ESSAYS ON MACROECONOMIC RISK IN FINANCIAL MARKETS

by

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Abstract

This thesis contains three essays. In the first essay, I provide new evidence on the failure of the Q theory of investment. The Q theory implies the state-by-state equivalence of stock returns and investment returns. However in the data, I find that investment and stock returns are negatively correlated. I also show that a production economy with time-to-build can explain these empirical facts. When I compute Q theory based investment returns on simulated data of the time-to-build model, they are uncorrelated with simulated stock returns, as in the data. Moreover, the model replicates the empirical negative correlation between stock returns and investment growth which some researchers have interpreted as evidence for irrational markets.

In the second essay, I analyze the equilibrium effects of investment commitment on asset prices when the representative consumer has Epstein-Zin utility. Investment commitment captures the idea that long-term investment projects require not only current expenditures but also commitment to future expenditures. The general equilibrium effects of investment commitment and Epstein-Zin preferences generate endogenously time-varying first and second moments of consumption growth and stock returns. As a result, the first and second moments of excess returns are endogenously counter-cyclical, excess returns are predictable, and the equity premium increases by an order of magnitude. This paper also offers novel empirical findings regarding the predictability of returns. In the real and simulated data, the lagged investment rate helps to forecast the mean and volatility of returns.

In the third essay, we embed a structural model of credit risk inside a consumption-based model, which allows us to price equity and corporate debt in a single framework. Our key economic assumptions are that the first and second moments of earnings and consumption growth depend on the state of the economy which switches randomly, creating intertemporal risk, which agents prefer to resolve quickly because they have Epstein-Zin-Weil preferences. Our model generates co-movement between aggregate stock return volatility and credit spreads, consistent with the data, and potentially resolves the equity risk premium and credit spread puzzles.

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Dedication

Für meine Eltern.

Statement of Co-Authorship

The third essay of my thesis (Chapter 4) is joint work with Harjoat S. Bhamra and Ilya A. Strebulaev. My contribution to the paper includes building and refining the theoretical model, gathering data and estimating the model, solving the model numerically, and writing parts of the final manuscript.

Chapter 1

Introduction

Macroeconomic risk is ubiquitous. Workers face unemployment risk, firms face changing market conditions, and stock market investors face the risk of variable stock returns. Further, macroeconomic risk and asset markets are highly related. Firstly, people trade stocks in asset markets where risks are priced. Secondly, people choose their actions, i.e., consumption and labor, facing macroeconomic risk and their prices. Thus, asset prices and the allocation of economic resources are tightly intertwined and it is important to gain a better understanding of these interdependencies, which is the goal of my thesis. My thesis contains three essays that explore these links. The first two essays study how firms optimally adjust their investment decisions when they face macroeconomic shocks and investment projects are not completed instantaneously. The last essay studies how macroeconomic risk jointly affects the equity risk premium and credit spreads.

A unifying element in the first two essays is the analysis of long-term investment projects. A common assumption in the literature is that real investment occurs instantaneously. This modeling assumption is typically made to simplify the problem but it contrasts with real life investment projects where completion is not instantaneous; for instance, Advanced Micro Devices (AMD) recently announced it would build a new computer chip plant in New York State with construction beginning in mid-2007 and planned completion in 2010, costing $3.2 billion.

The contribution of my first essay is twofold: First, I provide new evidence on the failure of the Q theory of investment. Second, I show that a production economy with time-to-build can explain these empirical facts. Specifically, the Q theory implies the state-by-state equivalence of stock and investment returns—returns from investment in capital reflect the firm's tradeoff between the investment's marginal costs and marginal benefits whereas stock returns are the consumer's tradeoff of investing in the stock market. Using aggregate US data, I find that there exists a realistic parameterization of the production and investment adjustment cost function such that empirical investment returns have the same mean and standard deviation as historical stock returns, supporting the Q theory. However, state-by-state equivalence also implies that investment and stock returns should

be perfectly correlated. I find this implication is clearly rejected by the data.

I show that a production economy with time-to-build can explain these empirical facts. Time-to-build captures the idea that investment projects are not completed instantaneously. Instead, firms have to allow for several quarters to expand capacity. When I compute Q theory based investment returns on simulated data from the time-to-build model, they are uncorrelated with simulated stock returns, as in the data. Moreover, the model replicates the empirical negative correlation between stock returns and investment growth. The time-to-build model therefore provides a rational explanation for an empirical fact which some researchers have interpreted as evidence for irrational markets.

In the second essay, I study the effect of long-term investment commitment on asset prices in a general equilibrium production economy. Investment commitment is distinct from traditional time-to-build which I analyze in the first essay. The time-to-build friction is about the lag between the investment decision and the time when the new project becomes productive. My focus is instead on the equilibrium consequences of investment projects which involve not only current expenditures but also commitment to future expenditures.

The model has two ingredients. The first one is a new and tractable specification of long-term investment commitment. The second one is Epstein-Zin preferences. The model produces a more realistic equity premium and asset pricing dynamics as a result of the following mechanism: After a positive technology shock, firms initiate new investment projects to take advantage of higher productivity. As a result, the commitment level increases. If an adverse shock follows, the household would like to lower investment; however, prior commitments oblige the firm to complete initiated projects. Moreover, since investment projects are not completed instantaneously, investment commitments depress consumption for several consecutive periods. With Epstein-Zin preferences, this effect increases the equity risk premium.

The cyclicality and predictability of returns arises in the model since investment commitment, similar to irreversible investment, is an asymmetric friction. It prevents firms from disinvesting but does not hinder firms from investing. This feature implies that the impact of the friction varies over the business cycle. As a consequence, my model endogenously generates counter-cyclical consumption growth volatilities and countercyclical first and second moments of expected excess returns, consistent with the empirical evidence.

I also present new empirical results supporting my model. In the model, a higher lagged investment means that the firm has initiated large investment projects in the past, which it is committed to complete even after adverse shocks. Thus, the model predicts a positive

relation between the lagged investment rate and the first and second moments of future returns. Using aggregate data, I find that there is a positive relation between the lagged investment rate and future returns and return volatility which is significant for the latter.

In the last essay, we study how macroeconomic risk impacts credit spreads. This paper is motivated by a growing body of empirical work which indicates that common factors may affect both the equity risk premium and credit spreads on corporate bonds. In particular, there is now substantial evidence that stock returns can be predicted by credit spreads, that movements in stock-return volatility can explain movements in credit spreads and that credit spread changes across firms are driven by a single factor.

Motivated by this empirical evidence, this paper aims to provide a unified consumption-based framework for resolving both the credit spread and equity risk premium puzzles. The credit risk puzzle refers to the finding that structural models of credit risk generate credit spreads smaller than those observed in the data when calibrated to observed default frequencies. We make two assumptions. First, there is intertemporal macroeconomic risk: the expected values and volatilities (first and second moments) of fundamental economic growth rates vary with the business cycle. Second, agents prefer intertemporal risk to be resolved sooner rather than later which can be captured by Epstein-Zin preferences.

Intertemporal macroeconomic risk combined with an aversion to it makes the state-price density jump upward in recessions. Jump risk impacts both credit spreads and stock returns, generating co-movement across markets. The stock-market risk premium and credit spreads increase as the agent's dislike for regime shifts increases. The model can generate realistically high credit spreads without raising actual default probabilities and leverage. This is crucial, because in the data expected default frequencies are very small and leverage is relatively low.

Chapter 2

Disentangling Investment Returns and Stock Returns: The Importance of Time-to-Build[1]

2.1 Introduction

The workhorse of the real investment literature is the Q-theory of investment based on continuous adjustment costs. In this paper, I provide new evidence on the failure of the Q theory. The Q theory implies the state-by-state equivalence of stock and investment returns—returns from investment in capital reflect the firm's tradeoff between the investment's marginal costs and marginal benefits whereas stock returns are the consumer's tradeoff of investing in the stock market.[2] Using aggregate US data, I find that there exists a realistic parameterization of the production and investment adjustment cost function such that empirical investment returns have the same mean and standard deviation as historical stock returns, supporting the Q theory. However, state-by-state equivalence also implies that investment and stock returns should be perfectly positively correlated. Yet in the data, they are negatively correlated.

The Q theory also predicts that investment growth and stock returns should be positively correlated. When discount rates fall, investment increases in value and thus firms increase investment. At the same time, stock prices appreciate, leading to positive comovement with investment growth. But, empirically, this correlation is negative contradicting the Q-theory. From an economic perspective, both findings suggest that firms *irrationally* reduce investments when the stock market signals good times.

The goal of the paper is to rationalize these findings in a general equilibrium model with production since it is important to be able to differentiate between the effect of irrational and rational market forces on real investment—a point nicely summarized by

[1] A version of this chapter will be submitted for publication. Kuehn, L.-A., Disentangling Investment Returns and Stock Returns: The Importance of Time-to-Build.

[2] The equivalence of these two returns is a crucial assumptions for many asset pricing models, for instance, Cox, Ingersoll, and Ross (1985), Gomes, Kogan, and Zhang (2003), Hall (2001), Jermann (1998, 2005), and Liu, Whited, and Zhang (2007).

Baker, Ruback, and Wurgler (2006):

> Of paramount importance are the real consequences of market inefficiency. It is one thing to say that investor irrationality has an impact on capital market prices, or even financing policy, which lead to transfers of wealth among investors. It is another to say that mispricing leads to underinvestment, overinvestment, or the general misallocation of capital and deadweight losses for the economy as a whole.

I show that a production economy with time-to-build can explain these empirical facts. When I compute Q theory based investment returns on simulated data from the time-to-build model, they are uncorrelated with simulated stock returns, as in the data. Moreover, the model replicates the empirical negative correlation between stock returns and investment growth. The time-to-build model therefore provides a rational explanation for an empirical fact, which some researchers have interpreted as evidence for irrational markets.

Time-to-build captures the idea that investment projects are not completed instantaneously. Instead, firms have to allow for several quarters to expand capacity. The first paper to consider time-to-build in a general equilibrium economy is Kydland and Prescott (1982). Their specification of time-to-build has two distinct features: First, it takes four quarters for an investment project to be finished and the costs are spread over this period according to some weights. Second, a new investment project increases the productive capital stock only after the investment project is completed and the total costs are incurred. In this paper, I find that the timing of the investment costs is critical to generating the observed negative correlation between investment growth and stock returns. When most costs are incurred at the end of the construction period of a new project, the correlation is negative. Importantly, the empirical investment literature supports this assumption. For instance, Koeva (2001) reports that in the first year roughly 10% and in the second year 90% of the costs are incurred.

To keep my model as simple as possible, I consider a two period time-to-build framework where the household has time-separable power utility. In a two period time-to-build model, there is only one free parameter. It determines when the investment costs are due—today or the period thereafter. The new project becomes online when all costs are incurred, i.e. three periods after the initial decision was made. Even though a two period time-to-build specification is simplifying, it is rich enough to generate interesting asset pricing implications.

In contrast to the traditional exchange economy, the supply of capital is endogenously

determined in a production economy and its elasticity determines risk premia. Time-to-build affects the elasticity of capital and thus risk premia in the following way. After a positive technology shock, firms initiate new investment projects to take advantage of higher productivity. Time-to-build implies that there is a lag between the investment decision and when the new project becomes productive. Consequently, the capital stock is fully inelastic in the short-term and excess returns are high. High expected excess returns lower prices and therefore lead to low current realized returns. Based on the empirical evidence, I assume that most investment costs are incurred in the period after the shock. Thus, both realized returns and investment expenditures are low in the current period. This mechanism leads to a positive correlation between returns and investments, contradicting the empirical evidence.

However, time-to-build also affects the risk-free rate. In the period of a positive shock, investment expenditures cannot increase immediately, but they do so with a lag, since most costs are incurred in the period after the shock. To ensure market clearing in equilibrium, consumption has to absorb the positive shock, but falls afterwards. This negative expected consumption growth necessitates a low risk-free rate because the household would like to sell the bond to smooth consumption over time. The low risk-free rate increases the stock price and realized returns. The effects of the risk-free rate and the risk premium on current realized returns work in opposite directions. I find, however, that the risk-free rate effect dominates the risk premium effect. As a result, realized returns and investments are negatively correlated, as in the data.

This general equilibrium mechanism is in stark contrast to partial equilibrium considerations where the pricing kernel is exogenous. In partial equilibrium models, time-to-build affects the riskiness of the firm and thus risk premia. In general equilibrium, the endogeneity of the pricing kernel implies that time-to-build also affects the risk-free rate, which is constant in a partial equilibrium model. This additional channel is crucial to generate the observed negative correlation between investment growth and realized equity returns.

In a general equilibrium model with time-to-build, stock returns reflect not only the value of its productive capital but also the value attributed to ongoing investment projects that have not become productive yet. As a result, average Q deviates from marginal Q, creating a wedge between investment and stock returns. This fact helps to explain the equity premium and the excess volatility puzzle because stock returns are not tied to the marginal rate of transformation of capital, i.e. investment returns.

The standard time-to-build model has the drawback that it implies an oscillating optimal investment policy. After a positive shock, it is optimal for the firm to iterate between

large and small investment projects, because it thereby smoothes the average investment costs over time. This implication is counterfactual. To overcome this drawback, I introduce adjustment costs into the model. The standard adjustment cost function, which is defined in terms of the investment rate, the ratio of investment to capital, does not solve the problem. In contrast, I assume that the firm faces adjustment costs in the investment growth rate. As a result, the firm is penalized for iterating between small and large projects, because in that case the investment growth rate varies greatly over time.

The time-to-build model with adjustment costs produces persistent investment growth after a positive technology shock, as observed in the data. The model is able to explain three empirical facts of aggregate data: (*i*) the negative comovement of stock returns and investment growth, (*ii*) the prolonged positive correlation between stock returns and future investment growth, and (*iii*) the negative correlation between investment growth and future stock returns.

Even though time-to-build is a highly realistic friction, it has not received much attention in the literature. A notable exception is, for instance, Cochrane (1991). He suggests that "If there are lags in the investment process, then investment will not rise for a few periods, but orders or investment plans rise immediately." Building on this intuition, Lamont (2000) shows that investment plans instead of actual investment forecast stock returns because plans are not affected by lags. Lettau and Ludvigson (2002) demonstrate that CAY forecasts investment growth at long horizons because the predictability of investment growth at short horizons might be distorted by investment lags. Carlson, Fisher, and Giammarino (2005) provide evidence that investment commitment in long-term projects is important to explain the dynamics of stock return betas around SEOs.

The empirical investment literature provides plenty of direct evidence for time-to-build. For instance, Mayer (1960) conducts a survey of 110 companies and finds that the average length of the time between the decision to build a plant and its completion is 21 months. Montgomery (1995a,b) uses survey data collected by the U.S. Department of Commerce to construct the completion pattern for nonresidential structures between 1961 and 1991. He finds that the construction period averages between 5 and 6 quarters. Further evidence is presented in Mayer (1960), Jorgenson and Stephenson (1967), Ghemawat (1984), and Koeva (2000).

My paper is closely related to the business cycle literature. In line with the equity premium puzzle in an endowment economy, Rouwenhorst (1995) demonstrates the failure of the standard RBC model to account for the equity premium. Jermann (1998) and, more recently, Boldrin, Christiano, and Fisher (2001) show that business cycle models can

generate a reasonable equity premium when they are enhanced with frictions. The key insight of these papers is that frictions in the capital market as well as modifications of preferences are necessary. Both papers rely on internal habit, yet Jermann (1998) includes capital adjustment costs and Boldrin, Christiano, and Fisher (2001) inter-sector capital and labor immobility. Boldrin, Christiano, and Fisher (2001) also show internal habit preferences combined with time-to-build generate a realistic equity premium. However, they do not try to explain the negative correlation of aggregate investments and stock returns.[3]

Time-to-build was introduced into a business cycle model by Kydland and Prescott (1982). They assume that it takes four quarters for an investment project to be finished. In contrast, in the standard RBC model investment increases next period's capital stock. More recently, Christiano and Todd (1996) argue that most of the investment costs are incurred at the end of the project. Their reasoning is that most investment projects begin with a lengthy planning period, which is less resource-intensive than the actual construction phase of the project. In contrast, Kydland and Prescott (1982) assume that the investment costs are spread evenly over the investment project horizon.[4] Christiano and Vigfusson (2003) employ frequency domain tools to estimate and test a business cycle with time-to-build. They confirm the importance of time-to-build to explain business cycle variations. In addition, they estimate that the business cycle model, which gives the best fit of the data, has most investment costs due at the end of the project. Similar results are contained in Zhou (2000) and Koeva (2001), who estimate the Euler equation of a time-to-build model on investment data.

This paper is also related to the literature trying to explain the cross-sectional predictability of returns by their book-to-market ratio. In partial equilibrium models, Berk, Green, and Naik (1999), Carlson, Fisher, and Giammarino (2004), Zhang (2005), and Cooper (2006) explore the investment decision of dynamically optimizing firms. These papers differ in the investment frictions to generate a realistic cross-section of returns; yet they only consider instantaneous investment. In a general equilibrium model, Kogan (2001) analyzes the effects of irreversible investment on stock returns. Kogan (2004) adds an upper bound on the rate of investment to ensure that investment cannot occur instantaneously. His specification, however, does not allow to vary the timing of the investment

[3]More recent contributions are Danthine and Donaldson (2002), Lettau (2003), Christiano and Fisher (2003), and Guvenen (2004).

[4]Christiano and Todd (1996) call their specification time-to-plan because of the different timing of the investment costs compared to the assumption of Kydland and Prescott (1982). In this paper, I use the words time-to-build and time-to-plan synonymously, even though time-to-plan would be more precise.

costs, which is the crucial element of this paper.

The effect of time-to-build on the exercise thresholds of real options has been examined by Majd and Pindyck (1987), and Bar-Ilan and Strange (1996). These authors find that investment lags reduce the option value of waiting because of the opportunity costs of waiting for the new productive asset.

The road map of the paper is as follows. In Section 2.2, I test the Q theory of investment by estimating investment returns. In Section 2.3, I present the time-to-build model and derive its optimality conditions. Section 2.4 contains simulation results and the asset pricing implication. Section 2.5 concludes.

2.2 Empirical Findings

The goal of this section is to estimate investment returns based on the widely used capital adjustment cost framework. Capital adjustment costs are the key ingredient of the Q theory of investment. This empirical exercise is therefore a test of the Q theory.

In the first section, I derive the investment return based on concave adjustment costs in a partial equilibrium setting. In the second section, I estimate investment returns for gross investment, nonresidential and residential investment. In the last section, I also analyze the correlation between stock returns and investment growth rates.

2.2.1 A Neoclassical Production-Based Asset Pricing Model

Production-based asset pricing models are derived from firm's optimal investment decision. The firm's problem can be stated as maximizing firm value P_t by optimally choosing future real investment I_t, i.e.

$$P_t = \max_{\{I_{t+s}\}_{s=1}^{\infty}} \mathbb{E}_t \sum_{s=1}^{\infty} \frac{\Lambda_{t+s}}{\Lambda_t} D_{t+s} \qquad D_t = Y_t - I_t - w_t N_t \tag{2.1}$$

where Λ_t denotes the pricing kernel and D_t the dividend payment to the stock holder. Dividends are defined as the residual payment after subtracting investment expenditures I_t and labor costs, which are hours-worked N_t times wage rate w_t, from output Y_t. Output is determined by a constant returns to scale Cobb-Douglas production function F

$$Y_t = Z_t F(K_t, N_t) \qquad F(K_t, N_t) = K_t^{\alpha} N_t^{1-\alpha} \tag{2.2}$$

where K_t denotes capital, Z_t an exogenous technology shock, and α the capital share of production.

Firms incur adjustment costs when they invest. Adjustment costs reduce tomorrow's capital and thus the law of motion for capital is

$$K_{t+1} = (1-\delta)K_t + G(I_t, K_t) \tag{2.3}$$

where δ denotes depreciation and G is a constant returns to scale function given by

$$G(I_t, K_t) = \left(\frac{a_1}{1-1/\xi} \left(\frac{I_t}{K_t} \right)^{1-1/\xi} + a_2 \right) K_t \tag{2.4}$$

This function is concave in I and decreasing in K and thus captures the notion that it is more costly to change the capital stock quickly. The parameter ξ is the elasticity of investment-capital ratio with respect to marginal Q and controls the concavity of the function. The concavity of G also implies irreversibility of investment because G is not defined for negative I_t. As noted by Hayashi (1982), this feature does not affect the dynamics, since optimal investment is never negative at the aggregate level.

As a consequence of the capital adjustment costs, a Tobin's Q interpretation arises. Marginal Q is the ratio of the marginal value of an additional unit of capital Λ_t^K over the price of a unit of capital Λ_t. Marginal Q, denoted by q_t, is defined as

$$q_t = \frac{\Lambda_t^K}{\Lambda_t} \tag{2.5}$$

where Λ_t^K is the Lagrange multiplier on (2.3) and therefore the price of capital.

Investment should take place when $q_t > 1$ and destruction of capital when $q_t < 1$. Adjustment costs prevent the firm from adjusting the capital stock every period to its optimal level resulting in a time-varying marginal Q which deviates from unity.

The solution of (2.1) can be characterized by the Euler equation

$$\mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} R_{t+1}^I = 1 \tag{2.6}$$

where

$$R_{t+1}^I = \frac{MPK_{t+1} + q_{t+1}(G_2(I_{t+1}, K_{t+1}) + (1-\delta))}{q_t} \tag{2.7}$$

defines the investment return on capital; $MPK_{t+1} = Z_{t+1}F_1(K_{t+1}, N_{t+1})$ is the marginal product of capital and G_i denotes the partial derivative of G with respect to its i-th element. The investment return is the ratio of marginal productivity plus capital gains tomorrow divided by marginal costs and therefore reflects the firm's intertemporal tradeoff of investing.

One of the standard neoclassical assumptions is constant returns to scale (CRS). This innocuous-looking assumption has, however, major asset pricing implications. Hayashi

(1982) proves in a non-stochastic setting and Abel and Eberly (1994) in a stochastic setting that when production function and adjustment cost function are both CRS, marginal Q equals Tobin's average Q. Thus, firm value is given by

$$P_t = q_t K_{t+1} \tag{2.8}$$

This equation says that firm value is the value of capital in terms of the numeraire times the amount of capital. Substituting (2.8) into the investment return (2.7) and assuming that labor is paid its marginal product, it follows that

$$R^I_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = R^E_{t+1}$$

and hence the investment return on capital is equal to the stock return R^E_{t+1} *state by state.* This equivalence was first noticed by Restoy and Rockinger (1994).

The only paper that tries to test this implication is Cochrane (1991). In contrast to Cochrane (1991), I assume a different functional form for the adjustment cost function. This specification is used more often in theoretical work, e.g., Jermann (1998). Consequently, my empirical analysis can be understood as a direct test of these models. The results in this paper are not driven by the functional form of the adjustment cost. The main driving force is the relation between investment expenditures and stock prices.

Different adjustment cost functions, or more generally, different production-based asset pricing models, result in different functional forms for investment returns. This fact implies that investment returns are not uniquely identified but are model dependent, whereas stock returns are not. Nevertheless, one can rebut production-based asset pricing models for their implications for investment returns. That is precisely done here.

2.2.2 Data

To construct an investment return time-series, I use aggregate quarterly US NIPA data covering the period 1947 Q1 until 2004 Q4. Output Y_t is real GDP minus government expenditure. I compute the investment return based on real gross private domestic investment and the two subcategories real private nonresidential and residential fixed investment. I leave out the third subcategory which is changes in private inventories.

Gross private investment comprises nonresidential and residential fixed investment as well as changes in private inventories. Nonresidential fixed investment comprises structures as well as equipment and software. Nonresidential structures are, for instance, new constructions (e.g., hotels or mining explorations), improvements to existing structures, and brokers' commissions of sales of structures. Nonresidential equipment and software

are purchases by private business of new machinery, equipment, furniture, vehicles, and computer software. Residential fixed investment consists, for instance, of new construction of permanent single and multi family units, improvements to housing units, and brokers' commissions on the sale of residential property.

Cochrane (1991) computes the investment return based on gross investment and Lamont (2000) focuses on nonresidential fixed investment because "excluding residential investment is arguably more appropriate for relating investment and stock returns, because most of the residential capital stock is not traded on equity markets" (p. 2729). Recently, however, the effects of housing has been linked to the stock market, see e.g. Piazzesi, Schneider, and Tuzel (2007) or Lustig and Nieuwerburgh (2005).

Further, residential and nonresidential behave very differently over the business cycle. Residential investment is known to lead the cycle whereas nonresidential investment tends to lag the cycle.

The capital stock time-series is constructed by aggregating investment following (2.3). I set the initial value of the capital stock equal to the Bureau of Economic Analysis (BEA) value reported for 1946. Since by assumption the production function (2.2) features constant returns to scale, the marginal product of capital can be estimated from

$$MPK_t = \alpha \frac{Y_t}{K_t} \tag{2.9}$$

Inherent in MPK_t is a technology shock and labor. By making use of the constant return to scale assumption, I do not need to estimate separately the technology shock and work with labor data. In contrast, Cochrane (1991) assumes a constant marginal product of capital and therefore ignores any influences coming from the technology shock.

2.2.3 Investment Returns

Investment returns depend on the capital share α, depreciation rate δ, and the adjustment cost parameter ξ. The goal of this section is to find a set reasonable parameter values so that the first and second moments of real investment returns match the first and second moments of real stock returns—as predicted by the Q theory. Generally, I find that investment returns are sensitive to the capital share α and adjustment cost parameter ξ but insensitive to the depreciation parameter δ which I set to the common quarterly value of 2.5%.[5]

Table 2.1 reports the annualized first and second moments of investment returns computed for gross investment (Panel A and B), residential (Panel C) and nonresidential investment (Panel D). In Panel A and B, the gross investment return is computed with

[5]See for instance Cooley and Prescott (1995).

a capital share of $\alpha = 0.2$ and $\alpha = 0.3$, respectively. Nonresidential investment returns (Panel C) and residential investment returns (Panel D) are both computed with a capital of $\alpha = 0.1$. The last column is the annualized real return of the quarterly value-weighted CRSP index. I exclude data prior to 1955 since the estimated investment return time series are extremely volatile.

For gross investment returns (Panel A and B), two effects are noticeable: First, the mean level of investment returns increases with the capital share α. A higher capital share implies a higher marginal product of capital via (2.9) which in turn determines the mean level of investment returns since q is 1 in the long-run. Second, the adjustment cost parameter ξ affects mainly the investment return volatility. Specifically, the investment return volatility increases with adjustment costs. As a result, the parameter choice of $\alpha = 0.2$ and $\xi = 0.55$ results in an annualized mean investment return of 7.89% and standard deviation of 17.19% compared with the mean CRSP return of 7.86% and standard deviation of 17.14%. Hence, there exists a realistic parameter set such that the first and second moments of investment returns equal the ones of stock returns—supporting the Q theory.

To match the mean stock return, I lower the capital share to $\alpha = 0.1$ for nonresidential (Panel C) and residential investment returns (Panel D). Otherwise, the mean investment return would be too large because the capital stock in each subcategory is smaller than the capital stock for gross investment resulting in an increase of the marginal product of capital (2.9) which controls the mean return.

For a capital share $\alpha = 0.1$ and adjustment cost parameter $\xi = 0.45$, the mean nonresidential investment return and stock return are almost identical, however, the standard deviation of the nonresidential investment return is considerably lower than the standard deviation of stock returns. In the case of residential investment with $\alpha = 0.1$ and $\xi = 0.55$, the standard deviations match perfectly; yet the mean residential investment return is higher than the mean stock return.

Cochrane (1991) is only able to match the mean of gross investment returns and stock returns. The standard deviation of investment returns is roughly half the standard deviation of stock returns. The fact that I can match first and second moments whereas Cochrane (1991) only matches the first moment is the result of the adjustment cost function. In the specification of Cochrane (1991), the elasticity of the investment-capital ratio is fixed at two whereas here the adjustment cost parameter ξ affects the elasticity and thus the concavity of the function.

In the following, I focus on gross investment returns based on $\alpha = 0.2$ and $\xi = 0.55$,

nonresidential investment returns based on $\alpha = 0.1$ and $\xi = 0.45$, and residential investment returns based on $\alpha = 0.1$ and $\xi = 0.55$ because these parameter choices give the best fit in terms of matching the first and second moments of investment and stock returns.

In Figure 2.1-2.3, I plot the annual gross investment return, nonresidential, and residential investment return, respectively. The solid blue line in each figure is the annual real CRSP return and the dashed black line the investment return. Especially for gross investment returns, it is visible that the volatility prior to 1955 is fairly large. Importantly, gross investment returns and nonresidential investment returns seem to lag stock returns. This feature of the data contrasts with the Q-theory and is further analyzed in the next section.

2.2.4 Correlation of Investment Returns and Stock Returns

The last row of each panel of Table 2.1 presents the contemporaneous correlation of quarterly investment and stock returns, $\rho = \text{Corr}(R_t^E, R_t^I)$. As shown above, the production-based asset pricing model implies that investment returns and stock returns have to be identical state by state. Hence, all moments have to be equal and, more importantly, the two time series have to be perfectly positively correlated.

For gross investment returns, I find that even though the first and second moments of investment and stock returns almost perfectly match, they are contemporaneously *uncorrelated*, invalidating the model, $\rho = -4.10\%$. The result is even stronger for nonresidential investment returns, $\rho = -8.95\%$. Yet residential investment returns are positively correlated with stock returns, $\rho = 18.19$.

In Table 2.2, I report the correlation of real stock returns (CRSP value-weighted) with gross investment returns, nonresidential investment returns, and residential investment returns at k leads and lags, i.e., $\rho_k = \text{Corr}(R_t^E, R_{t+k}^I)$. The t-statistic of the null hypothesis of zero correlation is reported in parenthesis below the estimate.

Even though the returns are contemporaneously uncorrelated, gross investment and stock returns have a correlation of almost 34% when gross investment returns are shifted by half a year forward. The correlation of real stock returns with gross investment returns is significant at the 5% level at 1 to 3 quarters leads. Thus, gross investment returns lag stock returns.

The lag of nonresidential investment returns is even more pronounced (second column). The correlation pattern is humped shaped indicating that stock returns have a prolonged effect on nonresidential investment. The correlation peaks at 31% after shifting investment returns half a year forward. The correlation of real stock returns with nonresidential

investment returns is significant at the 5% level at 2 to 5 quarters leads.

Residential investment returns behave differently (third column). They are contemporaneously positively and at the 5% level significantly correlated. Residential investment returns lag stock returns by only one quarter, $\rho_1 = 41\%$. The correlation of real stock returns with residential investment returns is significant at the 5% level at 1 and 2 quarters leads.

Cochrane (1991) reports a contemporaneous correlation of quarterly returns of 24.1%. He obtains this positive correlation because he shifts stock returns by a half quarter so that they go from center to center of each quarter. His reasoning for the adjustment is that investment is a quarterly aggregate and stock prices are point to point. I do not make this adjustment because the correlation pattern between investment and stock returns cannot be resolved by this simple shift. Further, Lamont (2000) does not follow the timing convention of Cochrane (1991) as well.

2.2.5 Investment Growth and Stock Returns

What drives the correlation pattern between investment returns and stock returns? To this end, I report the correlation of real equity returns (CRSP value-weighted) with log gross investment growth, log nonresidential investment growth, and log residential investment growth at k leads and lags, i.e., $\rho_k = \text{Corr}(R_t^E, \Delta \log I_{t+k})$, in Table 2.3. The t-statistic of the null hypothesis of zero correlation is reported in parenthesis below the estimate. Comparing the correlations of investment returns and stock returns (Table 2.2) with the correlations of investment growth with stock returns (Table 2.3), it is striking how similar the magnitudes are. Thus, most of the variation of investment returns is driven by investment growth.

One of the main puzzles in the investment literature is the negative contemporaneous correlation between gross investment growth and stock price changes (first column) because it contradicts the Q theory of investment. A decline in expected returns raises marginal Q and as a result investment should increase. At the same time, stock prices increase due to lower expected returns. Hence, investment and stock prices should be contemporaneously positively correlated. Yet empirically, the contemporaneous correlation is negative, contradicting the Q theory.

The contemporaneous negative comovement is even stronger for firm (nonresidential) investment (second column). Lamont (2000) nicely summarizes the point with "The significant negative contemporaneous covariation is particularly puzzling since it seems to suggest that firms perversely cut investment when stock prices go up".

Even though the contemporaneous correlation is negative, stock returns are positively correlated with future gross as well as nonresidential investment growth. Thus, the impact of high stock returns leads to positive investment growth for a prolonged period. The highest correlation between future gross investment growth and stock returns occurs at half a year lag, $\rho_2 = 34.2\%$.

Another important feature of gross as well as nonresidential investment growth is the negative correlation with future stock returns, i.e., $\text{Corr}(R_t^E, \Delta \log I_{t-k}) < 0$. Thus, high gross and nonresidential investment growth predict low stock returns.

The correlation pattern between residential investment growth and stock returns is very different (third column). The contemporaneous correlation is positive and, therefore, accords with the Q theory. The correlation peaks with one quarter lag at 41.2%.

These findings suggest that it is important to differentiate between nonresidential and residential investment because they behave very differently. Nonresidential investment is mainly firm investment and residential investment is mainly housing expenditures by households. Firms seem to react too slowly to good news as conveyed by positive stock returns and thus the decision process lags behind. Residential investment, which is mainly housing expenditures by households, reacts quickly to positive returns and lags by one quarter.

One reason for the slow reactions of firms to good news might be adjustment costs. In good times, firms want to expand their production capacity to meet higher demands. Continuous adjustment costs, as assumed in the Q-theory of investments, slow the adjustment process because firms are penalized for high investment rates. However, continuous adjustment costs still imply a positive correlation between investment growth and stock returns. Thus, only non-continuous adjustment costs might be the cause.

In addition to facing adjustment costs, firms also need time to expand capacity. When the economy enters a boom, firms may decide to build a new factory, for instance. In standard models, this new capacity is available in the next period. Yet as documented above, the investment process takes on average one and a half years. In the following section, I show that time-to-build can cause the negative correlation.

2.3 Model

The goal of the model is to explain the correlation pattern between Q-theory investment returns and stock returns and the correlation pattern between investment growth rates and stock returns. To this end, I solve a stochastic general equilibrium with production

where capital needs time-to-build. The model is similar to Kydland and Prescott (1982) and Christiano and Todd (1996).

2.3.1 Household

The representative household maximizes expected lifetime utility over consumption

$$\max_{\{C_t, s_t, B_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t) \tag{2.10}$$

where $\beta \in (0,1)$ denotes the individual discount rate, C_t consumption, and u a time-separable utility function. Since the goal of the model is not to solve the equity premium, I assume power utility

$$u(C_t) = \frac{1}{1-\gamma} C_t^{1-\gamma}$$

where γ is the coefficient of relative risk aversion. The household can buy a risky claim on the firm's dividend stream and a risk-free bond B_t such that the time t budget constraint reads

$$C_t + s_t P_t + B_t \leq s_{t-1}(P_t + D_t) + R_{ft} B_{t-1} + l_t N_t \tag{2.11}$$

where s_t is the fraction of the firm owned by the household, P_t the stock price, D_t the dividend payment, l_t the wage rate, and N_t the amount of time working.

2.3.2 Firm

Firms own the economy's real capital and decide about investments. Their objective is to maximize expected firm value P_t by making optimal real investment decisions X_t:

$$P_t = \max_{\{X_{t+s}\}_{s=1}^{\infty}} \mathbb{E}_t \sum_{s=1}^{\infty} \frac{\Lambda_{t+s}}{\Lambda_t} D_{t+s} \qquad D_t = Y_t - I_t - l_t N_t \tag{2.12}$$

where Λ_t denotes the pricing kernel and D_t the dividend payment to the stock holder. Dividends are defined as the residual payment of output after subtracting investment expenditures I_t and labor costs $l_t N_t$. Output is determined by a Cobb-Douglas production function F

$$Y_t = Z_t F(K_t, N_t) \qquad F(K_t, N_t) = K_t^{\alpha} N_t^{1-\alpha} \tag{2.13}$$

where K_t denotes capital, Z_t an exogenous technology shock, and α the capital share of production.

Contrary to the model in Section 2.2.1, I assume that the current investment decision X_t becomes productive 2 periods later. The law of motion for capital is therefore

$$K_{t+2} = (1-\delta) K_{t+1} + X_t \tag{2.14}$$

where X_t is the investment decision at time t but not the cost. The investment costs I_t are a weighted average of past investment decisions

$$I_t = wX_t + (1-w)X_{t-1} \tag{2.15}$$

where the weight w determines the timing of the costs. When $w = 1$, firms incur the full cost within the same period whereas firms incur the cost a period later when $w = 0$. The specification of the the investment costs (2.15) follows Kydland and Prescott (1982) and Christiano and Todd (1996).

Previous papers have assumed four quarters time-to-build. Since the asset pricing implications are driven by the timing of the costs, I have simplified the standard time-to-build model to 2 quarters. As a result, only a single parameter, namely w, determines the timing of the investment costs.

Output is subject to a technology shock Z_t which follows an AR(1) processes

$$\ln Z_t = \rho \ln Z_{t-1} + \varepsilon_t$$

with $\varepsilon_t \sim \mathcal{N}(0, \sigma)$.

2.3.3 Equilibrium Returns

The household's first order condition[6] with respect to s_t gives the standard Lucas Euler equation for stock returns

$$u'(C_t) = \beta \mathbb{E}_t u'(C_{t+1}) R^E_{t+1}$$

where

$$R^E_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}$$

denotes the return on equity. Note that in equilibrium $N_t = 1$ since labor does not enter the utility function. The price of the consumption numeraire is $\Lambda_t = u'(C_t)$ and thus the pricing kernel is

$$M_{t+1} = \beta \frac{\Lambda_{t+1}}{\Lambda_t} \tag{2.16}$$

The risk-free rate is given by $1/R^f_t = \mathbb{E}_t M_{t+1}$.

[6] A competitive rational expectations equilibrium is a sequence of allocations $\{C_t, K_t\}_{t=0}^{\infty}$ and a price system $\{\Lambda_t, P_t\}_{t=0}^{\infty}$ such that: (1) given the price system, the representative household maximizes (2.10) s.t. (2.11); (2) given the price system, the representative firm maximizes (2.12) s.t. (2.14); (3) the good market clears: $Y_t = C_t + I_t$; (4) the stock market clears $s_t = 1$. By Walras' law, the labor market clears as well.

Since the firm's optimality conditions are non-standard, I derive them explicitly. The firm's Lagrange function is

$$\begin{aligned} \max \mathbb{E}_0 \sum_{t=0}^{\infty} & \quad \beta^t \Lambda_t \{ Z_t K_t^{\alpha} N_t^{1-\alpha} - w X_t + (1-w) X_{t-1} - w_t N_t \} \\ + & \quad \beta^t \Gamma_t \{ (1-\delta) K_{t+1} + X_t - K_{t+2} \} \end{aligned}$$

where Γ_t is the Lagrange multiplier on (2.14) and therefore the price of capital. The first order conditions with respect to X_t, K_{t+2}, N_t are

$$\Gamma_t = w\Lambda_t + (1-w)\beta \mathbb{E}_t \Lambda_{t+1} \tag{2.17}$$

$$\Gamma_t = \beta^2 \mathbb{E}_t \Lambda_{t+2} Z_{t+2} \alpha K_{t+2}^{\alpha-1} N_{t+2}^{1-\alpha} + \beta \mathbb{E}_t \Gamma_{t+1} (1-\delta) \tag{2.18}$$

$$l_t = Z_t (1-\alpha) K_t^{\alpha} N_t^{-\alpha} \tag{2.19}$$

Since time-to-build is an investment friction, a Tobin's Q interpretation arises. Marginal Q is the ratio of the marginal value of an additional unit of capital, Γ_t, divided by the price of a unit of capital, Λ_t. Dividing the FOC (2.17) by Λ_t yields marginal Q, denoted by q_t,

$$q_t = \frac{\Gamma_t}{\Lambda_t} = w + (1-w)\beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \tag{2.20}$$

Consequently, marginal Q is a weighted average of the investment costs where the weight is given by the risk-free rate. Marginal Q is time-varying because parts of the investment costs are incurred in the future and these sure costs have to be discounted at the risk-free rate.

Substituting (2.17) into (2.18) results in the firm's Euler equation

$$\Lambda_t = \beta \mathbb{E}_t \Lambda_{t+1} R_{t+1}^I$$

where the the investment return is defined by

$$R_{t+1}^I = \frac{\beta \mathbb{E}_{t+1} \frac{\Lambda_{t+2}}{\Lambda_{t+1}} Z_{t+2} F_1(K_{t+2}, N_{t+2}) + q_{t+1}(1-\delta)}{q_t} \tag{2.21}$$

The investment return R_{t+1}^I defines the firm's intertemporal tradeoff of investing one more unit of capital. It is the ratio of tomorrow's marginal benefits of investing one additional unit of capital divided by today's marginal costs. Since the marginal product of an additional unit of capital occurs in 2 periods and is thus risky, it has to be discounted using the pricing kernel.

Due to constant returns to scale, firm value P_t can be solved analytically and is given by

$$\begin{aligned} P_t &= \mathbb{E}_t \sum_{s=1}^{\infty} M_{t+s} D_{t+s} \\ &= q_t K_{t+2} - (1-w) X_t \mathbb{E}_t M_{t+1} + \mathbb{E}_t M_t Z_{t+1} F(K_{t+1}, N_{t+1}) \end{aligned} \tag{2.22}$$

Under time-to-build, the equivalence of marginal and average Q breaks down and hence leads to a deviation of investment and stock returns. To see this note that average Q is defined as

$$Q_t = \frac{P_t}{K_{t+2}} \tag{2.23}$$

Substituting (2.22) into (2.23) and comparing the resulting expression with (2.20), it follows that $Q_t \neq q_t$. Altuğ (1993) first noticed that time-to-build leads to a divide between average and marginal Q.

The difference between marginal and average Q arises because firm value (2.22) reflects not only the value of productive capital $q_t K_{t+2}$ but also the value of unfinished investment projects. More specifically, firm value has to be reduced by future investment costs, which surely occur because of past investment decisions. The second term of (2.22) captures these costs. The third term is discounted future profits due to past investment decisions.

2.3.4 Calibration

Table 2.4 summarizes the parameters' choice. These values are similar to Cooley and Prescott (1995) and correspond to quarterly frequency. The household discounts future consumption at an annual rate of 3 percent implying $\beta = 1.03^{-1/4}$. I set the coefficient of relative risk aversion equal to 5 so that the results are not driven by extreme risk aversion. The capital share of production is $\alpha = 0.36$. The quarterly depreciation rate of capital, δ, equals 0.025 implying 10% annual depreciation. The technology shock is mean zero with autocorrelation ρ of 0.95 and standard deviation σ of 0.007 percent. I solve the model with a second order perturbation around the simulated mean of the model, following Collard and Juillard (2001).

2.4 Results

Since the asset pricing implications are driven by the timing of the investment costs, I present three model versions with different timing. In Model A, I assume that the total investment costs are incurred in the period after the investment decision, i.e., $w = 0$. Thus,

the firm decides to invest in the future but does not incur any costs in the current period. Following Kydland and Prescott (1982), in Model B the investment costs are spread evenly over the two periods, i.e., $w = 0.5$. The specification of Kydland and Prescott (1982) results in an oscillating investment decision because firms thereby smooth the average investment costs. In Model C, I enhance the two period time-to-build with continuous adjustment costs. Consistent with the empirical investment literature, I assume that some investment costs are incurred in the current period, i.e., $w = 0.2$. This specification results in a very realistic correlation pattern between investment growth and stock returns.

2.4.1 Model A: $w = 0$

To explore the timing of events, I first analyze the impulse response functions generated by the model. Figure 2.4 depicts the impulse response functions of consumption C_t, risk-free rate R_t^f, the investment decision X_t and investment expenditures I_t, the realized stock return R_t^E and expected stock return $\mathbb{E}_t[R_{t+1}^E]$ after a one percent technology shock in period one.

Because the firms wants to take advantage of higher productivity after a positive shock, the investment decision rises. The investment expenditures have the same impulse response pattern as the investment decision but they lag one period due to time-to-build. The aggregate resource constraint implies that consumption has to absorb the positive shock because investment expenditures lag by one period. Consequently, consumption is initially high but then reverts back.

This negative expected consumption growth rate after the shock suggests that the household wants to buy the bond to smooth the consumption stream. But since the bond is in zero net supply, the risk-free rate has to adjust and therefore is below the steady state. The low risk-free rate also depresses the expected stock return. Moreover, the risk premium component of the expected return is not high enough to offset the negative risk-free rate effect.

The low risk-free rate results in a high stock price and thus a high realized return. The realized stock return falls after the initial peak because the firm now incurs the investment costs. This effect leads to the negative comovement of stock returns and investment growth. Table 2.7 (Model A) reports the correlation of stock returns and investment growth, $\text{Corr}(R_t^E, \Delta \log I_{t+k})$, based on simulating the model. The first column of Table 2.7 (Data) is the correlation between the return of the value-weighted CRSP index and nonresidential investment growth. Consistent with the data, stock returns and investment growth are negatively correlated contemporaneously and positively correlated with a lag.

Missing, however, is the prolonged positive effect.

Another goal of the model is to replicate the correlation pattern between stock returns and Q-theory investment returns. Table 2.6 reports the correlation between stock returns $R^E_{t-1,t}$ and misspecified investment returns $R^I_{t-1,t}$ based on the adjustment costs model (2.7). The adjustment cost parameter ξ is set to same value used in the empirical part, $\xi = 0.45$. The Q-theory based investment returns are misspecified on simulated data since investment returns depend on the investment frictions. The true investment return underlying this economy is given by (2.21).

In column labeled Model A, misspecified Q-theory investment returns are contemporaneously negatively correlated with stock returns and positively correlated at one lag, similar to the data. The response of Q-theory investment returns is delayed by one period because time-to-build delays the response of investment expenditures which are the main determinants of Q-theory investment returns.

Even though the expected return is low on impact of the shock, the risk premium is positive. Since a positive technology cannot be absorbed by investment, prices must change. Therefore, asset prices are volatile and sensitive to to market-wide uncertainty, i.e., returns have higher systematic risk. The stock holder therefore receives a compensation for the investment risk in terms of a positive expected excess return, which can be seen in Table 2.5. Specifically, even though the household has power utility, the model generates almost a one percent excess return.

In the standard RBC model, stock and investment returns are identical state by state. To generate a high stock return in such a model, the marginal rate of transformation of capital, i.e. investment return, has to be very volatile as well. This close tie between stock and investment returns is broken in a model with time-to-build because average Q diverge from marginal Q. Here, investment returns can remain smooth but stock returns are volatile. Further, the model generates a high standard deviation of the stock return, alleviating the excess volatility puzzle.

2.4.2 Model B: $w = 0.5$

Following Kydland and Prescott (1982), the investment costs are now spread evenly over the two periods, i.e. $w = 0.5$. When firms incur some of the investment cost in the same period as the investment decision, the dynamics are more complex. Figure 2.5 depicts the impulse response functions of consumption C_t, risk-free rate R^f_t, the investment decision X_t and investment expenditures I_t, the realized stock return R^E_t and expected stock return $\mathbb{E}_t[R^E_{t+1}]$ after a one percent technology shock in period one.

A striking feature of these impulse response function is the oscillation. This pattern is not unique to two periods of time-to-build, but also apparent in the graphs reported by Christiano and Todd (1996) with 4 periods time-to-build. The rationale behind the dynamics is the objective of a smooth consumption stream for the household, which necessitates a smooth dividend stream. Since the firm cannot initially smooth output because of time-to-build of the capital stock, it wants to dampen the investment costs.

As the technology shock is highest on impact, the firm wants to incur most of the costs at time 1 and thus the investment decision is above the steady state. In the second period, the firm decides to invest only the steady state level $\bar{X}$. As a result, the investment costs are smooth

$$I_1 = 0.5X_1 + 0.5\bar{X} \;=\; I_2 = 0.5\bar{X} + 0.5X_1$$

Hence, oscillating between large and small investment projects is optimal.

Following the same reasoning as above, the negative expected consumption growth rate after the initial shock results in a low risk-free rate. Since consumption growth oscillates after the shock, the risk-free rate oscillates as well.

The expected stock return is again low on impact of the shock, which is induced by the low risk-free rate. This implies that the risk premium component does not offset the negative risk-free rate effect. Another feature of the expected stock return impulse response is its fluctuation, which is caused by the dividend process. Since the stock price is the present value of an oscillating dividend stream with next period's dividend having the highest weight, the stock price fluctuates as well.

The low expected return causes a high stock price and high realized return but, in the model specification with $w = 0.5$, the investment costs are high as well. As a result, investment growth and stock return are positively correlated, contradicting empirical facts. Table 2.7 and 2.2 present the correlation between investment growth and stock returns and the correlation between misspecified investment returns and stock returns, respectively (column Model B). It is apparent, that the standard time-to-build specification generates an unrealistic correlation pattern between investment growth and stock returns and investment returns and stock returns. Since firms incur half of the investment costs in the initial period, stock returns are positively correlated with investment growth and Q-theory investment returns.

The second column of Table 2.5 summarizes the moments of data generated by this specification. Most importantly, the mean excess return with $w = 0.5$ is lower than with $w = 0$. The reason is that with $w = 0.5$ half of the investment costs are already incurred

in the initial period of the project. Consequently, consumption does not have to absorb the entire shock, but investments expenditures can adjust as well. This effect reduces the volatility of consumption growth and therefore the equity premium.

Concluding, time-to-build does not necessarily imply a negative correlation between investment growth and stock returns. The timing of the investment costs are the crucial determinant of the asset pricing implications.

2.4.3 Model C: Time-to-Build with Adjustment Costs

As soon as firms incur some of the costs in the initial period, it is optimal for the firm to oscillate between high and low investment decisions. To remedy this feature, I enhance the time-to-build specification with continuous adjustment costs. Q-theory based investment models assume continuous adjustment costs in the investment rate, I_t/K_t, so that firms are penalized for quick capital adjustment. This specification, however, does not solve the oscillation because the investment costs are fairly smooth. The adjustment costs employed here penalize the firm for switching between high and low investment decisions. The adjustment costs are therefore a function of the investment decision *growth rate* X_t/X_{t-1}. This adjustment cost function induces more realistic firm behavior.

Adjustment costs reduce the 2 periods ahead capital stock and thus the new law of motion for capital is

$$K_{t+2} = (1-\delta)K_{t+1} + \left(1 - S\left(\frac{X_t}{X_{t-1}}\right)\right) X_t$$

where S is a concave function in X_t/X_{t-1}

$$S(x) = \frac{\chi}{2}\left(e^{x-1} + e^{-(x-1)} - 2\right)$$

A similar specification has been used by Christiano, Eichenbaum, and Evans (2005). Since it is unrealistic to assume that no investment costs are due in the initial period, I assume $w = 0.2$. Further, I set the adjustment costs parameter $\chi = 1$.

As a result of the adjustment costs, the impulse response functions of the model are smooth; see Figure 2.6. The impulse responses of the investment decision and investment costs are hump-shaped because the firm would otherwise incur high adjustment costs. As before, the expected stock return and risk-free rate are low on impact of the shock, resulting in a high realized return. In Table 2.7 (Model C), the correlation between investment growth and stock returns is depicted. This table shows that the model is able to replicate three features of investment and stock market data: First, investment growth is negatively

correlated with future stock returns; second, the contemporaneous correlation between investment growth and stock return is negative; and third, stock returns are positively correlated with future investment growth. Table 2.2 (Model C) shows that the correlation between stock returns and Q-theory investment returns also matches the observed pattern in the data.

A drawback of this model is that consumption growth is too volatile, causing a high standard deviation of the risk-free rate. The third column of Table 2.5 summarizes the moments of the model. The model further generates a considerable standard deviation of the stock return and a small excess return which is larger than in the standard time-to-build with $w = 0.5$.

2.5 Conclusion

The findings in this paper support the importance of time-to-build in order to jointly explain stock market and investment data. At the aggregate level, investment growth and equity returns are negatively correlated. This empirical fact contradicts the Q theory of investment and has therefore been interpreted as evidence for irrational markets. However, a general equilibrium model, in which investment projects are not completed instantaneously, can explain this negative correlation, because the model endogenously generates dynamics in the risk premium and risk-free rate. These dynamics are crucial to replicate the negative correlation between investments and returns.

Time-to-build induces risk dynamics because it affects the elasticity of capital. A partial equilibrium model would therefore predict high expected returns at the investment date. This effect reduces current prices and realized returns. Since investment expenditure are initially low, too, investment growth and returns are positively correlated, which is inconsistent with the data.

In general equilibrium, however, the pricing kernel is endogenous and its dynamics determines the risk-free rate. Since most investment expenditures are due at the end of the project, consumption has to absorb positive shocks and is therefore initially high but falls afterwards. A negative expected consumption growth rate necessitates a low risk-free rate because the agent would like to sell the bond to smooth consumption over time. The low risk-free rate increases current prices and realized returns. After a positive shock, investment growth is initially low, because most expenditures are due in later periods, but realized returns are high. Thus, the model generates a negative correlation between investment growth and return, consistent with the data.

A extension of the two period time-to-build specification with costly adjustment is able to explain three phenomena: First, the negative correlation of investment growth and future stock returns; second, the negative contemporaneous comovement of stock returns and investment growth; and third, the prolonged positive correlation of stock returns and future investment growth.

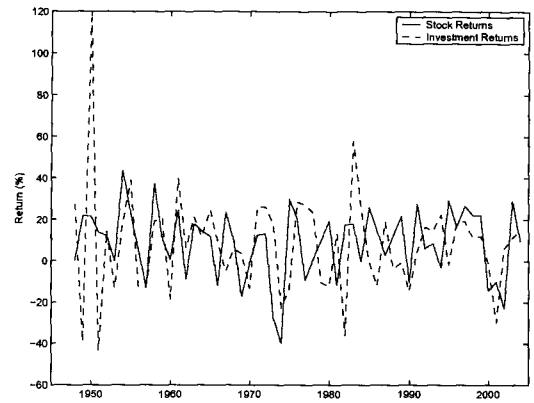

Figure 2.1: Annual gross private investment returns ($\alpha = 0.2, \xi = 0.55$) and stock returns

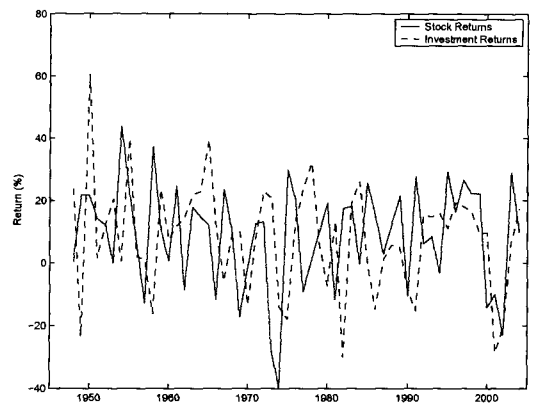

Figure 2.2: Annual nonresidential fixed investment returns ($\alpha = 0.1, \xi = 0.45$) and stock returns

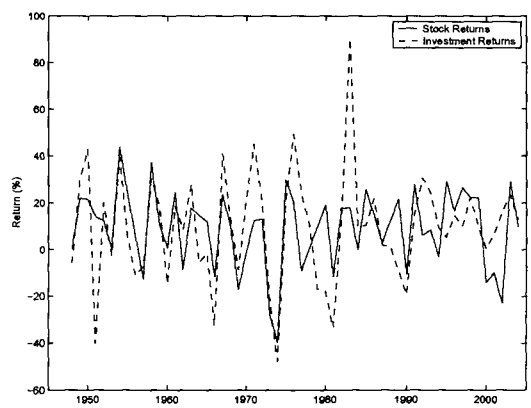

Figure 2.3: Annual residential fixed investment returns ($\alpha = 0.1, \xi = 0.55$) and stock returns

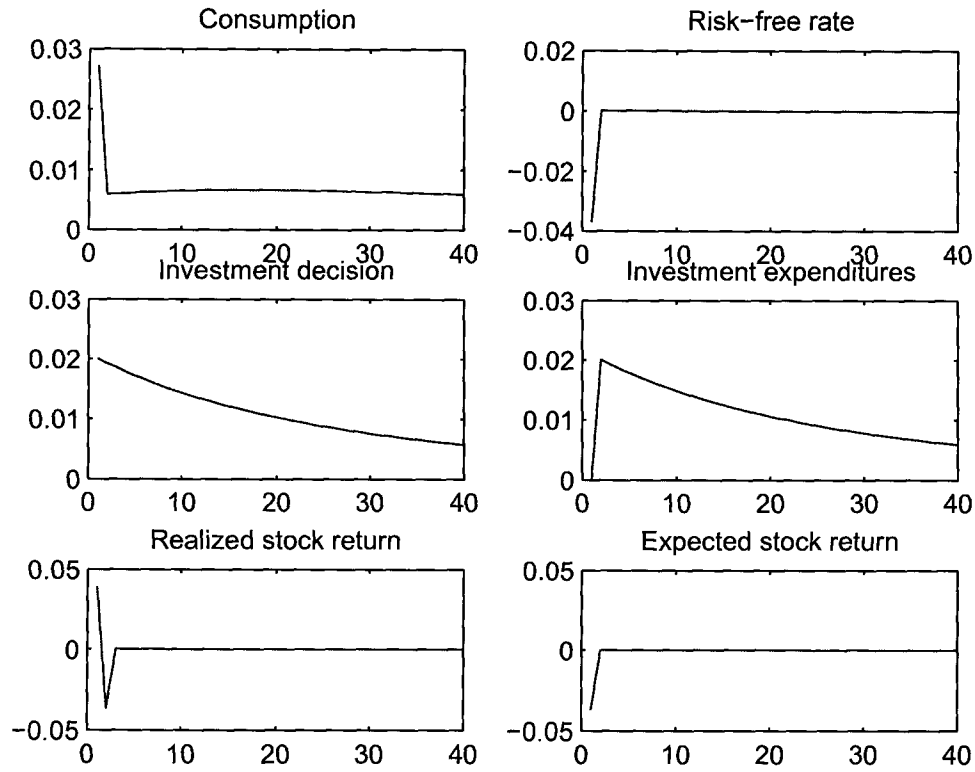

Figure 2.4: Impulse response functions of consumption C_t, risk-free rate R_t^f, investment decision X_t, investment expenditures I_t, realized stock return R_t^E and expected stock return $\mathbb{E}_t[R_{t+1}^E]$ when $w = 0$

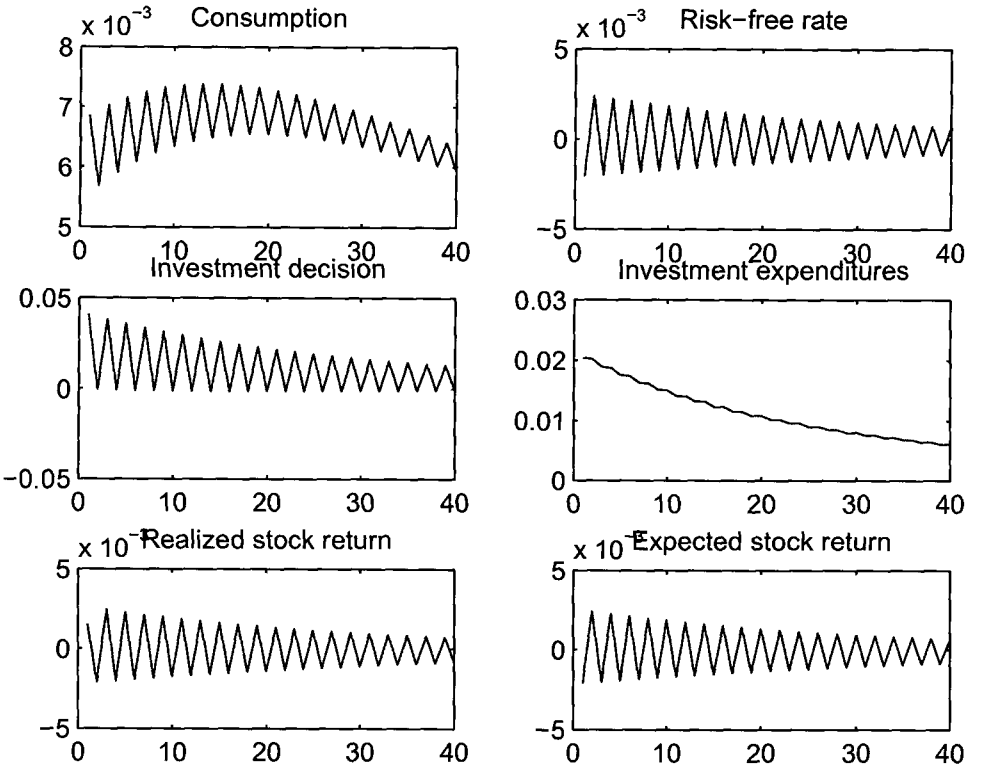

Figure 2.5: Impulse response functions of consumption C_t, risk-free rate R_t^f, investment decision X_t, investment expenditures I_t, realized stock return R_t^E and expected stock return $\mathbb{E}_t[R_{t+1}^E]$ when $w = 0.5$

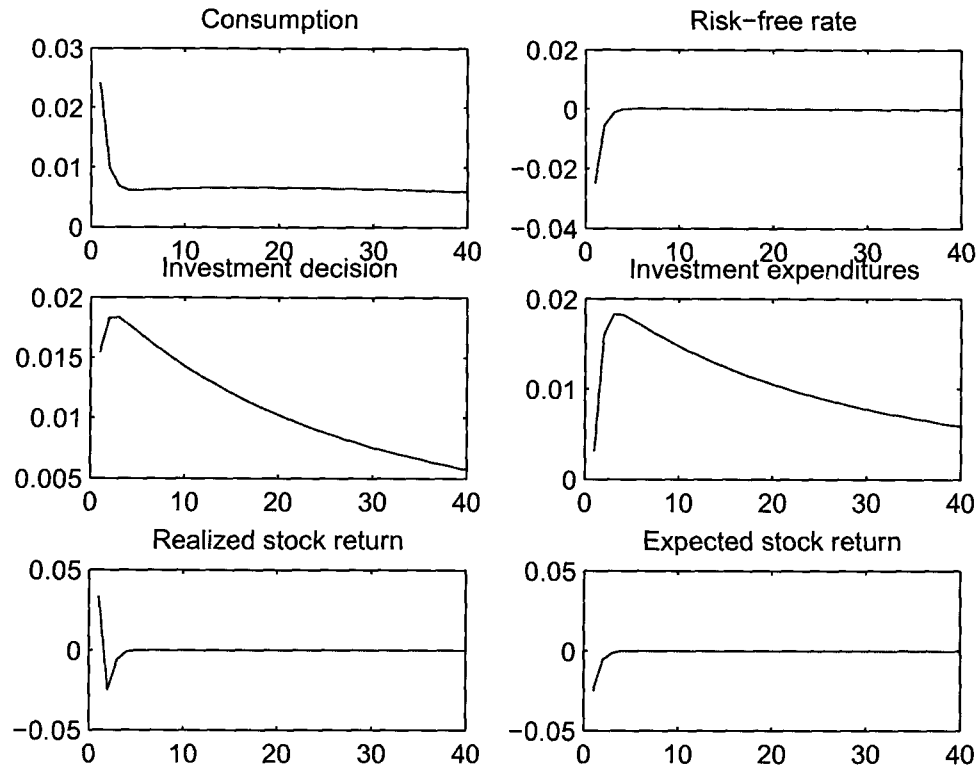

Figure 2.6: Impulse response functions of consumption C_t, risk-free rate R_t^f, investment decision X_t, investment expenditures I_t, realized stock return R_t^E and expected stock return $\mathbb{E}_t[R_{t+1}^E]$ with adjustment costs $\chi = 1$ and $w = 0.2$

Table 2.1: Investment Returns

This table reports the annualized mean and standard deviation of real investment returns based on real gross private investment (Panel A and B), real private nonresidential (Panel C) and residential fixed investment (Panel D). The last row of each panel is the contemporaneous correlation of investment and stock returns in percent, $\text{Corr}(R_t^E, R_t^I)$. The last column is the annualized mean and standard deviation of the quarterly real return of the CRSP value-weighted index. The sample period is 1955-2004.

Panel A: Gross private investment ($\alpha = 0.2$)				
ξ	10	2	0.55	CRSP
Mean	6.39	6.42	7.89	7.86
Std. Dev.	1.14	4.85	17.19	17.14
Correlation	-8.53	-5.30	-4.10	
Panel B: Gross private investment ($\alpha = 0.3$)				
ξ	10	2	0.55	CRSP
Mean	14.36	13.50	12.18	7.86
Std. Dev.	1.30	4.94	17.33	17.14
Correlation	-9.07	-5.09	-3.71	
Panel C: Nonresidential investment ($\alpha = 0.1$)				
ξ	10	2	0.45	CRSP
Mean	3.55	4.10	7.41	7.86
Std. Dev.	1.00	2.66	10.91	17.14
Correlation	-12.83	-10.71	-8.95	
Panel D: Residential investment ($\alpha = 0.1$)				
ξ	10	2	0.55	CRSP
Mean	7.90	7.92	9.06	7.86
Std. Dev.	2.10	5.00	17.16	17.14
Correlation	7.74	16.57	18.19	

Table 2.2: Correlation of Investment Returns and Stock Returns

This table shows the correlation (in percent) of stock returns with gross investment returns, nonresidential investment returns, and residential investment returns at k leads and lags, $\rho_k = \text{Corr}(R_t^E, R_{t+k}^I)$, $k = -4, -3, ..., 3, 4$. The t-statistic of the null hypothesis of zero correlation, $H_0 : \rho_k = 0$, is reported in parenthesis. Correlation coefficients, which are significant at the 5% level, are marked with a *. The sample period is 1955-2004.

k	Gross Inv.	Non-resid. Inv.	Resid. Inv.
-4	-2.93	-8.40	-2.40
	(-0.41)	(-1.17)	(-0.33)
-3	-13.93*	-4.37	-9.09
	(-1.96)	(-0.61)	(-1.28)
-2	-6.30	-12.24	-0.49
	(-0.88)	(-1.73)	(-0.07)
-1	-2.85	-11.61	0.89
	(-0.40)	(-1.64)	(0.12)
0	-4.10	-8.95	18.19*
	(-0.58)	(-1.27)	(2.60)
1	18.61*	11.35	41.09*
	(2.66)	(1.60)	(6.33)
2	33.96*	31.08*	27.21*
	(5.06)	(4.58)	(3.96)
3	22.12*	24.33*	9.91
	(3.17)	(3.50)	(1.39)
4	1.88	22.36*	-2.69
	(0.26)	(3.20)	(-0.37)

Table 2.3: Correlation of Investment Growth and Stock Returns

This table reports the correlation (in percent) of stock returns with gross investment growth, nonresidential investment growth, and residential investment growth at k leads and lags, $\rho_k = \text{Corr}(R_t^E, \Delta \ln I_{t+k})$, $k = -4, -3, ..., 3, 4$. The t-statistic of the null hypothesis of zero correlation, $H_0 : \rho_k = 0$, is reported in parenthesis. Correlation coefficients, which are significant at the 5% level, are marked with a *. The sample period is 1955-2004.

k	Gross Inv.	Non-resid. Inv.	Resid. Inv.
-4	-3.64	-8.29	-3.50
	(-0.51)	(-1.16)	(-0.49)
-3	-14.58*	-4.86	-9.88
	(-2.06)	(-0.68)	(-1.39)
-2	-5.74	-12.62	-1.84
	(-0.81)	(-1.78)	(-0.26)
-1	-3.90	-12.60	-1.65
	(-0.55)	(-1.78)	(-0.23)
0	-6.04	-10.72	15.84*
	(-0.85)	(-1.52)	(2.26)
1	17.51*	10.49	41.22*
	(2.50)	(1.48)	(6.35)
2	34.20*	31.22*	28.03*
	(5.09)	(4.60)	(4.09)
3	23.50*	24.28*	11.30
	(3.38)	(3.49)	(1.59)
4	2.64	22.48*	-0.06
	(0.37)	(3.21)	(-0.01)

Table 2.4: Benchmark Calibration

This table reports the benchmark parameter choice. β denotes the household's discount rate and γ the household's relative risk aversion; ρ is the autocorrelation and σ the standard deviation of the technology shock; α is the capital share and δ the depreciation rate of capital.

Parameter	Symbol	Value
Preferences:		
Discount rate	β	$1.03^{-1/4}$
Risk aversion	γ	5
Technology:		
Autocorrelation	ρ	0.95
Std. deviation	σ	0.007
Production:		
Capital elasticity	α	0.36
Depreciation	δ	0.025

Table 2.5: Moments

This table reports the moments of simulated models: In Model A, all project costs are due in the second period, i.e., $w = 0$. In Model B, the costs are spread evenly over the two period construction period, i.e. $w = 0.5$. In Model C, the firm has to pay 20% of the costs in the initial period and, in addition, faces capital adjustment costs in the investment growth rate. R^E denotes the return on equity, R^f the risk-free rate, and SD its standard deviation. All asset pricing moments are annualized. $\sigma(.)$ denotes the standard deviation, $\mathbb{E}[.]$ the unconditional mean and Corr(.,.) the correlation between two variables.

Variable	Model A	Model B	Model C
$\mathbb{E}[R^E]$	3.242	2.929	3.163
$\sigma(R^E)$	7.593	1.601	5.209
$\mathbb{E}[R^E - R^f]$	0.728	0.007	0.224
$\mathbb{E}[R^f]$	2.514	2.922	2.940
$\sigma(R^f)$	7.610	1.601	8.306
$\sigma(C)/\sigma(Y)$	0.641	0.593	0.641
$\sigma(I)/\sigma(Y)$	4.639	2.171	2.091
$\text{Corr}(C, Y)$	0.935	0.958	0.949
$\text{Corr}(I, Y)$	0.960	0.979	0.968

Table 2.6: Model-based Correlation of Investment Returns and Stock Returns
This table reports the correlation (in percent) of stock returns with gross investment growth, nonresidential investment growth, and residential investment growth at k leads and lags, $\rho_k = \text{Corr}(R_t^E, R_t^I)$, $k = -4, -3, ..., 3, 4$. The t-statistic of the null hypothesis of zero correlation, $H_0 : \rho_k = 0$, is reported in parenthesis. The sample period is 1955-2004. In Model A, all project costs are due in the second period, i.e., $w = 0$. In Model B, the costs are spread evenly over the two construction periods, i.e., $w = 0.5$. In Model C, the firm has to pay 20% of the costs in the initial period and, in addition, faces capital adjustment costs in the investment growth rate. All model versions are simulated for 5000 periods and investment returns are based on Equation (2.7) with $\xi = 0.45$.

k	Data	Model A	Model B	Model C
-4	-2.93	3.38	2.31	-0.04
	(-0.41)			
-3	-13.93	-0.47	0.29	-0.53
	(-1.96)			
-2	-6.30	-1.10	2.07	-3.71
	(-0.88)			
-1	-2.85	-1.01	1.14	-24.43
	(-0.40)			
0	-4.10	-67.72	-8.22	-41.25
	(-0.58)			
1	18.61	73.33	21.59	66.58
	(2.66)			
2	33.96	0.71	-22.46	10.94
	(5.06)			
3	22.12	0.78	21.11	1.20
	(3.17)			
4	1.88	-0.16	-21.70	0.10
	(0.26)			

Table 2.7: Model-based Correlation of Investment Growth and Stock Returns
This table reports the correlation (in percent) of stock returns with gross investment growth, nonresidential investment growth, and residential investment growth at k leads and lags, $\rho_k = \text{Corr}(R_t^E, \Delta \ln I_{t+k})$, $k = -4, -3, ..., 3, 4$. The t-statistic of the null hypothesis of zero correlation, $H_0 : \rho_k = 0$, is reported in parenthesis. The sample period is 1955-2004. In Model A, all project costs are due in the second period, i.e., $w = 0$. In Model B, the costs are spread evenly over the two construction periods, i.e., $w = 0.5$. In Model C, the firm has to pay 20% of the costs in the initial period and, in addition, faces capital adjustment costs in the investment growth rate.

k	Data	Model A	Model B	Model C
-4	-8.29	3.43	12.52	0.11
	(-1.16)			
-3	-4.86	-0.26	-9.91	-0.53
	(-0.68)			
-2	-12.62	-0.99	12.92	-3.58
	(-1.78)			
-1	-12.60	-0.59	-9.55	-24.46
	(-1.78)			
0	-10.72	-69.02	2.95	-42.83
	(-1.52)			
1	10.49	72.45	11.05	65.52
	(1.48)			
2	31.22	0.90	-11.53	11.10
	(4.60)			
3	24.28	1.01	11.10	1.51
	(3.49)			
4	22.48	0.13	-11.38	0.44
	(3.21)			

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Chapter 3

Asset Pricing with Real Investment Commitment[7]

3.1 Introduction

Most real investment is not instantaneous. For example, expanding capacity in a manufacturing process can take several years: the semiconductor manufacturer AMD recently announced it would build a new $3 billion computer plant in New York State with construction beginning in mid-2007 and planned completion 3 years later. Long-term investment projects are in addition costly to undo because of contractual agreements between sellers and suppliers. While airlines may agree to buy several new planes now, it will take several years for them to be delivered and paid for. In addition, the cancelation rates are very small, indicating severe punishment for breaches. Consequently, investments in long-term projects implies a commitment on the part of firms.[8]

In this paper, I study the effect of long-term investment commitment on the cyclicality, predictability and level of excess returns in a general equilibrium production economy. Investment commitment is distinct from traditional time-to-build, e.g. Kydland and Prescott (1982).[9] The time-to-build literature focuses on the lag between the investment decision and the time when the new project becomes productive. My focus is instead on the equilibrium consequences of investment projects which involve not only current expenditures but also commitment to future expenditures.[10]

[7]A version of this chapter will be submitted for publication. Kuehn, L.-A., Asset Pricing with Real Investment Commitment.

[8]Empirical evidence for commitment is the following: Boeing reports small cancelations rates of 0.4%, 1.8%, 2.6% and 0.6% for the years 2003-2006 on their website. Based on 106 projects, Koeva (2000) finds that only one project was canceled because of a change in demand and nine projects were delayed due to technical problems. Kling and McCue (1987) write "What is so perplexing is that, in spite of the supply situation, office construction has continued unabated. [...] The first quarter of 1985 saw office construction proceed at a $30 billion annual rate, notwithstanding the high current vacancy rates." Similarly, MacRae (1989) notes that new power plants continue being built even in the presence of excess supply.

[9]The effect of time-to-build on the exercise thresholds of real options has been examined by Majd and Pindyck (1987) and Bar-Ilan and Strange (1996).

[10]The effect of investment commitment on discount rates was first recognized in a capital budgeting context by Black and Rice (1995). More recently, Carlson, Fisher, and Giammarino (2005) show how investment commitment affects risk dynamics in a partial equilibrium real options framework.

My paper contributes to the literature on general equilibrium asset pricing in production economies, e.g. Jermann (1998), and to the literature explaining return predictability by the book-to-market ratio, e.g. Berk, Green, and Naik (1999), Kogan (2004) and Gomes, Kogan, and Zhang (2003).[11] I depart from the previous asset pricing literature along two dimensions. First, I drop the assumption of instantaneous investment and model the commitment of long-term investment projects. Second, I embed this investment friction in a general equilibrium model where the representative agent has Epstein-Zin preferences. The combination of these two assumptions generates (a) conditional first and second moments of excess returns that are counter-cyclical relative to consumption growth, (b) excess returns that are predictable by price-scaled variables, and (c) an increase of the equity premium by an order of magnitude. Furthermore, the model generates novel empirical implications regarding the predictability of stock returns, which find support in the data.

The first building block of my model is a new and tractable specification of long-term investment projects. A common assumption is that investment occurs instantaneously. A more realistic assumption is that an investment decision results in a series of both expenditures and capital increases over time. The trouble is, however, that when investment projects take place over time, the distribution of initiated projects becomes a high dimensional state variable and renders the problem intractable. The approach I take solves this problem by modelling investment projects as perpetual, with costs declining geometrically over time. This formulation allows me to summarize the total costs of prior projects with a single additional state variable—lagged investment expenditures. In addition, I assume that firms cannot undo previous investment decisions. Initiated projects therefore represent commitment, and I call the resulting capital friction *investment commitment.*

This investment commitment friction generates interesting dynamics in consumption and prices. After a positive technology shock, firms initiate new investment projects to take advantage of higher productivity. As a result, the commitment level increases. After an adverse shock, the household would like to lower investment to smooth consumption over time. However, prior commitments oblige the firm to complete initiated projects, so that consumption is constrained by the history of investment decisions, making a negative shock worse after a boom than a bust. Moreover, commitments in long-term projects are not satisfied immediately, thereby depressing consumption for several consecutive periods. Thus, a negative shock adversely affects consumption in the current as well as future periods.

[11] Recent work includes Carlson, Fisher, and Giammarino (2004), Cooper (2006), Gala (2005) and Zhang (2005).

These shocks to future consumption are priced when the agent has Epstein-Zin preferences, which is the rationale for the second building block of my model. Unlike with power utility, an agent with Epstein-Zin preferences cares about the path of future consumption, so shocks to future consumption are priced. With instantaneous investment, technology shocks cause fluctuations primarily in current consumption growth. However, with investment commitment and Epstein-Zin preferences, adverse shocks also cause (priced) fluctuations in future consumption growth, leading to a significantly larger equity premium. This economic mechanism is closely related to the notion of long-run risk by Bansal and Yaron (2004). They show that exogenously time-varying first and second moments of consumption growth rates paired with Epstein-Zin utility can generate a realistic equity premium. Investment commitment generates similar dynamics endogenously in a production economy.[12]

To demonstrate that investment commitment has not only a momentary but a long lasting impact on consumption, I gauge the persistence of consumption volatility. In simulated data from the commitment model, a high book-to-market ratio, caused by previous commitments, forecasts high consumption volatility for several quarters. This evidence is consistent with the data and illustrates that the commitment friction provides a strong internal propagation mechanism.

Another advantage of Epstein-Zin preferences is that they achieve a separation of relative risk aversion and elasticity of intertemporal substitution. With power utility, high risk aversion implies a low elasticity of intertemporal substitution because the two concepts are inversely related. To generate a realistic risk premium, the household has to be highly risk averse. At the same time, for investment commitment to have a significant impact on quantities, the household needs to have a high elasticity of intertemporal substitution. Both are not possible with power utility. Only when the agent is willing to delay consumption over time (high elasticity of intertemporal substitution) does she invest a larger fraction of her wealth in real assets. With such a parameterization, the commitment level rises dramatically in good times leading to a frequently binding commitment constraint when the economy switches into a recession. Conversely, with a low elasticity of intertemporal substitution, the commitment constraint has little impact on consumption and prices, and the equity premium remains low.

[12]Some recent papers belonging to the long-run risk literature are the following: Bansal, Dittmar, and Lundblad (2005) and Hansen, Heaton, and Li (2005) examine empirically whether size and book-to-market sorted stock returns covary with the conditional first moment of consumption growth; Kiku (2006) explains the value premium and Bhamra, Kuehn, and Strebulaev (2007) the credit spread puzzle in a long-run risk model.

The cyclicality and predictability of returns arises in the model since investment commitment, similar to irreversible investment,[13] is an asymmetric friction. It prevents firms from disinvesting but does not hinder firms from investing. This feature implies that the impact of the friction varies over the business cycle—consumption as well as prices are only affected by adverse shocks. As a consequence, my model endogenously generates countercyclical consumption growth volatilities and countercyclical first and second moments of expected excess returns, consistent with the empirical findings of Kandel and Stambaugh (1990). In contrast, symmetric frictions, such as convex adjustment costs, impact the dynamics of investment in good and bad times. First and second moments of stock returns are counterfactually acyclical in these settings.

While research has made significant progress towards understanding the effects of irreversible investment at the micro level (see Dixit and Pindyck (1994) for a survey), very little is known about the aggregate consequences of non-convex investment frictions.[14] One reason is that irreversible investment only impacts the dynamics of a model when the volatility of shocks is unrealistically large. Empirically, this corresponds to the finding that aggregate and sectoral level investment rates are (always) positive, implying that the irreversibility constraint is not binding. Even at the firm level, the fraction of zero or negative investment rates is 3.6%—a marginal amount.[15] This paper demonstrates that a non-convex friction can be binding even in an aggregate model and can thereby impact aggregate consumption and prices. The constraint of the commitment friction binds because the lower bound of investment is not zero, as in the standard irreversible investment friction, but history dependent. Minimum investment depends on the firm's prior investment decisions, increasing as the firm initiates multi-period projects and falling as the firm completes projects.

I also present new empirical results supporting my model. My empirical findings complement recent work by Xing (2006), who finds a negative relation between investment and future returns.[16] Because of investment commitment, lagged investment arises as a state variable in my model. Controlling for the current book-to-market ratio and investment rate, my model predicts a *positive* relation between the lagged investment rate and the

[13]See, for instance, Barnett and Sakellaris (1998) and Abel and Eberly (2002), who report empirical evidence of nonconvex capital friction.

[14]The aggregate consequences of nonconvex capital frictions is an ongoing debate in the literature. Early papers, for instance Veracierto (2002), Thomas (2002), Khan and Thomas (2003), reach the conclusion that mirco lumpiness does not matter for aggregate quantities. More recently, Bachmann, Caballero, and Engel (2006) reach the opposite conclusion.

[15]See Figure 3.1 for more details.

[16]More evidence is contained in Titman, Wei, and Xie (2004), Anderson and Garcia-Feijóo (2006) and Polk and Sapienza (2006).

first and second moments of future returns. Higher lagged investment means that the firm has initiated large investment projects in the past, which it is committed to complete. Using aggregate data, I find that there is a positive relation between the lagged investment rate and future returns and return volatility which is significant for the latter.

This paper is closely related to the investment-based asset pricing literature studying general equilibrium effects of investment frictions.[17] Kogan (2001, 2004) analyzes the effects of irreversible investment on stock returns in a two-sector general equilibrium model. More recently, Novy-Marx (2007) provides a model of irreversible investment in the cross-section. Contrary to this paper, he assumes that investment occurs instantaneously and the household has power utility. Panageas and Yu (2006) study the delayed response of consumption to major technology innovations.

My paper is also closely related to the business cycle literature. In line with the equity premium puzzle in an endowment economy, Rouwenhorst (1995) demonstrates the failure of the standard business cycle model to account for the equity premium. Jermann (1998) and, more recently, Boldrin, Christiano, and Fisher (2001) show that business cycle models can generate a reasonable equity premium when they are enhanced with frictions. The key insight of these papers is that frictions in the capital market as well as modifications of preferences are necessary. Both papers rely on internal habit and not Epstein-Zin preferences. Further, Jermann (1998) includes convex capital adjustment costs and Boldrin, Christiano, and Fisher (2001) inter-sector capital and labor immobility. Because of internal habit preferences, these papers generate a risk-free rate volatility which is too large compared to the data. My commitment model requires a high elasticity of intertemporal substitution, which has the additional benefit of generating a realistic risk-free rate volatility. Jermann (1998) and Boldrin, Christiano, and Fisher (2001) also do not analyze the cyclicality of financial and real quantities and return predictability.

More recently, Kaltenbrunner and Lochstoer (2006) and Campanale, Castro, and Clementi (2007) embed Epstein-Zin preferences in a production economy and analyze its impact on stock returns. These papers rely, however, on convex adjustment costs and therefore cannot generate countercyclical moments and predictable returns. Croce (2006) studies the welfare costs of long-run risk in a production economy.

Motivated by the cross-sectional predictability of stock returns by their book-to-market

[17]Early empirical contributions to this literature are Cochrane (1991, 1996). More recently, Lamont (2000) shows that investment plans, instead of actual investment, forecast stock returns because plans are not affected by investment lags. Lettau and Ludvigson (2002) demonstrate that CAY forecasts investment growth at long horizons. They argue that the predictability of investment growth at short horizons might be distorted by investment lags.

ratio (found by Fama and French (1992)), Berk, Green, and Naik (1999) build a partial equilibrium model and Gomes, Kogan, and Zhang (2003) a general equilibrium model of optimal firm behavior to explain this empirical regularity. My model also replicates this empirical fact and, in addition, shows that investment commitment provides a role for new predictor variables.

Commitment has been of interest in other strands of the literature, too. Chetty and Szeidl (2005) model consumption commitment directly as a friction of the consumption process, thereby providing a foundation for habit formation. Eisfeldt and Rampini (2006) consider the effect of committed dividend payments on corporate liquidity demand.

The paper has the following structure: In Sections 3.2 and 3.3, I explain the optimization problem of the firm and household, respectively. Section 3.4 contains the numerical results of the model. Section 3.5 concludes.

3.2 Firm

Before going into the details of the models, I summarize the model economy. It consists of two agents: the representative household and firm. The firm owns the capital stock of the economy and chooses optimal real investment. The household trades in the stock and bond market and receives dividend income from holding the firm's stocks. In equilibrium, the household has to hold all stocks and the risk-free bond is in zero net supply.

The firm's objective is to maximize firm value P_t by making optimal real investment decisions I_t:

$$P_t = \max_{\{I_{t+s}\}_{s=1}^{\infty}} \mathbb{E}_t \sum_{s=1}^{\infty} M_{t+s} D_{t+s} \qquad D_t = Y_t - I_t$$

where M_t denotes the stochastic discount factor and D_t the dividend payment to the shareholder. Dividends are defined as the residual payment after subtracting investment I_t from output Y_t. Output is determined by a Cobb-Douglas production function F

$$Y_t = Z_t^{1-\alpha_1} K_t^{\alpha_2}$$

where K_t denotes capital, Z_t an exogenous technology shock, and α_2 the capital share of production.

The technology shock follows a geometric random walk

$$\frac{Z_{t+1}}{Z_t} = \exp\{g + \varepsilon_{t+1}\}$$

where g is the growth rate of the economy and ε_t is an i.i.d. process with mean zero and standard deviation σ. In many production economies the technology shock follows an

AR(1) process. The resulting dynamics of the model are then of course partly driven by the exogenous shock process. In contrast, all dynamics in this model arise endogenously from the investment friction because the shock process follows a random walk.

The crucial ingredient of a production economy is the investment friction. In response to demand shocks to the economy, prices and the supply of capital change. Depending on the elasticity of capital, prices react more or less strongly. In the extreme case of an exchange economy, allocations are fixed (i.e. fully inelastic) and shocks are absorbed by price variation. In a production economy without an investment friction, the supply of capital is fully elastic and prices are as volatile as real capital. Investment frictions reduce the elasticity of capital and therefore cause prices to be more volatile than capital, which is necessary to generate interesting asset pricing dynamics.

3.2.1 Non-Instantaneous Investment

In this paper, I entertain the realistic friction that investment projects are not completed instantaneously. Before going into the details of the investment friction studied in this paper, I first provide a general framework for non-instantaneous investment. Consider a firm that decides about a new investment project of size X_t in period t. In the most general form, non-instantaneous investment has two modeling implications: First, investment expenditures in period t are a function of all previous investment project choices:

$$I_t = f_t(X_t, X_{t-1}, ..., X_0) \tag{3.1}$$

Second, next period's productive capital stock is a function of current capital and initiated projects:

$$K_{t+1} = (1-\delta)K_t + g_t(X_t, X_{t-1}, ..., X_0) \tag{3.2}$$

The functional form of f determines when the firm has to pay for a new project and g determines when a new project adds to the productive capital stock.

The instantaneous investment model is a special case, in which the firm incurs the total costs of the new project in the current period, i.e. $I_t = X_t$, and the new project becomes productive in the next period, i.e. $K_{t+1} = (1-\delta)K_t + X_t$.

The first paper to consider non-instantaneous investment in a production economy is Kydland and Prescott (1982). They label their investment friction time-to-build. It has two distinct features: First, it takes four quarters for an investment project to be finished and the costs are spread over this period according to the weights $w_1, ..., w_4$. This implies that

$$I_t = f(X_t, X_{t-1}, X_{t-2}, X_{t-3}) = w_1 X_t + w_2 X_{t-1} + w_3 X_{t-2} + w_4 X_{t-3}$$

where $w_1 + ... + w_4 = 1$.

Second, a new investment project increases the productive capital stock only after the investment project is completed and the total costs are incurred:

$$K_{t+1} = (1-\delta)K_t + g(X_{t-3}) = (1-\delta)K_t + X_{t-3}$$

The idea of the time-to-build friction is to capture a gestation lag between the investment decision and when it becomes productive. Importantly, there is no restriction placed on the investment decision X_t—it can be *negative* so that the firm can achieve any desired investment expenditure level I_t in a given period. The gestation lag shows up in the law of motion of capital. The time-to-build friction has the drawback of too many state variables. Specifically, with four periods time-to-build, the model has five state variables, namely, K_t, K_{t+1}, K_{t+2}, K_{t+3} and the technology shock—the curse of dimensionality shows up.[18]

3.2.2 Investment Commitment

To have a tractable specification of investment commitment, I make two assumptions: First, I place more restrictions on the dynamics for investment expenditures (3.1). Second, I drop the gestation lag idea of time-to-build and assume that partially completed projects add to the capital stock, i.e. $f = g$ in Equations (3.1) and (3.2).

My first assumption is that investment projects are perpetual, with costs declining geometrically over time. To gain a better understanding of this assumption, consider the case where the firm initiates a project of size X_0 at time 0 and nothing thereafter, i.e. $X_1 = X_2 = ... = 0$. In each period, the firm pays a fraction of the total project costs according to the weights $(\frac{1-w}{w})w$, $(\frac{1-w}{w})w^2$, $(\frac{1-w}{w})w^3$, etc. Thus, investment expenditures are given by

$$I_0 = \left(\frac{1-w}{w}\right) wX_0 \quad I_1 = \left(\frac{1-w}{w}\right) w^2X_0 \quad I_2 = \left(\frac{1-w}{w}\right) w^3X_0 \quad \ldots$$

As a consequence, the firm incurs the total costs of X_0 over the infinite future, i.e. $I_0 + I_1 + I_2 + ... = X_0$. This follows since the weights, normalized by $(\frac{1-w}{w})$, add up to 1.[19]

In general, the firm initiates new projects every period and the investment expenditures at time t are a weighted sum of all ongoing projects with respective costs X_{t-s}:

$$I_t = \frac{1-w}{w}\left(wX_t + w^2X_{t-1} + ... + w^{t+1}X_0\right) \tag{3.3}$$

[18]Kydland and Prescott (1982) solve the model with a linear-quadratic approximation and Christiano and Todd (1996) use a log-linear approximation. Both methods rely on the certainty equivalence and therefore are not applicable to study excess returns. Another drawback of the model are jigsaw-like impulse response functions which do not resemble empirical estimates; see e.g. Christiano and Todd (1996).

[19]Note that the weights are a geometric series, i.e. $\sum_{s=1}^{\infty} w^s = \frac{w}{1-w}$ if $|w| < 1$.

For instance, suppose the firm initiates a project at time 0 and 1. The investment expenditures at time 1, I_1, then consists of two terms: $I_1 = (\frac{1-w}{w})(wX_1 + w^2X_0)$. The first term, wX_1, relates to the just initiated period 1 project. It requires current expenditures of $(1-w)X_1$. The second term captures current costs of last period's projects X_0, amounting to $(1-w)wX_0$. In Equation (3.3), $w \in (0,1)$ determines the degree of commitment. High w means that few of the investment costs are incurred in the current period and therefore commitment is high.

The capital expenditure equation (3.3) suggests that the solution is history dependent. However, at time t, I_{t-1} captures all information about previous investment decisions because the weights in Equation (3.3) decay geometrically over time—in contrast, in the standard time-to-build formulation the weights are free parameters. The geometric weighting implies that the investment expenditures follow a recursive law of motion

$$\begin{aligned} I_t &= (1-w)X_t + \frac{1-w}{w}\left(w^2X_{t-1} + w^3X_{t-2} + ... + w^{t+1}X_0\right) \\ &= (1-w)X_t + wI_{t-1} \end{aligned} \tag{3.4}$$

Consequently, the current capital stock K_t and last periods investment expenditures I_{t-1} are the only endogenous state variables. The reduction in the number of endogenous state variables significantly reduces the computational complexity of the model.

Without any constraint on X_t, the investment expenditure law (3.4) would not impose any restriction on the investment behavior of the firm. To ensure that current investment decision are irreversible in the future and therefore represent commitment, I impose the constraint that firms can only initiate projects of non-negative size

$$X_t \geq 0 \tag{3.5}$$

Substituting (3.5) into (3.4) gives a lower bound on investment expenditures

$$I_t \geq wI_{t-1} \tag{3.6}$$

which I will call the *commitment constraint.*[20] It captures the notion that firms cannot reduce investment expenditures more quickly than at the rate at which they complete unfinished projects. The lower bound on investment expenditures (3.6) is similar to the irreversibility constraint, $I_t \geq 0$. However, there are two major differences:

[20]The recursive law for investment looks similar to a habit model in consumption. The difference is that habit implies a lower bound on consumption because consumption has to remain above the habit level. In contrast, commitment constraint (3.6) implies an upper bound on consumption because of the aggregate resource constraints.

First, the lower bound in this model depends on the decisions of the firms. When firms decide to invest more after a good technology shock, the investment bound increases and reduces the flexibility of the firm in bad times. The bound is therefore an endogenous outcome of the model and *state-dependent.* Second, it is well-known that the irreversibility constraint is never binding in aggregate models because optimal investment is never negative due to the small standard deviation of the aggregate technology shock. The commitment constraint, however, is binding in this model when w is large enough.

The main friction in the time-to-build model of Kydland and Prescott (1982) comes from the gestation lag in the law of motion for capital. Since the focus of this paper is investment commitment, my second assumption is a one period lag in the capital accumulation equation

$$K_{t+1} = (1-\delta)K_t + I_t \tag{3.7}$$

The only free parameter in the commitment friction is w. Because the investment expenditures decrease geometrically over time, the half-life of a project is[21]

$$H = \frac{-\ln(2)}{4\ln(w)} \tag{3.8}$$

That means, after H years, the firm has incurred half the project's total costs. As a result, w can be calibrated to the data because we can observe the length of projects. In contrast, the free parameter of convex adjustments costs has been subject to much controversy because estimates seem unrealistically large.[22]

3.2.3 Firm's Optimality Conditions

In this section, I derive the firm's optimality conditions and characterize the firm's value function. The firm's value function is

$$V(K_t, I_{t-1}, Z_t) = \max_{I_t} \{Z_t^{1-\alpha_1} K_t^{\alpha_2} - I_t + \mathbb{E}_t M_{t+1} V(K_{t+1}, I_t, Z_{t+1})\} \tag{3.9}$$

subject to

$$K_{t+1} \geq (1-\delta)K_t + I_t \tag{3.10}$$

$$I_t \geq wI_{t-1} \tag{3.11}$$

[21]The project half-life in quarters is defined as time τ when

$$\frac{1-w}{w}(w + w^2 + ... + w^\tau) = \frac{1}{2}$$

[22]See Gilchrist and Himmelberg (1995) and Erickson and Whited (2000) for recent evidence.

The next three propositions summarize important properties of the firm's value function:

Proposition 1 *There is a unique continuous function $V : \mathcal{K} \times \mathcal{I} \times \mathcal{Z} \rightarrow \mathbb{R}$ satisfying (3.9).*

Proposition 2 *The value function is continuously differentiable in its first and second argument.*

Proposition 3 *For each $Z \in \mathcal{Z}$, $V(\cdot, \cdot, Z) : \mathcal{K} \times \mathcal{I} \rightarrow \mathbb{R}$ is strictly increasing (decreasing) in its first (second) argument and strictly concave.*

All proofs are contained in Appendix A.1. In the following, let V_i denote the derivative of the value function with respect to its i-th element. Propositions 1 and 2 are standard results. Proposition 3 tells us that the value function is decreasing in lagged investment, since higher lagged investment reduces the feasible choice set of the firm. Moreover, the envelope condition with respect to lagged investment is

$$V_2(K_t, I_{t-1}, Z_t) = -w\mu_t$$

where μ_t denotes the multiplier on the commitment constraint (3.11) and thus measures the shadow costs of commitment. This envelope condition says that the slope of the value function with respect to lagged investment measures the shadow cost of commitment. A binding commitment constraint raises the economic shadow costs μ and causes additional curvature in the value function.

In Appendix A.1, I show that the firm's optimality conditions are

$$q_t = \mathbb{E}_t M_{t+1}(Z_{t+1}^{1-\alpha_1}\alpha_2 K_{t+1}^{\alpha_2-1} + q_{t+1}(1-\delta)) \tag{3.12}$$

$$q_t = 1 - \mu_t + w\mathbb{E}_t M_{t+1}\mu_{t+1} \tag{3.13}$$

where q_t denotes the Lagrange multiplier on (3.10), which is the shadow value of capital and usually termed marginal Q. Equation (3.12) equates the marginal costs of investment (left side) with the expected marginal benefit of investment (right side). Equation (3.13) determines marginal Q in terms of the current and expected shadow costs of commitment μ_t and $\mathbb{E}_t M_{t+1}\mu_{t+1}$, respectively. In the standard irreversible investment model ($w = 0$), $q_t = 1 - \mu_t$ and marginal Q is falling in the severity of the binding constraint as measured by μ. Commitment gives rise to a second term, $\mathbb{E}_t M_{t+1}\mu_{t+1}$, which captures the effect that the commitment constraint is not fixed, but state-dependent. This term is not contained in the irreversible investment model because the lower bound on investment expenditures

is constant at zero. It is important to note that this additional term enters positively into the expression for marginal Q and therefore *lowers* it. The reason is that μ measures the economic costs of a binding commitment constraint under the assumption that the constraint is fixed at the current level. Yet the level of the constraint falls at the rate w at which the firm completes initiated projects. This reduction of the commitment level is captured by the term $\mathbb{E}_t M_{t+1}\mu_{t+1}$.

Under a linear production function, I can derive an expression for firm value and the stock price in terms of endogenous variables.

Proposition 4 *With a linear production function, $\alpha_2 = 1$, the cum-dividend firm value is given by*

$$V_t = Z_t^{1-\alpha_1} K_t + q_t(1-\delta)K_t - \mu_t w I_{t-1} \tag{3.14}$$

and ex-dividend firm value, i.e. stock price, is

$$P_t = q_t K_{t+1} - w I_t \mathbb{E}_t M_{t+1}\mu_{t+1} \tag{3.15}$$

Equation (3.15) of Proposition 4 states that the stock price contains two terms. The first one measures the value of capital and the second one captures the value of committed expenditures. Since $w > 0$ in the commitment model, marginal and average Q deviate from each other, where average Q (or market-to-book ratio) is defined as

$$MB_t = \frac{P_t}{K_{t+1}} = 1 - \mu_t + \frac{K_{t+1} - I_t}{K_{t+1}} w \mathbb{E}_t M_{t+1}\mu_{t+1} \tag{3.16}$$

Equation (3.16) implies that the market-to-book ratio is less than one when the commitment constraint is binding, $\mu_t > 0$. The last term captures the fact that the commitment constraint is not constant, but state-dependent.

Breaking the equivalence of marginal and average Q has interesting implications for stock return predictability. In the standard irreversible investment model, $w = 0$, the equivalence of marginal and average Q holds. As a result, investment and stock returns are identical state-by-state[23] and given by

$$R_{t+1} = \frac{Z_{t+1}^{1-\alpha_1} + MB_{t+1}(1-\delta)}{MB_t}$$

This equation implies that the market-to-book ratio is a sufficient statistic for expected returns since Z_{t+1} follows a random walk. Without an investment friction, the market-to-book ratio is constant and so are expected returns. A time-varying market-to-book ratio

[23]The equivalence of investment and stock returns under CRS also holds in the convex adjustment cost model and Liu, Whited, and Zhang (2007) test this implication. Yet it does not hold in the commitment model; see Appendix A.1 for details.

causes capital gains in stock returns. These capital gains represent time-variation in the marginal cost of capital, which measure the slope of the firm's value function with respect to current capital, i.e $V_1(K_t, I_{t-1}, Z_t)$.[24]

In the model with commitment, a second source for capital gains in stock returns arises, namely, time-variation in the marginal cost of committed investment expenditures, μ_t. Proposition 4 implies that stock returns are given by

$$R_{t+1} = \frac{Z_{t+1}^{1-\alpha_1} + q_{t+1}(1-\delta) - \overline{IR}_t w \mu_{t+1}}{MB_t} \tag{3.17}$$

where $\overline{IR}_t = I_t/K_{t+1}$ is the adjusted investment rate. Expected returns are now a function of the marginal cost of capital, i.e. $V_1(K_t, I_{t-1}, Z_t)$, and the marginal cost of commitment, i.e. $V_2(K_t, I_{t-1}, Z_t)$. Since the econometrician does not observe marginal Q, a natural proxy is the market-to-book ratio. Substituting the market-to-book ratio (3.16) into (3.17) gives

$$R_{t+1} = \frac{Z_{t+1}^{1-\alpha_1} + (1-\delta)MB_{t+1} + \overline{IR}_{t+1}(1-\delta)w\mathbb{E}_{t+1}M_{t+2}\mu_{t+2} - \overline{IR}_t w \mu_{t+1}}{MB_t} \tag{3.18}$$

which suggests a relationship between returns and the current book-to-market ratio as well as current and lagged investment rate. The coefficient on the book-to-market ratio measures the effect of the marginal cost of capital on returns, after controlling for the cost of commitment. Controlling for the marginal cost of capital and the current investment rate, the coefficient on the lagged investment rate measures the effect of higher committed expenditures and therefore the marginal cost of investment commitment. In the following, I set $\alpha_1 = \alpha_2$.

3.3 Household

The representative household maximizes recursive utility over consumption following Kreps and Porteus (1978), Epstein and Zin (1989), and Weil (1989):

$$U_t = \left\{(1-\beta)C_t^{\rho} + \beta\left(\mathbb{E}_t U_{t+1}^{1-\gamma}\right)^{\rho/(1-\gamma)}\right\}^{1/\rho} \tag{3.19}$$

where C_t denotes consumption, $\beta \in (0,1)$ the rate of time preference, $\rho = 1 - 1/\psi$ and ψ the elasticity of intertemporal substitution (EIS), and γ relative risk aversion. Implicit in the utility function (3.19) is a CES time aggregator and CES power utility certainty equivalent.

Epstein-Zin preferences provide a separation of the elasticity of intertemporal substitution and relative risk aversion. These two concepts are inversely related when the agent

[24] For details, see Equation (A.1) in Appendix A.1 and note that $q_t = MB_t$.

has power utility. Intuitively, the EIS measures the agent's willingness to postpone consumption over time and this concept is well-defined even under certainty. Relative risk aversion measures the agent's aversion to atemporal risk (across states). Separating these two concepts is crucial for the results of this paper.[25]

The last piece of the model is the household's budget constraint. The household can buy a risky claim on the firm's dividend stream and a risk-free bond such that the return to his portfolio is

$$R^w_{t+1} = s_t \frac{P_{t+1} + D_{t+1}}{P_t} + (1 - s_t) R^f_{t+1}$$

where R^w_{t+1} is the return on wealth, s_t the fraction of wealth invested in the risky asset, P_t the stock price, D_t the dividend payment, and R^f_t the risk-free rate. Then, the budget constraint reads

$$W_{t+1} = (W_t - C_t) R^w_{t+1} \tag{3.20}$$

where W_t denotes wealth.

3.3.1 Elasticity of Intertemporal Substitution

The magnitude of the EIS plays an important role for the model's implications regarding the dynamics and magnitude of excess returns. Specifically, the commitment constraint binds frequently only when the EIS is large enough. This section therefore contains a discussion on how the EIS affects the consumption policy function.

I show in Appendix A.2 that the consumption-wealth ratio is given by

$$\varphi_t = \frac{A_t}{1 + A_t} \qquad A_t = \left(\hat{\mu}_t^{\rho} \frac{\beta}{1 - \beta} \right)^{1/(\rho - 1)}$$

where $\hat{\mu}_t = \left(\mathbb{E}_t [\phi_{t+1} R^w_{t+1}]^{1-\gamma} \right)^{1/(1-\gamma)}$ and ϕ_t is the utility-wealth ratio defined in Equation (3.19). Because the fraction of wealth which is not consumed has to be invested, this equation also determines the investment policy in real capital.

In Figure 3.2, I plot the consumption-wealth ratio φ as a function of the EIS for a given value of $\hat{\mu}$. The solid line corresponds to the case when $\hat{\mu} = 0.1$ and the dashed line when $\hat{\mu} = 10$. Two effects are notable: First, for a given $\hat{\mu}$, the consumption-wealth ratio is falling in the EIS, implying that a larger fraction of wealth is invested in the risky asset, which here is real capital. More precisely, the consumption-wealth ratio is strictly falling in the EIS if and only if $\ln \hat{\mu} > \ln((1 - \beta)/\beta)$. This inequality has an intuitive interpretation. First note that the right hand side of the inequality is approximately the logarithm of

[25] Another important feature of Epstein-Zin preferences is the endogenous preference for early or late resolution of uncertainty.

the rate of time preference when beta is close to one.[26] Consequently, as long as the log (certainty equivalent) return on wealth is larger than the rate of time preference, an agent who is willing to postpone consumption (high EIS), invests a larger fraction of her wealth in the risky asset.[27]

Second, when the EIS is larger than one, an increase in expected returns, i.e. a jump from the solid to the dashed line, lowers the consumption-wealth ratio because the intertemporal substitution effect dominates the income effect. In a production economy, an increase in expected returns corresponds to a positive technology shock. An agent with an EIS larger than one wants to take advantage of higher productivity and consequently invests a larger fraction of her wealth in capital.

3.3.2 Household's Equilibrium Conditions

The household's first order condition with respect to s_t gives the standard Lucas Euler equation for stock returns

$$1 = \mathbb{E}_t[M_{t+1}R_{t+1}] \tag{3.21}$$

where

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} \tag{3.22}$$

denotes the return on equity. Note that in equilibrium the household has to hold all shares, i.e. $s_t = 1$. Further, the risk-free rate asset is in zero net supply and determined by $R^f_{t+1} = 1/\mathbb{E}_t M_{t+1}$.

The pricing kernel based on Epstein-Zin preferences is[28]

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{\rho-1} \left(\frac{U_{t+1}}{\left(\mathbb{E}_t U_{t+1}^{1-\gamma}\right)^{1/(1-\gamma)}}\right)^{1-\gamma-\rho} \tag{3.23}$$

An important property of this pricing kernel is its dependence on the agent's value function. Intuitively, since the value function is the present value of future utility from consumption, an agent with Epstein-Zin preferences cares about her future consumption path and the

[26]The rate of time preference is usually set around 0.99.

[27]See Bhamra and Uppal (2006) for more details on Epstein-Zin preferences.

[28]Because of the homogeneity of the utility function (3.19), the pricing kernel can also be written in terms of the return on wealth:

$$M_{t+1} = \beta^{\theta} \left(\frac{C_{t+1}}{C_t}\right)^{-\theta/\psi} (R^w_{t+1})^{-(1-\theta)}$$

where $\theta = \frac{1-\gamma}{1-1/\psi}$ measures the departure of the agent's preferences away from the time-additive expected utility framework and $\theta = 1$ is the power utility case.

value function measures the agent's satisfaction thereof. Because the agent cares about future consumption, shocks to future consumption growth are priced. In contrast, when the agent has power utility, the pricing kernel depends only on marginal utility. Accordingly, the agent evaluates risks at different dates in isolation and shocks to the consumption path are not priced.

Applying a log-linear approximation to the pricing kernel, excess returns are given by

$$\begin{aligned} E_t[R_{t+1}] - R^f_{t+1} \approx & -(\rho - 1)\mathrm{Cov}_t(R_{t+1}, \Delta c_{t+1}) \\ & -(1 - \gamma - \rho)\mathrm{Cov}_t\left(R_{t+1}, \ln U_{t+1}\right) \end{aligned} \tag{3.24}$$

where lower case letters denote the logarithm of capital letters. This equation says that excess returns are high when stock returns are correlated with either consumption growth or the agent's value function. In a production economy, the value function has to be approximated numerically. In an endowment economy, an approximate closed-form solution is possible as demonstrated by Bansal and Yaron (2004). They show that shocks to expected consumption growth and the variance of consumption growth are priced when the agent has Epstein-Zin preferences.

3.4 Model Results

In this section, I calibrate the model and then solve it numerically as described in Appendix A.3.

3.4.1 Calibration

In Table 3.1 the parameters choice is summarized. These values are similar to Cooley and Prescott (1995) and Boldrin, Christiano, and Fisher (2001) and correspond to a quarterly frequency. The growth rate of the economy g is set to 0.4%. The capital share of production is $\alpha = 0.4$. The quarterly depreciation rate of capital δ is set to 2% implying 10% annual depreciation. The innovations to the technology shock, ε_t, are mean zero with standard deviation σ of 0.03 percent.

The last parameter of the calibration is the commitment parameter w. As shown with Equation (3.8), w determines the half-life of a project. The empirical investment literature makes it possible to calibrate w because it contains results regarding the duration of investment projects. Several papers have estimated the length of investment projects; however, estimates vary strongly across industries. For instance, Mayer (1960) conducts a survey of 110 companies and finds that the average length of the time between the decision

to build a plant and its completion is 21 months. MacRae (1989) notes that investments in a power generating plant typically takes 6 to 10 years to complete; Pindyck (1991) observes that investment lags in the aerospace and pharmaceutical industries are comparable. In a more recent study, Koeva (2000) reports lead times between 13 months for the rubber industry and 86 months for utilities. Based on the empirical evidence, I choose a project half-life of $H = 3.5$ years, implying $w = 0.95$.

3.4.2 Stock Market Moments

Figures 3.3 and 3.4 show the value function and consumption policy function, respectively, for the base case parameterization when the shock $Z_t = 1$. The commitment constraint is clearly visible as a kink in the consumption policy function since a binding commitment constraint imposes an upper bound on consumption. Specifically, when the economy is hit by an adverse shock, the agent would like to lower investment, but a binding commitment constraint prevents the firm from doing so. Consequently, consumption falls by more than is optimal.

In Table 3.2, I present numerical results for the stock market. I simulate the economy for 5,000 periods and report unconditional moments. Instead of presenting results only for the full model, I consider 4 cases: low and high EIS, as well as commitment and no commitment. I do not report results for the power utility case because the mean and the volatility of the risk-free rate are unrealistically large. This comparison facilitates a better understanding of each model component. In Model A and B, the agent has low and high EIS, respectively, but there is no investment commitment. Model C and D contain commitment and the agent has low and high EIS, respectively. Following Bansal and Yaron (2004), I choose risk aversion of 10 and EIS of 0.5 and 1.5.

Model A and B do not generate realistic first and second moments of stock returns, which is the equity premium puzzle of Mehra and Prescott (1985). Combining the commitment friction with a low EIS agent (Model C) still does not generate a large risk premium because the commitment constraint is seldom binding. The reason is that after a positive shock an agent with a low EIS does not invest much because of the strong desire to smooth consumption over time. With low investment in good times, the commitment constraint does not increase much and therefore binds occasionally in recessions. In contrast, an agent with a high EIS wants to take advantage of higher productivity and is willing to accept more variation of consumption growth over time as explained in Section 3.3.3.3.1. As a result, the commitment constraint rises dramatically following good productivity, leading to a commitment constraint that binds more frequently when the economy switches into

a recession. As a result, the full model (Model D) generates a risk premium of 3.5%—a marked increase over the model without commitment (Model B) and low EIS (Model C).

The equity premium generated by the full model (D) is lower than common estimates of around 6%. What explains this discrepancy? First, in the real world, equity is a leveraged claim on the firm's profit and this model does not contain financial leverage. Following Gomes, Kogan, and Yogo (2007), the data implies a leverage-free risk premium of only 3.5% which is arguably a better benchmark for my model.[29] Accordingly, Bhamra, Kuehn, and Strebulaev (2007) show that a significant portion of the equity premium can be explained with financial leverage. Second, using the Gordon growth model, Fama and French (2002) find that the average expected equity premium for the period 1872-2000 is 3.5%. By the law of large numbers, the average expected and average realized excess return should converge in the long run. However in finite samples, this does not have to hold. More importantly, Fama and French (2002) argue that the equity premium estimate based on the Gordon growth model is closer to the true expected value than the estimate from average returns because of smaller standard errors of the former. Similarly, Donaldson, Kamstra, and Kramer (2007) estimate the ex ante equity premium with simulated method of moments of the dividend discount model and also find 3.5%.

The finding that the EIS has a strong impact on the equity premium contrasts with Tallarini (2000) who finds that only risk aversion, but not the EIS, affects the risk premium. I reach the opposite conclusion because his model does not contain any investment friction.

A high EIS is also necessary to achieve a realistic risk-free rate volatility. In Models A and C, the volatility of the risk-free rate is at least twice as high as in the data. A high risk-free rate volatility is also the major shortcoming of internal habit based explanations of the equity premium puzzle such as Jermann (1998) and Boldrin, Christiano, and Fisher (2001). Internal habit preferences increase the curvature of the utility function, thereby raising risk aversion and lowering the EIS. A low EIS means that the household is very eager to smooth consumption over time. To achieve this goal, the household's demand for the risk-free bond is high, especially in recessions. But the supply is perfectly inelastic, since the bond is in zero net supply. To accommodate these demand swings, the risk-free

[29]The expected return of a leveraged portfolio, which is long V dollars in the market portfolio and short bV dollars in the risk-free asset, is given by

$$\mathbb{E}[R^L] = \frac{1}{1-b}\mathbb{E}[R] - \frac{b}{1-b}R^f$$

where R^L is the levered market return. Using US data as reported in Table 3.2 and a leverage ratio of $b = 0.52$ for the post-war sample (Gomes, Kogan, and Yogo (2007)) implies that the leverage-free risk premium $\mathbb{E}[R] = 3.5\%$.

rate has to adjust accordingly, resulting in a large risk-free rate volatility.

Even though Campanale, Castro, and Clementi (2007) rely on recursive preferences, their choice of a low EIS has the same counterfactual implication regarding the risk-free rate volatility. In the commitment model (D) with a high EIS, however, the volatility of the risk-free rate is 0.89% compared to 0.97% in the data. Thus, the magnitude of the equity premium is not achieved at the cost of a high risk-free rate volatility.

The Sharpe ratio, $\mathbb{E}[R - R^f]/\sigma(R - R^f)$, is lowest in the low EIS model without commitment (A) and highest in the full model (D). In high EIS commitment model, it is 0.37, compared to 0.33 in the data. In the last row of Table 3.2, I report the volatility of time aggregated annual consumption growth, $\sigma(\Delta \ln C^a)$. It varies between 2.1% and 2.9%, compared to 2.9% in the data. The reason for the equity premium puzzle is the low volatility of consumption growth. It is therefore important that models, which try to explain the equity premium, do not exceed empirical estimates of the consumption volatility.

As a robustness check, I also compute the equity premium for other project half-lifes. The equity premium increases in the half-life of projects because the frequency of a binding commitment constraint increases, too. When the project half-life is only 2 years, the equity premium falls to 2.8%. When the project half-life increase to 5 years, the equity premium reaches 4%.

3.4.3 Decomposing Excess Returns

Since one goal of the model is to endogenously generate shocks to future expected consumption growth and/or future consumption volatility, I use Equation (3.24) for excess returns to gauge the strength of the model. Figure 3.5 shows the conditional covariance between returns and log consumption growth, i.e. the first component of excess returns in Equation (3.24), and Figure 3.6 depicts the conditional covariance between returns and the value function, i.e. the second component of excess returns in Equation (3.24), as a function of lagged investment for different levels of the capital stock. In both figures, the solid line corresponds to a low capital stock level, the dashed line to a medium capital stock level, and the dotted line to a high capital stock level.

Both components of the excess return are flat when the commitment constraint is not binding but strictly increasing in lagged investment when the commitment constraint becomes binding. This demonstrates that commitment endogenously generates a loading on the second risk factor, thereby causing a failure of the standard power utility consumption CAPM. Bansal and Yaron (2004) show that excess returns are constant in their long-run

risk model when there are no shocks to consumption volatility. The fact that excess returns are state-dependent in the commitment model indicates that a binding commitment constraint causes time-varying consumption volatility. I explore this channel in more detail in the next section. Another interesting observation is that the covariance of returns with the value function is an order of magnitude larger than the covariance of returns with consumption growth.

As shown in Section 3.2.3.2.3, the fact that the commitment constraint is not constant introduces a second term into the expression for marginal Q in Equation (3.13). This term shows up as second kink for high levels of lagged investment, thereby reducing the slope at which excess returns increase with lagged investment.

3.4.4 Consumption Volatility

A negative technology shock causes a negative shock to current consumption growth and leads to an increase in the volatility of consumption growth when the commitment constraint binds. But more importantly, the effect of investment commitment on consumption is not restricted to the instantaneous impact but is long-lasting. In particular, I will show that the conditional consumption volatility is high for several periods after the adverse shock. This evidence illustrates that the commitment friction provides a strong internal propagation mechanism, in particular, since the model is driven by a (geometric) random walk.

To gauge the strong internal propagation mechanism, I first estimate a GARCH(1,1) process for consumption data to have a measure of the conditional consumption volatility. In simulated data, a binding commitment constraint can be identified by a high book-to-market ratio as shown with Equation (3.16).

In Panel A of Table 3.5, I report estimates of a GARCH(1,1) process for consumption growth based on real data and simulated data using quasi maximum likelihood. A GARCH(1,1) process postulates the following process for the conditional variance of log consumption growth

$$\sigma_t^2 = \omega_0 + \omega_1 \varepsilon_{t-1}^2 + \omega_2 \sigma_{t-1}^2$$

where ε_t is the demeaned consumption growth rate. Consumption is real non-durable plus service expenditures at quarterly frequency for the period 1947-2006. In the data, both the ARCH component w_1 and the GARCH component w_2 are significant at the 1% level. The model generates a similar result. The volatility of consumption growth is explained well by a GARCH(1,1) process, but with a higher coefficient on the ARCH component w_1

and lower one on the GARCH component w_2. This means that the instantaneous effect is stronger in the model than in the data and the consumption volatility is more persistent in the data than in the model.

Panel B reports forecasting regressions of future consumption volatilities, σ_{t+s}, on the current log book-to-market ratio BM_t using the estimated time-series from the GARCH(1,1). As a proxy for the aggregate book-to-market ratio I use the average value-weighted book-to-market ratio taken from Kenneth French's website.[30] I use the quarterly return of the value-weighted CRSP index to compute quarterly book-to-market ratios. In the data, the current book-to-market ratio is positively related with current consumption volatility, implying that consumption volatility is counter-cyclical. Further, a high book-to-market ratio forecasts a high consumption volatility over the next several quarters.[31] The model generates similar results. A binding commitment constraint causes an increase in the book-to-market ratio and an increase in consumption volatility. Similar to the data, a high book-to-market ratio also causes high consumption volatility for the next quarters. This implies that the effect of a binding commitment constraint is not momentary, but instead causes a *persistent* shock to the volatility of the equilibrium consumption process. This mechanism is exactly the second building block of the long-run risk of Bansal and Yaron (2004) and the reason why the commitment model outperforms the standard production economy without commitment. In the model, the effect is again less persistent than in the data.

Panel C presents forecasting regressions of the future log book-to-market ratio BM_{t+s} on the current consumption volatility σ_t. The data shows that a rise in consumption volatility predicts a fall in asset prices. Bansal and Yaron (2004) show analytically that an EIS greater than one is necessary for this relation to hold—a condition satisfied in the model. In line with the data, high consumption volatility forecasts a high book-to-market ratio.

3.4.5 Cyclicality and Persistence

This section examines the cyclicality and persistence of moments of returns and consumption growth. To gain a a better understanding of the cyclicality of stock returns, I rewrite the Euler equation (3.21) in terms of the risk-free rate, R^f_{t+1}, the conditional correlation

[30] Specifically, I use the data of the 3-portfolio sort and weight the average value-weighted book-to-market ratio of each tercile with their respective weights of 30%, 40% and 30% to get a proxy for the aggregate book-to-market ratio.

[31] This evidence is consistent with Kandel and Stambaugh (1990), Bansal, Khatchatrian, and Yaron (2005) and Bansal and Yaron (2004) who use the price-dividend and price-earnings ratio.

between the pricing kernel and future stock returns, $\text{Corr}_t(M_{t+1}, R_{t+1})$, the conditional volatility of the pricing kernel, $\sigma_t(M_{t+1})$ and the conditional volatility of stock returns, $\sigma_t(R_{t+1})$, i.e.

$$\mathbb{E}_t[R^e_{t+1}] \quad = \quad -R^f_{t+1}\text{Corr}_t(M_{t+1}, R_{t+1})\sigma_t(M_{t+1})\sigma_t(R_{t+1})$$

The main determinants of the cyclicality of stock returns are the conditional volatility of the pricing kernel and the conditional volatility of stock returns. Less important is the effect of the risk-free rate; it tends to be procyclical because it is mainly governed by the expected growth rate of consumption. The conditional correlation between the pricing kernel and future stock returns is irrelevant because the model is driven by a single shock and thus these two variables are conditionally perfectly (negatively) correlated.

The standard real capital friction is convex adjustment costs, employed, for instance, by Jermann (1998), Kaltenbrunner and Lochstoer (2006), and Campanale, Castro, and Clementi (2007). A major drawback of this friction is that excess returns tend to be *procyclical.* The problem is that convex adjustment costs have opposing effects on the conditional volatility of the pricing kernel and stock returns.

On the one hand, convex adjustment costs can potentially give rise to countercyclical consumption growth volatility.[32] As productivity falls, investment must be reduced by increasing rates if consumption were to fall at a constant rate. However, convex adjustment costs prevent this from happening, resulting in countercyclical consumption growth volatility. Accordingly, the volatility of the pricing kernel is countercyclical, leading to countercyclical excess returns.

On the other hand, convex adjustment costs also result in procyclical stock return volatility. The reason is the following: Under convex costs, firms incur adjustment costs when they disinvest as well as invest. At the aggregate and sectoral level, investment is always positive. Hence, convex costs impact quantities and prices in booms, leading to procyclical stock return volatility.[33] The second channel usually dominates the first one because the volatility of stock returns is an order of magnitude larger than the volatility of the pricing kernel. Hence, excess returns tend to be procyclical. The investment commitment friction overcomes this drawback because it impacts quantities and prices only in recessions. Consequently, the volatility of the pricing kernel and stock returns are both countercyclical and therefore excess returns are as well.

[32] A point made by Gomes, Kogan, and Zhang (2003) and Cooper and Priestley (2005).

[33] In a convex adjustment costs model with constant returns to scale, marginal Q equals average Q, i.e. $P_t = q_t K_{t+1}$.

To assess the cyclicality of the model, Table 3.3 reports the (unconditional) correlation of realized log consumption growth, $\Delta \ln C_t$, with the conditional volatility of consumption growth, $\sigma_t(\Delta \ln C_{t+1})$, conditional excess returns, $\mathbb{E}_t[R_{t+1}] - R^f_{t+1}$, conditional stock return variance, $\mathrm{Var}_t(R_{t+1})$, conditional consumption beta, β_t, and market price of risk, λ_t. This table shows the cyclicality for all four model versions as in Table 3.2. Models A and B have no commitment whereas Models C and D have commitment; Models A and C have a low EIS whereas Models B and D have a high EIS.

In the models without commitment (Model A and B) the conditional volatility of consumption growth, conditional excess returns and the conditional volatility of stock returns are procyclical—contradicting empirical facts. Conditional consumption betas and the market price of risk are procyclical as well.

In the models with commitment (Model C and D) the conditional volatility of consumption growth, conditional excess returns and the conditional volatility of stock returns are countercyclical, consistent with the empirical evidence in Kandel and Stambaugh (1990). Conditional consumption betas are also countercyclical, which is consistent with Ang, Chen, and Xing (2006) downside beta risk story. The market price of risk is only slightly countercyclical. These countercyclicalities are caused by the asymmetric nature of the commitment friction.

Table 3.4 reports the (unconditional) autocorrelation of the price-earnings ratio, P_t/E_t, the book-to-market ratio, K_{t+1}/P_t, conditional excess returns, $\mathbb{E}_t[R_{t+1}] - R^f_{t+1}$, conditional stock return volatility, $\sigma_t(R_{t+1})$, conditional consumption beta, β_t, expected consumption growth, $E_t[\Delta \ln C_{t+1}]$, and conditional variance of consumption growth, $\mathrm{Var}_t(\Delta \ln C_{t+1})$ of the commitment model.

Two interesting results emerge: First, in the model, price-scaled ratios, such as the price-dividend and book-to-market ratio, are highly persistent. The same finding applies to the conditional first and second moments of stock returns. Second, conditional first and second moments of consumption growth are also highly persistent, even though the economy is driven by a random walk shock. In their long-run model, Bansal and Yaron (2004) assume that the first-order autocorrelation of expected consumption growth and variance of consumption growth is 0.94 and 0.96, respectively. The commitment model generates an autocorrelation of 0.51 and 0.49, respectively. It therefore provides partial justification of their exogenously assumed consumption process.

3.4.6 Predictability

The predictability of stock returns is another important feature of the data. As shown in Section 3.2, in the standard irreversible investment model the book-to-market ratio is a sufficient statistic for expected returns. In the investment commitment model, the book-to-market is not a sufficient statistic for returns, thus giving rise to additional predictor variables. Specifically, since investment is not completed instantaneously, lagged investment arises as an additional state variable in the model. Accordingly, the lagged investment rate helps to forecast returns. The model predicts that, controlling for the current book-to-market ratio and investment rate, there is *positive* relation between the lagged investment rate and the first and second moments of future returns. Higher lagged investment means that the firm has initiated large investment projects in the past which it is committed to complete in the future.

Table 3.7 reports time-series regressions of the conditional excess return, $\mathbb{E}_t[R^e_{t+1}]$ (Panel A), conditional volatility of stock returns, $\mathrm{Var}_t(R_{t+1})^{1/2}$ (Panel B), and future realized excess returns, R^e_{t+1} (Panel C) on the book-to-market ratio, and current and lagged investment rate. I simulate 50 years of data 100 times and report cross-simulation averages. On simulated data (Regression 1), the book-to-market ratio is positively related with expected returns (Panel A) and realized excess returns (Panel C), consistent with Fama and French (1992). Further, the current investment rate (Regression 2) forecasts lower expected and realized excess returns, consistent with Xing (2006). Panel B shows that future stock return volatility is positively related to the book-to-market ratio (Regression 1) and negatively to the current investment rate (Regression 2).

In univariate Regression 3 of Panel A, B and C, the lagged investment rate is not significant at the 5% level. But Regressions 4 and 6 in Panel A, B, and C confirm the intuition that lagged investment is positively related with future returns and return volatility after controlling for the current book-to-market ratio and/or current investment rate.

To test these predictions, I use the following quarterly data: Excess returns are the difference between the return on the value-weighted CRSP index and the 90-day treasury bill; the investment rate is constructed using (3.7) based on real nonresidential investment data coming from the BEA-NIPA. Table 3.8 reports the results. In Panel A, I regress future realized excess returns on the current and lagged investment rate. As a proxy for the conditional return volatility, I use the absolute value of excess returns which I regress on the same predictor variables in Panel B. In both Panels, the lagged investment rate is positively related with future returns and return volatility; yet it is only significant at the

5% level for return volatility.

The statistical properties of return predictability regressions has to be evaluated with care (e.g. Stambaugh (1999), Lewellen (2004), Boudoukh, Richardson, and Whitelaw (2006) and Ang and Bekaert (2007)). The reason is that conventional tests of the predictability of stock returns are invalid, i.e., reject the null too frequently, when the predictor variable is persistent and its innovations are highly correlated with returns. These two conditions are typically satisfied when the predictor variable is a price-scaled variable such as the dividend yield. To gain a better understanding, consider the following setup:

$$\begin{aligned} r_t &= \alpha + \beta x_{t-1} + \varepsilon_t \\ x_t &= \phi + \rho x_{t-1} + \mu_t \end{aligned}$$

where r denotes returns and x_t the dividend yield. Since an increase in price leads to a decrease in the dividend yield, the residuals are negatively correlated. Consequently, ε_t is correlated with x_t and one assumption of OLS is violated. Moreover, Lewellen (2004) shows that β estimates are biased by $\gamma(\hat{\rho} - \rho)$ where $\gamma = \text{Cov}(\varepsilon, \mu)/\text{Var}(\mu)$. For the dividend yield, Lewellen (2004) reports an auto-correlation of 0.997 and $\text{Corr}(\varepsilon, \mu) = -0.96$. As a result, standard estimates and tests are potentially invalid. However, since the investment rate is *not* a price-scaled variable, the same critic applies to a much lesser extent. In particular, the correlation between innovations to excess returns and the investment rate is very small, i.e. $\text{Corr}(\varepsilon, \mu) = -0.09$.

Another aspect is return predictability at long horizons. Challenging the view that stock returns follow a random walk, Campbell and Shiller (1988) and Fama and French (1988) show that the dividend yield forecasts stock returns and the explanatory power increases with the horizon. I replicate their finding within the model and Table 3.6 reports the results. Consistent with empirical facts, a high price-earnings ratio predicts lower future returns and the R^2 of the regressions is increasing with the horizon. Previous production economy models, such as Jermann (1998) and Boldrin, Christiano, and Fisher (2001), have not been able to replicate this empirical fact.

3.5 Conclusion

In this paper, I explore the asset pricing implications of investment commitment in a general equilibrium economy, where the household has Epstein-Zin preferences. A common assumption in literature is that investment occurs instantaneously. A more realistic assumption is that investment projects last for many periods and require current expen-

ditures as well as commitment to future expenditures. The contribution of this paper is to provide a tractable specification of investment commitment and gauge its general equilibrium effects on asset prices.

In equilibrium, consumption and investment are determined jointly and, as a result, the investment commitment friction impacts the equilibrium consumption process. With standard convex or non-convex investment frictions, investment occurs instantaneously and the impact of the friction is momentary. In contrast, investment commitment also impacts the *distribution* of *future* consumption growth rates, because commitments in long-term projects are not satisfied immediately. As a consequence, investment commitment generates time-varying first and second moments of consumption growth. Since the household has Epstein-Zin preferences, this effect gets priced, leading to a significant larger equity premium and return volatility. The same mechanism underlies the long-run risk model of Bansal and Yaron (2004). Whereas they assume an exogenous consumption process with time-varying first and second moments, these dynamics arise endogenously in a production economy with investment commitment.

Investment commitment is also an asymmetric friction. It affects consumption mainly after adverse shocks, when firms would like to reduce investment. Consequently, first and second moments of expected excess returns are endogenously counter-cyclical relative to consumption growth. Furthermore, the model generates novel empirical implications regarding the predictability of stock returns which find support in the data. Lagged investment arises as state variable in the model, capturing the amount of committed expenditures. The model predicts that, ceteris paribus, times with a higher lagged investment rate are riskier because the consequences of adverse shocks are more severe. Using aggregate data, I find that there is a positive relation between the lagged investment rate and future returns and return volatility which is significant for the latter.

Figure 3.1: Investment Rate Histogram

This figure shows the histogram of the investment rate measured as ratio of capital expenditures (data128) over lagged property, plant, and equipment (data8). The light bars represent the investment rate when its numerator is reduced by sales of property, plant, and equipment (data107).

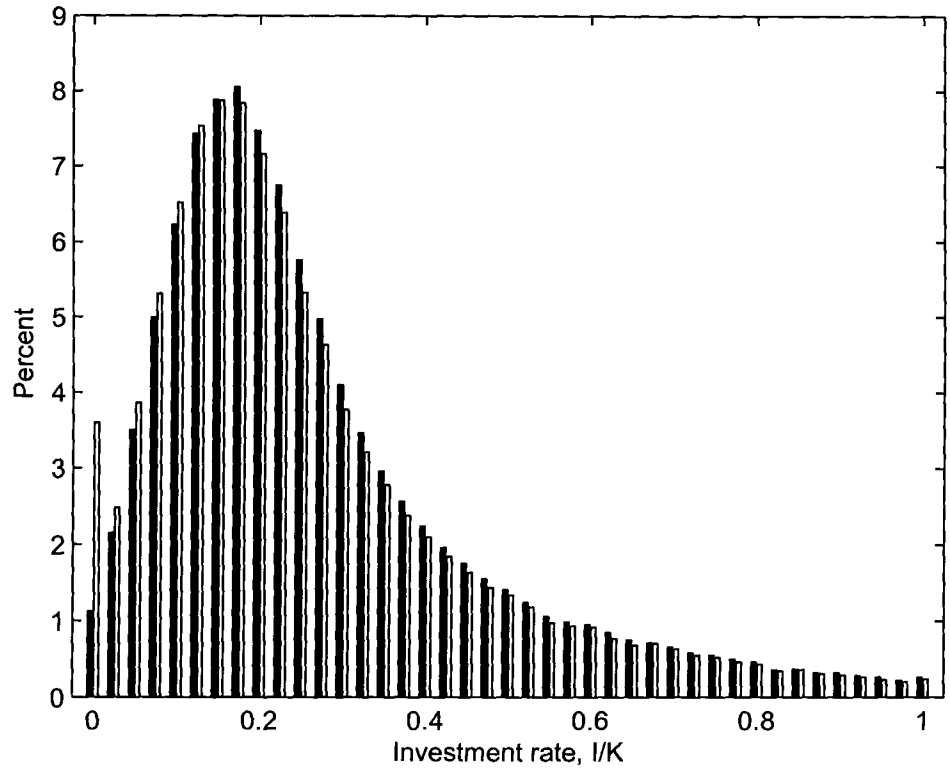

Figure 3.2: Consumption-Wealth Ratio

This figure depicts the consumption-wealth ratio φ as a function of EIS for a constant $\hat{\mu}$. The solid line represents $\hat{\mu} = 0.1$ and the dashed one $\hat{\mu} = 10$.

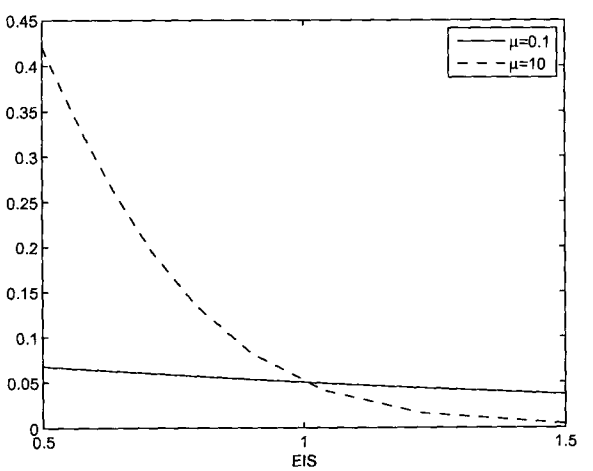

Figure 3.3: Value function

This figure shows the social planer's value function, $V(K_t, I_{t-1}, Z_t)$, as a function of capital, K_t, and lagged investment, I_{t-1}, at $Z_t = 1$.

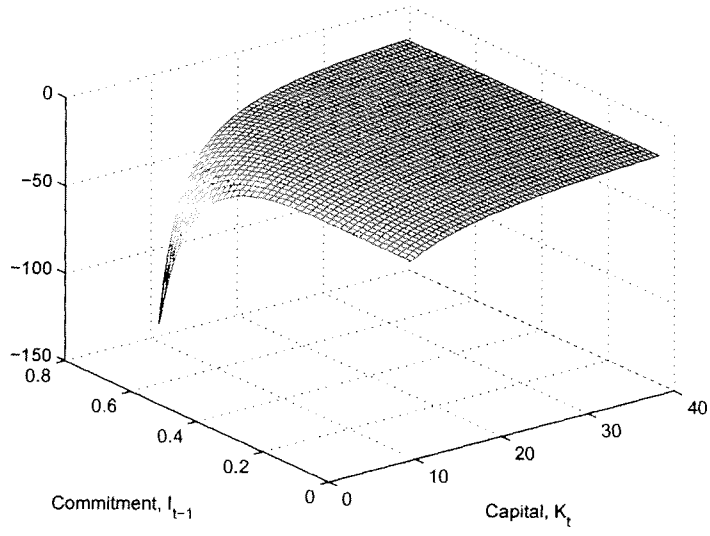

Figure 3.4: Consumption policy function

This figure shows the consumption policy function, $C_t = C(K_t, I_{t-1}, Z_t)$, as a function of capital, K_t, and lagged investment, I_{t-1}, at $Z_t = 1$.

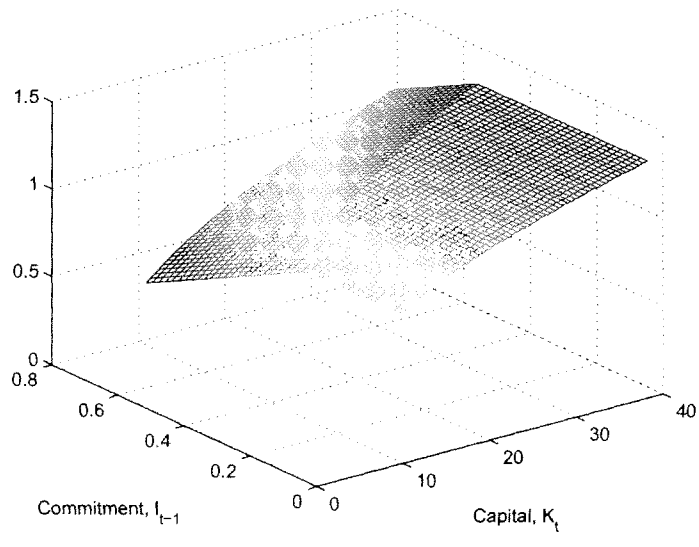

Figure 3.5: Covariance with Consumption Growth

This figure shows the conditional covariance between returns and log consumption growth, i.e.

$$-(\rho - 1)\mathrm{Cov}_t(R_{t+1}, \Delta c_{t+1})$$

as a function of lagged investment for different levels of the capital stock. The solid line corresponds to a low capital stock level, the dashed line to a medium capital stock level, and the dotted line to a high capital stock level.

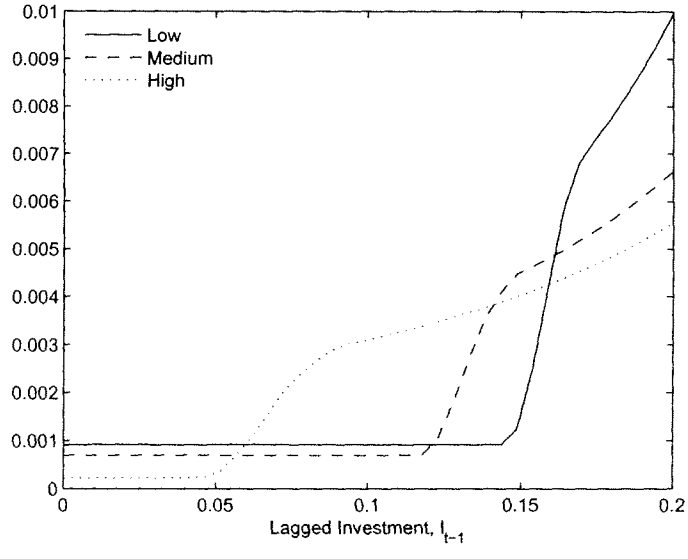

Figure 3.6: Covariance with the Value Function

This figure shows the conditional covariance between returns and the value function, i.e.

$$-(1-\gamma-\rho)\mathrm{Cov}_t\left(R_{t+1}, \ln U_{t+1}\right)$$

as a function of lagged investment for different levels of the capital stock. The solid line corresponds to a low capital stock level, the dashed line to a medium capital stock level, and the dotted line to a high capital stock level.

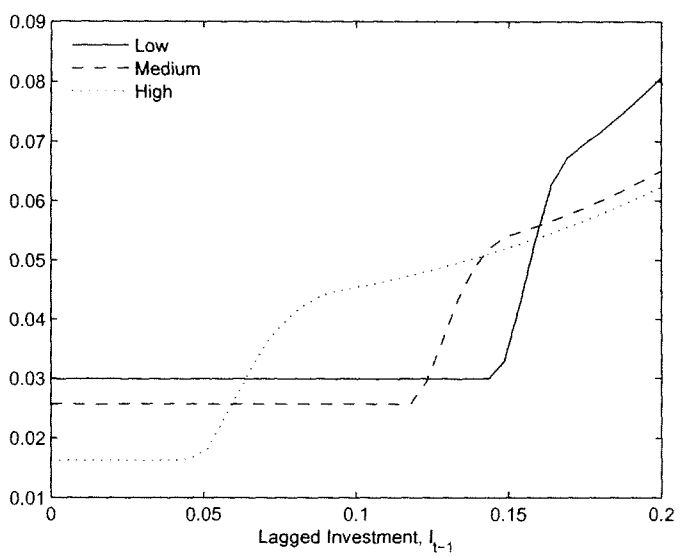

Table 3.1: Parameters of the Benchmark Calibration
This table summarizes the benchmark calibration of the model. All values are quarterly.

Parameter	Symbol	Value
Preferences:		
Discount rate	β	0.995
Risk aversion	γ	10
EIS	ψ	1.5
Technology:		
Growth rate	g	0.004
Std. deviation	σ	0.03
Production:		
Capital elasticity	α	0.4
Depreciation	δ	0.02
Commitment	w	0.95

Table 3.2: Model Comparison

This table presents the unconditional moments generated by four versions of the model: Model A and B have no commitment and a low and high EIS, respectively; Model C and D have commitment and and a low and high EIS, respectively. In the table, $\mathbb{E}[R - R^f]$ and $\sigma(R - R^f)$ denote the average excess return and excess return volatility, $\mathbb{E}[R^f]$ and $\sigma(R^f)$ the average risk-free rate and risk-free rate volatility, $\sigma(\Delta \ln C^a)$ the volatility of annual log consumption growth. All moments are annualized. The data column is taken from Bansal and Yaron (2004) and their sample period is 1929-1998.

			Model			
Parameter		Data	A	B	C	D
Risk aversion	γ		10	10	10	10
EIS	ψ		0.5	1.5	0.5	1.5
Project half-life	H (years)		0	0	3.5	3.5
Variable	Statistic					
Excess return	$\mathbb{E}[R - R^f]$ (%)	6.33	0.10	0.23	0.61	3.47
	$\sigma(R - R^f)$ (%)	19.42	0.45	0.78	1.86	9.29
Sharpe-Ratio	$\mathbb{E}[R - R^f]/\sigma(R - R^f)$	0.33	0.23	0.30	0.33	0.37
Risk-free rate	$\mathbb{E}[R^f]$ (%)	0.86	1.56	0.36	2.98	1.20
	$\sigma(R^f)$ (%)	0.97	2.54	1.49	2.09	0.89
Consumtion growth	$\sigma(\Delta \ln C^a)$ (%)	2.93	2.54	2.19	2.09	2.95

Table 3.3: Cyclicality

This table reports the (unconditional) correlation of realized log consumption growth, $\Delta \ln C_t$, with the conditional volatility of consumption growth, $\sigma_t(\Delta \ln C_{t+1})$, conditional excess returns, $\mathbb{E}_t[R_{t+1} - R^f_{t+1}]$, conditional stock return volatility, $\sigma_t(R_{t+1})$, conditional consumption beta, β_t, and market price of risk, λ_t. Each model is simulated for 5,000 periods.

		Model			
Parameter		A	B	C	D
Risk aversion	γ	10	10	10	10
EIS	ψ	0.5	1.5	0.5	1.5
Project half-life	H (years)	0	0	3.5	3.5
Variable	Statistic				
Cond. consumption volatility	$\sigma_t(\Delta \ln C_{t+1})$	0.09	0.05	-0.15	-0.47
Cond. risk premium	$\mathbb{E}_t[R_{t+1} - R^f_{t+1}]$	0.14	0.39	-0.21	-0.46
Cond. return volatility	$\sigma_t(R_{t+1})$	0.13	0.00	-0.05	-0.47
Cond. consumption beta	β_t	0.13	0.28	-0.07	-0.47
Cond. market price of risk	λ_t	0.13	0.51	-0.14	-0.28

Table 3.4: Persistence

This table reports the (unconditional) auto-correlation of the price-earnings ratio, P_t/E_t, the book-to-market ratio, K_{t+1}/P_t, conditional excess returns, $\mathbb{E}_t[R_{t+1}] - R^f_{t+1}$, conditional stock return variance, $\mathrm{Var}_t(R_{t+1})$, conditional consumption beta, β_t, expected consumption growth, $E_t[\Delta \ln C_{t+1}]$, conditional variance of consumption growth, $\mathrm{Var}_t(\Delta \ln C_{t+1})$ of the benchmark model D based on 5,000 periods.

		Lags (quarters)				
Variable	Statistic	1	2	3	4	8
Price-earnings ratio	P_t/E_t	0.97	0.93	0.90	0.87	0.74
Book-to-market ratio	K_{t+1}/P_t	0.92	0.86	0.81	0.77	0.66
Excess return	$\mathbb{E}_t[R_{t+1}] - R^f_{t+1}$	0.44	0.22	0.10	0.04	0.04
Return volatility	$\mathrm{Var}_t(R_{t+1})$	0.48	0.24	0.11	0.06	0.05
Consumption beta	β_t	0.45	0.22	0.09	0.04	0.04
Risk-free rate	R^f_t	0.56	0.39	0.30	0.27	0.22
Consumption growth	$\mathbb{E}_t(\Delta \ln C_{t+1})$	0.51	0.33	0.23	0.19	0.15
	$\mathrm{Var}_t(\Delta \ln C_{t+1})$	0.49	0.25	0.12	0.07	0.06

Table 3.5: Consumption Volatility

Panel A reports estimates of a GARCH(1,1) process for consumption growth, i.e. the conditional variance of log consumption growth is

$$\sigma_t^2 = \omega_0 + \omega_1 \varepsilon_{t-1}^2 + \omega_2 \sigma_{t-1}^2$$

Panel B reports forecasting regressions of future consumption volatilities on the current log book-to-market ratio and Panel C forecasting regressions of the future log book-to-market ratio on the current consumption volatility. Data: Consumption data is real non-durable plus service expenditures at quarterly frequency for the period 1947-2006. The aggregate book-to-market ratio is from Kenneth French's website. t-statistics are reported in parenthesis and based on Newey-West with 8 lags.

	Data		Model	
Panel A: GARCH(1,1) Estimates				
ω_1	0.17		0.26	
	(15.51)			
ω_2	0.79		0.43	
	(3.47)			
Panel B: Forecasting σ_{t+s}	Slope	R^2	Slope	R^2
$s = 0$	0.28	0.35	0.29	0.83
	(4.65)			
$s = 1$	0.28	0.34	0.18	0.32
	(4.40)			
$s = 2$	0.28	0.35	0.12	0.14
	(4.33)			
$s = 3$	0.28	0.34	0.08	0.06
	(4.07)			
$s = 4$	0.27	0.31	0.05	0.03
	(3.75)			
Panel C: Forecasting $\ln BM_{t+s}$	Slope	R^2	Slope	R^2
$s = 1$	1.29	0.37	0.02	0.27
	(6.98)			
$s = 2$	1.30	0.37	0.01	0.11
	(6.78)			
$s = 3$	1.28	0.36	0.01	0.05
	(6.16)			
$s = 4$	1.25	0.34	0.01	0.02
	(5.81)			

Table 3.6: Long-Run Predictability

This table presents predictability regression of future excess returns on the log price-earnings ratio, $pe_t = \ln P_t/Y_t$,

$$R^e_{t+s} = a + b\, pe_t + \varepsilon_{t+s}$$

at quarterly frequency. Standard errors are corrected using Newey-West with 8 lags. Data: The price-earnings ratio is from Robert J. Shiller's website. The excess return is the CRSP value-weighted return and the risk-free rate is the 90-day T-Bill. The data is at quarterly frequency spanning the period 1926-2006.

	Data		Model	
s	Slope	R^2	Slope	R^2
1	-4.12	0.02	-6.74	0.01
	(-1.94)			
2	-7.08	0.04	-13.14	0.02
	(-2.30)			
3	-10.12	0.05	-20.73	0.04
	(-2.47)			
4	-15.16	0.08	-28.22	0.06
	(-2.48)			
8	-25.34	0.12	-59.15	0.12
	(-2.59)			
12	-32.55	0.14	-93.12	0.17
	(-2.59)			

Table 3.7: Conditional First and Second Moments of Returns

This table presents regressions of the conditional first and second moments of stock returns on accounting information. In Panel A, I regress the conditional excess return, $\mathbb{E}_t[R^e_{t+1}]$, on the current book-to-market ratio, $BM_t = K_{t+1}/P_t$, and the current and lagged investment rate, $IR_t = I_t/K_t$ and $IR_{t-1} = I_{t-1}/K_{t-1}$, respectively. Panel B and C contain the same regression specification but the dependent variable is the conditional variance of stock returns, $\mathrm{Var}_t(R_{t+1})^{1/2}$, and future realized excess returns, R_{t+1}, respectively. I simulate 100 times 50 years of data and report cross-simulation averages. All regressions are at quarterly frequency and standard errors are corrected using Newey-West with 8 lags.

	Const.	BM_t	IR_t	IR_{t-1}	R^2
	Panel A: Expected Excess Returns				
1	-0.10	0.15			0.75
	(-20.02)	(20.57)			
2	0.00		-0.01		0.01
	(6.18)		(-0.62)		
3	0.00			0.01	0.01
	(4.93)			(1.03)	
4	0.00		-0.44	0.44	0.18
	(6.16)		(-8.44)	(9.23)	
5	-0.11	0.15	-0.05		0.80
	(-23.85)	(24.42)	(-2.81)		
6	-0.10	0.15	-0.13	0.09	0.80
	(-21.47)	(21.96)	(-4.89)	(4.13)	
	Panel B: Cond. Volatility of Returns				
1	-0.24	0.35			0.69
	(-16.40)	(17.00)			
2	0.01		-0.11		0.03
	(9.96)		(-2.45)		
3	0.01			-0.03	0.01
	(8.63)			(-0.72)	
4	0.01		-1.17	1.10	0.20
	(10.43)		(-8.97)	(9.19)	
5	-0.26	0.37	-0.18		0.78
	(-21.55)	(22.58)	(-4.55)		
6	-0.25	0.36	-0.42	0.24	0.79
	(-19.39)	(20.29)	(-6.04)	(4.30)	

	Const.	BM_t	IR_t	IR_{t-1}	R^2
	Panel C: Realized Excess Returns				
1	-0.25	0.35			0.05
	(-4.36)	(4.39)			
2	0.00		-0.10		0.00
	(3.08)		(-1.48)		
3	0.00			-0.00	0.00
	(1.75)			(-0.01)	
4	0.00		-1.58	1.53	0.03
	(2.35)		(-5.19)	(5.15)	
5	-0.26	0.37	-0.19		0.06
	(-4.30)	(4.33)	(-2.06)		
6	-0.23	0.33	-0.88	0.73	0.07
	(-3.72)	(3.75)	(-3.03)	(2.43)	

Table 3.8: Short-Run Predictability

This table reports time-series regressions of future realized excess returns (Panel A) and the absolute value of excess returns (Panel B) on the current and lagged investment rate. Data: CRSP value-weighted return, 90-day t-bill rate, real nonresidential investment.

Const.	IR_t	IR_{t-1}	R^2
	Forecasting R^e_{t+1}		
0.10	-2.71		0.01
(2.18)	(-1.75)		
0.10	-10.53	7.92	0.02
(2.18)	(-1.53)	(1.21)	
	Forecasting $\lvert R^e_{t+1}\rvert$		
0.04	0.83		0.00
(1.08)	(0.75)		
0.03	-11.50	12.48	0.05
(0.89)	(-2.55)	(2.60)	

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Chapter 4

The Levered Equity Risk Premium and Credit Spreads: A Unified Framework[34]

4.1 Introduction

A growing body of empirical work indicates that common factors may affect both the equity risk premium and credit spreads on corporate bonds. In particular, there is now substantial evidence that stock returns can be predicted by credit spreads,[35] and that movements in stock-return volatility can explain movements in credit spreads. In credit risk, Collin-Dufresne, Goldstein, and Martin (2001) show that credit spread changes across firms are driven by a single factor. These results suggest that there is "*overlap between the stochastic processes for bond and stock returns*" (Fama and French (1993, p. 26, our emphasis)).

The existence of common factors indicates that the two well-known puzzles, the equity risk-premium puzzle and the credit risk puzzle[36], are inherently linked. Motivated by this empirical evidence, this paper aims to provide a unified consumption-based framework for resolving both the credit spread and equity risk premium puzzles. Similar to Bansal and Yaron (2004), we make two assumptions. First, there is intertemporal macroeconomic risk: the expected values and volatilities (first and second moments) of fundamental economic growth rates vary with the business cycle, which is modeled by a regime-switching process. Second, agents prefer intertemporal risk to be resolved sooner rather than later.

Specifically, we price corporate bonds in a consumption-based asset pricing model with a representative agent. In particular, we assume aggregate consumption consists of wages paid to labor and firms' earnings, and the division between wages and earnings is exogenous.

[34]A version of this chapter will be submitted for publication. Bhamra, H.S. and Kuehn, L.-A. and Strebulaev, I.A., The Levered Equity Risk Premium and Credit Spreads: A Unified Framework.

[35]See Chen, Roll, and Ross (1986), Keim and Stambaugh (1986), Campbell (1987), Fama and French (1989), Fama and French (1993) Campbell and Ammer (1993).

[36]The credit risk puzzle refers to the finding that structural models of credit risk generate credit spreads smaller than those observed in the data when calibrated to observed default frequencies. Recent evidence is presented in Eom, Helwege, and Huang (1999), Ericsson and Reneby (2003) and Huang and Huang (2003).

Earnings are divided into coupon payments to bondholders and dividends to equityholders. Capital structure is chosen optimally by equityholders to maximize firm value which implies the endogeneity of both coupons and dividends. In addition, equityholders choose a default boundary to maximize equity value so that the default boundary is also endogenous. Thus, in our model, the prices of equity and debt are not only linked by a common state-price density, but they are also affected by the optimal leverage and default decisions. Essentially, we embed the contingent-claim models of Fischer, Heinkel, and Zechner (1989) and Leland (1994) inside an equilibrium consumption-based model.[37] We call the resulting framework a *structural-equilibrium* model.[38]

We then introduce intertemporal macroeconomic risk into our structural-equilibrium model to capture a common macroeconomic factor that underlies both expected excess stock returns and credit spreads. Modelling of intertemporal macroeconomic risk hinges on several critical and intuitive features. Firstly, the properties of firms' earnings growth change with the state of the economy, with expected growth lower in recessions and volatility lower in booms. Secondly, the properties of consumption growth also change with the state of the economy. As expected, first moments are lower in recessions, whereas second moments are higher. We model switches in the state of the economy via a Markov chain.[39] Thirdly, we assume that the representative agent cares about the intertemporal composition of risk. In particular, she prefers uncertainty about the future to be resolved sooner rather than later.[40] In essence, she is averse to uncertainty about the future state of the economy. We model this by assuming that the representative agent has Epstein-Zin-Weil preferences.

The representative agent, of course, does not use actual probabilities to compute prices. Instead, she uses risk-neutral probabilities. It is well-known that for a risk-averse agent, the risk-neutral probability of a bad event occurring exceeds its actual probability. In the context of our model, asset prices will depend on the risk-neutral probability (per-unit time)

[37]Since in contingent-claim models the state-price density is not linked to consumption, the asset prices they produce are completely divorced from macroeconomic variables, such as aggregate consumption. Consequently, these models alone cannot be used to find a macroeconomic explanation for a common factor behind stock returns and credit spreads.

[38]The germ of this idea is contained in within Goldstein, Ju, and Leland (2001). They state that their EBIT-based model can be embedded inside a consumption-based model, where the representative agent has power utility, though they do not investigate how credit spreads depend on the agent's risk aversion.

[39]See Hamilton (1989). For applications which study the stock market see Cecchetti, Lam, and Mark (1993), Whitelaw (2000), Calvet and Fisher (2008), Calvet and Fisher (2007) and Hansen, Heaton, and Li (2006). For option pricing applications see Jobert and Rogers (2006). The Markov chain approach to modelling intertemporal macroeconomic risk is related to the long-risk model of Bansal and Yaron (2004).

[40]Kreps and Porteus (1978, p. 186) explain the intuition for modelling preferences in this way via a coin-flipping example: "If ... the coin flip determines your income for the next two years, you probably prefer to have the coin flipped now [instead of later] ..."

of the economy moving from boom to recession. Increasing the risk-neutral probability of entering a recession increases the average duration of recessions in the risk-neutral world. When the average time spent in recessions in the risk-neutral world increases, it is intuitive that risk premia will go up. If the agent prefers earlier resolution of uncertainty, the risk-neutral probability of entering a recession exceeds the actual probability. Consequently, the agent prices assets as if recessions last longer than is actually the case, which raises risk premia.

The same mechanism delivers high credit spreads. To see the intuition, observe that the credit spread on default risky debt can be written as

$$s = r\frac{lq_D}{1 - lq_D},$$

where r is the risk-free rate, l is the loss ratio for the bond (which gives the proportional loss in value if default occurs) and q_D is the price of the Arrow-Debreu security which pays out 1 unit of consumption at default. Empirically, both the risk-free rate and loss ratio are too low to explain credit spreads. Thus, any economic channel which generates realistic credit spreads must raise the value of the Arrow-Debreu default claim. One of the novel results of this paper shows how q_D can be decomposed into three factors, each with an economically intuitive meaning:

$$q_D = \mathcal{T}\,\mathcal{R}\,p_D,$$

where p_D is the actual probability of default, $\mathcal{T}$ is a downward adjustment for the time value of money and $\mathcal{R}$ is an adjustment for risk. Actual default probabilities are small. Our decomposition then tells us that the value of the Arrow-Debreu default claim will be high if the risk adjustment, $\mathcal{R}$, and the time adjustment, $\mathcal{T}$, are high. So, why are they high in our model?

It is well known from Weil (1989) that using Epstein-Zin-Weil preferences makes it possible to obtain a low risk-free rate, simply by increasing the elasticity of intertemporal substitution. When the risk-free rate is low, the discount factor associated with the time value of money will be high. Therefore, the time-adjustment factor, $\mathcal{T}$, is high. This happens even if there is no intertemporal macroeconomic risk. Combining intertemporal macroeconomic risk with Epstein-Zin-Weil preferences increases the risk-neutral probability of entering a recession, which increases the risk-adjustment factor, $\mathcal{R}$. Thus, our model can generate high credit spreads, while keeping the actual probability of default low, as observed in the data.

Since the same economic mechanism increases both credit spreads and the risk premium, co-movement arises naturally between equity and corporate bond market values.

In particular, our model generates co-movement between credit spreads and stock return volatility as observed by Tauchen and Zhou (2006).

We now preview our quantitative results. For the benchmark case of relative risk aversion equal to 10 and an elasticity of intertemporal substitution of 1.5, the model delivers a 10-year BBB-AAA credit spread of 88 basis points, when the model-implied 10-year actual default probability is realistically low. The levered equity risk premium is 4.2% and levered equity volatility is 27%, which are close to empirical estimates[41]. Finally, the optimal static leverage ratio is 46% for BBB firms, lower than in most models of static capital structure (e.g. Leland (1994)).

Our framework also delivers a number of testable implications. From an asset pricing perspective, one important implication concerns the cyclicality of the default boundary. When expressed in cash flow terms the default boundary is countercyclical. But for values of risk aversion and elasticity of intertemporal substitution which generate a realistic equity premium and credit spread, the *asset-value* default boundary is procyclical. In contrast, Chen, Collin-Dufresne, and Goldstein (2006) show that in asset value terms, a habit formation model with i.i.d. consumption growth must have a countercyclical default boundary to generate a realistic credit spread. Thus, empiricists can study the default boundary to determine whether a model with intertemporal macroeconomic risk and Epstein-Zin-Weil preferences or a model with i.i.d. consumption and habit-formation preferences offers the more plausible framework for jointly resolving the equity risk premium and credit spread puzzles.

There are also myriad implications for corporate financing. For example, defaults cluster and also can occur simply because of worsening macroeconomic conditions, despite there being no change in earnings. Financing decisions are subject to hysteresis effects, i.e. the timing of past financing decisions influences default and leverage decisions, even though our model is fully rational. When capital structure is chosen in recessions, the optimal leverage ratio is lower. However, once leverage has been chosen, market leverage is lower in booms as in Korajczyk and Levy (2003). Taken together this implies that capital structures across firms co-move and the macroeconomic conditions at previous financing dates are cross-sectional determinants of current leverage ratios.

In the remainder of the introduction we discuss the relationship between our paper and the existing literature. On one side, our paper inherits features of structural models of credit risk (Merton (1974), Fischer, Heinkel, and Zechner (1989), Leland (1994), Goldstein, Ju, and Leland (2001), and Hackbarth, Miao, and Morellec (2006)). On the other side, our

[41] See Mehra and Prescott (1985), Weil (1989) and Hansen and Jagannathan (1991)

model is deeply indebted to a different set of forebears: consumption-based asset pricing models (Lucas (1978) and in particular Bansal and Yaron (2004)).

We now discuss several papers with which our paper is particularly close. The contingent-claims, structural model that our paper is most closely related to is Hackbarth, Miao, and Morellec (2006).[42] They also study the influence of macroeconomic factors on credit spreads. Importantly, they are the first to show that changing macroeconomic factors imply a countercyclical earnings default boundary. There are several key differences between our models. Firstly, Hackbarth, Miao, and Morellec do not use a state-price density linked to consumption and therefore do not study the equity risk premium and its relation to credit spreads. Also, their model does not allow them to check the size of actual default probabilities. Finally, while we study the impact of macroeconomic factors on both cash flows and discount rates, Hackbarth, Miao, and Morellec focus purely on the cash flow channel by assuming that firms' earnings levels jump down in recessions.

A second closely related paper is Chen, Collin-Dufresne, and Goldstein (2006). They study a pure consumption-based model and use two distinct mechanisms to resolve the equity risk premium and credit spread puzzles. The first mechanism is habit formation, which makes the marginal utility of wealth high enough in bad states so that the equity risk premium puzzle is resolved. This does not resolve the credit spread puzzle, because actual default probabilities and thus credit spreads are procyclical. To remedy this, Chen, Collin-Dufresne, and Goldstein use a second mechanism: they force the asset-value default boundary to be *exogenously countercyclical.* There are several key differences between our models. First, we only need one economic mechanism to generate a realistic credit spread and risk premium, as outlined above. Second, the asset-value default boundary in our model is *endogenously procyclical.* Third, the risk premium in our model is directly affected by default risk creating a *levered* risk premium and capital structure is endogenous. Finally, we obtain closed-form solutions for asset prices, which are natural extensions of the formulae in Leland (1994) and Lucas (1978).

Another related paper on credit spreads is David (2007). David prices corporate debt in a model where expected earnings growth rates and expected inflation follow a Markov switching process and are unobservable. This framework generates realistic credit spreads. Since David (2007) focuses on corporate bonds alone, he does not study the equity risk

[42] We have recently become aware of contemporaneous, but independent, work by Chen (2007), who uses a similar modelling framework to this paper. Chen (2007) seeks to resolve the low-leverage and credit spread puzzles, but does not address the issues of co-movement between bond and stock markets and the equity premium puzzle.

premium or co-movement between stock and bond markets.[43] Furthermore, he does not endogenize corporate financing decisions.

Our paper is not the first to consider default in a consumption-based model (see e.g. Alvarez and Jermann (2000), and Kehoe and Levine (1993)). These papers focus on default from the viewpoint of households. They assume households have identical preferences, but are subject to idiosyncratic income shocks. Households can default on payments in the same way that people cannot always pay back credit card debt or a mortgage. Chan and Sundaresan (2005) consider the bankruptcy of individuals in a production framework, looking at its impact on the equity risk premium and the term structure of risk-free bonds. Unlike the above papers, which look at personal bankruptcy, we look at firm bankruptcy and the pricing of corporate debt.

The remainder of the paper is organized as follows. Section 4.3 describes the structural-equilibrium model with intertemporal macroeconomic risk and Epstein-Zin-Weil preferences. Section 4.4 explores the implications of the model for pricing corporate debt and levered equity and develops an intuitive decomposition for the Arrow-Debreu default claim. In Section 4.5, we first calibrate the model and then we strip down the model to see which assumptions drive which results. We conclude in Section 4.6. Proofs and other additional material are contained in the Appendices.

4.2 An Example

Before describing the details of our model, we give a simple example showing how the credit spread is related to the price of the Arrow-Debreu security, which pays out a unit of consumption at default. Initially we do this when macroeconomic conditions *do not* affect default probabilities. We then extend this example to the case when default probabilities *do* increase in bad states together with bankruptcy costs and show how this can explain a substantial part of the credit spread puzzle.

First, we assume for simplicity that the economy *cannot* change state, thereby ruling out the possibility that default is more likely in bad states as well as any associated risk premium,

Consider a perpetual consol bond which pays c units of consumption per unit time until default, which occurs at some random time, τ_D. If default occurs the bondholder receives

[43]Since David (2007) restricts the state-price density to one that can be obtained from a representative agent with power utility, the shifts in growth rates are not priced. Clearly, it is possible to use this framework to study the equity risk premium. But it would be only be possible to generate a realistic premium with very high risk aversion.

a fraction, α, of the unlevered after-tax value of the firm. Let $A(X_D)$ be the after-tax unlevered value of the firm at default, which occurs when $X \leq X_D$. One can then express the bond price as

$$\frac{c}{r}(1 - q_{D,t}) + q_{D,t}\alpha A(X_D)$$

or

$$\frac{c}{r}(1 - q_D l),$$

where $\frac{c}{r}$ is the price of the risk-free perpetual consol bond which pays c units of consumption per unit time and $q_{D,t}$ is the price of the Arrow-Debreu claim that pays out a unit of consumption at default and $l = \frac{\frac{c}{r} - \alpha A(X_D)}{\frac{c}{r}}$ is the loss ratio in bond value if default occurs. The bond's credit spread, s, is the difference between its yield and the yield on the equivalent risk-free security, so

$$s = r\frac{q_D l}{1 - q_D l}. \tag{4.1}$$

Solving (4.1) for q_D gives

$$q_D = \frac{s}{l(r + s)}. \tag{4.2}$$

If the yields over treasuries of 10-year BBB and AAA debt are 190 and 51 b.p.'s, respectively, the amount of the BBB-Treasury yield spread caused by credit risk can be approximated by the BBB-AAA spread, which is 139 b.p. The loss ratio for 10-year BBB debt is 0.41 and the 10-year risk-free rate is 6.70% . Hence, (4.1) implies that $q_D = 0.29$.

Like any Arrow-Debreu security which pays out if some event occurs, q_D is just the probability of the event, adjusted upwards for risk and downwards for time (because of discounting). Hence,

$$q_D = \mathcal{T}\mathcal{R}p_D, \tag{4.3}$$

where $\mathcal{T}$ is a downwards time-adjustment, $\mathcal{R}$ is an upwards risk-adjustment and p_D is the actual default probability. The estimated actual default probability for 10 year BBB debt is 4.54%, so the product of the time and risk-adjustments is $\mathcal{T}\mathcal{R} = 6.39$. This simple calculation implies the existence of a substantial premium for default risk, so that the risk adjustment $\mathcal{R}$ is large *and* a low risk-free rate so that the time adjustment $\mathcal{T}$ is not too small.

However, if we assume that aggregate consumption and firm earnings growth are i.i.d. and agents have Epstein-Zin-Weil preferences, then it is not possible to generate a sufficiently high time and risk adjustment unless risk aversion is absurdly high. This is the credit spread puzzle.

In this paper we offer a potential resolution of the credit spread puzzle by assuming that the expected growth rates and volatilities of growth rates for consumption and earnings are time-varying, with lower expected growth rates and higher volatilities in bad times. Consequently, default is more likely in bad times, precisely when state prices are high and loss rates are high. This covariation increases the time and risk adjustments without increasing the average values of actual default probabilities. To see this note that the price of Arrow-Debreu claim that pays off a unit of consumption if default occurs before time T, conditional on the current state of the economy being i, is given by

$$q_{D,i,T-t} = E_t \left[\frac{\pi_{\tau_D}}{\pi_t} 1_{\{t \leq \tau_D < T\}} du | \nu_t = i \right], \tag{4.4}$$

where the state of the economy at time t is ν_t and π is the state-price density. Hence,

$$q_{D,i,T-t} = E_t \left[\frac{\pi_{\tau_D}}{\pi_t} | \nu_t = i \right] E_t \left[1_{\{t \leq \tau_D < T\}} du | \nu_t = i \right] + Cov_t \left[\frac{\pi_{\tau_D}}{\pi_t}, 1_{\{t \leq \tau_D < T\}} du | \nu_t = i \right]. \tag{4.5}$$

When defaults are more likely in bad times, $1_{\{t \leq \tau_D < T\}}$ is more likely to be equal to one when the state-price density is high, which increases the covariance in the above expression and hence the risk adjustment. Since covariation between the state-price density and the actual default probability caused by greater likelihood of defaults in bad times increases the probability of early default under the risk-neutral measure, time discount factor is less severe, leading to a higher time adjustment.

In addition to having more defaults in bad times, another essential ingredient in increasing the time adjustment is a low risk-free rate. We achieve this by using the state-price density of a representative agent with Epstein-Zin-Weil preferences. Because these preferences allow the separation of preferences over time from preferences over states, we can keep the risk-free rate low by assuming the agent has a low aversion to non-smooth intertemporal consumption (high EIS) and is hence willing to pay alot to save now via risk-free bonds in order to consume later. This leads to a higher risk-free bond price and hence a low risk-free rate.

4.3 Model

In this section we introduce the structural-equilibrium model with intertemporal macroeconomic risk. The basic idea is simple: we embed a contingent claims model inside a representative agent consumption-based model. Debt and levered equity are valued using the state-price density of the representative agent. Two consequences of this modelling approach are worth noting. Credit spreads depend on the agent's preferences and aggregate

consumption, which is not the case in pure structural models. The equity risk premium is affected by default risk, which is not the case in pure consumption-based models.

4.3.1 Aggregate Consumption and Firm Earnings

The are N firms in the economy. The output of firm n, Y_n, is divided between earnings, X_n, and wages and other human capital income, W_n, paid to workers. Aggregate consumption, C, is equal to aggregate output. Therefore,

$$C = \sum_{n=1}^{N} Y_n = \sum_{n=1}^{N} X_n + \sum_{n=1}^{N} W_n.$$

We model aggregate consumption and individual firm earnings directly, and thus aggregate wages are just the difference between aggregate consumption and aggregate earnings.[44]

Aggregate consumption, C, is given by

$$\frac{dC_t}{C_t} = g_t dt + \sigma_{C_t} dB_{C,t}, \tag{4.6}$$

where g is expected consumption growth, σ_C is consumption growth volatility and $B_{C,t}$ is a standard Brownian motion.

The earnings process for firm n is given by

$$\frac{dX_{n,t}}{X_{n,t}} = \theta_{n,t} dt + \sigma^{id}_{X,n} dB^{id}_{X,n,t} + \sigma^{s}_{X,n,t} dB^{s}_{X,t}, \tag{4.7}$$

where θ_n is the expected earnings growth rate of firm n, and $\sigma^{id}_{X,n}$ and $\sigma^{s}_{X,n}$ are, respectively, the idiosyncratic and systematic volatilities of the firm's earnings growth rate. Total risk, $\sigma_{X,n}$, is given by $\sigma_{X,n} = \sqrt{(\sigma^{id}_{X,n})^2 + (\sigma^{s}_{X,n})^2}$. The standard Brownian motion $B^{s}_{X,t}$ is the systematic shock to the firm's earnings growth, which is correlated with aggregate consumption growth:

$$dB^{s}_{X,t} dB_{C,t} = \rho_{XC} dt, \tag{4.8}$$

where ρ_{XC} is the constant correlation coefficient. The standard Brownian motion $B^{id}_{X,n,t}$ is the idiosyncratic shock to firm earnings, which is correlated with neither $B^{s}_{X,t}$ nor $B_{C,t}$.

4.3.2 Modelling Intertemporal Macroeconomic Risk

We assume that the first and second moments of macroeconomic growth rates are stochastic and governed by a non-stationary distribution, which converges to a long-run (also known as steady-state, stationary or ergodic) distribution. Specifically, we assume that g_t, θ_t,

[44] In assuming so we follow such papers as Kandel and Stambaugh (1991), Cecchetti, Lam, and Mark (1993), Campbell and Cochrane (1999) and Bansal and Yaron (2004).

$\sigma_{C,t}$ and $\sigma^s_{X,t}$ depend on the state of the economy, ν_t, which is either 1 or 2. Hence, the conditional expected growth rate of consumption, g_t, can take two values, g_1 and g_2, where g_i is the expected growth rate when the economy is in state $i \in \{1, 2\}$ and, similarly, for θ_t, $\sigma_{C,t}$ and $\sigma^s_{X,t}$.[45] State 1 is the low state and state 2 is the high state. The state changes according to a 2-state Markov chain, defined by λ_i, $i \in \{1, 2\}$, which is the probability per unit time of the economy leaving state i.[46] Since the first moments of fundamental growth rates are procyclical and second moments are countercyclical, we assume that $g_1 < g_2$, $\theta_1 < \theta_2$, $\sigma_{C,1} > \sigma_{C,2}$ and $\sigma^s_{X,1} > \sigma^s_{X,2}$.

The Markov chain gives rise to uncertainty about the future moments of consumption growth. This will only impact the state-price density if the representative agent cares about the intertemporal distribution of risk. To ensure this, we assume the representative agent has the continuous-time analog of Epstein-Zin-Weil preferences.[47] Consequently, the representative agent's state-price density at time-t, π_t, is given by (see Appendix B.2 for the derivation)

$$\pi_t = \left(\beta e^{-\beta t}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} C_t^{-\gamma} \left(p_{C,t} e^{\int_0^t p_{C,s}^{-1} ds}\right)^{-\frac{\gamma-\frac{1}{\psi}}{1-\frac{1}{\psi}}}, \tag{4.9}$$

where β is the rate of time preference, γ is the coefficient of relative risk aversion (RRA), and ψ is the elasticity of intertemporal substitution under certainty.[48] The Epstein-Zin-Weil agent cares about the intertemporal distribution of risk. Therefore, she cares whether news about consumption growth and hence future consumption is bad or good. Her state-price density then depends on the value of the claim to aggregate consumption per unit consumption, i.e. the price-consumption ratio, p_C. When $\gamma > 1/\psi$, bad news about consumption growth decreases the price-consumption ratio and leads to an increase in the state-price density. The intertemporal distribution of risk is also affected by how quickly news arrives. The rate of arrival of news is governed by the rate at which the distribution for the Markov chain converges to its long-run distribution. This rate is given by $p = \lambda_1 + \lambda_2$. A smaller p means news is arriving more slowly. When $\gamma > 1/\psi$, the agent prefers intertemporal risk to be resolved sooner rather than later, so a fall in p raises risk prices and increases the state-price density. An agent with time-separable preferences (e.g. $\gamma = 1/\psi$, power utility) does not care about the intertemporal distribution of risk, so the

[45] To ensure idiosyncratic earnings volatility, σ^{id}_X, is truly idiosyncratic, we assume it is constant and thus independent of the state of the economy.

[46] The extension to $L > 2$ states does not provide any further economic intuition and is straightforward.

[47] The continuous-time version of the recursive preferences introduced by Epstein and Zin (1989) and Weil (1990) is known as stochastic differential utility, and is derived in Duffie and Epstein (1992).

[48] Schroder and Skiadas (1999) provide a proof of existence and uniqueness for an equivalent specification of stochastic differential utility.

price-consumption ratio does not enter into her state-price density.

The switching probabilities per unit time, λ_i, $i \in \{1,2\}$ are not directly relevant for valuing securities. We must account for risk by using the risk-neutral switching probabilities per unit time, which we denote by $\hat{\lambda}_i$, $i \in \{1,2\}$. The following proposition relates the risk-neutral to the actual switching probabilities (per unit time) via the state-price density.

Proposition 1 *The risk-neutral switching probabilities per unit time are related to the actual switching probabilities per unit time by the risk-distortion factor, ω,*

$$\widehat{\lambda}_1 = \lambda_1 \omega^{-1}, \tag{4.10}$$

$$\widehat{\lambda}_2 = \lambda_2 \omega, \tag{4.11}$$

where ω measures the size of the jump in the state-price density when the economy shifts from the good state to the bad state, i.e.

$$\omega = \left. \frac{\pi_t}{\pi_{t-}} \right|_{\nu_{t-}=2,\nu_t=1}. \tag{4.12}$$

The size of the risk-distortion factor depends on the representative agent's preferences for resolving intertemporal risk: $\omega > 1$ ($\omega < 1$), if the agent is averse to (likes) intertemporal macroeconomic risk ($\gamma > 1/\psi$ and $\gamma < 1/\psi$, respectively) and $\omega = 1$, if the agent is indifferent to intertemporal macroeconomic risk ($\gamma = 1/\psi$).

The risk-neutral switching probabilities per unit time are related to the actual switching probabilities by the risk distortion factor, ω, which also gives is the price of risk, $\omega - 1$, associated with a change in the state of the economy from good to bad. The intuition is as follows. Switching from the good state to the bad state is bad news for consumption growth. When $\gamma > 1/\psi$ the agent dislikes this bad news, leading to a positive risk price ($\omega > 1$) and an upward jump in her state-price density.[49] Since the risk price is higher, the risk-neutral probability per unit time of switching from the good state to the bad state is higher then the actual probability, i.e. $\widehat{\lambda}_2 > \lambda_2$. Similarly, when considering the probability of moving from the bad state to the good state, $\widehat{\lambda}_1 < \lambda_1$. Since risk-neutral probabilities are used for pricing instead of actual ones, it follows that securities are priced *as if the bad state lasts longer and the good state finishes earlier* when $\gamma > 1/\psi$. With time-separable preferences such as power utility ($\gamma = 1/\psi$), the agent is indifferent to the temporal distribution of risk and changes in the moments of consumption are no longer

[49] To distinguish between the state of the economy before and after the jump, denote the time just before the jump occurs by $t-$, and the time at which the jump occurs by t. We give precise definitions of the left and right limits, ν_{t-} and ν_t, respectively in Equations (B.5) and (B.6) in the Appendix.

priced and so the risk price is zero ($\omega = 1$). Hence, the state-price density is not affected by news about future consumption growth and the risk-neutral switching probabilities are the same as the actual switching probabilities.

Since the state-price density jumps up in the bad state, asset returns contain a premium for jump risk. In particular, the presence of jump components forces the stochastic processes for bond and stock returns to overlap, a feature of the data observed by Fama and French (1993). Also, as long as jump risk is priced, there is a jump-risk component in credit spreads, which co-moves with the jump component in stock return volatility, as documented in Tauchen and Zhou (2006).[50]

4.3.3 Quantifying Long-Run Risk

Changes in fundamental growth rates are driven by a non-stationary distribution, which eventually converges to its long-run distribution (also called the stationary or ergodic distribution).[51] Intuition suggests that when this convergence occurs more slowly, there will be more long-run risk in the economy. Our model uses a Markov chain to model the non-stationary distribution. The Markov chain converges to its long-run distribution at the rate $p = \lambda_1 + \lambda_2$. Hence, the half-life of the Markov chain, given by

$$t_{1/2} = \frac{\ln 2}{p}, \tag{4.13}$$

is a quantitative measure of the amount of long-run risk in the economy. When this half-life is longer, there is more long-run risk.

We can also quantify the price impact of long-run risk via the risk-neutral half-life, which accounts for the effect of the agent's preferences. The risk-neutral half-life is computed using the rate of convergence of the Markov chain to its long-run distribution under the risk-neutral measure, i.e. $\hat{p} = \hat{\lambda}_1 + \hat{\lambda}_2$.

4.4 Asset Valuation

In this section we derive the prices of all assets in the economy and investigate the properties of credit spreads and the equity premium.

[50] In our model the jumps in prices are endogenous, because we derive prices by valuing cash flows with a state-price density, which depends on preferences and consumption. This is in contrast with previous contingent-claims models of debt featuring jumps in prices, such as Cremers, Driessen, and Maenhout (2006).

[51] One could use a number of ways to model this behavior, such as the Ornstein-Uhlenbeck process, the Cox-Ingersoll Ross process or a Markov chain. The advantage of the Markov chain approach is that it allows us to derive asset prices in closed-form without using the loglinear approximation of Campbell and Shiller.

To introduce benchmark corporate securities, equity and debt, we follow standard EBIT-based models of capital structure (see Goldstein, Ju, and Leland (2001)) where the earnings of a firm, X, is split between a coupon, c, promised to debtholders in a perpetuity and a dividend, $X - c$, paid to equityholders. If corporate income is taxed at the rate η, the after-tax distribution to equityholders is $(1-\eta)(X-c)$. Equityholders have the right to default which they exercise if earnings drop below a certain earnings level which we call the default boundary. As we discuss in detail in Section 4.4.4, the default boundary is endogenously state-dependent. For now we assume that default occurs in state i if $X \leq X_{D,i}$, $i \in \{1,2\}$. Upon default, bondholders receive what can be recovered of the firm's assets in lieu of coupons, i.e. a fraction α_t of the after-tax present value of the firm's earnings.

Since the value of any corporate security, such as debt and equity, can be written in terms of the prices of a set of Arrow-Debreu default claims, we now derive expressions for the values of these fundamental securities.

4.4.1 Arrow-Debreu Default Claims

The Arrow-Debreu default claim, denoted by $q_{D,ij,T-t}$, is the value of a unit of consumption paid upon default if the current state is i, default occurs before time T and the state at the moment of default is j. In other words, if the current date is t and the current state is i, and earnings hit the boundary $X_{D,j}$ from above for the first time in state j at some time time before T, one unit of consumption will be paid that instant. Since each Arrow-Debreu default claim is effectively a digital put, their values can be derived by solving a system of ordinary differential equations

$$E_t^{\mathbb{Q}}[dq_{D,ij,T-t} - r_i q_{D,ij,T-t} dt] = 0,\ i,j \in \{1,2\}. \tag{4.14}$$

Intuitively, the above conditions hold because of the no-arbitrage restrictions. Appendix B.3 provides a formal proof. For the case when $T \to \infty$, which we denote as $q_{D,ij,t}$, closed-form solutions can be obtained and are given in (B.3) in Appendix B.3.

The next proposition shows that the price of the Arrow-Debreu default claim can be decomposed into several intuitive components.

Proposition 2 *The price of the Arrow-Debreu default claim, $q_{D,ij,T-t}$, can be written as*

$$q_{D,ij,T-t} = p_{D,ij,T-t}\mathcal{T}_{ij,T-t}\mathcal{R}_{ij,T-t}, \tag{4.15}$$

where $p_{D,ij,T-t}$ is the actual probability of default occurring in state j before time T, conditional on the current state being i: $p_{D,ij,T-t} = \Pr(t < \tau_D \leq T | \nu_t = i, \nu_{\tau_D} = j)$, where τ_D

is the default time and $\mathcal{T}_{ij,T-t}$ is a time-adjustment factor given by the weighted average discount factor:

$$\mathcal{T}_{ij,T-t} = \int_t^T E_t^{\mathbb{Q}}\left[e^{-\int_t^s r_u du}\right] h_{ij}(s)ds, \tag{4.16}$$

where $E_t^{\mathbb{Q}}\left[e^{-\int_t^s r_u du}\right]$ is the time-t discount factor for risk-free cash flows arriving at time $s > t$ and the weight $h_{ij}(s)$ is risk-neutral density for default times, conditional on the current state being i and the state at default being j. $\mathcal{R}_{ij,T-t}$ is a risk-adjustment factor given by

$$\mathcal{R}_{ij,T-t} = \frac{\widehat{p}_{D,ij,T-t}}{p_{D,ij,T-t}}, \tag{4.17}$$

where $\widehat{p}_{D,ij,T-t}$ is the risk-neutral probability of defaulting in state j before time T, conditional on the current state being i.

Previous literature on credit spreads (e.g. Chen, Collin-Dufresne, and Goldstein (2006)) notes that to resolve the credit spread puzzle, risk-neutral default probabilities must be high, while actual default probabilities must be low. As the above decomposition shows, *Arrow-Debreu default claims* must be relatively high while actual default probabilities are low. That can be achieved via a high risk-adjustment factor, leading to a high risk-neutral default probability *and* a high time-adjustment factor.

To understand the intuition behind the time-adjustment factor, note that it performs two functions. First, it acts as a standard discount factor, since default is going to happen in the future. To see this assume that default can only occur at date T (as in Merton (1974)). Then the risk-neutral probability of defaulting before date s conditional on default occuring by time T is zero unless $s = T$.Therefore, the associated risk-neutral density is zero for $s \neq T$ and has all its probability mass concentrated at $s = T$. Hence, (4.16) reduces to the standard time-discount factor

$$\mathcal{T}_{ij,T-t} = E_t^{\mathbb{Q}}\left[e^{-\int_t^T r_u du}\right].$$

But in our model the timing of default is uncertain and this is where the second function of the time adjustment appears. If default could only occur at two times, T_1 and $T_2 > T_1$ the time-discount factor would be the risk-neutral expectation of the weighted average of the standard time-discount factors, $E_t^{\mathbb{Q}}[e^{-\int_t^{T_1} r_u}]$ and $E_t^{\mathbb{Q}}[e^{-\int_t^{T_2} r_u}]$, where the weights are the risk-neutral probabilities of default occurring at dates T_1 and T_2, respectively (both probabilities are conditional on default occurring to ensure they sum to one). The intuition is that when the agent thinks default under the risk-neutral measure is more likely at the earlier date T_1, more weight is put on the larger discount factor, $E_t^{\mathbb{Q}}[e^{-\int_t^{T_1} r_u}]$, and the

time-adjustment gets larger. When default can occur at any time, the two weights are replaced by a continuum of weights, given by the conditional risk-neutral default density, h_{ij}, as shown in (4.16).

The intuition behind the risk adjustment is straightforward: it is just a risk premium for default and is described as such in prior literature (see Berndt, Douglas, Duffie, Ferguson, and Schranz (2005) and Almeida and Philippon (2007)).

4.4.2 Abandonment Value

The firm's state-conditional liquidation, or abandonment value, denoted by $A_{i,t}$, is the after-tax value of the unlevered firm's future earnings, when the current state is i:

$$A_{i,t} = (1-\eta)X_t E_t\left[\int_t^\infty \frac{\pi_s X_s}{\pi_t X_t} ds \middle| \nu_t = i\right], \text{ for } i \in \{1,2\}. \tag{4.18}$$

The liquidation value in (4.18) is a function of the current earnings level and is time-independent, $A_{i,t} = A_i(X_t)$. The next proposition derives the value of A_i in terms of fundamentals of the economy.

Proposition 3 *The liquidation value in state $i \in \{1,2\}$ is given by*

$$A_i(X_t) = \frac{(1-\eta)X_t}{r_{A,i}}, \tag{4.19}$$

where

$$r_{A,i} = \overline{\mu}_i - \theta_i + \frac{(\overline{\mu}_j - \theta_j) - (\overline{\mu}_i - \theta_i)}{\widehat{p} + \overline{\mu}_j - \theta_j}\widehat{p}\widehat{f}_j,\ j \neq i, \tag{4.20}$$

and

$$\overline{\mu}_i = r_i + \gamma\rho_{XC}\sigma^s_{X,i}\sigma_{C,i}, \tag{4.21}$$

is the discount rate in the standard Gordon growth model.

To understand the intuition behind the discount rate (4.20), note that if the economy stays in state i forever, the discount rate reduces to the standard expression

$$r_{A,i} = \overline{\mu}_i - \theta_i. \tag{4.22}$$

If the economy is, say, in state 1 (recession), then the discount rate (4.20) is obtained by adjusting $\overline{\mu}_1 - \theta_1$ downwards by the amount, $\frac{(\overline{\mu}_2-\theta_2)-(\overline{\mu}_1-\theta_1)}{\widehat{p}+\overline{\mu}_2-\theta_2}\widehat{p}\widehat{f}_2 < 0$, to account for time spent in state 2 (boom) at future times. The magnitude of the adjustment increases with the growth rate in the boom state, θ_2, and the risk-neutral probability per unit time of switching into state 2, $\widehat{\lambda}_1$.

4.4.3 Credit Spreads and the Levered Equity Risk Premium

In this section, we provide closed-form expressions for corporate debt and levered equity prices. We then use these expressions to derive credits spreads and the levered equity risk premium.

The generic value of debt at time t, conditional on the state being i, denoted by $B_{i,t}$, is given by

$$B_{i,t} = E_t\left[\int_t^{\tau_D} \frac{\pi_s}{\pi_t} c ds \middle| \nu_t = \nu_i\right] + E_t\left[\frac{\pi_{\tau_D}}{\pi_t}\alpha_{\tau_D} A_{\tau_D} \middle| \nu_t = i\right], i \in \{1,2\}. \tag{4.23}$$

The first term in (4.23) is the present value of a perpetual coupon stream until default occurs at a random stopping time τ_D. The second term is the present value at time t of the asset recovery value the debtholders successfully claim upon default, where $\alpha_t \in \{\alpha_1, \alpha_2\}$ is the date t recovery rate. We show (see Proof of Proposition 4, Appendix B.3) that (4.23) reduces to

$$B_{i,t} = \frac{c}{r_{P,i}}\left(1 - \sum_{j=1}^{2} l_{ij,t} q_{D,ij,t}\right), \tag{4.24}$$

where

$$l_{ij,t} = \frac{\frac{c}{r_{P,j}} - \alpha_j A_j\left(X_{D,j}\right)}{\frac{c}{r_{P,i}}}$$

is the loss ratio at default, when the current state is i and default occurs in state j. The first factor in (4.24) is the price of the equivalent riskless consol bond, $c/r_{P,i}$, and the second factor is a downward adjustment for default risk, where $l_{ij,t}q_{D,ij,t}$ is the present value of the loss ratio. The discount rate for a riskless perpetuity when the current state is i is given by

$$r_{P,i} = r_i + \frac{r_j - r_i}{\widehat{p} + r_j}\widehat{p}\widehat{f}_j, j \neq i. \tag{4.25}$$

Note that $r_{P,i}$ is not equal to the risk-free rate in state i, r_i, because the risk-free rate is expected to change in the future whenever the state of the economy switches.[52]

The next proposition gives the corporate bond spread in terms of the discount rate for a risk-free perpetuity, loss ratios and Arrow-Debreu default claims. Note that we define the credit spread as the yield on corporate debt less the yield on an equivalent risk-free security of the *same maturity*, thus ensuring that the credit spread is not falsely inflated by a term spread.

[52]Note that (4.25) can be obtained from the formula for the discount rate for a stochastically growing cash flow, (4.20), by replacing the Gordon growth model discount rate, $\overline{\mu}_i$, with the risk-free rate, r_i and setting the expected growth rate of earnings, θ_i, equal to zero.

Proposition 4 *The credit spread in state i, $s_{i,t}$, is given by*

$$s_{i,t} = \frac{c}{B_{i,t}} - r_{P,i} = r_{P,i} \frac{\sum_{j=1}^{2} l_{ij,t} q_{D,ij,t}}{1 - \sum_{j=1}^{2} l_{ij,t} q_{D,ij,t}}, \; i \in \{1,2\}. \tag{4.26}$$

The above proposition tells us that credit spreads are affected by three components: a risk-free rate, loss rates, and Arrow-Debreu default claims. To summarize the economic intuition we developed so far, existing empirical evidence suggests that both risk-free rates and loss rates are too low. Therefore, the only way to generate realistic credit spreads is via higher prices for Arrow-Debreu default claims. However, since Proposition 2 shows that Arrow-Debreu default claims are just actual default probabilities adjusted for time and risk and actual default probabilities are low, the time- and risk-adjustment factors alone can generate realistic spreads.

Current levered equity value is given by the expected present value of future cashflows less coupon payments up until bankruptcy, conditional on the current state:

$$S_{i,t} = (1-\eta) E_t \left[\int_t^{\tau_D} \frac{\pi_s}{\pi_t} (X_s - c)\, ds \middle| \nu_t = i \right], \; i \in \{1,2\}.$$

We can show (see Proof of Proposition 5, Appendix B.3) that the above equation simplifies to give

$$S_{i,t} = A_i(X_t) - (1-\eta)\frac{c}{r_{P,i}} + \sum_{j=1}^{2} q_{D,ij} \left[(1-\eta)\frac{c}{r_{P,j}} - A_j(X_{D,j}) \right], \; i \in \{1,2\}. \tag{4.27}$$

The first two terms in the above equation are the present after-tax value of future cashflows less coupon payments, if the firm were never to default. The last term accounts for the fact that upon default shareholders no longer have to pay coupons to bondholders and at the same time they lose the rights to any future cash flows from owning the firm's assets.

In the next proposition we derive the levered equity risk premium of an individual firm.

Proposition 5 *The conditional levered equity risk premium in state i is*

$$\mu_{R,i} - r_i = \gamma \rho_{XC} \sigma_{R,i}^{B,s} \sigma_{C,i} + \Lambda_i, \; i \in \{1,2\}, \tag{4.28}$$

where Λ_i is the jump risk-premium in state i, given by

$$\Lambda_i = \begin{cases} (1-\omega^{-1})\sigma_{R,1}^{P}\lambda_1, & i = 1 \\ (1-\omega)\sigma_{R,2}^{P}\lambda_2, & i = 2 \end{cases}, \tag{4.29}$$

and

$$\sigma_{R,i}^{P} = \frac{S_j}{S_i} - 1,\ i \in \{1,2\},\ j \neq i \tag{4.30}$$

is the volatility of stock returns caused by Poisson shocks, $\sigma_{R,i}^{B,s}$ *is the systematic volatility of stock returns caused by Brownian shocks*

At first blush, one might expect the levered equity risk premium to be larger than the unlevered risk premium, simply because the act of paying coupons leaves behind less dividends for equity holders. But introducing leverage into a firm does not simply reduce dividend payments; it also brings in default risk. Intuitively, default risk increases the value of the option to default, which increases equity value and hence decreases the risk premium.

4.4.4 Optimal Default Boundary and Optimal Capital Structure

Equityholders maximize the value of their default option by choosing when to default and also choose optimal capital structure. Intuitively, the endogenous default boundary depends on the current state of the economy, i.e. there is a set of default boundaries $X_{D,i}$, $i \in \{1,2\}$, where $X_{D,i}$ is the default boundary when the economy is in state i. The default boundaries satisfy the following two standard smooth-pasting conditions:

$$\left.\frac{\partial S_i(X)}{\partial X}\right|_{X=X_{D,i}} = 0,\ i \in \{1,2\}. \tag{4.31}$$

In Appendix B.3 we prove that the default boundary is weakly countercyclical, i.e. $X_{D,1} \geq X_{D,2}$.

Equityholders choose the optimal coupon to maximize firm value at date 0. There are two important features to note. First, by maximizing firm value equityholders internalize debtholders' value at date 0. However, in choosing default times they ignore the considerations of debtholders. This feature creates the basic conflict of interest between equity and debtholders, which is standard in the optimal capital structure literature. Second, the optimal coupon depends on the state of the economy at date 0. To make this clear, we denote the date 0 coupon choice by $c_{i,0}$, where i is the state of the economy at date 0. Therefore equityholders choose the coupon to maximize date 0 firm value, $F_{i,0} = B_{i,0} + S_{i,0}$, i.e.

$$c_{i,0} = argmax\, F_{i,0}(c)$$

The choice of optimal default boundaries will depend on the coupon choice. This implies hysteresis in the sense that the default boundaries not only depend on the current state of

the economy, but also on its initial state. For simplicity we omit this in the notation for the default boundaries.

4.5 Empirical Implications

In this section we analyze the quantitative implications of the model. We start by obtaining conditional estimates of parameter values. Then, in Sections 4.5.3 and 4.5.4, we see whether our model can resolve the credit spread and equity risk premium puzzles. We also look at the term-structure of credit spreads (Section 4.5.5) and cross-market co-movement, the cyclicality of credit spreads (Section 4.5.7).

4.5.1 Calibration

To calibrate parameter values we use aggregate US data at quarterly frequency for the period from 1947Q1 to 2005Q4. Consumption is real non-durables plus service consumption expenditures from the Bureau of Economic Analysis. Earnings data are from S&P and provided on Robert J. Shiller's website. We delete monthly interpolated values and obtain a time-series at quarterly frequency. The personal consumption expenditure chain-type price index is used to deflate the earnings time-series. Unconditional parameter estimates are summarized in Table 4.1. With intertemporal macroeconomic risk, we need conditional estimates, and their calibrated values are are given in Table 4.1. We now discuss the calibration exercise in more detail.

We obtain estimates of λ_1, λ_2, g_1, g_2, θ_1, θ_2, $\sigma_{C,1}$, $\sigma_{C,2}$, $\sigma^s_{X,1}$, $\sigma^s_{X,2}$ and ρ_{XC} by maximum likelihood. The approach is based on Hamilton (1989) and details specific to our implementation are summarized in Appendix B.1.

Andrade and Kaplan (1998) report default costs of about 20% of asset value. Moreover, Thorburn (2000), Altman, Brady, Resti, and Sironi (2002) and Acharya, Bharath, and Srinivasan (2007) find that bankruptcy costs, $1 - \alpha_t$, are countercyclical, i.e., $\alpha_1 < \alpha_2$. Consequently, we assume $\alpha_1 = 0.7$ and $\alpha_2 = 0.9$. The annualized rate of time preference, β, is 0.01. The corporate tax rate, η, is set at 15%.

One goal of this paper is to study the term-structure of credit spreads as well as cross-sectional differences in credit risk. To this end, we calibrate the idiosyncratic earnings volatility so that the model-implied 5 year and 10 year default probability is consistent with the data of AAA and BBB firms. Moody's reports actual default probabilities of 0.106% and 0.555% at 5 year and 10 year for AAA firms and 2.037% and 4.732% at 5 year and 10 year for BBB firms. To match the time-series of actual default probabilities, we

vary the idiosyncratic earnings volatility between 5 year debt and 10 year debt, i.e., we set $\sigma^{id}_{AAA,5y} = 16\%$, $\sigma^{id}_{AAA,10y} = 10\%$, $\sigma^{id}_{BBB,5y} = 29.3\%$ and $\sigma^{id}_{BBB,10y} = 21\%$. To generate cross-sectional differences across firms, we scale the drift and systematic volatility of the earnings process. Specifically, we set the drift and systematic volatility of BBB firms equal to the estimates of aggregate earnings as reported in Table 4.1 and halve the values for AAA firms.

4.5.2 Arrow-Debreu Default Claims

Proposition 4 links the credit spread to the prices of Arrow-Debreu default claims which in turn depend on the time and risk-adjustment, 2. Therefore, we first focus on understanding their behavior and then use that understanding to explain the implications of our model for corporate bond prices and corporate financing decisions.

Table 4.2 shows how intertemporal macroeconomic risk affects risk-distortion factor, ω, the convergence rate of the Markov chain to its long-run risk-neutral distribution, $\widehat{p}$, the long-run risk-neutral distribution, $(\hat{f}_1, \hat{f}_2)$, the risk-adjustment R_i, and time-adjustment T_i of perpetual BBB debt for three values of the EIS parameter, ψ: 0.1, 0.75, and 1.5. It is also important to be clear about the importance of the size of ψ for our model's implications, because empirical estimates of its magnitude differ widely.[53] We do not vary RRA in Table 4.2 since the impact is tiny (results are available upon request).

The results of this paper are driven by the fact that the risk-distortion factor increases in the EIS. When $\psi = 0.1$, the risk-distortion factor, ω, equals 1 because the representative agent is indifferent to whether intertemporal uncertainty is resolved sooner rather than later. When $\psi = 0.5$, $\omega = 1.377$ and for $\psi = 1.5$, $\omega = 1.424$, reflecting an increasing preference for the early resolution of intertemporal uncertainty. This is reflected in more long-run risk as measured by $\widehat{p}$, because $\widehat{p}$ decreases as ω increases. Observe that this is purely a preference based effect, because while $\widehat{p}$ falls, the convergence rate of the Markov chain under the actual probability measure, p, is constant.[54] The increased preference for the early resolution of intertemporal uncertainty also shifts more weight on the recession state, as measured by $\hat{f}_1$.

Table 4.2 also shows that both the risk adjustment and time-adjustment increase with the EIS. These two effects raise the value of Arrow-Debreu default claims and hence credit spreads. The risk-adjustment increases with the EIS since in a larger distortion factor

[53]Hansen and Singleton (1982), Attanasio and Weber (1989), Vissing-Jorgensen (2002) and Guvenen (2006) estimate that $\psi > 1$, whereas Hall (1988) estimates that ψ is much less than one.

[54]Note that $\widehat{p} = p(\omega^{-1} f_1 + \omega f_2)$, where $f_i = \frac{\lambda_j}{p}$, $j \neq i$ and $p = \lambda_1 + \lambda_2$. It follows that $\widehat{p} < p$ and $\widehat{p}$ is decreasing in ω provided that the average duration of recessions is less than booms, i.e. $f_1 < f_2$.

leads to higher state prices in recessions and more long-run risk. When the EIS equals 0.1, the risk adjustment is just above 1. Increasing the EIS to 1.5 increases risk adjustments to around 1.4. The time-adjustment increases with the EIS since the EIS lowers the risk-free rate. When $\psi = 0.1$ the time adjustment is below to 0.5. Increasing ψ to 1.5 ensures that time-adjustments are around 0.8.

In all following results, we fix EIS at 1.5 and relative risk aversion at 10. These are also the values suggested by Bansal and Yaron (2004).

4.5.3 Results Summary

Table 4.3 summarizes the main results of this paper. We report credit spreads, actual and risk-neutral default probabilities, Arrow-Debreu default claim prices and leverage ratios for 5 year debt (Panel A) and 10 year debt (Panel B). The numbers in parenthesis are historical estimates for the US. Actual default probabilities come from Moody's and the BBB-AAA spread from Duffee (1998). Throughout the paper, we compute credit spreads as the difference between the yields of defaultable bonds relative to non-defaultable bonds with the same maturity.

The fact that the standard structural bond pricing model cannot explain high credit spreads, given low actual default probabilities, is called the credit spread puzzle. In our model, credit spreads are soley determined by default risk. In reality, taxes and liquidity affects credit spreads as well. A natural benchmark is therefore the spread between BBB and AAA debt which should be mainly affected by default risk difference between BBB and AAA firms. Duffee (1998) estimates the BBB-AAA spread for short-term debt (2-7 years maturities) to be 75b.p. and for medium term debt (7-15 years maturities) to be 70b.p. Since our model generates 74b.p. for 5 year debt and 88b.p. for 10 year debt, we are able to resolve the credit spread puzzle.

Another way to study the credit spread puzzle to look at the level of BBB credit spreads and to adjust the BBB spread for the default component. Duffee (1998) reports that the BBB credit spread is 142b.p. for short-term debt and 147b.p. for medium term debt. Based on CDS data, Longstaff, Mithal, and Neis (2005) estimate that the default component of BBB credit spreads is 71%. Hence, the BBB credit spread due to default risk is 101b.p. for short-term debt (2-7 years) and 104b.p. for medium term debt (7-15 years). Our model generates 78b.p. and 100b.p. for 5 and 10 year debt.

To gain a better understanding of credit spreads, we can use our decomposition of Arrow-Debreu default claims. Credit spreads are determined the value of Arrow-Debreu default claims which depend on the time and risk-adjustment. Unfortunately, there does

not exist empirical estimates of the time-adjustment but the literature provides estimates of the risk-adjustment. The risk-adjustment is the ratio of risk-neutral to actual default probabilities. For 10 year debt, Almeida and Philippon (2007) report 2.07 for AAA firms and 4.00 for BBB. Our model produces a risk-adjustment 2.26 for AAA and 1.75 for BBB. Thus, we are able to replicate the risk-adjustment for AAA firms but not for BBB firms. Another estimate of the risk-adjustment is provided by Berndt, Douglas, Duffie, Ferguson, and Schranz (2005) who report an average risk-adjustment of 2.757.

In general, there are two patterns in the risk-adjustments: first, the risk-adjustment increases with the maturity; second, the risk-adjustment is higher for AAA firms than for BBB firms. The first implication finds support in Almeida and Philippon (2007); yet the second implication is at odds with the data. The reason for this result is that we calibrate AAA to have lower idiosyncratic volatility than BBB firms which seems reasonable. As a result, debt financing is cheaper for AAA firms than for BBB and therefore AAA firms lever up. Importantly, higher leverage also implies higher risk-adjustments. However, in the data AAA firms have lower leverage ratios than BBB firms. For instance, Kisgen (2006) reports an average leverage ratio of 39% for AAA firms and 49% for BBB firms. Our model predicts 53% for AAA firms and 43% for BBB firms.

In Panel C of Table 4.3, we report the unlevered and levered equity premium and levered equity volatility for AAA and BBB firms by setting the idiosyncratic volatility equal to zero.

Leverage has two effects on the risk premium. First, the dividend payment to equity holders is reduced by the coupon. Second, the probability of default shifts value from debtholders to equityholders. The act of paying coupons increases the risk premium. Default risk, however, increases the value of equity via introducing a default option and thus decreases the premium. Overall, the coupon effect dominates the default risk effect and leverage increases the risk premium from 0.79% to 2.73% for AAA firms and from 1.91% to 4.2% for BBB firms. Our model also generates realistic equity volatility of 18% for AAA firms and 27% for BBB firms.

4.5.4 Stripping Down the Model: What Causes What?

Next, we show how our each of our modelling assumptions impacts the credit spread and the equity risk premium. We strip down the model by removing intertemporal macroeconomic risk in the first and second moments of earnings and consumption growth and the representative agent's preference about the resolution of that risk over time. This leaves us with Model 1a, where aggregate consumption growth and earnings growth is i.i.d. and the

representative agent has power utility. We then rebuild the model piece-by-piece. In Model 1b, we introduce Epstein-Zin-Weil preferences. In Model 2, we add Markov switching in the first and second moments of earnings growth, but not to consumption growth. And finally, in Model 3 (a and b), we rebuild the model fully by having Markov switching in the first and second moments of consumption growth.

To get a fair comparison across models, we recalibrate the idiosyncratic earnings volatility across models to match the actual 5 year default probability of BBB firms, which is 2.04%. Moreover, we choose the coupon so that every model version has a 43% leverage ratio. Table 4.4 summarizes our results for the corporate bond and equity markets. Note, that we report the results for 5 year corporate debt in Panel A.

Bond Market

When there is no intertemporal macroeconomic risk, using Epstein-Zin-Weil preferences instead of power utility (moving from Model 1a to Model 1b) increases credit spreads. This increase stems purely from an increase in the time-adjustment factor in the Arrow-Debreu default claim: separating relative risk aversion from the EIS allows us to reduce the risk-free rate.

Adding Markov switching to the first and second moments of earnings growth, but not consumption growth (moving from Model 1b to Model 2) does not impact the size of the credit spread much at all, because these switches are not correlated with the state-price density, and are hence not priced. This is summarized by the fact that the risk-distortion factor, ω, is 1.

Introducing switching in the moments of consumption growth (moving from Model 2 to Model 3) leads to a significant increase in the spread. Switches in the moments of earnings growth are now correlated with the state-price density ($\omega > 1$, so the state-price density jumps up whenever expected earnings growth/earnings growth volatility jumps down/up), so they are priced into credit spreads. This is reflected by an increase in the risk-adjustment factor.

In summary, two assumptions lie behind the model's ability to generate high prices for Arrow-Debreu default claims, without increasing actual default probabilities. The first is the use of Epstein-Zin-Weil preferences, which increases the time-adjustment factor by lowering the risk-free rate. The second is the assumption of switching in the first and second moments of both earnings and consumption growth rates (intertemporal macroeconomic risk). When the representative agent is averse to the delayed resolution of intertemporal risk ($\gamma > 1/\psi$), she dislikes intertemporal macroeconomic risk, which is reflected in an

increased risk-adjustment factor.

Equity Market and Risk-Free Rate

Introducing Epstein-Zin-Weil preferences when there is no intertemporal macroeconomic risk decreases the risk-free rate. This, however, has no impact on risk premia (see Kocherlakota (1990)). While switching in the moments of earnings growth makes the price-earnings ratio procyclical, which implies risk premia are countercyclical, risk premia remain small. Finally, introducing switching in the first and second moments of consumption growth increases risk premia since upward jumps in the state-price density are now correlated with downward jumps in price-earnings ratios, creating a jump-risk premium. We can see this clearly in the behavior of the risk-distortion factor, which is now greater than 1. That implies that the risk-neutral probability of switching from a boom to a recession is now higher than the actual probability, which is reflected in a higher risk premium.

4.5.5 Term-Structure

We have shown our model can match historical BBB-AAA credit spreads at maturities of 5 and 10 years *as well as* historical default probabilities at those maturities. But to achieve this, we chose different values for the idiosyncratic earnings volatility, σ_X^{id}, at 5 and 10 year maturities. In this section, we investigate how well our model can match both the term structure of credit spreads and actual default probabilities by keeping σ_X^{id} constant across maturities.

Figure 4.2 depicts the term-structure of actual default probabilities, $p_{D,i,T}$, of BBB firms ($\sigma_X^{id} = 25\%$) conditional on the initial state of the economy. In addition, the figure also contains the term-structure of BBB rated debt as reported by Moody's. We find that our model cannot match the term structure of credit spreads. The reason is that the shape of the term structure of model implied actual default probabilities is different from the observed term structure. As a result, the 5 year implied default probability is too low, whereas the 10 year implied default probability is too high. Consequently, our model implies 5 year credit spread is too low, while the implied 10 year credit spread is too high. Figure 4.5 depicts the resulting term-structure of credit spreads, $s_{i,T}$. Even though recession actual default probabilities are higher than boom ones, $p_{D,1,T} > p_{D,2,T}$, the reverse is true for credit spreads, $s_{1,T} < s_{2,T}$. The reason is that firms issue more debt in booms than in recessions which increase the leverage ratio and thus credit spreads.

The term structure of implied default probabilities is a function of the cash flow process and the default boundary. Since adjusting the default boundary will not change the shape

of the term structure, it follows that the model implied shape of the term structure of implied default probabilities is a consequence of the stochastic process we have assumed for the cash flow process and has nothing to do with our choice of preferences or consumption process. Thus, we would face the same problem if we had assumed i.i.d. consumption growth with habit formation as in Chen, Collin-Dufresne, and Goldstein (2006). Hence, the only way to obtain a more realistic term structure for default probabilities would be to use an alternative cash flow process. One possibility would be to introduce stochastic idiosyncratic volatility, which is mean-reverting. One could model this via assuming that idiosyncratic volatility is subject to Markov regime-shifts. But to ensure that the volatility is truly idiosyncratic, one would have to assume that the Markov chain driving the regime shifts is uncorrelated with the Markov we already have, which drives the state of the economy.

In addition to providing a term structure of credit spreads and actual default probabilities, our model also gives a term structure of Arrow-Debreu default claims and risk-neutral probabilities. Hence, we can obtain the term structure of risk and time adjustments. Figure 4.1 depicts the term-structure of actual, $p_{D,i,T}$, and risk-neutral default probabilities, $\widehat{p}_{D,i,T}$, as well as Arrow-Debreu default claim prices, $q_{D,i,T}$, when the initial state is 1 (top panel) and 2 (bottom panel). For maturities up to 10 years, risk-neutral default probabilities exceed Arrow-Debreu default claim prices ($\widehat{p}_{D,i,T} > q_{D,i,T}$) and actual default probabilities ($\widehat{p}_{D,i,T} > p_{D,i,T}$). This is a consequence of time discounting and the pricing of risk. Moreover, Arrow-Debreu default claim prices exceed actual default probabilities ($q_{D,i,T} > p_{D,i,T}$). However, for very long maturities, Arrow-Debreu default claim prices are *less than* actual default probabilities (see Figure 4.4). The reason is that at very long maturities, time discounting is very severe, making the time adjustment $\mathcal{T}$ and hence the price of the Arrow-Debreu default claim smaller. The risk adjustment increases slightly over time. Therefore the product of the time and risk adjustment is decreasing at very large maturities, ensuring that $q = \mathcal{T}\mathcal{R}p < p$. This has an important consequence for the term structure of credit spreads—it creates a hump shape (see Figure 4.5).

4.5.6 Business Cycle vs Long-Run Risk

The calibration on which our results are based assumes that that the Markov regime shifts occur at a business cycle frequency. This is quite different from the calibration used in Bansal and Yaron, where growth rates and volatilities are subject to much lower frequency shifts, motivating the use of the term long-run risk. Clearly our 'Business Cycle' calibration assumes that cash flows and consumption contain less long-run risk than the 'Long-Run

Risk' calibration of Bansal and Yaron. We can measure long-run risk by the half-life $t_{1/2}$ of the Markov chain that drives regime-shifts. In this section we study the impact of increasing long-run risk firstly on the credit spread for perpetual debt and secondly on the term structure of credit spreads. That enables us to study how the pricing implications of our model change as we move from our original business cycle calibration to a long-run risk calibration, which is closer to Bansal and Yaron.

The credit spread on *perpetual* debt increases with more long-run risk. The reason is that more long-run risk leads to greater demand for long-dated risk-free assets, and so the perpetual risk-free rate falls. This increases the time-adjustment factor and hence the price of the perpetual (infinite maturity) Arrow-Debreu default claim rises, as shown in Figure 4.6.

In contrast, the credit spread on 5 year corporate debt *decreases* when there is more long-run risk. The same results holds for 10 year debt which we do not report to save space. This is a consequence of optimal corporate financing decisions. More long-run risk increases expected bankruptcy costs making it optimal to reduce leverage as time-variation in cash flow and consumption growth rates becomes more persistent. Hence, default boundaries fall as long-run risk increases, leading to lower 5 year actual default probabilities as shown in Figures 4.7. The price of 5 year Arrow-Debreu default securities is then lower with more long-run risk.

4.5.7 Comovement and Cyclicality

In this section we are interested whether our model can replicate the comovement between markets, the cyclicality of credit spreads, credit spread volatility and default probabilities. Table 4.5 summarizes the results. All results in this table are cross-simulation averages based on 100 panels each containing 3000 firms simulated for 5 years at monthly frequency.

Both earlier work by Fama and French (1993) and more recent work by Tauchen and Zhou (2006) finds evidence of comovement between bond and stock market variables. Tauchen and Zhou regress Moody's AAA and BBB credit spread on the jump-component of aggregate stock-return volatility and find significant regression coefficients of 1.50 and 1.92, respectively. To replicate Tauchen and Zhou's findings for BBB credit spreads we simulate 5 years of monthly data and then regress the equally-weighted credit spread on the value-weighted jump component of stock-return volatility for each panel in line with Tauchen and Zhou's empirical exercise. Table 4.5 (Panel A) shows that our model generates a mean regression coefficients of 1.76 which is very close to the result Tauchen and

Zhou obtained from actual data.[55]

Our model's results hinge crucially on intertemporal macroeconomic risk, which creates a jump factor in both stock return volatility and credit spreads. Because we model intertemporal macroeconomic risk via jumps in the drift and diffusion components of consumption and earnings growth, both equity and debt values jump. Jumps in equity value always create a jump component in stock return volatility, but jumps in debt value are not always priced in the credit spread. The jumps are priced if and only if there are correlated jumps in the state-price density, i.e., the risk-distortion factor, ω, is not equal to one. If we want credit spreads to move up at the same time as volatility, the state-price density must jump up when the economy shifts into recession, i.e., $\omega > 1$.

Another important implication of our model is the cyclicality of default probabilities, credit spreads and credit spread volatilities. Bonds do not default independently of each other. Empirically, defaults are correlated across firms, i.e., they cluster at certain times. To gauge whether our model can replicate this regularity, we report the correlation of default frequency and consumption growth (Panel B). The actual estimate is based on annual real non-durable plus service consumption and annual annual issuer-weighted global corporate default rates as reported by Moody's for the period 1930-2005. In the data, default rates are higher in recessions when consumption growth is low. Our model replicates this finding for two reason: First, the drift of the earnings process is lower and its volatility is higher in recessions. Second, and more importantly, defaults cluster because the default boundary jumps upward in recessions causing instantaneous default of firms below the default boundary. This effect prevails even though these firms might have received a positive earnings shock.

The cyclicality of the credit spread level is reported in Panel C. The actual estimate is based on quarterly observations of real non-durable plus service consumption and the difference between Moody's BBB and AAA yield. Both, in the data and on simulated data, credit spreads are higher in recessions when consumption growth is low. This countercyclicality is driven by countercyclical Arrow-Debreu default claims. Arrow-Debreu default claims are the product of time adjustment, risk adjustment and actual default probabilities. Introducing switching in the moments of earnings growth makes actual default probabilities countercyclical. Introducing switching in the moments of consumption growth makes the time adjustment countercyclical and the risk adjustment procyclical. The time adjustment is countercyclical because the risk-free rate is procyclical. In our benchmark calibration,

[55]The regression coefficients we obtain from our simulated data are expressed in daily units so they can be compared directly with the empirical estimates in Tauchen and Zhou.

we find that the instantaneous risk-free rate is 1.74% in recessions and 3.65% in booms. The risk adjustment is procyclical with respect to the current state of the economy. While this may initially seem surprising, this is a direct consequence of the countercyclicality of the actual default probabilities and our definition of the risk-adjustment factor as a ratio, $\mathcal{R}_i = \widehat{p}_{D,i}/p_{D,i}$. Higher systematic earnings volatility in recessions implies that the actual default probability is countercyclical. While risk-neutral default probabilities also rise in recessions, they must increase by a lesser proportion to ensure that they remain less than 1. Therefore, the ratio $\widehat{p}_{D,i}/p_{D,i}$, which defines the risk adjustment, is lower in recessions. Overall, the Arrow-Debreu default claims remain countercyclical.

This is in contrast with Chen, Collin-Dufresne, and Goldstein (2006), where the Arrow-Debreu default claims and hence credit spreads are procyclical, unless the asset-value default boundary is countercyclical. The reason for this stems from the time-adjustment factor. Chen, Collin-Dufresne, and Goldstein (2006) use Campbell-Cochrane habit formation to model preferences. Therefore, the risk-free rate is constant, implying that the time adjustment is constant. Furthermore, the risk adjustment is highly procyclical. Consequently, the Arrow-Debreu default claims and hence the credit spread are procyclical, unless an exogenously countercyclical asset-value default boundary is imposed.

The cyclicality of the credit spread volatility is reported in Panel D. As a proxy for the conditional credit spread volatility, we use daily data of Moody's BBB minus AAA yield and compute the quarterly realized volatility. Both, in the data and on simulated data, credit spreads are more volatile in recessions when consumption growth is low.

4.6 Conclusion

We develop a theoretical framework that jointly prices corporate debt and equity in order to deliver a unified understanding of what drives the equity risk premium, credit spreads, and optimal financing decisions. To this end, we embed a structural model of credit risk with optimal financing decisions inside a representative agent consumption-based model. To study a common economic mechanism that affects both credit and equity markets, we introduce intertemporal macroeconomic risk by allowing the first and second moments of consumption and earnings growth processes to switch randomly. Furthermore, we ensure the representative agent dislikes these regime shifts, by assuming she has Epstein-Zin-Weil preferences and prefers uncertainty to be resolved sooner rather than later.

Intertemporal macroeconomic risk combined with an aversion to it makes the state-price density jump upward in recessions, leading to jump-risk premia in asset returns. Jump

risk impacts both credit spreads and stock returns, generating co-movement between credit spreads and the jump component of stock-market return volatility. The stock-market risk premium increases as the agent's dislike for regime shifts increases. The model can generate realistically high credit spreads without raising actual default probabilities and leverage. This is crucial, because in the data expected default frequencies are very small and leverage is relatively low. In essence, the framework can drive a wedge between the value of the Arrow-Debreu default claim and actual default probabilities. To show this, we develop a novel understanding of the intuition behind the Arrow-Debreu default claim. We decompose the claim into three components: the actual default probability, a risk adjustment and a time adjustment. To increase credit spreads both the risk and time adjustments must be larger than in standard structural models. We show how incorporating intertemporal macroeconomic risk achieves this goal.

Our model also generates a number of testable corporate finance implications relating to the effect of macroeconomic conditions on default and optimal financing decisions and can give rise to co-movement in the time-variation of capital structure and default clustering. Importantly, for parameter values which generate a realistic equity premium and credit spread, the asset-value default boundary is procyclical. This is in contrast with Chen, Collin-Dufresne, and Goldstein (2006), who show that in a habit-formation model with i.i.d. consumption growth, the asset value default boundary has to be countercyclical to obtain realistic credit spreads. Hence, empirical studies of the default boundary would offer an appealing way to judge whether long-run risk or habit-formation models are more promising for jointly pricing debt and equity.

This paper is only a first step towards the development of a fully-fledged consistent framework for pricing corporate equity and debt and the unification of existing asset pricing and corporate finance paradigms. Interesting possibilities for further research include studying the effects of default on consumption and introducing heterogeneous agents to distinguish between equity and debtholders.

Figure 4.1: Term-Structure of Arrow-Debreu Default Claims, Actual and Risk-Neutral Default Probabilities

This figure shows the term-structure of actual, $p_{D,i,T}$, and risk-neutral default probabilities, $\widehat{p}_{D,i,T}$, as well as Arrow-Debreu default claim prices, $q_{D,i,T}$, of BBB firms ($\sigma_X^{id} = 25\%$) when the initial state is 1 (top panel) and 2 (bottom panel). Leverage is chosen optimally and so are the default boundaries.

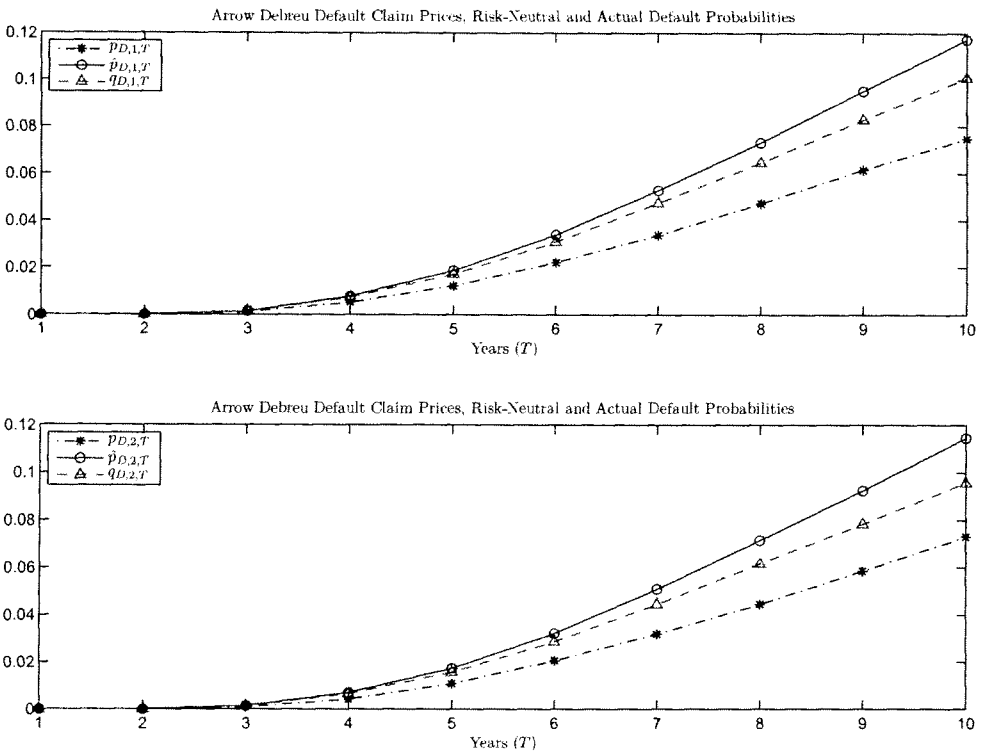

Figure 4.2: Term-structure of Actual Default Probabilities

This figure shows the term-structure of actual default probabilities, $p_{D,i,T}$ of BBB firms ($\sigma_X^{id} = 25\%$) when the initial state is 1 (blue solid line) and 2 (red dashed line). Leverage is chosen optimally and so are the default boundaries. The black line is the term-structure of BBB firms as reported by Moody's.

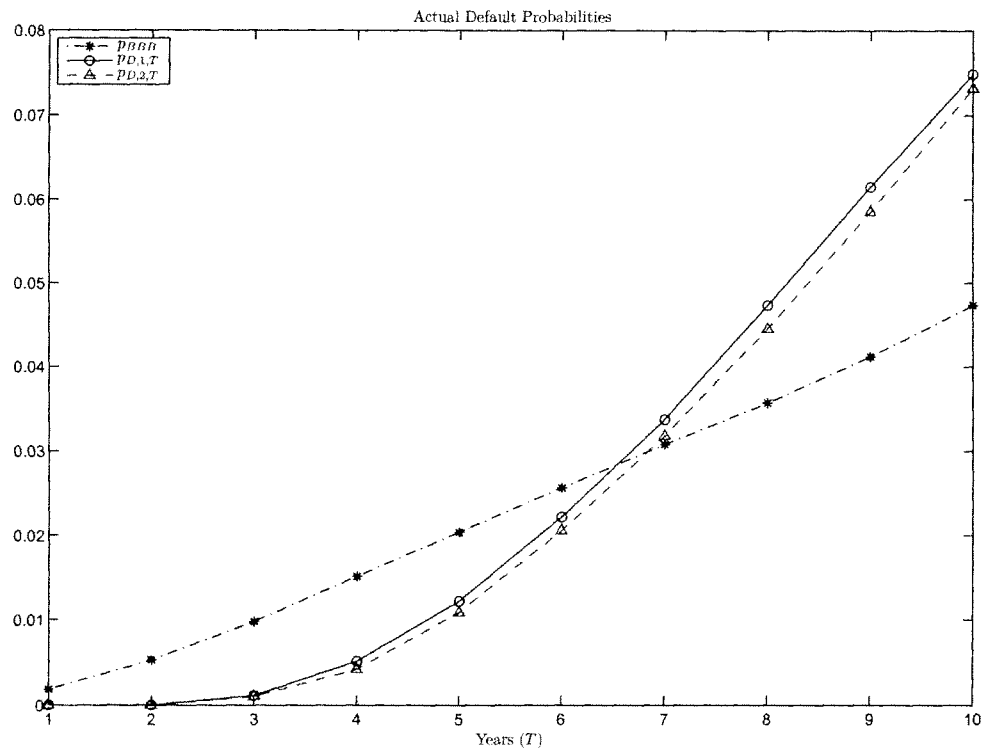

Figure 4.3: Term-Structure of Credit Spreads

This figure shows the term-structure of credit spreads, $s_{i,T}$ of BBB firms ($\sigma_X^{id} = 25\%$) when the initial state is 1 (blue solid line) and 2 (red dashed line). Leverage is chosen optimally and so are the default boundaries.

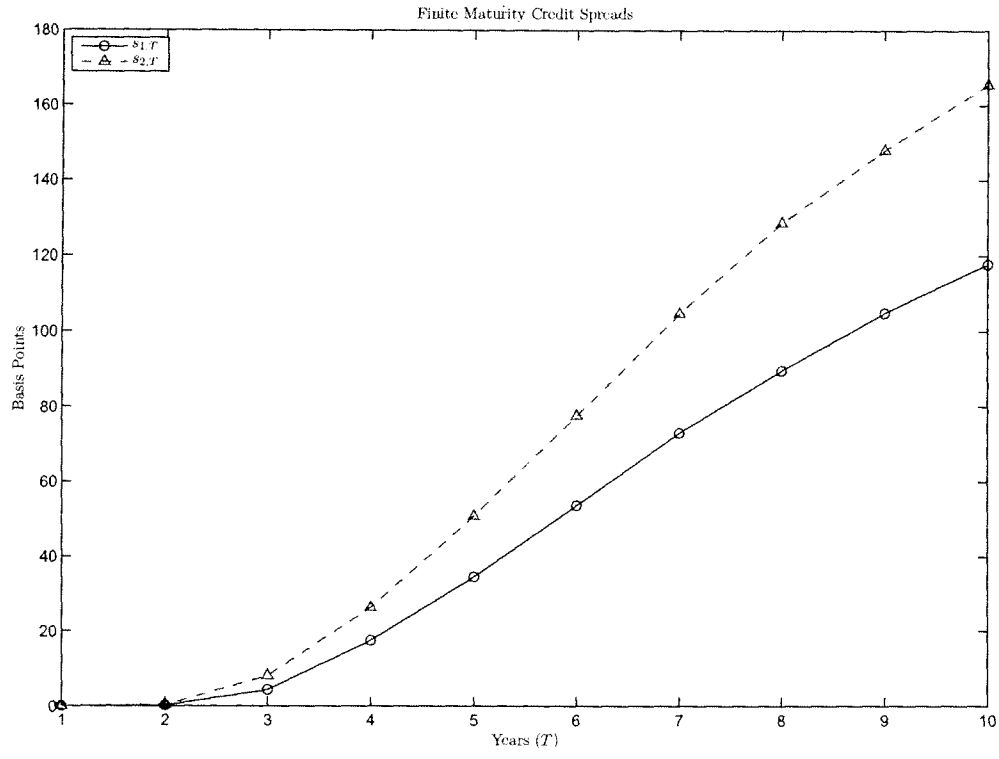

Figure 4.4: Term-Structure of Arrow-Debreu Default Claims, Actual and Risk-Neutral Default Probabilities

This figure shows the term-structure of actual, $p_{D,i,T}$, and risk-neutral default probabilities, $\widehat{p}_{D,i,T}$, as well as Arrow-Debreu default claim prices, $q_{D,i,T}$, of BBB firms ($\sigma_X^{id} = 25\%$) when the initial state is 1 (top panel) and 2 (bottom panel). Leverage is chosen optimally and so are the default boundaries.

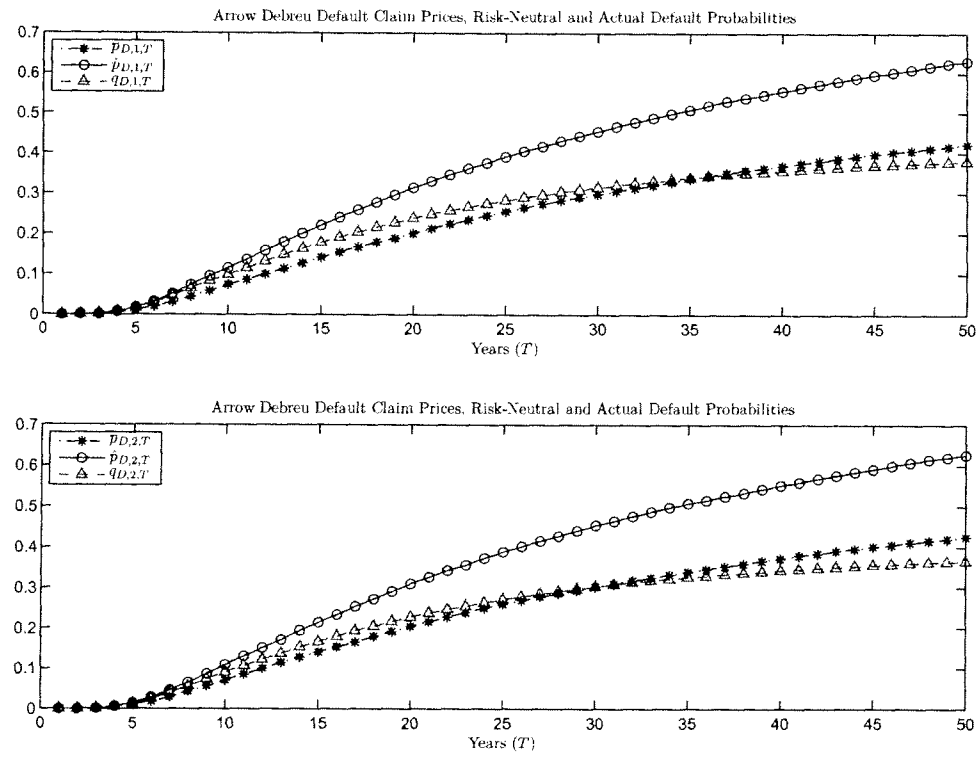

Figure 4.5: Term-Structure of Credit Spreads

This figure shows the term-structure of credit spreads, $s_{i,T}$ of BBB firms ($\sigma_X^{id} = 25\%$) when the initial state is 1 (blue solid line) and 2 (red dashed line). Leverage is chosen optimally and so are the default boundaries.

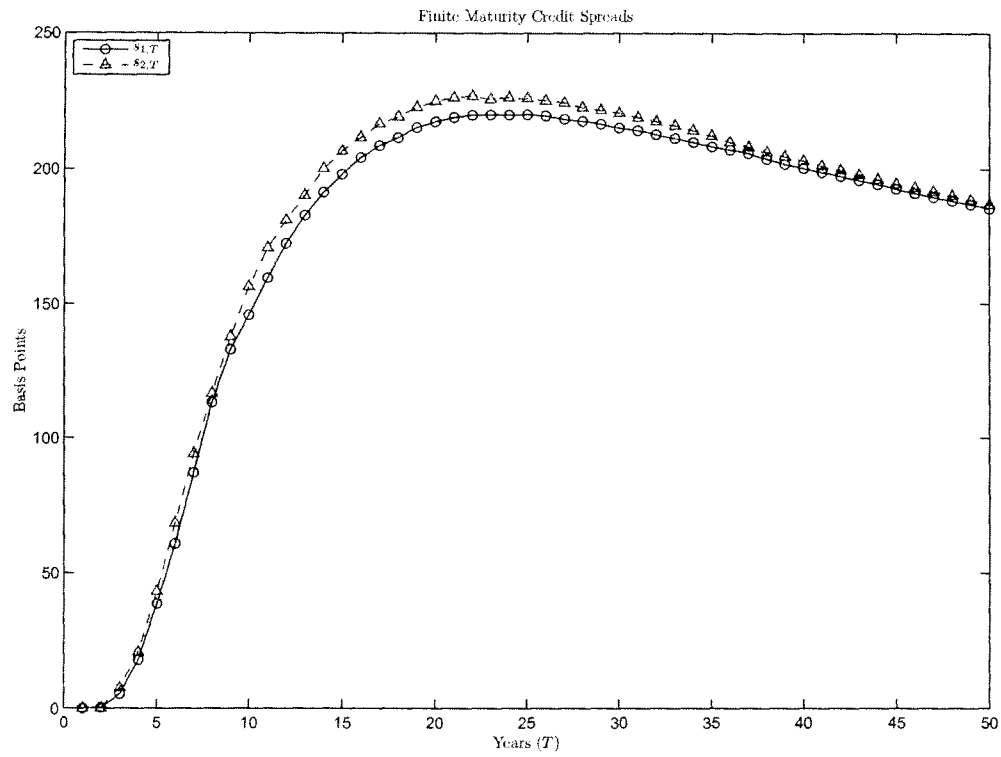

Figure 4.6: Perpetual Arrow-Debreu Default Claim Prices, Risk-Neutral and Actual Default Probabilities

This figure shows actual, $p_{D,i}$, and risk-neutral default probabilities, $\widehat{p}_{D,i}$, as well as Arrow-Debreu default claim prices, $q_{D,i}$, for the infinite maturity case as a function of the actual half-life of the Markov chain, $t_{1/2}$, when the initial state is 1 (top panel) and 2 (bottom panel). Leverage is chosen optimally and so are the default boundaries.

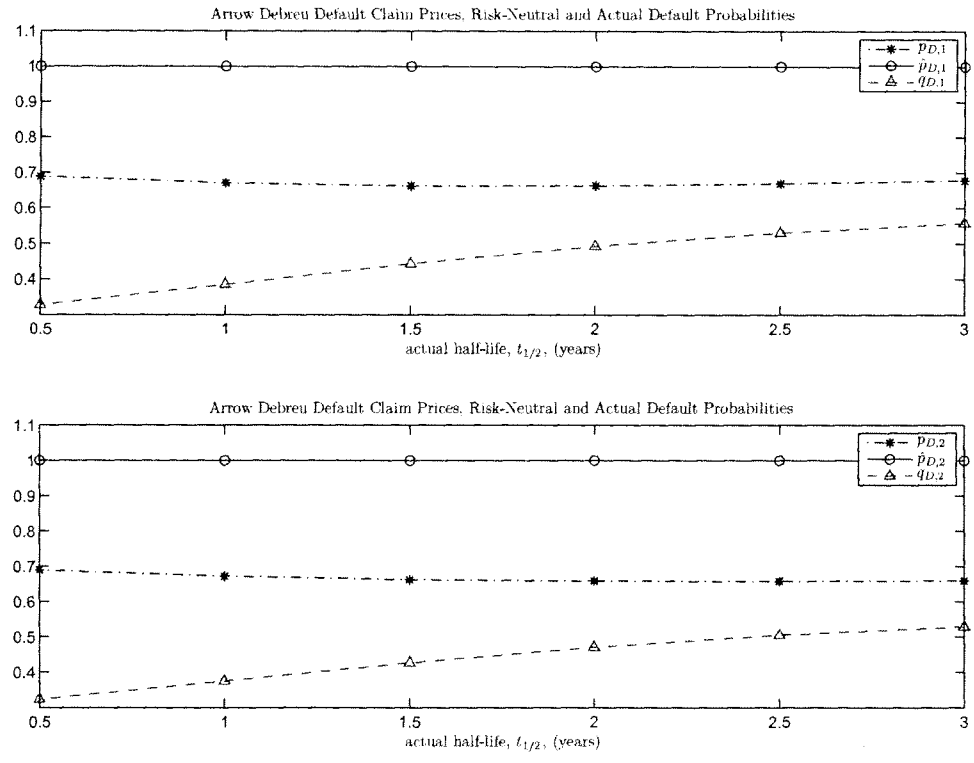

Figure 4.7: 5 Year Arrow-Debreu Default Claim Prices, Risk-Neutral and Actual Default Probabilities

This figure shows 5 year actual, $p_{D,i,5}$, and risk-neutral default probabilities, $\widehat{p}_{D,i,5}$, as well as Arrow-Debreu default claim prices, $q_{D,i,5}$, for BBB firms ($\sigma_X^{id} = 21\%$) as a function of the actual half-life of the Markov chain, $t_{1/2}$, when the initial state is 1 (top panel) and 2 (bottom panel). Leverage is chosen optimally and so are the default boundaries.

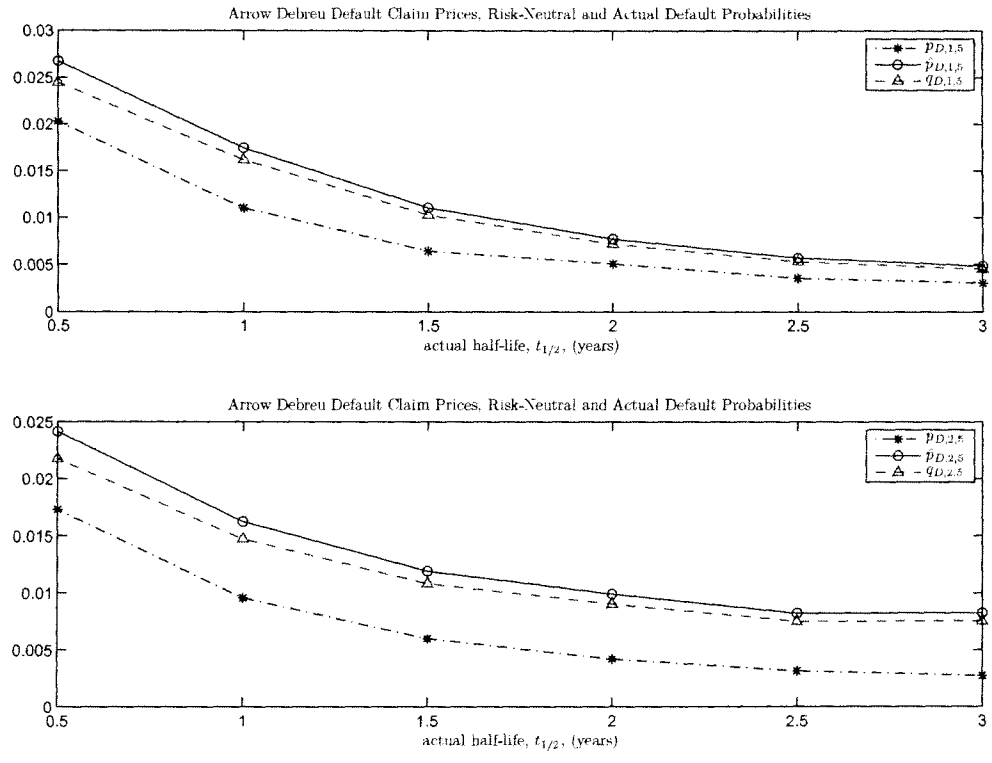

Table 4.1: Parameter Estimates

To calibrate the model to the aggregate US economy, we use quarterly real non-durable plus service consumption expenditure from the Bureau of Economic Analysis and quarterly earnings data from Standard and Poor's, provided by Robert J. Shiller. The personal consumption expenditure chain-type price index is used to deflate nominal earnings. All estimates are annualized and based on quarterly log growth rates for the period from 1947 to 2005.

Panel A: Unconditional Estimates		
	Mean	Std. dev.
Real consumption growth	0.0333	0.0099
Real earnings growth	0.0343	0.1072

Panel B: Unconditional Estimates			
Parameter	Symbol	State 1	State 2
Consumption growth rate	g_i	0.0141	0.0420
Consumption growth volatility	$\sigma_{C,i}$	0.0114	0.0094
Earnings growth rate	θ_i	-0.0401	0.0782
Earnings growth volatility	$\sigma^s_{X,i}$	0.1334	0.0834
Correlation	ρ_{XC}	0.1998	0.1998
Actual long-run probabilities	f_i	0.3555	0.6445
Actual convergence rate to long-run	p	0.7646	0.7646
Annual discount rate	β	1%	1%
Tax rate	η	15%	15%
Recovery rate	α_i	70%	90%

Table 4.2: Long-Run Risk

This table contains the risk-distortion factor ω, the convergence rate of the Markov chain to its long-run risk-neutral distribution $\widehat{p}$, the long-run risk-neutral distribution $(\hat{f}_1, \hat{f}_2)$, the risk-adjustment R_i, and time-adjustment T_i of perpetual debt for risk aversion of 10 and three values of the elasticity of intertemporal substitution, ψ.

	ω	$\hat{p}$	$\hat{f}_1$	$\hat{f}_2$	R_1	R_2	T_1	T_2
$\psi = 0.1$	1.000	0.765	0.355	0.645	1.219	1.201	0.461	0.360
$\psi = 0.75$	1.377	0.732	0.511	0.489	1.412	1.405	0.757	0.723
$\psi = 1.5$	1.424	0.733	0.528	0.472	1.487	1.483	0.824	0.805

Table 4.3: Summary Table

This table reports the results for the corporate bond market implied by the model. The coupon and default boundary are chosen optimally at date zero. Calibration: $\sigma^{id}_{AAA,5y} = 16\%$, $\sigma^{id}_{AAA,10y} = 10\%$, $\sigma^{id}_{BBB,5y} = 29.3\%$, $\sigma^{id}_{BBB,10y} = 21\%$. Numbers in parenthesis are US estimates: the BBB-AAA spread comes from Duffee (1998) and actual default probabilities from Moody's.

	AAA	BBB	BBB-AAA
Panel A: 5 year Bonds			
Credit spread (b.p.)	4.21	78.17	73.97
			(75.00)
Actual def. probs. (%)	0.12	1.99	
	(0.11)	(2.04)	
Risk-neutral def. probs. (%)	0.19	2.89	
AD def. claims (%)	0.18	2.64	
Risk-adjustment	1.56	1.46	
Time-adjustment	0.91	0.91	
Leverage (%)	52.82	43.18	
Panel B: 10 year Bonds			
Credit spread (b.p.)	11.82	100.18	88.36
			(70.00)
Actual def. probs. (%)	0.58	4.87	
	(0.56)	(4.73)	
Risk-neutral def. probs. (%)	1.31	8.51	
AD def. claims (%)	1.10	7.19	
Risk-adjustment	2.28	1.75	
Time-adjustment	0.84	0.84	
Leverage (%)	58.77	46.49	
Panel C: Equity Market			
Unlevered equity premium (%)	0.79	1.91	
Levered equity premium (%)	2.73	4.20	
Levered equity volatility (%)	18.05	27.16	

Table 4.4: Model Comparison

This table provides a comparison between stripped down versions of our model. In Models 1a and 1b there is no intertemporal risk, i.e., the first and second moments of consumption and earnings growth rates do not switch. In Model 2 the first and second moments of earnings growth switch but the first and second moments of consumption growth do not. In Model 3 the first and second moments of both earnings and consumption growth switch. In Model 1a the representative agent has power utility whereas in Models 1b, 2 and 3 she has Epstein-Zin-Weil utility. All numbers in the Bond Market panel (Panel A) refer to 5 year debt.

	Model 1a	Model 1b	Model 2	Model 3a	Model 3b
Risk aversion	10.00	10.00	10.00	10.00	10.00
EIS	0.10	0.75	0.75	0.75	1.50
Idio. volatility (%)	21.00	26.00	25.80	27.50	29.00
Panel A: Bond Market (5 year debt)					
Credit spread (b.p.)	11.36	24.89	43.04	66.16	73.95
Actual def. probs. (%)	1.97	2.03	1.99	1.95	1.91
Risk-neutral def. probs. (%)	2.11	2.15	2.05	2.80	2.77
AD def. claims (%)	0.59	1.75	1.67	2.39	2.53
Risk-adjustment	1.07	1.06	1.04	1.43	1.45
Time-adjustment	0.28	0.81	0.81	0.85	0.91
Leverage (%)	43.00	43.00	43.00	43.00	43.00
Panel B: Equity Market					
Unlev. equity premium (%)	0.32	0.32	0.30	1.43	1.91
Levered equity premium (%)	0.52	0.52	0.49	2.66	3.27
Levered equity volatility (%)	17.59	17.59	21.82	20.32	21.26
Locally risk-free rate (%)	33.81	5.33	5.33	5.23	2.97
Distortion Factor	1.00	1.00	1.00	1.38	1.42

Table 4.5: Comovement

We simulate 100 panels each containing 3000 firms for 5 years at monthly frequency. In Panel A, we regress the equally-weighted credit spread on the value-weighted jump component of levered equity volatility. Panel B reports the correlation between actual default probabilities and consumption growth, Panel C the correlation between BBB credit spreads and consumption growth, and Panel D the correlation between BBB credit spread volatilities and consumption growth. Standard errors are reported in parenthesis.

	Model	Data
Panel A: Cross-Market Comovement		
Slope	1.76	1.92
	(0.15)	(0.02)
Panel B: Default Clustering		
Correlation	-0.19	-0.48
	(0.01)	
Panel C: Credit Spread Cyclicality		
Correlation	-0.62	-0.19
	(0.03)	
Panel D: Credit Spread Volatility Cyclicality		
Correlation	-0.70	-0.13
	(0.03)	

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Chapter 5

Conclusion

In this thesis, I examine how macroeconomic risk translates into asset prices within two settings: (1) In the first and second essay, I study the asset pricing implications of firms making optimal real investment decisions when they face macroeconomic shocks and investment projects are not completed instantaneously. (2) In the third essay, we study how macroeconomic risk impacts credit spreads.

The first and second essay are motivated by the observation that most real investment is not instantaneous. For example, expanding capacity in a manufacturing process can take several years. Even though the duration of investment projects is an intuitive concept, there is only little theoretical research about its economic consequences. The reason for this void in the literature is that when investment projects take place over time, the distribution of initiated projects becomes a high dimensional state variable and renders the problem intractable. One contribution of the second essay is to provide a new specification of long-term investment projects which makes the problem tractable.[56]

In the second essay, I employ the new specification of long-term investment projects to analyze the asset pricing implications of investment commitment. Intuitively, investment commitment impacts the distribution future consumption growth rates, because commitments in long-term projects are not satisfied immediately. The household, having Epstein-Zin preferences, dislikes shocks to future consumption growth rates, leading to a significantly larger equity premium and return volatility.

My thesis also provides directions for further research. Investment commitment is only one special case of the non-instantaneous investment framework. Generally, investment projects can be characterized by two features: First, by the timing when firms pay for new projects; second, by the timing when new projects become productive. In the second essay, I make the restrictive assumption that the timing of the costs coincides with the timing of when new projects becomes productive. In future research, it would be interesting to break this link. More precisely, the firm's optimization problem would then depend on one state variable characterizing the amount of committed expenditures and another one

[56]See Kydland and Prescott (1982) for an early contribution in this area.

describing how much capital will become productive in future periods. This exercise might lead to new insights regarding the risk dynamics of firms.

Another interesting research project is to extend my model to a cross-section of firms.[57] Preliminary research shows that the lagged investment rate contains information about returns and return volatility in cross-sectional Fama-MacBeth regressions.[58] This empirical evidence suggests that long-term investment projects impact stock returns at the firm level. In contrast, the empirical results in my second essay are at the aggregate level.

My third essay develops a theoretical framework that jointly prices corporate debt and equity in order to deliver a unified understanding of what drives the equity risk premium, credit spreads, and optimal financing decisions. To this end, we embed a structural model of credit risk with optimal financing decisions[59] inside a representative agent consumption-based model. We introduce intertemporal macroeconomic risk by allowing the first and second moments of consumption and earnings growth processes to switch randomly.[60] Furthermore, we ensure the representative agent dislikes these regime shifts, by assuming she has Epstein-Zin preferences and prefers uncertainty to be resolved sooner rather than later.

It is also important to mention what our model does not do. It does not account for the impact of default on consumption, because we model consumption as an exogenous process.[61] Furthermore, our model ignores the impact of agency conflicts on the state-price density, because the state-price density in our model is the marginal utility of wealth of the representative agent.[62] Incorporating these two important effects is beyond the scope of this thesis and left for future research.

[57] See, for example, Gomes, Kogan, and Zhang (2003).

[58] My empirical findings complement recent work by Xing (2006), who finds a negative relation between current investment and future returns.

[59] See, for example, Leland (1994).

[60] Our approach is similar to Bansal and Yaron (2004).

[61] Alvarez and Jermann (2000) and Chan and Sundaresan (2005) offer a promising approach.

[62] Albuquerue and Wang (2008) provide a recent contribution.

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Appendix A

Asset Pricing with Real Investment Commitment

A.1 Firm

Optimality conditions: Optimal firm behavior can be characterized by studying the firm's first-order conditions. The first-order conditions with respect to K_{t+1} and I_t and the Kuhn-Tucker condition are

$$\begin{aligned} -q_t + \mathbb{E}_t M_{t+1} V_1(K_{t+1}, I_t, Z_{t+1}) &= 0 && \text{(A.1)} \\ -1 + q_t + \mu_t + \mathbb{E}_t M_{t+1} V_2(K_{t+1}, I_t, Z_{t+1}) &= 0 && \text{(A.2)} \\ \mu_t(I_t - wI_{t-1}) &= 0 \end{aligned}$$

where q_t is the multiplier on (3.10) and thus the shadow value of capital usually termed marginal Q. μ_t is the multiplier on the commitment constraint (3.11) and thus the shadow costs of commitment. The Kuhn-Tucker condition guarantees that either $\mu_t \geq 0$ or $(I_t - wI_{t-1}) \geq 0$ holds.

The envelope conditions with respect to the two endogenous state-variables, current capital and lagged investment expenditures, are

$$\begin{aligned} V_1(K_t, I_{t-1}, Z_t) &= Z_t^{1-\alpha_1} \alpha_2 K_t^{\alpha_2 - 1} + q_t(1-\delta) && \text{(A.3)} \\ V_2(K_t, I_{t-1}, Z_t) &= -w\mu_t && \text{(A.4)} \end{aligned}$$

Combining the first-order conditions (A.1) and (A.2) with the envelope conditions (A.3) and (A.4) gives the Equations (3.12) and (3.13) in the main text.

Proofs: In the proofs, I drop the time index t and denote next period's variables with a $'$ and last period's variable with a $^-$. For the proofs of Proposition 1-2, I need the following assumptions: Let the shock $Z \in \mathcal{Z} = [\underline{Z}, \bar{Z}]$ and the transition function Q satisfies the Feller property. Let the capital stock $K \in \mathcal{K} = [0, \bar{K}]$ where $\bar{K}$ solves $\bar{Z}\,\bar{K}^{\alpha} - \delta = 0$. Let investment $I \in \mathcal{I} = [0, \bar{K}]$.

The feasible policy correspondence is

$$\Gamma(K, I^-) = \{(K', I) : K' \in \mathcal{K} \text{ and } I \in [wI^-, \bar{K}]\}$$

Proof of Proposition 1: The set $\mathcal{K} \times \mathcal{I}$ is non-empty, compact and convex. Because F is continuous with compact domain, it is bounded. Note that $\Gamma(K, I^-)$ is nonempty, compact-valued, and continuous. Thus the Assumptions 9.4-9.7 in Stokey and Lucas (1989) (hereafter SL) are satisfied. The proposition follows from Theorem 9.6 in SL.

Proof of Proposition 2: From Theorem 9.10 in SL follows that for each (K', I) in the interior of $\mathcal{K} \times \mathcal{I}$, V is continuously differentiable. Sargent (1980) extends this result to corner solutions.

Proof of Proposition 3: To prove monotonicity of the value function, simply redefine the problem in terms of $\bar{I} = -I$. The redefined value function satisfies $v(K, \bar{I}^-, Z) = V(K, I^-, Z)$. The new correspondence $\Gamma(K, \bar{I}^-) = \{(K', \bar{I}) : K' \in \mathcal{K} \text{ and } \bar{I} \in [-\bar{K}, -w\bar{I}^-]\}$ is increasing in $(K', \bar{I})$. Thus the Assumptions 9.4-9.9 in SL are satisfied. The monotonicity of v follows from Theorem 9.7 in SL which implies that V is strictly increasing (decreasing) in its first (second) argument.

Because $\alpha \in (0, 1)$, F is strictly concave. Because $\mathcal{K} \times \mathcal{I}$ is convex, so is Γ. Thus, Assumption 9.10-9.11 in SL are satisfied. The concavity of V follows from Theorem 9.8 in SL.

Proof of Proposition 4: Firm value can also be stated in the Lagrange form

$$\begin{aligned} V_t &= Z_t^{1-\alpha_1} K_t - I_t + q_t((1-\delta)K_t + I_t - K_{t+1}) + \mu_t(I_t - wI_{t-1}) \\ &+ \mathbb{E}_t M_{t+1} \left\{ Z_{t+1}^{1-\alpha_1} K_{t+1} - I_{t+1} + q_{t+1}((1-\delta)K_{t+1} + I_{t+1} - K_{t+2}) + \mu_{t+1}(I_{t+1} - wI_t) \right\} + \ldots \end{aligned}$$

To simplify the Lagrange equation, multiply (3.12) with K_{t+1} and (3.13) with I_t

$$\begin{aligned} q_t K_{t+1} &= \mathbb{E}_t M_{t+1} (Z_{t+1}^{1-\alpha_1} + q_{t+1}(1-\delta)) K_{t+1} \\ q_t I_t &= I_t - \mu_t I_t + w \mathbb{E}_t M_{t+1} \mu_{t+1} I_t \end{aligned}$$

Next plug the modified first-order conditions into the Lagrange function. Firm value then simplifies to

$$V_t = Z_t^{1-\alpha_1} K_t + q_t(1-\delta)K_t - \mu_t w I_{t-1}$$

The firm's stock price is

$$\begin{aligned} P_t &= V_t - D_t \\ &= Z_t^{1-\alpha_1} K_t + q_t(1-\delta)K_t - \mu_t w I_{t-1} - (Z_t^{1-\alpha_1} K_t + (1-\delta)K_t - K_{t+1}) \\ &= q_t K_{t+1} - w I_t \mathbb{E}_t M_{t+1} \mu_{t+1} \end{aligned}$$

which proves the claim.

Investment vs. Stock Returns: Investment commitment also causes a wedge between investment and stock returns. To see that the equivalence here does not hold, note that equation (3.12) defines the investment return. With a linear production function, the investment return simplifies to

$$R^I_{t+1} = \frac{Z^{1-\alpha_1}_{t+1} + q_{t+1}(1-\delta)}{q_t} \tag{A.5}$$

The investment return R^I_{t+1} defines the firm's intertemporal tradeoff of investing one more unit of capital. It is the ratio of tomorrow's marginal benefits of investing one additional unit of capital divided by today's marginal costs.

By contrast, stock returns are

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{Z^{1-\alpha_1}_{t+1} K_{t+1} + q_{t+1}(1-\delta)K_{t+1} - \mu_{t+1} I_{t+1}}{q_t K_{t+1} - w I_t \mathbb{E}_t M_{t+1} \mu_{t+1}}$$

The difference between investment and stock returns arises because the former reflects only the tradeoff between marginal costs and benefits of *new* investment projects. The latter, however, is the return to the entire firm value which also includes the value of committed expenditures. Breaking the equivalence between investment and stock returns is important for explaining stock returns since investment returns are tied to the intertemporal tradeoff of real capital which does not fluctuate much.

A.2 Household

The homogeneity of the utility function (3.19) and the linearity of the budget constraint (3.20) imply that U is also homogeneous (scale invariant) in wealth, i.e.

$$U(W_t, \xi_t) = \phi(\xi_t) W_t = \phi_t W_t \tag{A.6}$$

where ξ_t is a vector of state variables coming from the production side of the economy. For the same reason, the consumption policy function is proportional in wealth, i.e. $C_t = \varphi(\xi_t) W_t = \varphi_t W_t$, where φ_t is the consumption-wealth ratio.

Substituting the guess for U (A.6) and the budget constraint (3.20) into the utility function (3.19) yields

$$U(W_t, \xi_t) = \max_{C_t} \{(1-\beta) C_t^\rho + \beta \hat{\mu}_t^\rho (W_t - C_t)^\rho\}^{1/\rho}$$

where $\hat{\mu}_t = \mathbb{E}_t[(\phi_{t+1} R^w_{t+1})^\alpha]^{1/\alpha}$.

The first-order condition with respect to consumption C_t is

$$(1-\beta) C_t^{\rho-1} = \beta \hat{\mu}_t^\rho (W_t - C_t)^{\rho-1}$$

Combing the first-order condition with the guess for the consumption policy function gives the solution for the consumption-wealth ratio

$$\varphi_t = \frac{A_t}{1+A_t} \qquad A_t = \left(\hat{\mu}_t^{\rho}\frac{\beta}{1-\beta}\right)^{1/(\rho-1)}$$

The derivative of the consumption-wealth ratio with respect to the EIS is

$$\frac{\partial\varphi}{\partial\psi} = -\frac{((1-\beta)\beta)^{\psi}\mu^{1+\psi}(\ln\mu - \ln(1-\beta) + \ln\beta)}{((1-\beta)^{\psi}\mu + \beta^{\psi}\mu^{\psi})^2}$$

which is positive if and only if $\ln\mu - \ln(1-\beta) + \ln\beta > 0$.

A.3 Numerical Solution Method

The model is solved by imposing a competitive equilibrium and market clearing. The following definition states this precisely.

Definition 1 *A competitive rational expectations equilibrium is a sequence of allocations $\{C_t, K_t\}_{t=0}^{\infty}$ and a price system $\{\Lambda_t, P_t\}_{t=0}^{\infty}$ such that: (i) given the price system, the representative household maximizes (3.19) s.t. (3.20); (ii) given the price system, the representative firm maximizes (3.9) s.t. (3.10) and (3.11); (iii) the good market clears $Y_t = C_t + I_t$; (iv) the stock market clears $s_t = 1$.*

Because of the welfare theorems, the decentralized economy and the social planner give the same optimal allocations. Since solving the latter is computationally easier, I first solve the planner's problem for optimal quantities. Second, given quantities I use the Euler equation to solve for prices.

The planner's problem is

$$J(K_t, I_{t-1}, Z_t) = \max_{C_t}\left\{(1-\beta)C_t^{\rho} + \beta\mathbb{E}_t[J(K_{t+1}, I_t, Z_{t+1})^{\alpha}]^{\rho/\alpha}\right\}^{1/\rho}$$

s.t.

$$\begin{aligned}
Z_t^{1-\alpha}K_t^{\alpha} &= C_t + I_t \\
K_{t+1} &= (1-\delta)K_t + I_t \\
I_t &= (1-w)X_t + wI_{t-1} \\
X_t &\geq 0 \\
\frac{Z_{t+1}}{Z_t} &= \exp\{g + \varepsilon_{t+1}\}
\end{aligned}$$

Because the technology follows a geometric random walk, the problem has to be reformulated so that it is difference stationary. Stationary variables are denoted with a hat, i.e. $\hat{C}_t = C_t/Z_t$, $\hat{K}_t = K_t/Z_{t-1}$, $\hat{I}_t = I_t/Z_t$, $\hat{X}_t = X_t/Z_t$, $\hat{J}_t = J_t/Z_t$. The stationary planner's problem is

$$\hat{J}\left(\hat{K}_t, \hat{I}_{t-1}, \tfrac{Z_t}{Z_{t-1}}\right) = \max_{\hat{C}_t} \left\{ (1-\beta)\hat{C}_t^{\rho} + \beta \mathbb{E}_t \left[e^{\alpha(g+\varepsilon_{t+1})} \hat{J}\left(\hat{K}_{t+1}, \hat{I}_t, \tfrac{Z_{t+1}}{Z_t}\right)^{\alpha} \right]^{\rho/\alpha} \right\}^{1/\rho}$$

s.t.

$$\begin{aligned}
e^{-\alpha(g+\varepsilon_t)}\hat{K}_t^{\alpha} &= \hat{C}_t + \hat{I}_t \\
\hat{K}_{t+1} &= (1-\delta)\hat{K}_t e^{-(g+\varepsilon_t)} + \hat{I}_t \\
\hat{I}_t &= (1-w)\hat{X}_t + w\hat{I}_{t-1}e^{-(g+\varepsilon_t)} \\
\hat{X}_t &\geq 0 \\
\frac{Z_{t+1}}{Z_t} &= \exp\{g + \varepsilon_{t+1}\}
\end{aligned}$$

After having solved for quantities, I use the Euler equation to solve for the equilibrium price function $P_t = P(K_t, I_{t-1}, Z_t)$

$$P_t = \mathbb{E}_t M_{t+1}(D_{t+1} + P_{t+1})$$

As before, define the stationary variable $\hat{D}_t = D_t/Z_t$ and $\hat{P}_t = P_t/Z_t$. The Euler equation in terms of stationary variables is

$$\hat{P}_t = \mathbb{E}_t M_{t+1} e^{g+\varepsilon_{t+1}}(\hat{D}_{t+1} + \hat{P}_{t+1})$$

The model is solved on a 3-dimensional discrete grid. The aggregate shock ε_t is modeled as a Markov chain with 5 states. The grid for capital and lagged investment has 80 and 40 elements, respectively. The planner's value function is solved with modified policy iteration and bicubic interpolation for a choice vector with 5,000 elements. Given the value function and optimal allocations, the stationary price functional is solved numerically as a fixed point problem on the same grid as the value function with bicubic interpolation.

Appendix B

The Levered Equity Risk Premium and Credit Spreads: A Unified Framework

B.1 Calibration Details

To estimate a Markov switching model as described in Hamilton (1989), we start by assuming that aggregate consumption, C, is given by

$$c_{t+1} = \Delta \log C_{t+1} = g_i - \frac{1}{2}\sigma_{C,i}^2 + \sigma_{C,i}\varepsilon_{C,t+1}, \tag{B.1}$$

and aggregate earnings, $X = \sum_{n=1}^{N} X_n$, by

$$x_{t+1} = \Delta \log X_{t+1} = \theta_i - \frac{1}{2}(\sigma_{X,i}^s)^2 + \sigma_{X,i}^s\varepsilon_{X,t+1}, \tag{B.2}$$

where shocks to earnings growth and consumption growth are normally distributed with zero mean and unit variance and correlation of ρ_{XC}. Equation (B.1) is simply a discretized version of (4.6). To justify (B.2), we prove that if $\theta_{n,i} = \theta_i$ and $\sigma_{X,n,i}^s = \sigma_{X,i}^s$ for all $i \in \{1,2\}$ and $n \in \{1,\ldots,N\}$, and there exists some $\epsilon > 0$ independent of N, such that $\frac{X_n}{X} \leq \frac{\epsilon}{N}$ for all $n \in \{1,\ldots,N\}$, then

$$\lim_{N\to\infty} \frac{dX}{X} = \theta_i dt + \sigma_{X,i}^s dB_X^s. \tag{B.3}$$

The proof proceeds by noting that

$$\frac{dX}{X} = \theta_i dt + \sigma_{X,i}^s dB_X^s + \sum_{n=1}^{N} \frac{X_n}{X}\sigma_{X,i,n}^{id} dB_{X,n}^{id},$$

from which it follows that

$$Var_t\left(\frac{dX}{X}\right) = (\sigma_{X,i}^s)^2 + \sum_{n=1}^{N}\left(\frac{X_n}{X}\right)^2\left(\sigma_{X,n}^{id}\right)^2 \leq (\sigma_{X,i}^s)^2 + \frac{1}{N}\epsilon^2(\max_n \sigma_{X,n}^{id})^2,$$

which implies that

$$\lim_{N\to\infty} Var_t\left(\frac{dX}{X}\right) = (\sigma_{X,i}^s)^2.$$

Equation (B.3) follows. Thus, (B.2) is justified under the given assumptions if the number of firms N is large. As a consequence, we can obtain the aggregate levered equity risk premium by setting $\sigma^{id}_{X,n} = 0$. The joint normal distribution for earnings growth and consumption growth denoted by Φ

$$\Phi(x_t, c_t|\nu_t = i, \Omega_{t-1}; \Gamma) = \frac{1}{2\pi\sigma^s_{X,i}\sigma_{C,i}\sqrt{1-\rho^2_{XC}}} \exp\{-1/2(\varepsilon^2_{X,t}+\varepsilon^2_{C,t}-2\rho_{XC}\varepsilon_{X,t}\varepsilon_{C,t})/(1-\rho^2_{XC})\}.$$

where Ω_t denotes the set of all observations up to time t and Γ the set of unknown parameters. Having obtained our parameter estimates by maximizing the log-likelihood, we must obtain the parameters of the continuous-time Markov chain from the estimated discrete-time transition matrix

$$P = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix},$$

where $P_{ij} = P(\nu_{t+1} = \theta_j|\nu_t = \theta_i)$ is the probability of switching from state i to j within a quarter. To do this note that the matrix of quarterly transition probabilities, P, is related to the generator of the continuous-time chain

$$\Lambda = \begin{pmatrix} -\lambda_1 & \lambda_1 \\ \lambda_2 & -\lambda_2 \end{pmatrix}$$

by

$$e^{\Lambda\frac{1}{4}} = P.$$

Using standard techniques from linear algebra, we can show that

$$e^{\Lambda\frac{1}{4}} = \begin{pmatrix} f_1 & f_2 \\ f_1 & f_2 \end{pmatrix} + \begin{pmatrix} f_2 & -f_2 \\ -f_1 & f_1 \end{pmatrix} e^{-\frac{1}{4}p}, \tag{B.4}$$

Where $p = \lambda_1 + \lambda_2$ and $f_i = \frac{\lambda_i}{p}$, $i \in \{1, 2\}$. Equating (B.4) with P implies after some algebra:

$$\begin{aligned} f_1 &= \left(1 + \frac{P_{12}}{P_{21}}\right)^{-1} \\ p &= -4\ln\left(1 - \frac{P_{12}}{1-f_1}\right). \end{aligned}$$

B.2 Derivation of the State-Price Density

First, we introduce some notation related to jumps in the state of the economy. Suppose that during the small time-interval $[t - \Delta t, t)$ the economy is in state i and that at time t

the state changes, so that during the next small time interval $[t, t+\Delta t)$ the economy is in state $j \neq i$. We then define the left-limit of ν at time t as

$$\nu_{t-} = \lim_{\Delta t \to 0} \nu_{t-\Delta t}, \tag{B.5}$$

and the right-limit as

$$\nu_t = \lim_{\Delta t \to 0} \nu_{t+\Delta t}. \tag{B.6}$$

Therefore $\nu_{t-} = i$, whereas $\nu_t = j$, so the left- and right limits are not equal. If some function E depends on the current state of the economy i.e. $E_t = E(\nu_t)$, then E is a jump process which is right continuous with left limits, i.e. RCLL. If a jump from state i to $j \neq i$ occurs at date t, then we abuse notation slightly and denote the left limit of E at time t by E_i, where i is the index for the state. i.e. $E_{t-} = \lim_{s \uparrow t} E_s = E_i$. Similarly $E_t = \lim_{s \downarrow t} E_s = E_j$. We shall use the same notation for all processes that jump, because of their dependence on the state of the economy.

Using simple algebra we can write the normalized Kreps-Porteus aggregator in the following compact form:

$$f(c, v) = \beta \left(h^{-1}(v)\right)^{1-\gamma} u\left(c/h^{-1}(v)\right),$$

where

$$\begin{aligned} u(x) &= \frac{x^{1-\frac{1}{\psi}} - 1}{1 - \frac{1}{\psi}}, \psi > 0, \\ h(x) &= \begin{cases} \frac{x^{1-\gamma}}{1-\gamma}, & \gamma \geq 0, \gamma \neq 1. \\ \ln x, & \gamma = 1. \end{cases} \end{aligned}$$

The representative agent's value function is given by

$$J_t = E_t \int_t^\infty f(C_t, J_t)\, dt. \tag{B.7}$$

Theorem 1 *The state-price density of a representative agent with the continuous-time version of Epstein-Zin-Weil preferences is given by*

$$\pi_t = \begin{cases} \left(\beta e^{-\beta t}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} C_t^{-\gamma} \left(p_{C,t} e^{\int_0^t p_{C,s}^{-1} ds}\right)^{-\frac{\gamma - \frac{1}{\psi}}{1-\frac{1}{\psi}}}, & \psi \neq 1 \\ \beta e^{-\beta \int_0^t [1+(\gamma-1)\ln(V_s^{-1})] ds} C_t^{-\gamma} V_t^{-(\gamma-1)}, & \psi = 1 \end{cases}. \tag{B.8}$$

When $\psi \neq 1$, the price-consumption ratio in state i, $p_{C,i}$, satisfies the nonlinear equation system:

$$p_{C,i}^{-1} = \overline{r}_i + \gamma\sigma_{C,i}^2 - g_i - \left(1 - \frac{1}{\psi}\right)\lambda_i \left(\frac{\left(p_{C,j}/p_{C,i}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} - 1}{1-\gamma}\right), \; i \in \{1,2\}, \; j \neq i. \tag{B.9}$$

where

$$\overline{r}_i = \beta + \frac{1}{\psi}g_i - \frac{1}{2}\gamma\left(1 + \frac{1}{\psi}\right)\sigma_{C,i}^2, \; i \in \{1,2\}. \tag{B.10}$$

When $\psi = 1$, define V via

$$J = \ln(CV). \tag{B.11}$$

Then V_i satisfies the nonlinear equation system:

$$\beta \ln V_i = g_i - \frac{\gamma}{2}\sigma_{C,i}^2 + \lambda_i \frac{\left(V_j/V_i\right)^{1-\gamma} - 1}{1-\gamma} \; i \in \{1,2\}, \; j \neq i. \tag{B.12}$$

Proof of Theorem 1

Duffie and Skiadas (1994) show that the state-price density for a *general* normalized aggregator f is given by

$$\pi_t = e^{\int_0^t f_v(C_s,J_s)dt} f_c\left(C_t, J_t\right), \tag{B.13}$$

where $f_c(\cdot,\cdot)$ and $f_v(\cdot,\cdot)$ are the partial derivatives of f with respect to its first and second arguments, respectively, and J is the value function given in (B.7). The Feynman-Kac Theorem implies

$$f\left(C_t, J_{t-}\right)\big|_{\nu_{t-}=i} dt + E_t\left[dJ_t \middle| \nu_{t-} = i\right] = 0, \; i \in \{1,2\}.$$

Using Ito's Lemma we rewrite the above equation as

$$0 = f\left(C, J_i\right) + CJ_{i,C}g_i + \frac{1}{2}C^2 J_{i,CC}\sigma_{C,i}^2 + \lambda_i\left(J_j - J_i\right), \tag{B.14}$$

for $i, j \in \{1,2\}$, $j \neq i$. We guess and verify that

$$J = h(CV), \tag{B.15}$$

where V_i satisfies the nonlinear equation system

$$0 = \beta u\left(V_i^{-1}\right) + g_i - \frac{1}{2}\gamma\sigma_{C,i}^2 + \lambda_i\left(\frac{\left(V_j/V_i\right)^{1-\gamma} - 1}{1-\gamma}\right), \; i,j \in \{1,2\}, \; j \neq i. \tag{B.16}$$

Substituting (B.11) into (B.13) and simplifying gives

$$\pi_t = \beta e^{-\beta\int_0^t\left[1+\left(\gamma-\frac{1}{\psi}\right)u(V_s^{-1})\right]dt} C_t^{-\gamma} V_t^{-\left(\gamma-\frac{1}{\psi}\right)}. \tag{B.17}$$

When $\psi = 1$, the above equation gives the second expression in (B.8). We rewrite (B.16) as

$$\beta\left[1+\left(\gamma-\frac{1}{\psi}\right)u\left(V_i^{-1}\right)\right]=\overline{r}_i-\left(\gamma-\frac{1}{\psi}\right)\lambda_i\left(\frac{\left(V_j/V_i\right)^{1-\gamma}-1}{1-\gamma}\right)-\left[\gamma g_i-\frac{1}{2}\gamma\left(1+\gamma\right)\sigma_{C,i}^2\right], \tag{B.18}$$

$i, j \in \{1,2\}, j \neq i$, where $\overline{r}_i$ is given in (B.10). Setting $\psi = 1$ in (B.18) gives (B.12).
To derive the first expression in (B.8) from (B.17) we prove that

$$V_i = \left(\beta p_{C,i}\right)^{\frac{1}{1-\frac{1}{\psi}}}, \psi \neq 1. \tag{B.19}$$

We proceed by considering the optimization problem for the representative agent. She chooses her optimal consumption (C^*) and risky asset portfolio (φ) to maximize her expected utility

$$J_t^* = \sup_{C^*,\varphi} E_t \int_t^\infty f\left(C_t^*, J_t^*\right) dt.$$

Observe that J^* depends on optimal consumption-portfolio choice, whereas the J defined previously in (B.11) depends on exogenous aggregate consumption. The optimization is carried out subject to the dynamic budget constraint, which we now describe. If the agent consumes at the rate, C^*, invests a proportion, φ, of her remaining financial wealth in the claim on aggregate consumption (the risky asset), and puts the remainder in the locally risk-free asset, then her financial wealth, W, evolves according to the dynamic budget constraint:

$$\frac{dW_t}{W_{t-}} = \varphi_{t-}\left(dR_{C,t} - r_{t-}dt\right) + r_{t-}dt - \frac{C_{t-}^*}{W_{t-}}dt,$$

where $dR_{C,t}$ is the cumulative return on the claim to aggregate consumption. We define $N_{i,t}$ as the Poisson process which jumps upward by one whenever the state of the economy switches from i to $j \neq i$. The compensated version of this process is the Poisson martingale

$$N_{i,t}^P = N_{i,t} - \lambda_i t.$$

It follows from applying Ito's Lemma to $P = p_C C$, that the cumulative return on the claim to aggregate consumption is

$$dR_{C,t} = \frac{dP_t + C_t dt}{P_{t-}} = \mu_{R_C,t-}dt + \sigma_{C,t-}dB_{C,t} + \sigma_{R_C,t-}^P dN_{\nu_{t-},t}^P,$$

where

$$\begin{aligned}
\mu_{R_C,t-}\big|_{\nu_{t-}=i} &= \mu_{R_C,i} = g_i + \frac{1}{2}\sigma_{C,i}^2 + \lambda_i\left(\frac{p_{C,j}}{p_{C,i}} - 1\right) + \frac{1}{p_{C,i}}, \\
\sigma_{C,t-}\big|_{\nu_{t-}=i} &= \sigma_{C,i}, \\
\sigma_{R_C,t-}^P\big|_{\nu_{t-}=i} &= \sigma_{R_C,i}^P = \frac{p_{C,j}}{p_{C,i}} - 1,
\end{aligned}$$

for $i \in \{1,2\}$, $j \neq i$. The total volatility of returns to holding the consumption claim, when the current state is i, is given by

$$\sigma_{R_C,i} = \sqrt{\sigma_{C,i}^2 + \lambda_i \left(\sigma_{R_C,i}^P\right)^2}.$$

Note that C^* is the consumption to be chosen by the agent, i.e. it is a control, and at this stage we cannot rule out the possibility that it jumps with the state of the economy. In contrast, C is aggregate consumption, i.e. the dividend received by an investor who holds the claim to aggregate consumption. Because aggregate consumption, C, is continuous, it's left and right limits are equal, i.e. $C_{t-} = C_t$.

The system of Hamilton-Jacobi-Bellman partial differential equations for the agent's optimization problem is

$$\sup_{C^*,\varphi} f\left(C_{t-}^*, J_{t-}^*\right)\big|_{\nu_{t-}=i} dt + E_t\left[dJ_t^* \,|\, \nu_{t-} = i\right] = 0,\ i \in \{1,2\}.$$

Applying Ito's Lemma to $J_t^* = J^*\left(W_t, \nu_t\right)$ allows us to write the above equation as

$$\begin{aligned} 0 \;=\; & \sup_{C_i^*,\varphi_i} f\left(C_i^*, J_i^*\right) + W_i J_{i,W}^* \left(\varphi_i\left(\mu_{R_C,i} - r_i\right) + r_i - \frac{C_i^*}{W_i}\right) + \frac{1}{2} W_i^2 J_{i,WW}^* \varphi_i^2 \sigma_{R_C,i}^2 + \\ & + \lambda_i \left(J_j^* - J_i^*\right),\ i \in \{1,2\},\ j \neq i. \end{aligned}$$

We guess and verify that

$$J_t^* = h\left(W_t F_t\right),$$

where F_i satisfies the nonlinear equation system

$$0 = \sup_{C_i^*,\varphi_i} \beta u\left(\frac{C_i^*}{W_i F_i}\right) + \left(\varphi_i\left(\mu_{R_C,i} - r_i\right) + r_i - \frac{C_i^*}{W_i}\right) - \frac{1}{2}\gamma \varphi_i^2 \sigma_{R_C,i}^2 + \lambda_i \left(\frac{\left(F_j/F_i\right)^{1-\gamma} - 1}{1-\gamma}\right),$$

$i \in \{1,2\}$, $j \neq i$. From the first order conditions of the above equations, we obtain the optimal consumption and portfolio policies:

$$\begin{aligned} C_i^* &= \beta^{\psi} F_i^{-(\psi-1)} W_i,\ i \in \{1,2\}, \\ \varphi_i &= \frac{1}{\gamma} \frac{\mu_{R_C,i} - r_i}{\sigma_{R_C,i}^2},\ i \in \{1,2\}. \end{aligned}$$

The market for the consumption good must clear, so $\varphi_i = 1$, $W_i = P_i$, $C_i^* = C$ (and thus $J = J^*$). Note that this forces the optimal portfolio proportion to be one and the optimal consumption policy to be continuous. Hence

$$\mu_{R_C,i} - r_i = \gamma \sigma_{R_C,i}^2,$$

and

$$p_{C,i} = \beta^{-\psi} F_i^{1-\psi}. \tag{B.20}$$

The above equation implies that for $\psi = 1$, $p_{C,i} = 1/\beta$. The equality, $J = J^*$, implies that $CV_i = WF_i$. Hence, $F_i = p_{C,i}^{-1} V_i$. Using this equation to eliminate F_i from (B.20) gives (B.19). Substituting (B.19) into (B.17) and (B.18) gives the expressions in (B.8) for $\psi \neq 1$ and (B.9), after some algebra.

B.3 Proofs

Proof of Proposition 1

We start by proving that the state-price density satisfies the stochastic differential equation

$$\left.\frac{d\pi_t}{\pi_{t-}}\right|_{\nu_{t-}=i} = -r_i dt + \left.\frac{dM_t}{M_{t-}}\right|_{\nu_{t-}=i}, \tag{B.21}$$

where M is a martingale under $\mathbb{P}$ such that

$$\left.\frac{dM_t}{M_{t-}}\right|_{\nu_{t-}=i} = -\Theta_i^B dB_t + \Theta_i^P dN_{i,t}^P, \tag{B.22}$$

r_i is the risk-free rate in state i given by

$$r_i = \begin{cases} \bar{r}_1 + \lambda_1 \left[\frac{\gamma - \frac{1}{\psi}}{\gamma - 1} \left(\omega^{-\frac{\gamma-1}{\gamma-\frac{1}{\psi}}} - 1 \right) - \left(\omega^{-1} - 1 \right) \right] & , i = 1 \\ \bar{r}_2 + \lambda_2 \left[\frac{\gamma - \frac{1}{\psi}}{\gamma - 1} \left(\omega^{\frac{\gamma-1}{\gamma-\frac{1}{\psi}}} - 1 \right) - (\omega - 1) \right] & , i = 2 \end{cases}, \tag{B.23}$$

and

$$\omega_2 = \omega_1^{-1} = \omega,$$

where ω is the solution of

$$G(\omega) = 0, \tag{B.24}$$

and

$$G(x) = \begin{cases} x^{-\frac{1-\frac{1}{\psi}}{\gamma-\frac{1}{\psi}}} - \dfrac{\bar{r}_2 + \gamma\sigma_{C,2}^2 - g_2 + \lambda_2 \frac{1-\frac{1}{\psi}}{\gamma-1} \left(x^{\frac{\gamma-1}{\gamma-\frac{1}{\psi}}} - 1 \right)}{\bar{r}_1 + \gamma\sigma_{C,1}^2 - g_1 + \lambda_1 \frac{1-\frac{1}{\psi}}{\gamma-1} \left(x^{-\frac{\gamma-1}{\gamma-\frac{1}{\psi}}} - 1 \right)}, & \psi \neq 1 \\ \ln x^{\frac{1}{\gamma-1}} - \dfrac{g_2 - \frac{1}{2}\gamma\sigma_{C,2}^2 + \lambda_2(x-1)}{g_1 - \frac{1}{2}\gamma\sigma_{C,1}^2 + \lambda_1(x^{-1}-1)}, & \psi = 1 \end{cases}. \tag{B.25}$$

Θ_i^B is the market price of risk due to Brownian shocks in state i, given by

$$\Theta_i^B = \gamma\sigma_{C,i}. \tag{B.26}$$

and Θ_i^P is the market price of risk due to Poisson shocks when the economy switches out of state i:

$$\Theta_i^P = \omega_i - 1. \tag{B.27}$$

We begin the proof by noting that if we define

$$\omega_i = \left.\frac{\pi_t}{\pi_{t-}}\right|_{\nu_{t-}=i,\nu_t=j}, j \neq i, \tag{B.28}$$

then (B.8) implies that

$$\omega_i = \begin{cases} \left(\frac{p_{C,j}}{p_{C,i}}\right)^{-\frac{\gamma-\frac{1}{\psi}}{1-\frac{1}{\psi}}}, & \psi \neq 1. \\ \left(\frac{V_j}{V_i}\right)^{-(\gamma-1)}, & \psi = 1. \end{cases} \tag{B.29}$$

The above equation implies that $\omega_2 = \omega_1^{-1}$, so we can set $\omega_2 = \omega_1^{-1} = \omega$, where ω is determined below. Using (B.29) we can rewrite (B.9) and (B.12) as

$$p_{C,i} = \frac{1}{\overline{r}_i + \gamma\sigma_{C,i}^2 - g_i + \lambda_i \frac{1-\frac{1}{\psi}}{\gamma-1}\left(\omega_i^{\frac{\gamma-1}{\gamma-1/\psi}} - 1\right)}, i \in \{1,2\}, \tag{B.30}$$

and

$$\beta \ln V_i = g_i - \frac{1}{2}\gamma\sigma_{C,i}^2 + \lambda_i \frac{\omega_i - 1}{1-\gamma}, i \in \{1,2\}, \tag{B.31}$$

respectively. Therefore, from (B.29) and the above two equations it follows that ω is the solution of Equation (B.24). Ito's Lemma implies that the state-price density evolves according to

$$\begin{aligned} \frac{d\pi_t}{\pi_{t-}} &= \frac{1}{\pi_{t-}}\frac{\partial \pi_{t-}}{\partial t}dt + \frac{1}{\pi_{t-}}C_t\frac{\partial \pi_{t-}}{\partial C_t}\frac{dC_t}{C_t} + \frac{1}{2}\frac{1}{\pi_{t-}}C_t^2\frac{\partial^2 \pi_{t-}}{\partial C_t^2}\left(\frac{dC_t}{C_t}\right)^2 \\ &\quad + \lambda_{\nu_{t-}}\frac{\Delta\pi_t}{\pi_{t-}}dt + \frac{\Delta\pi_t}{\pi_{t-}}dN_{\nu_{t-},t}^P, \end{aligned} \tag{B.32}$$

where

$$\Delta\pi_t = \pi_t - \pi_{t-}.$$

The definition (B.28) implies

$$\left.\frac{\Delta\pi_t}{\pi_{t-}}\right|_{\nu_{t-}=i,\,\nu_t=j} = \omega_i - 1,\ j \neq i.$$

Together with some standard algebra that allows us to rewrite (B.32) as

$$\left.\frac{d\pi_t}{\pi_{t-}}\right|_{\nu_t=\nu_i} = -\left(\kappa_i + \gamma g_i - \frac{1}{2}\gamma\left(1+\gamma\right)\sigma_{C,i}^2 + \lambda_i\left(1-\omega_i\right)\right)dt - \gamma\sigma_{C,i}dB_{C,t} + \left(\omega_i - 1\right)dN_{i,t}^P.$$

Comparing the above equation with (B.21), which is standard in an economy with jumps, gives (B.26) and (B.27), in addition to

$$r_i = \kappa_i + \gamma g_i - \frac{1}{2}\gamma(1+\gamma)\sigma_{C,i}^2 + \lambda_i(1-\omega_i),$$

where

$$\kappa_i = \begin{cases} \beta\left[1 + \left(\gamma - \frac{1}{\psi}\right)\frac{(\beta p_{C,i})^{-1}-1}{1-\frac{1}{\psi}}\right], & \psi \neq 1 \\ \beta\left[1 + (\gamma-1)\ln\left(V_i^{-1}\right)\right], & \psi = 1 \end{cases}. \tag{B.33}$$

We use Equations (B.30) and (B.31) to eliminate $p_{C,i}$ and V_i from (B.33) to obtain

$$\kappa_i = \begin{cases} \overline{r}_i - \left(\gamma - \frac{1}{\psi}\right)\lambda_i\left(\frac{\omega_i^{\frac{\gamma-1}{\gamma-1/\psi}}-1}{1-\gamma}\right) - \left[\gamma g_i - \frac{1}{2}\gamma(1+\gamma)\sigma_{C,i}^2\right], & \psi \neq 1 \\ \overline{r}_i + \lambda_i(\omega_i - 1) - \left[\gamma g_i - \frac{1}{2}\gamma(1+\gamma)\sigma_{C,i}^2\right], & \psi = 1 \end{cases}, \tag{B.34}$$

so

$$r_i = \begin{cases} \overline{r}_i - \left(\gamma - \frac{1}{\psi}\right)\lambda_i\left(\frac{\omega_i^{\frac{\gamma-1}{\gamma-1/\psi}}-1}{1-\gamma}\right) + \lambda_i(1-\omega_i), & \psi \neq 1. \\ \overline{r}_i, & \psi = 1. \end{cases} \tag{B.35}$$

Taking the limit of the upper expression in the above equation gives the lower expression, so (B.23) follows. The total market price of consumption risk in state i accounts for both Brownian and Poisson shocks, and is thus given by

$$\Theta_i = \sqrt{\left(\Theta_i^B\right)^2 + \lambda_i\left(\Theta_i^P\right)^2},\ i \in \{1,2\}. \tag{B.36}$$

Because the Poisson and Brownian shocks in (B.22) are independent and their respective prices of risk are bounded, M is a martingale under the actual measure $\mathbb{P}$. Thus M defines the Radon-Nikodym derivative $\frac{d\mathbb{Q}}{d\mathbb{P}}$ via

$$M_t = E_t\left[\frac{d\mathbb{Q}}{d\mathbb{P}}\right].$$

It is a standard result (see Elliott (1982)) that,

$$\widehat{\lambda}_i = \lambda_i E_t\left[\frac{M_t}{M_{t-}}\middle|\nu_{t-} = i, \nu_t = j\right],\ j \neq i.$$

The jump component in $d\pi$ comes purely from dM. Thus, using (B.28), we can simplify the above expression to obtain

$$\widehat{\lambda}_i = \lambda_i\omega_i,$$

which implies (4.10) and (4.11).

We deduce the properties of the risk distortion factor, ω, from the properties of the function g defined in (B.25). We restrict the domain of G to $x > 0$. First we consider the case where $\psi \neq 1$. We assume that the price-consumption ratios, $p_{C,i}$, $i \in \{1,2\}$ are strictly positive. Therefore, G is continuous. We observe that if G is monotonic, then by continuity, $G(1)$ and $G'(1)$ are of the same sign iff $\omega < 1$ and $G(1)$ and $G'(1)$ are of different signs iff $\omega > 1$. Clearly, in both cases, ω is unique. To establish monotonicity note that

$$\begin{aligned} G'(x) &= -\frac{1-\frac{1}{\psi}}{\gamma-\frac{1}{\psi}}\left[x^{-\frac{1-\frac{1}{\psi}}{\gamma-\frac{1}{\psi}}-1}+\frac{1}{\left(\overline{r}_1+\gamma\sigma_{C,1}^2-g_1+\lambda_1\frac{1-\frac{1}{\psi}}{\gamma-1}\left(x^{-\frac{\gamma-1}{\gamma-\frac{1}{\psi}}}-1\right)\right)^2}\right. \\ &\quad\cdot\left(\overline{r}_1+\gamma\sigma_{C,1}^2-g_1+\lambda_1\frac{1-\frac{1}{\psi}}{\gamma-1}\left(x^{-\frac{\gamma-1}{\gamma-\frac{1}{\psi}}}-1\right)\lambda_2 x^{\frac{1/\psi-1}{\gamma-1/\psi}}\right. \\ &\quad\left.\left.+\left(\overline{r}_2+\gamma\sigma_{C,2}^2-g_2+\lambda_2\frac{1-\frac{1}{\psi}}{\gamma-1}\left(\omega^{\frac{\gamma-1}{\gamma-\frac{1}{\psi}}}-1\right)\right)\lambda_1 x^{-\frac{1/\psi-1}{\gamma-1/\psi}-2}\right)\right] \end{aligned}$$

The above equation implies that for $x > 0$, if $p_{C,1}$ and $p_{C,2}$ are strictly positive, then $G'(x)$ does not change sign. Therefore, G must be monotonic. Now we use the following expressions:

$$G(1) = 1 - \frac{\overline{r}_2+\gamma\sigma_{C,2}^2-g_2}{\overline{r}_1+\gamma\sigma_{C,1}^2-g_1},$$

and

$$G'(1) = -\frac{1-\frac{1}{\psi}}{\gamma-\frac{1}{\psi}}\left[1+\frac{(\overline{r}_1+\gamma\sigma_{C,1}^2-g_1)\lambda_2+(\overline{r}_2+\gamma\sigma_{C,2}^2-g_2)\lambda_1}{(\overline{r}_1+\gamma\sigma_{C,1}^2-g_1)^2}\right],$$

to relate the signs of $G(1)$ and $G'(1)$ to the properties of the agent's preferences. Note that $G'(1) < 0$, $(G'(1) > 0)$ iff $\frac{1-\frac{1}{\psi}}{\gamma-\frac{1}{\psi}} > 0$, $\left(\frac{1-\frac{1}{\psi}}{\gamma-\frac{1}{\psi}} < 0\right)$. We assume that $\overline{r}_i + \gamma\sigma_{C,i}^2 - g_i > 0$ for $i \in \{1,2\}$, which is equivalent to assuming that if the economy were always in state i, then the price-consumption ratio would be positive. Simple algebra tells us that $\overline{r}_i+\gamma\sigma_{C,i}^2-g_i = \beta+\left(\frac{1}{\psi}-1\right)\left(g_i-\frac{1}{2}\gamma\sigma_{C,i}^2\right)$. We know that $g_1-\frac{1}{2}\gamma\sigma_{C,1}^2 < g_2-\frac{1}{2}\gamma\sigma_{C,2}^2$. Therefore $G(1) < 0$, $(G(1) > 0)$ iff $\psi > 1$, $(\psi < 1)$. Consequently, $G(1)$ and $G'(1)$ are of the same sign iff $\gamma < 1/\psi$ and $G(1)$ and $G'(1)$ are of different signs iff $\gamma > 1/\psi$. It then follows that $\omega > 1$ iff $\gamma > 1/\psi$ and $\omega < 1$ iff $\gamma < 1/\psi$, assuming that $\psi \neq 1$.

Similarly, when $\psi = 1$, if we assume that $V_i > 0$ for $i \in \{1,2\}$, then we can prove that: $\omega > 1$ if $\gamma > 1$ and $g_i - \frac{1}{2}\gamma\sigma_{C,i}^2$, $i \in \{1,2\}$ are of the sign and $\omega > 1$ if $\gamma < 1$ and $g_i - \frac{1}{2}\gamma\sigma_{C,i}^2$, $i \in \{1,2\}$ are of opposite sign. Now, if $\gamma < 1$, then $\overline{r}_1 + \gamma\sigma_{C,1}^2 - g_1 > 0$ implies $g_1 - \frac{1}{2}\gamma\sigma_{C,1}^2 > 0$, which means $g_i - \frac{1}{2}\gamma\sigma_{C,i}^2$, $i \in \{1,2\}$ cannot be of opposite sign. Therefore, $\omega > 1$ iff $\gamma > 1$.

So, for $\psi > 0$, $\omega > 1$ iff $\gamma > 1/\psi$ and $\omega < 1$ iff $\gamma < 1/\psi$. It follows that $\omega = 1$ iff $\gamma = 1/\psi$.

Proof of Proposition 2

No-arbitrage principle gives (4.14), which using Ito's Lemma can be rewritten as the following ordinary differential-equation system:

$$\frac{dq_{D,ij,t}}{dX_t}\widehat{\theta}_i X_t + \frac{1}{2}\frac{d^2 q_{D,ij,t}}{dX_t^2}\sigma_{X,i}^2 X_t^2 + \widehat{\lambda}_i\left(q_{D,kj,t} - q_{D,ij,t}\right) = r_i q_{D,ij,t},\ i,j \in \{1,2\}, k \neq i, \quad \text{(B.37)}$$

where

$$\sigma_{X,i} = \sqrt{(\sigma_X^{id})^2 + (\sigma_{X,i}^s)^2} \quad \text{(B.38)}$$

is total earnings growth volatility in state i and

$$\widehat{\theta}_i = \theta_i - \gamma\rho_{XC,i}\sigma_{X,i}^s\sigma_{C,i}$$

is the risk-neutral earnings growth rate in state i. The definitions of the payoffs of the Arrow-Debreu default claims give us the following boundary conditions:

$$q_{D,ij}(X) = \begin{cases} 1, & j = i,\ X \leq X_{D,i} \\ 0, & j \neq i,\ X \leq X_{D,i} \end{cases}. \quad \text{(B.39)}$$

Value-matching and smooth-pasting give us the remaining boundary conditions: for $j \in \{1,2\}$

$$\begin{aligned} \lim_{X \downarrow X_{D,j}} q_{D,2j} &= \lim_{X \uparrow X_{D,j}} q_{D,2j}, \\ \lim_{X \downarrow X_{D,j}} q'_{D,2j} &= \lim_{X \uparrow X_{D,j}} q'_{D,2j}. \end{aligned}$$

Expressing (B.37) in matrix form gives:

$$\left(\frac{1}{2}\begin{bmatrix} \sigma_{1,X}^2 & 0 \\ 0 & \sigma_{2,X}^2 \end{bmatrix} X^2 \frac{d^2}{dX^2} + \begin{bmatrix} \widehat{\theta}_1 & 0 \\ 0 & \widehat{\theta}_2 \end{bmatrix} X \frac{d}{dX} - \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} + \begin{bmatrix} -\widehat{\lambda}_1 & \widehat{\lambda}_1 \\ \widehat{\lambda}_2 & -\widehat{\lambda}_2 \end{bmatrix}\right) \cdot \begin{bmatrix} q_{D,11} & q_{D,12} \\ q_{D,21} & q_{D,22} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \quad \text{(B.40)}$$

From (B.39) it follows that

$$q_{D,ij}|_{X=X_{D,i}} = \begin{cases} 1, & j = i \\ 0, & j \neq i \end{cases}. \quad \text{(B.41)}$$

We first solve (B.40) subject to the conditions above for the region $X > X_{D,1}$. We seek solutions of the form

$$q_{D,ij} = h_{ij} X^k \; i, j \in \{1, 2\}.$$

Hence,

$$\left(\frac{1}{2} \begin{bmatrix} \sigma_{X,1}^2 & 0 \\ 0 & \sigma_{X,2}^2 \end{bmatrix} k(k-1) + \begin{bmatrix} \widehat{\theta}_1 & 0 \\ 0 & \widehat{\theta}_2 \end{bmatrix} k + \begin{bmatrix} -\widehat{\lambda}_1 - r_1 & \widehat{\lambda}_1 \\ \widehat{\lambda}_2 & -\widehat{\lambda}_2 - r_2 \end{bmatrix} \right) \cdot \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \tag{B.42}$$

A solution of the above equation exists if

$$\det \left(\frac{1}{2} \begin{bmatrix} \sigma_{X,1}^2 & 0 \\ 0 & \sigma_{X,2}^2 \end{bmatrix} k(k-1) + \begin{bmatrix} \widehat{\theta}_1 & 0 \\ 0 & \widehat{\theta}_2 \end{bmatrix} k + \begin{bmatrix} -\widehat{\lambda}_1 - r_1 & \widehat{\lambda}_1 \\ \widehat{\lambda}_2 & -\widehat{\lambda}_2 - r_2 \end{bmatrix} \right) = 0.$$

Therefore, k is a root of the quartic polynomial

$$\left[\frac{1}{2} \sigma_{X,1}^2 k(k-1) + \widehat{\theta}_1 k + \left(-\widehat{\lambda}_1 - r_1 \right) \right] \left[\frac{1}{2} \sigma_{X,2}^2 k(k-1) + \widehat{\theta}_2 k + \left(-\widehat{\lambda}_2 - r_2 \right) \right] - \widehat{\lambda}_2 \widehat{\lambda}_1 = 0, \tag{B.43}$$

which is the characteristic function of (B.40). The above quartic has 4 distinct real roots, two of which are positive, provided that $\sigma_{X,i}$, r_i, $\widehat{\lambda}_{ij} > 0$ for $i \in \{1, 2\}$ and $j \neq i$ (see Guo (1999)). Therefore the general solution of is

$$q_{D,ij} = \sum_{m=1}^{4} h_{ij,m} X^{k_m},$$

where k_m is the m'th root (ranked in order of increasing size, accounting for sign) of (B.43). To ensure that $q_{D,ij}$, $i, j \in \{1, 2\}$ are finite as $X \to \infty$, we set $h_{ij,3} = h_{ij,4} = 0$, $i, j \in \{1, 2\}$, so we use only the two negative roots: $k_1 < k_2 < 0$. From equation (B.42), it follows that

$$\frac{h_{21,m}}{h_{11,m}} = \frac{h_{22,m}}{h_{12,m}} = \epsilon(k_m), \; m \in \{1, 2\},$$

where

$$\epsilon(k) = -\frac{\widehat{\lambda}_2}{\frac{1}{2}\sigma_{X,2}^2 k(k-1) + \widehat{\theta}_2 k + \left(-\widehat{\lambda}_2 - r_2 \right)} = -\frac{\frac{1}{2}\sigma_{X,1}^2 k(k-1) + \widehat{\theta}_1 k + \left(-\widehat{\lambda}_1 - r_1 \right)}{\widehat{\lambda}_1}.$$

Therefore

$$\begin{aligned} q_{D,1j} &= \sum_{m=1}^{2} h_{1j,m} X^{k_m}, \; j \in \{1, 2\}, \\ q_{D,2j} &= \sum_{m=1}^{2} h_{1j,m} \epsilon(k_m) X^{k_m}, \; j \in \{1, 2\}. \end{aligned}$$

We now solve (B.40) subject to the relevant boundary conditions for the region $X_2 < X \leq X_1$. We know

$$\begin{aligned} q_{D,11} &= 1, \\ q_{D,12} &= 0. \end{aligned}$$

Therefore

$$\left(\frac{1}{2}\sigma_{X,2}^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{d^2}{dX^2} + \widehat{\theta}_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{d}{dX} - \left(\widehat{\lambda}_2 + r_2\right) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) \begin{bmatrix} q_{D,21,t} \\ q_{D,22,t} \end{bmatrix} + \begin{bmatrix} \widehat{\lambda}_2 \\ 0 \end{bmatrix} = 0.$$

We can show (using the same method we used to solve (B.40)) that the general solution of the above equation is

$$\begin{aligned} q_{D,21} &= \frac{\widehat{\lambda}_2}{r_2 + \widehat{\lambda}_2} + s_{1,1}X^{j_1} + s_{1,2}X^{j_2}, \\ q_{D,22} &= s_{2,1}X^{j_1} + s_{2,2}X^{j_2}, \end{aligned}$$

where j_i, $i \in \{1, 2\}$ are the roots of the quadratic

$$\frac{1}{2}\sigma_{X,2}^2 j\,(j-1) + \widehat{\theta}_2 j - \left(\widehat{\lambda}_2 + r_2\right) = 0,$$

such that $j_1 < j_2$. In summary

$$q_{D,11} = \begin{cases} \sum_{m=1}^{2} h_{11,m}X^{k_m}, & X > X_{D,1}. \\ 1, & X_{D,2} < X \leq X_{D,1}. \\ 1, & X \leq X_{D,2}. \end{cases}$$

$$q_{D,12} = \begin{cases} \sum_{m=1}^{2} h_{12,m}X^{k_m}, & X > X_{D,1}. \\ 0, & X_{D,2} < X \leq X_{D,1}. \\ 0, & X \leq X_{D,2}. \end{cases}$$

$$q_{D,21} = \begin{cases} \sum_{m=1}^{2} h_{11,m}\epsilon\left(k_m\right)X^{k_m}, & X > X_{D,1}. \\ \frac{\widehat{\lambda}_2}{r_2+\widehat{\lambda}_{21}} + \sum_{m=1}^{2} s_{1,m}X^{j_m}, & X_{D,2} < X \leq X_{D,1}. \\ 0, & X \leq X_{D,2}. \end{cases}$$

$$q_{D,22} = \begin{cases} \sum_{m=1}^{2} h_{12,m}\epsilon\left(k_m\right)X^{k_m}, & X > X_{D,1}. \\ \sum_{m=1}^{2} s_{2,m}X^{j_m}, & X_{D,2} < X \leq X_{D,1}. \\ 1, & X \leq X_{D,2}. \end{cases}$$

To find the 8 constants: $h_{11,1}, h_{11,2}, h_{12,1}, h_{12,2}, s_{1,1}, s_{1,2}, s_{2,1}, s_{2,2}$, we use the following 8

boundary conditions:

$$\begin{aligned} q_{D,11}|_{X=X_{D,1}} &= 1,\, q_{D,12}|_{X=X_{D,1}} = 0, \\ \lim_{X\uparrow X_{D,1}} q_{D,21} &= \lim_{X\downarrow X_{D,1}} q_{D,21},\, \lim_{X\uparrow X_{D,1}} q_{D,22} = \lim_{X\downarrow X_{D,1}} q_{D,22}, \\ \lim_{X\uparrow X_{D,1}} q'_{D,21} &= \lim_{X\downarrow X_{D,1}} q'_{D,21},\, \lim_{X\uparrow X_{D,1}} q'_{D,22} = \lim_{X\downarrow X_{D,1}} q'_{D,22}, \end{aligned}$$

and

$$q_{D,21}|_{X=X_{D,2}} = 0,\, q_{D,22}|_{X=X_{D,2}} = 1.$$

The first set being applied at $X = X_{D,1}$ and the second set at $X = X_{D,2}$. The 8 boundary conditions give 8 linear equations, which can be solved in closed-form to give $h_{11,1}, h_{11,2}, h_{12,1}, h_{12,2}, s_{1,1}, s_{1,2}, s_{2,1}, s_{2,2}$.

We obtain $\{\widehat{p}_{D,ij}\}_{i,j\in\{1,2\}}$ and $\{p_{D,ij}\}_{i,j\in\{1,2\}}$, by setting $r_1 = r_2 = 0$, and $r_1 = r_2 = 0$, $\gamma = 1/\psi = 0$, respectively.

Proof of Proposition 3

Suppose the economy is currently in state i. Then, the risk-neutral probability of the economy switching into a different state during a small time interval Δt is $\widehat{\lambda}_i \Delta t$ and the risk-neutral probability of not switching is $1 - \widehat{\lambda}_i \Delta t$. We can therefore write the unlevered firm value in state i as

$$A_i = (1-\eta)X\Delta t + e^{-(\overline{\mu}_i - \theta_i)\Delta t}\left[\left(1 - \widehat{\lambda}_i \Delta t\right) A_i + \widehat{\lambda}_i \Delta t A_j\right],\ i,j \in \{1,2\},\ j \neq i. \quad \text{(B.44)}$$

We take the limits of (B.44) as $\Delta t \to 0$, to obtain

$$0 = (1-\eta)\, X - (\overline{\mu}_i - \theta_i) A_i + \widehat{\lambda}_i \left(A_j - A_i\right),\ i \in \{1,2\}, j \neq i.$$

To obtain the solution of the above linear equation system, we define

$$p_i = \frac{1}{(1-\eta)} \frac{A_i}{X},$$

the before price-earnings ratio in state i. Therefore

$$\left(diag\left(\overline{\mu}_1 - \theta_1, \overline{\mu}_2 - \theta_2\right) - \widehat{\Lambda}\right) \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \text{(B.45)}$$

where $diag\left(\overline{\mu}_1 - \theta_1, \overline{\mu}_2 - \theta_2\right)$ is a 2×2 diagonal matrix, with the quantities $\overline{\mu}_1 - \theta_1$ and $\overline{\mu}_2 - \theta_2$ along the diagonal and

$$\widehat{\Lambda} = \begin{pmatrix} -\widehat{\lambda}_1 & \widehat{\lambda}_1 \\ \widehat{\lambda}_2 & -\widehat{\lambda}_2 \end{pmatrix}$$

is the generator matrix of the Markov chain under the risk-neutral measure. Solving (B.45) gives (4.20), if $\det\left(diag\left(\overline{\mu}_1-\theta_1,\overline{\mu}_2-\theta_2\right)-\widehat{\Lambda}\right)\neq 0$. We now define $P_i^X=p_iX$, the before-tax value of the claim to the earnings stream X in state t. Hence, from the basic asset pricing equation

$$E_t\left[\frac{dP^X+Xdt}{P^X}-rdt\middle|\nu_{t-}=i\right]=-E_t\left[\frac{dM}{M}\frac{dP^X}{P^X}\middle|\nu_{t-}=i\right],$$

we obtain the unlevered risk premium:

$$E_t\left[\frac{dP^X+Xdt}{P^X}-rdt\middle|\nu_{t-}=i\right]=\gamma\rho_{XC}\sigma^s_{X,i}\sigma_{C,i}dt-\left(\widehat{\lambda}_i-\lambda_i\right)\left(\frac{p_j}{p_i}-1\right)dt, i\in\{1,2\},\ j\neq i.$$

Applying Ito's Lemma,

$$dP^X=p_idX_t+\lambda_i\left(p_j-p_i\right)dt+\left(p_j-p_i\right)dN^P_{i,t},\ i\in\{1,2\},\ j\neq i.$$

Thus, the unlevered volatility of returns on equity in state i is given by

$$\sigma_{R,i}=\sqrt{\sigma^2_{X,i}+\lambda_i\left(\frac{p_j}{p_i}-1\right)^2},\ j\neq i,$$

where $\sigma_{X,i}$ is defined in (B.38).

Proof of Proposition 4

First we show that (4.24) holds. The central part of our proof consists of proving that

$$E_t\left[\int_t^{\tau_D}\frac{\pi_s}{\pi_t}ds\middle|\nu_t=i\right]=\frac{1}{r_{P,i}}-\sum_{j=1}^{2}\frac{q_{D,ij}}{r_{P,j}},\tag{B.46}$$

where

$$r_{P,i}=\left(E_t\left[\int_t^{\infty}\frac{\pi_s}{\pi_t}ds\middle|\nu_t=i\right]\right)^{-1},\tag{B.47}$$

and

$$E_t\left[\frac{\pi_{\tau_D}}{\pi_t}\alpha_{\tau_D}A_{\tau_D}\left(X_{\tau_D}\right)\middle|\nu_t=i\right]\ =\ \sum_{j=1}^{2}\alpha_jA_j\left(X_{D,j}\right)q_{D,ij}.\tag{B.48}$$

Using the above result, (4.24) follows immediately from (4.23). First, we observe that

$$q_{D,ij,t}=E_t\left[\Pr\left(\nu_t=i|\,\nu_{\tau_D}=j\right)\frac{\pi_{\tau_D}}{\pi_t}\middle|\nu_t=i,\nu_{\tau_D}=j\right].\tag{B.49}$$

To prove (B.46), we note that

$$E_t\left[\int_t^{\tau_D}\frac{\pi_s}{\pi_t}ds\middle|\nu_t=i\right]=E_t\left[\int_t^{\infty}\frac{\pi_s}{\pi_t}ds\middle|\nu_t=i\right]-E_t\left[\frac{\pi_{\tau_D}}{\pi_t}\int_{\tau_D}^{\infty}\frac{\pi_s}{\pi_{\tau_D}}ds\middle|\nu_t=i\right],$$

and conditioning on the event $\{\nu_{\tau_D} = j\}$, we obtain

$$E_t\left[\frac{\pi_{\tau_D}}{\pi_t}\int_{\tau_D}^{\infty}\frac{\pi_s}{\pi_{\tau_D}}ds\middle|\nu_t = i\right] = \sum_{j=1}^{2} E_t\left[\Pr\left(\nu_{\tau_D} = j|\,\nu_t = i\right)\frac{\pi_{\tau_D}}{\pi_t}\int_{\tau_D}^{\infty}\frac{\pi_s}{\pi_{\tau_D}}ds\middle|\nu_t = i, \nu_{\tau_D} = j\right].$$

What happens from date τ_D onwards is independent of what happened before, so

$$\begin{aligned} & E_t\left[\Pr\left(\nu_{\tau_D} = j|\,\nu_t = i\right)\frac{\pi_{\tau_D}}{\pi_t}\int_{\tau_D}^{\infty}\frac{\pi_s}{\pi_{\tau_D}}ds\middle|\nu_t = i, \nu_{\tau_D} = j\right] \\ = \; & E_t\left[\Pr\left(\nu_{\tau_D} = j|\,\nu_t = i\right)\frac{\pi_{\tau_D}}{\pi_t}\middle|\nu_t = i, \nu_{\tau_D} = j\right] E_t\left[\int_{\tau_D}^{\infty}\frac{\pi_s}{\pi_{\tau_D}}ds\middle|\nu_{\tau_D} = j\right]. \end{aligned}$$

Therefore

$$\begin{aligned} E_t\left[\int_t^{\tau_D}\frac{\pi_s}{\pi_t}ds\middle|\nu_t = i\right] &= E_t\left[\int_t^{\infty}\frac{\pi_s}{\pi_t}ds\middle|\nu_t = i\right] \\ -\sum_{j=1}^{2} E_t\left[\Pr\left(\nu_{\tau_D} = j|\,\nu_t = i\right)\frac{\pi_{\tau_D}}{\pi_t}\middle|\nu_t = i, \nu_{\tau_D} = j\right] & E_t\left[\int_{\tau_D}^{\infty}\frac{\pi_s}{\pi_{\tau_D}}ds\middle|\nu_{\tau_D} = j\right]. \end{aligned} \tag{B.50}$$

Conditional on being in state i, the value of a claim which pays one risk-free unit of consumption in perpetuity is $E_t\left[\int_t^{\infty}\frac{\pi_s}{\pi_t}ds\middle|\nu_t = i\right]$, so the discount rate for this perpetuity, $r_{P,i}$, is given by (B.47). Consequently, (B.50) implies

$$E_t\left[\int_t^{\tau_D}\frac{\pi_s}{\pi_t}ds\middle|\nu_t = i\right] = \frac{1}{r_{P,i}} - \sum_{j=1}^{2}\frac{E_t\left[\Pr\left(\nu_t = i|\,\nu_{\tau_D} = j\right)\frac{\pi_{\tau_D}}{\pi_t}\middle|\nu_t = i, \nu_{\tau_D} = j\right]}{r_{P,j}}. \tag{B.51}$$

Using the definition of the Arrow-Debreu default claim, $q_{D,ij}$, given in (B.49), (B.46) follows. We do not have to evaluate $r_{P,i}$ from scratch based on (B.47), because we can infer its value from (4.20), by setting $\theta_i = \sigma_{X,i} = \rho_{XC,i} = 0$, $\forall i \in \{1, 2\}$ to obtain (4.25). To prove (B.48), we condition on the event $\{\nu_{\tau_D} = j\}$ to obtain

$$E_t\left[\frac{\pi_{\tau_D}}{\pi_t}\alpha_{\tau_D}A_{\tau_D}\left(X_{\tau_D}\right)\middle|\nu_t = i\right] = \sum_{j=1}^{2}\alpha_j A_j\left(X_j\right)E_t\left[\Pr\left(\nu_{\tau_D} = \nu_j|\,\nu_t = i\right)\frac{\pi_{\tau_D}}{\pi_t}\middle|\nu_t = i, \nu_{\tau_D} = j\right].$$

Using (B.49) to simplify the above expression we obtain (B.48). The credit spread in state i is

$$s_i = y_i - r_i = \frac{c}{B_i} - r_{P,i}. \tag{B.52}$$

Substituting (4.24) into the above equation and simplifying gives (4.26).

Proof of Proposition 5

Using the same approach we used to derive (4.24), we can derive (4.27). Applying Ito's Lemma to (4.27), we obtain

$$dR_t|_{\nu_{t-}=i} = \left.\frac{dS_t + (1-\eta)\left(X_t - c\right)dt}{S_{t-}}\right|_{\nu_{t-}=i} = \mu_{R,i}dt + \sigma_{R,i}^{B,id}dB_{X,t}^{id} + \sigma_{R,i}^{B,s}dB_{X,t}^{s} + \sigma_{R,i}^{P}dN_{i,t}^{P},$$

where

$$\mu_{R,i} = \frac{A_i(X) + \sum_{j=1}^{2}\left[Xq'_{D,ij}\theta_i + \frac{1}{2}X^2 q''_{D,ij}\sigma^2_{X,i}\right]\left[(1-\eta)\frac{c}{r_{P,j}} - A_j\left(X_{D,j}\right)\right]}{S_i}$$
$$+\left(\frac{S_k}{S_i} - 1\right)\lambda_i + \frac{A_i(X)}{S_i},\ k \neq i.$$

The idiosyncratic volatility of stock returns caused by Brownian shocks is

$$\sigma^{B,id}_{R,i} = \frac{\partial \ln S_{i,t-}}{\partial \ln X_t}\sigma^{id}_{X,i},\ i \in \{1,2\}$$

The systematic volatility of stock returns caused by Brownian shocks is

$$\sigma^{B,s}_{R,i} = \frac{\partial \ln S_{i,t-}}{\partial \ln X_t}\sigma^{s}_{X,i},\ i \in \{1,2\},$$

where

$$\frac{\partial \ln S_{i,t-}}{\partial \ln X_t} = \frac{\frac{A_i(X_t)}{X_t} + \sum_{j=1}^{2} q'_{D,ij}\left[(1-\eta)\frac{c}{r_{P,j}} - A_j\left(X_{D,j}\right)\right]}{S_{i,t-}/X_t},\ i \in \{1,2\}$$

is the elasticity of levered equity with respect to earnings.

Therefore,

$$-E_t\left[dR\frac{d\pi}{\pi}\middle|\nu_{t-} = i\right] = \left\{\gamma\rho_{XC,i}\sigma^{B,s}_{R,i}\sigma_C - \sigma^{P}_{R,i}\left(\omega_i - 1\right)\lambda_i\right\}dt,$$

and because the levered equity risk premium is given by

$$E_t\left[dR - rdt|\,\nu_{t-} = i\right] = -E_t\left[dR\frac{d\pi}{\pi}\middle|\nu_{t-} = i\right],$$

we obtain (4.28). Overall levered stock return volatility in state i is given by combining the variances from Brownian and Poission shocks given by

$$\sigma_{R,i} = \sqrt{\left(\sigma^{B,id}_{R,i}\right)^2 + \left(\sigma^{B,s}_{R,i}\right)^2 + \lambda_i\left(\sigma^{P}_{R,i}\right)^2},\ i \in \{1,2\},$$