As an alternative measure, we directly regress the realization duration time on housing characteristics. We then use this estimated duration time as a proxy for waiting cost. Here, a higher expected duration time implies a lower waiting cost. We get very similar results when we use Cox hazard rate. The results are available upon request.
Table 2.2
Selling Price and Loss/Gain Exposures: OLS Results
Dependent Variable: Residual from Stage 1 Hedonic Regression

<table>
<thead>
<tr>
<th>Explanatory variables</th>
<th>Model 1 (OLS)</th>
<th>Model 2 (OLS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loss (std)</td>
<td>0.3638 ***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0123)</td>
<td></td>
</tr>
<tr>
<td>Loss^2 (std)</td>
<td>-0.1609***</td>
<td>-0.0706***</td>
</tr>
<tr>
<td></td>
<td>(0.0143)</td>
<td>(0.0046)</td>
</tr>
<tr>
<td>Gain (Std)</td>
<td></td>
<td>0.0457**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0036)</td>
</tr>
<tr>
<td>Gain^2 (Std)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Last Residual</td>
<td>0.1305***</td>
<td>0.1392***</td>
</tr>
<tr>
<td></td>
<td>(0.0034)</td>
<td>(0.0036)</td>
</tr>
<tr>
<td>Months since Last Purchase (std)</td>
<td>-0.0001***</td>
<td>-0.0001***</td>
</tr>
<tr>
<td></td>
<td>(0.00002)</td>
<td>(0.00002)</td>
</tr>
<tr>
<td>Moving Hazard Rate (std)</td>
<td>-0.3971***</td>
<td>-0.3971***</td>
</tr>
<tr>
<td></td>
<td>(0.0661)</td>
<td>(0.0661)</td>
</tr>
<tr>
<td>Constant (std)</td>
<td>0.0633***</td>
<td>-0.0395***</td>
</tr>
<tr>
<td></td>
<td>(0.0035)</td>
<td>(0.0038)</td>
</tr>
<tr>
<td>Neighborhood Fixed Effect</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td># observations</td>
<td>23273</td>
<td>23273</td>
</tr>
<tr>
<td>R^2</td>
<td>0.1670</td>
<td>0.1059</td>
</tr>
</tbody>
</table>

Note: 1) * Significant at 0.10 level. ** Significant at 0.05 level. *** Significant at 0.01 level.

2) Loss is defined as \((\delta - \delta^t + \epsilon_n)^t\), and gain is defined as \(|(\delta - \delta^t + \epsilon_n)^t|\).

3) Months since Last Purchase and Moving Hazard Rate are regressed in the first stage hedonic equation.

In Model 1, we essentially redo the empirical exercise as done by Genesove and Mayer [2001], namely, to look at the change of selling price with respect to seller’s potential loss.
exposure, which is measured as the positive part of the difference between the prior transaction price and the current period estimated hedonic price. The dependant variable is the residual from stage 1 hedonic regression, which represents a quality controlled selling price. Our results are consistent with prior findings in the literature. For example, we find the same pricing behavior as found in Genesove and Mayer [2001], i.e., people tend to ask a higher price when subject to a potential loss. In addition, the coefficient associated with the square term of loss is negative, which is consistent with predication of marginal decreasing markup.  

The residual from the last sale, which controls the potential un-observable house quality, shows a significant positive effect on the current selling price, which is intuitive. A positive last residual means the seller was willing to pay a higher than expected market price when she purchased that unit. Hence, it is very likely that the house may have some un-observable quality premium, which may make the current selling price high. In stage 1 hedonic regression, we include our estimated moving hazard rate as a measure of waiting cost. Being consistent with our theory, we find that people with a higher probability of moving tend to sell their houses at lower prices. Furthermore, we add months since last purchase as a proxy for loan to value ratio, which is unavailable in our dataset, also. It is reasonable to argue that, roughly speaking, the longer you stay in your house, the lower your loan to value ratio should be since you keep paying out your mortgage through time. Again, our finding is consistent with this argument. We find a significantly negative coefficient, which predicts that holding other factors equal, the longer the seller has lived in that house, the smaller the markup she would require in the market.

More important is our finding in Model 2. As explained in section 2.2, a procedure, like that in Model 1, has a very limited power on identifying either loss aversion effect or diminishing sensitivity effect. To test these two effects, we focus our attention on sellers with low reference values. Our theory predicts that, if people have asymmetric value function, diminishing sensitivity and low reference values, they should mark down their selling price in a marginally decreasing way. Since a low reference value means a

21 Genesove and Mayer [2001] talk about the potential issue of endogeneity in the measure of Loss. In an earlier draft of this paper, we also use differences in market price index as an instrument for Loss, and the results are quite similar.
potential gain instead of a loss, we perform a procedure, which is almost the opposite of Model 1, i.e., to regress selling price on the amount of prospective gains instead of losses. Consistent with the prediction of our model, we find significant evidence that a seller with a lower reference value tends to sell her house at a lower price, and also in a marginally decreasing way. Combined with our finding in Model 1, we are able to get an S shape bidding price curve, which is only consistent with effect of loss aversion and marginal diminishing sensitivity from prospect theory. Finally, the coefficients for other control variables have expected signs and similar magnitude as in Model 1 as explained in the last paragraph.

Our two stage testing process in Table 2.2 has potential problems. As discussed in Genesove and Mayer [2001], when considering the effect of loss aversion on transaction prices, we need to simultaneously estimate the expected market value and a seller’s reference value. Nevertheless, we only have one set of information: the realized transaction price. To estimate both values using the same set of information, a one stage process is more appropriate. Meanwhile, by definition, Loss and Gain are defined upon the difference between prior transaction price $P_{it}$ and the current market value $V_{it}$, which implies that $\text{Loss}_{it} = (P_{it} - V_{it}) = \delta_i - \delta_t + e_{it}$. Since the dependant variable in the second stage is the residual from stage 1 hedonic regression, our two stage specification implies that the residual term from hedonic regression will be correlated with the residual term in the last observation, which also generates an autocorrelation problem in stage 1 hedonic regression. To overcome this limitation, we use the nonlinear least square method and receive all estimation done in one stage. In particular, we estimate:

$$P_{it} = X_{it}\beta + \delta_i + \gamma f(Loss_{it}) + \lambda Lastresid + e_{it}$$  \hspace{1cm} (3.6)

where $Loss_{it} = P_{it} - V_{it} = P_{it} - X_{it}\beta - \delta_i$, and $\gamma$ is the coefficient vector associated with reference value specifications in $f$. The model is solved by the Maximum Likelihood method, and the results are presented in Table 2.3.
<table>
<thead>
<tr>
<th>Explanatory variables</th>
<th>Model 3 (NLLS)</th>
<th>Model 4 (NLLS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loss (std)</td>
<td>0.4805***</td>
<td>0.1463***</td>
</tr>
<tr>
<td></td>
<td>(0.0120)</td>
<td>(0.0103)</td>
</tr>
<tr>
<td>Loss^2 (std)</td>
<td>-0.3198***</td>
<td>0.1394**</td>
</tr>
<tr>
<td></td>
<td>(0.0119)</td>
<td>(0.0059)</td>
</tr>
<tr>
<td>Gain (Std)</td>
<td></td>
<td>-0.1463***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0103)</td>
</tr>
<tr>
<td>Gain^2 (Std)</td>
<td>0.1394**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0059)</td>
<td></td>
</tr>
<tr>
<td>Last Residual</td>
<td>0.3083***</td>
<td>0.3886***</td>
</tr>
<tr>
<td></td>
<td>(0.0023)</td>
<td>(0.0080)</td>
</tr>
<tr>
<td>Months since Last Purchase (std)</td>
<td>-0.0002***</td>
<td>-0.0005***</td>
</tr>
<tr>
<td></td>
<td>(0.00002)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>Moving Hazard Rate (std)</td>
<td>-0.3560***</td>
<td>-0.4530***</td>
</tr>
<tr>
<td></td>
<td>(0.0499)</td>
<td>(0.1097)</td>
</tr>
<tr>
<td>Constant (std)</td>
<td>11.9158***</td>
<td>12.1852***</td>
</tr>
<tr>
<td></td>
<td>(0.0686)</td>
<td>(0.1483)</td>
</tr>
<tr>
<td>Neighborhood Fixed Effect</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td># observations</td>
<td>23273</td>
<td>23273</td>
</tr>
</tbody>
</table>

Note: 1) * Significant at 0.10 level. ** Significant at 0.05 level. *** Significant at 0.01 level.

2) Loss is defined as \((\delta_i - \delta_i + e_{it})^+\), and gain is defined as \(|(\delta_i - \delta_i + e_{it})^-|\).

Our results in Table 2.3 are largely consistent with what we found in the OLS regression case. The coefficients on Loss and quadratic loss both increase in magnitude. They are now more consistent with what was reported by Genesove and Mayer [2001] for sold
properties in their sample.\footnote{They report the coefficient on Loss to be 0.49 and on Loss^2 to be -0.29 for properties that were sold eventually, but on the asking price instead of the realized transaction price. See Table V in Genesove and Mayer [2001].} Again, we observe an increasing and concave bidding price pattern for sellers with high reference values, and a decreasing and concave pattern for a lower reference value case. The waiting cost effect is very persistent and also has a similar magnitude as in OLS regression. Being different from findings in Table 2.2, we find a much stronger effect from the unobserved house quality on a realized housing transaction price. For both cases, the coefficients associated with Last Residual are more than twice the size of what we found in Table 2.2. The coefficient for Last Residual implies that a 1% increase in a seller’s initial purchasing price will cause a 0.3 to 0.4% increase in the current selling price after controlling the effect of reference value.

In Models 1 to 4, we essentially separate sellers into two categories: those subject to perspective losses and those subject to perspective gains. In each regression, we censor one group’s Loss/Gain exposure as zero and look at the behavior of the other group. This is the standard method in the literature when researchers try to test loss aversion effect. Since in previous literature people only focused on the loser side, they wanted to isolate the effect on gainers, but excluded other information. That is why they chose censoring gainer’s reference exposure to zero instead of dropping them completely. However, the justification for this censoring treatment is unclear. Next, we choose not to censor seller’s loss and gain exposures and estimate the reference value effect for both groups jointly. We still call this variable as Loss, and a negative value of Loss means a positive gain. To test for the S shape of the selling price curve, we add up to the fourth power for Loss in our regression.
| Table 2.4 |
| Selling Price and Loss/Gain Exposures: A Joint Estimation |

<table>
<thead>
<tr>
<th>Explanatory variables</th>
<th>Dependent Variable: Residual from Stage 1 Hedonic Regression</th>
<th>Dependent Variable: Log of Transaction Price</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model 5 (OLS)</td>
<td>Model 6 (NLLS)</td>
</tr>
<tr>
<td>Loss (std)</td>
<td>0.1363 *** (0.0065)</td>
<td>0.1364 *** (0.0067)</td>
</tr>
<tr>
<td>Loss^2 (std)</td>
<td>0.0591*** (0.0074)</td>
<td>0.0536*** (0.0063)</td>
</tr>
<tr>
<td>Loss^3</td>
<td>-0.0465*** (0.0045)</td>
<td>-0.0643*** (0.0041)</td>
</tr>
<tr>
<td>Loss^4</td>
<td>-0.0126*** (0.0017)</td>
<td>-0.01564*** (0.0014)</td>
</tr>
<tr>
<td>Last Residual</td>
<td>0.2866*** (0.0062)</td>
<td>0.3523*** (0.0063)</td>
</tr>
<tr>
<td>Months since Last Purchase (std)</td>
<td>-0.0001*** (0.00002)</td>
<td>-0.0002*** (0.0001)</td>
</tr>
<tr>
<td>Moving Hazard Rate (std)</td>
<td>-0.3971*** (0.0661)</td>
<td>-0.4614*** (0.0846)</td>
</tr>
<tr>
<td>Constant (std)</td>
<td>0.0419*** (0.0059)</td>
<td>12.1840*** (0.1130)</td>
</tr>
<tr>
<td>Neighborhood Fixed Effect</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td># observations</td>
<td>23273</td>
<td>23273</td>
</tr>
<tr>
<td>R^2</td>
<td>0.1640</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Note: 1) * Significant at 0.10 level. ** Significant at 0.05 level. *** Significant at 0.01 level.

2) Loss is defined as \( \delta_s - \delta_t + e_{is} \), without censoring.

3) In Model 5, Months since Last Purchase and Moving Hazard Rate are regressed in the first stage hedonic equation.
Except Loss related variables, other coefficients remain consistent with our prior findings. Furthermore, the four coefficients associated with Loss are all significant, although the curvature is difficult to see directly. To delve deeper into the joint curvature of selling price on reference values, we plot the predicted selling price, conditional to different loss/gain exposures in Figure 2.8.

**Figure 2.8: The Joint Curvature of Selling Price**

From Figure 2.8 we do observe a clear S shape selling price curve with an increasing reference value. The x-axis ranges from -1 to 1.2\(^2\), which covers 90% of the observations in the sample. Compared with our theoretical prediction in Figure 2.6, this pattern is only consistent with the case that has loss aversion and diminishing sensitivity in the value function. Hence, we are able to support the prospect theory, which confirms the existence of reference dependence and loss aversion effect.

The last predication from our theory is the price dispersion effect. If we interpret the difference between the realized transaction price and expected market price as a noise, our theory predicts that the higher the expected market price is, the smaller the noise should be since there would be fewer and fewer losers in the market. To test this prediction, in each quarter, we first compute the variance of the stage 1 hedonic residuals and then regress it on the level of price index in that given quarter. One point to note is that our theory has different implications on repeat sellers and sellers for new house. Repeat sellers generally

\(^{23}\) Which refers to about 40% of potential loss and 172% of potential gain.
bought their houses at different time and hence subject to different initial purchase prices, even after controlling from quality. In another word, there is bigger heterogeneity in repeat sellers’ reference values. As a result, we should expect the existence of price dispersion for repeat sellers. However, for sellers of new house, this may not be true. Firstly, new house is typically sold by real estate developers, instead of individual households. And the prospect theory is more relevant for individual’s decision making process. Secondly, even with the assumption that real estate developers follow the exactly same decision process as individual sellers, we should still expect little price dispersion effect for these sellers. The reason is simple. Conditional on a given quarter, developers should face very similar cost in terms of construction materials, financing etc, which are possible candidate for the reference values of real estate developers. Hence these sellers should have very similar reference values, if any. However, conditional on a common \( v_i \), they will also have same asking price. In this case, our model predicts no price dispersion effect. Accordingly, to test price dispersion effect, we calculate two variances for hedonic residuals in each quarter. One is for the repeat-sellers and the other is for the new house transactions. Table 2.5 reports the regression results for both groups.

<table>
<thead>
<tr>
<th>Table 2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price-dispersion Effect</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dependent Variable: Variance of Hedonic Residuals in Each Quarter</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Variables</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Constant</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>House Price Index</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td># observations</td>
</tr>
<tr>
<td>( R^2 )</td>
</tr>
<tr>
<td>( \rho(\sigma^2, Index) )</td>
</tr>
</tbody>
</table>

Note: 1) * Significant at 0.10 level. ** Significant at 0.05 level. *** Significant at 0.01 level.

2) The level of base quarter house price index is 100.

Consistent with the prediction of our theory, for repeat sellers, the coefficient for house price index is negative significant at 1%, which means when market price get higher, the
market noise (measured by the variance of hedonic residuals) becomes smaller. This negative relationship is also supported by the correlation measure between variance and price index level. Meanwhile, as expected, we don’t find similar dispersion effect for new house transactions. The price noise for those houses seems to be insensitive to the market price level.

2.4. Further Implication on the Repeat-sales Index

One of the most commonly used methods to construct house price index is the repeat-sales method. This technique only examines transactions in which the same house was sold more than once during the time period under examination. However, one key implication from our theory is that for those repeat sellers who face a potential loss, they tend to ask a higher than expected market price and wait longer in the market. It causes problems when we calculate the price difference for two sequential transactions, which are then used as the dependent variable in repeat-sales regression. This idea is better explained by the figure below.

**Figure 2.9: Fishing Effect in Repeat-Sales Data**

Suppose we observe in the dataset a negative price pair in which a seller who purchases a house in period 1 at $10 sells her house in period 5 at $8. The observed data tells us that this seller realizes a 20% loss from period 1 to period 3. However, the loss we observe is a realized one. Since this seller experiences a loss in the transaction, it is very likely that the
observed loss is a result after she has fished on the market. Therefore, it may be a biased reflection of true market dynamics. The true market could have dropped 30% from period 1 to period 4, but we cannot observe it from data. Following this intuition, we could identify two potential biases if we use repeat-sales data directly. One is the level bias due to the standard fishing or discounting effect and this will generate an upward bias when we observe a negative price pair and either upward or downward biases for positive price pairs. The other bias is a lagging bias due to the timing effect after fishing. As we already know, if a seller asks for a higher than the expected market price, the expected waiting time for her to sell becomes longer, which causes our data to be a lagged indicator of true market dynamics. One instant implication from these two biases is that repeat-sales index tends to underestimate the market volatility and, therefore, the risk level of housing market. To check this prediction, Figure 2.10 plots both hedonic and repeat-sales price indices for the Vancouver single-family housing market during our sampling period.

**Figure 2.10: Hedonic and Repeat-Sales Index**

Due to our hedonic index, including information from both repeat-sellers and first time transactions, we expect hedonic index to have a better measure of true market volatility compared to repeat-sales index. To compare the volatility from both indices, we compute the standard deviation from each of them. The SDEV for hedonic index is 152.33.

\[24\] For gainers, we may expect a downward bias since they may sell at a lower than expected market price.
Consistent with our prediction, the SDEV from repeat-sales index is only 139.85, which is 8.2% lower than the SDEV of the hedonic index. To summarize, we do find evidence that the repeat-sales index generally underestimates the true market volatility.

2.5. Concluding Remarks

This paper provides a critical judgment on the current literature on testing loss aversion behavior in housing market, as lead by Genesove and Mayer [2001]. We build a simple search model, which enables us to examine the impact from three components of prospect theory separately. We conclude that both findings in Genesove and Mayer [2001] are evidence that support a reference-dependence value function. Therefore, they are valid tests of the first component in the prospect theory. However, finding one does not have a necessary relationship with loss aversion, nor does finding two have a necessary relationship with diminishing sensitivity in the value function. Hence, there is a conceptual mismatch between two empirical findings and their theoretical counterparts.

We further propose that a value function, which is fully consistent with prospect theory, should imply sellers’ asking prices to increase convexly when reference values are low and to increase concavely when they are high. In addition, there should be a significant slope increase among sellers who are around break-even positions. Using a dataset of Vancouver single-family transactions, we are able to find evidence that supports all predictions from prospect utility.

Finally, our model also helps to explain the positive price-volume relationship and why the price dispersion in a cold market is bigger than in a hot market.
References


CHAPTER 3: Bidding Heterogeneity, Signaling Effect and Its Implications on House Seller’s Pricing Strategy

3.1. Introduction

An intriguing finding in the housing market is that, when facing a common housing market environment, sellers, for qualitatively similar units, may set very different prices; and provided the units are sold successfully, may get very different selling proceeds. For example, Genesove and Mayer [2001] provides evidence that after controlling for the quality heterogeneity of house goods, different sellers’ asking prices or the realized transaction prices could differ by about 20%. Additional findings on this price dispersion effect in housing market include, but are not limited to, Leung et al, [2006], Sun [2007], and Zheng et al, [2008]. To rationalize this phenomenon, a theoretical model needs to address two questions: firstly, facing the same market condition, why sellers may have incentive to ask for different prices for the same house good; secondly, why sellers can actually sell the qualitatively similar house goods at different prices.

Inspired by the seminal work on prospect theory by Kahneman and Tversky [1979] and Tversky and Kahneman [1991, 1992], Sun [2007] looks at the impact of reference value on house sellers’ pricing strategy and provides a potential answer on the different asking prices across sellers. However, the second question still remains to be answered. The fact that it is actually possible for sellers to sell a house at very different prices implies that potential buyers should have heterogeneous valuations on a given house good. Then, an important research question here is that what is the impact of this bidding heterogeneity on seller’s pricing strategy, in particular when the extent of this bidding heterogeneity changes. As a follow on research of Sun [2007], the purpose of this paper is to look at the source of the varying bidding heterogeneity and the impact of this varying heterogeneity

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25 A version of this chapter will be submitted for publication. Sun, H. Bidding Heterogeneity, Signaling Effect and Its Implications on House Seller’s Pricing Strategy.

26 Other factors such as different opportunity cost of waiting may also help generate the heterogeneous asking prices. See Sun [2007] for the discussion on it.
on a seller’s optimal pricing strategy. As pointed by Zheng et al, [2008], the research on the role of bidding heterogeneity on optimal seller pricing strategy is very limited.

The prior research that is most related with this study is Zheng et al, [2008]. Using a search model, the authors show that a greater dispersion in bidding price leads to a higher asking price and they provide some empirical evidence on it. However, their research has some limitations. First, from the theoretical perspective, their modeling part doesn’t provide insight on the factors that may generate the varying bidding heterogeneity. The authors simply assume this variation exists when they examine the seller’s optimal pricing strategy. Second, their empirical measure on the varying bidding heterogeneity, the standard deviation of the delisting prices of sold properties in different geographic submarkets, has some drawbacks. Following the model, each delisting price is determined by the extent of bidding heterogeneity. Therefore, using the second moment of a pool of the delisting prices as a measure of bidding heterogeneity causes the endogeneity problem when they regress the delisting price on this standard deviation measure, which is essentially the base of their empirical test. Meanwhile, since they use the whole sampling period when calculating this second moment, they inevitably face the problem of using future information to estimate the current outcome. Third, the reference-dependence effect is abstracted in their study. In another word, when facing the same bidding distribution, their model predicts a common optimal asking price across sellers, on which much counter evidence has been provided in the literature.

Despite a similar research focus as in Zheng et al, [2008], this study aims to contribute to the literature in several aspects with the caution to avoid the above-mentioned limitations. Firstly and the most importantly, unlike Zheng et al, [2008], we explicitly identify one source of the varying bidding heterogeneity in the model, i.e., the signaling effect when market participants can observe the transaction prices of other units that are close comparables for the unit that a current seller wants to sell.

Acknowledging the fact that in the multi-unit residential market, units in the same building serve as close substitutes for each other, we show that the recent transaction price
on a unit in the same building generates two signaling effects: first, it generates positive spatio-temporal autocorrelation with the subsequent price for the units in the same building; second, after observing the prior price, the heterogeneity of potential buyers’ willingness to pay decreases. The intuition is that, as the available housing goods become more comparable to each other, the transaction prices of other units may convey useful information on the valuation of a particular unit for sale on the market, which helps to reduce the disagreement on the willingness to pay across subsequent buyers and hence reduce the sellers’ bidding heterogeneity on that unit.

Adopting a simplified version of the search framework proposed in Sun [2007], our model predicts that sellers’ optimal asking prices would decrease when the potential buyers’ bidding heterogeneity decreases, which is consistent with the theoretical prediction of Zheng et al, [2008]. Using a geo-coded dataset on condominium transactions in Singapore from 1990 to 2001, we show that the spatio-autocorrelation on the quality-controlled price of one unit and the previously transacted units in the same building is significantly higher than with that of the units in neighboring buildings.

Meanwhile, using a two order spatio-temporal autoregressive model as proposed by Sun, Tu and Yu [2005], we find that, after controlling for the autocorrelation effect, sellers tend to mark down their asking prices by at least 1.6% if a recent transaction has occurred within the same building, which is consistent with the effect from the decreasing bidding heterogeneity among potential buyers. Since we use the incidence of a transaction in the same building within a very short time period (one month) as an indicator for a signal, our measure on the varying bidding heterogeneity is less likely to face the endogeneity problem as in Zheng et al, [2008].

Secondly, in the model we explicitly control for the impact of reference values on seller’s optimal pricing strategy. And our study re-affirms Sun [2007]’s finding that sellers’ optimal asking prices would increase with their reference values, which generates heterogeneous asking prices across sellers, and it is absent in the model of Zheng et al, [2008].
Another unique feature of our model is that, when decomposing potential buyers’ willingness to pay, we introduce a component which we call the substitution premium. We expect that in a heterogeneous housing market, potential buyers’ willingness to pay should also depend on the availability of finding a different house good which is a close substitute to a particular housing unit. Nevertheless, we don’t have any prior belief on to which direction would this substitution effect go. Our empirical finding suggests that potential buyers’ willingness to pay for any particular house unit is negatively related with the number of house substitutes in the market. This is consistent with a story of decreasing opportunity cost for buyers to sacrifice a house good when outside substitutes are easier to find. Think of the case in which a buyer finds one unit that is attractive to him. If it is relatively easy for him to find another unit that is highly substitutable on the market, the potential cost of sacrificing the current shopping opportunity decreases and hence we should expect a lower willingness to pay. This can be contrasted to a case in which a potential buyer would have difficulty finding another substitute for this particular house unit. As a result, provided that the house goods on the market are equally accessible to all potential buyers, the mean level of the bidding distribution will decrease. As our theory predicts that the seller’s optimal asking price would decrease with the mean level of potential buyers’ bidding distribution, it implies that a seller’s optimal asking price should decrease with the empirical measure on this substitution premium.

Furthermore, our model predicts that the sellers’ optimal asking prices should also increase with the exogenous arrival rate of potential buyers, which is intuitive.

Our empirical findings are again consistent with the above mentioned model predictions. For example, we find significant evidence of the reference dependence effect. In addition, using a proxy measure on the exogenous arrival rate of potential buyers, we find sellers’ asking prices tend to decrease with this measure, as predicted by the model.

The structure of the paper is as follows: Section 3.2 first lays out the model for a seller’s decision problem which incorporates reference-dependent utility as a special case. It then introduces a signaling mechanism and talks about its impact on potential buyers’ bidding distribution. The section ends by a discussion on the model’s empirical predictions on a
seller’s optimal pricing strategy, when the parameter values such as the varying bidding heterogeneity and arrival rate, etc, changes. Section 3.3 proposes a testing procedure and discusses the empirical findings. Section 3.4 concludes the paper.

3.2 Model Setup and Results.

3.2.1 General Setup
We adopt an analytical framework which is simplified from Sun [2007]. Consider a large housing market with a countable infinite number of sellers and potential buyers. Potential buyers arrive in independent Poisson process. During the time window of $\Delta t$, a buyer arrives with the probability of $B\Delta t$. As a result, no buyer arrives with the probability of $1 - B\Delta t$, and more than one buyer arrives with a probability that is smaller than the order of $\Delta t$.

To simplify our analysis, we assume that house goods are structurally identical. Consistent with the prior literature on search model in the housing market such as Williams [1998], Sun [2007] and Zheng et al, [2008], we assume that buyers’ bidding prices follow some distribution with mean $\mu$ and variance $\sigma^2$, and it is common knowledge to all sellers. In addition, to take use of the nice property of joint normal distribution to illustrate the signaling mechanism in the next subsection, we further assume that this distribution is actually normal and we use $G(p; \mu, \sigma^2)$ to stand for the C.D.F of this normal distribution$^{27}$. In section 3.2.2 we will have a detailed discussion on the formation and evolvement of this $G$ distribution.

The seller $i$ then chooses an asking price $r_i$ for her house. If $p_i \geq r_i$, the transaction is closed and the buyer pays $r_i$. Nevertheless, instead of drawing utility directly from selling proceeds, seller $i$ will evaluate it relative to an inherited reference value, $v_i$. Therefore, $v_i$ is explicitly treated as an un-sunk cost opposite the classical model$^{28}$, in which all prior

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$^{27}$ For applications of this search model using other distributions, see Sun [2007].

$^{28}$ The classical model is actually a special case of our general reference-dependent utility, i.e., when $v_i = 0$ for all i. In this case, sellers are maximizing the expected selling proceeds.
costs are sunk and hence irrelevant to the current decision. Collectively, the seller i’s objective is to solve the following maximization problem:

\[
U^{v,c}(G; \tilde{G}) = \max_{\epsilon} e^{-\beta\Delta t} \left\{ B(1-G(r; \mu, \sigma^2))\Delta t(r_i - v_i) + (1-B(1-G(r; \mu, \sigma^2))\Delta t)[E[U^{v,c}(\tilde{G})] - c\Delta t]\right\} + o(\Delta t)
\]

(2.1)

The first observation is that \( U^{v,c}(G; \tilde{G}) \) is a function of the current bidding distribution \( G \). Meanwhile, we use \( \tilde{G} \) to emphasize that this bidding distribution is actually a state variable which may evolve over time. In our model, the only way \( \tilde{G} \) could change is by receiving a price signal, which we will discuss in the next subsection. Here the superscript on \( U \) means that, the seller’s reference value (\( v_i \)) and the waiting cost (\( c \)) are treated as exogenous parameters in seller i’s maximization problem.

Over the time interval \( \Delta t \), the seller maximizes the discounted expected payoff from the following decision problem. First, it is possible that one buyer, who is willing to pay \( r_i \) will arrive. In this case, if the transaction is completed, the seller draws a reference-dependent utility, \( r_i - v_i \). The first term in the bracket of equation 2.1 measures this effect. Likewise, \( 1-B(1-G(r_i))\Delta t \) refers to the probability of remaining unsold and rolling over to the next \( \Delta t \) time interval. In this case, the seller incurs a waiting cost, \( c\Delta t \), and will repeat the decision problem, which yields \( E[U^{v,c}(\tilde{G})] \). The reason of using \( E[U^{v,c}(\tilde{G})] \) is because potential buyers bidding distribution, \( G \), may have changed when a seller rolls over to the next \( \Delta t \) time interval and hence she must draw an expected utility. We should note that the terms in brackets correctly consider all possible events that could happen during period \( \Delta t \). The other events can only occur with the probability of a smaller order than \( \Delta t \). Those terms are collected as \( o(\Delta t) \). We now discuss the signaling mechanism and its implication on the evolvement of potential buyers’ bidding distribution \( G \).

### 3.2.2 An Illustration of the Signaling Effect

In this study, we decompose the willingness to pay from potential buyers on house unit \( i \) into three components. The first component is the fundamental value of housing, \( P_i \), and
we assume it is common knowledge for all buyers. The second component reflects a common substitution premium, $\theta_i(N)$, for all potential buyers when they inspect house unit $i$. Here $N$ stands for the number of house goods that are close substitute for house unit $i$. Empirically, we use the total number of house units in each condominium building as a measure of $N$. As discussed in the introduction, $\theta_i$ depends on the uniqueness of the house good and is treated as an exogenous parameter in our model. Whether $\theta_i$ should be smaller or bigger with a larger $N$ is an empirical question. Upon this point the readers may have noted that the first two components are common values for all potential buyers. In the third component, we introduce a source of heterogeneous willingness to pay across buyers, i.e., the personal tastes from potential buyer $j$ on unit $i$, $\sigma \varepsilon_{i,j}$. Here $\varepsilon_{i,j}$ is distributed as standard normal and $\sigma$ measures the extent of the valuation heterogeneity across buyers. Furthermore, we assume $\varepsilon_{i,j}$ is private information only for potential buyer $j$.

Mathematically, we can express the willingness to pay from potential buyer $j$ for house unit $i$ as:

$$P_{i,j} = \overline{P}_i + \theta_i + \sigma \varepsilon_{i,j}$$  \hspace{1cm} (2.2)

Hence, without any signal, potential buyers’ willingness to pay is normally distributed with mean $\overline{P}_i + \theta_i$ and variance $\sigma^2$, and we assume seller $i$ is aware on this distribution.

As mentioned in section 3.2.1, in our model house goods are structurally identical. Then the only factor that can differentiate house goods is location. Now consider the case in a multi-unit residential market. The fact that units in the same building share a common parcel of land then imply that units in the same building should share a common fundamental value, i.e., $\overline{P}_i = \overline{P}_t$ if unit $i$ and $t$ are in the same building. By similar argument, we also expect that $\theta_t = \theta_i$ and $\varepsilon_{i,j} = \varepsilon_{t,j}$ if unit $i$ and $t$ are in the same building.
We are now ready to discuss the signaling effect in a multi-unit residential market. Before we illustrate the mechanism, we need to assume the correlation structure on the private information, $\varepsilon_{i,j}$. In particular, we assume potential buyers’ personal tastes are positively correlated, i.e.,

$$\text{Cov}(\varepsilon_{i,j_1}, \varepsilon_{i,j_2}) = \sigma^2$$

for potential buyers $j_1$ and $j_2$. The assumption that potential buyers’ personal valuations are positively correlated is not unreasonable. One stylized example is the well-known winners curse effect from the auction literature. The fact that by realizing lower willingness to pay from other buyers make winner worry about paying too much exactly reflects the idea that his/her personal valuation should be positively correlated with other buyers’ valuation, in particular when other buyers’ valuations are observable in the market. Now, suppose that within one building, the willingness to pay of a potential buyer $j$ on unit $i$, $p_{i,j}$, is revealed to the market. We use lower case $p$ to emphasize that $p_{i,j}$ is a realization of random variable $P_{i,j}$. Then, conditional on observing $p_{i,j}$, other buyers may update their personal valuations accordingly. In particular, we assume that collectively, potential buyers as a pool will update their heterogeneous willingness to pay following the Bayesian rule$^{29}$. We thus get the following proposition:

**Proposition 1.** Suppose $p_{i,j}$ is a random realization of $P_{i,j}$. Upon observing $p_{i,j}$, the conditional bidding distribution ($G_{p_{i,j}}$) of potential sellers for house unit $i$ is normal and has the following properties:

$$E[P_{i,j} | p_{i,j} = p_{i,j}] = E[P_{i,j} | \overline{P}_i + \theta_i + \sigma \varepsilon_{i,j}] = \overline{P}_i + \theta_i + \frac{\sigma^2}{\sigma} \varepsilon_{i,j}$$

(2.3)

$$\text{Var}[P_{i,j} | p_{i,j} = p_{i,j}] = \text{Var}[P_{i,j} | \overline{P}_i + \theta_i + \sigma \varepsilon_{i,j}] = \sigma^2 - \frac{\sigma^4}{\sigma^2} < \sigma^2$$

(2.4)

$^{29}$ Alternatively, we can choose to interpret this updating process from the seller’s perspective. In this case, we assume seller $i$’s prior belief on the bidding distribution $G$ is given by equation 2.2. And upon observing a revealed willingness to pay, $p_{i,j}$, she then updates and forms a posterior belief following the Bayesian rule.
**Proof:** In appendix.

Two instantaneous results can be learned from Proposition 1. Firstly, equation 2.3 implies that the posterior mean on the bidding distribution is positively correlated with the observed signal, $p_{i,j}$. A higher revealed willingness to pay implies a higher personal valuation from buyer $j$, as reflected by a bigger value of $\sigma_{i,j}$. Due to the positive correlation among personal valuations across potential buyers, upon observing a higher realized value of $\sigma_{i,j}$, other buyers then infer that it is more likely that the units in that building are more valuable, as reflected by the higher value of the conditional mean. In the next section, we will show that, compared to the prior mean value, an increase in the posterior mean level on the willingness to pay distribution will induce a universal mark up on the asking price for sellers with any reference values. This then generates a positive correlation between the prior realized transaction price for one unit and the subsequent transaction prices for other units in the same building.

Secondly, upon observing a signal, the bidding heterogeneity among potential buyers also decreases, as shown by equation 2.3 due to the shrinking variance. This standard noise reduction process is intuitive. As long as other buyers’ revealed willingness to pay conveys meaningful information on adjusting a buyer’s personal valuation, the opportunity of observing this revealed willingness to pay should make subsequent buyers have less divergent opinions on their personal valuations, and the law of one price seems to become more and more relevant.

As a side note, we need to mention that using inter-correlated personal valuations is only one way to illustrate this signaling process. For example, we can alternatively assume that due to the imperfect information, potential buyers cannot perfectly learn the expected house value, i.e., $\overline{P}_i + \theta_i$. Rather, what they observe is some private noisy signal on it, which equals to $\overline{P}_i + \theta_i + \sigma_{i,j}$; and we assume this signal is inter-correlated across potential buyers and the whole distribution is common knowledge to sellers. The
collectively updating process is again as described in this section. For those readers that are not used to the updating process from potential buyers’ side, we can also choose to interpret this updating process from the seller’s perspective. In this case, we assume seller i’s prior belief on the bidding distribution G is given by equation 2.2. And upon observing a revealed willingness to pay, \( p_{i,j} \), she then updates and forms a posterior belief following the Bayesian rule. All these interoperations will lead to the same mathematical implications as outlined in Proposition 1. The key intuition here is that, as long as the signal conveys meaningful information to the market participations, we should expect a kind of noise reduction process as we discussed in this section.

We now discuss the solving strategy for the proposed search model and look at the impact from the positive correlation and decreasing heterogeneity on a seller’s pricing strategy.

### 3.2.3 Modeling Results

To solve for seller i’s maximization problem as in equation 2.1, we need to first characterize the expected rolling-over utility, \( E[U^{v,c}] \), beyond the current \( \Delta t \) time interval. During this short period, the only concern is whether the state variable, G, will change or not. Following Proposition 1, G will change upon receiving a signal, with both a mean shifting and variance reduction. As a result, suppose the current state bidding distribution is G, then

\[
E[U^{v,c}] = U^{v,c}(G; \tilde{G}) \times \text{Prob(No Signal)} + E[U^{v,c}(G_{1 \text{Signal}}; \tilde{G})] \times \text{Prob(1 Signal)} \\
+ E[U^{v,c}(G_{2 \text{Signals}}; \tilde{G})] \times \text{Prob(2 Signals)} + \ldots
\]

(2.5)

where \( G_{1 \text{Signal}} \) stands for the updated bidding distribution upon getting one signal; and likewise for \( G_{2 \text{Signals}} \). Following our definition, to observe a signal, a necessary condition is that two events must occur simultaneously: firstly, within the same building, there is at least one competing seller \( i' \) who is selling her house; secondly, at least one potential buyer will show up to inspect seller \( i' \)’s house. Now suppose that the total number of units in the building including house unit i is n, and let’s make the following assumption:
Assumption 1: At any decision point, there is no competing seller ex-ante in the same building. Nevertheless, each owner in the same building may choose to become a seller, following an independent Poisson process. During the time window of $\Delta t$, the switching probability is $\tau \Delta t$.

Following this assumption, during period $\Delta t$, the probability that at least one competing seller will show up is $(n-1)\tau \Delta t$. Meanwhile, as discussed above, for any house unit, a potential buyer arrives in another independent Poisson process $B \Delta t$. Hence, during period $\Delta t$, the probability that there is one competing owner chooses to become a seller and she also attracts a potential buyer is $(n-1)\tau B \Delta t^2$. Since it is a necessary condition for observing a signal, we know that, during period $\Delta t$, $\text{Prob}(1 \text{ Signal})$ is at the magnitude of $\Delta t^2$. Accordingly, $\text{Prob}(2 \text{ Signals})$ is at the magnitude of $\Delta t^4$. Put differently, during period $\Delta t$, $\text{Prob}(\text{At Least 1 Signal}) = \alpha(\Delta t)$.

As a result, $\text{Prob}(\text{No Signal}) = 1 - \text{Prob}(\text{At Least 1 Signal}) = 1 - o(\Delta t)$. Equation 2.5 then implies that $E[U^{v,c}] = U^{v,c}(G; \tilde{G}) + o(\Delta t)$. Substituting it into equation 2.1, we get the following result:

$$
U^{v,c}(G; \tilde{G}) = \max_{r_i} e^{-\beta \Delta t} \left\{ B(1-G(r_i; \mu, \sigma^2)) \Delta t (r_i - v_i) + (1-B(1-G(r_i; \mu, \sigma^2)) \Delta t) [U^{v,c} - c \Delta t] \right\} + o(\Delta t)
$$

Equation 2.6 implies that, during period $\Delta t$, as the probability of receiving a signal is at the magnitude of $o(\Delta t)$, a seller will maximize as if the rolling-over bidding distribution, $G$, remains unchanged. However, provided that a signal accrures, although the chance is tiny, unsuccessful seller $i$ will maximize based on the updated bidding distribution from the next round maximization process. As you will see, this myopic-seller treatment greatly simplifies the analysis below as we can treat the state variable $G$ as fully exogenously given. Note that it is not a unique assumption that is made only in this study. For example, Zheng et al, [2008] also implicitly assume that the rolling-over $G$ distribution remains...
unchanged in their search model. The resulting solution for equation 2.6 is presented in Proposition 2.

**Lemma 1.** The equilibrium asking price, \( r^*_i \), for equation 2.6 is characterized by solving the following non-linear equation:

\[
\frac{g(r^*_i; \mu, \sigma^2)}{1-G(r^*_i; \mu, \sigma^2)} = \frac{B[1-G(r^*_i; \mu, \sigma^2)] + \beta}{\beta(r^*_i-v_i) + c}
\]

where \( g(r^*_i; \mu, \sigma^2) \) is the normal density function.

**Proof:** In appendix.

The left hand side of equation 2.7 is the hazard function, and Proposition 2 says that seller i should choose an optimal selling hazard rate, as determined by the right hand side of equation 2.7. As discussed in introduction, we are interested in four key parameters: 1) the reference value; 2) the mean of the bidding distribution \( G \); 3) the variance of the bidding distribution \( G \); 4) the exogenous arrival rate. Proposition 2 summaries the comparative static results on the interested parameters:

**Proposition 2.** The equilibrium asking price, \( r^*_i \), has the following properties:

\[
\frac{\partial r^*_i}{\partial \sigma} = \frac{r^*_i - \mu + \sigma[\beta(v_i - \mu) - c]}{\beta B[1-G(r^*_i; \mu, \sigma^2)] + \beta \cdot 2\sigma^2 - \frac{1-G(r^*_i; \mu, \sigma^2)}{g(r^*_i; \mu, \sigma^2)}(r^*_i - \mu)}
\]

Furthermore,

\[
\frac{\partial r^*_i}{\partial v_i} > 0, \quad \frac{\partial r^*_i}{\partial \mu} > 0 \quad \text{and} \quad \frac{\partial r^*_i}{\partial B} > 0.
\]

---

30 We can further strengthen our defense on Assumption 1, although heuristically. Now suppose that, during the rolling over period \( \Delta t \), the probability of observing a signal is non-neglectable. As discussed before, our interest is to look at the effect of changing bidding heterogeneity on a seller’s pricing strategy. And as shown by Zheng et al., [2008] and also by our later numerical simulation, holding other factors constant, a smaller bidding variance across potential buyers induces seller to ask for a lower price; and we also know that a signal will reduce the variance of future bidding distribution. Therefore, if rolling over to the next period will subject to a non-trivial chance of getting a signal and hence facing a tighter bidding distribution, the attractiveness of rolling-over to the next \( \Delta t \) period should be less, due to the discounting and variance-reduction effects. If it is true, it will in turn induce seller i to ask for a lower price today so as to increase the hazard rate of a successful sale now. Fortunately, the implied effect here is a lower asking price, which is in the same direction as when we use the myopic-seller treatment in this paper.
Proof: In appendix.

The above proposition proposes a rich set of empirical implications. Let’s discuss those comparative static results that with deterministic signs first. The fact that \( \frac{\partial r_i^*}{\partial v_i} > 0 \) implies that the seller’s asking price is increasing in reference value. Genesove and Mayer [2001] examine sellers’ behavior in the housing market and find that compared to potential gainers, a seller subject to bigger potential loss (hence high reference value) tends to set a higher asking price and to obtain a higher transaction price if the house is sold. Therefore, our model prediction is consistent with this empirical finding. Since the reference value is regarded as an un-sunk cost by sellers, the seller will ask for more compensation with a higher the reference value.

The finding of \( \frac{\partial r^*}{\partial \mu} > 0 \) is also intuitive. Knowing the fact that potential buyers are willing to pay more in aggregate, as shown by a bigger \( \mu \), a seller’s optimal asking price should be increasing, which is hardly surprising. This result has two empirical implications in multi-unit residential market: firstly, within the same building, the realized transaction prices should have positive spatio-temporal autocorrelation. In section 3.2.2 we have shown that, conditional on observing a revealed willingness to pay, the prior private signal for that buyer, \( \epsilon_{i,j} \), is now a common knowledge. And equation 2.3 says that the updated mean of the bidding distribution is shifted with the level of \( \epsilon_{i,j} \). Therefore, a higher realized transaction price, which is associated with a higher draw on the value of \( \epsilon_{i,j} \), will lead the updated bidding distribution a higher mean. According to the fact that \( \frac{\partial r_i^*}{\partial \mu} > 0 \), it in turn will lead the subsequent seller in the same building to market up her asking price. Secondly, even without any signal, \( \frac{\partial r_i^*}{\partial \mu} > 0 \) also implies that seller i’s asking
price should be decreasing when potential buyers have a lower value on $\theta_i$, a component in the mean value of bidding distribution $G$.

The last deterministic comparative static result is on the exogenous arrival rate, $B$. Intuition suggests that the effect from an increasing arrival rate should imply a higher selling hazard rate conditional on any given level of asking price. As a result, asking for a higher price is less costly for seller $i$. Our finding of $\frac{\partial r^*}{\partial B} > 0$ confirms this intuition. In equilibrium, a seller’s optimal asking price does increase with the exogenous arrival rate. In other words, when the market becomes more active, sellers tend to mark up their asking prices due to the expanded confidence of being able to sell their houses within a fixed period of time.

To understand the effect from the varying bidding heterogeneity, we need to know the sign of $\frac{\partial r^*}{\partial \sigma}$. Unfortunately, equation 2.8 gives us little clue on its sign, which is jointly determined by the values of $r^*_i$ and $v_i$. As a result, we choose to look at the pricing gradient numerically. Note that with specifications on $G$ and $B$, we can substitute them into equations 2.7. Then for given values of $v_i$, $c$ and $\beta$, we can solve the system numerically and obtain $r^*_i$. In the following numerical illustration, we assume $\mu = 0.5$, $\beta = 0.05$ and $c = 0.5^{31}$. We look at the pattern of $r^*_i$ with different values of $v_i$ and $\sigma$ and present the results in Figure 3.1.

---

31 Our results are robust with different parameter values.
From figure 3.1, with a more heterogeneous pool of potential buyers, holding other factors constant, a seller tends to increase her asking price. This is true for all possible reference values. The intuition is that if, according to a seller’s belief, potential buyers are more heterogeneous in terms of their preferences, then the probability of meeting a buyer who is willing to pay a higher matching premium becomes larger, provided that the optimal asking price is not in the lower tail of the normal bidding distribution. In this case, it is more attractive for a seller to ask a higher price in the market. And our numerical results based on a wide range of parameter values all confirm that the equilibrium asking prices seems to be consistently in the upper tail of the bidding distribution, i.e., not significantly lower than 0.5. Therefore, we should expect a higher asking price associated with a bigger bidding variance.

Our findings in Figure 3.1 raise an important problem when we want to test for the potential signaling effect. Suppose the price of a recently transacted unit is high. It then generates two effects. On one hand, a subsequent seller may update her belief with a
higher mean of the bidding distribution and mark up the optimal asking price, which we refer to as the autocorrelation effect. On the other hand, the signal also makes the variance of subsequent buyers’ bidding distribution smaller, which induces the seller to mark down her asking price. We refer to this as the varying bidding heterogeneity effect. Since the autocorrelation effect and varying bidding heterogeneity effect result in opposite forces on a seller’s pricing strategy, we need to isolate these two effects empirically in order to perform any meaningful test on the validity of the price signal.

In summary, Proposition 2 and the numerical result imply that, holding other factors constant: 1) A seller’s optimal asking price increases with her reference value; 2) A seller’s optimal asking price increases in the mean of the bidding distribution. Due to the fact that \( \theta_i \), substitution premium, is a component in the mean of \( G \), it implies that a seller’s optimal asking price also increases with \( \theta_i \). 3) A seller’s optimal asking price increases in the exogenous arrival rate. 4) A seller’s optimal asking price increases in the variance of the bidding distribution. We now move on to discuss the empirical findings on testing these model predictions.

3.3 Empirical Tests

To test for the above mentioned model predictions, we use condominium transaction data from Singapore. Section 3.3.1 introduces our dataset. Section 3.3.2 discusses our measures for the changing of bidding heterogeneity and exogenous arrival rate. Section 3.3.3 proposes our testing procedures and section 3.4.4 reports the modeling results.

3.3.1 Data

The data used in this paper is an augmented version of the data used by Sun, Tu and Yu [2005]. The period covered runs from January 1, 1990 to December 31, 2001. In the Singapore private housing market, 40\% of the buildings are condominiums while 25\% are apartments, 20\% are terraced houses and 15\% are semi-detached houses or bungalows. The data with hedonic characteristics used in this paper was collected from the Singapore

\[32\] Sun, Tu and Yu [2005] use data from January 1 1990 to December 31 1999 instead on estimation.
Institute of Surveyors and Valuers (SISV) transaction database. Before analysis, we deleted the transactions which did not have accurate information regarding postal code, the date of Temporary Occupation Permit, transaction price, the floor level of the unit and some hedonic characteristics that will be used later in our estimation. We also deleted those transactions that are identified as “same” unit but which have inconsistent structural characteristics. The amended data set has 51047 observations.

A nice feature of this data set is that in Singapore, each building block corresponds to one postal code. The geo-statistical information in our dataset includes X-Y coordinates for each building. Each transaction record is associated with a set of variables comprising hedonic characteristics of the condominium project, neighborhood amenities, as well as details in relation to the sale of the unit. The statistics regarding some of the key variables in the data are given in Table 3.1.

<table>
<thead>
<tr>
<th>Variables [1]</th>
<th>Mean</th>
<th>Std</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y(S$)</td>
<td>865,582</td>
<td>511,361.2</td>
</tr>
<tr>
<td>Area (sqm)</td>
<td>135.96</td>
<td>53.36</td>
</tr>
<tr>
<td>Age (year)</td>
<td>1.09</td>
<td>5.67</td>
</tr>
<tr>
<td>Level</td>
<td>7.27</td>
<td>5.59</td>
</tr>
<tr>
<td>Total Units</td>
<td>401.59</td>
<td>319.70</td>
</tr>
<tr>
<td>Units_Building</td>
<td>100.66</td>
<td>78.31</td>
</tr>
<tr>
<td>Dist to Top Secondary School (km)</td>
<td>2.06</td>
<td>1.32</td>
</tr>
<tr>
<td>Dist to Junior College (km)</td>
<td>4.08</td>
<td>2.22</td>
</tr>
<tr>
<td>Dist to MRT (km)</td>
<td>1.38</td>
<td>0.85</td>
</tr>
<tr>
<td>Dist to CBD (km)</td>
<td>8.54</td>
<td>4.20</td>
</tr>
<tr>
<td>Sample size</td>
<td>51047</td>
<td></td>
</tr>
</tbody>
</table>

Note: 1): The definition of all variables is given in Table 1 of the Appendix.
2): Out of all the condominium projects in the dataset, 42.84% were 99-year leaseholds while the rest were either 999-year leaseholds or freehold. 69.85% had barbecue facilities, 63.94% had a gymnasium, 21.48% had a jacuzzi, 60.48% had a sauna, 94.76% had a swimming pool and 77.79% had a tennis court.
3.3.2 Key Variables and Their Measurement

3.3.2.1 Reference Value

As is consistent within the literature, we measured the individual seller’s reference values as the difference between the prior purchasing price and the current expected market price, which was estimated from some econometric specifications. In this paper, we adopt two kinds of specifications. One is the traditional hedonic regression with a detailed set of structural and neighborhood characteristics. Alternatively, we also use a two order spatio-temporal autoregressive model which has been proposed by Sun, Tu and Yu [2005] to control for the spatio-temporal autocorrelation effect so as to test for the effect from decreasing bidding heterogeneity. We will discuss these two approaches in section 3.3.3. We expect the seller’s asking price to be increasing with her reference value.

3.3.2.2 Substitution Premium for Potential Buyers

To measure the substitution availability for a particular unit for sale, we use the total number of units in the same building as a proxy. Ideally, house units in the same building have almost identical structural characteristics as well as location amenities. Hence, we should expect these units to be close substitutes for each other. Unfortunately, we don’t have this information in our dataset; rather, what we have in the data is the total number of units in a project. Since we do have postal code information which can help us identify each building in a project, we divide the total number of units by the number of buildings in the project and use it (Units_Building) as our final measure on this substitution availability. The sign associated with Units_Building would tell us whether potential buyers are willing to pay more or less when it is relatively easier for them to find other substitutable house goods.

3.3.2.3 Exogenous Arrival Rate

In reality, the arrival rate for potential buyers is extremely hard to measure. In this paper, for each observation, we track the transactions of the prior one month and calculate the average distance between the current unit and all other transacted units within the last 30 days (AvgDist). We use this average distance as a rough proxy for the exogenous arrival rate. The idea is that, if the unit is spatially closer to the other units that have been recently
sold on the market, we should expect a higher arrival rate, probably due to the existence of a changing “hot area” from time to time, which is driven by the launch of new projects on the market or other factors. If such a hot area exists, then we should expect units that are closer to the hot area to be able to attract more potential buyers within a given period of time, cateris paribus. To some extent, we do find the clustering phenomena within the data in terms of the location distribution of transacted housing units. For example, throughout our sampling period, the average building distance is 8.63 km with a standard error of 5.15 km. Nevertheless, the monthly average building distance is only 6.89 km with a standard error of 4.55 km. Clearly, within each month, the transacted units are distributed in a more clustered fashion, as the average building distance is about 20% closer than the average building distance in the whole sample. If our measurement is a good proxy for the exogenous arrival rate, we should expect the seller’s asking price to decrease with the monthly average building distance due to a lower arrival rate.

3.3.2.4 Varying Bidding Heterogeneity

To measure any potential variations on buyers’ bidding heterogeneity, we define a dummy variable called “Signal”. It equals to one if during the last 30 days, at least one transaction has occurred in the same building. Also using Singapore condominium data, Hwang and Quigley [2004] estimate that the temporal correlation of condominium transaction prices is around 30 days. Therefore, we treat the transaction price that has occurred within one month as a valid signal for what potential buyers are willing to pay in the current period, and assume that sellers should update their beliefs accordingly. Following our theory, after controlling for the spatio-temporal autocorrelation effect, we expect the seller’s asking price to decrease with buyers’ bidding variance, and hence on our Signal measure.

3.3.3 Empirical Method

Define unit i’s expected log market value at time t as:

\[ V_{it} = f(X_{it}, t). \] (4.1)

33 The monthly average building distance is weighted by the number of transactions.
34 As a robustness check, we also tried different time windows ranging from 10 days to 45 days and found similar results.
where \( X_i \) is a vector of house characteristics and \( t \) stands for time. In reality, we cannot observe this expected market value. Instead, what we observe is the transaction price at time \( t \), in log form, we express it as:

\[
P_{it} = f(X_i, t) + e_{it} = V_{it} + e_{it}
\]

(4.2)

where the additional component \( e_{it} \) is the amount that is over or under-paid by the buyer.

In the theory part, we assume that housing units are ex-ante identical in terms of their structural characteristics and quality, which is obviously violated in reality. To control for the quality difference in real data, we perform a two-stage process. In stage 1, we run a hedonic regression, through which we can recover the expected market price for each unit. We then subtract the expected market prices of different housing units and only focus on the residual term, \( e_{it} \), which measures a seller’s heterogeneous selling prices after controlling for the quality difference. In stage 2, we perform tests on our model predications, using \( e_{it} \) as the measure of the net asking price.

The key predictions from our theory are that \( e_{it} \) depends on the seller’s heterogeneous reference value. We measure it by a variable called \( \text{Ref}_{ist} \). As is typically done in the literature, we use the original purchase price \( P_{is} \) at time \( s \) as reference value and, hence, \( \text{Ref}_{ist} \) is defined as the difference between prior transaction price and the current expected value that is estimated in a stage 1 regression:

\[
\text{Ref}_{ist} = (P_{is} - \hat{V}_{it}) = f(X_i, s) - f(X_i, t) + e_{is}.
\]

(4.3)

To be consistent with Genesove and Mayer [2001] and Sun [2007], the potential loss is captured by \((\text{Ref}_{ist})^+\), and the potential gain is therefore \(|(\text{Ref}_{ist})^-|\). Together with our measures on other key variables, we then specify our stage 2 regression as:

\[
e_{it} = \alpha + \beta_1 \text{Ref}_{ist} + \beta_2 \text{Units}_{Building} + \beta_3 \text{Signal} + \beta_4 \text{AvgDist} + e_{it}
\]

(4.4)
Before running stage 2 regression, we need to get $e_{it}$, the quality adjusted asking price. In this study, we use two approaches to measure the expected market price: the traditional hedonic model and a two order spatio-temporal autoregressive model (2STAR).

### 3.3.3.1 Traditional Hedonic Model

In the traditional hedonic model, we employ three types of variables. The first relates to the physical structure such as level, floor area, etc. The second group is comprised of dummy variables associated with condominium attributes such as the availability of a barbecue pit, sauna, swimming pool etc. The third consists of variables related to the location of the neighborhood such as distance to good primary or secondary schools, MRT, CBD, etc. In addition, we use quarterly time indicator variables in the hedonic model. The model is estimated using the OLS method. The outcome of the traditional hedonic model is presented in Table A.2 of the appendix. The traditional hedonic model shows an $R^2$ of 0.7005. All hedonic independent variables are significant and have the expected sign.

### 3.3.3.2 A Two Order Spatio-temporal Autoregressive Model (2STAR)

One challenge of using a traditional hedonic model is the existence of spatio-temporal autocorrelation. In order to test for decreasing bidding heterogeneity, we need to control for the spatio-temporal autocorrelation that was also generated by a recent price signal. To accomplish this, we use a spatio-filtering process which has been proposed by Sun, Tu and Yu [2005]. We refer readers to this study on the detail of this econometric treatment.35 The optimum space and time lags are determined by Geographically Weighted Regression (GWR) as explained by Sun, Tu and Yu [2005].

The outcome of the 2STAR model is presented in Table A.3 of the appendix. The optimum orders for the spatial and temporal weight matrices are 16 and 20 respectively. Compared with the traditional hedonic model, 2STAR has a much better fit ($R^2$ is

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35 As in Sun, Tu and Yu [2005], the first 1000 observations are dropped, covering the transactions in Year 1990 and the earlier part of 1991. This is to ensure that each observation has enough neighborhood comparatives.
boosted from 0.7007 to 0.8305) and all structural characteristics have come up with the expected signs.

3.3.4 Empirical Results

3.3.4.1 Evidence of the Autocorrelation Effect

Table 3.2 and Figure 3.2 show the pairwise spatio-temporal autocorrelation\(^\text{36}\) for the first 16 lagged residuals of the traditional hedonic and 2STAR models. One important finding is that, in the traditional hedonic model, the first lag autocorrelation, which refers to observations within the same building, is much higher than the second lag autocorrelation. This finding is consistent with our expectations of a signaling effect. Meanwhile, the second lag of autocorrelation is close to the first lag of autocorrelation estimated in the market for a single family (see, Pace et al., [1998b], with estimated first lag autocorrelation around 0.4). As in the multi-unit residential market, the second nearest building is equivalent to the first lag in the market for the single family market, and this finding implies the existence of a common neighborhood effect. As the distance between buildings become larger, the commonality of the neighborhood environment decreases; and this is reflected in the decaying process of the spatio-temporal autocorrelation along spatial lags.

Interestingly, after adopting a spatial-filtering process, as is done in the 2STAR model, we effectively reduce the pairwise autocorrelation across residuals. After incorporating the spatial information into the estimating process, we find no significant evidence regarding the existence of spatio-temporal autocorrelations in any lags, even when considering units within one building.

\(^{36}\) For each unit, we track the first 16 nearest buildings that have prior transactions to calculate the autocorrelation between residuals.
### Table 3.2 Spatio-temporal Autocorrelations of the Residuals

<table>
<thead>
<tr>
<th></th>
<th>Lag 1</th>
<th>Lag2</th>
<th>Lag3</th>
<th>Lag4</th>
<th>Lag5</th>
<th>Lag6</th>
<th>Lag7</th>
<th>Lag8</th>
<th>Lag9</th>
<th>Lag10</th>
<th>Lag11</th>
<th>Lag12</th>
<th>Lag13</th>
<th>Lag14</th>
<th>Lag15</th>
<th>Lag16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hedonic</td>
<td>0.646</td>
<td>0.3908</td>
<td>0.3373</td>
<td>0.2498</td>
<td>0.3002</td>
<td>0.1857</td>
<td>0.2097</td>
<td>0.1706</td>
<td>0.1765</td>
<td>0.176</td>
<td>0.1374</td>
<td>0.1281</td>
<td>0.1132</td>
<td>0.1276</td>
<td>0.1206</td>
<td>0.1096</td>
</tr>
<tr>
<td>2STAR</td>
<td>-0.0118</td>
<td>0.0305</td>
<td>0.0309</td>
<td>0.0124</td>
<td>0.0226</td>
<td>0.0093</td>
<td>0.0145</td>
<td>0.005</td>
<td>-0.0017</td>
<td>-0.0122</td>
<td>0.0026</td>
<td>-0.0011</td>
<td>0.0088</td>
<td>-0.0083</td>
<td>-0.014</td>
<td>0.0068</td>
</tr>
</tbody>
</table>

### Figure 3.2 Spatio-temporal Autocorrelations of the Residuals

![Graph showing spatio-temporal autocorrelations of the residuals](image)

- **Hedonic** line
- **2STAR** line

The graph illustrates the correlation between residuals at different spatial lags, showing the spatio-temporal autocorrelations for Hedonic and 2STAR models.
3.3.4.2 Results in the Second Stage Regression

Table 3.3 presents the results of the second stage regression, using both the traditional hedonic model and the 2STAR model as measurements regarding the expected market price. The dependent variable is the residual from the stage one regression, which can be interpreted as a quality controlled selling price. Before going to the results, we want to point out a measurement problem on two of our explanatory variables, Units_Building and Signal. As discussed in section 3.4.4, Units_Building represents the total number of units in the same building; and we define Signal dummy equals to 1 if there has been at least one transaction that has occurred within the last 30 days. It is obvious that the likelihood that we could find one comparable transaction should be highly influenced by the total number of units in that building. If there are only a few units in one building, we would be less likely to be able to observe a comparable transaction easily and hence the values of the Signal dummy for most of the observations in that building will equal to zero.

Acknowledging the strong positive correlation between Units_Building and Signal, we examine four model specifications. In cases 1 and 2, we only include one of them separately in the second stage regression. In case 3, we pool them together to look at the combined effects. In case 4, we restrict our sample to those observations where there are at least 100 units in the same building\textsuperscript{37}. By doing this, we aim to isolate the potential problem of availability restriction on observing a signal ex-ante and hence try our best to mitigate this problem of co-linearity\textsuperscript{38}.

Our results are largely consistent with the theoretical predictions. For example, in both the hedonic and 2STAR models, we find significant evidence of the reference dependence effect in all cases. As found in Genesove and Mayer [2001], sellers tend to mark up the price when they also have higher reference values. However, by comparing the two models, we find that, after controlling for the spatio-temporal autocorrelation in the stage one residuals, the extent of the reference dependence becomes much more moderate. Using the full sample, as in cases 1 to 3, we consistently find that the impact of reference value shrinks by more than one half after we

\textsuperscript{37} 100 corresponds to the mean value of Units_Building in our sample, see Table 1.

\textsuperscript{38} Our finding is robust when we restrict the sample by median number of units in a building, which is 84.66.
### Table 3.3 Stage Two Regression Results

**Dependent Variable:** Residual from Stage One Regression

<table>
<thead>
<tr>
<th>Explanatory variables</th>
<th>Traditional Hedonic</th>
<th>2STAR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1</td>
<td>Case 2</td>
</tr>
<tr>
<td>Reference (std)</td>
<td>0.1385 ***[1]</td>
<td>0.1339 ***</td>
</tr>
<tr>
<td></td>
<td>(0.0089)</td>
<td>(0.0091)</td>
</tr>
<tr>
<td>Units Building[2] (std)</td>
<td>-0.0682***</td>
<td>-0.0668***</td>
</tr>
<tr>
<td></td>
<td>(0.0037)</td>
<td>(0.0040)</td>
</tr>
<tr>
<td>Signal (std)</td>
<td>-0.0013</td>
<td>-0.0018*</td>
</tr>
<tr>
<td></td>
<td>(0.0010)</td>
<td>(0.0010)</td>
</tr>
<tr>
<td>AvgDist[3] (std)</td>
<td>-0.0024***</td>
<td>-0.0027***</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>Residual from Last Sale (std)</td>
<td>0.0576***</td>
<td>0.0635***</td>
</tr>
<tr>
<td></td>
<td>(0.0112)</td>
<td>(0.0114)</td>
</tr>
<tr>
<td>Quarters Since Last Purchase (std)</td>
<td>-0.0024***</td>
<td>-0.0027***</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>Constant (std)</td>
<td>0.1749***</td>
<td>0.1399***</td>
</tr>
<tr>
<td></td>
<td>(0.0107)</td>
<td>(0.0109)</td>
</tr>
<tr>
<td># observations</td>
<td>7187</td>
<td>7187</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.1065</td>
<td>0.0710</td>
</tr>
</tbody>
</table>

Note: 1): * Significant at 0.10 level. ** Significant at 0.05 level. *** Significant at 0.01 level.

2): Per 100 units.

3): Per kilometer.
control for the autocorrelation effect.

Furthermore, we find that sellers tend to mark down that price when there are more units in the same building. This is consistent with a story of decreasing opportunity cost for buyers to scarify a house good when outside substitutes are easier to find. Think of the case in which a buyer finds one unit that is attractive to him. Ideally, house units in the same building have almost identical structural characteristics as well as location amenities. Hence, we should expect these units to be close substitutes for each other. As a result, for a building that has more units, sellers may have relatively weaker bargaining power since it is easier for a potential buyer to find another unit which is a good substitute for the unit seller i owns. In this case, we may expect that the seller would ask a lower price due to the bigger substituting flexibility for potential buyers. This empirical finding is robust for different specifications. Although the mark down margin appears to be smaller when we control for the autocorrelation effect, we should note that the finding is still economically significant. For example, the smallest effect that we find is in case 4 of the 2STAR model. Even in this case, the seller will mark down the price by 0.88% for an extra one hundred units in the building\(^{39}\). Given that the average transaction price is S$865,582 in our sample, this number then stands for S$7,617, which is clearly non-trivial.

In terms of the Signaling effect, we consistently find a negative coefficient on it, which is as expected. Although the coefficient appears to be insignificant when we pool Signal with Units_Building and use the full sample, we argue that this may be driven by the colinearity problem, as we discussed earlier. Actually, when we isolate the availability restriction, as done in case 4, the negative coefficient on Signal becomes significant at the 0.01 level in the hedonic model and becomes significant at the 0.05 level in the 2STAR model. After controlling for the autocorrelation effect and using the restricted sample which is less subject to the availability constraint, a seller tends to mark down the price by 1.6% if a recent price signal exists, which is consistent with the prediction of decreasing bidding heterogeneity on potential buyers.

\(^{39}\) As shown in Table 1, the average number of units in a building is 100.6611 and the standard deviation is 78.31.
The two models give similar results when testing for the arrival rate effect. In the traditional hedonic model, the coefficient is negative except in case 3, although it is insignificant in all cases. Nevertheless, the coefficient becomes negative and significant once we control for the autocorrelation effect. A negative coefficient is consistent with our expectations. If the average distance is a reasonable proxy for the arrival rate, we should expect sellers whose units are further from the “hot areas” in the market to mark down their asking prices. In the 2STAR model and with the use of a full sample, we find that sellers tend to mark up their asking price by 0.38% with every 1km decrease of the distance between the units they own and the other recently transacted units\(^{40}\), which accounts for $3,289.

In addition to the four key variables that we have discussed so far, we include two additional control variables, as suggested within the prior literature. The first variable, the residual from the last sale, aims to control for the potential un-observability in house quality. It shows a significant positive effect on the current selling price in all regressions, which is as expected. A positive last residual means that the seller was willing to pay a higher than the expected market price when she purchased that unit. Hence, it is very likely that the house may have some un-observable quality premium, which may make the current selling price high. Furthermore, we add quarters since the last purchase as a proxy for loan to value ratio, which is unavailable in our dataset. It is reasonable to speculate that, roughly speaking, the longer you stay in your house, the lower your loan to value ratio should be since you keep paying out your mortgage through time. Again, our finding is consistent with this argument. We find a significantly negative coefficient in all regressions, which predicts that holding other factors equal, the longer the seller has lived in that house, the smaller the markup she would require in the market.

\(^{40}\) In our sample, the mean of AvgDist is 9.2467km, with stand error of 2.3724km. The smallest AvgDist is 2.9027km and the largest AvgDist is 21.5919km.
3.4 Concluding Remarks

In this paper we examine individual seller’s pricing strategies in a general search model with a reference dependent utility. Specially, we look at the signaling effect on a seller’s optimum pricing behavior when potential buyers can observe the transaction prices of other units that are considered close substitutes for the unit that a seller wants to sell. To accomplish it, we split the willingness of a potential buyer to pay into three parts: fundamental value, substitution premium and inter-correlated personal valuations. Acknowledging the fact that units in the same building serve as close substitutes for each other, we show that the recent transaction price on a unit in the same building generates two signaling effects. First, it generates positive spatio-temporal autocorrelation with the subsequent price for the units in the same building; second, after observing the prior price, the bidding heterogeneity among potential buyers decreases.

Our empirical results largely support the theory predictions. Consistent with the prior literature, we find significant evidence regarding the reference dependence effect, i.e., sellers tend to mark up their prices if they also have higher reference values. With regard to the signaling test, we show evidence that in the multi-unit residential market, the spatio-temporal autocorrelation among units in the same building is significantly higher than with units in neighboring buildings. Meanwhile, after controlling for the autocorrelation effect, sellers tend to mark down their asking prices if a recent transaction has occurred within the same building, which reflects the signaling effect on the decreasing bidding heterogeneity among potential buyers. Furthermore, with an increasing amount of available house goods that can serve as close substitutes for a particular unit for sale, sellers would tend to mark down the price accordingly, thus reflecting the effect of better substitution opportunities for potential buyers. Finally, using our proxy measure of the exogenous arrival rate for potential buyers, we find moderate evidence that sellers’ asking prices tend to decrease with the arrival rate, as predicted by our theory.
References


CHAPTER 4: MONITORING HETEROGENEITY, STOCHASTIC FORCING CONTRACT AND THE BOUNDARY OF ADVISOR CHOICE IN REAL ESTATE INVESTMENT TRUSTS

4.1. Introduction

One puzzling question in real estate literature involves the justification for the external managerial structure in REIT firms. It is widely agreed that managing real estate investments through external advisors in REITs generates a larger conflict of interest between shareholders and agents than does managing real estate investments through internal advisors. For example, Howe and Shilling [1990] find that, after controlling for the firm’s characteristics and risk levels, externally advised REITs performed worse than did the general stock market up to the late 1980s. Another convincing study by Capozza and Seguin [1998] also shows that in their sampling period, externally advised REITs underperformed internally advised REITs by more than 7%. In addition to having an inferior stock performance, Ambrose and Linneman [2001] found that externally advised REITs in general also incur higher financial expenses. Because of the potentially high agency costs associated with the external advisor structure, REIT industry has experienced a significant trend in converting to be internally advised since 1986, when the private-letter rulings from the IRS first allowed REIT firms to do so. According to Chan, Erickson and Wang [2003], up until the year 2000, more than 87% of U.S REITs chose to convert to an internal advisor structure. Despite a broad consensus among academics on the inferiority of external management, the fact that a non-trivial number of REITs keep to be externally managed seems controversial. This phenomenon is particularly relevant when we look at the emerging REITs markets, such as in Asia, since their emergence in 2001.

41 A version of this chapter will be submitted for publication. Sun, H. Monitoring Heterogeneity, Stochastic Forcing Contract and The Boundary of Advisor Choice in Real Estate Investment Trusts.

42 See Chapter 4 of Chan, Erickson and Wang [2003] for a comprehensive review on the agency problems associated with REIT’s advisor types.
Interestingly and also puzzlingly, most of REIT companies in Asia markets choose to be externally managed, despite the serious concerns on the agency problem as mentioned above. For example, as of 2005, all REITs in Singapore are externally managed (Sing [2005]). Furthermore, in Japan the proportion of external REITs accounts for around 50%\(^43\), at least.

As pointed by Chan, Erickson and Wang [2003], theoretical research on rationalizing external advisor structure is very important. Nevertheless, from author’s very limited knowledge, the study on the optimal advisor choice is still absent in the literature. As an early attempt, this paper looks at the REITs advisor choice problem and aims to provide a theoretical justification of the potential appeal of the external managerial structure. In particular, the agency problem is examined through focusing on the power of monitoring and forcing contract on the improvement of the advisor’s equilibrium effort supply. We then identify conditions under which external management may be optimal. One important factor that determines optimal management structure is REITs shareholders’ monitoring ability. We show that in general, for both types of advisors, an increased monitoring power will increase their optimal effort. Furthermore, we argue that by choosing external structure, shareholders may enjoy a monitoring advantage, compared with internal management. We motive this argument from three aspects in the modeling part. The most important aspect, as will be discussed in detail in later section, is the dual role for an external advisory firm. On one hand, advisory firm serves as an agent for a REIT company and gets compensation from REIT shareholders; on the other hand, it is also the principal who in turn compensates external advisor it sends to the REIT company. We show how, by correctly specifying a compensation mechanism, REIT shareholder can free-ride on advisory firm’s superior monitoring ability; and it may induce higher effort levels from external advisors, despite a higher agency cost associated with this management structure. Acknowledging the potential heterogeneity of monitoring power between internal and external advisors and across REIT firms, the boundary of the optimal advisor type is then

\(^{43}\) It is only a roughly estimated number but we guess should stand for a lower bound. Following UBS Asian REITs Report by Neo [2005], as of the end of 2004, there are at least 18 externally managed REITs in Japan. And following Ooi, Ong and Neo [2007], as of 2007, the total number of REITs in Japan is 40.
derived, and a numerical illustration on the advisor choice problem is presented in the modeling section.

In addition to the advisor choice problem, this study also contributes to the literature by investigating the optimal compensation scheme for REIT advisors in general. A set the comparative static results on optimal contract are proposed, which has clear empirical implications. Our analytical results also show clear implications regarding the power of retrospective monitoring and forcing contract in improving production efficiency and shareholders’ welfare. Furthermore, we compare the difference between a fixed and stochastic forcing contract. We show that with imperfect performance measures, the stochastic forcing contract always dominates the fixed one in increasing shareholders’ profit. Finally, we examine the impact of shareholder monitoring on the advisor’s welfare. Our theory proposes that the monitoring power from shareholders may increase advisor’s equilibrium utility when the power gets below a certain threshold level, despite the fact that is also restricts the advisor’s flexibility regarding shirking. As a result, it could be in the advisor’s self-interest to help the shareholders monitor her effort level. Put differently, an advisor, independent of its type, prefers moderate to weak monitoring, but is averse to intense monitoring.

The remainder of this paper is organized as follows. In section 4.2, we begin with an introduction regarding difference between internal and external managerial structures. We then briefly review the theoretical literature that is related to this study. Section 4.3 presents the model and discusses its welfare implications. Section 4.4 examines the sub-optimality of the fixed forcing contract under imperfect performance measures. Section 4.5 points to several directions of future extensions. Section 4.6 concludes the paper.

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44 We will explain in detail the content of these two contracts in later sections. Simply speaking, a fixed forcing contract is a contract involving a fixed regular wage, penalty wage, and a prescribed effort level. In general, agents receive regular wages unless they are captured for shirking, in which case a penalty wage is applied. In contrast, in the stochastic forcing contract, a regular wage is not fixed; rather it is based on some performance measure, which will be discussed thoroughly in the modeling section.
4.2. Research Background

4.2.1 Internal and External Advisor

Within the top management team, the advisor plays a key role in making business decisions such as determining property dispositions, acquisitions and signing contracts with property-related service providers and lower tier property managers. The internal advisor structure is very similar to a standard operating company, i.e., advisor is an in-house employee and hence directly affiliated with a REIT company. Being different, the external advisor structure refers to a case in which an REIT firm appoints an outside entity, such as a mortgage bank or a professional advisory firm, to conduct its routine operations. In this case, external advisor is affiliated with advisory firm instead of REIT company; and more importantly, it is legal for external advisor to manage more than one REITs at the same time. Not surprisingly, the controversial phenomenon that advisor and its affiliated entities are allowed to participate in dealing and the possibility for an advisory firm to manage more than one REIT makes the concern of self-dealing particularly relevant. Prior to 1986, external management was mandatory by law. After 1986, REITs were allowed to manage their assets and business decisions on a self-operating basis. It brings REITs the freedom to self-appoint internal advisors and makes internally-managed REITs more closely resemble traditional operating companies. The difference between two types of management can be best explained by the following Figure.
4.2.2 Related Literature

Our theory has close links with two branches of the literature. The first involves research on the effects of monitoring and forcing contract. Unlike the standard agency model, which assumes un-observability for an agent’s effort level, the theory on forcing contract assumes a partial observability of the agent’s efforts. For example, there may be a positive probability that the principal can detect an agent’s shirking behavior through monitoring. In such a situation, a new contract type, called a ‘forcing contract’ is proposed. A forcing contract is characterized by a standard wage and a penalty wage. If no shirking behavior is detected, the agent gets a standard wage; instead, if the agent is detected to have engaged in shirking behavior, a penalty wage is applied, which is typically set to zero, and refers to the action of dismissal. The application of such measures is popular in the literature of labor economics. For example, Shapiro and Stiglitz [1984] apply this idea in an efficiency wage model. One conclusion that is relevant to this study is that with positive monitoring

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45 Shirking behavior refers to the fact that agent spends less effort than the prescribed level set by the principal.
power, the principal can induce a higher effort level from the agent, thus increasing efficiency in the economy\textsuperscript{46}.

The second branch involves research on optimal compensation schemes and the agency problem in the REIT industry. Despite a vast literature on empirical evidence regarding the sub-optimality of externally advised REITs\textsuperscript{47}, theoretical studies on the agency problem and the choice of advisor types in REITs are surprisingly scarce. The only research that sketches the connection between potential agency costs and compensation structures is that of Solt and Miller [1985], which points out the potential moral hazard resulting from the problem of the information asymmetry between REITs advisors and shareholders.

\textbf{4.3. Model Setup}\textsuperscript{48}

\textbf{4.3.1 Case 1: No Monitoring Power}

We first present our model in the standard principal-agent context, in which risk-neutral shareholders of an REIT firm appoint a risk-averse advisor to make investment and managerial decisions. The effort level taken by the advisor is not observable to the shareholders.

As is popular in the contract theory literature, we assume a linear compensation package for the advisor:

\[ w = \alpha + \beta V \]  \hspace{1cm} (3.1)

where \( \alpha \) refers to the fixed wage and \( \beta V \) stands for a performance-related component in the compensation package. The magnitude of \( \beta \) measures the intensity of the incentive component in the overall package, and \( V \) is a performance measure regarding the advisor’s effort, such as the value of the firm. Typically, it is difficult to accurately

\textsuperscript{46} The efficiency wage model also implies a trade-off between raising the equilibrium wage level and the saved monitoring cost. Since we abstract the monitoring cost in this study, we will not discuss implications regarding that dimension here.

\textsuperscript{47} See our discussion in the introduction section and the incomplete reference list from there.

\textsuperscript{48} The modeling framework in this section is largely followed from Silberberg and Suen [2001].
measure an agent’s output. To reflect this imperfect performance measure, we assume $V$ to be a continuously and (weakly) concave function of the advisor’s effort plus some noise:

$$V = V(x) + \epsilon$$ (3.2)

where $\epsilon$ is normally distributed with mean zero and variance $\sigma^2$, and $x$ is the effort level provided by the advisor. Furthermore, we assume that both internal and external advisors have the same productivity function. Rather, we model the bigger conflicts of interest associated with external advisors through the channel of cost function. In section 5, we will drop this common productivity assumption and discuss the impact of potential differences in productivity levels on the optimal choice of an advisor.

A risk-averse advisor works for REIT shareholders. As usually specified in the literature, the REIT advisor has a constant absolute risk aversion (CARA) utility which is given by:

$$u(Z) = -\exp(-2rZ)$$ (3.3)

where $2r$ is the coefficient of risk aversion, and $Z$ is the uncertain net income of the advisor.

We further assume that the advisor’s net income $Z$ is determined by following:

$$Z_j = w_j - C_j(x) = \alpha_j + \beta_j V(x) - C_j(x) + \beta_j \epsilon$$ (3.4)

where $j$=i, e refers to the cases of internal and external advisors separately. And $C_j(x)$ is the cost incurred by a type-j advisor\(^{49}\) when providing $x$ units of effort. As usual, we assume $C_j(x)$ is increasing and convex in $x$. The first best solution should be the effort level ($x^*_j$) in which the marginal productivity is equal to the marginal cost, i.e.,

$$V'(x^*_j) = C'_j(x^*_j).$$

To reflect the empirical fact that an external managerial structure has bigger conflicts of interest, we assume $C'_e(x) > C'_i(x)$ for all non-zero $x$. This assumption implies that, conditional on the same effort level, an external advisor is always more reluctant to increase her marginal effort supply than is an internal advisor. This hence reflects the idea that compared with an internal advisor, the incentive for shirking (i.e.,

\(^{49}\) Measured in pecuniary terms.
putting in lower effort) for an external advisor is stronger due to a larger benefit from cost savings. Obviously, shirking is detrimental to shareholders since it decreases the value of the firm.

A nice property of CARA utility is that with a normal risk, this preference can be equivalently represented by the mean-variance preference:

\[
 u(Z) = E[Z] - rVar[Z] \tag{3.5}
\]

Hence, given a compensation package, type-j advisor then chooses \( x \) to maximize:

\[
 u_j^* = \max_{x_j} \alpha + \beta V(x_j) - C_j(x_j) - r \beta^2 \sigma^2
\]

(3.6)

The first order condition is:

\[
 \beta V'(x_j) = C'_j(x_j) \tag{3.7}
\]

The solution in equation 3.7, \( x_j^*(\beta) \), is the traditional second best solution when the advisor’s effort is not observable. Intuitively, there is an efficiency loss as the optimal effort level in this case is always smaller than the first best solution. This is due to the fact that the advisor only enjoys the \( \beta \) portion of the marginal benefit, but must bear all of the marginal cost.

Total differentiating equation 3.7, we get:

\[
 \frac{\partial x_j^*}{\partial \beta} = \frac{-V'(x_j^*)}{\beta V'(x_j^*) - C_j'(x_j^*)} \tag{3.8}
\]

Equation 3.8 is always positive, which is intuitive. An increase in the incentive component in the total compensation package should induce the advisor to put in greater effort.

The shareholders’ objective is to choose a compensation package that would maximize the expected net profits:

\[
 U_j^* = \max_{(\alpha, \beta)} -\alpha + (1 - \beta)V(x) \tag{3.9}
\]

subject to

\[
 \alpha + \beta V(x) - C_j(x) - r \beta^2 \sigma^2 \geq 0 \quad (\text{IR}_j)
\]

\[
 x_j = x_j^*(\beta) \quad (\text{IC}_j)
\]
The first constraint is called the individual rationality constraint (IR). It states that the advisor’s expected utility from working should be no worse than her outside utility, which is normalized to zero. The second constraint is the incentive compatibility constraint (IC), which states that when facing the compensation package that is offered by shareholders, it is in the type-j advisor’s self-interest to provide the induced effort.

From the standard principal-agent model, we know that in equilibrium, (IR) constraint must bind. When we substitute this result into equation 3.9, we get:

\[ U^*_j = \operatorname{Max}_{\beta_j} \left( V(x_j^*(\beta)) - C_j(x_j^*(\beta)) - r \beta^2 \sigma^2 \right) \]  

(3.10)

The \( \beta_j^* \) in the optimal linear contract can be solved through the first order condition:

\[ \left[ V(x_j^*(\beta)) - C_j(x_j^*(\beta)) \right] \frac{\partial x_j^*}{\partial \beta} - 2r \beta \sigma^2 = 0 \]  

(3.11)

From the binding (IR), \( \alpha_j^* \) can be solved accordingly:

\[ \alpha_j^* = r \beta_j^2 \sigma^2 - \beta_j^* V(x_j^*) + C_j(x_j^*) \]  

(3.12)

Now we can summarize the equilibrium result on advisor choice in Proposition 1.

**Proposition 1.** With unobservable effort and common productivity, it is always optimal for REIT shareholders to choose an internal instead of an external advisor.

**Proof.** From equation 3.7, we know \( \frac{V(x_j^*)}{C_j(x_j^*)} = \frac{1}{\beta} \). Since \( C_j(x) > C_j^*(x) \) for all \( x \), it is true that for any positive \( \beta \), \( x_j^*(\beta) > x_j^*(\beta^*) \). Meanwhile, since \( C_j(0) = 0 \), equation 3.10 implies that \( U_j^* > U_e^* \) as \( U_j^* > U_j^*(\beta_j^*, \beta_j^*) > U_e^*(\beta_e^*, \beta_e^*) = U_e^* \). **End.**

The conclusion from Proposition 1 is hardly surprising. Compared with an external advisor, an internal advisor is in all ways identical except through having a lower cost on providing effort. Hence it makes no sense for shareholders to appoint an external advisor.
4.3.2 Case 2: Heterogeneous Monitoring Power

4.3.2.1 General Model

Now we consider the effects on monitoring on the choice of an optimal advisor. In this case, a new type of contract, a forcing contract, is proposed\(^{50}\). Under a forcing contract, in addition to specifying a regular compensation package \(w\), shareholders also prescribe an expected level of effort which they want an advisor to supply. Here, instead of assuming a fully unobservable effort, we assume shareholders can, at no cost\(^{51}\), detect a type-\(j\) advisor’s shirking behavior with some level of probability, \(\pi_j\). The advisor gets regular \(w\) if no shirking is detected; otherwise, she is paid 0, which is the penalty wage. In this section, we only consider the case of a stochastic forcing contract in which \(w\) is not fixed but is rather contingent on the performance measure \(V\). In section 4, we will look at the case of a fixed forcing contract and discuss the differences between these two types of contracts.

Although the ability to detect shirking does not mean that shareholders can directly observe an advisor’s effort, a stronger monitoring power does imply that the noise in the performance measure should be smaller when there is a higher power in terms of monitoring is present. In the extreme case, when \(\pi_j = 1\), i.e., shareholders can perfectly detect any deviation in effort, the measurement error on effort supply must be zero. To reflect this, we assume the performance measure \(V(x)\) under monitoring power \(\pi_j\) to be:

\[
V_{\pi_j} = V(x) + \epsilon_{\pi_j} \tag{3.13}
\]

where \(\epsilon_{\pi_j}\) is normally distributed with a mean zero and variance \((1-\pi_j)^\gamma \sigma^2\) for \(\gamma > 0\). This specification is very general. When \(\pi_j = 0\), equation 3.13 reduces to equation 3.2 as in case 1; when \(\pi_j = 1\), the error term vanishes. And parameter \(\gamma\) controls the speed of noise reduction when the monitoring power improves.

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\(^{50}\) See section 4.1.3 in Bolton and Dewatripont [2005] for a brief introduction on a forcing contract.

\(^{51}\) In a more general case, monitoring should incur some cost. Abstracting the monitoring cost reduces one dimension of the trade-off and greatly simplifies our discussion. Furthermore, in this case, no economic insight is lost.

\(^{52}\) Note that we don’t require shareholders to observe the actual effort level. All we require is their ability to detect a deviation from the prescribed effort by an advisor.
Given $\pi_j$, now the shareholder’s objective becomes:

$$U_j^{\ast}(\pi_j) = \max_{\alpha, \beta, x} -\alpha + (1 - \beta)V(x)$$

(3.14)

subject to

$$\alpha + \beta V(x) - C_j(x) - r \beta^2 (1 - \pi_j)^{\gamma} \sigma^2 \geq 0 \quad \text{(IR}_2)$$

$$\alpha + \beta V(x) - C_j(x) - r \beta^2 (1 - \pi_j)^{\gamma} \sigma^2 \geq (1 - \pi_j) \left[ \alpha + \beta V(\hat{x}_j(\beta; \pi_j)) - C_j(\hat{x}_j(\beta; \pi_j)) - r \beta^2 (1 - \pi_j)^{\gamma} \sigma^2 \right] \quad \text{(IC}_2)$$

Compared with equation 3.9, (IR) constraint remains unchanged. However, (IC) constraint becomes different: it now states that it should be optimal for an advisor to obey the prescribed effort level set by the shareholders instead of choosing an effort level based on her self-interest. A further look tells us that (IR$_2$) is redundant if (IC$_2$) holds. To further illustrate, consider any possible compensation package $(\alpha, \beta)$. An advisor makes a two-step decision conditional on this given package. Firstly, she must decide whether to take the offer or not. If she declines the offer, she then provides zero unit of effort and also gets zero utility. However, (IC$_2$) implies that the shareholders must provide a package which would guarantee acceptance by the advisor. The reason is, as shown in equation 3.6, $\hat{x}_j(\beta; \pi_j)$ gives the unconstrained maximum of $\alpha + \beta V(x) - C_j(x) - r \beta^2 (1 - \pi_j)^{\gamma} \sigma^2$. Since $0 \leq (1 - \pi_j) \leq 1$, under (IC$_2$), both sides must be non-negative. Hence, (IR$_2$) is automatically satisfied and the advisor will always accept the offer whenever (IC$_2$) holds. Secondly if she takes the offer, she needs to decide the level of effort supply; and (IC$_2$) states that it is optimal for the advisor to obey the prescribed effort.

The lagrangian for this maximization problem can be written as:

$$-\alpha + (1 - \beta)V(x) - \lambda[(\alpha + \beta V(x) - C_j(x) - r \beta^2 (1 - \pi_j)^{\gamma} \sigma^2$$

$$- (1 - \pi_j) \left[ \alpha + \beta V(\hat{x}_j(\beta; \pi_j)) - C_j(\hat{x}_j(\beta; \pi_j)) - r \beta^2 (1 - \pi_j)^{\gamma} \sigma^2 \right]}$$

(3.15)

Taking first order conditions for $\alpha$, $\beta$ and $x$ and rearranging terms give us:

$$-1 - \lambda \pi_j = 0$$

(3.16)
\[(1-\pi_j)[V(x) - V(\hat{x}_j(\beta; \pi_j))] = 2\pi_j r \beta (1-\pi_j)^{\gamma} \sigma^2 \]  \hspace{1cm} (3.17)

\[
\frac{V'(x)}{C_j'(x)} = \frac{1}{\pi_j + (1-\pi_j)\beta} 
\]  \hspace{1cm} (3.18)

For non-zero \(\pi_j\), equation 3.15 implies that \((IC_2)\) must bind. Hence we can solve \(\alpha\) from \((IC_2)\):
\[
\alpha = \frac{(1-\pi_j)(\beta V(\hat{x}_j) - C_j(\hat{x}_j)) + \pi_j r \beta^2 (1-\pi_j)^\gamma \sigma^2 - \beta V(x) + C_j(x)}{\pi_j} 
\]  \hspace{1cm} (3.19)

We summarize our results in the following proposition:

**Proposition 2.** With monitoring power \(\pi_j\), the optimal forcing contract is fully characterized by equation 3.17 to 3.19. Furthermore, provided that the incentive component is positive\(^{53}\), shareholders will always require a higher effort level than would be evidenced by the advisor’s self-interest level (\(\hat{x}_j(\beta)\)). Finally, the optimal advisor type is determined by \(\Max \{U_i^{**}(\pi_i), U_e^{**}(\pi_e)\}\).

**Proof.** Obvious from the above derivation and equation 3.17 \hspace{1cm} \textit{End.}

The results up to this point are general since we assume very little with regard to the functional forms of \(V(x)\) and \(C_j(x)\). However, in order to arrive at more concrete implications such as the sign of some comparative static results and the boundary of optimal advisor types, we need to further specify our model. To achieve this, from now on we will consider a linear productivity and quadratic cost structure regarding the advisor. In particular, we assume \(V(x) = px\) and \(C_j(x) = c_j x^2\). Substituting them into equations 3.17 to 3.19, we get the following results:

**Proposition 3.** When \(\pi_j \in [0,1)\), the optimal linear forcing contract for a type-\(j\) advisor is in the following form:
\[
\hat{x}_{j}^{**} = \frac{p[p^2 + 4\pi_j r c_j (1-\pi_j)^{\gamma-1} \sigma^2]}{2c_j[p^2 + 4rc_j(1-\pi_j)^{\gamma-1} \sigma^2]} > 0 
\]  \hspace{1cm} (3.20)

\(^{53}\) That is, \(\beta > 0\).
\[
\beta^*_j = \frac{p^2}{p^2 + 4rc_j(1 - \pi_j)^{-1} \sigma^2} > 0
\]  
(3.21)

\[
\alpha^*_j = \frac{p^2[4rc_j(1 - \pi_j)^{-1} \sigma^2 - p^2][4\pi_jrc_j(1 - \pi_j)^{-1} \sigma^2 + p^2]}{4c_j[p^2 + 4rc_j(1 - \pi_j)^{-1} \sigma^2]^2}
\]  
(3.22)

If \( \pi_j = 1 \), then \( x^*_j = \frac{p}{2c_j}, \beta^*_j = 0 \) and \( \alpha^*_j = \frac{p^2}{4c_j} \).

**Proof.** Straightforward algebra. **End.**

Proposition 3 gives us an explicit form for the optimal forcing contract. When \( \pi_j = 0 \), it reduces to the second best solution in case 1. In the opposite, when \( \pi_j = 1 \), we reach the first best case in which the marginal productivity equals to the marginal cost of the advisor. In this case, since shareholders can fully infer an advisor’s effort, there is no informational rent left over for the advisor.

After some algebraic manipulation, we can get the following comparative static results:

**Lemma 1.**

\[
\frac{\partial \beta^*_j}{\partial \pi_j} = \frac{2[p^2 + 4rc_j(1 - \pi_j)^{-1} \sigma^2]p(1 - \pi_j)^{-1} \sigma^2}{[p^2 + 4rc_j(1 - \pi_j)^{-1} \sigma^2]^2} > 0
\]  
(3.23)

\[
\frac{\partial \beta^*_j}{\partial \gamma} = \frac{-4p^2rc_j(1 - \pi_j)^{\gamma} \sigma^2}{[p^2(1 - \pi_j) + 4rc_j(1 - \pi_j)^{\gamma} \sigma^2]^2} \begin{cases} 
< 0 & \text{if } \gamma < 1 \\
0 & \text{if } \gamma = 1 \\
> 0 & \text{if } \gamma > 1
\end{cases}
\]  
(3.24)

\[
\frac{\partial \alpha^*_j}{\partial \sigma} = \frac{-4p^3r\sigma(1 - \pi_j)^{\gamma}}{[p^2 + 4rc_j(1 - \pi_j)^{-1} \sigma^2]^2} < 0
\]  
(3.25)

\[
\frac{\partial \beta^*_j}{\partial \sigma} = \frac{-8p^2rc_j\sigma(1 - \pi_j)^{-1}}{[p^2 + 4rc_j(1 - \pi_j)^{-1} \sigma^2]^2} < 0
\]  
(3.26)
\[
\frac{\partial x_j^{**}}{\partial p} = \frac{4rc_j\sigma^2(1-\pi_j)^{\gamma-1}[p^2(3-\pi_j)+4\pi_r r\sigma^2(1-\pi_j)^{\gamma-1}]+p^4}{2c_j[p^2+4rc_j(1-\pi_j)^{\gamma-1}\sigma^2]^2} > 0
\]  
(3.27)

\[
\frac{\partial \beta_j^{**}}{\partial p} = \frac{8p rc_j\sigma^2(1-\pi_j)^{\gamma-1}}{[p^2+4rc_j(1-\pi_j)^{\gamma-1}\sigma^2]^2} > 0
\]  
(3.28)

\[
\frac{\partial x_j^{**}}{\partial c_j} = \frac{p\{8rc_j\sigma^2(1-\pi_j)^{\gamma-1}[p^2+2\pi_r rc_j\sigma^2(1-\pi_j)^{\gamma-1}]+p^4\}}{2c_j[p^2+4rc_j(1-\pi_j)^{\gamma-1}\sigma^2]^2} < 0
\]  
(3.29)

\[
\frac{\partial \beta_j^{**}}{\partial c_j} = \frac{-4p^2rc_j\sigma^2(1-\pi_j)^{\gamma-1}}{[p^2+4rc_j(1-\pi_j)^{\gamma-1}\sigma^2]^2} < 0
\]  
(3.30)

Lemma 1 generates a rich set of predictions. The effect of monitoring power is shown by equations 3.23 and 3.24. As shareholders have higher monitoring power, they can make use of this power and induce a higher effort supply from the advisor. The potential benefit from shirking drops due to the higher risk of being caught by shareholders. Meanwhile, as monitoring power gradually increases, the change in the incentive component depends on the value of \(\gamma\), which determines the speed of the increasing accuracy in the performance measure. When \(\gamma > 1\), monitoring is also effective as a device of noise reduction since the variance becomes smaller when \(\pi_j\) gets bigger. As a result, shareholders want to increase the incentive component. Nevertheless, when \(\gamma < 1\), an increasing level in the monitoring power generates two opposite effects on error reduction. This can be seen by looking at the variance of the measurement error, \((1-\pi_j)^{\gamma}\sigma^2\). On one hand, when \(\pi_j\) gets bigger, the multiplicative term \((1-\pi_j)^{\gamma}\sigma^2\) becomes smaller and hence provides a positive error reduction effect; on the other hand, \(\gamma < 1\) scales up this term and enlarges the variance. Therefore, the offset forces imply that when \(\gamma < 1\), monitoring is likely to play a minor role in error reduction. When the detecting benefit dominates the error reduction benefit, it will substitute for the role of the incentive component and hence decrease \(\beta\). Equations 3.25 and 3.26 illustrate the importance of improving the accuracy in performance measurement. When \(\sigma\) is big\(^{54}\), the performance measure reveals less information on the advisor’s true efforts. In this case, shirking becomes more attractive and shareholders are less willing to

\(^{54}\) The effect of a risk-aversion coefficient is very similar to the change of \(\sigma\), and we do not discuss it here.
pay using incentive components as the output measure reveals little information on the effort-induced\(^{55}\). Finally, equations 3.27 to 3.30 reflect the impact of productivity and effort cost with regard to an equilibrium contract. As the advisor becomes more productive, the correlation between her effort supply and the stochastic firm value becomes higher. As firm value now becomes a better indicator of the advisor’s effort, it is optimal for shareholders to put more weight on the incentive component. Contrarily, when it is more costly for an advisor to provide additional effort, in order to induce one unit of extra effort, shareholders need to give up a larger share of their profit. Furthermore, this system limits the power of using an incentive component as an effective device.

With the contract solution from Proposition 3, we can calculate the expected profit for REIT shareholders when appointing a type-\(j\) advisor:

**Proposition 4.** The maximized expected profit for REIT shareholders with type-\(j\) advisor is:

\[
U_j^{**} = \frac{p^2(p^2 + 4\pi_j rc_j (1-\pi_j)^{-1}\sigma^2)}{4\sigma_j [p^2 + 4rc_j (1-\pi_j)^{-1}\sigma^2]} \tag{3.31}
\]

Meanwhile,\(^{55}\)

\[
\frac{\partial U_j^{**}}{\partial \pi_j} = \frac{[\gamma p^2 + 4rc_j (1-\pi_j)^{-1}\sigma^2]p^2r(1-\pi_j)^{-1}\sigma^2}{[p^2 + 4rc_j (1-\pi_j)^{-1}\sigma^2]^2} > 0 \tag{3.32}
\]

\[
\frac{\partial U_j^{**}}{\partial \sigma} = \frac{-2p^4r\sigma(1-\pi_j)^{\gamma}}{[p^2 + 4rc_j (1-\pi_j)^{-1}\sigma^2]^2} < 0 \tag{3.33}
\]

Finally, the choice of the optimal advisor is determined by \(\text{Max}\{U_i^{**}, U_e^{**}\}\). (Rule 1)

**Proof.** \(U_i^{**}\) can be obtained by directly substituting equations 3.20 to 3.22 into equation 3.14. Here, the selection rule is obvious. \(\textbf{End.}\)

Proposition 4 sheds light on the significance of a forcing contract. We can see from equation 3.32 that the welfare of shareholders is raised when they have a stronger

\(^{55}\) A well-known example involves the limitation of using profit as a performance measure in compensating managers in the oil industry. The variation in terms of oil price is largely independent of the manager’s effort, but may have a significant impact on realized profit.
monitoring power, and thus the production efficiency increases from the effort channel. Likewise, equation 3.33 implies the significance of a more accurate performance measure. Intuitively, a more noisy measure (hence a bigger σ) will have the effect of dropping shareholders’ welfare.

It should be clear from equation 3.31 that the rule regarding the selection of optimal advisor type depends in great part on whether the heterogeneity on monitoring power between internal and external advisors exists. Put differently, is it reasonable to expect shareholders to be able to detect shirking behavior more effectively from an external advisor than from internal one? Given the fact that external advisors inherently have a bigger conflict of interest, if shareholders cannot get a monitoring advantage (i.e., \( \pi_e > \pi_i \)) over an external advisor, there is no scope for this type of management to be preferable.

We believe that the existence of a monitoring advantage over external advisors is not an unreasonable assumption. We rationalize this argument from the following three aspects:

**Aspect 1 for \( \pi_e > \pi_i \): The dual-role of the external advisory firm**

In the model, we don’t differentiate between an external advisory firm and an external advisor. In practice, the external advisor is affiliated with a professional advisory firm. Therefore, the professional advisory firm plays a dual-role in this process. This dual-role relationship and the different compensation is illustrated in Figure 4.2.
From Figure 4.2, on one hand, an external advisory firm serves as an agent of REIT shareholders in providing an advisory service; on the other hand, the advisory firm is the principal for the advisor it sends out. And in practice, the compensation mechanisms for the two structures are quite different. As in the typical operating company, REIT shareholders will compensate internal advisor directly as an in-house employee. Nevertheless, as documented by Chan, Erickson and Wang [2003], a REIT firm is in general more likely to compensate an external advisory firm rather than an external advisor directly. This double-side principal-agent relationship is quite interesting as we can show that by correctly specifying a compensation mechanism for external advisor structure, REIT shareholders can use the compensation package as a monitoring device. To see it, we need to think about a richer model which incorporates the dual-role explicitly by emphasizing two potential factors: first, the professional advisory firm may have a different risk-tolerance than an individual advisor, generating a space of risk-sharing. Secondly, compared to REIT shareholders, a professional advisory firm may possess a superior ability to monitor the behavior of individual advisors. This assumption is reasonable as advisory firm is in some sense an insider and providing managerial service is its core business. Furthermore, as typically has more than one advisor employed, it is also easier for the firm to benchmark individual advisor’s effort with the pool of other peer
advisors. With these two assumptions, we can propose a mechanism through which REIT shareholder could get a free ride with regard to this superior monitoring power, even though shareholders have no advantage on monitoring external advisor’s effort themselves. Suppose: 1) REITs shareholders have a common monitoring ability, $\pi_i$, on both types of advisors. 2) Advisory firm is risk-neutral. 3) Advisory firm has monitoring power $\pi_e > \pi_i$.

**Lemma 2.** Under the above three assumptions, REIT shareholders can fully extract external advisory firm’s monitoring advantage by setting $\beta=1$ and $\alpha=-U_e^{**}$.

**Proof.** Use backward looking. With monitoring power $\pi_e$, the biggest possible benefit is $U_e^{**}$, as shown by Proposition 4. Plug in $\beta=1$ and $\alpha=-U_e^{**}$, it is easy to see that the expected equilibrium profit for advisory firm is 0, the outside utility of not providing advisory service. Hence the advisory firm will be indifferent in taking this contract or not. Meanwhile, advisory firm can replicate REITs shareholders by compensating external advisor by setting $\beta_e=\beta_e^{**}$, $\alpha_e=\alpha_e^{**}$ and $x_e=x_e^{**}$. Following Proposition 3, external advisor will take this contract, too. 

*End.*

From this illustration we can see that the equilibrium utility for REIT shareholders by choosing external management is $U_e^{**}$, the utility as if shareholders themselves have superior monitoring power $\pi_e$, instead of $\pi_i$. This example is more or less extreme as we neglect the possibility that advisory firm may collude with external advisor to pursue some private benefits. Therefore, more realistically, we expect to see a trade-off between the informational rent and private benefits and hence decrease the free-ride rent. However, we hope the intuition on this free-riding effect is clear through this example.

**Aspect 2 for $\pi_e > \pi_i$ : The reputation cost for external advisor**

By assuming $\pi_e > \pi_i$, we do not necessarily require that REIT shareholders have the ability to detect shirking behavior more effectively on an external advisor than on an internal advisor. Rather, it can be interpreted as a proxy for other factors. All in all, the mathematical implication of a higher value on $\pi$ is nothing but a reduced shirking benefit.
Recognizing this, we can justify $\pi_e > \pi_i$ in many guises. For example, this assumption is fully consistent with the scenario in which REIT shareholders have no superior monitoring power over an external advisor; rather, once detected in shirking, an external advisor may also incur an extra cost with regard to their reputation\textsuperscript{56}. This is very likely to be the case. As discussed in section 2, an external advisor typically comes from a professional advisory firm or another institution that has a close relationship with the real estate market. Furthermore, it is reasonable to expect that being caught for misbehavior not only hurts the reputation of the advisor herself, but also the reputation of the professional company that the external advisor is affiliated with. The assumption of $\pi_e > \pi_i$ captures this effect naturally. To see this clearly, we assume that the extra cost in reputation for an external advisor is $R$, and that REIT shareholders have the same monitoring power $\pi_i$ on detecting any misbehavior.

For an internal advisor, the IC constraint is the same as before:

$$\alpha + \beta V(x) - C_i(x) - r \beta^2 (1 - \pi_i)^\gamma \sigma^2 \geq (1 - \pi_i) \left[ \alpha + \beta V(x_e (\beta; \pi_i)) - C_i(x_e (\beta; \pi_i)) - r \beta^2 (1 - \pi_i)^\gamma \sigma^2 \right]$$

However, for external advisor, the IC constraint is:

$$\alpha + \beta V(x) - C_e(x) - r \beta^2 (1 - \pi_i)^\gamma \sigma^2 \geq (1 - \pi_i) \left[ \alpha + \beta V(x_e (\beta; \pi_i)) - C_e(x_e (\beta; \pi_i)) - r \beta^2 (1 - \pi_i)^\gamma \sigma^2 \right] - \pi_i R$$

The difference here is that for an external advisor, the expected penalty wage becomes $-\pi_i R$ instead of 0. Equivalently, we can re-write the right hand side of the above inequality as:

$$(1 - \pi_i) \left[ \alpha + \beta V(x_e (\beta; \pi_i)) - C_e(x_e (\beta; \pi_i)) - r \beta^2 (1 - \pi_i)^\gamma \sigma^2 \right] - \pi_i R = (1 - \pi_e) \left[ \alpha + \beta V(x_e (\beta; \pi_e)) - C_e(x_e (\beta; \pi_e)) - r \beta^2 (1 - \pi_e)^\gamma \sigma^2 \right]$$

\textsuperscript{56} Or bigger if we assume an internal advisor also incurs a similar cost once caught.
By solving equation 3.34 for $\pi_e$, the only unknown parameter, it is straightforward to observe that $\pi_e > \pi_i$ must hold, and this is implied by the fact that $\hat{x}_e(\beta; \pi_e) = \hat{x}_e(\beta; \pi_i)$.

Interestingly, Howe and Shilling [1990] find that REITs advised by more well-known external advisors showed better performance than did those run by less reputable ones. Our model can help explain these results since the more well-known advisors also derive lower benefits from shirking due to their greater potential for loss of reputation if caught for misbehavior. Thus, ex-ante, shareholders can induce higher levels of effort from them, and this is reflected by better firm performance, cateris paribus.

**Aspect 3 for $\pi_e > \pi_i$: More concrete targets on monitoring external advisor**

In addition to the above two motivations, which don’t depend on the “real” monitoring advantage, we may also expect that REIT shareholders can actually detect an external advisor’s shirking more effectively. There is a vast literature that examines the typical types of misbehavior (called shirking in our model) that has been attributed to REIT advisors. Chan, Erickson and Wang [2003] have presented an in-depth discussion on this issue. To summarize, external advisors are likely to misallocate revenue if they are in charge of several REITs. Furthermore, self-dealing with a parental company or closely related entities is another common behavior that external advisors have been known to partake in. Hence, in comparison with internal advisors, who are often treated as in-house employees, shareholders should have clear methods through which to monitor external advisors in order to be able to perceive their potential abuses. Secondly, as pointed out by Chan, Erickson and Wang [2003], self-advised REITs hold a strong resemblance to operating companies. In comparison to companies with externally advised structures, in self-advised REITs, the boundary between parties within a company is in general vaguer. Since they form stronger team-production relationships, it is more difficult to isolate one agent’s effort level from that of other agents. Thirdly, in general, investors are likely to have the perception that external advisors are more inclined to misbehave. Expecting that ex-ante, the incentive for monitoring an external advisor may be stronger than for monitoring an internal one. Furthermore, conditional on the same effort level but a low level of realized firm value, investors may be more likely to suspect shirking if the
company is run by an external advisor. In this sense, shareholders may experience a “bias” with regard to noise tolerance in favor of an internal advisor. Assuming such an expectation regarding an external advisor, the level of effective monitoring power could be strengthened. This intuition is supported by the findings of Ambrose and Linneman [2001], who argue that external advisors continually try to improve their efficiency so as to keep their positions. Shareholders may induce higher effort from external advisors through the use of threats of replacement.

After providing the above three motivations on the monitoring heterogeneity between internal and external structures, we are now ready to give a concrete illustration on the optimal advisor choice problem. To do it, we assume $c_i = 5$, $r=2$, $\gamma=0.5$ and $\sigma=10$. Further, we set $\frac{c_e}{c_i}=1.5$. Unless state otherwise, we maintain these parameter values in all of the following numerical exercises. We plot the results in Figure 4.3, which is followed by Rule 1 on optimal advisor choice.

**Figure 4.3 The Region of Optimal Advisor Type**

![Figure 4.3 The Region of Optimal Advisor Type](image-url)
In figure 4.3, the 45 degree plane refers to the case of common monitoring power, and the red plane gives us the boundary of the optimal advisor type. In the area situated above this boundary plane, an external advisor is considered preferable. Below it lies the area which indicates that an internal advisor should be chosen instead. The fact that the boundary plane always lies above the 45 degree plane implies that without monitoring heterogeneity, internal management is always superior to external management. In addition, we observe that the required monitoring gap, i.e., the vertical distance between the two planes increases when p increases. This too is intuitive. Recall from equation 3.2 that 

\[ V = V(x) + \varepsilon = px + \varepsilon \]. Hence, a higher marginal productivity (bigger p here) means a lower noise to output ratio, which implies that our performance measure can more accurately reflect an advisor’s true effort. It, in turn, implies that the benefit from monitoring decreases. Meanwhile, appointing an internal advisor can generate a larger benefit to shareholders due to the lower cost in effort. Not surprisingly, to induce a higher level of effort from external advisors, shareholders require a larger monitoring advantage in order to choose the external management structure.57

4.3.2.2 Welfare Implications for Shareholders and REITs Advisors

Since the existence of monitoring power restricts an advisor’s likelihood of shirking, a positive question to ask is whether it is detrimental to her welfare. Put differently, increasing the level of monitoring power could result in a mutual benefit for both shareholders and advisor. Furthermore, as discussed in the last section, for shareholders, an improvement in performance measure can serve a similar function as inducing more effort from an advisor. Clearly, both devices should improve the shareholders’ welfare, as shown in Proposition 4. However, do they have different impacts on the advisor’s welfare? We summarize our conclusion on this issue in the following proposition:

**Proposition 5.** The type-j advisor’s equilibrium utility is in the following form:

\[
\begin{align*}
    u_{j}^{**} &= \frac{4p^{2}\pi_{j}^{2}\epsilon_{j}(1-\pi_{j})^{2\gamma-1}\sigma^{4}}{\left[p^{2}+4\epsilon_{j}(1-\pi_{j})^{2\gamma-1}\sigma^{2}\right]} \quad (0,1) \\
    &= 0 \quad \text{if } \pi_{j} = 0 \text{ or } 1
\end{align*}
\]

(3.35)

57 By a similar argument, we can show that changing \( \sigma \) has an opposite effect from changing \( p \).
Furthermore,

\[
\frac{\partial u_{j}^{**}}{\partial \pi_{j}} = \frac{r^{2}c_{j}(1-\pi_{j})^{2}\pi_{j}^{-1}[4rc_{j}(1-\pi_{j})^{-1}(1-2\pi_{j})\sigma^{2} + p^{2}(1-2\gamma\pi_{j})]}{[p^{2} + 4rc_{j}(1-\pi_{j})^{-1}\sigma^{2}]^{3}} \tag{3.36}
\]

\[
\frac{\partial u_{j}^{**}}{\partial \sigma_{j}} = \frac{16p^{4}\pi_{j}r^{2}\pi_{j}^{-1}(1-\pi_{j})^{2}\pi_{j}^{-1}}{[p^{2} + 4rc_{j}(1-\pi_{j})^{-1}\sigma^{2}]^{3}} \begin{cases} > 0 & \text{if } \pi_{j} \in (0,1) \\ \leq 0 & \text{if } 4rc_{j}(1-\pi_{j})^{-1}(1-2\pi_{j})\sigma^{2} + p^{2}(1-2\gamma\pi_{j}) \leq 0 \end{cases}
\]

\[
\begin{cases} = 0 & \text{if } \pi_{j} = 0 \text{ or } 1 \\
\end{cases}
\]

**Proof.** Substituting equation 3.20 to 3.22 into equation 3.5 gives us the equilibrium utility for an advisor. The comparative static results involve straightforward algebra. **End.**

Proposition 5 provides very different welfare implications from the two devices used to stipulate effort, \(\pi_{j}\) and \(\sigma\). In particular, provided that the monitoring power is below a certain threshold, increases in monitoring ability always increase an advisor’s equilibrium utility. Hence, within this range, facilitating monitoring is also in the interest of the advisor, since investors must transfer some rents from the increasing production efficiency to the advisor. This has significant implications regarding corporate governance for REITs. Since a better design of performance measure is only at cost to advisors under imperfect monitoring, it is this difference that emphasizes the distinction between detecting misbehavior and observing true effort. Overall, the objective for compensation design is to try to pay an advisor every dollar for her contribution to the effort-induced output only, but no more. This can be achieved by creating a finer measure of effort-induced output, i.e., a lower \(\sigma\). Nevertheless, it also decreases the advisor’s insurance demand. As a result, shareholders can extract rents that were previously enjoyed by advisors through decreasing their fixed wage component, which is a free lunch to the advisor and is ex-post independent of her effort. Instead, as long as monitoring is not perfect, the lack of observability of the advisor’s effort level makes it difficult for shareholders to decide how much exactly to pay. In this case, shareholders must sacrifice some rents in order to compensate for the higher income risks faced by the advisor.
As a demonstration, we plot the simulated welfare for an advisor under different levels of monitoring power and uncertainty in Figure 4.4. Specifically, we set \( \sigma = 10 \) when changing \( \pi_i \) and set \( \pi_i = 0.5 \) when changing \( \sigma \). We also set \( p = 30 \). The other parameters are maintained the same as before.

**Figure 4.4 Welfare Implication**

![Graph showing welfare implications](image)

### 4.4. The Sub-optimality of a Fixed Forcing Contract

Our previous discussion shows that in equilibrium, the advisor will obey the prescribed effort level set by shareholders. One related question to ask is: why should shareholders pay an advisor upon the noisy performance measure \( V \) if they already know that in equilibrium, the effort-induced output is \( V(x) \)? Wouldn’t it be optimal for shareholders to bear all the risk due to their risk-neutrality? We show in this section that, as long as \( \sigma \) is not zero, a stochastic forcing contract as in Case 2 always dominates a fixed forcing
contract. To illustrate this, consider the case in which shareholders propose a fixed forcing contract. Now their objective function becomes:

\[ U_{j, \text{fixed}}(\pi_j) = \max_{(\alpha, \beta, x)} -\alpha + (1-\beta)V(x) \quad (4.1) \]

subject to

\[ \alpha + \beta V(x) - C_j(x) \geq 0 \quad (\text{IR}_j) \]
\[ \alpha + \beta V(x) - C_j(x) \geq (1-\pi_j) \left[ \alpha + \beta V(x_j) \right] \quad (\text{IC}_j) \]

Compared with equation 3.9, the IC constraint changes. Now the advisor faces a contract with a fixed wage of \( \alpha + \beta V(x) \), which is based on a prescribed effort level. Nevertheless, the trade-off for the advisor also differs significantly: on one hand, she can either follow the prescribed effort level and enjoy the utility of \( \alpha + \beta V(x) - C_j(x) \); on the other hand, she can choose to shirk by not putting in any effort at all, and the expected utility is \( (1-\pi_j) \left[ \alpha + \beta V(x_j) \right] \). Clearly, in the absence of uncertainty on wage income, shirking becomes more attractive due to its greater insurance benefits. Intuition tells us that to induce enough effort in this case, shareholders must offer a much higher wage than in the case of a stochastic contract. This is confirmed by the following proposition.

**Proposition 6.** Assume \( V(x) = px \) and \( C_j(x) = c_j x^2 \). The solutions for equation 4.1 are:

\[ x_{j, \text{fixed}}^{**} = \frac{\pi_j p}{2c_j} \quad (4.2) \]
\[ w_{j, \text{fixed}}^{**} = \frac{\pi_j p^2}{4c_j} \quad (4.3) \]

Meanwhile, shareholders’ utility is:

\[ U_{j, \text{fixed}}^{**} = \frac{\pi_j p^2}{4c_j} \quad (4.4) \]

In this case, a stochastic forcing contract always dominates a fixed contract:

\[ U_{j, \text{fixed}}^{**} - U_{j, \text{fixed}}^{**} = \frac{p^4 (1-\pi_j)}{4c_j [p^2 + 4rc_j (1-\pi_j) \gamma^{-1} \sigma^2]} \geq 0 \quad (4.5) \]
And a fixed forcing contract outperforms the second best solution $U_j^{*}$ only when

$$\pi_j > \frac{p^2}{p^2 + 4c, r\sigma^2} = \beta_j^{*}.$$ 

**Proof.** The solution part follows exactly the same steps as in Case 2 in section 4. After arriving at the solutions, it is straightforward to derive and remaining conclusions. **End.**

### 4.5. Future Extensions

#### 4.5.1 Converging Cost Structure

In this mode, we assume that an external advisor’s marginal cost is always higher than that of an internal advisor. This assumption may be highly unfair to an external advisor. In the case that the advisor’s efforts are perfectly observable, we would expect both internal and external advisors to perform at least equally well. If this is not the case, we must make the assumption that an external advisor is less capable, which is hardly related with any agency problem. In some sense, our examination from section 3 was a worst case scenario for an external advisor. Even in such an extreme situation, we are still able to find a positive region in which an external advisor is preferred. Hence, it makes sense to expect some level of this robustness in our findings.

One way to relax this assumption is to change our specification on $\theta$. In stead of assuming it as a constant, we can specify $\theta = \theta(\pi_e)$, where $\theta(1) = 1$ and $\theta'(\pi_e) < 0$. In this case, an external advisor still has a higher effort cost, but the gap tends to decrease and eventually converge and vanish when shareholders can perfectly detect any misbehavior, i.e., $\pi_e = 1$. The analytical process is very similar to what we have done in section 3. Obviously, in this case, the external advisor’s conflict of interest declines as monitoring power increases. Hence, we would expect that external management would more likely be a preferred choice than it was in section 3. Modeling this way will further strengthen our proposed rationale supporting external management.

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58 See equation 3.10.
4.5.2 Costly Monitoring
In this study, we have held the assumption that monitoring power is exogenously given and that it incurs no cost for shareholders. The reason that we have chosen this type of model was to simplify our analysis and to isolate the implication of monitoring power on the equilibrium compensation scheme from other factors. Of course, in reality, we would expect some cost to be associated with monitoring. Our model can easily be extended to include the reality of costly monitoring. To maintain the heterogeneity on monitoring power, we need to assume two different monitoring cost structures for the two types of advisors. In particular, to prevent external advisors from being fully dominated, we need to either assume that monitoring external advisors can be achieved more efficiently than monitoring internal advisors, or we need to introduce a component dealing with reputation cost, as we have discussed in section 3. Once we introduce monitoring cost, there will be an extra layer of trade-off between the efficiency gain in forcing contract and the increased monitoring cost. Nevertheless, provided that there is a big enough efficiency gain, we can again derive conditions under which shareholders would prefer to choose the external managerial structure.

4.5.3 Heterogeneous Productivity
This, in some sense, mirrors differential effort cost. The extension in this direction is straightforward. The greatest challenge here is to justify why we should expect any productive advantage associated with an external advisor.

4.5.4 Mixed Forcing Contract
The reason a fixed forcing contract is sub-optimal is because of the absence of uncertainty in the agent’s wage income. As a result, the advisor is granted too much insurance benefits when shirking takes place. Hence, an interesting question to ask is whether it is possible for shareholders to continue offering insurance benefits to agents, but to do so only when they are very unlikely to shirk? One potential way to achieve this is to use a mixed forcing contract. To implement a mixed forcing contract in a dynamic context, shareholders can offer the advisor two wage menus. The primary menu is a stochastic forcing contract, which would function as the default menu for the advisor. During each period, a second
menu of a fixed forcing contract is also made available to the advisor on condition of meeting some specific criteria set by the shareholders. These criteria may be contingent on realized output or some other effort indicators, and would be reviewed by the shareholders after each period. It is possible that by correctly specifying the transferring criteria, the optimal response for the agent would not be to shirk, but to maximize the possibility of transferring to the fixed menu and thus providing the prescribed level of effort. By using this method, shareholders could also achieve their goal of paying exactly according to the advisor’s efforts rather than having to rely on an imperfect performance measure. Furthermore, the agent’s welfare may also increase due to the reduced level of uncertainty in their income.

4.6. Concluding Remarks

In this paper, we present a simple theory regarding the choice of optimal forcing contract and advisor type. We emphasize the importance of monitoring power on the improvement of an advisor's optimal level of effort. Our model may help explain the well-known puzzle concerning external advisors in the real estate literature. We show that if there is potential heterogeneity with regard to monitoring power between internal and external advisors, shareholders can make use of this heterogeneity and induce a higher level of effort from an external advisor. As a result, when the gap on monitoring power is large enough, it could still be optimal for shareholders to maintain an external managerial structure, despite the fact that external advisors have a bigger incentive to shirk and extract money from REIT shareholders. We motivate the rationale for expecting a monitoring advantage over external management from three aspects: the dual-role of external advisory firm, a bigger reputation cost associated with external advisor and more concrete monitoring targets with external advisor. We believe the dual-role of advisory firm deserves more attention; and we propose a compensation mechanism and show how REIT shareholders can use compensation as a monitoring device and extract advisory firm’s superior monitoring ability. We then demonstrate the advisor choice problem by explicitly deriving an optimal selecting rule which identifies the boundary of optimal advisor type.
In addition, we examine the welfare implication of monitoring power for both shareholders and advisors. Our results imply that when shareholders can better monitor an advisor's misbehavior, both parties' welfare could increase under some conditions. Thus, the advisor as agent may not necessarily be averse to a better monitoring environment. Furthermore, shareholders can enjoy the whole surplus through the use of a better performance measure. When measurement error decreases, the effort supply from an advisor increases, and the equilibrium utility for the advisor keeps dropping.

Although the primary focus in this study is to examine the effect of monitoring heterogeneity on advisor choice, we also look at other sources of heterogeneity including differences in productivity, etc. on explaining advisor choice. It is easy to see that conditional on the same effort level and if an external advisor also has a higher productivity, it is possible that under certain conditions the optimal choice for some REIT firms is to remain externally advised.

This research aims to provide a theory on REITs advisor choice and optimal compensation design. And we hope the study can help provide implication on how to rationalize external advisory structure, in particular with its remained popularity in the fast-growing emerging REITs market such as in Asia.
References


CHAPTER 5: CONCLUSION

In my dissertation, I try to address two important research topics in real estate. The first is to model the individual house seller’s pricing process. In particular, I look at the role of reference-dependence on a seller’s decision making process. The reason that reference-dependence may turn out to be an important factor is due to the intriguing finding that in the housing market, sellers of qualitatively similar units may set very different prices, even when facing the common housing market environment. This phenomenon is inconsistent with the classic-utility model, as in the classic case the objective function for sellers should be the same if they face the same market environment and the maximization horizon. Hence, to justify this phenomenon, sellers should be maximizing some objective functions that are individual-specific, which renders the well-known prospect theory particularly relevant. Following this theory, instead of drawing utility directly from financial wealth, people draw utility over gains and losses relative to a reference value such as prior acquisition price or an initial endowment. In other words, sellers are not maximizing the expected selling proceeds in this case. Furthermore, the fact that it is actually possible for a seller to sell a house at very different prices implies that potential buyers should have heterogeneous valuations on a given house good.

These two ingredients lead us to consider a standard search model in which house sellers face a pool of potential buyers who have heterogeneous valuations on the house good. In chapter 2, I treat the extent of heterogeneity as exogenous so as to purify the effect of reference-dependence on a seller’s pricing strategy and its implications on loss aversion behavior. When comparing my model predictions with the stylized empirical evidence on loss aversion in the literature as presented by Genesove and Mayer [2001], I find that the proposed evidence actually has a very limited power on testing loss aversion. In particular, Genesove and Mayer [2001] report that, in the housing market: 1) compared to potential gainers, a seller subject to bigger potential loss sets a higher asking price and obtains a higher transaction price if the house is sold (finding one); and 2) the marginal mark up declines with the size of seller’s potential loss exposure (finding two). Genesove and
Mayer interpret finding one as a test of loss aversion effect and finding two as a test of diminishing sensitivity in value function.

Nevertheless, in our model, I show that both findings are evidence only supporting a reference-dependent value function. Therefore, they are valid tests of the first component in the prospect theory. However, neither does finding one have a necessary relationship with loss aversion, nor does finding two have a necessary relationship with diminishing sensitivity. For example, I show that an increasing and concave relationship between a seller’s asking price and her potential loss exposure is fully consistent with a value function that has symmetric response between losses and gains, and a marginal increasing sensitivity in both dimensions. As a result, there is a conceptual mismatch between two empirical findings and their theoretical counterparts.

Acknowledging the weak testing power of both findings one and two on loss aversion, I argue that, to test for loss aversion and marginal diminishing sensitivity, a more important way to examine this is to look at the behavior of sellers who have lower reference values. In particular, a value function that is fully consistent with prospect theory implies an S shape asking price curve along reference values, i.e., sellers’ asking prices increase convexly when reference values are small and increase concavely when they are big. In addition, there is a significant slope increase among sellers who are around break-even positions. Using a dataset from Vancouver single-family transactions, I find evidence that is consistent with the predictions made by prospect theory.

In contrast, in chapter 3, I look at the impact of potential buyers’ changing bidding heterogeneity on the equilibrium pricing strategy of house sellers. I model house sellers’ pricing behavior under two market conditions: first, sellers have reference-dependent utility; second, the house good is relatively less heterogeneous, such as in the multi-unit residential market. I contribute to the literature by extending chapter 2’s analytic framework on the pricing strategy of house sellers to look at the signaling effect on a seller’s optimum pricing behavior when potential buyers can observe the transaction prices of other units that are close substitutes for the unit that a current seller wants to sell.
The intuition is that, as the available housing goods become more comparable to each other, the transaction prices of other units on the market may convey more useful information on the fundamental value of the unit that a buyer is inspecting.

Acknowledging the fact that in the multi-unit residential market, units in the same building serve as close substitutes for each other, I show that the recent transaction price on a unit in the same building generates two signaling effects: first, it generates positive spatio-temporal autocorrelation with the subsequent price for units in the same building; second, after observing the prior price, the heterogeneity of a potential buyer’s willingness to pay decreases, which in turn induces house sellers to mark down their asking price.

Using a geo-coded dataset on condominium transactions in Singapore from 1990 to 2001, I find significant evidence of the reference dependence effect. Furthermore, I show that the spatio-temporal autocorrelation among units in the same building is significantly higher than with units in neighboring buildings. Meanwhile, using a two order spatio-temporal autoregressive model as proposed by Sun, Tu and Yu [2005], I find that, after controlling for the autocorrelation effect, sellers tend to mark down their asking prices by at least 1.6% if a recent transaction has occurred within the same building. This reflects the effect from the decreasing bidding heterogeneity among potential buyers.

In my last essay, I switch to the area concerning REIT corporate governance. Chapter 4 proposes a model which examines the power of monitoring and forcing contract on improving managerial efficiency. I put particular focus on its implications regarding the choice of advisor type used by REITs. This question has long been a puzzling one in the real estate literature. Our model provides a theoretical justification regarding the potential appeal of the external managerial structure, which is usually regarded as being inferior to internal managerial structure. I show that, for both types of advisors, increasing levels of monitoring power will increase their equilibrium effort under a stochastic forcing contract. Furthermore, I am able to specify the range within which an improved monitoring power is pareto-optimal for both REIT shareholders and advisors. One significant implication is that, as agents, it may also be to the benefit of advisors to be better monitored. A crucial
driving force regarding advisor choice is the heterogeneity on monitoring power between internal and external advisors and across REIT firms. Provided that the gap of monitoring power is large enough between internal and external advisors, shareholders could make use of the heterogeneity, and induce higher effort levels from external advisors. Finally, I compare the difference between fixed and stochastic forcing contracts. Our findings show that with their imperfect performance measures, stochastic forcing contracts always dominate fixed ones.
References

## Appendix: Appendix for Chapter 3

Table A.1 Variable Definitions in the Traditional Hedonic Model in Chapter 3

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>Dwelling transaction price (Unit: S$)</td>
</tr>
<tr>
<td>Area</td>
<td>Floor area in each condominium flat (unit: m$^2$)</td>
</tr>
<tr>
<td>Age</td>
<td>The age of the condominium project (Unit: year)</td>
</tr>
<tr>
<td>Level</td>
<td>The floor level where the flat is (Unit: number)</td>
</tr>
<tr>
<td>Tenure</td>
<td>Dummy variable with ONE indicating 999 years’ leasehold or freehold, otherwise ZERO.</td>
</tr>
<tr>
<td>Total Units</td>
<td>Total number of dwelling units in the condominium project (Unit: number)</td>
</tr>
<tr>
<td>Units_Building$[^1]$</td>
<td>Total number of dwelling units in a particular building (Unit: number)</td>
</tr>
<tr>
<td>Dis to Secondary School</td>
<td>Distance to the 1$^{st}$ nearest top 10 secondary schools (Unit: km)</td>
</tr>
<tr>
<td>Dis to Junior College</td>
<td>Distance to the 1$^{st}$ nearest top 10 junior college (Unit: km)</td>
</tr>
<tr>
<td>Dis to MRT</td>
<td>Distance to the nearest MRT station (Unit: km)</td>
</tr>
<tr>
<td>Dis to CBD</td>
<td>Distance to the central of CBD (Unit: km)</td>
</tr>
<tr>
<td>Barbecue</td>
<td>Dummy Variable, 1 if condo project has barbecue area and 0 if not.</td>
</tr>
<tr>
<td>Gymnasium</td>
<td>Dummy Variable, 1 if condo project has gymnasium and 0 if not.</td>
</tr>
<tr>
<td>Jacuzzi</td>
<td>Dummy Variable, 1 if condo project has Jacuzzi and 0 if not.</td>
</tr>
<tr>
<td>Sauna</td>
<td>Dummy Variable, 1 if condo project has Sauna and 0 if not.</td>
</tr>
<tr>
<td>Swimming pool</td>
<td>Dummy Variable, 1 if condo project has swimming pool and 0 if not.</td>
</tr>
<tr>
<td>Tennis court</td>
<td>Dummy Variable, 1 if condo project has tennis court and 0 if not.</td>
</tr>
</tbody>
</table>

Note: 1) This is not available in the raw dataset and the measurement process is explained in section 3.2.4.
Table A.2: Results for the Traditional Hedonic Model in Chapter 3

<table>
<thead>
<tr>
<th>Explanatory variables [1]</th>
<th>OLS Estimates</th>
<th>STD</th>
<th>P-level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>12.4136</td>
<td>0.0326</td>
<td>0.0000</td>
</tr>
<tr>
<td>Level</td>
<td>0.0045</td>
<td>0.0002</td>
<td>0.0000</td>
</tr>
<tr>
<td>Area</td>
<td>0.0053</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Age</td>
<td>-0.0121</td>
<td>0.0003</td>
<td>0.0000</td>
</tr>
<tr>
<td>Tenure</td>
<td>0.1647</td>
<td>0.0028</td>
<td>0.0000</td>
</tr>
<tr>
<td>Dis to MRT</td>
<td>-0.0038</td>
<td>0.0015</td>
<td>0.0100</td>
</tr>
<tr>
<td>Dis to CBD</td>
<td>-0.0244</td>
<td>0.0004</td>
<td>0.0000</td>
</tr>
<tr>
<td>Dis to Secondary School</td>
<td>-0.0101</td>
<td>0.0011</td>
<td>0.0000</td>
</tr>
<tr>
<td>Dis to Junior College</td>
<td>-0.0153</td>
<td>0.0006</td>
<td>0.0000</td>
</tr>
<tr>
<td>Barbecue area</td>
<td>0.0294</td>
<td>0.0029</td>
<td>0.0000</td>
</tr>
<tr>
<td>Gymnasium</td>
<td>0.0226</td>
<td>0.0028</td>
<td>0.0000</td>
</tr>
<tr>
<td>Jacuzzi</td>
<td>0.0507</td>
<td>0.0031</td>
<td>0.0000</td>
</tr>
<tr>
<td>Sauna</td>
<td>0.0359</td>
<td>0.0027</td>
<td>0.0000</td>
</tr>
<tr>
<td>Swimming pool</td>
<td>0.1154</td>
<td>0.0057</td>
<td>0.0000</td>
</tr>
<tr>
<td>Tennis court</td>
<td>0.0255</td>
<td>0.0033</td>
<td>0.0000</td>
</tr>
<tr>
<td>Quarterly Dummy [2]</td>
<td>Not Reported</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.7005</td>
<td></td>
<td></td>
</tr>
<tr>
<td># observations</td>
<td>51047</td>
<td></td>
<td></td>
</tr>
<tr>
<td># Variables</td>
<td>62</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: 1) The dependent variable is the $Y=\log(\text{price})$, the definition of all variables are given by Table A.1 in the appendix.
2) The base quarter is the first quarter of 1990.
Table A.3 2STAR Model Estimates in Chapter 3

<table>
<thead>
<tr>
<th>Explanatory variables</th>
<th>OLS Estimates</th>
<th>STD</th>
<th>P-level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.0560</td>
<td>0.0910</td>
<td>0.5383</td>
</tr>
<tr>
<td>Level</td>
<td>0.0056</td>
<td>0.0002</td>
<td>0.0000</td>
</tr>
<tr>
<td>Area</td>
<td>0.0045</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Age</td>
<td>-0.0064</td>
<td>0.0002</td>
<td>0.0000</td>
</tr>
<tr>
<td>Tenure</td>
<td>0.0644</td>
<td>0.0030</td>
<td>0.0000</td>
</tr>
<tr>
<td>W1×Level</td>
<td>-0.0038</td>
<td>0.0002</td>
<td>0.0000</td>
</tr>
<tr>
<td>W1×Area</td>
<td>-0.0024</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>W2×Level</td>
<td>-0.0029</td>
<td>0.0004</td>
<td>0.0000</td>
</tr>
<tr>
<td>W2×Area</td>
<td>-0.0012</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>W3×Age</td>
<td>0.0062</td>
<td>0.0003</td>
<td>0.0000</td>
</tr>
<tr>
<td>W3×Tenure</td>
<td>-0.0404</td>
<td>0.0039</td>
<td>0.0000</td>
</tr>
<tr>
<td>T×Level</td>
<td>-0.0027</td>
<td>0.0008</td>
<td>0.0005</td>
</tr>
<tr>
<td>T×Area</td>
<td>-0.0023</td>
<td>0.0001</td>
<td>0.0000</td>
</tr>
<tr>
<td>T×Age</td>
<td>0.0072</td>
<td>0.0008</td>
<td>0.0000</td>
</tr>
<tr>
<td>T×Tenure</td>
<td>-0.0486</td>
<td>0.0110</td>
<td>0.0000</td>
</tr>
<tr>
<td>W1×Y</td>
<td>0.5933</td>
<td>0.0043</td>
<td>0.0000</td>
</tr>
<tr>
<td>W2×Y</td>
<td>0.3302</td>
<td>0.0060</td>
<td>0.0000</td>
</tr>
<tr>
<td>T×Y</td>
<td>0.4419</td>
<td>0.0123</td>
<td>0.0000</td>
</tr>
<tr>
<td>ST×Level</td>
<td>-0.0006</td>
<td>0.0013</td>
<td>0.6169</td>
</tr>
<tr>
<td>ST×Area</td>
<td>0.0016</td>
<td>0.0002</td>
<td>0.0000</td>
</tr>
<tr>
<td>ST×Age</td>
<td>-0.0047</td>
<td>0.0009</td>
<td>0.0000</td>
</tr>
<tr>
<td>ST×Tenure</td>
<td>0.0086</td>
<td>0.0100</td>
<td>0.3859</td>
</tr>
<tr>
<td>TS×Level</td>
<td>0.0014</td>
<td>0.0012</td>
<td>0.2667</td>
</tr>
<tr>
<td>TS×Area</td>
<td>0.0005</td>
<td>0.0001</td>
<td>0.0018</td>
</tr>
<tr>
<td>TS×Age</td>
<td>-0.0044</td>
<td>0.0011</td>
<td>0.0001</td>
</tr>
<tr>
<td>TS×Tenure</td>
<td>0.0002</td>
<td>0.0134</td>
<td>0.9865</td>
</tr>
<tr>
<td>ST×Y</td>
<td>-0.1147</td>
<td>0.0129</td>
<td>0.0000</td>
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<tr>
<td>TS×Y</td>
<td>-0.2502</td>
<td>0.0164</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

\[ R^2 \] = 0.8305

Optimum order: q=16, p=20

# observations: 51047

# Variables: 28

Note: 1) The dependent variable is Y=log (price), all variables are defined by Table 1 in the appendix.
2) The first 1000 observations (covering 1990 and the early part of 1991) were cut from the sample for prior validation of the spatial and temporal weight matrices.
3) W1×Level is the multiplication of building effect matrix W1 and the variable of “Level”. It indicates the spatial lag of the variable “Level” at building level. The similar interpretation applies to W1×Area, TW1×Level and TW1×Level. Tenure and age are omitted as within one building, all units have the same tenure and age.
4) T×X is the multiplication of temporal effect matrix T and the variable of X where X represents Level, Area, Age or Tenure. It indicates the temporal lag of the variable. The similar interpretation applies to W1×T×X, TW1×X and W2×T×X that indicates the spatial (at building level)-temporal lags, the temporal-spatial lags (at neighborhood level) and the spatial-temporal lags (at neighborhood level) of X separately, where X represents Level, Area, Age or Tenure.
5) W1×Y is the multiplication of building effect matrix W1 and the dependent variable Y. It indicates the spatial lag of the dependent variable at building level. The similar interpretation applies to W2×Y, T×Y, W1×T×Y, W1×T×Y, TW1×Y, TW2×Y.
6) Interpret in a similar way as in [3] and [4].
Proof of Proposition 1

First note that unit \( i \) and \( i' \) are in the same building. As discussed above, we have \( \bar{P}_i = \bar{P}_{i'} \), \( \theta_i = \theta_{i'} \) and \( \varepsilon_{i',j} = \varepsilon_{i,j} \) for all \( j \). Accordingly, \( p_{i',j} = \bar{P}_i + \theta_{i'} + \sigma_{\varepsilon_{i',j}} = \bar{P}_i + \theta_i + \sigma_{\varepsilon_{i,j}} \).

Meanwhile, we know that \( \sigma_{\varepsilon_{i',j}}, \sigma_{\varepsilon_{i,j}} \) are joint normal with \( \text{Cov}(\sigma_{\varepsilon_{i',j}}, \sigma_{\varepsilon_{i,j}}) = \sigma^2 \).

Following the properties of conditional distribution for Bivariate normal\(^{59}\), if \( Y \) and \( X \) are jointly normal with common mean \( \mu \) and variance \( \sigma^2 \) for their marginal distributions, and if \( \text{Cov}(X,Y) = \sigma_{X,Y}^2 \), then:

\[
E[Y \mid X=a] = \mu - \frac{\sigma_{X,Y}^2}{\sigma^2} (a - \mu), \quad \text{Var}[Y \mid X=a] = \sigma^2 - \frac{\sigma_{X,Y}^4}{\sigma^2},
\]
and the distribution of \( Y \) conditional on \( X=a \) is also normal. In our model, \( \mu = \bar{P}_i + \theta_i \), \( \sigma_{X,Y}^2 = \sigma^2 \), \( a = \bar{P}_i + \theta_i + \sigma_{\varepsilon_{i,j}} \), and the notation on the variance of marginal distribution is still \( \sigma^2 \). Substituting \( \mu \), \( \sigma_{X,Y}^2 \), \( a \) and \( \sigma^2 \) with their counterparts in our model, we can get equation 2.3 and 2.4 directly.

\(^{59}\) A reference for the following results is on Page 83 of Greene [2000].
Proof of Lemma 1

By Taylor expansion on $e^{-\beta t}$, equation 2.6 can be rewritten as:

$$0 = \max_{r_i} e^{-\beta t} \left\{ B(1-G(r_i; \mu, \sigma^2))\Delta t[r_i - v_i + c\Delta t - U^{*,r,c}] - c\Delta t \right\} - \beta U^{*,r,c} \Delta t \quad \text{(A.1)}$$

Dividing the above equation by $\Delta t$, taking limit as $\Delta t \to 0$ and re-organizing the terms gives us

$$U^{*,r,c} = \frac{B[1-G(r_i^*; \mu, \sigma^2)](r_i^* - v_i) - c}{B[1-G(r_i^*; \mu, \sigma^2)] + \beta} \quad \text{(A.2)}$$

Now taking the first order condition with respect to $r_i$, we get

$$1-G(r_i^*; \mu, \sigma^2) - g(r_i^*; \mu, \sigma^2)[r_i^* - v_i - U^{*,r,c}] = 0 \quad \text{(A.3)}$$

Substituting equation A.2 into equation A.3 gives us equation 2.7, and a general proof for the existence of a unique solution for this type of searching problem can be found in Williams [1998].
Proof of Proposition 2

Total differentiating equation A.3 with respect to $r^*_i$ and $\sigma$, and using the fact that equation 2.7 holds in equilibrium, we get the following expression after some algebraic manipulations:

$$\frac{\partial r^*_i}{\partial \sigma} = \frac{r^*_i - \mu + \frac{\sigma[\beta(v_i - \mu) - c]}{\{B[1 - G(r^*_i; \mu, \sigma^2)] + \beta\} \cdot [2\sigma^2 - 1 - G(r^*_i; \mu, \sigma^2)] (r^*_i - \mu)}}{\sigma},$$

which is equation 2.8. Similarly by total differentiating with corresponding parameters, we can show that

$$\frac{\partial r^*_i}{\partial v_i} = \frac{\beta \sigma^2}{\{B[1 - G(r^*_i; \mu, \sigma^2)] + \beta\} \cdot [2\sigma^2 - 1 - G(r^*_i; \mu, \sigma^2)] (r^*_i - \mu)} = \frac{\beta \sigma^2}{2g(r^*_i; \mu, \sigma^2)} \cdot \frac{\sigma^2(1 - G(r^*_i; \mu, \sigma^2))}{\{B[1 - G(r^*_i; \mu, \sigma^2)] + 2\sigma^2 - 1 - G(r^*_i; \mu, \sigma^2)\} (r^*_i - \mu)}(A.4)$$

$$\frac{\partial r^*_i}{\partial \mu} = \frac{\frac{\sigma^2[1 - G(r^*_i; \mu, \sigma^2)]}{\{B[1 - G(r^*_i; \mu, \sigma^2)] + \beta\} \cdot [2\sigma^2 - 1 - G(r^*_i; \mu, \sigma^2)] (r^*_i - \mu)}}{\sigma^2[1 - G(r^*_i; \mu, \sigma^2)](A.5)}$$

We now prove that $\frac{\partial r^*_i}{\partial v_i} > 0$, $\frac{\partial r^*_i}{\partial \mu} > 0$ and $\frac{\partial r^*_i}{\partial B} > 0$. First note that equation A.4 to A.6 share the same denominator, the multiplicative of two terms, $B[1 - G(r^*_i; \mu, \sigma^2)] + \beta$ and $2\sigma^2 - 1 - G(r^*_i; \mu, \sigma^2)$. By definition, $B[1 - G(r^*_i; \mu, \sigma^2)] + \beta > 0$. If we can show that $2\sigma^2 - 1 - G(r^*_i; \mu, \sigma^2) (r^*_i - \mu) > 0$, it would be clear that equation 2.8 and equation A.4 to A.6 must be all positive. We now prove it is actually the case. To see it, let’s look at $\frac{\partial r^*_i}{\partial B}$ first. From the envelope theorem, $\frac{\partial U^{v,c}}{\partial B} = \frac{(1 - G(r^*_i; \mu, \sigma^2))(\beta(r^*_i - v_i) + c)}{B[1 - G(r^*_i; \mu, \sigma^2)] + \beta}$. From equation 2.7, we know that $\beta(r^*_i - v_i) + c = \{B[1 - G(r^*_i; \mu, \sigma^2)] + \beta\} \cdot \frac{1 - G(r^*_i; \mu, \sigma^2)}{g(r^*_i; \mu, \sigma^2)} > 0$. 

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As a result, \( \frac{\partial U^{v,c}}{\partial B} > 0 \). From equation A.3, \( \frac{g(r^*_i; \mu, \sigma^2)}{1 - G(r^*_i; \mu, \sigma^2)} = \frac{1}{r^*_i - v_i - U^{v,c}} \), for all B.

Therefore, \( \frac{\partial r^*_i}{\partial B} > 0 \) must hold to maintain this equality, due to the fact that the normal hazard function is upward sloping.

Since \( \frac{\partial r^*_i}{\partial B} > 0 \), we have proved that \( 2\sigma^2 - \frac{1 - G(r^*_i; \mu, \sigma^2)}{g(r^*_i; \mu, \sigma^2)} (r^*_i - \mu) > 0 \) must hold. It in turn implies that \( \frac{\partial r^*_i}{\partial v_i} > 0 \) and \( \frac{\partial r^*_i}{\partial \mu} > 0 \), which completes our proof.