Essays in Open-Economy Macroeconomics

by

Ke Pang

B.A., Peking University, 2001
M.A., Memorial University, 2002

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
Doctor of Philosophy

in

The Faculty of Graduate Studies
(Economics)

The University Of British Columbia
(Vancouver)
December, 2008
© Ke Pang 2008
Abstract

This dissertation addresses three issues in international macroeconomics. The first chapter examines optimal portfolio decisions in a monetary open-economy DSGE model. In a complete market environment, Engel and Matsumoto (2005) find that sticky price can generate equity home bias. However, their result is sensitive to the structure of the financial market. In an incomplete market environment, we find “super home bias” in the equilibrium equity portfolio, which casts doubt on the ability of sticky price in describing the observed equity portfolios. We further show that introducing sticky wages helps to match the data. The second chapter analyzes the welfare impact of financial integration in a standard monetary open-economy model. Financial integration may have negative effects on welfare if integration occurs in the presence of nominal price rigidities and constraints on the efficient use of monetary policy. The reason is that financial integration leads to excessive terms of trade volatilities. From a policy perspective, the model implies that developing economies that are experiencing financial integration may attempt to alleviate the welfare cost of integration by stabilizing the exchange rate. This prediction is consistent with the widespread reluctance to following freely floating exchange rates among these economies. On the other hand, for advanced economies that have the ability to operate efficient inflation targeting monetary policies, financial integration is always beneficial. Thus, the model accounts for the observed acceleration in cross-border asset trade among advanced economies in the early 1990s as it was mainly the industrial countries that switched to an inflation targeting regime at the time. The third chapter uses an open-economy neoclassical growth model to explain the saving and investment behavior of the U.S. and a group of other OECD countries. We find that while the model explains investment
Abstract

quite well, it tends to overpredict U.S saving and underpredict saving in the rest of the world. We show that the closed-economy version of the model also predicts saving accurately but that is only because it imposes equality between saving and investment. In effect, the model explains investment not saving behavior.
# Table of Contents

Abstract ................................................. ii

Table of Contents ........................................ iv

List of Tables ............................................. vii

List of Figures ........................................... viii

Acknowledgments ........................................... x

Dedication ................................................ xi

1 Equity Home Bias and Nominal Rigidity .............................. 1
  1.1 Introduction .......................................... 1
  1.2 The Model ............................................. 4
    1.2.1 Consumers ........................................ 5
    1.2.2 Firms ............................................. 8
    1.2.3 Financial Sector .................................. 10
    1.2.4 Market Clearing ................................ 10
    1.2.5 Equilibrium ..................................... 10
  1.3 Portfolio Decisions .................................. 11
    1.3.1 Solution Method .................................. 11
    1.3.2 Risk Sharing ...................................... 12
    1.3.3 Flexible Price Case: $\kappa = 0$ .................. 14
    1.3.4 Sticky Price Case: $\kappa = 1$ ................... 15
  1.4 A Model with Sticky Wage .............................. 19
  1.5 Conclusion .......................................... 22
# Table of Contents

## 2 International Financial Integration and Monetary Policy  
2.1 Introduction ..................................................... 28  
2.2 The Model ...................................................... 33  
  2.2.1 Consumers .................................................. 34  
  2.2.2 Firms ....................................................... 38  
  2.2.3 Financial Sector ............................................ 39  
  2.2.4 Monetary Rules ............................................. 39  
  2.2.5 Market Clearing ............................................ 41  
  2.2.6 Equilibrium ............................................... 41  
2.3 Solution Method .............................................. 41  
  2.3.1 Optimal Portfolios ....................................... 41  
  2.3.2 Welfare .................................................... 42  
2.4 Model Solution .............................................. 45  
  2.4.1 Exchange Rate, Terms of Trade, and Risk Sharing ...... 46  
  2.4.2 Volatility of the Terms of Trade ......................... 47  
  2.4.3 Efficiency of the Terms of Trade ....................... 48  
  2.4.4 Welfare ................................................... 49  
2.5 Policy Analysis .............................................. 54  
  2.5.1 Exchange Rate Targeting Rules ......................... 54  
  2.5.2 Producer Price Targeting Rule .......................... 56  
  2.5.3 Financial Integration and Monetary Policy ............. 57  
2.6 Conclusion ................................................... 60  

## 3 Explaining Saving Behavior  
3.1 Introduction ................................................... 69  
3.2 The Model ..................................................... 71  
3.3 Equilibrium and Solution ..................................... 74  
3.4 Data and Calibration ......................................... 76  
3.5 Results ........................................................ 78  
3.6 Conclusion .................................................... 80  

Bibliography ....................................................... 93
Appendices

A Appendix for Chapter 1 ........................................ 100  
A.1 First-Order Approximation .................................. 100  
A.2 Asset Returns, Exchange Rate, and Terms of Trade .......... 101  
A.3 Log-linearized Sticky Wage Model .......................... 103  

B Appendix for Chapter 2 ........................................ 105  
B.1 A Symmetric Steady State .................................. 105  
B.2 First-Order Approximation .................................. 105  
B.3 Proof of Proposition 1 ....................................... 107  
B.4 Proof of $E(\hat{L}) + \frac{1}{2}E(\hat{L}^2) = 0 + O(\epsilon^3)$ .............. 108  
B.5 Computing the Consumption Equivalent Welfare Measure .... 109  

C Statement of Co-Authorship ................................. 111
List of Tables

1.1 Optimal Portfolio Holdings with Flexible Prices . . . . . . . . 14
1.2 Optimal Portfolio Holdings with Sticky Prices . . . . . . . . 15
1.3 Share of Domestically Owned Home Equities with Sticky Prices
    and Flexible Wages . . . . . . . . . . . . . . . . . . . . . . . 18
1.4 Optimal Portfolio Holdings with Sticky Wages . . . . . . . . 20
1.5 Share of Domestically Owned Home Equities with Sticky Prices
    and Sticky Wages . . . . . . . . . . . . . . . . . . . . . . . . . 21
2.1 Targeting Regimes . . . . . . . . . . . . . . . . . . . . . . . 41
2.2 Second Moments and Welfare with Money Targeting Rules . 50
2.3 Second Moments and Welfare with Exchange Rate Targeting
    Rules . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 55
2.4 Second Moments and Welfare with Price Targeting Rules . . 57
2.5 Average Inflation and Central Bank Characteristics . . . . . 59
3.1 Steady State Values of the Exogenous Drivers . . . . . . . . . 77
List of Figures

1.1 Share of Domestically Owned Home Equities with Flexible Wage and Linear Disutility in Labor ........................................ 24
1.2 Share of Domestically Owned Home Equities with Flexible Wage and Quadratic Disutility in Labor .............................. 25
1.3 Share of Domestically Owned Home Equities with Sticky Wage and Cut-off Value 0.9 ................................. 26
1.4 Share of Domestically Owned Home Equities with Sticky Wage and Cut-off Value 0.75 ........................................ 27

2.1 International Financial Integration: Industrial Group and Developing Countries Group, 1970-2004 .......................... 62
2.2 International Financial Integration: Explicit Inflation Targeting Countries ............................................................. 63
2.3 International Financial Integration: Implicit Inflation Targeting Countries ............................................................ 64
2.4 Second Moments and Welfare with Money Targeting Rules ................................................................. 65
2.5 Exchange Rate Response Coefficients with Incomplete Markets ............................................................. 66
2.6 Consumption Equivalent Measures with Money Targeting Rules ..................................................... 67
2.7 Welfare Comparison ................................................................. 68

3.1 U.S. Saving and Investment - Closed Economy ........................................ 82
3.2 U.S. Saving and Investment - Benchmark ........................................ 83
3.3 U.S. Saving and Investment - Only TFP ........................................ 84
3.4 U.S. Saving and Investment - All Except TFP ................................ 85
3.5 U.S. Saving and Investment - Only Government Expenditure .... 86
3.6 U.S. Saving and Investment - Only Population Growth ................ 87
List of Figures

3.7 U.S. Saving and Investment - Only Depreciation Rate . . . . 88
3.8 U.S. Saving and Investment - Only Capital Income Taxes . . 89
3.9 U.S. Saving and Investment - Only Labor Income Taxes . . . 90
3.10 Net Investment Rates vs TFPF Growth Rates - Benchmark . 91
3.11 Net Saving Rates vs TFPF Ratios - Benchmark . . . . . . . 92
Acknowledgments

I am extremely grateful to my supervisor, Prof. Michael B. Devereux for advice, guidance and encouragement.

I am also greatly indebted to other members of my supervisory committee, Profs. Amartya Lahiri, Henry Siu and Viktoria Hnatkovska for research direction and discussion.

I would like to thank Profs. Paul Beaudry and Anji Redish for valuable suggestions and discussions at various stages of this dissertation. Sincere thanks are also extended to my colleagues, faculty and staff members at UBC for their support.

I thank the seminar participants at Wilfrid Laurier University, Ryerson University, Louisiana State University, West Virginia University, the SOEGW II conference, and the 2006, 2007 and 2008 meetings of the Canadian Economics Association for helpful comments.

I am solely responsible for any error or misinterpretation.
Dedication

To my parents and Jianfeng.
Chapter 1

Equity Home Bias and Nominal Rigidity

1.1 Introduction

Typical investors hold too little of their wealth in foreign assets relative to the predictions of standard financial and macroeconomic theory. According to French and Poterba (1991) [29] and Tesar and Werner (1995) [67], the percentages of aggregate stock-market wealth invested in domestic equities at the beginning of the 1990s were well above 90% for the US and Japan, and around 80% in the UK and Germany. Based on the portfolio data from Kraay et al (2005) [44], Kollmann (2005) [43] shows that the average locally owned capital share for 17 OECD countries is 91% in 1997. More recently, Heathcote and Perri (2007) [38] report that foreign assets accounted for only around 25% of the total value of the assets owned by the U.S. residents over the period 1990-2004. This widespread lack of diversification across countries, named equity home bias, has become a major empirical puzzle in international finance.

In a complete financial market environment, Engel and Matsumoto (2005) [24] find that the presence of nominal price rigidity can help explain the equity home bias puzzle, because it generates a negative correlation between the labor income and the profit of domestic firms with respect to productivity shocks. Within a similar open economy macro framework, this paper examines the impact of two additional frictions on the optimal equity portfolios. First, incomplete financial market. Second, nominal wage rigidity.

We find that in an incomplete market environment, the labor income
Chapter 1. Equity Home Bias and Nominal Rigidity

and the profit of domestic firms are negatively correlated in response to not only productivity shocks but also monetary shocks.\(^1\) Therefore, the equilibrium portfolios require an aggressive investment position in domestic equities. This “super home bias” result weakens the ability of sticky price alone in describing the observed equity holdings. Furthermore, introducing sticky wage can help match the data. The reason is that the labor income and the profit of domestic firms become positively correlated with respect to monetary shocks when wages are pre-set. Hence, it is optimal for households to hold some positive amount of foreign equities. With incomplete financial markets and nominal rigidities in both the goods price and the wage rate, this model predicts home bias for a wide range of parameterization that are often used in the macro literature.

There is a large literature that seeks resolutions to the equity home bias puzzle.\(^2\) Potential explanations range from barriers to international capital movements to frictions that justify the observed portfolios as optimal risk management decisions. This paper builds on one thread of the study that focuses on the importance of hedging against non-traded labor income risk.

Baxter and Jermann (1997) \(^4\) show that returns to human capital are positively correlated with returns to domestic equities but not with returns to foreign equities. As the labor income risk is non-diversifiable and the labor income accounts for more than half of the total income, investors should take large short positions in domestic assets. The implied equity portfolios are more foreign-weighted than what a classical endowment economy, such as Lucas (1982) \(^5\), would predict. Indeed, as long as non-traded labor income is more correlated with the domestic stock market than with the foreign stock market, the puzzle becomes even worse. In the flexible price case of our model, we corroborate the above results and show that they are identical in the sense that there is perfect pooling (i.e. each country receives half of the world output) eventually.

Jermann (2002) \(^41\) characterizes optimal international portfolios in a

---

\(^1\) We assume money supply shocks in this paper. However, results are identical if assume money demand shocks.

\(^2\) See Lewis (1999) \(^49\) for a comprehensive survey on this literature.
multi-country general equilibrium model with endogenous labor-leisure choice and with preferences that are nonseparable over consumption and leisure. The return on human capital and the return on domestic equity are still positively correlated. However, with consumption and leisure being substitutes, consumption is highly valued in periods when work effort is high. Therefore, a domestic claim provides the right hedge. Heathcote and Perri (2007) [38] studies a two-goods general equilibrium model with investment. In their framework, each country specializes in production of a final good that uses both local and imported intermediate inputs. Following a positive domestic productivity shock, home output and demand for labor increase. Home investment goes up as well, which reduces the dividend paid on home equities. Thus, the labor income is negatively correlated with the return to domestic equities. Home bias arises because domestic stocks make a good hedge against non-diversifiable labor income risk.

The empirical evidence on the correlation of returns to human capital and domestic equities is mixed. Baxter and Jermann (1997) [4] use annual OECD data for Japan, Germany, UK, and US from 1960 to 1993, and find that human capital returns are highly correlated with domestic capital returns. However, Bottazzi, Pesenti, and van Wincoop (1996) [7] use slightly different measures based on annual OECD data from 1970 to 1992, and find that human capital returns are negatively correlated with domestic capital returns for most OECD countries except US. Therefore, it is not clear whether home bias is puzzling or not by just looking at the unconditional correlation. As discussed above, what important here is the conditional correlation between returns to human capital and returns to domestic capital. Gali (1999) [32] and Gali and Rabanal (2004) [33] find that the conditional correlation between labour hours and productivity is negative in response to technology shocks, while the unconditional correlation is positive. These results are further confirmed by Francis and Ramey (2005a, 2005b) [27] [28] and Rotemberg (2003) [58].

This paper is also related to the research that focus on the importance of non-traded risk due to non-traded consumption goods. Earlier papers in this literature see Stockman and Dellas (1989) [62], Tesar (1993)
Chapter 1. Equity Home Bias and Nominal Rigidity

[43] generates portfolio home bias in an endowment economy with home bias in consumption and complete markets. Hnatkovska (2005) [39] shows that equity home bias can arise naturally in the presence of non-diversifiable non-traded consumption risk, consumption home bias, and incomplete asset markets. She employs a numerical approximation method to solve for endogenous portfolio choices. This paper follows Devereux and Sutherland (2006a) [20] and uses a second-order approximation method to solve for the optimal portfolio composition.

The paper is organized as follows. Section 1.2 presents the model. Section 1.3 describes the solution to the model and compares the equilibria under different market configurations. Section 1.4 analyzes the effect of sticky wage. Sections 1.5 concludes.

1.2 The Model

The world is assumed to exist for a single period and to consist of two countries, which will be referred to as the home country and the foreign country. Each country is populated by agents who consume a basket of home and foreign produced goods. Each agent, using a linear technology in labor, is a monopoly producer of a particular differentiated product. The world population is normalized to have a measure of one. Home agents are indexed $h \in [0, \frac{1}{2}]$ and foreign agents are indexed $f \in [\frac{1}{2}, 1]$.

A fraction $\kappa$ of agents in each country set prices before the realization of shocks. They are contracted to meet demand at pre-fixed prices. Other agents in the economy can set prices after shocks are realized. All prices are assumed to be set in the currency of producers. Thus, there is full exchange rate pass-through to prices paid by consumers. Agents supply homogeneous labor.\(^4\) Prior to the realization of shocks, they can trade in a range of financial assets. The financial market structure and the payoff

\(^4\)We first look at the effect of nominal price rigidity. Nominal wage rigidity will be introduced later.
to each asset are defined in Section 2.3. Each country faces two types of shocks: productivity and money supply shocks.

The detailed structure of the home country is described below. The foreign country has an identical structure. Where appropriate, foreign variables are denoted with an asterisk.

1.2.1 Consumers

All agents in the home country have utility functions of the same form

\[ U = \frac{C^{1-\rho}}{1-\rho} + \chi \log \frac{M}{P} - \eta \frac{L^{1+\psi}}{1+\psi} \quad (1.1) \]

where \( C \) is the consumption index, defined across all home and foreign goods, \( M \) denotes the end-of-period nominal money holding, \( P \) is the consumer price index (CPI), \( L \) is the labor supply, \( \rho (\rho \geq 0) \) is the coefficient of relative risk aversion, \( \chi \) is the coefficient of real balance, \( \eta \) is the coefficient of labor supply, and \( \psi (\psi \geq 0) \) is the elasticity of labor supply.

There are two stages to the household decision problem. Before shocks are realized, households choose portfolio positions out of available assets to maximize expected utility, \( E[U(C, M/P, L)] \), subject to

\[ \sum_{k=1}^{n} \alpha_k = 0 \quad (1.2) \]

where \( \alpha_k \) represents the real holding of asset \( k \), and \( n \) is the total number of assets. All real variables in this paper are defined in terms of home consumption basket. Each country starts with zero net wealth.

After shocks are realized, households choose consumption, labor supply, and money balances, in order to maximize ex-post utility, \( U(C, M/P, L) \), subject to

\[ M + PC = M_0 + P_H Y_H + P \sum_{k=1}^{n} \alpha_k r_k + T \quad (1.3) \]

where \( M_0 \) is the initial nominal money holding, \( P_H \) is the aggregate price
Chapter 1. Equity Home Bias and Nominal Rigidity

of home produced goods, \( r_k \) is the real aggregate rate of return on asset \( k \), and \( T \) is a lump-sum government transfer. \( Y_H \) is the world demand for aggregate home produced goods

\[
Y_H = \frac{1}{2} \left( \frac{P_H}{P} \right)^{-\theta} (C + C^*) \tag{1.4}
\]

where \( \theta \) is the elasticity of substitution between home and foreign goods.

Firms’ revenues are used to pay wages and profits

\[
P_H Y_H = wL + \Pi \tag{1.5}
\]

where \( w \) and \( \Pi \) denote the wage and the profit (dividend), respectively.

Here, home agents first receive all profits from domestic firms. Then, if an international equity market exists, claims to home profits may be transferred to foreign consumers via trade in equity shares.\(^5\)

Moreover, we define \( Y \) as the home aggregate real production income

\[
PY = P_H Y_H \tag{1.6}
\]

Combined with the government budget constraint

\[
M - M_0 = T \tag{1.7}
\]

we can rewrite home household’s budget constraint (1.3) in real terms

\[
C = Y + \sum_{k=1}^{n} \alpha_k r_k \tag{1.8}
\]

\( M \) is an i.i.d. stochastic money supply shock with \( E(\log M) = 0, \text{Var}(\log M) = \sigma_M^2 \), and \( \log M \in [-\epsilon, \epsilon] \).

\(^5\)Alternatively, it can be assumed that profits proceed directly to shareholders. There is no fundamental difference between the two modeling approaches.
The consumption index $C$ for home agents is defined as

$$C = \left(\frac{1}{2}\right)^{\frac{1}{\phi-1}} \left( C_H^{\frac{\phi-1}{\phi}} + C_F^{\frac{\phi-1}{\phi}} \right)^{\frac{\phi}{\phi-1}} \tag{1.9}$$

$C_H$ and $C_F$ are indices of home and foreign produced goods

$$C_H = \left[ \left(\frac{1}{2}\right)^{\frac{1}{\phi}} \int_0^{\frac{1}{2}} C_H(h)^{\frac{\phi-1}{\phi}} dh \right]^{\frac{\phi}{\phi-1}} \tag{1.10}$$

$$C_F = \left[ \left(\frac{1}{2}\right)^{\frac{1}{\phi}} \int_{\frac{1}{2}}^1 C_F(f)^{\frac{\phi-1}{\phi}} df \right]^{\frac{\phi}{\phi-1}} \tag{1.11}$$

where $\phi$ ($\phi > 1$) is the elasticity of substitution between individual home (or foreign) goods.

The aggregate consumer price index for home households is

$$P = \left(\frac{1}{2}\right)^{\frac{1}{1-\theta}} \left( P_H^{1-\theta} + P_F^{1-\theta} \right)^{\frac{1}{1-\theta}} \tag{1.12}$$

where $P_H$ and $P_F$ are the price indices for home and foreign produced goods, respectively

$$P_H = \left[ 2 \int_0^{\frac{1}{2}} P_H(h)^{1-\phi} dh \right]^{\frac{1}{1-\phi}}, \quad P_F = \left[ 2 \int_{\frac{1}{2}}^1 P_F(f)^{1-\phi} df \right]^{\frac{1}{1-\phi}} \tag{1.13}$$

The law of one price implies that $P_H(h) = P_H^* S$ and $P_F(f) = P_F^* S$ for all $h$ and $f$. An asterisk indicates that the price is in foreign currency and $S$ is the nominal exchange rate defined as the domestic price of foreign currency. Because there is no home bias in consumption, purchasing power parity (PPP) holds, i.e. $P = SP^*$.

Given prices and the total consumption $C$, home consumers’ optimal demands for home and foreign goods are

$$C_H = \frac{1}{2} \left( \frac{P_H}{P} \right)^{-\theta} C, \quad C_F = \frac{1}{2} \left( \frac{P_F}{P} \right)^{-\theta} C \tag{1.14}$$
Chapter 1. Equity Home Bias and Nominal Rigidity

\[ C_H(h) = 2 \left[ \frac{P_H(h)}{P_H(h)} \right]^{-\phi} C_H, \quad C_F(f) = 2 \left[ \frac{P_F(f)}{P_F} \right]^{-\phi} C_F \quad (1.15) \]

The remaining first order conditions are

\[
\begin{align*}
E \left( r_1 C^{-\rho} \right) &= E \left( r_n C^{-\rho} \right) \\
E \left( r_2 C^{-\rho} \right) &= E \left( r_n C^{-\rho} \right) \\
&\vdots \\
E \left( r_{n-1} C^{-\rho} \right) &= E \left( r_n C^{-\rho} \right) \\
\eta \psi &= \frac{w C^{-\rho}}{P} \quad (1.17) \\
\chi &= \frac{C^{-\rho}}{P} \quad (1.18)
\end{align*}
\]

Equation (1.16) indicates that portfolio choices are optimal only when the expected returns on all assets are equalized in terms of utility. Equations (1.17) and (1.18) are the standard intra-temporal labor-leisure choice function and the money demand function.

1.2.2 Firms

Firms engage in monopolistic competition. Each produces specific goods indexed by \( h \) with a linear technology in labor \( Y_H(h) = AL(h) \). \( A \) is an i.i.d. stochastic technology shock with \( E(logA) = 0, \ Var(logA) = \sigma_A^2, \) and \( \logA \in [-\epsilon, \epsilon] \). By assumption, a fraction \( \kappa \) of firms have to set prices in advance and the rest can set prices after shocks are realized.

The profit maximization problem of a pre-set price firm \( i \) is

\[
Max \ E\{D(i) \left[ P_{pre,H}(i) - \frac{w}{A} \right] \left[ Y_{pre,H}(i) + Y_{pre,H}^*(i) \right] \}
\]

where \( D(i) \) is the stochastic discount factor for firm \( i \), \( Y_{pre,H}(i) \) and \( Y_{pre,H}^*(i) \) are the demand for good \( i \) from the home and foreign markets

\[
Y_{pre,H}(i) = \left[ \frac{P_{pre,H}(i)}{P_H} \right]^{-\phi} \left( \frac{P_H}{P} \right)^{-\theta} C \quad (1.19)
\]
Chapter 1. Equity Home Bias and Nominal Rigidity

\[ Y_{pre,H}^*(i) = \left[ \frac{P_{pre,H}^*(i)}{P_H^*} \right]^{-\phi} \left( \frac{P_H^*}{P^{*\text{e}}} \right)^{-\theta} C^* \quad (1.20) \]

Because firms of each type are all alike, they will set identical prices in equilibrium. Hence, the optimal pre-set price of home goods is

\[ P_{pre,H} = \frac{\phi}{\phi - 1} \frac{E[Dx^H_X]}{E[D^H]} \quad (1.21) \]

\( X_H \) represents the demand for home produced goods

\[ X_H = P^\phi_H \left( \frac{P_H}{P} \right)^{-\theta} (C + C^*) \]

where we have applied the law of one price and PPP in CPI.

The profit maximization problem of a flexible price firm \( j \) is

\[ \text{Max } D(j) \left[ P_{flx,H}(j) - \frac{w}{A} \right] \left[ Y_{flx,H}(j) + Y_{flx,H}^*(j) \right] \]

where \( D(j) \) is the stochastic discount factor for firm \( j \), \( Y_{flx,H}(j) \) and \( Y_{flx,H}^*(j) \) are the demand for good \( j \) from the home and foreign markets

\[ Y_{flx,H}(j) = \left[ \frac{P_{flx,H}(j)}{P_H^*} \right]^{-\phi} \left( \frac{P_H}{P} \right)^{-\theta} C \quad (1.22) \]

\[ Y_{flx,H}^*(j) = \left[ \frac{P_{flx,H}^*(j)}{P_H^*} \right]^{-\phi} \left( \frac{P_H}{P^{*\text{e}}} \right)^{-\theta} C^* \quad (1.23) \]

The optimal flexible price of home goods is

\[ P_{flx,H} = \frac{\phi}{\phi - 1} \frac{w}{A} \quad (1.24) \]

The price index for home produced goods can be rewritten as

\[ P_H = \left[ \kappa P_{pre,H}^{1-\phi} + (1 - \kappa) P_{flx,H}^{1-\phi} \right]^{1-\phi} \quad (1.25) \]
1.2.3 Financial Sector

This paper examines two different asset market configurations: (1) Bond and Equity Economy (FBE), in which home and foreign nominal bonds and equities can be traded internationally; (2) Equity Economy (FE), in which only home and foreign equities are allowed for trade. Whether each financial market is complete depends on the nature of the rest of the model.

Nominal bonds represent a claim on a unit of currency. Equities represent a claim on aggregate profits. The real aggregate rate of return on each asset is defined as following

\[ r_B = \frac{1}{q_B P} \]  
\[ r_{B*} = \frac{Q}{q_{B*} P^*} \]  
\[ r_E = \frac{\Pi}{q_E P} \]  
\[ r_{E*} = \frac{Q \Pi^*}{q_{E*} P^*} \]

where \( q_k \) is the real price of asset \( k \) and \( Q \) is the real exchange rate, i.e. \( Q = \frac{S P^*}{P} \).

1.2.4 Market Clearing

The goods market clearing condition is

\[ AL = \frac{1}{2} \left( \frac{P_H}{P} \right)^{-\theta} (C + C^*) \]  

The asset market clearing condition is

\[ \alpha_k = -\alpha_k^*, \quad k = 1, 2, \ldots, n \]

1.2.5 Equilibrium

The equilibrium comprises a set of prices, \( P, P^*, P_H, P_F, P_{pre,H}, P_{pre,F}, P_{flx,H}, P_{flx,F}, w, w^*, r_k, S, \) and a set of quantities \( C, C^*, L, L^*, \Pi, \Pi^* \).
Chapter 1. Equity Home Bias and Nominal Rigidity

\( Y_H, Y_F^*, Y, Y^*, \alpha_k, \alpha_k^*, M, M^* \), which solves a system of equations (1.4)-(1.6), (1.12), (1.16)-(1.18), (1.21), (1.24)-(1.25), (1.30), and their foreign counterparts, as well as (1.8), (1.26)-(1.29), and (1.31), given productivity and money supply shocks, \( A, A^*, M, \) and \( M^* \).

1.3 Portfolio Decisions

1.3.1 Solution Method

Using the method developed by Devereux and Sutherland (2006a) [20], the equilibrium portfolio choices are solved to a second-order accuracy. The second-order approximation of home portfolio selection equations (1.16) around the non-stochastic steady state are given by

\[
\begin{align*}
E[(\hat{r}_1 - \hat{r}_n) + \frac{1}{2}(\hat{r}_1^2 - \hat{r}_n^2) - \rho \hat{C}(\hat{r}_1 - \hat{r}_n)] &= 0 + O(\epsilon^3) \\
E[(\hat{r}_2 - \hat{r}_n) + \frac{1}{2}(\hat{r}_2^2 - \hat{r}_n^2) - \rho \hat{C}(\hat{r}_2 - \hat{r}_n)] &= 0 + O(\epsilon^3) \\
&\vdots \\
E[(\hat{r}_{n-1} - \hat{r}_n) + \frac{1}{2}(\hat{r}_{n-1}^2 - \hat{r}_n^2) - \rho \hat{C}(\hat{r}_{n-1} - \hat{r}_n)] &= 0 + O(\epsilon^3)
\end{align*}
\]

Because shocks are symmetrically distributed in the interval \([-\epsilon, \epsilon]\), \( O(\epsilon^n) \) represents residuals of an equation approximated to order \( n - 1 \).

Foreign agents face a set of portfolio selection equations similar to (1.16)

\[
\begin{align*}
E\left(\frac{r_1}{QC^\ast}\right) &= E\left(\frac{r_n}{QC^\ast}\right) \\
E\left(\frac{r_2}{QC^\ast}\right) &= E\left(\frac{r_n}{QC^\ast}\right) \\
&\vdots \\
E\left(\frac{r_{n-1}}{QC^\ast}\right) &= E\left(\frac{r_n}{QC^\ast}\right)
\end{align*}
\]

\footnote{By Walras’ Law, there is one equation redundant in the system. The foreign agent’s budget constraint is dropped.}

\footnote{Hereafter, \( \hat{x} = \log(X) - \log(\bar{X}) \).}
Chapter 1. Equity Home Bias and Nominal Rigidity

whose second-order approximation is given by

\[ E[(\hat{r}_1 - \hat{r}_n) + \frac{1}{2}(\hat{r}_1^2 - \hat{r}_n^2) - \rho \hat{C}^*(\hat{r}_1 - \hat{r}_n)] = 0 + O(\epsilon^3) \]

\[ E[(\hat{r}_2 - \hat{r}_n) + \frac{1}{2}(\hat{r}_2^2 - \hat{r}_n^2) - \rho \hat{C}^*(\hat{r}_2 - \hat{r}_n)] = 0 + O(\epsilon^3) \]

\[ \vdots \] (1.34)

\[ E[(\hat{r}_{n-1} - \hat{r}_n) + \frac{1}{2}(\hat{r}_{n-1}^2 - \hat{r}_n^2) - \rho \hat{C}^*(\hat{r}_{n-1} - \hat{r}_n)] = 0 + O(\epsilon^3) \]

Note that PPP has been applied here. By subtracting equations (1.32) from (1.34), the set of equations to solve for equilibrium portfolios is obtained

\[ E[\rho(\hat{C} - \hat{C}^*)\hat{r}_{x}] = 0 + O(\epsilon^3) \] (1.35)

It is written in a vector form with \( \hat{r}_{x}' = [\hat{r}_1 - \hat{r}_n, \hat{r}_2 - \hat{r}_n, \ldots, \hat{r}_{n-1} - \hat{r}_n] \).

Condition (1.35) contains only the second moments of endogenous variables, indicating that solving the portfolio choice to a second-order accuracy only requires the first-order solution of the non-portfolio part of the model. This is because second-order accurate second moments can be computed from first-order solutions for realized variables. Finally, firms’ discount factors do not appear in equation (1.35) or any other equilibrium condition up to the first-order. Hence, they do not affect the solution of optimal portfolio choices. The log-linearization of the model is presented in Appendix A.1.

1.3.2 Risk Sharing

Use the log-linearized home CPI,

\[ \hat{P} = \frac{1}{2}(\hat{P}_H + \hat{P}_F^* + \hat{S}) + O(\epsilon^2) \] (1.36)

we can express home real GDP as

\[ \hat{Y} = \frac{1-\theta}{2}(\hat{P}_H - \hat{P}_F^* - \hat{S}) + \frac{1}{2}(\hat{C} + \hat{C}^*) + O(\epsilon^2) \]
Chapter 1. Equity Home Bias and Nominal Rigidity

\[ \frac{1 - \theta}{2} \hat{\tau} + \frac{1}{2} \left( \hat{C} + \hat{C}^* \right) + O(\epsilon^2) \]  

(1.37)

where \( \tau \) is the home country’s terms of trade — defined as the price of exports relative to the price of imports. Because there is no aggregate uncertainty at the world level, country-specific income risks all come from the terms of trade fluctuations.

Moreover, the log-linearized home budget constraint is

\[ \hat{C} = \hat{Y} + \tilde{\alpha}' \hat{r}_x + O(\epsilon^2) \]  

(1.38)

where \( \tilde{\alpha} = \frac{\bar{\alpha}}{\bar{Y}} \). Hence, the consumption difference between home and foreign countries can be written as

\[ \frac{1}{2} (\hat{C} - \hat{C}^*) = \frac{1 - \theta}{2} \hat{\tau} + \tilde{\alpha}' \hat{r}_x + O(\epsilon^2) \]  

(1.39)

Clearly, households’ consumption risks originate from uncertainties in their real income, which are further caused by variations in the terms of trade. For instance, home country’s terms of trade deteriorate, owing to certain fundamental shocks, \( \hat{\tau} < 0 \). Home goods become cheaper relative to foreign goods. Given that home and foreign goods are substitutes, the world demand shifts towards home goods. Home output increases, \( \hat{Y}_H > 0 \), which is called the expenditure switching effect. However, home goods are sold at a relatively lower price and foreign goods cost relatively more. Whether home GDP rises or falls in real terms depends on the strength of this substitution across countries. If the expenditure switching effect is strong enough (\( \theta > 1 \)), home real GDP increases. The exact opposite takes place in the foreign country. As a result, the portfolio decision is essentially about how to optimally hedge against the terms of trade fluctuations. Agents can diversify away at least part of the consumption risks by trading assets across borders — as long as the returns on these assets are somehow correlated with the terms of trade.

Now it is possible to state the optimal portfolio choices under each financial market configuration. The model is entirely symmetric, which implies
that in the FB economy, agents in both countries will have bond holdings that sum to zero, and in the FBE economy, their equity holdings and bond holdings will separately sum to zero. Thus, for the home country, we have $\tilde{\alpha}_{FE,E} + \tilde{\alpha}_{FE,E} = 0$ in the FE economy, and $\tilde{\alpha}_{FE,B} + \tilde{\alpha}_{FE,B} = 0$, $\tilde{\alpha}_{FE,E} + \tilde{\alpha}_{FE,E} = 0$ in the FBE economy.

As the aim is to understand the effect of nominal price rigidity on the equilibrium equity holdings, we will focus on the two extremes, in which goods prices are either completely flexible ($\kappa = 0$) or completely sticky ($\kappa = 1$). Any intermediate case with $0 < \kappa < 1$ can be easily inferred from these two scenarios.

1.3.3 Flexible Price Case: $\kappa = 0$

Table 1.1 describes the optimal portfolio holdings in the FE and FBE economies when goods prices are fully flexible. The equilibrium asset holdings are identical between the two economies. In this case, the excess return of foreign equities relative to home equities is $\hat{r}_{E^*} - \hat{r}_E = (\theta - 1)\hat{\tau} + O(\epsilon^2)$.

As long as equities can be traded internationally, households achieve perfect consumption risk sharing.

<table>
<thead>
<tr>
<th></th>
<th>FE</th>
<th>FBE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{\alpha}_{FE,E^*}$</td>
<td>$\frac{1}{2}$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\tilde{\alpha}_{FBE,B^*}$</td>
<td>$0$</td>
<td>$\frac{1}{2}$</td>
</tr>
</tbody>
</table>

Here, money is neutral. Money supply shocks have no real effect on the economy, as changes in the prices and the exchange rate cancel out completely. The terms of trade respond only to productivity shocks. There is no need to trade nominal bonds. With the optimal equity positions, households essentially receive half of each country’s output, which corresponds to the “full diversification” prediction by Lucas (1982) [50].

---

8Detailed derivation see Appendix A.2.
Chapter 1. Equity Home Bias and Nominal Rigidity

The share of domestically owned home equities is given by

\[ \delta_E = 1 - \frac{\bar{E}^*}{q_E} = 1 - \frac{\bar{E}^*}{\Pi \bar{P} \bar{Y}} \cdot \zeta \]

\[ \zeta \] is the share of labor income in total output at the non-stochastic steady state. When prices are fully flexible, \( \delta_E = \frac{1-2\zeta}{2(1-\zeta)} \). As labor income generally accounts for about two thirds of the total income, \( \delta_E = -\frac{1}{2} < 0 \). In other words, home households should not only diversify their portfolios and hold foreign equities, but indeed take a short position in their domestic equities. This reproduces the well-known Baxter and Jermann (1997) [4] result that households should aggressively invest in foreign equities to hedge their non-tradable labor income from productivity shocks, because the labor income and the profit of domestic firms are highly correlated.

1.3.4 Sticky Price Case: \( \kappa = 1 \)

Table 1.2 describes the optimal portfolio holdings in the FE and FBE economies when goods prices are all pre-set. The two economies accomplish different degrees of consumption risk sharing. To understand the intuition behind the above optimal portfolios, we need to find out how the economy responds to various shocks in autarky.

As all prices are pre-set and output is demand-determined, productivity shocks have no effect on firm revenue. The only changes occur are associated with the allocation between labor income and profit. For example, if home firms experience a positive productivity shock, they need less labor to produce the same quantity of goods. Labor income decreases, but firm profit increases at the same time. If home agents hold one hundred percent of their own firms, their income as well as consumption are unaffected. In other words, the default equity position, or a complete home bias in equities,
Chapter 1. Equity Home Bias and Nominal Rigidity

provides a perfect hedge against productivity shocks. Hence, money supply shocks are the only source of income uncertainties left in the economy.

FBE Economy

With fixed goods prices, the terms of trade move in the opposite direction of the exchange rate up to first-order. The excess return of foreign bonds relative to home bonds can be expressed as \( \hat{r}_{B^*} - \hat{r}_B = -\hat{\tau} + O(\epsilon^2) \). Households can fully insure themselves against money supply shocks by holding the right amount of nominal bonds (\( \hat{\alpha}_{FBE,B^*} = \frac{1-\theta}{2} \)). For instance, if home experiences a positive money supply shock, home currency would depreciate, causing home terms of trade to deteriorate. Home real GDP increases when \( \theta > 1 \). Home households should lend in the home currency denominated bond and should borrow in the foreign currency denominated bond, \( \hat{\alpha}_{FBE,B^*} < 0 \). In this way, the gain in terms of real production income is balanced by a negative payment from bond holdings (when home currency depreciates). In short, home and foreign nominal bonds provide a perfect hedge against money supply shocks. As long as they are allowed for trade, the financial market is complete.

Here, we confirm the Engel and Matsumoto (2005) [24] result that in the presence of sticky prices, the returns to workers and those to firm owners become negatively correlated in response to productivity shocks, leading to a home bias in investors’ portfolios. However, this result also relies on the fact that consumption risk sharing is perfect. It may no longer be true when the financial market is incomplete.

FE Economy

Generally speaking, home and foreign equities alone cannot fully hedge against productivity and money supply shocks at the same time. The portfolio decisions become a lot more complicated as only two individual assets are available to deal with four competing hedging tasks. The exact equity holdings depend on the relative size between the two types of shocks and on
the value of structural parameters.

Following the literature, we set the share of labor income equal to two thirds \( \zeta = \frac{2}{3} \). Let the shocks be one percent of their steady state level \( \sigma_A^2 = \sigma_M^2 = 0.0001 \). Figure 1.1 shows five subsets of the share of domestically owned home equities \( \delta_E \) when the disutility of labor is assumed to be linear \( \psi = 0 \) and the relative risk aversion coefficient \( \rho \) and the elasticity of substitution between home and foreign goods \( \theta \) are both set to be within the range of \([0, 5]\).

The first thing to note is that when \( \theta > 1 \), home households take a long position in domestic equities \( \delta_E > 1 \). The intuition is as following. Productivity shocks are hedged perfectly if households hold one hundred percent of their domestic equities. Therefore, how to hedge against money supply shocks using home and foreign equities in the FE economy determines the equilibrium portfolios.

In response to a positive home money supply shock, home currency depreciates. Demand for home goods goes up. The demand for labor and the wage rate increases at home, so is the firm’s revenue when \( \theta > 1 \). Given \( \zeta = \frac{2}{3} \), the increase in labor cost is so significant that the dividend moves in the opposite direction to the firm’s revenue. Hence, it is optimal for home households to aggressively invest in their domestic equities \( \delta_E > 1 \) or take a short position in foreign equities \( \tilde{\alpha}_{FE,E*} < 0 \). In this way, the gain in terms of real production income is balanced by a negative payment from equity holdings.

Now let us look at the case when \( \theta < 1 \). Again, suppose the home country experiences a positive money supply shock, home currency depreciates. Home goods become relatively cheaper than foreign goods. Demand for home goods goes up, and so are the demand for labor and the wage rate at home. If \( \theta < 1 \), the expenditure switching effect is weak relative to the change in price. Thus, home real GDP decreases. The dividend on home equities drops too. It is optimal for home households to invest in foreign equities. Moreover, the smaller the expenditure switching effect \( \theta \) is, the higher the share of foreign equities in home households’ portfolios.

Figure 1.2 repeats the above exercise, but assumes the disutility of labor
Chapter 1. Equity Home Bias and Nominal Rigidity

is in a quadratic form ($\psi = 1$). It is easy to see that the results are not sensitive to this modification.

The elasticity of substitution between home and foreign goods ($\theta$) is a key parameter that determines the equilibrium equity holdings. It has a wide range of estimates in the international economics literature, ranging from 1.2 to 21.4 as reviewed by Obstfeld and Rogoff (2000a) [53]. The number often used in the macroeconomics study is between 1 and 2, following Backus, Kehoe and Kydland (1994) [3] and Chari, Kehoe and McGrattan (2002) [10]. A value smaller than unity is seldom used except Stockman and Tesar (1995) [63] set the elasticity of substitution between traded and non-traded goods equal to 0.44 and Heathcote and Perri (2002) [37] set the elasticity of substitution between intermediate goods equal to 0.9. Table 1.3 describes the equilibrium share of domestically owned home equities in the FE economy with $\zeta = \frac{2}{3}$, $\sigma_A^2 = \sigma_M^2 = 0.0001$ and $\rho = 1.5$.

Following the most typical calibration in the macro literature, we find that the labor income and the return on domestic equities are negatively correlated with respect to both the productivity shocks and the money supply shocks. Therefore, the optimal portfolios involve a substantial short position in foreign equities (see the last column in Table 1.3). This “super home bias” result casts doubt on the ability of sticky price in characterizing the observed equity portfolios.

Wage adjustments are likely to have strong implications for the correlation between the returns to workers and the returns to firm owners. In fact, there are more extensive empirical evidence for sluggish wages than that for

<table>
<thead>
<tr>
<th>$\delta_{FE,E}$</th>
<th>$\theta = 0.44$</th>
<th>$\theta = 0.9$</th>
<th>$\theta = 1.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi = 0$</td>
<td>0.7445</td>
<td>0.9624</td>
<td>1.1957</td>
</tr>
<tr>
<td>$\psi = 1$</td>
<td>0.8284</td>
<td>0.9805</td>
<td>1.0818</td>
</tr>
</tbody>
</table>
Chapter 1. Equity Home Bias and Nominal Rigidity

sticky prices. We are going to examine the effect of nominal wage rigidity in the next section.

1.4 A Model with Sticky Wage

Following Obstfeld and Rogoff (2000b) [52], each worker acts like a monopolistic supplier of a distinctive variety of labor services. Workers set wages (in their domestic currency) before shocks are realized. Each firm uses workers of every type and the elasticity of substitution among varieties of labor is given by $\mu$. The other parts of the model are kept same as before.

The production function for aggregate home firms is

$$Y_H = AL = A \left[ \frac{1}{2 \mu} \int_0^1 \frac{1}{2} l(z) \frac{\mu-1}{\mu} dz \right]^{\frac{2}{\mu}}$$

which implies that their demand for agent $z$’s labor is

$$l(z) = \left[ \frac{w(z)}{w} \right]^{-\mu} L$$

Before shocks are realized, agent $z$ sets his wage to maximize his expected utility

$$E \left[ U(c(z), \frac{m(z)}{P}, l(z)) \right]$$

subject to

$$m(z) + P \sigma = m_0(z) + w(z)l(z) + \pi(z) + P \sum_{k=1}^n \alpha_k(z)r_k + t(z)$$

---

9See, for example, Altonji and Devereux (2000) [1], Fehr and Goette (2005) [26], and Goette et al (2007) [34].

10Because the real marginal consumption value of wage is not equal to the marginal disutility of working when wages are pre-set, only sufficiently small shocks are considered here to avoid a further discussion of voluntary participation constraint. This concern also applies to sticky goods prices, as firms may have to operate under negative profits if a big negative productivity shock hits the economy.
Chapter 1. Equity Home Bias and Nominal Rigidity

In a symmetric equilibrium, all agents will choose the same wage rate

\[ w = \frac{\eta\mu}{\mu - 1} E \left[ \frac{L^{1+\psi}}{LPC} \right] \]  

(1.42)

The wage equation (1.42) takes the place of the intratemporal labor-leisure choice equation (1.17) in the original set of equilibrium conditions. Table 1.4 describes the optimal portfolio holdings in the FE and FBE economies in the presence of nominal wage rigidities.

Table 1.4: Optimal Portfolio Holdings with Sticky Wages

<table>
<thead>
<tr>
<th>( \kappa = 0 ) :</th>
<th>( \mu_{FE,E}^* )</th>
<th>( \mu_{FBE,B}^* )</th>
<th>( \mu_{FBE,E}^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>FE</td>
<td>( \frac{1}{2} )</td>
<td>0</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>FBE</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \kappa = 1 ) :</th>
<th>( \mu_{FE,E}^* )</th>
<th>( \mu_{FBE,B}^* )</th>
<th>( \mu_{FBE,E}^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>FE</td>
<td>( \frac{1}{2} \frac{(\theta-1)^2(\zeta-1)^2\sigma^2}{\sigma^2_A + (\theta-1)^2(\zeta-1)^2\sigma^2_M} )</td>
<td>( \frac{1-\theta}{2} \sigma^2_A + (\theta-1)^2(\zeta-1)^2\sigma^2_M )</td>
<td>0</td>
</tr>
<tr>
<td>FBE</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Sticky Wage and Flexible Price Case

When wages are fixed but goods prices are flexible, the excess return of foreign equities relative to home equities is \( \hat{r}_{E}^* - \hat{r}_E = (\theta - 1)\tau + O(\epsilon^2) \). Therefore, the portfolio decisions are identical to the case where both wages and prices are flexible. Consumption risk sharing is perfect so long as home and foreign equities are allowed for trade.\(^{11}\)

\(^{11}\)Detailed derivation see Appendix A.3

\(^{12}\)One thing different here is that money has real effect on the economy now. In response to a positive home money supply shock, home currency depreciates but home goods prices are not adjusted due to fixed wage rates. Home terms of trade deteriorate and home real GDP becomes relatively higher than foreign real GDP. Although the equilibrium asset positions are not affected by this additional friction, the welfare changes. In fact, the welfare is lower because the two countries produce at different levels when they are
Chapter 1. Equity Home Bias and Nominal Rigidity

Sticky Wage and Sticky Price Case

As shown in Appendix A.3, the excess return of foreign bonds relative to home bonds can be expressed as \( \hat{r}_B - \hat{r}_B = -\hat{\tau} + O(\epsilon^2) \). Therefore, the presence of nominal wage rigidity has no effect on the optimal portfolios in the FBE economy. Households will still hold one hundred percent of their domestic equities to hedge against the productivity shocks, and borrow (lend) in foreign (home) nominal bonds to hedge against the money supply shocks when \( \theta > 1 \).

In the FE economy, home and foreign equities cannot simultaneously hedge against the productivity shocks and the money supply shocks. The consumption risk sharing is imperfect in this case. Once again, let \( \zeta = \frac{2}{3} \) and \( \sigma_A^2 = \sigma_M^2 = 0.0001 \). Table 1.5 describes the equilibrium share of domestically owned home equities when \( \rho = 1.5 \).

Table 1.5: Share of Domestically Owned Home Equities with Sticky Prices and Sticky Wages

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( \delta_{FE,E} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.44</td>
<td>0.5067</td>
</tr>
<tr>
<td>0.9</td>
<td>0.9956</td>
</tr>
<tr>
<td>1.5</td>
<td>0.9483</td>
</tr>
</tbody>
</table>

For \( \theta > 1 \), we no longer have the “super home bias” result as in the case with flexible wages. Recall that with fixed goods prices, households’ default equity positions provide a perfect hedge against the productivity shocks. What is important is how home and foreign equities are used to hedge against the money supply shocks. In response to a positive home money supply shock, home currency depreciates. Demand for home goods goes up, so are the demand for labor and the firm’s revenue (when \( \theta > 1 \)). Remember that nominal wages are fixed. So even the labor income accounts for two thirds of the total income, the increase in labor cost is not big enough to reduce home firms’ profits. The dividend on home equities is positively correlated with the labor income. Hence, it is optimal for home households equally productive.
to diversify their portfolios and invest in foreign equities ($\tilde{\alpha}_{FE,E^*} > 0$). In this way, the gain in terms of real production income is balanced by a negative payment from equity holdings.

Figure 1.3 shows five subsets of the share of domestically owned home equities ($\delta_E$) when the relative risk aversion coefficient ($\rho$) and the elasticity of substitution between home and foreign goods ($\theta$) are both set to be within the range of $[0, 5]$. For most values of $\theta \in [0.8, 2]$, the model with sticky wages and sticky goods prices infers optimal equity portfolios that match the data quite well. With incomplete financial markets, the model predicts home bias for a wide range of parameterization, especially when the cut-off value for home bias is set to be 0.75 in stead of 0.9 (see Figure 1.4).

1.5 Conclusion

This paper analyzes the optimal portfolio decisions in a monetary open economy macro framework with a particular attention to the completeness of financial markets and the presence of nominal rigidities in goods prices and in wages.

With complete financial markets and sticky prices, the model generates a complete home bias in equities because the return to human capital and the return to domestic firms are negatively correlated with respect to productivity shocks. However, this result is sensitive to the configuration of financial markets. With incomplete financial markets, the model produces a “super home bias” result. As the return to human capital and the return to domestic firms are negatively correlated with respect to both the productivity shocks and the monetary shocks, the optimal portfolios actually require households to take a short position in foreign equities rather than diversify their asset holdings. Therefore, sticky prices alone cannot fully explain the observed portfolios.

Introducing sticky wages will help match the data because the return to human capital and the return to domestic firms become positively correlated with respect to monetary shocks in this case. In fact, the model with incomplete financial markets and nominal rigidities in both goods prices and
wages predicts equity home bias for a wide range of parameterization used in the macro literature.

An interesting direction for future research is to conduct welfare analysis for models incorporating endogenous portfolio choices, especially those with incomplete financial markets. Portfolio decisions can be very complicated as a limited set of assets often need to deal with many competing hedging tasks in these environments. Such models may have intriguing policy implications. After all, the real world is far from having perfect risk sharing.
Figure 1.1: Share of Domestically Owned Home Equities with Flexible Wage and Linear Disutility in Labor
Figure 1.2: Share of Domestically Owned Home Equities with Flexible Wage and Quadratic Disutility in Labor
Figure 1.3: Share of Domestically Owned Home Equities with Sticky Wage and Cut-off Value 0.9
Figure 1.4: Share of Domestically Owned Home Equities with Sticky Wage and Cut-off Value 0.75
Chapter 2

International Financial Integration and Monetary Policy

2.1 Introduction

Cross-country gross asset positions have increased dramatically in the past few decades. As shown in Lane and Milesi-Ferretti (2006) [46], the ratio of the sum of foreign assets and liabilities to GDP (IFIGDP) has increased from 45 percent to over 300 percent in industrial countries and from 40 percent to 150 percent in developing countries (between 1970 and 2004).\(^\text{13}\)

During the 1970s and 1980s, the increase in cross-border asset trade was fairly stable, and the two country groups had similar trends in international financial integration. IFIGDP reached 100 percent in 1987 for both groups. However, since the early 1990s, an acceleration of cross-border asset trade has taken place in industrial countries, while developing countries have failed to pick up the pace. Clearly, the early 1990s is a decisive period to focus on for explaining the observed cross-section and time-series differences in the evolution of financial integration.

New Zealand adopted the first inflation targeting regime in 1990, quickly followed by other developed countries. According to Rose (2006) [57], Goodfriend (2003) [35], Ito and Mishkin (2004) [40], and Wyplosz (2006), all

\(^{13}\)See Figure 2.1. The industrial countries include Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, and United States.
Chapter 2. International Financial Integration and Monetary Policy

the twenty three industrial countries classified by Lane and Milesi-Ferretti (2006) [46] target inflation explicitly or implicitly. Therefore, the entire industrial country group essentially has adopted the same monetary strategy since the early 1990s, which is exactly the time when the acceleration in cross-border asset trade began. Is this just a coincidence? Figure 2.2 shows that there is sharp trend change in international financial integration for the eight explicit inflation targeting countries around the year they adopted the policy: New Zealand 1990, Canada 1991, UK 1992, Australia 1993, Sweden 1993, Switzerland 2000, Norway 2001, Iceland 2001. The twelve Euro zone countries target inflation jointly. Figure 2.2 clearly shows that the trend in international financial integration starts to change after 1996, when the Euro zone countries get ready for the formal launch of Euro. Denmark implements a fixed exchange rate policy vis-a-vis the Euro. Hence, similar change is observed in Denmark as well. The United States has been implicitly targeting inflation since early 1990s. The only exception is Japan. It is not clear the trend in international financial integration has surged in Japan. Given the decade long deflation, Japan is probably the only industrial country that does not target inflation in any way. Therefore, all the evidences indicate that the link between the acceleration of cross-border asset trade and the introduction of inflation targeting is not random.

According to the De Facto Classification of Exchange Rate Regime and Monetary Policy Framework published by IMF (2006), only 17 out of 122 developing countries classified by Lane and Milesi-Ferretti (2006) [46] have adopted an inflation targeting regime.\(^{14}\) Most of these countries started to adopt such a regime after the 1997 Asian Financial Crisis. The majority of developing countries still use exchange rates as a nominal anchor when implementing monetary policies. This phenomenon is well-known and has been termed “fear of floating”.\(^{15}\)

Overall the industrial country group and the developing country group

\(^{14}\)The developing countries that adopt an inflation targeting regime are Brazil, Chile, Colombia, Czech Republic, Hungary, Israel, Korea, Mexico, Peru, Philippines, Poland, South Africa, Thailand, Indonesia, Romania, Slovak Republic, and Turkey.

participate at very different levels in international financial markets. The two groups also adopt very different monetary strategies. This paper examines the interaction between financial integration and monetary policy by addressing following three questions: (1) Is financial integration always welfare-improving? (2) Is there a case for fixed exchange rate regimes to be favorable as financial openness increases? (3) Does the monetary policy regime have an impact on the desirability of financial integration?

In a simple two-country monetary general equilibrium model with nominal price rigidity,\textsuperscript{16} this paper explores a number of financial market structures, in which agents optimally choose portfolio positions that comprise familiar assets (such as nominal bonds and equities). Conditional on the range of assets and other aspects of the model, the financial market can be in autarky, incomplete, or complete. By contrasting the financial autarky with the complete or incomplete financial market, this paper unifies the analysis of full and partial financial integrations. Moreover, four simple monetary rules are examined: a money targeting rule, a unilateral exchange rate peg, a bilateral exchange rate peg, and a producer price targeting rule. The money targeting rule simply involves a constant money supply, representing a passive floating exchange rate regime and corresponding to the policy recommended by Friedman (1953) \cite{friedman1953}. The producer price targeting rule, which represents an active floating exchange rate regime, corresponds approximately to inflation targeting. By analyzing the choice of financial integration (given the rule of monetary policy) and the choice of monetary policy (given the structure of financial market), this paper also reveals the interaction between monetary policy and financial globalization.

The main finding of this paper is that in a sticky price world with a passive floating exchange rate regime, financial integration reduces welfare. The reason is that financial integration leads to an increase in the terms of trade volatility, which is already excessive from a welfare standpoint, in the

\textsuperscript{16}This type of model, which is often referred to as the new open economy macroeconomics (NOEM) framework, has become a standard tool of policy analysis in open economy macroeconomics. See Obstfeld and Rogoff (1995, 2000) \cite{obstfeld1995, obstfeld2000}, Corsetti and Pesenti (2001, 2005) \cite{corsetti2001, corsetti2005}, Devereux and Engel (2003) \cite{devereux2003}, and others. For a survey of the NOEM literature, see Lane (2001) \cite{lane2001}.
presence of nominal price rigidities and constraints on the efficient use of monetary policy. Active monetary policy certainly has a role in improving welfare. Pegging exchange rates eliminate the excessive terms of trade adjustment. Hence, fixed exchange rate regimes become more appealing than a money targeting rule to a country that has some access to international asset markets. Nonetheless, a producer price targeting rule removes the sticky price distortion and replicates the flexible price equilibrium. This active floating exchange rate policy, combined with perfect consumption risk sharing, brings the economy to the first-best. Thus, by targeting inflation, countries will benefit the most from financial globalization.

This paper suggests that the preferred depth of financial integration varies across countries that have different monetary regimes. Countries with a passive floating exchange rate choose to stay in the financial autarky. Countries with a fixed exchange rate are indifferent to financial openness. Only those countries that can successfully target inflation will gain from financial integration. As a result, inflation targeting countries should be the most financially open economies in the world. It was mainly the industrial countries that switched to an inflation targeting regime in the early 1990s. Hence, this paper accounts for the observed acceleration in cross-border asset trade among these countries. This paper also justifies the observed slow progress of developing countries in the process of financial globalization, as most developing countries implement a de facto fixed exchange rate regime.

In order to implement efficient inflation targeting, countries need to meet several initial conditions, such as having a central bank that has enough independence, accountability, and transparency, having well developed domestic financial markets and sufficient financial stability, and having adequate research and statistic resources in forecasting inflation. While industrial countries seem to have no problem fitting into an inflation targeting regime, developing countries may find it significantly challenging. In addition, financial and monetary shocks are believed to be much larger in developing countries. It is rational for developing countries to adopt fixed exchange rate regimes first to alleviate the welfare cost of financial integration before they are ready to adopt inflation targeting. Accordingly, this paper provides
This paper compliments the literature that examines the impact of financial integration by paying particular attention to the interaction between financial globalization and monetary policies that are popular in practice. A large literature in international finance analyzes the size of gains from financial integration. As models in this literature are frictionless other than imperfect risk sharing, they all imply positive gains from financial integration. However, in an environment with more than one distortion, financial integration may not always be beneficial. In a multi-country model with endogenous growth, Devereux and Smith (1994) [22] shows that increased international risk sharing discourages precautionary saving, which reduces the growth rate. Hence, welfare in complete financial markets is actually lower than that in financial autarky. Tille (2005) [68] also finds that integration is not universally beneficial in a model similar to ours. As he assumes a unity elasticity of substitution between home and foreign goods, the attention is focused on the degree of exchange rate pass-through. Instead, this paper solves for a general elasticity of substitution between home and foreign goods, which is a critical parameter that determines the welfare impact of integration. We find loss from integration when this elasticity is greater than unity, a case that is more empirically relevant and indicates that terms of trade play a key role in welfare evaluations. Engel (2001) [23] studies how the degree of exchange rate pass-through and the degree of risk sharing affect the choice between fixed and floating exchange rate regimes. However, he does not compare welfare between different asset market structures. All the above authors overlook the interaction between financial integration and monetary policy. They also ignore the issue of partial financial integration for tractability reasons.

This paper is also related to the literature regarding the analysis of en-

---

17 Factors that have been emphasized in the “fear of floating” literature include liability dollarization, lack of central bank credibility, exchange rate pass-through effect, concern about sudden stop, and the mercantilist view.

dogenous portfolio choices in dynamic general equilibrium models with complex asset markets. Devereux and Sutherland (2007) [21] are the first to analyze monetary policy when international portfolio decisions are endogenous. While they explore the impact of monetary policy on country asset positions in both complete and incomplete financial market environments, they limit their examination to a producer price targeting rule and to the impact of the stance of monetary policy on equilibrium asset holdings. They do not compute welfare and do not study other types of monetary policies. They also do not link policy analysis with the observed trend in cross-border asset trade.

The paper proceeds as follows. Section 2.2 describes the structure of the model. Section 2.3 presents the solution method for optimal portfolio choice and welfare. Section 2.4 discusses the impact of financial integration. Section 2.5 analyzes the welfare of alternative money supply rules. The paper concludes with a brief summary and with suggestions for subsequent research.

2.2 The Model

The world is assumed to exist for a single-period and to consist of two countries, which will be referred to as the home country and the foreign country. Each country is populated by agents who consume a basket of home and foreign produced goods. Each agent, using a linear technology in labor, is a monopoly producer of a particular differentiated product. The world population is normalized to have a measure of one. Home agents are indexed \( h \in [0, \frac{1}{2}] \) and foreign agents are indexed \( f \in [\frac{1}{2}, 1] \).

Agents in each country set prices before the realization of shocks and the setting of monetary policies. They are contracted to meet demand at prefixed prices. All prices are assumed to be set in the currency of producers.

\[\text{A list of related papers are authored by Devereux and Sutherland (2006a, 2006b) [19, 20], Engel and Matsumoto (2005) [24], Evans and Hnatkovska (2005) [25], and Kollmann (2005) [43].}\]
Thus, there is full exchange rate pass-through to prices paid by consumers. Agents supply homogeneous labor. Prior to the realization of shocks, they can trade in a range of financial assets. The financial market structure and the payoff to each asset are defined in Section 2.2.3. The timing between asset trade and monetary policy does not affect the welfare of each scenario considered in this paper. Each country faces two types of shocks: productivity and money demand shocks. A production subsidy is introduced to precisely offset the friction of imperfect competition. Therefore, only two types of distortions remain: nominal price rigidity and incomplete consumption risk sharing.

The detailed structure of the home country is described below. The foreign country has an identical structure. Where appropriate, foreign variables are denoted with an asterisk.

2.2.1 Consumers

All agents in the home country have utility functions of the same form

\[
U = \frac{C^{1-\rho}}{1-\rho} + \chi \log \frac{M}{P} - \eta \frac{L^{1+\psi}}{1+\psi},
\]

where \(C\) is the consumption index, defined across all home and foreign goods, \(M\) denotes the end-of-period nominal money holding, \(P\) is the consumer price index (CPI), \(L\) is the labor supply, \(\rho\) \((\rho > 0)\) is the coefficient of relative risk aversion, \(\eta\) is the coefficient of labor supply, \(\psi\) \((\psi \geq 0)\) is the elasticity of labor supply, and \(\chi\) is an i.i.d. stochastic money demand shock, with \(E(\log \chi) = 0\), \(Var(\log \chi) = \sigma^2\), and \(\log \chi \in [-\epsilon, \epsilon]\).

There are two stages to the household decision problem. Before shocks are realized, households choose portfolio positions out of available assets to maximize expected utility, \(E \left[ U(C, \frac{M}{P}, L) \right] \), subject to

\[
\sum_{k=1}^{n} \alpha_k = 0
\]

where \(\alpha_k\) represents the real holding of asset \(k\), and \(n\) is the total number.
Chapter 2. International Financial Integration and Monetary Policy

of assets. All real variables in this paper are defined in terms of home consumption basket. Each country starts with zero net foreign asset.

After shocks are realized, households choose consumption, labor supply, and money balances, in order to maximize ex-post utility, \( U(C, \frac{M}{P}, L) \), subject to

\[
M + PC = M_0 + (1 + \gamma)P_H Y_H + P \sum_{k=1}^{n} \alpha_k r_k + T
\]  

(2.3)

where \( M_0 \) is the initial nominal money holding, \( \gamma \) is the rate of production subsidy, \( P_H \) is the aggregate price of home produced goods, \( r_k \) is the real aggregate rate of return on asset \( k \), and \( T \) is a lump-sum government transfer. \( Y_H \) is the world demand for aggregate home produced goods

\[
Y_H = \frac{1}{2} \left( \frac{P_H}{P} \right)^{-\theta} (C + C^*)
\]  

(2.4)

where \( \theta \) is the elasticity of substitution between home and foreign goods. Although empirical works suggest a wide range of estimations for this elasticity, most researchers agree that a greater than unity elasticity is more empirically relevant.

Firms’ revenues are used to pay wages and profits

\[
(1 + \gamma)P_H Y_H = wL + \Pi
\]  

(2.5)

where \( w \) and \( \Pi \) denote the wage and the profit (dividend), respectively.

Here, home agents first receive all profits from domestic firms. Then, if an international equity market exists, claims to home profits may be transferred to foreign consumers via trade in equity shares.\(^{20}\)

Moreover, we define \( Y \) as the disposable home aggregate real production

\(^{20}\)Alternatively, it can be assumed that profits proceed directly to shareholders. There is no fundamental difference between the two modeling approaches. However, all firms have to be domestically owned in the financial autarky. It is easier to compare various financial market structures by assuming all profits are received by domestic agents first.
income

\[ PY = P_H Y_H \]  \hspace{1cm} (2.6)

Combined with the government budget constraint

\[ M - M_0 = \gamma P_H Y_H + T \]  \hspace{1cm} (2.7)

we can rewrite home household’s budget constraint (2.3) in real terms

\[ C = Y + \sum_{k=1}^{n} \alpha_k r_k \]  \hspace{1cm} (2.8)

The consumption index \( C \) for home agents is defined as

\[ C = \left[ \left( \frac{1}{2} \right)^{\frac{1}{\theta-1}} \left( C_H^{\frac{\theta-1}{\theta}} + C_F^{\frac{\theta-1}{\theta}} \right) \right]^{\frac{\theta}{\theta-1}} \]  \hspace{1cm} (2.9)

\( C_H \) and \( C_F \) are indices of home and foreign produced goods

\[ C_H = \left[ \left( \frac{1}{2} \right)^{\frac{1}{\phi}} \int_{1/2}^{1} C_H(h)^{\frac{\phi-1}{\phi}} dh \right]^{\frac{\phi}{\phi-1}} \]  \hspace{1cm} (2.10)

\[ C_F = \left[ \left( \frac{1}{2} \right)^{\frac{1}{\phi}} \int_{1/2}^{1} C_F(f)^{\frac{\phi-1}{\phi}} df \right]^{\frac{\phi}{\phi-1}} \]  \hspace{1cm} (2.11)

where \( \phi (\phi > 1) \) is the elasticity of substitution between individual home (or foreign) goods.

The aggregate consumer price index for home households is

\[ P = \left( \frac{1}{2} \right)^{\frac{1}{1-\theta}} \left( P_H^{1-\theta} + P_F^{1-\theta} \right)^{\frac{1}{1-\theta}} \]  \hspace{1cm} (2.12)

where \( P_H \) and \( P_F \) are the price indices for home and foreign produced goods,
Chapter 2. International Financial Integration and Monetary Policy

respectively

\[ P_H = \left[ 2 \int_0^1 P_H(h)^{1-\phi} dh \right]^{\frac{1}{1-\phi}}, \quad P_F = \left[ 2 \int_{\frac{1}{2}}^1 P_F(f)^{1-\phi} df \right]^{\frac{1}{1-\phi}} \quad (2.13) \]

The law of one price implies that \( P_H(h) = P_H^*(h)S \) and \( P_F(f) = P_F^*(f)S \) for all \( h \) and \( f \). An asterisk indicates that the price is in foreign currency and \( S \) is the nominal exchange rate defined as the domestic price of foreign currency. Because there is no home bias in consumption, purchasing power parity (PPP) holds, \( i.e. \ P = SP^* \).

Given prices and the total consumption \( C \), home consumers’ optimal demands for home and foreign goods are

\[ C_H = \frac{1}{2} \left( \frac{P_H}{P} \right)^{1-\theta} C, \quad C_F = \frac{1}{2} \left( \frac{P_F}{P} \right)^{1-\theta} C \quad (2.14) \]

\[ C_H(h) = 2 \left[ \frac{P_H(h)}{P_H} \right]^{1-\phi} C_H, \quad C_F(f) = 2 \left[ \frac{P_F(f)}{P_F} \right]^{1-\phi} C_F \quad (2.15) \]

The remaining first order conditions are

\[ E (r_1 C^{-\rho}) = E (r_n C^{-\rho}) \]
\[ E (r_2 C^{-\rho}) = E (r_n C^{-\rho}) \]
\[ \vdots \]
\[ E (r_{n-1} C^{-\rho}) = E (r_n C^{-\rho}) \]

\[ \eta L^\psi = \frac{wC^{-\rho}}{P} \quad (2.17) \]
\[ \frac{\chi}{M} = \frac{C^{-\rho}}{P} \quad (2.18) \]

Equation (2.16) indicates that portfolio choices are optimal only when the expected returns on all assets are equalized in terms of utility. Equations (2.17) and (2.18) are the standard intra-temporal labor-leisure choice function and the money demand function.

37
2.2.2 Firms

Firms engage in monopolistic competition. Each produces specific goods indexed by \( h \) with a linear technology in labor \( Y_H(h) = AL(h) \). \( A \) is an i.i.d. stochastic technology shock with \( E(\log A) = 0 \), \( \text{Var}(\log A) = \sigma_A^2 \), and \( \log A \in [-\epsilon, \epsilon] \).

The profit maximization problem of a firm \( h \) is

\[
\text{Max } E\{D(h) \left[(1 + \gamma)P_H(h) - \frac{w}{A}\right]\} \left[Y_H(h) + Y_H^*(h)\right]
\]

where \( Y_H(h) \) and \( Y_H^*(h) \) are the demand for goods \( h \) from the home and foreign markets

\[
Y_H(h) = \left[\frac{P_H(h)}{P_H^*}\right]^{\phi} \left(\frac{P_H}{P}\right)^{-\theta} \left(\frac{P_H}{P}\right) - \theta \left(\frac{C}{C^*}\right)
\]

\[
Y_H^*(h) = \left[\frac{P_H^*(h)}{P_H^*}\right]^{\phi} \left(\frac{P_H^*}{P^*}\right)^{-\theta} \left(\frac{P_H^*}{P^*}\right) - \theta \left(\frac{C^*}{C^*}\right)
\]

Because firms are all alike, they will set identical prices in equilibrium. Hence, the optimal price of home goods is

\[
P_H = \frac{\phi}{(\phi - 1)(1 + \gamma)} \frac{E\left[D\frac{w}{A}X_H\right]}{E\left[D X_H\right]}\]

\( X_H \) represents the demand for home produced goods

\[
X_H = P_H^\phi \left(\frac{P_H}{P}\right)^{-\theta} (C + C^*)
\]

where we have applied the law of one price and PPP in CPI. When \( \gamma = \frac{1}{\phi - 1} \), the distortion created by monopoly is completely offset and the average output is at its first-best level.

\( D \) is the stochastic discount factor for home firms. In the financial autarky and with complete financial markets, \( D = \frac{C^*}{P^*} \). Under incomplete financial markets, it would be natural to assume that firms will evaluate profits at their shareholders’ discount rate. However, this would require
knowing the portfolio holdings in the first place, as home and foreign agents' discount factors are different (if consumption risk sharing is imperfect). This issue turns out to be irrelevant to the solution of portfolio choices, given the order of accuracy considered in this paper. It may become a concern for solving welfare. This point will be discussed below.

2.2.3 Financial Sector

This paper examines four different asset market configurations: (1) Financial Autarky (FA), in which no assets can be traded across countries; (2) Bond Economy (FB), in which home and foreign nominal bonds are allowed for trade; (3) Equity Economy (FE), in which home and foreign equities can be traded; (4) Bond and Equity Economy (FBE), in which both nominal bonds and equities are allowed for trade. As long as the returns on different assets are not fully correlated, trading assets can increase the degree of international risk sharing. Whether the financial market is complete depends on its structure and on the nature of the rest of the model.

Nominal bonds represent a claim on a unit of currency. Equities represent a claim on aggregate profits. The real aggregate rate of return on each asset is defined as following

\[
\begin{align*}
    r^B &= \frac{1}{q^B P} \\
    r^{B^*} &= \frac{Q}{q^{B^*} P^*} \\
    r^E &= \frac{\Pi}{q^E P} \\
    r^{E^*} &= \frac{Q\Pi^*}{q^{E^*} P^*}
\end{align*}
\]

where \(q_k\) is the real price of asset \(k\) and \(Q\) is the real exchange rate, \(i.e.\) \(Q = \frac{S_{P^*}}{P^*}\).

2.2.4 Monetary Rules

We consider four different policies: a money targeting rule (MT), a un-
lateral (or one-sided) exchange rate targeting rule (UERT), a bilateral (or cooperative) exchange rate targeting rule (BERT), and a producer price targeting rule (PPT). In each regime, the money supply is targeted on some easily observable variables, such as exchange rate and price index.

With a constant money supply, the exchange rate will respond endogenously to domestic and foreign disturbances. Hence, the money targeting rule represents a passive floating exchange rate regime. The unilateral peg involves one country (w.o.l.g., the home country) adjusting its money supply to achieve a fixed exchange rate target. Under the bilateral peg, both countries target the same level of exchange rate. When the home currency depreciates, the home money supply contracts and the foreign money supply expands at the same time. The producer price targeting rule stabilizes the domestic producer price level, which eliminates the relative price distortion when some prices cannot be adjusted in the short-run. Such a policy represents a type of active floating exchange rate regime. Because all prices are assumed to be pre-fixed, the price targeting is defined in terms of stabilizing the prices that producers would choose if prices were fully flexible. The policy equations are in the following form

\[ M = \tilde{M} \left( \frac{S}{\bar{S}} \right)^{\delta_s} \left( \frac{P_H}{\bar{P}_H} \right)^{-\delta_p} \]  \hspace{1cm} (2.26)

\[ M^* = \tilde{M}^* \left( \frac{S}{\bar{S}} \right)^{\delta_s^*} \left( \frac{P_F^*}{\bar{P}_F^*} \right)^{-\delta_p^*} \]  \hspace{1cm} (2.27)

where \( S, P_H \) and \( P_F^* \) are the target levels of exchange rate and home and foreign producer prices. “Flexible” prices are given by

\[ P^{XH}_H = \frac{\phi}{(\phi - 1)(1 + \gamma)} \frac{w}{A} \]

\[ P^{XF}_F = \frac{\phi}{(\phi - 1)(1 + \gamma)} \frac{w^*}{A^*} \]  \hspace{1cm} (2.28)

Table 2.1 summarizes all the regimes. Hereafter, \( \dot{x} = log(x) - log(\bar{x}) \), where \( \bar{x} \) is the non-stochastic steady state value of variable \( x \).
Table 2.1: Targeting Regimes

<table>
<thead>
<tr>
<th>Regime</th>
<th>Targets</th>
<th>Policy Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money</td>
<td>$M = M^* = 0$</td>
<td>$\delta_s = \delta^<em>_s = \delta_p = \delta^</em>_p = 0$</td>
</tr>
<tr>
<td>Unilateral Exchange Rate</td>
<td>$\hat{S} = 0$</td>
<td>$\delta_s \to \infty, \delta^<em>_s = \delta_p = \delta^</em>_p = 0$</td>
</tr>
<tr>
<td>Bilateral Exchange Rate</td>
<td>$\hat{S} = 0$</td>
<td>$\delta_s = \delta^<em>_s \to \infty, \delta_p = \delta^</em>_p = 0$</td>
</tr>
<tr>
<td>Producer Price</td>
<td>$\tilde{P}_H = \tilde{P}_F^X = 0$</td>
<td>$\delta_s = \delta^<em>_s = 0, \delta_p = \delta^</em>_p \to \infty$</td>
</tr>
</tbody>
</table>

2.2.5 Market Clearing

The goods market clearing condition is

$$AL = \frac{1}{2} \left( \frac{P_H}{P} \right)^{-\theta} (C + C^*)$$

(2.29)

The asset market clearing condition is

$$\alpha_k = -\alpha^*_k, \quad k = 1, 2, \ldots, n$$

(2.30)

2.2.6 Equilibrium

The *equilibrium* comprises a set of prices, $P, P^*, P_H, P_F^*, w, w^*, r_k, S$, and a set of quantities $C, C^*, L, L^*, \Pi, \Pi^*, Y_H, Y_F^*, Y, Y^*, \alpha_k, \alpha^*_k, M, M^*$, which solves a system of equations (2.4)-(2.6), (2.12), (2.16)-(2.18), (2.21), (2.29), and their foreign counterparts, as well as (2.8), (2.22)-(2.27), and (2.30), given productivity and money demand shocks, $A, A^*, \chi, \chi^*$.

2.3 Solution Method

2.3.1 Optimal Portfolios

Using the method developed in Devereux and Sutherland (2006a) [20], the equilibrium portfolio choices are solved to a second-order accuracy. From

\[\text{By Walras’ Law, there is one equation redundant in the system. The foreign agent’s budget constraint is dropped.}\]
the second-order approximations of home and foreign portfolio selection equations around the non-stochastic steady state, we obtained the set of equations to solve for equilibrium portfolios

$$E[\rho(\hat{C} - \hat{C}^* )\hat{r}_x] = 0 + O(\epsilon^3) \quad (2.31)$$

It is written in a vector form with $$\hat{r}'_x = [\hat{r}_1 - \hat{r}_n, \hat{r}_2 - \hat{r}_n, \cdots, \hat{r}_{n-1} - \hat{r}_n]$$.

Condition (2.31) contains only the second moments of endogenous variables, indicating that the optimal portfolio allocation is generally indeterminate in a first-order approximation of a model. Moreover, it shows that solving the portfolio choice to a second-order accuracy only requires the first-order solution of the non-portfolio part of the model. This is because second-order accurate second moments can be computed from first-order solutions for realized variables. Finally, firms’ discount factors do not appear in equation (2.31) or any other equilibrium condition up to the first-order. Hence, they do not affect the solution of optimal portfolio choices. The log-linearization of the model is presented in Appendix B.2.

### 2.3.2 Welfare

It is not possible to derive an exact analytical expression for welfare in this model. A second-order accurate solution for welfare is necessary. It requires solving the model as a second-order approximation around the non-stochastic steady state.

A second-order approximation of the welfare measure is given by

$$\tilde{U} = E \left[ \hat{C}^{1-\rho} \left( \hat{C} + \frac{1-\rho}{2} \hat{C}^2 \right) - \eta\hat{L}^{1+\psi} \left( \hat{L} + \frac{1+\psi}{2} \hat{L}^2 \right) \right] + O(\epsilon^3) \quad (2.32)$$

where $$\tilde{U}$$ is the deviation of welfare from the non-stochastic equilibrium. $$\hat{C}$$ and $$\hat{L}$$ are the steady state values of consumption and labor supply. Welfare

---

22The non-stochastic symmetric steady state is described in Appendix B.1. More details about the portfolio solution method can be found in Chapter 1 of this thesis.

23This has been well addressed in the literature. For example, see Collard and Juillard (2001) [15], Kim and Kim (2003) [42], Schmitt-Grohe and Uribe (2004) [59].
is increasing in the expected level of consumption but is decreasing in the variance of consumption, in the expected level of labor and in the variance of labor.

Equilibrium conditions that are log-linear are ready to use. The budget constraint, the consumer price index, the pre-fixed price, and the real production income equations require second-order approximations. They are given by

\[
\hat{C} = \hat{Y} + \hat{\alpha} \hat{r}_x + \lambda_C + O(\epsilon^3)
\]

\[
\hat{C}^* = \hat{Y}^* - \hat{\alpha} \hat{r}_x + \lambda_{C^*} + O(\epsilon^3)
\]

\[
\hat{P} = \frac{1}{2}(\hat{P}_H + \hat{P}_F + \hat{S}) + \lambda_P + O(\epsilon^3)
\]

\[
\hat{P}^* = \frac{1}{2}(\hat{P}_H^* + \hat{P}_F^* - \hat{S}) + \lambda_{P^*} + O(\epsilon^3)
\]

\[
\hat{Y} = (1 - \theta)(\hat{P}_H - \hat{P}) + \frac{1}{2}(\hat{C} + \hat{C}^*) + \lambda_Y + O(\epsilon^3)
\]

\[
\hat{Y}^* = (1 - \theta)(\hat{P}_F^* - \hat{P}^*) + \frac{1}{2}(\hat{C} + \hat{C}^*) + \lambda_{Y^*} + O(\epsilon^3)
\]

\[
\hat{L} = -\hat{A} - \theta(\hat{P}_H - \hat{P}) + \frac{1}{2}(\hat{C} + \hat{C}^*) + \lambda_L + O(\epsilon^3)
\]

\[
\hat{L}^* = -\hat{A}^* - \theta(\hat{P}_F^* - \hat{P}^*) + \frac{1}{2}(\hat{C} + \hat{C}^*) + \lambda_{L^*} + O(\epsilon^3)
\]

where \( \hat{\alpha} = \frac{\bar{\alpha}}{\bar{Y}} \). \( \lambda \) summarize all the second-order terms.\(^{24}\)

\[
\lambda_P = \lambda_{P^*} = \frac{1 - \theta}{8}(\hat{P}_H - \hat{P}_F^* - \hat{S})^2
\]

\[
\lambda_Y = \lambda_{Y^*} = \frac{1}{8}(\hat{C} - \hat{C}^*)^2 = \lambda_L = \lambda_{L^*}
\]

\[
\lambda_C = \left[\hat{\alpha}_{B^*} \hat{r}_{B^*} - \hat{\alpha}_B + \hat{\alpha}_{E^*} \hat{r}_{E^*} - \hat{\alpha}_E\right] \hat{Y} + \frac{1}{2} \hat{\alpha}_{B^*} \hat{r}_{B^*}^2 - \hat{\alpha}_B^2 + \frac{1}{2} \hat{\alpha}_{E^*} \hat{r}_{E^*}^2 - \hat{\alpha}_E^2\]

\[
- \frac{1}{2} \left[\hat{\alpha}_{B^*} \hat{r}_{B^*} - \hat{\alpha}_B + \hat{\alpha}_{E^*} \hat{r}_{E^*} - \hat{\alpha}_E\right]^2 + \frac{1}{2} \hat{\alpha}_{B^*} \hat{r}_{B^*} - \hat{\alpha}_B + \frac{1}{2} \hat{\alpha}_{E^*} \hat{r}_{E^*} - \hat{\alpha}_E
\]

\(^{24}\) This solution technique has been employed in a series of papers. For instance, Sutherland (2004) [64] and Senay and Sutherland (2005) [60].
A country’s terms of trade are defined as the price of its exports in terms of its imports, i.e. $\tau = 1/\tau^* = \frac{P_H}{P_F}$. Hence, $E(\lambda_P)$ and $E(\lambda_{P^*})$ correspond to the variance of terms of trade. $E(\lambda_Y)$, $E(\lambda_{Y^*})$, $E(\lambda_L)$, and $E(\lambda_{L^*})$ indicate the degree of consumption risk sharing across countries. Moreover, $\hat{\alpha}_{B^*,0} = \hat{\alpha}_{E^*,0} = 0$ in a single-period model. The exact form of $\lambda_C$ and $\lambda_{C^*}$ depends on the asset market structure. $E(\lambda_C)$ and $E(\lambda_{C^*})$ represent the variance and covariance between production income and financial income.

Firms’ discount factors drop out in the first-order approximation of the model. Hence, they have no influence on the optimal portfolio decisions. However, the discount factors appear in the second-order approximation of the model through the pricing function. In order to compute welfare appropriately, it is important to specify the discount factors clearly.

If the financial market is complete, the marginal utility of one unit of currency is equalized between home and foreign agents. Thus, it does not matter whether the discount factors follow the home or the foreign owners or a combination of them. If there is no financial market at all, the discount factors follow the domestic agents automatically. Therefore, the only case that needs to be approached with caution is where financial markets exist but are incomplete. If firms are held by both the home and the foreign agents and assumed to evaluate profits at their shareholders’ discount rate, it becomes quite complicated to solve the model, especially in a multiple-period or infinite-horizon dynamic environment. However, in the single-period model, we can show that the results are robust to this assumption.\(^{25}\)

For simplicity, the discount factors are assumed to always follow the domestic owners. That is, $D = \frac{C^* - \rho}{F}$. As a result, second-order approximations of the

\(^{25}\)Results are available upon request.
Chapter 2. International Financial Integration and Monetary Policy

Pre-fixed prices are given by

\[
\hat{P}_H = E(\psi \hat{L} + \hat{P} + \rho \hat{C} - \hat{A}) + \lambda_{P_H} + O(\epsilon^3) \tag{2.45}
\]

\[
\hat{P}_F = E(\psi \hat{L}^* + \hat{P}^* + \rho \hat{C}^* - \hat{A}^*) + \lambda_{P_F} + O(\epsilon^3) \tag{2.46}
\]

where \(\lambda_{P_H}\) and \(\lambda_{P_F}\) represent the risk premium that home and foreign firms build into their pre-fixed prices

\[
\lambda_{P_H} = \frac{1}{2} E \left[ (\psi \hat{L} + \hat{P} + \hat{Y} - \hat{A})^2 - (\hat{Y} - \rho \hat{C})^2 \right] \tag{2.47}
\]

\[
\lambda_{P_F} = \frac{1}{2} E \left[ (\psi \hat{L}^* + \hat{P}^* + \hat{Y}^* - \hat{A}^*)^2 - (\hat{Y}^* - \rho \hat{C}^*)^2 \right] \tag{2.48}
\]

Equations (2.33)-(2.48) plus other log-linear equilibrium conditions allow us to express the approximated welfare (2.32) in terms of \(\lambda\). The expectations of these second-order terms contain only the second moments of variables in the model. Because second-order accurate second moments can be computed from first-order solutions for the realized values of endogenous variables, analytical solutions for welfare can be easily obtained. In addition, home and foreign countries have symmetric structures and are subject to i.i.d. shocks with zero means. Thus, the two countries are ex-ante identical and have same expected utilities.

2.4 Model Solution

This section abstracts from policy issues and only looks at the solution under a money targeting rule. Section 2.5 will give a complete analysis of monetary policy. We start with discussing the relationship between the exchange rate, the terms of trade, and the risk sharing condition. Then, we show that there are two key aspects of the terms of trade in determining the welfare. One is the volatility of the terms of trade and the other is the efficiency of the terms of trade adjustment. By efficiency, we mean whether the terms of trade move in the right direction to achieve efficient relative price adjustments with respect to country-specific shocks. More importantly, both the volatility and the efficiency of the terms of trade depend on the
financial market structure.

### 2.4.1 Exchange Rate, Terms of Trade, and Risk Sharing

From the PPP and the money demand equations, we can write the exchange rate as

\[
\hat{S} = (\hat{M} - \hat{M}^*) - (\hat{\chi} - \hat{\chi}^*) - \rho(\hat{C} - \hat{C}^*) \tag{2.49}
\]

In this model, people hold money because real balances are a part of the utility function. The role of money can be rationalized as capturing the benefits of saving time when conducting transactions. The exchange rate is basically the relative price of two countries’ currencies; it is factored according to the relative money supply and demand in each country. Money supply is given by the monetary policy rule that is adopted by each country. Money demand shocks are indeed preference shocks, which are meant to capture the technology innovations that alter the usefulness of money balances. Other things being equal, the more that agents consume, the more demand for real balances. It is easy to see that higher money supply makes a currency depreciate, while higher money demand makes a currency appreciate.

Home terms of trade in log-linear form are given by

\[
\hat{\tau} = \hat{P}_H - \hat{P}_F^* - \hat{S} \tag{2.50}
\]

Combined with the log-linearized home CPI and the log-linearized aggregate demand for home produced goods, home real GDP can be written as

\[
\hat{Y} = \frac{1 - \theta}{2} \hat{r} + \frac{1}{2}(\hat{C} + \hat{C}^*) + O(\epsilon^2) \tag{2.51}
\]

Together with the log-linearized home budget constraint,

\[
\hat{C} = \hat{Y} + \hat{\alpha}'\hat{r}_x + O(\epsilon^2) \tag{2.52}
\]

\footnote{With money targeting rules, $\hat{M} = \hat{M}^* = 0.$}
the consumption difference between home and foreign can be expressed as

\[
\frac{1}{2} (\hat{C} - \hat{C}^*) = \frac{1 - \theta}{2} \hat{\tau} + \hat{\sigma} \hat{r}_x + O(\epsilon^2) \tag{2.53}
\]

Clearly, households’ consumption risks originate from real income risks, which indeed all come from the terms of trade fluctuations because there is no aggregate uncertainty at the world level. Agents can diversify away at least part of the consumption risks by trading assets across borders — as long as the returns on these assets are somehow correlated with the terms of trade.

2.4.2 Volatility of the Terms of Trade

The exchange rate equation (2.49), the terms of trade equation (2.50), and the consumption difference equation (2.53) are crucial for us to understand how the terms of trade would adjust in the process of financial integration. Suppose that home terms of trade deteriorate owing to certain fundamental shocks, \( \hat{\tau} < 0 \). Home goods become cheaper relative to foreign goods. Given that home and foreign goods are substitutes, the world demand shifts towards home goods. Home output increases, which is called the expenditure switching effect. However, home goods are sold at a relatively lower price and foreign goods cost relatively more. Whether home GDP rises or falls in real terms depends on the strength of this substitution across countries. If the expenditure switching effect is strong enough (when \( \theta > 1 \)), home real GDP increases. In the lack of consumption risk sharing, home agents consume more relative to foreign agents. As shown in equation (2.49), home currency appreciates in this case, flattening the initial deterioration in home terms of trade. The income effect of the initial terms of trade movement triggers an additional exchange rate adjustment that stabilizes the terms of trade. Once international asset markets open, the cross-country consumption difference diminishes and even disappears, if the market is complete. Hence, the higher the degree of consumption risk sharing, the smaller the damping effect from the exchange rate. As a result,
financial integration increases the terms of trade volatility when $\theta > 1$.\textsuperscript{27} Moreover, this result does not hinge on the degree of nominal price rigidity in the economy.\textsuperscript{28}

**Proposition 1** When $\theta > 1$, terms of trade become more volatile as financial markets liberalize.

**Proof:** See Appendix B.3 and the above discussion.

### 2.4.3 Efficiency of the Terms of Trade

Efficient terms of trade adjustment should respond only to productivity shocks. In particular, a country’s terms of trade should deteriorate if the country experiences a positive productivity shock relative to another country. The reason is straightforward: shocks to productivity are real shocks; they affect the comparative advantage of each country in producing substitutable goods. It is efficient to allow the country that has a better technology to produce more. This country’s terms of trade have to worsen, in order that the demand for its goods increases. In contrast, shocks to money demand are nominal shocks. Any response of the terms of trade with respect to money demand shocks represents a cost to efficiency. The real allocation of consumption and labor must be suboptimal if two countries, with the same technology and preference, produce at different levels.

In all sticky price environments, terms of trade adjust through exchange rates. If a country’s currency depreciates regarding a positive productivity shock, the efficiency of world production will increase. Nevertheless, if a country’s currency appreciates regarding a positive productivity shock or if the value of a currency changes in any way regarding a money demand shock, the efficiency of world production will decrease.

\textsuperscript{27}The result will be reversed if $\theta < 1$.

\textsuperscript{28}Preliminary empirical results confirm that the volatility of terms of trade does response positively to the degree of international financial integration for most G7 countries, after controlling for trade openness, GDP volatility, and the volatility of commodity prices.
2.4.4 Welfare

The general solution is very complicated and can only be addressed numerically. Here, we focus on a simplified version of the model, in which utility is logarithmic in consumption and linear in labor, i.e. $\rho = 1$ and $\psi = 0$. This special case has interpretable analytical expressions and allows us to discuss the important and intuitive features of the model. Relaxing the above simplifying assumptions will not change the results qualitatively.

When $\rho = 1$ and $\psi = 0$, we can show that $E(\hat{L}) + \frac{1}{2}E(\hat{L}^2) = 0 + O(\epsilon^3)$.\(^{29}\)

In this case, welfare only depends on the expected level of consumption. It also indicates that welfare is not affected by monopolistic distortion, regardless of the existence of production subsidy.

When all prices are sticky, each country’s welfare is given by

$$\tilde{U} = E(\hat{C}) = \tilde{U}^* = E(\hat{C}^*)$$

$$= -\frac{1}{2}(\lambda_{PH} + \lambda_{PF}) - E(\lambda_P) + O(\epsilon^3) \quad (2.54)$$

$\lambda_{PH}$ and $\lambda_{PF}$ are the risk premiums included in the pre-fixed prices. Other things being equal, a higher level of risk faced by producers makes them charge a higher price ex-ante, which means a lower level of expected output and a lower level of expected consumption. Therefore, the pricing risk premium has a negative effect on welfare.

$\lambda_P$ stems from the non-log-linearity of CPI and $E(\lambda_P) = \frac{1-\theta}{\theta} Var(\hat{\tau})$.

When $\theta > 1$, CPI is concave regarding the price of home and foreign goods. Any volatility in the relative price of home and foreign goods reduces the expected cost of the consumption basket. Intuitively speaking, when home and foreign goods are substitutable with each other, agents can spend less by switching expenditure towards a set of goods that is cheaper ex-post. Therefore, the volatility of terms of trade affects welfare positively.

However, the volatility of terms of trade also affects the risk of setting prices in advance. Such risks are caused by the fact that firms cannot freely adjust their prices after knowing the state of the economy. Given the struc-

\(^{29}\)Proof see Appendix B.4.
ture of the economy, firms know in advance how the exchange rate would respond to different shocks, ex-post. If the exchange rate facilitates efficient adjustment in production, the risk of the inability to change prices after shocks are realized is smaller. Firms will charge a lower price, ex-ante, leading to an increase in expected consumption. On the other hand, if the exchange rate causes inefficient adjustment in production, the pricing risk is higher. Firms will charge a higher price, ex-ante, leading to a reduction in expected consumption. As asset markets liberalize, the volatility of the exchange rate goes up. Welfare will fall if financial integration causes excessive adjustments in terms of trade.

**Financial Autarky v.s. Complete Markets**

When all prices are sticky, output is demand-determined. Productivity shocks have no effect on firm revenue. The only changes occur are associated with the allocation between labor income and profit. If home agents hold one hundred percent of their own firms, their income as well as consumption are independent of productivity shocks. In other words, the default equity position, or a complete home bias in equities, provides a perfect hedge against productivity shocks. Money demand shocks are the only source of income risks. As Home and foreign nominal bonds provide a perfect hedge against money demand shocks, the financial market is complete in the bond and in the bond equity economies. Table 2.2 summarizes the key second moments and welfare results in the financial autarky and in the complete market.

Table 2.2: Second Moments and Welfare with Money Targeting Rules

<table>
<thead>
<tr>
<th></th>
<th>Financial Autarky</th>
<th>Bond and Bond Equity Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Var}(S) )</td>
<td>( \frac{2}{\theta^2} \sigma_X^2 )</td>
<td>( 2 \sigma_Y^2 )</td>
</tr>
<tr>
<td>( \lambda_{P_H} = \lambda_{P_F} )</td>
<td>( \frac{\sigma^2}{\theta} \sigma_X + \frac{1}{2} \sigma_A^2 )</td>
<td>( \frac{\theta}{2} \sigma_X^2 + \frac{1}{2} \sigma_A^2 )</td>
</tr>
<tr>
<td>( E(\lambda_{P}) )</td>
<td>( \frac{1-\theta}{\theta^2} \sigma_X^2 )</td>
<td>( \frac{1-\theta}{1-\theta} \sigma_X^2 )</td>
</tr>
<tr>
<td>( \hat{U} = E(\hat{C}) )</td>
<td>( -\frac{2\theta^2 - \theta + 1}{4\theta^2} \sigma_X^2 - \frac{1}{2} \sigma_A^2 )</td>
<td>( -\frac{\theta+1}{4\theta^2} \sigma_X^2 - \frac{1}{2} \sigma_A^2 )</td>
</tr>
</tbody>
</table>
Because productivity shocks receive perfect hedging in both the financial autarky and the complete market, exchange rates react solely to money demand shocks. Any induced adjustment on real allocations is inefficient. Increased exchange rate volatility simply means bigger distortion and higher pricing risk. Let \( \sigma_A^2 = \sigma_X^2 = 0.0001 \). Figure 2.4 shows that the pricing risk is clearly higher under the complete market when \( \theta > 1 \). Although the variance of consumption is reduced due to enhanced consumption risk sharing, financial integration is still costly because it leads to a lower level of consumption. As the first-moment effect dominates the second-moment effect, welfare is lower under the complete market.

**Proposition 2** With complete nominal price rigidity and money targeting rules, perfect international consumption risk sharing reduces welfare when \( \theta > 1 \).

**Proof:** From the last row of Table 2.2, \( \tilde{U}_{FB/FBE} - \tilde{U}_{FA} = \frac{\theta^2 + 1}{\theta^2} (1 - \theta) \sigma_X^2 < 0 \) if \( \theta > 1 \).

**Incomplete Market**

When all prices are sticky, international risk sharing is generally incomplete in the equity economy. The exchange rate responds to both productivity and money demand shocks. Portfolio diversification may raise or reduce welfare depending on deeper parameters in the model, especially the steady state labor income share \( \zeta \). In particular, we find a hump-shape relationship between the value of \( \zeta \) and welfare in the equity economy.

When \( \zeta = 0 \), each equity has a payoff equal to the GDP of the corresponding country. Agents are indeed trading two state-contingent assets. Agents can perfectly hedge against money demand shocks by holding half of each country’s equities. In this case, markets are complete, and the exchange rate responds to money demand shocks only. Thus, the equity economy yields the same welfare as the complete market. This result holds for any \( \theta \).
When $\zeta > 0$, money demand shocks affect both a firm’s revenue and its labor cost. In response to a positive home demand shock, both the firm’s revenue and the wage rate decrease in the home country. If $\zeta$ is small, the dividend on equity moves in the same direction as the firm’s revenue. Hence, it is optimal for home households to hold some foreign equities, i.e. $\tilde{\alpha}_{FE,E^*} > 0$. On the other hand, if $\zeta$ is big, the reduction in wages is so significant that the dividend on equity moves in the opposite direction to the firm’s revenue. Hence, it is optimal for home households to take a short position in foreign equities, i.e. $\tilde{\alpha}_{FE,E^*} < 0$. Once households start trading equities, exchange rates (or terms of trade) are influenced by productivity shocks. In this case, productivity shocks will have real income effects on the economy.

As $\zeta$ increases, the welfare first rises and then falls. The intuition to this hump-shape relationship lies in how the terms of trade respond to different shocks. Up to first-order, the exchange rate in the equity economy is given by

$$
\hat{S}_{FE} = s_1(\hat{A} - \hat{A}^*) + s_2(\hat{\chi} - \hat{\chi}^*)
$$

(2.55)

$$
s_1 = \frac{2\zeta \tilde{\alpha}_{FE,E^*}}{(1 - \zeta)[\theta + 2\tilde{\alpha}_{FE,E^*}(1 - \theta)]}
$$

(2.56)

$$
s_2 = \frac{2\zeta \tilde{\alpha}_{FE,E^*} - 1 + \zeta}{(1 - \zeta)[\theta + 2\tilde{\alpha}_{FE,E^*}(1 - \theta)]}
$$

(2.57)

where $\tilde{\alpha}_{FE,E^*} = \frac{(1-\theta)(1-\zeta)[(1-\theta)(1-\zeta)+\theta\zeta]\sigma^2_{\chi}}{2(\theta\zeta^2\sigma^2_{A}+(1-\theta+\theta\zeta)(1-\theta)(1-\zeta)+\theta\zeta^2\sigma^2_{A})}$. Figure 2.5 plots the response coefficient of the exchange rate to each type of shock against the value of labor income share. We assume productivity and money demand shocks are equally volatile and that the elasticity of substitution between home and foreign goods is equal to 1.5, following Backus, Kehoe, and Kydland (1993) [2]. Similar results hold for any $\theta > 1$.

With respect to a positive home demand shock, home currency always appreciates. While with respect to a positive home productivity shock, home households experience a loss from their portfolio holdings, if $\tilde{\alpha}_{FE,E^*} > 0$. 

52
In this case, home currency depreciates. On the contrary, home currency appreciates if $\tilde{\alpha}_{FE,E^*} < 0$.

It is already known that any terms of trade adjustment due to monetary shocks cause inefficient movement in production. Therefore, the smaller the absolute value of $s_2$, the less there is of real distortion. Moreover, when all prices are sticky, nominal exchange rates act as a relative price adjuster to real shocks. However, it is beneficial only if the exchange rate moves in the right direction. For example, if the home country experiences a positive productivity shock, a depreciation of home currency will help lower the relative price of home goods and shift world demand towards home produced goods. World production thus becomes more efficient. In contrast, world production is definitely more distorted if home currency appreciates; this is the case when $\zeta$ is greater than a certain level.

International consumption risk sharing is usually imperfect in an equity economy, but it still trims down some of the income effect on exchange rates. Terms of trade are more volatile in the equity economy than that in the financial autarky. Whether portfolio diversification is welfare improving depends on the parameterization of the model, particularly on the value of $\zeta$. If $\zeta$ is big enough, the exchange rate reacts inefficiently to both productivity and money demand shocks. Hence, increased international risk sharing makes people worse off. Instead, the exchange rate reacts efficiently to productivity shocks if $\zeta$ is small. There exists potential gain from diversification. The net effect between efficient and inefficient terms of trade adjustments determines whether people will be better off.

**Proposition 3** With complete nominal price rigidity and money targeting rules, imperfect international risk sharing may either raise or reduce welfare, as a condition of the value of structural parameters in the model.

**Proof:** See the above discussion.

This paper illustrates the welfare effect by computing the consumption equivalent measure. This shows how much consumption would have to be given up under segmented markets to lead to the welfare level observed un-
der complete markets. Appendix B.5 gives the details of the derivations. Figure 2.6 plots the consumption equivalent measure (under each market configuration) against the value of the labor income share. As discussed, welfare is lower in the complete market than that in the financial autarky. The rank of incomplete market relies on the value of $\zeta$. In the macroeconomics literature, the labor income share is normally set equal to two thirds, which implies that the equity economy generates a welfare lower than both the financial autarky and the complete market.

2.5 Policy Analysis

Financial integration may reduce welfare because increased consumption risk sharing can cause excessive terms of trade volatility in the presence of nominal price rigidity. This quite surprising result suggests that active monetary policy may help to improve welfare as asset markets liberalize.

2.5.1 Exchange Rate Targeting Rules

If the nominal exchange rate is fixed, there will be no idiosyncratic income risk. Hence, welfare with each type of exchange rate targeting rule is independent of the financial market structure. Nevertheless, a cooperative peg welfare dominates a one-sided peg due to the difference in the induced aggregate money supply, which affects the aggregate demand for goods. When all prices are sticky, output is demand-determined. The output volatility depends on the volatilities of relative price and aggregate demand. The bigger the output volatility, the higher the pricing risk faced by producers. Although the terms of trade are fully stabilized with a one-sided peg, the aggregate money supply varies so much that the pricing risk is indeed the same as that under a money targeting rule in the financial autarky. For a cooperative peg, there are not only no terms of trade adjustment, but no aggregate money supply change. As shown in Table 2.3, row 3, the pricing risk is smaller under a bilateral peg, i.e. $\lambda_{URT}^{UP} > \lambda_{URT}^{REP}$. Undoubtedly, the cooperative peg dominates the unilateral peg, as can be
seen from the last row of Table 2.3, i.e. \( \tilde{U}^{BERT} > \tilde{U}^{UERT} \).

Table 2.3: Second Moments and Welfare with Exchange Rate Targeting Rules

<table>
<thead>
<tr>
<th></th>
<th>One-sided Peg</th>
<th>Cooperative Peg</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Var(\hat{S}) )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \lambda_{P_H} = \lambda_{P_F} )</td>
<td>( \frac{1}{2} \sigma^2 \chi + \frac{1}{2} \sigma^2_A )</td>
<td>( \frac{1}{4} \sigma^2 \chi + \frac{1}{2} \sigma^2_A )</td>
</tr>
<tr>
<td>( E(\lambda_P) )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \tilde{U} = E(\hat{C}) )</td>
<td>( -\frac{1}{2} \sigma^2 \chi - \frac{1}{2} \sigma^2_A )</td>
<td>( -\frac{1}{4} \sigma^2 \chi - \frac{1}{2} \sigma^2_A )</td>
</tr>
</tbody>
</table>

For any positive \( \theta \), the cooperative peg also dominates the money targeting rule in both the financial autarky and the complete markets, i.e. \( \tilde{U}^{BERT} > \tilde{U}^{MT} \). The ranking between the money targeting rule and the unilateral peg depends on the structure of the financial markets and the value of \( \theta \). Consider the case \( \theta > 1 \). In the financial autarky, the money targeting rule dominates the unilateral peg, i.e. \( \tilde{U}^{MT}_{FA} > \tilde{U}^{UERT} \). Although the two policies have the same level of pricing risks, the money targeting rule allows some terms of trade volatility, which helps to lower the average cost of a consumption basket consisting of all home and foreign produced goods. When the financial market is complete, the terms of trade become too volatile and the pricing risk is much higher under a money targeting rule. In this case, the money targeting rule is dominated by the unilateral peg, i.e. \( \tilde{U}^{UERT} > \tilde{U}^{MT}_{FB/FBE} \). Similar results hold for the incomplete market with standard calibration.

**Proposition 4** Welfare of an exchange rate targeting rule is independent of the financial market structure. A cooperative peg always dominates a unilateral peg and also dominates a money targeting rule for all \( \theta > 0 \). A unilateral peg is dominated by a money targeting rule in the financial autarky but dominates a money targeting rule in an open financial market when \( \theta > 1 \).

**Proof:** From the last rows of Table 2.2 and Table 2.3, \( \tilde{U}^{BERT} - \tilde{U}^{UERT} = \frac{1}{4} \sigma^2 \chi > 0; \tilde{U}^{BERT} - \tilde{U}^{MT}_{FA} = \frac{\theta^2 - \theta + 1}{4 \sigma^2} > 0, \forall \theta > 0; \tilde{U}^{BERT} - \tilde{U}^{MT}_{FB/FBE} = \)
\[ \frac{\theta}{4} > 0, \forall \theta > 0; \tilde{U}^{UERT} - \tilde{U}^{MT}_{FA} = \frac{1 - \theta}{4\theta^2} < 0 \text{ if } \theta > 1; \tilde{U}^{UERT} - \tilde{U}^{MT}_{FBE/FBE} = \frac{\theta - 1}{4} > 0, \text{ if } \theta > 1. \]

In a non-coordinated game between home and foreign monetary authorities, both authorities will choose not to intervene in the financial autarky, if the alternative policy is to operate a one-sided peg. However, when there is some international risk sharing, a passive floating exchange rate regime admits too many inefficient terms of trade adjustment. Each countries is better off by pegging its currency to the other. Once both countries do so, they in fact end up with a bilateral peg without any coordination. Therefore, financial market structures do matter for the choice of exchange rate regimes. Financial integration will be beneficial if countries are allowed to adjust their policies, even within a set of restricted exchange rate targeting rules. Moreover, stabilizing excessive terms of trade volatility can improve welfare, providing a new angle on the analysis of fixed vs floating exchange rate regimes.

### 2.5.2 Producer Price Targeting Rule

An active floating exchange rate regime, such as the producer price targeting rule, will eventually dominate the money targeting rule and the fixed exchange rate regimes. With a producer price targeting rule, prices are still one hundred percent pre-fixed, but they are set at a level that is optimal, even after shocks are realized. In other words, firms will stick to the prices they have chosen even when they are allowed to readjust the prices after the state of the economy is revealed. As shown in Table 2.4, row 3, there is no pricing risk at all, i.e. \( \lambda^{PPT}_{P/F} = \lambda^{PPT}_{P/F} = 0 \). Hence, a price targeting rule actually replicates the flexible price equilibrium. The only difference compared to a flexible price environment is that the relative price modification is made through nominal exchange rates rather than by prices themselves. There is no fundamental difference on real allocations or on any other aspect. Furthermore, markets are complete in the equity economy, if monetary authorities target producer prices directly. Therefore, the welfare of a producer
price targeting rule only depends on whether asset markets exist. As long as some assets (nominal bonds or/and equities) are available for trade, international consumption risk sharing is perfect. In brief, producer price targeting and financial integration bring the economy to the first-best. As shown in the last row of Table 2.4, financial integration is always welfare improving once the sticky price distortion is removed, \( \tilde{U}_{PPT}^{FB/FE/FBE} > \tilde{U}_{FA}^{PPT} \).

Table 2.4: Second Moments and Welfare with Price Targeting Rules

<table>
<thead>
<tr>
<th>Financial Autarky</th>
<th>Bond, Equity, and Bond Equity Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Var(\hat{S}) )</td>
<td>( \frac{2}{\theta^{2}} \sigma^{2}_{A} )</td>
</tr>
<tr>
<td>( \lambda_{P_{H}} = \lambda_{P_{F}} )</td>
<td>0</td>
</tr>
<tr>
<td>( E(\lambda_{P}) )</td>
<td>( \frac{1-\theta}{\theta^{2}}\sigma^{2}_{A} )</td>
</tr>
<tr>
<td>( \tilde{U} = E(\tilde{C}) )</td>
<td>( \frac{\theta-1}{4\theta^{2}}\sigma^{2}_{A} )</td>
</tr>
</tbody>
</table>

Let \( \sigma^{2}_{A} = \sigma^{2}_{x} = 0.0001 \) and \( \theta = 1.5 \). Figure 2.7 plots the consumption equivalent measure of each targeting rule under each market configuration against the value of labor income share. Now it is possible to state the following proposition.

**Proposition 5** A producer price targeting rule dominates the money targeting and the exchange rate targeting rules in each financial market configuration. With either bonds or equities available for trade, a producer price targeting rule delivers the first-best outcome.

**Proof:** From the last row of Table 2.4, \( \tilde{U}_{FB/FE/FBE}^{PPT} - \tilde{U}_{FA}^{PPT} = \frac{(\theta-1)^{2}(1+\theta)}{4\theta^{2}} > 0, \forall \theta > 0. \)

2.5.3 Financial Integration and Monetary Policy

The producer price targeting rule generates the highest welfare in each fi-
nancial environment. Therefore, given the process of financial integration, it should be the choice of monetary policy for all countries. However, inflation targeting is quite a sophisticated policy to conduct in practice. It requires a mandate to pursue an inflation objective, the accountability of the central bank, macroeconomic and financial stability, and well-functioning domestic asset markets (especially the bond market). Deep and liquid markets contribute to the effective execution of the policy operation in the sense that there will be appropriate market-based monetary instruments and that the transmission mechanism is clear. The central bank should also be able to give credible inflation forecasting in order to carry out such a policy. Industrial countries seem to have no problem in qualifying for all these conditions. It may be hard for developing countries to meet even some of these conditions.

Table 2.5 is constructed based on the Central Bank Studies Survey by Fry et al (2000) [31]. This survey is conducted for 93 central banks in 1998 and is revised in 1999. The data for the average inflation between 1997 - 1998 are drawn from Table A.1 of Fry et al (2000) [31], and the data for the indices of independence, accountability, and transparency are obtained from Table A.5-A.7. These scores represent percentages of the maximum. There are twenty two industrial countries and seventy one developing countries in this dataset. According to Rose (2006)[57], Australia, Canada, New Zealand, Sweden, and United Kingdom are explicit inflation targeters by 1999. As we have discussed at the beginning of this paper, the other seventeen industrial countries target inflation implicitly. At the same time, six developing countries, Chile, Czech Republic, Israel, Korea, Mexico, and Poland, implement inflation targeting.

Among the industrial countries, the explicit inflation targeters experience lower inflation than the implicit targeters. Their central banks also have higher degrees of independence, accountability, and transparency. Similar patterns hold in the developing countries, except that the central banks which target inflation seem to have a slightly lower score of accountability than those which do not. However, the difference is tiny (less than 1%)

---

31 Use the classification of Lane and Milesi-Ferretti (2006) [46].
Table 2.5: Average Inflation and Central Bank Characteristics

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Industrial</td>
<td>0.0196</td>
<td>0.8400</td>
<td>0.7143</td>
<td>0.7457</td>
</tr>
<tr>
<td>explicit - inflation targeter</td>
<td>0.0131</td>
<td>0.8553</td>
<td>0.9333</td>
<td>0.8771</td>
</tr>
<tr>
<td>implicit - inflation targeter</td>
<td>0.0216</td>
<td>0.8355</td>
<td>0.6458</td>
<td>0.7047</td>
</tr>
<tr>
<td>Developing</td>
<td>0.2721</td>
<td>0.7030</td>
<td>0.7171</td>
<td>0.5401</td>
</tr>
<tr>
<td>inflation targeter</td>
<td>0.1007</td>
<td>0.8297</td>
<td>0.7083</td>
<td>0.7713</td>
</tr>
<tr>
<td>non-inflation targeter</td>
<td>0.2880</td>
<td>0.6913</td>
<td>0.7179</td>
<td>0.5188</td>
</tr>
</tbody>
</table>

compared to the differences observed in other categories. These empirical evidences support the view that, in order to carry out efficient inflation targeting, it is necessary for all countries to build up their institutional infrastructures. For developing countries, this process may take some time.

Fry et al (2000) [31] reports scores for financial stability (Table A.8) as well. Not surprisingly, industrial countries are more stable than developing countries; inflation targeters are more stable than non-inflation targeters. Considering the fact that financial and monetary shocks are larger and more frequent in developing countries, it may be rational for developing countries to target the exchange rate first. Such a policy is easy to formulate, and it eliminates the excessive terms of trade adjustment. Hence, it is more favorable than a passive floating exchange rate regime to a country that has some access to international asset markets. Once the above prerequisites are met (at least to some extent), developing countries can proceed by pursuing the optimal inflation targeting rule. Therefore, financial integration and the lack of readiness for complex policy actions could be the reasons why most developing countries do not float the way they announce.

Given the nature of monetary policy, what is the choice of financial integration? Countries with a money targeting rule are better off in the financial autarky. Countries with exchange rate targeting rules are indifferent to the asset market structures. Only countries with an inflation targeting rule
gain from financial integration. Thus, the most financially open economies should be those who can successfully target inflation. This paper provides an explanation for the acceleration of cross-border asset trade observed in industrial countries during the early 1990s because most industrial countries started an inflation targeting regime around that time. This model also rationalizes the developing countries’ slow progress in the process of financial globalization because most developing countries are implementing a de facto fixed exchange rate regime.

2.6 Conclusion

This paper analyzes the welfare impact of financial integration in a standard monetary open economy model with nominal price rigidity. Results show that terms of trade become more volatile when international consumption risk sharing increases. Financial integration may reduce welfare if the integration leads to excessive terms of trade adjustment. This can happen when financial market segmentation is not the only distortion in the economy. The existence of nominal price rigidity allows monetary shocks to affect the terms of trade, which is at a cost to efficiency.

A fixed exchange rate regime is indeed more favorable than a passive floating exchange rate regime because fixed exchange rates eliminate the excessive terms of trade volatility. Hence, this model implies that developing economies that are experiencing financial integration may attempt to alleviate the welfare cost of integration by stabilizing the exchange rate. This prediction is consistent with the widespread reluctance to following freely floating exchange rates among these economies, a phenomenon that has been well documented in the “fear of floating” literature. On the other hand, for advanced economies that have the ability to operate efficient, inflation targeting monetary policies, financial globalization is always beneficial. The model thus predicts that advanced economies that have introduced inflation targeting as a monetary policy rule should experience a deeper level of financial globalization. Most industrial countries started an inflation targeting regime in the early 1990s, which explains the acceleration of cross-border
asset trade in industrial countries since then. Given that most developing countries still conduct a de facto fixed exchange rate regime, the slow progress of developing countries in the process of financial globalization is not surprising.

An interesting direction for future research is to test the empirical implications suggested by this model. Testable hypotheses have been developed about the impact of international consumption risk sharing on terms of trade volatility and about the interaction between monetary policy and financial integration. First, financial integration increases the terms of trade volatility, which is independent of whether there is nominal price rigidity and of how welfare changes. Second, inflation targeting countries are more financially open and integrated. Last but not least, the coexistence of financial globalization and lack of institutional infrastructure and financial stability may be the reasons why developing countries prefer not to float their currencies.
Figure 2.1: International Financial Integration: Industrial Group and Developing Countries Group, 1970-2004

Note: Ratio of sum of foreign assets and liabilities to GDP, 1970-2004.
Source: Lane and Milesi-Ferretti (2006)
Chapter 2. International Financial Integration and Monetary Policy

Figure 2.2: International Financial Integration: Explicit Inflation Targeting Countries

- Australia
- United Kingdom
- Canada
- Switzerland
- New Zealand
- Sweden
Chapter 2. International Financial Integration and Monetary Policy

Figure 2.3: International Financial Integration: Implicit Inflation Targeting Countries

- United States
- Japan
- Denmark
- Euro Zone
Figure 2.4: Second Moments and Welfare with Money Targeting Rules

- Consumption Equivalent Measure
- Pricing Risk
- Expected Level of Consumption
- Variance of Consumption
- Labor Force
- Variance of Labor

Chapter 2. International Financial Integration and Monetary Policy
Chapter 2. International Financial Integration and Monetary Policy

Figure 2.5: Exchange Rate Response Coefficients with Incomplete Markets

The figure illustrates the exchange rate response coefficients with respect to productivity shocks (s1) and monetary shocks (s2) for different steady state labor income shares. The graph shows two curves: one solid line representing s1 with respect to productivity shocks and a dashed line representing s2 with respect to monetary shocks.
Figure 2.6: Consumption Equivalent Measures with Money Targeting Rules
Figure 2.7: Welfare Comparison
Chapter 3

Explaining Saving Behavior

3.1 Introduction

There has been some recent work on explaining the evolution of saving rates in Japan and the U.S. (see in particular Imrohoroglu et al (2006, 2008) [11, 12]). This work finds that when the observed time-series pattern of total factor productivity (TFP) is fed as an exogenous driving process into the standard neoclassical growth model, it generates a time series pattern of saving which matches the data in these countries reasonably well. Moreover, the model does an even better job of explaining saving rates once the list of exogenous driving processes is augmented to include population growth, government consumption, depreciation, and factor income tax shocks.

In this paper we show that the ability of these models to explain saving rates is due to the assumption that the model economy is closed to international goods and factor trade. This assumption forces saving to equal investment every period. We demonstrate that the model actually does a relatively good job of explaining investment rates in these countries. The reason they also appear to do well in explaining saving is due to the closed economy assumption that saving equals investment and the data fact that national saving and domestic investment rates are not very different.

The main question of this paper is “Can a model in which saving behavior is dictated by the logic of the standard neoclassical growth model explain the time series and cross-sectional pattern of saving observed in the data?” We construct a model of the world economy comprised of two regions – the US and a group of OECD countries (called the rest of the world (ROW)).

The group of OECD countries are Austria, Canada, Belgium, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Netherlands, Norway, Portugal,
Chapter 3. Explaining Saving Behavior

model is the standard one-sector neoclassical growth model with complete markets, production, and investment. Exogenous drivers include measured TFP, population growth, government consumption, depreciation, and taxes on capital and labor incomes. We find that a closed-economy version of the model does an excellent job of reproducing investment as well as saving rates in the U.S. In the two-region version on the other hand, the model continues to match the U.S. investment rate but it fails to reproduce the path of US saving rates either in levels or in its time series pattern. The model is similarly unable to predict saving in the rest of the world. Therefore, the answer to our central question is “no, the neoclassical model with TFP movements cannot explain saving behavior.” The model explains investment not saving.

The key intuition for our findings is that, in an open economy environment, the presence of international consumption risk sharing delinks saving from investment. Basically, investment decisions are based on the future return on capital. Hence, it is the growth rate of TFP in each country that drives the evolution of investment. On this margin, there is not much difference between a closed-economy version and an open-economy version of the neoclassical growth model. That is why both versions of the model can predict investment quite well. However, saving decisions rely on the risk sharing structure. When there exists some international risk sharing, the more productive country will run a current account surplus. In other words, this country will save more than its own investment. Thus, it is the relative level of TFP between the two countries that determines the behavior of saving. Given that the U.S. is consistently more productive than the ROW in the data, it is unsurprising to find that the model overpredicts the U.S. saving and underpredicts saving in the ROW. We conclude that explaining saving, especially the large cross-sectional and time series variations in them, remains a challenge for the profession.33

Spain, Sweden, Switzerland, and United Kingdom.

The next section lays out the baseline model. Section 3.3 defines the equilibrium, presents the stationary form of the model, and states the algorithm. Section 3.4 describes the data and the calibration strategy. Section 3.5 reports results of the basic data accounting exercise wherein we simulate the model under each of the exogenous drivers individually and also all of them together in order to assess the relative importance of each factor. The last section concludes.

3.2 The Model

We consider the basic one good, two-country open economy model. We label the two countries Home and Foreign. At every date \( t \), Home and Foreign are inhabited by \( N_t \) and \( N_t^* \) identical agents. The lifetime welfare of each agent at Home is given by:

\[
U = \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - h_t)
\]  
(3.1)

where

\[
u(c_t, 1 - h_t) = \log c_t + \alpha \log(1 - h_t), \quad \alpha > 0
\]  
(3.2)

c is consumption and \( h \) denotes the fraction of time devoted to work. Preferences are identical across countries so there is an identical utility function for Foreign residents.

Output of the single good in both countries is produced using an identical production technology which uses capital and labor:

\[
Y_t = K_t^\theta (a_t N_t h_t)^{1-\theta}
\]  
(3.3)

where \( \theta \) is the income share of capital, \( K_t \) is aggregate physical capital, \( N_t \) is total population, and \( a_t \) denotes labor augmenting productivity. There is a corresponding production technology for output abroad which takes as inputs \( K_t^* \) and \( a_t^* N_t^* h_t^* \). Note that per capita output at Home can be written
Chapter 3. Explaining Saving Behavior

as

\[ y_t = k_t^\theta (a_t h_t)^{1-\theta} \tag{3.4} \]

where \( y_t = Y_t/N_t \) and \( k_t = K_t/N_t \).

In terms of notational convention, throughout the paper we shall denote all Foreign variables with stars. Variables with capital letters denote aggregate variables, lower case letters denote per capita variables \((z = \frac{Z}{N})\), and lower case letters with tildes on top denote variables per efficiency units of labor \((\tilde{z} = \frac{Z}{aN})\).

Capital accumulates at Home according to

\[
K_{t+1} = (1 - \delta_t) K_t + X_t \tag{3.5}
\]

\[
I_t = X_t + \phi K_t \left( \frac{X_t}{K_t} - \psi \right)^2 \tag{3.6}
\]

where \( I_t \) denotes gross investment. Thus, we are allowing for adjustment costs in the capital accumulation process whenever \( \phi > 0 \). Similarly, for Foreign we have:

\[
K^*_{t+1} = (1 - \delta^*_t) K^*_t + X^*_t \tag{3.7}
\]

\[
I^*_t = X^*_t + \phi K^*_t \left( \frac{X^*_t}{K^*_t} - \psi^* \right)^2 \tag{3.8}
\]

There is a government in each country which consumes \( G_t \) and \( G^*_t \) in period \( t \)

\[
G_t = g_t Y_t \tag{3.9}
\]

where \( g_t \) is the government consumption as a share of total output. A corresponding equation holds for Foreign.

The aggregate resource constraint for the world dictates that total private consumption plus total investment plus total public consumption in
Chapter 3. Explaining Saving Behavior

Home and Foreign must equal total world output. Hence,

\[ C_t + I_t + C^*_t + I^*_t = (1 - g_t) Y_t + (1 - g^*_t) Y^*_t \] (3.10)

Moreover, the aggregate government budget constraint is given by

\[ g_t Y_t + g^*_t Y^*_t = \tau_{h,t} w_t H_t + \tau_{k,t} (r_t - \delta_t) K_t + \tau^*_{h,t} w^*_t H^*_t + \tau^*_{k,t} (r^*_t - \delta^*_t) K^*_t + T_t \] (3.11)

where \( w_t \) is the real wage. \( r_t \) is the rate of return on capital. \( \delta_t \) is the depreciation rate. \( \tau_{h,t} \) and \( \tau_{k,t} \) are tax rates on labor and capital incomes respectively. There is no international factor trade.

We set up the problem as a planning problem. In particular, the social planner maximizes:

\[ V = \sum_{t=0}^{\infty} \beta^t \left[ N_t u (c_t, 1 - h_t) + N^*_t u (c^*_t, 1 - h^*_t) \right] \] (3.12)

subject to equations (3.2)-(3.3), (3.5) - (3.6) and their foreign counterparts, (3.10), and (3.11). We choose to set up the problem in this way because the planning problem provides the frictionless paradigm. This corresponds to the case of complete markets which is the benchmark model in the literature.\(^{34}\) Note that the social planner pays factors at their competitive market rates, takes the tax regime in each country as given, and keeps equation (3.11) balanced with a lump sum tax \( T_t \) in every period.

Finally, the net saving rate is defined as

\[ \text{sav}_t = \frac{(1 - g_t) Y_t - C_t - \delta_t K_t}{Y_t - \delta_t K_t} \] (3.13)

\(^{34}\)Note that we could have instead set up the decentralized problem with firms in each country choosing inputs, households choosing labor supply and their portfolio allocation between domestic capital, domestic real bonds and foreign real bonds, and governments financing their consumption needs through the given tax regimes. That decentralized problem would induce identical allocations to the planning problem we analyze here.
with the net investment rate defined as

\[ inv_t = \frac{X_t + \phi \left( \frac{X_t}{K_t} - \psi \right)^2 - \delta_t K_t}{Y_t - \delta_t K_t} \]  

(3.14)

### 3.3 Equilibrium and Solution

#### Equilibrium

Given the fiscal policy \( \{G_t, G^*_t, \tau_{h,t}, \tau^*_{h,t}, \tau_{r,t}, \tau^*_{r,t}\} \), the equilibrium comprises allocations \( \{C_t, C^*_t, H_t, H^*_t, X_t, X^*_t, K_{t+1}, K^*_{t+1}\} \) such that following conditions are satisfied:

- the international risk sharing equation
- home and foreign intratemporal Euler equations which describe the labor leisure choice in each country
- home and foreign intertemporal Euler equations which characterize the investment decision in each country
- home and foreign capital accumulation equations
- the world resource constraint

#### Stationary Representation

The model has a unique steady state if written in terms of per efficiency units.

\[ S_{a,t} h_t \tilde{c}_t = h^*_t \tilde{c}^*_t \]  

(3.15)

\[ \frac{\alpha h_t \tilde{c}_t}{1 - h_t} = (1 - \theta)(1 - \tau_{h,t})(\tilde{k}_t)^\theta \]  

(3.16)

\[ \frac{\alpha h^*_t \tilde{c}^*_t}{1 - h^*_t} = (1 - \theta)(1 - \tau^*_{h,t})(\tilde{k}^*_t)^\theta \]  

(3.17)
\[
\frac{\gamma_t h_{t+1} \tilde{c}_{t+1}}{\beta h_t \tilde{c}_t} \left[ 1 + 2 \phi \left( \frac{\tilde{x}_t}{\tilde{k}_t} - \psi \right) \right]
\]

\[
= 1 + (1 - \tau_{k,t+1}) \left[ \theta (\tilde{k}_{t+1})^{\phi-1} - \delta_{t+1} \right]
+ \phi \left( \frac{\tilde{x}_{t+1}}{\tilde{k}_{t+1}} - \psi \right) \left[ \frac{\tilde{x}_{t+1}}{\tilde{k}_{t+1}} + \psi + 2(1 - \delta_{t+1}) \right]
\]

(3.18)

\[
\frac{\gamma_t^* h_{t+1}^* \tilde{c}_{t+1}^*}{\beta h_t^* \tilde{c}_t^*} \left[ 1 + 2 \phi \left( \frac{\tilde{x}_t^*}{\tilde{k}_t^*} - \psi^* \right) \right]
\]

\[
= 1 + (1 - \tau_{k,t+1}^*) \left[ \theta (\tilde{k}_{t+1}^*)^{\phi-1} - \delta_{t+1}^* \right]
+ \phi \left( \frac{\tilde{x}_{t+1}^*}{\tilde{k}_{t+1}^*} - \psi^* \right) \left[ \frac{\tilde{x}_{t+1}^*}{\tilde{k}_{t+1}^*} + \psi^* + 2(1 - \delta_{t+1}^*) \right]
\]

(3.19)

\[
\gamma_{n,t} \gamma_t h_{t+1} \tilde{k}_{t+1} = h_t \left[ (1 - \delta_t) \tilde{k}_t + \tilde{x}_t \right]
\]

(3.20)

\[
\gamma_{n,t}^* \gamma_t^* h_{t+1}^* \tilde{k}_{t+1}^* = h_t^* \left[ (1 - \delta_t^*) \tilde{k}_t^* + \tilde{x}_t^* \right]
\]

(3.21)

\[
S_{n,t} S_{a,t} h_t \left[ (1 - g_t) (\tilde{k}_t)^{\phi} - \tilde{c}_t - \tilde{x}_t - \phi \tilde{k}_t \left( \frac{\tilde{x}_t}{\tilde{k}_t} - \psi \right) \right]^2
\]

\+

\[
h_t^* \left[ (1 - g_t^*) (\tilde{k}_t^*)^{\phi} - \tilde{c}_t^* - \tilde{x}_t^* - \phi \tilde{k}_t^* \left( \frac{\tilde{x}_t^*}{\tilde{k}_t^*} - \psi^* \right) \right]^2 = 0
\]

(3.22)

\( \gamma_{n,t} \) is the population growth rate, \( \gamma_{n,t} = \frac{N_{t+1}}{N_t} \). \( \gamma_t \) is the TFP factor (TFPF) growth rate, \( \gamma_t = \frac{a_{t+1}}{a_t} \). Following the literature, the adjustment cost is specified such that there is no cost occurred at the steady state. Hence, \( \psi = \bar{\gamma}_n \bar{\gamma} - 1 + \bar{\delta} \). Note that variables denoted with bars represent the corresponding steady state value. To be consistent with balanced growth, the population growth rates and the TFPF growth rates have to be equal across countries in the steady state (\( \bar{\gamma}_n = \bar{\gamma}_n^* \) and \( \bar{\gamma} = \bar{\gamma}^* \)). In addition, we assume \( \bar{\delta} = \bar{\delta}^* \), which implies that \( \psi = \psi^* \).

\( S_{a,t} \) is the relative TFPF between home and foreign, \( S_{a,t} = \frac{a_t}{a_t^*} \). \( S_{n,t} \) is the relative population size between the two countries, \( S_{n,t} = \frac{N_t}{N_t^*} \). Once \( S_{a,0} \) and \( S_{n,0} \) are given, the entire series of \( S_{a,t} \) and \( S_{n,t} \) can be derived recursively from the sequences of \( \{ \gamma_t, \gamma_{n,t} \}_{t=0}^\infty \)

\[
S_{a,t} = \frac{\gamma_t}{\gamma_t^*} S_{a,t-1}
\]

(3.23)
Chapter 3. Explaining Saving Behavior

\[ S_{n,t} = \frac{\gamma_{n,t}}{\gamma^*_t} S_{n,t-1} \]  \hspace{1cm} (3.24)

**Algorithm**

Our strategy is to compute the transition dynamics of an open economy given the initial capital-output ratio in each country. We obtain our solution by numerically solving a system of non-linear equations. The baseline two-country model has eight endogenous variables with eight associated equilibrium conditions at each date. We allow 200 periods for the economy to converge to the steady state. We stack all the equations together and solve the resulting system. Thus, the stacked system has 1,600 equations and 1,600 unknowns. The sparsity of the Jacobian matrix makes solving such a huge nonlinear system practical. The Newton-Raphson algorithm is much more efficient and stable than the more commonly used shooting method to solve such models. This is especially true in our two-country context where there are more than one state variable.

### 3.4 Data and Calibration

The data for exogenous driving forces\(^{35}\) are from Imrohoroglu et al (2008). The sample period is 1960-2004. Hence, the economy converges to the steady state in 2159.\(^{36}\) The depreciation rates are assumed to be the same across countries. The steady state values for these exogenous driving forces, which are also the values for 2005-2159, are set to be the followings (see Table 3.1). To be consistent with balanced growth, \(\psi = \psi^* = 0.0786\).

\(^{35}\) \{\(\gamma_t, \gamma^*_t, \gamma_{n,t}, \gamma^*_{n,t}, g_t, g^*, \delta_t, \delta_{h,t}, \delta^*_{h,t}, \delta_{k,t}, \delta^*_{k,t}\)\}\(^{2004}\) \(t=1960\)

\(^{36}\) Imrohoroglu et al (2008) also examine an open economy model with international trade of risk-free bond. They conclude that the model can still predict saving and investment rates reasonably well. However, that is because they assume an unrealistically small size for the ROW relative to the U.S. In other words, although they look at a two-country world, one region (the ROW) is so small that it has very limited effect on the big country (the U.S.). Therefore, their exercise does not differentiate from the closed-economy case very much. Once the actual relative size between the ROW and the U.S. is fed in, the results become similar to ours.
In order to show that the standard neoclassical growth model explains investment rather than saving, we keep the calibration as close as possible to Imrohoroglu et al (2008). The model has four invariant parameters. The labor disutility scale parameter $\alpha$ is set to 1.4480 so that the steady state fraction of time spent working equals to one thirds in the U.S. The subjective discount factor $\beta$ is set to 0.9759 so that the steady state capital output ratio is 3.2 in the U.S. The capital share of output $\theta$ is set to 0.4, which is the average capital share in the U.S. for the period 1960-2004. Furthermore, the adjustment cost parameter $\phi$ is set to 2 to match the volatility of investment in the U.S. The initial capital output ratio for the U.S. is set to 3.5, while the initial capital stock of the ROW is set to be 1.55 times that in the U.S.

The steady state population ratio between the U.S. and the ROW $\bar{S}_n$ is set to 0.55, which matches the actual relative size of population between the two countries in 2004. The steady state TFPF ratio between the U.S. and the ROW $\bar{S}_o$ is set to 0.9383 so that the steady state consumption output ratio is 0.5884 in the U.S. This assures that there is no current account deficit at the steady state. Moreover, the initial relative TFPF between the U.S. and the ROW $S_{o,1960}$ is set to 1.5922 to be consistent with the exogenous series for TFPF growth rates $\{\gamma_t, \gamma^*_t\}_{t=1960}^{2159}$ and the fact that the steady state is reached in 2159 according to equation (3.23). Similarly, the initial relative population between the U.S. and the ROW $S_{n,1960}$ is set to 0.3955 to satisfy equation (3.24) given $\{\gamma_{n,t}, \gamma^*_{n,t}\}_{t=1960}^{2159}$. 

<table>
<thead>
<tr>
<th>Parameter</th>
<th>US</th>
<th>ROW</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\gamma}$</td>
<td>1.0184</td>
<td>1.0184</td>
</tr>
<tr>
<td>$\bar{\gamma}_n$</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>$\bar{g}$</td>
<td>0.16</td>
<td>0.18</td>
</tr>
<tr>
<td>$\bar{\delta}$</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>$\bar{\tau}_k$</td>
<td>0.42</td>
<td>0.34</td>
</tr>
<tr>
<td>$\bar{\tau}_h$</td>
<td>0.29</td>
<td>0.38</td>
</tr>
</tbody>
</table>
Chapter 3. Explaining Saving Behavior

3.5 Results

We now show our basic quantitative results. We shall treat the U.S. as the home country and the ROW as the foreign country in all our simulations. We report eight sets of results: (1) the benchmark case with all shocks; (2) only TFP shocks in which all the other exogenous driving forces are set to their steady state values; (3) all except TFP shocks; (4) only government expenditure shocks; (5) only demographic shocks; (6) only depreciation shocks; (7) only capital income tax shocks; (8) only labor income tax shocks.

We start by studying the closed economy case for the U.S. This is done by making the two countries symmetric, i.e. U.S. vs U.S.. We apply the U.S. time series for exogenous variables to both countries in the model. This implies that the relative levels of all the exogenous variables is unity throughout the simulation. Alternatively, we can assume the U.S. is huge compared to the ROW. If we set $\bar{S}_n = 10000$ and adjust the corresponding $S_n,1960$, the results will be identical. These two approaches effectively makes the model a closed economy. Figure 3.1 plots the model generated U.S. net saving to net GDP ratio and net investment to net GDP ratio with respect to the data. The star line represents the data, while the solid line represents the model prediction. clearly, the closed economy version does a good job of explaining both the saving and the investment rates for the U.S. Thus, the model replicates the principal findings of Imrohoroglu et al (2008). However, it is hard to tell which one is fitted better, given the fact that saving and investment are not very different in the data.

Next we study the open economy case by reverting to the two-country model. Here we use the region specific time series for the exogenous variables for the U.S. and the ROW. Figure 3.2 plots the benchmark net saving rate and the net investment rate with respect to the data when all the exogenous variables are used. Figure 3.3 plots the net saving rate and the net investment rate with respect to the data when we only use the TFP series as exogenous drivers. In this and the following counterfactual experiments, the other exogenous variables are held constant at their mean levels during
the sample period which are also their assumed steady state levels. Figure 3.4 plots the net saving rate and the net investment rate with respect to the data when all exogenous variables except the TFP series are used. Figure 3.5 plots the net saving rate and the net investment rate with respect to the data when only government expenditures vary across countries and over time. Figure 3.6 plots the demographically driven net saving rate and net investment rate with respect to the data. Figure 3.7 plots the net saving rate and the net investment rate with respect to the data when only depreciation rate series are used. Figure 3.8 plots the net saving rate and the net investment rate with respect to the data driven by capital income taxes. Finally, Figure 3.9 plots the net saving rate and the net investment rate with respect to the data driven by labor income taxes.

The above simulations tell a similar story. In all the experiments, the predicted investment rate is close to that in the data. This is true for both the U.S. and the ROW. In addition, TFP explains most of the dynamics of the model, which is consistent with the previous literature. Saving, on the other hand, is a different story. The model tends to overpredict saving for the U.S. and underpredict saving for the ROW in all the simulations. In effect the observed saving behavior is unexplained by the model.

We interpret these results as saying that the neoclassical model does a good job of predicting investment behavior. It is far less accurate in predicting saving behavior. The intuition is as following. Investment decisions are based on the future return on capital. Hence, the higher the TFP growth rate, the more incentives for investing. There is not much international factor entering into this decision. Figure 3.10 plots the two countries' benchmark net investment rates with respect to their corresponding TFPF growth rate. We can see that the U.S. TFPF growth rate is slightly trending up over the sample period. The model also predicts the U.S. investment going up. For the ROW, the TFPF growth rate first rises in 1960s, but descends since the early 1970s. Thus, the model predicts the ROW investment climbing up first, then crawling down.

In the presence of international consumption risk sharing, it is efficient to let the more productive country produce more and share its output with
the less productive country. Alternatively speaking, the country with a higher TFP level should become a net exporter. Figure 3.11 plots the two countries’ benchmark net saving rates with respect to their TFPF ratios. Clearly, each country’s saving rate is highly correlated with its TFP level relative to that of the other country. The correlation coefficient is 0.975 for the U.S. and 0.9 for the ROW. Both are significant at any conventional level of significance. As the U.S. is more productive than the ROW (especially in 1960s and early 1970s), the model overpredicts the U.S. saving rate and generates huge current account surplus for the U.S. in 1960s.\footnote{Notice that the series for TFPF ratios is backed out from the actual series for TFPF growth rates and the calibrated steady state level of TFPF ratio. The resulting relative TFPF has exactly the same pattern as the actual TFPF ratio, but smaller values. In the data, U.S. is much more productive than the ROW throughout the sample period. Hence, the model would have generated even larger current account surplus for the U.S., if we fit in the actual TFPF ratio.} On the flip side, the model underpredicts the ROW saving rate and generates huge current account deficit for the ROW.

Therefore, the model fails to predict saving is highlighted by the fact that when saving and investment become delinked from each other in an open economy environment, the model underperforms on the saving margin while its investment prediction remains relatively unchanged. The reason the closed economy neoclassical model is able to explain saving is that it imposes an identity between saving and investment. As the model predicts investment accurately and because the gap between saving and investment is relatively small in the data, the model generated saving tracks the saving data reasonably well.

### 3.6 Conclusion

The closed economy neoclassical growth model has proved to be a worthy workhorse model for understanding and explaining a number of issues in macroeconomics. However, the model has trouble explaining some features of open economies. One example of this is the current account. The standard version of the open economy neoclassical model has difficulty in
Chapter 3. Explaining Saving Behavior

explaining the observed behavior of the current account. In this paper we have shown that this difficulty is the mirror image of the fact that while the model can predict investment behavior reasonably accurately, it does not explain saving. Hence, its current account predictions are often inaccurate. We have illustrated this fact using the example of the U.S. We show that while the investment predictions of the closed and open economy versions of the neoclassical model are similar and, crucially, very close to the data, the saving predictions are not. In particular, the open economy version of the model tends to overpredict the U.S. saving and underpredict saving in the ROW. The closed economy version of the model does well in explaining saving only because it imposes equality between saving and investment and the fact that saving and investment are relatively close in the data. Effectively, the closed economy model explains investment rather than saving. We consider the observed saving behavior a puzzle and we hope to address the saving puzzle in future work.
Figure 3.1: U.S. Saving and Investment - Closed Economy
Figure 3.2: U.S. Saving and Investment - Benchmark
Chapter 3. Explaining Saving Behavior

Figure 3.3: U.S. Saving and Investment - Only TFP
Chapter 3. Explaining Saving Behavior

Figure 3.4: U.S. Saving and Investment - All Except TFP
Chapter 3. Explaining Saving Behavior

![Figure 3.5: U.S. Saving and Investment - Only Government Expenditure](image-url)
Chapter 3. Explaining Saving Behavior

Figure 3.6: U.S. Saving and Investment - Only Population Growth
Chapter 3. Explaining Saving Behavior

Figure 3.7: U.S. Saving and Investment - Only Depreciation Rate
Chapter 3. Explaining Saving Behavior

Figure 3.8: U.S. Saving and Investment - Only Capital Income Taxes

- U.S. Saving Data
- ROW Saving Data

- U.S. Investment Data
- ROW Investment Data
Chapter 3. Explaining Saving Behavior

Figure 3.10: Net Investment Rates vs TFPF Growth Rates - Benchmark
Chapter 3. Explaining Saving Behavior

Figure 3.11: Net Saving Rates vs TFPF Ratios - Benchmark

- U.S. TFPF / ROW TFPF
- ROW Saving / ROW TFPF
- U.S. Saving / U.S. TFPF
- ROW Saving / U.S. TFPF
Bibliography


manuscript, Middle East Technical University and University of St. Andrews, 2005.


Appendix A

Appendix for Chapter 1

A.1 First-Order Approximation

The system consists of equations (1.5), (1.6), (1.12), (1.16)-(1.18), (1.21),
(1.24)-(1.25), (1.30), and their foreign counterparts, as well as (1.8), (1.26)-(1.29),
and (1.31). Some of the equilibrium conditions are log-linear in themselves, such as the wage equations, the money demand equations, the flexible prices, and the asset market clearing condition.

\[
\dot{w} = \dot{P} + \rho \dot{C} + \psi \dot{L} \quad \text{(A.1)}
\]
\[
\dot{w}^* = \dot{P}^* + \rho \dot{C}^* + \psi \dot{L}^* \quad \text{(A.2)}
\]
\[
\dot{M} = \dot{P} + \rho \dot{C} \quad \text{(A.3)}
\]
\[
\dot{M}^* = \dot{P}^* + \rho \dot{C}^* \quad \text{(A.4)}
\]
\[
\dot{P}_{flx,H} = \dot{w} - \dot{A} \quad \text{(A.5)}
\]
\[
\dot{P}_{flx,F} = \dot{w}^* - \dot{A}^* \quad \text{(A.6)}
\]
\[
\bar{\alpha} = -\bar{\alpha}^* \quad \text{(A.7)}
\]

where \(\bar{\alpha} = \frac{\bar{\alpha}}{\bar{\alpha}^*}\). First-order approximations of the other equilibrium conditions are given by

\[
\dot{P}_{pre,H} = 0 + O(\epsilon^2) \quad \text{(A.8)}
\]
\[
\dot{P}_{pre,F} = 0 + O(\epsilon^2) \quad \text{(A.9)}
\]
\[
\dot{P}_H = \kappa \dot{P}_{pre,H} + (1 - \kappa) \dot{P}_{flx,H} + O(\epsilon^2) \quad \text{(A.10)}
\]
\[
\dot{P}_F = \kappa \dot{P}_{pre,F} + (1 - \kappa) \dot{P}_{flx,F} + O(\epsilon^2) \quad \text{(A.11)}
\]
\[
\dot{P} = \frac{1}{2}(\dot{P}_H + \dot{P}_F + \dot{S}) + O(\epsilon^2) \quad \text{(A.12)}
\]
\[ \hat{P}^* = \frac{1}{2} (\hat{P}_H + \hat{P}_F^* - \hat{S}) + O(\epsilon^2) \] 
\[ \hat{\Pi} = \frac{1}{1 - \hat{\zeta}} \left[ \hat{P} + \hat{Y} - \hat{\zeta}(\hat{w} + \hat{L}) \right] + O(\epsilon^2) \] 
\[ \hat{\Pi}^* = \frac{1}{1 - \hat{\zeta}} \left[ \hat{P}^* + \hat{Y}^* - \hat{\zeta}(\hat{w}^* + \hat{L}^*) \right] + O(\epsilon^2) \] 
\[ \hat{Y} = (1 - \theta)(\hat{P}_H - \hat{P}) + \frac{1}{2}(\hat{C} + \hat{C}^*) + O(\epsilon^2) \] 
\[ \hat{Y}^* = (1 - \theta)(\hat{P}_F^* - \hat{P}^*) + \frac{1}{2}(\hat{C} + \hat{C}^*) + O(\epsilon^2) \] 
\[ \hat{L} = -\hat{A} - \theta(\hat{P}_H - \hat{P}) + \frac{1}{2}(\hat{C} + \hat{C}^*) + O(\epsilon^2) \] 
\[ \hat{L}^* = -\hat{A}^* - \theta(\hat{P}_F^* - \hat{P}^*) + \frac{1}{2}(\hat{C} + \hat{C}^*) + O(\epsilon^2) \] 
\[ \hat{C} = \hat{Y} + \hat{\alpha}'\hat{r}_x + O(\epsilon^2) \] 
\[ \hat{r}_B = -\hat{P} + O(\epsilon^2) \] 
\[ \hat{r}_B^* = \hat{Q} - \hat{P}^* + O(\epsilon^2) \] 
\[ \hat{r}_E = \hat{\Pi} - \hat{P} + O(\epsilon^2) \] 
\[ \hat{r}_E^* = \hat{Q} + \hat{\Pi}^* - \hat{P}^* + O(\epsilon^2) \]

\( \hat{\zeta} \) is the share of labor income at the non-stochastic steady state, \( \hat{\zeta} = \frac{\hat{\bar{w}}\hat{\bar{L}}}{\hat{\bar{\Pi}} + \hat{\bar{w}}\hat{\bar{L}}} \).
\( \hat{r}_x \) is the vector of relative returns between each asset and the reference asset \( n \), \( \hat{r}_x = [\hat{r}_1 - \hat{r}_n, \hat{r}_2 - \hat{r}_n, \ldots, \hat{r}_{n-1} - \hat{r}_n] \).

### A.2 Asset Returns, Exchange Rate, and Terms of Trade

From equations (A.1)-(A.4), (A.12)-(A.19), and (A.21)-(A.24), the excess return on foreign bonds and foreign equities can be written as

\[ \hat{r}_{B^*} - \hat{r}_B = \hat{S} + O(\epsilon^2) \] 
\[ \hat{r}_{E^*} - \hat{r}_E = \hat{\Pi}^* - \hat{\Pi} + \hat{S} + O(\epsilon^2) \]

\[
= \frac{1}{1 - \hat{\zeta}} \left\{ -\hat{\zeta}\hat{S} + [\theta - 1 - \theta\hat{\zeta}(\psi + 1)]\hat{\tau} + \hat{\zeta}(\hat{M} - \hat{M}^*) - \hat{\zeta}(\psi + 1)(\hat{A} - \hat{A}^*) \right\} + O(\epsilon^2) \]
Appendix A. Appendix for Chapter 1

Flexible Price Case: $\kappa = 0$

When goods prices are fully flexible,

\[
\hat{P}_H = \hat{w} - \hat{A} \quad \text{(A.27)}
\]
\[
\hat{P}_F^* = \hat{w}^* - \hat{A}^* \quad \text{(A.28)}
\]

Substitute equations (A.27)-(A.28) into the definition of the terms of trade and equation (A.26), we have

\[
\hat{r}_E^* - \hat{r}_E = (\theta - 1)\hat{\tau} + O(\epsilon^2) \quad \text{(A.29)}
\]

It is easy to see that achieving perfect consumption risk sharing up to first-order ($\hat{c} - \hat{c}^* = 0 + O(\epsilon^2)$) requires households hold half of foreign equities ($\hat{\alpha}_{E^*} = \frac{1}{2}$). In this case,

\[
\hat{S} = \hat{M} - \hat{M}^* + O(\epsilon^2) \quad \text{(A.30)}
\]
\[
\hat{\tau} = -\frac{1 + \psi}{1 + \psi \theta} (\hat{A} - \hat{A}^*) + O(\epsilon^2) \quad \text{(A.31)}
\]

Sticky Price Case: $\kappa = 1$

When goods prices are all pre-set,

\[
\hat{P}_H = 0 + O(\epsilon^2) = \hat{P}_F^* \quad \text{(A.32)}
\]

which implies that $\hat{\tau} = -\hat{S} + O(\epsilon^2)$. From equation (A.26), we have

\[
\hat{r}_E^* - \hat{r}_E = \frac{1}{1 - \zeta} \{[\theta - 1 + \zeta - \theta \zeta (\psi + 1)]\hat{\tau} + \zeta (\hat{M} - \hat{M}^*) - \zeta (\psi + 1)(\hat{A} - \hat{A}^*)\} + O(\epsilon^2) \quad \text{(A.33)}
\]

There is complete risk sharing in the FBE economy. Hence, the exchange rate as well as the terms of trade respond only to money supply shocks up to first-order ($\hat{S} = \hat{M} - \hat{M}^* + O(\epsilon^2)$). However, the consumption risk sharing
Appendix A. Appendix for Chapter 1

is generally incomplete in the FE economy. In this case, the exchange rate and the terms of trade will react to both the productivity shocks and the money supply shocks.

A.3 Log-linearized Sticky Wage Model

As wages are chosen in advance of production and consumption, log-linearized wage equations will substitute for the labor-leisure choice functions (A.1)-(A.2) in the set of log-linearized equilibrium conditions

\[
\begin{align*}
\dot{w} &= 0 + O(\epsilon^2) \quad (A.34) \\
\dot{w}^* &= 0 + O(\epsilon^2) \quad (A.35)
\end{align*}
\]

From equations (A.3)-(A.4), (A.12)-(A.19), A.21)-(A.24), and (A.34)-(A.35), the excess return on foreign bonds and foreign equities can be written as

\[
\begin{align*}
\hat{\tau} &- \hat{\tau}_B = \hat{S} + O(\epsilon^2) \\
\hat{\tau}^* &- \hat{\tau}_E = \hat{\Pi}^* - \hat{\Pi} + \hat{S} + O(\epsilon^2) \\
&= \frac{1}{1-\zeta}\{-\zeta \hat{S} + \theta(1-\zeta) - 1\hat{\tau} - \zeta(\hat{\tau}_B - \hat{\tau}_B^*)\} + O(\epsilon^2)
\end{align*}
\]

Flexible Price Case: \( \kappa = 0 \)

When goods prices are fully flexible,

\[
\begin{align*}
\hat{P}_H &= \hat{A} \\
\hat{P}_E^* &= \hat{A}^*
\end{align*}
\]

Substitute equations (A.38)-(A.39) into the definition of the terms of
trade and equation (A.37), we have
\[
\hat{r}_{E^*} - \hat{r}_E = (\theta - 1)\hat{\tau} + O(\epsilon^2) \quad (A.40)
\]

Similar to the case with flexible wages, achieving perfect consumption risk sharing up to first-order \((\hat{c} - \hat{c}^* = 0 + O(\epsilon^2))\) requires households hold half of foreign equities \((\tilde{\alpha}_{E^*} = \frac{1}{2})\). Here,
\[
\hat{S} = \hat{M} - \hat{M}^* + O(\epsilon^2) \quad (A.41)
\]
\[
\hat{\tau} = -(\hat{M} - \hat{M}^*) - (\hat{A} - \hat{A}^*) + O(\epsilon^2) \quad (A.42)
\]

When goods prices are flexible, the portfolio choices are the same whether wages are fixed or not. However, it is interesting to see that the terms of trade respond not only to the productivity shocks, but also the money supply shocks due to sticky wages. This clearly will affect the welfare.

**Sticky Price Case: \(\kappa = 1\)**

When goods prices are all pre-set, we have \(\hat{\tau} = -\hat{S} + O(\epsilon^2)\). Thus, the FBE economy generates the same results as before and both the exchange rate and the terms of trade respond only to money supply shocks up to first-order \((\hat{S} = \hat{M} - \hat{M}^* + O(\epsilon^2))\). While from equation (A.37), we know
\[
\hat{r}_{E^*} - \hat{r}_E = (\theta - 1)\hat{\tau} - \frac{\zeta}{1 - \zeta}(\hat{A} - \hat{A}^*) \quad (A.43)
\]
In general, the consumption risk sharing is going to be imperfect in the FE economy.
Appendix B

Appendix for Chapter 2

B.1 A Symmetric Steady State

In a non-stochastic steady state, there is no shock, \( \bar{A} = \bar{A}^* = \bar{z} = \bar{z}^* = 1 \). Note steady state values are marked by overbars.

Since returns on all assets are equal, asset holdings are indeterminate in a non-stochastic steady state. However, asset positions are often well defined in a stochastic environment. Devereux and Sutherland (2006a) [20] prove that the solution of their method corresponds to a stochastic equilibrium portfolio allocation that is arbitrarily close to the non-stochastic equilibrium.

All the prices are equal and the steady state exchange rate \( \bar{S} = 1 \). The steady state values of other variables are

\[
\bar{Y} = \bar{Y}^* = \bar{L} = \bar{L}^* = \bar{C} = \bar{C}^* = (\eta)^{1 - \frac{1}{\rho + \phi}}
\]

(B.1)

where we have applied \( \gamma = \frac{1}{\phi - 1} \) given the production subsidy exactly offsets the distortion due to monopoly power.

B.2 First-Order Approximation

The system consists of equations (2.5), (2.6), (2.12), (2.16)-(2.18), (2.21), (2.29), and their foreign counterparts, as well as (2.8), (2.22)-(2.27), and (2.30). Some of the equilibrium conditions are log-linear in themselves, such as the wage equations, the money demand and supply equations, the virtual price of each country’s goods, and the asset market clearing condition.

\[
\hat{w} = \hat{P} + \rho \hat{C} + \psi \hat{L}
\]

(B.2)
Appendix B. Appendix for Chapter 2

\begin{align*}
\hat{w}^* &= \hat{P}^* + \rho \hat{C}^* + \psi \hat{L}^* \\
\hat{M} &= \hat{\chi} + \hat{P} + \rho \hat{C} \\
\hat{M}^* &= \hat{\chi}^* + \hat{P}^* + \rho \hat{C}^* \\
\hat{M} &= -\delta_s \hat{S} - \delta_p \hat{P}^X \\
\hat{M}^* &= \delta_s \hat{S} - \delta_p \hat{P}^*_X \\
\hat{P}^X &= \hat{w} - \hat{A} \tag{B.21}
\end{align*}

where \( \tilde{\alpha} = \frac{\bar{\alpha}}{Y} \). First-order approximations of the other equilibrium conditions are given by

\begin{align*}
\hat{P}_H &= 0 + O(\epsilon^2) \\
\hat{P}_F^* &= 0 + O(\epsilon^2) \tag{B.11} \\
\hat{P} &= \frac{1}{2}(\hat{P}_H + \hat{P}_F^* + \hat{S}) + O(\epsilon^2) \tag{B.12} \\
\hat{P}^* &= \frac{1}{2}(\hat{P}_H + \hat{P}_F^* - \hat{S}) + O(\epsilon^2) \tag{B.13} \\
\hat{\Pi} &= \frac{1}{1-\zeta} \left[ \hat{P} + \hat{Y} - \zeta (\hat{w} + \hat{L}) \right] + O(\epsilon^2) \tag{B.14} \\
\hat{\Pi}^* &= \frac{1}{1-\zeta} \left[ \hat{P}^* + \hat{Y}^* - \zeta (\hat{w}^* + \hat{L}^*) \right] + O(\epsilon^2) \tag{B.15} \\
\hat{Y} &= (1-\theta)(\hat{P}_H - \hat{P}) + \frac{1}{2}(\hat{C} + \hat{C}^*) + O(\epsilon^2) \tag{B.16} \\
\hat{Y}^* &= (1-\theta)(\hat{P}_F^* - \hat{P}^*) + \frac{1}{2}(\hat{C} + \hat{C}^*) + O(\epsilon^2) \tag{B.17} \\
\hat{L} &= -\hat{A} - \theta(\hat{P}_H - \hat{P}) + \frac{1}{2}(\hat{C} + \hat{C}^*) + O(\epsilon^2) \tag{B.18} \\
\hat{L}^* &= -\hat{A}^* - \theta(\hat{P}_F^* - \hat{P}^*) + \frac{1}{2}(\hat{C} + \hat{C}^*) + O(\epsilon^2) \tag{B.19} \\
\hat{C} &= \hat{Y} + \tilde{\alpha} \hat{r}_x + O(\epsilon^2) \tag{B.20} \\
\hat{r}_B &= -\hat{P} + O(\epsilon^2) \tag{B.21} \\
\hat{r}_B^* &= \hat{Q} - \hat{P}^* + O(\epsilon^2) \tag{B.22} \\
\hat{r}_E &= \hat{\Pi} - \hat{P} + O(\epsilon^2) \tag{B.23}
\end{align*}
$$\hat{r}_{E^*} = \hat{Q} + \hat{P}^* - \hat{P}^* + O(\varepsilon^2)$$ (B.25)

$\zeta$ is the share of labor income at the non-stochastic steady state, $\zeta = \frac{\bar{w}\bar{L}}{\bar{w}\bar{L} + \bar{w}\bar{L}}$.

$\hat{r}_x$ is the vector of relative returns between each asset and the reference asset $n$, $\hat{r}_x' = [\hat{r}_1 - \hat{r}_n, \hat{r}_2 - \hat{r}_n, \cdots, \hat{r}_{n-1} - \hat{r}_n]$.

### B.3 Proof of Proposition 1

Here, we focus on the case with all flexible prices and the case with all sticky prices. In addition, we just compare the financial autarky with the complete financial markets. Intermediate cases, such as only a fraction of firms can set prices freely and the international risk sharing is imperfect, can be inferred easily from the extreme cases shown below.

#### Sticky Price Case

From equations (2.50), (B.11)-(B.12), we have

$$\hat{\tau} = -\hat{S} + O(\varepsilon^2)$$ (B.26)

Substitute equation (2.53) into (2.49) and use (B.26), we get

$$\hat{\tau} = \frac{\hat{\chi} - \hat{\chi}^*}{1 + \rho(\theta - 1)} \text{ financial autarky}$$ (B.27)

$$= \hat{\chi} - \hat{\chi}^* \text{ complete financial markets}$$ (B.28)

Thus, the volatility of the terms of trade are given by

$$\text{Var}(\hat{\tau}) = \frac{2\sigma^2_{\chi}}{[1 + \rho(\theta - 1)]^2} \text{ financial autarky}$$ (B.29)

$$= 2\sigma^2_{\chi} \text{ complete financial markets}$$ (B.30)

As $\rho > 0$, the terms of trade are more volatile under complete financial markets than that under financial autarky when $\theta > 1$.  

107
Flexible Price Case

In this case, $\hat{P}_H = \hat{w} - \hat{A}$ and $\hat{P}_F^* = \hat{w}^* - \hat{A}^*$. Combined with the wage equations (B.2)-(B.3) and the labor demand equations (B.19)-(B.20), we have

\[
\hat{r} = \frac{(1 + \psi)(\hat{A} - \hat{A}^*)}{1 + \rho(\theta - 1) + \psi \theta} \quad \text{financial autarky} \tag{B.31}
\]

Thus, the volatility of the terms of trade are given by

\[
Var(\hat{\tau}) = \frac{2(1 + \psi)^2 \sigma_A^2}{[1 + \rho(\theta - 1) + \psi \theta]^2} \quad \text{financial autarky} \tag{B.33}
\]

\[
= 2 \sigma_A^2 \quad \text{complete financial markets} \tag{B.34}
\]

As $\rho > 0$ and $\psi \geq 0$, the terms of trade are more volatile under complete financial markets than that under financial autarky when $\theta > 1$.

B.4 Proof of $E(\hat{L}) + \frac{1}{2} E(\hat{L}^2) = 0 + O(\epsilon^3)$

When $\rho = 1$ and $\psi = 0$, equations (2.45), (2.46), (B.4), and (B.5) imply that

\[
\hat{P}_H = E(\hat{M}) + \lambda_{PH} + O(\epsilon^3), \quad \hat{P}_F^* = E(\hat{M}^*) + \lambda_{P_F^*} + O(\epsilon^3) \tag{B.35}
\]

Combined with (2.35) and (2.36), we can show that

\[
E(\hat{C} + \hat{C}^*) = -(\lambda_{PH} + \lambda_{P_F^*}) - 2E(\lambda_P) + O(\epsilon^3) \tag{B.36}
\]

Furthermore, equations (2.33), (2.34), (2.37), and (2.38) indicate

\[
\lambda_Y = (1 - \theta)\lambda_P - \frac{1}{2}(\lambda_C + \lambda_{C^*}) + O(\epsilon^3) \tag{B.37}
\]
Substitute (B.36) and (B.37) into equations (2.39) and (2.40), we have

\[ E(\hat{L} + \hat{L}^*) = -(\lambda_{P_H} + \lambda_{P_F}) - E(\lambda_C + \lambda_{C^*}) + O(\epsilon^3) \]  

(B.38)

Moreover, substitute (B.11), (B.12), (B.13), (B.14), (B.17), (B.18), (B.21), and its foreign counterpart into equations (2.43), (2.44), (2.47), and (2.48), we get

\[ \lambda_C + \lambda_{C^*} = -\tilde{\alpha}\hat{r}_x(\theta - 1)\hat{S} - (\tilde{\alpha}\hat{r}_x)^2 + O(\epsilon^3) \]  

(B.39)

\[ \lambda_{P_H} = \frac{1}{2} E \left[ \left( \frac{1}{2}\hat{S} + \hat{C} - \hat{A} \right)^2 - 2\tilde{\alpha}\hat{r}_x \left( \frac{1}{2}\hat{S} + \hat{C} - \hat{A} \right) \right] + O(\epsilon^3) \]  

(B.40)

\[ \lambda_{P_F} = \frac{1}{2} E \left[ \left( -\frac{1}{2}\hat{S} + \hat{C}^* - \hat{A}^* \right)^2 + 2\tilde{\alpha}\hat{r}_x \left( -\frac{1}{2}\hat{S} + \hat{C}^* - \hat{A}^* \right) \right] + O(\epsilon^3) \]  

(B.41)

Note that \( \tilde{\alpha}\hat{r}_x = \frac{1 - \theta}{\hat{r}} \hat{S} + \frac{1}{2}(\hat{C} - \hat{C}^*) \) up to first order. Finally, with equations (B.19) and (B.20), we can show that

\[ E(\hat{L} + \hat{L}^*) + \frac{1}{2} E(\hat{L}^2 + \hat{L}^{*2}) = \tilde{\alpha} E[\hat{r}_x(\hat{C} - \hat{C}^*)] + O(\epsilon^3) \]  

(B.42)

Recall the equilibrium portfolio equation (2.31), it is obvious that the right hand side of (B.42) equals zero. In other words, expected changes in the first and second moments of labor exactly offset each other. Given the two countries are ex-ante symmetric, we know \( E(\hat{L}) + \frac{1}{2} E(\hat{L}^2) = 0 \) up to a second-order accuracy. Therefore, when \( \rho = 1 \) and \( \psi = 0 \), each country’s utility is simply represented by their expected level of consumption, \( i.e. \bar{U} = E(\hat{C}) \).

### B.5 Computing the Consumption Equivalent Welfare Measure

This section gives the details of the derivation of the consumption equivalent measure \( \omega \). Consider the money targeting rule in the complete market as the reference scenario, denoted by \( r \), and an alternative scenario, denoted
Appendix B. Appendix for Chapter 2

by s. The expected utility in case r is given by

\[ U^r = \frac{(C^r)^{1-\rho}}{1-\rho} - \eta \frac{(L^r)^{1+\psi}}{1+\psi} \]  

where \( C^r \) and \( L^r \) are the corresponding consumption and labor supply. Similarly, the expected utility in case s is given by

\[ U^s = \frac{(C^s)^{1-\rho}}{1-\rho} - \eta \frac{(L^s)^{1+\psi}}{1+\psi} \]

\( \omega \) is defined as the fraction of consumption that an agent in scenario s would be willing to give up in order to make her indifferent between this and the reference scenario r. Thus, \( \omega \) can be derived from the following equality

\[ \frac{[(1-\omega)C^s]^{1-\rho}}{1-\rho} - \eta \frac{(L^s)^{1+\psi}}{1+\psi} = \frac{(C^r)^{1-\rho}}{1-\rho} - \eta \frac{(L^r)^{1+\psi}}{1+\psi} \]  

(B.45)

Let \( V_c \) denote the utility of consumption and \( V_l \) denote the disutility of labor, i.e. \( V_c = \frac{C^{1-\rho}}{1-\rho} \), \( V_l = \eta \frac{L^{1+\psi}}{1+\psi} \). Note \( U = V_c - V_l \). Equation (B.45) can be rewritten as

\[ (1-\omega)^{1-\rho}V^s_c = U^r + V^s_l \]  

(B.46)

When \( \rho \neq 1 \), we have

\[ \omega = 1 - \left( \frac{U^r + V^s_l}{V^s_c} \right)^{\frac{1}{1-\rho}} \]  

(B.47)

When \( \rho = 1 \), it is even easier

\[ \omega = 1 - \exp(U^r - U^s) \]  

(B.48)

Given that we compute utility accurately up to second-order, i.e. \( U = \bar{U} + \tilde{U} \), we restrict attention to an approximation of \( \omega \) that is accurate up to second order and omits all terms of order higher than two.
Appendix C

Statement of Co-Authorship

The third chapter in this thesis reports the results of a joint research with Profs. Michael B. Devereux and Amartya Lahiri.

The author took central roles in all stages of the research, such as identifying the research question, developing the model, acquiring the data set, and conducting the numerical simulation.