TARGETED PERSUASIVE ADVERTISING

by

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Abstract

The three essays of this thesis consider a firm's choice of advertising campaign when advertising may be conditioned on the preferences of individual consumers. In essay one, I show that a monopolist will use such advertising to turn sub-marginal consumers, who are not quite willing to pay for the good, into marginal consumers who are indifferent to paying for the good or going without it. The second essay considers the use of targeted advertising in duopoly, when one of the firms does not have access to advertising. I find that advertising will target those consumers most likely to switch to the non-advertising firm. Each firm sets a price just high enough to capture the consumers on either side of the advertising 'barrier'. The third essay looks at targeted advertising in the context of Canadian public health. When the goals of government and industry are aligned, advertising by the firm may be an alternative superior to government advertising in the form of a public health education campaign.

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For my father

1. Introduction

All advertising is targeted. When a florist places a sidewalk sign, she targets passers-by. Television commercials for a nationally-advertised detergent target consumers who watch a particular channel at a certain time. No message has a truly universal audience. Where, when and how an advertisement is placed determines the nature of its audience in ways that are often predictable. A firm placing a roadside billboard knows it will be viewed by motorists on that road traveling in one of two possible directions. This information, along with knowledge of the points the road passes through, may be used to make the message on the board appealing to its audience.

Though advertising is targeted, the advertiser is not required to make use of this fact. Mass advertising is useful when the goal is to increase awareness of a new product, or to communicate indisputable properties of the good in question (such as price, weight or colour). In such a case, consumers are alike at a fundamental level: they are aware of the existence and objective characteristics of the product, or they are not. Strategic targeting recognizes that consumers do not generally have access to all of a product's characteristics. The same consumer who knows that Acme detergent exists in powder form may not be aware that a box of the detergent costs five dollars.

When an audience is known by more than a single general trait, it is possible to customize an advertising message to appeal to its specific characteristics. If a florist knows that passers-by on her shop's street enjoy classical music and like the colour purple, she may use this information to create a sign that appeals to that particular demographic. Any good has a very large number of characteristics that a consumer may be informed of. Not all of a product's traits may be expounded in one message. Knowledge of consumer characteristics allows the advertiser to choose those bits of information that are most likely to have the desired effect. In the present example, our florist may choose a sign indicating that her shop sells purple flowers which look lovely next to a grand piano. This sort of advertising, in which known characteristics of the intended audience determine the shape of the advertising message, is known as 'targeted advertising'.

Advances in communication technology have made it possible for advertisers to build databases of detailed consumer data. Web sites track the movement and behaviour of visitors; supermarket scanners collect purchase data and consumers voluntarily fill out thorough online surveys for a chance to win a prize, or have their opinion heard. Once, the chief difficulty in targeting was gathering enough information to make customization of the advertising message worthwhile. Now that this information is readily available, the difficulty lies in deciding who to target, to what extent, and in what fashion. The following three essays examine this decision.

Accurately targeting consumers can be difficult and costly. Some information, such as a consumer's name, address and phone number, are readily available. Other information, such as taste in clothing or favourite sports team, is more difficult to obtain and was until recently out of reach for most firms. In what follows, I assume that advertising is costly: it costs at least a dollar in advertising to raise a consumer's willingness to pay for a good by a dollar. If this were not the case, advertisers would spend as much as possible on advertising, since an extra dollar in expenditure would bring in more than an extra dollar in revenue.

The first of the three essays looks at the choices faced by a monopolist. Targeted advertising can be used to persuade consumers to pay a price higher than that they would otherwise find acceptable. Though a single product is sold to many consumers, by customizing the advertising message, the experience of consuming the product is transformed. Consider a canned version of an ethnic food that is sold to both members of the relevant ethnic group, and others. Advertisements to the ethnic group may use visual cues and copywriting to emphasize the traditional and old-fashioned nature of the product. Meanwhile, ads to non-members may describe the food as exotic and daring. If these ads are properly targeted, in that the advertising message matches the tastes and desires of its audience, they may make both of these groups equally willing to pay a high price for the product. This is of clear advantage to a firm unable to price-discriminate.

Suppose a non-price-discriminating firm is sole producer of a good for which consumers have varying willingness to pay. In traditional economic theory, the firm may use advertising to inform consumers of the existence of the product, its characteristics and the price at which it is sold. Once all consumers are aware of the product and its price, the firm does not benefit from further advertising. Targeted advertising, however, is useful even when all consumers are fully informed. Suppose the firm sets a high price, in the sense that some consumers prefer to go without the good than to pay for it. By advertising to these consumers, the firm may raise their valuation of the good to the point where they are just willing to pay the stated price.

I show that a monopolist will use such advertising to turn sub-marginal consumers, who are not quite willing to pay for the good, into marginal consumers who are indifferent to paying for the good or going without it. Consumer tastes, defined in the sense understood by the general public, are unchanged - indeed, targeted advertising works by appealing to these tastes, as in the example above. The monopolist's advertising leads to a group of consumers with a similar willingness to pay for the product. This may be either an advantage or a barrier for a potential entrant. If an entrant is able to capture one of these consumers, it captures all of them. If it fails to capture one, then it will fail to capture the rest.

The second essay considers the use of targeted advertising in duopoly, when one of the firms does not have access to advertising. I find that the advertiser will use its campaign to erect a barrier against price-based poaching by its rival. Advertising will target those consumers most likely to switch to the other firm, and raise their willingness to pay by an amount substantially greater than that which is needed to convince them to buy the advertised product. These newly-loyal consumers lower the non-advertiser's gains from price competition. The end result is that each firm sets a price just high enough to capture the consumers on either side of the advertising 'barrier'. The advertiser's profits are, as they must be, higher than in the benchmark case of a duopoly with no advertising. Surprisingly, the non-advertiser's profits will often be higher than those of the advertiser. This happens because it reaps most of the benefits of decreased price competition, while having to incur none of the costs.

The third essay looks at targeted advertising in the context of public health. I find that when the goals of government and industry are aligned, advertising by the firm may be an alternative superior to government advertising in the form of a public health education campaign. Consider the case of preventive medicine. Consumers differ in their susceptibility to a preventable disease. Once the disease manifests, the government pays for curative care. Consumers are responsible for paying for preventive care, which is sold by a monopolist. Unless the government can commit to an advertising strategy that targets consumers with a low risk of illness, the firm will set a high price and hold the high-risk consumers hostage. As a result, government's ability to advertise may reduce coverage of preventive treatment, and raise its cost - exactly the opposite of its intended effect. If government commits to targeting low-risk consumers, then it provides an incentive for the firm to set a low price and capture them.

Targeted advertising is viewed with suspicion by both firms and consumers. In these three essays, I show that when used properly, costly targeted advertising is not only profitable for the advertiser, but often also for its rival and other agents in the economy.

2. Targeted persuasive advertising[1]

2.1. Introduction

"*An important aspect of marketing practice is the targeting of consumer segments for different promotional activity*." (Rossi *et al.* 1996)

The door-to-door salesman has returned in electronic form. Thanks to the internet and improvements in data collection, modern marketers are once more able to tailor a sales pitch to an individual customer. A web-based retailer may offer different storefronts and products to customers with different tastes, and a company can send its clients e-mails with enticements to purchase in keeping with their recorded preferences. At present, this 'direct marketing' technology is in its adolescence, and spam (a common term for unsolicited commercial e-mail) is still largely a hit or miss affair - men receive enticements to breast enlargement and teenagers are asked if they wish a low interest rate on their mortgage. There is, however, every reason to believe that the accuracy of these methods will increase, and the current extent of their use bears witness to firms' faith in their efficacy. (Rossi *et al.* 1996) In 1999, direct marketing accounted for well over half of all marketing expenditures in the United States (Economist 1999).

The mechanics of data gathering via the internet is already impressive. Cookies, small bits of code transmitted by web sites, already allow advertisers to track consumers' browsing patterns, and legal, ubiquitous spyware uses methods similar to those of computer viruses to gather detailed information on the content and use of a computer on which they are installed, then transmit it to its home firm. The number of such programs has become a nuisance in its own right, and Spybot[2], a program designed to keep track of and eliminate spyware, had over 11,000 programs in its search database as of January

[1] A version of this chapter will be submitted for publication.

[2] Hosted at http://www.safer-networking.org/

2004. These electronic researchers, the data collection and analysis programs, have numerous advantages over their flesh and blood kin. They do not sleep, take days off or earn wages. And most importantly to many firms, they do precisely what they're told.

This begs the question - what is it, exactly, that firms will do with these possibilities, and this information? The companies themselves are still uncertain of how best to exploit the possibilities. They have gathered vast amounts of data on consumer habits and preferences, but are still at a loss as to how to use it. There is some worry, particularly noticeable among privacy advocates (Economist 2003) and European governments (Economist 2000) that powerful corporations will abuse their ability to influence consumers in such a way that diversity and competition will be compromised. Multinationals selling the same good to different cultures, they reason, will wish to do what they can to shape consumer tastes into a homogeneous mass favourable to the sale of their product.

The key issue causing the worry is that firms to not need to use their wealth of information to find a suitable market for their products, given preferences and the characteristics of the good. Instead, like the salesmen of yore, the electronic marketers can change their approach to suit the consumer, convincing them to buy something they would otherwise have done without. The French government is not worried that McDonald's will take the business of all its hamburger-loving clientele. It is worried that the fast food chain will use its wiles to charm the French away from *pate de fois gras* and to the Big Mac, convincing them in the process that they prefer fountain drinks to wine. When a firm knows with some accuracy what stimuli a consumer responds to, it can use these to make the customer fit the good - but only if it can target its messages: while Ginger may buy the soda endorsed by a prominent rap artist, it is not entirely clear that Aunt Maude will be likewise enticed.

Despite clear concern about the problem, there have been few detailed analyses (in the economic literature, at least) of the strategies involved in direct marketing. Indeed, there has been relatively little work done on persuasive advertising of any sort, as opposed to informative advertising. This neglect is partly the legacy of George Stigler and Gary Becker's influential and eloquent assertion that "tastes, at least when held by an adult, are not capable of being changed by persuasion." (Stigler and Becker 1977). Economic discourse on advertising has instead focused on its role as a means for disseminating information.

Figure 2.1: Copy from a cell phone ad campaign

> "The imagery from our advertising campaigns is so popular we've designed a screensaver calendar so you can continue to enjoy them right from your desktop."

Source: http://www.telusmobility.com

Models of informative advertising assume that a consumer enters the world not knowing about the products within it, and must be informed of their characteristics and price through the issuing of advertisements. This is their sole purpose. While information is undoubtedly an important part of marketing (brand recognition, 'a name you can trust'), a cursory glance at actual advertisements will show that much of what is seen in them is not directly related to the features of the product in question. Abercrombie and Fitch, a clothing store, has run a marketing campaign starring notably unclothed and attractive ladies and gentlemen. A BC cell phone vendor's ads feature pigs, and little else.

The idea behind persuasive advertising is that a firm can somehow pay for advertisements that raise the valuation of a product in the eyes of its consumers. (A typology of the ways in which advertising may affect a consumer of differentiated products may be found in (von der Fehr and Stevik 1998).) An Abercrombie and Fitch shopper is enticed by the store's ad not because it reveals much about their clothing, but because the positive emotions experienced when viewing the ads are transferred to the company that created them, and the products that they sell. Most models of persuasive advertising assume that ads are sent to the market as a whole, or to a sizeable portion of that market.

Economists have scarcely looked at targeted advertising, that is, a framework where the ads sent depend upon the characteristics of individual consumers. The most thorough paper on the phenomenon to date (Esteban *et al.* 2001) focuses on the case of a monopolist engaged in informative advertising. The firm may choose between advertising cheaply to a large number of consumers with mixed tastes, or paying more to advertise in a medium with a smaller audience whose tastes are more favourable to its product. In equilibrium, the monopolist chooses a higher level of advertising than is socially optimal.

As may be expected, marketers are more prolific in their writing about targeting. The technical discussion has focused largely on price discrimination and coupon issue[3]. This is presumably because the psychology-inspired marketing language commonly used to write about persuasive advertising and its effects is difficult to reconcile with the more mathematical approach taken by this subject. Examples of the genre are a recent research note (Chen and Iyer, 2002) and an article on the profitable use of purchase history data (Rossi *et al.* 1996). The former paper uses a framework similar to the present one to address customized pricing under costly information acquisition.

The absence of an analysis of the strategic uses of persuasive advertising is an important gap in our knowledge, since as D.P.T. Young (Young 2000) has argued, the existence of persuasive advertising complicates the matter of defining a market, and must be taken into account when formulating regulatory policy. If persuasive advertising is targeted,

[3] Given the possibility of perfect price discrimination through targeting, why should a firm bother with persuasive advertising? One reason is the possibility of arbitrage. If consumers may communicate and trade with each other (possibly through a centralized system such as e-bay), then these coupons and discounts could be arbitraged to the issuing firm's detriment. Persuasive advertising does not have this problem, and this may explain much of its appeal even to those firms best placed to used 'customized pricing'. The relative merits of each technique of consumer targeting will vary with the type of product under consideration.

then this provides firm with a flexible tool with which to manipulate preferences and through them, profits. Young suggests that the market power of a firm may be usefully measured not only by the ability to maintain a significant price increase, but also by its capability to gain an advantage over its competitors by *creating* an asymmetry in demand.

In the following pages I endeavour to show that, under plausible conditions, direct marketing by one of two duopolists can be worse for consumers than a monopoly. Targeted ads can be used as a precise tool to facilitate collusion; a very small and judicious marketing campaign may leave both firms better off than in the absence of advertising. Even when the non-advertising firm is worse off, as would be expected, it will do as well as a monopolist with its given market share.

My approach is most closely related to the economic literature on persuasive advertising and product differentiation surveyed in (von der Fehr and Stevik 1998) and the marketing literature on customized pricing exemplified by (Chen and Iyer 2002). As in both traditions, I use the Hotelling (Hotelling 1929) framework as a basis for analysis. I differ from the former in using targeting, and the latter in using advertising.

Section 2.2 restates the Hotelling model and introduces terminology. Section 2.3 examines the advertising choice of a monopolist with direct marketing technology. Section 2.4 analyses the case of duopoly, with only one firm advertising, and section 2.5 concludes and lists directions for further work.

2.2. The Hotelling model

The framework for the analysis will be a version of Hotelling's 1929 model of differentiated products. Two firms are located at opposite ends of a line of length one. Firm 1 is at address zero, and Firm 2 is at address one. A unit mass of consumers is uniformly distributed along this line, and they are arranged according to their preferences for the goods. The utility for the consumer at address r from each firm's product is given by the expressions $U_r^1(p_1, r) = V - p_1 - tr$, for firm 1's product, and $U_r^2(p_2, r) = V - p_2 - t(1-r)$ for firm 2's product. Here, V is an intrinsic valuation, t is a positive taste parameter (Hotelling's 'transport cost'), and p_i is the price charged by firm i. Each consumer buys one of good 1, good 2, or an outside good providing zero utility.

Producers have no marginal or fixed costs. Profits are given by

$$\pi_1 = p_1 x$$
$$\pi_2 = p_2(1-x)$$

where x is the address at which a consumer is indifferent between good one and good two, and weakly prefers both to the outside good. Consumers to the right of x will buy from Firm 2, and consumers to the left of x will buy from Firm 1. This boundary point is defined by

$$V - p_1 - tx = V - p_2 - t(1-x)$$
$$x = \frac{1}{2}\left(1 + \frac{p_2 - p_1}{t}\right)$$

and is subject to the standard constraint that the utility of goods 1 and 2 at x be weakly greater than that of the outside good, so that the market is covered.

Solving the first order conditions, we find

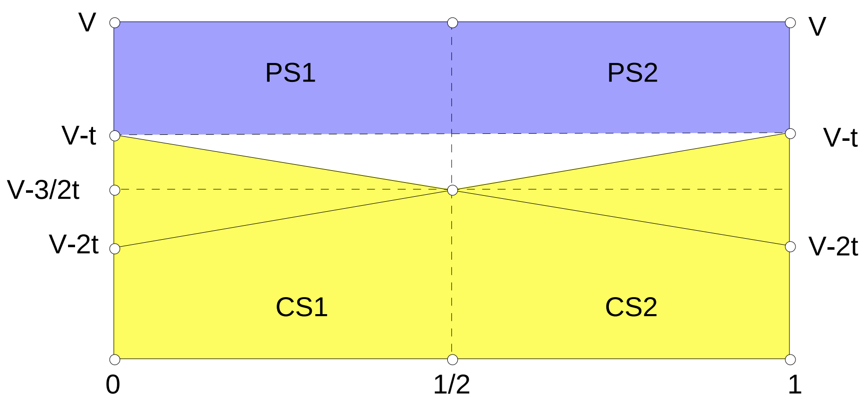
$$p_1 = p_2 = t$$

$$x = \frac{1}{2}$$

$$\pi_1 = \pi_2 = \frac{t}{2}$$

And the constraint becomes $V - \frac{3}{2}t \geq 0$, or $\frac{V}{t} \geq \frac{3}{2}$. This means that transport costs must be sufficiently small with respect to the intrinsic valuation of the good for all consumers to buy one of the two varieties offered.

Figure 2.2: Consumer and producer surplus in the Hotelling model



Producers' surplus is t, and consumers' surplus is given by

$$CS = \int_0^{1/2} (V - t - tr)dr + \int_{1/2}^{1} (V - t - t(1-r))dr = V - \frac{5}{4}t$$

The case of monopoly is solved similarly, save that the rival supplies the outside good. Assuming firm 1 is the monopolist, the monopoly's market share x^M is given by

$$V - p_1^M - tx^M = 0$$

$$x^M = \frac{V - p_1^M}{t}$$

The superscript M denotes monopoly values. The firm's profits are given by

$$\pi^M = p_1^M x^M$$

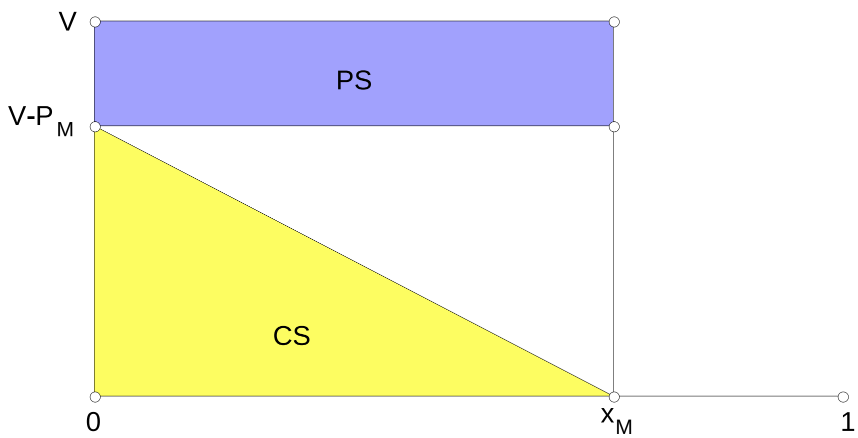
Solving the first-order conditions, we find

$$p_1^M = \frac{V}{2}$$

$$x^M = \frac{V}{2t}$$

$$\pi^M = \frac{V^2}{4t}$$

Figure 2.3: Producer and consumer surplus for a Hotelling monopolist



In this case, the market need not be covered. It will be covered if and only if $\frac{V}{t} \geq 2$. For $\frac{V}{t} > 2$, the monopolist has the entire line as its market share, and both its price and profits are equal to V-t, the indifference price for the consumer at address 1.

Given $\frac{V}{t} \leq 2$, that is, that $x^M \leq 1$, consumers' surplus is given geometrically by

$$CS_B^M = \frac{1}{2}\left(V - \frac{V}{2}\right)\left(\frac{V}{2t}\right) = \frac{V^2}{8t}$$

Consumers' surplus is equal to half of producer's surplus. The 'B' denotes a benchmark value.

Total surplus is

$$TS_B^M = \frac{3}{8}\frac{V^2}{t}$$

For $\frac{V}{t} > 2$, consumers' surplus is found by similar methods to be $\frac{t}{2}$.

The above models will be the benchmarks against which the effects of advertising will be compared.

2.3. Advertising by a monopolist

Definition: *Targeted Persuasive Advertising is a technology that allows a firm to pay cM_r, $c>1$, to raise the valuation of its good to the consumer at address r by M_r.*

As an example, suppose a given consumer is willing to pay up to 6 for a good whose price is 10. Without advertising, the consumer would decline to buy this good. Now suppose that the producer has access to advertising as above, and the cost parameter, c, is equal to 2. By paying 8, the firm is able to raise the consumer's reservation price by 4.

The consumer now values the good at 10, and will buy it, resulting in a profit of 2 for the company.

Let us look at TaPA in the context of a Hotelling monopoly. Consumer utility is now given by

$$U_M^A(r) = V - p_M^A - tr + M_r(r)$$

where p_M^A denotes the price charged by the firm. The monopolist's profits are

$$\pi_M^A = p_M^A x_M^A - c\int_0^1 M_r(r)dr$$

The monopolist's task is to set the price choose an advertising scheme.

Proposition 2.1: *For a given price, optimal advertising involves raising all negative-valued preferences to zero until an address* $b \leq 1$. That is,

$$M_r^*(r) = \begin{matrix} \mathrm{A}(r) & a < r < b \\ 0 & otherwise \end{matrix}$$

$$\mathrm{A}(r) \equiv -\left(V - p_M^A - tr\right)$$

The points a and b are defined by

$$V - p_M^A - ta = 0$$

$$cM_r^*(b) = p_M^A$$

The intuition behind this is as follows. Given a price such that the monopolist does not cover the market, it will always be profitable to advertise to the customers just to the right of the indifference point. As in the example above, by topping up preferences to the point of indifference, the monopolist can gain the whole of the price as revenue for the cost of the advertising. At some point, called b, the cost of advertising will be weakly greater than its benefit. The firm will not advertise beyond this point. It will also forego advertising to those consumers whose valuations are higher than the price it charges, since this would be a wasteful expenditure.

Lemma 2.1: *If* $M_r^*(r) > 0$*, then* $U_M^A(r) = 0$ *(that is,* $M_r^*(r) = \mathrm{A}(r)$*).*

Proof: Proof by contradiction.

Suppose

$$U_M^A(r) > 0 \text{ and } M_r^*(r) > 0$$

for some r. The consumer at r will buy the good. Further, there exists some $\varepsilon > 0$ such that $U_M^A(r) - \varepsilon > 0$, and so $M_r^*(r)$ could fall to $M_r^*(r) - \varepsilon$ and the consumer at r would still buy the good at the given price. Since this would result in an increase in profits for the monopolist, the stated case cannot be optimal.

Now suppose that

$$U_M^A(r) < 0 \text{ and } M_r^*(r) > 0.$$

In this case, the consumer will not buy the good. Since the same result could be obtained at zero cost by not advertising, this cannot be optimal. Hence, if $M_r^*(r) > 0$, it must be the case that $U_M^A(r) = 0$ identically, *q.e.d.*.

Corollary 2.1: *If there exists an* $0 \le a \le 1$ *such that* $U_M^A(a) = 0$ *when* $M_r(a) = 0$*, then* $M_r^*(r) = 0$ *for all* $r \le a$.

This follows from the above and the fact that the level of advertising can never be negative. The corollary establishes the left end point for the support of the advertising campaign.

What of the right end point?

Lemma 2.2: *If* $c\mathrm{A}(r) > p_M^A$*, then* $M_r^*(r) = 0$.

Proof: Suppose $c\mathrm{A}(y) > p_M^A$ for some $0 \le y \le 1$, and $M_r^*(r) > 0$. Then reducing $M_r^*(r)$ to zero would increase firm profits, and so it cannot be optimal. Since advertising cannot be negative, the lemma holds.

Lemma 2.3: *If* $c\mathrm{A}(r) < p_M^A$ *and r>a, then* $M_r^*(r) > 0$

Suppose not. That is, suppose that r>a and $M_r^*(r) = 0$. Since $U_M^A(r)$ falls with r, by the definition of a, we must have $U_M^A(r) < 0$. That is, the consumer at r will not buy the monopolist's product. The monopolist could increase its profits by $\left(p_M^A - c\mathrm{A}(r)\right) > 0$ by paying for the amount of advertising dictated in Lemma 2.1. Hence, zero advertising at this point cannot be optimal.

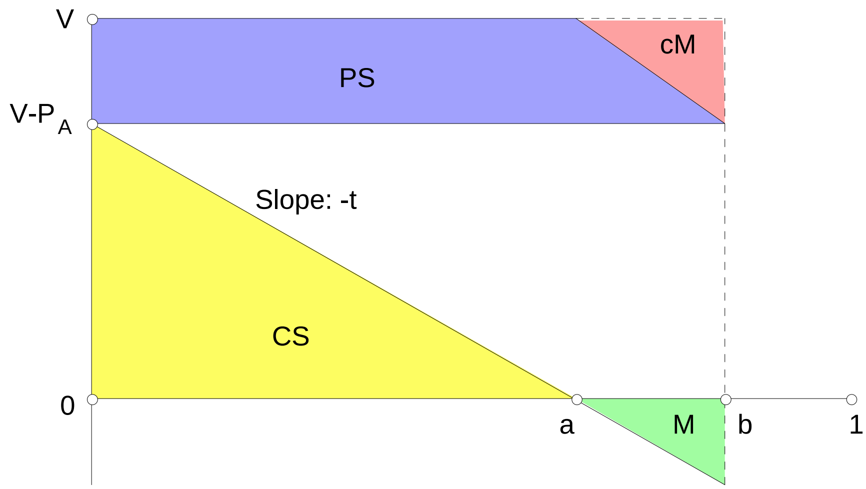
Corollary 2.2: *The point b defined by* $cM_r^*(b) = p_M^A$ *is the right end point of the support of the advertising campaign.*

From Lemma 2.1, we see that $M_r^*(r)$ rises with r, and so if there exists a $0 \le b \le 1$ such that $cM_r^*(b) = p_M^A$, then using Lemma 2.2, $M_r^*(r) = 0$ for all r>b.

Proof of proposition 2.1: From Lemma 2.1, we see that within the support of advertising, it is optimally $A(r)$. From Corollary 2.1 and Lemma 2.3, we see that the left end point of the support is the point a, and Corollary 2.2 gives us the right end-point, b.

Given this advertising scheme, what does it look like?

Figure 2.4: Advertising by a monopolist



The monopolist's profits are given by

$$\pi_M^A = p_M^A b - c\int_a^b M_r^*(r)dr$$

Substituting for a, b and $M_r^*(r)$ turns this into a function of p_M^A:

$$\pi_M^A\left(p_M^A\right) = \frac{p_M^A}{t}\left(V + \left(\frac{1-2c}{2c}\right)p_M^A\right)$$

Solving the first-order conditions, we find

$$p_M^{A^*} = \frac{Vc}{2c-1}$$

$$\pi_A^{M^*} = \frac{1}{2}\frac{V^2}{t}\frac{c}{2c-1}$$

$$a^* = \frac{V}{t}\frac{c-1}{2c-1}$$

$$b^* = \frac{V}{t}\frac{c}{2c-1}$$

These are reasonable results. The price is positive and greater than the no ad monopoly price for c>1. The support is such that a<b, and profits are greater than monopoly profits in the benchmark case. In order to have b<1, we require

$$\frac{V}{t} < 2 - \frac{1}{c}$$

The value of the right hand side ranges between 1 and 2, and fits well with previous restrictions on the ratio between intrinsic valuation and transport costs.

Note that $a \leq x_M \leq b$. That is, the support of the advertising campaign brackets the benchmark monopoly market share, with more of the market being covered than in the absence of ads. That a is lower than x_M follows from the price being higher when ads are present. A higher price will, in the absence of ads, drive the indifferent consumer towards the origin. That more of the market is covered follows from advertising making it possible to raise prices without lowering them for all consumers, in a manner similar to price discrimination[4].

[4] Similar, but not congruent. Since advertising is particular to a consumer, it can be used in situations where arbitrage among customers would make price discrimination impossible. This may be the case in a computer-literate society where customers have access to an auction network where they may trade amongst themselves.

Geometrically, we find

$$CS_A^M = \frac{a}{2}\left(V - p_M^A\right) = \frac{1}{2}\frac{V^2}{t}\left(\frac{c-1}{2c-1}\right)^2$$

For c>1, this will always be less than the surplus in the case of no ads. Total surplus is

$$TS_A^M = \frac{1}{2}\frac{V^2}{t}\frac{\left(3c^2 - 3c + 1\right)}{\left(2c-1\right)^2}$$

Surprisingly, even given that advertising is costly, total surplus rises from the benchmark monopoly case:

$$TS_A^M - TS_B^M = \frac{1}{8}\frac{V^2}{t}\frac{1}{\left(2c-1\right)^2} > 0$$

The increases in price and market share dominate the effects of a costly transfer of surplus. The reason for this is, once more, the same effect illustrated by the numerical example used to introduce the advertising technology. While c>1 and thus advertising is costly, the firm only needs to 'top up' a consumer's preferences, which by construction is cost-effective on the entirety of the support.

It is interesting to consider the fraction of the monopolist's market that will consist of indifferent consumers.

The fraction of the monopoly's market that is advertised to, and which is thus indifferent and obtains zero surplus, is inversely related to the costs of advertising:

$$\frac{b-a}{b} = \frac{1}{c}$$

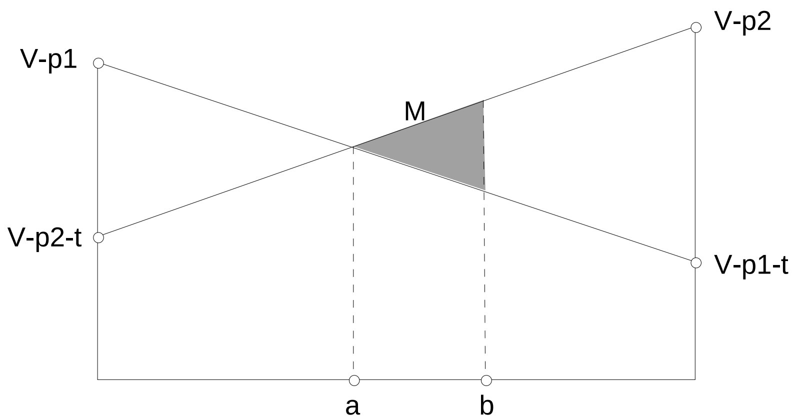
Unless persuasive advertising is exceedingly costly, the clientele of the advertising monopolist will consist chiefly of indifferent consumers.

2.4. Barriers

When the advertising firm needs to take into account a rival as well as the outside good, the situation becomes more complicated.

At first blush, it might seem that the case of duopoly, with only one firm advertising, is a straightforward extension of the case of an advertising monopoly. Instead of providing zero surplus at the margin, the advertiser must provide a positive surplus equal to that granted by its rival at that address.

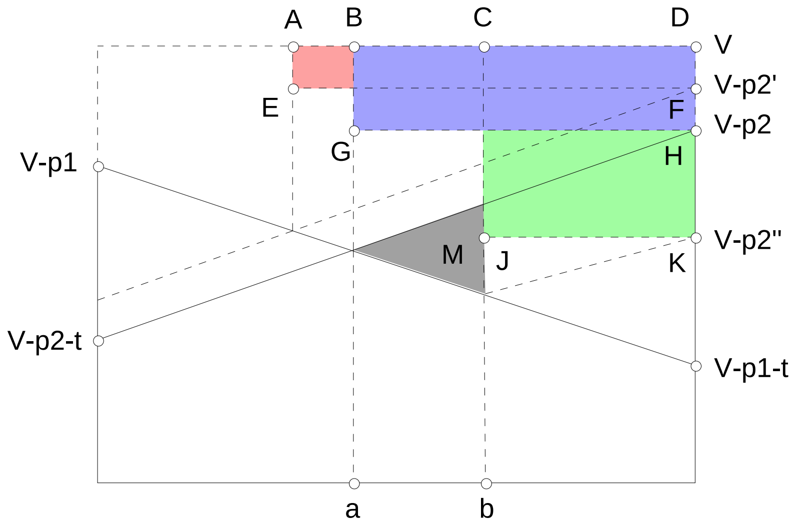
Figure 2.5: Monopoly-style advertising in duopoly



Points a and b are chosen as in the previous section. In what is now a two-period game, advertising is set in the first period and prices are simultaneously determined in the second.

Unfortunately, this procedure will not, in general, provide an equilibrium. Faced with a situation such as that shown above, the non-advertising firm (Firm 2) will have an incentive to deviate. This deviation may take two forms, as illustrated below:

Figure 2.6: Profitable deviations from monopoly-style advertising



Faced with M and starting from p2, Firm 2 may either lower its price to (say) p2' so as to bypass Firm 1's advertising and gain market share, or raise its price to p2'' while keeping to its new market share. In the former case, its profit is the rectangle ADFE, and in the latter, CDKJ. One or both of these will, in general, provide greater profit than that from p2, that is, BDHG. Note that in the case of the price falling to p2', or indeed to any level below p2, Firm 1's advertising, M, is entirely wasted. The consumers in the support of the advertising (and beyond) will prefer Firm 2's good to Firm 1's. It is not enough, then, for Firm 1 to react to a given p2, as a monopolist reacts to a given outside good. It must take these strategic effects into account if its persuasion is to have a positive impact on profits.

Given the incentive for Firm 2 to deviate, the advertising firm will wish to engage in advertising for two different reasons: first, to prevent its rival from price-cutting its way past the advertising campaign, and secondly, to exploit the territory so secured through the methods of the previous section. That is to say, the first aim of an advertising scheme should be to persuade Firm 2 to restrict itself to the interval (b,1]. With this accomplished, Firm 1 may act as a monopolist on [0,b]. The former motive is the *barrier* motive for advertising; the second, the *top-up* (in that preferences of consumers are topped up to some minimum level if they fall below it).

In the analysis that follows, I will concern myself only with the construction of a barrier and ignore the top-up motive. This is done because the intuition for top-up is exceedingly similar to that of the monopoly case, so that there is little to be gained by introducing it here. Moreover, I will show that even in the absence of top-up advertising, Firm 1 can do far better and consumers far worse than in the case of no advertising. Adding the possibility of indifference-inducing advertising on the blocked-off interval would only strengthen these results.

2.4.1. Mirror prices

How is this barrier created? Consider the case of no advertising. For any given p_1 such that all consumers weakly prefer good 1 to the outside good, the profits of Firm 2 are quadratic in p_2, or equivalently (and more usefully) in the market share of Firm 1, $x(p_1,p_2)$.

$$\pi_2(p_1, p_2) = p_2(1 - x(p1, p2))$$

The indifference point is defined by the condition

$$V - t(x) - p_1 = V - t(1 - x) - p_2$$

from which we see that

$$p_2 = p_1 + t(2x - 1)$$

Given Firm 1's price, Firm 2's price is a linear function of x. Using the above and substituting for p_2 as a function of x, we have

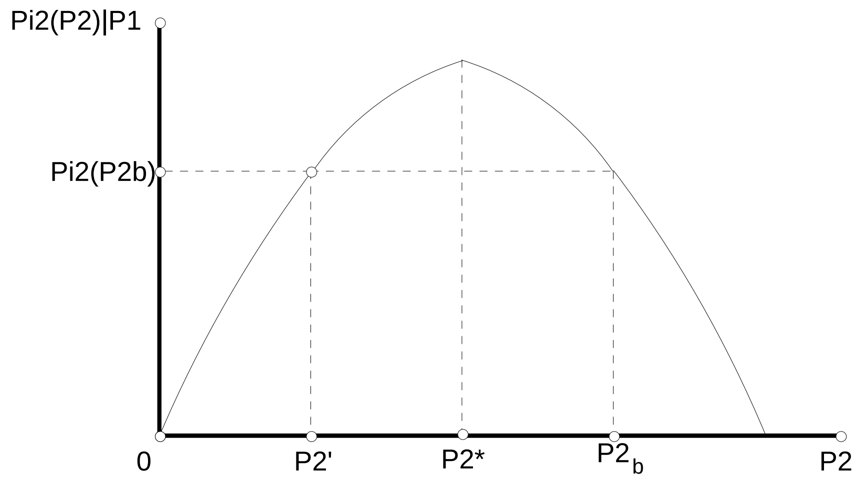
$$\pi_2(x(p_1,p_2),p_1) = (p_1 - t) + (3t - p_1)x(p_1,p_2) - 2tx(p_1,p_2)^2$$

The equivalent expression in prices is

$$\pi_2(p_2) \mid p_1 = \frac{1}{2}\left(1 + \frac{p_1}{t}\right)p_2 - \frac{1}{2t}p_2^2$$

For a fixed p_1, these are quadratic in x and p_2, respectively, as shown below:

Figure 2.7: A quadratic profit function



$$p_2^* = \frac{1}{2}(t + p_1)$$

The diagram in term of x is similarly shaped.

This implies that, given $\pi_2 \geq 0$,

$$\forall p_2 > p_2^* \;\; \exists p_2^{'}(p_2) < p_2 \text{ s.t. } \pi_2(p_2^{'}(p_2)) \mid p_1 = \pi_2(p_2) \mid p_1$$

For any price greater than Firm 2's optimal price, there exists a lower, 'mirror' price at which the firm can earn an equal profit by trading price for market share[5]. The lower mirror price implies a lower x and hence a lower market share for Firm 1.

The 'mirror price' result continues to hold in the case where p_1 does not cover the consumer line, but only within certain restrictions. Suppose that p_1 is such that (only) consumers on (b,1] , 0<b<1, prefer the outside good to good 1. Then

$$V - tb - p_1 = 0$$
$$p_1 = V - tb$$

For $x \le b$, the diagram is the same as above. For x>b, however, it no longer applies. Since consumers to the right of b prefer the outside good, Firm 2 can act as a monopolist on that interval[6].

The graph has a kink at b, as shown below. Before b, the duopoly profit function applies, and after b, the monopoly profit function is relevant.

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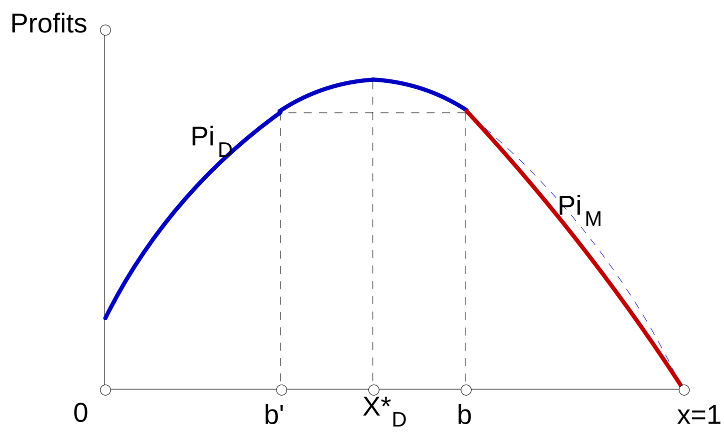
[5] At this lower price, and given p_1, Firm 2's market share may well be constrained to be unity. This does not change the existence of the mirror price, though it will somewhat alter its calculation.

[6] An example might be helpful. Suppose that Firm 1's price is such that the consumer at address 1/3 is indifferent between good 1 and the outside good. That means that all consumers to the right of 1/3 will prefer the outside good to good 1. Now think of Firm 2, located at address 1. The firm must decide, given Firm 1's price, which market share it desires (or equivalently, what price it wishes to charge). If it chooses a market share between 2/3 and 1, say, 7/8, then it must provide the consumer at address 1/8 with the same, positive surplus she would obtain from consuming Firm 1's good. Since Firm 2 cannot advertise or price-discriminate, this determines the price of its good. What if Firm 2 instead chooses a market of ½? The consumers at address ½, and all the consumers with a higher address, prefer the outside good to good 1. That means that Firm 2 only has to provide this consumer with the zero surplus she would obtain from the outside good, her most preferred alternative. By choosing a market share on which good 1 is considered inferior to the outside good, Firm 2 can price as a monopolist constrained to that interval would, and extract the entire surplus from the marginal consumer.

Figure 2.8: A kinked profit function



It is easily verified that when $p_1 = V - tb$, the graphs cross at b[7]. Of course, when x=1, and the market share of Firm 2 is zero, the monopoly and duopoly profit functions are also zero. The slope of the duopoly function at b is less negative than its monopoly counterpart at b, and so provided that the kink is to the right of the optimal monopoly market share (on the descending portion of the quadratic), there will be a mirror market share (equivalently, a mirror price) for b<x<1. Additionally, the profit at b will be higher than any profit from a higher x. Since the optimal monopoly share $x_M^{2*} = 1 - \frac{1}{2}\frac{V}{t}$, we require that $b > 1 - \frac{1}{2}\frac{V}{t}$. Recall that our benchmark duopoly model assumes that V/t>3/2. This being the case, at its most binding, this restriction asks that b>1/4.

2.4.2. Relevance of the mirror price

If the advertising firm needs to constrain Firm 2, it will be because it wishes its rival to obtain a smaller than optimal market share, that is, $x > x_D^{2*}$. Were this not the case, Firm

[7] $\pi_M^D = (V - t(1-x))(1-x)$, and $\pi_D^2(p_1, x) = ((2x-1)t + p1)1 - x$. When $p_1 = V - tb$ and x=b, both expressions are equal to $(V - t(1-b)(1-b))$.

2 would automatically accommodate, albeit leaving the market not entirely covered. The implications for the construction of a barrier are clear.

Suppose the support of the advertising campaign is the interval [a,b], a<b<1. Given the purpose of the barrier, which is to cordon a segment of the consumer line for Firm 1, Firm 2 has two choices. It may accommodate and accept a market share between 0 and 1-b, or it may attempt to price-cut its way past the ads. Let the optimal price for Firm 2 to set while accommodating be called p_2^*. Firm 2 will be willing to lower its price to any level up to and including the mirror price, $p_2^{'}$, since any such successful undercutting will yield a profit weakly greater than the best outcome from accommodation. This *threat price* represents the non-advertiser's most harmful credible attempt at ignoring the ads.

To prevent this undercutting, Firm 1 must provide consumers on the support of the advertising campaign equal to the surplus they would receive from Firm 2, should it charge the threat price. This gives us the height of the barrier. What of its extent?

Since advertising is costly, Firm 1 will wish to keep the length of the barrier at the minimum level necessary for it to be effective.

The forces governing the shape of the barrier are now known, and we may proceed with more formality to its construction.

2.4.3. The setting

As before, a unit mass of consumers is uniformly distributed along a line of unit length. The advertising firm, Firm 1, is located at address 0. Its rival, Firm 2, is at address 1. Consumers have Hotelling preferences for each good, as given earlier.

$$U_r^1(r) = V - tr - p_1$$

$$U_r^2(r) = V - t(1-r) - p_1$$

Each consumer buys one of either Firm 1's good, Firm 2's good, or an outside good providing a utility of zero.

Definition: *Monopoly profits at r*

Monopoly profits for Firm at r are defined as $\pi_1^M(r) \equiv (V - tr)r$ for Firm 1, and $\pi_2^M(r) \equiv (V - t(1-r))(1-r)$ for Firm 2. They are equal to the profits obtained by a non-advertising monopolist bound to a marginal consumer[8] at address r, who it leaves indifferent to the outside good.

Lemma 2.4: $\pi_i^M(r)$ *is equal to the maximum no-advertising duopoly profit with an indifference point at r.*

Proof: Given a marginal consumer at r (and assuming the market is covered), the consumer at r is indifferent between the goods provided by firm 1 and firm 2:

$$U_r^1(r) = U_r^2(r)$$

Expanding the left-hand side, we have

$$V - tr - p_1 = U_r^2(r)$$

$$p_1 = V - tr - U_r^2(r)$$

[8] By marginal consumer, I mean the consumer whose address marks the boundary of a given firm's market share. The marginal consumer is indifferent between the good of the firm in question and the next-best alternative.

giving us a duopoly profit for firm 1 of

$$\pi_1^D(r) = p_1 r = \left(V - tr - U_r^2(r)\right)r$$

But, given the existence of the outside good,

$$\min U_r^2(r) = 0$$

and so

$$\max \pi_1^D(r) = (V - tr)r = \pi_1^M(r)$$

By symmetry, the same is true for Firm 2.

2.4.4. Construction of the barrier

The firms play a two-stage game. In the first stage, Firm 1 sets up the barrier. In the second, the two firms set prices simultaneously.

For the purposes of stage 2, the barrier is an interval [a,b], with a<b<1, which's consumers are not available to Firm 2. The length of the barrier is defined as $k \equiv b - a$.

We begin with stage 2, and proceed by backwards induction.

Proposition 2.2: *Given* $\frac{1}{3}\frac{V}{t} < b < \frac{1}{2}\frac{V}{t}$[9].

[9] The right-hand-side restriction merely says that b is less than that address which yields maximum monopoly profits. While intuitively plausible and supported by the results of numerical simulations, it remains for the moment a conjecture and assumption, rather than a proven result.

$$p_1 = V - tb$$
$$p_2 = V - t(1-b)$$

is a Nash equilibrium of the stage 2 game.

Proof:

Part 1: Firm 1's best response to $p_2 = V - t(1-b)$ *is to set* $p_1 = V - tb$

Suppose $p_2 = V - t(1-b)$. Then, all consumers on the interval [0,b) prefer the outside good to good 1. It is easily seen that

$$\frac{d}{dr}\pi_1^M(r) = V - 2tr > 0 \quad \forall r < \frac{1}{2}\frac{V}{t}$$

Combining this with Lemma 2.4, we find that Firm 1 will prefer to set $p_1 = V - tb$ and earn $\pi_1^M(b)$ to setting any higher price (and hence lower address of indifferent consumer).

What of lower prices, and indifferent consumers on the interval (b,1]? Given $p_2 = V - t(1-b)$, the indifference point between Firms 1 and 2 for $p_1 < V - tb$ is defined by

$$V - t(b+\varepsilon) - p_1 = V - t(1-(b+\varepsilon)) - (V - t(1-b)) \quad 0 < \varepsilon < 1-b$$

This yields

$$p_1 = V - tb - 2t\varepsilon$$

and the profits to firm 1 when extending past b are given by

$$\pi_1^D(b+\varepsilon) = p_1(b+\varepsilon) = (V - tb - 2t\varepsilon)(b+\varepsilon)$$

The loss from extending past b is then given by

$$L(b,\varepsilon) \equiv \pi_1^M(b) - \pi_1^D(b+\varepsilon) = \varepsilon(2t\varepsilon - V + 3tb)$$

$$L(b,\varepsilon) > 0 \ \ \forall b > \frac{1}{3}\frac{V}{t}, \varepsilon > 0$$

and since

$$\frac{\partial L}{\partial \varepsilon} = 3tb + 4t\varepsilon - V > 0 \quad \forall b > \frac{1}{3}\frac{V}{t}, \varepsilon > 0$$

Firm 1 will always prefer charging setting $p_1 = V - tb$ and earning $\pi_1^M(b)$ to all other outcomes, given $p_2 = V - t(1-b)$.

Part 2: Firm 2's best response to $p_1 = V - tb$ *is* $p_2 = V - t(1-b)$.

When $p_1 = V - tb$, all consumers to the right of b prefer the outside good to Firm 1's good.

By construction (through the definition of the purpose of the barrier), when $p_1 = V - tb$, firm 2 will prefer to set $p_2 = V - t(1-b)$ to charging any lower price.

Note that

$\pi_2^M(b) < \max \pi_2^M(r),\ 0 < r < 1$

$b > \arg\max \pi_2^M(r),\ 0 < r < 1$

If this were not the case, there would be no need for the barrier, as Firm 2 would automatically accommodate and not be tempted to extend into Firm 1's chosen turf.

It is easily found that

$$r^* \equiv \arg\max \pi_2^M(r) = 1 - \frac{1}{2}\frac{V}{t}$$

and

$$\frac{d}{dr}\pi_2^M(r) = 2t(1-r) - V < 0 \quad \forall r > r^*$$

Combined with Lemma 2.4, we see that firm 1 can earn no profit higher than $\pi_2^M(b)$ by charging a higher price (and obtaining an indifference point to the right of b).

When $p_1 = V - tb$, Firm 2 will not wish to set a price lower or higher than $p_2 = V - t(1-b)$, *q.e.d.*.

Stage 1: Construction of the barrier

As in previous sections, targeted persuasive advertising allows Firm 1 to pay c>1 to raise preferences for its good by a mass of 1[10].

The purpose of the barrier is to eliminate the incentive for Firm 2 to extend into Firm 1's chosen turf, the interval [0,b), given that firm 1 does not itself stray. More precisely, the length of the barrier has to be such that

[10] For instance, to raise preferences for good 1 by 1 on the interval [0.3,0.5] would cost cx(0.5-0.3)x1 = 0.2c.

$$\max_{b \le r < 1} \pi_2^M(r) \ge \max_{0 < r < b2} \pi_2^k(r)$$

where $\pi_2^k(r)$ are the profits obtained by Firm 2 given the existence of a barrier of length k, and that $p_1 = V - tb$.

If $p_1 = V - tb$, the indifference point x between firms 1 and 2 for an arbitrary p_2 is defined by

$$V - tx - (V - tb) = V - t(1 - x) - p_2$$

This gives us $p_2(b, x)$, allowing us to construct

$$\pi_2^k(b, k, x) \equiv p_2(b, x)(1 - x - k)$$

We can solve the first-order conditions for $x*(b, k)$[11], and substitute this back into $\pi_2^k(b, k, x)$ to obtain

$$\max \pi_2^k(b, k) = p_2(b, x*(b, k))(x - k)$$

Setting this equal to $\pi_2^M(b)$, we can solve for

$$k*(b) = \min_k \left| \{\max \pi_2^k(b, k) = \pi_2^M(b)\} \right.$$

[11] I assume that 0<r*<b-k. This will be true if 3-3b+2k<V/t<3-2k+b. Since generally k/b will be small (that is what makes the barrier effective), this may be loosely interpreted as restricting V/t to be less than 3 and greater than 3-3b. Note that for the benchmark Hotelling duopoly case to be valid, we require V/t>3/2. Numerical simulation shows that these constraints are seldom binding.

This is the optimal length for the barrier, insofar as it is the shortest, and therefore cheapest, length of consumers which must be removed from Firm 2's set of possible customers to entice it to stay on its own turf, when Firm 1 does the same.

The consumers in the blocked interval [b-k,b] must be targeted with persuasive advertising to such an extent that Firm 2 cannot profitably undercut its way past the barrier.

Define

$$\pi_2^D(b,x) \equiv p_2(b,x)(1-x)$$

where $p_2(b,x)$ is as above. This is the profit that Firm 2 can expect after successfully undercutting past the block, for $p_1 = V - tb$. The equation

$$\pi_2^D(b,x) = \pi_2^M(b)$$

has two solutions for x, one of which is x=b. The other solution is x_T, the threat point at which Firm 2 charges the lowest price it is willing to offer to overcome the barrier.

$$x_T = \left. \begin{matrix} \min \\ r \end{matrix} \right| \left\{ \pi_2^D(b,x) = \pi_2^M(b) \right\}$$

The corresponding price, which I will call the *threat price*, p_T, is given by

$$p_T(b) \equiv p_2(b, x_T(b))$$

Consumers in the barrier interval must be given a surplus, varying with their address r, of

$$V - t(1-r) - p_T(b)$$

Prior to advertising, for $p_1 = V - tb$ they receive

$$V - tr - (V - tb)$$

The amount to be 'topped up' by advertising is the difference between the two:

$$\tau(b,r) \equiv V - p_T b + t(2r - (1+b))$$

and the cost of the barrier is the cost of providing this top up over the length of the barrier interval:

$$\vartheta(b) = \int_{b-k(b)}^{b} \tau(b,r)dr$$

Firm 1's problem then becomes one of finding the b to maximize

$$\pi_1(b) = (V - tb)b - \vartheta(b)$$

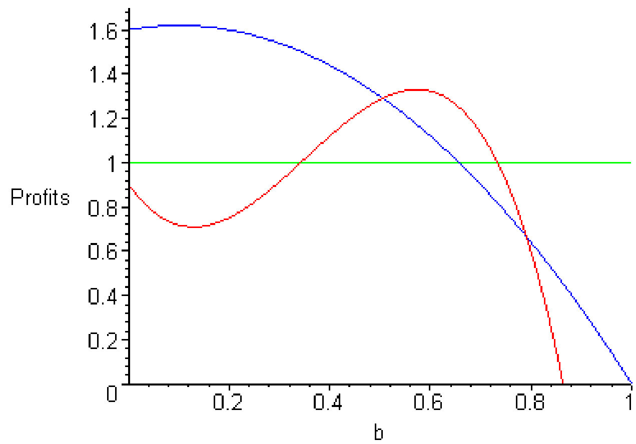
This is easily solved numerically. An example follows, with all results given to two significant figures.

2.4.5. Numerical example

Suppose V=1.8, t=1, and c=2.5. The optimal b is then 0.57. This is within the restrictions mentioned above. Firm 2's threat point is 0.31, which is safely between 0 and b-k, since k=0.03.

Figure 2.9: Results of the numerical simulation

Barrier	Firm 1	Firm 2
Profits	0.67	0.59
Price	1.2	1.4
Market share	0.57	0.43
Benchmark	Firm 1	Firm 2
Profits	0.50	0.50
Price	1.0	1.0
Market share	0.50	0.50

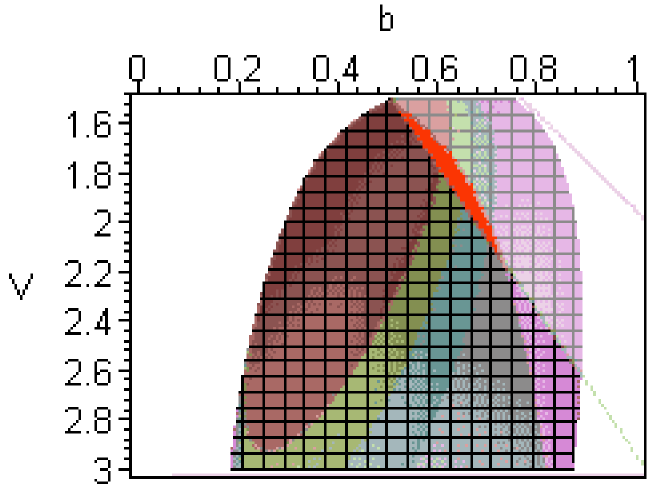


The diagram on the right shows profits for the example as a function of b, in units of benchmark (Hotelling duopoly) profits. The green line is a visual aid for determining market shares at which advertising is profitable. The red line shows profits for Firm 1, and the blue line for Firm 2.

Compared to the benchmark, both firms benefit, despite Firm 2's losing market share. The reason that Firm 2 sees its profits increase from the benchmark case is that given the barrier, it is free to monopolize what's left of the consumer line after Firm 1 has staked out its turf (which it also monopolizes).

The barrier is rather inexpensive – its costs of construction are only 5.6% of Firm 1's profits, and it extends through 5.3% of Firm 1's turf.

Figure 2.10: Regions compatible with a barrier (simulation)



The figure above shows, given t=1, the range of b's and V's for which Firm 1's profits after the barrier are greater than the benchmark duopoly profits, for c=1 to 8. The cost parameter is decreased through five equal intervals in chromatic order, with red being 8 and purple being 1. As costs increase, the viable area shrinks to a subset of its predecessor. The darker areas are those in which Firm 2's profits also increase. The lighter areas are those in which b lies between V/3t and V/2t. Note that the upper constraint never binds. The bright red segment shows the intersection of these last two regions, where b is within the assumed interval, and Firm 2's profits increase from the benchmark case. Though this is a small region, Firm 2 will never be entirely driven out of business through advertising, or indeed have less than a 10% market share.

The graphs below give some idea of the magnitudes involved. Again, the height of the graph shows profits in units of Hotelling duopoly profits. Costs have been fixed at 2.5, and t=1.

Figure 2.11: Firm 1 profits (simulation)

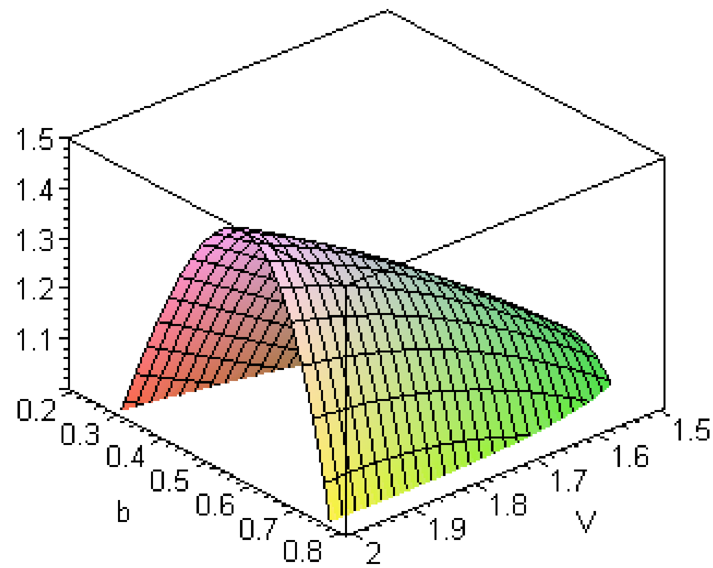
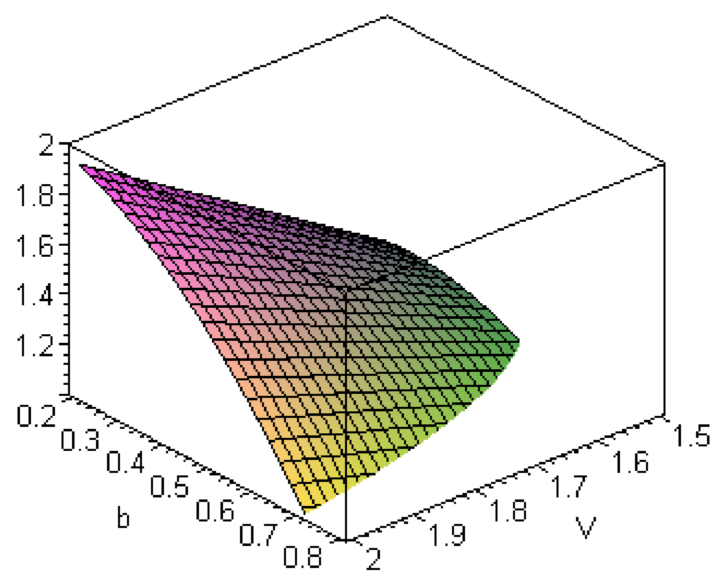


Figure 2.12: Firm 2 profits (simulation)



Back to our example. What happens to surplus?

Table 2.1: Consumer and producer surplus (simulation)

	Consumers' surplus	Producers' surplus	Total surplus
Barrier	0.29	1.2	1.49
Hotelling duopoly	0.55	1.0	1.55
Hotelling monopoly	0.41	0.81	1.22

For our test case, creating a barrier through targeted advertising is even worse for Consumer Surplus than a Hotelling Monopoly – this is not entirely surprising, given the double monopolization. Total surplus is somewhat higher than that in a Hotelling monopoly, but lower than a Hotelling duopoly's. Numerical simulations suggest these relationships are quite general.

2.5. Conclusion

In the preceding sections, we have seen that when given the ability to mould consumer preferences, a monopolist will tend to equalize the willingness to pay of its least valuable customers. The addition of a rival selling a differentiated product makes this more difficult. In this case, a firm using targeted advertising will set up a barrier dividing the consumer line into two neat segments. The advertising firm will monopolize the choice cut, and its rival will monopolize the rest. Its rival may even, under plausible conditions, benefit from the other firm's advertising campaign, relative to the case of an ad-less duopoly. This outcome will be worse for consumers even than a true monopolist (sans advertising). There is cause, then, to believe that there can be too much information. Privacy advocates are justified in fearing that personal data and purchasing histories gathered by firms may be used to their detriment.

It is interesting that in the duopoly case analyzed, the advertising used may seem to observers to be of an entirely different type. Since the outcome predicts that marginal consumers will be bombarded with advertising while others receive less or none, it may look like a campaign to convince people to switch. Instead, it is better interpreted as a signal of intent, and commitment to stay on one side of the boundary thus marked, and let its rival take the rest. This is seen in the real world in Apple Computer's popular 'switch' campaign. Apple Computer holds 2 to 3% of the market for computer sales (Fried 2003). Its 'Switch' advertising campaign consists of intensive, often-emotional ads targeted towards those people who are (barely) in the PC camp. The campaign consists of vignettes about people who have switched to an Apple computer and are happy because of it. Targeting of the ads can take place not only by choosing where these ads are placed (as banners on web sites devoted to Apple vs. PC comparisons, for instance) but by tailoring the stars of the stories to appeal specifically to the desired demographic. Despite its small market share, Apple computers are rather more expensive than their PC counterparts. All this is consistent with and explained by barrier advertising, which predicting heavy advertising to the marginal consumers and a monopolization of the captive customer base. The reason that the other 98% of computer buyers aren't similarly gouged is that by comparison with the makers of the Macintosh, PC manufacturers are a competitive fringe.

Figure 2.13: Copy from Apple's 'switch' campaign

"Yes, I'm a PC guy that switched to Mac."

- Aaron Adams, Windows LAN administrator

Source: http://www.apple.com/switch/

The model is also useful as an instance of what may happen when a bricks-and-mortar retailer, that is, one without an Internet presence, competes with a rival who only operates online. The online retailer is better-placed to have precise information on its clients, since it can examine and record every aspect of their visits and choices (and beyond, with spyware). The offline store will have more difficulty in retrieving this information, if it may do so at all. Thus, the online store will be able to target its advertising directly, whereas its bricks-and-mortar counterpart will have to make do with no ads, mass advertising, or perhaps crude targeting (ads appealing to large groups such as teens, mothers, the elderly, etc.).

With some modifications, the story also applies to the motivating example referenced in the introduction, of a multinational company entering a new market, and using its accumulated knowledge of consumer behaviour to alter existing preferences in its favour. It is not clear that the local rivals of the multinational, even if (especially if?) they cannot afford a similar advertising technology, will fare poorly. They may even benefit with comparison to the no-advertising case, where the newcomer 'plays fair'.

Targeted advertising may be used as a tool to grant monopoly power not only to the firm implementing it, but to its rivals. What, then, should be done about it? Privacy regulations may help. The United States already has an opt-in system for telephone marketing, and it could be used as the blueprint for a system to deal with the release and use of all sorts of consumer information by firms. The problem with this is that consumers are likely to opt in to any such program - under barrier advertising, the heavily targeted consumers are very well treated, indeed. Another idea would be to ban direct marketing, but this, too, might fail. Barrier advertising is very robust to 'trembling hand' errors, and does not require precise information to improve the advertising firm's profits over those earned in its absence. (Though of course precise information is needed to reach the optimal point.) By following Apple's lead and advertising heavily to the desired boundary, a 'second-best' outcome may be reached - perhaps making the non-advertising firms worse off in the process.

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3. Costly targeted persuasive advertising[12]

3.1. Introduction

What type of message is an advertisement? Until recently the exemplar was a poster or informative billboard – a fact sheet that everyone in the community had an opportunity to look at. This has changed, thanks to advances in consumer research, changes in the nature of urban spaces and the increasing importance of the internet. Advertisers now know the traits of potential customers and possess the technology to contact them directly through telephone, postal service or electronic mail. When consumers can be reached individually, it is tempting to tailor the product message to the characteristics of the intended recipient.

This happens in both the physical and on-line worlds. 'Junk mail', or unsolicited commercial matter, sent to residents in a particular area code often appeals to the demographics of the region. Immigrant-heavy areas may receive messages in several languages, and rich homes might see more ads for luxury products. Online, electronic messages or 'e-mail' can potentially be customized to take advantage of personality traits of the addressee gleaned from a trail left during their internet wanderings.

The internet epitomizes the current state of consumer 'addressability' (Silk et al. (2001)). An individual's tastes and movements can be tracked continuously for as long as they remain online. It is possible for a firm to directly observe what a consumer is looking for, what companies she visited and where she eventually made her purchase[13] (AOL Citation). This information can then be used to create ads that are appealing to that

[12] A version of this chapter will be submitted for publication.

[13] See for example Orlowski (2006).

specific individual and place them in her path (Emerging citation). For example, a cat lover might see a feline mascot selling her a soft drink, while a pigeon fancier receives a more feathery advertisement. This differs from traditional mass advertising, in which the message is crafted to appeal to as many people as possible.

The target audience of advertisements is a concern even in the absence of detailed consumer information. Since the end of the Second World War, North American cities have lost their public areas to suburban migration (Satterthwaite, 2001). It is no longer possible to place a billboard at a prominent downtown corner and assume that most people in the city will see it. Network television, which came into being at roughly the same time as the suburbs, has become a replacement downtown of sorts. However, the multiplicity of television channels in North America since the introduction of cable – typically dozens in any given area – make it difficult to mass advertise to the extent that was possible in the 1930s. Those few broadcasts that do draw in a substantial fraction of the population, such as the Superbowl, can charge extraordinary premiums to advertisers. Most modern ads are perforce targeted, in the sense that any given advertising medium, such as a television program or magazine, is seen only by a small subset of the population.

How may a firm use this market segmentation to its advantage? A product message that caters to a particular consumer's likes and dislikes will be more effective than a generic advertisement, but also more costly. The more versions of an ad that must be crafted, the dearer it becomes. Moreover, it must be sent to the correct individual. As was written of direct mailing, if the "message does not reach the proper targets, it has little chance of being effective." (Bult and Wansbeek, 1995) The information gathering and processing required are in themselves expensive.

If there *is* an advantage to targeted advertising, this raises the possibility of an 'informational divide' between firms. Those with well-developed information and distribution networks will be able to engage in a kind of marketing unavailable to other companies. How does this affect non-advertisers?

The following paper examines the case of duopoly where one of the firms can advertise to individual consumers with heterogeneous tastes. We investigate which consumers will be targeted by the advertiser, and to what extent. Also of interest is the impact of this campaign on the profits of both firms, relative to the case of no advertising.

Advertising in our model is persuasive, in the sense that it raises a consumer's willingness to pay for a product, independently of the good's price or quality. This is meant to capture the potential for a product message to cater to an individual's tastes and preferences through color, style, choice of mascot and other elements not directly related to the good sold. For example, one televised tobacco ad from the early 1950s consists entirely of animated cigarettes square-dancing while a narrator extols the virtues of the brand in question (Madacy Entertainment, 2002). Another features a cartoon cigarette singing a love song to a shapely cigar. More recently, a Canadian telecommunications firm has promoted its cell phone plans with charismatic wildlife. The 'fun' aspect of ads can be used to create positive associations in the mind of the consumer between the product and potentially unrelated experiences. When consumer tastes are perfectly known, these may be chosen for maximum effect.

3.2. The approach

We wish to know how a firm may best customize its advertising, when customization is costly. The two aspects of customization are *targeting* – choosing a subset of consumers who will be exposed to advertising – and the *degree of persuasion* – the extent to which an ad appeals to an individual, increasing their willingness to pay for the featured product.

The benchmark is Hotelling's model of horizontal product competition (Hotelling, 1929). Using this model as a base has several advantages. It is well understood, and has seen previous use in the persuasive advertising literature (Bloch and Manceau, 1999), allowing for an easy comparison of results. The duopoly it describes allows for interaction between a 'have' advertiser and a non-advertising (but otherwise equal) 'have-not' rival.

Two firms sell a product at either end of a line of length one. Consumer heterogeneity is represented by identifying each consumer with an address on the unit line. This address is that of a consumer's ideal good, and an individual's distance from a firm indicates the disutility of buying a less-preferred variety.

The game is in two stages. In the first, the advertiser customizes and pays for its campaign, which is thereafter perfectly observable. It chooses the location of the campaign, how many consumers it will advertise to, and the degree to which the willingness to pay of the affected individuals is increased. Advertising is inefficient by design. The advertiser must pay more than a dollar to raise a consumer's willingness to pay, by one dollar. This represents the costs of information and customization needed to appeal to a particular individual.

In the second stage, both firms set prices simultaneously, taking the campaign as given. Consumers make their purchase decision after both prices are revealed, and they have been affected by advertising.

The results are surprising. Under general conditions, targeted persuasive advertising can be used as a way to divide the consumer line between the two firms. The targeted segment forms a boundary. Each firm acts as a monopolist over the consumers on its side of the campaign, so that the consumers at the edges of the targeted segment receive no surplus from their purchase. Despite the advertising firm's high price, its rival cannot profitably deviate from this outcome. The consumers on the campaign have been heavily advertised to, so that it is not worthwhile for the non-advertiser to set a price low enough to capture them. This 'barrier' of customers with a very high willingness to pay weakens price competition in the second stage, when it is taken as a given. Put differently, in the first stage, the advertiser may pay to induce this particular Nash equilibrium in the second stage.

Generally, in such a case, the *non-advertiser* will profit more than the advertising firm. This is because it receives the benefit of lessened competition without having to pay for the campaign. Both firms do better than in the benchmark Hotelling case.

For advertising sufficiently expensive, we will not see this 'double monopoly'. The required campaign will be too costly. Instead, we will see small amounts of advertising that lead to an outcome very similar to that in the original Hotelling model. The differences lie in a slight increase in price and decrease in market share for the advertiser, and fall in price and rise in market share for its rival. The changes in price dominate the changes in market share, and the non-advertiser is unambiguously worse off than in the benchmark case of no advertising. This 'Faux Hotelling' outcome will *always* be profitable for the advertiser, regardless of advertising cost. This is a function of the infinite divisibility of the consumer line as presented in the model. No matter how high the advertising cost parameter rises, a sufficiently small segment of consumers can be found that will make it profitable.

Advertising also depends on the degree of product differentiation. High product differentiation makes advertising more effective. When differentiation is low, and consumers do not much mind buying a good very different from their preferred variety, a campaign is less effective. It is too easy for the non-advertiser to undercut the effect of advertising by charging a lower price. At high levels of differentiation, we will see the split in the consumer line mentioned above.

3.3. Related research

Traditionally, economists have treated advertising as an *informative* message. Its purpose is to inform consumers of the existence of a product, its price and characteristics. In this vein, Grossman and Shapiro (1984) examined the impact of advertising cost on differentiated product competition, and Schmalensee (1978) analyzed the role of advertising in an economy where product quality was uncertain. The current paper differs in that consumers are fully informed of product qualities and price. The role of advertising is to increase an individual's willingness to pay for a product through the use of a customized message.

This falls into the category of persuasive advertising. There is a small literature on this subject, which treats advertising as a message that changes a consumer's perception of a product in a way not necessarily related to the good's characteristics. Von der Fehr and Stevik (1998) provide a useful summary of the various types of persuasive advertising. Bloch and Manceau (1999) examine persuasive advertising in the Hotelling model, but their ads are not targeted, and shift the entire distribution of consumers toward the advertised firm. The approach of Koh and Leung (1992) is closer to that in the current paper. They examine the case of horizontal competition where both firms can advertise, and do not allow targeting. Results vary with the specification of advertising.

Most work on targeted advertising belongs to the marketing literature and deals with direct mail campaigns, e.g. Bult and Wansbeek (1995). An exception is the work of Esteban et al. (2001). They examine a monopolist's use of targeted advertising and find that it will be lower than under mass advertising, with adverse welfare effects.

Stigler and Becker (1977) argue that "tastes neither change capriciously nor differ importantly between people". This view is espoused by Johnson and Myatt (2006), who do, however, allow advertising to reveal that a good is particularly compatible with existing preferences. For example, an ad may reveal to a consumer that a car is 'sporty'. This type of information is called 'hype' by Johnson and Myatt, as opposed to more mundane details on price and objective quality, which are called 'real information'. Advertising in the current model may be interpreted as a firm 'hyping' a product to an individual consumer, with the particular characteristics (sporty, classic, elegant) revealed depending on the consumer's tastes. This is also consistent with Anderson and Renault (2006), who find an advertiser may not wish to reveal all product features in their messages, but rather 'just enough' to overcome a consumer's reservation price.

The present model shares similarities with those used in the customized pricing literature, and in particular with Chen and Iyer (2002). They examine targeted price discrimination by duopolists in a model of horizontal competition, and find that equilibrium profits generally increase with product differentiation, except when targeting is very costly, when they fall. Schaeffer and Zhang (1995) look at the targeting of discount coupons by duopolists in a model of spatial competition, and conclude that the equilibrium is a costly prisoner's dilemma. Shor and Oliver (2006) study the use of online coupons, finding that arbitrage in the form of coupon repositories and imperfect targeting (ads that annoy viewers) can offset gains from market segmentation. Neither of these are problematic for targeted persuasive advertising, as the increase in willingness to pay is tied to a specific consumer.

3.4. The model

Two firms exist on either end of a line of length 1, in a Hotelling (1929) setup. Firm 0 is at address 0 and Firm 1 is at address 1. A unit mass of consumers is uniformly distributed along the line. Each firm sells the same good, but consumers prefer to buy from the firm closest to them. A given consumer will buy one unit from either Firm 0, Firm 1, or an outside firm providing zero utility[14]. The two firms set prices simultaneously, after which each consumer makes her purchase decision.

The game takes place in two stages. In Stage 1, Firm 1 chooses an advertising campaign (defined below) and pays for it. In Stage 2, this advertising is observed by both firms, who then simultaneously set their prices. After this, consumers observe prices and purchase from the supplier providing them with highest utility.

[14] A real-world example of the setup of the model exists in the retail market for flour in 1950s North America. Consumers have a choice between heavily-advertised Robin Hood flour (Firm 1), Red Seal flour, which while not advertised is sold in its own distinctive packaging (Firm 0), and flour from the bulk bin (the outside supplier). In this case, the 'distance' is psychological, not physical, since all three flours are available in the same supermarket – and are functionally identical.

3.4.1. Stage 2

Preliminaries

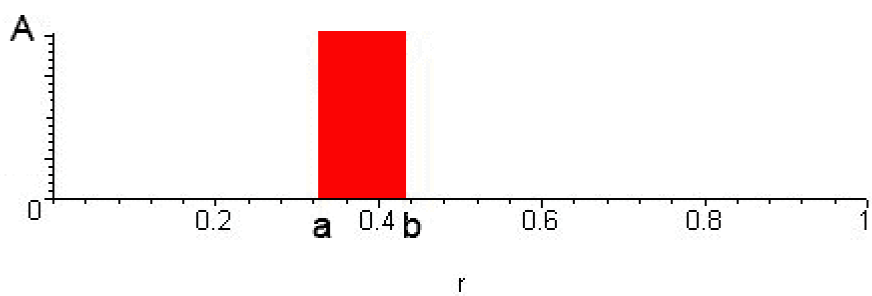
We first examine Stage 2, holding advertising fixed and searching for equilibria to the pricing game. An equilibrium in prices is a set of prices (p_0,p_1) such that, for a given advertising campaign, neither firm has an incentive to deviate.

The advertising campaign may be thought of as a slice of the unit line on which consumers have a strong preference for Firm 1's good. It will be shown that under certain conditions there exists a unique Nash equilibrium in prices such that each firm's market share is equal to the consumers on its side of the campaign.

Before beginning the analysis, it will be useful to establish a few basic definitions.

Definition 3.1: An *advertising campaign* consists of a sub-interval of the consumer line $[a,b]$, $0 \leq a \leq b \leq 1$, and a height A, $A \geq 0$. The length of the campaign is labelled k, $k \equiv b\text{-}a$.

Figure 3.1: An advertising campaign



The advertising campaign is fixed and observable at the beginning of Stage 2, and affects consumers in the same way as a decrease in price.

Definition 3.2: The *utility* of the consumer at address r from purchasing from Supplier j at price p_j is labelled $U_j(r,p_j)$.

$$U_0(r, p_0) = V - tr - p_0$$

$$U_1(r, p_1) = \begin{matrix} V - t(1 - r - p_1) + A & r \in [a, b] \\ V - t(1 - r - p_1) & otherwise \end{matrix}$$

Consumers like advertising, dislike high prices and prefer to buy from a Firm located 'near' them. The concept of location may be interpreted literally – as in the case of two corner stores at either end of a block, one of which sends personalized fliers to houses on a particular segment of the street. Distance may also be seen as a psychological penalty from consuming a variety different to the consumer's ideal choice. A consumer who prefers soggy cereal is in this sense distant from a firm that produces crunchy cereal.

Following Hotelling (1939),V and t are constants, representing an intrinsic value of the good and a transport cost, respectively. The prices set by Firms 0 and 1 are p_0 and p_1, respectively.

It will be useful to have notation for the location of the marginal consumer – the consumer that, in the absence of a campaign, would define the Firm's market share.

Definition 3.3: The *indifference point* z_j is the address of the consumer indifferent between Firm j and her next-best supplier in the absence of advertising. More formally,

Let a=b, so that the support of the campaign is the empty set. Then

$U_j(z_j) \equiv \max(0, U_i(z_j))$

for j=0,1 and $i = |j - 1|$.

Previous models of differentiated product competition have used price as the choice variable. For reasons explored below[15], in the present case it is more convenient to phrase the problems in terms of market share.

[15] The firm's reaction function has a discontinuous derivative. The points of discontinuity have a simple interpretation in terms of the address of an indifferent consumer. This simplicity is hidden if the problem is solved in terms of prices.

Given the actions of its rival, a firm's reaction may equivalently be phrased in terms of price, or in terms of the location of the indifference point. This is important, because changes in the trade-off between price and market share are obscured in price space, but easily visible when the discussion is couched in terms of indifference points.

To see this, suppose that Firm 1's price, $p_1>0$, is given. For any positive value of p_1, there exists a portion of the consumer line that prefers Good 1 to the outside good. The remainder of consumers prefer the outside good to Good 1.

This is illustrated below, in which consumer utility from consumption of Good 1 is plotted. Consumers prefer Good 1 to the outside good if and only if they are to the right of the point labelled r_1, at which $U_1(r,p_1)=0$.

Figure 3.2: Consumer utility from good 1

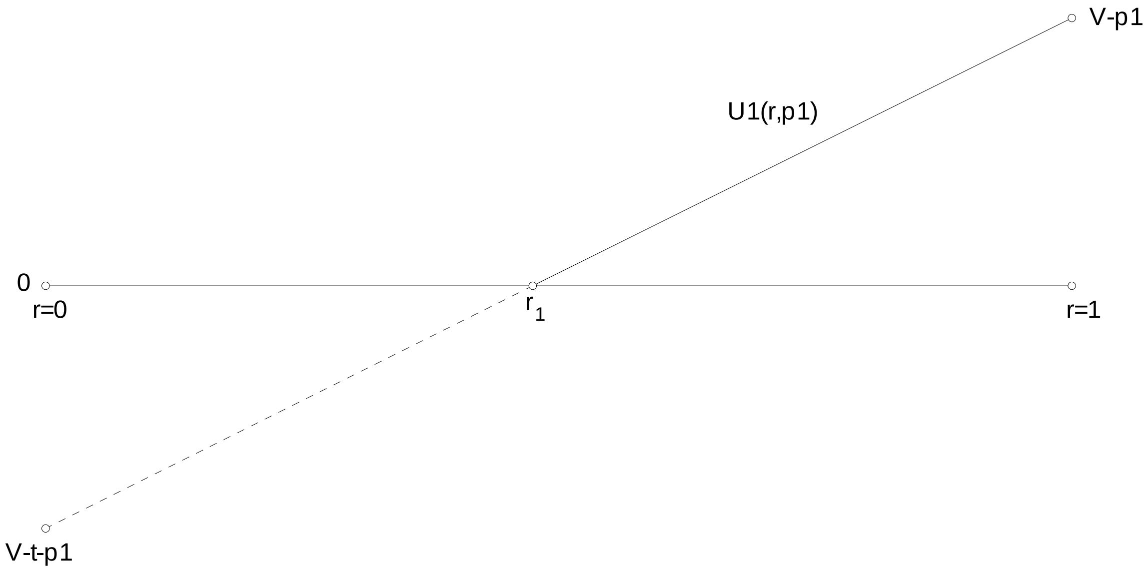


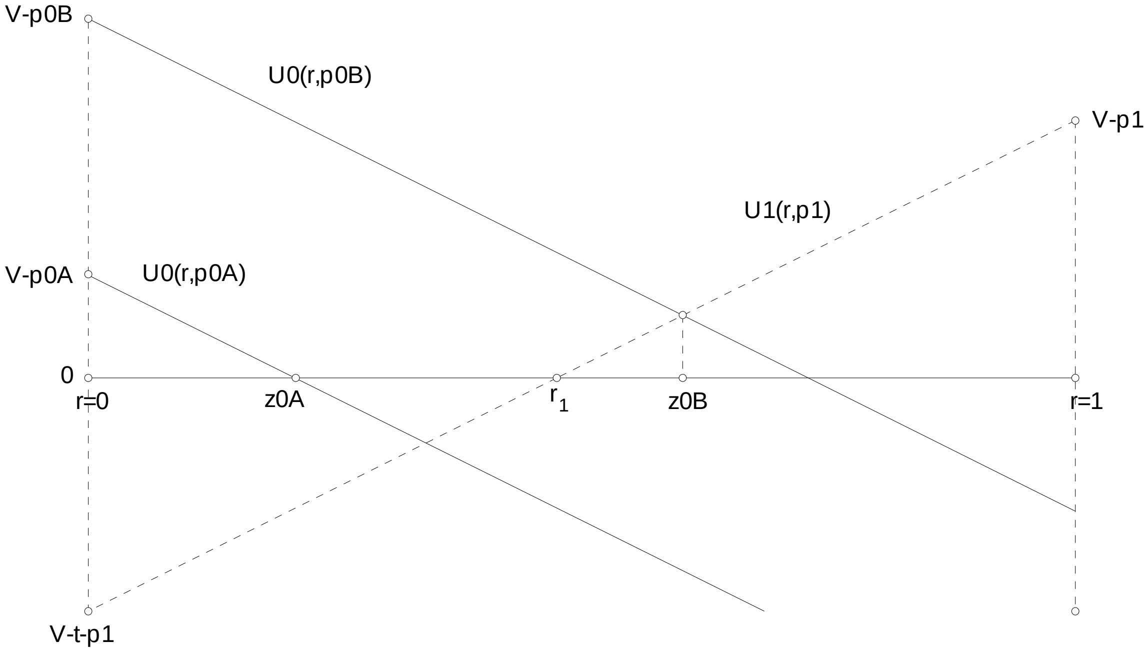
Figure 2: Utility of the consumer at address r from consumption of good 1 at price p_1.

Since a change in price is equivalent to a vertical translation of the line representing utility, each $r_1 \in (0,1)$ is associated with a unique value of p_1.

Now consider Firm 0's reaction to its rival's price, thus represented. Similarly to the above situation, there is a one-to-one correspondence between the address of the indifferent consumer and Firm 0's price.

If Firm 0's price is sufficiently high, such as the price p0A in the diagram below, then the marginal consumer of Good 0 – at z0A in the graph - is indifferent between Good 0 and the outside good. Firm 0 extracts maximum surplus, in a fashion similar to a monopolist who cannot price-discriminate. This happens for all prices such that the consumer at r_1 prefers the outside good to Good 0.

Figure 3.3: The effect of an increase in price on an indifference point



Now suppose that Firm 0's price is low enough that the consumer at r_1 prefers Good 0 to the outside good, as is the case when p_0=p0B in the diagram. Then the indifference point will be to the right of r_1. All consumers, and in particular the indifferent consumer, will obtain a positive surplus from consumption of their preferred good. The market will be covered.

Geometrically, when p_1 is given we are holding the line representing consumer utility from consumption of good 1, fixed. A change in p_0 is equivalent to a vertical translation of the $U_0(r,p_0)$ line. The higher p_0, the lower the line. Straight lines which are not parallel cross once. Thus, for a particular value of p_0 and given p_1, there is a unique point of intersection between $U_0(r,p_0)$ and $U1(r,p_1)$. It is also unique in the sense that a different value of p_0 (for a given p_1) will lead to a different intersection. A higher p_0 leads to an intersection closer to r=0, and a lower p_0 one closer to r=1. Thus, given p_1, there is a 1:1 mapping between p_0 and address at which $U0(r,p_0)=U1(r,p_1)$.

The economic intuition is that given its rival's price, a firm's market share will fall as its own price rises. Since the rival's price is held fixed, any rise in own price will lead to a loss in customers, and so there is a unique mapping between price and market share.

The same intuition applies when the relevant competitor is the provider of the outside good, the only difference being that this good provides zero utility to every consumer.

All together, as long as p_1 (or r_1) is taken as given, we can express Firm 0's choices in terms of either prices, or indifference point. Each indifference point will be linked to a unique value of p_0. The presence of an advertising campaign does not change this, because the indifference point is defined as Firm 0's market share for a set of prices (p_0,p_1), in the *absence* of a campaign.

This representation clarifies changes in the trade-off between price and market share. Let z_0 be the indifference point for Firm 0. When $z_0<r_1$ (as when p_0=p0A above), Firm 0 extracts the entire surplus of the marginal consumer, because it need only match the surplus granted by the outside good. If Firm 0 were to choose a $z_0>r_1$ (say, by setting p_0=p0B), then it would have to provide the consumer at that z_0 a positive utility, to match that she would earn from consumption of Good 1. If z_0 happens to be interior to the advertising campaign, then Firm 0's actual market share will be lower than z_0. This is explained in detail below.

The correspondence between price and indifference point can be made explicit. Let Firm i have an indifference point $z_i \in [0,1]$. By definition, the utility of the consumer at z_i from Good i must be equal to that of the next-best supplier. If Firm i's rival is Firm j, then we must have, at z_i,

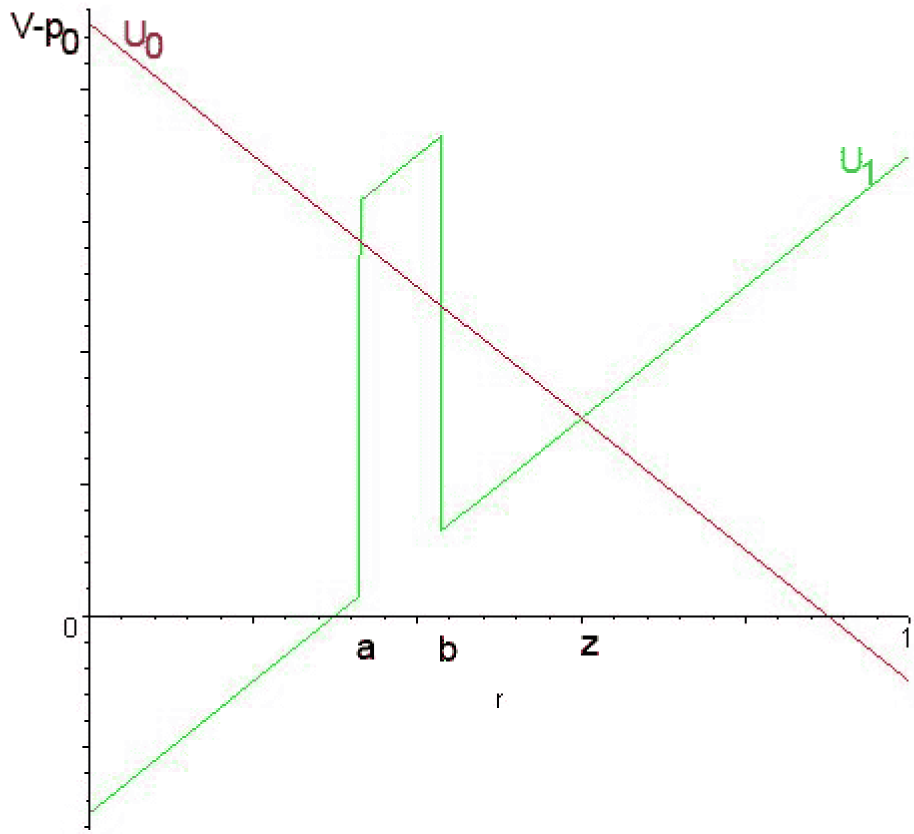
$$U_i(z_i,p_i)=\max(U_j(z_i,p_j),0)$$

This expression can be solved for $p_i(z_i,p_j)$.

Definition 3.4: Let p_i be given. Firm j is said to be *pricing at z_j* when $p_j=p_j(z_j,p_i)$, where $p_j(z_j,p_i)$ is the price by Firm j that leads to an indifference point z_j when its rival prices at p_i. In particular,

$$p_0(z_0,p_1) = \min(p_1 + t(1 - 2z_0), V - tz_0)$$
$$p_1(z_1,p_0) = \min(p_0 - t(1 - 2z_1), V - t(1-z_1))$$

Figure 3.4: Hotelling utilities, the indifference point and the barrier for arbitrary parameter values, finite A and $p_0<p_1$. Note that $z_0=z_1$.



The profit function

Let the advertising campaign be fixed, and the price of Firm i's good be given. Furthermore, assume that advertising intensity A is high enough that it is not worthwhile for Firm 0 to try to capture consumers along the campaign. That is, assume that advertising has made Good 1 very desirable to the affected customers. Firm 0 cannot price-discriminate, and to obtain the custom of these consumers, it would have to lower its price to an undesirable extent.

This advertising introduces a discontinuity in the trade-off between price and market share. As a result, although continuous, Firm j's profit function, $\pi_j(z_j, p_i)$, has a discontinuous derivative. (The exact expressions are shown in a table below.)

For example, when $z_0<a$, by raising z_0 (hence lowering its price) Firm 0 will increase its market share. However, when $a<z_0<b$, Firm 0 will gain no additional market share by slightly raising z_0, since all the consumers on [a,b] will buy from Firm 1 if the advertising campaign is effective.

A second discontinuity is afforded by the address of the consumer indifferent between the outside supplier, and Firm i. This point divides the consumer line into two segments – on one, $U_i<0$ and Firm j provides zero utility to the marginal consumer, as a monopolist would. On the second, $U_i>0$, and Firm j must provide positive utility to the marginal consumer.

The possible locations of the three points of discontinuity – a,b and $r|U_i(r,p_i)=0$ – divide the consumer line into five possible regimes, listed below.

Definition 3.5: A *regime* for Firm j is the largest interval along which the derivative (with respect to z_j) of Firm j's profit function, given its rival's price, adopts a particular, continuous form.

Given the price of its rival, the profit function for Firm j in each regime is concave and quadratic[16]. In some cases, the maximand of the function depends on k, the length of the barrier.

This piecewise nature of the profit function makes it important to have explicit notation for values restricted to a particular regime or subset of the unit line.

Notation

Notation takes the general form ${}_f^g X_j^{R**}$. The expression under consideration is the X of Firm j. The range of z_j is restricted to [f,g] and the relevant regime is R. One asterisk means that it is an optimal value for Firm j, given the actions of its rival. Two asterisks mean that this is a Nash equilibrium value.

For example, ${}_0^a\pi_1^*$ are maximum profits for Firm 1, given p_0, when $z_1 \in [0,a]$.

Where no range is specified, as in π_1^*, a global value (subject to the assumptions of the model) is implied.

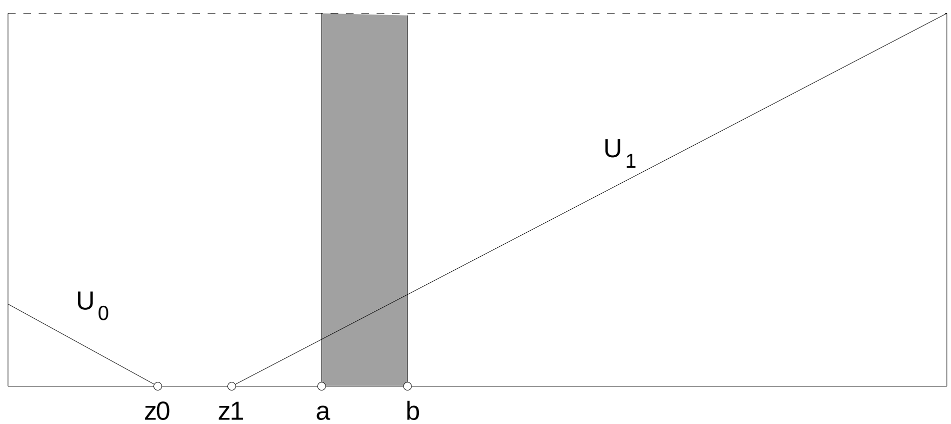
It will also be helpful to have notation that allows us to refer to a specific regime.

The table below lists all possible regimes and their associated profit functions. The abbreviations listed (D,DB,B,M,MB) will be used for the rest of the paper. The profit functions are written under the assumption that consumers on [a,b] prefer Good 1 to Good 0. This assumption will be justified and explored in detail later in the paper.

[16] This is proven below, in Lemma 1.

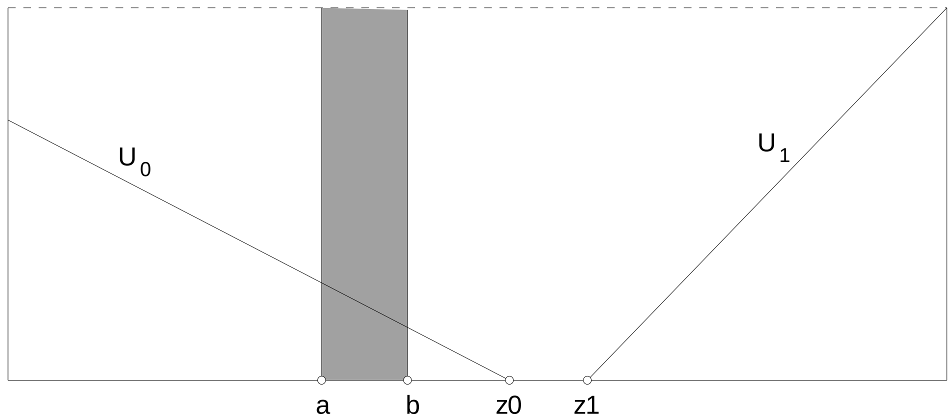
The first column, 'support', shows the condition under which $\pi_j(z_j, p_i)$ will take the specified form. For example, when $z_j \leq a$ and $U_j(z_j, p_j) = 0$, we are in the Monopoly (M) regime. In this regime, the advertising campaign is irrelevant, and the marginal consumer receives zero utility from consuming her preferred good. Given these conditions, the consumer at z_j obtains negative utility from Firm i's good, and so Firm j need only match the utility provided by the outside good. The advertising campaign does not enter into Firm 0's profit function in this regime, because all consumers on $[0,z_0]$ prefer Firm 0 to Firm 1 when $z_0<a$. It does not enter Firm 1's profit function because if $z_1<a$, then the consumers on [a,b] would prefer Firm 1 to Firm 0 even in the absence of advertising.

Figure 3.5: An example of a (z_0,z_1) pair in the M regime.



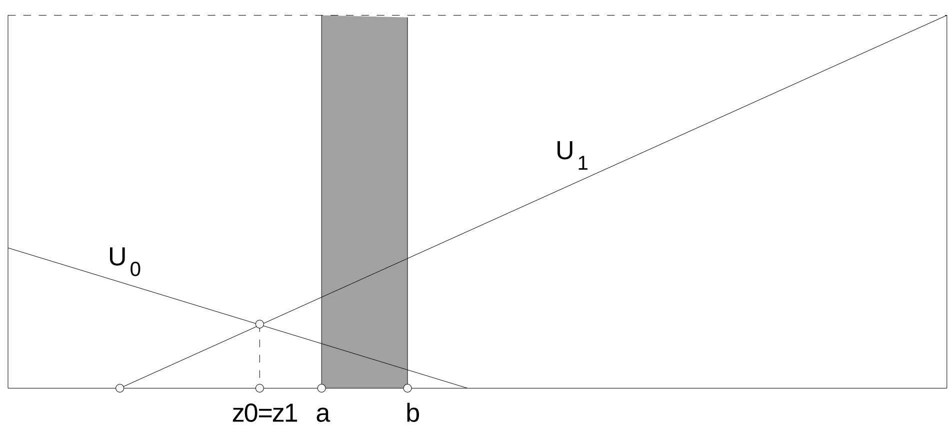
The other regimes are as follows. The Monopoly, Barrier (MB) regime is similar to the monopoly regime in that the marginal consumer receives zero utility. However, $z_j \geq b$. Consider Firm 0. In the absence of advertising, its market share would be z_0. and all consumers on $[0,z_0]$ would buy Good 0. Since $z_0 \geq b$, [a,b] is interior to $[0,z_0]$, and so Firm 0's market share is only z_0-k. The consumers on the campaign prefer to buy from Firm 1, and the campaign is of length k. Similarly, in the absence of advertising and under these conditions, Firm 1's market share would only be $(1-z_1)$. Given enough advertising (a high enough A), consumers on [a,b] who would have bought from the rival firm or outside supplier at the given prices will buy from Firm 1, instead.

Figure 3.6: An example of a (z_0,z_1) pair in the MB regime.



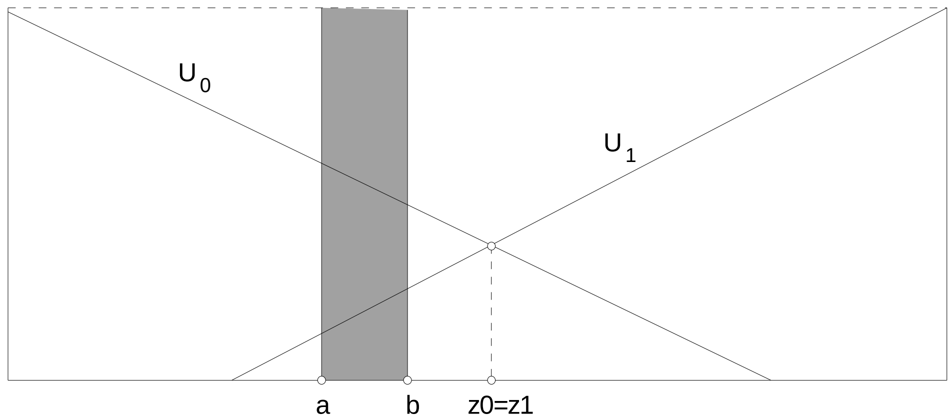
The Duopoly (D) regime is that of the standard Hotelling model. Advertising is irrelevant, and the marginal consumer receives positive utility from her preferred good.

Figure 3.7: An example of an indifference point in the D regime.



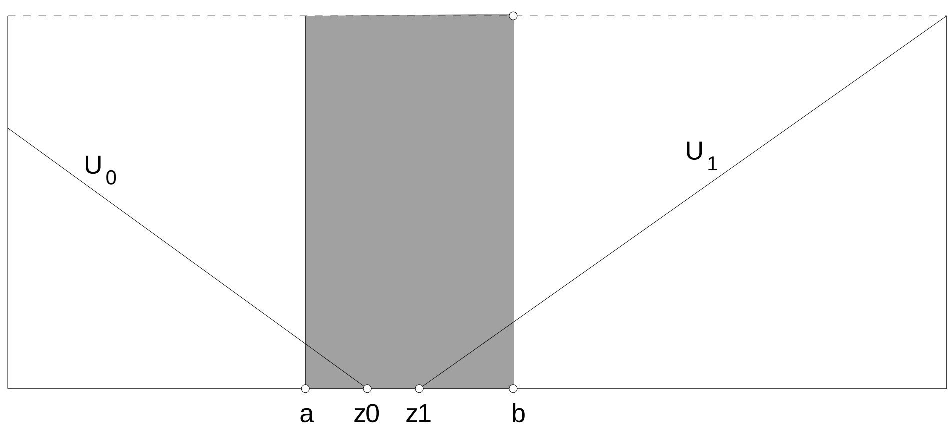
The Duopoly (DB) regime transfers the consumers along the advertising campaign from Firm 0 to Firm 1.

Figure 3.8: An example of an indifference point in the DB regime.



When z_j is internal to the advertising campaign, we are in the Barrier (B) regime. Each z_j in the Barrier regime corresponds to a different p_j, but to the same market share. For Firm 0, this will be a, and for Firm 1, this will be (1-a).

Figure 3.9: An example of a (z_0,z_1) pair in the B regime.



An example of what the complete profit function looks like is shown below.

Figure 3.10: Firm 0 profits and the corresponding regimes, for arbitrary p_1 and parameter values.

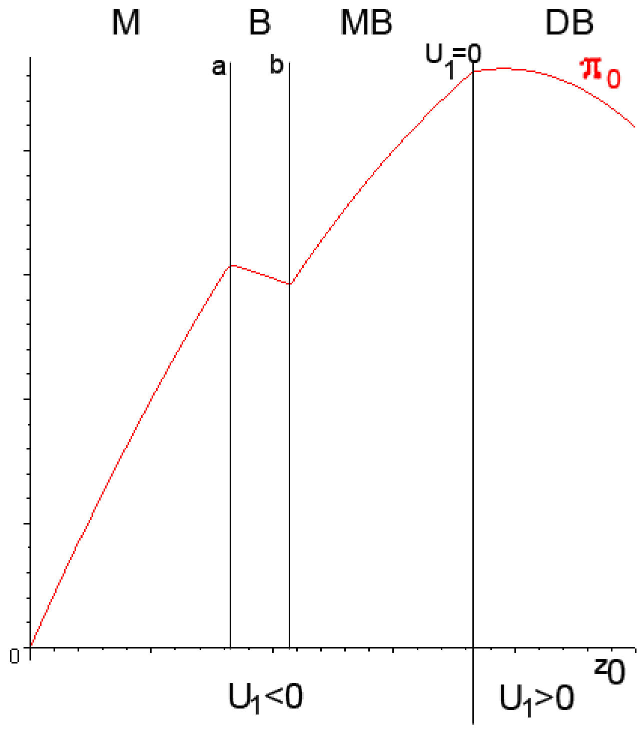


Table 3.1: Notation

Regime	Support	Profit dunctions[17]	Maximands[18]	Second derivative
D: Duopoly	$0 \le z_j \le a$ and $U_j(z_j, p_j) > 0$	$\pi_0^D(z_0, p_1) = p_0(z_0, p_1)z_0$ $\pi_1^D(z_1, p_0) = p_1(z_1, p_0)(1 - z_1)$	$z_0^{D*} = \frac{1}{4}\left(1 + \frac{p_1}{t}\right)$ $z_1^{D*} = \frac{1}{4}\left(3 - \frac{p_0}{t}\right)$ $x^{D**} = \frac{1}{2}$	$\frac{\partial^2 \pi_0^D}{\partial z_0^2} = -4t$ $\frac{\partial^2 \pi_1^D}{\partial z_1^2} = -4t$
DB: Duopoly, Barrier	$b \le z_j \le 1$ and $U_j(z_j, p_j) > 0$	$\pi_0^{DB}(z_0, p_1) = p_0(z_0, p_1)(z_0 - k)$ $\pi_1^{DB}(z_1, p_0) = p_1(z_1, p_0)(1 - z_1 + k)$	$z_0^{DB*} = \frac{1}{4}\left(\frac{p_1}{t} + 1 + 2k\right)$ $z_1^{DB*} = \frac{1}{4}\left(3 - \frac{p_0}{t} + 2k\right)$ $z^{DB**} = \frac{1}{2} + \frac{2}{3}k$	$\frac{\partial^2 \pi_0^{DB}}{\partial z_0^2} = -4t$ $\frac{\partial^2 \pi_1^{DB}}{\partial z_1^2} = -4t$
B: Barrier	$a \le z_j \le b$	$\pi_0^B(z_0, p_1) = p_0(z_0, p_1)a$ $\pi_1^B(z_1, p_0) = p_1(z_1, p_0)(1 - a)$	$z_0^{B*} = a$ $z_1^{B*} = b$	
M: Monopoly	$0 \le z_j \le a$ and $U_j(z_j, p_j) = 0$	$\pi_0^M(z_0) = p_0(z_0)z_0$ $\pi_1^M(z_1) = p_1(z_1)(1 - z_1)$	$z_0^{M*} = \frac{1}{2}\frac{V}{t}$ $z_1^{M*} = 1 - \frac{1}{2}\frac{V}{t}$	$\frac{\partial^2 \pi_0^M}{\partial z_0^2} = -2t$ $\frac{\partial^2 \pi_1^M}{\partial z_1^2} = -2t$
MB: Monopoly, Barrier[19]	$z_j \ge b$ and $U_j(z_j, p_j) = 0$	$\pi_0^{MB}(z_0) = p_0(z_0)(z_0 - k)$ $\pi_1^{MB}(z_1) = p_1(z_1)(1 - z_1 + k)$	$z_0^{MB*} = \frac{1}{2}\left(\frac{V}{t} + k\right)$ $z_1^{MB*} = \frac{1}{2}\left(2 - \frac{V}{t} + k\right)$	$\frac{\partial^2 \pi_0^{MB}}{\partial z_0^2} = -2t$ $\frac{\partial^2 \pi_1^{MB}}{\partial z_1^2} = -2t$

[17] Assuming consumers on [a,b] prefer Firm 1. See ‘Assumptions’, below.

[18] There is no guarantee that the maximands fall within the support of their regime. All are found via the usual first-order conditions, save those for the Barrier regime. These are derived in a lemma below.

[19] There is no Nash equilibrium in this regime or in M except when V/t=1.

The entire profit function for Firm j, regardless of regime, is denoted $\pi_j(z_j)$.

All but one of the individual regime profit functions is concave and quadratic in z_j, as proven below.

Lemma 3.1: Let p_0 be taken as given by Firm 1, and p_1 by taken as given by Firm 0. Then the regime-specific profit functions are quadratic and concave with respect to z_i, except for those corresponding to the Barrier regime, which are linear.

Proof:

Let p_j be given. The expressions for $p_i(z_i,p_j)$ are linear in z_i:

$$p_0(z_0,p_1) = \min(p_1 + t(1 - 2z_0),V - tz_0)$$
$$p_1(z_1,p_0) = \min(p_0 - t(1 - 2z_1),V - t(1-z_1))$$

By inspection, this makes the profit functions in all regimes save B a product of two linear functions, giving us a quadratic function in z_j. The second derivatives with respect to zj of the regime-specific profit functions are negative whenever t is positive. In the case of the B regime, we have a linear function, $p_i(z_i,p_j)$ multiplied by a constant (a, or (1-b)). This gives us a linear function, q.e.d.

■

We are interested in finding the Nash equilibrium to the Stage 2 pricing game, given a,k and A.

Definition 3.6: An equilibrium to the pricing game is a pair (z_0,z_1) such that for given (z,k,A) neither firm has an incentive to deviate. That is, $(z_0,z_1)=(z_0^*, z_1^*)$.

For the moment, a few assumptions are needed.

Assumption 3.1: *A monopolist covers the market*. This happens when $V/t \geq 2$.

Discussion: This is a strong assumption, made to ensure there is enough competition to make the problem interesting. The ratio V/t may be thought of as a measure of the importance of product differentiation. When V/t is low, transport costs and individual tastes are very important relative to the good's 'intrinsic value'. Thus, for very low V/t there will be little competition, the market will not be covered, and each firm will sell the good at a high price to the consumers nearest their location.

Assumption 3.2: A is sufficiently high that, it is not worthwhile for Firm 0 to capture consumers on [a,b].

Discussion: Recall that A is the advertising *intensity* – the amount by which the utility of an individual consumer, from consuming the advertiser's good, is raised. If advertising intensity is very low, then it is easy for the non-advertiser to undercut it by charging a slightly lower price than in the absence of the campaign. By making this assumption, we can focus on the interesting case where the campaign meaningfully changes the choices available to the non-advertiser[20]. The exact conditions under which this assumption will be true will be solved for in the analysis of Stage 1, the advertising game.

Assumption 3.3: Prices are non-negative and at least one consumer will, in the absence of advertising, (weakly) prefer Firm j to the outside supplier. $0 \leq p_j \leq V$

[20] If Assumption 3.2 is violated, the analysis of both stages is trivial. In Stage 1, Firm 1 pays for advertising. In Stage 2, we have one of two results. The first is the Hotelling outcome, which happens at very low values of A, such that the campaign is undercut at Hotelling prices. The end result is that Firm 0 earns Hotelling profits, and Firm 1 earns less than this, because of the cost of advertising. The second happens for slightly higher A and involves lower prices for both firms, as Firm 0 reacts to the campaign by a price that will undercut it. The Nash equilibrium also involves a lower price for Firm 1, as its best response to a low price by Firm 0. In neither case is positive advertising optimal.

Discussion: This is needed because of Assumption 3.2. It prevents nonsensical equilibria in the pricing game, such as Firm 1, the advertiser, charging an unreasonably high price to the consumers on [a,b]. When A is finite (as it must be when it is paid for by a profit-maximizing firm), Firm 1 cannot charge an arbitrarily high price to consumers on the campaign and hope to retain them. Because advertising is costly by design (it costs more than one dollar to raise a consumer's utility by one dollar), when this assumption is violated, Firm 1's profits for the game as a whole are negative.

Finally, it will be interesting to consider those cases in which each firm chooses a marginal consumer on its side of the campaign. This is an outcome similar to that when V/t is very low and there is no advertising.

Incentive constraints

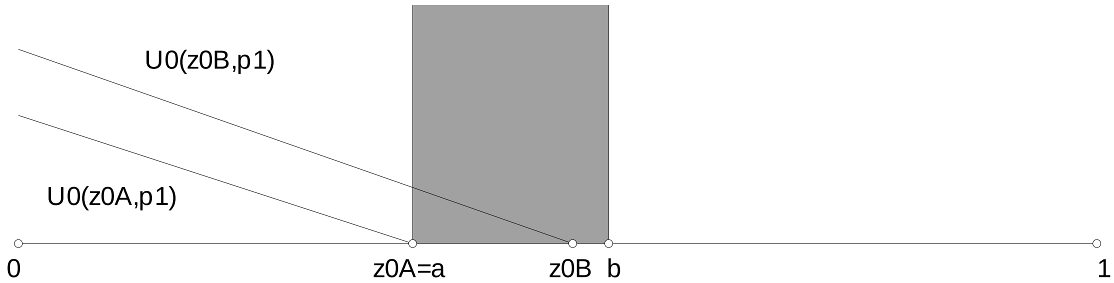
Firm j's incentive constraint, ICJ, is the requirement that it earn more profits on its side of the campaign support than by straying beyond it.

$$\text{IC0: } {}_0^a\pi_0^* \geq {}_b^1\pi_0^* \iff z_0^* \in [0, a]$$

$$\text{IC1: } {}_0^a\pi_1^* \leq {}_b^1\pi_1^* \iff z_1^* \in [b, 1]$$

A simple result, which will be very useful in what follows, is that it is never optimal for either firm to set its indifference point strictly within the advertising campaign. Consider the following situation:

Figure 3.11: Two different values of z_0 resulting in the same market share.



Here, two different possible values of z_0 are shown. One is z_0=a, the left end-point of the campaign, and the other has z_0 be an arbitrary point within the campaign. The straight lines are consumer utility from consuming Firm 0's good, for each choice of z_0. For a given p_1, a rise in z_0 is equivalent to a fall in price. The length z_0 *would* be Firm 0's market share, in the absence of advertising. However, because of Assumption 3.2, the consumers on [a,b] belong to Firm 1. Therefore, Firm 0's market share when z_0=z0B is only equal to a. Both of the z_0 shown in the diagram grant Firm 0 a market share of a. However, the z0B requires a lower price than z0A, and thus lower profits. Since the only characteristic of z0B is that it is inside the campaign, it is therefore generally true that setting z_0=a is preferable to setting z_0 to be any point strictly within (a,b).

A similar argument applies for Firm 1. When Assumption 3.2 holds, the consumers along the campaign 'belong' to Firm 1, and it is not necessary for that firm to lower its price (by lowering z_1) in order to appeal to them alone.

These results are summarized in the following lemma.

Lemma 3.2: Let Assumption 3.2 hold. Then it is not optimal for either firm to set their indifference point strictly within the support of the advertising campaign. Specifically, ${}_a^b z_0^*$=a and ${}_a^b z_1^*$=b.

Proof: Suppose a and b are given, and A satisfies Assumption 3.2. Let $r \in (a,b)$.

Consider Firm 0's choice of indifference point for any given p_1. When z_0=a, Firm 0's market share is a and price is $p_0(a,p_1)$. When z_0=r, Firm 0's market share is only a. By definition, in the absence of advertising consumers on $[0,z_0]$ prefer Good 0 to Good 1. However, because Assumption 3.2 holds, consumers on [a,b] prefer to buy from Good 1. Therefore Firm 0's market share is (r-(r-a))=a. Its price is $p_1(r,p_1)$. But $p_1(r,p_1)< p_0(a,p_1)$ for r>a. Since z0=a provides the same market share as z0=r, but at a higher price, then Firm 0 profits when z_0=a must be higher than Firm 0 profits when z0=r. Therefore Firm 0 will prefer to set z0=a to any other point within the campaign support, and ${}_a^b z_0^*$=a.

Now consider Firm 1's choice of indifference point for any given p_0. When z_1=b, Firm 0's market share is (1-a). In the absence of advertising, consumers on $[z_1,1]$ prefer Good 1 to Good 0. However, because Assumption 3.2 holds, consumer son [a,b] also prefer Good 1 to Good 0. Therefore Firm 1's market share when z_1=b is (1-b) + (b-a) = 1-a. If z_1=r, then Firm 1's market share is still 1-a. However, its price is lower, because for a given p_0, $p_1(z_1,p_0)$ rises with z_1. Thus Firm 1 profits are higher when z_1=b than for any $z_1 = r \in (a,b)$, and ${}_a^b z_1^*$=b , q.e.d.■

Finding the equilibrium

To find the pricing game equilibrium, we will consider each possible state of the two incentive constraints, IC0 and IC1. In any equilibrium, it must be true that both constraints are satisfied, both are violated, or one is satisfied and the other is violated. Each of these possible states places certain restrictions on the location of each firm's indifference point. For example, if IC0 is satisfied, then Firm 0 earns maximum profits by staying on its side of the campaign support, and in equilibrium we must see z_0<a. If IC1 is violated, then Firm 1 must prefer to set z_1<b. Lemma 3.2 helps us narrow the location farther, because in this case, it tells us Firm 2 will not set z_1 interior to [a,b]. Therefore when IC0 is satisfied and IC1 is violated, both z_0 and z_1 must be on [0,a].

The figure below shows possible equilibrium locations for z_0 and z_1 in each case. They are equilibrium positions in the sense that, out of equilibrium, it is entirely possible to have, say, $z_1<b$ when IC1 is satisfied. However, this cannot be true in equilibrium, because if IC1 is satisfied, then by definition Firm 1 can earn higher profits by setting $z_1 \geq b$. There is no guarantee that an equilibrium actually *exists*. However, *if* it exists, and satisfies a particular configuration of incentive constraints, then this implies restrictions on the location of indifference points.

Figure 3.12: Constraint satisfaction by location of z_j.

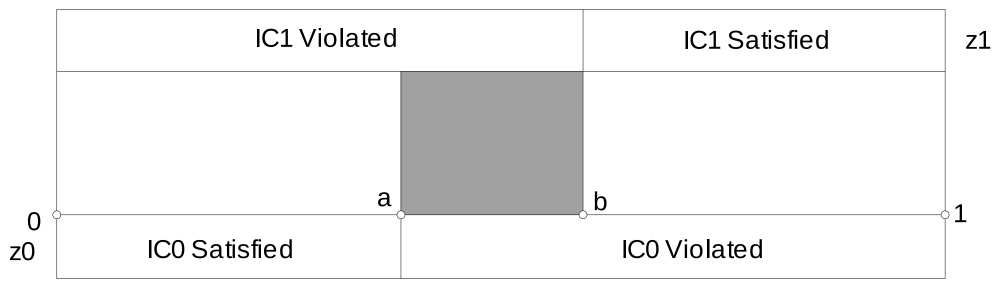
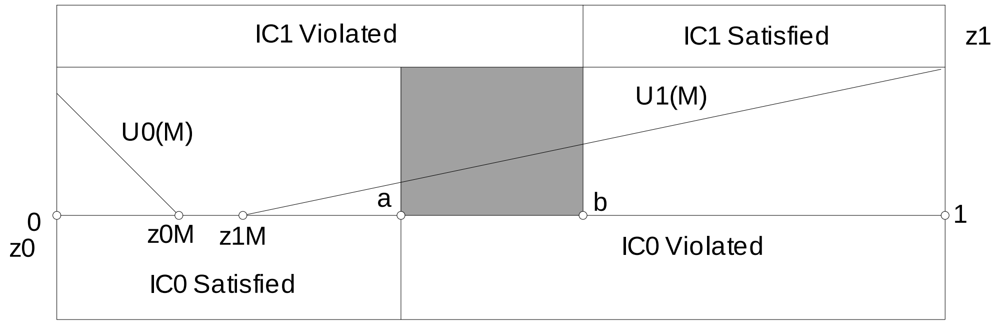


Figure 3.12 illustrates the regions in which each constraint is violated or satisfied. For example, if in equilibrium, Firm 0 chooses z_0=b, then IC0 is violated, because that constraint requires that Firm 0 obtain maximum profits by choosing an indifference point no greater than a.

By knowing where z_0 and z_1 must lie, relative to the campaign, we also know what regimes it is possible for the equilibrium to lie in. Take our example of IC0 satisfied and IC1 violated, above. Looking back at the table of regimes, we see there are *two* possible regimes where both z_0 and z_1 are on [0,a]. They are the Monopoly (M) regime, and the Duopoly (D) regime.

An equilibrium in the M regime would have to look as in the diagram below. The marginal consumer for each firm receives zero utility. In the diagrams below, the bracketed letters next to the utility function labels are a reminder of what regime we are dealing with.

Figure 3.13: An arbitrary candidate for an M-regime equilibrium.



However, this cannot be an equilibrium. By assumption, a monopolist covers the market. By Lemma 3.1, we know that the profit function in the M regime is quadratic and concave. Therefore there cannot be any 'gap' between z_0 and z_1 – we will see the whole market covered. But if in equilibrium $z_0=z_1$, and (z_0,z_1) are elements of [0,a], then we are *also* in the D regime – or rather, at the boundary point between the D and M regimes.

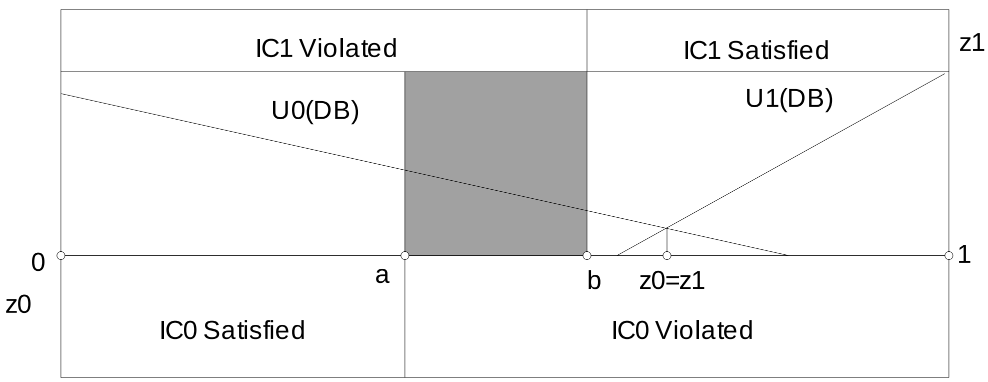
Thus, if there is an equilibrium, it must be in the D regime. Solving the first-order conditions shows us that there is a unique Nash equilibrium in this regime, listed in the table of regimes. It is at $z_0=z_1=1/2 + 2k/3$. If k is such that this value does not fall on [0,a], then there is no equilibrium in the D regime. When there is no equilibrium in the D regime, there can be no equilibrium for the case where IC0 is satisfied, and IC1 is violated[21].

The following two theorems carry out this reasoning for all possible states of IC0 and IC1. We find that when one or both of the constraints are violated, one of two pricing equilibria are possible. The first is the standard Hotelling equilibrium in the D regime, in which advertising is rendered irrelevant by the location of the indifference point. The second is similar to the Hotelling outcome, in that the marginal consumer receives positive utility, but is in the DB regime. Thus, consumers along the campaign are

[21] A 'corner solution' is not possible, because there are no real corners – only transitions into other regimes. This is true even at the end-points of the consumer line. Given our restriction on prices and transport costs, there can be no equilibrium with z_i=0 or 1, because each firm will always be able to obtain at the very least the consumer at its location, by setting a sufficiently low but positive price.

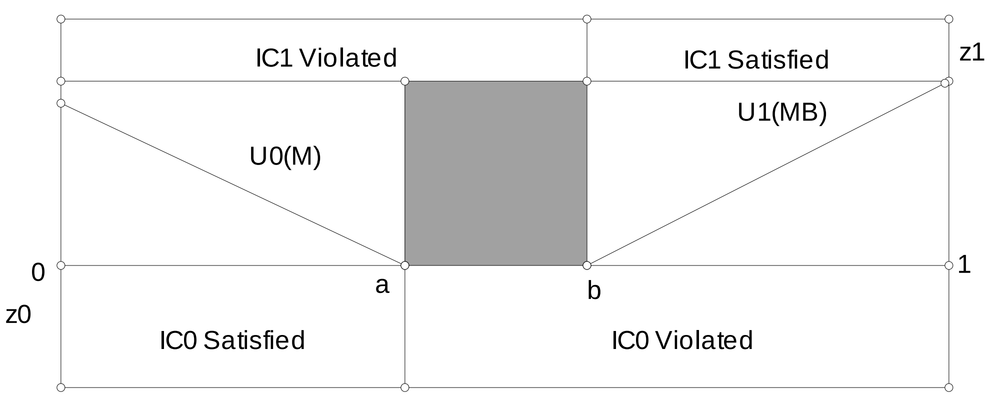
essentially transferred from Firm 0 to Firm 1. The latter equilibrium will be called a *Faux Hotelling* equilibrium. It is also possible that neither of these equilibria exist, if the Nash equilibrium indifference point does not lie within the correct regime.

Figure 3.14: An arbitrary candidate for an equilibrium in the DB regime.



Theorem 3.2 looks at what happens when both constraints are satisfied. In this case, each firm stays to its own side of the campaign, by the definition of the constraints. Thus, Firm 0 is in the M regime, and Firm 1 is in the MB regime. Because a monopolist covers the market, and the profit functions are quadratic and concave in z_i, we will see z_0=a and z_1=b. This is called a *Barrier* equilibrium, because the advertising campaign acts as a wall separating the firms from each other.

Figure 3.15: A barrier equilibrium



Theorem 3.1: Let $k>0$ and suppose that IC0 and IC1 are not simultaneously satisfied. Then there exists at most one equilibrium to the pricing game.

The *Hotelling equilibrium* $(z_0,z_1)=(1/2,1/2)$ exists if and only if $a \geq 1/2$.

The *Faux Hotelling Equilibrium*, $(z_0,z_1)=(z^{DB**}, z^{DB**})$ exists if and only if $k \leq 3/4$ and $a \leq ½ - k/3$.

When $½-k/3<a<1/2$, there exists no equilibrium in which at least one constraint is violated. (Though there may exist equilibria in which both constraints hold.)

Proof: In four parts. We first consider the case where both constraints are violated, then look at violations of a single constraint. Finally, we consider the case where none of these applies.

i. Let both IC0 and IC1 be violated. Then $z_0 \in (a,1]$ *and* $z_1 \in [0,b)$. This can only be true if $z_0 \in (a,b)$ and $z_1 \in (a,b)$. By Lemma 3.1, it is never optimal for Firm j to set $z_j \in (a,b)$, and so this cannot be an equilibrium.

ii. Let IC0 hold and IC1 be violated. Then $z_0 \in [0,a]$ and $z_1 \in [0,b)$. By Lemma 3.1, we may rule out $z_1 \in (a,b)$, and so $z_1 \in [0,a]$. The market is covered by assumption. Therefore we are in the D regime[22], and the only possible equilibrium is the Hotelling equilibrium. If $a<1/2$, this equilibrium does not exist.

iii. Let IC0 be violated and IC1 hold. Then $z_0 \in (a,1]$ and $z_1 \in [b,1]$. By Lemma 3.1, we may rule out $z_0 \in (a,b)$ and so $z_0 \in [b,1]$. The market is covered by assumption, therefore we are in the DB regime, and the only possible equilibrium is the Faux Hotelling equilibrium. For this equilibrium to exist,

[22] More explicitly, the M regime can be ruled out by the following argument. Suppose $z_0<z1$. This cannot be an equilibrium, since a monopolist covers the market and Firm 0's profit function is quadratic in z_0. Firm 0 will strictly prefer to set $z_0 \geq z_1$, leading us into the D regime.

we must have $b \le z^{DB**} \le 1$. Since z^{DB**}=1/2+ 2k/3 and b=a+k, $b \le z^{DB**}$ when $a \le$ ½ - k/3 and $z^{DB**} \le 1$ when $k \le 3/4$.

iv. Let ½-k/3<a<1/2. Then it cannot be the case that IC1 is violated and IC0 holds, because the Hotelling equilibrium point does not lie in the D regime. It also cannot be the case that IC0 is violated and IC1 holds, because the only Nash equilibrium for the DB regime profit functions does not lie within the DB regime. When both constraints are violated, we cannot have an equilibrium. Therefore, by exhaustion, given this constraint on a, there cannot exist an equilibrium with one or both constraints violated, q.e.d.

■

Thus, one of two equilibria is possible when both constraints are not simultaneously satisfied: a symmetric Hotelling equilibrium, or an asymmetric Faux Hotelling equilibrium.

If advertising is costly, the Hotelling equilibrium is clearly inefficient, since it may be achieved costlessly by not advertising. The Faux Hotelling equilibrium uses the campaign as a simple transfer of market share, and leaves the marginal consumer with positive surplus.

Theorem 3.2: Let IC0 *and* IC1 be satisfied. Then the *Barrier equilibrium*, at $(z_0,z_1)=(a,b)$, is the unique equilibrium to the pricing game.

Proof: When both constraints are satisfied, $z_0 \in [0,a]$ and $z_1 \in [b,1]$. Hence z_0 is in the M regime, and z_1 is in the MB regime. Profit functions for both firms are concave and quadratic in z, and a monopolist covers the market. Therefore ${}_0^a z_0^* = a$ and ${}_b^1 z_1^* = b$, q.e.d.

■

In a Barrier equilibrium, we have what amounts to a double monopoly – each firm's marginal consumer[23] receives zero surplus.

Theorems 1 and 2 have told us what sorts of equilibria are *allowable*. They have *not* told us what equilibria we will actually see, since their results depend on particular states (satisfied, not satisfied) of the incentive constraints. There is no guarantee, for instance, that both constraints will ever be simultaneously satisfied, and so we may never see a Barrier equilibrium.

The next lemma shows that for a sufficiently long campaign support, it is always possible to satisfy at least *one* of the constraints. The conditions under which both constraints will be satisfied will be examined closely in the next section, during the analysis of the advertising game.

Lemma 3.3: It is always possible to find a length k'<1 such that when k=k', at least one incentive constraint is satisfied. In particular,

Given any $a \in [0,1)$, there exists a k'<(1-a) such that IC0 is satisfied iff $k \geq k'$.
Given any $b \in (0,1]$, there exists a k'<b such that IC1 is satisfied iff $k \geq k'$.

Proof: In the Appendix.

3.4.2. Stage 1

In Stage 1, the advertising firm, Firm 1, chooses the location and intensity of its advertising campaign, and pays for it. An equilibrium in the first stage consists of a triplet (a,k,A) satisfying the conditions for a subgame perfect Nash equilibrium. Recall that a is the left end-point of the campaign support, k is its length, and A is the intensity

[23] When I speak of the marginal consumer, I refer to z_j. It is understood that consumers on [a,b] belong to Firm 1.

by which the willingness to pay for Firm 1's good is boosted, for consumers along the campaign.

We have already seen that, depending on which of the incentive constraints (if any) are satisfied, one of three equilibria may result in Stage 2, or none at all. Where an equilibrium exists, we may have a Hotelling, Faux Hotelling, or Barrier equilibrium.

In what follows we will examine which choices of advertising will lead to a given equilibrium in Stage 2. Firm 1 must weigh the profits from this equilibrium against the cost of the required advertising.

The advertiser's profit maximization problem in Stage 1 can thus be broken into two parts. The first is to select the type of equilibrium that is desired in Stage 2: Hotelling, Faux Hotelling or Barrier. We will find, for example, that for any given starting point 'a' there is a size of campaign that will ensure a Barrier equilibrium in the second period. The second part of the choice is to choose the location of the campaign, given the class of equilibrium desired. The required size mentioned above will depend on 'a'. It may be that it is possible for Firm 1 to ensure Barrier equilibria in which it has a market share of ½, or of ¾, but that the latter is far more expensive in terms of advertising cost, making the former more profitable.

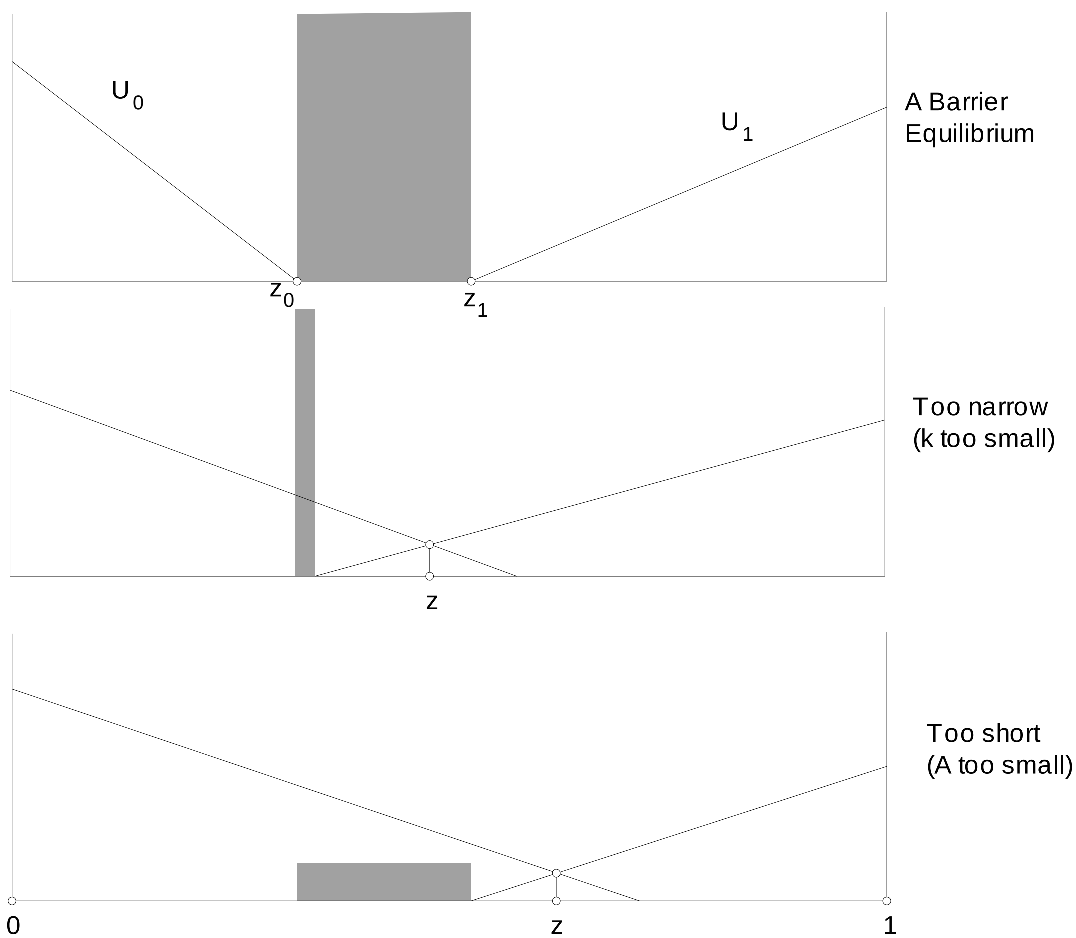
It will be shown that under general conditions, when advertising is employed, it will be used to induce a Barrier equilibrium in Stage 2, with both incentive constraints satisfied. Once the *type* of induced Stage 2 equilibrium has been chosen, Firm 1 must decide on its exact location – that is, the starting point of the campaign. This is a standard optimization over one parameter.

Firm 1 will never pay for advertising that induces a Hotelling outcome, because it can obtain this equilibrium for free by not advertising. We saw above that when advertising is sufficiently costly, it will prefer the profits from not advertising to a Faux-Hotelling output. That leaves us with the possibility of a Barrier equilibrium.

For an advertising campaign to work as a barrier – that is, to induce compliance with the incentive constraints – it must be of sufficient height and breadth. It must be high enough that Firm 0 will not find it worthwhile to capture the consumers on the campaign. (That is, it must be high enough that our Assumption 3.2 is satisfied). It must be long enough so that neither firm finds it worthwhile to capture consumers on its far side, due to the fall in price that it would require.

In the following diagram, Firm 1 is pricing at b, as it would in a Barrier equilibrium. The advertising campaign is denoted by the shaded area, and Firm 0's best response to the *given* campaign, and Firm 1 pricing at b, is shown.

Figure 3.16: Different ways of undercutting a campaign.



Suppose that Assumption 3.2 still holds, and it is not worthwhile for Firm 0 to capture consumers *on* the advertising support. In the following theorem, we find the minimum campaign length needed to ensure that, given Firm 1 pricing at b, Firm 0 does not wish to capture consumers *beyond* the far side of the campaign, on [b,1]. That is to say, we find the minimum length of k that will prevent outcomes such as that in the middle diagram above.

Symmetrically, we also find the minimum k needed to dissuade Firm 1 from capturing consumers on [0,a] when Firm 0 is pricing at a. We must find this because advertising costs are sunk in Stage 2, and Firm 1 does not take them into account when making its *pricing* decision.

Theorem 3.3: Let Assumption 3.2 hold. The incentive constraints for both firms can only be simultaneously satisfied if the advertising campaign is long enough to sufficiently dampen competition. In particular, suppose $(z_0,z_1)=(a,a+k)$ describes a Barrier equilibrium to the pricing game for a given $a\in[0,1)$. Then it must be the case that $k\geq k_{min}$, where $k_{min}=\max(k_0,k_1)$ and

$$k_0 = \begin{cases} 0 & \dfrac{V}{3t}\leq a \\ k_0^{'} & a\leq \dfrac{V}{3t} \;\; and\; k_0^{'}\leq \dfrac{1}{3}\left(4-\left(\dfrac{V}{t}+a\right)\right) \\ \max\left[k_0^{"},0\right] & otherwise \end{cases}$$

$$k_0^{'} \equiv a+\frac{V}{t}-2\sqrt{2a\left(\frac{V}{t}-a\right)}$$

$$k_0^{"} = \frac{1}{2}\left(3-a-\frac{V}{t}+\sqrt{\left(\frac{V}{t}\right)^2-2(1+a)\frac{V}{t}+5a^2-2a+1}\right)$$

$$k_1 = \begin{cases} 0 & a\leq 1-\dfrac{V}{3t} \\ \dfrac{\left(\dfrac{V}{t}-3(1-a)\right)^2}{8(1-a)} & a\geq \max\left(1-\dfrac{V}{3t},\dfrac{V}{t}-3\right) \\ \max\left[\dfrac{a}{1-a}\left(\dfrac{V}{t}+a-3\right),0\right] & a\leq \dfrac{V}{t}-3 \end{cases}$$

Proof: A sketch of the proof follows. The full proof is found in the Appendix.

IC0 and IC1 are satisfied in a Barrier equilibrium. From Theorem 3.3, we know that for a given a there exists a minimum k such that IC0 holds, and for a given b a minimum k such that IC1 holds. Noting b=a+k, it is possible to rephrase both of these conditions in terms of a given a.

The required values of k may be solved for by equating each firm's profits under a barrier equilibrium to those at the most profitable deviation, given the actions of its rival. Doing so yields the expressions shown. IC0 is satisfied when $k \geq k_0$, and IC1 when $k \geq k_1$. If k should be lower than either of the two, then one of the constraints would be violated. Therefore it must be true that in a Barrier equilibrium, $k \geq k_{min}$.

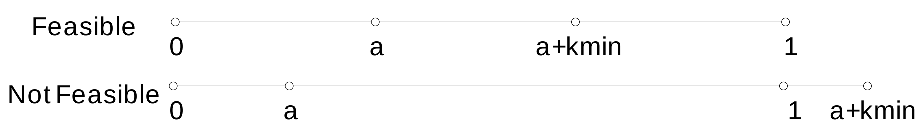
■

The above theorem gives us the minimum length required of the campaign so that, if Assumption 3.2 holds, neither firm will wish to deviate from a Barrier equilibrium by capturing consumers on the far side of the campaign.

The theorem does not, however, guarantee that a campaign of size k_{min} will 'fit' on the consumer line. That is, it is possible that there are choices of a for which no Barrier equilibrium is possible.

Because of this, it is important to introduce the notion of the subset of addresses on the consumer line that can support a barrier equilibrium when used as the left end-point, a, of a campaign (if Assumption 3.2 holds). This will be true whenever $a+k_{min} \leq 1$ - when the minimum value for the right end-point is on the unit line.

Figure 3.17: Feasible and non-feasible addresses



Definition 3.6: The *Feasible Set*, F, is a subset of the unit line containing all values of a for which $k_{\min}(a) \leq (1-a)$. F has left end-points f_1 and f_2, so that F=[f_1,f_2]. The size of the feasible set is described by $f \equiv f_2 - f_1$.

We have referred to k_{min} as a 'minimum' length of a campaign that, for a given a, will support a Barrier equilibrium when Assumption 3.2 holds. It is true that for k=k_{min}(a), both IC0 and IC1 are satisfied, and a Barrier equilibrium exists. However, it has not been shown explicitly that this continues to be true for k>k_{min}. This is not a trivial question, because b is a function of k (since we took a as the choice variable), and Firm 1 prices at b in a Barrier equilibrium.

The following lemma shows explicitly that any k>k_{min} will also satisfy both incentive constraints when the firms price at the boundaries of the campaign (as they will in a Barrier equilibrium).

Lemma 3.4: Suppose Assumption 3.2 holds. Also, let 0<a<1 and the firms price at the boundaries of the campaign, as in a Barrier equilibrium. Then both incentive constraints are satisfied for any k greater than k_{min}(a) and less than (1-a).

That is, if $a \in F$, Assumption 3.2 holds and (z_0,z_1)=(a,a+k), then if $k \geq k_{min}$, IC0 and IC1 are satisfied.

Proof:

Let k_0=k_1. Then the lemma is trivially true.

From Theorem 3.3, we know that when both constraints are satisfied, we have a Barrier equilibrium. If k=k_j, then Firm j's incentive constraint is satisfied. We consider below what happens when k>k_j, given Firm j's rival prices as in a Barrier equilibrium. Firm j may 'accommodate' to a Barrier equilibrium, or deviate from it to an equilibrium in the D regime.

Let $k_0 > k_1$, and suppose $k = k_0$. Firm 1's accommodation profits are greater than when $k = k_1$. Accommodation market share is unchanged at (1-a), but accommodation prices are higher, since in a Barrier equilibrium Firm 1 prices at $z_1 = a + k$. Maximum deviation profits are unchanged from the case where $k = k_1$. Therefore, Firm 1 will accommodate to a barrier equilibrium, and IC1 will hold, whenever $k \geq k_1$.

Let $k_1 > k_0$, and suppose $k = k_1$. Firm 0's accommodation profits are unchanged from the case where $k = k_0$. It can be shown that

$$\frac{\partial_{a+k}{}^{1}\pi_0^{D*}}{\partial k} < 0 \text{ whenever } (k-a) \leq \frac{V}{t}$$

By assumption, $V > t$ and $k < (1-a)$, and so Firm 0 maximum deviation profits fall with k. The loss of market share from a greater k overpowers the effect of a rise in p_1. Therefore IC0 holds for $k > k_0$.

■

We can combine the results we have so far into a single statement. Suppose Assumption 3.2 holds. When a given address r on the consumer line is a member of the Feasible set, then it is possible to support a Barrier equilibrium in the second stage by setting the campaign endpoints at r and r+k, where k is at least equal to k_{min} – as long as Assumption 3.2 holds.

Assumption 3.2 – that it is not worthwhile for Firm 0 to capture the consumers on Firm 1's campaign support - has played a major role in our results regarding the Barrier equilibrium. The next theorem calculates the lowest value of A that is needed for this assumption to hold in such a situation.

Theorem 3.4: Let $a \in F$ and $k \geq k_{min,}$, and suppose Firm 1 prices at (a+k). Then there exists a finite value for A such that Firm 0 cannot profitably violate its incentive constraint, and Assumption 3.2 is satisfied. This value is no higher than

$$A_{\min} \equiv V - t(a-k) - p_0^m$$

where

$$(V - ta)a \equiv p_0^m \min\left\{\frac{1}{2}\left(\frac{V - p_0^m}{t} + a + k\right), 1\right\}$$

Proof:

Consider the situation where both incentive constraints are satisfied in the pricing game. Then we have a Barrier equilibrium and $(z_0,z_1)=(a,a+k)$.

Suppose A=0, so the advertising campaign has no effect, and allow Firm 0 to deviate from $z_0=a$, with z_1 remaining at a+k.

The regimes for Firm 0 are M from 0 to z_1 and D from z_1 to 1.

Since a monopolist covers the market, $\pi_0(a+k, a+k) > \pi_0(a, a+k)$. Therefore the *lowest* Firm 0 price that will ensure a profitable deviation is in the D regime, and may involve a corner solution with a price that implies $z_0^D > 1$.

Let this minimum Firm 0 price be p_0^m. We require deviation profits to be equal to reservation profits when $p_0 = p_0^m$. Because $\pi_0^D(z_0)$ is quadratic and concave in z_0, and p_0 falls with z_0, Firm 0 will not wish to set a lower price than p_0^m. Reservation profits are (V-ta)a, and the deviation market share is either z_0^D, or unity, if $z_0^D > 1$. Thus

$$(V-ta)a \equiv p_0^m \min\{z_0^D(p_0^m.p_1),1\}$$

The exact expression for p_0^m is derived in the Appendix. The term $z_0^D(p_0^m.p_1)$ is the address of the indifferent consumer when $(p_0,p_1)=$ $(p_0^m.p_1)$. Since Firm 1 is pricing at a+k, p_1=V-t(1-a+k). At the indifference point, the utility from both goods is equal, so

$$V-tz_0^D-p_0^m=V-t(1-z_0^D)-p_1$$

Solving,

$$z_0^D(p_0^m.p_1)=\frac{1}{2}\left(\frac{V-p_0^m}{t}+a+k\right)$$

Now let A be such that $U_1(r,p_1)>U_0(r,p_0^m)$ for all $r\in$[a,a+k]. Since U_0 falls with p_0, this is also true for all $p_0>p_0^m$. Firm 0 has no incentive to deviate from (z_0,z_1)=(a,a+k). Because $k\geq k_{min}$, Firm 0's incentive constraint is satisfied whenever the consumers on [a,a+k] prefer Firm 1, and they are prefer Firm 1 for all prices at which Firm 0 could profitably undercut the campaign.

Suppose A is such that $U_1(r,p_1)<U_0(r,p_0^m)$ for all $r\in$[a,a+k]. Then there exists a price above p_0^m to which Firm 0 may profitably deviate and undercut the barrier.

Solving $U_1(r,p_1)=U_0(r,p_0^m)$ for A when $r\in$[a,a+k], we obtain

$$A=V-t(a-k)-p_0^m$$

When A meets this value, all consumers on the campaign support weakly prefer Firm 1 for all prices to which Firm 0 could profitably deviate by undercutting the campaign, q.e.d.

■

There is nothing in the solution that relies on the specifics of the Barrier equilibrium. We can also use this method to find the minimum advertising intensity A needed to support a Faux Hotelling equilibrium.

Theorem 3.4.b: Let k>0 and suppose Firm 1 prices as it would in a Faux Hotelling equilibrium. Then there exists a finite value for A such that Firm 0 cannot profitably capture the consumers on [a,b], satisfying Assumption 3.2. This value is no higher than

$$A_{\min}^{FH} = 2t\left(1 - a + \frac{k}{3}\right) - p_0^{mFH}$$

where

$$\frac{t}{18}(2k-3)^2 \equiv p_0^{mFH} \min\left\{1 + \frac{k}{3} - \frac{p_0^{mFH}}{2t}, 1\right\}$$

Proof: As above. The Faux Hotelling equilibrium corresponds, where it exists, to the equilibrium in the DB regime. This equilibrium has a single indifference point at $z^{DB**} = \frac{1}{2} + \frac{2}{3}k$, found by solving the first-order conditions and listed on the regime table. From the first order conditions, we find Firm 0's price is $t\left(1 - \frac{2}{3}k\right)$. Its market share, when Assumption 3.2 holds, is $z^{DB**} - k$. Firm 0's profits in such an equilibrium are $\frac{t}{18}(2k-3)^2$. As in Theorem 6.a, we require deviation profits to be equal to equilibrium profits at the mirror price. In a Faux Hotelling equilibrium, $p_1^{FH**} = t\left(1 + \frac{2}{3}k\right)$ (from the first-order conditions). At the indifference point, we will have the utility from both goods be equal:

$$V - tz_0^D - p_0^{mFH} = V - t(1 - z_0^D) - p_1^{FH**}$$

Substituting for p_1 and solving,

$$z_0^D\left(p_0^{mFH}, p_1^{FH**}\right) = 1 + \frac{k}{3} - \frac{p_0^{mFH}}{2t}$$

There is no guarantee that this is less than one. Once this constraint has been accounted for, our condition that deviation profits be equal to equilibrium profits at the mirror price becomes

$$\frac{t}{18}(2k-3)^2 = p_0^{mFH} \min\left\{1 + \frac{k}{3} - \frac{p_0^{mFH}}{2t}, 1\right\}$$

We must ensure that A is such that at the mirror price, no consumer on [a,b] is captured by Firm 0.

$$V - tr - p_0^{mFH} \le V - t(1-r) - p_1^{FH**} + A_{\min}^{FH} \text{ for } r \in [a, a+k]$$

This will be true for all the required addresses if it is true for r=a, since as we move along the consumer line, pre-advertising utility from Good 0 falls, and that from Good 1 rises. Substituting r=a and $p_1^{DB**} = t\left(1 + \frac{2}{3}k\right)$ and solving, we find

$$A_{\min}^{FH} = 2t\left(1 - a + \frac{k}{3}\right) - p_0^{mFH}$$

q.e.d.

■

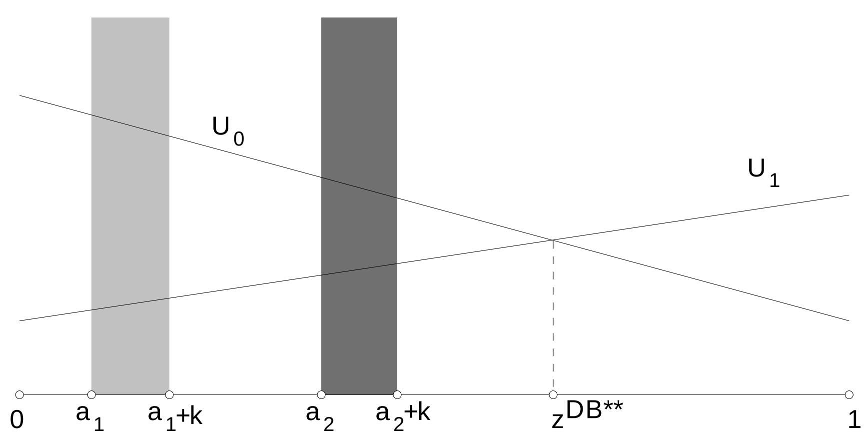
This is a good place to take a look at the Faux Hotelling equilibrium in its entirety. We have a formula for the required height of its campaign, and conditions on when it will exist, and when it will yield greater than Hotelling profits for the advertiser.

The faux Hotelling equilibrium

The location of the advertising campaign pays a far less important role in a Faux Hotelling equilibrium than in a Barrier equilibrium. In the latter case, firms price at the campaign borders. In a Faux Hotelling outcome, prices are independent of the *location* of the campaign, a. All that matters is the *length* of the campaign, k, and that z^{DB**}, the equilibrium indifference point, falls within the appropriate regime.

The diagram below shows two different locations for a campaign of length k. Either one will lead to the *same* Faux Hotelling equilibrium – that is, the same equilibrium indifference point z^{DB**} - provided that its height A is such that Assumption 3.2 is satisfied.

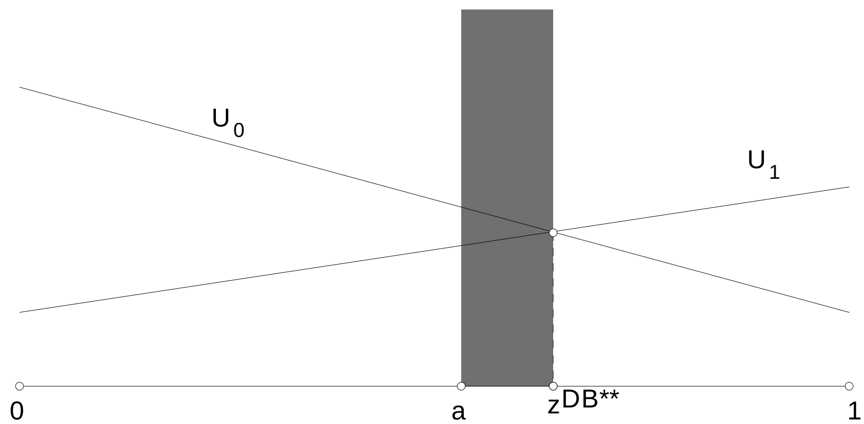
Figure 3.18: Different campaign locations leading to the same faux Hotelling equilibrium.



The only time that campaign location enters consideration is when calculating the necessary *height* to prevent the undercutting of the ads, as in Theorem 4.b. The nearer the campaign is to the advertiser, the less intensive the ads need to be[24]. Consumers located near Firm 1 have a pre-existing preference for Good 1 over Good 0, and do not need to be 'persuaded' of its merits as forcefully as consumers near Firm 0.

The implication is that if Firm 1 wishes to induce a Faux Hotelling equilibrium in Stage 2, it is best to place the campaign adjacent to the equilibrium indifference point, so that $a+k = z^{DB**}$.

Figure 3.19: A campaign adjacent to the DB-regime equilibrium indifference point.



However, by Lemma 3.1, we know that this cannot be an equilibrium. Given Firm 1's price, Firm 0 could deviate to z_0=a and earn a higher profit than at z_0=b(=z^{DB**}).

Instead, a must be chosen so that Firm 0 cannot profitably deviate to a z_0<b when Firm 1 prices as in a Faux Hotelling equilibrium.

A Faux Hotelling equilibrium must fall (by definition) in the DB regime. Solving the first-order conditions for the DB regime profit function, we find that at the Nash equilibrium in this regime, $p_1^{DB**} = t\left(1+\frac{2}{3}k\right)$. At this price, the utility of the consumer at address r from Good 1 is

$$U_1\left(r, p_1^{DB**}\right) = V - t(1-r) - p_1^{DB**}$$

[24] This may be seen algebraically in the statement of Theorem 6.b. Note that 'a' only appears once, with a negative coefficient.

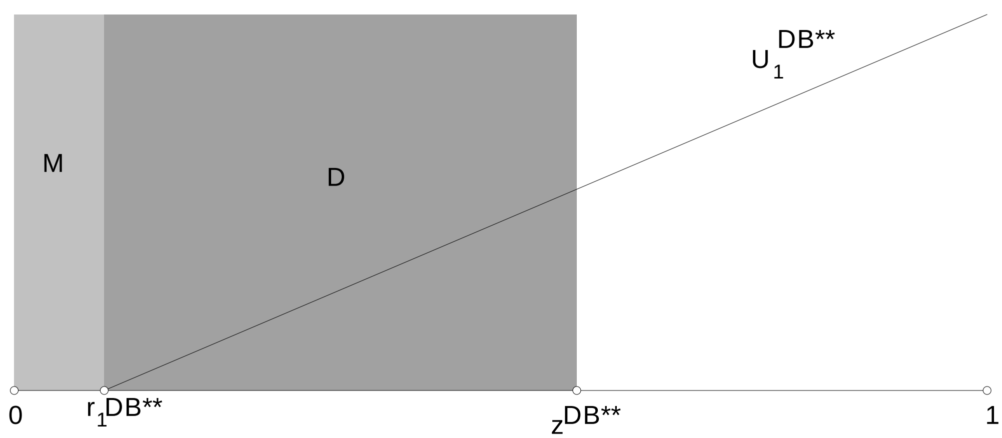
From which we see that the address of the consumer indifferent between the outside good and good 1 is

$$r_1^{DB**} \equiv \left(2 - \frac{V}{t}\right) + \frac{2}{3}k$$

The first term is never positive, because of our maintained assumption of a covered market under monopoly. The second term is small for small k. What this means is that most consumers on [0,a] have a positive valuation of Good 1. Therefore, Firm 0's most profitable deviation is likely to be in the D regime (but may also be in the M regime).

The figure below illustrates the situation. In a Faux Hotelling equilibrium, Firm 0 must wish to set $z_0=z^{DB**}$. Possible deviations include choosing z_0 such that the marginal consumer has a lower, but positive valuation – in the dark shaded 'D' area – or choosing a z_0 that gives the marginal consumer of Good 0, zero utility – in the lightly shaded 'M' area. If $r_1^{DB**} < 0$, then we only have the D regime.

Figure 3.20: A possible division of regimes when Firm 1's price is p_1^{DB**}.



Given Firm 1's price, Firm 0's profit function in either regime is quadratic and concave in z_0 (Lemma 3.1). If we set a equal to the lowest-valued address that yields a profitable deviation, Firm 0 will accommodate to a Faux Hotelling equilibrium – the remainder of the profitable deviations are now to the right of a, and either in the B or DB regimes.

In the following demonstration, we will assume that k is small, so that only the D regime need be considered[25].

We need to solve for the lowest non-negative value of z_0 such that

$$\pi_0^D\left(z_0, p_1^{DB^{**}}\right) = \pi_0^{DB^{**}}\left(z^{DB^{**}}, p_1^{DB^{**}}\right)$$

Substituting the expressions from above and the regime table, then solving, we find this happens when z_0 is equal to

$$a^{FH} \equiv \frac{1}{2} + \frac{1}{6}\left(k - \sqrt{3k(6-k)}\right)$$

When k=0, this falls to ½ , as expected. This is to be our choice of a, and in the limit of k=0, a Faux Hotelling equilibrium is a standard Hotelling equilibrium $z_0=z_1=1/2$.

We now have a characterization of Firm 1's Faux Hotelling profits as a function of a single variable.

The cost of advertising is equal to the cost parameter, c, times the area of the campaign – its width, k, by its height, A. Combining all our information thus far, Firm 1's total profits from a Faux Hotelling equilibrium outcome may be written entirely in terms of k:

$$\pi_1^{FH^{**}}(k) = p_1^{DB^{**}}(k)\left(1 - z^{DB^{**}}(k) + k\right) - cA_{\min}^{FH}\left(a^{FH}(k), k\right)k$$

Once the appropriate substitutions are made, this function exhibits several convenient properties.

$$\pi_1^{FH**}(k) = \left(\frac{1}{2}\left(1+\frac{2}{3}k\right)^2 - 2ck\sqrt{k\left(2-\frac{1}{3}k\right)}\right)t$$

It is independent of the intrinsic valuation of the good, V. The transport cost only determines the *level* of profits for a given k, without affecting the profit-maximizing choice of k. It may be thought of as a scaling parameter. Finally, profits unambiguously fall with advertising cost, as expected.

These properties allow us to quickly find the *maximum* profits that a Faux Hotelling equilibrium can provide the advertiser, in a best-case scenario. Because advertising is costly, the 'best case' involves setting c=1. Solving the first-order conditions[26] numerically, we find that profits are highest when $k \approx 0.03$, and that $\pi_1^{FH**}(0.03) = 0.506t$. This represents scarcely more than a 1% increase over Hotelling profits of t/2. For costlier advertising, the situation is worse – but maximum advertiser profits in a Faux Hotelling equilibrium will *always* be higher than Hotelling profits.

To see this, consider the first derivative of the Faux Hotelling profit function with respect to k. Arranged by powers of k, it is

$$\frac{d\pi_1^{FH**}(k)}{dk} = k^2\frac{2ct}{\sqrt{18k-3k^2}} + kt\left(\frac{4}{9} - \frac{6c}{\sqrt{18k-3k^2}}\right) + \frac{t}{3}\left(2 - c\sqrt{18k-3k^2}\right)$$

The limit of the derivative as k approaches 0 is

$$\lim_{k\to 0}\frac{d\pi_1^{FH**}(k)}{dk} = 0 + 0 + \frac{t}{3}(2-0) = \frac{2}{3}t$$

[26] The second derivative of $\pi_1^{FH**}(k)$ is negative when k=0.07, with a value of -5.4t.

When k is exactly zero, we are back in the standard Hotelling case. Therefore, an advertiser will *always* prefer a Faux Hotelling equilibrium to not advertising. Profits for both firms and the optimal Faux Hotelling campaign length are shown below. In this situation, the non-advertiser is unambiguously worse off than in the absence of advertising. The campaign is a simple capture of consumers. As advertising costs rise, Firm 0's profits rise asymptotically toward Hotelling profits as Firm 1's fall toward it.

Figure 3.21: Profits in a faux Hotelling equilibrium (numerical simulation)

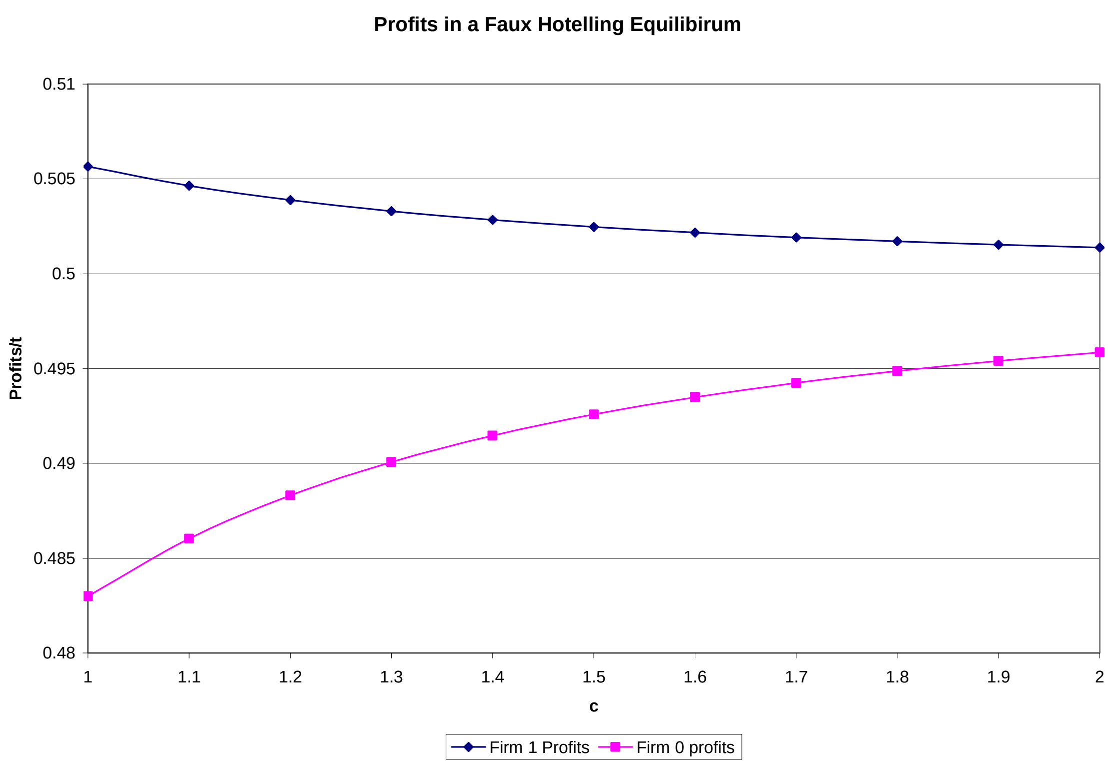
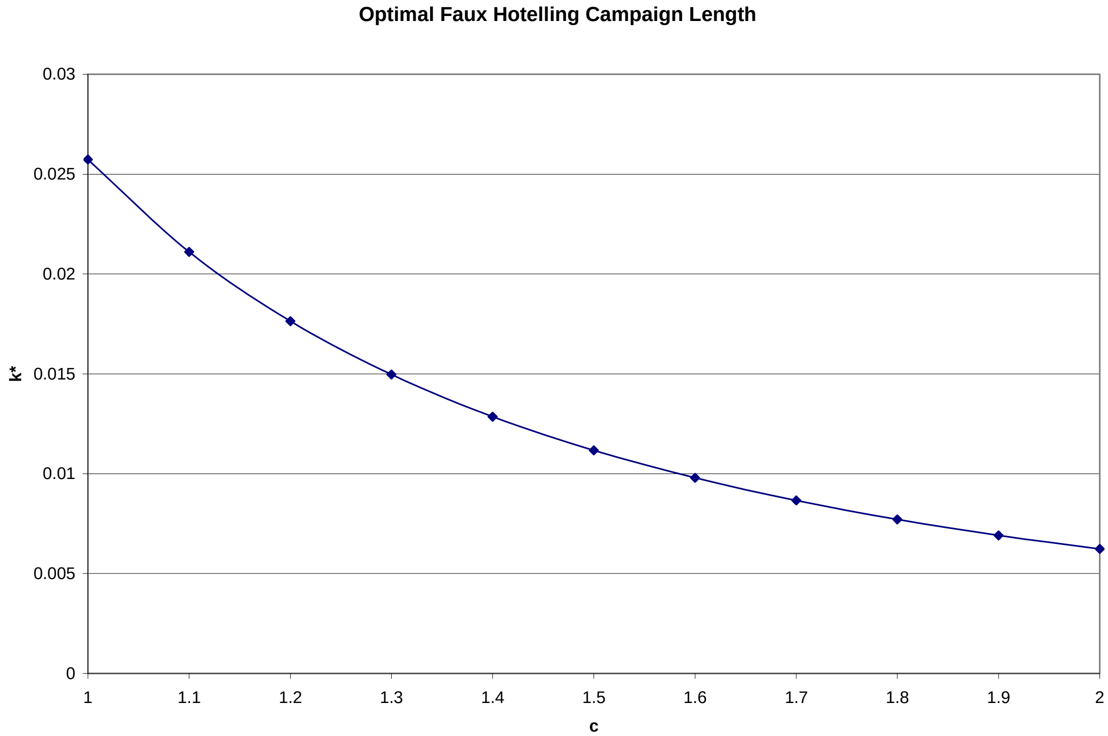


Figure 3.22: Optimal faux Hotelling campaign length (numerical simulation)



The key to the Faux Hotelling equilibrium's profits is the possibility of infinitely-fine *targeting*. Even when advertising is exceedingly costly, the advertiser can target a miniscule segment of just the right consumers and earn small (but positive) profits. If instead of being a continuum, the consumer line consisted of discrete individuals, then there would be a finite value for the cost parameter above which Faux Hotelling advertising was not worthwhile.

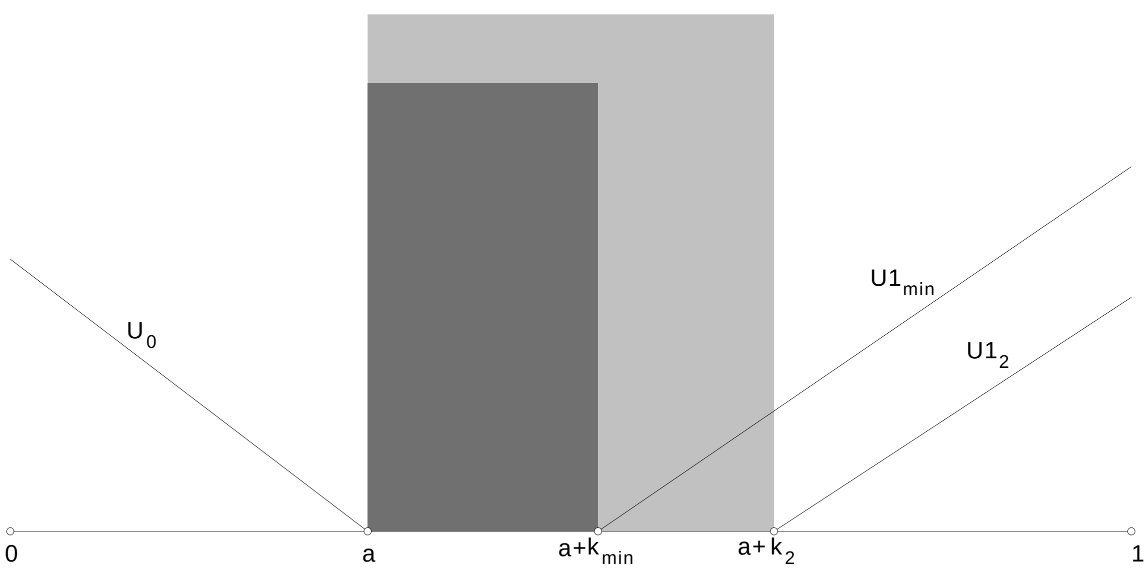
The barrier equilibrium

In a Barrier equilibrium, it is campaign *location* that matters. The starting point of the campaign, a, determines the length of campaign necessary to support a Barrier equilibrium, $k_{min}(a)$. The minimum campaign intensity that will prevent undercutting of the ads by Firm 0 is a function of both k and a – $A_{min}(a,k)$.

It is important to make a distinction between the campaign length in a barrier equilibrium, and $k_{min}(a)$. The campaign must be at least of length $k_{min}(a)$, but it may also be longer. The benefit of a longer campaign is a higher price with constant market share. The cost comes directly from the additional advertising required. Not only is the campaign longer, it must also be taller. A higher price for Firm 1 means a greater incentive for Firm 0 to deviate to a lower price

The diagram below shows two possible barrier equilibria for the same value of a.

Figure 3.23: Minimum, and greater-than-minimum campaigns leading to a barrier equilibrium.



The dark shaded area is the minimum campaign size required. It allows Firm 1 to price at $a+k_{min}$. If Firm 1 wishes to price at $a+k_2$ (a higher price), then it must make the campaign longer, and increase its height to account for the greater threat of undercutting by Firm 0. The new campaign is the sum of the dark and the light shaded areas. As might be expected, the opportunities to profit from a larger barrier are small when advertising is costly.

In what follows, we will assume that $k=k_{min}(a)$ in a Barrier equilibrium. This will underestimate advertiser profits, while allowing for a much simpler exposition. Our main result will be that Barrier profits are generally superior to either Hotelling or Faux Hotelling profits, even at very high cost parameters. That these Barrier profits are a lower bound only emphasizes the finding.

When we let $k=k_{min}(a)$, Firm 1's profit function in a Barrier equilibrium is

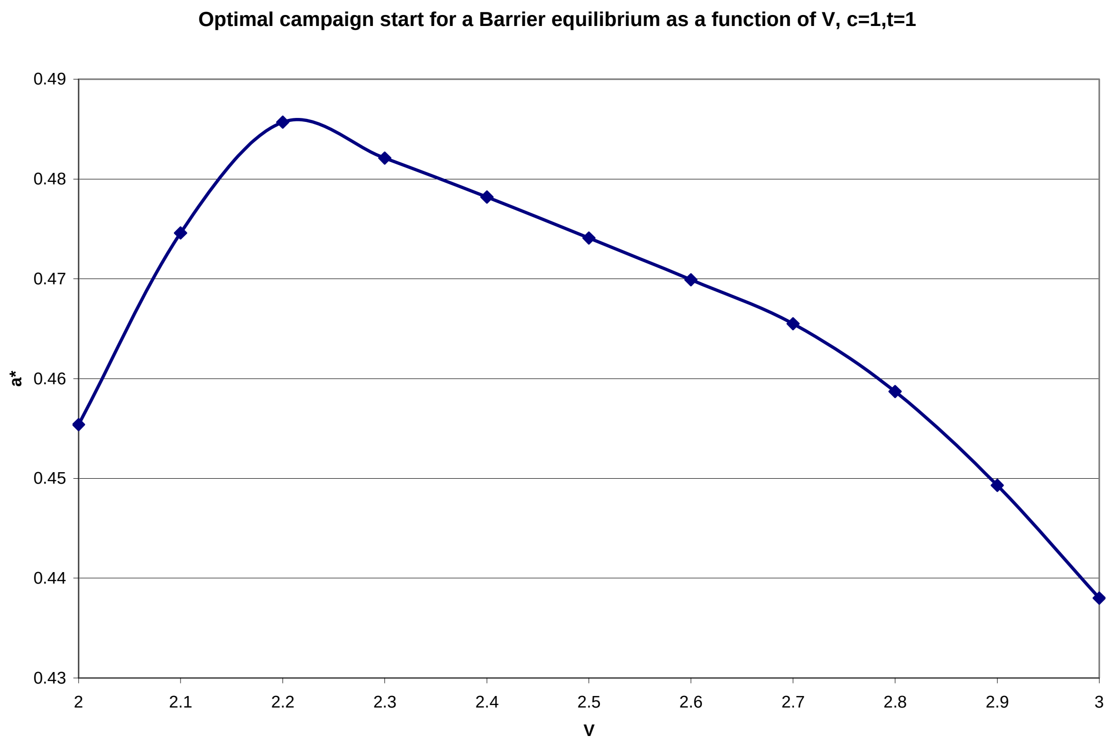
$$\pi_1(a) = (V - t(1 - a + k_{\min}(a)))(1-a) - ck_{\min}(a)A_{\min}(a, k_{\min}(a))$$

This is entirely a function of a. The first term is that price which leaves the consumer at b (=a+k) with zero utility from Good 1, times Firm 1's market share of (1-a). The second term is the price of the campaign – cost parameter by length by height.

If Firm 1 wishes to induce a Barrier Equilibrium in Stage 2, then it will choose the value of a that maximizes the above profit function as the starting point of its campaign. The height will be $A_{min}(a)$, and its length will be $k_{min}(a)$.

The graph below shows optimal values of a for a Barrier equilibrium, with advertising and transport costs held constant, and inherent value, V, allowed to change.

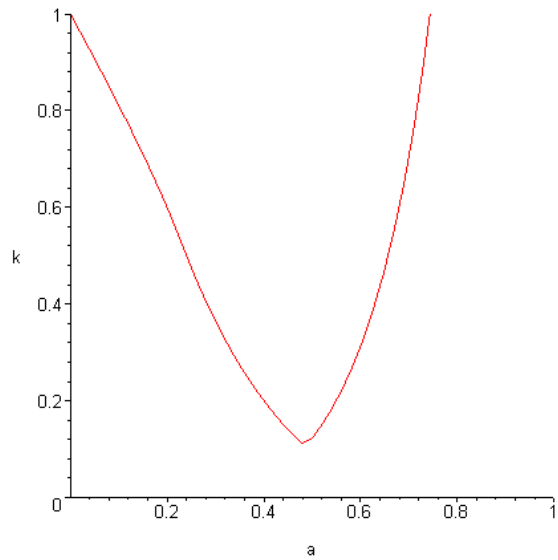
Figure 3.24: Optimal campaign start for a barrier equilibrium (numerical simulation)



There are several features of note. First, the values are fairly close to ½, indicating a campaign near the middle of the consumer line. Secondly, the graph is concave, and appears to have a discontinuity at around V=2.25.

These characteristics are due to the need for satisfying both incentive constraints simultaneously. Below is a plot of $k_{min}(a)$ for V=2.2. Firm 0's incentive constraint is very binding at low values of a, but becomes easier to satisfy as a rises, as its profits from a Barrier equilibrium also rise. Firm 1's incentive constraint becomes more difficult to satisfy at higher values of a, because its own market share from a Barrier equilibrium is (1-a). Thus, $k_{min}(a)$ is very high near the ends of the consumer line, and reaches a minimum near ½. This means that a campaign that supports a Barrier equilibrium is cheaper to erect near the middle of the line than elsewhere.

Figure 3.25: Minimum campaign length for a barrier equilibrium as a function of campaign location for V=2.2, t=1 and c=2.

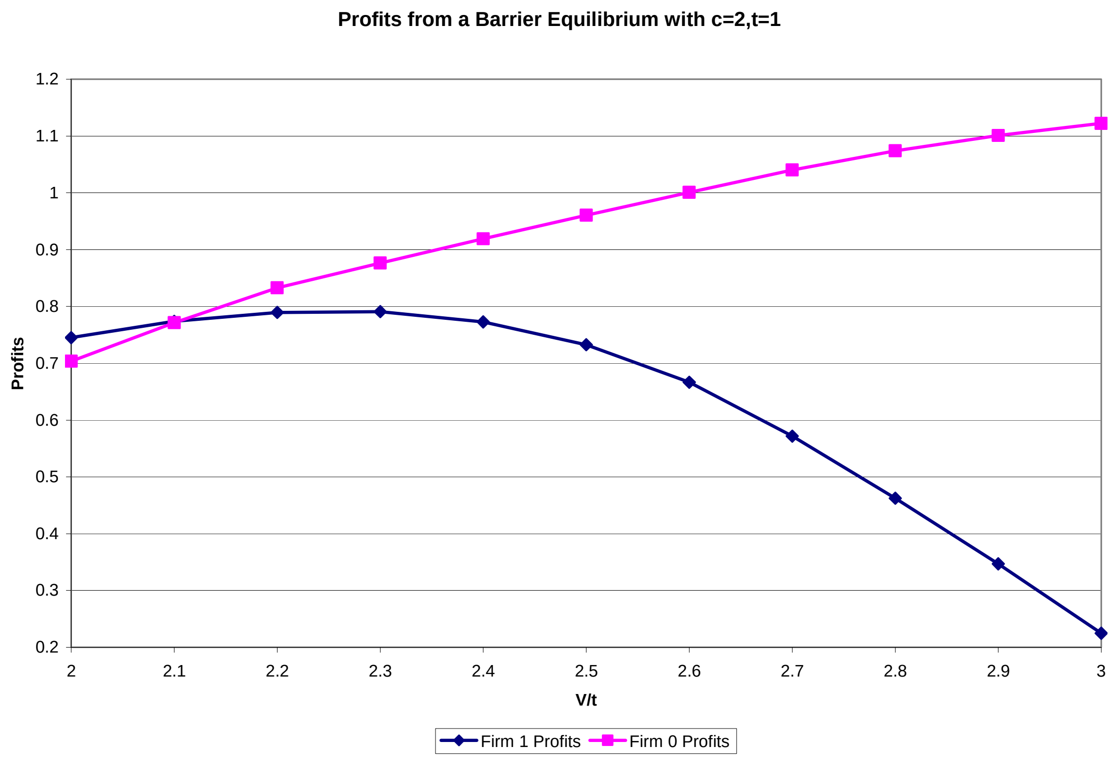


The value of $k_{min}(a)$ increases with V, and at higher values of V it will dominate the shape of the profit function through its impact on the (negative) cost parameter. That is, the point of minimum advertising cost becomes more important than the point of maximum revenue. At c=2, this happens when V is about 2.25.

When t is held constant, a rise in V is equivalent to a *decrease* in the importance of product differentiation. Intuitively, when product differentiation is of little importance, it is very difficult for the advertiser to prevent undercutting of its campaign. One ‘brand’ is as good as another – the V for both firms is identical, and when individual tastes are of little importance, it is nearly as easy to capture a consumer near a rival firm than one farther away.

As might be expected, once cost effects dominate, advertiser profits fall with V. For V sufficiently small, the advertiser earns profits considerably higher than Hotelling profits. More surprisingly, the *non-advertiser* generally does *better*. This is shown in the graph below.

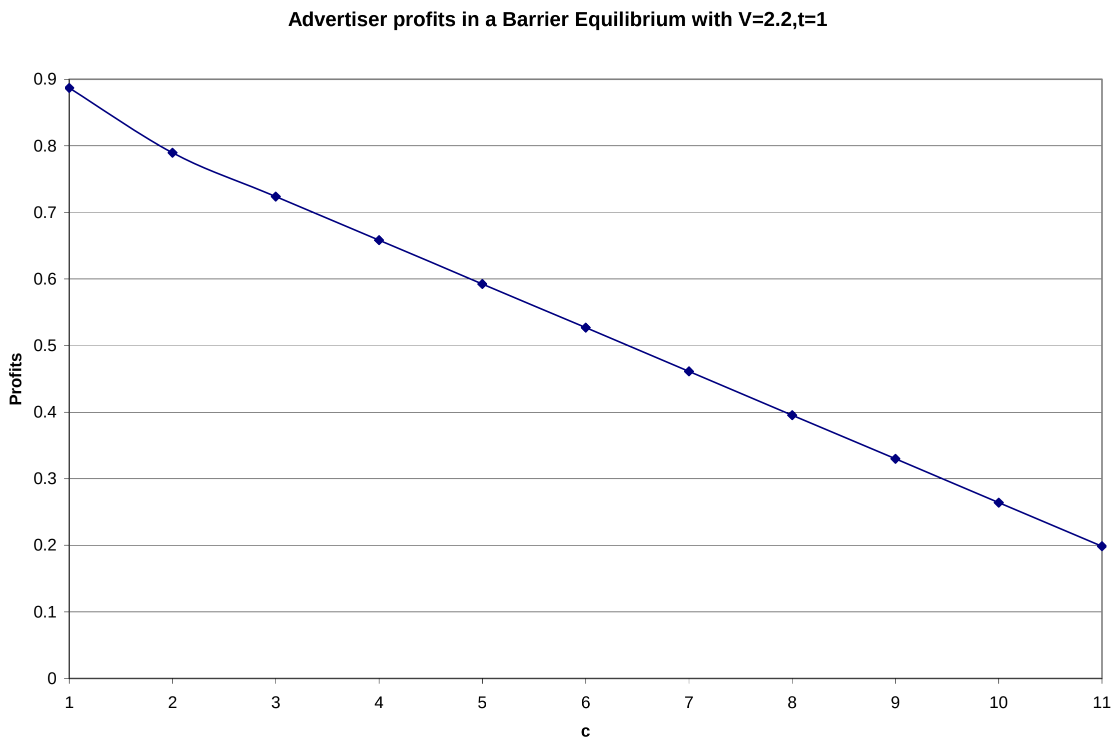
Figure 3.26: Profits from a barrier equilibrium (numerical simulation)



The reasons for this are twofold. First, as previously mentioned Firm 1 will generally advertise near the middle of the consumer line, due to the discontinuity in $k_{min}(a)$. This means that the market shares of both firms are roughly equal. Secondly, the non-advertiser obtains all the benefits of decreased price competition at *zero cost*. Except at very low values of V, this overwhelms any advantage Firm 1 might have due to a higher price or slightly higher share of sales.

The advertiser's profits fall with rising costs, but the non-advertiser's remain constant. A higher advertising cost parameter lowers the level of profits, but does not change the location of a*.

Figure 3.27: Advertiser profits in a barrier equilibrium (numerical simulation)



Results

In Stage 1, Firm 1 is faced with two choices – the *type* of equilibrium it wishes to induce in Stage 2, and the exact *location* of the campaign that it creates to secure the outcome. The first choice is simple. There are only three types of equilibria possible – Hotelling, Faux Hotelling and Barrier. The Faux Hotelling equilibrium yields better than Hotelling profits at all times. Under general conditions – for low c, and low V – the Barrier equilibrium outperforms the Faux Hotelling equilibrium in terms of advertiser profits. Thus, for a constant transport cost t, we will see Barrier advertising when V is low, and Faux Hotelling advertising when V is high. When advertising is very costly, we will only observe Faux Hotelling equilibria.

The *location* of the campaign will generally be near the middle of the consumer line. This is because the incentive constraints which determine campaign size are easiest to satisfy in this region.

Taking a Hotelling equilibrium with no advertising as a benchmark, the non-advertiser will be better off in a Barrier equilibrium and worse off in a Faux Hotelling equilibrium. In the first, the firm reaps the benefits of weakened price competition, with none of the costs. In the second, the campaign is used by the advertiser as a means to capture infra-marginal consumers.

3.5. Conclusion

Modern advertising is costly, and it is increasingly difficult to reach more than a small fraction of a good's consumers with one message. As information technology improves, consumers are also better informed about the price and qualities of existing products, even before being reached by an advertising message.

Heavy advertisement of universally recognized products, such as Coca-Cola or McDonald's fast food, suggests that there is a role for costly advertising even once consumers are aware of product price and characteristics. The content of such advertisements is often only marginally relevant to the featured product, and is frequently tailored to appeal to the market segment to which it is presented.

If this form of advertising is important, it raises the question of whether there might be an 'informational divide' favouring companies with the ability to obtain information on individual consumers and the wealth to customize advertising messages according to this data.

In the preceding pages, we examined this situation by introducing costly, targeted persuasive advertising by one firm into Hotelling's model of differentiated product competition in duopoly. The advertiser was allowed to pay $c>1$ to raise an individual consumer's willingness to pay by 1. An advertising campaign was restricted to a continuous segment of consumers who saw their willingness to pay increased by a non-negative constant A.

Once advertising was in place and paid for, both firms simultaneously set prices, after which consumers made their purchase decisions.

It was found that when product differentiation is sufficiently important, the advertiser can use a campaign to divide the consumer line, with each firm monopolizing those consumers on its side of the advertising campaign. When product differentiation is not very important, the required campaign becomes too expensive to be profitable. We instead see some slight advertising to consumers near the middle of the line.

The *non-advertiser* will benefit from advertising of the first type. Since it obtains the benefit of drastically weakened competition with none of the cost, it is in fact possible for the non-advertiser to do *better* than its advertising rival, in equilibrium.

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4. Targeted advertising of preventive medicine in Canada[27]

4.1. Introduction

Canada has a publicly funded universal health care system. When a Canadian citizen falls ill, the government pays for medical treatment. If a Canadian citizen is healthy, but at risk of illness, she must pay for preventive treatment out of her own pocket. Both pharmaceutical firms and the government make use of advertising to convince citizens to spend money on preventive treatment.

This paper studies the interaction between public health education campaigns and direct-to-consumer advertising (DTCA) of preventive treatment in the Canadian setting. Canada boasts universal health care, paid for and managed by provincial governments. The province pays for medical treatment, but does not in general subsidize measures meant to prevent illness. Such treatment must be paid for by consumers. Though health care is funded through taxes, an individual patient's treatment has a negligible impact on total health care costs. Consumers therefore treat their health-related tax burden as fixed. When deciding whether or not to purchase preventive treatment, the consumer does not take into account the financial cost of treating the illness once it manifests. This leads consumers to under-purchase prevention relative to the government's objectives. The government can try to correct this under-supply through public health education campaigns. Both the government and the pharmaceutical firm manufacturing the treatment have an incentive to increase the population's willingness to pay for prevention. However, the firm's profit-maximizing objective may conflict with the government's desire to reduce health care expenditure. High profits may not be compatible with a high level of prevention, and the firm may use its advertising campaign to raise prices and possibly lower coverage.

[27] A version of this chapter will be submitted for publication.

The Canadian government's traditional response to this conflict has been strict regulation of direct-to-consumer advertising by pharmaceutical firms. Among many other restrictions, an advertiser is allowed to either name the product, *or* its function – never both[28]. The former practice is referred to as 'reminder' advertising, and fulfils the informative function of reminding consumers of the product's existence. The latter is 'disease' advertising. Its purpose is to bring to the attention of consumers the perils of a particular disease, in the hopes that they will then be willing to pay for its prevention. By separating product identification from product characteristics, this legislation addresses the frequently-raised concerns that pharmaceutical advertising will lead to over-use of medication and higher prices.

In this paper, I consider an alternate approach to public health, in which DTCA may be used as a complement to or substitute for a public health education campaign. The interaction between public and private disease advertising has been largely ignored, and it is possible that through an appropriate choice of policy the government may be able to harness DTCA as a tool with which to achieve its cost-minimizing objective. For example, by targeting low-risk consumers with its advertising, the government may induce a pharmaceutical firm to set a low price in order to capture this low end of the market[29], increasing coverage of preventive treatment.

The problem of public versus private advertising is quite general. In the present paper I restrict myself to the targeted advertising of a patented preventive treatment of a non-infectious disease in Canada. The treatment of preventable diseases is an increasingly important component of health care expenditure in Canada, as recognized by several recent government reports[30]. Heart disease, certain cancers and hepatitis are among those illnesses that can be avoided through appropriate lifestyle choices, vaccination or prophylactic use. Despite the availability of preventive treatment, incidence of these diseases remains high, in part because such treatments must usually be paid for by the

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[29] For full details, see Section 4.3.2 below.

[30] For example, (Health Council of Canada, 2007)

patient. In recent years, patented vaccines for hepatitis and the human papillomavirus (HPV) have been the subject of intensive advertising campaigns by their manufacturers. Although Canada has strict laws concerning the style and content of such ads, it is not clear that these can be effectively enforced. In 1997, the United States relaxed its own DTCA legislation, leading to a surge in pharmaceutical advertising. Many of the ads produced in the United States' more favourable legal climate are seen by Canadian audiences, who receive American channels as part of a cable package or over the air. Further difficulties rise from the prominence of internet advertising, which allows for the private transmission of targeted advertising. Both vaccines mentioned above inoculate the patient against infectious diseases. When a disease is contagious, the decision to invest in prevention is complicated by the endogeneity of infection risk[31]. The present paper assumes that infection risk is exogenous, and therefore the preventable disease is non-infectious.

There is potential for conflict between the goals of patients, government and pharmaceutical firms. Curative health care is paid for by government, but preventive measures are usually paid for by consumers. When deciding whether or not to invest in prevention, Canadian citizens do not take into account the financial cost of treating the illness once it manifests. Similarly, when paying for health care the province has an incentive to rely on cost-effective treatments, regardless of any private discomfort or inconvenience that must be endured by the patient. In the case of preventable disease, government and private advertisers have similar goals. Their aim is to convince members of the population who would not otherwise do so, to engage in treatment. The difference lies in that the firm controls not only the advertising message, but the price of treatment, and profit-maximization may be consistent with higher prices and lower coverage than that desired by the government.

In this paper, I investigate whether it is possible for government to use private DTCA as a tool in its efforts to minimize health care expenditure in a manner consistent with the

[31] For a discussion of public and private treatment of an infectious disease, see (Geoffard and Philipson, 2001).

Canadian setting. I assume that all consumers are at risk of contracting a preventable disease. An individual's risk of infection is common knowledge. If a consumer falls ill, the government pays the medical bill. A healthy consumer may buy preventive treatment from a pharmaceutical firm. Both government and firm may pay to increase a consumer's awareness of the disease, and thereby increase her willingness to pay for prevention.

The model assumes a setting in which citizens, government and firms are perfectly informed about any given individual's risk of infection, but where a healthy consumer's perception of the personal cost of disease is subjective, and subject to change. This assumption may seem out of place, since it is tempting to attribute inadequate levels of prevention to consumer myopia or a lack of information. Exercise today, for example, involves a certain expenditure of time and effort in exchange for uncertain health benefits in the future. An individual at high risk of a disease for which there is a vaccine might under-estimate the true risk of infection. Some consumers may not be aware of the existence of preventive treatment. Advances in mass media and information technology make these scenarios increasingly unlikely. Supermarket racks are filled with magazines touting the latest diets and health supplements. Television talk shows interview fitness experts and disease survivors in between advertisements for the latest vitamin supplements. The internet provides a world of information in searchable format, and increasingly pharmaceutical advertisers target their messages to those consumers most likely to benefit from them. It is difficult for the modern Canadian consumer to plead ignorance regarding the existence of major preventable illnesses, and the steps which may be taken to avoid them.

I assume that advertising is costly and may be perfectly targeted. The advertiser is able to customize its message to an individual consumer. Ads are expensive, and it costs more than a dollar to raise willingness to pay by a dollar. Under this assumption, we will never see advertising to someone whose risk of contracting the disease is identically zero. This is true to the spirit of internet advertising and current marketing practices – it is possible for firms to obtain detailed information about the characteristics of individual consumers, and customize their marketing messages to them accordingly[32]. Such customization is costly (Schuh, 2000), which means it must be used strategically. The Canadian government's sources of information are different than those available to a private firm. However, demographic and other data allow for targeting advertising messages. Indeed, almost all advertising by the Canadian government is necessarily targeted. Canada is a bilingual, multi-cultural society. Messages sent by the government must be in the language spoken by their intended audience, and follow the correct cultural cues if they are to be effective[33].

My results hinge on the assumption that advertisements are able to influence a healthy consumer's assessment of the private cost of illness. There are many ways in which this might take place. If disease is perceived as something that may happen in the distant future, the framing of an advertisement's message may make the illness more immediate, and increase a consumer's willingness to pay for its prevention (Chandran *et al*, 2004). While consumers are assumed to know their risk of infection, this does not imply awareness of the consequences of an illness. For instance, it is possible for an individual to know that she has a 30% chance of developing colon cancer in the next ten years, without being aware of the pain and suffering that such a condition entails. Advertisements informing consumers of previously unknown negative outcomes may increase their willingness to engage in prevention.

I find that it is possible for public health campaigns to have an effect directly opposite to that intended, whether DTCA is allowed or forbidden. Under general circumstances, government-sponsored promotion of preventive treatment will lead to a higher price for such treatment, and lower coverage, than if the government had never advertised at all. Such an effect is not uncommon in campaigns intended to curtail vices such as gambling

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[32] For details, see (Zhang, 2004), (Gal-Or, 2005), (Montgomery, 1997) and (Rossi *et al*, 1996)

[33] For example, see (Larkin *et al*, 2007) on the need to tailor HIV awareness campaigns to the specific circumstances of aboriginal youth, (Messerlian and Derevensky, 2007) on customizing anti-gambling messages for young Canadian adults, and (Lagarde, 2004) on the complexities involved in a bilingual health education campaign.

(Messerlian and Derevensky, 2007) or under-age drinking (Ringold, 2002). However, these ‘boomerang effects’ are largely due to psychological factors and imperfect targeting. Most commonly, a government campaign condemning an activity will increase its attraction to rebellious youth. In the present model, advertising is perfectly targeted by assumption, and this cannot happen. An advertising message sent to youth will, by assumption, properly take their personal characteristics into account and avoid a boomerang effect. The reduction in coverage seen in the present paper is independent of psychology. There is a natural temptation for government to advertise to consumers with a high infection risk, since their expected medical costs are highest. If government succumbs to this temptation, the pharmaceutical firm will set a high price and hold these consumers for ransom. Under the conditions of the model, the rise in price is such that treatment coverage is lower than it would have been, had the government never advertised.

The model predicts that DTCA will always increase treatment coverage over the case in which such advertising is banned. If the distribution of infection risk is sufficiently smooth, then for any given price, a pharmaceutical firm will always have an incentive to use advertising to increase coverage. This is illustrated in a simple case by the following example. Suppose that advertising is very expensive, so that it costs ten dollars to raise a consumer’s willingness to pay by a dollar. Let the population consist of two individuals, one of whom is initially willing to pay \$20 for treatment, and another who is only willing to pay \$19. Treatment is sold for \$20. Although advertising is costly, the firm is willing to pay the required \$10 in advertising in order to bring in an extra \$20 in revenue. No matter how expensive advertising is, there is always a willingness to pay between 19 and 20 so that in our example, the cost of advertising is less than the additional revenue. This example ignores the interaction between the firm’s choice of price and its choice of advertising. Below, I show that when this is taken into account, the firm will always choose a treatment coverage greater than that which prevails in the absence of advertising. By implication, DTCA by the firm can be a superior alternative to advertising by the government. However, DTCA does increase the cost of treatment. This must be taken into account in any policy decision.

It is possible for a government-sponsored advertising campaign to substantially improve health care outcomes, in the form of lower treatment costs and higher coverage. For this to happen, the government must advertise to consumers with a low to medium risk of infection, and be able to refrain from advertising to consumers with a high risk of infection. By assumption, consumers are perfectly informed of their infection risk without the need for advertising. By advertising to high-risk consumers, government makes it profitable for the firm to raise its price. If it advertises to lower-risk consumers, the government encourages the firm to lower its price in order to capture more of the market. Consider the following situation. The population consists of two consumers. One consumer has a 100% risk of contracting a disease, and the other has only a 25% risk. The disease may be prevented by purchasing treatment from a monopolist. Both consumers believe that a sick person incurs $10 of pain and suffering. In the absence of advertising, the firm will charge $10, and preventive treatment will be sold only to the high-risk consumer. Now suppose that the government is willing to use advertising to increase the willingness to pay of one of the two consumers by $5. If it advertises to the high-risk consumer, the firm will charge $15 and coverage will be unchanged from the case of no advertising. If it advertises to the low-risk consumer, the price of treatment will go down to $7.50 and both consumers will purchase treatment.

The link between public and private disease advertising has been largely ignored[34]. There is a large literature on direct-to-consumer advertising of pharmaceuticals[35], and in particular a large number of papers referring to Canada's experience. Most of these deal with the advertising of curative treatments, whereas the focus of the current paper is on preventive treatments. In general, it is found that DTCA is able to influence physician prescription decisions, and such marketing both raises the price of pharmaceuticals and shifts demand toward newer, more heavily advertised drugs, independently of their medical benefits. The literature on targeted advertising is small, and deals mostly with

[34] For an exception, see (Geoffard and Philipson, 2001).

[35] For example, (Findlay, 2001), (Batchlor and Laouri, 2003) and (Brekke and Kuhn, 2006). This list is not exhaustive.

informative advertising[36]. Consumers are unaware of the price and existence of a product unless targeted. As a result, firms focus their advertising campaigns on those consumers with the highest likelihood of buying their product. This differs from the current model, in which consumers are assumed to be informed of the product's price and existence, and advertisers focus their efforts on marginal consumers. There has been some interesting work on American Health Maintenance Organizations (HMOs) and their coverage of preventive treatment[37]. Fundamental differences between the Canadian and American health care systems lead to substantially different results in the present paper. My focus is not on a shifting bundle of consumers who choose private health insurance providers, but on the interaction between public and private promotion of preventive treatment in the context of a government-funded universal provision of curative health care. The paper closest in spirit to the present one is (Rubin and Schrag, 1996), which examines agency problems arising from informative drug advertising. They find that HMOs will in general under-provide prescription drugs relative to the needs of their subscribers, but drug companies may use advertising to correct this agency problem.

The benefits of public advertising depend crucially upon the ability of the government to commit to an advertising strategy. If the government is capable of such commitment, then a public health education campaign will lead to high coverage at a low cost to consumers and government. If not, there will be a pseudo-'boomerang effect' as the pharmaceutical firm uses the government's willingness to advertise to hold high-risk consumers for ransom. These results are independent of whether or not there is a ban on advertising by the private sector. If the government abstains from advertising, the firm's campaign provides a middle path with prices, cost and coverage between the two choices provided by public advertising. Thus, if the government is not capable of commitment to a particular advertising campaign, it is best to allow the firm to advertise in its place[38].

[36] See (Gal-Or, 2005), (Iyer *et al*, 2005) and (Esteban *et al*, 2004).

[37] (Rubin and Schrag, 1996), (Gohmann, 2005), (Dor, 2004) and (Michelli and Heffley, 2002)

[38] If the government has complete control the price of preventive treatment, then the coverage-reducing consequences of a lack of commitment may be avoided. However, the presence of the large American

4.2. The model

Society consists of government, a pharmaceutical firm and a unit mass of consumers. Consumers are at risk of a preventable disease. Individuals differ only in their susceptibility to a non-infectious, preventable disease. An individual's infection risk, x, is common knowledge. This knowledge can be thought of in terms of the existence of a public database linking demographic characteristics to infection risk. Consumers are uniformly distributed on the unit line according to their infection risk. The number of consumers is normalized to unity.

Figure 4.1: Uniform distribution of consumers

A monopolist offers preventive treatment at a price p. If a consumer pays for preventive treatment, her risk of infection becomes zero. There are no side effects.

The government offers curative treatment at no charge. If a consumer falls ill, the government pays for her medical treatment and incurs a financial cost g. There is pain and suffering associated with the illness. The true extent of this private harm from illness

market to the south, and the popularity of cross-border shopping for pharmaceuticals constrain the Canadian government's ability to set the price of medication.

is only revealed to a consumer once the disease manifests. Initially, all consumers believe that the private cost of illness is equivalent to the loss of h dollars.

It is possible for either the firm or the government to engage in perfectly targeted disease advertising. An advertiser pays a fee a(x) to raise the perception of the private cost of illness of the consumer with infection risk x, by a(x). For example, suppose that the consumer with 48% risk of infection believes that the private harm from illness is 98 dollars. The government can pay 2 dollars to raise her estimation of the private harm of illness to 100 dollars. Advertising is additive in its effects. If a consumer is targeted with 2 dollars of advertising from the government and 3 dollars from the firm, her estimation of the private harm of the disease rises by 5 dollars.

Consumer utility is equal to the expected harm from illness, less the price paid for treatment. I assume that consumers are risk neutral.

The utility of an untreated consumer is

$$U(x) = -x(h + a^f(x) + a^g(x))$$

Here, x is the consumer's risk of infection, and h is her initial belief about the private cost of illness. This belief is common to all consumers. Advertisement by the firm and government are denoted a^f and a^g, respectively. Since advertising is targeted, it is written as a function of infection risk x.

The utility of a consumer who pays for treatment is independent of infection risk, and depends only on the price of treatment:

$$T(p) = -p$$

A consumer will pay for treatment if and only if $U(x)<T(p)$.

Government pays for the curative treatment of sick consumers, and for its own advertising campaign. Treatment of an ill consumer costs g dollars. The expected cost of illness from a consumer with infection risk x, is xg. The cost of advertising to the consumer at x is $a^g(x)$. The government's objective function is equal to the sum of its costs, and given by

$$G(p) = g\int_0^1 H(U(x) - T(p))xdx + \int_0^1 a^g(x)dx$$

Here, H( ) is the Heaviside step function and p is the price of preventive treatment.

The monopolist's profits are equal to its revenue from sales of preventive treatment less its advertising costs. By assumption, the preventive treatment is costless to manufacture. The firm's profit function is therefore

$$\Pi(p) = p\int_0^1 H(T(p) - U(x))dx - \int_0^1 a^f(x)dx$$

Here, $a^f(x)$ is the firm's expenditure on advertising to the individual with infection risk x.

Table 4.1: Notation

g	Cost to the government of treating a diseased individual.
h	Private cost of the disease, or awareness, prior to advertising.
x	Risk of infection. $0 \le x \le 1$
$a^g(x)$	Advertising by the government to the individual with infection risk x.
$a^f(x)$	Advertising by the firm to the individual with infection risk x.
$h + a^f(x) + a^g(x)$	Private cost of the disease to the individual with infection risk x after advertising
K_j, K_j^*	K in Case j. An asterisk denotes an equilibrium value. For example, p_3^* is the equilibrium price for Case 3. Cases are defined in Table 2, below.

Timing

Government begins by deciding upon the regulatory climate in which it and the firm will operate. It may choose whether or not to allow advertising by the firm, and whether or not to allow its own advertising. These choices are binding.

After this initial decision, the government may be either the first or second agent to move. When the government moves, it implements its advertising campaign, $a^g(x)$, provided that advertising by the government is not forbidden. When the firm moves, it implements its advertising campaign, $a^f(x)$, provided that such advertising is allowed. During its move, the firm also chooses the price of treatment, p.

The sequence of movement need not be interpreted literally. As in the Stackelberg model of quantity competition, being the first mover may be interpreted as an ability to commit to a particular strategy. An inability to commit places the government in the position of a follower.

Once firm and government finish their moves, consumers make their purchase decisions.

The disease will manifest after the consumer's purchase decision is made. The model does not look beyond the purchase of preventive treatment. It is to be understood that all costs associated with the future, such as the government's costs of treating the disease, are in present discounted terms.

Figure 4.2: Timing

There are six distinct combinations of order of movement and regulatory climate to be considered. I will analyze the sub-game perfect Nash equilibrium to each of these cases in turn, taking the regulatory climate and order of movement as given. For example, when analyzing the case where all advertising is allowed and the government moves second, I will assume that government does not have the option of moving first. This approach is consistent with the primary goal of this paper, which is to find and categorize the environments in which private advertising is beneficial to government and health care outcomes.

Table 4.2: Cases

		Firm		
Government		No ads	First mover	Second mover
	No ads	Case 1	Case 4	
	First mover	Case 2		Case 5
	Second mover	Case 3	Case 6	

In Section 3, I consider those cases in which the firm is not allowed to advertise. Section 4 deals with the remaining cases. Section 6 provides a summary of findings and concluding remarks.

4.3. Advertising by the firm is forbidden

In this section, I examine Cases 1, 2 and 3. They hold in common that all advertising is done by the government, when any advertising is done at all.

4.3.1 Case 1: no advertising

Advertising is forbidden to both the government and the firm.

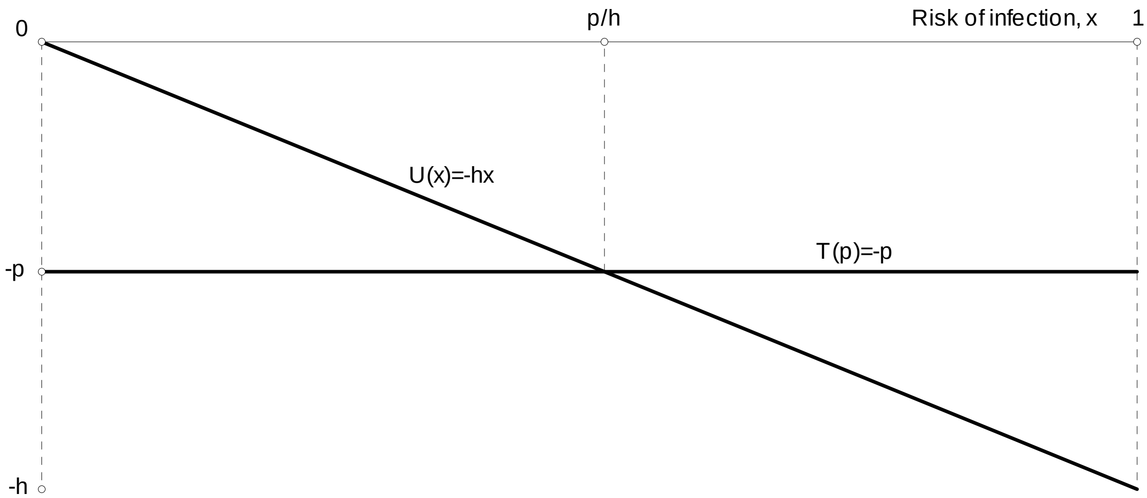
A unit mass of consumers is uniformly distributed along a line of length 1. Consumers differ only in their susceptibility to illness. Their address on the unit line corresponds to their chance of contracting the disease if left untreated.

Each consumer weighs her expected cost from illness, xh, against the cost of preventive treatment, p. If p<xh, the consumer pays for treatment. The consumer indifferent to treatment has infection risk

$$z_1 = \frac{p}{h}$$

Consumers pay for treatment if and only if their infection risk, x, is greater than z_1.

Figure 4.3: case 1 illustrated



A fraction $(1 - z_1)$ of consumers purchase treatment.

The firm's profits are equal to its revenue. The monopolist's profit function is given by

$$\Pi_1(p) = p(1 - z_1)$$

The profit-maximizing price is obtained by solving the firm's first-order conditions. Since there is no advertising in Case 1, this is the firm's only decision, and the profit-maximizing price is the Nash equilibrium:

$$p_1^* = \frac{h}{2}$$

This leads to profits of

$$\Pi_1^* = \frac{h}{4}$$

The indifferent consumer has an infection risk of

$$z_1^* = \frac{1}{2}$$

Since consumers with infection risks greater than ½ buy treatment, the government's costs consist only of the expected medical costs for consumers with x < ½. The government's objective function is therefore given by

$$G_1^* = g\int_0^{1/2} x dx = \frac{g}{2}$$

When neither the firm nor government advertise, treatment coverage will be 50%. Consumers pay for treatment if their infection risk exceeds 50%. The price of treatment is equal to half of the perceived private cost of illness, h. Government's expected costs are half of the cost of curative treatment for a single individual.

4.3.2 Case 2: ex-ante advertising by the government

Consider the two-stage game in which advertising by the firm is banned, the government advertises in the first stage and the firm sets the price of treatment in the second.

The government's advertising campaign consists of a complete specification $a^g(x)$ of its advertising expenditure to each consumer.

In the second stage, the firm observes the government's advertising, $a^g(x)$, and sets its price, p. The expected utility of a consumer who does not purchase treatment is

$$U_2(x) = -x\left(h + a_2^g(x)\right)$$

The utility of a consumer who purchases treatment is

$$T(p) = -p$$

Consumers will purchase treatment if and only if $U_2(x) < T(p)$, and so the firm's profits are given by

$$\Pi_2(p) = p\int_0^1 H(T(p) - U_2(x))dx$$

where H ( ) is the Heaviside step function. The firm's choice of price will therefore be conditional on the government's advertising campaign.

Now consider the government's choice of advertising in the first period. Let the price chosen by the firm in the second stage, upon observation of the government's campaign, be p^g. The government will not advertise to consumers who buy the treatment regardless of advertising. When the firm sets a price of p^g, the indifferent consumer is at p^g/h. The government will not advertise to consumers with $x>p^g/h$.

Lemma 4.1: Suppose government is the sole advertiser and first mover. Then in equilibrium all consumers advertised to have the same willingness to pay for treatment.

Proof: Let the price set by the firm in the second stage be p^g, and consider those x for which $a^g(x)>0$. If $U_2(x)<-p$ for some x, then the government would strictly prefer to set $U_2(x)=-p^g$, achieving treatment at a lower cost. If $U_2(x)>-p$ for some x, then the government may achieve the same result (non-treatment) by setting $a^g(x)=0$. ■

The government need only advertise to the extent that makes a targeted consumer indifferent to purchasing treatment. If it pays for more advertising, the additional expenditure is wasted, because the consumer would have bought treatment with less advertising. If it pays for too little advertising, the outlay is wasted because the consumer will not purchase treatment.

That is, $a^g(x)$ is such that U(x) = T(p) and

$$a_2^g(x) = \frac{p^g}{x} - h$$

The government's goal is to increase treatment coverage, and thereby lower its expected health care costs. It will never choose a coverage target less than ½, since it can achieve 50% coverage by not advertising (as shown in Section 3.1). It is impossible to reach total coverage, because no amount of advertising will convince the consumer with infection risk x=0 to purchase treatment. Coverage in this case will therefore lie strictly between ½ and 1.

The government's advertising campaign will target a continuous swathe of consumers. Let z_2 be the lowest infection risk for which $a_2^g(x) > 0$. Then it must be the case that $a_2^g(x) > 0$ for all x greater than z_2 and less than p^g/h. Recall that the government must pay g dollars in medical costs if a consumer contracts the disease. If $a^g(z_2)>0$, this implies that the government's expected benefit from advertising, z_2g, is greater than the cost of advertising to that consumer, $p^g/z_2 - h$. Since $(gx - (p^g/x - h))$ rises with x, this means that the benefit must exceed the cost for all $x>z_2$, as well. However, the government will not advertise to consumers with $x>p^g/h$, because they are willing to purchase treatment even without advertising.

Thus, in equilibrium, for a desired coverage of $(1-z_2)$, the government's advertising campaign must take the form

$$a_2^g(x,z_2) = \begin{matrix} \dfrac{p^g}{x} - h & z_2 < x < \dfrac{p^g}{h} \\ 0 & otherwise \end{matrix}$$

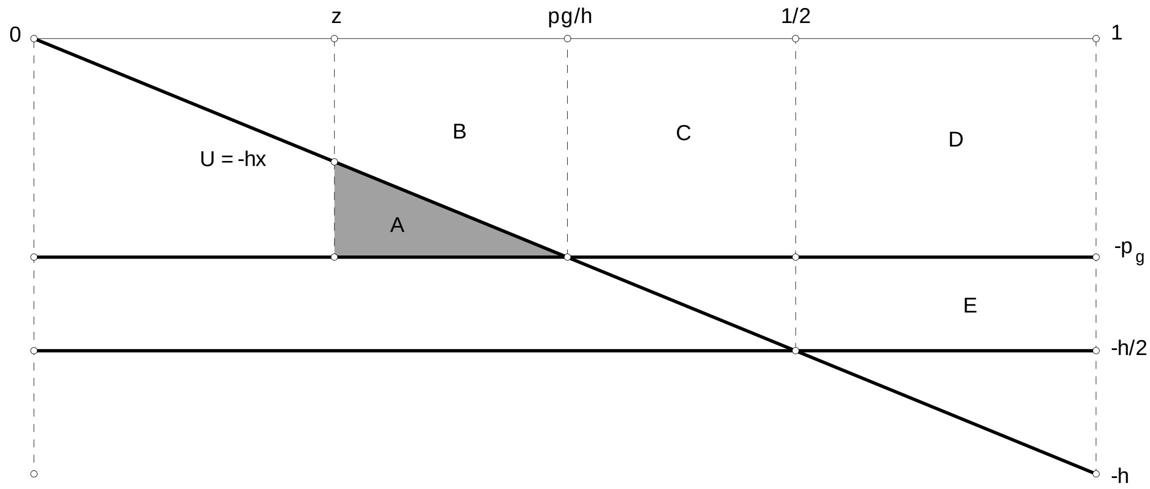
Now consider the firm's choice of price. The government does not advertise to consumers with x>1/2, so it is possible for the firm to choose p=h/2 and earn profits of h/4, as in Case 1. These are its reservation profits. If the firm chooses a price p^g<h/2, as required for coverage greater than ½, then the profit from doing so must at least be equal to the firm's reservation profits.

If the firm chooses p = h/2, its profits are h/4. If the firm chooses p = p^g, then U(x) > 0 for all x>z, and the firm's profits are equal to $p^g(1-z_2)$. When $p^g = h/(4(1-z_2))$, the two are equal. If the government desires coverage of $(1-z_2)$, it must therefore choose its campaign such that

$$a_2^g(x,z_2) = \begin{matrix} h\left(\dfrac{1}{4(1-z_2)x} - 1\right) & z_2 < x < \dfrac{1}{4(1-z_2)} \\ 0 & otherwise \end{matrix}$$

In the equilibrium for Case 1, the firm's surplus is equal to the sum of areas D and E in the diagram below. Through advertising, the government may raise awareness of the consumers on $[p^g/h, z_2]$ so that they are willing to pay for the treatment at a price p^g. The firm's surplus from setting a price p^g becomes A + B + C + D. If A + B + C is equal to E, then the firm will charge the lower price.

Figure 4.4: case 2



Equating A + B + C to E uniquely determines p^g as a function of z_2. In the limit of full coverage (that is, z_2=0), p^g = h/4. This is the lower limit for prices under ex ante government advertising. In no case will we see p^g>h/2, since this would mean lower coverage than in the case of no advertising.

The government chooses the amount of coverage that minimizes its objective function, which is the sum of advertising costs and expected health care costs from non-treatment. That is,

$$G_2(z_2) = g\int_0^{z_2} xdx + \int_{z_2}^{1}\left(\frac{1}{4(1-z_2)x} - h\right)dx$$

In equilibrium, coverage rises with the cost of medical treatment, g, as the government becomes more willing to invest in disease prevention. For a constant g, coverage falls with the intrinsic private cost of illness, h, as this increases the firm's reservation profits and makes advertising more expensive. As shown in the theorem below, for an appropriately large value of g, coverage may be brought arbitrarily close to 1.

When advertising by the firm is banned, government advertises and the government is the first mover, health care expenditure is less than in Case 1. Coverage is higher than in

Case 1, and the price of treatment is lower, but firm profits – and therefore total consumer expenditure – remain at the levels established in the equilibrium to Case 1.

Theorem 4.1: (case 2) Suppose government is the sole advertiser and first mover. Then the equilibrium price is lower than in Case 1, and treatment coverage is higher.

Proof: In the Appendix

4.3.3 Case 3: ex-post advertising by the government

In Case 3, advertising by the firm is forbidden, but advertising by the government is allowed. This regulatory environment is known to all agents.

The timing is such that the firm moves first, setting its price in full knowledge of the government's ability to advertise. It will therefore choose a higher price than in Case 1, where the government cannot advertise.

After the firm has set its price, the government implements its awareness campaign. Finally consumers make their purchase decisions.

The government takes the firm's price p as given. If it does nothing, then the indifferent consumer has infection risk p/h and treatment coverage is (1 – p/h). Through advertising, the government is able to raise the awareness of individual consumers so that they are indifferent to paying for treatment.

Letting $a^g(x)$ be government spending on advertising to the individual with infection x, the utility of an untreated consumer is

$$U_3(x) = -(h + a^g(x))x$$

The utility of a treated consumer is $T(p) = -p$.

The advertising required to make a consumer indifferent to treatment is $(p/x - h)$. The government will target all consumers who would not pay for treatment on their own, and for whom the benefit of treatment meets the cost of advertising. The expected benefit to the government of treating the consumer with infection risk x is in the form of forgone medical costs, xg.

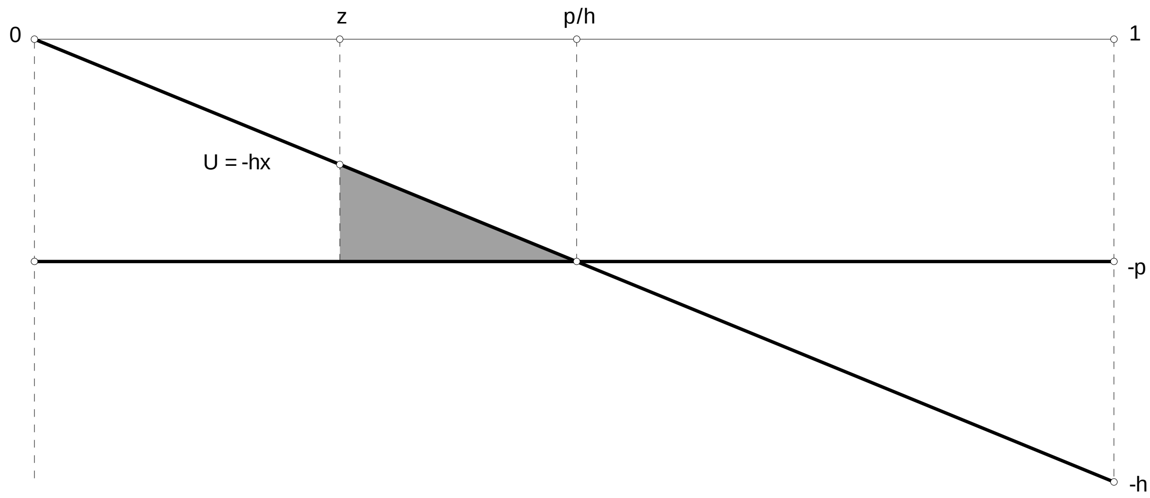
The government's surplus from advertising to the individual with infection risk x is $xg - p/x - h$. This rises with x.

The lowest risk infection targeted by government ads, z_3, will be that at which benefit is equal to cost, so that $gz_3 - p/z_3 - h = 0$. There are two solutions to this equation, only one of which is positive, and so

$$z_3(p) = \frac{\sqrt{h^2 + 4gp} - h}{2g}$$

Advertising is illustrated in the diagram below.

Figure 4.5: case 3



The firm has no advertising costs, and so its profits are equal to revenue:

$$\Pi_1(p) = p(1 - z_3(p))$$

The firm's first-order conditions can be solved for the profit-maximizing price, p_3^*. The full expression is presented in the proof of Theorem 4.2.

In the case of ex-post advertising by the government, the price of treatment is never less than in the case of no advertising. The limit of p_3^* as g tends to zero is h/2, the equilibrium price in the case of no advertising, and the profit-maximizing price rises with g.

This rise in price will lead to a pseudo-boomerang effect. For positive g, coverage is always lower than in the absence of advertising. As g ranges from zero to infinity, $z_3(p_3^*)$ ranges from ½ to 2/3. Since prices are never lower and coverage is never higher, total health care costs are never less than in the case of no advertising.

If the government cannot commit to advertising before the firm sets its price, then it is best to not advertise.

Theorem 4.2: (case 3) If the government is the sole advertiser and advertises after the firm sets its price, then in equilibrium the price of treatment is higher and coverage is lower than in Case 1.

Proof: In the Appendix.

4.4. Advertising by the firm is allowed

In this section, I examine Cases 4, 5 and 6, in which the firm is allowed to advertise.

4.4.1. Case 4: the firm as sole advertiser

Advertising by the firm will lead to higher prices and higher coverage than in Case 1, where neither government nor firm are allowed to advertise.

Consider the case where the firm is sole advertiser. The monopolist advertises, then sets its price, after which consumers make their purchase decisions.

The firm's targeted advertising campaign consists of a complete specification $a^f(x)$ of advertising expenditure as a function of infection risk.

If a consumer purchases preventive treatment from the firm at a price t, her utility is T(p) = -p. If a consumer does not purchase treatment, her expected utility is

$$U_4(x) = -x(h + a^f(x))$$

where x is the risk of infection. Consumers will buy treatment if and only if the benefits of treatment exceed the cost, which happens when $T(p)>U_4(x)$.

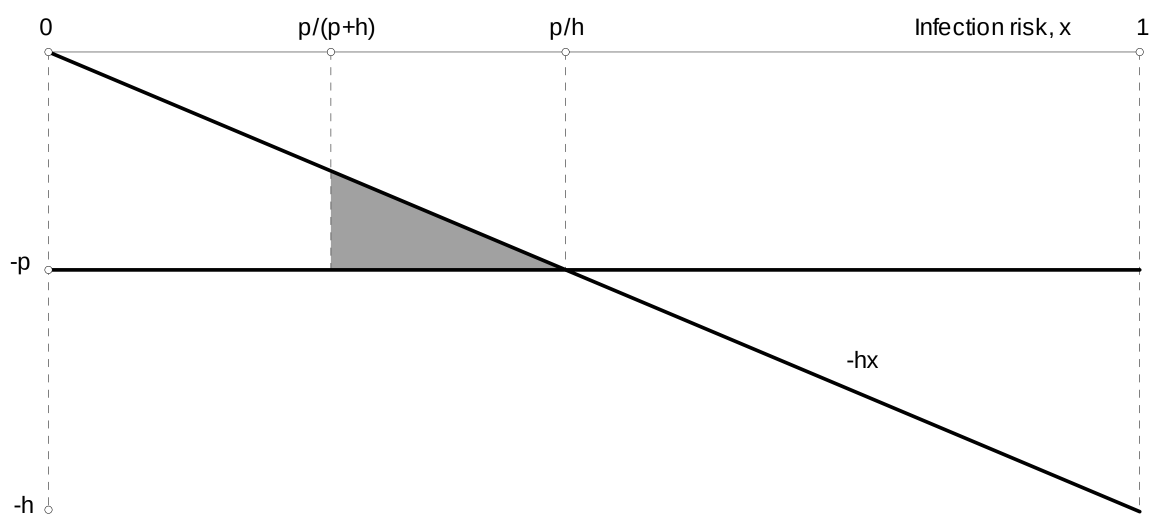
For consumers with $x>p/h$, $T(p)>U_4(x)$ even when $a^f(x) = 0$. There is no benefit to the firm from advertising to these high-risk individuals, since they will buy the product on their own.

Low-risk consumers with $x<p/h$ will not buy treatment unless they are advertised to. The firm has no incentive to advertise to them past the point that makes them indifferent to purchasing the product. As such, whenever advertising is positive, $U_4(x) = T(p)$ and $a^f(x) = p/x - h$

The firm will advertise to consumers as long as advertising produces additional revenue in excess of its costs. This is true whenever $a^f(x)<p$. That is, when $x>p/(p+h)$. The firm's campaign thus takes the form

$$a_4^f(p,x) = \begin{cases} \dfrac{p}{x} - h & \dfrac{p}{p+h} \le x \le \dfrac{p}{h} \\ 0 & otherwise \end{cases}$$

Figure 4.6: case 4



Since it has no production costs, the firm's profits are equal to its revenue minus advertising costs. All consumers with $x>p/(p+h)$ will pay for treatment. The firm must pay for advertisements sent to consumers with infection risks between $p/(p+h)$ and p/h.

The firm's profit function can therefore be written in terms of price, as

$$\Pi_4(p) = p \int_{\frac{p}{p+h}}^{1} dx - \int_{0}^{1} a^f(x)dx$$

$$\Pi_4(p) = p\left(1 - \frac{p}{p+h}\right) - \int_{\frac{p}{p+h}}^{\frac{p}{h}} \left(\frac{p}{x} - h\right)dx$$

Differentiating with respect to p and solving the first-order conditions yields a profit-maximizing value of p, p_4^*, equal to αh, where

$$\alpha \equiv \frac{1}{LambertW(1)} - 1$$

The Lambert W function is the transcendental inverse function of f(x) = xe^x, and α is about equal to 0.76.

The price obtained with the firm as sole advertiser is higher than the benchmark price of h/2 obtained in Case 1. When compared to Case 1, advertising by the firm raises the price of treatment by over 50%. However, for high medical costs, the price of preventive treatment in Case 4 is lower than the cost of treatment in Case 3, when the government is a follower and sole advertiser. In that situation, the price of treatment increases without bound as the cost of treating infected patients rises.

Although price of treatment rises linearly with the private cost of illness, coverage is independent of the value of h. When p = αh, p/(p+h) = α/(1+ α). This is a constant, equal to about 0.43, and coverage is thus always about 57%. In Case 1, only 50% of consumers purchase treatment. Advertising by the firm provides greater coverage than in the benchmark case.

Government costs are lower than in Case 1, because the increase in coverage is paid for entirely by consumers and the firm.

Theorem 4.3: (case 4) Suppose the firm is the sole advertiser. Then in equilibrium, price is αh and treatment coverage is $\frac{1}{1+\alpha}$ where $\alpha \equiv \frac{1}{LambertW(1)} - 1$.

Proof: In the Appendix.

4.4.2. Case 5: the firm as second advertiser

Consider the case where both the firm and the government are able to advertise, but the firm's advertising takes place after that of the government. In this situation, coverage will be greater than when the firm is the sole advertiser, though the price of treatment will not necessarily be lower.

It is a two-stage game. In Stage 1, the government advertises to consumers. In Stage 2, the firm advertises, and sets it price. Finally, consumers make their purchase decisions.

The utility of a treated consumer is T(p) = - p. The utility of an untreated consumer with infection risk x is

$$U_5(x) = -x(h + a^f(x) + a^g(x))$$

Here, $a^f(x)$ is the firm's advertising expenditure on the consumer with infection risk x, and $a^g(x)$ is the government's.

A consumer will purchase treatment if and only if $T(p)>U_5(x)$.

In stage 2, the firm observes the government's advertising choices before making its own advertising decision. It will not target consumers that will purchase the good without additional advertising. The firm will advertise to consumers who would otherwise not purchase treatment so long as the benefit of doing so exceeds the cost. The benefit of convincing the consumer with infection risk x to purchase treatment is equal to the revenue from an extra sale, p. The cost of advertising is equal to the amount of advertising needed to make the consumer indifferent to purchasing treatment at price p. Firm advertising thus takes the form

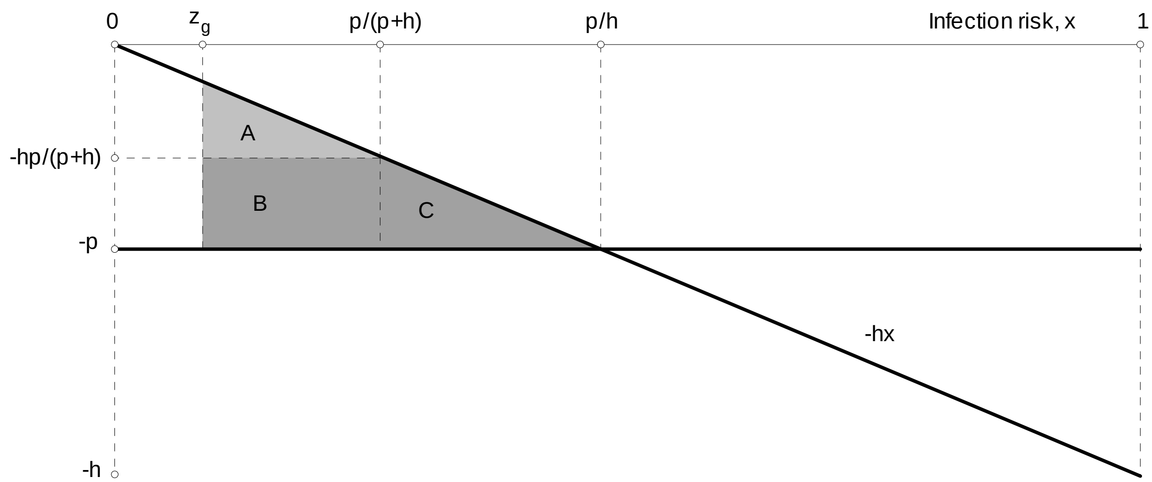
$$a_5^f(x) = \begin{cases} 0 & -x(h + a^g(x)) < -p \\ 0 & \dfrac{p}{x} - h - a^g(x) > p \\ \dfrac{p}{x} - h - a^g(x) & otherwise \end{cases}$$

The government's goal is to increase treatment coverage, and thereby lower its expected health care costs. It will never choose a coverage target less than $\frac{1}{1+\alpha}$ (about 57%), since by Theorem 4.3 it can achieve this coverage by not advertising. Total coverage is impossible, because the consumer with infection risk x=0 will never pay for treatment. Coverage in this case will therefore be strictly between $\frac{1}{1+\alpha}$ and 1. Consumers with an infection risk higher than $\frac{\alpha}{1+\alpha}$, the lowest infection risk that purchases treatment when the firm is the sole advertiser, will not be targeted by government advertising.

The government will not advertise to consumers that the firm is willing to fund entirely. When the government advertises, it will do so in such a way as to extract the firm's entire surplus.

This is illustrated in the diagram below – the government's contribution to awareness is area A, while that of the firm is B + C.

Figure 4.7: case 5



As established above, if x<p/(p+h), the cost of advertising required to bring a consumer to indifference exceeds the price of treatment. The government can use advertising to 'top up' willingness to pay in this region to the lowest level required for the firm to be willing to pay for advertising. This top-up is more expensive for individuals with lower infection risks. When $x>z_g$, the government's benefit from preventive treatment is greater than the cost of advertising. If $x<z_g$, the government is unwilling to pay for additional treatment..

The government's advertising campaign takes the form

$$a_5^g(x) = \begin{cases} \frac{p}{x} - h - p & z_g \leq x \leq \frac{p}{p+h} \\ 0 & otherwise \end{cases}$$

Where

$$\frac{p}{z_g} - h - p \equiv z_g g$$

The firm's advertising campaign will be

$$a_5^g(x) = \begin{cases} \frac{p}{x} - h & \frac{p}{p+h} \le x \le \frac{p}{h} \\ p & z_g \le x \le \frac{p}{p+h} \\ 0 & otherwise \end{cases}$$

Now consider the firm's choice of price. The government does not advertise to consumers with x> $\frac{\alpha}{1+\alpha}$, so it is possible for the firm to choose p=αh and earn profits of approximately h/3, as in Case 4. These are its reservation profits. If the firm chooses a lower price, then the profit from doing so must at least be equal to the firm's reservation profits.

While it is not possible to rule out prices lower than αh entirely, they are unlikely to form part of an equilibrium.

Suppose that in the second period, the firm sets a price p less than αh. If the government did not advertise, such a price would lead to profits less than those in Case 4. If the firm's profits are to be equal to reservation profits, then for some x the government must be providing advertising such that $a^g(x)>0$ and $p/x - h - a^g(x) < p$. That is, for some infection risk x, the government is paying for more advertising than is strictly necessary to convince the firm to capture that consumer. The government therefore has an incentive to deviate from its advertising strategy at that particular infection risk, weakening the stability of any possible equilibrium involving a price less than αh.

If we assume that the firm sets p= αh, as in Case 4, then all incentives are satisfied. The firm earns its reservation profits, and the government spends the minimum amount necessary on advertising. The price is by definition the same as when the firm is the sole advertiser, and coverage is necessarily higher.

4.4.3. Case 6: fhe firm as first advertiser

If the government cannot commit to advertising before the firm sets its price, then allowing advertising by the firm will increase coverage, but raise the price of treatment.

Consider the case where direct-to-consumer advertising by the firm is allowed, and the government advertises after the firm. Timing is as follows. In Stage 1, the firm sets its price and implements its advertising campaign. In Stage 2, the government takes the firm's advertising and price as given, and sets its own advertising campaign. Consumers then make their purchase decisions.

The utility of a treated consumer is T(p) = - p. The utility of an untreated consumer with infection risk x is

$$U_6(x) = -x(h + a^f(x) + a^g(x))$$

Here, $a^f(x)$ is the firm's advertising expenditure on the consumer with infection risk x, and $a^g(x)$ is the government's.

A consumer will purchase treatment if and only if $T(p)>U_6(x)$.

In stage 2, the government observes the firm's advertising choices before making its own advertising decision. It will not target consumers that will purchase the good without additional advertising. The government will advertise to consumers who would otherwise not purchase treatment so long as the benefit of doing so exceeds the cost. The benefit of convincing the consumer with infection risk x to purchase treatment is equal to the expected medical costs from leaving the individual untreated, xg. The cost of advertising is equal to the amount of advertising needed to make the consumer indifferent to purchasing treatment at price p. Government advertising thus takes the form

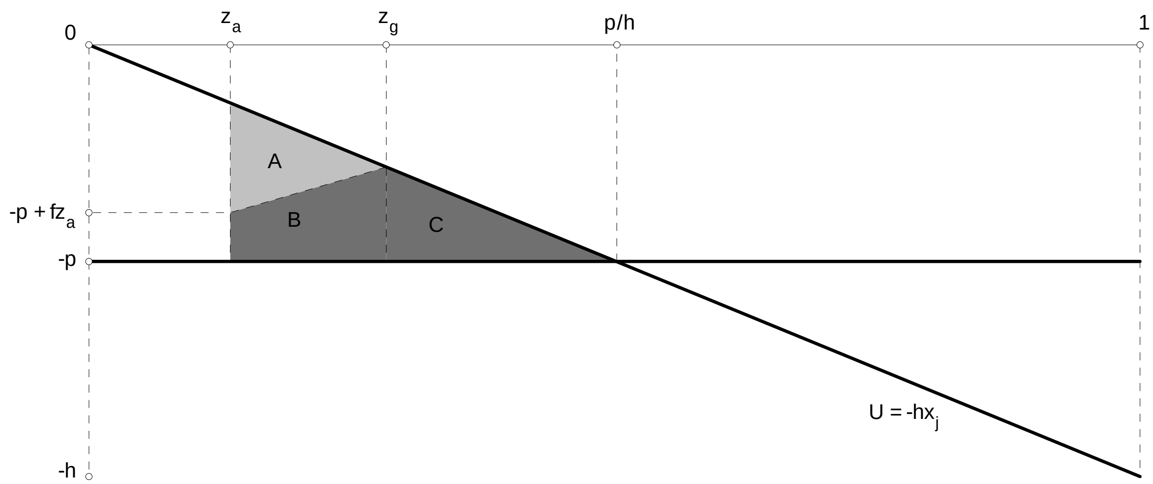
$$a_6^g(x) = \begin{cases} 0 & -x(h + a^f(x)) < -p \\ 0 & \frac{p}{x} - h - a^f(x) > xg \\ \frac{p}{x} - h - a^f(x) & otherwise \end{cases}$$

In stage 2, the government will advertise to consumers with $U_6(x)-a^g(x)>-p$ so long as the benefit of doing so, xg, exceeds the cost, $p/x - h - a^f(x)$.

In stage 1, the firm will not advertise to consumers with x>p/h, since they are willing to buy the treatment at price p. Neither will it advertise to consumers that the government is willing to fund entirely. When it does advertise, it will do so in such a way as to extract the government's entire surplus.

This is illustrated in the diagram below – the firm's contribution to awareness is area A, while that of the government is B + C.

Figure 4.8: case 6



When $x>z_g$, the government's benefit from preventive treatment is greater than the cost of advertising. If $x<z_g$, the government is unwilling to pay for additional treatment. The firm can use advertising to 'top up' willingness to pay in this region to the lowest level required for the government to be willing to pay for advertising. This top-up is more expensive for individuals with lower infection risks. When $x=z_a$, the cost of top-up is equal to the price of treatment, p.

The government's advertising campaign takes the form

$$a_6^g(x) = \begin{cases} \frac{p}{x} - h & z_g \le x \le \frac{p}{h} \\ gx & z_a \le x \le z_g \\ 0 & otherwise \end{cases}$$

The firm's advertising campaign is

$$a_6^f(x) = \begin{cases} \frac{p}{x} - h - gx & z_a \le x \le z_g \\ 0 & otherwise \end{cases}$$

The boundaries of each campaign are the points at which advertising cost becomes equal to the marginal benefit of coverage – z^g in the case of government, and z^a in the case of the firm:

$$\frac{p}{z_g} - h \equiv z_g g$$

$$\frac{p}{z_a} - h - g z_a \equiv p$$

All consumers with $x>z^a$ will purchase treatment at a price p. The firm's profit function is then equal to revenue minus advertising costs:

$$\Pi_6(p) = p(1 - z_a) - \int_{z_a}^{z_g} a^f(x)dx$$

It can be shown numerically that the profit-maximizing price is approximately α(f+h), implying treatment coverage between 0.43 and 0.56[39] - that is, between the coverage achieved by ex-post government advertising alone, and firm advertising alone. Coverage falls with medical costs g and rises with private costs h. When medical costs are high, the coverage-lowering effect of ex-post government advertising dominates. When they are low, the coverage-increasing effect of firm advertising is more important.

4.5. Conclusion

Public health education campaigns intended to increase coverage of preventive treatment may well have the opposite effect. If a pharmaceutical firm engages in direct-

[39] The first-order conditions must be solved numerically, and this result is most easily shown using the 'guess and verify' method. Take the derivative of the profit function with respect to p. Substitute p=0.75(h+g). Taking limits as h and f tend to zero and infinity, this is always positive. Now substitute p=0.78(h+g). Taking the same limits, in this case the derivative is always negative. The profit-maximizing price is never less than 0.75(h+g) and never greater than 0.78(h+g).

to-consumer advertising (DTCA), it will always increase coverage and price of treatment when compared to the case where it does not advertise. Whether government can improve on DTCA depends on the government's ability to commit to a targeted advertising strategy that excludes patients at high risk of illness. The model developed in this paper assumes that an individual's risk of infection is common knowledge. High-risk individuals are the most willing to pay for preventive treatment. Advertising to this demographic, and increasing their willingness to pay, induces the pharmaceutical firm producing the treatment to set a high price. The resulting increase in price is sufficiently high to reduce coverage when compared to the case of no advertising. If instead, the government targets consumers with lower infection risks, the firm will have an incentive to lower its price in order to capture this segment of the market. In this case, the price of treatment will be lower, and coverage higher, than they would be in the absence of advertising. This is true regardless whether or not DTCA is banned. However, the government's advertising strategy is of crucial importance. If the government cannot credibly exclude high-risk consumers from its advertising, then it will not be possible to obtain the lower prices and higher coverage mentioned above.

Several important assumptions distance the model in this paper from reality. The model assumes that the government is not a purchaser of preventive treatment. Targeted subsidies of preventive treatment are a subset of this model, and the results of this paper continue to hold[40] for any form of subsidy that varies linearly with infection risk. The model also assumes that no advertiser can revise its campaign. A more dynamic setting, in which firm and government are allowed to launch multiple advertising campaigns, is left for further work. Finally, the model assumes that preventive treatment is provided by a monopolist. Incorporating generic drugs and other competition into the model is an avenue for future research.

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[40] The original version of the model included a possibility of government purchases of preventive treatment, effectively a 100% targeted subsidy. The qualitative results thus obtained were not substantially different than those in the current paper.

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5. Conclusion

Traditionally, advertising has been thought of as an informative message displayed by firms to a large number of potential consumers in order to educate them about their product's characteristics. Recently, other aspects of advertising have moved to the forefront. Improvements in technology allow for the targeting of specific consumers with messages, and for collecting enough data on individuals so that said message may be tailored to suit their tastes and biases. This sort of data-collection and customization is often costly. The object of these chapters was to investigate the use of such advertising in a duopoly.

The case of a duopoly where only one firm is capable of advertising was investigated using a Hotelling (1929) framework. I found that when market differentiation and advertising cost are sufficiently low, the advertising firm will use targeted ads to weaken the incentive for price competition by denying a segment of consumers to a rival. The end result is a division of the consumer line into two 'turfs' on either side of a boundary marked by advertising, with each firm acting as a monopolist over its share of consumers. This raises profits for the advertiser over the no-advertising case, and generally the non-advertising rival will earn higher profits than the advertising firm. This happens because the non-advertiser enjoys the benefits of weakened competition, while not having to pay for the mechanism by which it is accomplished.

When advertising costs are high, the firm will instead engage in very slight advertising of a different nature. The outcome will be similar to that in Hotelling's model of spatial competition. It differs in a slightly higher price and lower market share for the advertising firm, and a lower price and market share for its rival. The price effect dominates, and the non-advertiser is unambiguously worse off than in the absence of advertising.

When both firms may advertise, the final form of each firm's advertising campaign depends strongly upon the particular circumstances of the firms' interactions. The market for preventive medicine in Canada was used as a specific example. Canada's universal health insurance system leads to the under-provision of disease prevention by consumers. Government and a pharmaceutical firm were seen as encouraging consumption of the same good – preventive treatment. Both were assumed to be capable of targeted advertising. The government was allowed to pay for disease awareness campaigns, while the pharmaceutical firm could engage in direct-to-consumer marketing. I showed that under certain assumptions, a public health education campaign designed to increase coverage of preventive treatment can have the opposite effect. This is due to the firm considering the government's health campaigns to be an inexpensive substitute for its own advertising. If the government advertises heavily to those at high risk of illness, then the pharmaceutical firm may profit from holding high-risk consumers for ransom.

The present work is only the sketch of a framework. To properly understand costly targeted persuasive advertising, many extensions must be made, including the following:

1. The possibility of mass advertising and coarse targeting of consumers (by interval, rather than address) must be introduced into the model. This is necessary both to study the implications of certain regulatory possibilities, and to ascertain how a firm which can advertise, but does not have perfect access to consumer information, might react to a rival so gifted.

2. Entry must be examined, both of advertising and non-advertising firms. The present study sheds no light on whether this sort of advertising may be used as a barrier to entry, or as an incentive to accommodation upon entry.

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Appendix

A.i. Quadratic utility

Throughout these essays, I have used a linear representation of consumer utility. This specification is not necessary for the results to stand. For instance, consider the case of quadratic utility, which has often been used in the past in Hotelling-type models.

The main result of the discussion of monopoly continues to hold. No matter how high the cost of advertising may be, a monopolist will always find it profitable to advertise to some extent. The crucial assumption is a smooth distribution of consumer tastes, not linear preferences. For the result to be true, the monopolist must be able to find a consumer whose tastes are close enough to the marginal consumer's to make advertising profitable. If the good sells for \$100 and advertising costs \$1000 per \$1 increase in willingness to pay, then the monopolist will advertise to consumers with an initial willingness to pay of \$99.90 to \$99.99.

More formally, suppose that as before, a unit mass of consumers is uniformly distributed on the unit line.

In the quadratic case, consumer utility is given by

$$U = V - p - tr^2$$

As before, V is an intrinsic valuation, p is the product's price and r is a measure of distance from the consumer's preferred good.

Let the cost of advertising be c. In the absence of advertising, the indifferent consumer has an address x, where x is given by

$$V - p - tx^2 = 0$$
$$x = \sqrt{\frac{V-p}{t}}$$

The cost of advertising to the consumer at r is cA, where A is the amount of advertising. To convince the consumer at r>x to purchase the good, the monopolist must spend an amount R on advertising, where

$$V - p - tr^2 + R = 0$$
$$R = tr^2 - V + p$$

The firm will advertise as the cost of advertising is equal to the price of the product. The left end-point of the campaign is x, and the right end-point is z, as defined below:

$$cR = p$$
$$c(tz^2 - V + p) = p$$
$$z = \sqrt{\frac{c(V-p)+p}{ct}}$$

As c tends to infinity, this value tends toward x. For all finite values of c, it is greater than x, q.e.d.

In a similar fashion, the chief results of the case of duopoly also hold under quadratic preferences. Details of barrier construction are somewhat different, and barriers are more expensive, but the intuition is the same: by creating a segment of extremely loyal consumers at the border of its market coverage, the advertiser may weaken price competition. When preferences are quadratic, consumers are more naturally 'loyal' than under linear preferences – there is a greater cost associated with traveling a given distance from their preferred good. This raises the cost of advertising to a continuous set of near-marginal consumers, as in a barrier. Under quadratic preferences, the per-unit cost of advertising above which there will be no advertising, is therefore somewhat lower.

A.ii. Spatial competition

The models in these essays may be interpreted as models of spatial competition. For example, consider a long commercial street at either end of which is a food wholesaler. There are a number of restaurants on the street, each of which prefers to buy their ingredients from the closest wholesaler. One of the wholesalers (in the case of only one firm advertising) sends a sales agent along the street to convince restaurant owners to buy from their store. The sales agent is paid by the hour, and must choose which restaurants to cultivate a relationship with. The 'barrier' represents restaurants in the middle of the street, which the salesman has spent a lot of time with and with whom she is now on friendly terms.

A.1. Proof of theorem 3.3

Notation

That part of z_j^{R*} that is independent of k is called F_j^R.

The campaign length for which z_j^{R*}=b is called $\bar{k}_j^R$.

Approach

Propositions 1, 1.a, 2 and 2.a establish the results for varying values of $U_j(r,p_i)$. The proof of Theorem 3.3 combines these results and fills in the gaps.

Proposition 1. Suppose $U_1(1,p_1) \leq 0$, so that all consumers except for those on [a,b] prefer the outside supplier to Firm 1. Then there exists a number $k_0<(1-a)$ such that for all $k \geq k_0$ IC0 is satisfied.

Proof:

In three parts. We proceed by looking in turn at each possible location of F_0^{MB}, and proving the existence of the required k'.

Let k=0 and suppose IC0 is satisfied. Then the result follows immediately. Therefore, we assume from now on that ${}_0^a\pi_0^* < {}_b^1\pi_0^*$ when k=0, and IC0 does not hold.

i. Let F_0^{MB}<a. Then z_0^{MB*}<b, and ${}_b^1 z_0^*$=b. By Lemma 3.1, $\pi_0(b) < \pi_0(a)$ for k>0, and so IC0 holds for all positive k.

ii. Let F_0^{MB}>1.Then z_0^{MB*}>1, and ${}_b^1 z_0^*$=1. By Lemma 3.2, when k=(1-a), IC0 holds. By assumption, when k=0, IC0 is violated. By the intermediate value theorem (IVT), there must exist a value of k, $k_0 \in (0,1-a)$ such that ${}_b^1\pi_0^* = {}_0^a\pi_0^*$. Since $\pi_0^{MB}(1)$ is decreasing in k, this means for all k>k_0, ${}_0^a\pi_0^* > {}_b^1\pi_0^*$, q.e.d.

iii. Let $F_0^{MB} \in [a,1]$.

a. Suppose $\bar{k}_0^{MB} \leq$ (1-a). When k=$\bar{k}_0^{MB}$, $z_0^{MB*} = b$ and so ${}_b^1 z_0^*$=b. By Lemma 3.1, $\pi_0(b) < \pi_0(a)$ for k>0, and so IC0 holds when k=$\bar{k}_0^{MB}$. By assumption, when k=0, IC0 is violated. By the IVT, there must exist a value of k, $k_0 \in (0, \bar{k}_0^{MB})$ such that ${}_b^1\pi_0^* = {}_0^a\pi_0^*$. Since $\pi_0^{MB}(b)$ is decreasing in k, this means for all k>k_0, ${}_0^a\pi_0^* > {}_b^1\pi_0^*$, q.e.d.

b. Let $\bar{k}_0^{MB}$>(1-a). Then when k=$\bar{k}_0^{MB}$, z_0^{MB*}>1. Now let $k = 2(1-F_0^{MB}) < (1-a)$, so that z_0^{MB*}=1.

i. Suppose in this case, ${}^{a}_{0}\pi^{*}_{0} > {}^{1}_{b}\pi^{*}_{0}$. By assumption, when k=0, ${}^{a}_{0}\pi^{*}_{0} <$ ${}^{1}_{b}\pi^{*}_{0}$. By the IVT, there must exist a value of k, $k_0 \in \left(0,2\left(1-F_0^{MB}\right)\right)$ such that ${}^{1}_{b}\pi^{*}_{0} = {}^{a}_{0}\pi^{*}_{0}$. Since $\pi_0^{MB}(1)$ is decreasing in k, this means for all k>k0, ${}^{a}_{0}\pi^{*}_{0} > {}^{1}_{b}\pi^{*}_{0}$, q.e.d.

ii. Suppose in this case, ${}^{a}_{0}\pi^{*}_{0} < {}^{1}_{b}\pi^{*}_{0}$. By Lemma 1, when k=(1-a), ${}^{a}_{0}\pi^{*}_{0}$ $> {}^{1}_{b}\pi^{*}_{0}$. By the IVT, there must exist a value of k, $k_0 \in \left(2\left(1-F_0^{MB}\right), 1-a\right)$ such that ${}^{1}_{b}\pi^{*}_{0} = {}^{a}_{0}\pi^{*}_{0}$. Since $\pi_0^{MB}(1)$ is decreasing in k, this means for all k>k0, ${}^{a}_{0}\pi^{*}_{0} > {}^{1}_{b}\pi^{*}_{0}$, q.e.d.

■

Proposition 1.a: Suppose $U_0(0,p_0)<0$, so that all consumers strictly prefer the outside supplier to Firm 0. Then there exists a number $k_1<b$ such that for all $k \geq k_1$, ${}^{a}_{0}\pi^{*}_{1} \leq {}^{1}_{b}\pi^{*}_{1}$ and IC1 is satisfied.

Proof:

Let k=0 and suppose that IC1 holds. Then the result follows immediately. Therefore, we assume from now on that IC1 is violated when k=0.

i. Let $z_1^{M*} \geq b$. Then ${}^{a}_{0}z^{*}_{1}$=a. By Lemma 1.a, $\pi_1(a) < \pi_1(b)$ for all k>0, and so ${}^{1}_{b}\pi^{*}_{1} > {}^{a}_{0}\pi^{*}_{1}$ for all positive k, q.e.d.

ii. Suppose $0 < z_1^{M*} < b$. Now let $a = n_1^{M*}$. Then ${}^{a}_{0}z^{*}_{1}$=a, and $k = b - n_1^{M*}$. By Lemma 1.a, $\pi_1(a) < \pi_1(b)$, and so ${}^{1}_{b}\pi^{*}_{1} > {}^{a}_{0}\pi^{*}_{1}$ when $k = b - n_1^{M*}$. Now suppose $b > k > b - n_1^{M*}$. Then ${}^{a}_{0}z^{*}_{1} = a < n_1^{M*}$. By Lemma 1.a, $\pi_1(a) < \pi_1(b)$ for all k>0, and so ${}^{1}_{b}\pi^{*}_{1} > {}^{a}_{0}\pi^{*}_{1}$ for all $k > b - n_1^{M*}$, q.e.d.

iii. Suppose $z_1^{M*} \leq 0$. Then ${}^{a}_{0}z^{*}_{1}$=0. By Lemma 3.2.a, when k=b, ${}^{1}_{b}\pi^{*}_{1} > {}^{a}_{0}\pi^{*}_{1}$. By assumption, when k=0, ${}^{1}_{b}\pi^{*}_{1} < {}^{a}_{0}\pi^{*}_{1}$. By the IVT, there must exist a value of k,

$k_1 \in (0, b)$ such that ${}_b^1\pi_1^* = {}_0^a\pi_1^*$. Given b, ${}_b^1\pi_1^*$ is increasing in k, and ${}_0^a\pi_1^*$ is independent of k. It follows that for all k>k$_1$, ${}_b^1\pi_1^* > {}_0^a\pi_1^*$, q.e.d.

■

Proposition 2: Let $U_1(a,p_1)>0$, so that all consumers on [a,1] strictly prefer Firm 1 to the outside supplier. Then there exists a number $k_0<(1-a)$ such that for all $k \geq k_0$, ${}_0^a\pi_0^* \geq {}_b^1\pi_0^*$ and IC0 holds.

Proof: In three parts. We proceed by looking in turn at each possible location of F_0^{DB}, and proving the existence of the required k'.

i. Let F_0^{DB}<a. Then z^{DB*}<b, and ${}_b^1x_0^*$=b. By Lemma 1, $\pi_0(b) < \pi_0(a)$ for k>0, and so ${}_0^a\pi_0^* > {}_b^1\pi_0^*$ for all positive k.

ii. Let F_0^{DB}>1.Then z^{DB*}>1, and ${}_b^1x_0^*$=1. By Lemma 3.2, when k=(1-a), ${}_0^a\pi_0^* > {}_b^1\pi_0^*$. By assumption, when k=0, ${}_0^a\pi_0^* < {}_b^1\pi_0^*$. By IVT, there must exist a value of k, $k_0 \in (0, 1-a)$ such that ${}_b^1\pi_0^* = {}_0^a\pi_0^*$. Since $\pi_0^{DB}(1)$ is decreasing in k, this means for all k>k$_0$, ${}_0^a\pi_0^* > {}_b^1\pi_0^*$, q.e.d.

iii. Let $F_0^{DB} \in [a,1]$.

  a. Suppose $\bar{k}_0^{DB} \leq$ (1-a). When k=$\bar{k}_0^{DB}$, $x^{DB*} = b$ and so ${}_b^1x_0^*$=b. By Lemma 1, $\pi_0(b) < \pi_0(a)$ for k>0, and so ${}_0^a\pi_0^* > {}_b^1\pi_0^*$ when k=$\bar{k}_0^{DB}$. By assumption, when k=0, ${}_0^a\pi_0^* < {}_b^1\pi_0^*$. By the IVT, there must exist a value of k, $k_0 \in (0, \bar{k}_0^{DB})$ such that ${}_b^1\pi_0^* = {}_0^a\pi_0^*$. Since $\pi_0^{DB}(b)$ is decreasing in k, this means for all k>k$_0$, ${}_0^a\pi_0^* > {}_b^1\pi_0^*$, q.e.d.

  b. Let $\bar{k}_0^{DB}$>(1-a). Then when k=$\bar{k}_0^{DB}$, x^{DB*}>1. Let $k = 2(1 - F_0^{DB}) < (1-a)$, so that x^{DB*}=1.

i. Suppose in this case, ${}_0^a\pi_0^* > {}_b^1\pi_0^*$. By assumption, when k=0, ${}_0^a\pi_0^* < {}_b^1\pi_0^*$. By the IVT, there must exist a value of k, $k_0 \in \left(0,2\left(1-F_0^{DB}\right)\right)$ such that ${}_b^1\pi_0^* = {}_0^a\pi_0^*$. Since $\pi_0^{DB}(x)$ is decreasing in k, this means for all k>k$_0$, ${}_0^a\pi_0^* > {}_b^1\pi_0^*$, q.e.d.

ii. Suppose in this case, ${}_0^a\pi_0^* < {}_b^1\pi_0^*$. By Lemma 1, when k=(1-a), ${}_0^a\pi_0^* > {}_b^1\pi_0^*$. By the IVT, there must exist a value of k, $k_0 \in \left(2\left(1-F_0^{DB}\right),1-a\right)$ such that ${}_b^1\pi_0^* = {}_0^a\pi_0^*$. Since $\pi_0^{DB}(1)$ is decreasing in k, this means for all k>k$_0$, ${}_0^a\pi_0^* > {}_b^1\pi_0^*$, q.e.d.

■

Proposition 2.a: Suppose $U_0(b,p_0)>0$, so that all consumers on [0,b] strictly prefer Firm 0 to the outside supplier. Then there exists a number $k_1<b$ such that for all $k \geq k_1$, ${}_0^a\pi_1^* \leq {}_b^1\pi_1^*$ and IC1 holds.

Proof:

iv. Let $z_1^{D*} \geq b$. Then ${}_0^a z_1^*$=a. By Lemma 1.a, $\pi_1(a) < \pi_1(b)$ for all k>0, and so ${}_b^1\pi_1^* > {}_0^a\pi_1^*$ for all positive k, q.e.d.

v. Suppose $0 < z_1^{D*} < b$. Now let $a = z_1^{D*}$. Then ${}_0^a z_1^*$=a, and $k = b - z_1^{D*}$. By Lemma 1.a, $\pi_1(a) < \pi_1(b)$, and so ${}_b^1\pi_1^* > {}_0^a\pi_1^*$ when $k = b - z_1^{D*}$. Now suppose $b > k > b - z_1^{D*}$. Then ${}_0^a z_1^* = a < z_1^{D*}$. By Lemma 1.a, $\pi_1(a) < \pi_1(b)$ for all k>0, and so ${}_b^1\pi_1^* > {}_0^a\pi_1^*$ for all $k > b - z_1^{D*}$, q.e.d.

vi. Suppose $z_1^{D*} \leq 0$. Then ${}_0^a z_1^*$=0. By Lemma 3.2.a, when k=b, ${}_b^1\pi_1^* > {}_0^a\pi_1^*$. By assumption, when k=0, ${}_b^1\pi_1^* < {}_0^a\pi_1^*$. By the IVT, there must exist a value of k, $k_1 \in (0,b)$ such that ${}_b^1\pi_1^* = {}_0^a\pi_1^*$. Given b, ${}_b^1\pi_1^*$ is increasing in k, and ${}_0^a\pi_1^*$ is independent of k. It follows that for all k>k$_1$, ${}_b^1\pi_1^* > {}_0^a\pi_1^*$, q.e.d.

■

Lemma 3: For any $a \in [0,1)$ there exists a number $k_0<(1-a)$ such that for all $k \geq k_0$ IC0 holds. For any $b \in [0,1)$, there exists a number $k_1<b$ such that for all $k \geq k_1$, IC1 holds.

Proof:

From Proposition 1, we know this is true when all consumers outside the barrier prefer the outside supplier to the rival supplier.

From Proposition 2, we know this is true when some consumers inside the barrier prefer the rival supplier to the outside supplier.

All that is left is to prove the result for the case where some or all consumers outside the barrier prefer the rival good to the outside good. We do this below in two steps: once for Firm 0, and once for Firm 1.

i. Let $U_1(r,p_1) \geq 0$ for $r \in [a,1]$ and suppose that k is greater than or equal to the value k_0 specified in Proposition 2. For any given $z_0>b$, Firm 0's price is equal to or lower than when $U_1(a,p_1)<0$. Therefore its profits on that interval are also equal or lower, and ${}_0^a\pi_0^* > {}_b^1\pi_0^*$, as required.

ii. Let $U_0(r,p_0) \geq 0$ for $r \in [0,b]$ and suppose that k is greater than or equal to the value k_1 specified in Proposition 2.a. For any given $z_1<a$, Firm 1's price is equal to or lower than when $U_0(a,p_0)<0$. Therefore its profits on that interval are also equal or lower, and ${}_b^1\pi_1^* > {}_0^a\pi_1^*$, as required.

■

A.2. Proof of theorem 3.4

Proposition 3: The minimum barrier length needed to satisfy Firm 0's incentive constraint is given by

$$k_0 = \begin{cases} 0 & \frac{V}{3t} \le a \\ k_0^{'} & a \le \frac{V}{3t} \quad and\ k_0^{'} \le \frac{1}{3}\left(4 - \left(\frac{V}{t} + a\right)\right) \\ \max\left[k_0^{"}, 0\right] & otherwise \end{cases}$$

$$k_0^{'} \equiv a + \frac{V}{t} - 2\sqrt{2a\left(\frac{V}{t} - a\right)}$$

$$k_0^{"} = \frac{1}{2}\left(3 - a - \frac{V}{t} + \sqrt{\left(\frac{V}{t}\right)^2 - 2(1+a)\frac{V}{t} + 5a^2 - 2a + 1}\right)$$

Proof: In four steps.

i. Given a, Firm 0's profit function is a continuous function with a discontinuous derivative.

$$\pi_0 = \begin{cases} \pi_0^M & 0 \le z_0 \le a \\ \pi_0^B & a \le z_0 \le b \\ \pi_0^{DB} & b \le z_0 \le 1 \\ p_0(z_0, z_1)(1-k) & z_0 \ge 1 \end{cases}$$

Regarding the final inequality in the above expression, $z_0^{DB*} \ge 1$ whenever

$$k \le \frac{1}{3}\left(4 - \left(\frac{V}{t} + a\right)\right)$$

Both π_0^M and π_0^{DB} are concave and quadratic in z_0. The intra-campaign profit function, π_0^B, is decreasing in z_0.

ii. Let k$\geq$0 and $a \geq z_0^{DB*}$. Then ${}_0^a\pi_0^* = a$ and ${}_b^1\pi_0^* = b$. But $\pi_0^B(a) \geq \pi_0^B(b)$, with equality only when k=0. Therefore, if $a \geq z_0^{DB*}$, for any non-negative value of k, Firm 0's incentive constraint is satisfied. It can be shown by solving the first-order conditions directly that $a \geq z_0^{DB*}$ whenever $a \geq \frac{V}{3t}$.

iii. Let $a < \frac{V}{3t}$ and $z_0^{DB*} \leq 1$, so that $a < z_0^{DB*} \leq 1$. Suppose k=0. Then ${}_0^a\pi_0^* = a$ and ${}_b^1 z_0^* = z_0^{DB*}$. But $\frac{d\pi_0^{DB}}{dz_0} > 0$ for $z_0 < z_0^{DB*}$, so $\pi_0^{DB*} > \pi_0^M(a)$ and ${}_0^a\pi_0^* < {}_b^1\pi_0^*$. Now suppose b=$z_0^{DB*}$. Then ${}_0^a\pi_0^* = a$ and ${}_b^1\pi_0^* = z_0^{DB*} = b$. Since π_0^B falls with z$_0$, we must have ${}_0^a\pi_0^* > {}_b^1\pi_0^*$. By the intermediate value theorem, there exists a value of k between 0 and $z_0^{DB*} - a$ such that ${}_0^a\pi_0^* \geq {}_b^1\pi_0^*$ for any k greater than or equal to this value. The requisite minimum value of k can be found by setting $\pi_0^{DB*} = \pi_0^M(a)$ and solving for k. This yields $k = a + \frac{V}{t} - 2\sqrt{2a\left(\frac{V}{t} - a\right)}$.

iv. Let $a < \frac{V}{3t}$ and $k > \frac{1}{3}\left(4 - \left(\frac{V}{t} + a\right)\right)$, so that z_0^{DB*}>1. Then ${}_b^1 z_0^* = 1$ and we find the threshold k by setting $\pi_0^{DB}(1) = \pi_0^M(a)$. This yields $k = k_0^{''}$, as above. For certain parameter values, this can be negative. Therefore, in this case the minimum require campaign length is $\max(k_0^{''}, 0)$, q.e.d.

■

Proposition 4: The minimum barrier length needed to satisfy Firm 1's incentive constraint is given by

$$k_1 = \begin{cases} 0 & a \leq 1 - \frac{V}{3t} \\ \frac{\left(\frac{V}{t} - 3(1-a)\right)^2}{8(1-a)} & a \geq \max\left(1 - \frac{V}{3t}, \frac{V}{t} - 3\right) \\ \max\left[\frac{a}{1-a}\left(\frac{V}{t} + a - 3\right), 0\right] & a \leq \frac{V}{t} - 3 \end{cases}$$

Proof: In four steps.

i. Firm 1's profit function is a continuous function with a discontinuous derivative.

$$\pi_1 = \begin{cases} p_1(z_0, z_1) & z_1 < 0 \\ \pi_1^D & 0 \leq z_1 \leq a \\ \pi_1^B & a \leq z_1 \leq b \\ \pi_1^{MB} & b \leq z_1 \leq 1 \end{cases}$$

Regarding the final inequality in the above expression, $z_1^{D*} \geq 0$ whenever

$$a \geq \frac{V}{t} - 3$$

Both π_1^{MB} and π_1^{DB} are concave and quadratic in n_1. The intra-campaign profit function, π_1^B, is increasing in n_1.

ii. Let $k \geq 0$ and $a \leq z_1^{D*}$. Then ${}_0^a\pi_1^* = a$ and ${}_b^1\pi_1^* = b$. But $\pi_1^B(a) \leq \pi_1^B(b)$, with equality only when k=0. Therefore, if $a \leq z_1^{D*}$, for any non-negative value of k, Firm 0's incentive constraint is satisfied. It can be shown by solving the first-order conditions directly that $a \leq z_1^{D*}$ whenever $a \leq 1 - \frac{V}{3t}$.

iii. Let $a > 1 - \frac{V}{3t}$, so that $a > z_1^{D*}$. Suppose k=0, and so a=b. Then ${}_0^a\pi_1^* = z_1^{D*}$ and ${}_b^1\pi_1^* = a = b$. But $\frac{d\pi_1^D}{dn_1} < 0$ for $z_1 > z_1^{D*}$, so $\pi_1^{D*} > \pi_1^M(b)$ and ${}_0^a\pi_1^* > {}_b^1\pi_1^*$. Now suppose b= z_1^{D*} and let a=0. Then ${}_0^a\pi_1^* = a$ and ${}_b^1\pi_1^* = z_1^{D*} = b$. Since π_1^B rises with n_1, we must have ${}_0^a\pi_1^* < {}_b^1\pi_1^*$. By the intermediate value theorem, there exists a value of k between 0 and z_1^{D*} such that ${}_0^a\pi_1^* \leq {}_b^1\pi_1^*$ for any k greater than or equal to this value.

iv. The requisite minimum value of k can be found by setting $\pi_1^{D*} = \pi_1^{MB}(a+k)$ and solving for k. This yields $k = \frac{\left(\frac{V}{t} - 3(1-a)\right)^2}{8(1-a)}$.

v. Let $a \leq \frac{V}{t} - 3$, so that $z_1^{D*} \leq 0$. Then ${}_0^a\pi_1^* = 0$ and we find the threshold k by setting $\pi_1^D(1) = \pi_1^{MB}(a+k)$, yielding $k_1 = \frac{a}{1-a}\left(\frac{V}{t} + a - 3\right)$. For certain parameter values, this may be negative, so when $a \leq \frac{V}{t} - 3$ the requisite minimum value of k is $k_1 = \max\left(\frac{a}{1-a}\left(\frac{V}{t} + a - 3\right), 0\right)$, q.e.d.

■

A.3. Full expression for p_0^m

Full derivation

Because $U_1(a+k, p_1) = 0$, p_1=V-t(1-(a+k)).

It can be shown that

$$z_0\left(p_0^m, p_1\right) = \frac{1}{2}\left(\frac{V}{t} + a + k\right) - \frac{p_0^m}{2t}$$

and

$$z_0\left(p_0^m, p_1\right) > 1$$

Whenever a>2-1/2t.

Combining expressions and solving,

$$p_0^m = \begin{matrix} p_0^{mA} & p_0^{mA} \geq V - t\left(2 - (a+k)\right) \\ (V - ta)a & elsewhere \end{matrix}$$

for

$$p_0^{mA} \equiv \frac{1}{t}\left(\frac{1}{2}\left(\frac{V}{t} + a + k\right) - \sqrt{\frac{1}{4}\left(\frac{V}{t} + a + k\right)^2 - 2a\left(\frac{V}{t} - a\right)}\right)$$

■

A.4. Proof of theorem 4.1

The game is in two stages. In the second, the firm sets its price, taking government advertising as given. In the first, the government advertises. An equilibrium consists of an advertising function $a_2^g(x)$ and price p from which neither firm nor government has an incentive to deviate.

By Lemma 1, government ads bring consumers with utility greater than $-p^g$ to a utility of $-p_g$, where g is a positive constant. The address of the highest consumer advertised to is $-p^g/h$. Let the address of the lowest consumer be z_2.

Since the firm does not advertise, profits are equal to revenue.
Given g and z_2, profits in the second stage are

$$\Pi_2(p) = \begin{array}{ll} p\left(1-\dfrac{p}{h}\right) & p > p^g \\ p(1-z_2) & z_2 h \leq p \leq p^g \\ p\left(1-\dfrac{p}{h}\right) & p < z_2 h \end{array}$$

Since sales are independent of price for $z_2h<p<p^g$, the firm will prefer $p=p^g$ to all other values of p in this range.

We know from the analysis of Case 1 that p(1-p/h) is at a maximum when p=h/2. When $z_2h<h/2<p^g$, the firm will choose p=h/2. Otherwise, it will choose p=h/2 when profits from such are higher than $p^g(1-z)$. That is,

$$p_2^* = \begin{cases} \frac{h}{2} & z_2 h \le \frac{h}{2} \le p^g \\ \frac{h}{2} & p^g(1-z_2) \le \frac{h}{4} \\ p^g & p^g(1-z_2) \ge \frac{h}{4} \end{cases}$$

In stage 1, the firm will not set $z_2>1/2$, since the firm is willing to supply these customers. It will not set $p^g>h/2$, since for any given z this raises advertising costs without increasing coverage.

For any given $z_2<1/2$, the government must set g such that

$$p^g(1-z_2) = h/4$$

This implies that

$$p^g = \frac{h}{4(1-z_2)}$$

The government will advertise to all consumers with addresses greater than or equal to z and utility greater than g – that is, all consumers on $[z_2, p^g/h]$.

Advertising is done in the amount just needed to make consumers indifferent to the good at price p^g.

$$a_2^g(x) = \frac{p^g}{x} - h$$

Government health care costs are equal to the expected medical costs of the untreated consumers:

$$G_2(z) = g\int_0^{z_2} x dx + \int_{z_2}^{1}\left(\frac{1}{4(1-z_2)x} - h\right)dx$$

The government seeks to minimize total costs.

The first-order conditions may be solved for z_2^*. The solution is not reproduced here for reasons of length.

When g=0, z_2^* is ½ as it is never worthwhile for the government to advertise, and we are back to the benchmark case. When h=0, z_2^*=0 as advertising is costless.

The value of z_2^* falls with g and rises with h, and thus so does the price p_2^*[41]. (A higher h raises advertising costs for x<p^g/h.) For positive g, coverage is always greater than ½ and price is always less than h/2. ■

[41] It may be verified that z_2^* depends only on the ratio h/f by making the substitution h=δf in the expression for $G_2(z)$. It is then easy to show that that for δ>0, z_2^* is independent of δ.

A.5. Proof of theorem 4.2

The game is in two stages. In Stage 1, the firm sets a price. In stage 2, the government advertises and consumers make their purchase decisions.

Stage 2

To make consumers indifferent between buying the product and not buying it, the government must set $a^g(x) = p/x - h$. It will not advertise to addresses greater than p/h. The lowest address advertised to, z_3, will have marginal benefit equal to advertising cost – the marginal benefit to the government from advertising is gx.

Solving, we find

$$z_3(p) = \frac{\sqrt{h^2 + 4gp} - h}{2g}$$

Firm profits in stage 1 are then

$$\Pi_3(p) = p(1 - z(p))$$

Solving the first-order conditions, we find that

$$p_3^* = \frac{1}{6g}\left(\frac{(2g+h)}{3}\left((2g+h) + 2\sqrt{(g+h)^2 - gh}\right) - h^2\right)$$

The limit of p_3^* as g → 0 is h/2, and this is greater than h/2 for g>0. When p=g+h, z=1, and so the price will be between h/2 and h+g. When h=0, p_3^*=4g/9. The limit of p_3^* as h→infinity is infinity. ■

A.6. Proof of theorem 4.3

Let the firm be the sole advertiser and first mover.

The utility of an untreated consumer who is not advertised to is –xh, where x is the risk of infection. The utility of a treated consumer is –p. Consumers with infection risk greater or equal to p/h are therefore willing to purchase treatment without being advertised to.

Consumers with x<p/h will not purchase treatment without being advertised to. The firm need only advertise to the extent that makes a consumer indifferent to purchasing treatment at price p. Since the utility of an untreated consumer is $U_4(x) = -x(h + a^f(x))$, this implies that when $a^f(x) > 0$, it takes the form $a^f(x) = p/x – h$.

Let $z_4 \leq \frac{p}{h}$ be the lowest infection risk targeted with advertising. The firm will advertise so long as the cost of doing so does not exceed revenue from an additional consumer, p. This means that at $a^f(z_4) = p$, and so $z_4 = p/(p+h)$. For $x>z_4$, $p>a^f(x)$. The firm will therefore advertise to all consumers on [p/(p+h),p/h].

A unit mass of consumers is uniformly distributed with respect to the risk of infection along the unit line. If all consumers with x>p/(p+h) purchase treatment, then a mass (1 – p/(p+h)) of consumers purchases treatment. Firm revenue is therefore p(1 – p/(p+h)).

The cost to the firm of its advertising campaign is

$$\int_0^1 a^f(x)dx = \int_{\frac{p}{p+h}}^{\frac{p}{h}} \left(\frac{p}{x} - h\right)dx$$

There are no costs of production, so firm profits are equal to revenue minus advertising costs:

$$\Pi_4(p) = p\left(1 - \frac{p}{h}\right) - \int_{\frac{p}{p+h}}^{\frac{p}{h}} \left(\frac{p}{x} - h\right) dx$$

This simplifies to

$$\Pi_4(p) = p\left(1 - \ln\left(1 + \frac{p}{h}\right)\right)$$

Solving the first-order conditions, we find this function is at a maximum when the price is equal to

$$p_4^* = \left(\frac{1}{LambertW(1)} - 1\right) h$$

If we define

$$\alpha \equiv \frac{1}{LambertW(1)} - 1$$

Then p_4^*= αh and coverage is $1 - \frac{p_4^*}{p_4^* + h} = 1 - \frac{\alpha h}{\alpha h + h} = \frac{1}{1+\alpha}$, as required. ■