ESSAYS IN HOUSING AND MACROECONOMY

by

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Abstract

Compared to the previous twenty years, residential investments in the US appear more stable after the mid-1980s. Chapter 2 explores key hypotheses regarding the underlying causes. In particular, it uses estimated DSGE models to examine whether a more responsive interest rate policy stabilizes the housing market by keeping inflation in check. These estimations indeed found a policy that has become more responsive over time. Counter-factual analysis confirms that the change stabilizes inflation as well as nominal interest rate. It does not, however, find the change in policy to have stabilizing effect on real economic activity including housing investment. It finds that smaller TFP shocks make modest contributions, while the biggest contributing factor to the fall in the housing volatility is a reduction in the sensitivity of the investment to demand variations.

Chapter 3 constructs a richly specified model for the housing market to examine the empirical relevance of various costs and frictions, including the investment adjustment cost, sticky construction costs, search frictions, and sluggish adjustment of house prices. Using the US national-level quarterly data from 1985 and 2007, we find that the gradual adjustment of house prices is the most important and irreplaceable feature of the model. The key to developing an optimization-based empirical housing model, therefore, is to provide a structural interpretation for the slow adjustment in house prices.

Chapter 4 uses US national-level time series of residential investment, price index of new houses, consumption and interest rate to explore whether the US, as a nation, experienced a drop in the price elasticity of supply of new housing. Maximum likelihood estimations with a simple stock-and-flow model found a statistically significant drop of the elasticity from 10 to 2.2, when the quarterly data between 1971 and 2007 are split at 1985. A richer model with mechanisms of gradual adjustment also indicates such a reduction, when existing knowledge about the adjustment parameters is incorporated in the analysis. For the Federal Reserve, an inelastic supply can be a source of concern, because policy-driven demand in housing market is more likely to trigger undesirable swings in prices.

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Dedication

To my mum, my brother, my father, my extended family. To D. Deng.

Chapter 1

Introduction

This dissertation explores housing market and its relation with the broader economy. The second and the fourth chapters focus on changes in the dynamic pattern of the US residential investments after the mid-1980s, and their policy implications. The third chapter augments a housing model with several features, and examines their empirical relevance. Chapter 3 bridges the gap between chapter 2, which uses a simple stock-flow model for housing investment, and chapter 4, which uses a relatively richer model for the same investment.

Chapter 2 looks at the reduction in the volatility of residential investments in the United States since the mid-1980s. The chapter provides a quantitative examination of underlying causes in the framework of a dynamic stochastic general equilibrium (DSGE) model. The objective is twofold: 1) to evaluate the hypotheses based on improvements in monetary policy; and 2) to detect and evaluate other contributing factors. The analysis does not find evidence that the estimated change in monetary policy contributed to stabilizing housing investments. The reason is that the estimated DSGE model does not have a strong link between nominal volatility and real volatility. We find that smaller TFP shocks make modest contributions. The biggest contributing factor is a reduction in the sensitivity of the investments to demand variations. The reduced sensitivity may be an indication of structural changes in the housing market. Chapter 4 explores one particular hypothesis: a reduction in supply elasticity of new housing.

Chapter 3 is motivated by the fact that the standard stock-flow model for housing investment, which chapter 2 uses, performs poorly in keeping track of the housing data and capturing their stylized patterns. This chapter augments the basic model with several costs and frictions, and examines the empirical relevance the newly added features. The new features come from both housing and macro literature. They include the investment and capital adjustment costs, sticky construction costs, and sluggish adjustment of house prices. Using quarterly US data between 1985 and 2007, and multiple empirical criterions, the chapter finds that the slow adjustment of house prices is the most important and irreplaceable feature of the model. The

chapter concludes that the key to developing an optimization-based empirical housing model is to provide a structural interpretation for the gradual adjustment in the house price.

Chapter 4 observes that the reduction in the volatility of housing investment in US since the mid-1980s is accompanied by a relative increase in the variability of prices. The relative increase appears at the high-frequency quarter-to-quarter changes, as well as at the low-frequency swing over housing cycles. The chapter sets out to examine if the supply of new houses in US, at the national level, has become less elastic. Using maximum likelihood estimations and time series of housing investment, price, consumption and interest rate, the chapter finds evidence to suggest a significant reduction of the elasticity of new supply in a simple stock-flow model. A richer model with mechanisms of gradual adjustment also suggests such a reduction, when knowledge about the parameter of gradual adjustments is incorporated in the estimations. An inelastic supply calls for greater attention to the housing market from the Federal Reserve when conducting its interest rate policy. With inelastic supply, fluctuations in housing demand driven by policy changes are more likely to trigger undesirable swings in house prices.

Chapter 2

Changes in Business Cycle Dynamics of Residential Investment in the U.S.: an Investigation with Estimated DSGE Models

2.1 Introduction

Since the mid-1980s, U.S. housing-related economic activity appears more stable than it had been in the preceding two decades. The standard deviation of the quarterly growth of residential fixed investment in the national income and product accounts (NIPA) was 6% between 1962 and 1985, and 2.4% between 1986 and 2006. The variability of other measures of housing investment, such as housing starts and issuance of housing permits, have also become smaller.[1]

Residential investment also appears less pro-cyclical. Historically, housing investment tends to fall before recessions and picks up during recoveries. The tendency of positive co-movement has been weaker in recent years. In the 1990 economic recession, the housing retrenchment was modest by historical standards. In the 2001 recession, the growth of residential investment was largely undisturbed.

What has caused these changes is not well understood. Many aspects of the US economy are different now than they were in the 1960s and 1970s. Some of these changes are likely to affect residential investments' business cycle properties.

One indication of the shifting economic landscape is the so-called "Great Moderation," or widespread reduction in macroeconomic volatility, that be-

[1] The reduction in variance is also found at the state level in housing permit data.

gan in the early-1980s. This included a 50% drop in the standard deviation of the quarterly output growth, a 2/3 reduction in the standard deviation of inflation, and smaller measures for volatility in almost all major NIPA aggregates (Table 3, Stock and Watson (2002)).

What produced the great moderation, and did it also affect residential investment? Three explanations have been suggested for the cause(s) of the great moderation. The first explanation is the smaller economic shocks, including less frequent disruptions of oil supply and smaller variance in Solow residuals, an empirical measure of productivity growth. The second explanation focuses on the apparent changes in Federal Reserve's monetary policy. The Federal Reserve is believed to have taken a more aggressive stand in fighting inflation since the early-1980s, and have succeeded in bringing down inflation and its variability. Because inflation volatility and real volatility tend to move together in the US and in other industrialized countries (Blanchard and Simon (2001)), the policy change is regarded as a likely factor behind the great moderation.[2] Finally, the third explanation focuses on structural changes in the US economy.[3]

For residential investment, both smaller economic shocks and the change in monetary policy are likely to have impacts. Oil shocks and productivity shocks affect all sectors. A reduction in their frequency and magnitude can reduce variance in housing market as well as the correlation across sectors. An improvement in the monetary policy, according to a conjecture in Taylor (2007), keeps inflation low and steady, thereby reducing boom-bust cycles and large swings in interest rates that can cause fluctuations in housing market. The monetary policy improvement, therefore, not only accounts for, but also ties together, the great moderation and the housing moderation. Empirically, residential investment is very sensitive to monetary policy shocks (Figure 3 in Bernanke and Gertler (1995a)). Such sensitivity highlights the importance to examine how the policy change have affected the housing sector.

Sector-specific structural changes in the housing market may also have played an important role. McCarthy and Peach (2002) describe the transformation of the US mortgage market in the past two decades with its banking

[2]See discussion in Bernanke (2004). Clarida *et al.* (2000) developed theoretical models that relate nominal volatility to real volatility.

[3]In the literature, structural changes that are suggested to be possible causes of the great moderation include improved management of business inventories, increased depth and sophistication of financial markets, deregulation, a shift away from manufacturing toward services, and increased openness to trade and international capital flows (see discussion in Bernanke (2004))

deregulation, mortgage securitization, and financial innovations, all of which have the potential to improve the resilience of housing financing system.

Important changes also occurred in the construction industry. The fall of unionism is one such example. The share of union in the construction workforce has dropped from 70% in the 1970s to 20% in the 1990s (see Allmon *et al.* (2000), also Thieblot (2001)). Unions in the industry were once known for their strong "hiring hall" practice, which often meant fixed compensation and workers queueing for job opportunities (see Abowd and Farber (1982) for their characterization of unions' hiring hall[4]). With fixed compensation, wages were unresponsive in the short-run, leaving employment and hours worked to do most of the adjustments. A reduction in the presence of union may lead to a more balanced response of wages and employments.

Finally, residential builders' business practices have also changed. The share of homes that are sold before construction starts has increased from 19% before 1981 to 36% in 1999 (Kahn (2000)). With a smaller share of speculative construction, builders can reduce inventory buildup in downturns, and therefore avoid slowing down too much compared to what they otherwise would have to.

This paper is an effort to quantitatively sort out the importance of competing explanations for the lower volatility of residential investment. It uses a structural macroeconomic model to explore the consequence of the change in monetary policy, and for other structural interpretations. Its choice of macro model belongs to the class of sticky-price DSGE models (or new neoclassical synthesis models.) This class of models combines Real Business Cycle models and New Keynesian features, thus admitting simultaneously the explanations based on economic shocks and those based on monetary policy. Because of the advantage, these models are popular in the literature that tries to understand the great moderation (for example Justiniano and Primiceri (2006), Leduc and Sill (2007), and Stock and Watson (2002)).

To capture possible changes that are specific to the housing sector, this paper uses a model fashioned along the line of Christiano *et al.* (2005) and Smets and Wouters (2003). This particular string of models employs rich specifications of nominal and real rigidity, whose parameters are important determinants of the dynamics of endogenous variables. A change in these rigidity parameters, if detected, is indicative of potential structural changes in the relevant sector. For housing sector, the key parameters to watch are

[4] Abowd and Farber (1982) does not focus on construction union. It studies the practice of union firing hall in general. Construction industry is one of their examples for strong practice of hiring hall.

those that govern the adjustment mechanism of the investment.

This paper follows a two-step approach for quantification analysis. In the first step, it estimates the model using split-samples to detect changes in parameters of the model. In the second step it performs counter-factual analysis based on the estimated changes. The estimation follows a Bayesian approach and uses Random Walk Metropolis algorithm to provide measures of statistical confidence. An and Schorfheide (2006) reviews the methodology in details. The appendix of this paper presents a brief description.

Existing literature has seen few effort to explain the observed change in residential investments using a structural business cycle model. One notable exception is Campbell and Hercowitz (2005). They construct a borrower-lender model in which credit-constrained households adjust labor supply to satisfy downpayment requirement when purchasing durable goods. They show that an easing of the borrowing constraint in the 1980s can explain a substantial fraction of the decline in the volatility of hours worked, GDP, and households' investments in durable goods. Their analysis is based on calibration and does not explore alternative explanations. Our paper is different in our use of estimation-based quantitative analysis to compare different hypotheses.

This paper is related to a growing literature that studies the interaction between the housing market and the broader economy in a DSGE model. Examples include Aoki *et al.* (2004), Iacoviello (2005a), Gammoudi and Mendesy (2005), and Pescatori and Mendicino (2005).

The organization of the paper is as follows: Section 2 documents the changes in the dynamic pattern of residential investment. It also presents evidence for smaller economic shocks and discusses the change in monetary policy and its implications for residential investment. Section 3 presents the linearized version of the model (The original model is in Appendix A). Section 4 provides an overview of the estimation strategy before presenting the findings, and Section 5 concludes.

2.2 Residential Investment in Business Cycles: Changes

The residential investment spending, which the paper focuses on, is from BEA's national income and product accounts (NIPAs). It consists mostly of homeowners' investments in private residential structures and landlord's investments in residential structures and equipment. It is about 5% of the total GDP. Even though the share of the investment is small, its volatility

is quite large, especially in the 1970s and the early 80s. The investment could swing, in the matter of a few quarters, by 2-3% of the GDP (Figure 2.2). This definition of housing investment does not include the purchase of consumer durables, which often comes with the purchase of residence units. When the value of consumer durable are included, the share of housing investment is larger than business sector's fixed investment in structure and equipment.

The fourth panel of Figure 2.1 plots the quarterly growth of residential fixed investment between 1963 and 2006. The period before the mid-1980s was significantly more volatile than the more recent period. The difference is visually apparent.

An alternative measure of residential investment is the series of housing starts published in monthly frequency by the Census Bureau. Figure 2.3 plots the % change of housing starts from the same month one year ago. The observation is similar to those from Figures 2.1, that the series was more volatile before the mid-1980s.

Figure 2.2 illustrates the weakened correlation between residential investments and business cycles. It plots residential investments as a share of potential GDP, together with the output gap from 1963Q2 to 2007Q1. For most of the sample, between the late-1960s and early-1990s, residential investment exhibits clear peaks and troughs, largely synchronized with those of the output gap, with residential investment usually leading the cycle. In the more recent period, movements of residential investment and output appear more detached from each other. The 2001 recession sent the GDP into prolonged stagnation. Residential investment, on the other hand, continued to grow steadily.

The lower panel of Table 2.1 shows the decline in the correlation coefficients between the growth of residential investment and the growth of total output. The growths are defined as % change from the same quarter one year ago. The sample is split into two groups: one from 1963Q1 to 1985Q4, and the other from 1986Q1 to 2006Q2. Four correlation coefficients are presented for each of the two sub-samples. These four coefficients measure the correlation between the growth of residential investment and the growth of GDP in the same-quarter, one-quarter-ahead, two-quarter-ahead and three-quarter-ahead, respectively. All four measures are lower in the more recent sample. For example, the correlation between housing investment growth and the three-quarter-ahead GDP growth is 0.71 in the pre-1986 sample, and 0.48 in the post-1986 sample.

Another piece of evidence that suggests a weaker correlation comes from the mortgage market. Figure 2.4 plots the mortgage flow (defined as the net

increase of household home mortgage debt outstanding) as a share of potential GDP, again with the output gap in the same panel. The observations are similar to those for the residential investments. The ups and downs of the mortgage flow largely match the output gap between the early-1970s and the late-1980s. Nevertheless, in 2001, they rose more quickly as the economy fell into recession and stagnation. The level of mortgage flow reached an unprecedented level in 2006 before falling in the later part of 2006 and in 2007. Fisher and Quayyum (2006) discuss what they call the "great turn-of-the-century" housing boom, attributing part of it to the development of a sub-prime mortgage market.

In the literature, Stock and Watson (2002) observed that the housing sector had the largest relative decline in volatility among major NIPA aggregates. Campbell and Hercowitz (2005) documented that household debt, including home mortgages, became less correlated with hours worked. Dynan and Sichel (2006) found that residential investment became less sensitive to changes in interest rates. McCarthy and Peach (2002) found that the investment's response to monetary policy shock became smoother over time.

Great Moderation: widespread reduction in output volatility

The reduction in the volatility of housing investment occurred at roughly the same period as the so-called great moderation. The great moderation is "one of the most striking features of the economic landscape over the past twenty years." (Bernanke (2004)) Figure 2.1, besides presenting the time series of housing investment, also plots the time series of GDP, consumption, and business investment, together with the federal funds rate and inflation. All, except for the business-fixed investment series, experienced a large decline in their volatility after the mid-1980s. Regarding the timing, Kim and Nelson (1999) estimated that 1984Q1 was the most likely date for a structural break. Besides the decline in variance, Stock and Watson (2002) found that GDP growth was easier to forecast, and that the decline in volatility was widespread across sub-components of output. Among the main NIPA aggregates that they reviewed, 21 of 22 had lower standard deviations in the post-1984 period (Table 3, in Stock and Watson (2002)). The decline in standard deviation is not uniform across the series. The largest relative decline in volatility happened with residential investments.

Smaller economic shocks

Several papers attribute the great moderation primarily to smaller oil price and productivity shocks.[5]

The first evidence of smaller shocks is seen in the oils shocks that become less frequent. The following information is from Table 4 of Hamilton (2003). It documents the military conflicts and revolutions that have disrupted the worlds' oil supply.

Date	Event	Drop in world production
Nov. 1973	Arab-Israeli War	7.8%
Dec. 1978	Iranian Revolution	8.9%
Oct. 1980	Iran-Iraq War	7.2%
Aug. 1990	Persian Gulf War	8.8%

In the seven years between 1973 and 1980, three major disruptions occurred, while after 1980, only one disruption took place.

Smaller TFP shocks were also reported. Leduc and Sill (2007) used input, output, and oil price information to measure the historical productivity growth in the US economy. They fed the measured productivity growth into a macro model and found that the change associated with the series explains most of the reduction in the variance of output growth.

Changes in monetary policy

Explanations based on the change in monetary policy are motivated by the view that the Federal Reserve pursued a more aggressive monetary policy in fighting inflation after Volcker became the chairman of the Federal Reserve system in 1979, and the policy change succeeded in bringing down inflation and its variability. The change in policy probably came from a better understanding of the expectational nature of the unemployment-inflation trade-off.

Clarida *et al.* (2000) find "striking difference" in the policy reaction function across time. In particular, they find that the response to inflation increased significantly from the pre-Volcker period to the Volcker-Greenspan era. They theorize that the policy change had a stabilizing effect in real activity by mitigating the effect of fundamental shocks and preventing self-fulfilling "sun-spot" shocks. The hypothesis receives significant attention in the literature. Many researchers confirm that the monetary policy became more responsive to inflation (For example, Ireland (2004), Boivin and Giannoni (2006), and Lubik and Schorfheide (2004)).

[5] For example Stock and Watson (2002), and Leduc and Sill (2007).

More controversial, however, is whether or not the change in monetary policy reduces real volatility. Stock and Watson (2002) and Leduc and Sill (2007) compared the shocks-base explanation and the policy-based explanation in quantitative models and attributed most of the great moderation to smaller shocks.

This paper will contribute to the discussion by including housing investment in a DSGE model. The inclusion of residential investment in the macro monetary model expands the investigation on the change in monetary policy. Empirically speaking, residential investment is a spending component that is most sensitive to monetary policy shocks. The investment drops quickly and sharply following a monetary tightening. The sensitivity appears to be greater than the business investment, and the structure investment in the business sector.[6] The latter observation is particularly interesting since residential investment and business structure investment both involve long-lived assets with similar characteristics (Edge *et al.* (2004)).

One explanation that was put forward by Edge *et al.* (2004) to explain the difference between residential investment and structure investment is that the former has a shorter project planning and completion time. Their calibrated business cycle model achieves some success in replicating the difference. Based on this hypothesis, however, it is not clear why residential investment has now shown a smoother response to monetary policy shocks (a finding in McCarthy and Peach (2002)). Has the planning and completion horizon of the investment become longer?

Another explanation for the high sensitivity of residential investment to monetary policy actions involves the possible existence of "banks-lending channel." The banks-lending view hypothesizes that when a monetary authority uses open market operations to implement monetary policy, it withdraws or injects reserve funds into the banking system. Because a reserve fund is needed by banks to satisfy reserve requirements, the fund availability affects the banks' ability to sell deposits and therefore their ability to make loans. An implication thus is that bank-dependent investors suffer more from monetary contraction than do those who have alternative sources of funds. As far as residential investment is concerned, households are the most bank-dependent, especially when the US mortgage market was dominated by the savings institutions in the 1960s and 1970s. The banks-lending channel may therefore explain the investment's high policy sensitivity.[7] Be-

[6] Housing investments' initial response in % terms is larger than that of business investments (Figure 3, Bernanke and Gertler (1995a)).

[7] Iacoviello and Minetti (2002) have discussions on banks-lending channel specific to the housing sector.

cause the strength of the channel depends on the tightness of constraints that are facing the banks, a relaxation of the reserve requirement reduces the channel's significance. Since the Federal Reserve lowered or eliminated many reserve requirements, this might explain why investment now exhibits a smoother response to monetary policy shocks.

Even if we take the policy sensitivity as given, how the change in monetary policy will affect the housing market is unclear. An aggressive monetary tightening, as a response to inflation, can send residential investment into a tailspin. An easing of monetary policy, for fear of deflation, can send housing market soaring. It is therefore possible that a more responsive policy makes the housing market more volatile. On the other hand, a more responsive policy can works through the channel of expectation, achieving stabilizing effect with better anchored inflation expectations. In other word, it can avoid boom-bust cycles in the economy and therefore the need for aggressive policy actions in the first place. It is the second channel that Taylor (2007) emphasizes when he hypothesizes the link between the change in policy and the volatility of the housing market.

Beside the indirect effect described above, the stabilization of inflation may have direct benefits in the housing sector as well. The sector has much to gain from stable inflation because it often uses long-term fixed-rate mortgages as a means of finance, which exposes it to inflation risks. Rudebusch and Wu (2004) construct a macro-finance model that interprets the latent factors of term structure as macroeconomic variables. One of the macro variables is the perception of investors about the inflation target, which is closely identified with the behavior of monetary policy. A policy that has a credible inflation target most likely will have a stabilizing effect on the long-maturity end of the yield curve, which, in turn, can make the decision on housing investment easier and smoother, since the incentive to time the cost will be reduced.

2.3 The Model Economy

This paper uses an DSGE model to tie together macroeconomic variables, residential investments, economic shocks, and monetary policy. The model jointly specifies the structure of the economy, the monetary policy reaction function, and the property of exogenous shocks.

The model involves a substantial number of parameters. For this reason, the estimations and statistical inferences are performed with the Bayesian approach and the Markov Chain Monte Carlo algorithms. The estimated

model in this paper achieves good empirical performance. For example, it substantially outperforms an unrestricted VAR in out-of-sample forecasts.

The theoretical model abstracts from long-term growth by assuming zero growth rate for population and productivity in the long run. Reflecting this imposed stationarity in the theoretical model, we detrend the time series of macroeconomic variables before using them to estimate the model. These macroeconomic variables are the total output, consumption, business investment, housing investment, inflation, and the nominal interest rate.

2.3.1 Households

Households' preference is identical to that in Smets and Wouters (2007) with one exception, that the definition of consumption in this paper is expanded to include the consumption of housing services. Households maximize expected life-time utility

$$\max_{(c_t, h_{t+1}, l_t, B_{t+1})} E_t \sum_{s=0}^{\infty} \Lambda_{t+s} \frac{(c_{b,t+s} - \lambda_b c_{b,t+s-1})^{1-\sigma_c}}{1-\sigma_c} \exp\left\{ \frac{\sigma_c - 1}{1+\sigma_l} l_{t+s}^{1+\sigma_l} \right\}$$

In the utility function, l_t indicates hours worked, and $c_{b,t}$ is the CES mixture of nondurable goods (c_t) and housing service (h_t). The consumption bundle is defined as

$$c_{b,t} = \left[c_t^{1-\gamma} + v_h h_t^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \tag{2.1}$$

The parameter γ ($0 < \gamma < \infty$) is the inverse of the elasticity of substitution. When $\gamma = 0$, nondurable goods c_t and housing service h_t are perfectly substitutable. As γ approaches ∞, the substitutability becomes zero, and we end up with the Leontief preference. The parameter v_h is one of the factors that determine the split between housing services and non-durable goods in households' total consumption expenditure. This model defines the unit of housing service by normalizing that one unit of housing capital provides one unit of service. Therefore h_t is also the stock of housing capital.[8]

The momentary utility function also features consumption habit formation. The external habit is captured by the parameter λ_b, which is between zero and one. When λ_b is zero, utility depends on the level of consumption.

[8] Households' utility function should have also included a preference for real monetary balance. In the equilibrium, however, the demand for money is met by monetary authority, who chooses an interest rate target and supply as much amount of money as necessary to achieve the target. As the result there is no need to model money in utility explicitly, as long as the preference for real monetary balance is separable to others arguments.

When λ_b is one, utility depends on changes in the consumption. When λ_b is between zero and one, both the level and the change affect utility.

The term $\Lambda_{c,t}$ indicates the weight that households assign to the utility in period t. As the result, the discount factor between two periods is $\frac{\Lambda_{c,t+s}}{\Lambda_{c,t}}$. The dynamics of $\Lambda_{c,t}$ are captured by

$$\frac{\Lambda_{c,t+1}}{\Lambda_{c,t}} = \beta e^{z_{\chi c,t}}$$

A positive shock to $z_{\chi c}$ at time t raises households' preference on current-period utility over that in the future. For this reason, the stochastic component in $z_{\chi c,t}$ is interpreted as consumption preference shocks.

Households' budget constraint is:

$$\begin{aligned} w_t l_t + \frac{B_{t-1}}{P_t} + \pi_{h,t} \quad \geq \quad & \frac{T_t}{P_t} + c_t + \frac{B_t}{1+R_t}\frac{1}{P_t} \\ & + q_{h,t} e^{z_{mh,t}} \left[h_{t+1} - (1-\delta_h) h_t \right] - m_h q_{h,t} e^{z_{mh,t}} h_t \end{aligned}$$

The left hand side of the budget constraint sums together the sources of revenues in period t. The first term, or $w_t l_t$, is the employment income expressed in real terms. The second term, or $\frac{B_{t-1}}{P_t}$, is the real value of bonds reaching maturity in period t (since the model has one-period bonds only, all bonds purchased in period $t-1$ mature in period t). The last term, or $\pi_{h,t}$, is the total amount of profit distributed to households by businesses in the economy.

The right hand side of the budget constraint is the expenditure lay-out. The first term, $\frac{T_t}{P_t}$, is the lump-sum tax collected by the government. The second term, c_t, is the consumption expenditure. In the third term, B_t is the face value of discount bonds purchased in period t. The capital-cased R_t is the discount rate. Therefore $\frac{B_t}{1+R_t}\frac{1}{P_t}$ is the real investment expenditure in bonds. The fourth term is the total cost incurred to the households for expanding their stock of housing capital from h_t to h_{t+1}. The total per unit cost of the capital is $q_{h,t}e^{z_{mh,t}}$, with $q_{h,t}$ being the unit price, and $z_{mh,t}$ (which has a mean of zero) indicating the extra costs. This extra cost may include transaction costs and mortgage costs (to the extent the mortgage rate is different from the cost of capital), among others. Since $z_{mh,t}$ affects demand for housing capital, it is interpreted as housing demand shocks. The last term is a property tax m_h levied on the value of the housing capital. The tax rate affects the spread between the return from home ownership and the return from alternative investment in bonds. Technically, this spread gives us the degree of freedom that is necessary to calibrate the model according to

the observed data. More specifically, we can choose m_h so that the steady-state solution to the model replicates the share of housing rents (including the market rent and the imputed rent) in total output, which is reported in the NIPA. The steady state solution is presented later in the paper.

The consumption Euler is

$$\Xi_t = \beta e^{z_{\chi c,t}} E_t \left[(1 + r_{t+1}) \Xi_{t+1} \right] \tag{2.2}$$

In this Euler equation, the ex-post real interest rate is defined as the difference between the pre-set discount rate and the realized inflation between t and $t+1$, i.e,

$$1 + r_{t+1} = (1 + R_t) \frac{P_t}{P_{t+1}} \tag{2.3}$$

The marginal utility of consumption, in the absence of preference shocks, is denoted as Ξ_t,

$$\Xi_t = (c_{b,t} - \lambda_b c_{b,t-1})^{-\sigma_c} c_{b,t}^{\gamma} c_t^{-\gamma} \exp\left\{ \frac{\sigma_c - 1}{1 + \sigma_l} l_t^{1+\sigma_l} \right\}$$

The optimal condition for hours worked, whose derivation is shown in sub-section (A.1.1), is captured by

$$(c_{b,t} - \lambda_b c_{b,t-1}) c_{b,t}^{-\gamma} c_t^{\gamma} l_t^{\sigma_l} = w_t \tag{2.4}$$

It is helpful, especially when solving the steady state of the model, to note that equ(2.4) has an alternative expression $\left(1 - \lambda_b \frac{c_{b,t-1}}{c_{b,t}}\right) e_t l_t^{\sigma_l} = w_t$, where $e_t = c_{b,t}^{1-\gamma} c_t^{\gamma}$ is the total consumption expenditure including rent. To see why e_t is the total consumption expenditure, we simply need to use the definition of $c_{b,t}$ (i.e., equ(2.1)) to expand $c_{b,t}^{1-\gamma} c_t^{\gamma}$ into $c_t + v_h \left(\frac{c_t}{h_t}\right)^{\gamma} h_t$, and realize that it is the sum of nondurable consumption and the value of housing services expressed in units of nondurable goods. The value of housing services is $v_h \left(\frac{c_t}{h_t}\right)^{\gamma} h_t$, where h_t is the amount of housing services provided by h_t units of housing capital, and $v_h \left(\frac{c_t}{h_t}\right)^{\gamma}$ is the intratemporal marginal rate of substitution between consumption and housing services, or the relative price of housing services in units of nondurable goods.

The investment Euler on housing capital is

$$e^{z_{mh,t}} q_{h,t} \Xi_t = \beta e^{z_{\chi c,t}} E_t \left\{ \left[v_h \left(\frac{c_{t+1}}{h_{t+1}} \right)^{\gamma} + (1 - \delta_h - m_h) e^{z_{mh,t+1}} q_{h,t+1} \right] \Xi_{t+1} \right\} \tag{2.5}$$

2.3.2 Producers of final goods

To model nominal rigidity, this paper uses a framework that involves three parties in the production process of final goods: competitive producers of generic goods, monopolistically-competative retailers, and competitive producers of final goods. The setup is similar to that in Bernanke *et al.* (1999). The producers of generic goods sell their output to retailers, who package them into differentiated products and re-sell them at a markup. The producers of final goods combine differentiated products into final goods, with a technology that features imperfect substitutability.

Let's start with the producers of final goods. The profit maximization problem is

$$\max_{y_{i,t}} P_t y_t - \sum_i P_{i,t} y_{i,t}$$

subject to

$$y_t = \left(\sum_i (y_{i,t})^{\frac{1}{\lambda_f}} \right)^{\lambda_f}$$

In this problem, the variable y_t is the output in term of final goods. The variable $y_{i,t}$ is the input purchased from retailer i. The parameter $1 < \lambda_f < \infty$ indicates the difficulty of substitution. The price of product i is $P_{i,t}$. The aggregate level of price is P.

The optimal demand for product i is

$$\frac{y_{i,t}}{y_t} = \left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda_f}{\lambda_f - 1}}$$

The price of the final goods can be derived through the following relation

$$P_t = \sum_i P_{i,t} \frac{y_{i,t}}{y_t} = \left(\sum_i P_{i,t}^{\frac{1}{1-\lambda_f}} \right)^{1-\lambda_f}$$

2.3.3 Retailers

Retailer i can transform one unit of generic goods into one unit of intermediate goods, without incurring any cost. The retailer has monopolistic power over the supply of its product. Individual retailers face a Calvo-type "stickiness" in price setting. In each period, there is a chance ξ_p that a retailer will have to stick with its previously-set price. The chance of being stuck is random and history-independent. At the aggregate level, $(1 - \xi_p)$

proportion of retailers can change their prices if they want to. The remaining proportion ξ_p index their prices according to lagged inflation. Inflation indexation automatically adjusts the price from $p_{i,t-1}$ to $p_{i,t-1}\pi_{t-1}^{\iota_p}\pi_*^{1-\iota_p}$, where π_* is a steady-state inflation.

Retailers who have the option to change their prices face a profit maximization problem. Note that in the following representation of the problem, the subscript i is omitted, because all retailers will end up choosing the same new price. The profit maximization problem is

$$\max_{\widetilde{P}_t} E_t \sum_{s=0}^{\infty} (\xi_p)^s \frac{\Lambda_{c,t+s}\Xi_{t+s}}{\Lambda_{c,t}\Xi_t} \frac{1}{P_{t+s}} \pi_{t+s}^R$$

where the profit in period $t+s$ is

$$\pi_{t+s}^R = \left(\widetilde{P}_t \Pi_{l=1}^s \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p} - \frac{P_{t+s}}{x_{t+s}}\right) * \left(\frac{P_t}{\widetilde{P}_t \Pi_{l=1}^s \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p}}\right)^{\frac{\lambda_f}{\lambda_f - 1}} y_{t+s}$$

In the function that describe π_{t+s}^R, the first term is per-unit profit, and second term is the sales volume. The variable $\widetilde{P}_t$ is the new price to be chosen. The variable x_{t+s} is defined as the ratio of the aggregate price P_{t+s} over the price of generic goods $P_{g,t+s}$, or

$$x_{t+s} \equiv \frac{P_{t+s}}{P_{g,t+s}}$$

From the definition of x, it is clear that the price of input (i.e., generic goods) in period $t+s$ is $\frac{P_{t+s}}{x_{t+s}}$. The price of output in the same period, assuming that the retailer can not re-adjust its price since, is $\widetilde{P}_t \Pi_{l=1}^s \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p}$. The per-unit profit is the difference between the price of output and the price of input , or $\widetilde{P}_t \Pi_{l=1}^s \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p} - \frac{P_{t+s}}{x_{t+s}}$. The sale volume is determined by the demand from the final goods sector, or $\left(\frac{P_t}{\widetilde{P}_t \Pi_{l=1}^s \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p}}\right)^{\frac{\lambda_f}{\lambda_f - 1}} y_{t+s}$. The total profit in the period is the product of per-unit profit and the sales volume. The profit is then divided by P_{t+s} so it is expressed in real terms. Real profit is then discounted into the present value using $\frac{\Lambda_{c,t+s}\Xi_{t+s}}{\Lambda_{c,t}\Xi_t}$, which is the same as the discount factor used by households. The coincidence of discount factors comes from the assumption that households are the ultimate owner of retailers and their profit. Finally, $(\xi_p)^s$ is the probability that the retailer is stuck with $\widetilde{P}_t$ after s periods.

The first order condition satisfies (see sub-section(A.1.2) for the derivation)

$$\frac{\widetilde{P}_t}{P_t} = \lambda_f \frac{\sum_{s=0}^{\infty} \xi_p^s E_t \frac{\Lambda_{t+s}\Xi_{t+s}}{\Lambda_t \Xi_t} \left(\frac{P_{t+s}}{P_t G_{t+s}}\right)^{\frac{\lambda_f}{\lambda_f - 1}} y_{t+s} \frac{1}{x_{t+s}}}{\sum_{s=0}^{\infty} \xi_p^s E_t \frac{\Lambda_{t+s}\Xi_{t+s}}{\Lambda_t \Xi_t} \left(\frac{P_{t+s}}{P_t G_{t+s}}\right)^{\frac{\lambda_f}{\lambda_f - 1} - 1} y_{t+s}} \tag{2.6}$$

where $G_{t+s} = \Pi_{l=1}^{s} \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p}$.

The aggregate price P_t is the weighted average of newly-set prices and the past price.

$$P_t = \left[\xi_p \left(P_{t-1} \pi_{t-1}^{\iota_p} \pi_*^{1-\iota_p}\right)^{\frac{1}{1-\lambda_f}} + (1 - \xi_p) \widetilde{P}_t^{\frac{1}{1-\lambda_f}}\right]^{1-\lambda_f} \tag{2.7}$$

The linearized version of equ(2.6) and equ(2.7) gives rise to a forward looking Phillips curve with back-ward looking indexation term, or equ(2.37). Sub-section(A.1.3) shows the linearization process.

2.3.4 Producers of generic goods

Producers of generic goods use a Cobb-Douglas production technology with two inputs, capital service (k_t) and labor service (l_t). These producers own the capital and employ workers for their labor service. They take the stock of capital in the current period as given, but they have the option to increase or reduce capacity utilization (u_t) of the existing stock. Higher utilization incurs a higher cost (such as faster depreciation). The output, in the form of generic goods, is sold to retailers at a relative price of $\frac{1}{x_t}$ (or $\frac{P_t}{x_t}$ in nominal terms). Since these producers own the capital stock, they are responsible for investing in the stock. There is a market for new capital goods where additional capital can be purchased. The installation of new capital is costless, but the production of these goods features increasing marginal cost in the short run. Finally, these generic-goods producers are assumed to operate in a competitive environment.

The profit maximization problem is

$$\max_{(y_t, u_t, l_t, k_{t+1})} \frac{1}{P_t} E_t \sum_{s=0}^{\infty} \frac{P_{t+s} \pi_{t+s}^g}{\Pi_{l=0}^{s} (1 + R_{t+l-1} + m_k)}$$

where the variable π^g is the dividend payout, which is defined as the total revenue after the deduction of wage, capacity utilization cost, and invest-

ment cost in the capital stock, or

$$\begin{aligned}\pi_t &= \frac{1}{x_t}y_t - w_t l_t - \Phi_u(u_t)\,k_t \\ &\quad -q_{k,t}\left[k_{t+1} - (1-\delta)\,k_t\right]\end{aligned}$$

The production technology is Cobb-Douglas,

$$y_t = e^{z_{a,t}}\left(u_t k_t\right)^{\alpha} l_t^{1-\alpha} \tag{2.8}$$

The random term z_{at} in the production function is the total factor productivity (TFP).

Finally, in the objective function of the firms, there is a extra spread m_k in the discount factor. Similar to the use of m_h in households' budget constraint, this extra parameter m_k creates a necessary degree of freedom to calibrate the model's steady state to the long term averages observed in the national account. More specifically, one can choose m_k so that the model's steady state replicates the observed averages of the investment to output ratio, for any given level of depreciation rate and interest rate in the steady state. The literature, such as Greenwood and Hercowitz (1991), some times uses capital-income tax for similar purpose.

The optimality condition for labor demand is

$$w_t = \frac{1}{x_t}\frac{(1-\alpha)\,y_t}{l_t} \tag{2.9}$$

The optimality condition for capacity utilization is

$$\Phi_u'(u_t)\,u_t = \frac{1}{x_t}\frac{\alpha y_t}{k_t} \tag{2.10}$$

which relates the extent of utilization to the productivity of capital service. A higher productivity gives the firms the incentive to intensify the use of their capital stock. For this reason, the function $\Phi_u(.)$ is chosen such that $\Phi_u'(u_t)\,u_t$ is increasing with u_t. The utilization in the steady state is normalized to be one. It is further assumed that $\Phi_u(1) = 0$.

The investment Euler is

$$P_t q_{k,t}(1+R_t) = E_t P_{t+1}\left[\frac{1}{x_{t+1}}\alpha\frac{y_{t+1}}{k_{t+1}} - \Phi_u(u_{t+1}) + (1-\delta_k)\,q_{k,t+1}\right] \tag{2.11}$$

2.3.5 Producers of capital goods

Following the long tradition in DSGE models, it is assumed that the expansion of capital stock takes resources that are otherwise consumable. In this model, the production of new capital goods uses the consumable final goods as the input. This assumption means that the model abstracts from the likely heterogeneity, between sectors, in term of capital and labor intensity.

There is adjustment cost in the sector. The transformation technology, from final goods to capital goods, is assumed to be one for one in the steady state. So the value of capital goods is one in the long run. In the short-run, however, the price of new capital goods can deviate from unity, because of adjustment costs. There are at least two types of adjustment costs in the existing literature. One type of the cost increases with the level of investment (often normalized by the stock of capital). This is the capital adjustment cost (or CAC), which gives rise to the Tobin's Q in neo–classical models. Another type of adjustment costs increases with speed of changes in the level of investment. It is often called the investment adjustment cost (or IAC) in DSGE literature. IAC is popularized by Christiano *et al.* (2005), and is rapidly gaining popularity because of its ability to replicate the hump-shaped response of investment to economic shocks (see discussions in Smets and Wouters (2007)). The model in this paper uses a hybrid version of adjustment costs, which admits both CAC and IAC.

There are two types of capital goods in the economy: business capital goods and housing capital goods. Their production is similarly modeled. We will start with business capital, and then move to housing capital.

Producers of new business capital

For the producers of new business capital, the adjustment cost function is $\Phi_k\left(\frac{i_t}{i_{t-1}^{\varpi_k}}\right)$, where i is the amount of resource used in the production of the capital. The parameter ϖ_k has a value between zero and one. It indicates the weight between CAC and IAC. When $\varpi_k = 0$, the adjustment cost is increasing only in the level of current investment. It has nothing to do with the level of investment in the past. This is the case of pure CAC. If $\varpi_k = 1$, we have the case of pure IAC, when the adjustment cost is a determined by $\frac{i_t}{i_{t-1}}$, the level of current investment relative to the level of investment in the previous period. Finally, it is assumed that $\Phi_k\left((i^*)^{1-\varpi_k}\right) = 0$ and $\Phi_k''\left((i^*)^{1-\varpi_k}\right) = \varphi_k > 0$.

Finally, there is a stochastic component in the sector's productivity. For i_t amount of investment expenditure (in real terms) in the sector, the total amount of output (denoted by $y_{KI,t}$), is

$$y_{KI,t} = e^{z_{i,t}} i_t \left[1 - \Phi_k \left(\frac{i_t}{i_{t-1}^{\varpi_k}}\right)\right]$$

where $z_{i,t}$ is the investment-specific productivity shocks.

These producers' objective is to maximize the present value of expected profit, or

$$\max_{i_t} E_t \sum_{s=0}^{\infty} \frac{P_{t+s}}{\Pi_{l=0}^{s}\left(1 + R_{t+l-1}\right)} \left\{q_{k,t+s} y_{KI,t+s} - i_{t+s}\right\}$$

The optimality condition is

$$\begin{aligned} P_t &= P_t q_{k,t} e^{z_{i,t}} \left[1 - \Phi_k \left(\frac{i_t}{i_{t-1}^{\varpi_k}}\right) - \Phi_k' \left(\frac{i_t}{i_{t-1}^{\varpi_k}}\right) \frac{i_t}{i_{t-1}^{\varpi_k}}\right] \\ &\quad + E_t \left[\varpi_k \frac{P_{t+1} q_{k,t+1} e^{z_{i,t+1}}}{1 + R_{t+l-1} + m_k} \Phi_k' \left(\frac{i_{t+1}}{i_t^{\varpi_k}}\right) \frac{i_{t+1}^2}{i_t^{\varpi_k + 1}}\right] \end{aligned} \tag{2.12}$$

Producers of new housing capital

Producers of new housing capital also face adjustment costs in the short-run. The adjustment cost function is $\Phi_h \left(\frac{d_t}{d_{t-1}^{\varpi_h}}\right)$, where the subscript h differentiates the function from the one in the production of business capital. The weighting parameter between CAC and IAC is ϖ_h, which also has a subscript of h for differentiation. It is assumed that $\Phi_h \left((d^*)^{1-\varpi_h}\right) = 0$ and $\Phi_h'' \left((d^*)^{1-\varpi_h}\right) = \varphi_h > 0$. Finally, the amount of input in the housing sector is denoted as d_t.

This model assumes that supply shocks in the housing market is small, so that the productivity in the sector is not model as a stochastic variables. Instead the productivity is assumed constant. Empirical literature, such as Topel and Rosen (1988), suggests that majority of housing fluctuation is driven by demand factors.

The profit maximization problem in the sector is

$$\max_{d_t} E_t \sum_{s=0}^{\infty} \frac{P_{t+s}}{\Pi_{l=0}^{s}\left(1 + R_{t+l-1}\right)} \left\{q_{h,t+s} y_{HI,t+s} - d_{t+s}\right\}$$

such that

$$y_{HI,t} = d_{t+s}\left[1 - \Phi_h\left(\frac{d_t}{d_{t-1}^{\varpi_h}}\right)\right]$$

The optimality condition is

$$\begin{aligned} P_t &= P_t q_{h,t}\left[1 - \Phi_h\left(\frac{d_t}{d_{t-1}^{\varpi_h}}\right) - \Phi_h'\left(\frac{d_t}{d_{t-1}^{\varpi_k}}\right)\frac{d_t}{d_{t-1}^{\varpi_h}}\right] \\ &\quad + E_t\left[\varpi_h \frac{P_{t+1}q_{h,t+1}}{1+R_t}\Phi_h'\left(\frac{d_{t+1}}{d_t^{\varpi_h}}\right)\frac{d_{t+1}^2}{d_t^{\varpi_h+1}}\right] \end{aligned} \tag{2.13}$$

2.3.6 Monetary authority

The monetary authority uses the one-period discount rate as its policy tool. It sets an interest rate target and supply (or subtract) as much money as necessary until the interest rate is at the target. The demand for money comes from households, whose utility function should have included a real monetary balance. Under the assumption of separability in the preference for money and other inputs, it is not necessary to model the demand for money nor the supply, as long as the monetary authority respects the equality between money injections and transfers (see discussions in Bernanke *et al.* (1999) and Iacoviello (2005a))

The monetary authority is assumed to follow a forward-looking Taylor rule that responds to changes in output, and the expected inflation. In addition, the policy rule has a interest rate smoothing component. The policy rate in period t is

$$\begin{aligned} R_t &= (1-\rho_R)R_* + \rho_R R_{t-1} + (1-\rho_R)\rho_\pi(E_t\pi_{t+1} - \pi_*) \\ &\quad + (1-\rho_R)\rho_{\Delta y}(y_t - y_{t-1}) + \varepsilon_{R,t} \end{aligned} \tag{2.14}$$

where the variables R, π, and y denote interest rate, inflation and output, respectively. The R and π that have "*" subscript are the steady state values. The non-systemic component of policy rate is denoted as $\varepsilon_{R,t}$. The interest-rate smoothing parameter is ρ_R, which is between zero and one. The parameter ρ_π captures the response of the policy rate to inflation. The parameter $\rho_{\Delta y}$ captures the response to changes in output.

2.3.7 Government

Government collects taxes from households. It also issues discounted bonds. The government operates with the following budget constraint

$$g_t + \frac{B_{t-1}}{P_t} = \frac{T_t}{P_t} + m_h e^{z_{mh,t}} q_{h,t} h_t + \frac{1}{P_t} \frac{B_t}{1+R_t}$$

The left hand side is the expenditure. The first term $g_t = g_* e^{z_{g,t}}$ is the government spending. The spending fluctuates around a steady state level g_*. The stochastic component is $z_{g,t}$. The second term is the real value of bonds that the government has to honor in period t. The right hand side describes the sources of revenue. The first two terms are the tax revenue that include $\frac{T_t}{P_t}$ the net lump-sum tax, and $m_h e^{z_{mh,t}} q_{h,t} h_t$ the property tax (or subsidies). The last term $\frac{1}{P_t} \frac{B_t}{1+R_t}$ is the proceed from issuing new bonds.

2.3.8 Aggregate resource constraint and the accumulation of capital stock

Aggregating the resource constraints across households and that of the government, taking into account all the profit transfer from businesses to households, and the net transfer between households and the government, there exists an economy-wide resource constraint

$$y_t = c_t + i_t + d_t + g_t \tag{2.15}$$

In other words, total output is allocated between consumption c_t, business investment i_t, residential investment d_t, and government spending g_t. The cost of capacity utilization is assumed to be small and therefore drops out from the constraint.

The accumulation of capital stock follows

$$k_t = (1-\delta_k)\, k_{t-1} + e^{z_{i,t}} i_t \left[1 - \Phi_k \left(\frac{i_t}{i_{t-1}^{\varpi_k}} \right) \right] \tag{2.16}$$

and

$$h_t = (1-\delta_h)\, h_{t-1} + d_t \left[1 - \Phi_h \left(\frac{d_t}{d_{t-1}^{\varpi_h}} \right) \right] \tag{2.17}$$

2.3.9 Exogenous shocks

There are six exogenous shocks in this system. They all are approximated with AR(1) processes except for the consumption preference shock. Following Smets and Wouters (2007), the preference shocks are modeled as a

ARMA(1,1) process. The moving average term picks up the high-frequency fluctuations in the quarterly consumption series.

Let ρ_z be the auto-correlation coefficient of shock z, let $\varepsilon_{z,t}$ be a random draw from the standard normal distributions at time t, let $\sigma_{\varepsilon,z}$ be the standard deviation of innovations to z, we can describe the stochastic process of these shocks with the following set of equations:

The TFP shocks

$$z_{a,t} = \rho_a z_{a,t-1} + \sigma_{\varepsilon,a}\varepsilon_{a,t} \tag{2.18}$$

The investment-specific technology shock in the capital goods sector

$$z_{i,t} = \rho_i z_{i,t-1} + \sigma_{\varepsilon,i}\varepsilon_{i,t} \tag{2.19}$$

The consumption preference shocks

$$z_{\chi c,t} = \rho_{\chi c} z_{\chi c,t-1} + \sigma_{\varepsilon,\chi c}\varepsilon_{\chi c,t} - \mu * \sigma_{\varepsilon,\chi c}\varepsilon_{\chi c,t-1} \tag{2.20}$$

where μ is the moving average term.

The government expenditure shocks

$$z_{g,t} = \rho_g z_{g,t-1} + \sigma_{\varepsilon,g}\varepsilon_{g,t} \tag{2.21}$$

The monetary policy shocks

$$z_{R,t} = \rho_{zR,} z_{R,t-1} + \sigma_{\varepsilon,R}\varepsilon_{R,t} \tag{2.22}$$

The housing demand shocks

$$z_{mh,t} = \rho_{mh} z_{mh,t-1} + \sigma_{\varepsilon,mh}\varepsilon_{mh,t} \tag{2.23}$$

Among these shocks, TFP shocks and the investment specific technology shocks are regarded in the literature as being major sources of fluctuation. Several papers have found the moderation in the volatility of these shocks to be important for the great moderation. Leduc and Sill (2007) makes a case for the moderation in TFP shocks. Justiniano and Primiceri (2006) makes a case for investment shock. The government spending shocks and consumption preference shock belong to the minimum set of perturbations needed by this model to avoid singularity. Monetary policy shock is an intrinsic part of monetary models. The housing demand shock drives fluctuations in the housing sector.

2.3.10 Steady state of the model, and the calibration strategy

The model has no explicit solution. This paper relies on linear approximation around steady state to capture the model's short-run dynamics. The steady state is solved with the following steps.

The first step is to solve for the steady-state real interest rate from the Euler equation, or equ(2.2), in its steady state. The result is

$$r_* = \frac{1}{\beta} - 1$$

The second step is to calculate the investment to output ratio, or $\frac{i_*}{y_*}$, using the conditions that describe the equilibrium in the market for business capital goods. These conditions include $\frac{i_*}{k_*} = \delta_k$ (which is the steady-state counterpart of equ(2.16)), $q_{k,*} = 1$ (which is the steady-state counterpart of equ(2.12)), and $\frac{\alpha}{x_*}\frac{y_*}{k_*} = q_{k,*}\left(r + m_k + \delta_k\right)$ (which is the steady-state counterpart of equ(2.11). These three equations together give $\frac{i_*}{y_*}$ as

$$\frac{i_*}{y_*} = \frac{\alpha}{x_*}\frac{\delta_k}{r_* + m_k + \delta_k}$$

The next step is to solve for $\frac{d_*}{y_*}$ and $\frac{c_*}{y_*}$, which are the shares of housing investment and consumption of non-durable goods in the total output. For this purpose we can use the resource constraint $\frac{d_*}{y_*} + \frac{c_*}{y_*} = 1 - \frac{i_*}{y_*} - \frac{g_*}{y_*}$, and a linear relation between $\frac{d_*}{y_*}$ and $\frac{c_*}{y_*}$ that is captured by $\frac{d_*}{y_*}/\frac{c_*}{y_*} = \frac{d_*}{c_*} = \delta_h\left[\frac{1}{v_h}\left(r + m_h + \delta_h\right)\right]^{-\frac{1}{\gamma}}$. The resource constraint is self explanatory. On its right hand side $\frac{g_*}{y_*}$ is exogenously given, and $\frac{i_*}{y_*}$ has been solved in the previous steps. The linear relation between d_* and c_* can be derived by combining $v_h\left(\frac{c_*}{h_*}\right)^{\gamma} = q_{h,*}\left(r_* + m_h + \delta_h\right)$ (which is equ(2.5) in the steady state), $q_{h,*} = 1$ (equ(2.13) in the steady state), as well as $d_* = \delta_h h_*$ (equ(2.17) in the steady state). The solutions to $\frac{d_*}{y_*}$ and $\frac{c_*}{y_*}$ are

$$\frac{c_*}{y_*} = \frac{1 - \frac{g_*}{y_*} - \frac{i_*}{y_*}}{1 + \delta_h\left[\frac{1}{v_h}\left(r + m_h + \delta_h\right)\right]^{-\frac{1}{\gamma}}}$$

and

$$\frac{d_*}{y_*} = 1 - \frac{g_*}{y_*} - \frac{i_*}{y_*} - \frac{c_*}{y_*}$$

Knowing $\frac{c_*}{y_*}$ and $\frac{d_*}{y_*}$ (the latter of the two gives us $\frac{h_*}{y_*}$ through $d_* = \delta_h h_*$), we can sum together the consumption of non-durable goods and the consumption of housing services as the total consumption expenditure, expressed as a share of total output. The result is

$$\frac{e_*}{y_*} = \frac{c_*}{y_*} + \frac{(r_* + m_h + \delta_h)\, h_*}{y_*}$$

which corresponds to the item "private consumption expenditure" defined in the national account.

To calculate the hours worked in the steady state, we need to use the clearance condition in the labor market. The clearance condition is $(1-\lambda_b)\, e_* l_*^{\sigma_l} = (1-\alpha)\, \frac{1}{x_*} \frac{y_*}{l_*}$, from equ(2.9) and from the alternative expression for equ(2.4). Using this relation we can calculate l_* as

$$l_* = \left(\frac{1-\alpha}{1-\lambda_b} \frac{1}{x_*} \right)^{\frac{1}{1+\sigma_l}} \left(\frac{e_*}{y_*} \right)^{-\frac{1}{1+\sigma_l}}$$

Finally, using l_* we can calculate y^* using the production function, which gives us

$$y_* = \left(\frac{k_*}{y_*} \right)^{\frac{\alpha}{1-\alpha}} l_* = \left(\frac{1}{\delta_k} \frac{i_*}{y_*} \right)^{\frac{\alpha}{1-\alpha}} l_*$$

This economy has a balance growth path, even if labor productivity has a positive growth rate in the long run. Productivity growth, in essence, increases the amount of effective labor in the economy, which pushes up the total output by the same proportion, but leaving $\frac{i_*}{y_*}$, $\frac{c_*}{y_*}$, $\frac{d_*}{y_*}$, $\frac{e_*}{y_*}$, l_* constant.

The steady state solution clarifies the calibration strategy of the model. The objective of the calibration is to make sure the model's steady state replicates long-term averages observed in the data. In this paper, the averages I use are those for the real interest rate and various expenditure shares. More specifically, I first choose β to produce a desired r_*, then m_k for $\frac{i_*}{y_*}$, then m_h for $\frac{e_*}{y_*}$, and finally v_h for $\frac{c_*}{y_*}$. The calibration takes several parameters directly from the data, including δ_k, δ_h and $\frac{g_*}{y_*}$. Two other parameters are taken from the literature. One of them is x_*, the steady-state markup charged by retailers. This parameter is taken from Iacoviello (2005a). The parameter σ_l, which determines the labor supply elasticity, is taken from Smets and Wouters (2007). The reason why this parameter is not estimated is because the estimations do not include wage and employment as observed variables. The data therefore have limited information about σ_l.

Other parameters of the model come from estimations. The steady state of the model contains no information about parameters that have only short-run implication. Such parameters include those of the monetary policy

rule, those of nominal rigidity and real rigidity, and those that describe the properties of the exogenous shocks. These parameters have to come from estimation. The estimation is done using a linear state-space econometric system that approximates the model around its steady state. The following section presents the linearized system, and the appendix provides a brief description of the estimation method.

2.3.11 The linearized system

Let's first explain the notation in the linear system. All variables are modified to have a hat over their original symbols. For majority of them, hatted variables means percentage deviation from the steady-state level, i.e., $\widehat{y}_t = \ln\left(\frac{y_t}{y_*}\right)$, $\widehat{i}_t = \ln\left(\frac{i_t}{i_*}\right)$, etc. The exceptions are the interest rates $\widehat{R}_t$ and $\widehat{r}$, and the inflation $\widehat{\pi}$. The hats for these three variables indicate percentage-point difference from their steady-state counterparts. In other words, $\widehat{R}_t = R_t - R_*$, $\widehat{r}_t = r_t - r_*$, and $\widehat{\pi}_t = \pi_t - \pi_*$.

Starting from households' utility maximization. The consumption Euler, or equ(2.2), is linearized into

$$\begin{aligned} E_t\widehat{r}_{t+1} &= \left(\frac{\sigma_c}{1-\lambda_b} - \gamma\right) E_t\widehat{c}_{bt+1} - \left(\sigma_c\frac{1+\lambda_b}{1-\lambda_b} - \gamma\right)\widehat{c}_{bt} \\ &\quad +\sigma_c\frac{\lambda_b}{1-\lambda_b}\widehat{c}_{bt-1} + \gamma\left(E_t\widehat{c}_{t+1} - \widehat{c}_t\right) \\ &\quad -\left(\sigma_c - 1\right)\frac{w_* l_*}{e_*}\frac{1}{1-\lambda_b}\left(E_t\widehat{l}_{t+1} - \widehat{l}_t\right) - z_{\chi c,t} \end{aligned} \tag{2.24}$$

where $\frac{w^* l^*}{e^*}$ is the ratio of total wage and total private consumption expenditure, measured at the steady state.

The variable $\widehat{c}_b$ comes from linearizing equ(2.1),

$$\widehat{c}_{bt} = \kappa_c\widehat{c}_t + \left(1-\kappa_c\right)\widehat{h}_t \tag{2.25}$$

where the coefficient $\kappa_c = \frac{c_*}{e_*}$ is the share of non-durable consumption in total private consumption expenditure.

Equ(2.3), which defines the real interest rate, becomes

$$\widehat{R}_t - \widehat{\pi}_t \equiv \widehat{r}_t \tag{2.26}$$

The linear version of equ(2.4), which is the optimality condition for labor supply, becomes

$$\frac{1-\gamma\left(1-\lambda_b\right)}{1-\lambda_b}\widehat{c}_{bt} + \gamma\widehat{c}_t - \frac{\lambda_b}{1-\lambda_b}\widehat{c}_{bt-1} + \sigma_l\widehat{l}_t = \widehat{w}_t \tag{2.27}$$

The linear version of equ(2.5), which is the investment Euler on housing capital, becomes

$$\begin{aligned}\widehat{q}_{h,t} &= (1-\varepsilon_{h,1})\, E_t\left(\gamma\widehat{c}_{t+1}-\gamma\widehat{h}_{t+1}\right) \\ &\quad +\varepsilon_{h,1}\left(E_t\widehat{q}_{h,t+1}+E_t z_{mh,t+1}\right)-\left(E_t\widehat{r}_{t+1}+z_{mh,t}\right)\end{aligned} \tag{2.28}$$

where $\varepsilon_{h,1} \equiv \frac{1-\delta_h}{(r_*+\delta_h+m_h)+1-\delta_h}$.

The linear version of equ(2.8), which describes the total output as a function of inputs and productivity, becomes

$$\widehat{y}_t = (1-\alpha)\,\widehat{l}_t + \alpha\widehat{k}_t + \alpha\widehat{u}_t + z_{a,t} \tag{2.29}$$

The linear version of equ(2.9), which governs the optimal demand for labor, is now

$$\widehat{y}_t - \widehat{l}_t - \widehat{x}_t = \widehat{w}_t \tag{2.30}$$

The linear version of equ(2.10), which governs the optimal capacity utilization, is now

$$\widehat{u}_t = \frac{1-\psi}{\psi}\left(\widehat{y}_t - \widehat{k}_t - \widehat{x}_t\right) \tag{2.31}$$

The coefficient $\frac{1-\psi}{\psi}$ measures the response of utilization rate to rental price. The parameter ψ is between zero and one. When $\psi = 1$ the cost of varying the utilization is extremely high, so that change in the productivity of capital does not affect the utilization rate. The case of $\psi = 0$ is the other extreme, when changes in the productivity of capital has infinite impact on the utilization rate.

The linear version of equ(2.11), which is the investment Euler on business capital, is

$$\widehat{q}_{k,t} = (1-\varepsilon_{k,1})\, E_t\left(\widehat{y}_{t+1} - \widehat{k}_{t+1} - \widehat{x}_{t+1}\right) + \varepsilon_{k,1}E_t\widehat{q}_{k,t+1} - E_t\widehat{r}_{t+1} \tag{2.32}$$

where $\varepsilon_{k,1} \equiv \frac{1-\delta_k}{(r_*+\delta_k+m_k)+1-\delta_k}$.

The linear version of equ(2.12), which describes the supply of new business capital, is

$$\frac{1}{\varphi_k}\widehat{q}_{k,t} = \left(1+\beta\varpi_k^2\right)\widehat{i}_t - \varpi_k\widehat{i}_{t-1} - \varpi_k\beta E_t\widehat{i}_{t+1} - \left(1+\beta\varpi_k^2\right) z_{i,t} \tag{2.33}$$

The linear version of equ(2.13), which describes the supply of new housing capital, is

$$\frac{1}{\varphi_h}\widehat{q}_{ht} = \left(1+\beta\varpi_h^2\right)\widehat{d}_t - \varpi_h\widehat{d}_{t-1} - \varpi_h\beta E_t\widehat{d}_{t+1} \tag{2.34}$$

The total stock of business capital evolves according to

$$\widehat{k}_{t+1} = (1-\delta_k)\,\widehat{k}_t + \delta_k \widehat{i}_t + \delta_k z_{i,t} \tag{2.35}$$

The total stock of housing capital evolves according to

$$\widehat{h}_{t+1} = (1-\delta_h)\,\widehat{h}_t + \delta_h \widehat{d}_t \tag{2.36}$$

Linearizing equ(2.6) and equ(2.7) gives rise to a Phillips curve,

$$\widehat{\pi}_t - \iota_p \widehat{\pi}_{t-1} = \beta\left(E_t \widehat{\pi}_{t+1} - \iota_p \widehat{\pi}_t\right) - \frac{(1-\xi_p)(1-\xi_p\beta)}{\xi_p}\widehat{x}_t \tag{2.37}$$

The policy reaction function, or equ(2.14), is now

$$\widehat{R}_{t+1} = (1-\rho_R)\,\rho_\pi E_t \widehat{\pi}_{t+1} + (1-\rho_R)\,\rho_{\Delta y}\left(\widehat{y}_t - \widehat{y}_{t-1}\right) + \rho_R \widehat{R}_t + z_{R,t} \tag{2.38}$$

Finally, the economy-wide resource constraint, or equ(2.15), is now

$$\widehat{y}_t - \frac{c_*}{y_*}\widehat{c}_t - \frac{i_*}{y_*}\widehat{i}_t - \frac{d_*}{y_*}\widehat{d}_t - z_{g,t} = 0 \tag{2.39}$$

2.3.12 Constructing a state-space econometric model

The linearized model is a system of linear expectational difference equations, or

$$\begin{aligned} AE_tX_{t+1} + G_0E_tZ_{t+1} &= BX_t + G_1Z_t \\ Z_t &= \Phi Z_{t-1} + \Sigma\varepsilon_t \end{aligned}$$

The system can be solved numerically using the Blanchard-Khan type procedure modified by Klein (2000). The solution to the linearized model, barring indeterminacy, expresses jump variables in the system, or X_t^f, as functions of predetermined state variables X_t^b and the current state of stochastic processes Z_t,

$$\begin{aligned} X_{t+1}^b &= M_xX_t^b + M_zZ_t \\ X_t^f &= F_xX_t^b + F_zZ_t \end{aligned}$$

The solutions and the process of the shocks together define the law of motion. Once we choose a set of observed variables, we can define the observed variables from X_t^f and X_t^b and their lags to complete the state-space system.

The regression uses quarterly data from 1963Q1 to 2006Q2. Six observed variables are in the regression: the growth rate of output (with the value of housing service subtracted from GDP), private consumption expenditure (net of housing service), private non-residential investment, and residential investment; the level of effective federal funds rate; and the growth rate of GDP price index.

The measurement equations of the state space system are,

$$\begin{aligned} gr\_y_t - \gamma_y &= \widehat{y}_t - \widehat{y}_{t-1} \\ gr\_c_t - \gamma_c &= \widehat{c}_t - \widehat{c}_{t-1} \\ gr\_i_t - \gamma_i &= \widehat{i}_t - \widehat{i}_{t-1} \\ gr\_d_t - \gamma_d &= \widehat{d}_t - \widehat{d}_{t-1} \\ R_t - R &= \widehat{R}_t \\ \pi_t - \pi &= \widehat{\pi}_t \end{aligned}$$

Here $\gamma_y = \frac{\sum_{t=1963Q2}^{2006Q2} gr\_y_t}{173}$ is the average growth rate of real GDP (net of housing output) over the sample. Other terms, γ_c, γ_i and γ_d are similarly defined. $R = \frac{\sum_{t=1963Q2}^{2006Q2} R_t}{173}$ and $\pi = \frac{\sum_{t=1963Q2}^{2006Q2} \pi_t}{173}$ are the sample means of R and π.

2.4 Estimations and Counter-factual Analysis

2.4.1 Overview of the strategy: Bayesian approach, split sample estimation, and counter-factual analysis

Here we briefly discuss the key elements in the empirical strategy of the paper. The estimation and the statistical inference follow a Bayesian approach. The prior distributions of the usual DSGE parameters follow those of Smets and Wouters (2007). The priors for housing-related parameters are loosely specified. The paper uses Sims' maximization routine csminwel.m to estimate the mode of posterior distribution by maximizing the log of posterior function. It then uses a Random Walk Metropolis algorithm (RWM algorithm, described in detail in An and Schorfheide (2006)) to explore the posterior distribution around the mode.[9]

[9] This algorithm makes a large number of draws in the neighborhood of the mode in order to provide a complete picture of the posterior distribution. These draws, numbered at hundred of thousands for each of the multiple chains, provide the basis for researchers to approximate the percentiles of the distribution of parameters and of the transformation of parameters, such as the variance decomposition.

Figure 2.1 presents the in-sample fit of the one-step-ahead forecast by comparing the data to the prediction. Table 2.3 presents the out-of-sample forecast performance of the estimated model over an unrestricted VAR(1). Table 2.4 presents the variance decomposition to explore the source of fluctuations in the housing market. Table 2.5 presents the posterior distributions from the split sample estimations. The break point is 1986Q1, the same as the one used in McCarthy and Peach (2002), who use split-sample VAR estimations to capture the difference in the transmission of monetary policy to residential investment. Finally, an identical set of priors is used for the split-sample estimations, so that the difference in the posterior distribution comes from the data as much as possible.

After the estimations I use counter-factual analysis to pinpoint the most important changes. It is safe to guess that the split-sample estimations will detect more than one change in the parameters of the model. Changes can occur in the monetary policy rule, in the nominal and real rigidity parameters, and in the stochastic processes of disturbance. The question of interest is how these individual blocks of changes affect the variance of endogenous variables. To answer this question, this paper uses a sequence of counter-factual analysis. The sequential begins with a data-generating process (DGP), defined by the modes of the parameters estimated from the pre-86 data. It then replaces all parameters, block by block, until finally reaching a DGP defined by the post-86 estimates. In each step, the variance of the endogenous variables is recorded. A quantitative assessment of the contribution made by one particular change of parameters is the difference between the variance at the step when the change is made and those in the previous step. The result is recorded in Table 2.6.

2.4.2 Parameters to be estimated and their prior distributions

Four sets of parameters need to be estimated. They are gathered by groups, their prior distribution is presented in the four panels of Table 2.2.

The first is the "shock" parameters, including the autocorrelation coefficients, the moving average coefficients, and the standard deviations of the six stochastic processes. Following Smets and Wouters (2007), the prior distributions are harmonized as much as possible. This way the difference in the posteriors reflects the information from the data. All the autocorrelation coefficients are beta distributed with mean 0.5 and a standard error of 0.2. All standard deviations of innovations are assumed to follow an inverse gamma distribution with mean 0.3 and two degrees of freedom, which amounts to a

loose prior. The only moving average parameter for consumption preference shocks has a mean of 0.7 and a standard error of 0.15.

The second set of parameters are those in the monetary policy reaction function. The smoothing parameter is beta with mean 0.75 and standard error of 0.1. The response to inflation is Gamma with mean 1.7 and a standard error of 0.3. The response to output growth is Gamma with mean 0.12 and standard error of 0.05.

The third set of parameters contains all the nominal and real rigidity parameters. As for the prior distribution, the Calvo stickiness is Beta of 0.75[0.1], the adjustment cost parameter for capital investment is normal of 4[1.5], which is adopted from Smets and Wouters (2007).[10] I use the same distribution for the adjustment cost parameter of the housing investment, since the prior is rather loose. Other parameters in the group have to lie between zero and one. I assume them all are beta distributed with mean 0.5 and standard error of 0.15.

The last set has the preference parameters. The risk aversion parameter σ and the intratemporal elasticity of substitution between housing service and consumption, or γ, are assumed to be normally distributed with mean 2 and a large standard error of 0.4. The habit parameter λ_b is normal with 0.5[0.2].

2.4.3 Full-sample estimation and its in-sample fit

Here I use a full-sample estimation to provide a brief overview of the estimation before the split-samples. The full-sample estimation is also used to assess the in-sample fit of the model. Table 2.2 presents the mode, the mean, and the percentiles of the posterior distributions of parameters. The mode is from the maximization routine of Sims.[11] The percentiles are the sample statistics of 300,000 draws from a RWM algorithm with three chains.[12]

The first observation is that the data is very informative for the stochas-

[10] who in turn chooses the prior based on Christiano *et al.* (2005).

[11] Chris Sims' csminwel.m is for unconstrained maximization. Because some parameters in the model are clearly constrained, parameter transformation is used first so that csminwel.m can be applied. Transformation is used only in for estimation of the mode. For the MCMC draw, no transformation is used. An and Schorfheide (2006) describes the estimation steps in greater detail.

[12] Each of these chains is initiated with a random draw around the mode of the posterior distribution, using a variance covariance matrix that equals to 0.5 times of the inverse Hessian approximated at the mode. The jumping distribution uses a covariance matrix that is 0.17 of the inverse Hessian. This choice of scale generates a acceptance rate ranging from 22-27%.

tic processes of shocks. The posterior distributions of these parameters, presented in the top panel of Table 2.2 are quite different from the assumed priors. Furthermore, despite the harmonized prior distributions, the estimates vary significantly for different types of shocks. Among these different shocks, the productivity shocks are highly persistent. Persistence of the productivity shocks is a feature of DSGE models that assumes a deterministic trend of productivity (for example, see Smets and Wouters (2007)). Monetary policy shocks and consumption preference shocks, on the other hand, are far less persistent. The housing-specific shock is also persistent, though not as much as the TFP shocks.

The data is quite informative for the adjustment cost parameters for housing and capital investment, but not for the Calvo stickiness and capacity utilization. In the bottom panel, most of the preference parameters have posteriors that are similar to the assumed priors, except for the habit parameter.

Figure 2.1 provides a visual diagnostic of the estimated model's fit in one-step-ahead forecast. The figure plots the actual time-series of the six variables together with the forecast. First of all, the time series generally fit well, even for growth rates. This means that the model is quite capable of capturing the direction of movements, not just for levels. Another observation is that the series of forecasts is smoother than the actual data, gliding through the middle of the data's high frequency fluctuations. This suggests that the forecast achieves a satisfactory balance between keeping track of the trends and not over-reacting.

Table 2.4 provides a variance decomposition for the eighth-quarter-ahead forecast error. The lower panel is the corresponding standard errors from the MCMC draws.[13]This table shows that fluctuations in GDP growth and consumption growth are mostly driven by TFP shocks (70% and 64%, respectively); investment fluctuations is mainly driven by investment-specific shocks (63%); and housing fluctuations is driven by housing-specific shocks (64%), and to a lesser extent TFP shocks (28%). The main contributor to the movements in the Federal funds rate and inflation are the TFP shocks, investment shocks, and consumption preference shocks. The non-systemic

[13]Variance decomposition breaks down the variance of J-step ahead forecasting errors, for each of the six endogenous variables, to components attributable to each of the six shocks. That is why this table has the 6-by-6 matrix, with endogenous variables in the column, and individual shocks in the rows. These decompositions are expressed in shares. Each column sums to 100 by construction. The matrix in the upper panel of the table is the mean of these shares from the every 500th draw in the MCMC chain. The lower panel is the corresponding standard errors.

monetary policy shocks account for only a small part of the variance.

2.4.4 Out-of-sample forecast of the estimated model

Table 2.3 compares the performance of the DSGE model in out-of-sample forecast to that of an unrestricted VAR(1), as in Smets and Wouters (2007). The forecast target is an eight-quarter window within the period between 1990Q3 and 2006Q2. Both models are re-estimated yearly using the sample from the earliest date (1963Q2) up to the quarter immediately before the window of forecast. In other words, the data between 1963Q2 and 1990Q2 are used in the first round to estimate the model and make forecasts for 1990Q3-1992Q2. In the second round, the sample is expanded by four quarters, the model parameters are re-estimated, and projections are made for the next 8 quarters. The process continues until the end of the sample.

The Root Mean Square Error (RMSE) is used as the measure of accuracy. For quantity variables, the growth rate is converted to the level before the calculation of RMSE. The accuracy gain is measured by the percentage reduction in RMSEs.

For the four quantity series, the DSGE model out-performs the unrestricted VAR(1) for the entire forecast window. The percentage improvement is the largest for the forecast of GDP, consumption, and investment, peaking at levels from 30 to 50%, similar to those reported in Smets and Wouters (2007). For residential investment, the best performance is found at the second and the third quarter, with a 10-15% improvement, which gets somewhat smaller over time.

For nominal interest rate and inflation, a few early quarters show that the DSGE model is worse than the VAR. The loss is not large, however, with the biggest loss being -17% for federal funds rate forecast at the second quarter, followed by a -8% loss for inflation in the third quarter. Smets and Wouters (2007) found an almost identical loss of accuracy for the Federal funds rate, though not for inflation. The loss turns into a gain for both series from the fourth quarter on, reaching about a 20% gain by the eighth quarter for both nominal rate and for inflation.

2.4.5 Split-sample estimations

The split-sample estimations in this paper allow all parameters to change between split-periods. The estimation and the RWM algorithm are identical

to those for the full sample exercise and uses the same set of priors.[14]

Table 2.5 presents the posterior distributions for the two sub-samples side by side. A majority of the parameters are quite stable over the samples. Some are different. One way to measure the statistical significance of the difference is to see if, for the same parameter but in difference periods, the 95% percentile of the lower estimate is still smaller than the 5% percentile of the higher estimate (in other words, the "confidence intervals" do not overlap). With this criterion, there are only four parameter that experienced significant changes from pre-85 period to the post-85 period. They are the standard deviation of the innovations to TFP shocks (smaller in the more recent period); the standard deviation of the innovation to government spending shocks (smaller); standard deviations of monetary policy shocks (also smaller). These changes suggest that the economic shocks has become smaller over time. Interestingly, there is no evidence that housing-specific shocks are now smaller. The estimated auto-correlation of the shocks, as well as the standard deviation of the innovation to the shocks, are almost identical between the two samples.

The fourth, and the last significant change in parameter is an increase in the responsiveness of interest rate policy to inflation, as suggested in Taylor (2007), Clarida *et al.* (2000), and many other papers. The first-period estimate of the parameter is 1.4. The second-period estimate is 2.1.

Furthermore, albeit barely missing the bar for significance, the smoothing parameter of the monetary policy increases from 0.58 to 0.72. The smoother policy rule confirms the suggestion in Ireland (2004) that the Federal reserves appears to have abandoned "a stylized pattern of 'stop-go' policy, according to which an initial tightening is quickly reversed, presumably in an attempt to partially offset the negative effects on output".

The adjustment cost parameter of housing investment, or φ_h, increases from 1.63 to 3.58, although it is short of outright significance. A higher adjustment cost parameter in the model smoothens out the response of residential investment to demand shocks. Higher adjustment cost means a steeper short-run supply curve. With steeper supply in the short run, the incentive to spread out investment over time is higher. The equilibrium housing investment therefore exhibits reduced sensitivity to demand shocks as φ_h becomes higher.

A higher adjustment cost slows down changes in investments, and is likely

[14]The Hessian matrices are approximated separately for each sub-sample at their posterior modes. The jumping distribution uses a scale factor of 0.17 for both sub-samples, and it results in acceptance rate about 20-28% for individual chains.

to shift the adjustment mechanism toward house prices. Is there evidence that housing prices are now more responsive? According to Figure 2.5, the answer may be yes. The figure plots the level of investment with the log of price index. The latter is derived from Davis and Heathcote (2005). Between the late-1970s and the mid-1980s, the price index appears to be rather sluggish, with little movement despite the large swings in investments. From the mid-1980s, however, the price series and investment series move more closely with each other. For example, the price series follows the turns of the investment series in the mid-1980s, the early-1990s, and in 2006.

2.4.6 Breaking down the reduction in variance using counter-factual analysis

Table 2.6 reports a sequence of counter-factual analysis that are used to break down the reduction in the standard deviation of endogenous variables. The first column presents the unconditional standard deviation of the six variables calculated from the mode of the parameters from the pre-1986 estimation. The last column is the same set of standard deviations from the mode of the post-1986 estimation. Between these columns is a sequence of hybrid data generating processes created by replacing the parameters, block by block, in the following order. First, the monetary policy parameters are switched from the pre-1986 estimate to the post-1986 estimate. The resulting data generating process is called Hybrid-1. Next, all behavior parameters in Hybrid-1, except for housing adjustment cost parameters, are replaced with post-1986 estimates, to reach Hybrid-2. The housing adjustment cost parameters are then substituted (Hybrid-3), followed by the substitution of monetary policy shock parameters (Hybrid-4), TFP shock parameters (Hybrid-5), and the housing-specific shock parameters (Hybrid-6). The last step is to replace all remaining shock parameters. With this last substitution, we reach the post-1986 data-generating process. The difference in the level of standard deviation measures the contribution from the changes in each block of parameters.

Sources of moderation for residential investment Over the sequence of counter-factual analysis, there are only two significant reductions in the standard deviations of quarter-to-quarter growth of residential investment. The first one happens when moving from Hybrid 2 to Hybrid 3, namely replacing the adjustment cost parameters for housing investment from the pre-1986 estimates to the post-86 ones. The variability measure drops from 6% to 3.1%. The second largest reduction is is between Hybrid 4 and Hybrid

5, when substituting the parameters of the TFP process. The standard deviation drops from 3.1% to 2.5%.

Our estimate of higher adjustment cost is consistent with empirical findings in Dynan and Sichel (2006) regarding a reduced sensitivity of residential investments to changes in interest rates. They attributed the change to financial innovations or other developments in the mortgage market.

Without excluding financial innovation as a cause for the reduced sensitivity, we believe there are other likely alternatives. One possibility is better management of inventory, as discussed in the introduction. Another possibility is the fall in union's presence in the construction industry. Construction unions are known for strong union hiring halls, which often fix compensation and queues up workers for job opportunities. Such labor market structure suppresses wage response, at the cost of more volatile building activity. The fall in union's presence can switch the market towards more price adjustment, and less quantity adjustment.

There is no escaping from one more obvious possibility, that the supply becomes sluggish because it is more difficult, or takes longer time, to build new houses. A drop in supply capacity, either due to the lack of land, or due to a greater regulatory burden on the use of land, can prevent supply from keeping up with the demand. A recent article by Glaeser *et al.* (2005) coins the term "man-made scarcity" to describe the consequence of regulations in the US housing market. Regardless the cause, a reduced supply is likely to reduce the sensitivity of housing investments. On the other hand, lower supply elasticity increases the responsiveness of house prices, a prediction that seems to be supported by observations in Figure 2.5. Even though lower supply elasticity reduces investments' response to demand shocks under normal situations, it could potentially be a source of instability, such as when speculations on price movements kick in (see discussions in Malpezzi and Wachter (2005)).

It is difficult to evaluate alternative hypotheses for the reduced sensitivity of housing investment. There are two major hurdles in testing and comparing them. The first one is an overly simplified structure of the housing model. The model, as it is right now, lacks inventory, union, and does not have a upward-sloped supply curve, which would have created diverging relative prices that his paper, and others (for example Smets and Wouters (2007)), avoid. The other hurdle is the absence of housing prices in the set of observable variables. The absence of capital price is common when estimating DSGE model, but it does creates difficulty in interpretating the change in the adjustment process.

Source of moderation in other real volatility As for GDP and consumption, most of the moderation comes from smaller TFP shocks. From Hybrid-4 to Hybrid-5, when the TFP parameters are replaced, the standard deviation of output growth drops by 0.5%, which is large, considering that the total reduction is 0.61%. At the same time, the standard deviation of consumption drops by 0.57%, out of the total 0.6%. The standard deviation of business investment drops by 0.35%, out of a total reduction of 0.8%.

To summarize, according to the estimated models, smaller TFP shocks are the most important driving force for the moderation in the volatility of output and consumption.

Source of moderation in nominal volatility As for nominal variables, namely inflation and nominal interest rate, the drop in their standard deviations is mostly driven by the systematic changes in the monetary policy rule. The standard deviation of inflation drops by 50% as we replace the parameters of the policy rule. The nominal rates experiences a 40% drops as well. In comparison, smaller non-systematic monetary policy shocks did not lead to much reduction in the standard deviations of these two nominal variables (see the difference between Hybrid-3 and Hybrid-4). This suggests that the non-systematic part of the monetary policy is a small component of the story. The change that really matters is in the policy rule, presumably due to its ability to anchor expectations.

Changes in monetary policy: effective in stabilizing nominal variables but not the real variables A more responsive monetary policy has no stabilizing effect on the four quantity variables. If anything, the standard deviations increase slightly under the new policy rule (see the difference between Pre-86 and Hybrid-1 in Table 2.6). The finding that an more aggressive policy does not reduce real economic volatility is not unusual. Stock and Watson (2002) and Leduc and Sill (2007) draw similar conclusions regarding the effect of the policy changes on total output.

Since the policy change substantially reduces nominal volatility, and it is well-known that nominal volatility and real volatility tend to move together in the data (as in the case of US and other G-8 countries, see Blanchard and Simon (2001)), it is inevitable to ask if the estimated model fails to capture a link that should be there. If a link exists between nominal and real volatility, housing sector may be a place where such a link is important. The sector extensively uses long-term, nominally fixed-rate mortgages as means of financing. As a consequence, the cost of capital in the sector is directly

related to long-term interest rates. For households, it is the mortgage rates. For investors in mortgage debt, it is the ten-year treasury yield. Since these long-term rates are sensitive to medium and the long-term inflation risks, it is likely that a more aggressive interest rate policy, by keeping inflation expectation in check, can reduce cost fluctuations, thus having a stabilizing effect in housing market.

Because this paper uses a first-order approximation that removes variations in term premiums, it has very little to say to what extent the such variations can add to the link between nominal and real volatility. To admit time-varying term premium requires a higher order approximation. More importantly, the theory between premium and economic activity is not well understood. In fact, a theoretical model in Rudebusch *et al.* (2006) found that "there is no structural relationship running from the term premium to economic activity".

Empirically speaking, however, there is evidence that is indicative of a link between monetary policy, economic activity, and changes in premiums. On the one hand, macro-finance literature, such as Rudebusch and Wu (2004), finds a relation between the price of risks and macroeconomic variables, including those closely identified with monetary policy, such as inflation targets. One the other hand, the above-mentioned Rudebusch *et al.* (2006) finds evidence from their data that a decline in the term premium has typically been associated with stimulus to real economic activity,.

To summarize, it is an interesting and worthy research question whether the change in monetary policy has an extra effect on real volatility through it impact on term premium. But to pursue the hypothesis is beyond the scope of the current paper.

2.5 Conclusion

This paper estimates a sticky-price DSGE model that is expanded to include housing investments. The purpose is to investigate what could explain the fall in the volatility of US residential investment in the past two decades. It uses split-sample estimations, and counter-factual analysis, to compare three hypotheses: a monetary policy that becomes more responsive to inflation, smaller economic shocks, and structural changes in the housing sector. According to the conjecture in Taylor (2007), a more responsive monetary policy keeps inflation under control, thereby reduces boom-bust cycles and large swings in interest rates, which in turn, leads to a more stable housing market.

The analysis confirms only part of the hypothesis regarding monetary policy. We detect the policy changes, and confirm that the estimated change is capable of reducing the variance of nominal interest rates and inflation substantially. Nevertheless, we do not find the change to reduce the variability of real economic activity, including that of residential investments. The estimated DSGE model appears to have only weak connections between nominal and real volatility. The paper touches on the interesting but difficult question, on whether or not allowing for a time-invariant premium in term structure will enhance the link.

The paper finds that smaller economic shocks make a modest contribution in reducing the variance of the housing investment, whereas, the biggest contributing factor is an increase in the parameters of adjustment costs, which indicates a reduced sensitivity of housing investments. This suggests the possibility of structural change in the housing sector. The paper briefly discusses some hypotheses that are not very often covered in the literature. These include the decline of union in the construction industry, an increase of housing starts with pre-committed buyers, and the possibility of a reduced capacity to supply new housing.

2.6 Tables and Figures

Table 2.1: Descriptive statistics of observed variables between 1963-2006

	Mean	Std. Dev.	Std. Dev. '63Q2-85Q4	Std. Dev. '86Q1-'06Q2
GDP, Quarterly Growth	0.82	0.93	1.17	0.55
Private Consumption, Q. Growth	0.90	0.81	0.98	0.55
Business Investment, Q. Growth	1.24	2.27	2.50	1.98
Resid. Investment, Q. Growth	0.70	4.65	5.97	2.41
Federal Funds Rate	6.36	0.82	0.89	0.55
Inflation	3.87	0.60	0.65	0.23

[1]GDP, PCE and the Investment spendings are in real terms
[2]Total value of housing service is subtracted from GDP and PCE
[3]Data Source: GDP, PCE, and Investment series are from BEA Table 1.1.5. Total value of housing service is from BEA Table 2.3.5. Federal funds rate is from the Federal Reserve

Correlation Coefficient	1963Q1-1985Q4	1986Q1-2006Q2
between d_t/d_{t-4} and y_t/y_{t-4}	0.22	0.17
between d_t/d_{t-4} and y_{t+1}/y_{t-3}	0.49	0.30
between d_t/d_{t-4} and y_{t+2}/y_{t-2}	0.66	0.42
between d_t/d_{t-4} and y_{t+3}/y_{t-1}	0.71	0.48

[1]y_t is GDP net of housing service in quarter t; d_t is residential investment

Table 2.2: Parameters: prior and posterior distributions from the full-sample estimation

		Prior			Posterior		
	Type	Mean	Std.er	Mode	Mean	5%	95%
ρ: TFP Shocks	Beta	0.5	0.2	0.99	0.98	0.97	0.99
ρ: Investment Shocks	Beta	0.5	0.2	0.95	0.94	0.91	0.97
ρ: Gov.Spending Shocks	Beta	0.5	0.2	0.99	0.99	0.99	1.00
ρ: Monetary Shocks	Beta	0.5	0.2	0.35	0.32	0.20	0.46
ρ: Consumption Shocks	Beta	0.5	0.2	0.90	0.89	0.82	0.95
μ: Consumption Shocks	Beta	0.7	0.15	0.80	0.72	0.43	0.93
ρ: Housing Shocks	Beta	0.5	0.2	0.94	0.92	0.85	0.95
σ: TFP Shocks	I.G.[1]	0.3	2	0.83	0.90	0.71	1.17
σ: Investment Shocks	I.G.	0.3	2	0.65	0.72	0.54	0.88
σ: Gov.Spending Shocks	I.G.	0.3	2	0.62	0.63	0.57	0.68
σ: Monetary Shocks	I.G.	0.3	2	0.25	0.26	0.23	0.29
σ: Consumption Shocks	I.G.	0.3	2	0.17	0.23	0.13	0.44
σ: Housing Shocks	I.G.	0.3	2	0.42	0.70	0.36	1.46
		Prior			Posterior		
	Type	Mean	Std.er	Mode	Mean	5%	95%
Policy Rule: ρ_R	Beta	0.75	0.1	0.59	0.61	0.49	0.70
Policy Rule: ρ_π	G.[1]	1.7	0.3	1.47	1.57	1.41	1.79
Policy Rule: $\rho_{\Delta y}$	G.	0.12	0.05	0.16	0.18	0.10	0.28
		Prior			Posterior		
	Type	Mean	Std.er	Mode	Mean	5%	95%
Calvo Stickiness ξ_p	Beta	0.75	0.1	0.73	0.75	0.68	0.82
Adjustment Cost, φ_k	N.m.[1]	4	1.5	2.11	2.46	1.60	3.68
Adjustment Cost, ϖ_k	Beta	0.5	0.15	0.86	0.82	0.73	0.91
Adjustment Cost, φ_h	N.m.	4	1.5	2.04	2.60	1.61	4.03
Adjustment Cost, ϖ_h	Beta	0.5	0.15	0.69	0.69	0.57	0.81
Capacity Utilization ψ	Beta	0.5	0.15	0.48	0.51	0.46	0.58
Inflation Indexation ι_p	Beta	0.5	0.2	0.04	0.07	0.02	0.13
		Prior			Posterior		
	Type	Mean	Std.er	Mode	Mean	5%	95%
Preference: σ_c	N.m.	2	0.4	2.49	2.52	1.83	3.23
Preference: γ	N.m.	2	0.4	2.21	2.28	1.80	2.76
Preference: λ_b	Beta	0.5	0.2	0.31	0.31	0.21	0.43

[1]I.G. is the Inverse Gamma distribution; G. is the Gamma distribution; N.m. is the Normal distribution

[2]Preference parameter σ_l is set at 1.92. Since employment data is not included in the estimation, there is not much information for estimating parameter related to labor supply elasticity. 1.92 is the estimate in Smet and Wouters (2007)

Table 2.3: Compare the estimated DSGE model's performance in out-of-sample forecast with a VAR model

Panel a: Root Mean Square Error (RMSE) from a VAR(1) model

	Q-1	Q-2	Q-3	Q-4	Q-5	Q-6	Q-7	Q-8
y, Level	0.6	1.2	1.8	2.2	2.6	3.0	3.3	3.5
c, Level	0.6	1.2	1.7	2.0	2.2	2.6	2.8	3.0
i, Level	1.3	2.5	4.6	5.8	7.0	8.5	10.0	10.8
d, Level	2.7	5.8	8.0	9.1	8.8	9.2	9.3	9.3
R	0.1	0.2	0.3	0.3	0.4	0.5	0.5	0.6
π	0.2	0.2	0.2	0.3	0.3	0.4	0.3	0.5

Panel b: Percentage improvement of DSGE model over the VAR(1) in reducing the size of RMSE

	Q-1	Q-2	Q-3	Q-4	Q-5	Q-6	Q-7	Q-8
y, Level	16.6	37.7	35.9	39.3	36.1	40.0	43.2	45.7
c, Level	22.0	27.6	38.2	39.1	38.5	45.9	50.0	47.4
i, Level	0.5	33.8	31.1	27.8	25.6	27.3	24.5	23.5
d, Level	3.7	15.3	14.5	10.3	7.8	7.1	9.6	5.6
R	-6.2	-17.1	-4.5	3.8	9.9	13.8	17.2	19.9
π	7.1	10.6	-8.7	16.8	16.7	20.1	13.3	21.8

[1] y is GDP, c is consumption, i is business investment, d is residential investment, R is federal funds rate, π is inflation

Table 2.4: Variance decomposition of the 8-quarter ahead forecast error

Panel-a: Mean taken from the every 500th MCMC draws

	y_t/y_{t-1}	c_t/c_{t-1}	i_t/i_{t-1}	d_t/d_{t-1}	R_t	π_t
TFP Shocks	70.98	64.38	29.21	27.66	36.92	35.66
Investment Shocks	0.74	2.73	62.78	1.66	23.97	22.81
Gov.Spending Shocks	14.88	11.85	0.19	5.24	5.42	4.55
Monetary Shocks	7.61	6.91	5.46	1.05	6.58	5.27
Consumption Shocks	4.95	12.92	1.42	0.62	17.61	22.43
Housing Shocks	0.84	1.21	0.93	63.78	9.50	9.28

Panel-b: Standard error taken from the every 500th MCMC draws

	y_t/y_{t-1}	c_t/c_{t-1}	i_t/i_{t-1}	d_t/d_{t-1}	R_t	π_t
TFP Shocks	4.99	5.39	7.90	6.19	11.65	11.98
Investment Shocks	0.68	1.18	9.58	0.86	7.10	5.95
Gov.Spending Shocks	2.30	2.45	0.18	1.62	2.84	2.47
Monetary Shocks	1.83	1.75	1.90	0.45	1.61	2.53
Consumption Shocks	2.84	5.19	1.00	0.45	7.05	6.81
Housing Shocks	0.73	0.65	0.52	8.43	4.12	3.68

[1] y is GDP, c is consumption, i is business investment, d is residential investment, R is federal funds rate, π is inflation

Table 2.5: Posterior distributions from the split-sample estimations

	Sample: 1963Q2-1985Q4			Sample: 1986Q1-2006Q2		
	Mode	5%	95%	Mode	5%	95%
ρ: TFP Shocks	0.98	0.96	0.99	0.98	0.96	0.99
ρ: Investment Shocks	0.95	0.91	0.97	0.95	0.92	0.98
ρ: Gov.Spending Shocks	0.99	0.98	1.00	0.99	0.97	1.00
ρ: Monetary Shocks	0.29	0.13	0.47	0.51	0.38	0.59
ρ: Consumption Shocks	0.89	0.75	0.92	0.94	0.89	0.97
μ: Consumption Shocks	0.81	0.52	0.93	0.60	0.31	0.85
ρ: Housing Shocks	0.93	0.88	0.95	0.93	0.80	0.94
σ: TFP Shocks	1.09	0.90	1.56	0.54	0.46	0.71
σ: Investment Shocks	0.61	0.51	0.95	0.79	0.63	1.19
σ: Gov.Spending Shocks	0.73	0.66	0.83	0.45	0.40	0.51
σ: Monetary Shocks	0.32	0.29	0.38	0.13	0.11	0.17
σ: Consumption Shocks	0.20	0.16	0.50	0.10	0.08	0.22
σ: Housing Shocks	0.50	0.36	1.19	0.48	0.38	1.28
	Sample: 1963Q2-1985Q4			Sample: 1986Q1-2006Q2		
	Mode	5%	95%	Mode	5%	95%
Policy Rule: ρ_R	0.58	0.45	0.69	0.72	0.66	0.81
Policy Rule: rho_π	1.38	1.30	1.72	2.12	1.89	2.80
Policy Rule: $\rho_{\Delta y}$	0.14	0.08	0.25	0.09	0.05	0.19
Calvo Stickiness ξ_p	0.77	0.69	0.86	0.69	0.63	0.77
Adjustment Cost, φ_k	2.44	1.73	4.70	2.61	1.58	5.40
Adjustment Cost, ϖ_k	0.88	0.72	0.92	0.73	0.50	0.83
Adjustment Cost, φ_h	1.63	1.25	3.51	3.58	2.52	5.67
Adjustment Cost, ϖ_h	0.67	0.51	0.79	0.64	0.51	0.83
Capacity Utilization ψ	0.51	0.46	0.65	0.55	0.52	0.66
Inflation Indexation ι_p	0.06	0.03	0.20	0.09	0.04	0.30
Preference: σ_c	2.30	1.48	2.92	2.14	1.53	2.72
Preference: γ	2.11	1.69	2.68	2.08	1.67	2.57
Preference: λ_b	0.31	0.21	0.48	0.34	0.20	0.43

Table 2.6: Breaking down the reduction in variance

Standard deviations of endogenous variables under the estimated and hypothetical data generating process

	Pre-86	Hybrid-1	H-2	H-3	H-4	H-5	H-6	Post-86
y_t/y_{t-1}	1.28	1.33	1.26	1.26	1.22	0.76	0.76	0.65
c_t/c_{t-1}	1.44	1.51	1.49	1.61	1.57	1.00	1.00	0.85
i_t/i_{t-1}	2.68	2.73	1.94	1.95	1.90	1.55	1.55	1.88
d_t/d_{t-1}	6.01	6.08	6.02	3.12	3.10	2.53	2.48	2.39
R_t	1.19	0.74	0.74	0.78	0.73	0.62	0.62	0.56
π_t	0.95	0.47	0.61	0.61	0.54	0.48	0.48	0.40

[1] y is GDP, c is consumption, i is business investment, d is residential investment, R is federal funds rate, π is inflation

[2] Pre-86 DGP describes the estimated data generating process before 1986; Post-86 DGP, in the last column, describes the estimated data generating process after 1986

[3] Hybrid-1 replaces monetary policy parameters in Pre-86 with those from Post-86; Hybrid-2 replaces non-monetary, non-housing behavior parameters in Hybrid-1 with those from Post-86; Hybrid-3 replaces the housing adjustment cost parameters in Hybrid-2 with those from Post-86; Hybrid-4 replaces the monetary policy shocks parameters in Hybrid-3 with those from Post-86; Hybrid-5 replaces the TFP shocks parameters in Hybrid-4 with those from Post-86; Hybrid-6 replaces the housing-specific shocks parameters in Hybrid-5 with those from Post-86

Figure 2.1: Observable variables: The data and the one-step-ahead forecast

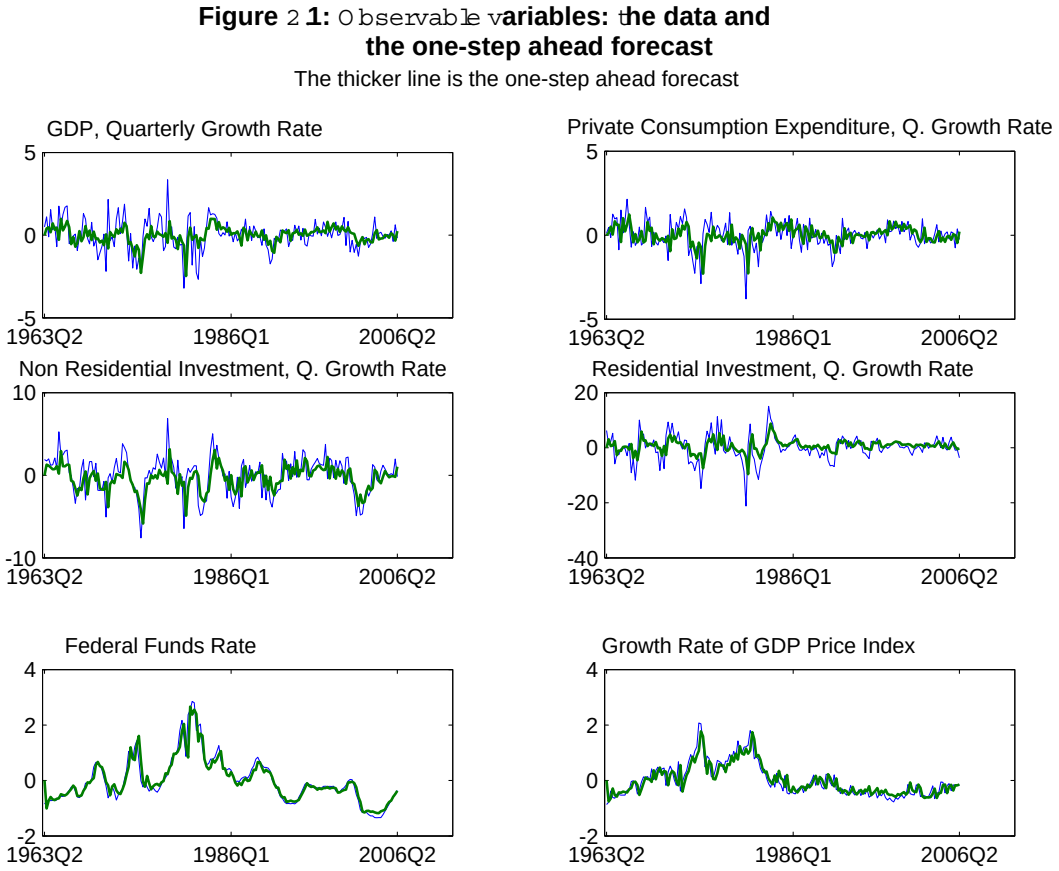
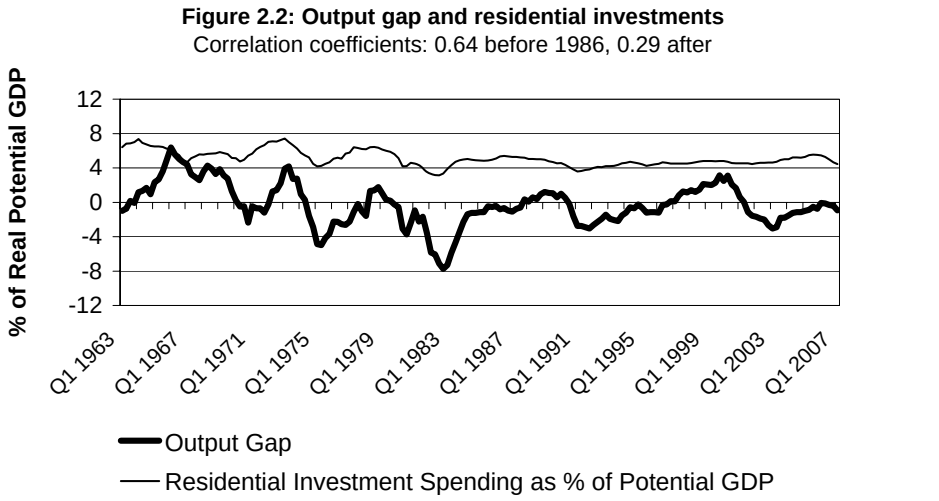
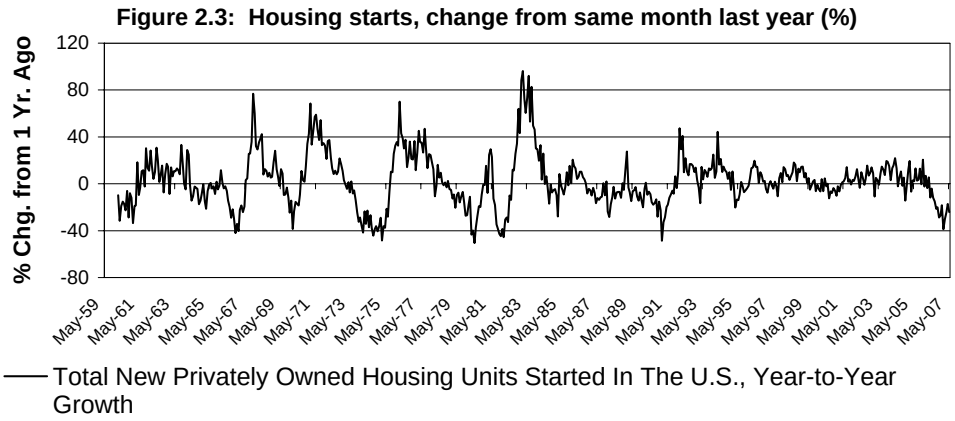


Figure 2.2: Output gap and residential investments



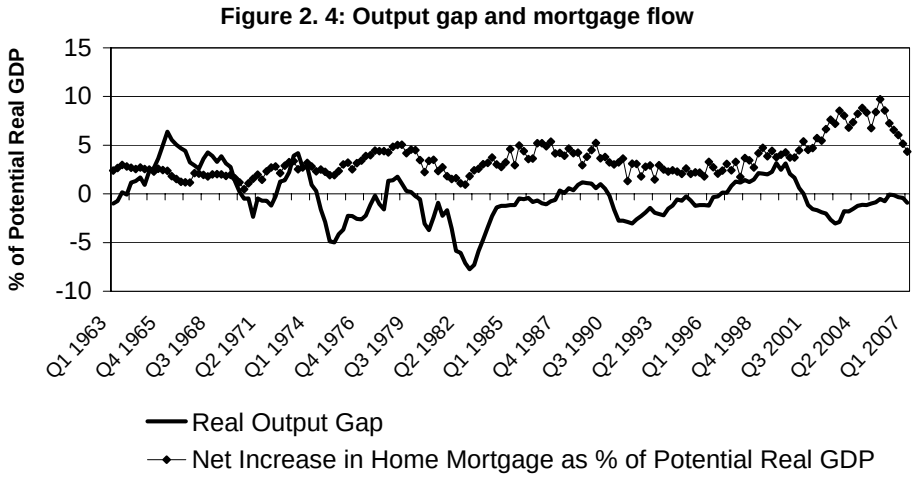
Data Source:
The real GDP and Investment is from BEA's GDP quantity index adjusted to dollar value.
Real Potential GDP is the Congressional Budget Office's Estimates. The series used in this figure is from Table 2-2 in "The Budget and Economic Outlook: Fiscal Years 2008 to 2017"

Figure 2.3: Housing starts, change from same month last year



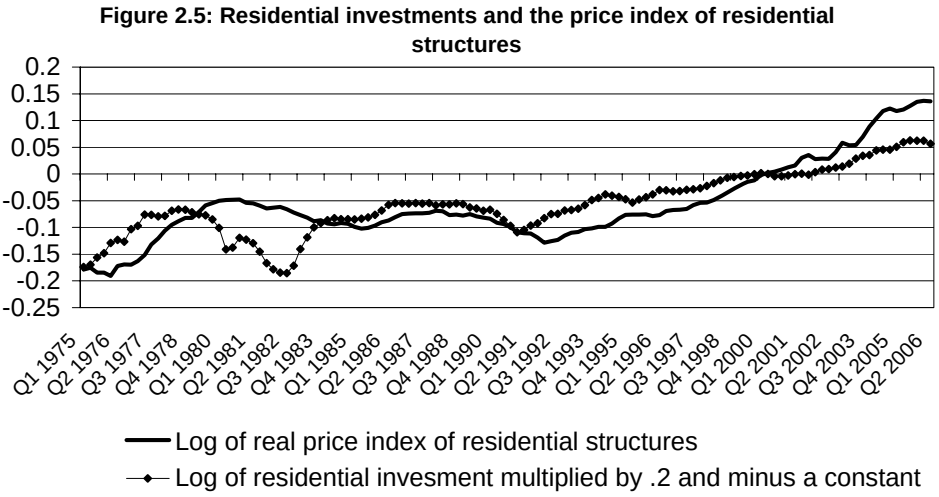
Data Source: U.S. Department of Commerce: Census Bureau
Assessed on June 27, 2007 at Economic Data - FRED®

Figure 2.4: Output gap and mortgage flow



Data Source
The real GDP and Investment is from BEA's GDP quantity index adjusted to dollar value.
Real Potential GDP is the Congressional Budget Office's Estimates. The series used in this figure is from Table 2-2 in "The Budget and Economic Outlook: Fiscal Years 2008 to 2017"
The flow of home mortgages is the net increase in liabilities of households and nonprofit organizations, or FA153165105 of the Federal Flow of Funds Account, that is deflated by GDP price index

Figure 2.5: Residential investments and the price index of residential structures



Data Source:
The real residential Investment is from BEA's GDP quantity index adjusted to dollar value.
The residential structure price index is from Davis and Heathcote (2005). Real series is derived by dividing the nominal series by the price index of GDP.

Chapter 3

Costs and Frictions in the Housing Market: a Macro and Quantitative Perspective

3.1 Introduction

Several recent papers have expanded DSGE (dynamic stochastic general equilibrium) macroeconomic models to include a housing sector (e.g., Aoki *et al.* (2004), Iacoviello (2005a), and Gammoudi and Mendesy (2005)). This development reflects the consensus that housing investments and house prices play prominent roles in macroeconomy.[15] Most of these papers describe the market with a standard stock-flow model that is unable to generate sufficient persistence in the growth of house prices.[16] There also exists ambiguity in terms of motivating the slow adjustment of capital stock.[17] This class of models performs poorly when applied to the data, especially when the house price is included in the estimation. The empirical weakness of the models presents a hurdle for developing DSGE-based macroeconometric models (i.e., Christiano *et al.* (2005) and Smet and Wouters(2003, 2007)) to cover the housing-macro interaction.

This paper constructs a richly specified stock-flow model for housing prices and investments. The model has detailed descriptions of the cost structure in the construction industry, differentiating the short-term and the

[15] At least four reasons explain why the housing sector is important for the macroeconomy. Housing investment is a big contributor to economic fluctuations (Leamer (2007)). The investment is also sensitive to monetary policy shocks (Chart 3 of Bernanke and Gertler (1995b)). House prices influence consumer spending (Greenspan (2001)), as well as financial stability. Trichet (2005) indicates the bust of a house price bubble can often bring more damage to the economy that the bust of a equity price bubble.

[16] For example, Gammoudi and Mendesy (2005) estimates a housing-DSGE model and find that the house prices respond much more rapidly to shocks in the model than in the data.

[17] For example, Aoki *et al.* (2004) motivates the slow adjustment through capital adjustment cost, while Gammoudi and Mendesy (2005) uses investment adjustment cost.

long-term supply elasticity, and admitting possible stickiness in the prices of construction inputs. It also features search frictions and gradual adjustment of house prices in part of the market. The paper estimates the model using maximum likelihood estimations and the US quarterly data between 1985 and 2007, for the purpose of testing the empirical relevance of different features in the model.

The goal of the paper is not to generate precise estimates of the numerous parameters, which is difficult given the complex interaction between various features of the model. Instead, the paper focuses on the loss of empirical capability when individual features is turned off from the full model. The empirical capability is measured by the likelihood function and other measures. The objective of the paper is to use the data to help determine the structure of the model, when the theory is ambiguous. This is one step in developing an optimization-based, yet empirically competent, housing model that may one day be built into a full-fledge DSGE model for macroeconomic analysis in places like central banks.

The full model in the paper includes several well-known costs and frictions. It has a long-run supply curve that is not necessarily flat, which brings into the model the complication of diverging relative prices.[18] The model also features short-term costs of adjusting construction activity, and adjusting the stock of housing capital. In addition, it admits possible rigidity of house price in the asset market. Most of these features are frequently found in empirical housing literature. For example, Topel and Rosen (1988) estimates a model of "internal adjustment costs" in construction industry, while DiPasquale and Wheaton (1994) estimates a mechanism of gradual price adjustment in house prices.

The model also include other features of interest. One of the new features is the demand for inventory, motivated by search frictions between homebuyers and residential builders. To deal with such frictions, builders use inventory to facilitate sales of new homes, which explains why, in the US, a majority of housing construction are started without committed buyers (see data in Kahn (2000)). The stock of inventory of new homes in the market is substantial, equivalent to half a year's sales on an ordinary day.

The model also features sticky construction cost, generated in the model by sticky movements in the prices of construction inputs. The modeling assumption is motivated by the heavy presence of unions in the construction industry, especially in the 60s and 70s, when unions' share of construction

[18] DSGE models, such as those in Smet and Wouters (2007), often abstract from upward sloped long run supply of investment to avoid diverging prices.

workforce was more than 70%. The share has subsequently fallen to about 20% in the 1990s (Allmon *et al.* (2000)). The presence of union affects how the industry responds to changes in market demand. This is due to construction unions' practice of "hiring hall", which often fixes workers' compensation, while at the same time queuing up workers for job opportunities. Such labor market structure makes construction activity more sensitive than it otherwise would be. If the cost is not responsive, output is likely to be volatile, especially given unions' mechanism of placing workers in and out of the queue.

The final model is rich and is likely over-parameterized relative to the information in the data. We therefore do not seek precise estimate for individual parameters. Instead, we test the importance of individual features by removing them from the full model. Smets and Wouters (2007) uses a similar method to test the importance of the features in their large scale DSGE macroeconomic model. We use maximum likelihood estimation on a linearized versions of the models. The observable variables include the real residential investments, the real price of new houses, the real consumption, and the real interest rates. The data are in quarterly frequency between 1985 and 2007Q2. Besides the log-likelihood statistics, we also compare different models' accuracy in forecasting individual time series in the short and in the medium run, their ability to replicate second moments of the data, and occasionally their ability to match empirical impulse responses.

The findings in the paper provide helpful information for modeling housing market. We find that the most important and irreplaceable feature of the model is the gradual adjustment of house price, a mechanism that is emphasized in DiPasquale and Wheaton (1994). In addition, the findings can be interpreted as cautious notes about the interpretation of investment adjustment cost (or IAC for short) that has become a staple feature of the latest DSGE models.[19] Groth and Khan (2006) also finds empirical evidence that questions the IAC cost structure. They are using the U.S. two-digit industry data while we are using aggregate U.S. housing data.

The paper is organized as follows. The remainder of this introduction presents the standard version of the stock-flow model and estimates it with MLE. The model's empirical performance is then discussed. Section 2 presents the full model. Section 3 estimates various versions of the model and discussed their empirical performance. Section 4 has more discussion about features in the model. Section 5 concludes.

[19] Investment adjustment cost is present in Christiano *et al.* (2005), Smet and Wouters (2003, 2007) and many others

3.1.1 The stock-flow model and its empirical performance

Macroeconomic models typically regard housing capital as durable goods and describe its pricing, investment and accumulation with a stock-flow model. In this type of model, the value of a house is determined in the asset market. It is the expected discounted value of the stream of services that it provides over its lifetime, minus maintenance and other costs. The discount factor is the cost of capital, adjusted for risk premiums. The level of investment is determined by intersecting the house price with an upward sloped supply schedule in construction industry (sometimes the increasing marginal cost is modeled as the result of adjustment costs). The flow of investment in each period expands the housing stock. The more recent application of stock-flow model in macro-housing models include Aoki *et al.* (2004), Gammoudi and Mendesy (2005), and Davis and Heathcote (2005).

The theoretical model

The first equation of a stock-flow model is the "stock" equation. It describes the clearance of the market for existing housing stock. The clearance takes the form of an non-arbitrage condition, in which the price of housing services (i.e., the dividend from housing asset) is determined in the rental market. If we use q_h as the notation for asset price, $R(.)$ for rental price, r_{t+1} for a risk-less rate of return, and m as the asset-specific risk premium, we can express the non-arbitrage condition as

$$(1 + r_{t+1} + m)\, q_{h,t} = E_t \left[R\left(c_{t+1}, h_{t+1}, z_{d,t+1}\right) + (1 - \delta_h)\, q_{h,t+1} \right] \tag{3.1}$$

Rental price $R(.)$ is determined by the clearance of the rental market. It is increasing in the total consumption c, which proxies wealth, but decreasing in the size of the existing housing stock h. The rental price is also affected by a demand shifter z_d.

This paper assumes a specific functional form for $R()$, that

$$R\left(c_t, h_t, z_{d,t}\right) = z_{d,t} \frac{c_t}{h_t}$$

This expression of rental price embeds the assumption of a Cobb-Douglas type utility function. The assumption of Cobb-Douglas utility guarantees a balance growth path. The data does support the hypothesis of balance growth, because the share of spending on residential investment tends to revert back to the average around 5-6% over the past 40 years, despite its volatile fluctuations in the short run.

The second equation is the "flow" equation. It describes residential builders' response to changes in house prices. These builders are assumed to operate with an upward-sloped marginal cost curve. In the equilibrium, builders optimally choose to produce at a level that equates the marginal cost to the price of their output. We can therefore back out the level of construction from the following equation

$$q_{h,t} = \frac{1}{z_{h,t}\left(1+\gamma_x\right)^t}\left(d_t\right)^{\psi} \tag{3.2}$$

In this equilibrium condition, the variable d_t is the equilibrium output of new houses. The parameter ψ determines the response of marginal cost to changes in the output. In the denominator, $(1+\gamma_x)^t$ is the housing-sector-specific (relative) level of productivity. The productivity will have a time trend if $\gamma_x \neq 0$.[20]. The marginal cost is also affected by a temporary cost shifter $z_{h,t}$

The stock of housing stock evolves according to

$$\left(1+\gamma_p\right) h_{t+1} = \left(1-\delta_h\right) h_t + \left(d_t + \delta_h h_t\right) \tag{3.3}$$

The parameter γ_p is the growth rate of the population. The total output from the construction industry is $(d_t + \delta_h h_t)$, within which $\delta_h h_t$ replenishes depreciations, and d_t is the new construction.

Similar versions of the stock-flow model underly most DSGE literature's modeling approach for investments in durable goods. In housing literature, the formalization of stock-flow model is often attributed to Poterba (1984).

Estimating the basic model

This section describes the estimation and the estimation results from the basic model. We use maximum likelihood estimation, for which we need to specify the stochastic processes of forcing variables.[21] In the current system, the forcing variables are the demand and cost shifters, as well as interest rate and consumption.

For the demand and cost shifters, the underlying processes are

$$\widehat{z}_{d,t} = \rho_{zd}\widehat{z}_{d,t-1} + \varepsilon_{zd,t} \tag{3.4}$$

[20] Since γ_x is relative to economy-wide technology, it can be either positive or negative.

[21] In fact, we use MLE with boundary constraints on the parameters being estimated. Imposing a boundary, but no prior within the range, our approach can be viewed as a naive version of the Bayesian approach.

$$\widehat{z}_{s,t} = \rho_{zs}\widehat{z}_{s,t-1} + \varepsilon_{zs,t} \tag{3.5}$$

where the hatted variables are expressed as percentage deviations. The random shocks $\varepsilon_{zd,t}$ and $\varepsilon_{zs,t}$ are draws from normal distributions $N\left(0, diag\left(\sigma^2_{\varepsilon,zd}, \sigma^2_{\varepsilon,zs}\right)\right)$.

The aggregate private consumption and interest rates are assumed to be exogenous to the housing market. The real interest rate is assumed to be stationary. Consumption moves around a deterministic time trend with positive growth γ_y. The cyclical components of consumption and real interest rate are not independent from each other, but follow auto-regressive processes with recursive timing assumptions. The recursive assumption is that consumption innovations affect interest rate in the current period, but interest rate shocks affect consumption with one lag. This assumption in timing is to identify the impulse responses to interest rate shocks from a recursive VAR, thus enabling the comparison between the model and the data in term of impulse responses.

Using $\widehat{c}_t$ to denote the percentage deviation of consumption from its trend level, and $\widehat{r}_t$ to denote the difference of (real) interest rate from its normal value, we can capture their stochastic processes with

$$\widehat{c}_t = \sum_{j=1}^{4} \rho_{cc,j}\widehat{c}_{t-j} + \sum_{j=1}^{4} \rho_{cr,j}\widehat{r}_{t-j} + \varepsilon_{c,t} \tag{3.6}$$

$$\widehat{r}_t = \rho_{rc,0}\widehat{c}_t + \sum_{j=1}^{2} \rho_{rc,j}\widehat{c}_{t-j} + \sum_{j=1}^{2} \rho_{rr,j}\widehat{r}_{t-j} + \varepsilon_{r,t} \tag{3.7}$$

where the random shocks $\varepsilon_{c,t}$ and $\varepsilon_{r,t}$ are drawn from normal distributions $N\left(0, diag\left(\sigma^2_{\varepsilon,c}, \sigma^2_{\varepsilon,r}\right)\right)$.

We now proceed to construct a linear state-space econometric model for the purpose of estimations. First we divide variables in equ(3.1), equ(3.2),and equ(3.3) by their respective growth rates in a balance-growth path, thus turning the system into a stationary form.[22] We then linearize the model around the steady state. The resulted linear system is combined with the exogenous stochastic processes to be solved using the solution method described in Klein (2000). The solution to the model describes the transition rule of the state vector in the system, and expresses the jump variables as functions of state variables. The next step is to construct a state-space system by appending to the solution an observation system, which expresses

[22] Of course, the growth rates for $q_{h,t}$, d_t, and h_t are endogenous. We start with assumed growth rates, and the solve the model to confirm that the conjecture is correct.

the observed data as functions of variables and parameters in the theoretical model. With the resulted state-space system and Kalman filter, we can evaluate the likelihoods of the observed time series, based on the assumed distributions. Estimating the model is to search for the combination of parameters that maximizes the likelihood.[23]

The observation equations are

$$\begin{aligned}
\ln\left(\frac{q_t^{new}}{q_{t-1}^{new}}\right) &= \widehat{q}_{h,t} - \widehat{q}_{h,t-1} + \gamma_2 \\
\ln\left(\frac{i_{h,t}}{i_{h,t-1}}\right) &= \left(1 - \frac{\delta_h}{\gamma_p + \gamma_1 + \delta_h}\right)\left(\widehat{d}_t - \widehat{d}_{t-1}\right) \\
&\quad + \frac{\delta_h}{\gamma_p + \gamma_1 + \delta_h}\left(\widehat{h}_t - \widehat{h}_{t-1}\right) + \gamma_1 \\
\ln\left(\frac{c_t}{c_{t-1}}\right) &= \widehat{c}_t - \widehat{c}_{t-1} + \gamma_y \\
R_t - \pi_t &= \widehat{r}_t + r
\end{aligned}$$

The left hand side of the system are the observed data. The Fisher price index of new houses from the Census Bureau is denoted with q_t^{new}. The quantity index of total residential investment (including maintenance, improvement and new construction) from the NIPA account is denoted with $i_{h,t}$. The real consumption from NIPA is c_t. The last equation expresses the real interest rate as the difference between the effective federal funds rate and the growth of the GDP price index, or $R_t - \pi_t$.[24]

The right hand side of the observation system are variables and parameters in the theoretical model. At the end of the first equation is the balance-path growth rate of house prices, $\gamma_2 = \frac{\psi\gamma_y - \gamma_x}{1+\psi}$. At the end of the second equation is the balance-path growth rates of the quantity of investment, $\gamma_1 = \frac{\gamma_y + \gamma_x}{1+\psi}$. Exogenous parameters in the system include γ_y, γ_p, and γ_x. They are the exogenous growth rate of consumption, population, and relative productivity in the construction sector, respectively. The second equation looks more complicated than others. The reason is that the investment data in NIPA includes both new construction and maintenance/improvements. As the result, the growth rate in the data corresponds to, in the model, a weighted average of the growth rate of new investments and the growth rate of the maintenance.

The parameters of estimation interest are (ψ, ρ_{zd}, ρ_{zs}, σ_{zd}^2, σ_{zs}^2, σ_y^2, σ_r^2). There are restrictions on the ranges of individual parameters during the

[23] Hansen and Sargent (2005) provides a detailed description of the method.

[24] All nominal series are turned into real series with the GDP price index as the deflator.

estimations, but there's no prior distributions within those ranges.[25] The parameters that describe the stochastic processes of consumption and interest rate are fixed during the estimations. They are fixed to the averages of estimates that are obtained from a set of trial estimations.[26]

The empirical performance of the model

The only behavior parameter in the basic model is the long-run supply elasticity, which is estimated to be about 2. Figures 3.1 and 3.2 assess the empirical performance of the estimated model. Figure 3.1 compares the model's one-step-ahead forecast with actual realizations. Figure 3.2 presents the comparison between theoretical and empirical impulse responses to interest rate shocks. The theoretical impulse response is from the estimated model, while the empirical responses are from a recursive VAR. The order of variables in the VAR is: consumption, interest, house price, and residential investment.

The comparison between one-step ahead forecast and the subsequent realization is shown in Figure 3.1. The comparison suggests that the model has a difficulty in keeping track of the time series of house prices and investments. The projected changes from the model have little persistence compared to what is observed from the data. The model fails to follow the direction of housing busts and booms, and does not reflect the deep and long decline of house price in the late-1980s. For example, the model continuously predicts house price deflation in the early-2000s, while in fact the prices grew significantly.

Figure 3.2 compares the theoretical and empirical impulse responses of house price and investment to interest rate shocks. The two plots on top are the impulse responses of consumption and interest rate to a 30-bp interest rate shock. The two plots at the bottom show the response of house price and investment. Each subplot has two lines. The thin line is the empirical impulse response and their 95% confidence bounds. The thick lines are IRF from the estimated model. For house price, the model describes a rapid response. The price drops immediately to the bottom and then increases

[25] The range restricts the supply elasticity to be non-negative, and auto-correlation to lie between zero and one. Finally standard deviations are strictly positive.

[26] More specifically, we first perform an initial round of estimations of the model, including the shock parameters. We run the estimations for all different specifications of the model that is presented later in the text. To fix the parameters that describe the shock processes, we take a simple average of all estimates of these parameters and use them for all estimations in the formal analysis. The reason for keeping these parameter unchanged is to focus the comparison to the difference between specifications of the model.

gradually. In the data, however, the price's trajectory is very sticky. It barely moves at all in the quarter of impact. Instead it moves slowly downward, and does not begin to rise until after several quarters. The response of investment to interest rate shock show similar failure of the model. In the data the investment drops gradually. In the model the investment drops immediately to the bottom.

To summarize, the model has difficulty in generating the degree of autocorrelation for house prices and residential investments observed in the data. The model's impulse responses to interest rate shocks are too fast compared to the sluggish movements found in a VAR. Finally, the estimated model significantly over-predicts the volatility of house prices. The growth rate of house price in the model has a standard deviation that is three times as large as that in the data. In the data the standard deviation is 0.74%. The same measure is 2.24% in the simulate series from the model.

3.2 The Augmented Model

This section discusses a model of housing investment with several new features. It is the "full model" of this paper. In the model, the structure of the economy is as follows. Four active players are in the market: homebuyers, the retail arm of residential builders, the construction arm of residential builders, and various contractors. Contractors provide necessary construction inputs (think labor, materials, components, and service of heavy equipment). The construction arm of residential builders works with contractors to create physical residential units. The retail arm purchases the completed units and is responsible for the sales. Households are buyers in the market. They have the alternative option of purchasing existing homes instead of newly constructed ones.

Five costs and frictions are added to the basic stock-flow model. Two of these are the sluggishness in the adjustment of prices, two others are costs in adjusting construction-related activity. The last one is search friction that creates demands for inventory.

We discuss, in greater detail, each of these new features in the following sections. A brief list of descriptions is provided here. One of the price adjustments is the rigidity in the asset market, reflecting the possibility that the houses adjust only slowly to changes in fundamentals. We point out that the asset value of a house can be different from the price of a newly constructed unit. A new house has to be purchased by households to generate rents. Before that, it stays in the inventory of residential builders. Its

price can be influenced by the stock of inventory. Holding all else constant, an increase in inventory reduces the price of new homes.

The second price rigidity is on the price of construction inputs, reflecting our concerns about the flexibility of the cost structure in the construction industry. Historically, the industry has had a high share of unionized workers in its workforce. The industry was well known for its strong practice of "union hiring hall". In these hiring halls, the wage was fixed but workers may have to wait in queue for job opportunities (see Abowd and Farber (1982)[27]). Such labor structures present an form of sticky wage that forces quantity to adjust more than it would if the price was fully flexible.

There is an investment adjustment costs in the housing market, which penalizes rapid changes in construction activity. Topel and Rosen (1988) calls it the "internal adjustment cost" of construction industry. Investment adjustment cost (or IAC) is the more familiar term in DSGE literature. There is another adjustment cost that is the capital adjustment cost (or CAC), which says it is costly for the economy to adjust the stock of housing (think building roads to connect housing and provide water and electricity). These two adjustment costs have different implications on the dynamics of investments. For example, IAC produces hump-shape response of investment, while IAC does not.

The last friction is search. Search friction makes holding inventory mandatory for residential builders, despite the cost of carrying it. Builders can however decide how much inventory they want to carry. The use of inventory in this model is similar to that in the model of sale-facilitating inventory in Bils and Kahn (2000). Furthermore, the option of using inventory as a storage tool give residential builders the tool to arbitrage, dynamically, for profit or to avoid loss.

3.2.1 The asset market

There is a fundamental price of houses. It is the solution to the difference equation describe in 3.1. Using $q_{*,t}$ to denote the fundamental price, we can rewrite the non-arbitrage condition into

$$(1 + r_{t+1} + m)\, q_{*,t} = E_t R\left(c_{t+1}, h_{t+1}, z_{d,t+1}\right) + E_t\left(1 - \delta_h\right) q_{*,t+1}$$

The price of existing homes in the asset market can deviate from the $q_{*,t}$ in the short run. Similar to DiPasquale and Wheaton (1994), the market

[27] Abowd and Farber (1982) does not focus on construction union. It studies the practice of union firing hall in general. Construction union is their example for strong practice of hiring hall.

price $q_{g,t}$ adjusts slowly according to

$$\frac{q_{g,t}}{q_{g,t-1}} = (1+\gamma_2)^{1-\tau}\left(\frac{q_{*,t}}{q_{g,t-1}}\right)^{\tau}$$

where the balance path growth rate of house price equals to γ_2 . The linearized version of this price adjustment equation is $\widehat{q}_{g,t} = (1-\tau)\,\widehat{q}_{g,t-1} + \tau\widehat{q}_{*,t}$, which is identical to that used in DiPasquale and Wheaton (1994). These authors find strong support for gradual adjustment of the house price from the statistical superiority of the model with lagged house prices.

The consensus from the literature on house price dynamics suggests that house prices exhibit a positive serial correlation in the short-run and a mean reversion in the long run (see Cho (1996) for a survey of the literature). Several mechanisms may create gradual adjustments, which can be classified into three classes. The first two classes are described in DiPasquale and Wheaton (1994) and in their references. One is learning under imperfect information, for example, time-consuming search that may result in stochastic sale time that would delay the seller's recognition that fundamentals have changed. The second is the existence of rule of thumb households that form expectations by looking at the past. The third class is related to credit and liquidity constraints. Krainer *et al.* (2005) describe an interesting model for the downward stickiness of house price. Their model involves search friction and a heterogenous willingness to pay for the same house. Hoping to find someone who is willing to pay a higher price, home owners may hold up the sales of their houses even if they recognize the downturn of the market. Past prices enter the current and future prices because they determines household's debt, or the extent of "debt overhang".

In DiPasquale and Wheaton (1994), the estimated speed of adjustment is 29% in the first year following a shock. This translates into a quarterly adjustment rate of slightly more than 7%. McCarthy and Peach (2002) use a similar adjustment mechanism in their vector error correction model, and estimate the adjustment rate to be 5% per quarter.

3.2.2 Supplier of construction inputs

This section describes the pricing decision of contractors and the equilibrium dynamics of construction cost. Contractors are suppliers of various construction inputs (think labor, construction materials and components, and service of heavy equipment). These inputs are not perfect substitutions. The optimal proportion of combination depends on the degree of

substitutability and relative price. As the result, individual suppliers face a demand that is decreasing in the relative price of their product.

We use monopolistic competition and infrequent price adjustment to motivate the possible stickiness in construction costs. We assume that the suppliers of different inputs have monopolistic rights to their own products. They preset a price before production (think labor unions entering a contract). Following the Calvo-type probabilistic approach, we assume individual suppliers have a random opportunity to re-negotiate a price contract in each period. The chance for re-optimizing is (1- ξ). Under these assumptions, the equilibrium features slowly adjusting construction costs.

In the equilibrium, there exists a mark-up between what is paid by residential builders and what would be called the "true" cost of building a house. The cost facing the builders is the actual cost plus a monopolistic markup charged by contractors,

$$c_{c,t} = \frac{d_t^{\psi}}{z_{h,t}\left(1+\gamma_x\right)^t} + x_{cc,t}$$

where the total cost incurred to the builder is $c_{c,t}$, the "true" marginal cost of production is $\frac{d_t^{\psi}}{z_{h,t}(1+\gamma_x)^t}$. The markup over the cost is $x_{cc,t}$. In the marginal cost function, the variable d is the level of new construction in period t. The marginal cost is increase in the output if $\psi > 0$. The marginal cost function has a trend component captured by $(1+\gamma_x)^t$, where the trend growth rate is γ_x. The marginal cost also has a cost shifter $z_{h,t}$.

Secondly, the markup responds to changes and expected changes in $c_{c,t}$. The following is the linearized version of the relation,

$$-\frac{(1-\xi)\left(1-\frac{1}{1+r}\xi\right)}{\xi} x_{cc,t} = \pi_{cc,t} - \frac{1}{1+r} E_t \pi_{cc,t+1}$$

The variable $\pi_{cc,t} = c_{c,t} - c_{c,t-1}$ is the price change. Since no nominal price is involved here, the change is in real terms. The parameter ξ is the Calvo stickiness parameter, which, in the theoretical model, literally means the chance that an individual supplier cannot change the price. Higher ξ means more stickiness.

When ξ becomes zero, the coefficient $\frac{(1-\xi)\left(1-\frac{1}{1+r}\xi\right)}{\xi}$ becomes infinity, which means $x_{cc,t}$ has to be zero at all time. In other words, the construction cost reflects exactly the true construction cost. In this case, the two equations are reduced into one, which is $c_{c,t} = \frac{d_t^{\psi}}{z_{h,t}(1+\gamma_x)^t}$.

3.2.3 Construction arm of residential builders

Besides the construction cost, the construction arm of residential builders also face a cost in adjusting construction activity. With the adjustment cost, the short-run supply elasticity is smaller than the long-run elasticity. Topel and Rosen (1988) finds that the response of housing starts to a given price change is gradual. Instead of jumping to a level where the new price intersects with the long term supply, housing starts move gradually to reach the new level. They attribute the difference to what they call "internal adjustment cost," a cost that penalizes rapid change in output. This internal cost can be interpreted as planning cost, or bottlenecks in supply chains. These costs may disappear in the long run, but will still have an impact on production and prices.

The builder's profit maximization problem is

$$\max_{d_{t+s}} \sum_{s}^{\infty} \frac{1}{\Pi_s \left(1 + r_{t+1+s}\right)} \left[q_{c,t+s} \left(1 - \Phi_1 \left(\frac{d_{t+s}}{d_{t-1+s}} \right) \right) d_{t+s} - c_{c,t+s} d_{t+s} \right]$$

where the level of output is d, the price of output received by the builder is q_c, the cost of capital is r. The adjustment cost function is $\Phi_1 \left(\frac{d_{t+s}}{d_{t-1+s}} \right)$. It is assumed that in the balance-growth path $\Phi_1 (.) = \Phi_1' (.) = 0$ and $\Phi_1'' (.) = \varphi_{hs}$. The construction industry faces a convex cost if $\varphi_{hs} > 0$, in which case the industry has the incentive to smoothen out its construction plan over time.

The optimal condition for the profit maximization problem is

$$\begin{aligned} 1 - \frac{c_{c,t}}{q_{c,t}} &= \Phi_1 \left(\frac{d_t}{d_{t-1}} \right) + \Phi_1' \left(\frac{d_t}{d_{t-1}} \right) \frac{d_t}{d_{t-1}} \\ &\quad - E_t \frac{1}{1 + r_{t+1}} \frac{q_{c,t+1}}{q_{c,t}} \Phi_1' \left(\frac{d_{t+1}}{d_t} \right) \left(\frac{d_{t+1}}{d_t} \right)^2 \end{aligned} \tag{3.8}$$

In DSGE literature, what Topel and Rosen (1988) call the "internal adjustment cost" is known as "Investment adjustment cost (IAC)". IAC is used in DSGE literature primarily for its ability to keep track of investment series and generate hump-shaped response to shocks.(Christiano *et al.* (2005), Smets and Wouters (2007), Justiniano and Primiceri (2006)). In housing models, Gammoudi and Mendesy (2005) use the adjustment cost to motivate slow stock adjustment of housing capital. From now on, I will follow the convention in DSGE literature and refer to such cost as investment adjustment cost.

Groth and Khan (2006) find that the evidence from US two-digit industry (1) does not support the IAC cost structure, (2) implies a very high elasticity of investment with respect to the shadow price of capital relative to aggregate estimates.

3.2.4 Retail arm of residential builders

This section focuses on the retail arm of a residential builder. One salient feature of the housing market is the large stock of inventory of new homes. According to the US Census Bureau, a typical day in the US will have 5.5 months' sales of houses looking for buyers. The stock of inventory is pro-cyclical with a lag, as it can takes a long time for inventory to clear after a housing slump. Maisel (1963) found that the inventory of unsold new homes has the most rapid impact on new starts, which he attributes to the carrying costs. The author also expressed concern about the possibility of a large backlog of unsold units discouraging lenders from lending to builders when needed. The current housing slump sees an unprecedented level of inventory buildup of unsold new homes.[28] Many now view it as a major concerns.[29] Housing inventory has not caught much attention in the macro-modeling of housing. Kahn (2000) provides a narrative description of the dynamics and composition of inventory in the housing market. Krainer (2006) found supply-to-sale ratio to be a better indicator than house price for predicting future investments. McCarthy and Peach (2002) included inventory stock in their estimated short-run housing supply equation.

Given the large cost of holding inventory, what can explain the need for such a large stock of inventory in the market? Production lag is one explanation, production smoothing is another. Residential construction, however, is more volatile than sales. It is therefore questionable whether production smoothing is the main justification for inventory in this market. Inventory is also useful for satisfying the diversity in demand. The housing market features heterogenous buyers and heterogenous houses. A successful trade requires a seller's goods to match reasonably well with the buyer's demands. Maintaining an inventory provides households with options. If search involves cost, then an inventory is valuable for its ability to increase the chance of a successful search.

[28] As the aftermath of the unprecedented housing boom, the inventory buildup has also exceeded all previous records. In terms of inventory to sales ratio, August 2007 had 7.5 months sales in stock; the average is 5.5. The ratio is still slightly lower than that seen in the early 1980s.

[29] For example by Bernanke in his Jacksonhole remark of August 2007 (Bernanke (2007)).

The paper captures the search friction with a "matching" function. Let the number of successful match be s_t, let the stock of inventory be v_t, and let households' search effort be b_t, the matching function is

$$s_t = v_t \left(1 - e^{-\frac{b_t}{v_t}}\right)$$

The matching function has the following features: 1. $s_t < v_t$, meaning that the number of sales will be strictly smaller than the stock; 2. Sales are increasing in both v_t and b_t ; 3. the matching function has a constant return to scale. From the builders' perspective, the probability that an individual house gets sold in a period t is $\frac{s_t}{v_t} = 1 - e^{-\frac{b_t}{v_t}}$.

Before modeling the investment decision in inventory, it may be helpful to describe in detail the flow of funds and goods: In period t , the retail arm of residential builders pays the construction arm for constructing new houses d_t. The price they pay is $q_{c,t}$ per unit. The houses are constructed during the period and becomes part of v_{t+1}, or the retailer's inventory in the next period. In period $t+1$, part of v_{t+1} will be sold. The remainder goes into period $t+2$ as inventory, together with new productions in $t+1$.

The retail arm of residential builders uses two instruments for profit maximization. One is $q_{h,t}$, the market price of new houses. The other instrument is d_t, the order of new constructions. The following is the maximization problem

$$\max_{(d_t, q_{h,t})} E_t \sum_{s=0}^{\infty} \left(\frac{q_{h,t+s} v_{t+s} \left[1 - e^{-\frac{b_{t+s}}{v_{t+s}}}\right] - q_{c,t+s} d_{t+s}}{\Pi_s \left(1 + r_{t+1+s}\right)} \right)$$

subject to

$$v_{t+1} = v_t e^{-\frac{b_t}{v_t}} + d_t \tag{3.9}$$

and

$$b_t = \arg\max_{b_t} \left(q_{b,t} - q_{h,t}\right) v_t \left(1 - e^{-\frac{b_t}{v_t}}\right) - b_t \left(\omega_b * q_{c,t}\right)$$

The first constraint describes the accumulation of the stock of inventory. The second constraint is the reaction function from households that have the alternative option to buy from the market of existing homes. A buyer arbitrages the price difference between new homes and existing homes. The variable $q_{b,t}$ is the willingness of a buyer to pay for a new house before taking into account search cost. For the buyer, the per-unit gain from a successful

match is the difference between the willingness to pay and the price, or $q_{b,t} - q_{h,t}$. The total gain is unit gain multiplied by the number of successful matches, or $v_t \left(1 - e^{-\frac{b_t}{v_t}}\right)$. The total gain minus total search cost is the net gain, which is the objective function of the buyer. The total search cost is $b_t (\omega_b * q_t)$. In this cost function, the search effort is b_t, and the cost per unit of effort is ω_b fraction of the price of the house.

With θ_t denoting $e^{-\frac{b_t}{v_t}}$, the buyer's optimality condition is

$$(q_{b,t} - q_{h,t})\, \theta_t = \omega_b * q_{c,t}$$

This optimality condition equates the expected marginal return to search effort (the left hand side of the equation) to the marginal cost of the effort. The return is increasing with the difference between $q_{b,t}$ and $q_{h,t}$. Meaning that the return is high when the market price is low relative to the buyer's valuation. The return is also high if θ_t is high, which indicates a "buyer's market": slow sale and large inventory.

Sellers take into account the relation between the price and the speed of sales when setting their price. In the equilibrium, they choose a level of price and sales ratio so that the allocation of total gain between seller and buyer satisfies:

$$\frac{q_{h,t} - q_{c,t}}{q_{b,t} - q_{h,t}} = \frac{(1 - \theta_t)}{\theta_t}$$

The left hand side of the equation is the ratio of "seller's markup" over "buyer's net gain". Seller's markup is the numerator $q_{h,t} - q_{c,t}$, or the difference between the sales price of a house and it acquisition cost. Buyer's net gain is the denominator, or $q_{b,t} - q_{h,t}$, the difference between the buyer's valuation and the sales price. From the left hand side we can see that the relative allocation is determined by θ_t. Recall that $\theta_t = e^{-\frac{b_t}{v_t}}$, the chance that a house fails to sell. A high θ_t indicates a buyer's market, which increases the buyer's net gain relative to seller's markup. A low θ_t indicates a seller's market, which increase the seller's markup relative to the buyer's net gain.

The two preceding equations can be put together to express $q_{h,t}$ and $q_{b,t}$ in term of $q_{c,t}$ and θ_t. They are

$$q_{h,t} - q_{c,t} = q_{c,t} \frac{(1 - \theta_t)}{\theta_t^2} \omega_b \tag{3.10}$$

and

$$q_{b,t} - q_{c,t} = q_{c,t} \frac{1}{\theta_t^2} \omega_b \tag{3.11}$$

The above two equations are in fact arbitrage conditions. Consider the case when there is no search fiction, i.e., $\omega_b = 0$ (zero search cost means infinite search effort and therefore immediate sales), we have $q_{h,t} = q_{c,t} = q_{b,t}$: the production cost equals the market price and the buyer's willingness to pay.

The last optimal condition is the Euler equation of investing in inventory

$$(1 + r_{t+1}) = E_t \left[\left(\frac{1}{\theta_{t+1}} - 1 \right)^2 \omega_b + 1 \right] \frac{q_{c,t+1}}{q_{c,t}} \tag{3.12}$$

Again, if we take out search friction from the model, we have $(1 + r_{t+1}) = E_t \frac{q_{c,t+1}}{q_{c,t}}$, which indicates a cost-less storage technology.

The Euler equation has two notable properties. One is that

$$\frac{\partial E_t \theta_{t+1}}{\partial r_{t+1}} < 0$$

which states that higher interest rate increase the cost of carrying inventory, thus reducing θ_t, meaning that the sales to inventory ratio has to increase to reduce the average holding time.

Secondly

$$\frac{\partial \theta_t}{\partial E_t \frac{q_{c,t+1}}{q_{c,t}}} > 0$$

which states that expected increases in replacement cost increase the θ_t. It other word, it reduces the speed of sales. This happens because sellers have the incentive to arbitrage over time: if the product's replacement cost is expected to increase, the incentive to sell the product today is lower, since expected appreciation remedies the cost of carrying the inventory.

3.2.5 Homebuyers

We now discuss $q_{b,t}$, the willingness of buyers to pay for a new house. It is defined as

$$q_{b,t} = \frac{q_{g,t}}{1 + \Phi_2 \left(\frac{d_t}{h_t} \right)}$$

The willingness to pay equals to the price of houses in the market for existing homes, or $q_{g,t}$, adjusted by an adjustment cost $\Phi_2(.)$. The adjustment cost, assumed to be paid by homebuyers, is increasing in the speed of development. This captures the short-term cost of connecting new development with roads and various types of services such as electricity, water, and cable. This cost, in essence, is the capital adjustment cost (or CAC) that is

commonly used in neo-classical models to generate Tobin's Q. In housing literature, Jud and Winkler (2003) define $\frac{q_{g,t}}{q_{h,t}}$ as the Tobin's Q in the housing market and find it positively correlated with residential investments. Technically, this paper assumes that $\Phi_2''(.) = \varphi_q \geq 0$ in the balance growth path. The findings in Jud and Winkler (2003) will be replicated with a positive φ_q.

3.2.6 The linearized model

Before applying maximum likelihood estimations to evaluate the maximized likelihood statistics associated with different versions of the model, the paper needs to construct a linear state-space econometric model. As the first step of the linearization, we stabilize the system by dividing variables that have a trend component with the corresponding trend. We then solve the stationarized system for its steady-state, and linearize the model around the solution. The resulted linear system is a system of difference equations that can be solved numerically. We then append a observational equation to relate the model with the observed data.

We use the same set of observed variables as the estimation of the basic model, i.e., the consumption growth, the (real) interest rate, the growth rate of (real) house price, and the growth rate of the quantity index of residential investment, taking into account the allocation between maintenance and new construction.

Here we present the linearized system only. A technical appendix can be requested from the author.

From the asset market

$$\begin{aligned}
\widehat{q}_{*,t} &= (r+\delta_h-\gamma_r)\, E_t\left(z_{d,t+1}+\widehat{y}_{t+1}-\widehat{h}_{t+1}\right) \\
&\quad + [1-(r+\delta_h-\gamma_r)]\, E_t\widehat{q}_{*,t+1} - r_{t+1} - z_{d,t} \\
\widehat{q}_{g,t} &= (1-\tau)\,\widehat{q}_{g,t-1} + \tau\widehat{q}_{*,t}
\end{aligned}$$

If asset price adjusts immediately, $\tau = 1$ and therefore $\widehat{q}_{g,t} = \widehat{q}_{*,t}$.

From the suppliers of construction inputs,

$$\begin{aligned}
\widehat{c}_{c,t} - \widehat{x}_{cc,t} &= \psi\widehat{d}_t - z_{s,t} \\
\beta E_t\pi_{cc,t+1} &= \pi_{cc,t} + \frac{(1-\xi)\left(1-\frac{1}{1+r}\xi\right)}{\xi} x_{cc,t}
\end{aligned}$$

From the construction arm of residential builders we have

$$\begin{aligned}\varphi_{hs}\widehat{d}_t &= \frac{1}{(1+\gamma_1)^2(1+\beta)}\left(\widehat{q}_{c,t}-\widehat{c}_{c,t}\right)\\ &\quad+\frac{\varphi_{hs}}{1+\beta}\widehat{d}_{t-1}+\varphi_{hs}\left(1-\frac{1}{1+\beta}\right)E_t\widehat{d}_{t+1}\end{aligned}$$

where $\beta=\frac{1}{1+r}$ is the steady-state discount factor. In the absence of investment adjustment cost (i.e., $\varphi_{hs}=0$), the equation is reduced into $\widehat{q}_{c,t}=\widehat{c}_{c,t}$.

From the retail arm of residential builders

$$\begin{aligned}\widehat{v}_{t+1} &= \frac{\theta}{1+\gamma_1}\left(\widehat{v}_t+\widehat{\theta}_t\right)+\left(1-\frac{\theta}{1+\gamma_1}\right)\widehat{d}_t\\ \widehat{r}_{t+1} &= E_t\left(\widehat{q}_{c,t+1}-\widehat{q}_{c,t}\right)-2\left(\frac{r-\gamma_2}{1-\theta}\right)E_t\widehat{\theta}_{t+1}\\ \widehat{q}_{h,t}-\widehat{q}_{c,t} &= -\left(1-\frac{1}{\frac{r-\gamma_2}{1-\theta}+1}\right)\left(\frac{\theta}{1-\theta}+2\right)\widehat{\theta}_t\\ \widehat{q}_{b,t}-\widehat{q}_{c,t} &= -\left(1-\frac{1}{\frac{r-\gamma_2}{(1-\theta)^2}+1}\right)2\widehat{\theta}_t\end{aligned}$$

where θ solves $\frac{\omega_b}{r-\gamma_2}=\left(\frac{\theta}{1-\theta}\right)^2$ and is interpreted as the chance that a house fails to sell in the steady state. Without search friction, the system becomes $\widehat{v}_{t+1}=\widehat{d}_t$, $\widehat{r}_{t+1}=E_t\left(\widehat{q}_{c,t+1}-\widehat{q}_{c,t}\right)$, and $\widehat{q}_{h,t}=\widehat{q}_{c,t}=\widehat{q}_{b,t}$ replace the system above, which would have been identical as setting the search cost parameter ω_b as zero. If storage technology is not allowed, then $\widehat{r}_{t+1}=E_t\left(\widehat{q}_{c,t+1}-\widehat{q}_{c,t}\right)$ drops out of the equation completely.

From a households's arbitrage condition

$$\widehat{q}_{b,t}=\widehat{q}_{g,t}-\varphi_q\left(\widehat{d}_t-\widehat{h}_t\right)$$

If the system has no capital adjustment cost, we will have $\varphi_q=0$ and $\widehat{q}_{b,t}=\widehat{q}_{g,t}$.

The accumulation of housing asset is

$$\widehat{h}_{t+1}=\frac{1}{1+\gamma_p+\gamma_1}\widehat{h}_t+\left(1-\frac{1}{1+\gamma_p+\gamma_1}\right)\left(\widehat{v}_t-\frac{\theta}{1-\theta}\widehat{\theta}_t\right)$$

when there is no search friction, $\theta=0$ and $\widehat{h}_{t+1}=\frac{1}{1+\gamma_p+\gamma_1}\widehat{h}_t+\left(1-\frac{1}{1+\gamma_p+\gamma_1}\right)\widehat{v}_t$

and $\widehat{v}_t = \widehat{d}_{t-1}$.

The system is completed by adding the four equations that describe the stochastic processes of exogenous shocks.

3.3 Compare the Estimated Models

This section discusses the empirical relevance of the new features added to the standard stock-flow model. The strategy is to compare the empirical performance of the full model to a set of smaller models. Each of these smaller models has a single feature that is turned off from the bigger one. Since the only difference between the full model and a smaller model is the elimination of one feature, the change in empirical performance reflects the empirical relevance of the feature that is turned off.

We examine "empirical performance" with four criterions, one for each of the following four sub-sections. The first sub-section looks at the log-likelihood statistics, the second sub-section at the accuracy of one-step-ahead forecasts for prices and for investments, the third one at the accuracy of medium-run forecast, and the last one at the model's ability to replicate second moments in the data

3.3.1 Log-likelihood statistics

This sub-section uses the fall in log-likelihood from the full model to the smaller model as a metric to measure the empirical importance of the cost/friction that is turned off. For statistical inference, we first use the log-likelihood ratio test (LRT) between the parental model and the smaller model. With LRT, twice the difference in log-likelihoods is compared to the 5% critical value of a Chi-squared distribution with one degree of freedom. An alternative statistical inference is the SIC (Schwarz Information Criterion). SIC will select the full model over the smaller one if the reduction in log likelihood is greater than $\frac{\ln(N)}{2}$.

Table 3.1 presents the results from the experiments. The first column of the table is from the full model. The estimate paints a picture of the housing market that has: a) a high degree of price rigidity in the asset market (the speed of adjustment is 12% per quarter); b) elastic long-term supply (about 6% increase in supply for 1% increase in price); c) short-run adjustment costs in both the supply side and the demand side; and d) construction costs that appear to be very flexible, 80% of the suppliers can adjust their price in one quarter.

Then, we start to turn off individual features from the full model. First, we look at the two price sluggishness. The first column assumes immediate adjustment in the asset market. The log-likelihood drops by more than 60. Twice this reduction is 120, which is well beyond the critical value for either the LRT or the criterion of the SIC, which are 3.8 and 4.9, respectively.[30] By turning off the sticky prices for construction inputs, the log-likelihood is reduced by 3, which is barely significant.

We proceed to the two quantity adjustment costs. By eliminating the IAC (or investment adjustment cost), the log-likelihood is reduced by 13. Eliminating the CAC (capital adjustment cost) leads to a reduction of 8.

In the estimations, the search friction parameter ω_b has been calibrated to the level so that, in the steady-state, the model generates a quarterly sales-to-inventory ratio of 0.55, the average ratio in the US market for new, single-family residential units. Slightly varying the value of this particular parameter does not change the likelihood statistics much. Of more importance, however, is the removal of the residential builders' option to invest in inventory. The removal forces builders to clear their productions in each period. Under this assumption, the estimation finds a radically different economy that fits the data equally well (the reduction in likelihood is almost zero). The estimates describe a rigid economy. More than 80% of construction suppliers do not have the option to reset their prices, and a large adjustment cost facing households. We believe that investment in inventory is a matter of fact, so we choose to work on the full model that allows this option.

The fit between the model's impulse response and the empirical response is shown in Figure 3.6. The fits improve compared to the simpler model (Figure 3.2).

3.3.2 One-step-ahead forecast

In Figure 3.3, we present a set figures to illustrate the match between the one-step-ahead forecast and the actual realization for house prices and for residential investment. The first column is for house price and the second column is for residential investment. The figures have five rows, each corresponding to one model. The ordering, at which the models are presented, is the same as that in Table 3.1. The full model is the top row. The second

[30] Schwarz information criterion (SIC) has the formula

$$SIC = -2\ln(L) + k\ln(N)$$

where $\ln(L)$ is log-likelihood an, k is number of parameter and N is number of obervation.

row assumes immediate adjustment of house price in the asset market. The third row removes sticky construction cost from the full model. The fourth row turns off IAC. The last row turns off CAC.

Figure 3.3 is helpful for understanding how individual frictions or costs affect the model's ability to keep track of house prices and residential investments. It demonstrates that the price rigidity in the asset market is, by far, the most important feature. Comparing row 2 to row 1, we see that taking away the price rigidity worsens the fit for both prices and investments. For example, in the housing boom during the early 2000s, the model without price rigidity forecasts a price drop instead of an increase. It also misses the investment boom during the same period. Again during the late 2006 and early 2007, the removal of price rigidity cause the model to miss the magnitude of price and investment drop.

The figure also suggests that the feature of sticky construction cost does not improve the fit for either prices or for investments. On the other hand, the two adjustment costs, IAC and CAC, both improve the fit of model in term of residential investment.

3.3.3 Medium-run forecast

This section examines the accuracy of different models in term of medium-run forecasts. In the exercise, the estimated models are made to stop at a given quarter and forecast the window of the next eight quarters, using only the current and the past data. For example, when the model stands at 2005Q2, it forecasts the house price and investment for every quarter between 2005Q3 and 2007Q2. We then compare the forecast to the actual realization, to derive a measure of forecast error. We repeat the exercise using the second quarter of every year between 1998 and 2005 as the stop-and-forecast point. The measure of forecast error is the Root Mean Square Error (RMSE), which is square root of the mean of squared errors. The RMSE from an estimated unrestricted VAR(2) is used as the benchmark for comparison.

Table 3.2 presents the results from the exercise, and shows that the estimated full model has the forecast ability that is comparable to an unrestricted VAR(2). In averaging over the time horizon of eight quarters, the estimated model has a RMSE that is 8% higher than that from a VAR(2) in the case of house price, and 1% higher in the case of residential investment. This does not sound impressive unless we consider that the standard stock-flow model has an RMSE that is 124% higher than VAR(2) for the price, and 52% higher for the investment.

Now we proceed to compare different specification of the housing models. The comparison shows that the gradual price adjustment in the asset market is the key to improving forecast accuracy. Removing rigidity in house price increases the RMSE substantially for both price and for investment (compare row 4 to row 3). Turing off IAC (investment adjustment cost),on the other hand, has little impact as long as CAC (capital adjustment cost) is present (compare row 5 to row3). Further turning off CAC, presented in row 6, has a greater impact for investments, but not for price.

An interesting comparison is between row 7 and row 2. Row 7 is from a model in which IAC is the only extra feature. This is similar to how Christiano *et al.* (2005) and Smet and Wouters (2007) model investments in their DSGE model. Row 2 corresponds to the full model. The comparison between row 7 and row 2 shows that, despite IAC's ability to improve the forecast for investments, it does virtually nothing to improve the forecast for house prices. This raises the interesting question about why investment adjustment cost fails to generate a better forecast for prices. Neither Christiano *et al.* (2005) nor Smet and Wouters (2007) have capital prices in their estimation. That is probably why the failure of investment adjustment cost on the price front is not reported.

Further exploration raises more questions about IAC. When the construction industry features IAC, the level of investment tends to move slowly. Similar gradual movements in the investment, however, can also be generated by a slowly adjusting house price, along an positive-sloped marginal-cost curve. Is it possible that the price rigidity would be mistaken as IAC when the price is assumed to adjust immediately?

3.3.4 Second moments

This subsection compares the second moments predicted by the models to those found in the data. We look at both the standard deviations and the correlation coefficients between price and investment.

The middle panel of Table 3.1 shows the standard deviation of the quarterly growth rate of the new house price and residential investment. The models are able to produce realistic-looking standard deviations as long as the price rigidity remains in the model. Once price rigidity is removed, the models predicts a volatility for house price that is far too high. This suggests that price rigidity is irreplaceable when it comes matching the volatility for house prices.

The bottom panel of Table 3.1 presents the correlation coefficients between the four-quarter growth in the price and the four-quarter growth in

the investment. From the data, the correlation coefficient is 0.5. In the simulated data from the estimated models, the correlation coefficients reach a height of 0.5 only when there is no rigidity in house prices. When there is rigidity, however, the coefficients drop to 0.15-0.27. We perform further experiments by stripping the model of all other features except for the rigidity in the asset price. The exercise confirms price rigidity by itself is the main source for the reduction in the price-investment correlation.

Why does rigidity in house price reduce the price-investment correlation? One explanation is that rigidity in house price slows down the response of investment to demand changes, thus reducing the volatility of the investment. To compensate the loss in investment's volatility, the model assigns more volatile supply shocks. Since demand shocks are the main source of positive co-movement, while supply shocks create negative co-movements, the correlation coefficients drops as the result of greater supply shocks. Variance decomposition provides supportive evidence. Without price rigidity, the contributions from demand shocks to the variance of the investment is equal to those from supply shocks. With rigidity, however, the contributions from demand shocks becomes smaller, while supply shocks dominate.

3.4 Some Discussions on the New Features of the Model

We now use simulation exercises to explore the properties of the models. A set of experiments were conducted to study the role of individual features in the model. The following is a summary of the findings:

3.4.1 Investment adjustment cost versus price rigidity

DSGE literature that emphasizes empirical performance often uses production inertia in the capital goods sector, i.e., IAC (investment adjustment cost), to keep track of the time series of investment. The existence of IAC makes it costly to change the level of production, thus giving producers the incentive to smoothen out their production plan. The result is greater extent of auto-correlation in the models' prediction. Such prediction matches data better than those from an alternative model without IAC. The analysis in this paper, however, raises questions on the use of IAC. The previous sections have shown that the IAC is not the only mechanism that can help the model to achieve better predictions on investment. Rigidity in house price also does.

Furthermore, rigidity in house price generates better forecast for house price as well. IAC, on the other hand, improves the forecast only for investment, but not for prices. The failure of IAC in providing better forecast of prices is rather unsettling. We might think that since a model with IAC gives better prediction about the future movements of investments, it must also has a better prediction about future prices. That does not appear to be the case.

Another often-cited advantage of using IAC in the model of investments is its ability to generate a hump-shape response of investment to shocks (Smets and Wouters (2007)). Price rigidity can replace IAC in this aspect to certain extent. Figure 3.4 presents the impulse response of the housing market to an interest rate change. One of the models has IAC as the only extra feature. The other model replaces IAC with price rigidity. Both models are seen to generate a slowly adjusting trajectory for housing investment. Price rigidity generates a smooth trajectory for price as well. In the case of IAC, however, price drops immediately to the bottom, contrary to the data. This again highlights IAC's inability to keep tract of house price, despite its apparent success in tracking investment. In comparison, the model with price rigidity succeeds in giving both a smoother movement in house prices, and a hump-shape investment. It is easy to see why this is the case. With price moving slowly, the construction industry responds gradually, in adjusting their construction activity.

3.4.2 Sticky construction costs, volatility of investments, and price rigidity

This subsection asks two questions: a). how does the stickiness in construction costs affect housing market's response to shocks? b). Is sticky cost responsible for price rigidity in the market? We find that sticky cost makes investment more sensitive, but it has little impact on the dynamics of price.

The observations are from Figure 3.5, which compares a simple stock-flow model's impulse responses, to an interest rate shock, with those from a model with sticky costs. The latter model has a Calvo stickiness at 0.65, which literally means the prices of 65% of construction input can not be re-adjusted in any a quarter. The figure shows that the stickiness in construction cost increases the immediate reaction of the investment by three times. It does not, however, affect the trajectory of price. In particular, it does not slow down the movement of house prices. The price still falls to the bottom at the period when the shock occurs.

The inability for sticky cost to create rigidity in house price shall not

be surprising. The quarterly flow of new homes into housing stock is small in relative size, accounting for only 0.6% of existing stock. As for house price in the theoretical model, it is the total unit of stock that counts. The rigidity in house price can not come from the construction industry, it must have more substantial sources.

3.4.3 Inventory and changes in interest rate

The option for residential builders to hold inventory increase the impact of an interest rate change on housing investment, especially when house price adjusts slowly. With gradually adjusting house price, an increase in interest rate has two effects on residential builders. First, higher interest rate increases the cost of carrying the inventory. Second, the price is expected to decline further. The expected depreciation imposes another layer of cost for holding the inventory. A residential builder therefore has the extra incentive to draw down inventories. The drawing down can be accomplished by two means: reducing price, or reducing orders for new investment. Builders in the model use both tactics. Figure 3.7 shows the impulse response, to an interest rate change, from two models. One of the model has sticky price and allows builder to invest/disinvest in inventory. The other model removes inventory from the model all together. The comparison shows that, when given the opportunity to use inventory to optimize inter-temporarily, residential builders react more strongly in their orders of new housing units. House price also drops, but only by a small amount.

3.5 Conclusion

This paper presents a richly specified stock-flow model for the housing market and empirically examines the importance of the different costs and frictions in the model. The model has detailed descriptions of the cost structure in the construction industry, differentiating the short-term and the long-term supply elasticity, and admitting possible stickiness in construction cost. It also features search frictions thus demand for inventories. Finally, it admits possible price rigidity in the asset market. We use several criteria to examine the empirical relevance, including the likelihood statistics, the model's forecast performance in the short run and in the medium-run, the model's ability to replicate second moments in the data, and the ability to match empirical impulses.

Using the US data between 1985Q1 and 2007Q2, we find that the gradual adjustment of house price in the asset market is by far the most impor-

tant friction. We conclude that the key for constructing an optimization-based housing model is to structurally interpret the slow adjustment of house prices.

In addition, the findings in the paper raise questions about the use of investment adjustment cost (IAC) in the model of investment. IAC is a staple feature of the some recent DSGE models. Its popularity primarily comes from its ability to keep tract of the time series of investment. The analysis in the paper, however, demonstrates that IAC is not capable of improving the models' fit and forecast of capital prices in the housing market, despite its apparent success in tracking housing investments. The paradox is rather unsettling and probably needs to be explained. In comparison, the sluggish adjustment of house prices improves the model's fit and forecasts of both prices and investments. The price rigidity is also able to generate hump-shape response of the investments even without IAC. It will be interesting to see how much of the perceived evidence for IAC is in fact the result of price rigidity.

3.6 Tables and Figures

Table 3.1: Testing the empirical importance of costs and frictions in the model

Panel -a: reductions in log-likelihood when features are turned off

Log-likelihood	316.65	376.89	319.71	329.68	324.73
Search friction*	Yes	Yes	Yes	Yes	Yes
Slope of the long run supply curve	0.170	0.157	0.145	0.231	0.740
Price rigidity in the asset market(Quarterly speed of adjustment)	0.123	off	0.123	0.110	0.049
Sticky input prices (Calvo stickiness)	0.208	0.174	off	0.001	0.104
Investment adjustment cost	0.107	0.064	0.104	off	4.722
Capital adjustment cost	0.441	0.388	0.460	0.278	off
(Reduction in maximum likelihood)*2		120.49	6.13	26.06	16.17
Critical value (5%) from Chi-square dist. of 1 degree of freedom:					3.84
SIC (Schwarz information criterion):					4.95

Panel -b: models' ability to match std dev. of prices and investments

Stdev.: house price (0.74 in data)	0.80	1.40	0.75	0.81	0.74
Stdev.: investments (2.5 in data)	2.37	2.81	2.43	2.47	2.37

Panel -c: testing whether the estimated models can match correlation coefficients found in the data between the four-quarter growth of house price and the four-quarter growth of investment

The price-investment corrcoef.	0.14	0.49	0.14	0.13	0.28
(0.5 in data)	[0.14]	[0.10]	[0.14]	[0.16]	[0.16]

[1]Search cost is calibrated so that the sales to inventory ratio is 0.55 in the steady state

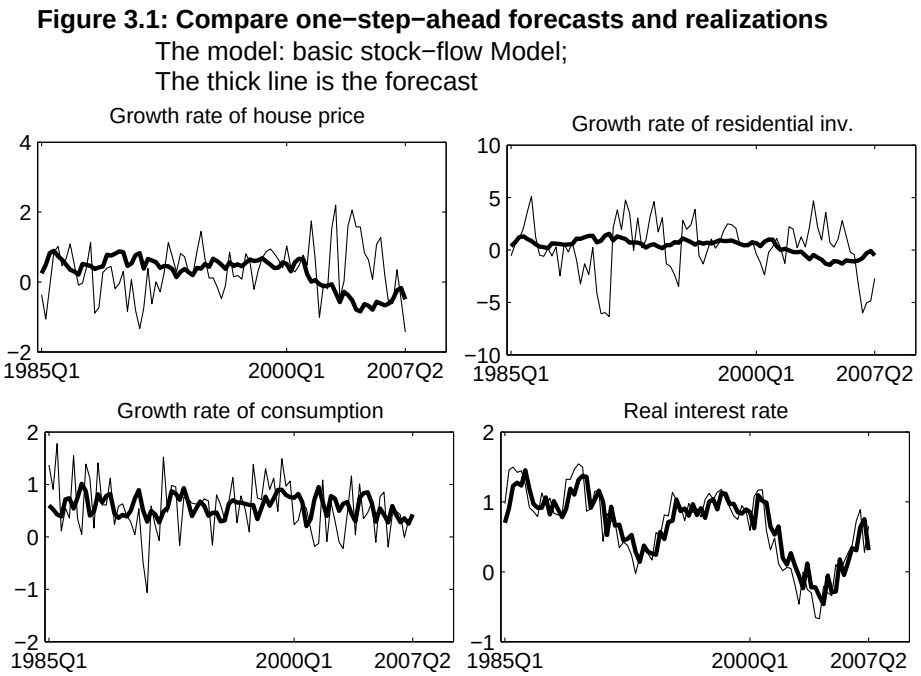
[2]The correlation coefficients of the estimated models are from simulations. Each simulation involves creating artificial data of comparable sample size(with early simulated data discarded). The table shows the mean and standard deviation from 200 simulations for each model.

Table 3.2: The performance of the estimated models in medium-run forecast compared to VAR(2)

RMSE (root mean square error) of a VAR(2)	**Q-1**	**Q-2**	**Q-3**	**Q-4**	**Q-5**	**Q-6**	**Q-7**	**Q-8**	
House price	0.7	1.2	1.3	1.4	1.5	2.1	2.6	3.2	
Resd. investment	1.4	2.7	3.5	3.6	5.0	6.9	8.8	10.3	
Percentage improvement over VAR(2), Negative value indicates worse forecast									Avg. Impv.
Row 2: Basic Stock-flow model									
House price	-42	-61	-131	-186	-188	-141	-136	-104	-124
Resd. investment	-89	-69	-63	-76	-61	-35	-16	-6	-52
Row 3: Full model									
House price	12	8	-8	-28	-14	-5	-16	-11	-8
Resd. investment	-6	-12	-8	-12	-8	4	12	19	-1
Row 4: Full model minus price rigidity in housing market									
House price	-23	-32	-77	-113	-105	-73	-74	-54	-69
Resd. investment	-56	-59	-56	-67	-54	-31	-14	-4	-43
Row 5: Full model minus IAC (investment adjustment cost)									
House price	19	12	2	-15	-2	3	-8	-3	1
Resd. investment	-20	-17	-16	-18	-9	1	9	17	-7
Row 6: Full model minus IAC and CAC (capital adjustment costs)									
House price	10	13	-2	-26	-13	-1	-9	-4	-4
Resd. investment	-41	-32	-27	-30	-21	-5	6	13	-17
Row 7: Model with IAC only									
House price	-41	-59	-129	-183	-184	-136	-132	-100	-121
Resd. investment	-33	-31	-31	-46	-41	-20	-6	2	-26

[1]How RMSE of forecast is calculated: The model stops at the second quarter of each year between 1998-2005, and uses current and past information to forecast the accumulated growth of price and investment in each of the future eight quarters. After that the model moves to the second quarter of the next year and repeat the exercise. We calculate the Root Mean Square Error (RMSE) for each of the eight quarters from the ten forecasts.
[2]Percentage improvement is defined as the reduction in RMSE relative to RMSE of the VAR(2)

Figure 3.1: Compare one-step-ahead forecasts and realizations

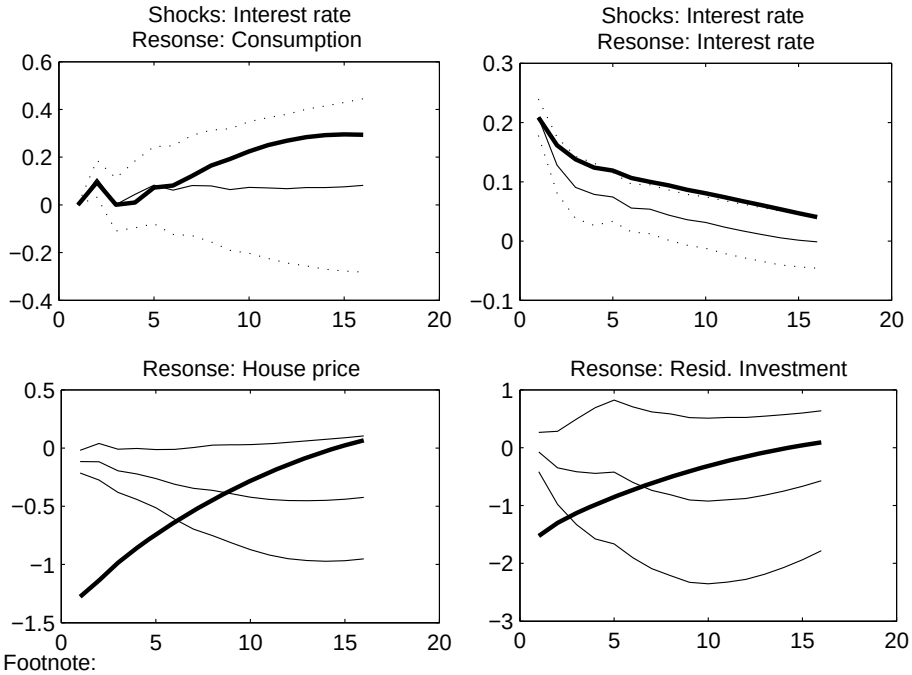


Footnote:
1. All units are in percentage points.
2. The growth rates are in quarterly frequency, yearly interest rate has been divided by four.

Figure 3.2: Compare impulse responses from the stock-flow model and those from the data

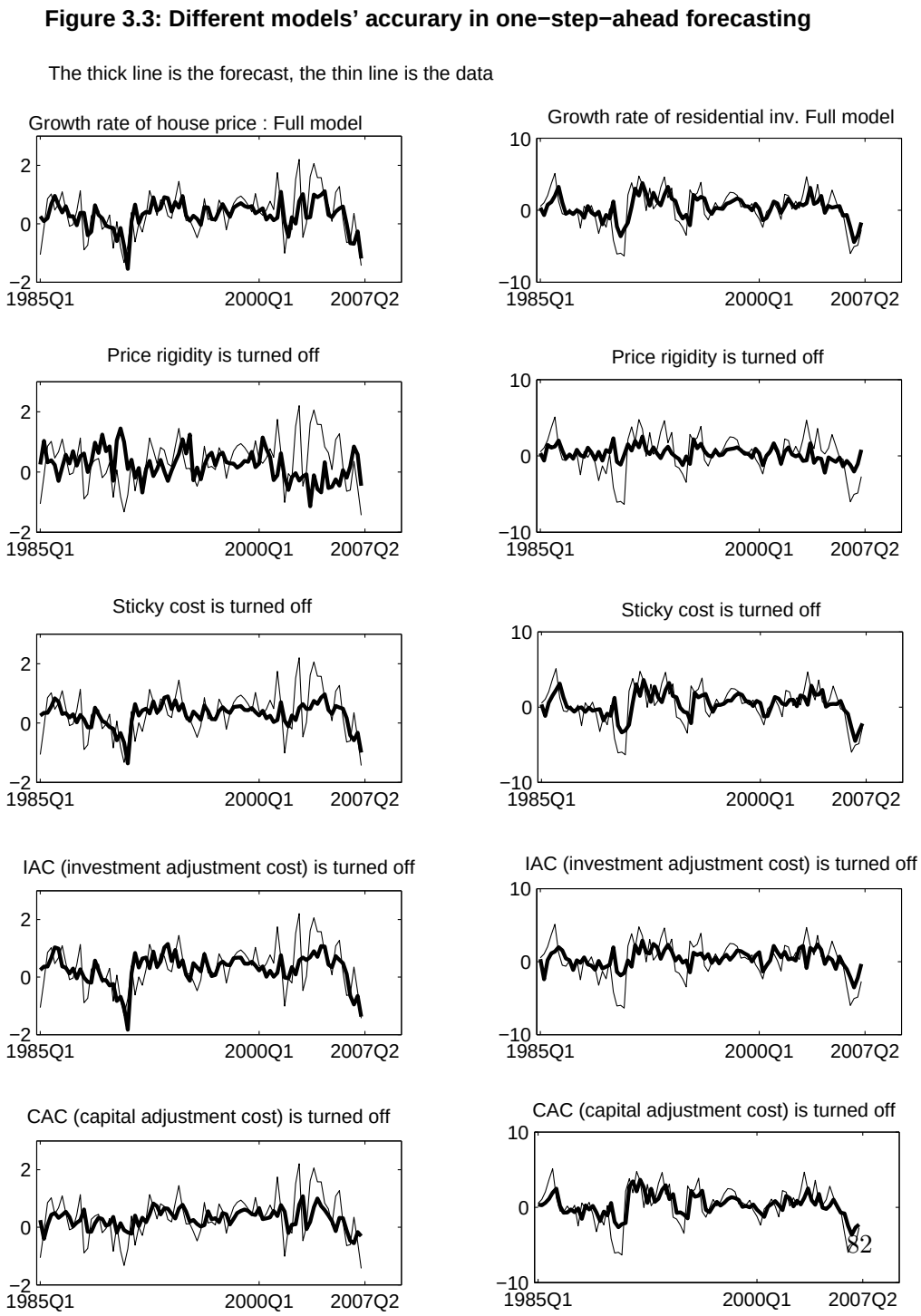
Figure 3.2: Compare impulse responses from the stock−flow model to those from the data

The thick lines are the IFR from the estimated model,
The thin lines are IRF from the data. Bounds indicate 95% confidence interval



Footnote:
1. All units are in percentage points.
2. The empirical impulse responses is from a recursive VAR. The order of variables is the same as the clock−wise order of plots.
3. Tme frame is in quarters

Figure 3.3: Compare different models for their accuracy in one-step-ahead forecasting



Footnote:
1. All units are in percentage points.
2. The growth rates are in quarterly frequency, yearly interest rate has been divided by four.

Figure 3.4: Impulse responses to an interest rate change : IAC V.S. rigidity in house price

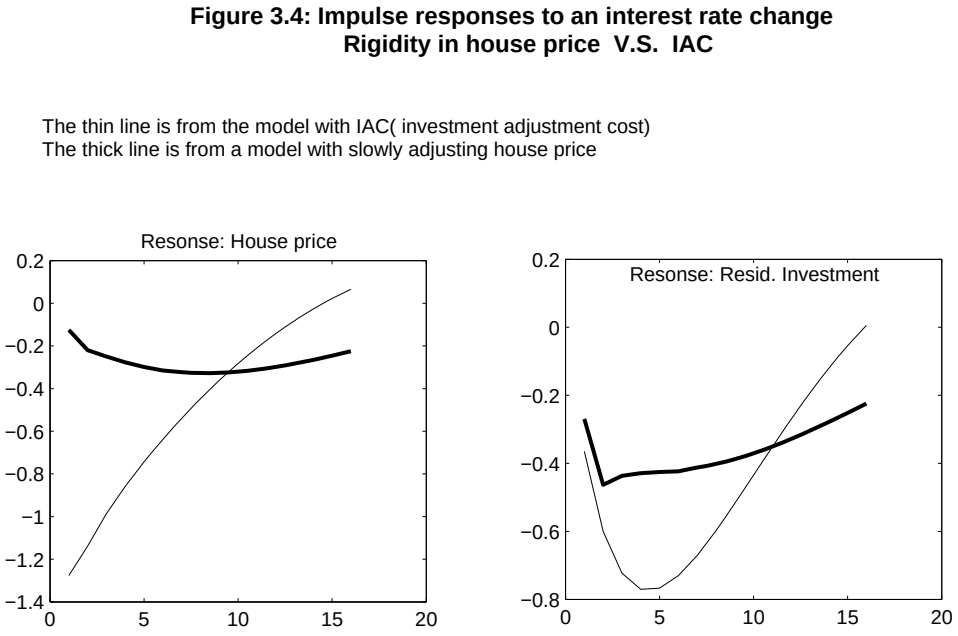


Figure 3.5: Impulse response to an interest rate change: sticky construction costs

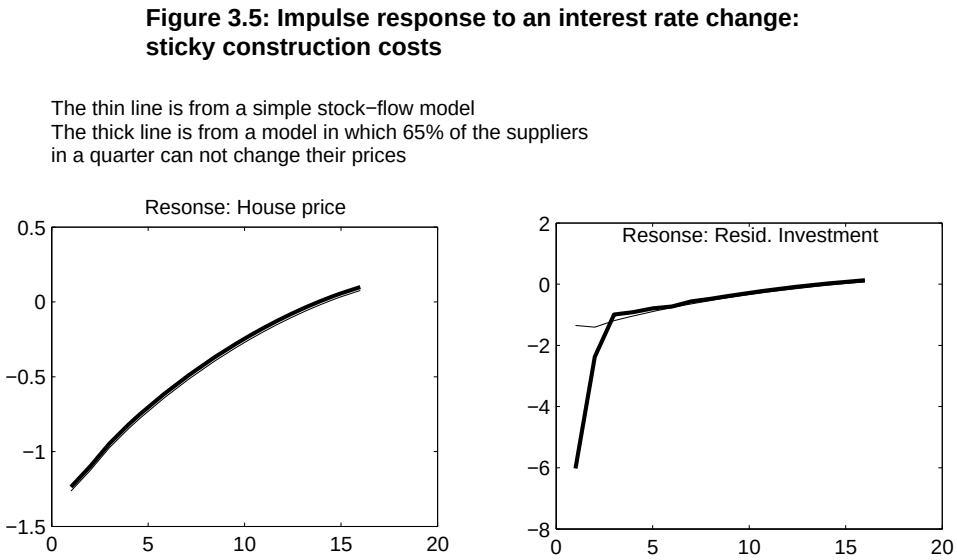
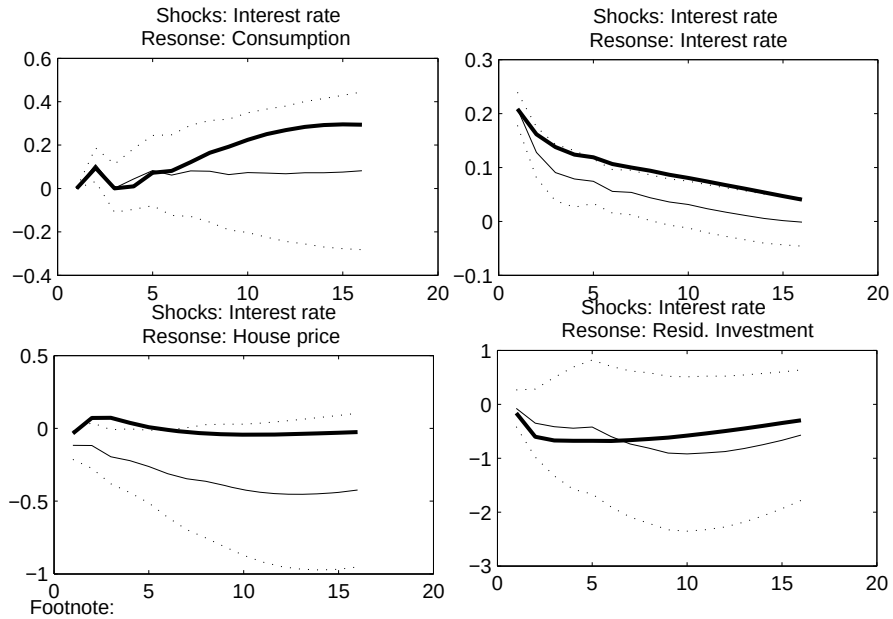


Figure 3.6: Compare impulse responses from the full model and those from the data

Figure 3.6: Compare impulse responses from the full model and those from the data

The thick line is IFR from the estimated model,
The thin lines are IRF from the data. Bounds indicate 95% confidence interval

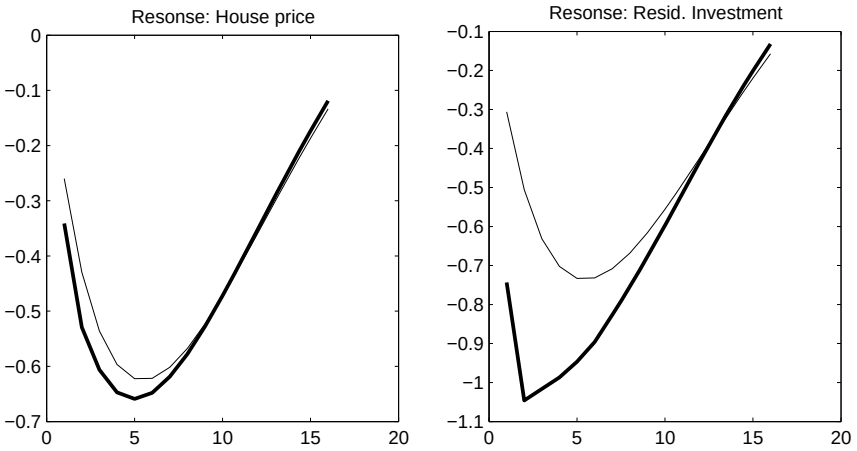


Footnote:
1. All units are in percentage points.
2. The empirical impulse responses is from a recursive VAR. The order of variables is the same as the clock−wise order of plots.
3. Tme frame is in quarters

Figure 3.7: Inventory and housing market's response to an interest rate shock

Figure 3.7: Inventory and market's responses to an interest rate shock

The thick line is a model in which builders can invest in the inventory
The thin line is the model that prohibits investment in the inventory



Footnote:
Price adjustment speed is 20% per quarter
When inventoty investment is allowed, the steady state sales to inventory ratio is about 0.55 a quarter
The remaining part of the model is from the estimated standard stock flow model

Chapter 4

Is there a Reduction in the Price Elasticity of the Supply of New Housing in the US?

4.1 Introduction

The US housing investments since the mid-1980s have become markedly less volatile and less sensitive compared to the earlier two decades. Figure 4.1 plots the time series of the quarterly growth rates of the investment from 1971 to the second quarter of 2007. The standard deviation of the percentage growth drops from 6% before 1985 to 2.5% after. Several papers, including McCarthy and Peach (2002), Dynan and Sichel (2006), and Bernanke (2007) find that housing investments have become less responsive to economic shocks, or exhibit smoother responses to them. Chapter 2 of this dissertation also finds housing investment to have a reduced sensitivity in an estimated Dynamic Stochastic General Equilibrium (DSGE) model.

During the same period, the US has seen a relative increase in the volatility of house prices. New home prices, more specifically the quarterly growth of inflation-adjusted prices of newly constructed houses, also show sign of moderation. The fall in their standard deviation, however, is smaller in proportion than that of the investments. As the result, the ratio of the standard deviations increases from 0.16 before 1985 to 0.3 after. Figure 4.1 presents the time series of prices and investments. The relative increase in the variability of the price series is visually notable.

Observations at a lower frequency also indicate a larger movement of house prices in the more recent period. During the latest upswing of housing cycle, the real price of new homes increased by 34% from the bottom in the early 1990s to the peak in 2005. Over the same upswing, the level of housing investments increased by 68% from the bottom to the peak. The ratio of price movement over investment movement is, therefore, 0.5. The identically defined ratio for the preceding two cycles are 0.08 (between the early and

the late 1980s) and 0.35 (between the mid and the late 1970s), respectively. Krainer (2005), when describing the 1990s experience, suggests "...the link between house prices and new residential construction shifted dramatically. Even though house prices shot up at the end of the decade, the response from the construction sector was unusually subdued."

These observations suggest that the supply of new houses may have become less elastic. Holding all else constant, a reduced capacity, or slower speed of new supply, suppresses the response of investments with greater responses of prices. In the extreme case when the supply is perfectly inelastic, all the adjustments in the market go through prices, and there will be no response from investments, as in the theoretical model of Iacoviello (2005a).

The possible change in supply conditions received little attention in the literature that involves interpreting the appearance of more stable housing investments. The reduced volatility and sensitivity of the investments are well documented. As possible explanations, the literature often refers to the perceived improvement in monetary policy or in the housing finance system. For example, Taylor (2007) suggests that the decline in volatility is "largely due to an improved monetary policy" that keeps inflation under control, which reduces the chances of boom-bust cycles and large swings in interest rates. Peek and Wilcox (2003) and Peek and Wilcox (2006) credit the improved resilience of the housing market to the expansion of a secondary market for mortgage debts, which they suggest has resulted in a more stable flow of mortgage credit. Dynan and Sichel (2006) credits financial innovations, and Bernanke (2007) suggest that "the easing of some traditional institutional and regulatory frictions seems to have reduced the sensitivity of residential construction to monetary policy".

It is important to take into account changes in supply conditions when trying to interpret the observed changes in the US housing market. Reduced supply capacity or speed has different implications than the explanations based on structural improvements. Explanations based on improvements have the implication that the housing sector has now become a smaller source of economic fluctuations. A lower supply elasticity, on the other hand, means that the prices are now doing more of the adjustments. In the case of extreme elasticity, housing market affects the real economy through residential construction. In the case of extreme inelasticity, housing market does so through property prices. One way for price to affect the real economy is through the so-called "wealth effect" on consumer spending, that higher prices increase aggregate consumption. This may happen because property values affect the perception of wealth. It is also likely that an increase in home value raises the collateral value of the asset, thus allowing credit

constrained home owners to borrow more and spend more. Iacoviello (2005a) provides a macro model that formalizes the collateral effect of housing prices. In his model housing market affects the economy only through prices.

Unlike explanations based on improvements in the housing finance system, a lower supply elasticity calls for greater attention to housing market when conducting monetary policy. Bernanke (2007) suggested that housing market may be "no longer so central to monetary transmission as it was" because housing construction is now less responsive to policy actions. He assumes that the reduced sensitivity is the result of easing frictions in the financing system. If the reduced sensitivity is the consequence of tighter supply conditions, however, the implication will be drastically different. Tighter supply leads to larger movement of house prices. Large movements of property value, in turn, can potentially trigger speculations and therefore excessive buildings. Malpezzi and Wachter (2005) makes a case that inelastic supply is the necessary condition for speculation to generate large real-estate cycles. In their paper speculation is defined as demand that is "...a positive function of recent price changes." To the extent that such speculation exists, the appearance of reduced sensitivity of construction could be a concern if it is accompanied by more responsive housing prices. A recent IMF report (World Economic Outlook, April 2008) urges central banks to pay greater attention to movements in house price, partly because "Recent innovations in housing finance markets have generally increased the impact of monetary policy on house prices." A reduction in supply elasticity has similar effect. It, too, translates policy-induced demand into movements in prices.

This paper uses US national-level data to estimate the supply elasticity of new housing. It estimates two versions of a housing model. The simpler version is a stock-flow model that is commonly used to study investments in durable goods. The second version is the same model augmented with mechanisms of gradual adjustments in housing prices and in construction activity. The paper then applies maximum likelihood estimations to the time series of residential investment from NIPA, and the price index of new, one-family houses from the Survey of Construction. The time series of consumption and interest rate are also included as forcing variables. The use of price index for new homes, instead of existing homes, reflects the paper's focus on the supply of new housing. The estimations uses split samples to detect changes in the elasticity parameters. The earlier sample covers the quarters between 1971 and 1984. The more recent sample is between 1985Q1 to 2007Q2.

The paper is organized as follows. Section 2 discusses the model and the estimation strategy. Section 3 presents the estimation results. Section

4 concludes.

4.2 The Model

This section presents the models used to estimate the supply elasticity. The model encompasses the stock-flow model laid out in Poterba (1984). It has two new features that Chapter 3 identifies as being the most empirically important. Chapter 3 augments the basic stock-flow model with several new features, including the sluggish adjustment of house prices, costs of adjusting the level of construction activity and housing stock, as well as sticky construction costs, among others. It then examines the drop in maximized log-likelihoods as individual features are turned off from the full model, so as to measure empirical relevance. It is found that the sluggish adjustment of house prices (as in DiPasquale and Wheaton (1994)) is the most important feature. Following behind is the cost of adjusting construction activity, which is also called "internal adjustment cost" in Topel and Rosen (1988). Reflecting these findings, these two features are built into the model in the current chapter.

4.2.1 Price of houses

The value of a house comes from its ability to provide housing service and possible capital gains. House price is the solution to a non-arbitrage condition in the housing market.

$$(1 + r_{t+1} + m)\, q_{*,t} = E_t R\left(c_{t+1}, h_{t+1}, z_{d,t+1}\right) + E_t\left(1 - \delta_h\right) q_{*,t+1} \tag{4.1}$$

In this equation, the price of a housing asset is q_*. The housing unit is normalized to produce one unit of service. The value of the unit of service is $R\left(.\right)$. The risk-free rate of return is r_{t+1}. The asset-specific risk premium is m. Further regarding the value of housing service, the (relative) price is determined by the clearance of the rental market. The price is increasing in the total consumption c, which proxies wealth and demand for housing, but decreasing in the size of the existing housing stock h, which is the total supply of housing. The rental price is also affected by a demand shifter z_d.

This paper assumes a specific functional form for $R\left(\right)$, that

$$R\left(c_t, h_t, z_{d,t}\right) = z_{d,t}\frac{c_t}{h_t}$$

In the background is a utility function, on the part of households, that has Cobb-Douglas feature. With such a utility function, the intra-temporal

marginal rate of substitution between consumption and housing services is just $z_{d,t}\frac{c_t}{h_t}$. The assumption of Cobb-Douglas utility guarantees a balance growth path, which does appear to be supported by the data. The share of spending on residential investment, over the past forty years, fluctuates significantly, but tend to revert back to a long term average that is about 5-6% of total GDP.

The demand shifter is assumed to follow exogenous stochastic process. The exact specification will be described later in the text.

The accumulation of housing stock follows

$$h_{t+1} = h_t + d_t \tag{4.2}$$

where the variable d_t denotes the net flow of new addition into existing housing stock.

4.2.2 Sluggish adjustment of house price

The price $q_{*,t}$ is the fundamental value of housing asset. In the short run, however, market price may deviate from the fundamental value. The evidence of sluggish adjustment has long been documented. For example, DiPasquale and Wheaton (1994) find that a model with lagged house price is statistically superior than a model without.

The following equation describes the adjustment process of house prices, where $q_{g,t}$ denotes the market prices,

$$\frac{q_{g,t}}{q_{g,t-1}} = (1+\gamma_2)^{1-\tau}\left(\frac{q_{*,t}}{q_{g,t-1}}\right)^{\tau} \tag{4.3}$$

This equation uses τ, which is between zero and one, to denote the speed of adjustment. The equation basically says that the growth rate of house price is a weighted average of γ_2 , which is the growth rate of house price in the balance-growth path, and the distance by which the fundamental value of housing asset exceeds the market price. The linearized version of the equation is $\widehat{q}_{g,t} = (1-\tau)\,\widehat{q}_{g,t-1} + \tau\widehat{q}_{*,t}$. When there is immediately adjustment of house prices, we have $\tau = 1$, which means $\widehat{q}_{g,t} = \widehat{q}_{*,t}$ for all t. The same adjustment mechanism is used in DiPasquale and Wheaton (1994).

4.2.3 Construction cost

This section describes the cost structure in the construction industry. Let d_t be the level of construction activity at the aggregate level, and let $c_{c,t}$ be

the cost of constructing a house, the relation between cost and construction is

$$c_{c,t} = \frac{d_t^{\psi}}{z_{h,t}} (1 + \gamma_x)^t \tag{4.4}$$

The cost structure has three elements. The first is an upward slope captured by d_t^{ψ} with $\psi > 0$, in which case the cost of construction is increasing with the level of construction. The second element is a trend in cost captured by $(1 + \gamma_x)^t$, with γ_x being the growth rate of exogenous costs. $\gamma_x < 0$ indicates a downward trend in cost while $\gamma_x > 0$ indicates a upward trend. The last element is mean-reverting cost shifter captured by variable $z_{h,t}$, which follows an exogenous stochastic process. The process will be described later in the text.

This paper takes the view that all three elements of the cost structure are necessary. The cost shifter $z_{h,t}$ captures exogenous and random elements such as the cyclical variations in commercial building and/or high way constructions, which compete for construction inputs. The parameter ψ, if positive, gives rise to a long-term supply elasticity that is less than infinity. The supply elasticity is an actively researched topic in empirical housing literature. Some early papers suggests that the supply of new housing is flat, i.e., $\psi = 0$ (See DiPasquale (1999) for a survey of the literature). The more recent papers, however, tend to estimate a positive ψ. Among them Topel and Rosen (1988) even find a modest supply elasticity of 3. Finally, the time trend $(1 + \gamma_x)^t$ captures long term technological progress (or relative regress). As mentioned above, the real price of new homes, even after adjusting for quality, has an upward trend in the long run. Part of the trend may come from expansion of investment along an upward slope supply curve. But it may also come from trends in unspecified costs, such as the cost of land, and the cost of obtaining regulatory approval for development. The model's inclusion of the exogenous time trend γ_x acknowledges the second possibility. The otherwise would be to fix the position of the supply curve over the course of four decades.

4.2.4 Adjustment cost in construction industry

According to Topel and Rosen (1988), the supply of new housing in the US is more elastic in the long run than it is in the short run. They attribute the difference to what they call the "internal adjustment cost" in the construction industry. Such a cost penalizes rapid adjustment in construction activity. Let's consider a hypothetical permanent increase of house price.

Without the "internal adjustment cost", construction activity jumps immediately to the level where the marginal cost equals to the new price. With the adjustment cost, construction activity approaches the higher level in a gradual manner. This explains why the long run supply elasticity is higher.

With internal adjustment cost, the price of houses does not always equal to the marginal cost of construction, even if the market is competitive. For one, a builder needs to be compensated for the adjustment costs. Secondly, the optimal level of construction is determined in a dynamic optimizing manner: If a builder expects an increase of house price in the future, it may increase current construction to the point where the cost is higher than the price. This happens because a higher construction activity today reduces the cost of construction in the future. Mathematically, the dynamic optimization problem is described by

$$\max_{d_t} E_t \sum_{s}^{\infty} \frac{1}{\Pi_s \left(1 + r_{t+1+s}\right)} \left[q_{c,t+s} \left(1 - \Phi_1 \left(\frac{d_{t+s}}{d_{t-1+s}} \right) \right) d_{t+s} - c_{c,t+s} d_{t+s} \right]$$

The objective function is the expected discounted value of current and future profits. The discount factor is $\frac{1}{\Pi_s(1+r_{t+1+s})}$. The periodical profit is inside the square bracket. In the profit function, the level of construction output is d_{t+s}, the price of output is $q_{c,t+s}$, the cost of construction cost per unit is $c_{c,t+s}$. The adjustment cost function is $\Phi_1 \left(\frac{d_{t+s}}{d_{t-1+s}} \right)$. It is assumed that in the balance-growth path $\Phi_1 (.) = \Phi_1' (.) = 0$ and $\Phi_1'' (.) = \varphi_{hs}$. The construction industry faces a convex cost if $\varphi_{hs} > 0$, in which case the industry has the incentive to smoothen out its construction plan over time.

The optimal condition for profit maximization is

$$\begin{aligned} 1 - \frac{c_{c,t}}{q_{c,t}} &= \Phi_1 \left(\frac{d_t}{d_{t-1}} \right) + \Phi_1' \left(\frac{d_t}{d_{t-1}} \right) \frac{d_t}{d_{t-1}} \\ &\quad - \frac{1}{1 + r_{t+1}} E_t \frac{q_{c,t+1}}{q_{c,t}} \Phi_1' \left(\frac{d_{t+1}}{d_t} \right) \left(\frac{d_{t+1}}{d_t} \right)^2 \end{aligned} \tag{4.5}$$

In the DSGE literature, what Topel and Rosen (1988) call the "internal adjustment cost" is known as "Investment adjustment cost (IAC)". IAC is used in DSGE literature primarily for its ability to keep track of investment series and generate hump-shaped response of investments to shocks. IAC is used in Christiano *et al.* (2005), Smets and Wouters (2007), and Justiniano and Primiceri (2006).

4.2.5 Market clearance

The market for new houses is cleared when the willingness to pay equals to the market price of a new house. The market price is denoted with $q_{c,t}$. The willingness to pay, on the other hand, equals to the price of existing homes under certain assumptions. These assumptions include the absence of preferential differentials, and the absence of frictions and additional costs associated with the purchase of new houses. The price of an existing house is denoted with q_{gt} as in equ(4.3), so we can describe the market clearance by

$$q_{c,t} = q_{g,t} \tag{4.6}$$

4.2.6 Reduce the model to a simple stock-flow model

The system, consisting of equ(4.1) to equ(4.6), describe the behavior part of the model. The system can be reduced into a simpler version if we impose $\varphi_{hs} = 0$, and $\tau = 1$. The former assumption excludes adjustment cost in the construction industry. The latter assumption excludes deviation of the price of a house from its fundamental value.

The simplified model is,

$$\begin{aligned}
(1 + r_{t+1})\, q_{g,t} &= E_t z_{d,t+1} \frac{c_{t+1}}{h_{t+1}} + (1 - \delta_h)\, E_t q_{g,t+1} \\
q_{g,t} &= \frac{d_t^{\psi}}{z_{h,t}} (1 + \gamma_x)^t \\
h_{t+1} &= h_t + d_t
\end{aligned}$$

which is just a three equation stock-flow model. The first equation is the stock equation, describing the formation of house prices as a discounted sum of the value of housing services. The second equation is the flow equation, describing the flow (or new investments) as a function of house price intersecting with an upward-sloped supply. The last equation is the accumulation equation, describing tomorrow's stock as a function of today's stock and flow.

4.2.7 The linearized system

Some variables in the model, namely the consumption, the housing prices and the investments, may have long-term trends in time. For this reason, the model, as it is presented in the previous sections, does not have a steady state. To achieve a steady state, we stabilize the system by dividing the

variables that have a theoretical trend with the corresponding level of the trend. This way we rewrite the system in a stationary form. We then solve the stationarized system for its steady-state. After that, we log-linearize the system around the steady-state. During the estimation, the trend components is added back to the model. Here we present the stationarized and linearized system,

In the balance growth path, the growth rate of residential investment in the balance-growth path is

$$\gamma_1 = \frac{\gamma_y - \gamma_x}{1+\psi}$$

The growth rate of house price in the balance-growth path is

$$\gamma_2 = \frac{\psi\gamma_y + \gamma_x}{1+\psi}$$

he linearized version of equ(4.1) is

$$\begin{aligned} \widehat{q}_{*,t} &= (r+\delta_h-\gamma_2)\left(E_t\widehat{z}_{d,t+1} + E_t\widehat{y}_{t+1} - E_t\widehat{h}_{t+1}\right) \\ &\quad + \left[1-(r+\delta_h-\gamma_2)\right] E_t\widehat{q}_{*,t+1} - \widehat{r}_{t+1} \end{aligned}$$

Equ(4.3) becomes

$$\widehat{q}_{g,t} = (1-\tau)\,\widehat{q}_{g,t-1} + \tau\widehat{q}_{*,t}$$

Equ(4.2) becomes

$$\widehat{h}_{t+1} = \frac{1}{1+\gamma_1}\widehat{h}_t + \left(1 - \frac{1}{1+\gamma_1}\right)\widehat{d}_t$$

Equ(4.4) is now

$$\widehat{c}_{c,t} = \psi\widehat{d}_t - \widehat{z}_{h,t}$$

Equ(4.5) is now

$$\begin{aligned} \varphi_{hs}\widehat{d}_t &= \frac{1}{(1+\gamma_1)^2\left(1+\frac{1}{1+r}\right)}\left(\widehat{q}_{c,t} - \widehat{c}_{c,t}\right) \\ &\quad + \frac{\varphi_{hs}}{1+\frac{1}{1+r}}\widehat{d}_{t-1} + \varphi_{hs}\left(1 - \frac{1}{1+\frac{1}{1+r}}\right)E_t\widehat{d}_{t+1} \end{aligned}$$

and finally the condition for market clearance, or equ(4.6), is

$$\widehat{q}_{c,t} = \widehat{q}_{g,t}$$

4.2.8 Stochastic processes

The following describes the stochastic processes that are the drivers of fluctuations in the housing market. The processes, in the order that they are presented, are housing demand shocks, housing supply shocks, and the processes of consumption and (real) interest rates:

$$\widehat{z}_{d,t} = \rho_{zd}\widehat{z}_{d,t-1} + \varepsilon_{zd,t} \tag{4.7}$$

$$\widehat{z}_{s,t} = \rho_{zs}\widehat{z}_{s,t-1} + \varepsilon_{zs,t} \tag{4.8}$$

$$\widehat{c}_t = \sum_{j=1}^{4} \rho_{cc,j}\widehat{c}_{t-j} + \sum_{j=1}^{4} \rho_{cr,j}\widehat{r}_{t-j} + \varepsilon_{c,t} \tag{4.9}$$

$$\widehat{r}_t = \rho_{rc,0}\widehat{c}_t + \sum_{j=1}^{2} \rho_{rc,j}\widehat{c}_{t-j} + \sum_{j=1}^{2} \rho_{rr,j}\widehat{r}_{t-j} + \varepsilon_{r,t} \tag{4.10}$$

4.2.9 Observable variables

The system of linear difference equations can be solved numerically. The solution can then be used to construct a state-space system of econometric model. This paper uses four observed variables in the estimations. They are the quarterly growth rate of consumption, the level of real interest rate, the growth rate of inflation-adjusted house price, and the growth rate of residential investment from the national account.

The relation between the observable data and the theoretical model is described by the following observation equations,

$$\begin{aligned}
\ln\left(\frac{q_t^{new}}{q_{t-1}^{new}}\right) &= \widehat{q}_{h,t} - \widehat{q}_{h,t-1} + \frac{\psi\gamma_y + \gamma_x}{1+\psi} \\
\ln\left(\frac{i_{h,t}}{i_{h,t-1}}\right) &= \left(1 - \frac{\delta_h}{\gamma_1 + \delta_h}\right)\left(\widehat{d}_t - \widehat{d}_{t-1}\right) \\
&\quad + \frac{\delta_h}{\gamma_1 + \delta_h}\left(\widehat{h}_t - \widehat{h}_{t-1}\right) + \frac{\gamma_y - \gamma_x}{1+\psi} \\
\ln\left(\frac{c_t}{c_{t-1}}\right) &= \widehat{c}_t - \widehat{c}_{t-1} + \gamma_y \\
R_t - \pi_t &= \widehat{r}_t + r
\end{aligned}$$

The left hand side of the system explains the use of the data. The inflation-adjusted Fisher price index of new houses from the Census Bureau is q_t^{new} in the first equation. The estimations use the growth rate of prices, or

$\ln\left(\frac{q_t^{new}}{q_{t-1}^{new}}\right)$. The quantity index of total residential investment (including maintenance, improvement and new construction) from the NIPA account is $i_{h,t}$ in the second equation. Again, the growth rate of the investment is used for estimations. The inflation-adjusted Private Consumption Expenditure from the NIPA is the consumption series. Its growth rate is $\ln\left(\frac{c_t}{c_{t-1}}\right)$ in the third equation. Finally, the real interest rate is defined as the difference between the effective federal funds rate and the growth of the GDP price index, or $R_t - \pi_t$ in the last equation.

The right hand side of the observation system are variables and parameters in the theoretical model. In the first equation, the growth rate of house price is expressed as the change in $\widehat{q}_h$ between t and t-1, plus a trend growth rate $\frac{\psi\gamma_y+\gamma_x}{1+\psi}$. The trend growth rate is added because $\widehat{q}_h$ is defined as the percentage deviation from trend.

The right hand side of the second equations is for the growth rate of the quantity of investment. It is constructed in a manner similar to that in the first equation, but with slight complication, because the investment in d is defined as new investment, while the data $i_{h,t}$ includes spending on maintenance/renovations. For this reason, the growth rate in the data is expressed as a weighted average of the growth rate of d_t (new investments) and the growth rate of $\delta_h h_t$ (the maintenance under the assumption of constant rate of depreciation δ_h). The share of maintenance is $\frac{\delta_h}{\gamma_1+\delta_h}$.

On the right hand side of the third equation, the growth rate of consumption is simply the change in $\widehat{c}$ plus the trend growth rate γ_y. On the RHS of the fourth equation, the the steady-state level of real interest rate equals to r.

During the estimations, the paper calibrates γ_y and r to the average levels in the sample. Allowing these two parameters to be part of the estimation does not change the result in noticeable way.

4.3 The Estimations

This section reports the estimations, focusing on the parameter of supply, or ψ. The estimations are performed in a basic stock-and-flow model as well as in a model with mechanisms of gradual adjustments. The basic model is the dominant way of modeling investments in durable goods in many macroeconomic models, so it is interesting to see what the model sees from the data. The model with gradual adjustment in price and construction is much more capable of tracking the dynamics of the time series of prices and investments. The additional features however makes estimating the

elasticity less precise, as we will see in the later discussion.

4.3.1 Estimating the basic model

The basic model has only two parameters to be estimated, not including those that describe the stochastic processes of exogenous shocks. One of these two parameters is ψ, the slope of supply curve. The other is γ_x, which is the growth rate of the exogenous component in construction cost. Table 4.1 presents the estimation results. Its first two columns have the estimated parameters from a routine of maximizing likelihood. The first column is for the sample between 1971Q1 and 1984 Q4. The second column is for the sample after 1984 until 2007Q2. The slope from the pre-85 estimation is 0.10, implying a supply elasticity of ten. The same parameter from the second estimation is 0.46, implying a supply elasticity slightly greater than 2. The growth rate of exogenous cost is 0.27% per quarter in the earlier sample, and 0.07% is for the more recent sample. The average growth rate of house price in both sample is about 0.33% per quarter. The estimation therefore suggests that most of the growth in house price before 1985 came from the growth in exogenous cost, while after 1985 the growth is coming from the increase of residential investment along a steeper supply curve. The third and the fourth column of the table test the robustness of these findings by setting the exogenous growth of costs to be zero. The findings are robust. The slope is higher in the second sample, increasing from 0.12 to 0.48, which of course means a movement from a higher to lower supply elasticity.

So far, the point estimates of supply elasticity fall within the range found in the literature. DiPasquale (1999) surveys the literature, and finds estimates ranging from infinity (Muth (1960), Follain (1979), and Stover (1986)), to between 4 to 14 in Malpezzi and Maclennan (2001) (published later after DiPasquale's review), to about 3 in Topel and Rosen (1988), to between 0.5 and 2.3 in Poterba (1984). The estimated elasticity in DiPasquale and Wheaton (1994), which takes into account gradual stock adjustment, is between 1.0 and 1.2.

Figure 4.2 provides a plot of maximized likelihoods along the dimension of ψ. This gives us an idea whether the estimated difference between the samples is significant or not. The range of ψ in the figure is from 0.01 to 1.4. Because ψ is the slope of the supply, a higher ψ indicates lower elasticity. The range of ψ in Figure 4.2 corresponds to a range of elasticity from 0.7 to 100, a reasonable range given the findings in the literature. The likelihood is re-maximized at each point of ψ, allowing other parameters to re-adjust

in search of a new maximum.

The figure shows that, the pre-85 log-likelihood peaks at about $\psi = 0.1$, and declines as ψ moves both to higher and lower level. When ψ increases to the level found in the second sample, the log-likelihood drops of the first sample by more than 20, a change that is well above 3.84, the 5% critical value of a Chi-squared distribution with one degree of freedom used in a likelihood ratio test. Similarly, the likelihood curve from the second sample assigns a significantly lower likelihood for the point estimate found in the earlier sample.

To summarize, the data before 1985 produces a point estimate of supply elasticity about 10. The data after 1985 produces an point estimate of 2.15. The fall in the elasticity appears statistically significant judging by likelihood curves.

4.3.2 Estimating the richer model: a MLE approach

The full model has two mechanisms of gradual adjustment. One of them is in housing prices. The other one is in construction activity. The adjustment mechanism in house prices brings into the model the parameter τ, which is the speed of adjustment. Higher τ indicates faster adjustment. Price adjusts immediately if $\tau = 1$. The mechanism of adjustment in construction activity brings the parameter φ_{hs}. Higher level of φ_{hs} indicates greater cost penalty to rapid changes in construction.

There is large improvement of fit from introducing the two adjustment mechanisms into the model. This is demonstrated by Figure 4.3 and 4.4, which plot the likelihood curves along the dimension of τ and φ_{hs}, respectively. The likelihood is re-maximized at each point on the horizontal axis, allowing other parameters to re-adjust in search of a new maximum. Figure 4.3 is for τ. Its horizontal axis starts from 0.01 and ends at 1. Moving from $\tau = 1$ to $\tau = 0.1$ increases the log-likelihood by 60 for the data of the earlier period, and by almost 100 for the data after 1985.

Figure 4.4 presents the likelihood curve along φ_{hs}. To choose a range for φ_{hs} , we turn to Smets and Wouters (2007), which has an identically defined parameter of adjustment cost, but for investment defined more broadly to include both business and housing investment. Their prior of this parameter is a normal distribution with a mean of 4, and a standard error of 1.5. We use the range between zero and 12 for φ_{hs}. It turns out that the likelihood curve plateaus after 1.5. Increasing φ_{hs} to beyond 1.5 does not change the likelihood in substantial way. Moving from $\varphi_{hs} = 0$ to $\varphi_{hs} = 1.5$, however, improves the likelihood by 10 for the data before 1985, and by 20 for the

data after.

Now moving on to ψ, the slope of supply. Figure 4.5 presents the likelihood curves along different values of ψ between 0.01 to 1.4. The pattern here is quite different from Figure 4.2, which is the earlier findings under the simpler model. Most importantly, the log-likelihood curves, for both samples, are quite flat over the entire range. The post-1985 sample slightly favors steeper supply, because the likelihood is increasing with ψ. But the minimum at ψ =0.01 and the maximum at ψ =1.4 is within 4 units of log-likelihood. The lack of difference in likelihood is even worse for the sample before 1985. Not only that the difference is small (smaller than one), but that there is no clear pattern whether the likelihood is increasing or decreasing in ψ.

The difficulty to estimate ψ is a result from short samples, and the inclusion of mechanisms for gradual adjustment. For the data before 1985, the main problem comes from the interaction between ψ and τ. Higher (lower) ψ indicates a steeper (flatter) supply, which boosts (suppresses) the response of house price for a given change in residential investments. A lower (higher) τ, on the other hand, indicates a slower (faster) adjustment speed of house price, which reduces (boosts) the response of price. As the result, a model with a high ψ and a low τ may look a lot like a model with a low ψ and a high τ. Figure 4.6 presents a contour chart of likelihood over the discretized (ψ, τ) two-dimensional space. It shows that there is a series of combination of (ψ, τ) that produces likelihood very close to the maximum. There exists a ridge on the surface of likelihood moving along from the combination of high ψ & low τ, to the combination of low ψ & high τ. It follows that one way to achieve a more precise estimate of ψ is to impose an prior belief about τ. In fact, if we limit the estimation to $\tau \geq 5\%$ (i.e., the gap between the market price for new houses and its fundamental value disappears at a rate greater or equal to 5% per quarter), most of the high ψ region will disappear, leaving ψ staying in the region that is less than 0.2, which indicates a high elasticity of supply.

For the data after 1986, the substitution between ψ and τ is less a problem. Instead the substitution between ψ and φ_{hs} matters more. φ_{hs} is the adjustment cost in construction industry. An extremely low ψ can be accommodated by a very high φ_{hs}. If we fix φ_{hs} at 4, the extremely low ψ become much less likely.

To summarize, with gradual adjustment in prices and construction activity, there is little hope to generate a meaningful estimate for the supply elasticity. On a brighter side, these experiments suggest that if there exists a set of prior regarding the likely range or distributions of τ and φ_{hs}, the

information will be very useful for estimating the parameter of interest, in this case the elasticity of supply.

It is noteworthy to point out that the simpler stock-flow model is just a extreme example, in which an econometrician sets $\tau = 1$ and $\varphi_{hs} = 0$ before running the regression.

4.3.3 Estimating the richer model: a Bayesian approach

As we find out from the preceding section, a MLE with minimum restriction makes it impossible to reach a meaningful estimate of the supply elasticity under the richer model. The simple stock-flow model gives a more precise estimate, but it arbitrarily sets τ and φ_{hs} to extreme values despite their empirical importance demonstrated in Figure 4.3 and Figure 4.4. This section seeks the middle road by adopting a Bayesian approach. Bayesian approach allows econometricians to incorporate prior knowledge for some or all parameters during the estimations. In this case, we try to incorporate existing knowledge about τ and φ_{hs} into the analysis. Instead of setting these parameters to extreme values as the simpler model does, a Bayesian approach summarizes existing knowledge about the parameter in the form of prior distributions.

As for τ, the speed of adjustment for house price, the estimate in DiPasquale and Wheaton (1994) is 29% in the first year following a shock, which translates into a quarterly adjustment rate slightly greater than 7%. Another paper, McCarthy and Peach (2002), estimates the speed to be 5%. We therefore assign this parameter a prior distribution that is Beta with a mean of 6% and a standard error of 1%.

The prior distribution for φ_{hs} follows the prior on a similar parameter in Smets and Wouters (2007), being normal with a mean of 4 and a standard error of 2 (larger than the 1.5 in the cited paper). It has to be acknowledged that the adjustment cost parameter in Smets and Wouters (2007) is for investment broadly defined, including both housing and business investment. The choice rather reflects the lack of understanding about this particular parameter. For this reason, the standard error is intentionally set at a higher value.

The same set of prior is used for both samples. Table 4.2 presents the estimation results. It shows that the supply elasticity in the sample before 1985 is closed to infinity (since ψ is approaching zero). Only the adjustment cost prevents an infinite jump of investment in response to changes in house prices. The elasticity estimated with data after 1985 is much lower, at about 1.25 (as an inverse to $\psi = 0.8$). There is no noticeable difference in

the adjustment cost parameter φ_{hs} between the two samples.

Figure 4.8 plots the log posterior function along ψ for the two samples. The posterior function clearly favors high elasticity in the earlier sample, and low elasticity in the sample after 1985.

4.4 Conclusion

This paper asks whether the US experienced a drop in the price elasticity of the supply of new housing. The comparison using the US national-level data, across time, appears to support the hypothesis of a fall in elasticity. Maximum likelihood estimations with the most basic form of the stock-and-flow model find an elasticity of 10 from the data between 1971Q1 and 1984Q4, and an elasticity of 2.15 between 1985Q1 and 2007Q2. In a richer model that incorporates mechanisms of gradual adjustment in house price and in construction activity, a MLE has difficulty in estimating the elasticity parameter. Nevertheless, when prior knowledge about the adjustment parameters is incorporated, likelihood-based estimations again favor a higher elasticity in the earlier sample, and a low elasticity in the more recent sample.

The lower supply elasticity may come from limited availability of land. An indication of land scarcity is that the price of land has grown much faster than the price of houses. According to Figures 1 in Davis and Heathcote (2005), the increase in the log of land's price is close to 3 between 1975 and 2006, while the log of home prices increased only by one. Why would a country with low population density like the US experience a land shortage? Glaeser *et al.* (2005) attribute the trend to what they call the "man-made scarcity": the "increasing difficulty of obtaining regulatory approval for building new homes" and "significant increase in the ability of local residents to block new projects and a change of cities from urban growth machines to homeowners' cooperatives." Whether regulations reduces elasticity is beyond the reach of the paper. The question may be better answered using more disaggregate data that also need to have information about the existence and types of regulations.

Regardless the cause for the reduction in supply elasticity, an inelastic supply is a source of concern for policy makers. Price movement can affect the broader economic by influencing consumer spending. To illustrate the quantitative impact of house price on consumption, I use the estimated model for the post-1986 data for an out-of-sample forecast. The forecast suggests that the next eight-quarters, following 2007Q2, will see a further

reduction of house price by 5% in real terms. This is probably at the conservative end of the predictions. Nevertheless, it still carries a sizable consumption effect. According to data compiled by Davis Morris and Heathcote, the aggregate market value of homes is $26.9 trillion dollars in the third quarter of 2007.[31] A 5% reduction in house price means a $1.35 trillion loss in housing wealth. If we adopt the wealth effect, estimated by Carroll *et al.* (2006), which is 9 cents per dollar in the long run, the loss in housing wealth will reduce households' spending by $120 billion, slightly less than 1% of the total GDP.[32] To put this into context, the housing slump contributes only a 1.2% reduction in GDP over the course of the 6 quarters, from 2006Q1 to 2007Q2. The indirect effect of house price on consumption, therefore, carries a size that is almost comparable to the direct effect.

Furthermore, Malpezzi and Wachter (2005) uses simulated models to show that an inelastic supply of new housing exacerbates the undesirable consequence from speculation, and is able to create a much larger price cycle than the alternative scenario with elastic supply. Large price cycle, in turn, can be destabilizing for the financial market and the economy. A reduced supply elasticity adds to the complexity facing the Federal Reserve. As they ease monetary policy, the resulted demand for housing investment is more likely to result in movement in house price, which has the potential of triggering an unwanted swings.

[31] The most up-to-the-date data is available from http://morris.marginalq.com/landdata.html. The cited number here is accessed on December 2007.

[32] The concept of "long-run" effect in Carroll *et al.* (2006) takes into account the serial correlation of consumption after the wealth shock. The authors' interpretation of their "long-run" is that "it really reflects the medium-run dynamics of consumption (over the course of a few years)" before endogeneity kicks in. Some often cited estimates, such as those in Case *et al.* (2005), is before taking into account the propagations. Case *et al.* (2005) find a 10% increase in housing wealth has the effect to increase households' consumption by 0.4%.

4.5 Table and figures

Table 4.1: Estimating the basic model

Samples	1971Q1- -1984Q4	1985Q1- -2007Q2	1971Q1- -1984Q4	1985Q1- -2007Q2
Log-likelihood	455.23	428.83	459.15	429.12
Parameters that describe the housing model and shocks				
ψ	0.10	0.46	0.12	0.48
γ_x	0.27	0.07		
$\rho_{\varepsilon,zd}$	0.99	0.96	0.97	0.97
$\sigma_{\varepsilon,zd}$	0.14	0.10	0.19	0.09
$\rho_{\varepsilon,zs}$	0.99	0.97	1.00	0.96
$\sigma_{\varepsilon,zs}$	1.39	1.93	1.45	1.95
Parameters that describe the joint stochastic process of consumption and interest rates				
$\rho_{cr,1}$	-0.16	0.46		
$\rho_{cr,2}$	0.02	-0.78		
$\rho_{cr,3}$	0.07	0.25		
$\rho_{cr,4}$	0.08	0.30	Same as	Same as
$\rho_{cc,1}$	1.24	0.94	the first	the second
$\rho_{cc,2}$	0.10	0.20	column[1]	column[1]
$\rho_{cc,3}$	-0.28	0.10		
$\rho_{cc,4}$	-0.06	-0.33		
$\rho_{rr,1}$	0.68	0.72		
$\rho_{rr,2}$	0.10	0.16		
$\rho_{rc,0}$	0.09	0.13		
$\rho_{rc,1}$	-0.07	-0.12		
$\rho_{rc,2}$	-0.04	-0.02		
$\sigma_{\varepsilon,c}$	0.80	0.43		
$\sigma_{\varepsilon,r}$	0.50	0.25		

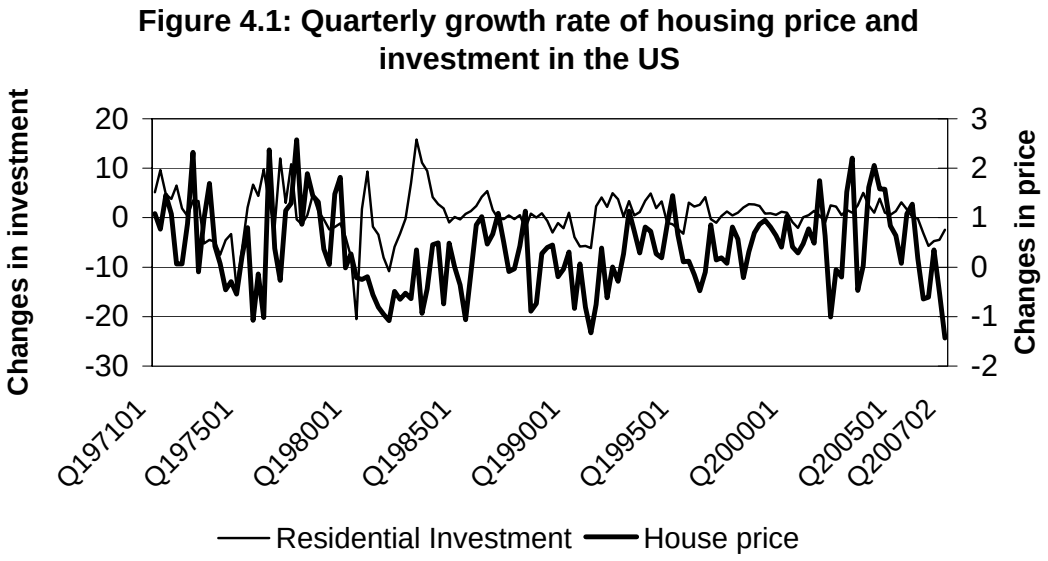
[1]The stochastic process for consumption and interest rate are estimated separately, the same set of point estimates are then used in all estimations.

Table 4.2: Estimating the model with gradual adjustment in prices and constructions

Samples		1971Q1- -1984Q4	1985Q1- -2007Q2
Log-posterior function		352.76	321.84
Parameter that describe the housing model and shocks			
	Prior distribution		
ψ	None	0.01	0.80
φ_{hs}	Normal(4,2)	5.33	4.70
τ	Beta(0.06,0.01)	0.06	0.06
γ_x	None	0.32	-0.13
$\rho_{\varepsilon,zd}$	None	0.38	0.49
$\sigma_{\varepsilon,zd}$	None	1.41	0.67
$\rho_{\varepsilon,zs}$	None	0.78	0.89
$\sigma_{\varepsilon,zs}$	None	2.59	2.04

[1]The stochastic process for consumption and interest rate uses the same point estimates as those in Table 4.1.

Figure 4.1: Quarterly growth rate of housing price and investment in the US



* The investment is the quantity index of residential investment in the US NIPA
* The house price is the Price Deflator (Fisher) Index of New, One-Family Houses Under Construction, from the Survey of Construction

Figure 4.2: Maximized log-likelihood at different values of ψ from the basic stock-flow model

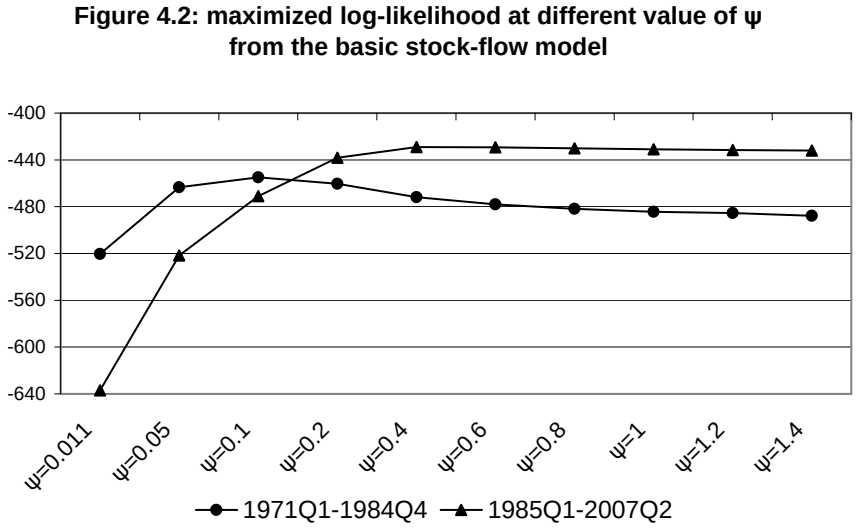


Figure 4.3: Maximized log-likelihood at different values of τ

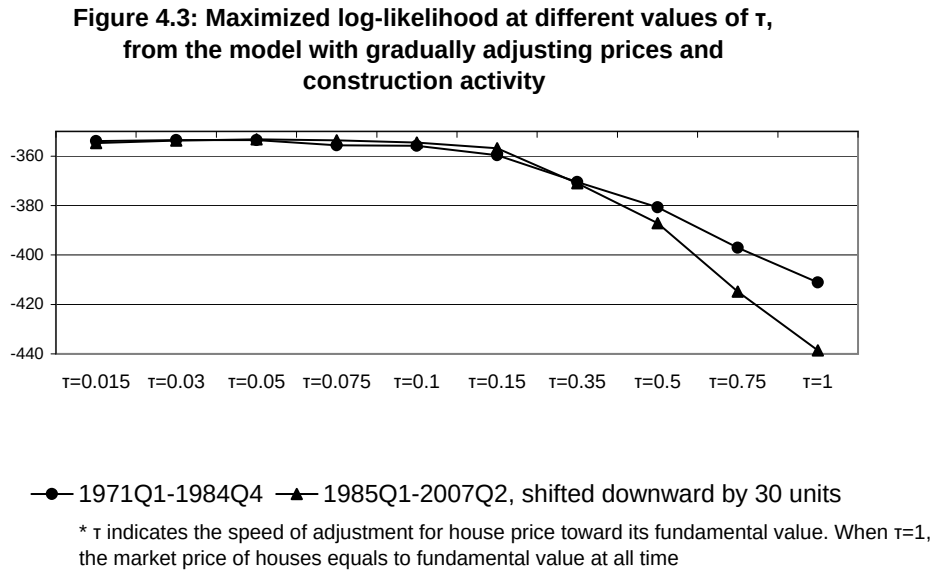


Figure 4.4: Maximized log-likelihood at different values of φ_{hs}

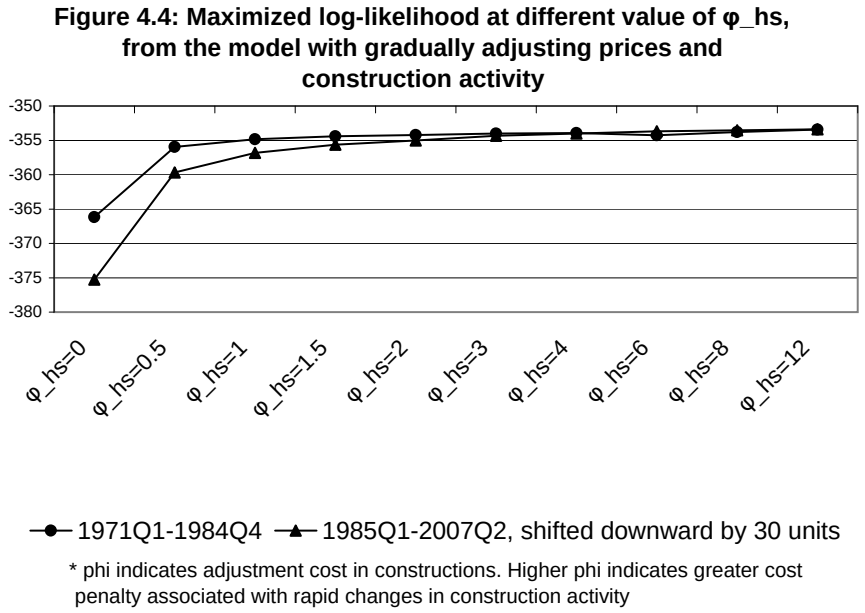


Figure 4.5: Maximized log-likelihood at different values of ψ

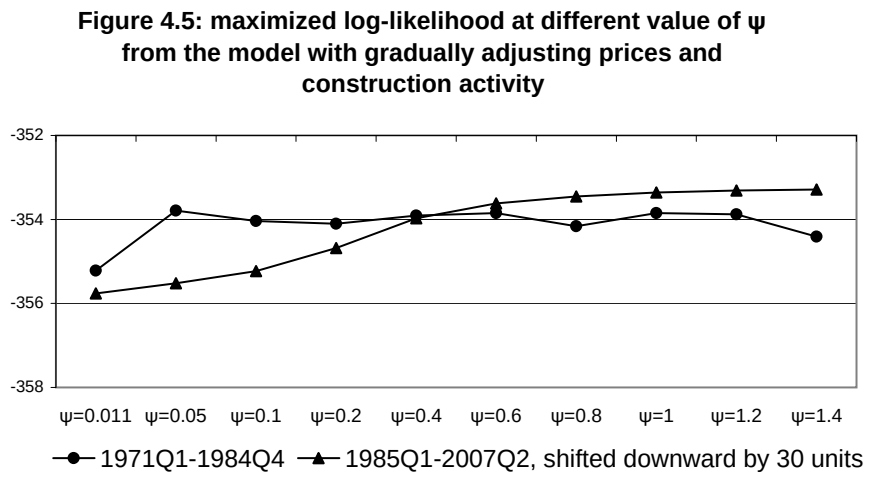


Figure 4.6: Contour chart of log-likelihood in (ψ, τ) space: 1971Q1-1984Q4

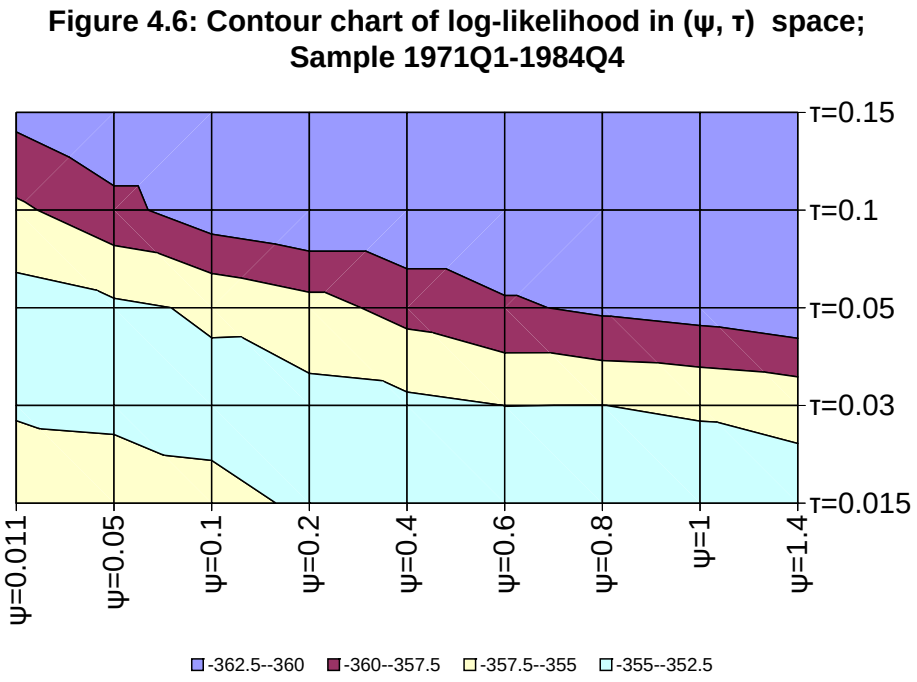


Figure 4.7: Contour chart of log-likelihood in (ψ, τ) space: 1985Q1-2007Q2

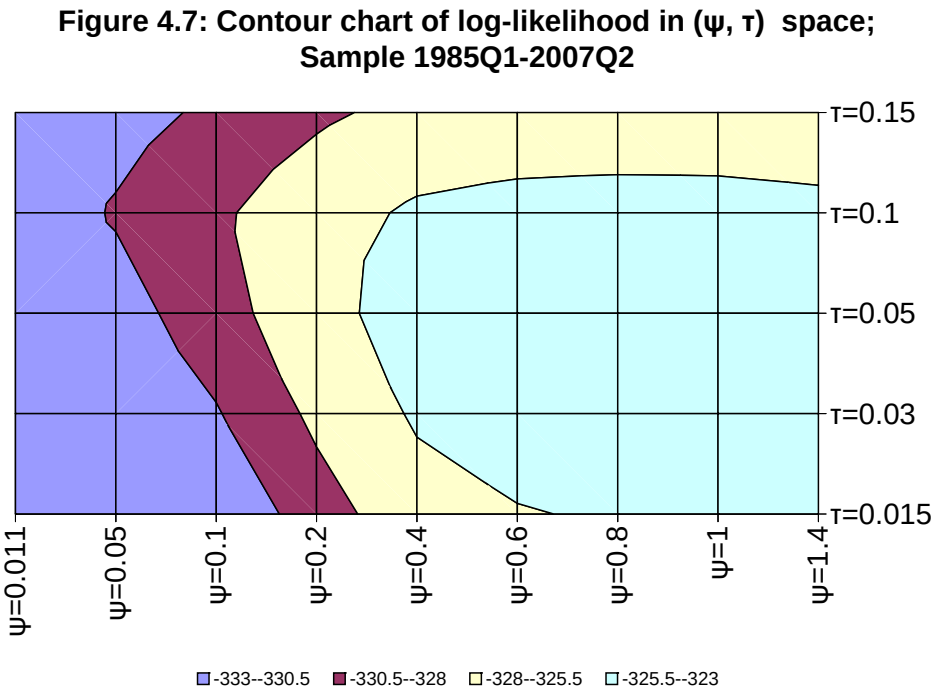
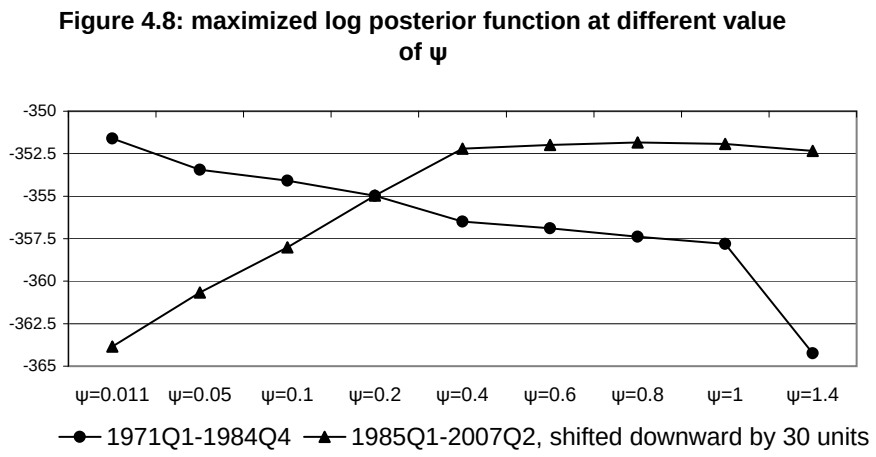


Figure 4.8: Maximized log posterior function at different value of ψ



Bibliography

Abowd, J. M. and Farber, H. S. (1982). Job queues and the union status of workers. *Industrial and Labor Relations Review*, **35** (3), 354–367, available at http://ideas.repec.org/a/ilr/articl/v35y1982i3p354-367.html.

Allmon, E., Haas, C. T., Borcherding, J. D. and Goodrum, P. M. (2000). U.s. construction labor productivity trends, 1970–1998. *Journal of Construction Engineering and Management*, **126** (2), 97–104.

An, S. and Schorfheide, F. (2006). *Bayesian analysis of DSGE models*. Tech. rep.

Aoki, K., Proudman, J. and Vlieghe, G. (2004). House prices, consumption, and monetary policy: a financial accelerator approach. *Journal of Financial Intermediation*, **13** (4), 414–435, available at http://ideas.repec.org/a/eee/jfinin/v13y2004i4p414-435.html.

Bernanke, B. S. (2004). The great moderation. *Remarks at the meetings of the Eastern Economic Association*.

— (2007). Housing, housing finance, and monetary policy: a speech at the federal reserve bank of kansas city's economic symposium, jackson hole, wyoming, august 31, 2007.

— and Gertler, M. (1995a). Inside the black box: The credit channel of monetary policy transmission. *Journal of Economic Perspectives*, **9** (4), 27–48, available at http://ideas.repec.org/a/aea/jecper/v9y1995i4p27-48.html.

— and — (1995b). Inside the black box: The credit channel of monetary policy transmission. *The Journal of Economic Perspectives*, **9** (4), 27–48.

—, — and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. In J. B. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics*, *Handbook of Macroeconomics*, vol. 1, *21*, Elsevier, pp. 1341–1393, available at http://ideas.repec.org/h/eee/macchp/1-21.html.

Bils, M. and Kahn, J. A. (2000). What inventory behavior tells us about business cycles. *American Economic Review*, **90** (3), 458–481, available at http://ideas.repec.org/a/aea/aecrev/v90y2000i3p458-481.html.

Blanchard, O. J. and Simon, J. A. (2001). The Long and Large Decline in U.S. Output Volatility. *SSRN eLibrary*.

Boivin, J. and Giannoni, M. P. (2006). Has monetary policy become more effective? *The Review of Economics and Statistics*, **88** (3), 445–462, available at http://ideas.repec.org/a/tpr/restat/v88y2006i3p445-462.html.

Campbell, J. R. and Hercowitz, Z. (2005). *The Role of Collateralized Household Debt in Macroeconomic Stabilization*. NBER Working Papers 11330, National Bureau of Economic Research, Inc, available at http://ideas.repec.org/p/nbr/nberwo/11330.html.

Carroll, C. D., Otsuka, M. and Slacalek, J. (2006). *How Large Is the Housing Wealth Effect? A New Approach*. Economics Working Paper Archive 535, The Johns Hopkins University,Department of Economics, available at http://ideas.repec.org/p/jhu/papers/535.html.

Case, K., Quigley, J. and Shiller, R. (2005). Comparing wealth effects: The stock market versus the housing market. *Advances in Macroeconomics*, **5** (1), 1235–1235, available at http://ideas.repec.org/a/bep/macadv/v5y2005i1p1235-1235.html.

Cho, M. (1996). House price dynamics: a survey of theoretical and empirical issues. *Journal of Housing Research*, **7** (2), 145–72.

Christiano, L. J., Eichenbaum, M. and Evans, C. L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, **113** (1), 1–45, available at http://ideas.repec.org/a/ucp/jpolec/v113y2005i1p1-45.html.

Clarida, R., Gal, J. and Gertler, M. (2000). Monetary policy rules and macroeconomic stability: Evidence and some theory. *The Quarterly Journal of Economics*, **115** (1), 147–180, available at http://ideas.repec.org/a/tpr/qjecon/v115y2000i1p147-180.html.

Davis, M. and Heathcote, J. (2005). *The Price and Quantity of Residential Land in the United States*. CEPR Discussion Papers 5333, C.E.P.R. Discussion Papers, available at http://ideas.repec.org/p/cpr/ceprdp/5333.html.

DiPasquale, D. (1999). Why don't we know more about housing supply? *The Journal of Real Estate Finance and Economics*, **18** (1), 9–23.

— and Wheaton, W. C. (1994). Housing market dynamics and the future of housing prices. *Journal of Urban Economics*, **35** (1), 1–27, available at http://ideas.repec.org/a/eee/juecon/v35y1994i1p1-27.html.

Dynan, E. D. W., Karen E. and Sichel, D. E. (2006). Can financial innovation help to explain the reduced volatility of economic activity? *Journal of Monetary Economics*, **53**, 123–150.

Edge, R. M., Laubach, T. and Williams, J. C. (2004). *Learning and shifts in long-run productivity growth*. Tech. rep.

Fisher, J. D. M. and Quayyum, S. (2006). The great turn-of-the-century housing boom. *Economic Perspectives*, (Q III), 29–44, available at http://ideas.repec.org/a/fip/fedhep/y2006iqiiip29-44nv.30no.3.html.

Follain, J. R. (1979). The price elasticity of the long-run supply of new housing construction. *Land Economics*, **55** (2), 190–199.

Gammoudi, M. A. and Mendesy, R. R. (2005). *Household Sector Financial Frictions in Canada*. Tech. rep.

Glaeser, E. L., Gyourko, J. and Saks, R. (2005). *Why Have Housing Prices Gone Up?* NBER Working Papers 11129, National Bureau of Economic Research, Inc, available at http://ideas.repec.org/p/nbr/nberwo/11129.html.

Greenspan, A. (2001). Monetary policy report to congress. *Board of Governors of the Federal Reserve System*.

Greenwood, J. and Hercowitz, Z. (1991). The allocation of capital and time over the business cycle. *Journal of Political Economy*, **99** (6), 1188–214, available at http://ideas.repec.org/a/ucp/jpolec/v99y1991i6p1188-214.html.

Groth, C. and Khan, H. (2006). *Investment Adjustment Costs: An Empirical Assessment*. Tech. rep., available at http://economics.ca/2007/papers/0207.pdf.

Hamilton, J. D. (2003). What is an oil shock? *Journal of Econometrics*, **113** (2), 363–398, available at http://ideas.repec.org/a/eee/econom/v113y2003i2p363-398.html.

Iacoviello, M. (2005a). House prices, borrowing constraints, and monetary policy in the business cycle. *American Economic Review*, **95** (3), 739–764, available at http://ideas.repec.org/a/aea/aecrev/v95y2005i3p739-764.html.

— (2005b). *House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle*. Tech. Rep. 3, available at http://ideas.repec.org/a/aea/aecrev/v95y2005i3p739-764.html.

— and Minetti, R. (2002). *The Credit Channel of Monetary Policy: Evidence from the Housing Market*. Boston College Working Papers in Economics 541, Boston College Department of Economics, available at http://ideas.repec.org/p/boc/bocoec/541.html.

Ireland, P. N. (2004). Technology shocks in the new keynesian model. *The Review of Economics and Statistics*, **86** (4), 923–936, available at http://ideas.repec.org/a/tpr/restat/v86y2004i4p923-936.html.

Jud, G. D. and Winkler, D. T. (2003). The q theory of housing investment. *The Journal of Real Estate Finance and Economics*, **27** (3), 379–92, available at http://ideas.repec.org/a/kap/jrefec/v27y2003i3p379-92.html.

Justiniano, A. and Primiceri, G. E. (2006). *The Time Varying Volatility of Macroeconomic Fluctuations*. NBER Working Papers 12022, National Bureau of Economic Research, Inc, available at http://ideas.repec.org/p/nbr/nberwo/12022.html.

Kahn, J. A. (2000). Explaining the gap between new home sales and inventories. *Current Issues in Economics and Finance*, (May), available at http://ideas.repec.org/a/fip/fednci/y2000imaynv.6no.6.html.

Kim, C.-J. and Nelson, C. R. (1999). Has the u.s. economy become more stable? a bayesian approach based on a markov-switching model of the business cycle. *The Review of Economics and Statistics*, **81** (4), 608–616, available at http://ideas.repec.org/a/tpr/restat/v81y1999i4p608-616.html.

Klein, P. (2000). Using the generalized schur form to solve a multivariate linear rational expectations model. *Journal of Economic Dynamics and Control*, **24** (10), 1405–1423, available at http://ideas.repec.org/a/eee/dyncon/v24y2000i10p1405-1423.html.

Krainer, J. (2005). Housing markets and demographics. *FRBSF Economic Letter*, (Aug 26), available at http://ideas.repec.org/a/fip/fedfel/y2005iaug26n2005-21.html.

— (2006). Residential investment over the real estate cycle. *FRBSF Economic Letter*, (Jun 30), available at http://ideas.repec.org/a/fip/fedfel/y2006ijun30n2006-15.html.

—, Spiegel, M. and Yamori, N. (2005). *Asset price declines and real estate market illiquidity: evidence from Japanese land values*. Tech. rep.

Leamer, E. E. (2007). *Housing IS the Business Cycle*. Working Paper 13428, National Bureau of Economic Research.

Leduc, S. and Sill, K. (2007). Monetary policy, oil shocks, and tfp: Accounting for the decline in u.s. volatility. *Review of Economic Dynamics*, available at http://ideas.repec.org/a/red/issued/05-13.html.

Lubik, T. A. and Schorfheide, F. (2004). Testing for indeterminacy: An application to u.s. monetary policy. *American Economic Review*, **94** (1), 190–217, available at http://ideas.repec.org/a/aea/aecrev/v94y2004i1p190-217.html.

Maisel, S. J. (1963). A theory of fluctuations in residential construction starts. *The American Economic Review*, **53** (3), 359–383.

Malpezzi, S. and Maclennan, D. (2001). The long-run price elasticity of supply of new residential construction in the united states and the united kingdom. *Journal of Housing Economics*, **10** (3), 278–306, available at http://ideas.repec.org/a/eee/jhouse/v10y2001i3p278-306.html.

— and Wachter, S. M. (2005). The role of speculation in real estate cycles. *Journal of real estate literature*, **13** (2), 143.

McCarthy, J. and Peach, R. W. (2002). Monetary policy transmission to residential investment. *Economic Policy Review*, (May), 139–158, available at http://ideas.repec.org/a/fip/fednep/y2002imayp139-158nv.8no.1.html.

Muth, R. (1960). The demand for non-farm housing. *in A.C.Harberger, ed., The demand for durable goods, University of Chicago Press, Chicago, 1960*.

Peek, J. and Wilcox, J. A. (2003). Secondary mortgage markets, gses, and the changing cyclicality of mortgage flows. In A. H. Chen (ed.), *Research in Finance Volume 20*, pp. 61–80.

— and — (2006). *Housing, Credit Constraints, and Macro Stability: The Secondary Mortgage Market and Reduced Cyclicality of Residential Investment*. Tech. Rep. 298, Fisher Center Working Papers, http://repositories.cdlib.org/iber/fcreue/fcwp/298.

Pescatori, A. and Mendicino, C. (2005). *Credit Frictions, Housing Prices and Optimal Monetary Policy Rules*. Money Macro and Finance (MMF) Research Group Conference 2005 67, Money Macro and Finance Research Group, available at http://ideas.repec.org/p/mmf/mmfc05/67.html.

Poterba, J. M. (1984). Tax subsidies to owner-occupied housing: An asset-market approach. *The Quarterly Journal of Economics*, **99** (4), 729–52, available at http://ideas.repec.org/a/tpr/qjecon/v99y1984i4p729-52.html.

Rudebusch, G. and Wu, T. (2004). A macro-finance model of the term structure, monetary policy, and the economy. *Proceedings*, (Mar), available at http://ideas.repec.org/a/fip/fedfpr/y2004imarx8.html.

Rudebusch, G. D., Sack, B. P. and Swanson, E. T. (2006). *Macroeconomic implications of changes in the term premium*. Tech. rep.

Smets, F. and Wouters, R. (2003). An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association*, **1** (5), 1123–1175, available at http://ideas.repec.org/a/tpr/jeurec/v1y2003i5p1123-1175.html.

— and — (2007). *Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach*. CEPR Discussion Papers 6112, C.E.P.R. Discussion Papers, available at http://ideas.repec.org/p/cpr/ceprdp/6112.html.

Stock, J. H. and Watson, M. W. (2002). *Has the Business Cycle Changed and Why?* NBER Working Papers 9127, National Bureau of Economic Research, Inc, available at http://ideas.repec.org/p/nbr/nberwo/9127.html.

Stover, M. E. (1986). The price elasticity of the supply of single-family detached urban housing. *Journal of Urban Economics*, **20** (3), 331–340.

Taylor, J. B. (2007). *Housing and Monetary Policy: a Paper presented at the 2007 Jackson Hole Conference*. Tech. rep.

Thieblot, A. (2001). The fall and future of unionism in construction. *Journal of Labor Research*, **22** (2), 287–306, available at http://ideas.repec.org/a/tra/jlabre/v22y2001i2p287-306.html.

Topel, R. H. and Rosen, S. (1988). Housing investment in the united states. *Journal of Political Economy*, **96** (4), 718–40, available at http://ideas.repec.org/a/ucp/jpolec/v96y1988i4p718-40.html.

Trichet, J.-C. (2005). Asset price bubbles and monetary policy. *Speech by Jean-Claude Trichet, Mas lecture, Singapore*.

Appendix A

Appendix for Chapter 2

A.1 Model Appendix

A.1.1 Households' optimality conditions

Labor supply

The marginal rate of substitution between consumption and hours worked can be defined as the ratio of the partial derivatives of the momentary utility function with respect to l_t over the derivative with respect to c_t. The consumption shock Λ_t drops out when taking the ratio,

$$w_t = \frac{(c_{b,t} - \lambda_b c_{b,t-1})^{1-\sigma_c} \exp\left(\frac{\sigma_c - 1}{1+\sigma_l} l_t^{1+\sigma_l}\right) l_t^{\sigma_l}}{(c_{b,t} - \lambda_b c_{b,t-1})^{-\sigma_c} c_{b,t}^{\gamma} c_t^{-\gamma} \exp\left\{\frac{\sigma_c - 1}{1+\sigma_l} l_t^{1+\sigma_l}\right\}} = (c_{b,t} - \lambda_b c_{b,t-1}) c_{b,t}^{-\gamma} c_t^{\gamma} l_t^{\sigma_l}$$

To reach the alternative expression for the labor market equilibrium in term of the Private Consumption Expenditure (E), we first need to define E

$$e_t \equiv c_t + v_h \left(\frac{h_t}{c_t}\right)^{-\gamma} h_t$$

where $v_h \left(\frac{h_t}{c_t}\right)^{-\gamma}$ is the rental price of housing asset. Intuitively one should be able to express e_t as the quantity of the mixture goods, $c_{b,t}$ (defined in equ(2.1)), divided by the relative price of $c_{b,t}$, or

$$e_t = \frac{c_{b,t}}{\left(\frac{c_{b,t}}{c_t}\right)^{\gamma}} = c_{b,t}^{1-\gamma} c_t^{\gamma}$$

It is straight forward to verify that the intuition is right because $c_{b,t}^{1-\gamma} c_t^{\gamma} = c_t + v_h \left(\frac{h_t}{c_t}\right)^{-\gamma} h_t$ once we expand $c_{b,t}$ according to equ(2.1)

Plugging $e_t = c_{b,t}^{1-\gamma} c_t^{\gamma}$ into the labor market equilibrium, or equ(2.4), gives us

$$w_t = \left(1 - \lambda_b \frac{c_{b,t-1}}{c_{b,t}}\right) e_t l_t^{\sigma_l}$$

or $\frac{w_t l_t}{e_t} = \left(1 - \lambda_b \frac{c_{b,t-1}}{c_{b,t}}\right) l_t^{1+\sigma_l}$, which in steady state becomes

$$\frac{w_* l_*}{e_*} \frac{1}{1-\lambda_b} = l_*^{1+\sigma_l}$$

Linearizing the optimality conditions

The linearized form of equ(2.1) is

$$\widehat{c}_{bt} = \kappa_c \widehat{c}_t + (1-\kappa_c)\,\widehat{h}_t$$

where κ_c is the steady state share of consumption in private consumption expenditure.

The linearized labor supply equation, or equ(2.4), is

$$\widehat{w}_t = \frac{1-\gamma(1-\lambda_b)}{1-\lambda_b}\widehat{c}_{bt} + \gamma\widehat{c}_t - \frac{\lambda_b}{1-\lambda_b}\widehat{c}_{bt-1} + \sigma_l \widehat{l}_t$$

The linearized Euler equation, or equ(2.2), is

$$1 = E_t \left[\begin{array}{c} (r_{t+1} + z_{\chi c,t}) - \sigma_c \left(\frac{1}{1-\lambda_b} c_{b,t+1} - \frac{1+\lambda_b}{1-\lambda_b} c_{b,t} + \frac{\lambda_b}{1-\lambda_b} c_{b,t-1} \right) + \gamma \left(c_{b,t+1} - c_{b,t} \right) \\ -\gamma \left(c_{t+1} - c_t \right) + (\sigma_c - 1) \frac{w_* l_*}{e_*} \frac{1}{1-\lambda_b} \left(\widehat{l}_{t+1} - \widehat{l}_t \right) \end{array} \right]$$

The term $(\sigma_c - 1) \frac{w_* l_*}{e_*} \frac{1}{1-\lambda_b} \left(\widehat{l}_{t+1} - \widehat{l}_t \right)$ approximates $\frac{\sigma_c - 1}{1+\sigma_l} \left(l_{t+1}^{1+\sigma_l} - l_t^{1+\sigma_l} \right)$ once we take into account the steady state solution of the labor supply equation, or $\frac{w_* l_*}{e_*} \frac{1}{1-\lambda_b} = l_*^{1+\sigma_l}$.[33].

With terms collected, the linearized Euler equation is

$$\begin{aligned} (r_{t+1} + z_{\chi c,t}) \quad = \quad & \left(\frac{\sigma_c}{1-\lambda_b} - \gamma \right) \widehat{c}_{bt+1} - \left(\sigma_c \frac{1+\lambda_b}{1-\lambda_b} - \gamma \right) \widehat{c}_{bt} + \sigma_c \frac{\lambda_b}{1-\lambda_b} \widehat{c}_{bt-1} \\ & + \gamma \left(\widehat{c}_{t+1} - \widehat{c}_t \right) - (\sigma_c - 1) \frac{w_* l_*}{e_*} \frac{1}{1-\lambda_b} \left(\widehat{l}_{t+1} - \widehat{l}_t \right) \end{aligned}$$

[33] More about the approximation: $\frac{\sigma_c - 1}{1+\sigma_l} \left(l_{t+1}^{1+\sigma_l} - l_t^{1+\sigma_l} \right) \simeq \frac{\sigma_c - 1}{1+\sigma_l} l_*^{1+\sigma_l} (1+\sigma_l) \left(\widehat{l}_{t+1} - \widehat{l}_t \right)$. The RHS can be written as $(\sigma_c - 1) \frac{w_* l_*}{e_*} \frac{1}{1-\lambda_b} \left(\widehat{l}_{t+1} - \widehat{l}_t \right)$ after canceling out $1 + \sigma_l$ and making use of the steady state solution $\frac{w_* l_*}{e_*} \frac{1}{1-\lambda_b} = l_*^{1+\sigma_l}$ of the labor supply equaiton.

A.1.2 Retailers' optimality conditions

For simplicity in notation, let's use G_{t+s} to denote $\Pi_{l=1}^{s}\pi_{t+l-1}^{\iota_p}\pi_*^{1-\iota_p}$. The derivative of the objective function with respect to $\widetilde{P}_t$ is

$$\begin{aligned} 0 &= \sum \xi_p^s \frac{\Lambda_{t+s}\Xi_{t+s}}{\Lambda_t\Xi_t}\frac{1}{P_{t+s}}\left[G_{t+s}y_{i,t+s} + \left(\widetilde{P}_t G_{t+s} - \frac{P_{t+s}}{x_{t+s}}\right)\frac{\partial y_{i,t+s}}{\partial \widetilde{P}_t}\right] \\ 0 &= \sum \xi_p^s \frac{\Lambda_{t+s}\Xi_{t+s}}{\Lambda_t\Xi_t}\frac{1}{P_{t+s}}\left[G_{t+s}y_{i,t+s} - \frac{\lambda_f}{\lambda_f-1}\left(\widetilde{P}_t G_{t+s} - \frac{P_{t+s}}{x_{t+s}}\right)\frac{y_{i,t+s}}{\widetilde{P}_t}\right] \end{aligned}$$

The square bracket can be rewritten as $y_{i,t+s}\left[G_{t+s} - \frac{\lambda_f}{\lambda_f-1}\left(\widetilde{P}_t G_{t+s} - \frac{P_{t+s}}{x_{t+s}}\right)\frac{1}{\widetilde{P}_t}\right]$, and then $y_{i,t+s}\frac{1}{\lambda_f-1}\left[-G_{t+s} + \frac{\lambda_f}{x_{t+s}}\frac{P_{t+s}}{\widetilde{P}_t}\right]$. If we plug this term back into the FOC, the equation becomes

$$0 = \frac{1}{\lambda_f-1}\sum \xi_p^s \frac{\Lambda_{t+s}\Xi_{t+s}}{\Lambda_t\Xi_t} y_{i,t+s}\frac{1}{P_{t+s}}\left[-G_{t+s} + \frac{\lambda_f}{x_{t+s}}\frac{P_{t+s}}{\widetilde{P}_t}\right]$$

Now we can move $\frac{1}{P_{t+s}}$ into the bracket and multiply both sides of the equation by $(\lambda-1)\,\widetilde{P}_t$, to have

$$0 = \sum \xi_p^s E_t \frac{\Lambda_{t+s}\Xi_{t+s}}{\Lambda_t\Xi_t} y_{i,t+s}\left[\frac{\widetilde{P}_t G_{t+s}}{P_{t+s}} - \frac{\lambda_f}{x_{t+s}}\right]$$

or

$$\widetilde{P}_t = \lambda_f \frac{\sum_{s=0}^{\infty}\xi_p^s E_t \frac{\Lambda_{t+s}\Xi_{t+s}}{\Lambda_t\Xi_t} y_{i,t+s}\frac{1}{x_{t+s}}}{\sum_{s=0}^{\infty}\xi_p^s E_t \frac{\Lambda_{t+s}\Xi_{t+s}}{\Lambda_t\Xi_t} y_{i,t+s}\frac{G_{t+s}}{P_{t+s}}}$$

We now can substitute $y_{i,t+s}$ with $\left(\frac{P_{t+s}}{\widetilde{P}_t G_{t+s}}\right)^{\frac{\lambda_f}{\lambda_f-1}} y_{t+s}$ to get

$$\widetilde{P}_t = \lambda_f \frac{\sum_{s=0}^{\infty}\xi_p^s E_t \frac{\Lambda_{t+s}\Xi_{t+s}}{\Lambda_t\Xi_t}\left(\frac{P_{t+s}}{G_{t+s}}\right)^{\frac{\lambda_f}{\lambda_f-1}} y_{t+s}\frac{1}{x_{t+s}}}{\sum_{s=0}^{\infty}\xi_p^s E_t \frac{\Lambda_{t+s}\Xi_{t+s}}{\Lambda_t\Xi_t}\left(\frac{P_{t+s}}{G_{t+s}}\right)^{\frac{\lambda_f}{\lambda_f-1}-1} y_{t+s}}$$

The only remaining thing to do is to divide both sides by P_t to get equ(2.6).

A.1.3 Phillips curve

First linearize the optimal price setting condition, or equ(2.6). In the process we can cancel out P_t from the RHS of the equation, and cancel out terms

that exist in both the numerator and the denominator of the RHS. The result is

$$\widehat{\widetilde{P}}_t = (1-\xi_p\beta)\, E_t \sum_{s=0}^{\infty} \xi_p^s \beta^s \left(\widehat{P}_{t+s} - \widehat{G}_{t+s} - x_{t+s}\right)$$

Moving this equation one period forward gives us

$$\begin{aligned}
E_t\widehat{\widetilde{P}}_{t+1} &= (1-\xi_p\beta)\, E_t \sum_{s=1}^{\infty} \xi_p^{s-1} \beta^{s-1} \left[\widehat{P}_{t+s} - \left(\widehat{G}_{t+s} - \widehat{G}_{t+1}\right) - x_{t+s}\right] \\
&= (1-\xi_p\beta)\, E_t \sum_{s=1}^{\infty} \xi_p^{s-1} \beta^{s-1} \widehat{G}_{t+1} \\
&\quad + (1-\xi_p\beta)\, E_t \sum_{s=1}^{\infty} \xi_p^{s-1} \beta^{s-1} \left(\widehat{P}_{t+s} - \widehat{G}_{t+s} - x_{t+s}\right) \\
&= \widehat{G}_{t+1} + (1-\xi_p\beta)\, E_t \sum_{s=1}^{\infty} \xi_p^{s-1} \beta^{s-1} \left(\widehat{P}_{t+s} - \widehat{G}_{t+s} - x_{t+s}\right) \\
&= \iota_p \widehat{\pi}_t + (1-\xi_p\beta)\, E_t \sum_{s=1}^{\infty} \xi_p^{s-1} \beta^{s-1} \left(\widehat{P}_{t+s} - \widehat{G}_{t+s} - x_{t+s}\right)
\end{aligned}$$

in the last equality $\iota_p\widehat{\pi}_t$ substitute $\widehat{G}_{t+1}$ from G's definition.

Now we can combine the expression for $\widehat{\widetilde{P}}_t$ and that for $E_t\widehat{\widetilde{P}}_{t+1}$ to derive $\widehat{\widetilde{P}}_t - \xi_p\beta E_t\widehat{\widetilde{P}}_{t+1}$,

$$\widehat{\widetilde{P}}_t - \xi_p\beta E_t\widehat{\widetilde{P}}_{t+1} = (1-\xi_p\beta)\left(\widehat{P}_t - x_t\right) - \xi_p\beta\iota_p\widehat{\pi}_t$$

The result of canceling out common term and setting $\widehat{G}_t = 0$ at time t.

We now move on to linearize equ(2.7). First we divide the equation by P_t on both sides and substitute $\frac{P_{t-1}}{P_t}$ with $\frac{1}{\pi_t}$. The linearized version of the resulting equation is

$$\widehat{\widetilde{P}}_t - \widehat{P}_t = \frac{\xi_p}{1-\xi_p}\left(\widehat{\pi}_t - \iota_p\widehat{\pi}_{t-1}\right)$$

Finally, we need to combine the linearized equ(2.6) and the linearized equ(2.7). It takes some tedious rearrangement, but the result is just equ(2.37) in the text.

A.2 Calibration

1). β is set at 0.982, implying a real net interest rate of 7% per year.

2). δ_k is set at 10% per year.

3). δ_h is set at 1.6% per year used in Davis and Heathcote (2005).

4). α the share of capital is set at 0.3 following usual calibration convention; x, the monopolistic markup is set at 1.05 following Iacoviello (2005b).

5). m_h and m_k are chosen so that in the steady state, the model features 61% consumption to output ratio, 12% investment to output ratio, 5% residential investment to output ratio (The remainder 22% of the output is treated as government consumption), and 15% of housing rent in total private consumption expenditure.

A.3 Kalman Filter and Likelihood Function

This section starts from a linear state space model that approximates the solution of the DSGE model around a unique stable rational expectations equilibrium. The objective is to illustrate how to construct a likelihood function when given a sample of observed endogenous variables and a set of parameters of the DSGE model. Readers can find a more detailed description of the process from Chapter 9 of Hansen and Sargent (2005). This section provides a compact treatment.

The state-space representation expresses the state vector x_t and the vector of observable y_t as linear functions of initial state and histories of orthogonal shocks:

$$\begin{aligned} x_{t+1} &= Ax_t + Cw_{t+1} \\ y_t &= Gx_t + v_t \end{aligned}$$

where $w_{t+1} \sim iidN(0, I_n)$ and $v_t \sim iidN(0, R)$. w represents structural shocks, and v is interpreted as the measurement errors. It is assumed that $Ew_{t+1}v_s' = 0$ for all t and s. The matrix A, C, G and R are constructed out of the parameters of the model, let's call it θ. The measurement error v_t is assumed to be zero is this paper.

For the purpose of estimation, the representation is not useful because these orthogonal shocks are not observable. We shall use the Kalman filter to obtain the "whitening filter" that is cast in terms of the observed variables and the parameters of the model. It is by using this whitening filter that we can calculate recursively a Gaussian likelihood function.

The whitening filter is described with

$$\begin{aligned} a_t &= y_t - GE_{t-1}x_t \\ E_t x_{t+1} &= AE_{t-1}x_t + A\Sigma_t G'(G\Sigma_t G' + R)^{-1} a_t \\ \Sigma_{t+1} &= A\Sigma_t A' + CC' - A\Sigma_t G'(G\Sigma_t G' + R)^{-1} G\Sigma_t A' \\ V_t &= G\Sigma_t G' + R \end{aligned}$$

where $K_t = A\Sigma_t G'(G\Sigma_t G' + R)^{-1}$ is the Kalman gain matrix, constructed as the parameters of the state space model and the matrix Σ_t. The latter is initiated from some matrix Σ_0 (more discussion follows later), and is updated according to the third equation.[34] From Σ_t we can calculate V_t, the covariance matrix of prediction error.

The filter accepts the data $\{y_t\}$ as the input and produces $\{a_t\}$ as the output. $\{a_t\}$ is the the one-step ahead prediction error of y_t. The process from $\{y_t\}$ to $\{a_t\}$ is called a "whitening" process, because $\{y_t\}$ is serially correlated and the $\{a_t\}$ is serially un-correlated, or "white". $\{a_t\}$ can be said to be a fundamental white noise for the $\{y_t\}$ process.

To construct the likelihood function for a sample of observations $\{y_t\}_{t=1}^T$, we need to feed $\{y_t\}_{t=1}^T$ into the filter described above with some initial $E_0 x_1$ and Σ_0. The output is a series of prediction error $\{a_t\}_{t=1}^T$, and a series of matrix $\{V_t\}_{t=1}^T$ that describe the conditional variance covariance of $\{a_t\}_{t=1}^T$. The log-likelihood function of $\{y_t\}_{t=1}^T$ given model's parameter is then

$$\ln L\left(\theta | y^T\right) = -\frac{n}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\ln\left(|V_t|\right) - \frac{1}{2}\sum_{t=1}^{T} a_t V_t^{-1} a_t'$$

To initiate the filter I use the unconditional variance of x_t as Σ_0 and the unconditional mean of x_t as $E_0 x_1$.

[34] The matrix Σ_{t+1} can be defined as the second moment of $x_{t+1} - E_t x_{t+1}$

$$\begin{aligned} x_{t+1} - E_t x_{t+1} &= A^o x_t + C w_{t+1} \\ &\quad - \left[A^o E_{t-1} x_t + K_t\left(G x_t + v_t - G E_{t-1} x_t\right)\right] \\ &= \left(A^o - K_t G\right)\left(x_t - E_{t-1} x_t\right) + C w_{t+1} - K_t v_t \end{aligned}$$

the second moment of $x_{t+1} - E_t x_{t+1}$ is

$$\begin{aligned} \Sigma_{t+1} &= \left(A^o - K_t G\right)\Sigma_t\left(A^o - K_t G\right)' + CC' + K_t R K_t' \\ &= A^o \Sigma_t A^{o\prime} + K_t G \Sigma_t \left(K_t G\right)' - A^o \Sigma_t \left(K_t G\right)' - K_t G \Sigma_t A^{o\prime} \\ &\quad + CC' + K_t R K_t' \end{aligned}$$

which can be rearranged into the updating rule after substituting in the Kalman gain matrix.

A.4 Bayesian Estimation and RWM Algorithm

Bayesian estimation maximizes an objective function that is defined as the sum of the log of the data likelihood and the log prior density. The set of the parameter that maximizes the objective function is the mode of the posterior distribution of the parameters. To explore the posterior distribution around the neighborhood of the mode, I use the Random Walk Metropolis (RWM) algorithm. More details can be found in An and Schorfheide (2006).

A.4.1 Maximizing posterior density to estimate the mode

Denoting the prior density by $p(\theta)$, the likelihood function by $L\left(\theta|y^T\right)$, the posterior density by $p\left(\theta|y^T\right)$, we have

$$p\left(\theta|y^T\right) = \frac{L\left(\theta|y^T\right)p(\theta)}{p\left(y^T\right)}$$

where $p\left(y^T\right) = \int_\theta L\left(\theta|y^T\right)p(\theta)\,d\theta$ is the marginal data density. Because $p\left(y^T\right)$ is independent of θ, it is sufficient, for the purpose of maximizing posterior density, to just maximize the log of the non-normalized posterior density $L\left(\theta|y^T\right)p(\theta)$, or

$$\max_\theta \ln\left(L\left(\theta|y^T\right)\right) + \ln\left(p(\theta)\right)$$

I use the maximization routine "cswell.m" written by Chris Sims for the maximum likelihood estimation of a DSGE model. The algorithm uses a fairly simple line search and randomly perturbs the search direction if it reaches a cliff caused by non-existence or non-uniqueness of the DSGE solutions. The latest version is available for download in Sims' website. Following An and Schorfheide (2006), I use parameter transformation to turn constraint maximization into unconstrained maximization in the search for mode. The transformation is NOT used in the RWM algorithm that will be described shortly, where the out-of-range draws are discarded instead.

A.4.2 Use RWM algorithm to explore the distribution around the estimated mode

RWM algorithm explores the posterior distribution of the model parameters in the neighborhood of the mode $\widetilde{\theta}$.

Step 1: Numerically maximize the non-normalized posterior density with respect to parameter vector θ to obtain the posterior mode $\widetilde{\theta}$.

Step 2: Compute $\widetilde{\Sigma}$, the negative of the inverse of the Hessian computed at the mode.

Step 3: Draw an initial parameter vector θ^0 from $N\left(\widetilde{\theta}, c_0^2\widetilde{\Sigma}\right)$.

Step 4: For $s = 1, ..., N_{sim}$, draw ϑ from the distribution $N\left(\theta^{(s-1)}, c^2\widetilde{\Sigma}\right)$, and jump from $\theta^{(s-1)}$ to $\theta^{(s)}$ with the following rule: $\theta^{(s)} = \vartheta$ with probability $\min\left\{1, r\left(\theta^{(s-1)}, \vartheta|y^T\right)\right\}$, and $\theta^{(s)} = \theta^{(s-1)}$ otherwise. The acceptance probability $r\left(\theta^{(s-1)}, \vartheta\right)$ is

$$r\left(\theta^{(s-1)}, \vartheta\right) = \frac{L\left(\vartheta|y^T\right) p\left(\vartheta\right)}{L\left(\theta^{(s-1)}|y^T\right) p\left(\theta^{(s-1)}\right)}$$

Step 5: Use the sample statistics from $\left\{\theta^{(s)}\right\}_s$ to approximate the posterior distribution such as the mean and the percentile. One can also approximate the posterior expected value of a parameter transformation $h(\theta)$ by $\frac{1}{N_{sim}}\sum_{s=1}^{N_{sim}} h(\theta^s)$. Examples of $h(\theta^s)$ include the variance decomposition of prediction errors.

In this paper the step sizes $c_0 = 0.3$, and c is chosen to produce an acceptance rate of about 25%.

A.5 Data

Real output and real expenditures are defined as the nominal dollar value divided by a single deflator that is the implicit price index of GDP. All aggregate values are divided by US population over age of 16. Inflation is defined as the quarterly growth rate of the implicit GDP price. Interest rate is the effective federal funds rate. Total output is defined as nominal GDP net of the value of housing service. The value of housing service is available in BEA's Table 2.3.5. Non-housing consumption expenditure is defined as the total Private Consumption Expenditure net of the value of housing service.

Appendix B

Appendix for Chapter 3

B.1 Data

The price index of new homes is the Price Deflator (Fisher) Index of New, One-Family Houses Under Construction, from the Survey of Construction, The survey collects information on the physical characteristics and prices of new, one-family houses. The value of land is subtracted from the sales price at a fixed proportion of about 11%.

The price index is the major component of the price index of residential investment of the NIPA. In fact, the two series (price for residential investment in the NIPA, and the Price Deflator of the Census Bureau for single-family houses) have a correlation of 0.94, in terms of quarter-to-quarter growth rate. Thus, they are hardly distinguishable.

The quantity index of residential investment is the chain type quantity index of residential investment from the NIPA of the BEA.

The consumption series is the total Private Consumption Expenditure from the NIPA.

The real interest rate is the difference between the effective federal funds rate and the growth rate of the GDP deflator.

B.2 Calibrated parameters

The depreciation rate of houses is calibrated from the BEA fixed asset tables (9.1, 9.2, and 9.3). The three tables show the Real Net Stock, Real Depreciation, and Real Investment of private residential fixed assets. These figures are available from 1990 on. The average yearly depreciation rate is calculated as a ratio of depreciation over stock, and is 1.59% for residential fixed asset.

The steady-state real interest rate (2.4% yearly rate) and the steady-state growth rate of consumption per capita (2% yearly) are averages from the data. The population growth rate is set at 1.2% per year; and the average growth rate of the population (aged 16 and over) is for the US between 1984

and 2007. The exogenous growth rate of sector-specific technology (γ_x) is calibrated to produce a 1.2% annual growth rate in the real house price for the estimated slope of the marginal cost curve ψ. The alternative of setting the exogenous growth rate to zero does not change the result in a noticeably different way.

B.3 Deriving first order conditions for the inventory model

Let $q_{b,t} = \frac{q_{g,t}}{1+\Phi_2\left(\frac{d_t}{h_t}\right)}$, households' optimal level of search effort satisfies the following condition

$$\left(q_{b,t} - q_{h,t}\right) e^{-\frac{b_t}{v_t}} = c q_{c,t} \tag{B.1}$$

The optimal condition, and equ(3.9) that describes the accumulation of inventory, are the two constraints in the profit maximization of the retail arm of residential builder.

The Lagrangian of residential builder's optimization problem is

$$\begin{aligned}
\pounds \quad = \quad & \max_{\left(d_t, v_{t+1}, q_{h,t}, b_t\right)} E_t \sum_{s=0}^{\infty} \left(\frac{q_{h,t+s} v_{t+s} \left[1 - e^{-\frac{b_{t+s}}{v_{t+s}}}\right] - q_{c,t+s} d_{t+s}}{\Pi_s \left(1 + r_{t+1+s}\right)} \right) \\
& + \sum_{s=0}^{\infty} \lambda_{1,t+s} \left(\frac{v_{t+s} e^{-\frac{b_{t+s}}{v_{t+s}}} + d_{t+s} - v_{t+1+s}}{\Pi_s \left(1 + r_{t+1+s}\right)} \right) \\
& + \sum_{s=0}^{\infty} \lambda_{2,t+s} \left(\frac{\left(q_{b,t+s} - q_{h,t+s}\right) e^{-\frac{b_{t+s}}{v_{t+s}}} - c q_{c,t+s}}{\Pi_s \left(1 + r_{t+1+s}\right)} \right)
\end{aligned}$$

The first order conditions are

$$\begin{aligned}
\frac{\partial \pounds}{\partial d_t} &= 0 = -q_t + \lambda_{1,t} = 0 \\
\frac{\partial \pounds}{\partial v_{t+1}} &= 0 = \frac{q_{h,t+1}}{1+r_{t+1}}\left[1 - e^{-\frac{b_{t+1}}{v_{t+1}}} - e^{-\frac{b_{t+1}}{v_{t+1}}}\frac{b_{t+1}}{v_{t+1}}\right] - \lambda_{1,t} \\
&\quad + \frac{\lambda_{1,t+1}}{1+r_{t+1}}\left(e^{-\frac{b_{t+1}}{v_{t+1}}} + e^{-\frac{b_{t+1}}{v_{t+1}}}\frac{b_{t+1}}{v_{t+1}}\right) \\
&\quad + \frac{\lambda_{2,t+1}}{1+r_{t+1}}\left((q_{b,t+1} - q_{h,t+1})\frac{e^{-\frac{b_{t+1}}{v_{t+1}}}}{v_{t+1}}\frac{b_{t+1}}{v_{t+1}}\right) \\
\frac{\partial \pounds}{\partial q_{h,t}} &= 0 = v_t\left(1 - e^{-\frac{b_t}{v_t}}\right) - \lambda_{2,t}e^{-\frac{b_t}{v_t}} \\
\frac{\partial \pounds}{\partial b_t} &= 0 = (q_{h,t} - \lambda_{1,t})\, e^{-\frac{b_t}{v_t}} - (q_{b,t} - q_{h,t})\,\lambda_{2,t}\frac{e^{-\frac{b_t}{v_t}}}{v_t}
\end{aligned}$$

From $\frac{\partial \pounds}{\partial d_t} = 0$,

$$\lambda_{1,t} = q_{c,t}$$

From $\frac{\partial \pounds}{\partial q_{h,t}} = 0$,

$$\lambda_{2,t}\frac{e^{-\frac{b_t}{v_t}}}{v_t} = \left(1 - e^{-\frac{b_t}{v_t}}\right)$$

which together with $\frac{\partial \pounds}{\partial b_t} = 0$ determines that

$$(q_{h,t} - q_{c,t})\, e^{-\frac{b_t}{v_t}} = \left(1 - e^{-\frac{b_t}{v_t}}\right)(q_{b,t} - q_{h,t}) \tag{B.2}$$

The equation has an alternative expression of $q_{b,t} - q_{h,t} = e^{-\frac{b_t}{v_t}}(q_{b,t} - q_{ct})$. It states that the net gain from the trade, defined as $q_{b,t} - q_{h,t}$, is split between developer and households. The share that goes to household, or $q_{b,t} - q_{h,t}$, is $e^{-\frac{b_t}{v_t}}$. The share is larger when inventory-to-sales is high.

The expression for $\frac{\partial \pounds}{\partial v_{t+1}} = 0$ can be simplified substantially, if we notice that the last term of the expression can be written as $\frac{1}{1+r_{t+1}}(q_{h,t+1} - q_{c,t+1})\, e^{-\frac{b_{t+1}}{v_{t+1}}}\frac{b_{t+1}}{v_{t+1}}$ using the expression for $\frac{\partial \pounds}{\partial q_{h,t}} = 0$ as well as equ(B.2). The simpler expression can then be plugged into $\frac{\partial \pounds}{\partial v_{t+1}} = 0$ to cancel out all terms that contains

$e^{-\frac{b_{t+1}}{v_{t+1}}}\frac{b_{t+1}}{v_{t+1}}$. The resulted equation is

$$q_{c,t}\left(1+r_{t+1}\right)=q_{h,t+1}\left(1-e^{-\frac{b_{t+1}}{v_{t+1}}}\right)+q_{c,t+1}\left(e^{-\frac{b_{t+1}}{v_{t+1}}}\right) \tag{B.3}$$

which is after substituting $\lambda_{1,t}$ away with $q_{c,t}$.

$$\begin{aligned}
q_{c,t}\left(1+r_{t+1}\right) &= \left(q_{h,t+1}-q_{c,t+1}\right)\left(1-e^{-\frac{b_{t+1}}{v_{t+1}}}\right)+q_{c,t+1} \\
&= m_{t+1}q_{c,t+1}\left(1-e^{-\frac{b_{t+1}}{v_{t+1}}}\right)+q_{c,t+1}, \text{ where } m_{t+1}=\frac{q_{h,t+1}-q_{c,t+1}}{q_{c,t+1}} \\
\left(1+r_{t+1}\right) &= \left[m_{t+1}\left(1-e^{-\frac{b_{t+1}}{v_{t+1}}}\right)+1\right]\frac{q_{c,t+1}}{q_{c,t}}
\end{aligned}$$

After solving Equ(B.2) and Equ (B.1) for $q_{h,t}$ and $q_{b,t}$ as a function of $q_{c,t}$ and $e^{-\frac{b_t}{v_t}}$, we can derive the equations in the text including equ(3.12).

Appendix C

Appendix for Chapter 4

C.1 Data

The price index of new homes is the Price Deflator (Fisher) Index of New, One-Family Houses Under Construction, from the Survey of Construction, The survey collects information on the physical characteristics and prices of new, one-family houses. The value of land is subtracted from the sales price at a fixed proportion of about 11%.

The price index is the major component of the price index of residential investment of the NIPA. In fact, the two series (price for residential investment in the NIPA, and the Price Deflator of the Census Bureau for single-family houses) have a correlation of 0.94, in terms of quarter-to-quarter growth rate. Thus, they are hardly distinguishable.

The quantity index of residential investment is the chain type quantity index of residential investment from the NIPA of the BEA.

The consumption series is the total Private Consumption Expenditure from the NIPA.

The real interest rate is the difference between the effective federal funds rate and the growth rate of the GDP deflator.

All nominal series are turned into real series with the GDP price index as the deflator.

C.2 Calibrated parameters

The depreciation rate of houses is calibrated from the BEA fixed asset tables (9.1, 9.2, and 9.3). The three tables show the Real Net Stock, Real Depreciation, and Real Investment of private residential fixed assets. The average yearly depreciation rate is calculated as a ratio of depreciation over stock. The result is 1.59% for residential fixed asset. The steady-state real interest rate (2.4% yearly rate) and the steady-state growth rate of consumption (3.6% yearly) are averages from the data.