VISCOELASTIC FLOWS WITHIN ECCENTRIC ROTATING CYLINDERS – JOURNAL BEARINGS

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Abstract

Experiments have shown that the addition of small amounts of long-chained polymer additives to a Newtonian fluid produces desirable lubricants. Additives added to oil make the fluid viscoelastic. The effect of viscoelasticity on lubrication characteristics has recently taken on added significance with the move to yet lower-viscosity lubricants for improved energy efficiency. Any factor influencing load-bearing capacity and wear is clearly of renewed importance. The general trend towards the usage of high performance lubricants and environmentally friendly products also support the design of new lubricants.

This thesis is aimed at investigating viscoelastic flows within eccentric rotating cylinders (practical application - journal bearings) using a commercial finite element software POLYFLOW. Numerous validations are performed and excellent agreements are achieved. Steady shear and small-amplitude oscillatory shear (SAOS) experiments are performed for specific lubricants including mineral-based and bio-based lubricants to characterize their rheological behavior. Experimental data are fitted by a viscoelastic constitutive model used for numerical simulations.

The effects of fluid viscoelasticity between eccentric rotating cylinders on the flow field and on the lubrication performances are revealed in 2D and 3D respectively. From 2D investigation, an increased load capacity on the inner cylinder is found to be achieved by increasing the viscoelasticity of flow. For the first time, to our knowledge, 3D results for an UCM (Upper-Convected Maxwell) fluid at steady state are presented and the flow patterns along the axial direction within the eccentric rotating cylinders are investigated. The viscoelastic effects of those lubricants on the journal bearing performances are revealed and compared at various temperatures. The modeling and numerical simulations used to predict the flow of lubricant in a journal bearing can generate important economic benefits. This research will lead to advanced predictive tools that can be used to improve the design of journal bearing and to propose new economically viable and environmentally friendly lubricants.
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Chapter 1  Introduction

1.1  Scope of thesis
The oil and gas industry is currently employing more than 50,000 people in Canada. Oil companies dedicate a large portion of their research and development efforts towards reducing emissions. Emissions from transportation vehicles of all types constitute the largest single contribution to greenhouse gases in Canada and are expected to have the highest growth rate over the next two decades. The minimization of greenhouse gas emissions to the levels set by Kyoto Protocol levels can only be achieved through innovative design which includes an optimal lubricant design or the design of new and better lubricant.

There are many challenges in developing the lubricants of the future. The study of journal bearing lubrication has, over the years, generated much interest from major oil industries, engine manufacturers and machinery manufacturers alike. In these applications it is important to predict and assess the performance of lubricant within a journal bearing with respect to wear and efficiency under a wide range of operating conditions. According to some studies, the direct cost of friction and wear can account for nearly 10% of the gross national product (GNP) in many industrial nations. Moreover, they estimate that cost savings of up to 1% of the GNP could be achieved simply by using the right lubricant [1].

It is important to understand the performance of the lubricants in more details. Lubricants in classical hydrodynamic lubrication analyses are assumed to behave as Newtonian fluids. Many practical lubrication applications may be found where the Newtonian fluid constitutive approximation is not a satisfactory engineering approach. Experiments have shown that the addition of small amounts of long-chained polymer additives to a Newtonian fluid produces desirable lubricants. Additives added to oil, for example, make the fluid viscoelastic. The effect of viscoelasticity on lubrication characteristics has recently taken on added significance with the move to yet
lower-viscosity lubricants for improved energy efficiency. Any factor influencing load-bearing capacity and wear is clearly of renewed importance. Therefore there are good practical reasons to investigate the general question of viscoelastic effect in lubrication. The general trend towards the usage of high performance lubricants and environmentally friendly products also support the design of new lubricant.

Even though the mineral-based lubricants are dominating the lubricant market and they can satisfy most performance requirements in various lubrication systems, they still encounter many problems in the future. The world now consumes in 6 weeks the amount of lubricants that was consumed in a full year in 1950, while mineral-based lubricants made from petroleum which are non-renewable and will be consumed in 50 years [2]. An emerging class of lubricants – bio-lubricants, is a good substitute of mineral-based lubricants. Bio-lubricants made from vegetable oils are renewable, nontoxic and non-polluting, and 85% of applications can be met by lubricants from bio-refinery. The demand and application of bio-lubricant are greatly increasing [3].

Today's challenge is to increase the lubrication efficiency, and to reduce the gas emissions by improving the design of the existing lubricants or to design new environmentally friendly lubricants. Before lubricants are selected, they should be assessed over a wide range of dynamic operating conditions which is an expensive and time consuming process. The modeling and simulation process to predict the flow of the lubricant in a journal bearing can generate important economic benefits. This research will lead to advanced predictive tools that can be used to improve the design of journal bearings and to propose new economically viable and environmentally friendly lubricants. The thesis will focus on the study of lubrication performance, using modeling and simulations, and experimental characterization. Suggestions and predictions for applications of lubricant can be given without a lubrication simulator apparatus.

1.2 Background
Lubrication is the technique employed to reduce wear of close surfaces with relative movement to each another. It is through interposing a substance called lubricant between
the surfaces to carry or to help carry the load between those two opposing surfaces. Lubrication has great influence on performance quality, working efficiency and life time of all mechanical systems. In the machines various mechanical elements require lubrication including bearings, gears, cylinders, chains, couplings, wire ropes, etc. These elements form various lubrication geometries in which the two surfaces move with respect to each other by rolling, sliding, advancing and receding, or combination of several motions.

The lubricant film between the two opposing surfaces can be a liquid, semi-fluid (greases), a solid, or exceptionally a gas. These lubricating substances are chosen to match various working requirements and operating conditions. The most commonly used lubricant is liquid which can carry the applied load mainly by pressure generated within the fluid due to the frictional viscous resistance to motion of the lubricating fluid between the surfaces.

Machinery has increased in complexity and technology to achieve higher production levels and expand to work under more severe operating conditions. These factors result in higher operating speeds, severe climate conditions and greater lubricant temperatures leading to better required lubrication performances. Along with increased environmental demands, higher safety requirements, and more maintenance-free operations, the research in this field is becoming more necessary and important.

1.2.1 Lubrication regimes
Machine elements are lubricated by interposing and maintaining fluid films between relatively moving surfaces which can decrease the friction and minimize actual contact between the surfaces. As the lubricant has the characteristic of strong conformability, subjected to shear stress it can deform very easily and generate very low resistance to shear. Therefore, the lubricant can separate the two surfaces in relative motion and reduce friction, wear or damage between these surfaces. Under various working conditions including rotating speed, temperature, pressure, load and lubricant characteristics, lubrication films present different forms which can have a great influence on lubricating capacity. As the lubrication film thickness between the two relatively
moving surfaces varies greatly in the real operation, several lubrication regimes arise. With increasing lubricant thickness, we can encounter different lubrication regimes (see figure 1.1): boundary (thin film), mixed, elastohydrodynamic and thick (hydrodynamic) lubrication [4].

![Figure 1.1 Schematic of lubrication regimes for journal bearing (Stribeck curve) [4].](image)

1. **Boundary lubrication**
   When surfaces run together under thin film conditions (boundary lubrication), the load is supported by both lubricant film and contact between surfaces. Under thin film lubrication, wear will become heavier and breakage of machine is easy to happen.

2. **Elastohydrodynamic lubrication**
   The application of hydrodynamic lubrication principle for rolling bodies, such as ball bearing or roller bearing, is elastohydrodynamic lubrication. The area of contact is extremely small, so the pressure applied by load conditions through the lubricating film is as high as 30,000 – 400,000 psi [5]. This high pressure results in elastic deformation on the surfaces.

3. **Hydrodynamic lubrication**
   The thick hydrodynamic film will separate two surfaces which are submerged in the lubricant and support the applied load on one surface such as journal surface. The
working principle of thick hydrodynamic film is illustrated in figure 1.2. [6].

![Diagram of thick hydrodynamic film](image)

**Figure 1.2 Thick hydrodynamic film [6].**

The upper surface is moving with velocity - U while the lower surface is stationary. The two relative moving surfaces form a wedge shape space. With the movement of the upper flat surface, the lubricant between two surfaces will be drawn into the wedge-shape space by internal friction of the lubricant. The drawing forces from internal friction are equal for draw lubricant in and discharge lubricant out while the cross-sectional area at the outlet is clearly smaller than the inlet of lubricant. This increasingly narrow shape of wedge restricts the flow of lubricant. The moving upper surface will compress the lubricant in the wedge space and force them through the restricted outlet. Under this condition, the pressure in the lubricant will rise. The risen pressure in the lubricant film will increase the flow at the restricted outlet and delay the flow at the inlet. However, the most important effect of the risen pressure in the wedge-shaped film is to support an applied load on the upper moving surface and keep the two surfaces to be separated completely. To this point, the thick hydrodynamic film is formed and perform lubrication medium between surfaces.

Basically the dominant events in lubrication problems involve pressure, viscosity and moving walls at a slight incline angle. The pressure gradients are generated by two
events. First, the moving wall on the side sweeps fluid into a narrowing passage through
the action of viscous shear forces. A local flow rate would be large where the gap
between two surfaces - \( h \) is large and small where \( h \) is small. This cannot happen,
because continuity demands that the overall flow rate be constant. Hence, the flow sets
up a pressure gradient that redistributes the fluid and maintains a constant rate.

Thick hydrodynamic film lubrication can reach coefficients of friction as low as 0.001.
In the thick hydrodynamic film lubrication, the load is distributed over a relatively large
area so that the pressure on the surfaces is relatively low. Typical pressures for this
lubrication are in the range of 50-300 psi. Such amount of pressure is not high enough to
significantly alter the surfaces. The lubricant films for thick hydrodynamic lubrication
are usually more than 25 \( \mu m \) [5]. In this case, the lubricant pressure is high enough to
support the journal and keep the journal and bearing from any metal to metal contact. At
this point, the only friction happened in the journal bearing is the lubricant viscous
resistance to the shear. Using a less viscous lubricant can obviously minimize the
friction in the journal bearing, but this also affects the minimum thickness of lubricant
film and load capacity. If the narrowest distance between two surfaces is smaller than the
largest asperities of the surfaces, the thick hydrodynamic film lubrication system will be
broken and heavy friction, wear or surfaces seizure might happen. Balancing these
factors to achieve optimal lubrication capacity in journal bearing makes the design very
challenging.

1.2.2  Bearings
Bearings are widely used elements in lubrication systems, and are devices that permit
constrained relative motion between two parts, typically rotation or linear movement.
Bearings can be classified based on the direction of applied load into journal bearings
and thrust bearings. Based on the bearing operation type we can have hydrodynamic,
hydrostatic, rubbing or rolling element bearings. Bearings can also be classified based
on geometric form, in stepped parallel surface, tilting pad or tapered land. We will
restrain this work at journal bearing.
Figure 1.3  Schematic of journal bearing.

The geometry of journal bearing consists of a stationary cylinder and a rotating journal which is the part of the shaft surrounded by the cylinder housing. The geometry is shown in figure 1.3. The space between the two surfaces in relative movement is filled with a lubricant. In this work, journal bearings are investigated and various lubricants’ properties related to lubrication performance are studied and compared.

One of the main problems related to journal bearings is to bear an important load while keeping low friction. The load capacity of journal bearings might be affected by lubricant characteristics, lubricant temperatures, operating conditions, relative surface speed, and clearance within the bearing.

1.2.3  Lubricants
Lubricants are normally composed of base oils and specific polymer additives which can enhance lubricating oils’ characteristics to meet operation requirements. In the finished lubricating oils, base oils are the most important component, representing from 70% to 99% in some special oils [5].

1.2.3.1  Base oils
Base oils characteristics have significant influence on the final products’ properties. The base oil could be a mineral oil, synthetic oil or more recently a vegetable oil.
- Mineral base oils are made from the more viscous portion of crude oil by removing the gas oil and lighter fractions through distillation process. Then two further processing methods are used to obtain base oils, including separation and conversion. The final base oils are different for hydrocarbon structure and heteroatoms. This can influence the characteristics of base oils such as viscosity
property or oxidative instability. Pollution from mineral base oils can occur through leaking, pipe breaching, equipment maintenance and waste discharge. Consumption of petroleum products can generate large amount of greenhouse gases or poisonous pollution.

- Synthetic base oils are manufactured from petroleum, combining low molecular weight compounds that have adequate viscosity for use as lubricants. A large proportion of synthetic base oil is either one or a few compounds. This can make the synthetic base oils obtain the properties of the best compounds in mineral base oils. The synthetic base oils can have extended service life time and handle a wider range of operation temperatures.

- Vegetable oils have better lubrication capacity than petroleum products - they are naturally more slippery and posses more polar characteristic which can make vegetable oils to adhere better to metal parts. The bio-based oils are made from vegetable resources such as sunflower seeds, canola, soybeans and rapeseed, which are readily biodegradable and nontoxic or very low toxic. Figure 1.4 illustrates the vegetable oils used for bio-lubricants. In any incidental spill, the bio-oils will not be harmful to the environments. Compared to mineral base oils, bio-base oils still have some shortcomings such as they are oxidized readily at extreme temperatures [7].

Figure 1.4  Bio-lubricants can be made directly from vegetable resource.
1.2.3.2 Additives

In order to meet various operation requirements, appropriate additives are properly blended with base oils to achieve desired performance characteristics of lubricants. Different additives have various specific functions, some impart absolutely new properties, some reduce the rate of undesirable changes, and some enhance properties already present. Some commonly used additives are viscosity index improvers, pour point depressants, defoamants, oxidation inhibitors, corrosion inhibitors, rust inhibitors and detergents.

1.3 Bio-lubricants

Bio-lubricants are manufactured from natural raw materials such as plant harvested from farms, forests and oceans. Even though the mineral-based lubricants are dominating lubricant market and they can satisfy most performance requirements in various lubrication systems, bio-lubricants provide a number of benefits comparing to mineral-based counterparts.

- Pollution from mineral-based lubricants can invade the environment through many possible ways including leaking, pipe breaching, equipment maintenance and waste discharge. Bio-lubricants are made from vegetable resources such as sunflower seeds, canola, soybeans and rapeseed, which are readily biodegradable and nontoxic or very low toxic. In any kind of incidental spill, the bio-based lubricating oils will not be harmful to the working environments.

- Bio-lubricant application can reduce emission of greenhouse gases. Consumption of petroleum products can generate large amount of greenhouse gases or poisonous pollution. The environment issues such as Kyoto Protocol provide great pressure to develop alternatives of petroleum products. The bio-based lubricants are the key part of the solution. More than 85% of applications can be met by lubricants from bio-refinery and they are good replacements of mineral oils. Studies have shown that the emissions, including nitrous oxide, hydrocarbons, particulates, volatiles, and other carbon products, can be significantly reduced by almost 300lbs annually in a average SUV-type vehicle driving at average conditions using a genetically modified oilseed crop [8].
- Bio-lubricants are plentiful and renewable. The world now consumes in 6 weeks the amount of lubricants that was consumed in a full year in 1950, while mineral-based lubricants made from petroleum are non-renewable and will be consumed out in about 50 years. However, bio-lubricant resources are plentiful. Only for soybean oils, the production is around 6.2 billion gallons every year in the world [7]. Vegetable resources for lubricating oils are enough to substitute mineral-based resources.

- Bio-lubricants have many other advantages such as: 1) low coefficient of friction and anti-wear protection - they are naturally more slippery and possess more polar characteristic which can make vegetable oils to adhere better to metal parts. 2) reduced combustibility – fire risk can be reduced greatly by the application of bio-lubricant. 3) energy efficient – reduce energy cost.

Although bio-lubricants have many natural advantages over mineral-based lubricant, they still have some shortcomings such as they are oxidized readily at extreme temperatures. Updated technologies allowed scientists to develop bio-lubricants that are better than their predecessors and meet performance requirements, such as polymer additives addition or genetic engineering for vegetables.

In general, with the increasing environmental demands and higher safe requirements, bio-lubricants gained more and more attention recently and become increasingly portion in lubricant market.

1.4 Literature review
A model for lubrication problem (journal bearings) is the flow between eccentric cylinders with inner cylinder rotating and the outer cylinder stationary. Numerous studies have been devoted to flow study in the annulus of eccentric cylinders, some of these studies consider this flow as a test problem due to the complexity (paragraph 1.4.1.). Others consider it as a model for lubrication study in journal bearings (paragraph 1.4.2).
1.4.1 Annuli flows

A study by John Vohr [13] experimentally obtained the transition rotating speed for onset of Taylor vortices of superlaminar Newtonian flows between eccentric rotating cylinders by the torque measurement. For Power-law fluids in eccentric annuli, a simplified solution of the equations of motion was presented by Yang L and Chukwu Ga [14]. Another study by Q. E. Hussain and M. A. R. Sharif [15] numerically investigated helical flow of Yield-Power-Law fluids in concentric and eccentric annuli with rotating inner cylinder using a finite volume algorithm. In addition, comparison between numerical calculations with experimental results for a fully developed laminar flow of Power-law and Cross fluids through eccentric annuli was presented by M. P. Escudier et al. [16] [17]. In a study by D. Grecov and J.-R. Clermont [18], domain decomposition and stream-tube method were applied to study the non-Newtonian flows including Upper Convective Maxwell (UCM) and integral models between eccentric cylinders. Furthermore, studies by A. N. Beris et al. [19] [20] and X.Huang et al. [21] investigated UCM fluids within eccentric rotating cylinders at low eccentricities in steady state (inertia neglected). Eccentric annuli for Newtonian and power-law fluids related to orbital motion or axially varying eccentricity of inner cylinder have been investigated [22] [23] as well.

1.4.2 Industrial application (lubrication)
Eccentric rotating cylinders associated to journal bearings were widely investigated by many researchers. The growing interest in considering materials with non-Newtonian properties in lubrication has led to various experimental and computational techniques for investigating such complex flows.
1.4.2.1 Experiments (Journal bearing)

In the following research papers, experiments are used to investigate the influence of the non-Newtonian lubricants on the flow between eccentric cylinders and ultimately on the lubrication capacities.

A. Berker et al. [24] experimentally assessed the effect of polymer additives in journal bearings for several multi-grade lubricants. In steady flow, laser Doppler velocimetry measured velocity profiles showed that the size of a rotating eddy adjacent to the stationary wall in the large gap could be significantly decreased by adding additives in lubricants. It was experimentally found that viscoelasticity of the multigrade oils did indeed produce a measurable and beneficial effect on lubrication characteristics such as load capacity at high eccentricity ratios by B. P. Williamson et al.[25] [26]. In addition, a weakly elastic shear-thinning fluid velocity was measured in an experiment rig of eccentric annulus with axial flow and journal rotation by J.M.Nouri and J.H.Whitelaw [27]. The effects of non-Newtonian characteristics were revealed such as the turbulence intensities in the region of smallest gap increased in Newtonian fluid and decreased in non-Newtonian fluid. Another study by Bekir Sadik Unlu and Enver Atik used a journal bearing wear test rig to measure friction coefficient for journal bearing under dry and lubrication (SAE 90, Newtonian) conditions [28].

1.4.2.2 Rheological characterization of lubricants

B. P. Williamson et al. [25] [26] investigated the rheological properties of some multi-grade oils by viscometry (glass capillary method for low shear rates and Ravenfield HTHS viscometer for high shear rates) using a Weissenberg Rheogoniometer, and a Lodge Stressmeter. They found shear thinning and relaxation time characteristics for these multigrade lubricating oils. A special rheometer [29] was designed and built up to study the rheology of lubricants at extremely high shear rates (up to $10^9 \text{s}^{-1}$). This rheometer had a magnetic disk drive configuration which sheared an ultrathin film of lubricant under relatively low sliding speeds. In traditional rheometers, the shear rates can’t be too high due to viscous heating. Two different types of perfluoropolyether lubricants were studied and showed shear thinning characteristics. Experimental measurements on lubricants done by S. Bair and colleagues ([30] [31] [32]) also showed
the non-Newtonian characteristics of lubricants at high shear rates and high pressures. Based on experimental data a few practical applications of lubricant models were described in [33], such as Cross equation for shear thinning, Vogel’s equation for variation of viscosity with temperature, and White-Metzner model for lubricant viscoelasticity. Viscosity-shear rheological results for a few bio-edible lubricants were also presented by W.B.Wan Nik et al. [34].

1.4.2.3 Numerical study
In a theoretical and computational framework, several studies investigated the influence of the lubricants viscoelasticity on the lubrication performances in eccentric rotating cylinders (journal bearings).

Using finite difference and bipolar coordinates methods, G. W. Roberts and K. Walters [35] found that a relaxation time of order $10^{-4} \text{s}^{-1}$ for UCM fluid can produce a practically important increase in load capacity at angular velocity of 250 rad/s. A study by K. Raghunandana and B.C.Majumdar [36] numerically investigated stability of journal bearing using a power-law pseudoplastic model developed by Dien and Elrod [37]. They found that the stability improves for Non-Newtonian lubricants with higher power law index. In addition, some simplified lubrication models for viscoelastic fluids were derived and studied for polynomial contact surfaces (UCM [38] and SPTT [39]). A numerical tool based on Lattice Boltzman method for Bird-Carreau models in the lubrication of plain contact surfaces was also developed by Woo Tae Kim et al [40]. Moreover, Couple stress fluids in journal bearings were studied based on modified Reynolds equation with non-slip boundary conditions and porous medium layer boundary conditions by Yan-Yan Ma et al [41] and A. A. Elsharkawy [42] respectively. Dynamic journal bearings were also investigated for Cross fluids [43] and couple stress fluids [44]. For 3D the Taylor instability in journal bearing micro-scale Newtonian flows was also studied by a commercial code, CFD-ACE+ [45].

The viscoelastic fluid effect on eccentric rotating cylinders (such as journal bearing) is a long-standing and attractive subject. However, the effect of fluid viscoelasticity for many different lubricants on the flow between eccentric rotating cylinders and on the
bearing performances is not yet completely elucidated. For numerical simulations, table 1.1 summarizes the current knowledge level based on the literature review.

Table 1.1  Numerical simulations in literature

<table>
<thead>
<tr>
<th></th>
<th>Newtonian</th>
<th>Inelastic models</th>
<th>Differential models</th>
<th>Integral models</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low $\lambda$</td>
<td>High $\lambda$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2D</td>
<td>steady state</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
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<td></td>
<td>transient</td>
<td>✓</td>
<td>×</td>
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<tr>
<td>3D</td>
<td>steady state</td>
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<td>×</td>
</tr>
<tr>
<td></td>
<td>transient</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
</tbody>
</table>

Note: $\lambda$ - relaxation time (s). ✓ - Already studied  × - Not studied.

The conclusions based on the literature review are as follows:
- 2D studies for differential with high relaxation time are missing from literature.
- Transient behavior for non-Newtonian fluids is not investigated in the literature.
- Experiments on a journal bearing simulator have been done, but only few rheological characterization experiments for lubricants were done. Especially only few numerical studies for viscoelastic flows in journal bearings simulate models with parameters obtained from rheological experiments for specific lubricants.
- For 3D investigations of eccentric rotating cylinders, Newtonian and few inelastic fluids are studied.

The published evidence for the beneficial effects of viscoelasticity in lubrication is still not conclusive, however the reliable evidence is supportive of the positive benefits of lubricant viscoelasticity. Based on the literature review, we can conclude that the study of the flow between eccentric rotating cylinders geometry for different types of fluids with complex behavior, present challenging problems:
- Difficulties to simulate the flow when the viscoelasticity increases - the non-linearity
of the system of equations will increase.
- Rheological characterization at high shear rates or high frequency rates.
- Difficulties to simulate 3D viscoelastic case problems.

1.5 Motivation and objectives
The global demand for finished lubricants was estimated at 37.1 millions tons in 2003. Unoptimized lubricants cause excessive wear of bearings, generate high running costs, and increase engine emissions. Consequently, it is highly beneficial both financially and from an environmental perspective to select the correct lubricant. Before lubricants are selected, they should be assessed over a wide range of dynamic operating conditions which is an expensive and time consuming process. The modeling and simulation process to predict the flow of the lubricant in a journal bearing can generate important economic benefits. This research will lead to advanced predictive tools that can be used to improve the design of journal bearing and to propose new economically viable and environmentally friendly lubricants. The progress made in lubricant research today will play an important part in safeguarding the environment for many generations to come. The thesis will focus on the study of lubrication performance, using parallel, high performance simulations, and experimental characterization.

The general objective of the proposed research is to study and predict the effect of viscoelastic properties of lubricant on the lubrication performance and to design new lubricants that are economically viable and environmentally friendly.
The specific objectives are:
1. Rheological characterization of different types of lubricants.
2. Constitutive equations development by fitting rheological experimental data.
3. Numerical simulation of the hydrodynamics between eccentric rotating cylinders.
4. Predictions and discussions of lubrication performance in journal bearings.

1.6 Thesis organization
The thesis is organized as follows:
Chapter 2 provides governing and constitutive equations used for the flow between eccentric cylinders. Analytical solutions including Reynolds lubrication theory and an
analytical solution for PTT model are presented. The solution of the model is given by POLYFLOW, a software designed primarily for viscoelastic fluid dynamic applications. The numerical methods adopted in this work are presented in chapter 3 including the numerical schemes for both 2D and 3D studies of eccentric rotating cylinders. In order to validate the numerical methods, numerical results are compared with analytical results, numerical results, and available experimental data from other published papers. Rheological characterization for mineral-based and bio-based lubricants is included in chapter 4. In chapter 5, the steady state and transient state numerical results for eccentric rotating cylinders with viscoelastic fluids and lubricants tested in rheological experiments are presented. Different variables related to the lubrication performances in journal bearings are characterized and discussed. Finally chapter 6 presents the conclusions, contributions to the research, recommendations and future work.
Chapter 2 Mathematical modeling

In this chapter, the mathematical model for the study of flow between eccentric rotating cylinders including governing and different constitutive equations is presented. Analytical solutions for Newtonian and PTT (Phan-Thien-Tanner) annulus flows are also provided, which will be used for the validations of numerical methods.

2.1 Governing equations
The flows between eccentric cylinders are studied in this work. Figure 2.1 presents the schematic of eccentric annulus geometry. A Cartesian system of coordinates is used and the center of outer cylinder is set as the origin. $\Omega$ is the angular velocity of inner cylinder and the outer cylinder is at rest. $R_i$ and $R_o$ are radii of inner and outer cylinders respectively. $e$ is the eccentricity. $\theta$ and $r$ are cylindrical coordinates.

![Flow domain within eccentric rotating cylinders.](image)

The governing equations for the annulus flows are presented as follows. Continuity equation for incompressible fluids [46]

\[ \nabla \cdot \mathbf{V} = 0 \] (2.1)

where $\mathbf{v}$ is velocity vector.
Continuity equation can be expressed in Cartesian coordinates:

\[
\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0
\]  

(2.2)

where \( v_x, v_y, v_z \) are the components of velocity.

Equation of momentum [46]

\[
\rho \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \right) = -\nabla p + \nabla \cdot \mathbf{\tau} + \rho \mathbf{g}
\]

(2.3)

where, \( \rho \) is density, \( p \) is pressure, and \( \mathbf{\tau} \) is stress tensor.

Equation of momentum can be expressed in Cartesian coordinates:

\[
\begin{aligned}
\rho \left( \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) &= -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \rho g_x \\
\rho \left( \frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \right) &= -\frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + \rho g_y \\
\rho \left( \frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) &= -\frac{\partial p}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} + \rho g_z
\end{aligned}
\]

(2.4)

where \( \tau_{ij} \) are the components of stress tensor.

For Newtonian fluids, the stress tensor can be expressed as follows.

\[
\begin{bmatrix}
\tau_{xx} & \tau_{xy} & \tau_{xz} \\
\tau_{yx} & \tau_{yy} & \tau_{yz} \\
\tau_{zx} & \tau_{zy} & \tau_{zz}
\end{bmatrix}
= \mu \cdot
\begin{bmatrix}
2 \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \\
\frac{\partial v_y}{\partial x} + 2 \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \\
\frac{\partial v_z}{\partial x} + \frac{\partial v_z}{\partial y} + 2 \frac{\partial v_z}{\partial z}
\end{bmatrix}
\]

(2.5)

where \( \mu \) is viscosity.

The momentum equation in general form can be simplified to Navier-Stokes used for Newtonian fluids. For a Newtonian fluid the viscosity is constant at constant temperature and pressure. Neglecting body force, the Navier-Stokes equation can be expressed as:
2.2 Constitutive models

Constitutive modeling represents the appropriate tensorial expressions for the stress in a flow as a function of deformation matching observed material behavior. Several constitutive models are presented below.

2.2.1 Generalized Newtonian constitutive models

For generalized Newtonian constitutive models, the relationship between the stress tensor and rate of the deformation tensor is as follows [46].

\[
\begin{bmatrix}
\gamma_{xx} & \gamma_{xy} & \gamma_{xz} \\
\gamma_{yx} & \gamma_{yy} & \gamma_{yz} \\
\gamma_{zx} & \gamma_{zy} & \gamma_{zz}
\end{bmatrix} = \mu(\dot{\gamma})
\begin{bmatrix}
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot 
\end{bmatrix}
\]

(2.7)

where \( \dot{\gamma} \) is the shear rate, \( \gamma_{ij} \) are the components of the rate of deformation tensor, and \( \mu(\dot{\gamma}) \) is the viscosity function of local shear rate.

In equation 2.7, the rate of deformation tensor can be calculated by equation 2.8.

\[
\begin{bmatrix}
\gamma_{xx} & \gamma_{xy} & \gamma_{xz} \\
\gamma_{yx} & \gamma_{yy} & \gamma_{yz} \\
\gamma_{zx} & \gamma_{zy} & \gamma_{zz}
\end{bmatrix} = \begin{bmatrix}
\frac{2}{\rho} \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \\
\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial y} + \frac{2}{\rho} \frac{\partial v_z}{\partial z} \\
\frac{\partial v_x}{\partial z} + \frac{\partial v_y}{\partial z} + \frac{2}{\rho} \frac{\partial v_z}{\partial z}
\end{bmatrix}
\]

(2.8)

The viscosity of generalized Newtonian is a function of shear rate. The generalized Newtonian constitutive equation is a simple constitutive model, but can capture the
non-Newtonian behavior with sufficient accuracy for inelastic fluids. The viscosity as a function of local shear rate can be identified by fitting with the viscosity experimental data. The function can have various forms in order to better fit the material’s characteristic. For example, the Carreau fluid viscosity function [47] is given by:

\[
\frac{\mu - \mu_0}{\mu_0 - \mu_\infty} = [1 + (\lambda \dot{\gamma})^2]^{n-1/2}
\]  

(2.9)

where \( \mu_0 \) and \( \mu_\infty \) are zero shear viscosity and the viscosity at high shear rates respectively, \( \lambda \) is relaxation time, and \( n \) is a material parameter.

The generalized Newtonian models rely on modeling shear viscosity to incorporate the non-Newtonian information. Therefore, there are some significant limitations for generalized Newtonian models. The generalized Newtonian models can not predict normal stress differences \( N_1 = \tau_{xx} - \tau_{yy} \) and \( N_2 = \tau_{yy} - \tau_{zz} \) caused by elastic effects in material. Since for generalized Newtonian models the stress is only a function of the instantaneous rate of deformation tensor, they also can not predict the effects depending on the history of rate of deformation tensor. Therefore, more complex constitutive models are required to represent viscoelastic behavior of a fluid.

### 2.2.2 Viscoelastic constitutive models

Viscoelastic constitutive models include memory effects caused by elasticity of material. The stress tensor is a function of the history of rate of deformation tensor instead of a function of instantaneous rate of deformation tensor. Many viscoelastic constitutive models are developed to fit various material behaviors. Appropriate choices for the viscoelastic model and related parameters can yield qualitatively and quantitatively accurate representations of viscoelastic behavior. Two viscoelastic models - Upper Convected Maxwell (UCM) model and Phan-Thien-Tanner (PTT) model are considered in this work. UCM model is used to represent viscoelastic fluids within eccentric rotating cylinders in numerical simulations, and PTT model is used in the validations of numerical methods.

UCM model is a differential generalization of Maxwell model for the case of large
deformations based on the upper-convected time derivative. The model can be written as [47]:

$$\dot{\tau} + \lambda \dot{\tau} = \mu_0 \dot{\gamma}$$  \hspace{1cm} (2.10)

where $\lambda$ is relaxation time, $\mu_0$ is zero shear viscosity, and $\dot{\tau}$ is upper-convected time derivative of stress tensor which is expressed as:

$$\dot{\tau} = \frac{\partial}{\partial t} \tau + \nabla \cdot \tau - (\nabla V)^T \cdot \tau - (\nabla V)$$  \hspace{1cm} (2.11)

UCM constitutive model incorporates memory effects of materials, but its viscosity is constant at various shear rates.

The PTT model is a simple quasi-linear viscoelastic constitutive equation, which is widely used in the simulations of polymer solutions flow. The complete form of PTT constitutive equation is [47]:

$$f(tr_\tau) \cdot \tau + \lambda \dot{\tau} + \frac{\xi}{2} \lambda (\dot{\gamma} \cdot \tau + \tau \cdot \dot{\gamma}) = \mu \dot{\gamma}$$  \hspace{1cm} (2.12)

The linearized form of stress function is:

$$f(tr_\tau) = 1 + \frac{\varepsilon_i \lambda}{\mu tr_\tau}$$  \hspace{1cm} (2.13)

where $\varepsilon_i$ is a parameter which is related to the slip between the molecular network and the continuum medium, and $\varepsilon_i$ is the elongation parameter.

PTT model also incorporates memory effects of material and has more parameters than UCM model. Its viscosity can change with the variation of shear rate. When $\xi$ is zero, PTT constitutive equation reduces to its simplified form (SPTT).

$$f(tr_\tau) \cdot \tau + \lambda \dot{\tau} = \mu \dot{\gamma}$$  \hspace{1cm} (2.14)

Solving the model equations, the hydrodynamics within the annulus of eccentric rotating cylinders is obtained. Based on different values for characteristic parameters, this flow could present different patterns.

2.3 Flow patterns in annuli of rotating cylinders

The flow between two cylinders (concentric or eccentric) could manifest instabilities
including parallel flow instability and centrifugal instability.

- Parallel flow instability is attributed to the high Reynolds number. There are a few definitions for Reynolds number. For the flow between cylinders Reynolds number – \( \text{Re} \) is defined as follows.

\[
\text{Re} = \frac{R \Omega (R_o - R_i) \rho}{\mu}
\]  

(2.15)

When \( \text{Re} \) exceeds a critical number, the flow will become unstable and turbulent with the further increase of \( \text{Re} \). It is generally accepted that the parallel flow instability will occur at Reynolds number around 2000 [6].

- Centrifugal instability is the instability driven by centrifugal force effects. In a rotating flow, viscous force has a stabilizing role by damping out perturbations. While inertial effects (centrifugal forces) destabilize the flow system. When the stabilizing factor (viscous force) is suppressed by the unstable factor (centrifugal force), the flow will encounter centrifugal instability. Taylor number \( T_a \) is a very important parameter for the stability criterion. It is a dimensionless quantity which is defined as the ratio of centrifugal force and viscous force. There are also a few definitions for \( T_a \). One definition for \( T_a \) is written as follows [6].

\[
T_a = \frac{2(1-\kappa)}{1+\kappa} \text{Re}^2
\]

(2.16)

where \( \text{Re} \) is the Reynolds number, and \( \kappa \) is the radius ratio expressed as \( \kappa = \frac{R_i}{R_o} \).

### 2.3.1 Concentric cylinders

For the flow within concentric cylinders when \( T_a \) is smaller than a critical value \( T_{a,c} \), the flow driven by the inner cylinder is purely circular Couette flow. The instabilities are not present under this condition. Perturbations in the flow are all damped out by viscous forces. If \( T_a \) is higher than \( T_{a,c} \), an axisymmetric instability will appear. The radial and axial velocity components both grow exponentially in time and saturate nonlinearly. The flow under this condition consists of axisymmetric vortices stacked on top of the other in the axial direction with radial inflows and outflows. This flow pattern is called Taylor
vortex flow which is a stable secondary flow pattern with large toroidal vortices forming in the flow. The currently accepted minimum value of $Ta_c$ is 1694.95 [6].

If $Ta$ is further increased, the Taylor vortex flow will present non-axisymmetric instabilities, and azimuthal waves rotate around the inner cylinder at some wave speed. This flow pattern is called wave vortex flow. A further increase of $Ta$ leads to a wider variety of flow patterns such as spiral vortex flow, twisting vortex flow and turbulent vortex flow, which have clearly defined stability boundaries (see figure 2.2) [4].

![Image](image.png)

Figure 2.2  Flow patterns. a-Taylor vortex flow, b-wavy vortex flow, c-spiral vortex flow, d-twisting vortex flow, e-turbulent vortex flow [4].

### 2.3.2 Eccentric cylinders

For the flow between eccentric cylinders, besides Reynolds and Taylor numbers, another dimensionless parameter will influence the flow stability– eccentricity ratio ($\varepsilon = \frac{e}{c}$, here $c = R_o - R_i$ and $e$ is eccentricity). DiPrima and Stuart define $Ta$ for the flow between eccentric rotating cylinders as follows [48].

$$Ta_{c_e} = 1695(1+1.162\frac{e}{R_i})(1+2.624\varepsilon^2)$$  \hspace{1cm} (2.17)

When $\varepsilon < 0.3$, if $Ta < Ta_{c_e}$ and $Re < Re_{cr}$, the flow between eccentric cylinders is Couette flow -like. However when $\varepsilon > 0.3$, $Ta < Ta_{c_e}$, and $Re < Re_{cr}$, a recirculation cell will
appear in two dimensional plane. Increasing of \( T_a \) over \( T_{ac} \), the Couette laminar flow will lose its stability, and cellular and toroidal vortices along the axis of cylinders will arise [6]. This toroidal vortex flow in eccentric cylinders is similar to the Taylor vortex flow between concentric cylinders.

### 2.4 Analytical methods for journal bearing

There are various available analytical solutions for lubrication problems including the classic theory and recently developed solutions.

#### 2.4.1 Lubrication theory for Newtonian fluids

Figure 2.3 presents the schematic of lubrication geometry. Based on the lubrication theory, the classical Reynolds theory of lubrication [6] can be derived assuming that:

1. Body force is negligible.
2. Pressure is constant across the thickness of the film.
3. Radius of curvature of surfaces is large compared to the film thickness.
4. Slip does not occur at boundaries.
5. The fluid is Newtonian.
6. The flow is laminar.
7. Fluid inertia is neglected comparing with viscous forces (produced by the high oil viscosity).

Reynolds equation can be expressed as follows.

\[
\frac{\partial}{\partial x} \left( \frac{h^3}{\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{\mu} \frac{\partial p}{\partial z} \right) = 6(U_1 - U_2) \frac{\partial h}{\partial x} + 6h \frac{\partial(U_1 + U_2)}{\partial x} + 12(V_2 - V_1) \tag{2.18}
\]

where \( h \) is the gap between two surfaces, \( U_1 \) and \( U_2 \) are the X-velocities of the surfaces respectively, and \( V_1 \) and \( V_2 \) are the Y-velocities of the surfaces respectively (see figure 2.3).
Writing Reynolds equation for the flow between eccentric rotating cylinders, the equation can be written as follows [6].

\[
\frac{\partial}{\partial x} \left( \frac{h^3}{\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{\mu} \frac{\partial p}{\partial z} \right) = 6 \frac{\partial}{\partial x} \left[ h(U_1 + U_2) \right] + 12(V_{2,t} - V_1)
\]  

(2.19)

where \( V_{2,t} \) is the Y-component of rigid body translation of the journal.

Set: \( U_0 = U_1 + U_2 \), \( V_0 = V_{2,t} - V_1 \), thus the equation 2.19 can be simplified as:

\[
\frac{\partial}{\partial x} \left( \frac{h^3}{\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{\mu} \frac{\partial p}{\partial z} \right) = 6U_0 \frac{\partial h}{\partial x} + 6h \frac{\partial U_0}{\partial x} + 12V_0
\]  

(2.20)

Based on the lubrication theory mentioned above, the dimensionless azimuthal velocity component between long eccentric cylinders (L/D>2 [6], here L and D are the length and diameter of the bearing respectively) can be predicted for a Newtonian fluid [24]:

\[
\frac{v_\theta}{V_0} = 1 - k - \frac{3\varepsilon(k - k^2)(2\cos \theta + 3\varepsilon \cos \theta)}{(2 + \varepsilon^2)(1 + \varepsilon \cos \theta)}
\]  

(2.21)

where \( v_\theta \)-azimuthal velocity;

\( V_0 \)-tangential speed of inner cylinder;

\( R_i \)-inner cylinder radius, \( R_o \)-outer cylinder radius;

\( k = (r - R)/(R_o - R_i) \) reduced radial coordinate;

\( \varepsilon = \frac{e}{R_o - R_i} \) eccentricity ratio, e-eccentricity, h-gap;

\( \theta \)-azimuthal angle (counter-clockwise starting from the widest gap).
This analytical solution for a Newtonian fluid within eccentric rotating cylinders will be used for the validation of numerical method in section 3.3.1.

2.4.2 The exact analytical solution for SPTT fluids
An exact analytical solution for SPTT fluids in purely tangential flow within a concentric annulus is developed in [49]. It is assumed that the flow is a purely tangential laminar flow at steady state, and body forces are neglected. The tangential velocity is \( v_{\theta} = v_{\theta}(r) \), and the radial and axial velocities are \( v_{r} = v_{z} = 0 \). The boundary conditions are non-slip on both cylinders. The dimensionless wall shear stress on the inner cylinder for a SPTT fluid is given by:

\[
\tau_{wi}^{*} = \frac{1}{6} \sqrt[3]{-108q + 12\sqrt{12p^3 + 81q^2}} - \frac{2p}{\sqrt[3]{-108q + 12\sqrt{12p^3 + 81q^2}}} 
\]  
(2.22)

where \( V_c = \frac{R_i + R_o}{2} |\Omega_o - \Omega_i| \) Characteristic velocity scale;
\( \delta = R_o - R_i \) Annular gap width;
\( \tau_c = \frac{\mu V_c}{\delta} \) Characteristic shear stress;
\( \tau_{wi}^{*} \) dimensionless wall shear stress comparing with \( \tau_c \);
\( We = \frac{2V}{\delta} \) Weissenberg number;
\( K = \frac{R_i}{R_o} \) radii ratio;
\( \beta = \frac{\Omega_o}{\Omega_i} \) angular velocity ratio;
\( p = \frac{3(1 - K^2)}{2\epsilon_i We^2(1 - K^6)} \);
\( q = \frac{6(1 - K)(\beta - 1)}{\epsilon_i We^2(1 - K^6)(1 + \kappa)|\beta - 1|} \).

This analytical solution for a SPTT fluid within concentric rotating cylinders will also be used for the validation of numerical method in section 3.3.2.
Chapter 3  Numerical methods

Numerical simulations of the mathematical model are performed using POLYFLOW software. POLYFLOW is a finite-element computational fluid dynamics software designed primarily for viscoelastic fluid dynamics applications. A Sun Fire X4450 X64 server with 4 × Quad-core Intel Xeon X7350 processor and 16 GB memory is used to run simulations by POLYFLOW.

3.1 Numerical tools

Figure 3.1 presents the workflow of POLYFLOW [50]. The mesh created in Gambit is imported into POLYDATA where a flow problem is set up. POLYMAT is used for analyzing experiment data by various constitutive models. Then the numerical simulations are performed in POLYFLOW and the results could be presented using different post-processing packages.

![Figure 3.1 Work flow of POLYFLOW [50].](image)

27
POLYFLOW based on finite-element method provides many numerical schemes for numerical simulations. The numerical methods chosen in this work are introduced as follows.

### 3.1.1 Finite element method

In finite element methods the unknown functions are approximated in some specific ways in terms of a finite number of parameters at the outset. These parameters are selected to satisfy the field and constitutive equations, and the boundary conditions. The finite element method is based on the following algorithm:

- The domain is divided into many non-overlapping domain elements and the unknown function is approximated over the domain by means of piecewise polynomials. On each domain element the nodes are identified, which may be the ends of the domain element or the ends together with a node in the middle of the domain element, etc.

- The approximation piecewise polynomial function for the corresponding domain element will be expressed by means of nodal values. These nodal values may be the direct value of the unknown function, the mean value of the unknown function over the domain element, or the derivatives of the unknown function at a node.

- The unknown function can be represented by the piecewise polynomial basing upon the selected nodal values over each domain element. The success of a finite element algorithm depends on the appropriateness of the approximation function and the rule for satisfying the equations [51].

There are different interpolation polynomials for various variables in POLYFLOW and the interpolation methods are presented in the following section.

### 3.1.2 Interpolation methods

The interpolation methods used in our numerical simulations, for different cases, are introduced as follows.

- Constant or linear pressure, and quadratic or mini-element velocity: These represent the approximation piecewise polynomial function in the domain element using finite element methods.
- Elastic-Viscous-Split–Stress, Streamline Upwinding method (EVSS SU): In EVSS method, the viscoelastic stress tensor $\tau$ is not directly used in the constitutive equation to solve flow problems. Instead, the constitutive equation is solved by splitting $\tau$ into a purely viscous component and an elastic one. The purely viscous component can be expressed as a Newtonian form. The elastic component is substituted into the constitutive equation and solved. The actual viscoelastic stress tensor can be recovered by combining both viscous and elastic results ([52-53]). Streamline Upwinding method adds an artificial term to the diffusion term of convection-diffusion differential equation to avoid the problem caused by crosswind diffusion.

- $4*4$ Streamline Upwinding method ($4*4$ SU): $4*4$ uses a higher order polynomial approximation in domain elements. For highly viscoelastic cases, $4*4$ Streamline Upwinding method is more effective and robust, but it is computationally expensive and time consuming.

### 3.1.3 Numerical integration methods

For transient cases, numerical integration methods are used. There are several integration schemes such as explicit Euler method, implicit Euler method, Galerkin method, and Crank-Nicolson method in POLYFLOW. The concepts for these integration methods are explained as follows.

In order to solve a first order differential equation, using forward finite difference method with first order error, the first order derivative for variable $x$ to variable $s$ can be expressed as:

$$x'_n = \frac{x_{n+1} - x_n}{\Delta s_n} = f(x_n)$$  \hspace{1cm} (3.1)

where $n$ is grid number in the discretized domain and $\Delta s_n = s_{n+1} - s_n$.

An approximation of $f(x_n)$ is created using the formula:

$$f(x_n) \approx \beta f(x_{n+1}) + (1 - \beta) f(x_n)$$ \hspace{1cm} (3.2)

where $\beta$ is a coefficient in the range of $0 \leq \beta \leq 1$.

Combine equations (3.1) and (3.2), we find the following expression for $x_{n+1}$.
\[ x_{n+1} = x_n + \Delta s_n [\beta f(x_{n+1}) + (1-\beta) f(x_n)] \] (3.3)

This equation can be used to solve first order differential equations. Different values of \( \beta \) result in different integration methods with different accuracy and stability attributes:

- \( \beta = 0 \) - explicit Euler method;
- \( \beta = 1 \) - implicit Euler method;
- \( \beta = 2/3 \) - Galerkin method;
- \( \beta = 1/2 \) - Crank-Nicolson method. These integration methods have different characteristics in numerical calculations.

- Explicit techniques are only conditionally stable. Using an explicit Euler method (first-order scheme), the time step must not exceed the time required for a trajectory to pass through an element.

- The implicit Euler method is also a first-order scheme. But the advantage of this scheme is that it does not cause oscillatory behavior, no matter how large the time-step size is.

- The Galerkin method (first-order scheme) is more accurate than the implicit Euler method. However, when the time step is large this scheme can generate oscillatory errors.

- The Crank-Nicolson method (second-order scheme) is the most accurate of the four methods available in POLYFLOW. But this integration scheme has more troublesome in oscillatory errors if the time step is large.

For cases simulated in this work, implicit Euler method and Galerkin method are used.

### 3.1.4 Evolution methods

Solving non-linear problems such as inertia terms or moreover viscoelastic constitutive equations, is very challenging. POLYFLOW provides evolution methods for non-linear problems. The evolution method is the most effective way to solve a non-linear system problem starting a numerical calculation from a simpler problem in which non-linearity is not as troublesome. The solution of the simpler problem can be used as the initial condition to solve a sequence of problems with increasing non-linearity. The process continues until the solution is reached for the original problem of high non-linearity. The general concept for evolution method in solving non-linearity problems can be expressed as follows.

- A critical parameter related to non-linearity (such as density or relaxation time) P
which value is \( C \) is found or suspected to be largely responsible for the non-linear nature of the original problem based on analytical theories or previous experience.

- The original problem is transformed to solve a sequence of problems. In each of this sequence of problems the critical parameter \( P \) is given a new value \( C*S \) in which \( S \) is a variable in the range of 0 to 1.

- The numerical calculation for the sequence of problems starts at \( S=0 \), where the solution is obtained for the problem of the parameter \( P \) value at \( C*0=0 \). Then the variable \( S \) is increased by a small amount \( \Delta S \) to \( S' = S + \Delta S \) and the next problem is the non-linear case at the parameter \( P \) value of \( C*S' \). This new problem will be calculated from the initial condition of previous solution at \( P \) value of \( C*0=0 \). If this new problem is solved, the procedure will continue by a larger increase step of \( S \), such as \( \Delta S_{\text{next}} = 1.5\Delta S \). On the other hand, if the calculation fails (divergence happens), the problem will return and a smaller increase step of \( S \) from \( S=0 \), such as \( \Delta S_{\text{next}} = 0.5\Delta S \), will be tried to create the new problem until the convergence can be reached.

It should be noticed that each problem solution can be used successfully as the initial condition for only the next problem with sufficient proximity to the initial condition. So there is a limit to the increase step of \( S \) for the next new problem. If the variable \( S \) is increased too much, it will result in divergence with the previous solution used as an initial condition.

Evolution method is implemented for our numerical calculations in POLYFLOW. Here the evolution procedure is employed having the option for the critical parameter \( P \) value which can be defined as \( C*f(S) \) in which \( f(S) \) is a pre-defined function and \( S \) can have any value.

### 3.2 Simulation setup

Various domain meshing methods and numerical schemes are tested and compared to obtain effective and efficient numerical systems for 2D and 3D numerical simulations.
3.2.1 2-D numerical study

For 2D geometry, mesh size independence is studied through load capacity results for Newtonian fluids. Figure 3.2 presents the mesh size independence study for eccentric cylinders \((R_o = 6\, \text{cm}, R_i = 5.084\, \text{cm}, \text{and} \, \epsilon = 0.7\) ) at angular velocity of inner cylinder \(\Omega_i = 5.24\, \text{rad/s}\) using a Newtonian fluid \((\mu = 0.3\, \text{Pa.s} \text{ and } \rho = 900\, \text{kg/m}^3)\). The geometry is shown in figure 3.4. Along the circumferential direction, the narrowest gap has the finest mesh. On the radial direction, the finest grid is near the outer cylinder. The variables’ (such as velocity) variations in these regions are important and the selected grading mesh is helpful to avoid divergence. Using eighty grids with the grading ratio of 1.01 along the semi-circumferential direction, we varied the grid number with the grading ratio of 1.05 along the radial direction in order to check mesh size independence. Different mesh cell numbers (one more grid along the radial direction generates 160 more cells in the domain) are tested to calculate the load capacity on the inner cylinder. Mesh independence is also checked for the grid number along the circumferential direction. Finally forty grids along the radial direction are used for this geometry. The mesh size independence checking is also performed for other studied geometries including concentric and eccentric cylinders to ensure the numerical accuracy.

Different 2D meshes are employed in this work. Figure 3.3 presents the mesh used in the simulation for concentric cylinders \((R_o = 6\, \text{cm and } R_i = 5.084\, \text{cm})\), which has equal spaces along the circumferential direction (one hundred and sixty grids) and grading spacing along the radial direction (forty grids with the grading ratio of 1.05).

Figure 3.4 presents the mesh used in the simulations for eccentric cylinders \((R_o = 6\, \text{cm}, \, R_i = 5.084\, \text{cm}, \, \epsilon = 0.7\) ). Forty grids with the grading ratio of 1.05 along the radial direction and eighty grids with the grading ratio of 1.01 along the semi-circumferential are used for meshing the domain. The recirculation zone could be formed in the wide gap of annulus, therefore the grading ratio along the radial direction for meshing need be carefully dealt with. Too high or too low grading ratio along the radial direction might cause numerical calculation failure for divergence. The mesh, based on grading spacing method along both circumferential and radial directions, is used for all eccentric cases.
Figure 3.2  2D mesh size independence check.

Figure 3.3  Finite element mesh for concentric cylinders.

Figure 3.4  Finite element mesh for eccentric cylinders $\varepsilon=0.7$
For UCM fluids in eccentric problems, the starting value for evolution or transient cases is zero. Non-slip boundary conditions are imposed on inner and outer cylinders. Implicit Euler method or Galerkin method for integration, and EVSS SU interpolation scheme for stress are applied for the cases of small relaxation time and lower angular velocity. As the relaxation time or angular velocity increases, evolution methods for journal angular velocity, density or relaxation time, and 4*4 SU interpolation scheme for stress are adopted in the numerical simulations to reach convergence. The numerical schemes for investigated cases are tabulated in table 3.1. The numerical convergence is $10^{-6}$.

3.2.2 3-D numerical study
In comparison with 2D numerical simulations, the computation complexity for 3D is increased greatly. Different meshes and numerical schemes are chosen to solve 3D cases for Newtonian and UCM fluids within eccentric rotating cylinders.

3.2.2.1 Newtonian fluids
For 3D geometry, mesh size independence is also studied through load capacity results for Newtonian fluids. Figure 3.5 presents the mesh size independence study for 3D eccentric cylinders ($R_o$=6 cm, $R_i$=5.084 cm, axial length $L$=8 cm, and $\varepsilon$=0.7) at angular velocity of inner cylinder $\Omega_i$=5.24 rad/s using a Newtonian fluid ($\mu$=0.3 Pa.s and $\rho$=900 kg/m$^3$). The geometry is shown in figure 3.6. Along the circumferential direction, the narrowest gap has the finest mesh. On the radial direction, the finest grid is near the outer cylinder and along the axial direction equal spaces are adopted. Using fifty grids with the grading ratio of 1.01 along the semi-circumferential direction and thirty grids with the grading ratio of 1.05 along the radial direction, we varied the grid number of equal intervals along the axial direction in order to check mesh size independence. Different mesh cell numbers (One more grid along axial direction will generate 3000 more elements in the 3D domain) are tested to calculate the load capacity on the inner cylinder. Mesh independence is also checked for the grid number along the radial and circumferential directions. The load capacity results calculated from 60*30*30 and 50*35*30 meshes are compared with 50*30*30 to ensure the mesh independence.
Due to the computation load (time-consuming) and memory requirement (limitation of memory) for 3D simulations, thirty equal space grids along the axial direction are used for meshing the domain in the numerical simulations of Newtonian flows. Figure 3.6 presents the mesh used for 3D simulations in Newtonian study (semi-circumferential: ratio 1.01, grid 50; radial: ratio 1.05, grid 30; and axial: ratio 1, grid 30). There are 90000 mesh elements in the 3D domain. Non-slip conditions are applied for inner cylinder, outer cylinder, and both axial ends for the cases of close journal bearing. On the other hand, periodic boundary conditions (with zero normal force) are imposed on axial ends for the cases of open journal bearing. For high angular velocity cases in Newtonian simulations, evolution methods for density and EVSS SU interpolation scheme for stress are used to reach numerical convergence. The numerical convergence is $10^{-4}$. 

**Figure 3.5  3D mesh size independence check.**
3.2.2.2 UCM fluids

Since the computation memory requirement for UCM fluid in 3D is more important, compared to Newtonian fluids, therefore a mesh with fewer elements is used for them.

Figure 3.7 presents the 3D mesh for the investigations of UCM fluids within eccentric cylinders ($R_o = 6\, \text{cm}$, $R_i = 5.084\, \text{cm}$, axial length $L = 8\, \text{cm}$, and $\varepsilon = 0.7$). The mesh is created with: semi-circumferential: ratio 1.01, grid 30; radial: ratio 1.05, grid 20; and axial: ratio 1, grid 20. There are 24000 mesh elements in the 3D domain. Non-slip boundary conditions are imposed on both cylinders, and periodic boundary conditions (with zero normal force) are used at both axial ends. In numerical simulations, evolution methods for angular velocity of inner cylinder and EVSS SU interpolation scheme for stress are used to reach numerical convergence. The numerical convergence is $10^{-4}$. 

Figure 3.6 Finite element mesh for 3D Newtonian at eccentricity ratio $\varepsilon = 0.7$. 

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The numerical schemes chosen in the 2D and 3D simulations are tabulated in table 3.1.

<table>
<thead>
<tr>
<th>Fluids</th>
<th>Numerical methods</th>
<th>Newtonian</th>
<th>UCM steady state</th>
<th>2D UCM transient</th>
</tr>
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<td></td>
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<td>high Ω</td>
<td>2D</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2D</td>
<td>3D</td>
<td>low Ω</td>
</tr>
<tr>
<td></td>
<td></td>
<td>low Ω</td>
<td>high Ω</td>
<td></td>
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<tr>
<td>Interpolation method</td>
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<tr>
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<td></td>
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<tr>
<td></td>
<td>Galerkin</td>
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<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

3.3 Numerical method validation

Different validations based on analytical solutions, experiment results and other numerical simulations are performed.
3.3.1 Comparison of velocity for Newtonian and Carreau fluids

The geometry parameters for a journal bearing and the properties of fluid are selected as the same as in the paper [24]: journal radius $R_i = 5.084$ cm, bearing radius $R_o = 6$ cm, and the eccentricity ratio $\varepsilon = 0.5$; the viscosity and density for a Newtonian fluid $\mu = 0.1$ Pa.s and $\rho = 900$ kg/m³, and the parameters for a Carreau fluid $\mu_e = 0.1$ Pa.s, $\mu_0 = 1.6$ Pa.s, $\lambda = 0.03$ s, and $n = 0.77$.

Figure 3.8 presents our numerical results for the Newtonian fluid, which are consistent with the analytical solution for a Newtonian fluid. Figure 3.9 shows experimental data for Newtonian and Carreau fluids provided by the paper [24]. In both figure 3.8 and figure 3.9, the solid lines are the same lines which are obtained from analytical solutions for a Newtonian fluid. Comparing with the experimental results, the numerical results for Newtonian and Carreau fluids show a very good agreement.

Figure 3.8  Comparison for dimensionless velocity in eccentric annuli. Solid line- analytical solution for Newtonian; *-Numerical results for Newtonian; o-Numerical results for Carreau.
Figure 3.9 Experimental results [24], solid line- analytical solution for Newtonian; open symbols-experiment results for Newtonian; solid symbols – experiment results for Carreau solution.

3.3.2 Comparison of wall shear stress for SPTT fluid

Figure 3.10 presents the computed wall shear stresses at low angular velocity which are in good agreement with analytical results developed by M. Mirazadeh [49] for purely tangential flow of a SPTT fluid within a concentric annulus ($R_i = 5.084$ cm and $R_o = 6$ cm). The parameters for SPTT fluid is chosen as: $\mu = 0.5$ Pa.s, $\rho = 900$ kg/m$^3$, $\lambda = 0.03$ s, and elongation parameter $\epsilon_i = 0.7$. There is an increasing deviation between the numerical results and analytical solutions as the angular velocity increases. This is probably caused by the assumptions used in the derivation of analytical solution - purely tangential flow and thin film theory.
Figure 3.10  Wall shear stress at the inner rotating cylinder versus angular velocity in concentric annulus for a SPTT fluid. o- analytical solution; *-numerical results.

3.3.3  Comparison of eddy center position for a Newtonian fluid
M. H. Chou [54] used a multi-grid finite difference approach to obtain the solution of the flow between eccentric rotating cylinders and C. Shu et al. [55], a differential quadrature method. Our computed polar angles giving the position of the eddy center are compared with the numerical results from ([54-55]). The geometry is $R_i=0.1$ m and $R_o=0.2$ m, and $\epsilon = 0.5$. The Newtonian fluid density is $\rho = 900$ kg/m$^3$ and its viscosity is $\mu = 0.3$ Pa.s.

Figure 3.11 shows the domain mesh (radial: grid number 60, grading ratio 1.02; semi-circumferential: grid number 80, grading ratio 1.01) and the stream function contour at Reynolds number 50. The radius ratio $K = \frac{R_i}{R_o} = 0.5$ and the Reynolds
number here is defined as \( \text{Re} = \frac{R \Omega (R_o - R_i) \rho}{\mu} \). The eddy center angle is measured from the widest gap (see figure 3.11).

Figure 3.11   Mesh and the stream function contour at Reynolds number 50.

The figure 3.12 presents the eddy center angles computed by finite element method (POLYFLOW) and the results from [54] and [55]. A very good agreement between all these three results is achieved.

Figure 3.12   Polar angle of eddy center versus Reynolds number (compared with [54] and [55]).
3.3.4  Comparison of load direction for an UCM fluid

The geometry used here is the same as in [20]: $R_o = 0.11$ m, $R_i = 0.1$ m, and $\varepsilon = 0.4$. The geometry ratio $\frac{R_o - R_i}{R_i}$ is 0.1. The UCM fluid viscosity is $\mu = 0.3$ Pa.s. The Deborah number is defined as $De = \lambda \Omega$ and the angular velocity is chosen as $\Omega = 5$ rad/s. Our numerical results for inner cylinder load directions are compared with the results obtained using Spectral element method (SEM) [20], body force and inertia being neglected. The domain mesh is shown in figure 3.13: semi-circumferential: grid number 120, grading ratio 1.005; radial: grid number 30, grading ratio 1.02.

![Figure 3.13 Mesh used for an UCM fluid.](image)

Figure 3.14 presents the comparison between my numerical results and SEM results for the directions of load versus Deborah number. Good agreement between our numerical results of load direction and the results from SEM method is obtained.

3.3.5  Comparison for a Newtonian fluid in 3D

The result for Newtonian Taylor vortex flow within a micro-scale annulus flow is compared to the numerical solution obtained using a commercial code CFD-ACE+ [45]. Figure 3.15 presents the geometry and mesh in this validation. The same geometry and mesh method as the paper [45] are used: concentric cylinders, $R_o = 2.54$ cm, $R_o - R_i = 0.0254$ cm, and length $L = 0.0508$ cm; the numbers of the nodes for meshing are 12*480*12 in radial, circumferential and axial direction respectively. The Newtonian fluid properties are also the same as the paper: $\mu = 0.099$ Pa.s and $\rho = 1048$ kg/m$^3$, and $\Omega = 60000$ rpm.
Figure 3.14  Direction of load versus Deborah number (compare with [20]).

Figure 3.15  Geometry and mesh used for 3D Newtonian fluid comparison.

Figure 3.16 presents Taylor vortex flow (Z-velocity contour in XZ plane at Y=0) within concentric cylinders for the 3D Newtonian fluid. Similar, but shifted Taylor vortex profiles are presented in [45]. In both simulations periodic boundary conditions at axial ends are used. However, in [45] boundary conditions definition is not presented. In our simulations, periodic boundary conditions (with zero normal force) are imposed on both axial ends. The different definition of the axial boundary conditions could explain the shift between flow profiles.
Figure 3.16  Taylor vortex flow (Z-velocity contour in XZ plane at Y=0) within concentric cylinders for 3D Newtonian fluid.

The validations of numerical schemes are performed for different cases, including the studies for Newtonian and viscoelastic fluids, and 2D and 3D. From the numerical results comparison with available analytical, numerical solutions, and experimental data, good agreements are obtained and the validation of the numerical scheme’s accuracy is achieved. The numerical schemes can be used confidently for our later numerical simulations of eccentric annulus flows.
Chapter 4  Rheology experimental results

Rheology is the study of the flow and deformation of material under the influence of applied stresses including shear stress and elongational stress. In order to characterize the non-Newtonian behavior, a rheological study is performed. Steady shear flow and small-amplitude oscillatory shear (SAOS) flow are employed to characterize rheological behavior of lubricants. Figure 4.1 illustrates the multiple functional rheometer (Bohlin C-VOR digital rheometer, Malvern Company) which is used to perform the rheological experiments in this work.

![Multi-functional Rheometer](image)

Figure 4.1  Multi-functional Rheometer

4.1  Rheological methods

Different flow tests such as steady shear, oscillatory shear, or elongation generating various flow patterns, could be used in order to characterize rheological behavior of the viscoelastic material. Different rheometer geometries such as parallel plates, cone and plate, or vane and cylinder can be selected to perform rheological tests. Test flows and geometries for rheological characterization are selected based on the material properties and specific application.
4.1.1 Viscometry (steady shear)

Steady shear flow can be produced by confining a fluid between two parallel plates and move one plate at a constant velocity. Figure 4.2 demonstrates the flow field in steady shear flow. The same flow can be created in a torsional disk flow, cone and plate torsional flow, or cup and bob Couette flow. In our rheological experiments, steady shear tests are used to find the variation of viscosity with shear rate for different lubricants.

![Flow field in steady shear flow](image)

Figure 4.2  Flow field in steady shear flow

Figure 4.3 illustrates schematic of a cone and plate geometry which is widely used in rheometers. In the cone and plate, the cone and plate squeeze the material by the rotation of the cone at a constant angular velocity with a fixed plate while measuring the torque generated by the tested fluid. In a limit of a small cone angle, the material between the cone and plate can have homogeneous strain, constant shear rate, and constant shear stress. The fluid viscosity at various shear rates can be calculated through the relation formula between torque and viscosity. Viscosity in cone and plate flow can be calculated from the formula [46]:

$$
\mu = \frac{3T\Theta_0}{2\pi R^2 \Omega}
$$

(4.1)

Where $\Theta_0$ is the cone angle, $T$ is the torque on the cone, $R$ is the radius of plate, and $\Omega$ is the angular velocity of the cone.
4.1.2 Small-amplitude oscillatory shear (SAOS) test

SAOS test is used to study complex fluids in rheometry. The velocity field of SAOS flow can be defined as below [46].

\[
\begin{align*}
  v_x &= \gamma(t) x = \gamma_0 \cos \omega t x \\
  v_y &= 0 \\
  v_z &= 0
\end{align*}
\]

where along the axis of a Cartesian system of reference (x, y, z), \(v_x, v_y,\) and \(v_z\) are the velocity components, \(\gamma(t)\) is the oscillatory shear rate, \(\gamma_0\) is the amplitude of the shear rate oscillation, and \(\omega\) is the frequency of the shear rate oscillation.

The flow is a shear flow, and the time dependent shear rate is a periodic function – cosine function. For a small shear strain \(\gamma\), it can be expressed as follows.

\[
\gamma(0, t) = \int_0^t \dot{\gamma}(t') dt' = \int_0^t \gamma_0 \cos \omega t' dt' = \gamma_0 \frac{\gamma_0 \sin \omega t}{\omega} = \gamma_0 \sin \omega t
\]

where \(\gamma_0\) is the amplitude of the oscillatory shear strain.
When the fluid is strained in a sinusoidal way, the shear stress will also be a sinusoidal wave of the same frequency as the strain wave. However, the shear stress usually is not in phase with the periodic strain wave. If $\varphi$ is defined as the angle difference between the strain wave and the stress response, the shear stress output would be,

$$\tau(t) = \tau_0 \sin(wt + \varphi) = \tau_0 \cos \varphi \sin wt + \tau_0 \sin \varphi \cos wt$$

(4.4)

where $\tau_0$ is the amplitude of the oscillatory shear stress.

Shear stress has two components. The first one is in phase with the strain wave, while the second one is in phase with the imposed shear rate. For viscous fluids, the shear stress response is proportional to the shear rate. For elastic materials, the produced shear stress, following Hooke’s law, is proportional to the imposed strain. Therefore, using SAOS flow as a test flow, we can identify both components of the stress – viscous and elastic. To determine the rheological behavior of a viscoelastic fluid, the SAOS test is suitable. Two important material functions are measured using SAOS flow:

Storage modulus (elastic shear stress)

$$G' = \frac{\tau_0 \cos \varphi}{\gamma_0}$$

(4.5)

Loss modulus (viscous shear stress)

$$G'' = \frac{\tau_0 \sin \varphi}{\gamma_0}$$

(4.6)

For purely viscous fluids, the shear stress is in phase with the shear rate, so storage modulus $G'$ is zero. For perfect elastic materials, loss modulus $G''$ is zero. For viscoelastic materials, both storage modulus and loss modulus are non-zero. They are generally functions of frequency.

The elastic modulus $G'$ and viscous modulus $G''$ in SAOS tests with a cone and plate geometry are calculated as follows [46]:

Storage modulus:

$$G' = \frac{3 \Theta_0 T_0 \cos \delta}{2\pi R^2 \phi_0}$$

(4.7)

Loss modulus:
\[ G'' = \frac{3\Theta_0 T_0 \sin \delta}{2\pi R^3 \phi_0} \]  

where \( \Theta_0 \) is the cone angle, \( T_0 \) is the amplitude of the torque on the cone, \( \delta \) is the phase difference between torque and torsional angle, and \( \phi_0 \) is the amplitude of torsional angle through which the cone oscillates.

### 4.2 Rheology experiment results

Two Mineral oil-base lubricants and a vegetable oil-base lubricant are characterized using test flows (steady shear and SAOS) in order to describe their non-Newtonian behavior. The mineral oil-base lubricants are used as test fluids, and they are already characterized in steady shear experiments in previous papers [26] and [30].

#### 4.2.1 Viscometry results

The rheological characteristics of two mineral oil-based lubricants – 20W50 and 10W40 and a canola oil-based bio-lubricant are determined. Through the thesis we will use for 20W50 and 10W40 the notations M1, respectively M2, and for canola oil bio-lubricant the notation B. The variation of viscosity with shear rate for lubricants has a great influence on the lubrication performance of journal bearing. The geometry used in steady shear test is cone and plate. This geometry requires the tested fluid to be confined well under the rotating cone for obtaining accurate viscosity data. For experiments at high shear rates, centrifugal forces overcome the surface tension forces and make the material splash out of the cone and plate geometry. As the viscosities of tested lubricants are all very low, this limitation greatly restricts the highest shear rate which can be obtained in the viscometry tests. However the viscosity at high shear rates is useful to be characterized for some applications in journal bearings, such as engine bearings. In order to test these lubricants at higher shear rates, different geometries are tried: 4°/40mm cone/plate, 1°/40mm cone/plate, 1°/60mm cone/plate and vane/cylinder. For 1°/40mm and 1°/60mm cone/plate, the lubricant films are too thin to maintain for the reason of low viscosity. The inertia effect restricts the upper limit of test shear rate for vane/cylinder geometry. Hence, stainless steel cone-plate (4°/40mm) is selected for the viscometry tests.
4.2.1.1 Mineral-based lubricants

Figure 4.4 and figure 4.5 present the viscometry results for M1 and M2 lubricants respectively. The tested viscosities for M1 and M2 are similar to available results ([26], [30]). The viscosities obtained in our experiments are within the range of lubricants viscosity, knowing that lubricants from different manufacturers have different viscosities. For both M1 and M2 lubricants, viscosity – shear rate tests were conducted up to a shear rate of 1000 s$^{-1}$. At high shear rates, the lubricants present shear thinning behavior. Figure 4.5 shows that the shear thinning behavior at high shear rates is more evident at 10°C rather than at higher temperatures for M2. At higher temperature 40°C and high shear rates the viscosity for both M1 and M2 reaches a plateau. Viscosity becomes almost constant at high temperatures. Our viscosity tests at very high shear rates for M1 and M2 lubricants were not performed due to the limitations imposed by the rheometer. From literature ([26], [30]), shear thinning can be expected at very high shear rates.

The steady shear experiments are performed for both M1 and M2 lubricants in the temperatures range of 10°C – 100°C. Figure 4.6 presents the viscosity of M1 and M2 as a function of temperature in the range of 10°C – 100°C for a shear rate of 50 s$^{-1}$. It can be observed that the M2 viscosity will change less throughout the whole range of tested temperatures than M1 viscosity.
Figure 4.4 Viscosity – shear rate tests for M1 lubricant at various temperatures.

Figure 4.5 Viscosity – shear rate tests for M2 lubricant at various temperatures.
4.2.1.2 Bio-lubricant

The bio-lubricant is based on canola oil and is produced by Greenland Corporation, Calgary. Figure 4.7 presents the relation between viscosity and shear rate at various temperatures, and figure 4.8 shows the variation of viscosity with temperature for B lubricant. The steady shear tests for B lubricant were conducted up to a shear rate of 1000 $s^{-1}$. B lubricant presents a slightly shear thickening behavior at low shear rates, which is increased with the increasing temperature. Increasing the shear rate up to 1000 $s^{-1}$, the viscosity will reach a plateau. From figure 4.8, it can be found that the viscosity nonlinearly decreases with the temperature increasing.
Figure 4.7  Viscosity as a function of shear rate for B lubricant at various temperatures.

Figure 4.8  Variation of viscosity for B lubricant with temperature.
4.2.2 SAOS results
In order to measure the viscoelastic characteristic of lubricants, SAOS test using cone and plate is applied for these lubricants. As the viscosities of tested lubricants are very low, so performing SAOS test at high frequencies need special techniques and apparatus [56]. Figure 4.9 illustrates the deformation of the sample in SAOS tests at low frequencies. While the oscillation frequency becomes very high, a sample having low viscosity is not able to maintain the behavior shown in figure 4.9 and the stress wave is attenuated in the sample by the inertia of sample as illustrated in figure 4.10. This phenomenon at high oscillation frequencies will cause the rheometer to lose accuracy.

On the other hand, in the oscillation tests, the inertia of the rheometer and the measuring system need to be compared with the response from the tested material. If inertia of the measuring system is too high compared to the total response from the tested material the rheometer loses accuracy.

![Figure 4.9 Oscillation at low frequencies](image1)
![Figure 4.10 Oscillation at high frequencies](image2)

Different geometries such as 4°/40mm cone/plate, 1°/40mm cone/plate, 1°/60mm cone/plate and vane/cylinder were used for the lubricants in SAOS tests and the results were compared. The 1°/60mm stainless steel cone-plate geometry gives stable results at higher frequencies compared to other geometries for SAOS tests. It should noted that the results of SAOS tests with 1°/60mm stainless steel cone-plate become unstable at the temperatures higher than 40°C and high frequencies, and only the stable results are presented in this thesis.

Linear viscoelastic tests for all samples are performed by amplitude sweeping tests in SAOS before the frequency sweeping SAOS experiments. For viscoelastic fluids, the
elastic modulus $G'$ and the viscous modulus $G''$ don’t change with the variation of the shear oscillation amplitude in linear viscoelastic ranges. The elastic and viscous moduli are only functions of shear oscillation frequencies. Figure 4.11 shows the variation of elastic and viscous modulii with shear oscillationary amplitude in the range of 1% - 50% for M2 lubricant at different frequencies for the temperature 10°C. In all SAOS experiments, the amplitude sweeping tests are performed firstly to ensure the frequency sweeping tests in linear viscoelastic regimes. From the figure 4.11, the elastic modulus and viscous modulus are constant in the oscillation amplitude range of 1% - 50% at various frequencies, which verifies that the frequency sweeping tests are in a linear viscoelastic regime.

![Figure 4.11 Linear viscoelastic tests for M2 lubricant at different frequencies (10°C)](image-url)

Figure 4.11  Linear viscoelastic tests for M2 lubricant at different frequencies (10°C)
4.2.2.1 Mineral-based lubricants

Figure 4.14 presents the variation of relaxation time with temperature for M1 and M2 lubricants. The relaxation times are derived from SAOS data fitting. Newtonian fluids relax instantaneously when flow stops, while many non-Newtonian fluids relax in a finite amount of time. Relaxation time is the material property, which characterizes the material’s stress relaxation after deformation. The relaxation time increases with temperature for both M1 and M2. The relaxation time of M2 is higher than M1 at the same temperature.

Figure 4.12 and figure 4.13 present the experimental data in SAOS for M1 and M2 lubricants respectively. These experimental data of viscous and elastic modulii are fitted by viscoelastic models to investigate the non-Newtonian characteristic of lubricant. For M1, the elastic modulus $G'$ and the viscous modulus $G''$ both increase with oscillation frequency, while the elastic modulus increases faster than the viscous one. From SAOS experimental data, it can be found that the crossing point between elastic modulus $G'$ and viscous modulus $G''$ shifts to the left with the increasing temperature. For M2, the variations of elastic modulus $G'$ and the viscous modulus $G''$ with oscillation frequency are similar to M1, while the frequency for the crossing point between elastic modulus $G'$ and viscous modulus $G''$ is smaller than for M1 at the same temperature. The experimental data for M1 and M2 are fitted by UCM model (the fitting was done using a specialized module, POLYMAT in POLYFLOW).
Figure 4.12  SAOS test (amplitude 1%) for M1 at four different temperatures (X1-frequency, Hz; Y1-modulus, Pa).
Figure 4.13  SAOS tests (amplitude 1%) for M2 at four temperatures (X1-frequency, Hz; Y1-modulus, Pa).

(a) $10^\circ C$

(b) $20^\circ C$

(c) $30^\circ C$

(d) $40^\circ C$
4.2.2.2 Bio-based lubricant

Figure 4.15 shows the experimental data of viscous and elastic modulii for B lubricant in SAOS tests. The elastic modulus $G'$ obtained from SAOS test increases with the oscillation frequency. The frequency for the crossing point between elastic modulus $G'$ and viscous modulus $G''$ is much smaller than for M1 and M2 at the same temperature.

Figure 4.16 presents the relaxation times derived by fitting experimental data with UCM model at various temperatures for B lubricant. Its relaxation time increases with temperature. Comparing to M1 and M2 lubricants, B lubricant has relatively high relaxation time at the same temperature.

The published evidence for the beneficial effects of viscoelasticity in lubrication is still not conclusive, but the reliable evidence which is available is supportive of the positive benefits of lubricant viscoelasticity. In chapter 5, we will characterize the effect of viscoelasticity on journal bearing performance.
Figure 4.15  SAOS tests (amplitude 1%) for the canola bio-lubricant at four temperatures (X1-frequency, Hz; Y1-modulus, Pa).
Figure 4.16  Variation of relaxation time of B lubricant with temperature.
Chapter 5  Numerical simulation results

The Newtonian and UCM fluids, and specific lubricants investigated in this chapter are listed in table 5.1. For these fluids, the numerical results for pressure, stresses, load capacity, and moment on the inner cylinders within eccentric annulus are presented.

Table 5.1  Fluids investigated in numerical simulations.

<table>
<thead>
<tr>
<th>Fluids</th>
<th>Viscosity (Pa.s)</th>
<th>Relaxation time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Newtonian</td>
<td>0.3</td>
<td>0</td>
</tr>
<tr>
<td>Newtonian</td>
<td>0.02</td>
<td>0</td>
</tr>
<tr>
<td>Newtonian</td>
<td>0.099</td>
<td>0</td>
</tr>
<tr>
<td>UCM</td>
<td>0.3</td>
<td>0.0003 – 0.15</td>
</tr>
<tr>
<td>UCM</td>
<td>0.02</td>
<td>0.0008</td>
</tr>
<tr>
<td>M1 lubricant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10°C</td>
<td>0.78</td>
<td>0.02</td>
</tr>
<tr>
<td>20°C</td>
<td>0.4</td>
<td>0.03</td>
</tr>
<tr>
<td>30°C</td>
<td>0.22</td>
<td>0.05</td>
</tr>
<tr>
<td>40°C</td>
<td>0.14</td>
<td>0.07</td>
</tr>
<tr>
<td>M2 lubricant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10°C</td>
<td>0.4</td>
<td>0.035</td>
</tr>
<tr>
<td>20°C</td>
<td>0.22</td>
<td>0.06</td>
</tr>
<tr>
<td>30°C</td>
<td>0.13</td>
<td>0.08</td>
</tr>
<tr>
<td>40°C</td>
<td>0.09</td>
<td>0.11</td>
</tr>
<tr>
<td>B lubricant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10°C</td>
<td>0.12</td>
<td>0.09</td>
</tr>
<tr>
<td>20°C</td>
<td>0.075</td>
<td>0.12</td>
</tr>
<tr>
<td>30°C</td>
<td>0.05</td>
<td>0.18</td>
</tr>
<tr>
<td>40°C</td>
<td>0.04</td>
<td>0.22</td>
</tr>
</tbody>
</table>
5.1 Fluids within eccentric rotating cylinders

Steady and transient states for the Newtonian and UCM fluids within eccentric rotating cylinders are investigated to reveal the viscoelastic effects on the flows.

5.1.1 Steady state

For the steady state of flow in eccentric annuli, 2D and 3D cases are considered and investigated. The viscoelastic characteristics of UCM fluids, the comparisons between 2D and 3D for a Newtonian fluid, and the difference between Newtonian and UCM fluids within an eccentric annulus in 3D are presented.

5.1.1.1 2D investigation

In the 2D investigations for the flows within eccentric rotating cylinders, the viscoelastic effects of UCM fluids are presented and discussed through various perspectives such as the variations of pressure and stresses profiles along the inner cylinder with viscoelasticity, and the variations of load capacity (related to journal bearing applications) and moment on the inner cylinder with viscoelasticity.

5.1.1.1.1 Contours and profiles

We recall that the investigated geometry in steady state study is eccentric rotating cylinders in which the inner cylinder rotates at a constant angular velocity with a fixed center position. UCM model is chosen to represent the viscoelastic fluids between the eccentric cylinders. The inner cylinder is rotating steadily clockwise at an angular velocity of 5.24 rad/s. Other parameters for UCM fluid properties and the geometry of eccentric cylinders are selected as follows: $\mu = 0.3 \text{ Pa.s}$, $\rho = 900 \text{ kg/m}^3$, $\lambda = 0.03 \text{ s}$, $R_o = 6 \text{ cm}$, $R_i = 5.084 \text{ cm}$, and $\epsilon = 0.7$.

Figure 5.1 presents the contours of velocity, stream function, shear stress and normal stresses for the UCM fluid. The recirculation zone within annular gap is well presented in contours of velocity-X and stream function.
In order to evaluate the viscoelastic effects of UCM fluid on the flow, various relaxation times varying from 0.0003 s to 0.1 s for UCM fluids are studied.
Figure 5.2 presents the pressure profiles along the inner cylinder for various relaxation times (\( \lambda = 0.0003s – 0.1s \)). The following can be observed:

- The profiles shift to left with the increase of relaxation time. This effect of relaxation time on pressure is very important for journal bearing performance, as the pressure around the inner cylinder is the main factor to generate load capacity in a journal bearing.

- When the relaxation time is small such as lower than 0.003 s, the pressure profile is close to anti-symmetric pressure profile for a Newtonian fluid.

- For a higher relaxation time – about 0.03 s, both the highest and lowest pressures increase, and the profile slightly shifts to the left. The pressure profile deviates slightly from anti-symmetry.

- For the relaxation time of 0.1 s, the pressure profile clearly shifts to the left and deviates from anti-symmetry strongly. The difference between extreme high and low pressures increases with relaxation time.

Figure 5.3 presents the profiles of shear stress along the inner cylinder for various relaxation times (\( \lambda = 0.0003s – 0.1s \)). The following can be observed:

- The profile of shear stress for a low relaxation time of 0.0003 s is very close to the symmetric shear stress profile for a Newtonian fluid.

- With the increase of relaxation time, viscoelasticity of UCM fluid is higher and influences the shear stress profile greatly. For the relaxation time of 0.003 s, the profile is still similar to Newtonian, but deviating from symmetry.

- While the relaxation time further increases, the shape of shear stress profile changes essentially. For the relaxation time of 0.03 s, the shear stress profile in the range of 3-4 radians inverses to downward direction. Especially for the relaxation time of 0.1 s, the lowest shear stress in the range of 3-4 radians decreases greatly, while the increase of the highest shear stress in the range of 2-3 radians is relatively low, comparing with the shear stress profile for the relaxation time of 0.03 s.

- The variation of shear stress profile with relaxation time evidently reveals the viscoelastic effects, which can affect both the load capacity and moment on the journal for journal bearing applications.
Figure 5.2  Pressures along the inner cylinder for the UCM fluids of various relaxation times 
($\lambda=0.0003 \text{ s}-0.1 \text{ s}$).

Figure 5.3  Shear stresses along the inner cylinder for the UCM fluids of various relaxation times 
($\lambda=0.0003 \text{ s}-0.1 \text{ s}$).
Figure 5.4 presents the profiles of normal stress differences along inner cylinder for various relaxation times ($\lambda = 0.0003s - 0.1s$). The following can be observed:

- The variation of relaxation time has a great influence on the profile of normal stress difference along the inner cylinder.
- For low relaxation times such as 0.0003 s and 0.003 s, the profiles of normal stress difference are very close, and the normal stress difference profiles are flat. As the relaxation time increases, the normal stress difference profile in the range of 2.5-3.5 radians becomes steeper, and the magnitudes of the highest and lowest normal stress difference becomes bigger.
- For the relaxation time of 0.1 s, the profile is greatly different from others. The highest value of normal stress difference reaches 500 Pa which is 2.5 times the highest value at the relaxation time of 0.03 s and nearly 10 times the highest value for the relaxation time of 0.003 s. The first normal stress difference profiles show an important viscoelastic effect at a relatively high relaxation time.

![Figure 5.4](image_url)

Figure 5.4  First normal stress differences along the inner cylinder for the UCM fluids of various relaxation times ($\lambda$=0.0003 s-0.1 s).
5.1.1.1.2 Load capacity and Moments

The Weissenberg number is a dimensionless number used in the study of viscoelastic flows which is the ratio of the relaxation time of fluid and a specific process time. It is defined as $We = \frac{\lambda V_c}{c}$, where $V_c$ is the characteristic velocity expressed as $V_c = \frac{R_i + R_o}{2} \Omega$ and $c$ is the radius difference expressed as $c = R_o - R_i$.

Figure 5.5 presents the variation of load capacity with Weissenberg number for Newtonian and UCM fluids. When the relaxation time of UCM fluid is lower than 0.03 s, the load capacity of Newtonian (\(\mu = 0.3 \text{ Pa.s}, \rho = 900 \text{ kg/m}^3\)) and UCM fluids are very close. With further increase of Weissenberg number (through increasing relaxation time), the load capacity on the inner cylinder gradually increases. The gradient of load capacity increases with Weissenberg number. It indicates that the load capacity can be greatly improved by increasing the relaxation time of UCM fluid. Higher load capacity can be achieved using UCM fluids rather than Newtonian fluids. This is a beneficial effect of viscoelasticity on journal bearing performance. High load capacity greatly decreases the contact opportunity between two relatively moving surfaces and make journal bearings work more efficiently and safely.

Figure 5.6 presents the moment on the inner cylinder versus Weissenberg number for Newtonian and UCM fluids. The moment increases with Weissenberg number. Comparing with the Newtonian fluid (\(\mu = 0.3 \text{ Pa.s}, \rho = 900 \text{ kg/m}^3\)), the viscoelasticity of UCM fluid can give a higher moment for Weissenberg numbers in the range of 1 - 4.75. This is an adverse side-effect for the performance of journal bearing. According to the literature [57] it’s expected that for higher Weissenberg numbers, the moment will decrease. In our numerical simulations the maximum Weissenberg number is 5, limited by POLYFLOW capabilities.
Figure 5.5  Load capacity on the inner cylinder versus Weissenberg number for Newtonian and UCM fluids.

Figure 5.6  Moment on the inner cylinder versus Weissenberg number for Newtonian and UCM fluids.
The friction coefficient is a dimensionless number used to characterize the friction. For journal bearing application, it can be calculated from the formula: \( C_f = \frac{F_f}{F_l} \).  

\( F_f \) and \( F_l \) are frictional force and load capacity on the inner cylinder respectively.

Figure 5.7 presents the friction coefficient versus Weissenberg number for Newtonian and UCM fluids. It can be found that the friction coefficient increases with Weissenberg number. This is a by-side effect for lubrication performance using UCM fluids comparing with the Newtonian fluid (\( \mu = 0.3 \text{ Pa.s, } \rho = 900 \text{ kg/m}^3 \)). For low Weissenberg numbers, the friction coefficient increases greatly, but the gradient of friction coefficient with Weissenberg number decreases for Weissenberg numbers between 1 and 3.5, and a nearly plateau region is reached for Weissenberg numbers in the range of 2.5–3.5.

For the cases presented in above paragraphs, the viscoelasticity is changed by varying the relaxation time, while the angular velocity maintains constant. In the following simulations, the viscoelasticity is changed due to the angular velocity variation, while the relaxation time is constant.

Figure 5.8 and figure 5.9 present the variation of load and moment with angular velocity on the inner cylinder for Newtonian and UCM (\( \lambda = 0.03 \text{s} \)) fluids. When the angular velocity increases, the load capacity and moment increase. The gradient of load capacity or moment with angular velocity gradually increases with the angular velocity. Comparing to the Newtonian fluid (\( \mu = 0.3 \text{ Pa.s, } \rho = 900 \text{ kg/m}^3 \)) which has a linear relationship between load capacity or moment and angular velocity, the differences for load capacity and moment of UCM fluid become significant at higher angular velocities. Using an UCM fluid, a much higher load capacity can be achieved at higher angular velocity, comparing with a Newtonian fluid. This is an important benefit for journal bearings operated at high angular velocities.
Figure 5.7  Friction coefficient versus Weissenberg number for Newtonian and UCM fluids.

Figure 5.8  Load capacity on the inner cylinder versus angular velocity for Newtonian and UCM ($\lambda=0.03s$) fluids.
5.1.1.1.3 Inertial effects
When the angular velocity of inner cylinder is increased, the inertial effect becomes more important. Inertial effects are taken into account in all numerical studies of this thesis.

Figure 5.10 presents stream function contours for an UCM fluid at two angular velocities of inner cylinder. These two stream functions results are calculated for the UCM fluid with viscosity $\mu =0.02$ Pa.s, $\rho =800kg/m^3$, and $\lambda =0.0008$ s at angular velocity of $5.24$ rad/s and $100$ rad/s respectively. The center of recirculation zone deflects to X-coordinate with the increasing angular velocity of inner cylinder. The angle of the center of recirculation zone within the annular gap at $100$ rad/s is much higher than that at $5.24$ rad/s. The recirculation zone size also increases with the increasing viscoelasticity caused by higher angular velocity.
Figure 5.10  Inertial effect for the UCM fluid ($\lambda=0.0008$ s) on eccentric rotating cylinders. (a) 5.24 rad/s, (b) 100 rad/s.
Figure 5.11 presents the angle of the recirculation zone center versus Reynolds number for Newtonian and UCM ($\lambda = 0.0008 \text{s}$) fluids. The Reynolds number is defined as $Re = \frac{f \lambda L^2}{\mu}$ with the length scale $L = \frac{R_0 + R_i}{2}$. Comparing to the Newtonian fluid ($\mu = 0.02 \text{ Pa.s, } \rho = 800 \text{kg/m}^3$), the angle of the recirculation zone center for the UCM fluid is lower in a wide range of Reynolds numbers. However at higher Reynolds numbers, the angles for both fluids reach almost the same value.

![Figure 5.11](image)

**Figure 5.11** The variation of recirculation zone center’s angle with Reynolds number for Newtonian and UCM ($\lambda = 0.0008 \text{s}$) fluids.

### 5.1.1.2 3D investigation

Steady states for Newtonian and UCM fluids within eccentric rotating cylinders are investigated in 3D numerical simulations as well. Results are presented in the following paragraphs.
5.1.1.2.1 Newtonian fluids (non-slip boundary)

In 3D simulations for the Newtonian fluid ($\mu = 0.3$ Pa.s, and $\rho = 900$ kg/m$^3$), non-slip boundary conditions are applied on all the boundaries – inner and outer cylinders, and both axial ends for the case of close journal bearing. The 3D geometry is created as follows: $R_o = 6$ cm, $R_i = 5.084$ cm, length $L = 8$ cm, and $\varepsilon = 0.7$.

Figure 5.12 presents the comparison of load capacities predicted by 2D and 3D simulations for the Newtonian fluid within the eccentric annulus. The finite length of the eccentric cylinders has an effect on load capacity. For both cases, the variation of load capacity with angular velocity is linear. However the slope of the line is slightly smaller compared to 2D case. This would lead to a significant difference in the load capacity at very high angular velocities.

![Figure 5.12 Load capacity for 2D and 3D for the Newtonian fluid within eccentric annulus.](image)

5.1.1.2.2 Newtonian fluids (periodic boundary)

A. The Newtonian fluid ($\mu = 0.3$ Pa.s and $\rho = 900$ kg/m$^3$) with the periodic boundary
condition (with zero normal force) at both axial ends is investigated in 3D simulations for the case of open journal bearing. The 3D geometry is the same as the 3D case in the section 5.1.1.2.1 and the angular velocity of inner cylinder is 5.24 rad/s.

Figure 5.13 presents the X-velocity contours at different axial positions. The X-velocity contours in the XY cross-sections at various axial positions exhibit evident differences. From the middle axial position (z=4 cm) to the end (z=8 cm) of eccentric cylinders, the contours of recirculation zones continuously break from outside to the middle, similar to the variation from the middle axial position (z=4 cm) to the other end (z=0 cm).

Figure 5.13  X-velocity contours (Ω=5.24 rad/s) for the Newtonian fluid in the XY cross-sections at different axial positions.
Figure 5.14 presents the Z-velocity contours for the Newtonian fluid at the YZ cross-section (X=0) for the widest and narrowest gaps respectively. Based on these plots a cellular flow is observed in the narrowest gap along the axial direction.

![Z-velocity contours](image)

**Figure 5.14** Z-velocity contours ($\Omega = 5.24$ rad/s) for the Newtonian fluid at YZ cross-section (X=0). (a) widest gap, (b) narrowest gap.

**B.** In this simulation, Taylor vortex flow within eccentric rotating cylinders is presented. The fluid is Newtonian ($\mu = 0.099$ Pa.s and $\rho = 1048$ kg/m$^3$) and periodic boundary conditions (with zero normal force) at both axial ends are used. The 3D geometry is created as follows: $R_o = 2.54$ cm, $R_i = 2.5146$ cm, length $L = 0.0508$ cm, and $\varepsilon = 0.2$. The angular velocity of inner cylinder is 6500 rad/s.

Figure 5.15 presents Z-velocity contours at YZ cross-section (X=0) for the widest and narrowest gaps respectively. The Taylor vortex flow is clearly observed in this eccentric annuli flow. According to DiPrima and Stuart’s [48] the definition for $Ta_{cr}$ within eccentric rotating cylinders is $Ta_{cr} = 1695(1 + 1.162 \frac{c}{R_i})(1 + 2.624\varepsilon^2)$. $Ta_{cr}$ for this eccentric annuli flow is 1900. For our simulation, $Ta$ is 1950 which is higher than $Ta_{cr}$. It can be concluded that our result is consistent with DiPrima and Stuart’s criterion, even if we didn’t study the flow within a range of Taylor numbers.
5.1.1.2.3 UCM fluids

The same UCM fluid as in section 5.1.1.1.1 ($\mu = 0.3 \text{ Pa.s}, \rho = 900 \text{ kg/m}^3, \lambda = 0.03 \text{ s}$) is used for the 3D study. The inner cylinder is rotating at an angular velocity of 5.24 rad/s. The 3D geometry is the same as for the 3D case in the section 5.1.1.2.1. Non-slip boundary conditions are imposed on both cylinders, and periodic boundary conditions (with zero normal force) are used at both axial ends.

Figure 5.16 presents X-velocity contours for the UCM fluid in XY cross-sections at different axial positions. In the wide gap, from the axial middle position ($z=4 \text{ cm}$) to the end ($z=8 \text{ cm}$), the recirculation size decreases and the break of recirculation contours occurs in the wide gap, similar to the variation from the middle axial position ($z=4 \text{ cm}$) to the other end ($z=0 \text{ cm}$). The recirculations configuration along the axial direction for UCM fluid is different compared to that for Newtonian fluid shown in figure 5.13.

Figure 5.17 presents Z-velocity contours for UCM fluid at YZ cross-section ($X=0$) for widest and narrowest gaps respectively. Flow instabilities occur in both widest and narrowest gaps. Comparing to the Newtonian fluid under the same boundary conditions (figure 5.14), no cellular flow appears in the narrowest gap.

Figure 5.15  Z-velocity contours ($\Omega = 6500 \text{ rad/s}$) for the Newtonian fluid at YZ cross-section ($X=0$). (a) widest gap, (b) narrowest gap.
Figure 5.16  X-velocity contours ($\Omega=5.24$ rad/s) for the UCM fluid ($\lambda=0.03$ s) in XY cross-sections at different axial positions.

Figure 5.17  Z-velocity contours ($\Omega=5.24$ rad/s) for the UCM fluid at YZ cross-section (X=0). (a) widest gap, (b) narrowest gap.
5.1.2 Transient state
For the start-up problem, UCM viscoelastic fluid (as a model of a multi-grade lubricant) is studied within eccentric rotating cylinders. The inner cylinder rotates at clockwise angular velocity 5.24 rad/s. Other parameters for the lubricant properties and the geometry of eccentric cylinders are the same as in steady state 2D study. The coordinates system is: the origin is the center of outer cylinder, Y-coordinates is pointing from the narrowest gap to the widest gap.

Figure 5.19 presents transient evolution of velocity-X at different points in widest and narrowest gaps shown in figure 5.18 for UCM fluids having two relaxation times (λ = 0.003 s, 0.03 s). The four figures clearly show the velocity-X evolution characteristics for UCM fluids at different points within the eccentric annulus in transient flow. For a low relaxation time of 0.003 s, the magnitude of velocity-X increases steadily with time until reaching steady state. For a relaxation time of 0.03 s, a velocity peak can be seen, which is even higher than the value at steady state, and the velocity evolution shows a fluctuating pattern. From figures 5.19 a and b, the directions of velocity-X are opposite, indicating that the recirculation occurs in the wide gap of eccentric cylinders. The variation of velocity with time shows viscoelastic characteristics which may influence the performance of journal bearings.

![Figure 5.18 Schematic showing the points considered in numerical simulations (fig. 5.19)](image-url)
Figure 5.19  Transient evolution of velocity-X at different points (unit: m) in widest (a, b) and narrowest (c, d) gaps for the UCM fluids of two relaxation times ($\lambda=0.003$ s, 0.03 s).
Figure 5.21 presents shear stresses in start-up on the inner cylinder at different points shown in figure 5.20 for UCM fluids having two relaxation times ($\lambda = 0.003\text{s}, 0.03\text{s}$). For a low relaxation time of 0.003 s, shear stresses have high starting values and gradually decrease until the steady state. However for the relaxation time of 0.03 s, the evolutions of shear stresses exhibit fluctuation characteristics.

![Figure 5.20 Schematic showing the points considered in numerical simulations (figures 5.21, 5.22).](image)

Figure 5.22 presents the evolutions of normal stress differences at different points shown in figure 5.20 for UCM fluids having two relaxation times ($\lambda = 0.003\text{s}, 0.03\text{s}$). At relaxation time of 0.003 s, normal stress differences start from high values and gradually decrease until the steady state. For a relaxation time of 0.03 s, the magnitudes of normal stress differences are much higher, due to viscoelastic effect. The fluctuation characteristics are clearly exhibit in the evolution of all normal stress differences for relaxation time of 0.03
Figure 5.21 Shear stresses in start up on the inner cylinder surface at different points (unit: m) for the UCM fluids of two relaxation times ($\lambda = 0.003$ s, 0.03 s).
Figure 5.22  Normal stress difference on the inner cylinder surface at different points (unit: m) for the UCM fluids of two relaxation times ($\lambda=0.003$ s, 0.03 s).
Figure 5.23 presents the transient (start-up) evolutions of load capacity on the inner cylinder for various relaxation times of UCM fluid. When the relaxation time is as low as 0.0003 s, the evolution of load capacity on the journal gradually increases until the steady state is reached. As the relaxation time increases to 0.003 s, there is a slight fluctuation characteristic in the evolution of load capacity. With the further increase of relaxation time to 0.03 s, the viscoelastic effect on the evolution of load capacity is evident and a clear fluctuation arises in the evolution process until the steady state is obtained. For higher relaxation time of 0.1 s, the scale and number of fluctuations are much greater, and it takes much more time to reach the steady state of flow, pointing out the dominant influence of elastic properties with respect to viscosity effects. These viscoelastic effects – fluctuation of load capacity in start-up - need a proper treatment to make journal bearings work properly.

![Figure 5.23 Transient load capacity on the inner cylinder in start-up flow for the UCM fluids of various relaxation times (\(\lambda=0.0003\) s - 0.1 s).](image)

Figure 5.24 presents the transient evolution (start-up) of moment on the inner cylinder
for the UCM fluid with a relaxation time of 0.03 s. The moment on the inner cylinder oscillates in the evolution process of start-up until the steady state is reached (t=0.25 s). The evolution shows an overshoot and an undershoot followed by smoother fluctuations. This behavior is a characteristic fingerprint of the viscoelasticity and should be considered in journal bearing start-up.

Figure 5.24 Transient moment on the inner cylinder in start-up flow for the UCM fluid (\(\lambda=0.03\) s).

Figure 5.25 presents in start-up the force on the inner cylinder at various eccentricity ratios (\(\varepsilon=0.6, 0.7 \text{ and } 0.8\)) for the UCM fluid (\(\lambda=0.03s\)). The trends for different eccentricities are similar. For the eccentricity of 0.7, the overshoot value is almost the same as at steady state, while for the eccentricity of 0.6, the steady state force is lower than the overshoot value. However for the eccentricity of 0.8, the steady state force is higher than the overshoot value.
Figure 5.25  Transient force on inner cylinder in start-up for the UCM fluid ($\lambda = 0.03$ s) at various eccentricity ratios ($\varepsilon = 0.6$, 0.7 and 0.8).

The steady state and transient state results for eccentric rotating cylinders with Newtonian and UCM fluids are investigated using POLYFLOW. In 2D investigation, the numerical results reveal that lubrication performances are greatly affected by the viscoelasticity of UCM fluid. Increased load capacity on the inner cylinder can be achieved by increasing viscoelasticity of flow. The 3D simulation for a Newtonian fluid in eccentric rotating cylinders reveals the load capacity, which is compared to the load capacity from 2D calculations. 3D results for the UCM fluid at steady state present the flow patterns along axial direction within the eccentric rotating cylinders, and show differences compared to 3D results for the Newtonian fluid under the same boundary conditions.
5.2 Lubricants in journal bearing
In this part of the thesis, the lubricant hydodynamic analysis of mineral-based lubricant (M1 and M2) and canola-based bio-lubricant (B) in journal bearings based on numerical simulations and rheological characterization is presented.

5.2.1 Mineral-based lubricants
Through the steady shear and SAOS experiments, the specific properties of viscosity and relaxation time for M1 and M2 multi-grade lubricants are obtained from rheological characterization. There are two parameters in UCM constitutive model – viscosity and relaxation time. As the viscosity for UCM model is constant with shear rate and the value derived by fitting SAOS data is higher than the value from the viscometry test, in order to better represent the lubricants by UCM model, viscosity is chosen from the viscometry experiments and relaxation time is derived by fitting the data of SAOS test. More complex model, such as PTT model which has more parameters and varying viscosity with shear rate, would need to be adopted to fit the experimental data (including viscometry and SAOS) completely. Table 5.2 gives the viscosities and relaxation times of M1 lubricant at various temperatures.

Table 5.2 Viscosities and relaxation times for M1 lubricant at various temperatures.

<table>
<thead>
<tr>
<th>M1 lubricant</th>
<th>10°C</th>
<th>20°C</th>
<th>30°C</th>
<th>40°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity(Pa.s)</td>
<td>0.78</td>
<td>0.4</td>
<td>0.22</td>
<td>0.14</td>
</tr>
<tr>
<td>Relaxation time (s)</td>
<td>0.02</td>
<td>0.03</td>
<td>0.05</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Figure 5.26 presents the profiles of pressure along the journal (Ω = 10 rad/s) for M1 lubricant at four temperatures. The following can be observed:
- The variations of pressure around the journal have similar profiles for all temperatures.
- The pressure profiles are not anti-symmetric for all temperatures.
- The magnitude of pressure maximum is higher than the magnitude of minimum pressure. The magnitude of maximum pressure is about 2 times minimum pressure.
at 40°C.
- The pressure profiles cross nearly one point without evident shift.
- The differences between extreme pressure values decrease and go closely with increasing temperature.

Figure 5.27 presents the profiles of shear stress around the journal ($\Omega=10$ rad/s) for M1 lubricant at four temperatures. The following can be observed:
- The shear stress profiles are not symmetric for all temperatures.
- The magnitudes of extreme shear stress decrease with the increasing temperature, and therefore with increasing viscoelasticity.
- The shear stress profiles in the range of 2.5-3.2 radians, for 10°C and 20°C, and 30°C and 40°C respectively, overlap with each other.
- By increasing the temperature the shear stress seems to be damped in magnitude, but not towards the Newtonian case.

![Figure 5.26](image_url)  
Figure 5.26  Pressure around the journal ($\Omega=10$rad/s) for M1 lubricant at various temperatures.
Figure 5.27  Shear stress around the journal ($\Omega =10\text{rad/s}$) for M1 lubricant at various temperatures.

Figure 5.28 presents the profiles of normal stress difference along journal ($\Omega =10\text{ rad/s}$) for M1 lubricant at various temperatures. The following can be observed:

- The two peak values of normal stress difference profile decrease with the increasing temperature.
- The shear stress profile at 10°C presents evident difference compared to other temperatures. The profiles at 30°C and 40°C nearly overlap.
- The profile in the range of 2.5-3.5 radians shifts to the left with the increasing temperature.
- Similar to the shear stress, by increasing the temperature the normal stress difference seems to be damped in magnitude, but not towards the Newtonian case.

Figure 5.29 presents the variation of load capacity with angular velocity at different temperatures for M1 lubricant, and figure 5.30 gives the variation of load capacity with angular velocity at 40°C. The following can be observed:

- The load capacity on journal decreases clearly at the same angular velocity with the increasing temperature, as the viscosity of M1 lubricant decreases with the
increasing temperature.

- At the temperature $10^\circ C$ and $20^\circ C$, the load capacities on journal both increase linearly with angular velocity, but the gradient of load capacity with angular velocity at $20^\circ C$ is much lower than at $10^\circ C$. This variation tendency is similar to a Newtonian fluid and shows that the viscosity of lubricant plays an important role on load capacity.

- As temperature increases, the relationship between load capacity and angular velocity gradually lose linearity, especially at temperature $40^\circ C$. The nonlinear relation is evidently shown in figure 5.30 in which the gradient of load capacity with angular velocity increases with the increasing angular velocity.

- The viscoelastic effect on load capacity becomes significant at higher temperature because of higher relaxation times. At temperature $40^\circ C$ and higher angular velocity, the viscoelastic effect can greatly compensate the loss on load capacity caused by viscosity decreasing with increasing temperature.

![Graph showing normal stress difference around the journal](image)

*Figure 5.28* Normal stress difference around the journal ($\Omega = 10$ rad/s) for M1 lubricant at various temperatures.
Figure 5.29  Variation of load capacity (M1) with angular velocity at different temperatures.

Figure 5.30  Variation of load capacity (M1) with angular velocity at 40° C.
The load capacities for M2 lubricant at different temperatures are also calculated in numerical simulations at various journal angular velocities. The results are compared to M1 lubricant, and performance characteristics for both M1 and M2 are revealed and investigated. Table 5.3 gives the viscosities and relaxation times of M2 lubricant at various temperatures.

<table>
<thead>
<tr>
<th>M2 lubricant</th>
<th>10°C</th>
<th>20°C</th>
<th>30°C</th>
<th>40°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity(Pa.s)</td>
<td>0.4</td>
<td>0.22</td>
<td>0.13</td>
<td>0.09</td>
</tr>
<tr>
<td>Relaxation time (s)</td>
<td>0.035</td>
<td>0.06</td>
<td>0.08</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Figure 5.31 presents the analysis of load capacity at various temperatures for M2 lubricant. The following can be observed:

- At 10°C the load capacity of M2 is linear with angular velocity and viscosity effect plays an important role on load capacity.
- Starting at 20°C, the load capacities vary non-linearly with angular velocity due to the increasing viscoelasticity effect.
- The load capacity at 40°C surpasses the load capacity at 30°C starting from the angular velocity of 7 rad/s. This better load capacity at high temperature is mainly caused by the higher viscoelasticity of lubricant which can generate a high gradient of load capacity with angular velocity of journal.
- M2 lubricant has some similar performance characteristics with M1 lubricant, but the viscoelastic effect of M2 lubricant on journal bearing is more important than for M1 lubricant at the same temperature.
5.2.2 Bio-based lubricants
UCM model is also chosen to represent B lubricant. Table 5.4 gives the viscosities and relaxation times obtained from rheological characterizations for B lubricant at various temperatures.

Table 5.4 Viscosities and relaxation times for B lubricant at various temperatures.

<table>
<thead>
<tr>
<th>B lubricant</th>
<th>10ºC</th>
<th>20ºC</th>
<th>30ºC</th>
<th>40ºC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity(Pa.s)</td>
<td>0.12</td>
<td>0.075</td>
<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
<td>Relaxation time (s)</td>
<td>0.09</td>
<td>0.12</td>
<td>0.18</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Figure 5.32 presents the profiles of pressure along journal (Ω=3 rad/s) for B lubricant at four temperatures. The following can be observed:

- The profiles distinctly deviate from the anti-symmetry of Newtonian fluids. The crossing point between the profile with the coordinate X shifts to left from 2.9 radians, and the magnitude of the maximum pressure is higher than the magnitude of the minimum value.
- All the profiles of pressure along the journal nearly cross one point and rotate clockwise with the increasing temperature.
- The difference between maximum and minimum pressures are dumped with the increasing temperature.
- The pressure profiles become closer with the increasing temperature – the profiles at $30^\circ C$ and $40^\circ C$ nearly overlap.

Figure 5.33 presents the profiles of shear stress around the journal ($\Omega=3$ rad/s) for B lubricant at four temperatures. The following can be observed:
- For all tested temperatures, the shear stress profiles deviate from symmetry (characteristic of a Newtonian fluid).
- The magnitudes of extremes shear stress decrease with the temperature increasing.
- The shear stress profile at $10^\circ C$ presents evident difference compared to the profiles at other three temperatures which are close together.

![Figure 5.32](image-url)  
**Figure 5.32** Pressure around the journal ($\Omega=3$ rad/s) for B lubricant at various temperatures.
Figure 5.33  Shear stress around the journal (Ω=3 rad/s) for B lubricant at various temperatures.

Figure 5.34 presents the profiles of normal stress difference along the journal (Ω=3 rad/s) for B lubricant at four temperatures. The following can be observed:

- The profiles are very close at the temperatures 20°C, 30°C and 40°C, except the profile at 10°C which is clearly different.
- The profile in the range of 2.5-3.5 radians slightly shifts to left with the increasing temperature.
- The maximum normal stress difference at 30°C surpasses the maximum value at 40°C.

Figure 5.35 presents the variation of load capacity with angular velocity for B lubricant at four temperatures. The following can be observed:

- The load capacity of B lubricant is linear with the angular velocity of journal in the range of 2-5 rad/s at 10°C.
- When the temperature increases, the viscoelasticity of B lubricant increases. Therefore, starting from 20°C, load capacity varies non-linearly with increasing
angular velocity.

At 30°C and 40°C, the viscoelastic effect of B lubricant on load capacity becomes more evident and the gradient of load capacity with angular velocity greatly increases with the increasing angular velocity. The load capacity at 40°C can surpass the load capacity at 30°C from the angular velocity of 3.5 rad/s. The viscoelastic effect of B lubricant on load capacity greatly compensates for the loss caused by lower viscosity especially at a relatively high temperature.

![Figure 5.34](image-url)  
**Figure 5.34** Normal stress difference around the journal ($\Omega$ = 3 rad/s) for B lubricant at various temperatures.
The viscoelastic effects of different lubricants on the journal bearing performances are revealed and compared at various temperatures. At high temperature, the high viscelasticity can compensate for the loss of load capacity caused by viscosity decreasing with the increasing temperature. The bio-lubricant presents higher viscoelasticity and as a consequence an increased load capacity for journal bearings.

Figure 5.35  Variation of load capacity (B lubricant) with angular velocity at various temperatures.
Chapter 6  Conclusions

The effect of viscoelasticity on lubrication characteristics has recently taken on added significance with the move to yet lower-viscosity lubricants for improved energy efficiency. Any factor influencing load-bearing capacity and wear is clearly of renewed importance, and there are therefore good practical reasons to investigate the general question of viscoelastic effects in lubrication. The effects of viscelastic fluids and specific lubricants including mineral-based lubricants and bio-lubricants on eccentric rotating cylinders (practical application - journal bearings) are investigated using modeling and simulations, and experimental characterization throughout the course of this thesis.

Numerical simulations for viscoelastic fluids flow within eccentric rotating cylinders are performed using a commercial finite element software POLYFLOW. In 2D investigation, the numerical results reveal that lubrication performances are greatly affected by the viscoelasticity of fluid. Increased load capacity on the inner cylinder can be achieved by increasing viscoelasticity of flow. The study for viscoelastic behavior of UCM fluid on eccentric rotating cylinders provides further detail information for pressure, shear stress and normal stress difference profiles along the inner cylinder. Therefore the viscoelastic effects on eccentric annulus flow are clearly revealed and compared for various relaxation times. The investigations for transient problems (start-up) in eccentric cylinders clearly show that viscoelastic behavior affects the lubrication performance. The evolution of load capacity and the torque present fluctuations when the relaxation time is high, caused by viscoelasticity. 3D study for the Newtonian fluid in eccentric rotating cylinders reveals load capacity which is compared to the load capacity from 2D calculations. 3D results for the UCM fluid at steady state present the flow patterns along axial direction within the eccentric rotating cylinders, which are compared to the 3D results for the Newtonian fluid under the same boundary conditions.

Steady shear and SAOS experiments are performed for specific lubricants including mineral-based lubricants and bio-based lubricants to characterize their rheological
behavior. Slight shear thinning phenomena appears for mineral-based lubricants at high shear rates, while the canola bio-lubricant has relative constant viscosity at the same temperature. In general, the viscosities of lubricant decrease nonlinearly with the temperature increasing. The viscoelastic characteristics of these lubricant samples are evidently revealed in SAOS experiments. The elastic moduli for all lubricants increase with the shear oscillation frequency. The relaxation time obtained from fitting the experiment data of viscous and elastic modulus increases with temperature.

The viscoelastic constitutive models fitting rheological experiment data are used for numerical simulations in order to investigate the viscoelastic effects of lubricant on journal bearing performances. The viscoelastic effects of these real lubricants on journal bearing performances are revealed and compared at various temperatures. At high temperature, the high viscoelasticity can greatly compensate for the loss of load capacity caused by viscosity decreasing with the temperature increasing. The bio-lubricant presents higher viscoelasticity and as a consequence an increased load capacity for journal bearings.

Based on the literature review (section 1.4), the effect of fluid viscoelasticity of lubricants on the flow between eccentric rotating cylinders and on the bearing performances is not yet completely elucidated. The results in this study fill some blank areas in the research on eccentric rotating cylinders, and make the contributions in the following aspects:

- Presents viscoelastic effects of UCM fluid at 2D steady state on eccentric rotating cylinders for high relaxation times.
- Presents viscoelastic effects on eccentric rotating cylinders for UCM fluid in 2D transient (start-up).
- Presents for the first time 3D simulations for UCM fluids within eccentric rotating cylinders at steady state.
- Presents the experimental characterization of the bio-lubricant rheological behavior.
- Presents numerical simulations of the performance of bio-lubricant in journal bearings.
Future investigations are required, such as higher Weissenberg number cases within the eccentric annuli, viscoelastic effects of PTT fluid which can better represent the rheological characterization of lubricants, on eccentric rotating cylinders, eccentric annuli cases with a dynamic rotating inner cylinder, and more 3D investigations.

The modeling and simulation processes used in this thesis to predict the flow of the lubricant in a journal bearing can generate important economic benefits. This research will lead to advanced predictive tools that can be used to improve the design of journal bearing and to propose new economically viable and environmentally friendly lubricants. The results in this thesis could be used to find the lubrication performances and select the proper lubricant for a specific lubricating system. Moreover, based on our results, design of new bio-lubricant could be done, as well as different recommendations for using of lubricants.
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