OPTIMIZATION OF THE KOOTENAY RIVER HYDROELECTRIC SYSTEM WITH A LINEAR PROGRAMMING MODEL

by

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ABSTRACT

The main objective of hydroelectric system optimization is to determine an operating policy for the best use of available resources. In order to find the optimum policy one should decide on the trade-off between the marginal value of water in the reservoirs and the amount of electricity produced electricity. The variability of natural inflows along with local and environmental limitations and special procedures and regulations, makes the decision making process more challenging for an operation planner of a reservoir system.

The region considered in this study is the Kootenay River System and it includes five hydroelectric plants and a canal. The main storage reservoir in this system is the Kootenay Lake, which is the largest natural lake in British Columbia, Canada. The operation of the Kootenay system is complex because day-to-day operation decisions should satisfy all existing rules and water treaties and agreements in the area. In addition, a power generation schedule should take advantage of electricity markets and meet local load demands.

This thesis developed a Linear Programming model to optimize the operation of the Kootenay system on daily timesteps for studies up to one year. Due to the operational complexities, the Kootenay system was not included in the optimization models developed at B.C Hydro (BCH). This model can be an extension to the existing "STOM" (Short Term Optimization Model) and Generalized Optimization Model "GOM", which have been successfully adopted by BCH to optimize the daily operation of its plants. This is the first optimization model for the operation of Kootenay System and was developed in accordance with all the existing international treaties and special constraints on this system.

Results obtained using the model have indicated that this model can successfully optimize the operation of the Kootenay system. Comparison of this model to the current operation method showed that with respect to all system's constraints and value of water, the optimization model can yield a higher value of electricity generation and it is expected that it will be added to the set of tools used by the system operation engineers for their daily operations.

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1 INTRODUCTION

1.1 BACKGROUND

B.C Hydro is the third-largest electricity producer in Canada with 33 generating facilities in several regions of British Columbia. More than 90% of B.C Hydro's generating facilities consist of hydroelectric powerplants with a total generating capacity of 10,700 megawatts (MW) and an average annual energy production of 48,000 GWh [1].

Figure 1.1 shows a schematic of B.C Hydro's different generation and load facility region.

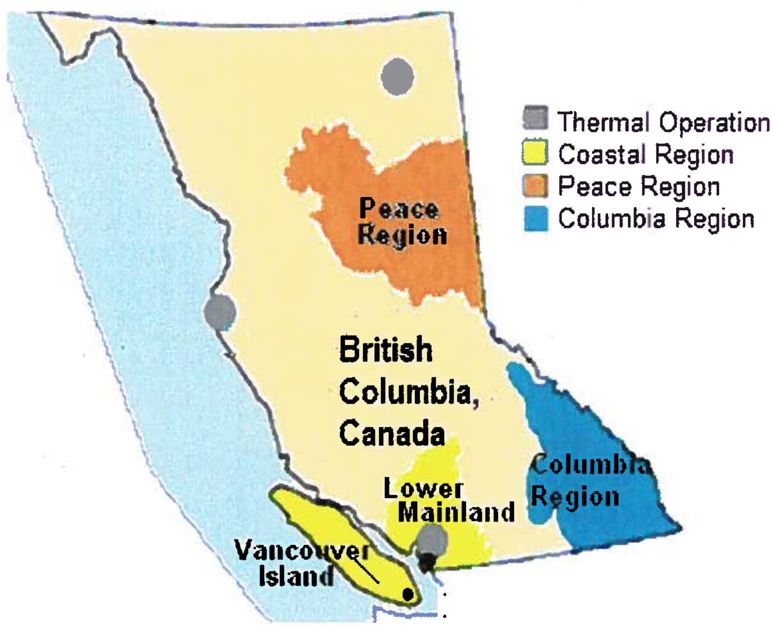
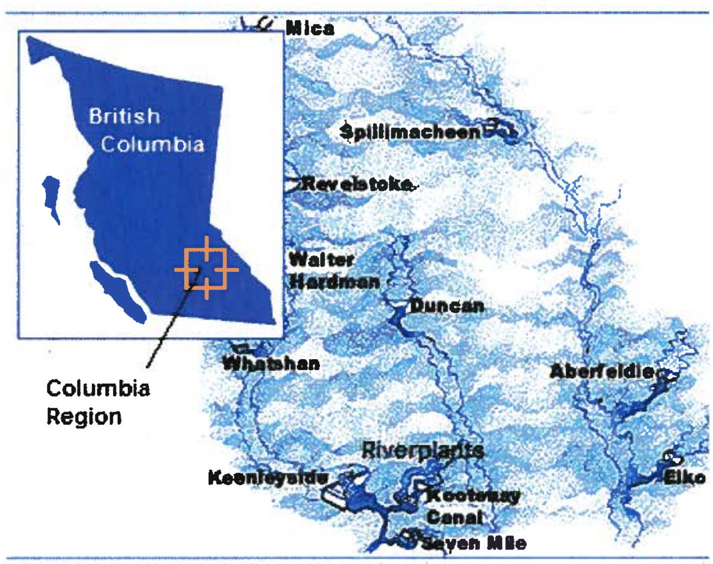


Figure 1.1 B.C. Hydro facility region [1]

The Columbia Generation Region consists of 11 hydroelectric plants with a total of 20 units with a total capacity of over 5,200 MW. These generating facilities are located in the upper Columbia region, the East and West Kootenay River and the Pend d'Oreille River [1]. Figure 1.2 shows a schematic of generation facilities in the Columbia Region. The Riverplants seen in the Figure are not owned but are operated by B.C Hydro on behalf of other electric utilities.

COLUMBIA



Area Facilities

Aberfeldie	Kootenay Canal	Shuswap
Duncan	Mica	Spillimacheen
Elko	Revelstoke	Walter Hardman
Keenleyside	Seven Mile	Whatshan

Riverplants: Cora Lynn, Upper Bonnington, Lower Bonnington, City of Nelson, South Slocan

Figure 1.2 B.C. Columbia Region [1]

The Kootenay Lake is the largest natural lake in British Columbia and is located between the Selkirk and the Purcell mountain ranges. The Kootenay Lake location can be seen in Figure 1.3.

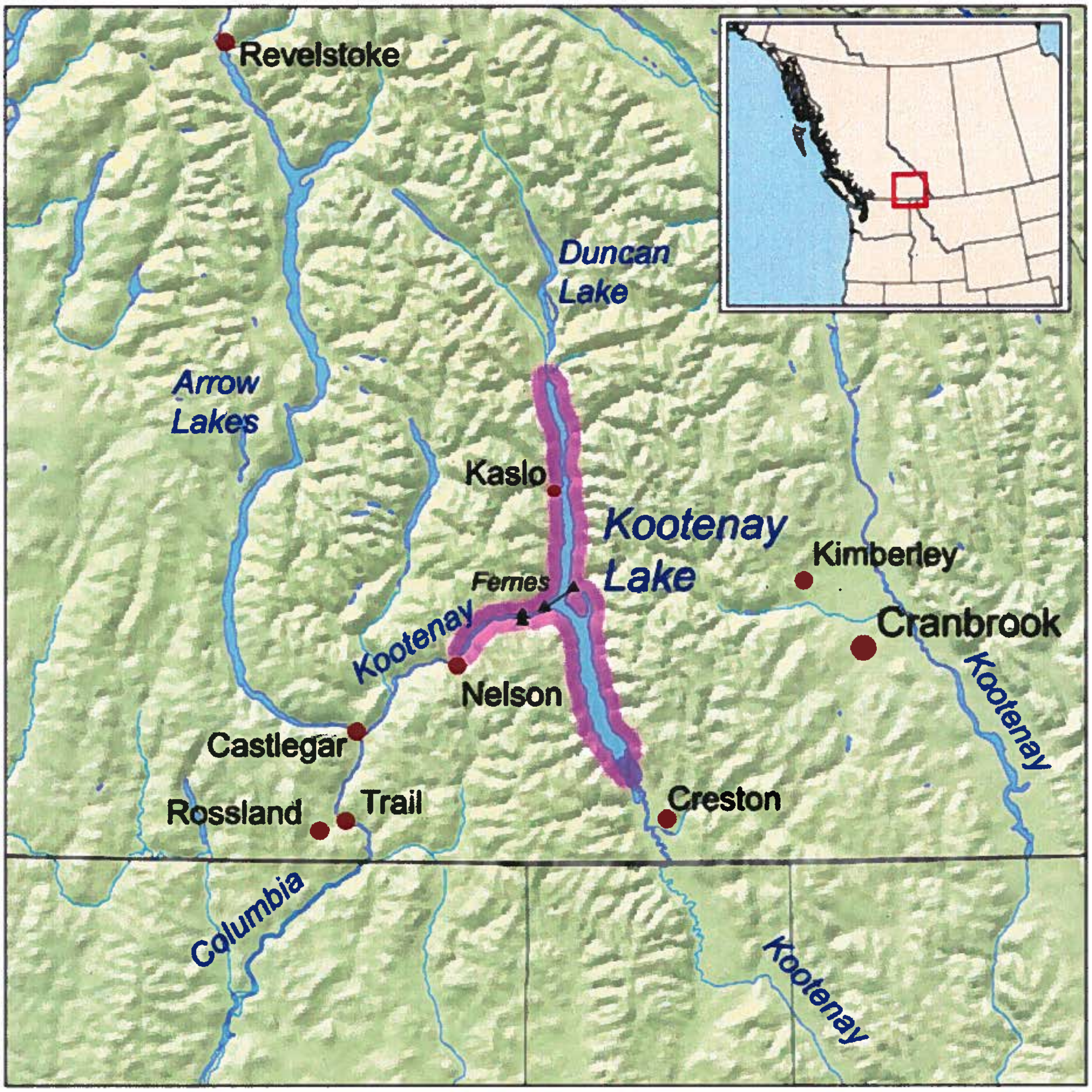


Figure 1.3 Kootenay Lake Location [Source: Wikipedia]

1.2 DESCRIPTION OF THE PROJECT

The Kootenay Lake outflow originally flows along the Kootenay River. After the completion of the Kootenay Canal in 1976 water was diverted from the Kootenay River to flow through the Kootenay Canal for power generation downstream of the Canal. In the current operation, lake outflow can be diverted into two parallel streams, the Kootenay Canal on the south side of the Kootenay River, and towards the Riverplants (the Upper Bonnington Dam, the Lower

Bonnington Dam and the South Slocan Dam) [4]. Part of the inflow to the lake is regulated by two upstream facilities, the Duncan and the Libby Dams. Water flowing into the Kootenay Canal and the Riverplants joins again downstream of the Kootenay Canal and the South Slocan Dam and then flows toward the Brilliant Dam as shown in Figure 1.4 [4].

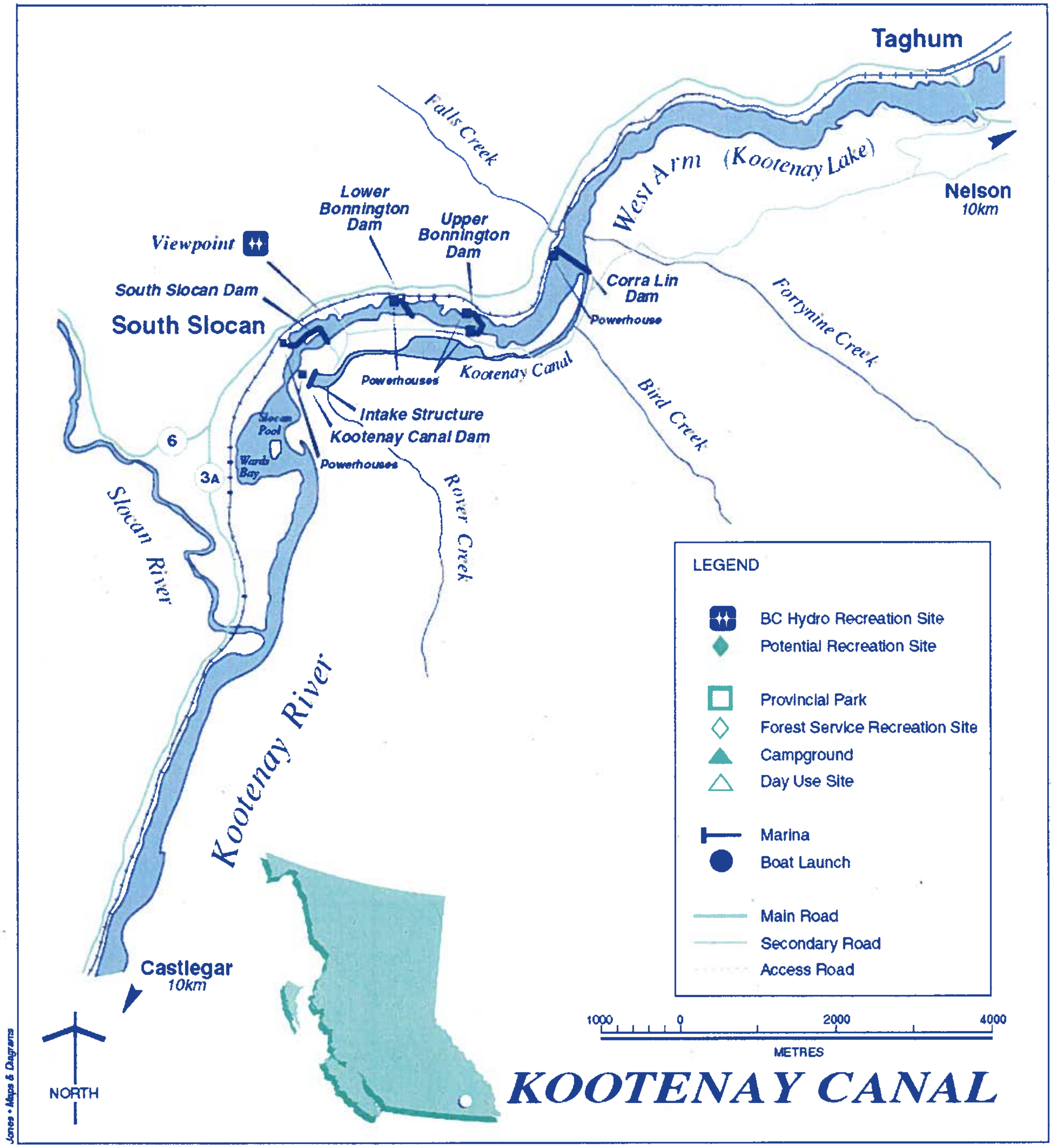


Figure 1.4 Kootenay Riverplants [1]

1.3 The KOOTENAY AREA GENERATING PLANTS

1.3.1 Model Configuration

The Kootenay Lake (KLK) is the main reservoir in the system and its elevation is an important parameter for this hydroelectric system. To operate the Kootenay System most of the constraints are imposed on the KLK level at Queen's Bay gauge, which is one of the two gauges used to monitor the lake level. The operating plants in this system consist of the Kootenay Canal plant, the Cora Lynn (COR), the Upper Bonnington (UBO), the Lower Bonnington (LBO), the South Slocan (SLC), and the Brilliant (BRD). The Brilliant Expansion (BRX) was recently completed (2007) as an extension of the generating units of the Brilliant Dam [3].

The natural local inflow into the Kootenay Lake (KLK) along with regulated inflow from the Duncan and Libby Dams are considered to be deterministic inputs in the model.

Figures 1.5 displays the location of the hydroelectric plants in the Kootenay system.

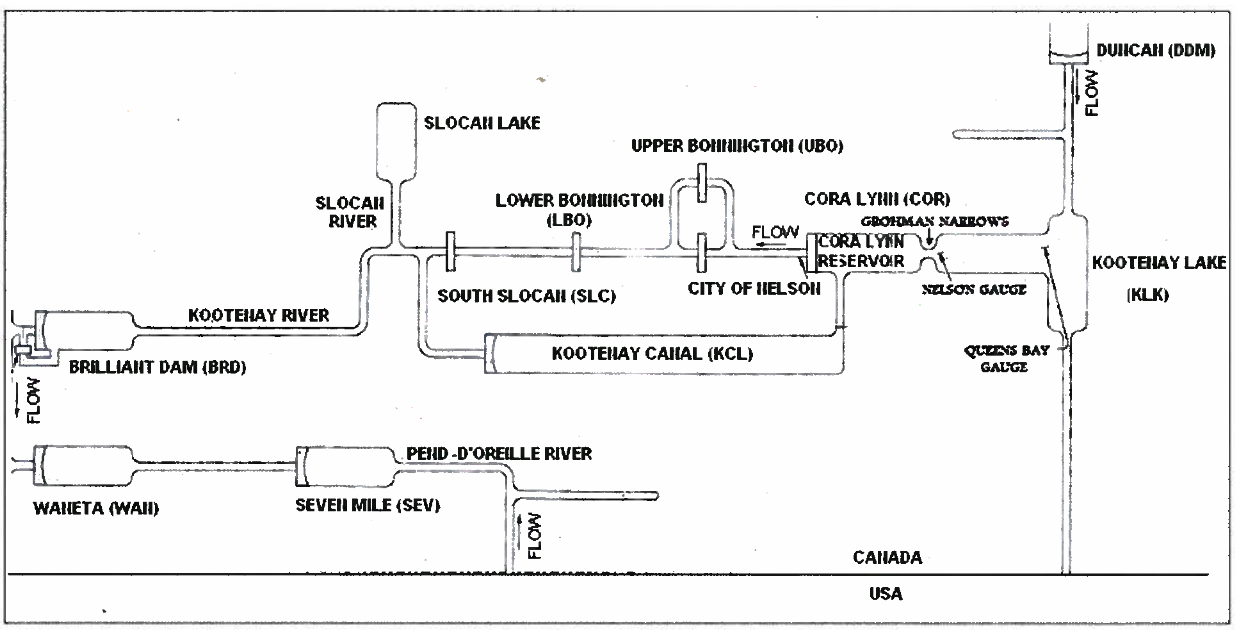


Figure 1.5 Kootenay Area Schematic Map [4]

1.3.2 Special Constraints on the Kootenay System

1.3.2.1 The Grohman Narrows Effect

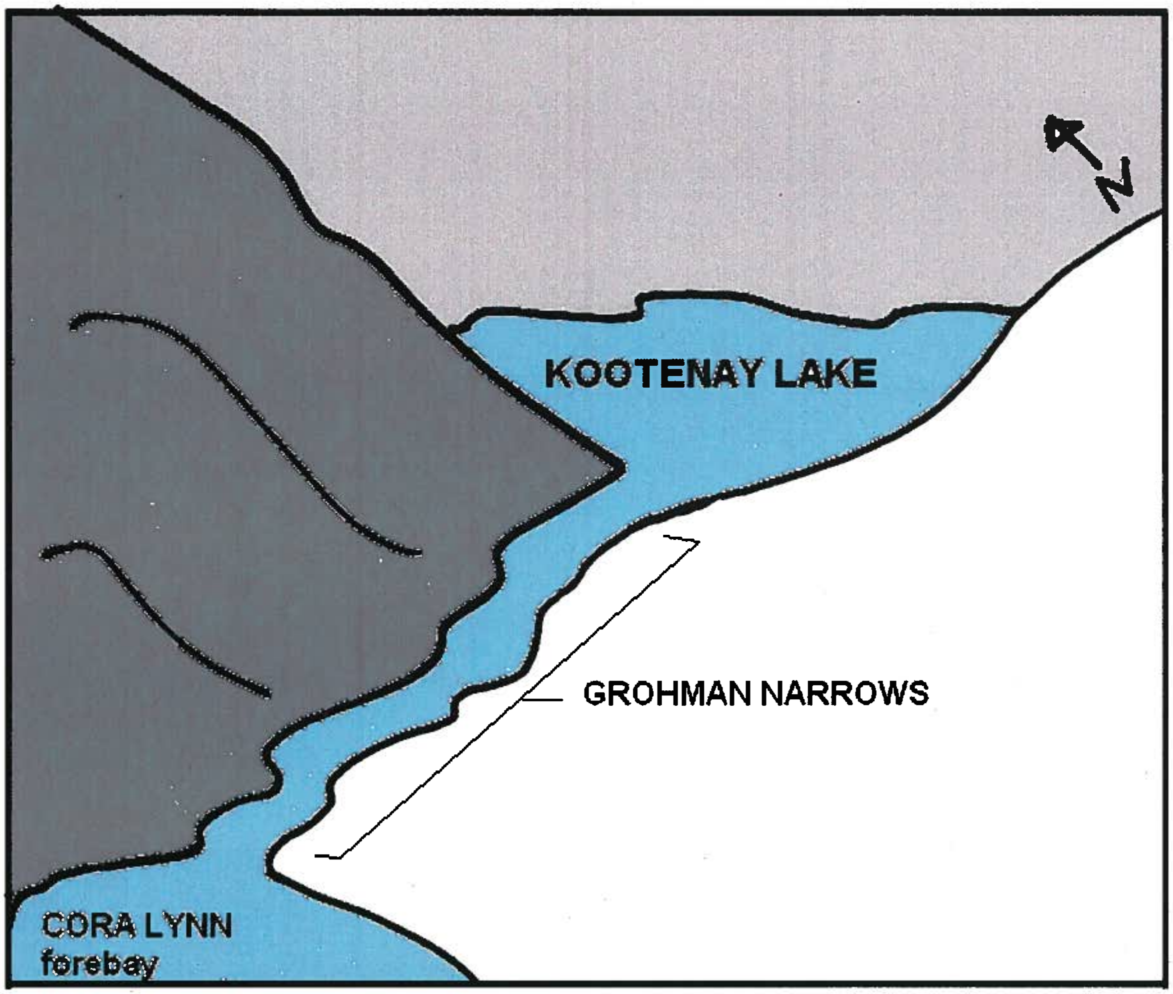


Figure 1.6 Grohman Narrows

The Grohman Narrows is a natural narrowing of the Kootenay River that restricts the outflows of the Kootenay Lake and causes considerable hydraulic losses between the Kootenay Lake and Cora Lynn forebay. The Grohman Narrows maximum discharge is a function of the Kootenay Lake level and the Cora Lynn reservoir level. The limitation caused by the Grohman Narrows is one of the main complexities that this optimization model had to resolve as will be explained in the following chapters.

1.3.2.2 The International Joint Committee Order (IJC) and the Canal Plant Agreement

The operation of the Kootenay System is governed by an "IJC Order of Approval" dated November 11, 1938. The IJC order limits the lake level at the Queen's Bay gauge (and at the Nelson gauge during certain periods and certain circumstances) to different levels during the year [3]. The Kootenay Lake must be operated according to a set of rules to stay less than the maximum water level according to IJC, as detailed in the following chapters.

The Kootenay System is also operated in accordance with the Canal Plant Agreement, which covers the operation of Fortis BC (FBC), Columbia Power Corporation (CPC) and BCH plants in the system [28].

1.4 GOALS AND OBJECTIVES

The main goal of this research project was to develop a hydroelectric scheduling system to assist the B.C Hydro operation planning engineers to mange the Kootenay hydroelectric power system. The scheduling system ensures that the operation engineers meet the system's physical and operational constraints while making the optimal trade-off between energy production and the value of water stored in the reservoirs. The following tasks were carried out to meet the above goals:

1- To develop an optimized operation schedule for the Kootenay Lake, the Kootenay Canal and the Riverplants, this research carried out a number of certain modifications and constraints to a Linear Programming Short Term Optimization Model "STOM" developed by Shawwash (2000). These modifications were made to capture the complex hydraulic configurations and special rules and regulations that govern the operation of the hydroelectric facility of river system.

2- The Model developed in this research project was then used to run several inflow scenarios which were selected from seventy simulation cases. The validity of model results were verified by the system operator and by comparing the model output to an existing simulation model for the Kootenay Lake and feedback from the Kootenay system operations engineers.

1.5 ORGANIZATION OF THE THESIS

This thesis is organized into six chapters. This **chapter** provided an introduction to the Kootenay system and outlined the objectives of the research. **Chapter 2** provides a literature review on the methods used for reservoir optimization problems. **Chapter 3** provides an overview of the Kootenay system, including the Kootenay Lake and the Kootenay River hydroelectric plants.

Chapter 4 describes the methodology and the mathematical formulation that were used to develop the optimization model for the operation scheduling of the Kootenay system. **Chapter 5** presents the results of applying the model to the operation scheduling of the Kootenay system and compares the results with the current simulation operation methods. This chapter also includes the result of a number of case studies that were performed. **Chapter 6** provides conclusions and recommendations for future research work.

2 LITERATURE REVIEW

The problem of optimizing the operation of a system of hydroelectric plants has been solved using different optimization techniques. Depending on the characteristics of the problem, different optimization methods are usually selected. Choosing an appropriate approach to deal with the problem and obtain reasonable results is an important step in any research project.

A literature review was performed on the use of optimization techniques and their implementation for single and multi-reservoir systems, either for short term or long term optimization problems. The most compatible technique for the purpose of this research project was chosen after studying different optimization methods.

Optimization techniques can be usually categorized into two main groups, deterministic and stochastic models [5]:

1) Deterministic Models: In a deterministic model, input data are determined and the model does not address uncertainty. Deterministic linear, integer, mixed integer and non linear programming are among the methods included in this group.
2) Stochastic Models: Stochastic models take into consideration the presence of some randomness in one or more of the input parameters or variables. Some well known stochastic optimization methods include stochastic dynamic programming, stochastic dual dynamic programming and stochastic linear programming techniques.

In this research project all inputs for the model are deterministic and therefore the main focus was on deterministic optimization techniques. However, a review was done on other optimization techniques to study the possibility of applying other methods to deal with the uncertain nature of some parameters in the scheduling problems such as electricity market price and the inflow into the reservoir for potential future improvements.

2.1 OPTIMIZATION TECHNIQUES

2.1.1 Linear Programming

Linear programming (LP) is a popular technique used to solve many optimization problems. It is said to be the most commonly applied form of *constrained optimization* [6]. Other optimization techniques such as binary, integer, and mixed-integer programming may be applied to highly nonlinear, non-convex terms in the objective function and constraints [7].
A linear program consists of parameters, variables, constraints and objective function(s) all expressed in linear expressions.

The advantages of using linear programming include: 1) the ability to efficiently solve large-scale optimization problems; 2) the convergence to a global optimal solution; 3) a first initial guess as an initial solution is not required to be specified by the user; 4) sensitivity analysis can be done on the output results; 5) the ease of structuring the problem and 6) the availability of low cost LP solvers [7].

AMPL is an example of a mathematical programming language for both linear and non linear models which is used to solve many reservoir optimization problems. Shawwash et al. (2000) developed a short term optimization model (STOM) using linear programming and AMPL to determine an optimal schedule for a set of hydroelectric plants. This model has been used by B.C Hydro's engineers to plan the hourly generation on a daily bases. Shawwash applied sensitivity analysis for further investigation of optimization results and concluded that the gain from applying the STOM model accounts for 0.25% to 1.0% of the optimized schedule by B.C Hydro [8].

Eschenbach et al. (2001) introduced a software system called "RiverWare" for modeling river basin operations. This software uses a linear goal programming method and piecewise-linear functions to approximate non-linear equations of a hydro system. The main decision variable in the model is the reservoir's outflow at each timestep. The continuity equation, the storage, the turbine discharge and the spilled water limits are the constraints in this model.

The user has the ability to define policy constraints and objectives through *RiverWare*'s constraint editor. The Tennessee Valley Authority uses *Riverware* to deal with the daily scheduling of multipurpose reservoir systems. The benefits of *Riverware* include the ability to consider flood control, hydropower generation needs, recreation purposes and environmental issues such as minimum flow requirements for aquatic habitat [9].

2.1.2 Integer and Mixed-integer Programming

Within a linear programming model we often find that we require a solution with integer variable values. Such problems are called integer programs. Models in which variables can take integer and non integer values are called mixed-integer programs. Integer programming is used for modeling and solving discrete optimization problems and is used in many real world applications. Hydro system operational problems attract the application of mixed integer programming methods because of the nature of these models. Mixed-integer programming has been successfully applied to unit commitment problems such as in hydroelectric plants [10].
Tang et al. (2007) used a mixed-integer and linear programming optimization approach to solve the maintenance scheduling problem for large scale hydro systems. He introduced a set of constraints to derive the maintenance schedule, for B.C Hydro system, and found the following on the characteristics of integer programming for the problem:

1) A maintenance schedule with efficient system operation defines the best mixed-integer models.

2) The outage scheduling of hydroelectric units can be influenced by variable head convergence.

3) Although there is no guarantee for a true optimal solution, the simple linear relaxation scheme that he developed is very efficient and its results seemed to solve a practical problem [11].

Kerr et al. (1997) studied the application of Integer Programming for short-term scheduling related to management and modeling issues. They introduced some of the managerial and modeling issues that are typically encountered in a hydro scheduling problem and discussed some heuristics to incorporate management priorities into the Integer Programming framework.

They found that Standard Mixed-Integer Programming (MIP) is a useful technique to deal with integer aspects of unit commitment problem, however it has some incompatibilities with managerial needs [10].

The main areas where Integer Programming does not appear to satisfy some of managers' needs such as time discretization, completeness, and focus on the key managerial aspects of the problem are discussed in their report. They found that by manipulating integer conditions performance in these areas could be improved and if mathematical modeling techniques could prioritize management needs then the results will be more acceptable [10].

Alavi (2003) developed the Rotational Energy Optimization Model (REOM) to derive the optimal electricity import for large storage hydroelectric systems including multiple units and complex hydraulic configurations for a short term planning horizon. The REOM consists of two main components: 1) An expert system that uses a set of rules on plant and unit operations to find the feasible system patterns; and 2) a mixed-integer programming algorithm to maximize the value of a hydro system import capability during low electricity market-price periods [12].

REOM is a hybrid system that implements integer programming, piecewise linear programming, expert systems and dynamic programming to solve the optimization problem. Alavi performed studies for four large hydroplants in the B.C Hydro system for a 24-hour study within a four-month period in 2002. He found that applying REOM would result in a more efficient operation schedule for the studied plants in the B.C Hydro. The REOM would enable the system operations engineer facing the rotational energy system constraint to determine the optimal decision on the value of imports with respect to the value of stored water in the reservoir [12].

2.1.3 Non- Linear Programming

The complexity of many hydropower optimization problems in some cases makes it impossible or very difficult to model the problem with piecewise-linear functions. In this case non-linear programming (NLP) could be the best way to solve the problem. Abdalla et al. (2007) stated that it is more common to apply LP and DP techniques to solve the reservoir optimization problems than using NLP methods. He concluded that this is because NLP methods are slow to converge and could easily arrive at non-optimal solutions, however, NLP methods may be applicable in cases where the objective function and/or some of the constraints cannot be realistically linearized [5].

Labadi et al. (1997) found that the most powerful NLP methods that could be used for hydroelectric power scheduling problem are the following:
1) the sequential linear programming (SLP); 2) the sequential quadratic programming (SQP); 3) the Projected Lagrangian method; 4) the method of multipliers (MOM) and 5) generalized reduced gradient method (GRG) [7].

Nonlinear programming can also be used in cases where the objective functions and the constraints are concave. W.S Chu et al. (1978) applied NLP to a similar case for a one-day to maximize the total hourly energy production. They used "Nonlinear Duality Theorems and Lagrangian procedures to solve the problem where the minimization of the Lagrangian is carried out by a modified gradient technique with an optimal determination routine" [14].
They indicated that there are many difficulties with the Lagrangian procedures when applied to real-life practical problems. They also found that the nonlinear programming problem can be simplified with the help of an efficient algorithm and that there is a need to specify an initial solution for this nonlinear programming method. They presented a numerical example and showed that if the objective function can be concisely defined, then without any approximation of the initial value for the decision variables, their algorithm can directly solve the nonlinear optimization problem [14].

An example of a large NLP optimization problem is a model of the Columbia River basin. Gagnon et al (1973) presented a numerical case involving "approximately 6000 variables, 4000 linear equations, 11000 linear and nonlinear inequality constraints and a nonlinear objective function". They found that a global optimum cannot be guaranteed because of the non-convex nature of the problem [15].

2.1.4 Dynamic Programming

Dynamic programming (DP) was originally developed by Richard Bellman in the 1940s. It develops an optimal solution through a backward recursion technique. In the dynamic programming method, the decision in each time step is made based on the outcome of each decision in the next stage (or timestep). In this method the end of the study is set to be the first stage and is used to initiate the decision making process in a backward recursion.
Yakowitz (1982) indicated that the DP method ranks second to linear programming among the optimization techniques for solving reservoir management problems [16]. One of the optimization problems that is regularly studied using DP is that of the reservoir filing problem. Korobova et al. (1968) applied DP in their study of a multi-annual regulation reservoir for the initial filling period. The objective was to create a set of rules on how to use the river flow in the initial reservoir filling period. Energy demand, hydroplant capacity and downstream constraints on the flow were given values in their model. However, river-flow forecasts were not accounted for in their study.

Applying a DP algorithm in this study, the authors found the following:

1) If the length of the computation period and the initial reservoir level is known then it is possible to find the optimum energy regime for the reservoir.

2) The dynamic programming method is especially useful when the power system, the energy output costs and the variations in reservoir storage have a non-linear relationship.

3) Acceptable DP solutions can be found within a zone with certain limits. These limits are typically defined by plant characteristics, operation rule curves, plant spillage regulations during high flow conditions and non-power requirements [17].

Dynamic programming was also used by Siu et al. (2001) to determine the optimal unit generation schedule of hydroelectric plants for the purpose of system operation at B.C Hydro. They solved the problem of Dynamic Unit Commitment and Loading (DUCL) with a procedure that integrated three algorithms: an expert system to eliminate infeasible solutions; a dynamic program to find the optimal static unit commitment for a given hydraulic condition, plant loading and the feasible unit combinations; and an integer programming model to solve a large-DUCL problem. This model and its component are regularly used for real time energy and capacity scheduling at B.C Hydro [18].

2.1.5 Stochastic Dynamic Programming

While in dynamic programming models the inputs and the immediate value of the decisions are known with certainty, a stochastic dynamic programming model (SDP) addresses uncertainty in some of inputs to the models.
A number of researchers carried compared stochastic and deterministic dynamic programming methods in the operation of the reservoir systems. Karamouz et al. (1987) compared a deterministic model with three components (DPR) to a stochastic model (SDP) for a reservoir operation problem. In their study, the DPR model consists of three components: a DP, a regression analysis and a simulation component. They used these methods to generate sets of operation rules. After studying three different cases, they found that the rules generated with SDP method was more effective than DPR model for small reservoirs whereas DPR model performed better for medium to large reservoir systems [19].

In a review of stochastic optimization methods for medium-term scheduling in hydro systems, Gjelsvik et al. (1996) stated that different stochastic techniques yield different results when used for long and mid term scheduling problems. They found that for long term scheduling SDP is a natural solution method, but that the aggregation of reservoirs that is usually necessary to solve long term problems does not permit modeling of individual reservoirs in details. In such problems, they found that the Stochastic Dual Dynamic Programming method was a more robust method to deal with the optimization of multi-reservoir systems [20].

Mazariegos (2006) applied dynamic and stochastic dynamic programming methods for allocation of load variability in hydro systems. The model considered the production costs of economic dispatch decisions for plants that are used for Automatic Generation Control. To determine the optimal allocation of within-the-hour load increment in the system a SDP model was used. The research also analyzed the seasonal probability of load forecasting errors, which was used to find the best dispatch regime for three of the largest plants in B.C Hydro's generation system [21].

2.1.6 Evolutionary and Heuristic Algorithms

Evolutionary Algorithms are another method to solve reservoir optimization problems. The most popular among heuristic methods are artificial neural networks and genetic algorithms (GA) [5]. Influenced by evolutionary biology, GA use different functions such as crossovers, mutation, and inheritance to arrive at optimal solution. Its parameters are in the form of strings and are called chromosomes. In each step "parental" chromosomes are combined randomly to generate a second set of parameters called "new generation". Accordingly, the fittest parameters (the best results) are chosen using a function called the "Roulette Wheel". After several iterations the generation produced is taken as the optimal solution. Several studies have been carried out on the potential use of genetic algorithms to solve reservoir optimization problems.

Oliveira et al. (1997) found that the power of genetic algorithms is based on a simple assumption: "The best solutions are more likely to be found in the regions of the solution space containing high proportions of good solutions and these regions can be found by sampling the entire solution randomly". They applied GA to two hypothetical hydro systems and showed that this can be a favourable method to solve complex objective functions in the reservoir optimization problem [22]. Momtahen et al. (2007) applied the GA method to a single reservoir system and compared it with other methods such as DP and SDP. They used GA to optimize policies given a set of parameters. They claimed that the main disadvantages of GA methods are that in spite of stable objective functions, there are a number of potential solutions to the problem, although it can also be regarded as an advantage, which gives the user the ability to choose a good policy among the optimal solutions [23].

Cai et al. (2001) combined genetic algorithms with linear programming to solve a non-linear water management problem [24]. The main idea in this approach was to specify a set of "complicating variables" and fix their values to simplify the problem into a linear programming model and improve the value of the fixed variables with the help of genetic algorithms. The result of the combined GA and LP method was then used as a starting point for a non-linear solver. The final result was then compared to a case in which some other initial input values were fed into the non-linear solver. Their comparison shows that the results are better in first method i.e. the GA and LP applied method [24].

2.1.7 Combing different Optimization Methods to Solve the Reservoir Operation Problems

Different optimization models in water resource management require different solution methodologies. Some reservoir optimization problems require discretization of the problem into stages. Over these stages the reservoir characteristics and limits may or may not be continuous. In some cases some limits can be relaxed over these stages. Guan et al. (1997) used a Lagrangian relaxation technique to find short term operating policy for a reservoir optimization problem. To make reservoir constraints decomposable over stages a set of multipliers were used. These multipliers were updated by a modified sub-gradient optimization algorithm. To optimize the operating states across the planning horizon dynamic programming was applied. A network algorithm was then used to find a feasible solution where operating states could be adjusted by heuristic rules. Not having to discretize the reservoir storage states is one of the main advantages of using this algorithm. When applied to a numerical example, this algorithm proved to be computationally efficient and provided satisfactory results along with a near optimal policy [25].

3 THE KOOTENAY RIVER SYSTEM

This chapter gives a general background on the Kootenay system and discusses some of its unique characteristics. It provides some information on the Kootenay Lake, the Cora Lynn (COR), the Upper Bonnington (UBO), the Lower Bonnington (LBO), the South Slocan (SLC), the Brilliant (BRD), the Brilliant Expansion (BRX) and the Kootenay Canal facilities (KCL). Several B.C Hydro's plant operation manuals were used as a reference for this chapter.

The Kootenay Lake is the main lake reservoir considered in this research. Part of the KLK outflow is regulated by the operation of 1) Lake Koocanusa behind the Libby Dam and powerplant in the United States, and 2) the Duncan Dam, a BCH dam which was built in 1960 and is regulated by the Columbia River Treaty between Canada and the U.S. [4].

3.1 KOOTENAY CANAL

3.1.1 History

The Kootenay Canal Project (KCL), constructed between 1972 and 1976, is located on the south side of the Kootenay River. The intake to the canal is to the left of the Cora Lynn Project on the Kootenay River, 14 km downstream of Nelson. The KCL powerhouse is 5 km downstream of the canal headworks [1].

Table 3.1 Kootenay Canal Facilities [Source: B.C Hydro, 2008]

Project Name:	Kootenay Canal (KCL)
Dam Name:	Kootenay Canal
Dam Height:	38 metres
Reservoir Name:	Kootenay Canal Headpond
Water Course:	Kootenay River
Upstream Project:	Duncan and Libby (U.S.)
Downstream Project:	Brilliant

The KCL has a maximum turbine capacity of 30,000 cubic feet per second (cfs) and 4 generating units with no spillgate facilities. The project was constructed for the purpose of power generation. In accordance with KCL's water license, BCH must allow for a minimum release of 5,000 cfs through the Riverplants before diverting any flows through the KCL. Outflows of more than 5,000 cfs are diverted to the KCL powerhouse because of the higher efficiency of its units. Once the KCL flows are at their maximum capacity, the balance of the water is then passed through the Riverplants turbines and discharge facilities.

3.2 THE CORA LYNN DAM

Built in 1932, the Cora Lynn Dam (COR) is located approximately 15 km downstream of the City of Nelson on the Kootenay River. The dam is used for the purpose of controlling the outflow from the Kootenay Lake and generating up to a maximum of 49 MW with three generation units. The maximum flow capacity of the COR powerplant is 12,600 cfs. The maximum flood flow that can pass through Cora Lynn dam is about 328,000 cfs (9,306 m^3/s). Figure 3.3 shows the location of the Cora Lynn Dam. The normal reservoir operating level is between 1,735.0 ft (529.0 m) and 1,745.0 ft (531.7 m) [4]. Table 3.2 shows the summary of plant characteristics [4].

Table 3.2 Cora Lynn plant summary [Source: B.C Hydro Operation Manuals, 2008]

General Information	
Plant Name:	Cora Lynn (COR)
Flow Capacity	12,600 cfs
Normal forebay operation	1,735 ft to 1,745ft
Water Course:	Kootenay River
Upstream Project:	Duncan and Libby (U.S.)
Downstream Project:	Upper Bonnington

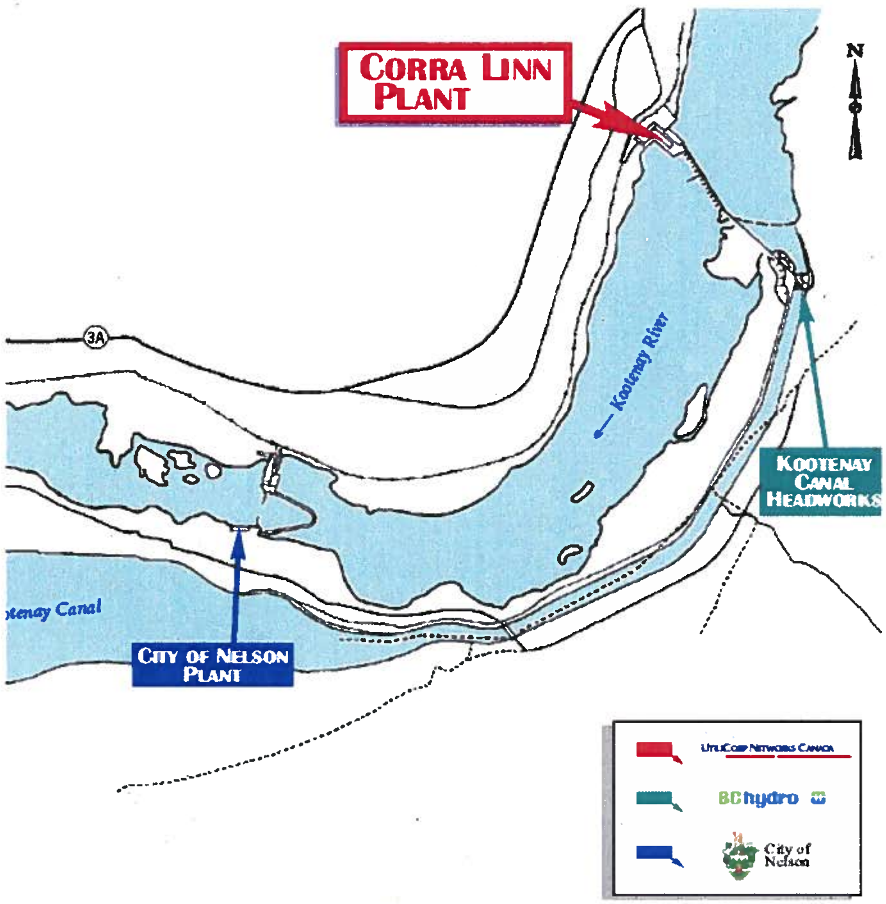


Figure 3.1 Cora Lynn Plant Location on the Kootenay River [Source: B.C Hydro Operation Manuals]

3.3 THE UPPER BONNINGTON

The Upper Bonnington (UBO) Dam was originally built in 1907 and now has six generation units. It was upgraded to a total maximum of 66 MW [26]. The maximum licensed turbine capacity of the UBO is 12,800 cfs and during most of the year it has a minimum inflow of 5,000 cfs. The maximum flow through the UBO without overtopping the dam is about 240,000 cfs according to B.C Hydro's operation orders. The UBO is a run-of-river plant with essentially zero storage capacity. The forebay at the UBO can vary about 1ft above and below 1,682 ft, but is considered constant for the purpose of power generation calculations. The City of Nelson shares the same forebay with UBO and has a license to use a small part of passing water through UBO for the purpose of power generation [4].

The UBO dam is downstream of COR and is the second dam on the Kootenay River below the Kootenay Lake. Figure 3.4 displays the location of the UBO Dam in relation to the other hydroelectric plants on the Kootenay River [4].

Table 3.3 shows the summary of plant characteristics.

Table 3.3 Upper Bonnington plant summary [Source: B.C Hydro Operation Manuals, 2008]

General Information	
Plant Name:	Upper Bonnington (UBO)
Flow Capacity	12,800 cfs
Normal forebay operation	1,682 ft
Water Course:	Kootenay River
Upstream Project:	Cora Lynn
Downstream Project:	Lower Bonnington

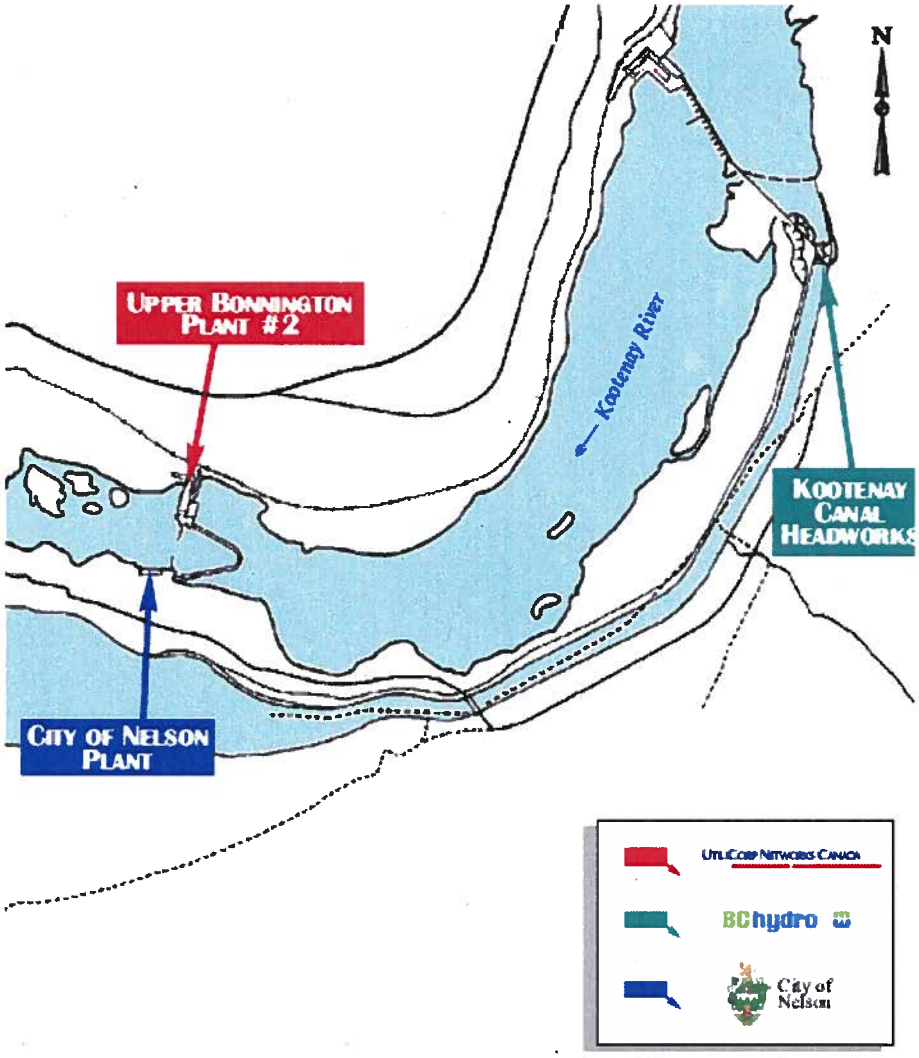


Figure 3.2 Upper Bonnington Plant Location on the Kootenay River [Source: B.C Hydro Operation Manuals]

3.4 THE LOWER BONNINGTON

The Lower Bonnington (LBO) Dam is located downstream of the Upper Bonnington and was originally built in 1897, then upgraded in 1925 [26]. The maximum licensed turbine capacity of LBO is 10,400 cfs with a maximum power generation of 44 MW with three generating units. The LBO is located on the Kootenay River approximately 18 km southwest of the City of Nelson as shown in Figure 3.5. The maximum normal operating forebay level of the Lower Bonnington is 1,610.7 ft (491.1 m) with no considerable variations [4].

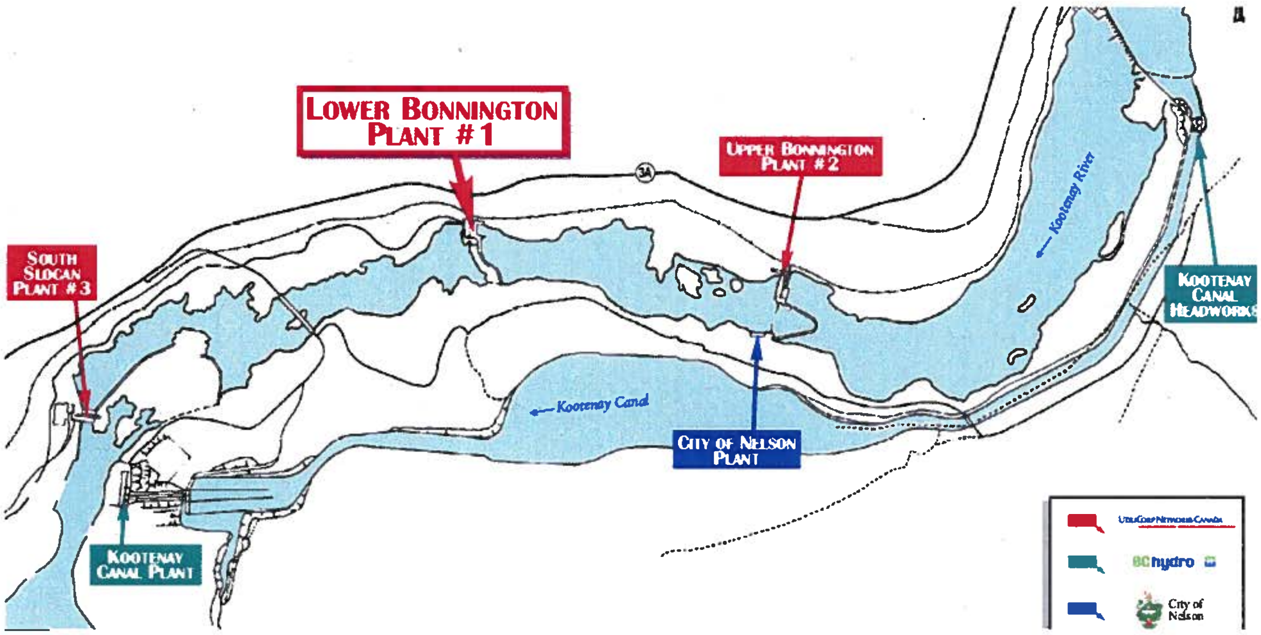


Figure 3.3 Lower Bonnington Location on the Kootenay River [Source: B.C Hydro Operation Manuals]

Table 3.4 shows the summary of plant characteristics.

Table 3.4 Lower Bonnington plant summary [Source: B.C Hydro Operation Manuals, 2008]

General Information	
Plant Name:	Lower Bonnington
Flow Capacity	10,400 cfs
Normal forebay operation	1,610.7 ft
Water Course:	Kootenay River
Upstream Project:	Cora Lynn
Downstream Project:	South Slocan

3.5 SOUTH SLOCAN DAM

The South Slocan Dam (SLC) is another run-of-river plant that was constructed in 1939 [26]. It has a maximum generating capacity of 54 MW with three generating units. The maximum licensed turbine flow through SLC power house is 10,500 cfs. Figure 3.5 shows the location of the SLC at 20 km south west of the City of Nelson [4].

Table 3.5 shows the summary of plant characteristics.

Table 3.5 South Slocan plant summary [Source: B.C Hydro Operation Manuals, 2008]

General Information	
Plant Name:	South Slocan
Flow Capacity	10,500 cfs
Normal forebay operation	1,543 ft
Water Course:	Kootenay River
Upstream Project:	Lower Bonnington
Downstream Project:	Brilliant

3.6 THE BRILLIANT DAM

The Brilliant Dam (BRD), constructed in 1944, is the furthest downstream of the five Riverplants on the Kootenay River. The BRD is owned by the Columbia Power Corporation and is operated according to the canal plant agreement between BCH, FBC, CPC and Teck Cominco Metals Ltd.(TCML).

Second to the Kootenay Canal, Brilliant is the most efficient of the other Riverplants. With a unit extension called the Brilliant Expansion Unit (BRX) completed in 2007, its generation capacity was increased to 276 MW. Brilliant has limited live storage. The water licenses allow for the storage of water between El.1,469.0 ft (447.9 m) and El. 1,479.0 ft (450.9 m)

Because of the IJC special regulation and the limited storage capacity at Brilliant, BRD often spills during high-flow periods. This especially happens during the freshet period (April to July). At this time of the year the other Riverplants also operate in a spill mode.

The operating rules for Brilliant specify a target minimum flow of 18 thousand cubic feet per second (kcfs) for most of the year and 16 kcfs for October and November. This rule guarantees the minimum flow required for downstream aquatic habitant. If water is insufficient to provide this flow, the Kootenay Lake storage does not have to be used to comply with Brilliant minimum flow rules but lake inflow must be passed on [4]. The BRD location on the Kootenay River is shown in Figure 3.6.

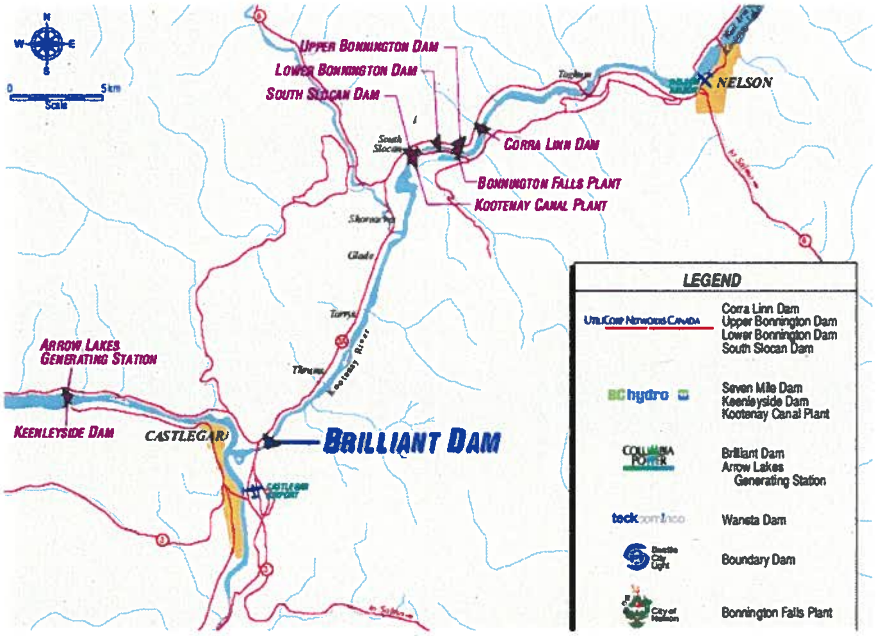


Figure 3.4 Brilliant Dam Location [Source: B.C Hydro operation manuals]

Table 3.6 Brilliant plant summary [Source: B.C Hydro Operation Manuals, 2008]

General Information	
Plant Name:	Brilliant (BRD)
Flow Capacity	21,600 cfs (36,00 cfs with expansion unit)
Normal forebay operation	1,469 ft-1,479 ft
Water Course:	Kootenay River
Upstream Project:	South Slocan
Downstream Project:	Columbia River

3.7 OPERATION OF KOOTENAY LAKE

3.7.1 International Joint Commission Order (IJC)

The operation of the Kootenay Lake is governed by several operational limits such as the 1938 IJC order that constraints the lake level [3]. In the Kootenay region there are different environmental issues associated with the lake level. Power demands should also be satisfied during periods of high energy demands. The problems on the Kootenay Lake are also compounded because the variations of the lake levels may affect the water level of Kootenai River in Idaho in the US [29].

Figure 3.7 shows how the Kootenay River changes to the "Kootenai River" on the other side of the Canada-US border. At the City of Wardner, BC the Kootenay River widens into the Lake Koocanusa behind the Libby Dam and then turns north in Idaho and it reenters Canada at Creston, British Columbia.

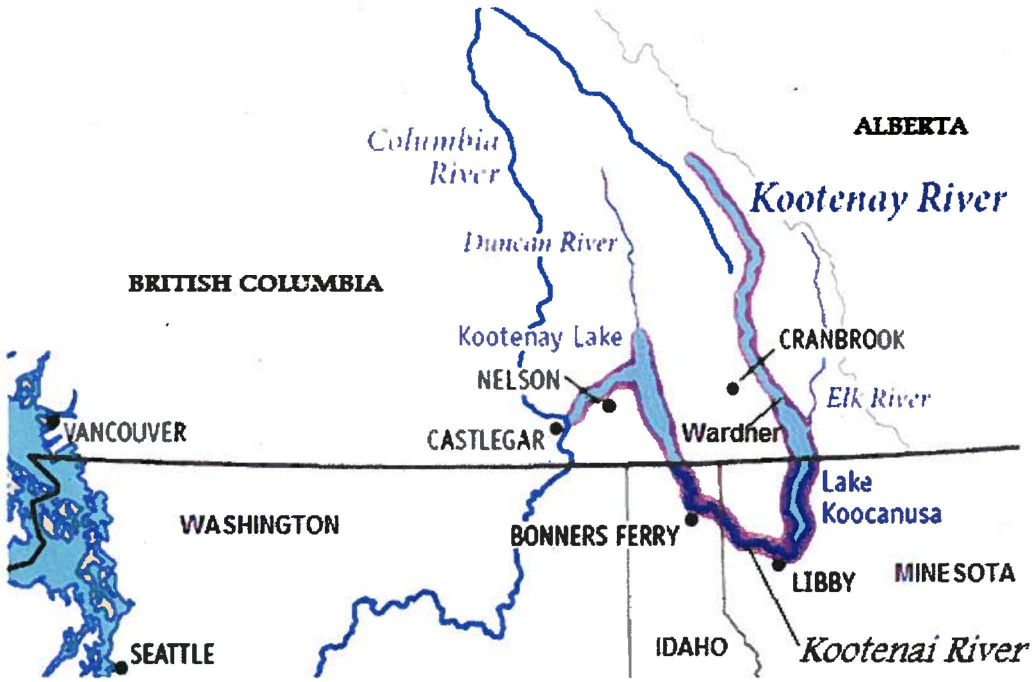


Figure 3.5 Kootenay River Location Map [Source: Wikipedia]

In November 1938 a number of rules were set out to regulate the Kootenay Lake operation. These rules were initially developed to avoid conflicts among stakeholders and to provide beneficial conditions without adverse effects to any of the interested parties.

<u>Outline of the IJC rule [3] :</u>

From the beginning of September to January 7th the maximum lake level at the Queen's Bay gauge is set at 1,745.32 ft. After that date the operation of the Kootenay Lake should meet the following target levels at the Queen's Bay gauge, and linear interpolation can be used for the following intermediate dates:

On February 1^{st} the IJC target level = 1,744.00 ft;

On March 1^{st} the IJC target level = 1,742.40 ft;

On April 1^{st} the IJC target level = 1,739.32 ft

From the beginning of April the IJC maximum level is set at 1,739.32 ft. at the Queen's Bay gauge until the start of freshet period is declared by the IJC Board of Control. After the beginning of freshet period the Kootenay Lake is regulated under a lowering formula and it must be operated so that the Queen's Bay level is at least at a specified margin (0.3 ft) below the level that would occur under the unregulated natural conditions, prior to excavation of the Grohman Narrows.

The margin of 0.3 ft is specified in order to reduce the chance of exceeding the IJC order for the maximum operating levels due to unforeseen storm events or operating contingencies [4].

After the lake peaks, if the Nelson gauge drops below 1,743.32 ft then the lake level must remain at or below this level (at Nelson gauge), or within a margin from it, until the end of August.

As shown in Figure 3.8 the black line is an example of the IJC upper rule curve typically used for the lake operation.

This constraint is one of the two main constraints on the Kootenay Lake. The operation of the Kootenay Riverplants and the Kootenay Canal is highly affected by this constraint.

Discharges from the lake (at any time of year) that result in exceeding the IJC upper level rule curve are unacceptable and must be avoided whenever possible.

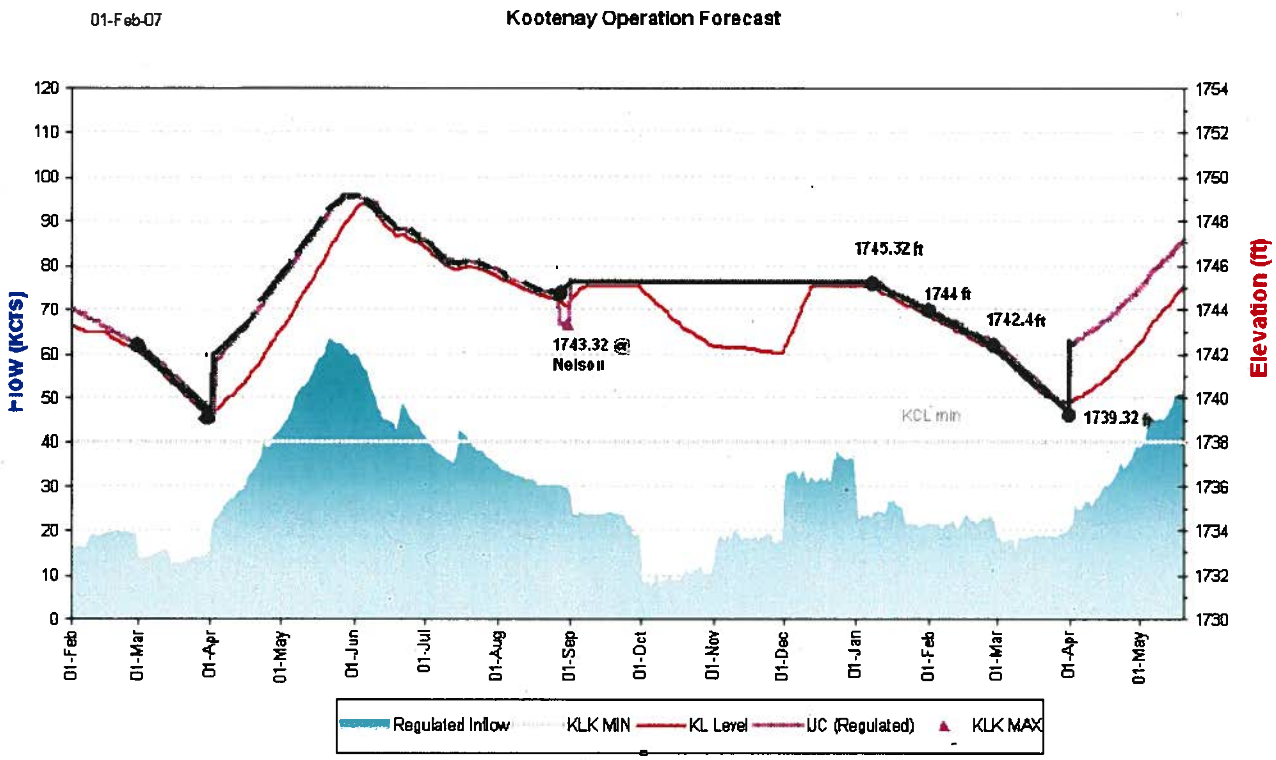


Figure 3.6 IJC Upper Rule Curve For the Year
[Source: B.C Hydro Kootenay operation spread sheet]

3.7.2 The Grohman Narrows restriction

The Grohman Narrows (GN) severely restricts outflows from the lake, especially at lower Kootenay Lake levels. The maximum outflow from the Kootenay Lake depends on the Cora Lynn reservoir level and the lake level. Figure 3.9 shows the relationship between the Kootenay Lake level, the Cora Lynn reservoir forebay level and the maximum outflow (kcfs) from the lake. This relationship is very important for operation planners to decide on the amount of outflow that can be released from the lake (through GN). It can also be interpreted as a headloss table where it shows the headloss between the Kootenay Lake level and Cora Lynn forebay level.

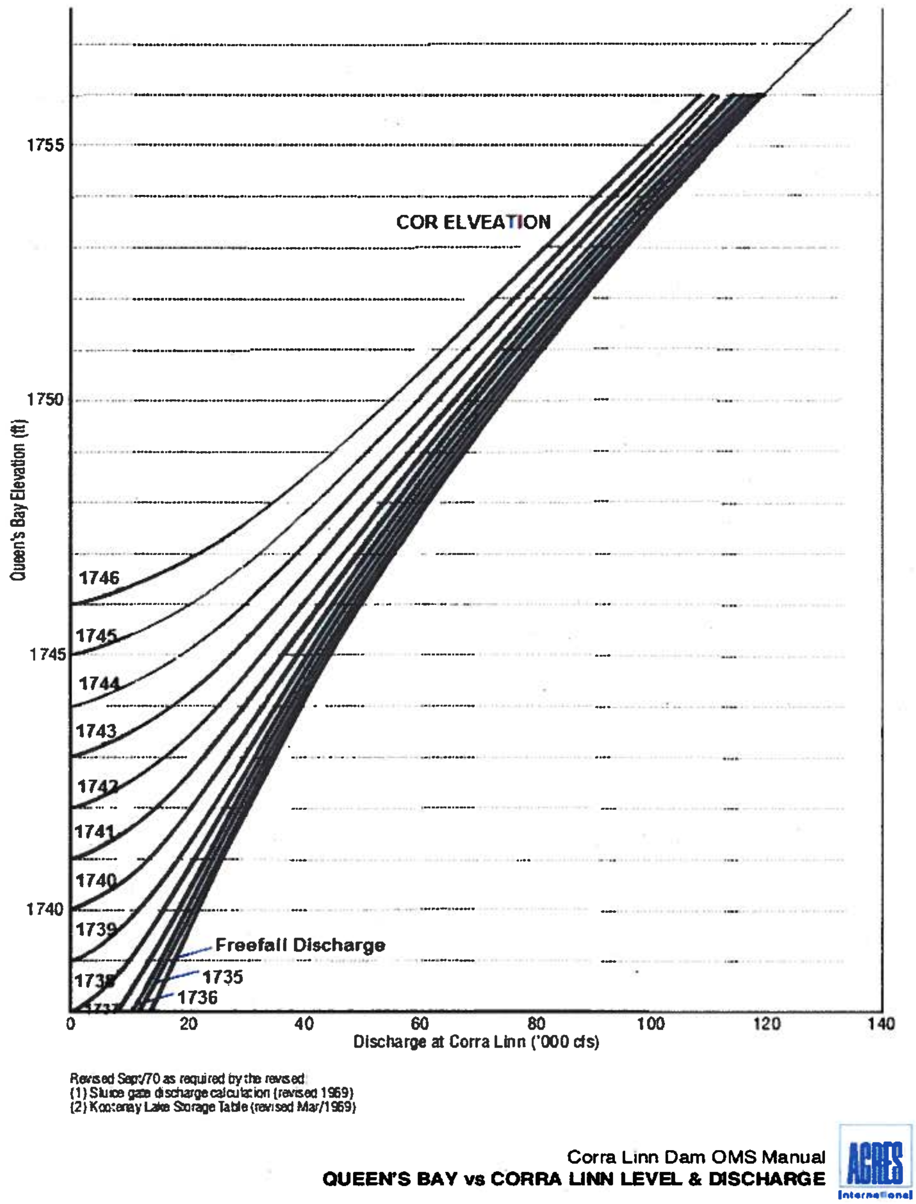


Figure 3.7 Queen's Bay vs Cora Lynn Level & Discharge
[Source: B.C Hydro operation manuals]

One of the complexities of this system is the ability to model the hydraulic losses between the Kootenay Lake level and the Cora Lynn reservoir. There are also headlosses between the Cora Lynn forebay and the Kootenay Canal forebay level at the canal headworks that should also be considered in the model.

3.7.3 The Canal Plant Agreement (CPA)

The Canal Plant Agreement (CPA) between B.C Hydro, Cominco Ltd. (now TCML), and West Kootenay Power (WKP) (now FBC) dated August 1, 1972, covers the operation of the Kootenay Riverplants, BRD on the Kootenay River, the Waneta Plant on the Pend d'Oreille River, and the Kootenay Canal Plant [3]. The Operating Procedures are developed by CPA operating committee for implementing the Canal Plant Agreement. They incorporate applicable parameters and legal obligations for the operation of all the facilities under the CPA [27].

4 MODELING METHODOLOGY

This chapter discusses the optimization model developed to operate the Kootenay Lake for a period of up to one year in a daily time step. The main model structure was adopted from *"STOM"* developed by Shawwash et al. (2000), which is a hourly short term linear optimization model developed for B.C Hydro's resource optimization.

4.1 INTRODUCTION

STOM is a resource optimization model that has been developed to determine the optimal hourly generation for the B.C Hydro system. It was developed using the AMPL software, which is a powerful mathematical programming language for linear and non-linear optimization problems. The formulation used in this research project is similar to STOM because of it would be easily incorporated into other general optimization models in B.C Hydro. The Kootenay system optimization is different from other B.C Hydro systems because of the special constraints and regulations and the complexity of including those constraints in a linear model. Different constraints and functions were added to the main structure of the STOM to better describe the Kootenay system characteristics.

4.2 MODEL STRUCTURE

4.2.1 Introduction to STOM formulation [Adapted from Shawwash (2000)]

4.2.1.1 Hydraulic Modeling of Reservoir Operations

i. Decision Variables and Model Constraints

In a hydropower facility the amount of discharge from a plant could be either the spilled flow or the turbine flow. The spilled flow is the part of flow that is not used for energy production and should be passed through spill facilities. The turbine flow is the portion of water used for power generation. The turbine maximum capacity and the reservoir storage limits specify the amount of water that can be used in each timestep. The operation of a hydro facility is also impacted by certain rules that are set by environmental, regulatory, navigational, and long-term planning requirements.

Hydro and power generation variables are the common variables in a hydro optimization model. The decision variables can be either independent or dependent. Independent variables are those that are searched for by the optimization algorithm, while the dependent variables are calculated by the model's equations [8].

The index "j" refers to plants in study, and "t" refers to timesteps.

- The independent turbine discharge variables, QT_{jt}, in cubic meters per second,
- The independent forced spill discharge variables, QS_{jt}, in cubic meters per second,
- The dependent total plant discharge variables, QP_{kt}, in cubic meters per second,
- The dependent reservoir storage variables, S_{kt}, in cubic meters per second for one day,
- The dependent plant generation variables, (G_{jt}), in megawatts for each hour [8].

ii. Representation of Hydroelectric Facilities

A typical hydroelectric generation system consists of sets of rivers, tributaries, reservoirs, powerhouses and additional hydraulic facilities such as intake structures, spillway gates and weirs. A system of hydroelectric plants could be connected in parallel or in series. Facilities that are serially connected are hydraulically connected because the outflow from the upstream plant is a part of the total inflow to the downstream plant's reservoir. Inflows to reservoirs may also be natural.

A matrix structure was used to capture the complex nature of inflows to and from the reservoirs in the B.C Hydro system. This matrix describes flow sources and destinations in a set of connected generating facilities. Several incidence matrices are used to describe the turbine and spill discharges and inflows from or to the reservoirs as follows. The QTR_{jk} and QSR_{jk} matrices describe the turbine and spill flows from hydroelectric facility j to a hydroelectric facility k ($j \in J$, $k \in K$: $j = k$). The index k represents the rows in the matrix while j represents the columns. Other matrices are used to describe the turbine UQT_{jk} and spill UQS_{jk} hydroelectric facility's inflows from facility j to facility k ($j \in J$, $k \in K$: $j \neq k$). An entry of '1' in the matrices indicates that a physical flow occurs from or between reservoirs, while '0' indicates no flows.

These simple yet powerful descriptions of the system have allowed the modeling of very complex patterns of flows between reservoirs. It has also allowed the model to be formulated in a general way dynamically by the user.

$$QTR_{jk} =$$

$$\begin{array}{ccccccc} & j & j+1 & j+2 & \cdots & \mathrm{J}-1 & \mathrm{J} \\ k & 1 & 0 & 0 & \cdots & 0 & 0 \\ k+1 & 0 & 1 & 0 & \cdots & 0 & 0 \\ k+2 & 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathrm{K}-1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \mathrm{K} & 0 & 0 & 0 & \cdots & \vdots & 1 \end{array}$$

In the QTR_{jk} matrix shown above, the index j and k represents the same facility, which gives rise to a square matrix. A turbine outflows from facilities j, $j+1$, and J are also discharged from facilities k, $k+1$, and K respectively. When the value in the matrix that corresponds to facility (J-1, K-1) is "0" in QTR_{jk}, there is no turbine discharge from the facility. Similar matrices are used to describe spill discharges from hydroelectric facilities, QSR_{jk} and upstream turbine $UQTR_{jk}$, and spilled discharge $UQSR_{jk}$ in to downstream [8].

iii. The Constraints

The constraints in the optimization model consist of hydro constraints and power generation constraints. Hydro constraints describe the hydraulic relationship in hydroplants and define the discharge limits such as the maximum and minimum turbine (QT^{Max}_{kt}, QT^{Min}_{kt}) or spill (QS^{Max}_{kt}, QS^{Min}_{kt}), capacity. Power constraints also define the limits on generation capacity and minimum generating limit of a generating plant.

The hydro constraints are:

- the matrices representing the turbine discharges from a reservoir,
$RT_{jkt} = QT_{kt} * QTR_{jk}$, (4.1)
- the matrices representing the spill discharges from a reservoir,
$RS_{jkt} = (QS_{kt}) * QSR_{jk}$, (4.2)
- the matrices representing the upstream turbine inflows to each reservoir,
$UT_{jkt} = QT_{jt} * UQT_{jk}$, (4.3)
- the matrices representing the upstream spill inflows to each reservoir,
$US_{jkt} = (QS_{kt}) * UQS_{jk}$ (4.4)
- the upper and lower bounds on turbine discharge from each reservoir,
$QT^{Min}{}_{kt} \leq QT_{kt} \leq QT^{Max}{}_{kt}$, (4.5)
- the upper and lower bounds on total spill discharges from a reservoirs,
$QS^{Min}{}_{kt} \leq QS_{kt} \leq QS^{Max}{}_{kt}$, (4.6)
- the total plant discharge (QP_{kt}) from each reservoir :
$QP_{kt} = QT_{kt} + QS_{kt}$, (4.7)
- the upper $QP^{max}{}_{kt}$ and lower bounds $QP^{min}{}_{kt}$ on total plant discharge from each reservoir,
$QP^{Min}{}_{kt} \leq QP_{kt} \leq QP^{Max}{}_{kt}$ (4.8)
- The mass-balance (continuity) equation for reservoirs, that couples the storage dependent decision variables across time. The storage value at each time step S_{kt+1} is equal to previous time step storage value S_{kt} plus total inflow to the plant minus total outflow. QRI_{kt} is the natural inflow to the plant at each timestep [8].

$$S_{k(t+1)} = S_{kt} + (-\sum_{j=1}^{J} RT_{jkt} - \sum_{j=1}^{J} RS_{jkt} + \sum_{j=1}^{J} UT_{jkt} + \sum_{j=1}^{J} US_{jkt} + QRI_{kt}) / 24 , \quad (4.9)$$

- the upper and lower bounds on each reservoir storage, storage values for each plant at each timestep is bounded by the maximum ($S^{Max}{}_{kt}$) and minimum ($S^{Min}{}_{kt}$) allowable storage levels [8].

$$S^{Min}{}_{kt} \leq S_{kt} \leq S^{Max}{}_{kt} . \quad (4.10)$$

4.2.1.2 Modeling Hydropower Generation and Generation Production Functions

Water stored in reservoirs is usually used for different purposes, mainly in this study for power generation. In a generating facility water is passed through penstocks, gates or valves, to turbines, which convert the kinetic energy of water into mechanical energy that in turn is converted into electrical energy by generators.

Generators are used for two primary control functions: power generation and frequency control, and they can be equipped with an automatic feedback control system to regulate frequency and load [8].

i. General Background on the Generation Production Function

Figure 4.1 illustrates a simplified production function of a typical plant with multiple units in the B.C Hydro system. This production function is developed using the assumption of optimal unit commitment. Production functions are simple and powerful because they can effectively summarize many details of the turbines, generators, and hydraulic structures in the plant.

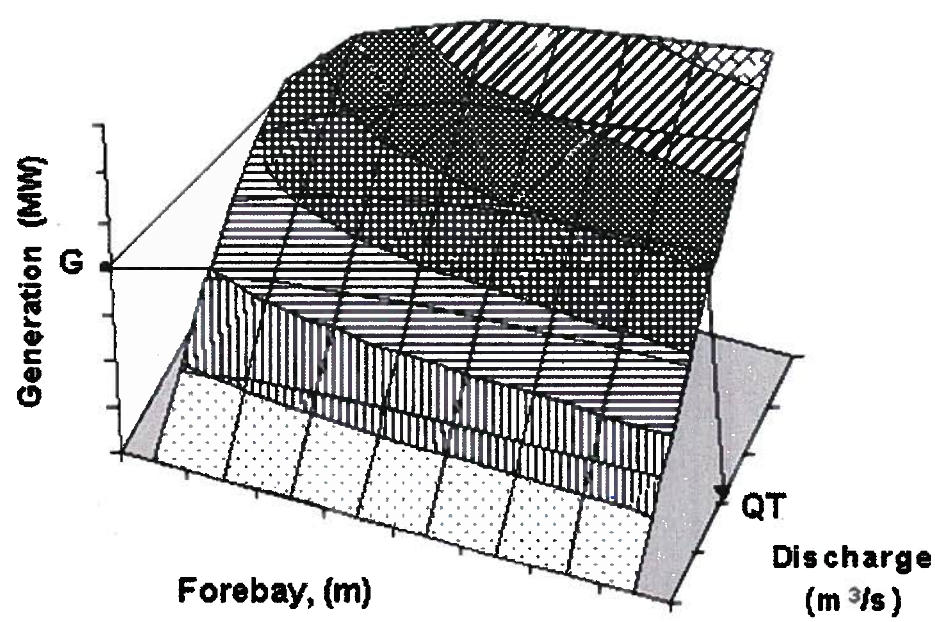


Figure 4.1 Production Function of a Hydroelectric Generating Plant [Source: Shawwash 2000]

The optimal transformation of the main input variables, water and forebay level, into power generation product is demonstrated in the production function of Figure 4.1. With the assumption of optimal unit commitment, each point on the production surface function represents the maximum generation that can be obtained for a given set of turbine water discharge and forebay level for the available number of units in the plant. Any from of wasteful or technically inefficient use of water and forebay is excluded in this production function.

The production function's shape, slope and smoothness are important parameters which describe the plant generation functions and determine the kind of optimization techniques that can be usefully applied. As shown in Figure 4.1 an isoquant is a locus on the production function of all equal levels of electric energy production. This shape of the production function shows the important fact that many different combinations of water turbine flow and forebay level inputs can result in the same level of electric energy production. Surface cuts provide information on the characteristics of the production function. The slope reflects the rate at which each of the inputs affects the outputs. Lastly, the smoothness of the cuts reflects whether there are any irregularities in input-output relationships, which could entail discontinuities in the production function [8].

ii. Main Features of the Hydroelectric Plant's Production Functions

Certain features of the plant production function that are suitable for use of the linear programming techniques were obtained with close examination and study of the B.C Hydro's hydroelectric generating facilities. Figure 4.2 is a production function for a plant with four units.

The following is a description of the main features of this typical generation production function:

1)As depicted in Figure 4.2, the effect of variation in forebay on the $G = f(Q)$ function for a given plant discharge is almost linear.

2) As a result of optimal unit commitment and switching between units some "bumps" exist in the $G = f(Q)$ curves; However the curves are generally smooth for each forebay level. Efficiency curves are demonstrated in Figure 4.2 to refer to the "bumps" in the $G = f(Q)$ as they are not clearly seen.

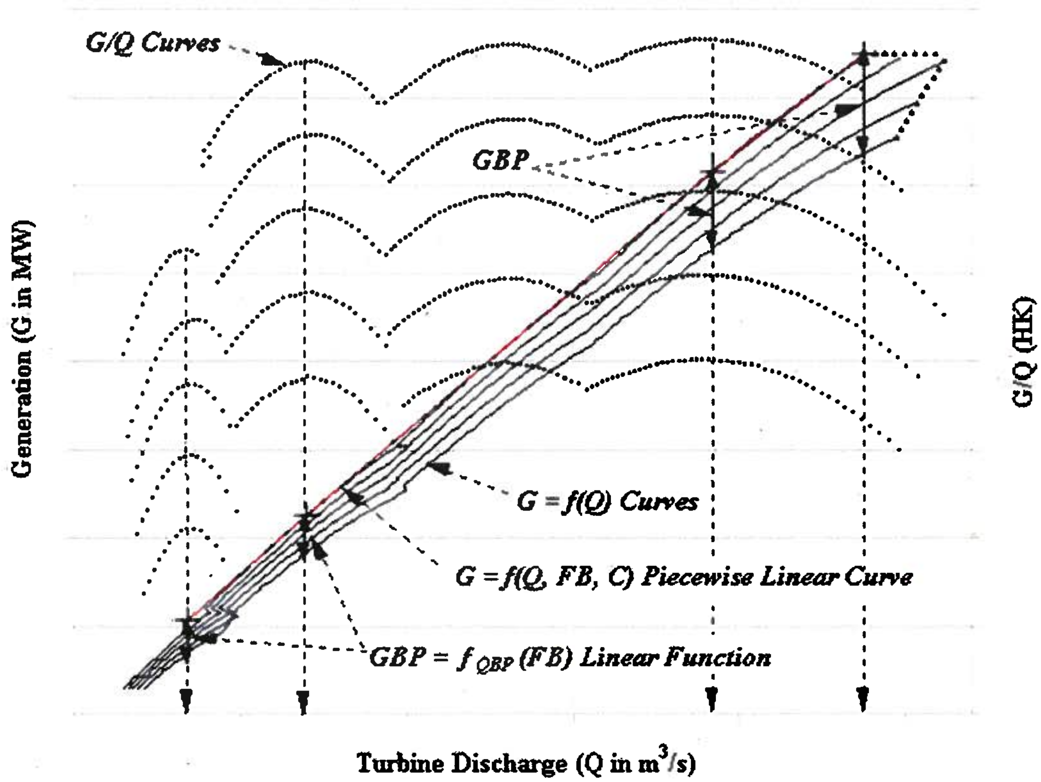


Figure 4.2 Typical Production Function for a Hydroelectric Plant
[Source: Shawwash 2000]

3) The $G = f(Q)$ curve for a given forebay is slightly concave, and in many instances is almost linear.

4) The peaks of the "bumps" in the efficiency curve represents local peak efficiency performance of the plant for a given plant generation range. These peaks result from operating one or a combination of units at their maximum efficiency, or optimal unit commitment.

5) The $G = f(Q)$ curves are almost linear between consistent ranges of plant discharge. This can be illustrated by taking a ruler and matching the curve for certain turbine discharge ranges.

6) The $G = f(Q)$ curves are not smooth near the plant's minimum operating ranges, which results from frequent switching between units due to the existence of inoperable generation zones for individual units. The inoperable zone results from excessive vibration, frequency problems, etc.

7) The $G = f(Q)$ are continuous except near the minimum operating ranges.

These features make it possible and easy to formulate a piecewise linear production function to output the plant generation. The required inputs are the turbine discharge, the forebay level and the unit availability. The production function for each plant consists of a family of piecewise linear curves that have been curve-fitted by a specialized procedure to accurately describe the plant generation at time-step t (G_{jt}) as a function of its forebay level, turbine discharge and unit availability [8].

The power generation function is given as a constraint equation in the model.

- the piecewise linear generation production function that calculates plant generation as a function of reservoir forebay FB_{jt}, turbine discharge QT_{jt} and unit combination C_{jt}s,

$$G_{jt} = f(FB_{jt}, QT_{jt}, C_{jt}), \tag{4.11}$$

The other *power* generation constraints, as discussed in Section 4.2.1.2, are:

- the upper and lower bounds on plant generation,

$$G^{Min}{}_{jt} \leq G_{jt} \leq G^{Max}{}_{jt}, \tag{4.12}$$

- the optional constraint that fixes the *LRB* (scheduled) generation for a plant to a fixed schedule [8].

$$G_{jt} = G^{LRB}{}_{jt}. \tag{4.13}$$

4.2.1.3 Maximize the Value of Resources Optimization Model

In a reservoir optimization problem the prime objective is to satisfy the local load demand and firm export/import contracts and then to make the optimal trade-off between present benefits, expressed as revenues from real-time spot energy sales, and the potential expected long-term value of resources, expressed as the marginal value of water stored in reservoirs [8].

Equation 4.14 lists the objective function considered in the STOM model for maximizing the value of resources:

Maximize:

$$+\sum_{m=1}^{M}\sum_{t=1}^{T} NSS_{mt}*NSSPrice_{mt}$$
$$+\sum_{k=1}^{K}(S_{kT}-STarget_{kT})*MVW_k*24*3600 \qquad (4.14)$$
$$-\sum_{t=1}^{T} GThert*TIC_t$$

The first term in Equation 4.14 represents the sum of revenues (or costs) accrued from net spot energy exports (or imports), given forecast hourly spot prices NSS price in \$/MWHr, in

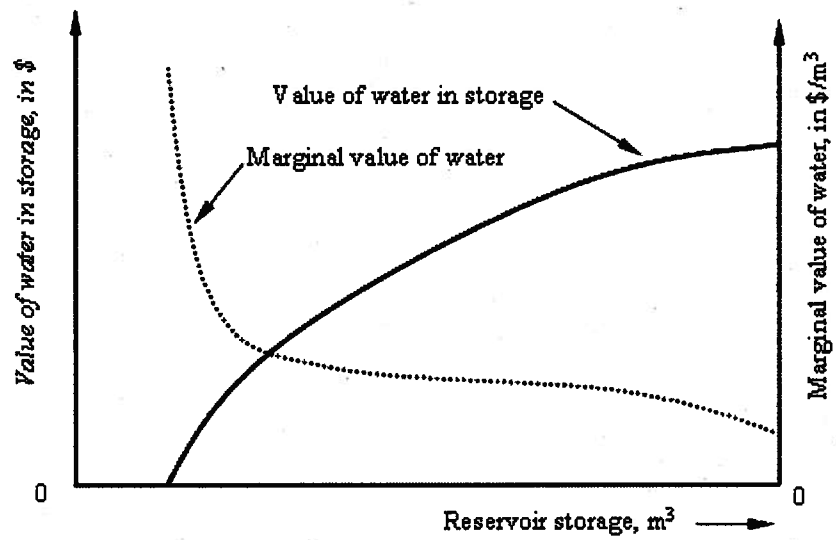


Figure 4.3 Value of Water in Storage and Marginal Value of Water for Timestep [Source: Shawwash 2000]

the U.S. and Alberta electricity markets. The second term represents the sum of the storage cost (or added storage value) of deviating from the terminal target storage level ($STarget_{kT}$) at the target hour (T). For each optimized reservoir, multiplying the difference between the optimized storage at the target hour (S_{kT}) and the target storage ($STarget_{kT}$) by the marginal value of water (MVW_k), in \$/m^3, yields its storage cost (or added storage value).

The marginal value of water and the target storage for each reservoir are predetermined from long and medium term optimization studies, which yield a water value function. Stochastic dynamic-programming and other models establish the value of water stored in reservoirs as a function of storage levels and the study duration, as illustrated in Figure 4.3. The derivative of the value of water function yields the marginal value of water for the duration of the planning horizon and for each storage state as seen in Figure 4.4 [8]

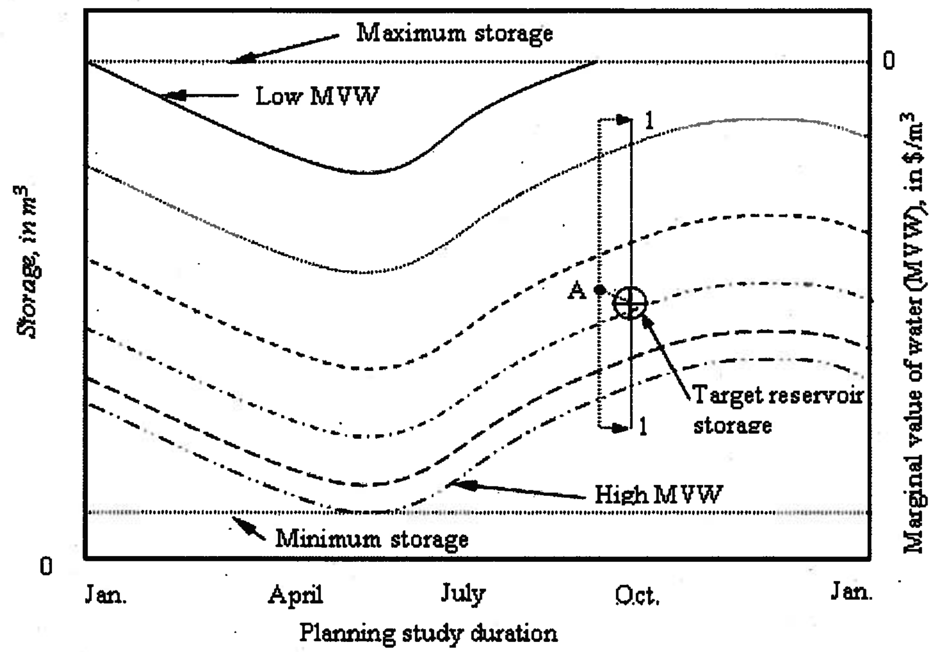


Figure 4.4 Marginal Value of Water as a Function of Storage and Time [Source: Shawwash 2000]

In the Kootenay model the objective function deals with the electricity generation and the value of stored water in the reservoir. For simplicity, revenues from real-time spot energy sales and import and/or export capacity is not considered in the objective. However, the objective function always yields the maximum energy production while maximizing for potential expected long-term value of resources.

Maximize:

Maximize:

$$+\sum_{m=1}^{M} G_{mt} * Price_t$$
$$+\sum_{k=1}^{K} (S_{kT} - STarget_{kT}) * MVW_k * 24 * 3.6 \qquad (4.15)$$

4.2.2 Kootenay Model Characteristics

The following abbreviations are used to explain the following modeling methodology of the model:

Table 4.1 List of Abbreviations in Model

TIME PARAMETERS	
T > 0	Number of production periods in days
initial	Initial time step in the analysis
start	Start date on which optimization starts
end	End day of optimization model
hours	Number of hours in each timestep
j,t	"j" Index used for plant, "t" index used for timestep
PLANTS/RESERVOIRS ABBREVIATIONS	
QBY	Queen's Bay Elevation
KLK	Kootenay Lake
COR	Cora Lynn plant
KCL	Kootenay Canal
UBO	Upper Bonnington
LBO	Lower Bonnington
SLC	South Slocan Dam
BRD	Brilliant Dam
BRX	Brilliant expansion unit
FB	Forebay Level

In addition to the STOM model constraints discussed in section 4.2.1.1 and 4.2.1.2, there are a number of specific constraints for the Kootenay system. As described in previous chapters, the main goal of this research was to:

1- Model the **IJC constraint** with the LP Model for the Kootenay system.
2- Model the **Nelson Gauge** constraint in the Model.
3- Model the **Grohman Narrows** as a constraint on the Kootenay Lake outflows and to account for **headloss** between the Kootenay Lake level and the Cora Lynn forebay, and subsequently from the Cora Lynn to the Kootenay Canal forebay level.
4- Model the special constraint on the Kootenay Lake outflow during the freshet period called **"freefall condition".**
5- Model the **Riverplants Production Functions** as they are slightly different from B.C Hydro's typical plants with storage capacity.
6- Include all other environmental constraints such as the **Brilliant Dam minimum inflow** to provide fish flow during the months of the year.

4.2.2.1 Modeling IJC Constraint in the Linear Programming Models

As described in the previous chapter, the IJC constraint plays an important role in the operation of the Kootenay Lake. The IJC Order issued by the UNC in 1938 governs the maximum elevation of the Kootenay Lake that is permitted at any point in time. Every year a fixed upper curve provided according to the IJC rule for the lake, is followed. This rule can be divided into two parts. First, the IJC rule for non-freshet period, and second, the IJC rule for the freshet period.

i. Modeling IJC upper rule curve for non- freshet period

IJC RULE:

From 01 September to 07 January: The maximum lake level measured at QBY: 1745.32 ft.
On February 1st the maximum lake level measured at QBY: 1,744.00 ft
On March 1st the maximum lake level measured at QBY: 1,742.40 ft
On April 1st the maximum lake level measured at QBY: 1,739.32 ft
A piecewise-linear function is formulated to the model to capture IJC rule curve during non-freshet period, with linear interpolation between the given dates as summarized in Table 4.2.

Table 4.2 IJC Level

date	IJC level in ft
01-Oct- 07 Dec	1745.32
31-Jan	1744
28-Feb	1742.4
01-Apr	1739.32
01-Sep	1745.32

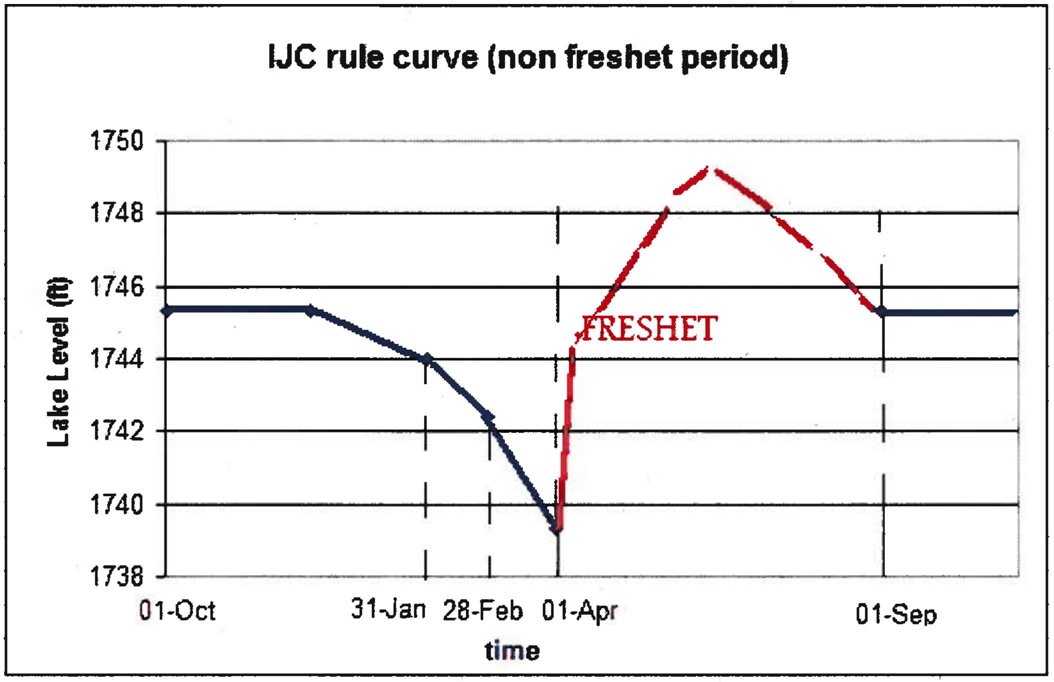


Figure 4.5 IJC Rule Curve (Linearized)

This model starts from the first day of a water year, Oct 1st. The beginning of the spring rise is given as an input to the model and is called the *freshetstart* date.

The AMPL language gives the ability to consider the IJC curve as a piecewise-linear constraint in the model. Figure 4.5 shows a schematic of the piecewise-linear function for the IJC order during the year.

ii. Modeling IJC upper rule curve for the freshet period

The IJC maximum lake level is set at 1,739.32 ft. at the Queen's Bay gauge until the "declaration of the spring rise" or (the start of the freshet) as declared by the IJC Kootenay Lake Board of Control. During this time, the Kootenay Lake must be operated with a lowering formula so that the lake level stays below a level that would occur under the original natural conditions prior to excavation of the Grohman Narrows. A margin of 0.3 ft from the IJC rule curve is usually chosen as a target to avoid exceeding the IJC curve by operation planers [27]. In order to calculate the natural lake level ($FB\ natural_t$), the natural outflow ($Q\ outflow\ Natural_{t-1}$) from the lake needs to be calculated with the absence of Grohman Narrows excavations.

The formulation outlined in this section was mainly derived from a simulation model that was developed by Louie et al. (1995) at BCH. This model is currently being used for the simulation of the Kootenay system within the B.C Hydro's Generation Resource Management.

$$FB\ natural_{t} = FB\ natural_{t-1} + (Q\ inflow_{t} - Q\ outflow\ Natural_{t-1}) / FDS_{t-1} \qquad (4.16)$$

Where *FDS* is the first derivative of the Storage-Elevation equation used in the simulation model and $Q\ inflow_t$ is the total inflow to the lake at each timestep [28].

FB Natural $_{(t)}$ denotes the forebay depending on the lake Natural level at Queens' Bay gauge at timestep $_{t-1,}$
The amount of outflow depends on the lake level at timestep $_{t-1}$ and it can be calculated using the following relationships from the simulation model [28].

a) IF ($FB\_natural_{t-1}$ <= 1,743.5 ft) then

$$Lake\ Natural\ Outflow_{t} = 0.027117*(FB\_natural_{t-1} - 1732.23)^{2.7128} \qquad (4.17)$$

b) IF ($FB\ natural_{t-1}$ > 1,743.5 ft) then

$$Lake\ Natural\ Outflow_{t} = 1.39547*(FB\_natural_{t-1} - 1737.27)^{1.4298})/(2*0.29027*FB\_natural_{t-1} - 956.9472) \qquad (4.18)$$

Both of these equations are non-linear and it is not possible to consider a direct conditional statement in a LP model to account for these constraints. However, it is possible to consider these conditional constraints with the help of a binary parameters.

Two parameters are considered to explain the natural outflow from the lake. If the *FB_ natural* is less than or equal to1,743.5 ft, the lake outflow is labeled *discharge1.* If *FB_natural* is greater than 1,743.5 ft, the lake outflow is called *discharge2.* In this model, "*discharge1*" and "*discharge2*" are both calculated with piecewise-linear functions of the natural lake level at the Queens' Bay gauge (in ft) as follows:

$$\text{let } discharge1_{klk,t+1} = \langle\langle\text{piecewise-linear}\rangle\rangle\ f(FB\_natural_{klk,t}) \tag{4.19}$$

$$\text{let } discharge2_{klk,t+1} = \langle\langle\text{piecewise-linear}\rangle\rangle\ f(FB\_natural_{klk,t}) \tag{4.20}$$

Two binary parameters, "A" and "1-A" are assigned as coefficients to each discharge formulation in the algorithm that calculates the natural forebay of the lake. The algorithm structure is explained in the following paragraphs.

In the beginning of the loop, "A" has the initial value of 1 and *discharge1* takes part in the calculation. At each timestep, the natural forebay is calculated and checked to see if it is below 1,745.3 ft, where "A" sets to zero and *discharge2* is used in the calculation as $(1-A)$ turns to 1.

The algorithm repeats the following three steps from time step *start* to *end*:

a) Set A =1;

b) If $FB\_natural_{klk,t-1}$ > (1,743.5 ft (531.4188 m)) then the value of A=0

c) The value of total lake outflow

$$Qtotal_{klk,t} = (QIR_{klk,t} - A * discharge1_{klk,t} - (1-A) * discharge2_{klk,t}) \tag{4.21}$$

Where $QIR_{klk,t}$ is the amount of the natural inflow to the lake at each timestep.
After the amount of natural outflow from the lake is calculated, the natural forebay can be calculated using the following formulations by Louie et al. (1995):

The value of natural forebay level is calculated as follows:

$$FB\_natural_{klk,t} = FB\_natural_{klk,t-1,} + (Qtotal_{klk,t} / FDS_{t-1}) \tag{4.22}$$

Where :

$$FDS_{(t-1)} = (2*0.29027* (FB\_natural_{klk,t-1} /0.3048)-956.9472))* 0.3048 \tag{4.23}$$

The critical period for lake operation starts in February and ends in Sep31 of the study year. The result of the natural lake level calculated in the model is then compared to the natural lake level developed by the "Operation Planning Department" at B.C Hydro as depicted in Figure 4.6. Any difference between the model result and the real values calculated using nonlinear functions, can be attributed to the use of piecewise-linear functions. However, increasing the number of pieces in the piecewise-linear function can reduce the error to less than 1% in most case but increase the number of parameters in the optimization model.

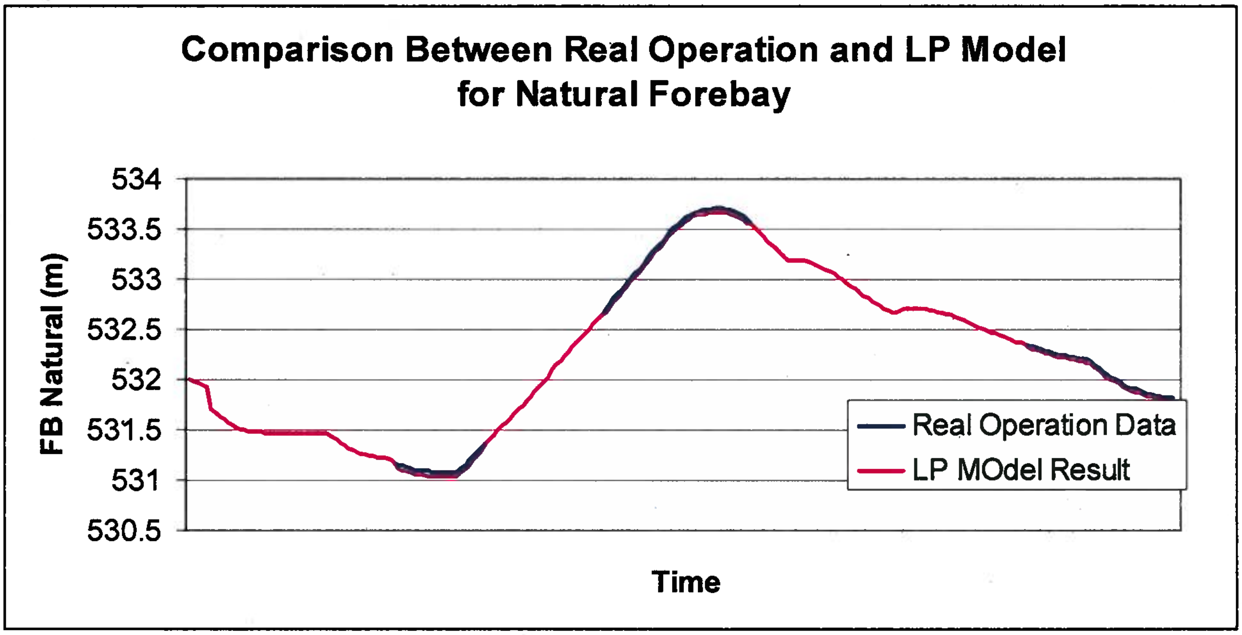


Figure 4.6 Natural Lake Level Example, Real Operation vs LP Model

After calculating the natural lake level, another formula is used to calculate the IJC maximum level [28]:

$$IJC\ level\ (\mathrm{ft}) = -0.00092*\ (FB\_natural\ -1700)^2 + (0.99363*(FB\_natural\ -1700) +1700.85663) \tag{4.24}$$

This formula is a polynomial and can be expressed in piecewise-linear form with very good approximation.

As explained in the previous paragraph, volume is one of the main variables in this model. The IJC upper rule curve is linked to the lake storage measured at the Queen's' Bay gauge.

The relationship between the Queen's Bay gauge level and the lake storage is calculated using the following equation as used in the simulation model [28].

Storage of the Kootenay Lake (kcfsd) =

$$\mathbf{A}^* (El^2) + \mathbf{B}^* El + \mathbf{C} \tag{4.25}$$

Where **A**= 0.290278, **B** = -956.947, **C** =786348.4

EL = lake elevation at QBY in ft.

After conversion to metric units, we can formulate a piecewise-linear curve that relates the lake volume to its FB. The storage value considered in the model is the *live* storage where the lake minimum storage is zero corresponding to 1,738 ft (529.7424 m) while the maximum storage corresponds to 1,755 ft (534.924 m).

iii. Calculation of the Lake Level at Nelson Gauge

The IJC rule states: "If the lake level drops below elevation 1,743.32.ft at the Nelson gauge subsequent to the freshet period, the lake must be held at or below this elevation until August 31st in order to provide improved drainage conditions upstream of the lake" [3].

In order to calculate the Nelson gauge level the following equation was used to calculate the difference between the Queens' Bay gauge level and the Nelson gauge level at each timestep as a function of the amount of outflow from the lake [28]:

$$\textit{Nelson gauge level}_t = \textit{FB natural}_t - (a + b * \textit{Q outflow}_t + d * \textit{Q outflow}_t^{\,2} + e * \textit{Q outflow}_t^{\,3}) \tag{4.26}$$

Figure 4.7 describes the second term in Equation 4.26, which is the difference between Queens' Bay gauge level and Nelson gauge level [28].

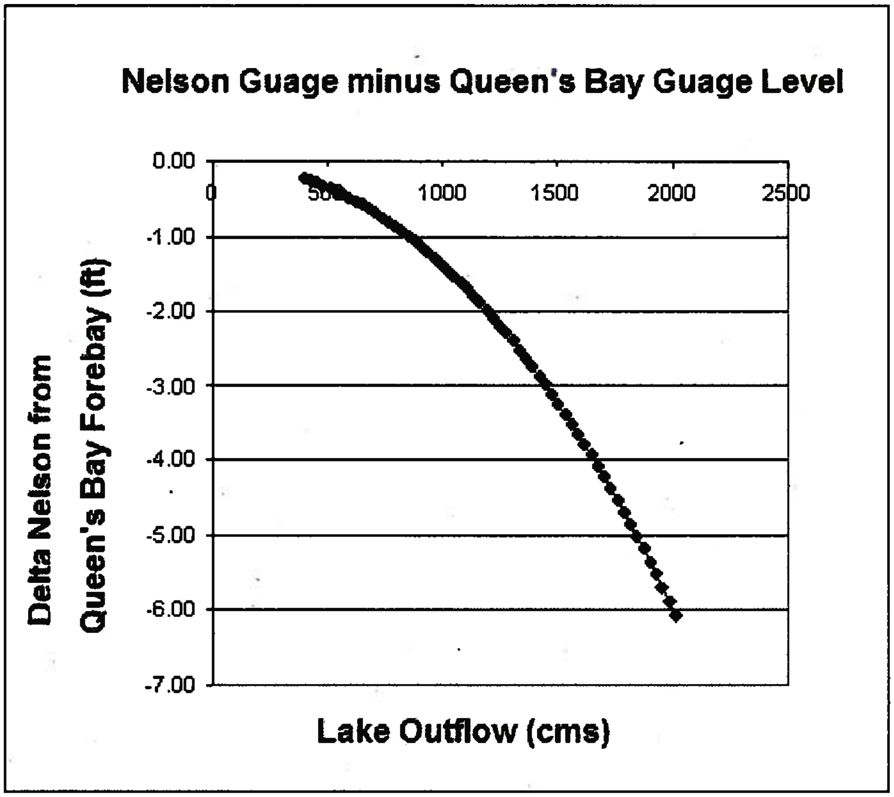


Figure 4.7 Nelson Gauge relation with Queens' Bay Gauge for Different Discharges

There is a loop in the model which tracks the Nelson level at each time step to check at which timestep the Nelson gauge drops below 1,743.32 ft (531.363 m). As explained previously in the IJC rules, if such a condition materializes the lake must be kept at this level until the end of August 31^{st} [3].

The main variable that controls the IJC constraint in the optimization model is the lake volume. In order to calculate the Nelson gauge level we need to calculate the volume that corresponds to 1,743.32 ft at Nelson gauge. The difficulty in calculating the Nelson corresponding volume is because the Nelson gauge level is a dependent variable of the lake level and the lake outflow. In order to deal with this problem several piecewise linear functions were used to calculate the lake level at Nelson. Equation 4.27 is a non-linear equation used to calculate the Nelson level in simulation model.

$$\text{Nelson Level (ft)} = \text{Lake level (ft)} - \mathbf{A} + \mathbf{B}{*}Q + \mathbf{C}{*}\, Q^2 + \mathbf{D}{*}Q^3 \tag{4.27}$$

Where A= 0.011792647, B = 0.002588961, C = 0.000917539, D = 0.00000345 and Q is the lake outflow.

The loop is included after each iteration of the model run. The Nelson gauge level is calculated for the next iteration.

$deltaNelson_{klk,t}$ is the different between Lake forebay level and Nelson guage level.

$$deltaNelson_{klk,t} = \mathrm{FB}_{\mathrm{klk,t}} - \text{Nelson gauge}_{\mathrm{t}} \tag{4.28}$$

$$deltaNelson_{klk,t} = \ll \text{piecewise-linear} \gg \mathrm{f}(QP_{\mathrm{klk,t}}) \tag{4.29}$$

$$Nelsonguage_{klk,t} = FB_{klk,t} + deltaNelson_{klk,t} \tag{4.30}$$

The Volume corresponding to the Nelson gauge $VNelson_{klk,t}$ is also calculated as follows:

$$VNelson_{klk,t} = \ll \text{piecewise-linear} \gg \mathrm{f}\,(\,Nelsonguage_{klk,t}\,) \tag{4.31}$$

A conditional statement is used in the model is used to flag when the Nelson gauge drops below 1,743.32 ft.

$$\text{if } Nelsonguage_{klk,t} \le 1743.32 \text{ ft (531.36 m) then let } nelsonstart = t \tag{4.32}$$

Because of the headlosses between Queens' Bay and Nelson, the Nelson gauge is always lower than Queens' Bay level. An easy way to control the lake level at Nelson gauge is to control the Queens' Bay elevation and limit it to certain level.

The Kootenay Lake level of 1,743.32 ft corresponds to a lake volume of 8,075.465 cmsd, and the margin of 0.32 ft in lake level is equal to 501.569 cmsd considering that the slope of Storage-Elevation curve as 5146.5:

Storage corresponding to 0.32 ft in level is equal to (0.32* 0.3048* 5146.5 =) 501.569 cmsd and is used in the Nelson constraint calculations.

Studies show that due to the higher energy market prices after freshet period, most of the water flowing into the reservoir is released and used for power generation. Therefore controlling the variation of the volume in the lake at Nelson can be successfully done without any further complication in the model.

The Nelson constraint is indeed a part of IJC rule curve from the *nelsonstart* day to Auguest 31^{st}. The *nelsonstart* day can happen any time after the "*peakday*" (the day at which the peak volume of the lake occurs). Calculating the *peakday* will be explained in details in section 4.2.2.4.

To include the Nelson constraint in IJC constraint the IJC1 constraint is divided in to three time periods and the following condition is implemented:

for timesteps from the *peakday* until *Auguest* 31^{st}

if $VIJCfreshet_{(t)} < 8175$ cmsd then the $VIJCfreshet_{(t)} = 8175$ cmsd (4.33)

subject to *IJC1*{ *t in freshetstart..**nelsonstart***}: $V_t \leq VIJCfreshet_t -$ margin (4.34)

subject to *nelson1*{ *t in **nelsonstart+1**..August 31*}: $V_t \leq VIJCfreshet_t$ (4.35)

subject to *nelson2*{*t in **nelsonstart.. August 31***}: $VIJCfreshet_t -$ margin $\leq V_t$ (4.36)

The margin as described above is typically set to volume corresponding to 0.3ft.

4.2.2.2 Modeling the Grohman Narrows Constraint

i. The Grohman Narrows and Headloss from KLK to COR reservoir level

The Grohman Narrows is another constraint that is very important in calculating the amount of outflow from the Kootenay Lake. There are certain relationships between the lake level, the Cora Lynn reservoir and the lake outflows. Figure 4.8 shows a set of curves that were used to calculate the effect of the Grohman Narrows in the model. The red line named as "A-A" is a horizontal section used to calculate the headlosses between lake level and Cora Lynn forebay for a constant lake level at Queen's bay at about 1,747ft. For each lake level value, the flow variations cause the Cora Lynn reservoir forebay to vary. Figure 4.9 shows the headlosses for lake levels at Queen's bay gauge for the discharges from 5 kcfs to 100 kcfs.

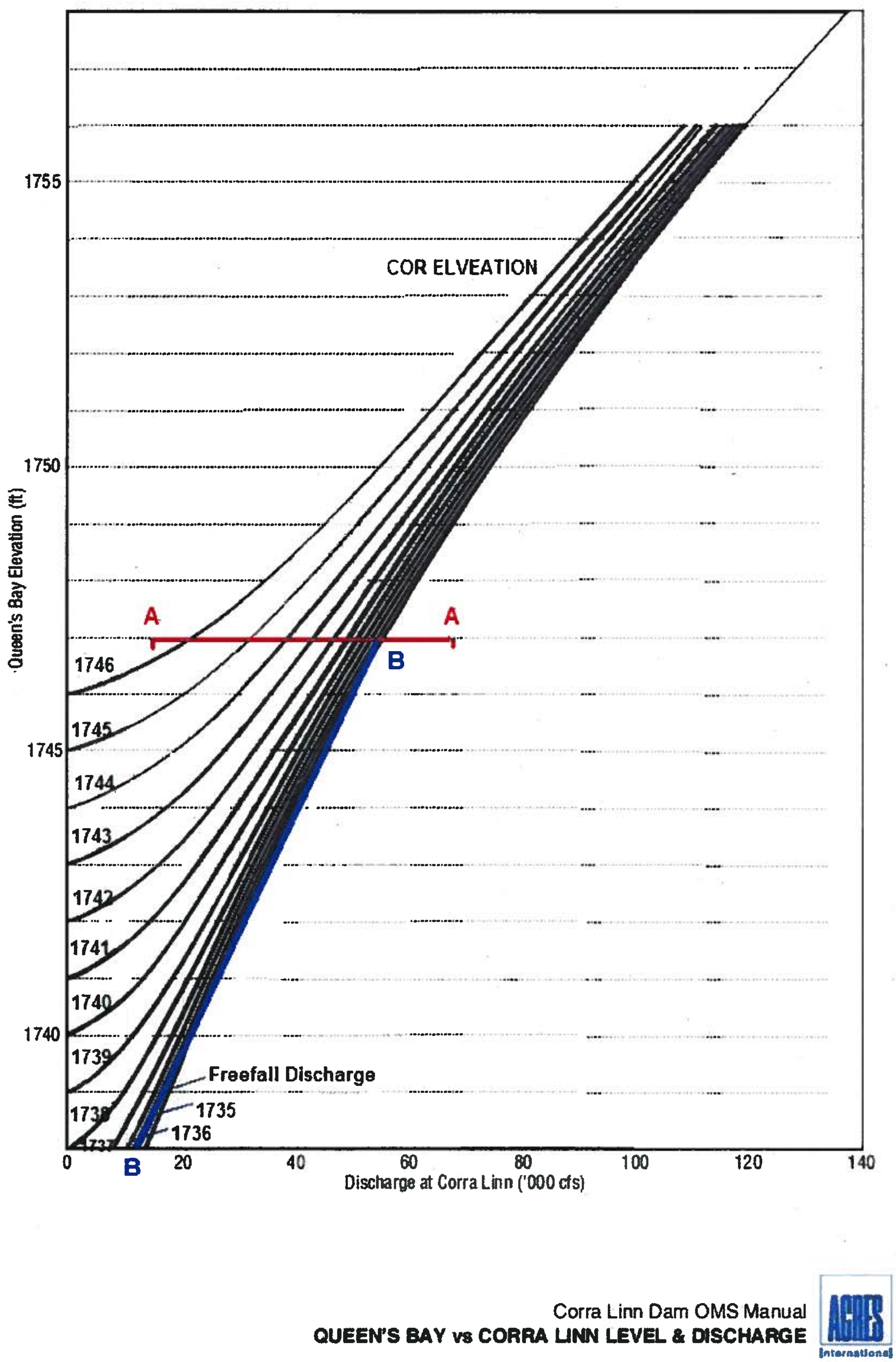


Figure 4.8 Linear Curves ,Queens's Bay vs Cora Lynn Level & Discharge

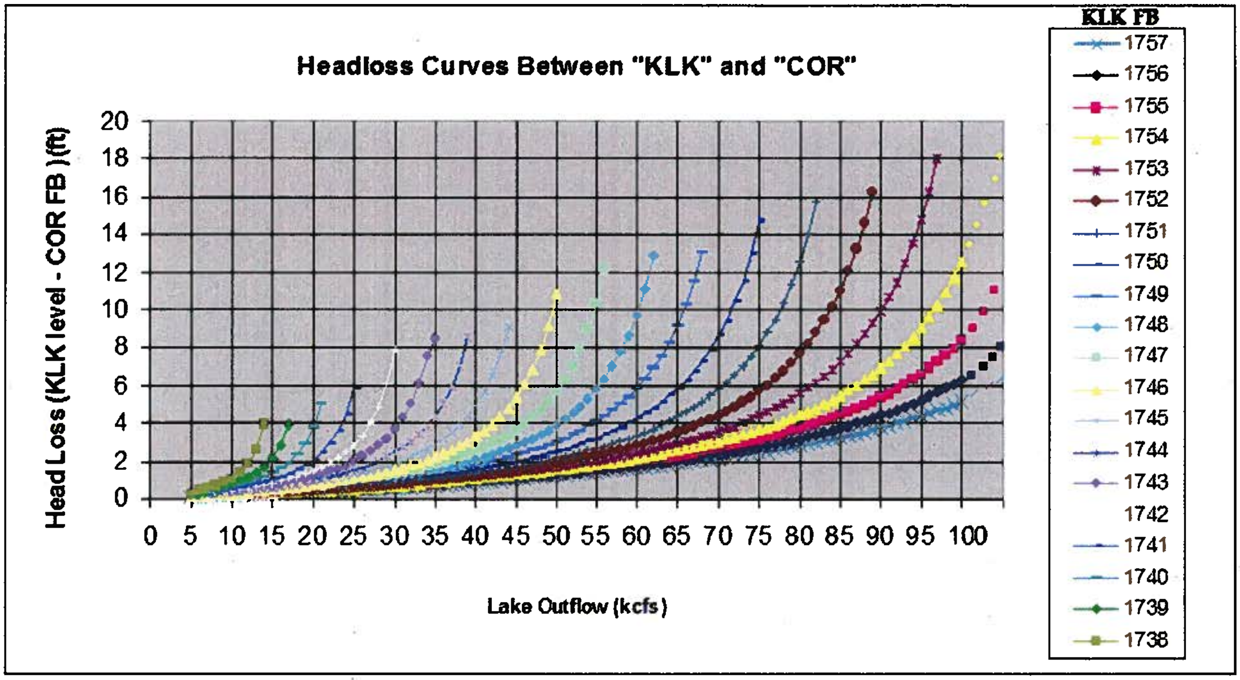


Figure 4.9 Headloss Function Linear Curves

The following formulation from the simulation model best describes the headloss between the Cora Lynn and the Kootenay Lake reservoir.

Headloss between the Kootenay Lake and the Cora Lynn reservoir [28]:

$$HLOSS = ((\mathbf{C1} * \text{el} + \mathbf{C2}) * \text{Q}) / (\mathbf{C3} * (\text{el} - \mathbf{C4})^2 - \mathbf{C5} - (\mathbf{k} * \text{Q})) \quad (4.37)$$

Where:

"el" is the Kootenay Lake elevation in "ft"

"Q" is the lake outflow in kcfs

k = 1.075, C1 =0.0872, C2 = -150.82, C3 =166.8, C4 =1726, C5 = 6357

To adjust for the headloss between the Kootenay Lake and the Cora Lynn reservoir the calculated FB in the model can be corrected using a look up table. To find the value for headloss between KLK and COR the amount of outflow from the lake *QP(t)* is required.

The table used in this model include several rows and columns. Table 4.3 is a summarized revision of the look-up table. The reason to consider a fairly large table is because headloss variations are very important for calculating COR forebay levels. Cora Lynn might not generate a large percentage of energy but its level is important as the Cora Lynn and the KCL plants share the same headwork.

Table 4.3 Headloss table from Kootenay Lake to Cora Lynn

	FLOW													
cms	141.58	424.75	453.07	481.39	622.97	651.29	877.82	906.14	1783.96	1812.3	2152.1	2180.4	2803.4	2831.7
cfs	5000	15000	16000	17000	22000	23000	31000	32000	63000	64000	76000	77000	99000	100000
Lake Level ft	Headloss (ft)													
1757	0.080	0.260	0.280	0.300	0.404	0.425	0.614	0.640	1.747	1.797	2.515	2.586	4.981	5.148
1756	0.083	0.271	0.291	0.312	0.422	0.445	0.647	0.674	1.908	1.966	2.820	2.908	6.107	6.351
1755	0.086	0.282	0.304	0.326	0.442	0.467	0.683	0.713	2.109	2.178	3.225	3.336	7.979	8.387
1754	0.089	0.295	0.318	0.341	0.465	0.491	0.724	0.757	2.366	2.450	3.788	3.937	11.715	12.586
1753	0.093	0.309	0.333	0.358	0.490	0.519	0.773	0.808	2.707	2.814	4.626	4.842		
1752	0.097	0.325	0.351	0.377	0.520	0.550	0.829	0.869	3.184	3.327	6.014	6.370		
1751	0.101	0.343	0.370	0.399	0.553	0.587	0.896	0.941	3.899	4.108	8.763	9.510		
1750	0.106	0.363	0.393	0.424	0.593	0.630	0.978	1.030	5.098	5.446				
1749	0.111	0.386	0.419	0.452	0.640	0.681	1.081	1.141	7.534079	8.282687				
1748	0.116	0.413	0.449	0.487	0.696	0.744	1.213	1.285						
1747	0.123	0.446	0.486	0.528	0.767	0.822	1.389	1.481						
1746	0.130	0.485	0.531	0.578	0.858	0.924	1.641	1.764						
1745	0.139	0.534	0.587	0.642	0.979	1.061	2.029	2.210						
1744	0.149	0.597	0.660	0.726	1.150	1.259	2.713	3.027						
1743	0.160	0.682	0.759	0.843	1.414	1.571	4.254	5.025						
1742	0.175	0.803	0.905	1.018	1.876	2.143								
1741	0.193	0.992	1.140	1.312	2.910	3.550								
1740	0.217	1.334	1.590	1.915										
1739	0.249	2.157	2.835	3.923										
1738	0.299													

The grey part in the table indicates the boundaries of infeasible ranges according to the maximum allowable outflow from the Grohman Narrows.

ii. Headloss lookup table formulation in AMPL

The lookup table was designed for integer values of lake outflow. For interpolation between integer values in the table another function is used. The closest lower (*Qtable1*) and upper (Qtable2) discharge integer values are found using the "truncate" function. These values are calculated to find the corresponding headloss values in the lookup table. The algorithm also truncates the FB levels in the table. Interpolation is used to calculate the headloss value for the given discharge and forebay level.

$$Qtable1_{klk,t} = (\mathrm{trunc}\ QP_{klk,t}) \tag{4.38}$$

$$Qtable2_{klk,t} = Qtable1_{klk,t} + 1 \tag{4.39}$$

$$\begin{aligned} Headloss\_calc1_{COR,t} &= \\ &((Headloss[(Qtable1_{klk,t}),\mathrm{trunc}((FB_{klk,t})+1)] - Headloss[Qtable1_{klk,t}\ \mathrm{trunc}(FB_{klk,t})]) * \\ &(FB_{klk,t} - \mathrm{trunc}(FB_{klk,t})) + Headloss[Qtabel1_{klk,t},\mathrm{trunc}\ (FB_{klk,t})]) \end{aligned} \tag{4.40}$$

The same formulation is repeated for Qtable2. After calculating *Headloss_calc1* for the lower bound and *Headloss_calc2* for the upper bound, the headloss values corresponding to a given FB levels are calculated.

Finally the following relationship is obtained:

$$FB_{COR,t} = FB_{klk,t} - Headloss\_calc_{COR,t} \quad (4.41)$$

In the freshet period when the releases are at the maximum allowable outflow levels, the Cora Lynn forebay level is fixed at the lowest possible level. .Therefore to calculate the correct values of the forebay levels for this specific condition we can fix the Cora Lynn forebay during the maximum outflow period (*freefall* period) to its lowest operable level as follows:

for {*freefall* period } { let $FB_{COR,t}$ = 528.828 m (1734 ft) (4.42)

iii. Headloss between the Cora Lynn and the Kootenay Canal Forebay Level

The problem in calculating the headloss between the Cora Lynn and the Kootenay Canal is less complicated because of the maximum limit of the Kootenay Canal flow is set at 30 kcfs. The equation used in simulation model is:

Headloss (*HLOSS*) between Cora Lynn reservoir and Kootenay Canal:
"El" is COR elevation in "ft"
"Q" is Kootenay Canal inflow in kcfs

$$HLOSS = (\mathbf{c9} + \mathbf{c1} * El + \mathbf{c2} * El^2) + (\mathbf{c3} + \mathbf{c4} * El + \mathbf{c5} * El^2) * Q + (\mathbf{c6} + \mathbf{c7} * El + \mathbf{c8} * El^2)*Q^2 \quad (4.43)$$

Headloss Equation Coefficients (COR to KCL)

C9 =	0
C1 =	-0.001208547
C2 =	7.02909E-07
C3 =	0.297189
C4 =	-0.000340171
C5 =	9.73417E-08
c6 =	2.20682E-05
c7 =	-2.51306E-08
c8 =	7.16E-12

Figure 5.10 depicts the Equation 4.43:

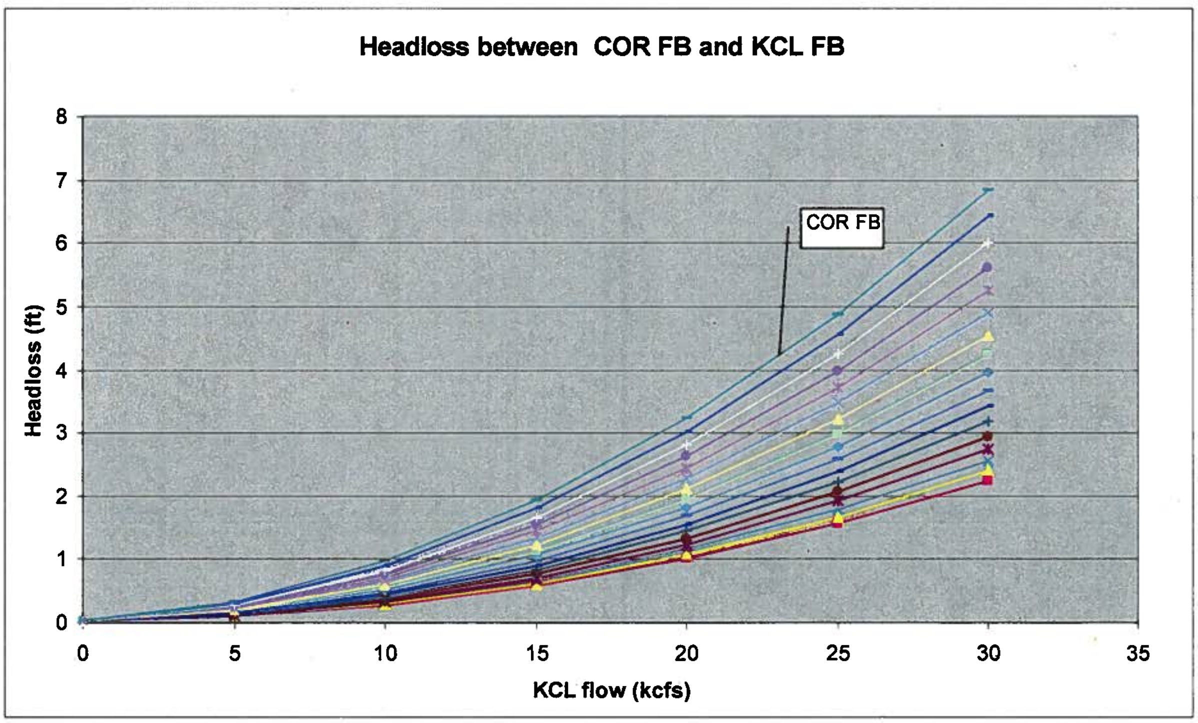


Figure 4.10 Headloss curves between Cora Lynn and Kootenay Canal

The approach used in section 4.2.2.2.ii is followed. A lookup table is used to find the headloss values between the Cora Lynn reservoir and the Kootenay Canal.

4.2.2.3 Freefall Constraint

Another important rule in operating the Kootenay system is to release the maximum possible discharge for as long as possible during the freshet period and before the lake reaches its peak level during the freshet period. *Freefall* is the condition when the Cora Lynn reservoir level is held at its lower level and the lake outflows were held at the maximum discharge as defined by the *freefall* curve, which represents the furthest curve in Figure 4.5. The *freefall* discharge is calculated using Equation 4.44 [28]:

$$\textit{Freefall discharge}\ (\text{kcsf}) = 0.0625*(FB_{klk,t}\ \text{ft}-1700)^2-0.625*(FB_{klk,t}\ \text{ft}-1700)-53.5 \qquad (4.44)$$

In order to include this formula in the model, a linear approximation of the corresponding left curve is used as shown by line B-B in Figure 4.8.

If we set the maximum discharge level to the mean annual maximum daily discharge from the Kootenay Lake the 56 kcfs (source:[3]) line A_B in Figure 4.8 is an acceptable approximation for the maximum lake outflow.
Under normal operation conditions, the *freefall* constraint in the model starts approximately ten days after the declaration of spring flow and continues until a certain date (*freeoff*) as detailed in the following section [27].

For *Freefall* period {t in *freshetstatr+10..freeoff*}:
$QP_t = (V_t * 0.08709 + 348.52)$ (calculated using line A-B in Figure 4.8)

This constraint forces the model to release the maximum freefall flow until the peak at the reservoir level is passed. An important question that could be asked is: "when can we stop the freefall and gain some head values to generate more power per unit of water?"

4.2.2.4 Predicting the best time to stay out of the freefall condition

Different runs with different inflow scenarios show that maintaining the lake outflows at the maximum *freefall* curve could result in a different peak lake level at different dates. Therefore it is important to find when the peak lake level occurs.

The yellow line in Figure 4.11 and 4.12 shows the maximum *freefall* flow, the light blue line is the lake outflow, the red line is the lake maximum upper rule (IJC), and the dark blue line is the lake elevation. The purple circle indicates the time in which the *freefall* condition is no longer binding.

Figure 4.11 portrays the resulting reservoir level if water is released from the lake prior to the maximum *freefall* date early in the freshet period. In this case, the releases are high during the freshet period and the operator will be violating the IJC rules by unnecessarily increasing the chance of flooding.

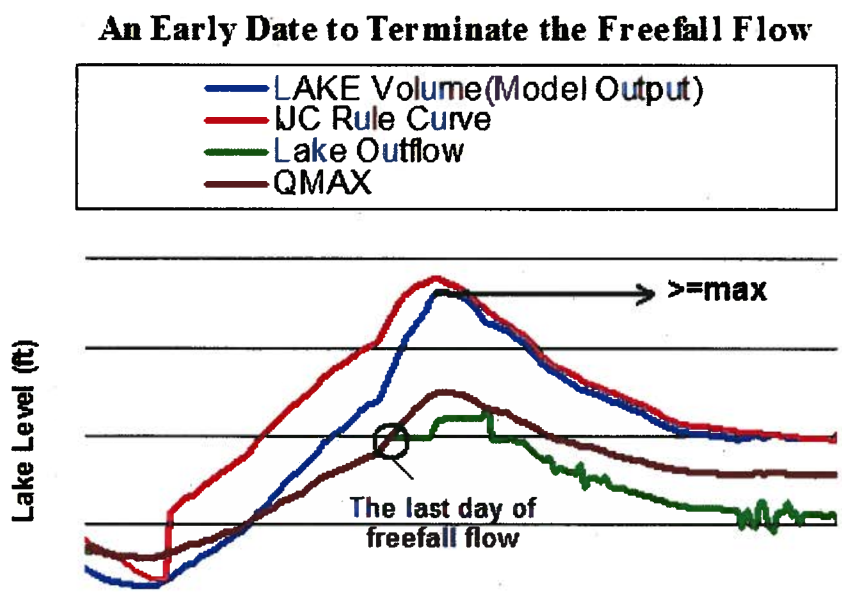


Figure 4.11 Kootenay Lake Peak Level

Figure 4.12 illustrates the resulting lake level if maximum flow release period is delayed until a later time than the previous case in Figure 5.9. In this case the lake level is excessively lower than its peak level and headlosses would be encountered because *freefall* condition was maintained for a longer period than necessary.

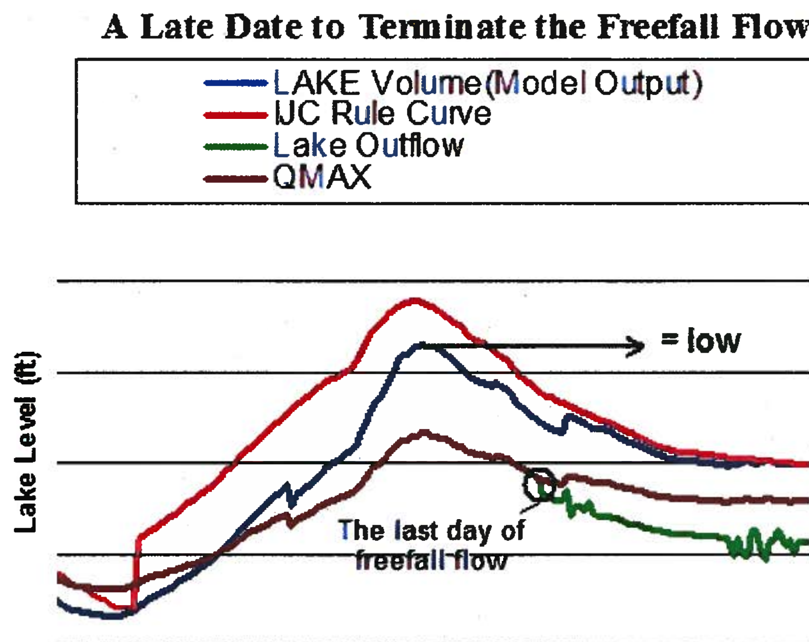


Figure 4.12 Kootenay Lake Peak Level

In this model the best time for reducing the *freefall* flow is derived iteratively. In the following the *"freeoff day"* refers to the day when we are allowed to terminate the *freefall* condition. Finding the date to reduce the maximum outflow depends on inflow scenarios as discussed in the following.

In real operations, maintaining the *freefall* period is based on the experience and is aided by a simulation model. Few days prior to the predicted peak the operations planner reduces the flow so that the maximum lake level possible (IJC Level minus margin) can be subsequently achieved [27].

(a) Estimating the *freefall* period in high-inflow scenarios

In this project high-inflow scenarios are those inflows which result in a simulated lake peak level higher than 1749.5 ft. The operation rule says that if the peak of the lake is higher than

1749.5 ft than lake outflows should be maintained on the *freefall* condition until the lake level reaches 1749.5 ft on the declining side of the lake level [27].

In order to model this rule the following procedures is used:

**Peak_rule_volume* refers to the volume corresponding to the peak target of 1,749.5 ft.

**Peakvolume* is the maximum lake volume in the freshet period.

let *peakvolume* = max $V_{klk,t}$ in the freshet period

To find the *peakday*

For every timesteps in the freshet period if $V_{klk,t}$ = *peakvolume* then let *peakday* = t

After *peakvoulme* is found, the flow scenario is determined. If *peakvoulme* is greater than 1,749.5ft the following algorithm is implemented. Four iterations are typically needed to ensure the correct *freeoff* day has been determined.

If *peakvolume* > *peak_rule_volume*

Then repeat followings while (*condition* is not equal to 1)

1) **Solve the optimization problem**

2) Call the *peak* routine;

3) Iteration =iteration +1

4) Stop if iteration =4

***Peak* Routine :**

The first guess for the *freeoff* day to start the iteration is the *peakday.*

let *freeoff* = *peakday*;

If *peakvolume* > *peak_rule_volume* then for the period of *peakday* until *August31* do the following :

1) let *peakslope* = $(V_{klk,t} - V_{klk,t+3}) * (V_{klk,t+3} - V_{klk,t+6})$

2) If $V_{klk,t}$ > *peak_rule_volume* and *peakslope* > 0 then let *freeoff*= *t*;

Peakslope is used to determine if we are on a decreasing slope after the peak level is passed. The number of days to be examined is usually up to seven days [27].This is done to avoid a premature termination of the *freefall* operation when a second peak could occur. Figure 4.13 shows a high-inflow scenario. In the absence of slope-check function the *freefall* termination can occur at any time between the lake peak level "A" and the second rise in the level at point "B". Checking the slope after the peak has passed guarantees that the *freefall* termination would occur after the fluctuations on the lake level is finished (after point C). Finally the model chooses point D as the best point to reduce the releases from maximum lake outflows.

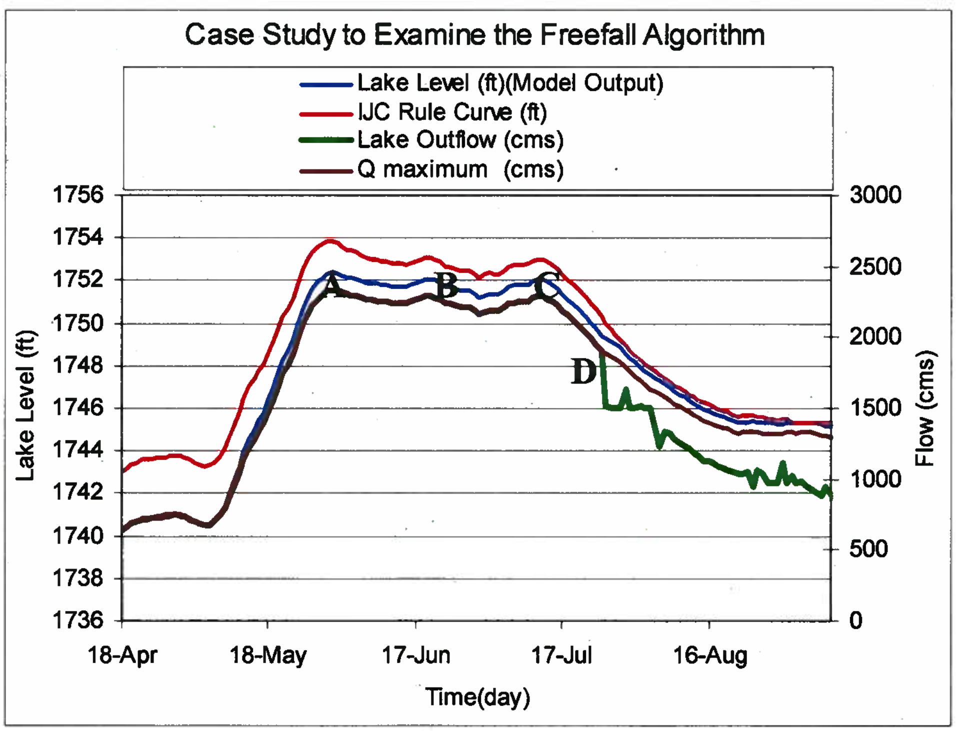


Figure 4.13 Freefall Period in a High-Inflow Scenario

Figure 4.14 shows another case in which a second peak occurs at the lake. It can be seen that the *freefall* period terminates when the lake level reaches 1,749.5 ft on the declining slope.

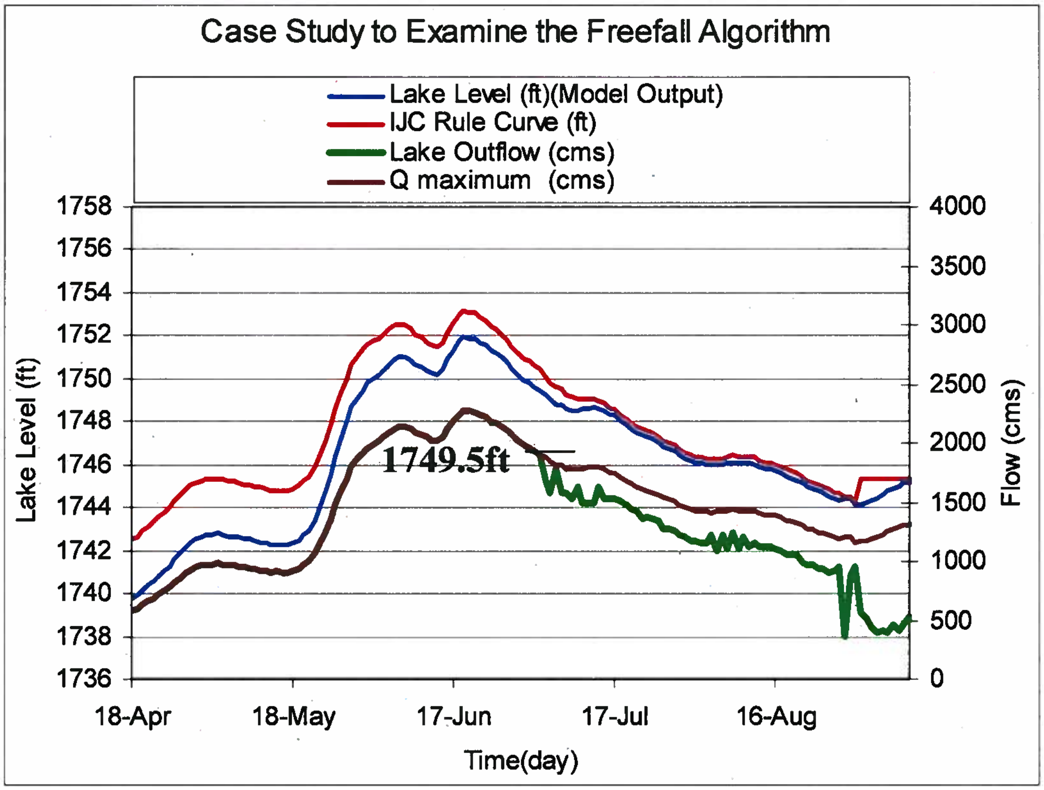


Figure 4.14 Freefall Period in a High-Inflow Scenario

The lake level and the lake slope are used to find the possibility of a higher level when the assumed peak level is passed [27].

For the cases with low-inflow scenarios or those high-flow scenarios which may change to low-flow scenarios during the iterations (probably because their peak values are only slightly higher than 1749.5 ft) the procedure outlined in (b) is applied.

(b) Estimating the *freefall* period in low-inflow scenarios

Low- inflow scenarios in this model are those in which the *peakvolume* of the lake is equal or less than the volume corresponding to 1,749.5 ft.

In these scenarios it is important to determine the *freeoff* day as it affects the peak level and therefore would result in different water head that is used for generation.

The iterations start with the assumption of *freeoff* day to be the same as the *peakday.* This assumption would result in a low *peaklevel* at the first iteration. In the algorithm the *freeoff* period is decreased in each iteration and the peaklevel is checked. The iterations stop where reducing the *freefall* period results in a lake peak level greater than 1,749.5 ft.

Let *freeoff = peakday;*

Repeat the iterations while (*peakvolume* < *peak_rule_volume*)
{
Let *freeoff = freeoff-1*
Solve the optimization problem;
Let *peakvolume* = max $V_{klk,t}$ in the freshet period
For {t in *freshetstart ..frehsetend (*freshet period*)*}
{If $V_{klk,t}$ = *peakvolume* then let *peakday* = **t**; }
}

4.2.2.5 Target Minimum Flow Constraint on the Kootenay System [4]

The target minimum flow as described in the previous sections are:

(Minimum flow targets: +/- 14 m^3/s (500 cfs)):

- December to September 18,000 cfs (510 m^3/s)
- October to November 16,000 cfs (453 m^3/s)

It should be noted that according to the rules described in section 3.6 the lake storage should not be used to satisfy these constraints. Therefore, the best way to describe this rule is to put the following constraints in the model:

Subject to *Brillinat1* {t in Oct01..Nov30}: $QP_{BRD,t}$ >= 453.0695455; (equal to 16000 cfs)
Subject to *Brillinat2* { t in Dec01..Sep31}: $QP_{BRD,t}$ >= min($QIR_{KLK,t}$,18000*0.3048^3);

The expression "min(*QIR (KLK,t)* , required flow)" indicates that if the inflow to the lake is less than the minimum required flow, then Brilliant outflow is not greater than the Kootenay Lake inflow. This constraint ensures that upstream storage is not used to satisfy the Brilliant plant minimum flow requirements when there is not enough water, as dictated by the system operation rules [3].

4.2.2.6 Unit Outages consideration

In real world operation generating units are regularly taken out of service for maintenance or are shutdown for other operational considerations. This impacts the entire plant generation and flow capacity and therefore these outages needs to be accounted in the optimization model.

This is achieved by representing the available units by "combos", where each combo represents the units' availability for each plant at each timestep. The equivalent decimal number to the binary values are used to represent the units on "on" or "off" states.
A procedure entitled "maxgen" adjusts the maximum generation and plant discharge based on unit outages for all timesteps in the study period. These are the same procedures used by the Generalized Optimization Model, "GOM" in B.C Hydro. In this file maximum generation and discharge capacity for the plant is calculated based on linear interpolation within the values given for each "combo" for each plant.

4.2.2.7 Optimization Model Structure

i. Input data and run files

A set of data files that define the characteristics of the system and specify the limits used by the optimization model are presented in Appendix A.

ii. General structure of LP Optimization Model

Figure 4.15 depicts the general algorithm structure considered in this model. The check box in the following Figure refers to the check for the convergence of the forebay levels.

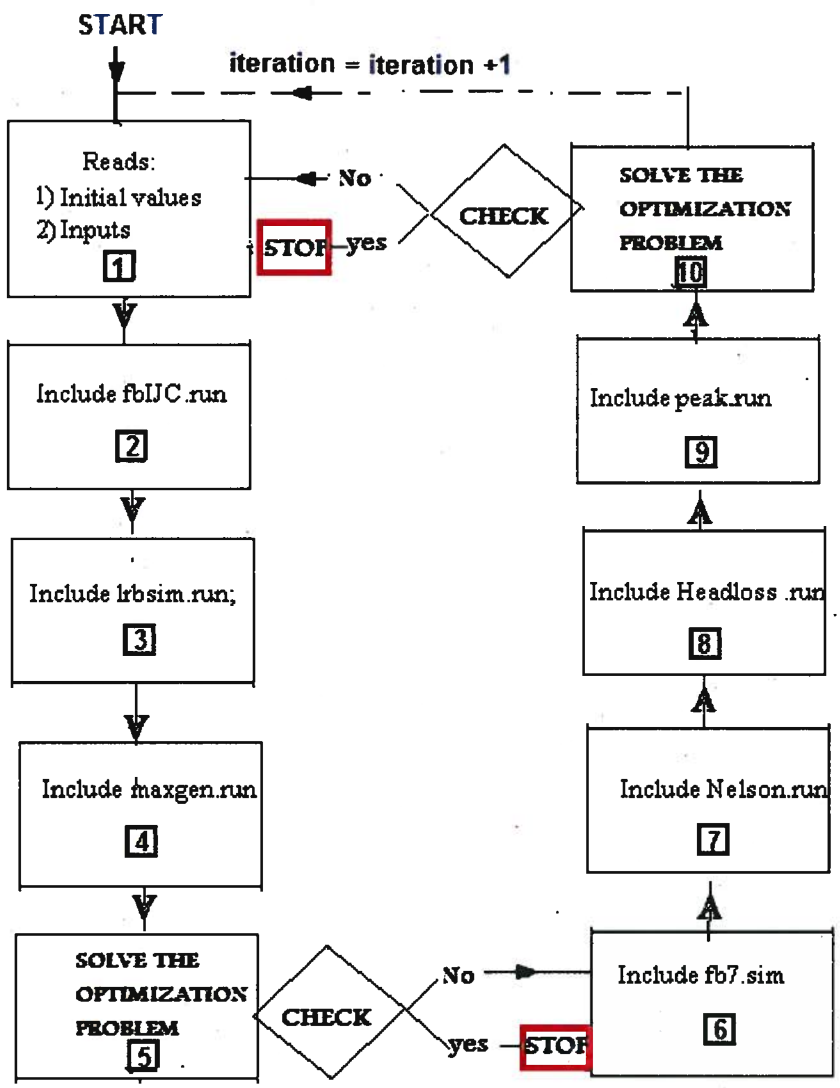


Figure 4.15 General Optimization Model Structure

In the first step of the algorithm the input data is read from the details provided by the user. The main inputs consist of inflow parameters, turbine limits, spill capacity, storage limits, plant generation functions and etc.

After the model checks the availability of correct input data, the second part considers the constraints. All the inputs provided must be within the provided limits. Initial conditions are also checked to make sure all the inputs are correct. In the second step, the natural forebay of the lake and IJC maximum limit are calculated and set as limits of the constraints. In the third step, the simulated parameters such as QP_LRB, FB_LRB and etc. are derived from the inputs. In the fourth step maximum flow and generation capacities of each plant is calculated given the outage schedule. The optimization problem is then solved and the variables values such as reservoirs volumes, turbine and spill flows are determined. Forebay levels are then calculated in step 6 using the storage- forebay relationship provided. The Nelson gauge level is then checked to see if there is any change in the Nelson constraint start date for the next iteration. The headloss equations are used to correct the forebay for Cora Lynn and the Kootenay Canal. In step 9, the peak routine is called to calculate the peak level and the *freefall* termination date. The calculated *freefall* period is replaced by the initial assumed period and the problem is solved to find the optimal values. A convergence check is performed to ensure the true optimal is achieved.

5 CASE STUDY

The methodology outlined in Chapter 4 is applied to different cases with different inflow scenarios to check the validity of the outputs. The results are also compared to the simulation that is currently used by B.C Hydro engineers for operation of the Kootenay system in B.C. This chapter presents the result of these case studies.

5.1 MODELING GENERATION PRODUCTION FUNCTIONS

One of the most important inputs to describe the plants' characteristics is generation production function. As previously described in section 4.2.1.2 the generation production functions (GPF) are described with the aid of piecewise linear functions. Section 5.1.1 describes the generation production function used for the plants under study in this research project.

5.1.1 The Kootenay Canal Plant Generation Production Curves

With the assumption of optimal unit commitment the generation production function for a hydroelectric plant can be developed.

Figure 5.1 displays the Kootenay Canal generation production curves using B.C Hydro database.

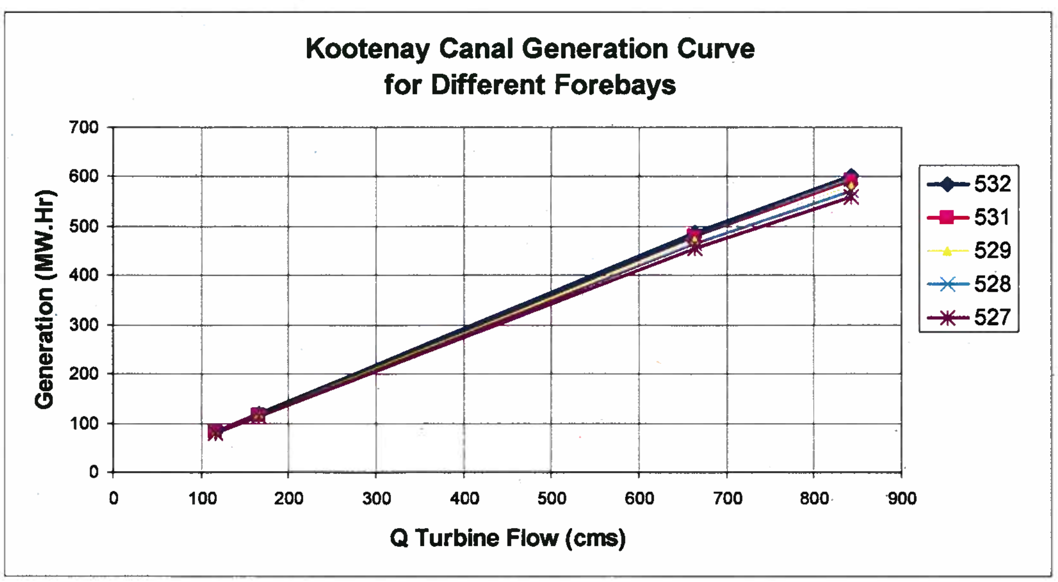


Figure 5.1 Kootenay Canal Generation Production Curves

5.1.2 Generating production functions for Kootenay Riverplants:

The Riverplants' generation- discharge relations were used to derive a set of generation coefficients similar to those used in STOM. For Riverplant, which are classified as run-of-river hydroplants in this study, the forebay level is assumed to be constant, with the exception of the Cora Lynn dam which has higher forebay level variations. The calculated generation in these plants (COR, UBO, LBO, SLC, BRD) were considered to be a function of discharge, head variations and their corresponding effect on the generation were subsequently used to derive the discharge-generation relations.

Table 5.1 is a sample of the tables that were used to calculate the generation production function for Riverplants. This table shows the distribution of the total flow at Cora Lynn dam among the units and the corresponding unit generation. The right column (P/H flows), shows the amount of flows passed through the turbines for generation. The difference between the total flow and P/H flow would yield the amount of spilled water. The FB column shows the forebay level corresponding to the data listed.

Table 5.1 Sample Riverplant's Discharge- Generation Table

TABLE CL-1	**FB El 1734.00**					
CORA LYNN						
PLANT OUTPUT VERSUS FLOW						
Unit 1	existing					
Unit 2	existing					
Unit 3	existing					
Max. Plant Flow (cfs) 12600 cfs						
Total Flow (cfs)	**FB (ft)**	**Unit 1 Flow (cfs)**	**Unit 2 Flow (cfs)**	**Unit 3 Flow (cfs)**	**Total Power (MW)**	**P/H Flow (cfs)**
0	1734.00	0	0	0	0.00	0
3500	1734.00	0	3500	0	12.09	3500
5000	1734.00	1250	3750	0	15.99	5000
5300	1734.00	2300	3000	0	16.68	5300
6000	1734.00	2600	3400	0	19.59	6000
7000	1734.00	3300	3700	0	23.31	7000
7500	1734.00	3700	3800	0	24.91	7500
9500	1734.00	2950	3550	3000	30.51	9500
10000	1734.00	3200	3600	3200	32.26	10000

10500	1734.00	3450	3700	3350	33.92	10500
10800	1734.00	3600	3700	3500	34.84	10800
12500	1734.00	3926	3886	4063	36.94	11875
13000	1734.00	3921	3881	4058	36.79	11859
13500	1734.00	3915	3875	4052	36.64	11843
14000	1734.00	3910	3870	4047	36.50	11828
16000	1734.00	3891	3851	4027	35.95	11769
18000	1734.00	3873	3833	4008	35.46	11714
20000	1734.00	3856	3816	3991	35.00	11663
30000	1734.00	3783	3745	3916	33.06	11444
40000	1734.00	3721	3683	3852	31.46	11256
50000	1734.00	3663	3626	3792	30.01	11081
90000	1734.00	3466	3431	3587	25.42	10484
100000	1734.00	3424	3389	3544	24.51	10357
110000	1734.00	3384	3349	3502	23.65	10235
120000	1734.00	3344	3310	3461	22.84	10116
200000	1734.00	3059	3028	3166	17.47	9253

Figures 5.2 shows the generation-discharge relationship derived from the Cora Lynn generation-discharge tables similar to Table 5.1. It can be seen that after certain discharge level (about 300 cms) the higher outflow results in lower power generation. This is because the spilled water will increase the tailwater level and reduce the head resulting in lower generation. Different lines in Figure 5.2 correspond to different forebay values for the Cora Lynn reservoir. It can be seen that higher forebay level generally results in higher generation values.

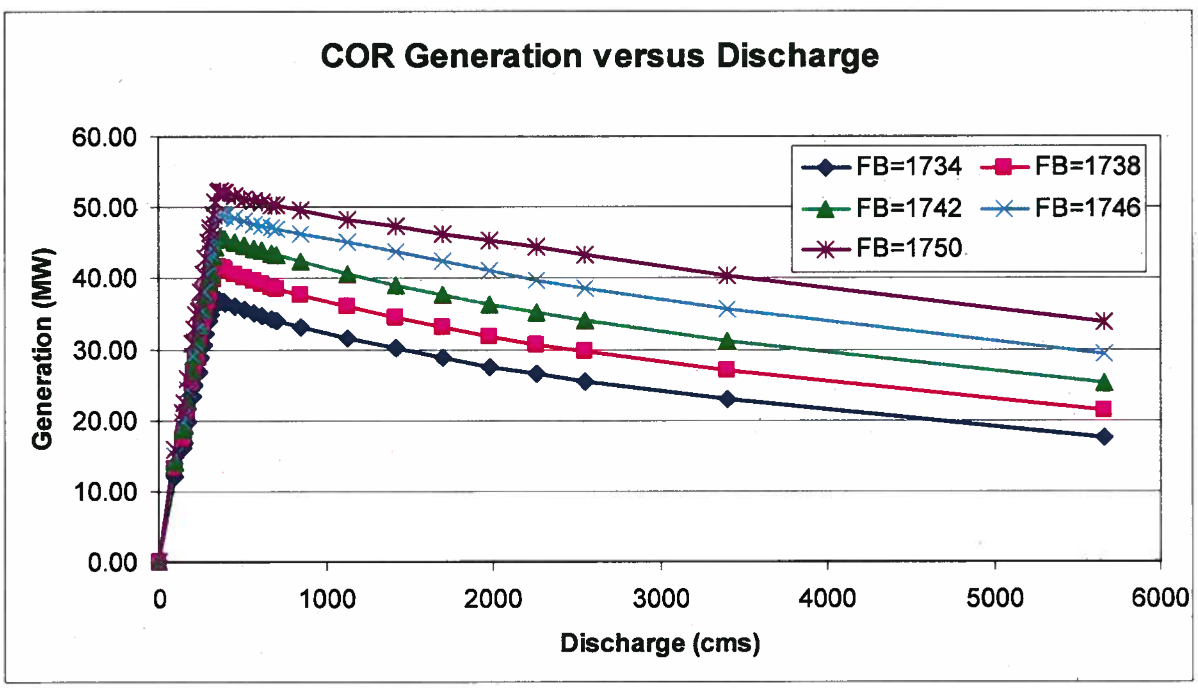


Figure 5.2 COR Generation vs Discharge Curve

Figures 5.3 to 5.6 show the generation-discharge graphs for the Riverplants in the Kootenay System.

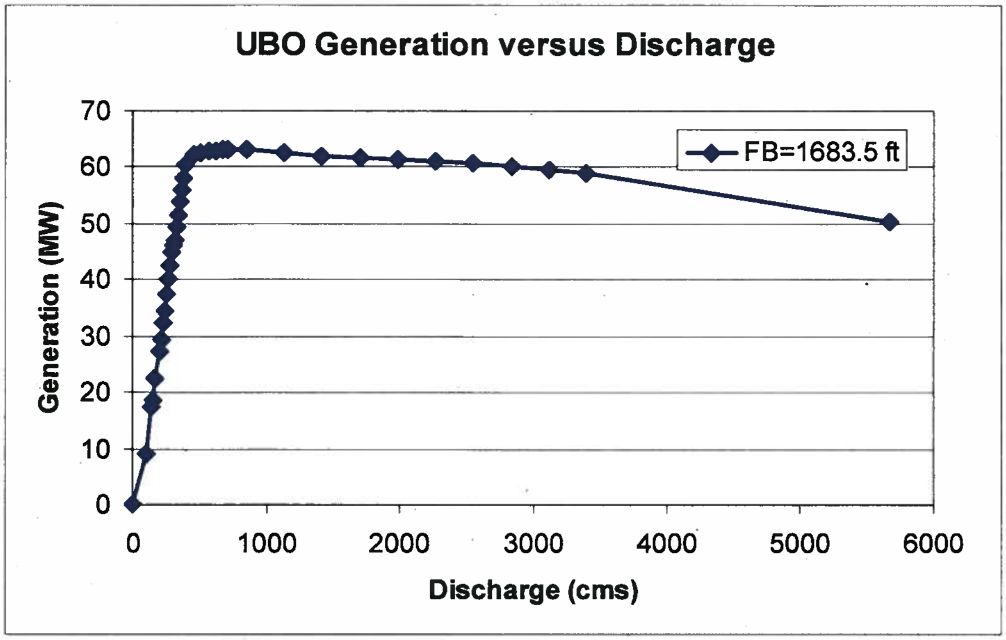


Figure 5.3 UBO Generation vs Discharge Curve

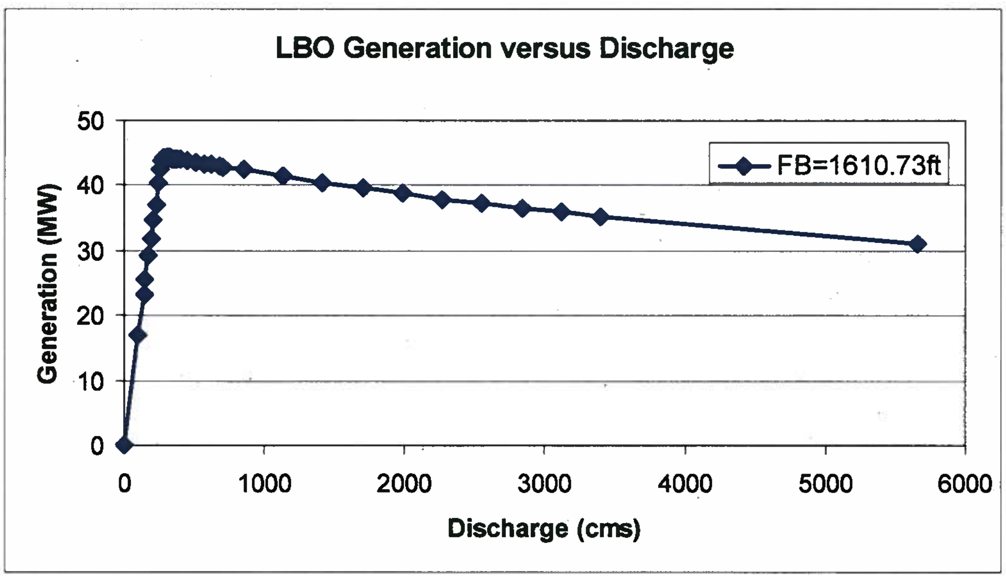


Figure 5.4 LBO Generation vs Discharge Curve

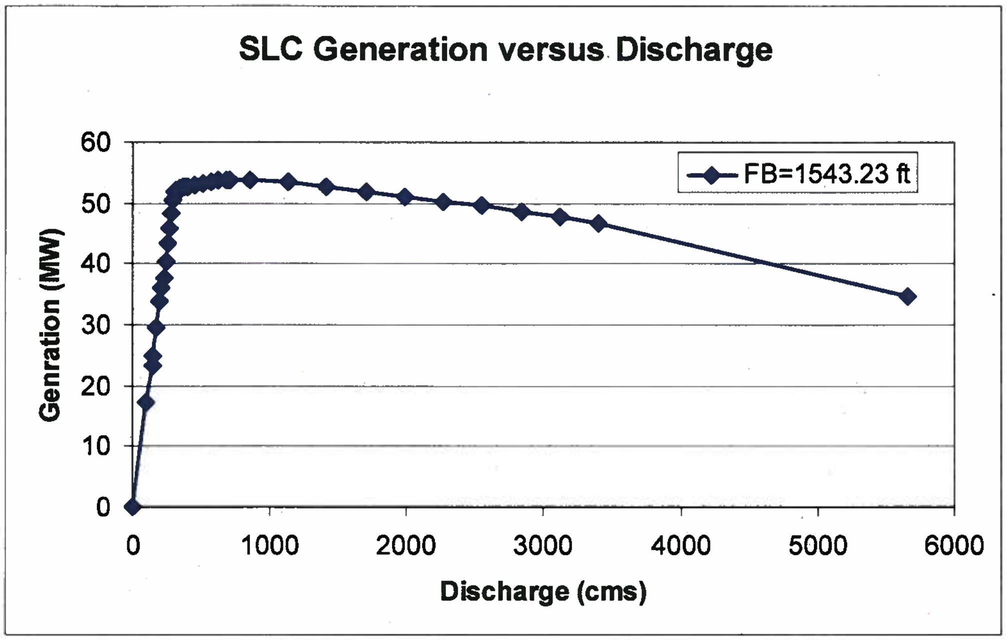


Figure 5.5 SLC Generation vs Discharge Curve

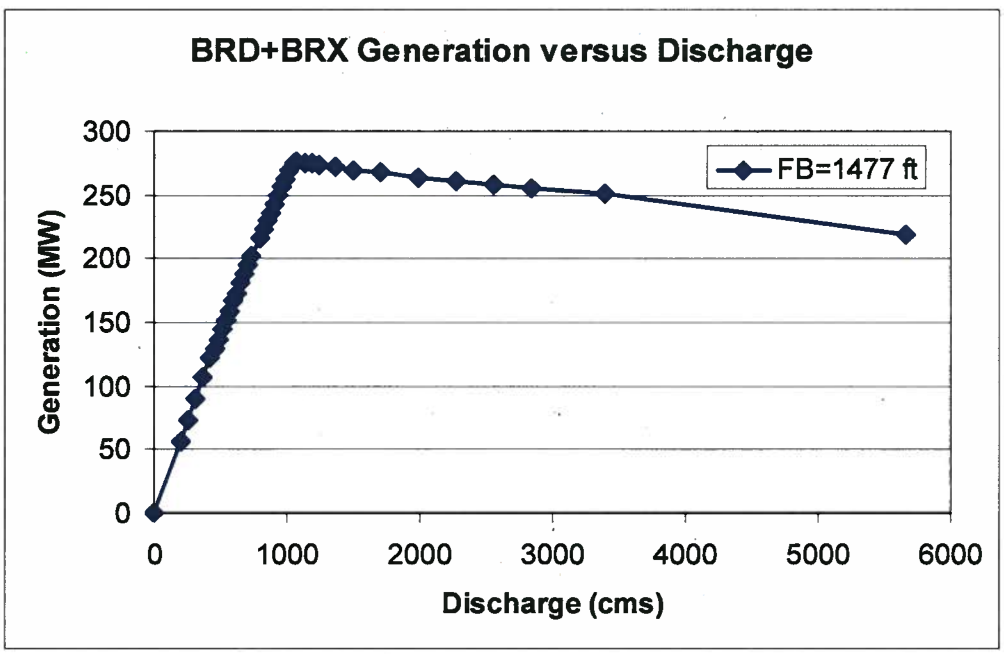


Figure 5.6 BRD(X) Generation vs Discharge Curve.

For a LP optimization problem the convexity of the production function is a necessity. Therefore a convex GPF was derived using these inputs. Minor changes were made to the functions to guarantee their convexity. The pink lines in the Figure 5.7 and 5.8 are the new points substituted for the real data to make these curves convex.

It can be noted that these minor adjustments do not result in considerable errors. A convexity check is also performed in the model to ensure this condition is enforced:

Check for all the plants in all timesteps if *slope (j,n,t)>= slope(j,n+1,t)*;

Where "j" is plant, "n" is the piecewise linear segment and "t" is timestep.

This is a check to determine if the slopes are decreasing. A decreasing order of a set of slopes in a piecewise linear function result in a convex function which is a requirement in a LP optimization model.

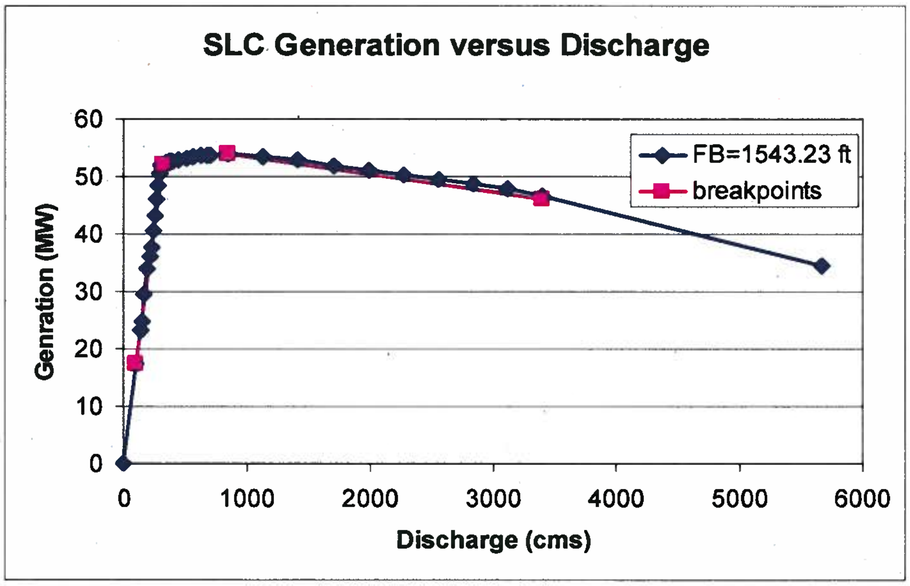


Figure 5.7 SLC Generation-Discharge Breakpoints

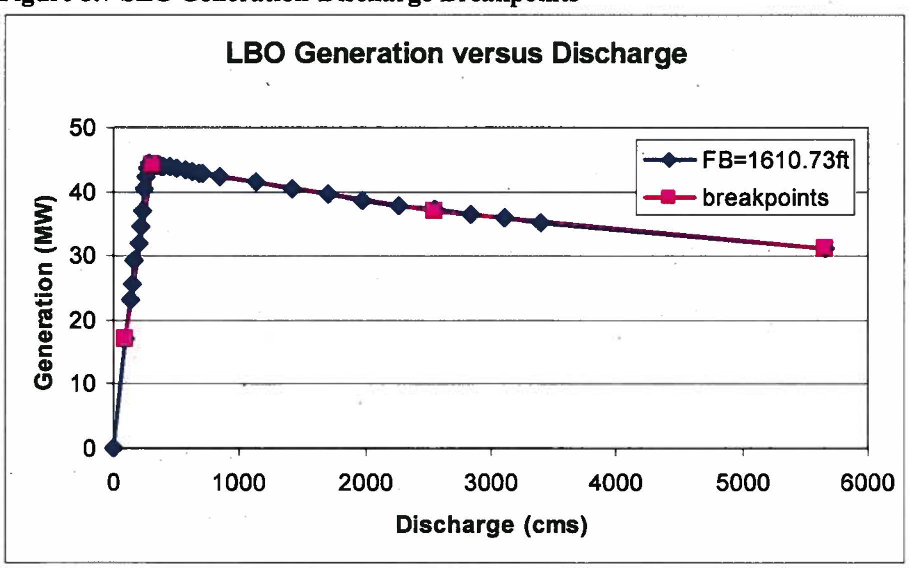


Figure 5.8 LBO Generation-Discharge Breakpoints

Table 5.2 shows the set of discharge breakpoints, slopes and generation breakpoints in the STOM model which are used to derive GPF, and a similar data set was used in this research.

Table 5.2 STOM Generation Production Function Coefficient [Source : Shawwash 2000]

Turbine Breakpoint	Generation Breakpoint (*m*) Coefficients	Generation Breakpoint (*c*) Coefficients
QBP_1	$GBPm_1$	$GBPc_1$
QBP_2	$GBPm_2$	$GBPc_2$
QBP_3	$GBPm_3$	$GBPc_3$
QBP_4	$GBPm_4$	$GBPc_4$

5.2 ADDITIONAL MODEL CONSTRAINTS

Additional constraints are those constraints that are not imposed by operation rules but may result in more reasonable outputs which are closer to real operation. These constraints are not usually used in the model, but can be used when more practical results are needed. The "FLOW-INCREMENT" is one of these constraints, and it restricts the rate of outflow increase or decrease for two subsequent time steps when the inflow condition does not require the operation planners to suddenly increase or decrease the outflow. Table 5.3 lists the flow increment constraints which are used for different dates in the model. "j" index is used for the plants and "t" index is used for the timesteps.

Table 5.3 Temporarily Constraints

Constraint Name	Time	Constraint
FLOW_INCREMENT1	t in start..freshetstart-1	$-60 <= (QP_{j,t+1} - QP_{j,t}) <= 60$
FLOW_INCREMENT11	t in Aug31..end-1	$-60 <= (QP_{j,t+1} - QP_{j,t}) <= 60$
FLOW_INCREMENT2	t in freshetstart..Aug31	$-60 <= (QP_{j,t+1} - QP_{j,t}) <= 60$
FLOW_INCREMENT22	t in nelsonstart+1..Aug31	$-30 <= (QP_{j,t+1} - QP_{j,t}) <= 30$

CASE STUDY

5.2.1 Base Case

In this case study the forecast regulated inflow to the lake for the period Feb 5th 2008 to Oct 1st 2008 was used as input to the model. The base-case input data are derived from operation planning spread sheets used for 2008 [27].

i. Initial Conditions

Table 5.4 Initial Forebays

Plant Name	FB(ft)	FB(m)
COR	1,742.3	531.0556
KCL	1,742.1	530.991
BRD	1,474.6	449.463
KLK	1,742.8	531.226
UBO	1,682	512.7
LBO	1,610.7	490.95
SLC	1,543.2	470.37

The lake natural level initial value is set to 1,742.18 ft (531.019 m).

ii. Lake Inflow

Figure 5.9 shows the lake inflow in this study which is equal to the sum of predicted natural inflow and the assumed regulated inflow from the Duncan and Libby dams for the period of Feb 5th 2008 to Oct 1st 2008.

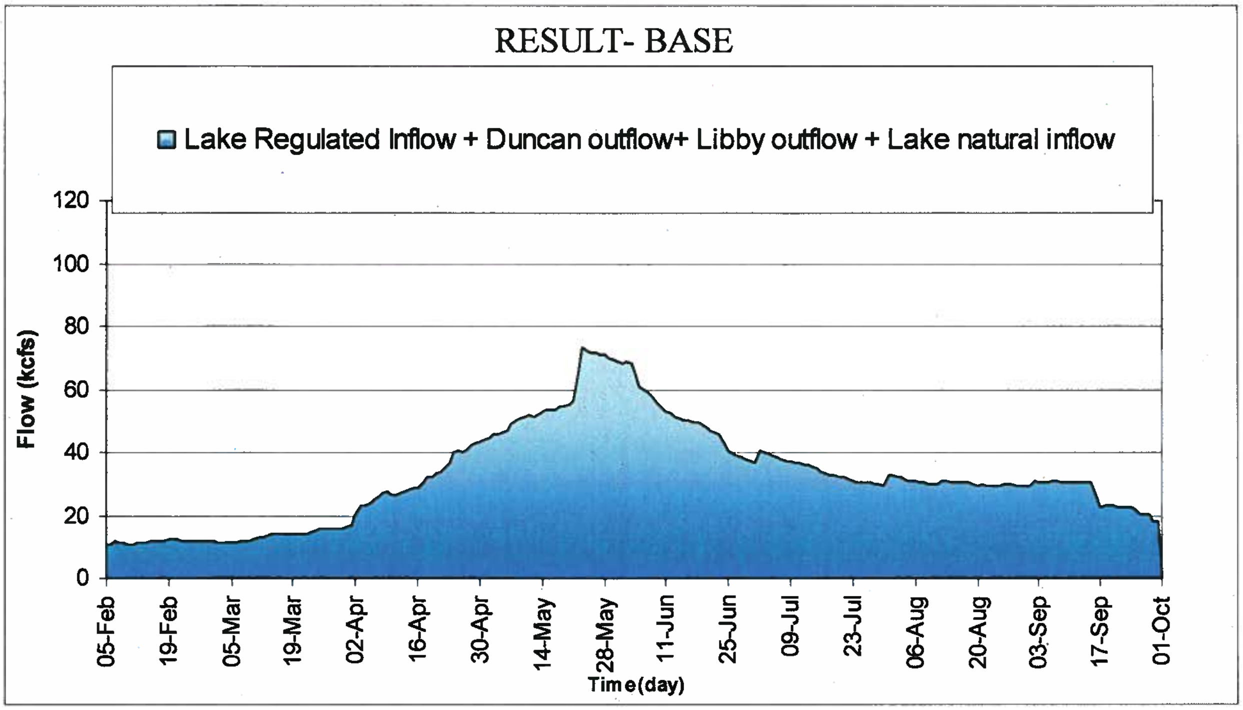


Figure 5.9 Lake Inflow (Base Case)

vi. Results

Figure 5.10 shows the optimized lake level based on the optimized value of the lake volume for the inflows in the base case. The results in this figure show that the lake storage is kept within a margin from the IJC constraint as required by the operation rules. It can also be seen that the Nelson gauge level drops below 1,743.32 ft in early August and it remains at that level for the rest of August based on the operation rules discussed in section 3.7.1.

Figure 5.11 shows the lake's outflow for the study period. According to the operation rules the lake outflows must be kept on the *freefall* condition until it is ensured that the peak lake level is not rising to a second peak in the same season.

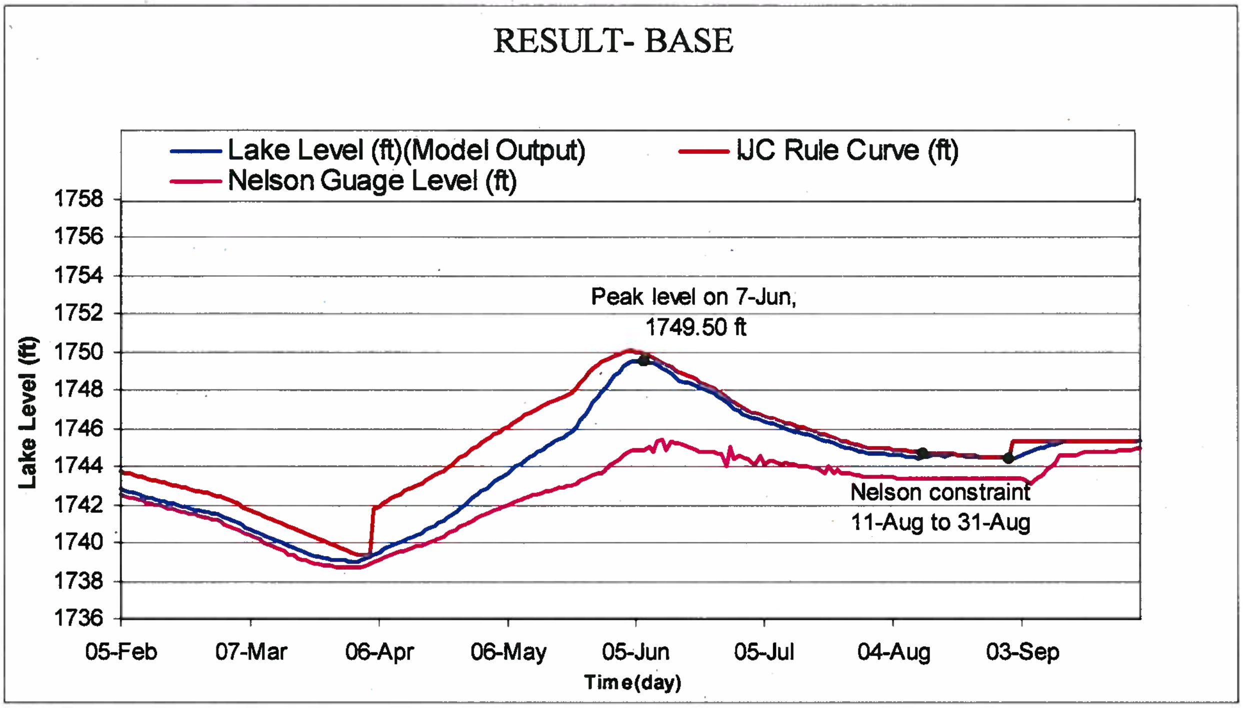


Figure 5.10 Nelson Gauge Result

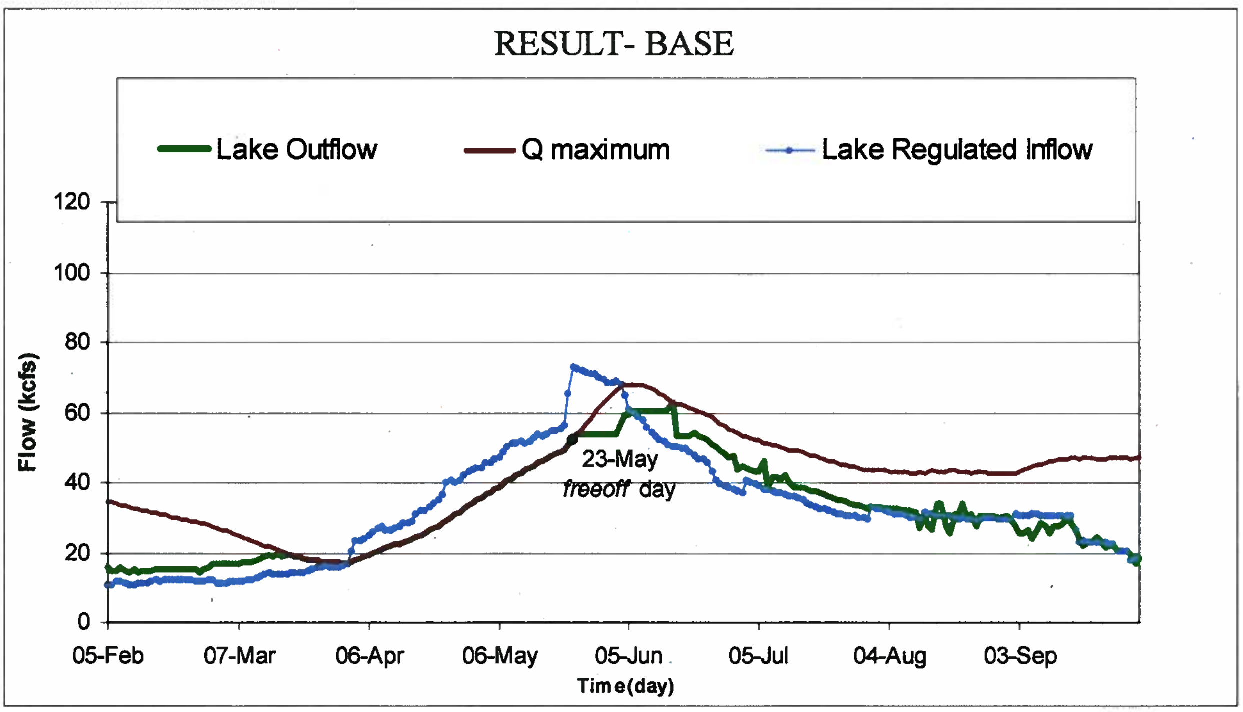


Figure 5.11 Optimized Lake Outflow vs Maximum Outflow

For this study, the algorithm finds 23rd of May 2008 as the date in which it terminates the *freefall* condition.

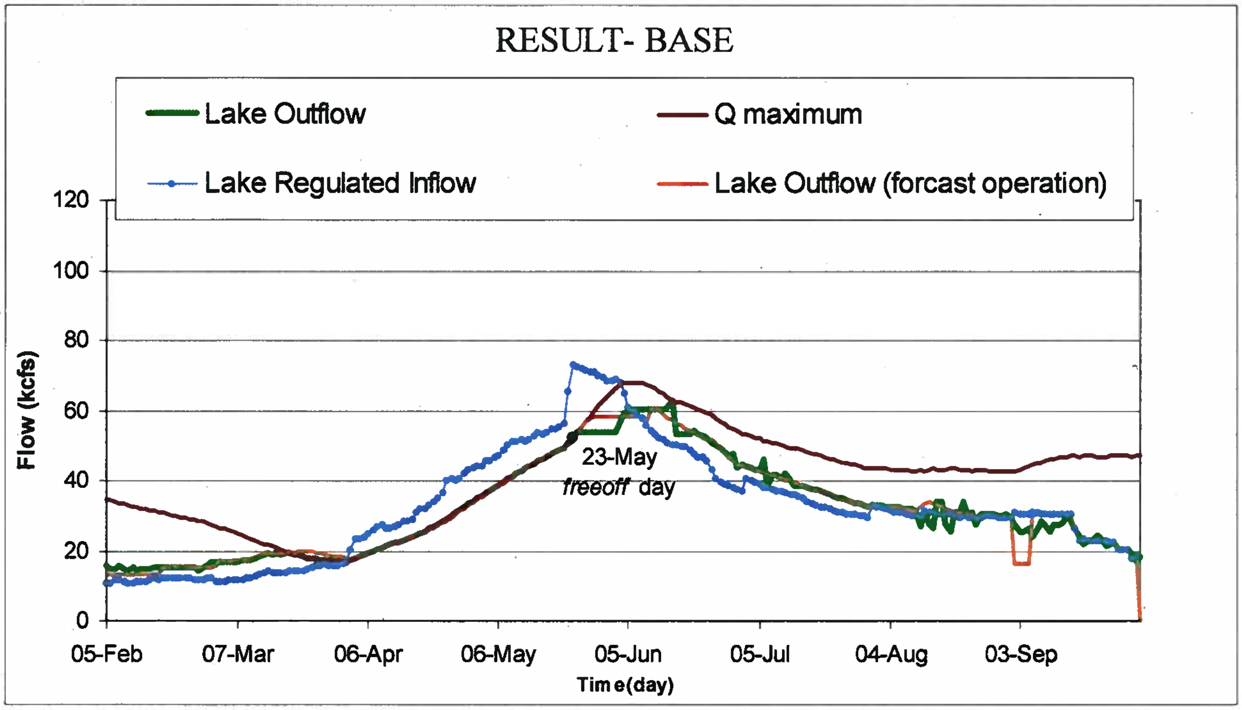


Figure 5.12 Optimized Outflow vs Simulated Operation

Figure 5.12 shows the expected operation of the forecasted inflow. Due to uncertainty associated with the natural inflows the operation planners usually keep the outflow constant for few days before the occurrence of the expected peak level as can be seen in Figure 5.12. This practice is followed to ensure that the peak inflow is passed [27].

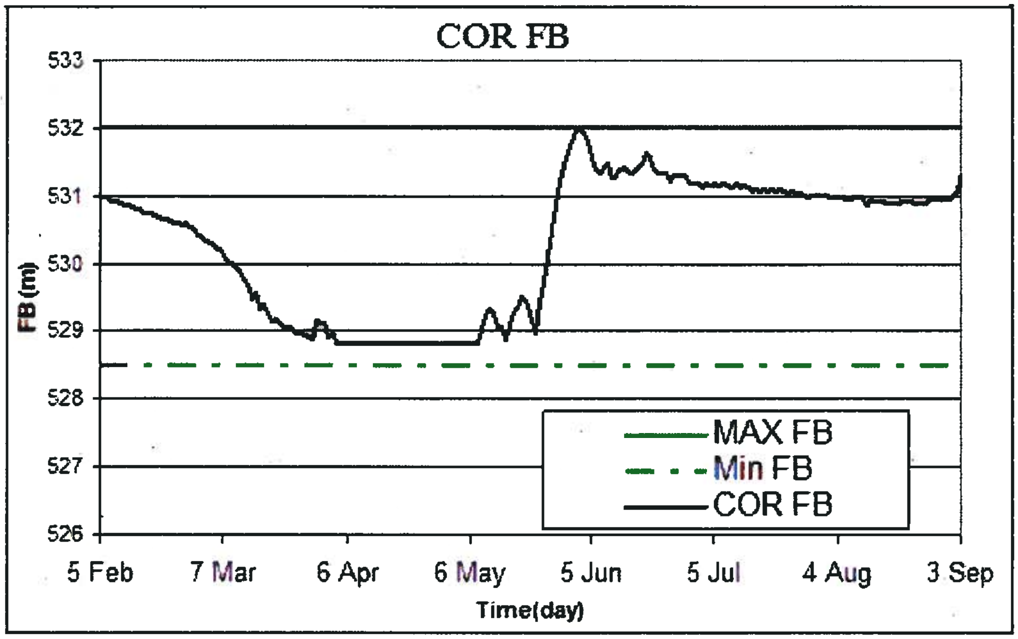


Figure 5.13 COR Forebay Result

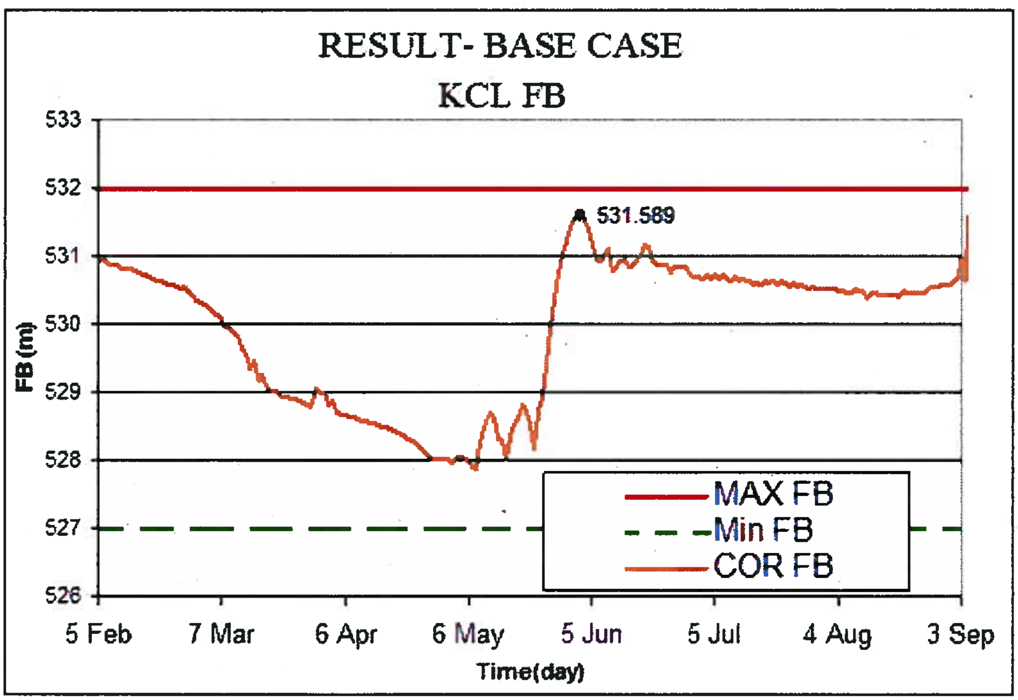


Figure 5.14 KCL Forebay Result

Figure 5.13 and 5.14 show the forebay levels for COR and KCL.

The two lines at the top and the bottom of forebay levels in Figure 5.13 and 5.14 show the minimum and the maximum forebay limits. The results show that the optimized COR and KCL forebays are within the limits and therefore feasible.

Figure 5.15 shows a comparison between KLK, KCL and COR forebay levels. It can be noted that from the first week of February until the first week of April, KLK level is decreasing, resulting in lowering of the COR and the KCL forebays. When the KCL forebay is compared to the COR forebay it can be seen that they follow the same pattern This is because both plants share the same headpond with only difference being the headloss from COR reservoir to KCL powerhouse results in a lower forebay level at KCL. It can also be seen that the forebay level of COR remain constant from the first week of April until the first week of May which is a typical of *freefall* period when Cora Lynn forebay is kept at its lowest level. The COR forebay will rise again after the termination of *freefall* period.When the lake peak level is passed, lake forebay would decrease and it results in the lowering of COR and KCL forebays as can be seen in Figure 5.15.

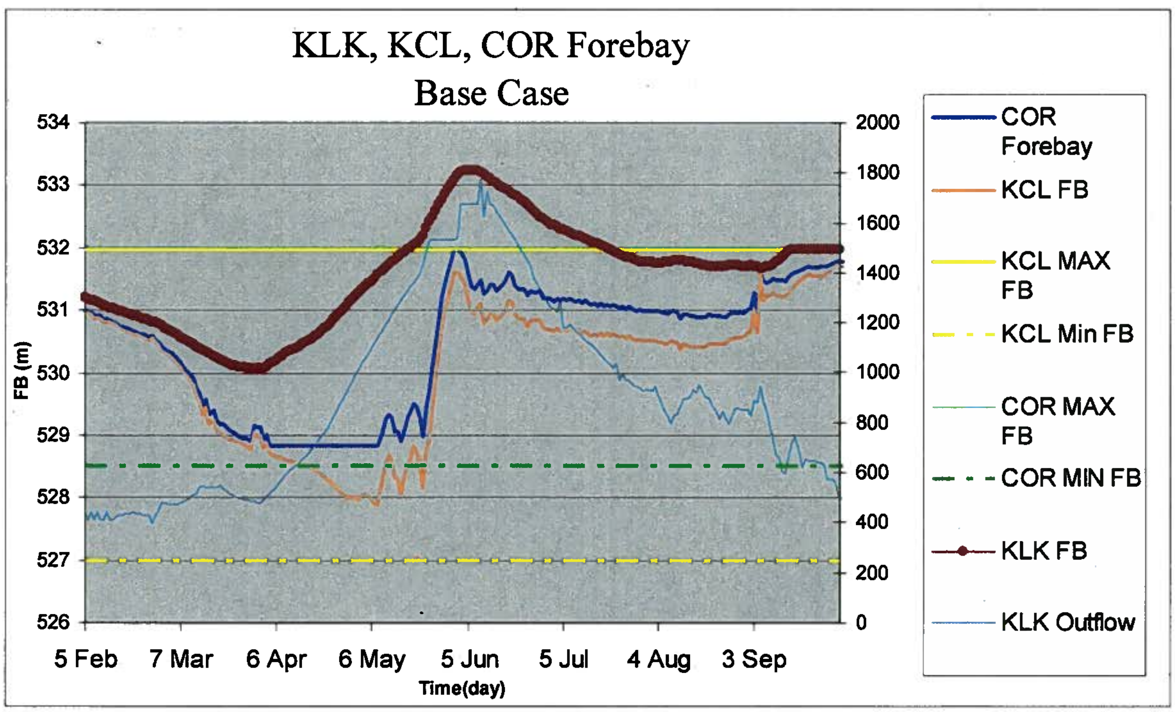


Figure 5.15 COR Forebay vs KCL Forebay

Figure 5.16 shows the Brilliant forebay level which fluctuates within its minimum and maximum level. The Brilliant forebay fluctuations are within two meters of its maximum forebay level and usually happens as a result of the minimum flow constraints imposed on Brilliant outflows (as explained in section 4.2.2.5) and it occurs in day-to-day operation [27].

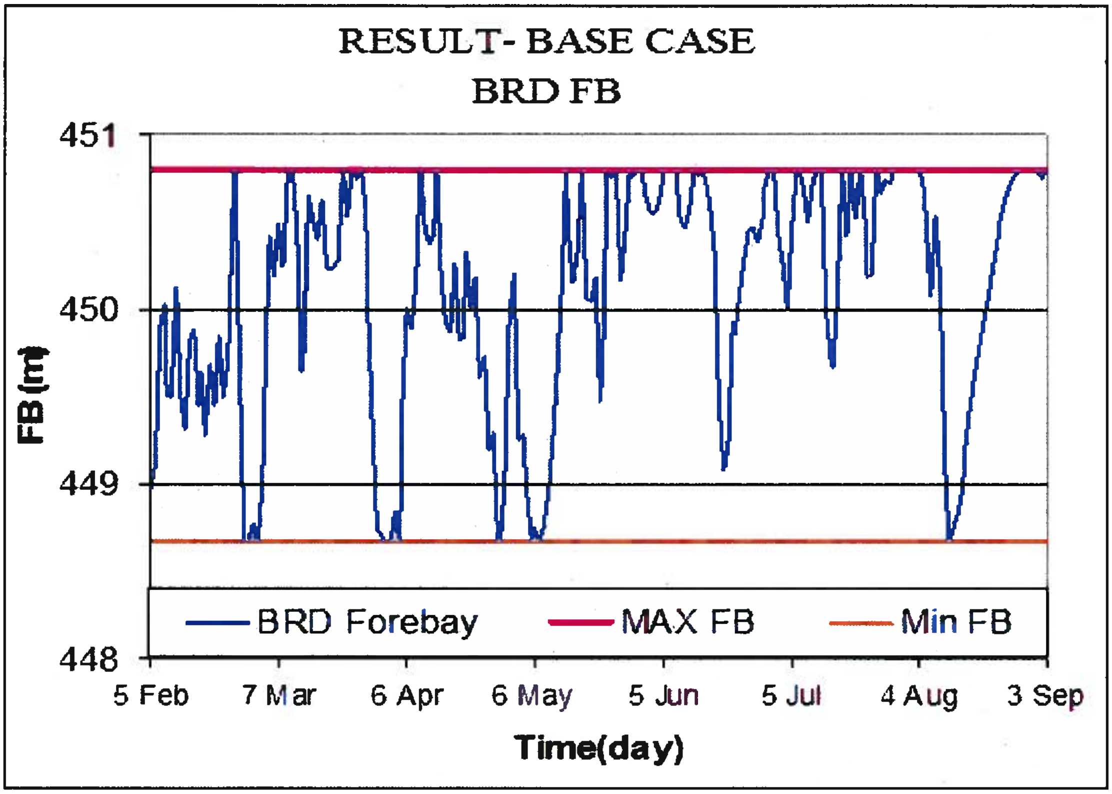


Figure 5.16 Brilliant Forebay Result

Figure 5.17 shows that UBO, LBO and SLC forebay levels change very slightly through out the study. As previously discussed, the Riverplants are considered to be "run-of-river" plants with essentially zero storage capacity and slight variation in forebay levels.

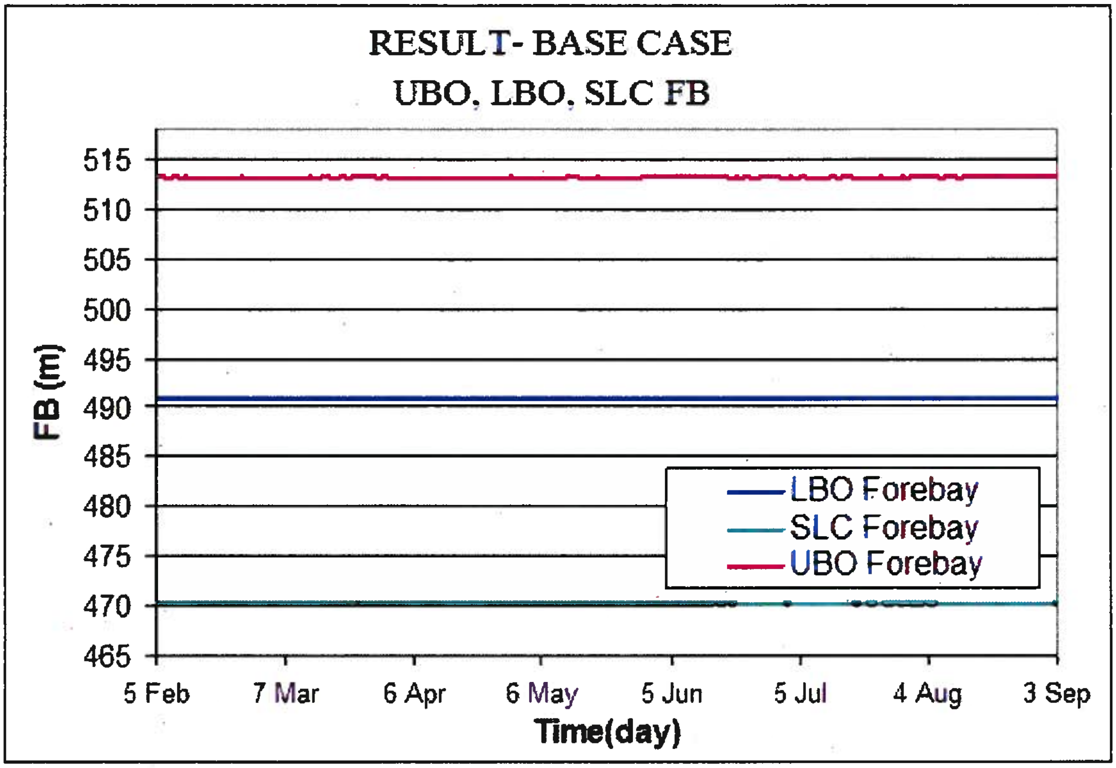


Figure 5.17 UBO, LBO,SLC Forebay Result

5.3 CASE STUDIES FOR REGULATED INFLOW SCENARIOS

The purpose of this section is to discuss the optimization model runs for five sets of regulated historical inflow data and illustrate the validity of the model and how well it adheres to the set of rules and constraints. The regulated inflow data into the Kootenay Lake for the five case studies were taken from a simulation study carried out by Engineer Herbert Louie of B.C Hydro Generation Resource Management Department. The study period starts from February1st to Sep 30th of the next year. Figure 5.18 shows the variation of inflows for 1929 to 1998.

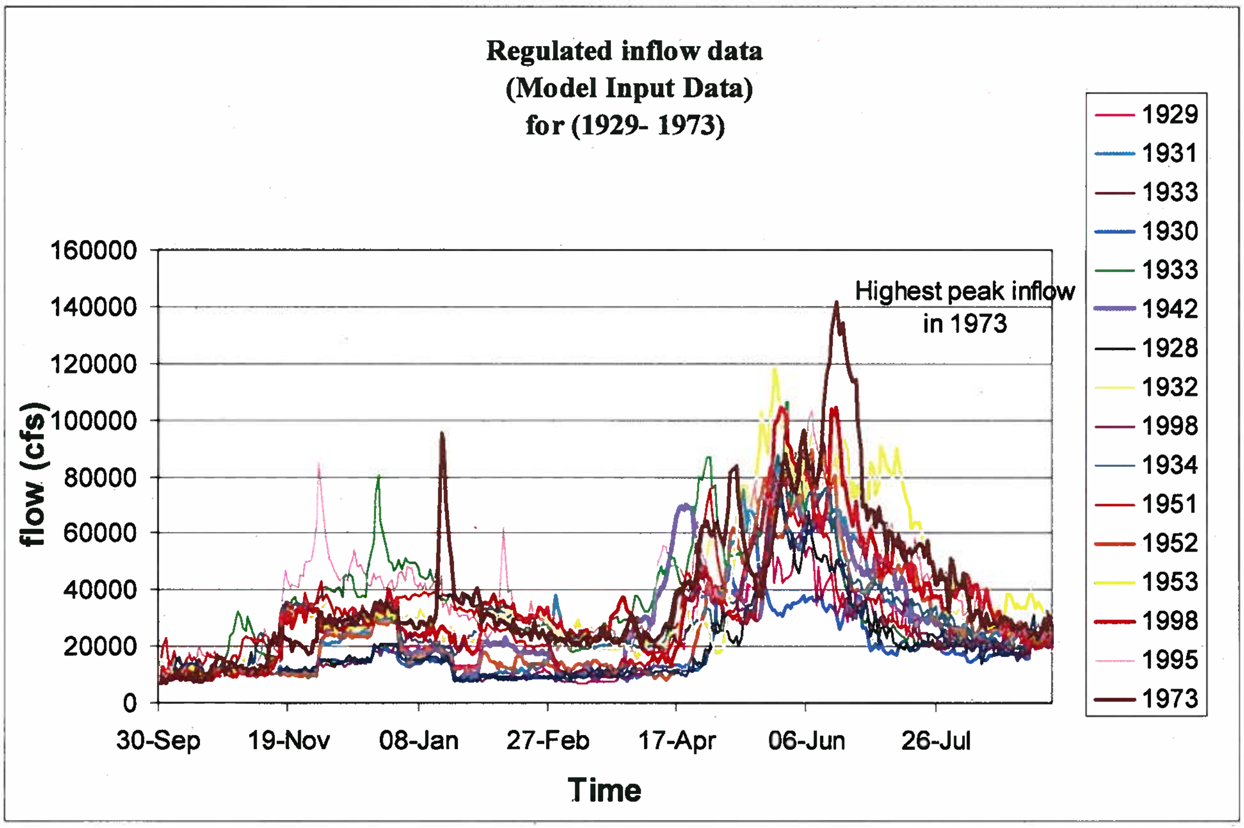


Figure 5.18 Historical Regulated Inflow Data

Figure 5.19 shows the variation of inflows for the past 80 years. It can be seen that inflows vary considerably from year to year with highest variation being in the freshet period and towards the end of winter. The lowest inflows occur in early fall and early spring.

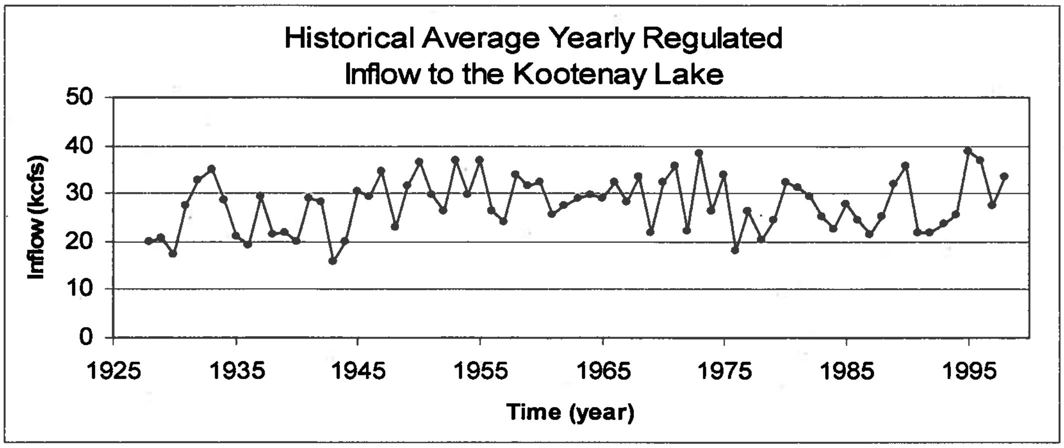


Figure 5.19 Historical Average Daily Inflow to the Kootenay Lake

Table 5.5 shows the ranking of the selected cases of the inflows between the seventy years record. .

Table 5.5 Historical Average Inflow Ranking

Year	Rank in higher inflow	Average cms	Average kcfs
1973	2	1082.449	38.22632
1933	9	986.4421	34.83587
1932	15	928.0084	32.77231
1928	67	559.6933	19.76538
1943	71	450.5901	15.91244

5.3.1 Case study- Inflow year 1928

The Kootenay optimization model was run using the regulated historical inflow records for the period Feb-Oct for 1928. The following paragraphs present the results and analysis for this case study.

Figure 5.19 shows that the average annual inflow to the lake in 1928 was within the lowest 10% average inflows for the past 70 years of inflow records. Figure 5.20 presents the optimized results for the lake levels, lake outflows and calculated values for the maximum Grohman Narrows outflows and the IJC rule curve. The simulation model results for 1928 inflow data are shown in Figure 5.21.

Comparing the optimization results to the simulation model output, it can be seen that the IJC upper rule curve and the lake level follow the same pattern in both models. It can also be seen that there is one peak lake level, equal to 1,747.723 ft, which has occurred on June 11th 1928.

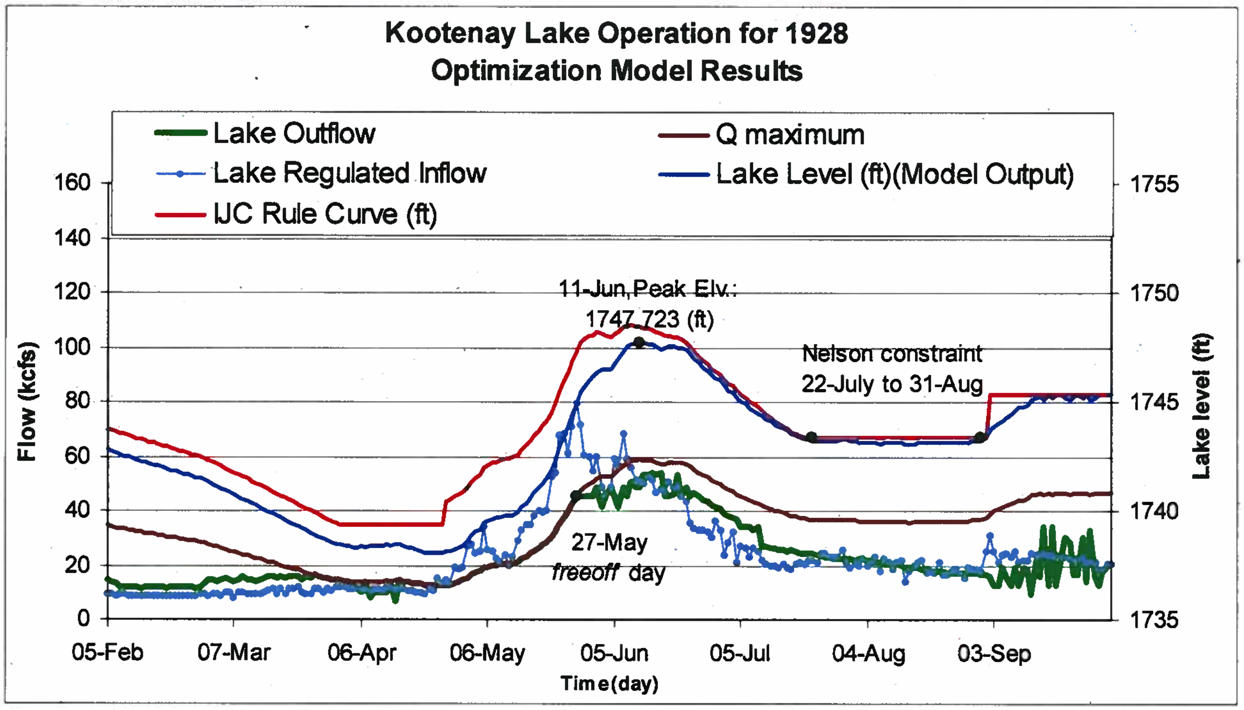


Figure 5.20 Optimization Model Result 1928

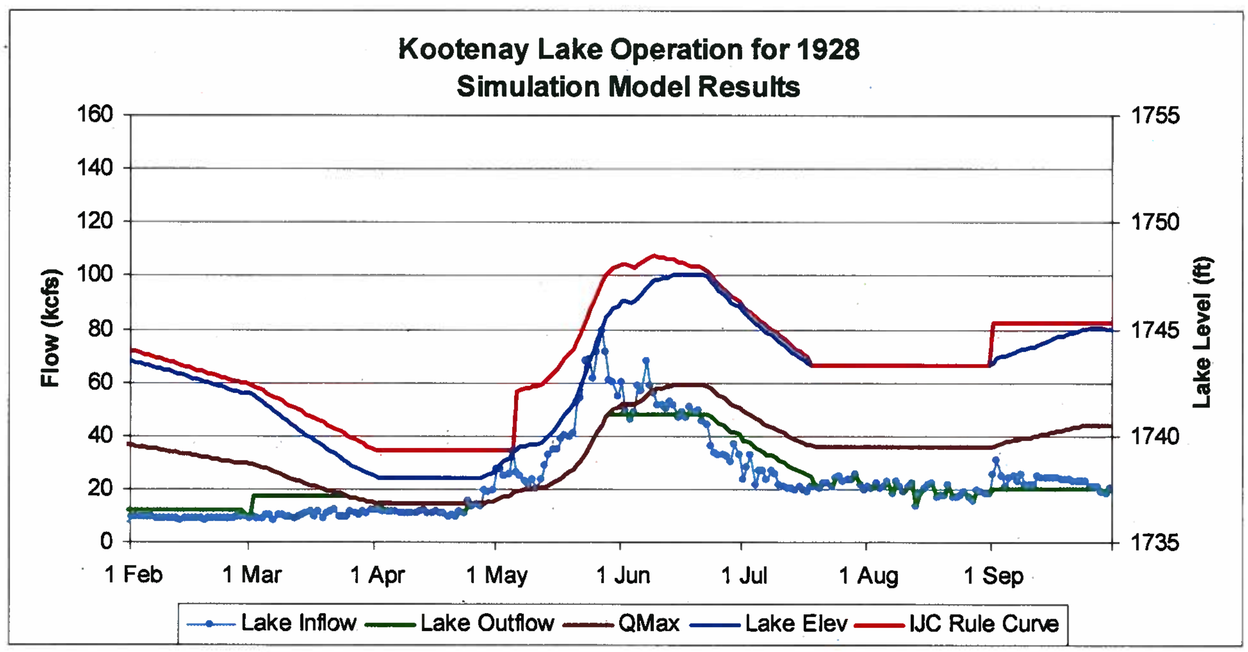


Figure 5.21 Simulation Model Result 1928 [28]

This year is categorized to be among one of the low-flow years, and the algorithm outlined in section 4.2.2.4.b was used to calculate the duration of *freefall* period. The terminating date for the *freefall* condition, or *freeoff* date, occurred on May 27th which is around the same date for simulation model results.

Because of the low inflows in this case, terminating the *freefall* condition at the end of May, which is an early date as compared to other years, does not increase the chance of flooding. Releasing the maximum outflow from the lake for a longer period in this study would only results in lower heads and therefore would result in less power generation and it would not be optimal.

Figure 5.20 and 5.21 show that the Nelson constraint also initiates around the same date in both models (see section 4.2.2.1.iii). In the simulation model the outflows during the Nelson constraint period (mid July to Aug 31st) is set equal to the total regulated inflow and the lake level is therefore held at a constant level. It should be noted that in the optimization model the lake levels are kept below the Nelson constraint level and can fluctuate within the 0.3 ft margin for power generation benefits, whereas the simulation model suggests a constant level during this period. Another important difference between two models is that in the simulation model the Queen's bay gauge level is assumed to be used as the control for the Nelson constraint and the Nelson gauge level is not calculated for the simulated operation [28]. The optimization model has the advantage of using and calculating the Nelson gauge levels, which enables the lake planners to consider this parameter to monitor the lake level at Nelson in their day-to-day operation.

It can also be seen that in the optimization results, the outflows fluctuate to take the best benefit of the energy prices. This factor is not considered in the simulation model. As shown in Figure 5.21, the simulated lake outflows are set equal to the lake inflows in several time periods or are set to average constant values. In the optimization model, the outflow fluctuations in addition to accurate calculation of control points, such as the Queen's bay gauge and the Nelson gauge, would result in more accurate results.

In general, both simulation and optimization model results for the 1928 inflow data show a typical operation for the Kootenay System with no special consideration or violation of model constraints.

5.3.2 Case study- Inflow year 1932

The input data sets for this study were retrieved from the historical inflow records of 1932. According to Figure 5.19 and Table 5.5 it can be seen that 1932 inflows are among the top %10 inflows for 70 years of record with the average of 32 kcfs (928 cms). In the following paragraphs the optimization results are discussed and compared to simulation model results.

Figure 5.22 and Figure 5.23 shows the optimization and simulation model results respectively. It can be seen that the IJC upper rule curve and the lake level follow the same pattern in both models.

This study is a one peak level scenario and the peak is equal to 1,752.82 ft which occurred on June 22nd. Because the peak level is higher than 1,749.5 ft, it is expected that the *freefall* terminating date occurs on the declining slope of the lake level after the peak is passed (see section 4.2.2.4.a). The optimization results show that the *freeoff* date occurs on July 2nd after the peak is passed, as expected.

It should be noted that in this year, because of the high inflows to the lake, the Nelson gauge does not drop below 1,743.32 ft and the IJC upper rule curve stay as the upper limit until the end of August in absence of the Nelson constraint.

It can also be seen that IJC rule is violated around the first week of April in both models. This condition may happen in high inflow scenarios when the maximum Grohman Narrows outflow is released. In Figure 5.23 the green line, the lake outflow, traces the brown line, the maximum Grohman Narrows outflow in April, which shows that the maximum outflow is released for the period when IJC is violated. According to the operation rules for high-inflow scenarios, exceeding the IJC rule curve is allowed in such cases [27].

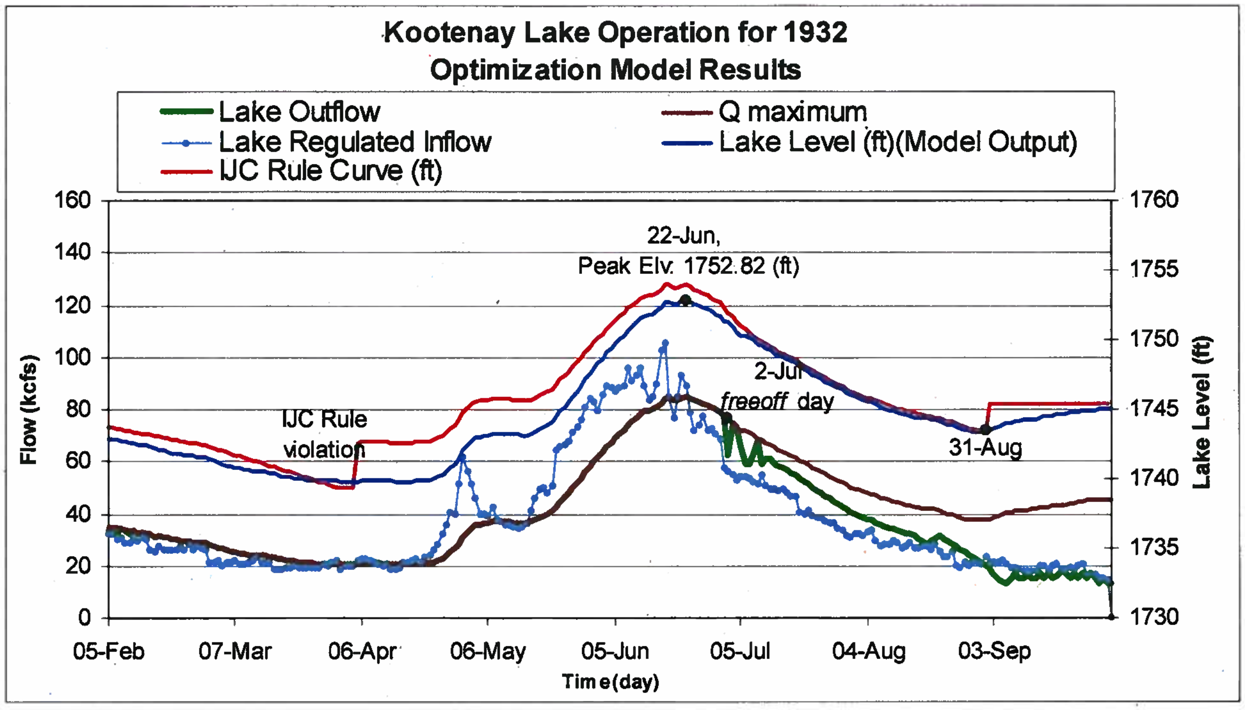


Figure 5.22 Optimization Model Result 1932

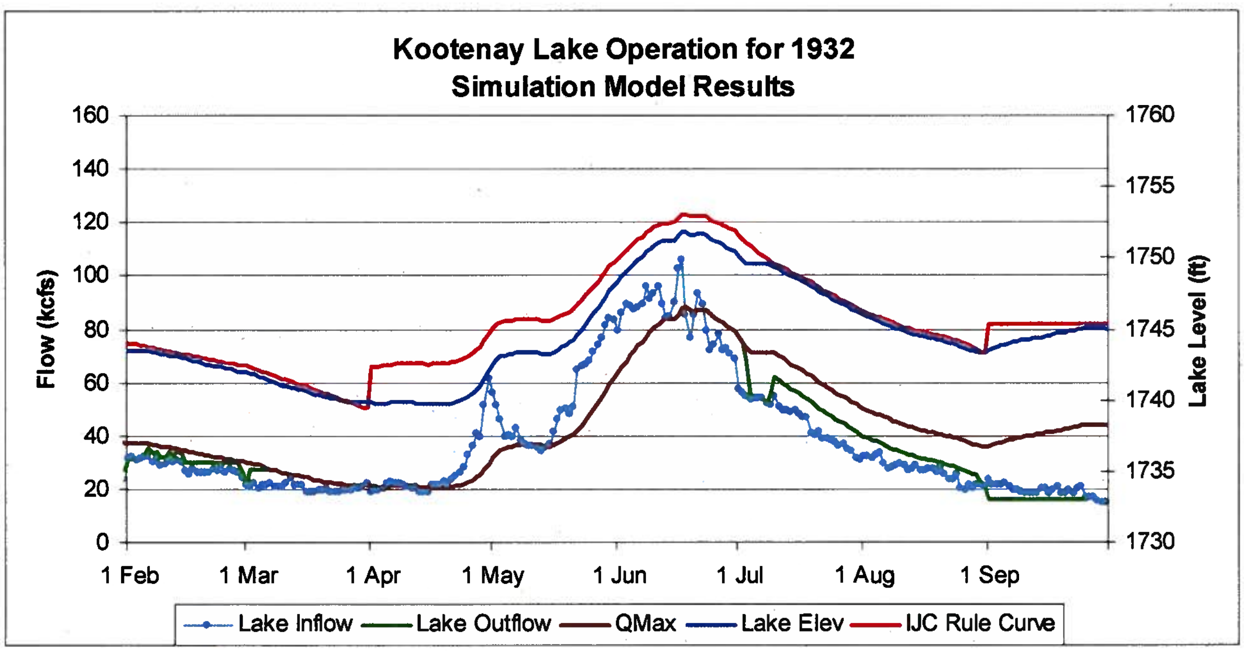


Figure 5.23 Simulation Model Result 1932 [28]

5.3.3 Case study- Inflow year 1933

The inflow year of 1933 was chosen for a case study because of the high variation of inflow values during the freshet period. This case is a good example for inflow conditions in which the first peak level can be mistaken for the main peak. Figure 5.24 shows the inflow data and optimization results for 1933.

In real operation, the uncertain nature of inflow always creates the risk for the second peak in lake level to occur. Monitoring the lake inflows with the aid of short term weather forecast help to predict the changes in lake level in day-to-day operation [27].

As can be seen in the results of optimization and simulation model, the IJC curve follows the same pattern as the lake level. The outputs from the model show that the *freefall* state is maintained until the second peak is passed. The termination date of *freefall* period occurs on June 13th as can be seen in Figure 5.24.

Although the average inflows in 1933 is ranked the 9th according to Table 5.5, the inflows decrease considerably after the freshet period causing the Nelson gauge to drop below 1,743.32 ft. This inflow pattern might occur after the freshet period and the day-to-day monitoring of the Nelson gauge level would help to keep the lake at the required level subsequent to the freshet. As can be seen in Figure 5.24 for the 1933 optimization results, the Nelson constraint limit the lake level from July 23th until the end of August.

Because of the high inflow prior to the start of the freshet period, IJC constraint is violated in the first week of April yet it is an allowable violation under the rules as explained in the case study 5.3.2. The same pattern can be seen in the simulation results as shown in Figure 5.25.

The start of freshet period is on June 4th in optimization model, which is slightly different than that derived using the simulation model. This date is declared by IJC Board of Control each year and is an assumed input for both models in this study [27].

As can be seen in this study, the variation of inflows may cause considerable changes in IJC and lake levels. The inflow records with two peak level are considered to be a more difficult operational circumstances as compared to typical conditions. In this study, the violation of the IJC rule curve due to high lake inflow, the two peak condition and the decrease in lake level after the freshet period, are a set of important factors that significantly influence operation of this system.

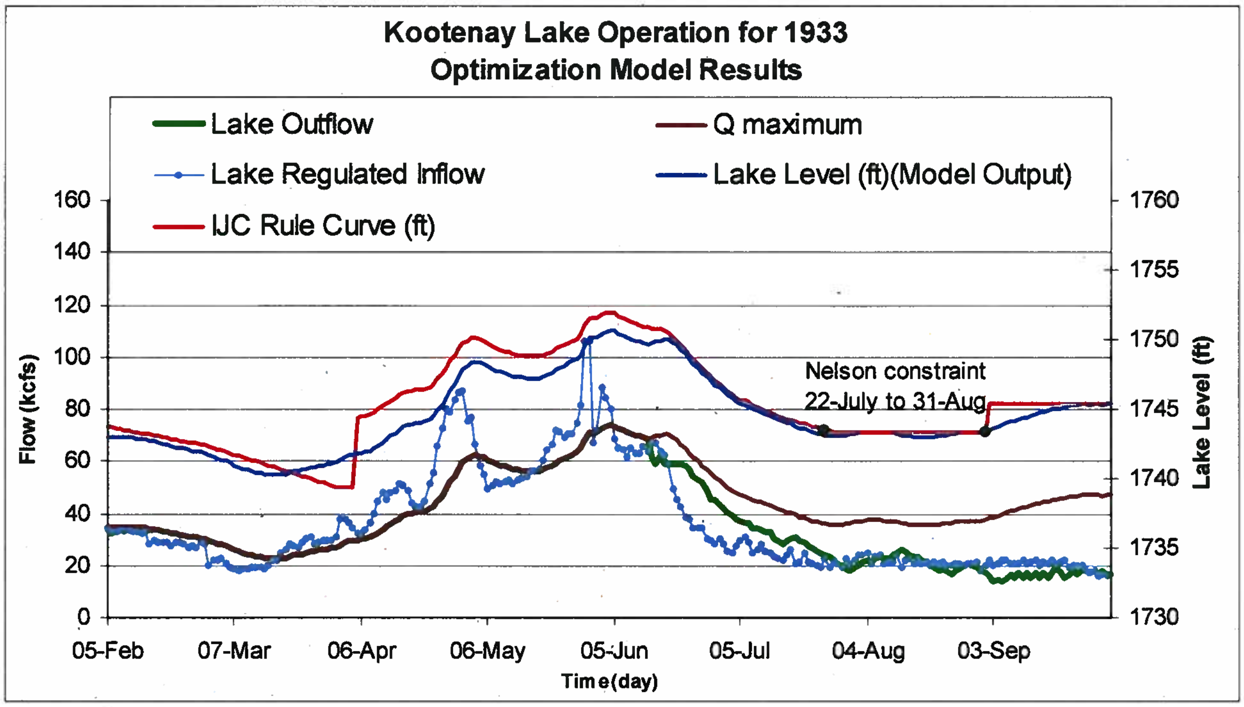


Figure 5.24Optimization Model Result 1933

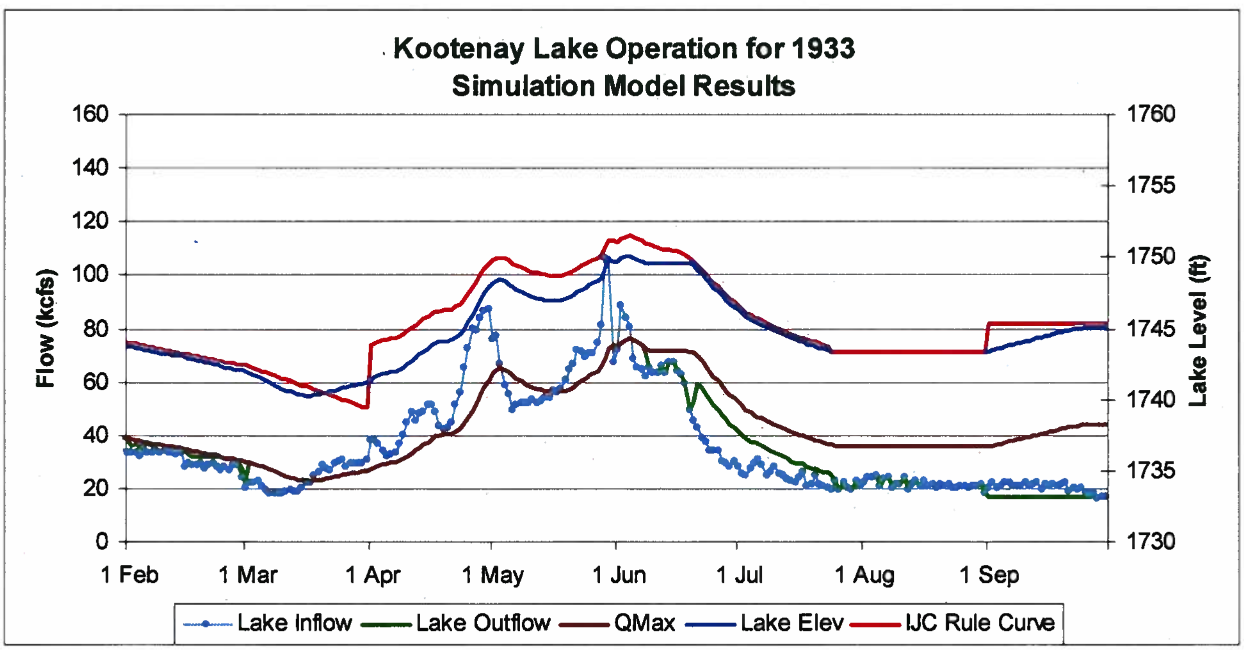


Figure 5.25 Simulation Model Result 1933 [28]

5.3.4 Case study- Inflow year 1943

This case shows the study done for the lowest inflow scenario for the past 80 years of records. Figure 5.26 shows the optimization results for this study. The IJC and peak level curves are all lower than the previous case studies. The start date of the freshet period is delayed to the end of April because of low inflow in this year. The peak level is 1,744 ft, which is considered as a low peak as compared to the rest of the cases.

The *freefall* flow, which is basically released to avoid flooding in the freshet period, could be terminated at any time during freshet because of low lake levels in this study. However a minimum duration of *freefall* flow was enforced for consistency with real operations where *freefall* period is maintained for a certain period to avoid the risk of potential flooding.

Figure 5.27 shows the simulation result for this case. In both models, it can be seen that the Nelson gauge drops below its limits early in summer. This long duration for the constraint to influence the lake level is unusual and only happens in very low flow scenario in which lake levels are substantially low.

The results of the study show that the rules and constraints are respected and the operation follows a typical mode.

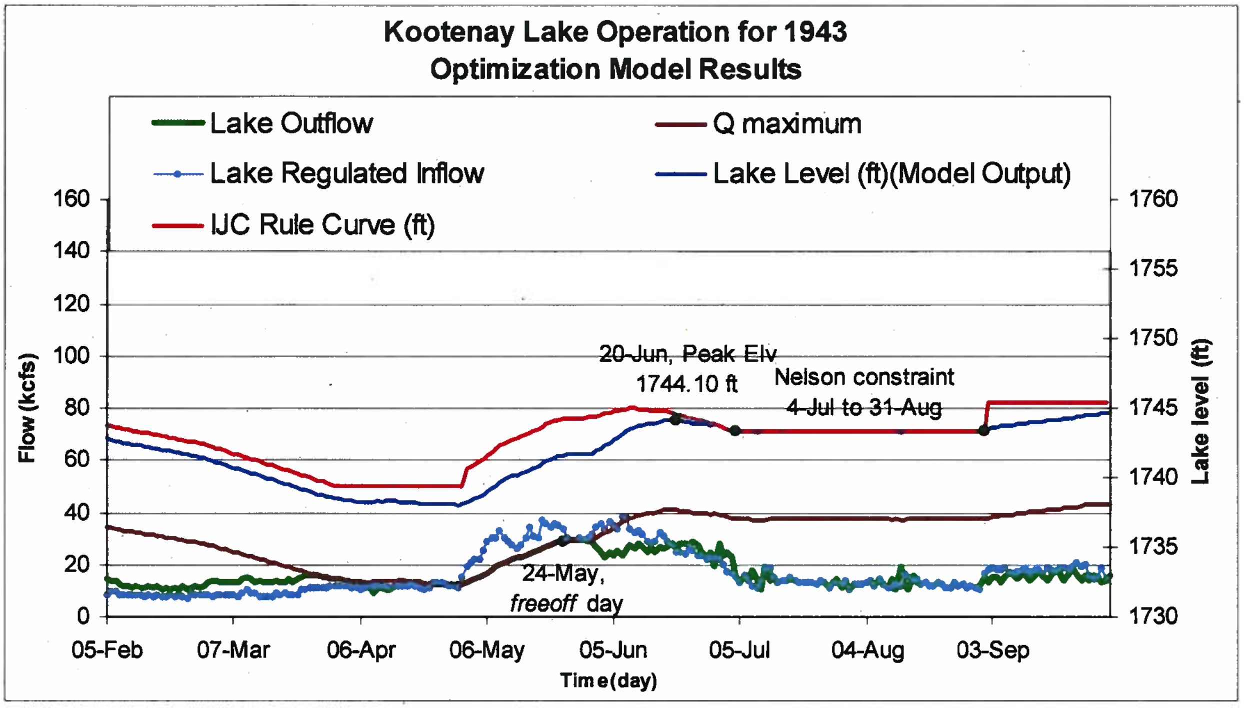


Figure 5.26 Optimization Model Result 1943

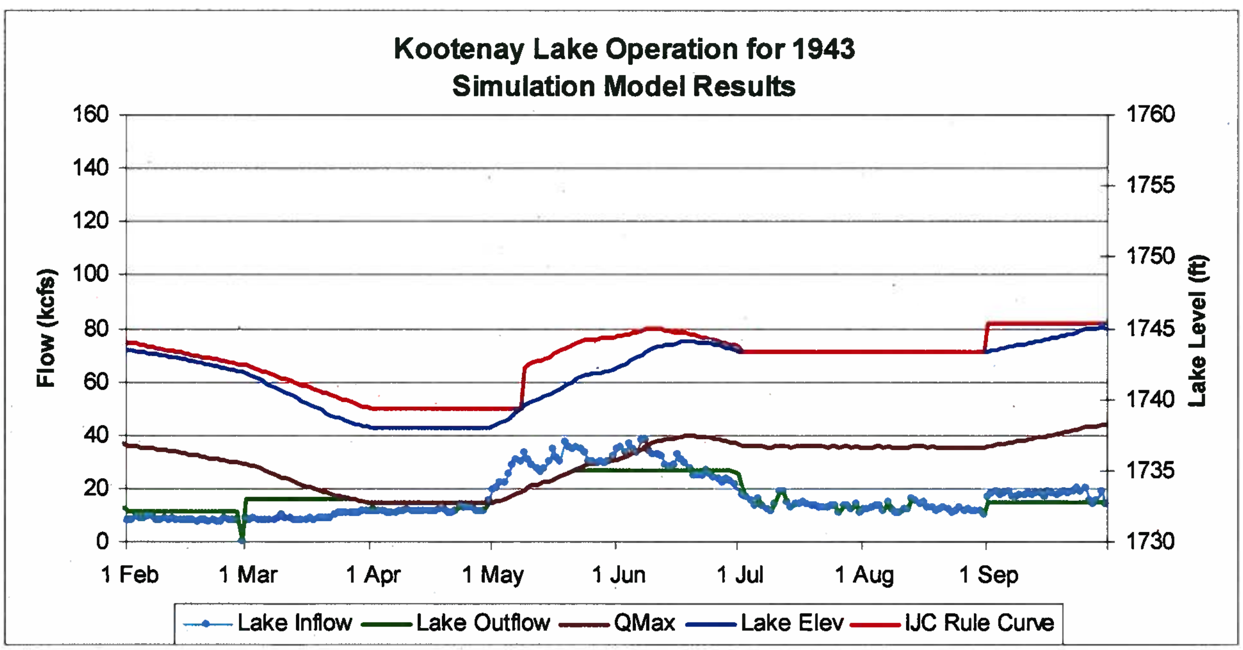


Figure 5.27 Simulation Model Result 1943[28]

5.3.5 Case study- Inflow year 1973

A case study was done for year 1973, a year that has the highest peak regulated inflow in the past 80 years. As can be seen in Figure 5.28, the sum of the lake regulated and natural inflows reached their highest value of about 141.25 kcfs (4000 cms) in the middle of June. The peak level was about 1,755 ft and it occurred on June 26th. This level is one of the highest peak levels for the past 80 years.

The *freefall* termination date was on July 10th, which is considered to be a late *freefall* date and is required to keep the lake level below IJC rule curve.

Because of the high amount of inflow the lake level at Nelson does not drop below 1743.32 ft and therefore the Nelson gauge constraint was not active in this study.

Figure 5.29 shows the simulation model results. Both models are consistent for the shape of IJC constraint on the lake level. The *freefall* duration also has the same duration in the optimization and the simulation models.

Although the inflows are high in this study, the variation of them does not create the chance for a second peak; therefore, operation of the lake follows a typical mode after the peak is passed.

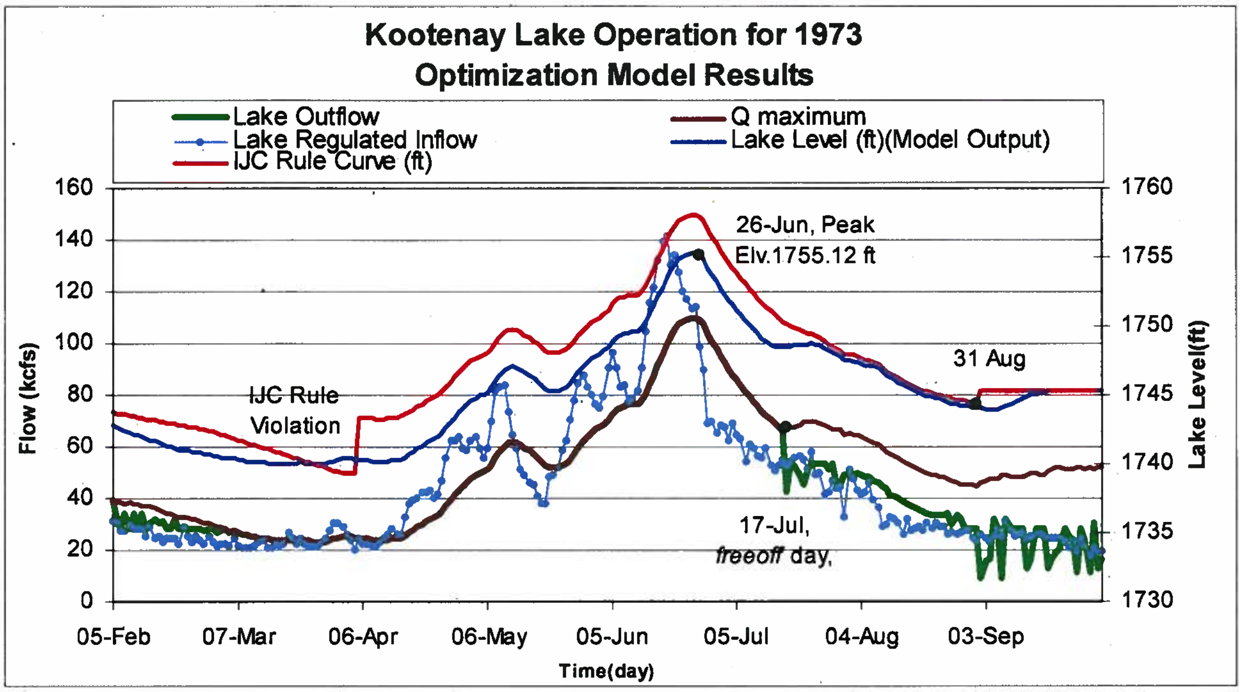


Figure 5.28 Optimization Model Result 1973

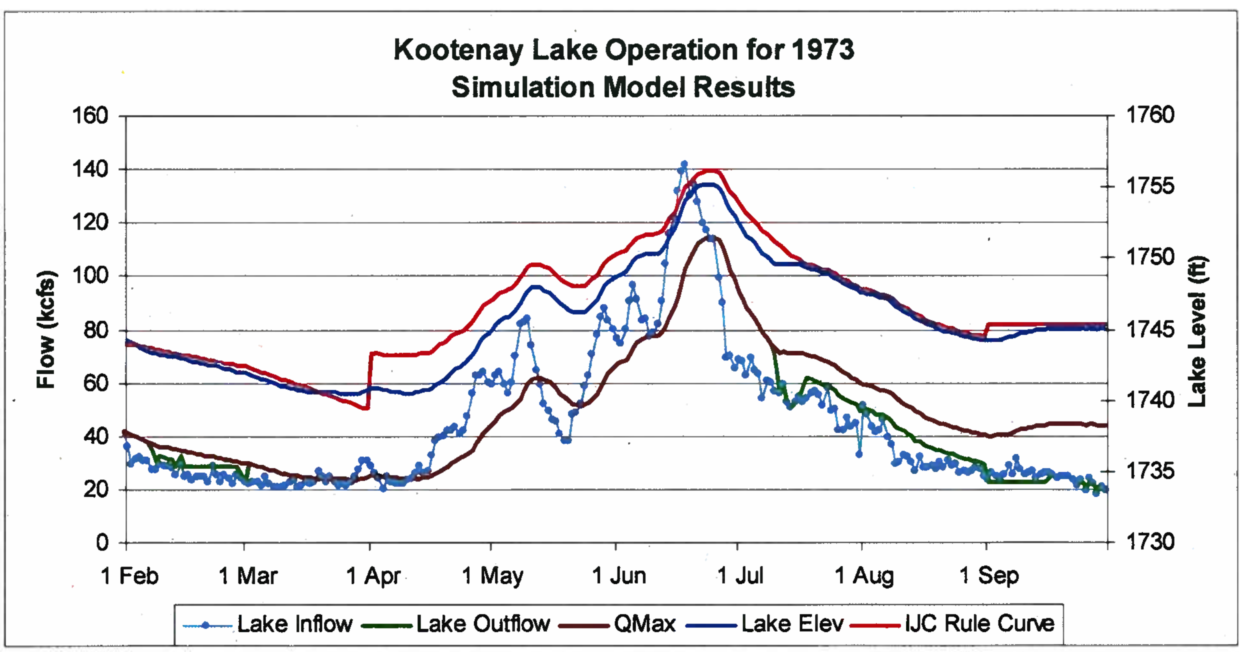


Figure 5.29 Simulation Model Result 1973 [28]

5.4 OBJECTIVE VALUE COMAPARISON

The Monetary value comparison between "Optimization Model" and "Simulation Model" (Base case):

The base case output was used as the main criteria to compare the performance of the optimization model. One way to compare the value of the "objective function" in the optimization model and actual operation is to compare the total amount of power in the study period and the value of stored water in the last time step as outlined in chapter 4.

The scheduled generation represented by "*QP_LRB*", was used as inputs into the model and the corresponding actual generation "*P_LRB*" was calculated using the simulated forebay levels.

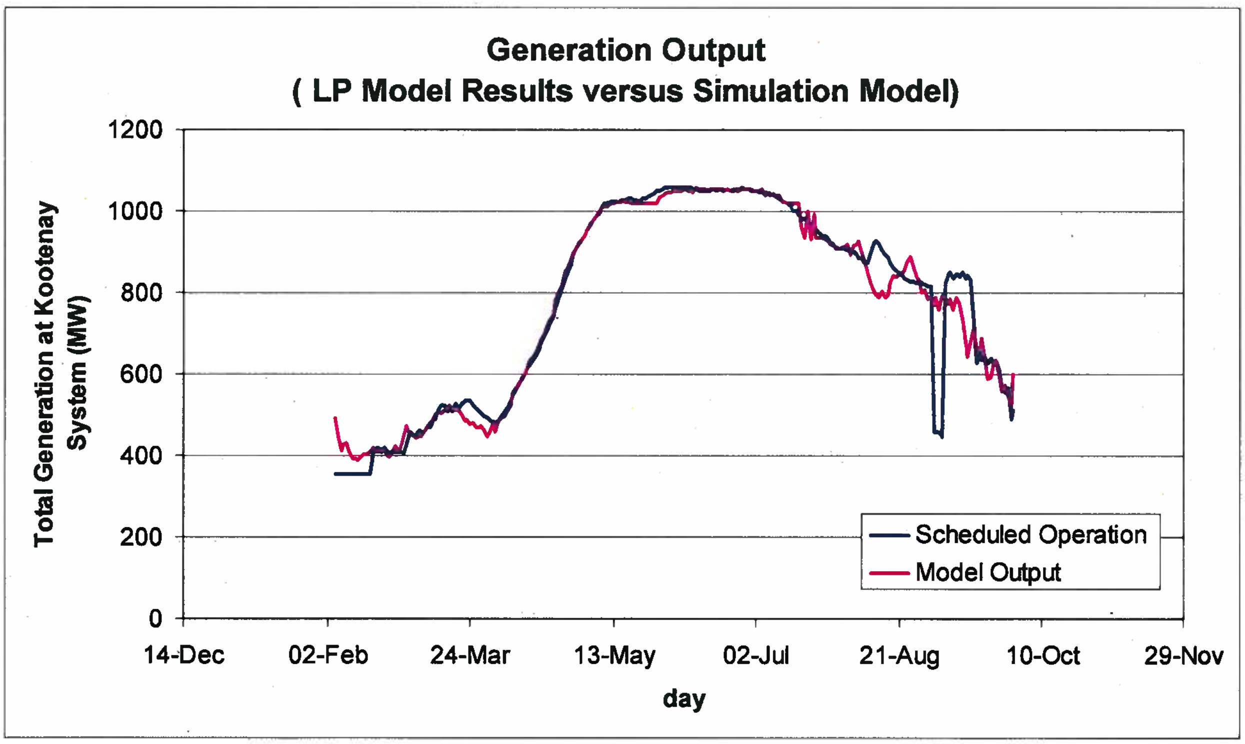


Figure 5.30 Energy MW Generation, Optimization Model vs Simulation Model

The total generation calculated from the scheduled operation for this period was equal to 185,490.1 MWHr.

The total generation calculated from the optimization model for base case was equal to 185,518.9 MWHr. Although the values are relatively close, the lake volume in the last time step for the LRB model was lower than the optimization model. (10,744.4 cmsd versus 11,231.33 cmsd).

With higher power generation in the LP Model and also higher storage value in the last time step, the LP model proved to yield better use of resources for energy production.

5.5 SENSITIVITY ANALYSIS (Base Case)

5.5.1 Sensitivity Analysis Information in STOM [Shawwash 2000]

Sensitivity analysis is known to be one of the major benefits of linear programming. Shadow prices, also known as dual prices or marginal prices, are the most basic outputs of sensitivity analysis. The rate of increase in the optimum objective value per unit increase in the value of the constraint is the shadow price of that constraint. The shadow price of a non-binding constraint is zero, as changing its value does not affect the objective function value. Shadow prices, along with other sensitivity analysis information, are very useful in finding optimal operating policies. They can also be used to compare alternative operating strategies and to determine the cost of limits imposed on the system (e.g., turbine, generation or tie line limits) [8].

5.5.1.1 The Shadow Price of the Generation Production Function.

The plant's incremental cost (in $/MWHr) is the shadow price of the generation production function of a generating facility. The incremental cost (IC) provides information on the cost of increasing production of the facility by one unit. The value of the IC and its permissible range depends on the generation level of the plant as well as the energy conversion rate of the piecewise linear segment. The value of IC may change slightly with variations in the forebay levels as the generation breakpoints in the piecewise linear functions depend on the plant's forebay level. Figure 5.31 illustrates the variation of IC for the Cora Lynn plant.

5.5.1.2 The Shadow Price of the Mass-Balance Equation.

The mass-balance equation in the model defines the hydraulic balance of a reservoir at each timestep. The shadow price of the storage mass-balance equation shows the incremental cost

($/cmsd) associated with per unit changes in the reservoir's storage. The shadow prices are illustrated in Figure 5.32, 5.33 and 5.34.

5.2.2 Sensitivity Analysis for the Kootenay Optimization Model (Base Case)

In the model under study the shadow prices of the generation production function are the same as the energy prices ($/MW). This is because the hydroplants in this study do not have noticeable forebay variations for the study period. Figure 5.31 shows the shadow prices for Cora Lynn plant.

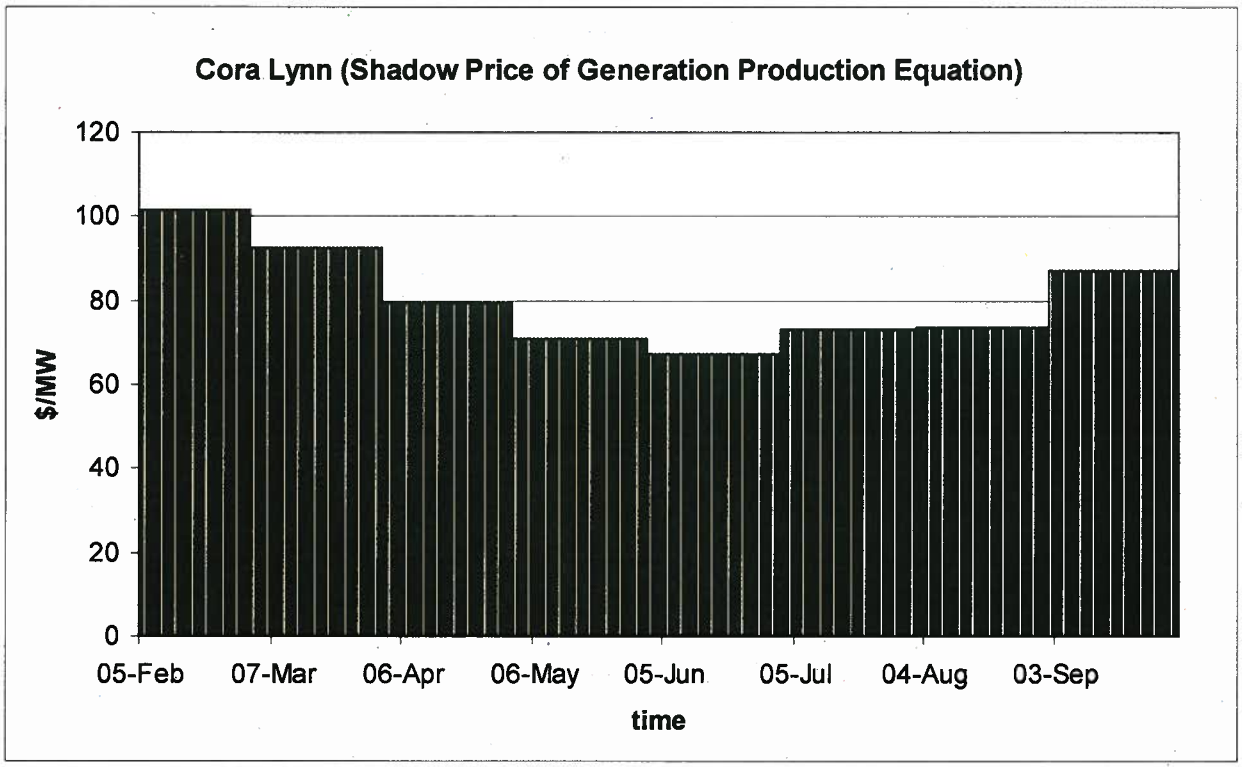


Figure 5.31 Shadow Price of Generation Production Equation for COR

Figure 5.32 shows the shadow price of the "STORAGE" Equation 4.9 in the model for the Kootenay Lake. In this figure the solid black part shows the dual price of the storage equation and the gray shaded bars show the market prices used in this study. It can be observed that the value of water is higher in periods with higher energy prices than in periods with lower energy prices. A good example of this is during the freshet period when the shadow price of the storage is zero. During this time water has no added value to the system since we are continuously spilling it from the reservoirs to comply with the IJC rule curve to avoid potential flood.

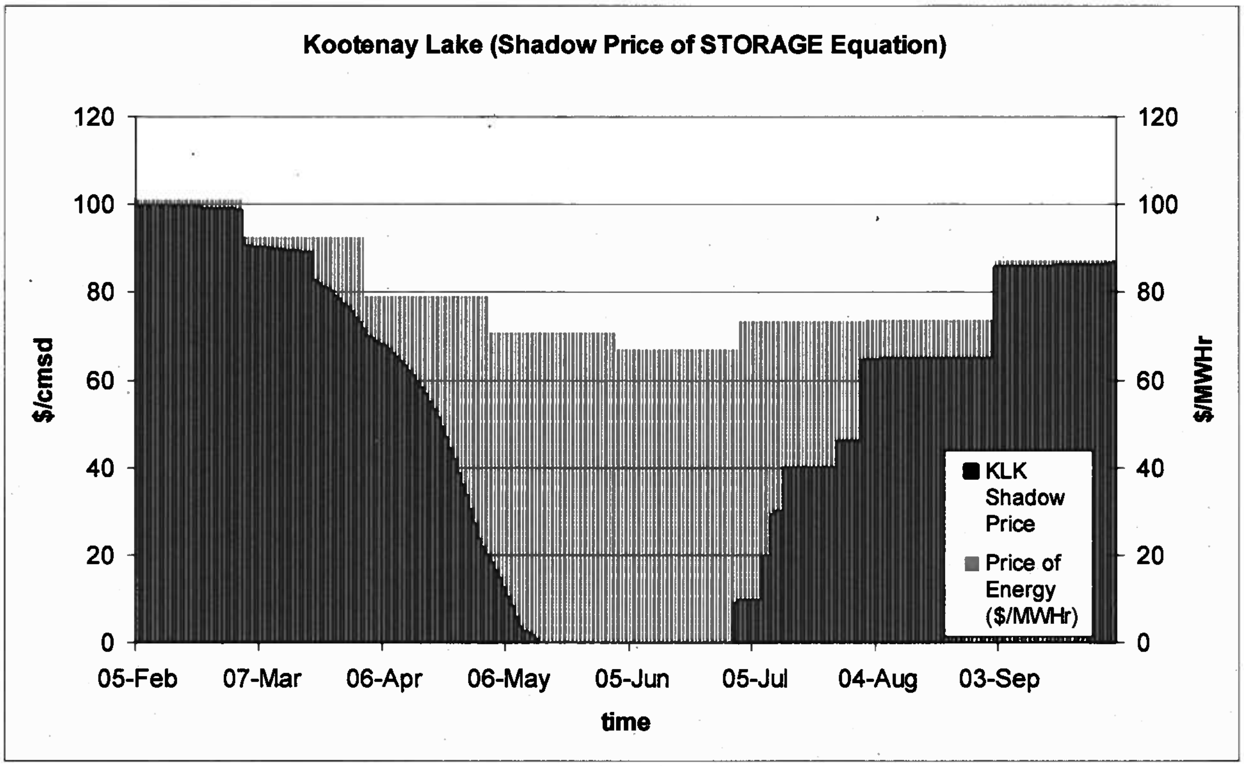


Figure 5.32 Shadow Price of "STORAGE" Equation for the Kootenay Lake.

Figure 5.33 and 5.34 show the shadow prices of STORAGE equation for the COR and Brilliant reservoirs. Dual prices are lower for plants with lower HK values. This is because the amount of generation per cubic meters for a plant with a lower HK is less than those with higher HK values. Similar to the Kootenay Lake shadow prices, the prices for COR and BRD during the high spill period is zero.

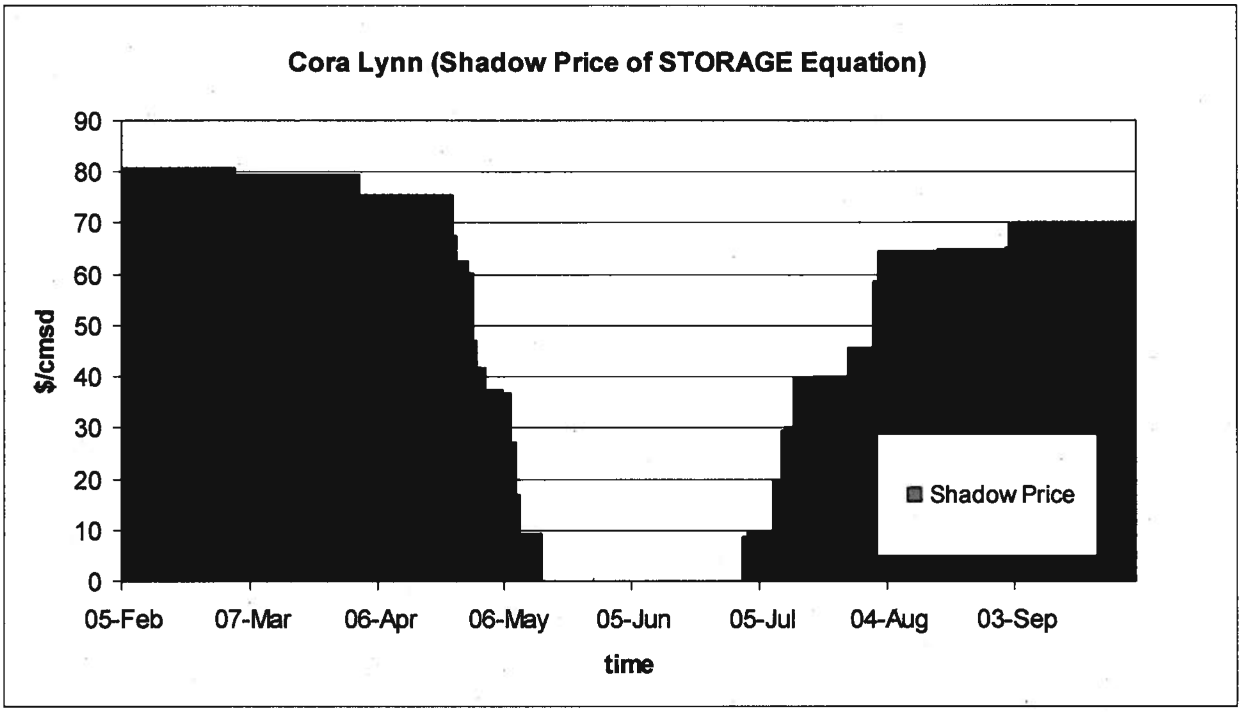


Figure 5.33 Shadow Price of "STORAGE" Equation for COR

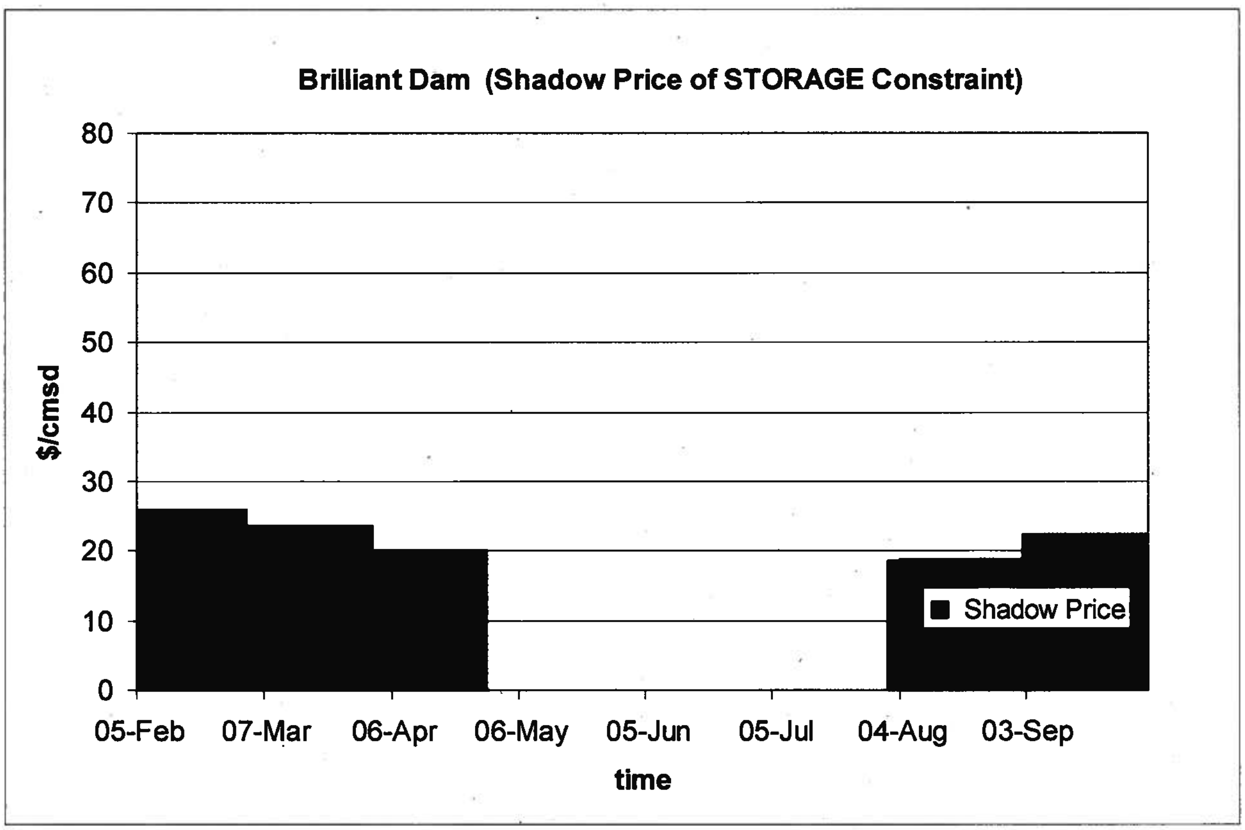


Figure 5.34 Shadow Price of "STORAGE" Equation for BRD

6 CONCLUSIONS AND RECOMMENDATIONS

6.1 CONCLUSIONS

Linear programming optimization models are powerful tools for their ability to efficiently solve large-scale optimization problems. In this research project the goal was to solve an optimization problem for the operation of a hydroelectric system with complex hydraulic configurations and a complex set of rules and procedures that are used to operate the system. The Linear Programming technique have proved to be an efficient tool in dealing with the complicated nature of the set of the Kootenay constraints and rules.

The Kootenay System optimization model is the first optimization model developed in B.C Hydro that deals with the special set of rules and regulations imposed on this system. Several case studies were carried out in this research and it shows that the results of different historical inflow scenarios outperformed those derived by the simulation model currently used for operation of this system. The model developed in this research follows the same context and uses the same structure as other B.C Hydro models (especially STOM) making the future use of this LP model simple to adapt in the future by B.C Hydro. However, this model is very sensitive to certain constraints and model inputs. To apply this model the user should make sure he/she has a good understanding of the system characteristics. When necessary, certain constraints should be added or omitted from the general model structure to keep the results closer to the desired operation. In special cases feasible results can be obtained by relaxing some of the constraints as discussed in the previous chapters. In addition, performing several studies will provide a good background on the Kootenay's specific constraints and on the sensitivity of the model to their variations. It is recommended that input data such as the freshet start date, the initial "*peakday*" guess and the "Nelson constraint" initial guess be provided depending on the inflow characteristics of the year under the study. Good knowledge of the hydro systems in the Kootenay area will help the interpretation of the model's outputs. The key point for the output of an optimization model is that a feasible model run does not guarantee that the results are applicable without review by the system operations engineer. The outputs should be analysed in further detail and appropriate changes to the constraint limits should be made until a satisfactory output is achieved.

6.2 RECOMMENDATIONS FOR FUTURE RESEARCH

Because of the time constraints on this research project, a number of simplified assumptions were made when necessary. The following provides a list of recommended tasks that can be taken to enhance this model.

1- In the developed optimization model, inflows are deterministic and certain, whereas in real operation, the operator deals with uncertain inflows. A "Stochastic Optimization Model" should be developed, possibly using a modified version of the model developed in this research.

2- The operation of the Duncan and Libby Dams and the set of rules and regulations governing their operation should be added to the existing model.

3- When units are shutdown in the Riverplants there are some limitations and costs associated in bringing them back to operation. These costs could be considered in outage studies and implemented in future models.

4- Proper "tail-water functions" for the Riverplants (COR, UBO, LBO, SLC, BRD) should be derived and implemented to more accurately model head impact on generation.

5- In this model a constant head (or FB) values were considered for all Riverplants. Variation in head values makes the model closer to actual operation and future model enhancement should consider head variation in the optimization model.

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APPENDIX A

Table A.1 includes the list of parameters used in this model.

Table A.2 includes the list of input data used in this model.

Table A.3 includes the list of run files used in this model.

Parameters used in this research project include:

- Time parameters, flow parameters, power parameters, reservoir parameters, piecewise- linear curve parameters and load resource balance parameters,
- Kootenay Model special parameters include :

Special time parameters, headloss parameters, IJC curve parameters, Nelson gauge parameter, and natural condition parameters.

There are a number of parameters used in this model and they are listed in Table A.1.

Table A.2 lists the data files used in the model and Table A.3 includes a list of run files used to run the model in AMPL.

Parameters:

Table A.1 Model Parameters

TIME PARAMETERS	
T > 0	Number of production periods in hours
initial	Initial time step in the analysis
start	Start date on which optimization starts
end	End day of optimization model
hours	Number of hours in each timestep
j,t	"j" Index used for plant, "t" index used for timestep
FLOW ETERS	
QIR jt>= 0	Local natural inflow to a reservoir in time step t,in m^3/s
QTMax jt >=0	Upper bounds on turbine flows from a plant in m^3/s
QTMin jt >=0	Lower bounds on turbine flows from a plant in m^3/s
QP_Max jt >= 0	Plant's outflows upper bounds in m^3/s (indexed over time)
QP_Min jt >= 0	Plant's outflows lower bounds in m^3/s (indexed over time)
QTIncr j >=0	Allowable increase (ramping) of turbine discharge, in m^3/s
QTDecr j<=0	Allowable decrease (ramping) of turbine discharge, in m^3/s
QSMax jt >=0	Upper bounds on turbine flows from a plant in m^3/s

QSMin jt >=0	Lower bounds on turbine flows from a plant in m^3/s
STORAGE PARAMETERS	
V max j	Reservoir Storage upper bound
V min j	Reservoir Storage lower bound
POWER PARAMETERS	
P max jt	Upper bound on plant generation at timestep t
P min jt	Lower bound on plant generation at timestep t
HEADLOSS PARAMETERS	
Headloss_calc jt	Headloss from Kootenay Lake to Cora Lynn in ft
Headlossk_calc jt	Headloss from Cora Lynn in ft to Kootenay Canal
"IJC" RULE CURVE PARAMETERS	
freshetstart	Declaration of Spring rise
freshetend	End of Freshet Period
IJCLEVEL t	Upper level constraint on Lake level for all the year
IJCfreshet t	Upper level constraint on Lake level during freshet period
VIJCfreshet t	Lake volume corresponding to IJC level during freshet period
VIJCyear t	Lake Volume corresponding to IJC level for all the year
V_natural t	Natural volume of the lake
FB_natural t	Natural level of the lake
Peakday	The day in which peak of the Lake happens
Freeoff	End of Freefall period
NELSON gauge PARAMETERS	
nelsonstart	Date on which Nelson gauge drops below 1743.32ft and becomes a constraint
DeltaNelson	The difference between Queen's Bay gauge level and Nelson Gauge
Nelsonguage	Lake level measured at Nelson

Table A.2 Model Data Files List

File Name	*Description*
CALCFB.dat	Specifies scheduled forebay levels for *jt*
CALCGMAX/MIN.dat	Specifies maximum/minimum generation for *jt*
CALCQMAX/MIN.dat	Specifies maximum/minimum flow for *jt*
QSMAX/MIN.dat	Specifies maximum/minimum spill for *jt*
Fbtarget.dat	Specifies target values for j at the last time step