Characterizing the Permeability and Dispersion of Flows through Compressible Wood-Chip Beds

by

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Abstract

During Kraft pulping, cellulose fibres are liberated from a lignin-matrix, found in a wood chip, by reaction at elevated temperature and pressure using an alkaline solution. Poor lignin removal is a deleterious effect that leads to downstream operational difficulties and decreased product quality. A number of research groups speculate that this is caused by the uneven distribution of the alkaline solution through the wood chip bed during reaction. As a result, the goal of this thesis is to characterize the ease by which fluid flows, and disperses, through wood chip beds. One of the open remaining scientific questions is understanding the effect of bed compressibility on the resulting flow patterns.

In the first portion of this work we present a methodology to characterize the permeability of a compressible bed of wood chips under mechanical load. We show that under the limiting condition of when the mechanical load is large in comparison to hydraulic pressure the equations of motion can be linearized and solved to produce an expression approximating the variation in porosity along the length of the bed. We show how this may be used, in conjunction with multiple linear regression, to estimate permeability of the bed. The usefulness of these estimates was then tested by predicting the pressure drop versus flow relationship for conditions outside the range of the linearized solution. Good agreement was obtained.

In the second portion of this work we present a methodology to characterize the axial dispersion of a solute during steady-flow through a compressible bed of wood chips under mechanical load. We use a non-invasive imaging technique, namely electrical resistance tomography (ERT), to visualize the uniaxial displacement of a salt solution. Here we demonstrate that under two limiting cases the porosity of the porous bed varies slowly in the flow-direction and to the lowest order can be considered a constant. This simplified the optimization routine we used to match the experimental data to the numerical results of the advection-diffusion equation. Using this, a methodology to estimate the axial dispersion is given by a minimization scheme.

Preface

A version of this thesis have been accepted for publication in The Canadian Journal of Chemical Engineering as two articles;

Alaqqad, M., C.P.J. Bennington and D.M. Martinez, "The Permeability of Wood-Chip Beds: The Effect of Compressibility," Can. J. Chem. Eng. (2011).

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My contributions include:

- Planning the experiments and designing the experimental setup
- Conducting all laboratory work and developing the predictive models
- Performing experimental data analysis
- Being the principal author of the manuscripts

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Nomenclature

Symbols

а	radius of the circular pipe	т
A, B	empirical constants	
a,b	empirical constants	
A_c	area of the compaction	m^2
A_s	surface area of the particle	m^2
$a_{o,1}$	empirical constants	
A _{piston}	area of the piston	m^2
С	concentration of the solute in the fluid	М
D	molecular diffusivity	m^2/s
d_c	diameter of the cylindrical particle	m
D_e	effective diffusivity (dispersion)	m^2/s

d_p	equivalent diameter of the particle	т
<i>d</i> ₃₂	Sauter mean diameter of the particle	т
$dp_{1,2}$	$_{2,3,4}$ pressure drop across four sections of the wood-chip column	Pa
d p _{scr}	een pressure drop across the screen plate	Pa
F	output force of the pneumatic cylinder	Ν
h	height of the cylindrical particle	m
k	permeability	m^2
L	length of the wood-chip bed	т
l	length of the parallelepipedal particle	т
L_o	initial bed length before compressing the cooked wood chips	т
L_p	bed length after compressing the cooked wood chips	т
т	empirical constant of the stiffness of the network	kPa
n	empirical constant	
р	fluid pressure	Pa
p_c	compaction pressure (mechanical load)	kPa
p_h	hydraulic pressure (total pressure drop)	Pa
p_s	solid-stress	Pa

p_{air}	air line pressure	kPa
Pe	Peclet number	
PF	power factor (efficiency) of the pneumatic cylinder	
r	arbitrary radius	m
r^2	correlation coefficient	
r_1	empirical constant of the flow resistance	$Pa.s/m^2$
<i>r</i> ₂	empirical constant of the flow resistance	$Pa.s^2/m^3$
<i>Re_{PM}</i>	Reynolds number for porous media	
S _V	specific surface area of the porous medium	m^2/m^3
Т	temperature of the fluid	$^{\circ}C$
t	arbitrary time	S
t	thickness of the parallelepipedal particle	m
и	local velocity	m/s
u _o	centreline velocity	m/s
v	superficial velocity	m/s
V_p	volume of the particle	m^3
w	width of the parallelepipedal particle	т

Subscripts		
Ζ	distance over which the image is averaged in the z-coordinate	т
<i>y</i> , <i>z</i>	coordinates in each individual ERT plane	
Y	distance over which the image is averaged in the y-coordinate	т
Y	Yield of the cook	
x	arbitrary vertical length	т

e exper	rimental
- 1	

- $_f$ final
- *i* initial

Greek Letters

β	empirical	constant

- δ dimensionless group
- ε porosity
- ε_c characteristic porosity
- ε_g gel porosity
- ε_{chip} volume fraction of the cooked wood chips
- γ dimensionless group

К	kappa number	
μ	dynamic viscosity of the fluid	Pa.s
ψ	sphericity shape factor	
ρ	density of the fluid	kg/m^3
${oldsymbol{ ho}_b^o}$	initial bulk packing density of the cooked wood chips	kg/m^3
$ ho_b^p$	bulk packing density of the cooked wood chips under compression	kg/m^3
$ ho_{chip}$	basic density of the uncooked wood chips	kg/m^3
$ ho_{chip}^{cook}$	basic density of the cooked wood chips	kg/m^3
σ	electrical conductivity of NaCl brine solution	mS/cm

 θ shape factor constant usually assigned the value of 5.5

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Dedication

To My Great Parents;

Omar Alaqqad, and Laila Alaqqad

And

My Lovely Wife;

Najat

And

My Brothers;

Emad, Iyad, and Alaa

And

My Friends, and Professors

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Chapter 1

Introduction

The focus of the present work is to develop a methodology to characterize the permeability and dispersion of a solute in a compressible porous medium. Although the goals of this work have relevance to a number of different scientific and industrial settings, such as the flow of groundwater through soil, this thesis, however, is motivated from one unit operation in the Kraft process in the pulp and paper industry, namely the chemical digester. Here, wood chips are cooked to remove lignin in large vessels typically 3 - 8 m in diameter and 25 - 60 m in height. The reaction solution is usually referred to as "white liquor", which is an aqueous solution of sodium hydroxide (*NaOH*) and sodium sulfide (*Na*₂*S*) and is in contact with the wood chips at approximately $170 \degree C$, $600 \ kPa$, for approximately 4 hours. To ensure good thermal and mass transfer to the wood chips during reaction, process liquors are circulated through the digester after being heated in an external heat exchanger (Smook, 1992). To achieve a high quality product, with small spatial variations in lignin content, the process demands that each chip receives equivalent treatment. In other words, liquor flow should be maintained uniform and at an optimal level so that the reactants can be supplied to the chip and that the bi-products removed. Unfortunately, this is not the case in most digesters as indications of non-uniformity, e.e. accelerated corrosion in the digester (Kiessling, 1995; Wensley, 1996), fluctuations in the exit kappa number, a measure of the extent of reaction (He *et al*, 1999), and temperature gradients (Pageau and Marcoccia, 2001) are evident. It is thought that spatial variations in the liquor flows result from chip softening as the reaction progresses. Chip softening results in a local compaction of the bed and the subsequent decrease in porosity limits penetration of the white liquor. This phenomenon is magnified as chip softening may cause a change in particle size distribution; this may also cause a local reduction in porosity (Lee, 2002). Characterizing this effect is one of the open remaining questions in this field and it is the primary scientific question addressed in this thesis.

The motivation and the major findings of these two studies in this thesis are presented in three chapters. In Chapter 2, we present a concise summary of the literature review in order to set our results in the context of other work. Here a description of the Kraft pulping process as well as a number of studies relating the behavior of wood chips-liquor interaction is given. A brief description of our main experimental tool, Electrical Resistance Tomography (ERT), is presented here as well. In Chapters 3 and 4, we present the methodology and the results for the measurement of permeability and axial dispersion in a packed column of wood chips under mechanical load.

Chapter 2

Literature Review

2.1 Background

Chemical pulping of wood chips is a method of pulp manufacture in which wood chips are treated with aqueous alkaline or acidic solutions. The aim of chemical pulping is to remove enough lignin that binds the wood fibers together, to give them the required characteristics for their derived use, and to do so at the lowest possible cost. Accordingly, the liberated fibers can now be used to produce paper. Kraft pulping is today the predominant process for producing chemical pulp. In Canada, 80% of all the chemical pulp produced is by this process while 85% of the worldwide chemical pulp is produced via this process. The chemical pulping process can be accomplished with either batch or continuous digesters.

Kraft pulping is a chemical process used to delignify wood chips by an aqueous solution of NaOH and Na_2S , referred to as "white liquor", at a high temperature reaching $170 \,^{\circ}C$ and a high pressure reaching $600 \, kPa$ for approximately 4 hours to produce pulp. Kappa number κ is used as a measure of the lignin content and decreases as lignin is removed as reaction proceeds. It is noteworthy that cooked chips have similar dimensions as the original uncooked chips except that cooked chips has less lignin content (lower kappa number) depending on the degree of cooking. To ensure good thermal and mass transfer to the wood chips during reaction, process liquors are circulated through the digester after being heated in an external heat exchanger.

Batch digesters range from 70 m^3 to 340 m^3 capacity, with a standard capacity of 170 m^3 to 230 m^3 for most modern mills. In batch cooking, the digester is filled with wood chips and enough white liquor is added to cover the chips. The contents are then heated to a predetermined schedule, usually by forced circulation of the cooking liquor through an external heat exchanger, as shown in Figure 2.1. Air and non-condensible gases generated during cooking are relieved through a pressure control valve at the top of the vessel. The maximum temperature, usually about 170 °*C*, is typically reached after 1.0-1.5 hours which allows for the cooking liquor to impregnate the chips. The cook is then maintained at the maximum temperature for up to 2 hours to complete the cooking reactions. After digestion, the contents are discharged into a blow tank where the softened chips are disintegrated into fibers; the off-vapors are condensed in a heat exchanger where water is heated for pulp washing (Smook, 1992).

Continuous digesters ranges from 6-9 m in diameter and 60-70 m in height with a typical capacity of 1000 tons per day of pulp (oven-dry basis). Most con-

tinuous digesters consist of three basic zones; an impregnation zone, one or more cooking zones, and a wash zone. A typical one-vessel Kamyr digester is shown in Figure 2.2. In continuous cooking, the wood chips are first carried through a steaming vessel where air and non-condensible gases are purged. The preheated chips and white liquor then enter the continuous digester where they move through the impregnation zone of an intermediate temperature (115 - 120 °C) to allow for uniform penetration of the chemical into the chips. As the chip mass moves through the cooking vessel, it enters the cooking zone where the mixture is heated to the cooking temperature, either by forced circulation of the cooking liquor through an external heat exchanger or by steam injection, and maintained at this temperature for up to 2 hours. Following completion of the cook, the chip mass enters the washing zone where hot spent liquor is extracted into a low-pressure tank where flash steam is generated for use in the steaming vessel. The pulp is usually quenched to below 100 °C with cool liquor to prevent mechanical damage to the fibers (Smook, 1992).

Over the past decades some significant modifications to conventional batch and continuous cooking in Kraft pulping have been developed, and these improved methods and technologies are rapidly being incorporated into new installations and retrofitted into older mills. The new methodology involves additional equipment and processing steps, but doesn't extend the cooking cycle (Smook, 1992). Current Kraft operating strategies rely on contacting the chips with liquors of different temperatures and compositions during cooking to optimize pulp quality and strength. This requires creation of uniform liquor flow throughout the chip column. Local compaction play a key role in this process due to the presence of interstitial spaces between the chips producing non-uniformity in the liquor flow. Indications of non-uniformity include accelerated corrosion in the digester (Kiessling, 1995; Wensley, 1996), fluctuations in the exit kappa number (He *et al*, 1999) and circumferential temperature gradients in the digester (Pageau and Marcoccia, 2001).

A number of researchers have measured the compressibility and flow resistance of wood-chip columns. He *et al* (1999) developed a 3-D coupled twophase computer model of a continuoss digester simulating the flow distribution and delignification process. Harkonen (1987) developed a 2-D flow model of the digester using an Eulerian volume-averaged two-phase flow approach to account for interactions between the liquor and chip phases and measured pressure drop in support of his modeling. Other researchers, including Lindqvist (1994); Lammi (1996); Wang and Gullichsen (1999); Lee (2002) have experimentally measured pressure drop in wood-chip beds as a function of flow velocity and void fraction for different chip furnishes and size distributions.



Figure 2.1: Schematic of a typical batch digester (Smook, 1992).



Figure 2.2: Schematic of a typical one vessel Kamyr digester (Luis *et al*, 2000).

2.2 Flow Resistance through Porous Media

Before we discuss the pertinent models relevant to flow through this class of material, it is instructive to first define the two parameters that will be used to characterize the resistance to flow in this work, i.e. the permeability k and the solid-stress p_s . We begin this discussion with permeability. Traditionally permeability is defined using Darcy's law

$$\frac{dp}{dx} = \frac{\mu}{k}v\tag{2.1}$$

which relates the change in pressure drop per unit length dp/dx required to maintain a superficial velocity v through a porous medium. The dynamic viscosity of the fluid is defined as μ . There is no mechanistic justification of the form of Equation 2.1 except for empirical evidence demonstrating its utility for most media under creeping flow conditions ($v \rightarrow 0$).

There are various experimental methods for determining k; see for example the summary given by Scheidegger (1957) or the references given therein. For incompressible media, these methods simply involve the measurement of the superficial flow rate of the fluid for a given pressure drop and comparing these data to Equation 2.1. It is well known that permeability is a function of the porosity ε and the most common expression outlining this dependency is given by the Kozeny-Carman relationship (Scheidegger, 1957), i.e.

$$k = \frac{\varepsilon^3}{\theta s_{\nu}^2 (1 - \varepsilon)^2} \tag{2.2}$$

where s_{ν} is the specific surface area of the particles; and θ is a constant usually ranges from 4.7 – 5.5. There are many other functional forms of permeability that have been developed for various ranges of volume fraction, particle geometry and network architecture. These models have been studied both theoretically and experimentally quite extensively (Jackson and James, 1986). Beyond this creeping flow limit, a number of authors have extended Darcy's law and advanced an equation of the form

$$\frac{dp}{dx} = \frac{\mu}{k}v + \frac{\beta}{\sqrt{k\varepsilon^{\frac{3}{2}}}}\rho v^2$$
(2.3)

where β is an empirical constant that must be determined from experiment, and ρ is the density of the fluid. This form of relationship is routinely found in a number of works and is generally referred to as the either the Ergun or Forchheimer relationship. In Equation 2.3, a characteristic dimension should be used to represent the particle size. Generally, the characteristic dimension is the diameter of a sphere having the same specific surface as the particle (Ergun, 1952; Comiti and Renaud, 1989; Niven, 2002), expressed as

$$s_{\nu} = \frac{A_s}{V_p} = \frac{\pi d_p^2}{\frac{1}{6}\pi d_p^3} = \frac{6}{d_p}$$
(2.4)

where A_s is the surface area of the particle, V_p is the volume of the particle, and d_p is the equivalent particle diameter. Substituting Equations 2.2 and 2.4 into Equation 2.3 results in the famous Ergun (1952) Equation, expressed as

$$\frac{dp}{dx} = A \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{\mu}{\psi^2 d_p^2} v + B \frac{(1-\varepsilon)}{\varepsilon^3} \frac{\rho}{\psi d_p} v^2$$
(2.5)

where *A* and *B* are empirical constants that must be determined experimentally, ψ is the sphericity shape factor. Sphericity is a measure of how spherical (round) an object is and it is defined as the ratio of the surface area of a sphere (with the same volume as the given particle) to the surface area of the particle, expressed as

$$\psi = \frac{\pi^{1/3} (6V_p)^{2/3}}{A_s} \tag{2.6}$$

It is noteworthy that A is dominant for laminar flows ($Re_{PM} \le 10$), and B is dominant for turbulent flows ($Re_{PM} \ge 1000$) where Re_{PM} is the modified Reynolds number for porous media, expressed as

$$Re_{PM} = \frac{d_p \rho v}{\mu (1 - \varepsilon)} \tag{2.7}$$

Values of *A* and *B* have been measured for several particles where it was found that they are functions of particle geometry and roughness (Ergun, 1952; Leva, 1959; MacDonald *et al*, 1979; Comiti and Renaud, 1989). A summary of the different studies, including the type and shape of the particles, is given in Table 2.1.

We now continue with the discussion of the second parameter, the solid-stress relationship $p_s(\varepsilon)$. The solid-stress is a property created by the connectivity and corresponding friction between the particles comprising the bed. If the frictional forces are sufficiently large, the bed may support an external load. If the load exceeds the compressive strength of the bed, the bed irreversibly densifies through a reduction of the void volume. The relationship between the applied pressure
Source	Material	Porosity	Empir	ical Parameters
		ε	A	В
Ergun (1952)	Spheres	_	150	1.75
Leva (1959)	Spheres	—	200	1.75
MacDonald et al (1979)	Spheres	-	180	1.8 - 4.0
Comiti and Renaud (1989)	Spheres, $d_p = 1.12 mm$	0.36	140	1.68
Comiti and Renaud (1989)	Spheres, $d_p = 4.99 mm$	0.36	142	1.59
Comiti and Renaud (1989)	Cylinders, $h/d_c = 5.49$	0.39	166	3.20
Comiti and Renaud (1989)	Plates, $t/l = 0.102$	0.46	216	12.2
Comiti and Renaud (1989)	Plates, $t/l = 0.209$	0.35	161	6.69

Table 2.1: Literature values for the flow resistance parameters *A* and *B* as defined by Equation 2.5.

and the void volume is defined as the solid-stress or compressibility function. This relationship is an implicit function of the particle geometry, packing and the orientation distributions (Toll, 1998). The solid-stress function usually obeys an empirical relationship of the form

$$p_s = f_3(\varepsilon) = m(\varepsilon_g - \varepsilon)^n \quad (m, n \ge 0)$$
(2.8)

where ε_g is the gel porosity, i.e. the porosity at which the network may first support an external load; and *m* and *n* are empirical constants that must be determined from experiment where *m* is the stiffness of the network. For a more detailed discussion of this function the reader is encouraged to examine the earlier works of Wilder (1960), Wrist (1964), Han (1969) or the more recent works by Vomhoff (1998) and Pettersson *et al* (2008).

For the particular case of flow through wood-chip beds, there are a limited

number of studies in which the permeability and compressibility functions are reported. As before, we shall report the values for permeability first and then move onto the compressibility function. Strictly speaking, the permeability for this material is never reported. Instead, we find that Equation 2.3 or or 2.5 is recast into the form

$$\frac{dp}{dx} = r_1 f_1(\varepsilon) v + r_2 f_2(\varepsilon) v^2$$
(2.9)

$$f_1(\varepsilon) = \frac{(1-\varepsilon)^2}{\varepsilon^3}$$
(2.10)

$$f_2(\varepsilon) = \frac{(1-\varepsilon)}{\varepsilon^3}$$
 (2.11)

through substitution of Equation 2.2 into Equation 2.3 and the parameters r_1 and r_2 are considered to be the flow resistance of the medium; they are determined by regression of a data set representing $(v_i, dp_i/dx)$. Physically, r_1 and r_2 represent the lumped effects of tortuosity, specific surface, and density and dynamic viscosity of the fluid, i.e.

$$r_1 = \mu \theta s_v^2 = A \frac{\mu}{\psi^2 d_p^2}$$
(2.12)

$$r_2 = \beta \rho \sqrt{\theta} s_v = B \frac{\rho}{\psi d_p}$$
(2.13)

A summary of the different studies, including the type of wood chips and degree of deliginification, is given in Table 2.2. As shown the data set is highly variable and the uncertainty in the estimate, as determined by the correlation coefficient, is large. It is interesting to note that for some estimates, the inertial component of the Ergun relationship, i.e. the second term in this relationship, is reported as a negative value. This is physically impossible. Thus, for these negative cases, we may argue that r_2 should not be different from zero due to the operation at near laminar regimes in such cases since Re_{PM} for flows through wood-chip beds usually lies within $50 \le Re_{PM} \le 2000$.

At this point, we attempt to gain insight into the problem by examining the literature for papermaking fibre suspensions. We do so as the wood chip is the precursor of the fibre suspension. For papermaking suspensions a fairly crude estimate of the permeability is of the order $k \sim 10^{-12} m^2$. We define this as a crude estimate as there is a vast literature with estimates of the permeability as a function of porosity for various fibre and filler particle or refining treatments,see *e.g.* Robetrson and Mason (1949), Ingmanson and Whitney (1954) or Lindsay and his co-workers (Lindsay, 1990; Lindsay and Brady, 1993a,b). We anticipate that the permeability of the wood chip to be larger than this value. In addition we find that at low superficial velocities, Robetrson and Mason (1949) demonstrate a linear relationship between pressure drop and superficial velocity and that a Kozeny-Carman type relationship described the data quite well. At higher velocities, Ingmanson and Whitney (1954) indicate deviations from this type of behavior and inertial effects are evident.

We now turn our attention to the characterization of the compressibility function describing wood-chip beds. In this literature, water saturated wood chips were subjected to a static uniaxial load from which the displacement of the bed was measured. All researchers report compressibility behavior of the same form

Source	Material	Empirical Parame		Corr. Coeff.
		r_1	r_2	r^2
		$(Pa.s/m^2)$	$Pa.s^2/m^3$	
Harkonen (1987)	Scandinavian Pine (distribution not specified)	0.05×10^{5}	3.9×10^{6}	_
Lindqvist (1994)	Scandinavian Pine + 6 mm slot 22.1% + 4 mm slot 44.2% + 2 mm slot 29.1% + 3 mm hole 4.6%	0.28×10^{5}	-0.13×10^{6}	0.36
Lindqvist (1994)	Scandinavian Pine + 6 mm slot 23.2% + 4 mm slot 46.3% + 2 mm slot 30.5%	0.51×10^{5}	-0.35×10^{6}	0.41
Lindqvist (1994)	Scandinavian Pine + $6 mm$ slot 60.3% + $4 mm$ slot 39.7%	0.06×10^{5}	-0.01×10^{6}	0.58
Lammi (1996)	Scandinavian Birch $+ 45 mm$ hole 0.6% $+ 8 mm$ slot 9.2% $+ 13 mm$ hole 51.2% $+ 7 mm$ hole 31.2% $+ 3 mm$ hole 6.4% $+$ fines 1.1%	0.68×10^{5}	2.47×10^{6}	0.89
Lammi (1996)	Eucalyptus Camaldulensis + 45 mm hole 1.1% + 8 mm slot 4.8% + 13 mm hole 78.8% + 7 mm hole 12.6% + 3 mm hole 2.2% + fines 0.5%	1.85×10^{5}	-0.03×10^{6}	0.91
Wang and Gullichsen (1999)	Scandinavian Pine + 7 <i>mm</i> hole 100%	$0.52 imes 10^5$	1.50×10^{6}	-
Wang and Gullichsen (1999)	Scandinavian Pine 4 <i>mm</i> thick, 40 <i>mm</i> length	0.82×10^5	-0.11×10^{6}	-
Lee (2002)	White Spruce + 3 mm hole 100%	0.11×10^5	13.78×10^{6}	0.76
Lee (2002)	White Spruce + 7 <i>mm</i> hole 75% + 3 <i>mm</i> hole 25%	0.66×10^{5}	4.72×10^{6}	0.82
Lee (2002)	White Spruce + 7 <i>mm</i> hole 87.5% + 3 <i>mm</i> hole 12.5%	0.60×10^{5}	4.57×10^{6}	0.74
Lee (2002)	White Spruce + 7 <i>mm</i> hole 100%	0.40×10^{5}	5.77×10^{6}	0.91

Table 2.2: Literature values for the flow resistance parameters r_1 and r_2 as defined by Equation 2.9. The uncertainty in the estimates are given by the correlation coefficients r^2 , if available. "+" means retained on the plate.

Source	Material	$m(\kappa)$		ϵ_{g}	п
		a_0	a_1		
		(kPa)	(kPa)		
Harkonen (1987)	Scandinavian Pine	$2.14 imes 10^{-1}$	3.57×10^{-2}	0.64	1.7
Lindqvist (1994)	Scandinavian Pine	$2.24 imes10^{-1}$	$4.48 imes 10^{-2}$	0.60	1.6
Lindqvist (1994)	Scandinavian Pine	$1.48 imes 10^{-1}$	$2.94 imes 10^{-2}$	0.62	1.3
Lindqvist (1994)	Scandinavian Pine	$1.86 imes10^{-1}$	$3.73 imes 10^{-2}$	0.65	1.4
Lammi (1996)	Scandinavian Birch	$1.78 imes10^{-1}$	$4.08 imes 10^{-2}$	0.59	1.8
Lammi (1996)	Eucalyptus Camald.	1.60×10^{-1}	3.46×10^{-2}	0.63	1.6
Wang and Gullichsen (1999)	Scandinavian Pine	$2.17 imes10^{-1}$	3.66×10^{-2}	0.66	1.8
Wang and Gullichsen (1999)	Scandinavian Pine	4.45×10^{-1}	6.56×10^{-2}	0.84	2.6

Table 2.3: Literature values for the compressibility parameters m, ε_g , and n as defined in Equation 2.8. These research groups indicate that m is a function of the degree of deliginification, i.e. the kappa number κ , and advance the equation $m(\kappa) = (a_o - a_1 \ln \kappa)^{-n}$.

as given by Equation 2.8. Here we find that the exponent *n* varies between 1.3 and 2.6 and that *m* varies according to the degree of deliginification of the chips (Harkonen, 1987; Lindqvist, 1994; Lammi, 1996; Wang and Gullichsen, 1999; Lee, 2002). There is some disagreement over the form of the gel point ε_g . Most authors in this literature indicate that this is simply the average sediment concentration of the wood chip pile; Lee (2002) indicates that this parameter varies also as a function of degree of deligninfication but with a function form slightly different than the body of work in the literature. A summary of the empirical models used to determine these parameters is given in Table 2.3.

2.3 Axial Dispersion through Porous Media

Dispersion is a term commonly found in the literature but its definition, especially for flow in porous media, is less clear. Before we discuss the pertinent literature relevant to dispersion in wood-chip beds, it is instructive to define this term for the simplest case of the behavior of a diffusing species in uniform Poiseuille flow in a uniform circular capillary tube. In a series of classic papers, Taylor (Taylor, 1953, 1954a,b) demonstrates that a simple shear flow can surprisingly increase the effective diffusivity. Shear, in this case, acts to smear out the concentration distribution in the direction of the flow, enhancing the rate at which the solute spreads in that direction. To highlight this work, Taylor considered fully-developed Hagen-Pouiselle flow in a circular pipe of radius *a*, i.e.

$$u(r) = u_o \left(1 - \left(\frac{r}{a}\right)^2 \right)$$
 (2.14)

where u and u_o are the local and centreline velocities. If the concentration of the diffusing species is denoted by c and its molecular diffusivity by D, the evolution of the concentration distribution in the pipe is governed by a linear advection-diffusion equation of the form

$$\frac{\partial c}{\partial t} + u(r)\nabla c = \nabla (D\nabla c)$$
(2.15)

In this classic analysis, Taylor re-expressed the concentration and velocity fields as the sum of a cross-sectional average (indicated by an overbar) and a deviation (indicated by a prime)

$$u(r) = \bar{u} + u'(r) \tag{2.16}$$

$$c(r,x) = \bar{c} + c'(r,x)$$
 (2.17)

and assumed that both $c' \ll \bar{c}$ and that the length scale of axial variation is much greater than *a*. By doing so, he shows that the advection-diffusion equation may be recast into the form

$$\frac{\partial \bar{c}}{\partial t} + \bar{u}\frac{\partial \bar{c}}{\partial x} = D_e \frac{\partial^2 \bar{c}}{\partial x^2}$$
(2.18)

where D_e is the effective diffusivity i.e. dispersion defined as

$$D_e = D\left(1 + \frac{1}{192} \left(\frac{2a\bar{u}}{D}\right)^2\right) \tag{2.19}$$

Taylor's analysis indicates that the effective diffusivity increases with the square of the average flowrate in the tube.

Since this classic work, a vast body of literature has been developed on the subject of dispersion, especially in porous media. It seems to be a reasonably well-established fact that the extent of mixing between the solvent and solute can be characterized by an advection-diffusion-type equation in which an axial dispersion coefficient appears in place of the usual molecular diffusivity (Danckwerts, 1953; Brenner, 1962; Levenspiel and Bischoff, 1963). The physics of the enhancement of diffusion is governed by a stochastic process arising from the subtle interplay

of advective velocity gradients, molecular diffusion, boundary layer effects, and tortuosity (Khrapitchev and Callagha, 2003).

There are various experimental methods for determining D_e . These methods simply involve the measurement of the concentration of a solute tracer as a function of time and comparing these data to analytical or numerical solutions of an advection-diffusion model. Measurements and correlations of axial dispersion in packed beds using gaseous or liquid solvents at ambient conditions have been extensively studied and reviewed by many researchers (Chung and Wen, 1968; Edwards and Richardson, 1968; Kehinde *et al*, 1983; Gunn, 1987; Tsotsas and Schlunder, 1988). As an example of this, for a brine solution flowing through a packed bed of glass spheres, it is reported in the review by Delgado (2006), that the effective diffusivity is related to the porosity ε of the bed according to

$$D_e = a\varepsilon^b \tag{2.20}$$

where *a* and *b* are empirical constants determined by experiment. For packed beds of glass spheres, Rumer (1962) reports values of *a* and *b* to be $2.7 \times 10^{-3} m^2/s$ and -1.11, respectively while Harleman and Rumer (1963) report values of $1.6 \times 10^{-4} m^2/s$ and -1.18, respectively. For the particular case of flow through woodchip beds in the pulp and paper industry, there are few, if any, studies in which the axial dispersion is reported (Michelsen and Foss, 1996). The only work that we could find directly related to this topic was that reported by Hradil *et al* (1993) for biomass conversion of wood chips into ethanol. The axial dispersion coefficient in such beds was found to be in the range of $D_e = 10^{-3} - 10^{-6} m^2/s$.

At this point, we attempt to gain insight into estimates of the diffusivity by examining the related literature, i.e. for dispersion through wood pulp fibre mats. We do so as the wood chip is the precursor of the pulp fibre. In this case, we find that the axial dispersion coefficient is of the order $D_e \sim 10^{-6} m^2/s$. We consider this as a crude estimate as there is a vast literature with estimates of the axial dispersion coefficient as a function of porosity and velocity for various fibre and filler particle or washing treatments, see *e.g.* Poirier *et al* (1988), Arora *et al* (2006) or Potucek and Miklik (2010).

2.4 Electrical Resistance Tomography

There are a number of experimental techniques currently available to visualize the motion of a fluid in a dense, opaque porous medium: X – or γ ray attenuation, acoustics, or nuclear magnetic resonance. In this work we use electrical resistance tomography (ERT), a methodology which has been recently applied to a number of engineering studies (Williams and Beck, 1995; Lucas *et al*, 1999; Bolton *et al*, 2004; Vlaev and Bennington, 2004, 2005, 2006; ITS, 2006). Recently, electrical resistance tomography (ERT) has gained increasing popularity due to its applications in the field of chemical and process engineering. ERT is a non-invasive technique that employs electrical measurements in order to obtain information about the contents of process vessels and pipelines such as concentration, temperature, velocity, and local void fraction in the measurement plane. Indeed, a number of researchers have used ERT successfully to image flow in packed beds (Lucas *et al*, 1999; Bolton *et al*, 2004). Any tomography imaging system that is based on electrical fields is called electrical tomography or soft field systems due to the sensitivity of measurement in the region of interest.

ERT has two main advantages. First, measurements are conducted non-invasively without disturbing the system. Second, the region of interest is captured along the periphery of the measurement plane achieving an accurate representation of the cross sectional area of that region of interest. It is noteworthy that such imaging techniques have fast data processing capabilities and have low cost compared to other hard field imaging techniques such as computed axial tomography (CAT Scan).

ERT images the spatial difference in conductivity through the region of interest using the tomographic sensors and data acquisition system. Accordingly, two-dimensional cross-sectional images of the region of interest are produced in real time by linear back projection algorithm used for image reconstruction. The ITS (Industrial Tomography Systems, Manchester, UK) P2000 system has eight sensor planes, each having 16 non-intrusive stainless steel electrodes located equidistantly around the vessel periphery (every 22.5°), enabling the region of interest to be measured as it is illustrated in Figures 2.3 and 2.4. Basically, ITS P2000 system injects current between a pair of adjacent electrodes and measures the resultant voltage difference between the remaining adjacent electrode pairs according to a pre-defined measurement protocol (ITS, 2006), see Figures 2.5a and 2.5b.



Figure 2.3: Photograph of ITS P2000 ERT data acquisition system (ITS, 2006).



Figure 2.4: Schematic diagram of electrode arrangement around the process vessel (ITS, 2006).



Figure 2.5: Adjacent current injection and resultant voltage measurement of the remaining adjacent electrodes. 2.5a The first current injection. 2.5b The second current injection (ITS, 2006).

The accuracy and repeatability of the voltage measurements made using the ERT system are $\pm 0.5\%$ and > 99.5%, respectively (ITS, 2006). Each plane is divided into a square 20 × 20 grid of pixels representing the interior cross-section of the vessel, for which some pixels lie on the outside of the vessel circumference; therefore, the reconstructed image is obtained from 316 of the 400 pixels, see Figure 2.6. It is noted that ERT has an excellent temporal resolution but achieves a spatial resolution of only 5 - 10% of the vessel diameter (Williams and Beck, 1995). Image reconstruction in each plane could also be distorted due to the three-dimensional nature of the electric field, which is not accounted for in the reconstruction of the two dimensional planar images. Despite these limitations, ERT has been successfully used to study flow in a wide range of applications including liquor flow in scale-model continuous digesters by Vlaev and Bennington



Figure 2.6: ERT image reconstruction grid (ITS, 2006).

(2004, 2005, 2006).

2.5 Research Objectives

The literature shows that permeability and dispersion have been widely studied for a wide range of chemical engineering systems. This class of problem has received considerable attention in the pulp and paper literature as permeability and dispersion have been related to the efficiency of a number of different unit operations, and are of great importance for flow through chip beds and consequently for digester design. However, very limited work has been published on permeability and dispersion through wood-chip columns and for most reported measurements compressibility effects have not been rigorously treated in these analyses (Harkonen, 1987; Hradil *et al*, 1993; Lindqvist, 1994; Lammi, 1996; He *et al*, 1999; Wang and Gullichsen, 1999; Lee, 2002). For the particular case of flow through wood-chip beds in the pulp and paper industry, there are few, if any, studies in which the axial dispersion is reported (Michelsen and Foss, 1996). The only work that we could find directly related to this topic was that reported by Hradil *et al* (1993) for biomass conversion of wood chips into ethanol. In other words, dispersion should be characterized intensively in wood-chip beds while visualizing the flow in these beds.

The objectives of this thesis are:

- To improve the understanding of flow resistance through compressible woodchip beds, and check the applicability of the Ergun Equation to describe the flow resistance in these beds.
- To develop a procedure to estimate permeability of compressible wood-chip beds in which compressibility effects are treated rigourously.
- To visualize the flow and investigate the flow uniformity by using electrical resistance tomography (ERT) while using salt as a passive scalar.
- To characterize axial dispersion and mixing through compressible woodchip beds.

Chapter 3

The Permeability of Wood-Chip Beds: The Effect of Compressibility

3.1 Introduction

In this chapter, the resistance to flow of a Newtonian fluid through a compressible porous bed of wood-chips under axial load is investigated experimentally. The essential difficulty of characterizing the behavior of this material is that densification of the chip-bed occurs by both the application of an external mechanical force as well as by a hydraulic force created by the motion of the fluid. This sets itself apart from the traditional flow through incompressible porous medium studies as the additional complication of wood-chip bed deformation is introduced into the problem description. Compressible in this case indicates that the wood-chip bed densifies by the reduction of interparticle volume as opposed to particle deformation. The reduction of interparticle volume may be associated with a change in particle orientation. We focus our efforts on the industrially-relevant case of understanding the flow in wood chips delignified to various kappa levels and estimate both the permeability and compressibility of these beds.

This chapter is organized into three subsections. In §3.2, we present the methodology to estimate the permeability relationship. Here we present an alternative solution method to the governing equations, which have been posed previously; this allows for a simpler means to estimate r_1 and r_2 through linear regression. In §3.3, we demonstrate the utility of the approach through measurements of the pressure drop per unit length required to maintain the flow through a wood-chip bed under mechanical load. Finally, a comparison of the novel methodology and the experimental measurements are made in §3.4.

3.2 A Method to Determine r_1 and r_2

We begin this analysis by considering the one-dimensional flow of fluid through a compressible porous bed of length L and loaded mechanically by a compaction pressure p_c , see Figure 3.1. The fluid is driven by a fluid pressure equal to a hydraulic pressure p_h , applied at the top of the bed; the fluid pressure at the bottom of the bed is considered to be zero. In a similar manner, we consider the solidstress on the wood chips to be equal to p_c at the top of the bed and $p_c + p_h$ at the bottom of the bed. For ease in the subsequent presentation, we define a characteristic porosity in the bed ε_c obtained by solving $p_c = f_3(\varepsilon_c)$ i.e. $\varepsilon(L)$ where the solid-stress on the wood chips at the top of the bed is equal to p_c .



Figure 3.1: Schematic of the geometry considered.

Some of the earliest and most significant studies on compressible porous media flows were conducted by Beavers and his coworkers (Beavers *et al*, 1981a,b), Burger and Concha (1998), Verhoff (1983), and Wakeman (1978). These are works with steady, one dimensional flow of non-compressible fluid through highly compressible material. Their analyses are not restricted to small deformations of flow within the Darcy regime (i.e. low velocity flow). Here it was observed that the gradient of solid-stress within the porous media is of equal and opposite magnitude to the fluid pressure gradient established in the flow

$$\frac{dp}{dx} = -\frac{dp_s}{dx} \tag{3.1}$$

Equation 3.1 can be obtained by the mathematical addition of the Eulerian volumeaveraged two-phase flow equations after neglecting the acceleration terms under the assumption of inviscid flow (Burger and Concha, 1998). With this, in combination with the Ergun equation and a compressibility relationship, formed a system of equations which could be integrated to compute the volumetric flowrate as a function of the solid-stress across finite porous materials. The boundary conditions, as introduced above, are

$$p(0) = 0$$
 $p_s(0) = p_c + p_h$ (3.2)

$$p(L) = p_h \qquad p_s(L) = p_c \qquad (3.3)$$

If we scale using

$$\bar{p} = \frac{p}{p_h}$$
 $\bar{p}_s = \frac{p_s - p_c}{p_h}$, $\bar{x} = \frac{x}{L}$ $\bar{\varepsilon} = \frac{\varepsilon}{\varepsilon_c}$ $\bar{f}_i = \frac{f_i(\varepsilon)}{f_i(\varepsilon_c)}$ (3.4)

the governing equations reduce to

$$\frac{d\bar{p}}{d\bar{x}} = -\frac{d\bar{p}_s}{d\bar{x}} \tag{3.5}$$

$$\frac{d\bar{p}}{d\bar{x}} = \bar{r}_1 \bar{f}_1(\bar{\varepsilon}) + \bar{r}_2 \bar{f}_2(\bar{\varepsilon})$$
(3.6)

$$\bar{p}_s = \frac{1}{\delta} \left(\bar{f}_3(\bar{\varepsilon}) - 1 \right) \tag{3.7}$$

where

$$\bar{r}_1 = \frac{r_1 f_1(\varepsilon_c) v L}{p_h} \quad \bar{r}_2 = \frac{r_2 f_2(\varepsilon_c) v^2 L}{p_h}, \quad \delta = \frac{p_h}{p_c}$$
(3.8)

Upon elimination of \bar{p}_s and \bar{p} , the governing equation becomes

$$-\frac{1}{\delta}\bar{f}_{3}'(\bar{\varepsilon})\frac{d\bar{\varepsilon}}{d\bar{x}} = \bar{r}_{1}\bar{f}_{1}(\bar{\varepsilon}) + \bar{r}_{2}\bar{f}_{2}(\bar{\varepsilon})$$
(3.9)

where the symbol ' refers to differentiation with respect to $\bar{\varepsilon}$. The boundary conditions are assigned as the solid-stress is known at both the bottom and top of the column. In our notation these conditions read

$$\bar{f}_3(\bar{\varepsilon}(0)) = 1 + \delta \tag{3.10}$$

$$\bar{f}_3(\bar{\epsilon}(1)) = 1$$
 (3.11)

As we have two conditions to satisfy a first-order ordinary differential equation, the extra condition will aid in determining the unknown resistances \bar{r}_1 and \bar{r}_2 .

Equation 3.9 can be integrated numerically using standard techniques. By doing so, the unknown resistances can be estimated using one of a number of different optimization schemes in conjunction with experimental data. However, for the special case where the hydraulic pressure is much less than the mechanical load, i.e. when $0 < \delta \ll 1$, it is possible to solve this relationship using asymptotic methods to develop an analytical expression which can then be used to determine the unknown resistance to motion using a simpler multiple regression technique.

This is shown below.

To begin, if we seek a solution of the form

$$\bar{\varepsilon} = \bar{\varepsilon}^{(0)} + \delta \bar{\varepsilon}^{(1)} + \dots \qquad (3.12)$$

$$\bar{r}_i = \bar{r}_i^{(0)} + \delta \bar{r}_i^{(1)} + \dots$$
 (3.13)

Equation 3.9 is reduced to two linear differential equations i.e.

$$-\bar{f}_{3}'(\bar{\varepsilon}^{(0)})\frac{d\bar{\varepsilon}^{(0)}}{d\bar{x}} = 0$$
(3.14)

$$-\bar{f}_{3}'(\bar{\boldsymbol{\varepsilon}}^{(0)})\frac{d\bar{\boldsymbol{\varepsilon}}^{(1)}}{d\bar{x}} = \bar{r}_{1}^{(0)}\bar{f}_{1}(\bar{\boldsymbol{\varepsilon}}^{(0)}) + \bar{r}_{2}^{(0)}\bar{f}_{2}(\bar{\boldsymbol{\varepsilon}}^{(0)})$$
(3.15)

with the boundary conditions

$$\bar{\varepsilon}^{(0)}(0) = 1 \qquad \qquad \bar{\varepsilon}^{(1)}(0) = \frac{1}{\bar{f}'_3(\bar{\varepsilon}^{(0)}(0))} \qquad (3.16)$$

$$\bar{\varepsilon}^{(0)}(1) = 1$$
 $\bar{\varepsilon}^{(1)}(1) = 0$ (3.17)

With this, the solution of this set of equations is given by

$$\bar{\varepsilon} = 1 + \delta \frac{1 - \bar{x}}{\bar{f}'_3(1)} + O(\delta^2) \tag{3.18}$$

subject to the constraint that

$$\bar{r}_1^{(0)} + \bar{r}_2^{(0)} = 1 \tag{3.19}$$

Equation 3.18 indicates that the first-order approximation of the porosity distribution is linear, with the porosity decreasing with increasing distance away from the top of the wood-chip bed. This analysis can be easily extended to obtain higherorder approximations. This is shown in Appendix H where the $O(\delta^2)$ term is reported. A comparison of the approximate analytical solution and the more accurate numerical solution are shown in Figure 3.2. Clearly this asymptotic methodology yields a fairly reasonable approximation of $\bar{\epsilon}(\bar{x})$ over the region $\delta < 0.3$

Before continuing we must comment on the utility of this asymptotic solution. There is a possibility of degenerative behavior as $\bar{f}'_3(\bar{\epsilon})$ approaches zero as the function $\bar{\epsilon} \to \bar{\epsilon}_g$. Under these conditions, the proposed series diverges. As a result, we must bound the useful range of solution to cases in which $\| \bar{f}'_3(1) \| \ge 1$. If we examine the form of this function, as given in Appendix I, we deduce that the useful range of this solution is given when $\bar{\epsilon}_g \ge n-1$.

Given a reasonable estimate of the variation of porosity along the length of the bed, we are now in a position to estimate the variation of the fluid pressure \bar{p} . Equation 3.6 may now be approximated using a similar series. If we seek a solution of the form

$$\bar{p} = \bar{p}^{(0)} + \delta \bar{p}^{(1)} + O(\delta^2)$$
(3.20)

Equation 3.6 reduces to

$$\frac{\partial \bar{p}^{(0)}}{\partial \bar{x}} = 1 \tag{3.21}$$

$$\frac{\partial \bar{p}^{(1)}}{\partial \bar{x}} = \frac{(1-\bar{x})}{\bar{f}'_{3}(1)} \left(\bar{r}^{(0)}_{1} \bar{f}'_{1}(1) + \bar{r}^{(0)}_{2} \bar{f}'_{2}(1) \right) + \bar{r}^{(1)}_{1} + \bar{r}^{(1)}_{2}$$
(3.22)



Figure 3.2: A comparison of the numerical (dashed line) and the asymptotic solution (circles) at various δ . The asymptotic solution is approximated using the first three terms in the series. This comparison is made with $m = 100 \ kPa$, n = 1.2, $\varepsilon_g = 0.54$, $r_1 = 3 \times 10^4 \ Pa.s/m^2$ and $r_2 = 1.9 \times 10^6 \ Pa.s^2/m^3$.

When integrated, the fluid pressure is given by

$$\bar{p} = \bar{x} + \delta \frac{(\bar{x} - \bar{x}^2)}{\bar{f}'_3(1)} \left(\bar{r}_1^{(0)} \bar{f}'_1(1) + \bar{r}_2^{(0)} \bar{f}'_2(1) \right) + O(\delta^2)$$
(3.23)

with the constraint that

$$\bar{r}_1^{(1)} + \bar{r}_2^{(1)} = -\frac{1}{2\bar{f}_3'(1)} \left(\bar{r}_1^{(0)} \bar{f}_1'(1) + \bar{r}_2^{(0)} \bar{f}_2'(1) \right)$$
(3.24)

At this point we will now turn our attention to the overall problem considered in

this section, i.e. an estimate methodology for \bar{r}_1 and \bar{r}_2 . Here, we will demonstrate how the two constraints, i.e. Equations 3.19 and 3.24, can be used in conjunction with linear regression of experimental data to estimate r_1 and r_2 . To do so, we add Equations 3.19 and 3.24, after first multiplying Equation 3.24 by δ , to yield

$$\left(\bar{r}_{1}^{(0)} + \delta\bar{r}_{1}^{(1)}\right) + \left(\bar{r}_{2}^{(0)} + \delta\bar{r}_{2}^{(1)}\right) = 1 - \frac{\delta}{2\bar{f}_{3}'(1)} \left(\bar{r}_{1}^{(0)}\bar{f}_{1}'(1) + \bar{r}_{2}^{(0)}\bar{f}_{2}'(1)\right) \quad (3.25)$$

Upon substitution of Equation 3.13, the equation above simplifies to

$$\left(1 + \delta \frac{\bar{f}_1'(1)}{2\bar{f}_3'(1)}\right)\bar{r}_1 + \left(1 + \delta \frac{\bar{f}_2'(1)}{2\bar{f}_3'(1)}\right)\bar{r}_2 = 1 + O(\delta^2)$$
(3.26)

which can also be expressed as

$$y = r_1 x_1 + r_2 x_2 \tag{3.27}$$

where

$$x_1 = \left(1 + \delta \frac{1}{2} \frac{(3 - \varepsilon_c)}{(1 - \varepsilon_c)} \frac{(\varepsilon_g - \varepsilon_c)}{\varepsilon_c n}\right) \frac{(1 - \varepsilon_c)^2}{\varepsilon_c^3} \nu$$
(3.28)

$$x_2 = \left(1 + \delta \frac{1}{2} \frac{(3 - 2\varepsilon_c)}{(1 - \varepsilon_c)} \frac{(\varepsilon_g - \varepsilon_c)}{\varepsilon_c n}\right) \frac{(1 - \varepsilon_c)}{\varepsilon_c^3} v^2$$
(3.29)

$$y = \frac{p_h}{L} \tag{3.30}$$

Equation 3.27 is in a form in which the unknowns r_1 and r_2 can be determined by multiple linear regression to a data set containing (v_i, p_{h_i}, L_i) .

3.3 Experimental Materials and Methods

Air dried (to approximately 90% solid content) Hemlock wood chips were used in this study. These chips were made by a disk chipper which produces a chip shaped as a parallelepiped, see Figures 3.3a and 3.3b. The chips were classified using a Wennberg ChipClassTM classifier (Weyerhaeuser), shown in Figures 3.4a and 3.4b. The accept chips were further classified according to thickness using 2 and 4 mm slotted screens. For each chip fraction, the average length, width and thickness were measured by hand by randomly selecting a sample consisting of 250 individual chips, see Appendix B. Only accept chips fraction with 2 - 4 mmthickness was used in this study to avoid any non-uniformity of the oversized and undersized chip fractions, see Table 3.1. In addition both the equivalent particle mean diameter d_p , defined as

$$d_p = 6\frac{V_p}{A_s} \tag{3.31}$$

as well as the sphericity ψ , are also given in the table to help characterize the wood chip further.



(a)



(b)

Figure 3.3: Geometry of disk chipper chips.



(a)



Figure 3.4: 3.4a Chip screening apparatus. 3.4b Chip size distribution.

Screen	Thickness	Width	Length	d_p	Ψ	Mass Fraction
	(mm)	(mm)	(mm)	(mm)		(%)
4 mm slot	$4.4(\pm 0.7)$	$19.1(\pm 8.5)$	$22.9(\pm 3.6)$	9.2	0.60	37.6
2 mm slot	$2.6(\pm 0.6)$	$14.7(\pm 6.5)$	$21.2(\pm 2.0)$	6.3	0.53	62.4
Total Mixture				7.1	0.56	100.0

Table 3.1: Characteristics and dimensions with standard deviations of the wood chips before cooking. The total mixture was composed of 37.6%(wt/wt) of the 4 *mm* slot chips with the balance of mixture comprised of the 2 *mm* slot chips. Both d_p as well as ψ for the total mixture are averages based upon the mass fraction of each species, see Appendices B and C.

Cooks of accept chips were done to κ numbers of 25, 53 and 80, which represents the variation of kappa number during industrial cooking, using a 28 *L* Werverk laboratory digester (FPInnovation, Vancouver). Cooks were made using conventional kraft pulping conditions, see Table 3.2. After the cook, we washed the cooked wood chips gently, while they are in the cooking basket of the digester, by running water for about 15 mins to remove any residual alkali, and then we transferred the washed wood chips into storing buckets. The washing and transferring processes were done very carefully in order to protect the dimensions of the cooked wood chips. Also before doing any experiments in our lab, we soaked and washed the cooked wood chips, while they are in our bed, with water until the effluent was clear. Our observations indicate that with lower kappa number chips, the longer the washing time that was required. Figures 3.5a-3.5d show a comparison of the wood chips preserved their dimensions.

Wood chips species	Hemlock (Accepts $2 - 4 mm$)
Wood chips charge (oven dry)	3.0 <i>O</i> . <i>D</i> . <i>kg</i>
Liquor to Wood ratio	4.5:1
Effective Alkali charge (EA)	17% as Na_2O
Sulfidity	26.95%
Heating rate	$1.11^{\circ}C/min$
Maximum cooking temperature	$170^{\circ}C$
Time to temperature	140 minutes
Time at temperature	To desired κ number

 Table 3.2: Cooking conditions using the laboratory digester.



(a) $\kappa = 180$

(b) $\kappa = 80$



(c) $\kappa = 53$

(**d**) $\kappa = 25$

Figure 3.5: Comparison of the accept chips before and after cooking to different κ numbers. Scale in the picture is in *cm*.

A schematic diagram of the test apparatus is shown in Figures 3.6 and 3.7. The column has an inside diameter of 15 *cm*, a height of 45 *cm* and was made from 1.25 *cm* thick Plexiglas. The column has a perforated plate (having an open area of 40% using 3 *mm* diameter holes) equipped by a piston in order to simulate a compaction pressure (0-14 *kPa*) and create uniform flow through the wood chips. The compressive pressure was controlled by a pneumatic air cylinder (American Cylinder®Co., Inc., Model 1500DVS-1", Peotone, Illinois). The pressure drop across the column was divided into four equal sections (11.25 *cm*) and measured by four differential pressure transducers (Omega®Engineering Inc., model PX2300-1DI, Laval, Quebec). For measuring the hydraulic pressure at the top of the Column, a gage pressure transducer (Omega®Engineering Inc., model PX01C1-015GI, Laval, Quebec) was used. For measuring the flow, a magnetic flow-meter (Rosemount®Measurement, model 8712, Irvine, CA) was used. In addition, a centrifugal pump (Cole-Parmer®Inc., Model K-07085-00, Concord, Ontario) was used to circulate the flow in the chip column.

The column diameter and height were chosen with consideration of industrial and previous researchers' designs. In addition, this column's diameter helped in creating uniform force by the piston as well as reducing the quantity of cooked wood chips used since we didn't have a limitless supply. Moreover, it is assumed that there is no channeling or other wall effects throughout the tests. According to Winterberg and Tsotsas (2000), wall effects can generally be neglected if the diameter of the packed bed is greater than ten times the diameter of the packing particles. Cooked wood chips at a prescribed κ were added carefully, to avoid chip



Figure 3.6: Schematic of the apparatus.

fracture as will as to avoid compaction in the chip column, into the test column to the initial height of 0.45 m. The packing was performed randomly. To start an experiment, water ($T = 23 \,^{\circ}C$, $\mu = 0.925 \times 10^{-3} Pa.s$, $\rho = 997.3 \, kg/m^3$) from the 25 *L* tank was pumped upward through the column to remove any trapped air and to fill the column with fluid. Then, flow was shifted through the flowmeter to the top of the column and was allowed to stabilize and reach steady state before taking pressure drop data. During an experiment, the chip packed bed was



Figure 3.7: Image of the apparatus.

compressed by the piston to give three compaction loads ranging from 0 to 14 kPa. The level of the compressive pressure was chosen to achieve porosities in the range of 0.2 - 0.5. This is the typical range found in industry and we determined this through literature values as well as in a pre-trial. The applied load was kept constant during an experiment while the flow rate was changed in increments up to 10 L/min. These ranges covered those encountered in operating digesters. Data was taken with increasing flow rate to avoid any hysteresis effects.

3.4 **Results and Discussion**

In this section the results will be presented in four subsections. In the first part, a brief description of the uncertainty of our measurement will be addressed by measuring the resistance to flow in our apparatus with a packed-bed of monodisperse spheres. By doing so, we are able to compare our apparatus to other estimates given in the literature. We then report the values of the empirical constants for the compressibility function p_s . Following this we turn our attention to the main results, that being on the estimates of r_1 and r_2 for wood chips at different κ using the methodology reported in §3.2. Finally, we attempt to validate these estimates by attempting to predict the pressure drop for a given flow rate, with knowledge of r_1 and r_2 under the conditions of no applied mechanical load.

To begin, in the first set of results, polypropylene spheres $(d_p = 1 \times 10^{-2} m, \rho = 905 kg/m^3)$ were used to verify our procedure. The results are shown in Figure 3.8 alongside the estimates given by the Ergun Equation (Equations 2.5). For the range of data considered, we find that A = 210 and B = 2.33 with the correlation coefficient estimated to be $r^2 = 0.99$ using Equation 2.5. The uncertainty in the experimental estimate stems from the range of pressures used in this case; we are at lower limit of the working range of the pressure transducers. In addition there is some variation in the estimates of A and B in the literature, see Table 2.1, due to, for example differences in the roughness of the particles, and as a result we consider our results shown comparable to those found in the literature.

At this point we turn our attention to characterizing the behavior of wood



Figure 3.8: The resistance to flow of water through a incompressible bed of polystyrene spheres. The data is compared to the literature values (solid line) to assess the uncertainty in our measurement system.

chip beds under uniaxial compression. Here we do so by measuring the change in bed height as a function of compression pressure p_c for beds that are water saturated. The experimentally measured data are shown in Figure 3.9a as circles with the dashed line representing the form of the function $f_3(\varepsilon)$ as defined in Equation 2.8. The estimated values of ε_g is 0.54; *n* was found to be 1.2 over the range of conditions tested. The constant *m* was found to vary as a function of κ where *m* was found to be 115,78, and 67*kPa* for κ_{80} , κ_{53} , and κ_{25} , respectively. This is shown in Figure 3.9b. The dashed line is the functional form for $m(\kappa)$ as advanced by Harkonen (1987). With the values of *m*, *n*, and ε_g determined we are now able to estimate the values of r_1 and r_2 through the methodology proposed in §3.2. The results of this analysis are presented in Figures 3.10a - 3.10c. The dashed line represents the regression of Equation 3.27 performed using the entire data set. For the range of data considered, we find that $r_1 = 3 \times 10^4 Pa.s/m^2$ and $r_2 = 1.9 \times 10^6 Pa.s^2/m^3$ with the correlation coefficient estimated to be $r^2 = 0.99$. These results are similar to those reported by Harkonen (1987). We also report *A* and *B* values since they are independent of fluid properties and particle diameter using Equations 2.12 and 2.13. For the wood-chips tests, we find that A = 475and B = 7.49 with the same correlation coefficient estimated to be $r^2 = 0.99$. It should be noted that we are able to do the regression over the entire data set as the shape of the chips does not change significantly over the range of κ .



Figure 3.9: The uniaxial compressibility of wood chips cooked to different κ numbers.


Figure 3.10: The measured pressure drop per unit length for a prescribed superficial velocity v at three different compaction pressures. The uncertainty in the estimate for p_h/L is approximately 0.3 kPa/m.

To test the usefulness of these estimates for r_1 and r_2 , we consider the case of flow through the packed bed without mechanical loading, i.e. $p_c = 0$. Here, with the values of r_1 and r_2 we should be able to *predict* the pressure drop per unit length for a prescribed superficial velocity. For this case, the asymptotic method outlined in §3.2 does not apply as $1/\delta \rightarrow 0$. What is required here is a numerical solution of the governing equations. The system of equations, in dimensional form, that are required to be solved is

$$\frac{dp_s}{dx} = -(r_1 f_1(\varepsilon)v + r_2 f_2(\varepsilon)v^2) \quad (p_s(L) = 0)$$
(3.32)

$$\varepsilon(x) = \varepsilon_g - \left(\frac{p_s(x)}{m}\right)^{\frac{1}{n}}$$
(3.33)

$$p_h = p_s(0) \tag{3.34}$$

Hence, with a prescribed superficial velocity v, Equation 3.32 can be integrated to find $p_s(x)$ and used in conjunction with Equation 3.34 to estimate the hydraulic pressure. This was performed in MATLAB using a Runge-Kutta scheme. The predictions for wood chips at three different κ are shown in Figures 3.11a - 3.11c; clearly a reasonable prediction was obtained.



Figure 3.11: The predicted pressured drop (dashed line), the experimentally measured values (circles), and the correlation without compressibility effects (solid line) for the case $p_c = 0$. The uncertainty in the estimate for p_h/L is approximately 0.3 kPa/m. The dashed line was calculated using the values of r_1 and r_2 determined above by numerical integration of the governing equations.

With the values of r_1 and r_2 estimated using the methodology given above, we are now able to estimate the permeability of the wood chip bed tested in this work. To do so we rearrange Equations 2.12 and 2.13 to find that

$$\theta s_v^2 = \frac{r_1}{\mu} \tag{3.35}$$

$$\beta = \frac{r_2}{\rho} \sqrt{\frac{\mu}{r_1}} \tag{3.36}$$

from which we can estimate the permeability using Equation 2.2. This is shown graphically in Figure 3.12a from which we observe that the order of magnitude of this function over the porosity range tested is approximately $k \sim 10^{-8} m^2$. What we find significant with these findings is that our results are dependent on porosity which in turn is formed as a combination of degree of delignification, i.e. κ number, and compaction pressure. We anticipated the possibility that the degree of delignification may affect permeability through changes in the specific surface; the chip may have a larger surface area through removal of the lignin. We argue that since this is not the case, the specific surface must be related solely to the geometry and orientation distribution of the chips in the compressible bed.

We continue the discussion of the results to attempt to highlight the effect of κ on the permeability. The degree of delignification affects permeability through the porosity; wood chips with different degree of delignification will compress to different porosities under the same external mechanical load. To characterize this, we report the relative permeability, i.e. the ratio of the permeabilities of a κ_{80} bed to that of a κ_{25} , compressed with equivalent mechanical loads, see Figure

3.12b. Under the industrially relevant loads, the relative permeability increases with increasing loads. This results as the κ_{25} chip-bed is more compressible, and compacts to a lower porosity, in comparison to the κ_{80} chip bed, at equivalent mechanical loading.

Finally, we compare our correlation to the literature for porosity of 0.35. The results are compared to Harkonen (1987) correlation since it is widely used in model digesters, as well as Lee (2002) correlation since Lee (2002) used similar chip size distribution to ours, i.e. 100 % accepts. We should note that Harkonen (1987) performed his experiments at a temperature $T = 170 \,^{\circ}C$ ($\mu = 0.068 \times 10^{-3} Pa.s$, $\rho = 885 \, kg/m^3$) using cooking liquor while Lee (2002) and our tests were made at $T = 23 \,^{\circ}C$ ($\mu = 0.925 \times 10^{-3} Pa.s$, $\rho = 997.3 \, kg/m^3$) using water. We corrected our r_1 and r_2 values for Harkonen (1987) conditions to get $r_1 = 2 \times 10^3 Pa.s/m^2$ and $r_2 = 1.7 \times 10^6 Pa.s^2/m^3$ with the same correlation coefficient estimated to be $r^2 = 0.99$. This is shown graphically in Figure 3.13 from which we observe that our correlation has lower flow resistances than the literature correlations for similar chip size distribution and under all conditions. This can be justified by the fact that we accounted for compressibility effects in our correlation. As a result, the literature correlations over estimate the pressure drop.



(b) The relative permeability of hemlock wood chips cooked to two different κ . The relative permeability is defined as the ratio of the permeability of a $\kappa = 80$ wood chip bed to that of a wood chip bed cooked to $\kappa = 25$.

Figure 3.12: 3.12a Estimate of the permeability of Hemlock wood chips using the Kozeny-Carman expression given by Equation 2.2. The tortuosity and the specific surface of the chips were estimated through use of Equations 2.12 and 2.13. 3.12b The ratio of the permeabilities of a κ 80 to κ 25 wood chips compressed at different loads.



Figure 3.13: A comparison of the pressure drop correlations between our correlation and the literature correlations for porosity of 0.35.

3.5 Summary and Conclusions

In this work we present a methodology to characterize the resistance to fluid motion through a porous compressible bed of wood chips. Here we advance the argument that under the limiting conditions of when the mechanical load is large in comparison to hydraulic pressure the equations of motion can be solved asymptotically to produce an expression approximating the variation in porosity along the length of the bed. With this, a methodology to estimate the resistance is given by multiple linear regression. In the experimental portion of the work we measured the resistance to flow of a bed of hemlock wood chips, cooked to three different κ , and compacted to three different p_c . With our methodology, we find that $r_1 = 3 \times 10^4 Pa.s/m^2$ and $r_2 = 1.9 \times 10^6 Pa.s^2/m^3$. The usefulness of these estimates were tested for the extreme case with no applied mechanical load. A reasonable prediction was obtained, thus verifying the utility of the methodology and the estimated resistances. After accounting for compressibility effects, we find that our correlation has lower flow resistances than the literature correlations.

Chapter 4

The Axial Dispersion in Wood-Chip Beds; ERT Visualization

4.1 Introduction

In this chapter, we characterize the dispersion of a solute flowing through a compressible porous bed of wood-chips under axial mechanical load. Here we examine the flow of brine solution through a compressible wood chip bed. We focus our efforts on the industrially-relevant case of understanding the flow in wood chips delignified to various extents, i.e. decreasing kappa numbers, and visualize the displacement of the brine solution using electrical resistance tomography. In §4.2, we present a methodology to estimate the axial dispersion coefficient. Here we present the governing equations and examine two special cases in which the porosity of the bed varies slowly in the axial direction. Under these limiting cases, the difficulty in estimating the diffusivity from experimental data is reduced significantly. In §4.3, we present the details of using electrical resistance tomography (ERT) to measure the evolution of the concentration profile of the displacement of a brine solution, with a different solution, from the wood chip bed, as well as the numerical methodology for estimating the axial dispersion. Here we use electrical resistance tomography (ERT), a non-invasive device, to reconstruct a tomographic image of the evolution of the concentration profile both spatially and temporally. In §4.4, we present qualitatively the images of the evolution of the concentration profiles as well as quantitative estimates of the axial dispersion coefficient.

4.2 The Methodology to Estimate *D_e*

In this section, we discuss the methodology required to image the evolution of the concentration profile in the wood-chip bed as well as the mathematical model used to estimate dispersion. This section is presented in three parts. In the first part, we present the model equations to describe our experimental conditions. The model presentation is relatively straightforward as the equations have been derived previously. Our contribution in this section is the scaling of the equations of motion, rendering them dimensionless, so that it is possible to see two limiting cases in which the porosity is nearly constant in the compressible bed. We do so as this directs our experimental efforts to measure the axial dispersion for cases which are relatively easy to analyze mathematically. Here we show two limiting cases which are experimentally achievable. In §4.2.1 and §4.2.2, we present asymptotic solutions for these limiting cases and demonstrate that to leading order the



Figure 4.1: Schematic of the geometry considered.

porosity is constant. We extend the solution and also present in these sections, the first-order approximations for porosity.

We begin this presentation of the governing equation by defining our geometry. We consider the one-dimensional flow of fluid through a compressible porous bed of length *L* and loaded mechanically by a compaction pressure p_c , see Figure 4.1. The fluid is driven by a fluid pressure equal to a hydraulic pressure p_h , applied at the top of the bed; the fluid pressure at the bottom of the bed is considered to be zero. In a similar manner, we consider the solid-stress on the wood chips to be equal to p_c at the top of the bed and $p_c + p_h$ at the bottom of the bed. In this experiment we consider the displacement of a fluid in the bed, with an initial concentration C_i ; at t > 0, a second fluid is introduced through the top at a concentration C_f . In this case, a fluid with a lighter density is displacing a fluid with a heavier density.

As the wood chip bed is compressible, that is, the porosity varies in the direction of the flow, we must consider both the evolution of the concentration distribution, as well as the spatial variations in the porosity and velocity fields for a complete description of this experiment. The literature in this area is vast and the reader should consult the classic works by Beavers and his coworkers (Beavers *et al*, 1981a,b), Burger and Concha (1998), Verhoff (1983), and Wakeman (1978) for a full description of the derivation of the equations describing the flow field or of the work by Poirier *et al* (1988) for the a description of the evolution of the concentration field. Using the notation defined in Chapter 3, the equations of motion for this geometry are

$$\frac{dp}{dx} = -\frac{dp_s}{dx} \tag{4.1}$$

$$\frac{dp}{dx} = f_1(\varepsilon)v + f_2(\varepsilon)v^2 \tag{4.2}$$

$$p_s = f_3(\varepsilon) \tag{4.3}$$

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x} \left(D_e \frac{\partial C}{\partial x} \right) - \frac{v}{\varepsilon} \frac{\partial C}{\partial x}$$
(4.4)

where p and p_s are the fluid pressure and the solid stress of the fluid and solid phases, respectively; ε is the porosity of the compressible wood-chip bed; v is the superficial velocity of the fluid; C is the concentration of a solute in the fluid; $f_1(\varepsilon)$ and $f_2(\varepsilon)$ are empirically determined coefficients related to the resistance of flow of the fluid; and $f_3(\varepsilon)$ is an empirically determined function that relates the applied solid stress to the local porosity. It is traditionally referred to as the compressibility relationship. These empirical relationships have been recently reported in Chapter 3 for wood-chip beds and are given as

$$f_1(\varepsilon) = r_1 \frac{(1-\varepsilon)^2}{\varepsilon^3} \tag{4.5}$$

$$f_2(\varepsilon) = r_2 \frac{(1-\varepsilon)}{\varepsilon^3} \tag{4.6}$$

$$f_3(\varepsilon) = m(\varepsilon_g - \varepsilon)^n \tag{4.7}$$

where r_1 , r_2 , m, n, and ε_g are experimentally determined constants. The boundary conditions, as introduced above, are

$$p(0) = 0$$
 $p_s(0) = p_c + p_h$ (4.8)

$$p(L) = p_h \qquad p_s(L) = p_c \qquad (4.9)$$

For ease in the subsequent presentation, we define a characteristic porosity in the bed ε_c obtained by solving $p_c + p_h = f_3(\varepsilon_c)$ i.e. $\varepsilon(0)$ where the solid-stress on the wood chips at the bottom of the bed is equal to $p_c + p_h$. If we scale the equations using

$$\bar{p} = \frac{p}{p_h}, \qquad \bar{p}_s = \frac{p_s - p_c}{p_h}, \qquad \bar{x} = \frac{x}{L}, \qquad \bar{\varepsilon} = \frac{\varepsilon}{\varepsilon_c}$$
$$\bar{f}_i = \frac{f_i(\varepsilon)}{f_i(\varepsilon_c)}, \qquad \bar{C} = \frac{C - C_i}{C_f - C_i}, \qquad \bar{t} = \frac{tv}{L\varepsilon_c}, \qquad \bar{D}_e(\bar{\varepsilon}) = \frac{D_e(\varepsilon)}{D_e(\varepsilon_c)} = \bar{\varepsilon}^b$$

and form the following dimensionless groups

$$Pe = \frac{vL}{\varepsilon_c D_e(\varepsilon_c)}, \quad \bar{r}_1 = \frac{r_1 f_1(\varepsilon_c) vL}{p_h}, \quad \bar{r}_2 = \frac{r_2 f_2(\varepsilon_c) v^2 L}{p_h}, \quad \delta = \frac{p_h}{p_h + p_c}$$

the governing equations reduce to

$$\frac{d\bar{p}}{d\bar{x}} = -\frac{d\bar{p}_s}{d\bar{x}} \tag{4.10}$$

$$\frac{d\bar{p}}{d\bar{x}} = \bar{r}_1 \bar{f}_1(\bar{\varepsilon}) + \bar{r}_2 \bar{f}_2(\bar{\varepsilon})$$
(4.11)

$$\bar{p}_s = \frac{1}{\delta} \left(\bar{f}_3(\bar{\varepsilon}) - 1 \right) + 1 \tag{4.12}$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} = \frac{1}{Pe} \frac{\partial}{\partial \bar{x}} \left(\bar{D}_e(\bar{\varepsilon}) \frac{\partial \bar{C}}{\partial \bar{x}} \right) - \frac{1}{\bar{\varepsilon}} \frac{\partial \bar{C}}{\partial \bar{x}}$$
(4.13)

where Pe is the Peclet number. Upon elimination of p_s and p, the governing equation for porosity becomes

$$-\frac{1}{\delta}\bar{f}_{3}^{\prime}(\bar{\varepsilon})\frac{d\bar{\varepsilon}}{d\bar{x}} = \bar{r}_{1}\bar{f}_{1}(\bar{\varepsilon}) + \bar{r}_{2}\bar{f}_{2}(\bar{\varepsilon})$$
(4.14)

Here the symbol ' refers to differentiation with respect to $\bar{\epsilon}$. The boundary conditions for Equation 4.14 are assigned as the solid-stress, and hence porosity, is known at both the bottom and top of the column. In our notation these conditions read

$$\bar{p}_s(0) = 1 \to \bar{\varepsilon}(0) = 1 \tag{4.15}$$

$$\bar{p}_s(1) = 0 \to \bar{\varepsilon}(1) = \bar{\varepsilon}_g - (\bar{\varepsilon}_g - 1)(1 - \delta)^{1/n}$$
(4.16)

For the advection-diffusion equation the initial and boundary conditions are those prescribed by Danckwerts (1953)

$$\bar{t} = 0:$$
 $\bar{C}(\bar{x}, 0) = 0$ (4.17)

$$\bar{x} = 0:$$
 $\frac{\partial C}{\partial \bar{x}} = 0$ (4.18)

$$\bar{x} = 1:$$
 $\frac{\bar{\varepsilon}D(\bar{\varepsilon})}{Pe}\frac{\partial C}{\partial \bar{x}} = \bar{C} - 1$ (4.19)

The objective in presenting these relationships is to develop a methodology for determining D_e , which is contained in the Peclet number Pe, which is robust and accurate. The complication in experimentally determining this variable is that this material is compressible and ε varies as a function of x. Clearly, the certainty of our estimate for D_e would increase if we could estimate the molecular diffusivity, though measurement of the evolution of the concentration of a solute, under conditions where the porosity is *nearly* constant. In the following two subsections, we will demonstrate two limiting cases, which are experimentally achievable, in which the porosity varies slowly in the direction of flow. We will then conduct our experiments under these conditions.

4.2.1 The First Limiting Case: $p_c \gg p_h$

Before we proceed it is interesting to note that there is a one-way coupling between Equations 4.14 and 4.13. Here the porosity is independent of the concentration field; the concentration field however is affected by the porosity. Because of this one-way coupling we can operate on Equation 4.14 independently of Equation 4.13. In this subsection, we will demonstrate that under this limiting case, the porosity varies slowly in *x*. Under the conditions that the mechanical load is much greater than the hydraulic pressure, we find that $0 < \delta \ll 1$. If we seek a solution of the form

$$\bar{\varepsilon} = \bar{\varepsilon}^{(0)} + \delta \bar{\varepsilon}^{(1)} + \dots \qquad (4.20)$$

Equation 4.14 is reduced to two linear differential equations i.e.

$$-\bar{f}'_{3}(\bar{\varepsilon}^{(0)})\frac{d\bar{\varepsilon}^{(0)}}{d\bar{x}} = 0$$
(4.21)

$$-\bar{f}_{3}'(\bar{\boldsymbol{\varepsilon}}^{(0)})\frac{d\bar{\boldsymbol{\varepsilon}}^{(1)}}{d\bar{x}} = \bar{r}_{1}\bar{f}_{1}(\bar{\boldsymbol{\varepsilon}}^{(0)}) + \bar{r}_{2}\bar{f}_{2}(\bar{\boldsymbol{\varepsilon}}^{(0)})$$
(4.22)

with the boundary conditions

$$\bar{\varepsilon}^{(0)}(0) = 1$$
 $\bar{\varepsilon}^{(1)}(0) = 0$ (4.23)

$$\bar{\epsilon}^{(0)}(1) = 1 \qquad \qquad \bar{\epsilon}^{(1)}(1) = \frac{(\bar{\epsilon}_g - 1)}{n} \qquad (4.24)$$

With this, the solution of this set of equations is given by

$$\bar{\varepsilon} = 1 + \delta \frac{(\bar{\varepsilon}_g - 1)\bar{x}}{n} + O(\delta^2)$$
(4.25)

subject to the constraint that

$$\bar{r}_1 + \bar{r}_2 = 1 + O(\delta^2) \tag{4.26}$$

We interpret Equation 4.25 to state that to the lowest-order, the porosity is constant and approximately equal to the characteristic porosity, i.e. $\varepsilon \approx \varepsilon_c$. The porosity gradient which must exist is small; its magnitude is proportional to the ratio of the hydraulic pressure to the compressive pressure. As we are able to conduct experiments with $p_h/p_c \rightarrow 0$, we consider ε to be a constant for the purpose of estimating *Pe*.

4.2.2 The second limiting case: $p_c = 0$ and $m \gg p_h$

Physically we interpret this case as one in which there is no applied mechanical load and that the applied hydraulic pressure is much less than the stiffness of the porous bed. In simpler terms, we consider the case of very slow flow through an unloaded bed. This case is quite subtle and to help highlight why the gradients are small we examine the boundary conditions given by Equation 4.16. As $\delta = 1$ under these conditions, this boundary condition reduces to $\bar{\varepsilon} = \bar{\varepsilon}_g$ which can be expressed as

$$\bar{\varepsilon} = \frac{\varepsilon_g}{\varepsilon_c} = \frac{1}{1 - \frac{1}{\varepsilon_g} \left(\frac{p_h}{m}\right)^{1/n}} \approx 1 + \frac{1}{\varepsilon_g} \left(\frac{p_h}{m}\right)^{1/n} + \dots$$
(4.27)

If $m \gg p_h$, we find that porosity at the upper and lower bounds of the bed, to the lowest order of approximation, are the same and equal to 1, see 4.15 to verify this argument. This indicates a regime in which the porosity gradients are small and achievable experimentally for the material we are using.

To formalize this argument and to rigourously deduce the form of the porosity

gradient, we define

$$\gamma = \frac{1}{\varepsilon_g} \left(\frac{p_h}{m}\right)^{1/n} \tag{4.28}$$

such that $0<\gamma \ll 1$ and seek a solution of the form

$$\bar{\varepsilon} = 1 + \gamma \bar{\varepsilon}^{(1)} + \dots \tag{4.29}$$

With this Equation 4.14 is reduced to

$$\frac{d\bar{\varepsilon}^{(1)}}{d\bar{x}} = 1 \tag{4.30}$$

where

$$\bar{f}'_{3}(1) = -\frac{n}{\bar{\epsilon}_{g}-1} = -\frac{n}{\gamma}$$
 (4.31)

subject to

$$\bar{r}_1 + \bar{r}_2 = n + O(\gamma)$$
 (4.32)

and

$$\bar{\varepsilon}^{(1)}(0) = 0$$
 $\bar{\varepsilon}^{(1)}(1) = 1$ (4.33)

With this, the solution of this set of equations is given by

$$\bar{\varepsilon} = 1 + \gamma \bar{x} + O(\gamma^2) \tag{4.34}$$

For both limiting cases, we find that to the lowest order of approximation, the porosity gradients are essentially constant and as a result, the linear advectivediffusion equation can be simplified, considerably to read

$$\frac{\partial \bar{C}}{\partial \bar{t}} = \frac{1}{Pe} \frac{\partial^2 \bar{C}}{\partial \bar{x}^2} - \frac{\partial \bar{C}}{\partial \bar{x}}$$
(4.35)

Under these conditions, we will estimate Pe by numerically integrating Equation 4.35 and comparing this to experimental data. The methodology for collecting the experimental data is given in the following section.

4.3 Experimental Materials and Methods

Air dried (to approximately 90% solid content) Hemlock wood chips were used in this study. These chips were made by a disk chipper which produces a chip shaped as a parallelepiped, see Figures 4.2a and 4.2b. The chips were classified using a Wennberg ChipClassTM classifier (Weyerhaeuser), shown in Figures 4.3a and 4.3b. The accept chips were further classified according to thickness using 2 and 4 *mm* slotted screens. For each chip fraction, the average length, width and thickness were measured by hand by randomly selecting a sample consisting of 250 individual chips, see Appendix B. Only accept chips fraction with 2 - 4 mmthickness was used in this study to avoid any non-uniformity of the oversized and undersized chip fractions, see Table 4.1. In addition both the equivalent particle mean diameter d_p , defined as

$$d_p = 6\frac{V_p}{A_s} \tag{4.36}$$

as well as the sphericity ψ , are also given in the table to help characterize the wood chip further.



(a)



(b)

Figure 4.2: Geometry of disk chipper chips.



(a)



Figure 4.3: 4.3a Chip screening apparatus. 4.3b Chip size distribution.

Screen	Thickness	Width	Length	d_p	Ψ	Mass Fraction	
	(mm)	(mm)	(mm)	(mm)		(%)	
4 mm slot	$4.4(\pm 0.7)$	$19.1(\pm 8.5)$	$22.9(\pm 3.6)$	9.2	0.60	37.6	
2 mm slot	$2.6(\pm 0.6)$	$14.7(\pm 6.5)$	$21.2(\pm 2.0)$	6.3	0.53	62.4	
Total Mixture				7.1	0.56	100.0	

Table 4.1: Characteristics and dimensions with standard deviations of the wood chips before cooking. The total mixture was composed of 37.6%(wt/wt) of the 4 *mm* slot chips with the balance of mixture comprised of the 2 *mm* slot chips. Both d_p as well as ψ for the total mixture are averages based upon the mass fraction of each species, see Appendices B and C.

Cooks of accept chips were done to κ numbers of 25, 53 and 80, which represents the variation of kappa number during industrial cooking, using a 28 *L* Werverk laboratory digester (FPInnovation, Vancouver). Cooks were made using conventional kraft pulping conditions, see Table 4.2. After the cook, we washed the cooked wood chips gently, while they are in the cooking basket of the digester, by running water for about 15 mins to remove any residual alkali, and then we transferred the washed wood chips into storing buckets. The washing and transferring processes were done very carefully in order to protect the dimensions of the cooked wood chips. Also before doing any experiments in our lab, we soaked and washed the cooked wood chips, while they are in our bed, with water until the effluent was clear. Our observations indicate that with lower kappa number chips, the longer the washing time that was required. Figures 4.4a-4.4d show a comparison of the wood chips preserved their dimensions.

Wood chips species	Hemlock (Accepts $2 - 4 mm$)
Wood chips charge (oven dry)	3.0 <i>O</i> . <i>D</i> . <i>kg</i>
Liquor to Wood ratio	4.5:1
Effective Alkali charge (EA)	17% as Na_2O
Sulfidity	26.95%
Heating rate	$1.11^{\circ}C/min$
Maximum cooking temperature	$170^{\circ}C$
Time to temperature	140 minutes
Time at temperature	To desired κ number

Table 4.2: Cooking conditions using the laboratory digester.



(a) $\kappa = 180$

(b) $\kappa = 80$



(c) $\kappa = 53$

(d) $\kappa = 25$

Figure 4.4: Comparison of the accept chips before and after cooking to different κ numbers. Scale in the picture is in *cm*.

There are a number of experimental techniques currently available to visualize the motion of a fluid in a dense, opaque porous medium: X – or γ ray attenuation, acoustics, or nuclear magnetic resonance. In this work we use electrical resistance tomography (ERT), a methodology which has been recently applied to a number of engineering studies (Williams and Beck, 1995; Lucas *et al*, 1999; Bolton *et al*, 2004; Vlaev and Bennington, 2004, 2005, 2006; ITS, 2006).

A schematic diagram of the test apparatus is shown in Figures 4.5 and 4.6. The column has an inside diameter of 15 *cm*, a height of 45 *cm* and was made from 1.25 *cm* thick Plexiglas. The column has a perforated plate (having an open area of 40% using 3 *mm* diameter holes) equipped by a piston in order to simulate a compaction pressure (0-14 *kPa*) and create uniform flow through the wood chips. The compressive pressure was controlled by a pneumatic air cylinder (American Cylinder®Co., Inc., Model 1500DVS-1", Peotone, Illinois). The pressure drop across the column was divided into four equal sections (11.25 *cm*) and measured by four differential pressure transducers (Omega®Engineering Inc., model PX2300-1DI, Laval, Quebec). For measuring the hydraulic pressure at the top of the Column, a gage pressure transducer (Omega®Engineering Inc., model PX01C1-015GI, Laval, Quebec) was used. For measuring the flow, a magnetic flow-meter (Rosemount®Measurement, model 8712, Irvine, CA) was used. In addition, a centrifugal pump (Cole-Parmer®Inc., Model K-07085-00, Concord, Ontario) was used to circulate the flow in the chip column.

The column diameter and height were chosen with consideration of industrial and previous researchers' designs. In addition, this column's diameter helped in



Figure 4.5: Schematic of the apparatus.

creating uniform force by the piston as well as reducing the quantity of cooked wood chips used since we didn't have a limitless supply. Moreover, it is assumed that there is no channeling or other wall effects, which was confirmed using ERT, for the experimental conditions tested. According to Winterberg and Tsotsas (2000), wall effects can generally be neglected if the diameter of the packed bed is greater than ten times the diameter of the packing particles.

With ERT, differences in electrical conductivity σ of the fluid are mapped



Figure 4.6: Image of the apparatus.

by conductivity sensors located on the periphery of the apparatus. With the tomograph used, an ITS P2000 (Industrial Tomography Systems, Manchester, UK), images are scanned along 8 different parallel planes, approximately 5.625 *cm* thick, to give an axial extent of approximately 45 *cm*. Each axial plane is made up of 20×20 pixels, 58 *mm*² formed from 16 conductivity probes separated by 22.5° along the circumference of the apparatus representing the interior cross-section of the vessel, for which some pixels lie on the outside of the vessel circumference. Before we process the conductivity signals, we need to convert the conductivity tomogram produced by ERT to a concentration tomogram since it is a 1:1 map as shown in Appendix G. We then processed the concentration signals to create two different reconstructed images. To help in this we define $C_e(x,y,z,t)$, where (y,z) are the coordinates in each individual ERT plane, to represent the experimental concentration of of the solute. In the first series of reconstructed images we transform the measurements in each plane to create a map $\langle C_e(x,y,t) \rangle$ from which we can examine the curvature in the advancing front. Specifically, $\langle C_e(x,y,t) \rangle$ is calculated using

$$\langle C_e(x,y,t)\rangle = \frac{1}{Z} \int_0^Z C_e(x,y,z,t) dz \tag{4.37}$$

where Z is the distance over which the image is averaged in the z-coordinate. In the second reconstructed images, we average the values at each plane to create a data point, i.e.

$$\langle\langle C_e(x,t)\rangle\rangle = \frac{1}{ZY} \int_0^Y \int_0^Z C_e(x,y,z,t) dy dz$$
 (4.38)

to estimate the extent of axial dispersion where *Y* is the distance over which the image is averaged in the y-coordinate.

To start an experiment, cooked wood chips at a prescribed κ were added carefully, to avoid chip fracture as will as to avoid compaction in the chip column, into the test column to the initial height of 0.45 m. The packing was performed randomly. Then the chip packed bed was compressed by the piston to a known load, somewhere in the range from 0 to 14 *kPa*. The level of the compressive pressure was chosen to achieve porosities in the range of 0.2 - 0.5. This is the typical range found in industry and we determined this through literature values as well as in a pre-trial. A 5.36×10^{-2} *M NaCl* salt solution ($T = 23 \,^{\circ}C$, $4 \, mS/cm$) was introduced by prescribing the inlet flowrate of 5 L/min and measuring the inlet pressure p_h . At t = 0, the valve was switched to the second tank, instead of the first tank, and a 2.52×10^{-2} *M NaCl* solution ($T = 23 \,^{\circ}C$, $2 \, mS/cm$) was introduced at the top of the column, at the same pressure and flowrate to displace the higher conductivity brine solution in the wood chip column. In this case a lower density solution displaces a higher density one to avoid any gravity currents. Table 4.3 shows a listing of the experimental conditions tested. Both limiting cases were tested using Hemlock wood chips cooked to three different κ . In total 12 different cases were tested and the conditions used are summarized in Table 4.3. These ranges covered those encountered in operating industrial digesters. Only the two limiting cases were tested. As shown cases under limiting conditions 1 is given when $\delta \rightarrow 1$; for the second limiting case $\gamma \rightarrow 0$.

To estimate the axial dispersion coefficient we formally define the case in which we want to minimize the function $S(D_e)$, which represents the L_2 norm of the difference between the experimental measurements and the solution of Equation 4.35. Formally, we state that we seek

$$\min S(D_e) = \|\bar{C}(\bar{x},\bar{t}) - \left\langle \left\langle \bar{C}_e(\bar{x},\bar{t}) \right\rangle \right\rangle \|$$
(4.39)

subject to

$$\frac{\partial \bar{C}}{\partial \bar{t}} = \frac{1}{Pe} \frac{\partial^2 \bar{C}}{\partial \bar{x}^2} - \frac{\partial \bar{C}}{\partial \bar{x}}$$
(4.40)

$$\bar{C}(0,\bar{t}) = \left\langle \left\langle \bar{C}_e(0,\bar{t}) \right\rangle \right\rangle \tag{4.41}$$

$$\bar{C}(1,\bar{t}) = \left\langle \left\langle \bar{C}_e(1,\bar{t}) \right\rangle \right\rangle \tag{4.42}$$

$$\bar{C}(\bar{x},0) = \left\langle \left\langle \bar{C}_{e}(\bar{x},0) \right\rangle \right\rangle \tag{4.43}$$

It should be noted that we are not using the (Danckwerts, 1953) conditions as indicated previously. Here we are using experimental data to represent both the boundary and initial conditions. By doing so we estimate D_e using the data from the six remaining ERT planes. This minimization problem was conducted using a MATLAB built-in function for unconstrained optimization, i.e. *fminbnd*. In addition Equation 4.40 was integrated using another built-in function in MATLAB, *pdepe*.

Limiting Case	к	т	p_c	p_h	L	ϵ_c	δ	γ	D_e	Pe	r^2
		(kPa)	(kPa)	(kPa)	(m)				(m^2/s)		
$p_c \gg p_h$	80	115	5.6	0.163	0.386	0.462	0.028	0.008	9.09×10^{-3}	0.43	0.997
	80	115	9.8	0.234	0.351	0.408	0.023	0.011	$2.01 imes 10^{-2}$	0.20	0.999
	80	115	14.0	0.391	0.330	0.370	0.027	0.016	$3.78 imes 10^{-2}$	0.11	0.999
	53	78	5.6	0.286	0.343	0.414	0.049	0.017	$9.18 imes 10^{-3}$	0.42	0.997
	53	78	9.8	0.545	0.309	0.354	0.053	0.030	3.31×10^{-2}	0.12	0.999
	53	78	14.0	1.736	0.277	0.280	0.110	0.078	$9.64 imes 10^{-2}$	0.05	0.999
	25	67	5.6	0.267	0.336	0.379	0.046	0.019	3.40×10^{-2}	0.12	0.999
	25	67	9.8	0.663	0.291	0.283	0.063	0.040	1.02×10^{-1}	0.05	1.000
	25	67	14.0	2.409	0.267	0.225	0.147	0.116	$2.16 imes 10^{-1}$	0.03	0.999
$m \gg p_h$	80	115	0.0	0.135	0.450	0.538	1.000	0.007	3.72×10^{-3}	1.06	0.989
	53	78	0.0	0.118	0.450	0.537	1.000	0.008	$3.17 imes 10^{-3}$	1.24	0.992
_	25	67	0.0	0.107	0.450	0.536	1.000	0.009	3.25×10^{-3}	1.21	0.992

Table 4.3: A summary of the experimental conditions tested with estimates of the axial dispersion coefficients.

4.4 **Results and Discussion**

We begin the presentation of the results by presenting a case representative of the phenomenon of the brine solution being displaced from the column. To aid in the clarity, we present a contour plot of $\langle C_e(x,y,t) \rangle$ at 6 different times, see Figures 4.7a - 4.7f. As shown, the column is initial one concentration and as time progresses, the more dense salt solution is swept vertically out through the bottom of the column. From these figures we see the evolution of the concentration and physically the dispersion coefficient represents the gradient of the concentration at the interface between the two fluids. In all cases measured, a larger concentration gradient existed in the vertical direction rather than in the horizontal direction; this indicates that we nearly achieved a one-dimensional displacement and wall effects are negligible. In all cases measured, channeling was not observed.

The evolution of the axial concentration profiles are shown for a number of cases in Figures 4.8a and 4.8b. Two dramatically different cases are shown in which in Figure 4.8b the porosity is much lower than in Figure 4.8a. What is evident in these images is that the concentration gradients for the conditions with the lower porosity are smaller than for those measured at the higher porosity. For the lower porosity case, the results appear seemingly like the idealized results of a well-stirred tank. The dashed lines in these figures represent the best fit for D_e as estimated using the optimization. We speculate that the results stem from the fact that there is greater tortuosity, created by more dead end channels, in the highly compressed bed. This may represent one source of the heterogeneity which is

found industrially.

Estimates of D_e for each case tested are shown in Table 4.3, along with the correlation coefficients. This is also shown in Figure 4.9a as a function of porosity for different kappa numbers. The trend in this data is similar to that indicated by Equation 2.20 and we find $a = 7.2 \times 10^{-4} m^2/s$ and b = -3.8. The estimates of D_e using Equation 2.20, with our experimentally determined a and b, are suitable for all industrial situations since the tested conditions covered those encountered in operating industrial digesters. We also report in Figure 4.9b that the axial dispersion is related to the inverse of the permeability, as measured in Chapter 3 where axial dispersion scales out with the inverse of the permeability.



Figure 4.7: Contour images of a representative ERT measurement with $\kappa = 80$, $p_c = 0$ kPa, and $\varepsilon = 0.54$.



Figure 4.8: The evolution of the concentration of the brine solution as a function of time at different elevations in the column. P1-P7 are ERT planes 1-7.


Figure 4.9: The estimated axial dispersion coefficient D_e with porosity ε and inverse of permeability k.

4.5 Summary and Conclusions

In this work we present a methodology to characterize axial dispersion through a porous compressible bed of wood chips while visualizing the flow using ERT. Here we demonstrate that under two limiting cases the porosity of the porous bed varies slowly in the flow-direction and to the lowest order can be considered a constant. This simplified the optimization routine we used to match the experimental data to the numerical results of the advection-diffusion equation. Using this, a methodology to estimate the axial dispersion is given by a minimization scheme. We find that for Hemlock chips, cooked to κ in the range 25-80, the axial dispersion followed an equation of the form $D_e = a\varepsilon^b$ where *a* and *b* were determined to be $7.2 \times 10^{-4} m^2/s$ and b = -3.8, respectively.

Chapter 5

Summary and Conclusions

In the first portion of this work we present a methodology to characterize the resistance to fluid motion through a porous compressible bed of wood chips. Here we advance the argument that under the limiting conditions of when the mechanical load is large in comparison to hydraulic pressure the equations of motion can be solved asymptotically to produce an expression approximating the variation in porosity along the length of the bed. With this, a methodology to estimate the resistance is given by multiple linear regression. In the experimental portion of the work we measured the resistance to flow of a bed of hemlock wood chips, cooked to three different κ , and compacted to three different p_c . With our methodology, we find that $r_1 = 3 \times 10^4 Pa.s/m^2$ and $r_2 = 1.9 \times 10^6 Pa.s^2/m^3$. The usefulness of these estimates was tested for the extreme case with no applied mechanical load. A reasonable prediction was obtained, thus verifying the utility of the methodology and the estimated resistances. After accounting for compressibility effects, we find that our correlation has lower flow resistances than the literature correlations.

In the second portion of this work we present a methodology to characterize axial dispersion through a porous compressible bed of wood chips while visualizing the flow using ERT. Here we demonstrate that under two limiting cases the porosity of the porous bed varies slowly in the flow-direction and to the lowest order can be considered a constant. This simplified the optimization routine we used to match the experimental data to the numerical results of the advection-diffusion equation. Using this, a methodology to estimate the axial dispersion is given by a minimization scheme. We find that for Hemlock chips, cooked to κ in the range 25-80 and compacted to p_c in the range $0 - 14 \, kPa$, the axial dispersion followed an equation of the form $D_e = a\varepsilon^b$ where a and b were determined to be $7.2 \times 10^{-4} \, m^2/s$ and -3.8, respectively.

Chapter 6

Recommendations for Future Work

The following recommendations are suggested for future work in flow uniformity on digester operations:

- Introduction of radial flow in the wood-chip column. This will allow for advancing 2-D/3-D models for estimating radial permeability and radial dispersion.
- Usage of high spatial resolution imaging techniques such as computed axial tomography (CAT-Scan) or nuclear magnetic resonance imaging (NMRI). This will allow for axial and radial porosity distribution measurements.
- CFD modeling, i.e. volume-averaged approach (RANS), with these new estimates of permeability and axial dispersion. This will account for compressibility effects when simulating the liquor flow through the digester.

• Industrial data of pressure drop and tracer studies are required to compare with our experimental data.

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Appendix A

Apparatus Design



Figure A.1: Schematic of the apparatus.



Figure A.2: Schematic of the top screen plate.



Figure A.3: Schematic of the top flange.



Figure A.4: Schematic of the top pipe section.



Figure A.5: Schematic of the pipe with ERT ring.



Figure A.6: Schematic of the ERT sensors.



Figure A.7: Schematic of the pipe with ERT and DP ring.



Figure A.8: Schematic of the bottom flange.



Figure A.9: Schematic of the ground ERT sensor.



Figure A.10: Schematic of the bottom screen plate.



Figure A.11: Schematic of the bottom cap.



Figure A.12: Side view of the apparatus.



Figure A.13: Top view of the apparatus.

Appendix B

Wood Chips Screening Analysis

Wood chips are produced by cutting the wood perpendicular to the grain direction with wedged knives. The average chip length, width, and thickness can vary due to the chipper setting depending on the structure of the wood. Common chip dimensions are 12 to 25 *mm* long, 12 to 25 *mm* wide, and 2 to 10 *mm* thick. Wood chips require screening before cooking to eliminate oversize pieces that cause heterogeneous delignification, and fines that easily overcook and increase flow resistance in liquor circulation (Gullichsen *et al*, 1992). Figure B.1 shows a common wood chip geometry and dimensions. Different thickness fractions were analyzed by randomly selecting 250 pieces of wood chips from the pile to determine their length, width, and thickness. The statistical analysis of different thickness fractions are tabulated in Table B.1 and B.2



Figure B.1: Typical chip geometry and dimensions.

Statistics	Length	Width	Thickness
	(mm)	(mm)	(mm)
Sample Size	250	250	250
Mean	21.21	14.69	2.76
Median	20.99	13.80	2.77
Mode	20.74	11.35	2.00
Standard Deviation	1.99	6.47	0.57
Sample Variance	3.97	41.89	0.32
Standard Error	0.13	0.41	0.04
Minimum	16.14	2.87	1.29
Maximum	31.89	39.73	4.08
Range	15.75	36.86	2.79
Sum	5302.88	3673.72	690.40
Confidence Level (95%)	0.25	0.80	0.07

Table B.1: Statistical analysis of 2 mm thickness fraction in accepts.



Figure B.2: 2mm thickness fraction in accepts.

Statistics	Length	Width	Thickness	
	(mm)	(mm)	(mm)	
Sample Size	250	250	250	
Mean	22.91	19.14	4.36	
Median	22.62	17.52	4.22	
Mode	22.04	17.95	4.03	
Standard Deviation	3.64	8.48	0.71	
Sample Variance	13.26	71.92	0.50	
Standard Error	0.23	0.54	0.04	
Minimum	2.17	4.95	2.21	
Maximum	50.63	57.78	6.38	
Range	48.46	52.83	4.17	
Sum	5726.92	4785.52	1090.25	
Confidence Level (95%)	0.45	1.05	0.09	

Table B.2: Statistical analysis of 4 mm thickness fraction in accepts.



Figure B.3: 4mm thickness fraction in accepts.

Appendix C

Calculation of Sauter Mean Diameter of Wood Chips

The method of Sauter mean diameter based on weight fraction was used to calculate the Sauter mean diameter d_{32} of wood chips.

Screen	Thickness	Width	Length	Mass Fraction
	(mm)	(mm)	(mm)	(x_i)
4 mm slot	$4.4(\pm 0.7)$	$19.1(\pm 8.5)$	$22.9(\pm 3.6)$	0.376
2 mm slot	$2.6(\pm 0.6)$	$14.7(\pm 6.5)$	$21.2(\pm 2.0)$	0.624

Table C.1: Average length, width, and thickness for mass fractions of accepts.

To determine d_{32} of wood chips (parallelepipedal shape), the specific surface

area S_v is used

$$S_{v} = \frac{A_{s}}{V_{p}} = \frac{2 \times (w \times t + w \times l + t \times l)}{l \times w \times t}$$
(C.1)

where A_s is the mean surface area of the particles, V_p is the volume of the particles, l is the mean length of the particles, w is the mean width of the particles, and t is the mean thickness of the particles.

For non-spherical particles, the equivalent particle diameter d_p can be written as (Comiti and Renaud, 1989):

$$d_p = \frac{6}{S_v} \tag{C.2}$$

For a wide range of size distribution, the Sauter mean diameter of particle can be given by (Coulson *et al*, 1978):

$$d_{32} = \frac{1}{\sum_{i} \frac{x_i}{d_{p,i}}}$$
(C.3)

Appendix D

Porosity Measurement

In the two-phase test column and at no flow, cooked chips occupy the volume fraction ε_{chip} and the remaining interstitial space is filled with liquid. This liquid occupies the characteristic void fraction (porosity) ε_c . As a result

$$\varepsilon_c + \varepsilon_{chip} = 1$$
 (D.1)

The basic density of uncooked wood chips ρ_{chip} was determined using a water displacement method (CPPA standard A.8P) to be 398.27 (oven dry kg)/(m^3 green volume). To account for the decreased density after cooking due to the loss of lignin, the chip density was multiplied by the cook yield Y to determine the cooked wood chips density ρ_{chip}^{cook}

$$\rho_{chip}^{cook} = Y \rho_{chip} \tag{D.2}$$

After adding the cooked wood chips to the test column, they have an initial bulk

packing density of ρ_b^o (volume occupied by the oven-dry mass of cooked wood chips added). To account for compression of the cooked chip column, a bulk packing density of ρ_b^p was used which varies according to Equation D.3

$$\rho_b^p = \rho_b^o(\frac{L_o}{L_p}) \tag{D.3}$$

where L_o is the initial bed length before compressing the cooked wood chips, and L_p is the bed length after compressing the cooked wood chips. The volume fraction of cooked wood chips becomes

$$\varepsilon_{chip} = \frac{\rho_b^p}{\rho_{chip}^{cook}} \tag{D.4}$$

By substituting Equation D.4 into Equation D.1 and rearranging to get

$$\varepsilon_c = 1 - \frac{\rho_b^p}{\rho_{chip}^{cook}} \tag{D.5}$$

or

$$\varepsilon_c = 1 - \frac{\rho_b^o(\frac{L_o}{L_p})}{\gamma \rho_{chip}} \tag{D.6}$$

Appendix E

Pneumatic Cylinder Desgin

Below shows the conversion from air pressure to actual force on the chip column

based on the area of the piston.

Power Factor (<i>PF</i>)	0.89
Piston Diameter (m)	$2.70 imes 10^{-2}$
Piston Area $A_{piston}(m^2)$	$5.72 imes 10^{-4}$
Compaction Plate Diameter (m)	0.1524
Compaction Area $A_c(m^2)$	1.820×10^{-2}

Air Line Pressure	Air Line Pressure	Cylinder Output Force	Compaction Pressure
(bar)	(kPa)	(N)	(kPa)
2.0	200	101.9	5.6
3.5	350	178.4	9.8
5.0	500	254.8	14.0

Tabl	e E	.1:	Calcu	lation	of	compaction	pressure.
------	-----	-----	-------	--------	----	------------	-----------

The pneumatic cylinder output force F can be given by

$$F = p_{air} \times PF \times A_{piston} \tag{E.1}$$

where p_{air} is the air line pressure, PF is the power factor (efficiency) of the pneumatic cylinder, and A_{piston} is the piston area.

The compaction pressure can be given then by

$$p_c = \frac{F}{A_c} = 0.028 \, p_{air} \tag{E.2}$$

where A_c is the compaction area. Figure E.1 shows the relationship between compaction pressure and air line pressure.


Figure E.1: Relationship between compaction pressure and air line pressure. Solid line is given by Equation E.2.

Appendix F

Pressure Drop Across Screen Plate

Before the test run, we need to determine pressure drop across the screen plate by measuring the screen plate resistance at empty column with different flow rates. The pressure drop across the screen plate as a function of superficial velocity is shown in Figure F.1. By simple curve fitting of the data using linear regression in MATLAB we determine the Equation of the fitting line to calculate pressure drop across the screen plate as a function of superficial velocity

$$dp_{screen}(kPa) = 0.0025v(mm/s) \tag{F.1}$$

During the test, the differential pressure transducer measures the pressure drops including; screen plate, hydrostatic water, and bed. In order to obtain the pressure drop across the bed only, we need to subtract pressure drop across screen plate, and hydrostatic pressure drop of water from the measured pressure drop.



Figure F.1: Pressure drop across the screen plate. Dashed line is given by linear regression fit in MATLAB to give Equation F.1.

Appendix G

Conductivity to Concentration Tomogram

Before the test run, we need to convert the conductivity tomogram produced by ERT to a concentration tomogram by measuring the conductivity of *NaCl* brine solution with different brine concentrations. The brine concentration as a function of brine conductivity is shown in Figure G.1. By simple curve fitting of the data using linear regression in MATLAB we determine the Equation of the fitting line to calculate brine concentration as a function of brine conductivity

$$C(M) = 0.0142\sigma (mS/cm) - 0.0032$$
 (G.1)



Figure G.1: Concentration of Brine Solution. Dashed line is given by linear regression fit in MATLAB to give Equation G.1.

Appendix H

Higher Order Estimates of $\varepsilon(x)$

We can extend the accuracy of the analysis in Chapter 3 by extending the number of terms of the form of the solution. If

$$\bar{\varepsilon} = \bar{\varepsilon}^{(0)} + \delta \bar{\varepsilon}^{(1)} + \delta^2 \bar{\varepsilon}^{(2)} \dots$$
(H.1)

$$\bar{r}_i = \bar{r}_i^{(0)} + \delta \bar{r}_i^{(1)} + \delta^2 \bar{r}_i^{(2)} \dots$$
 (H.2)

the $O(\delta^2)$ terms must satisfy

$$\begin{cases} -\bar{\varepsilon}^{(1)}\bar{f}_{3}^{\prime\prime}(1)\frac{d\bar{\varepsilon}^{(1)}}{d\bar{x}} - \bar{f}_{3}^{\prime}(1)\frac{d\bar{\varepsilon}^{(2)}}{d\bar{x}} = \\ \bar{\varepsilon}^{(1)}\left(\bar{r}_{1}^{(0)}\bar{f}_{1}^{\prime}(1) + \bar{r}_{2}^{(0)}\bar{f}_{2}^{\prime}(1)\right) + \bar{r}_{1}^{(1)} + \bar{r}_{2}^{(1)} \end{cases}$$
(H.3)

subject to

$$\bar{\boldsymbol{\varepsilon}}^{(2)}(0) = -\frac{1}{2} \frac{\bar{f}_{3}^{\prime\prime}(1)}{\left(\bar{f}_{3}^{\prime}(1)\right)^{3}} \qquad \qquad \bar{\boldsymbol{\varepsilon}}^{(2)}(1) = 0 \qquad (\text{H.4})$$

This relationship can be solved to yield

$$\bar{\varepsilon}^{(2)} = -\frac{1}{2} \frac{(\bar{x} - \bar{x}^2)}{\left(\bar{f}_3'(1)\right)^2} \left(\bar{r}_1^{(0)} \bar{f}_1'(1) + \bar{r}_2^{(0)} \bar{f}_2'(1)\right) - \frac{\bar{f}_3''(1)}{2\left(\bar{f}_3'(1)\right)^3} (1 - 2\bar{x} + \bar{x}^2) \quad (\text{H.5})$$

Appendix I

Summary of the Required Functions

Here is a summary of the required functions in Chapters 3 and 4:

$$\bar{f}_1(\bar{\varepsilon}) = \left(\frac{1 - \varepsilon_c \bar{\varepsilon}}{1 - \varepsilon_c}\right)^2 \frac{1}{\bar{\varepsilon}^3} \tag{I.1}$$

$$\bar{f}_2(\bar{\varepsilon}) = \left(\frac{1 - \varepsilon_c \bar{\varepsilon}}{1 - \varepsilon_c}\right) \frac{1}{\bar{\varepsilon}^3} \tag{I.2}$$

$$\bar{f}_3(\bar{\varepsilon}) = \left(\frac{\bar{\varepsilon}_g - \bar{\varepsilon}}{\bar{\varepsilon}_g - 1}\right)^n \tag{I.3}$$

$$\bar{f}_1'(1) = \frac{\varepsilon_c - 3}{1 - \varepsilon_c} \tag{I.4}$$

$$\bar{f}_2'(1) = \frac{2\varepsilon_c - 3}{1 - \varepsilon_c} \tag{I.5}$$

$$\bar{f}'_{3}(1) = -\frac{n}{\bar{\epsilon}_{g} - 1}$$
 (I.6)

$$\bar{f}_{3}^{\prime\prime}(1) = \frac{n(n-1)}{(\bar{\varepsilon}_{g}-1)^{2}}$$
(I.7)

Appendix J

MATLAB Codes

J.1 Compressibility Function Regression Code

The code written in MATLAB 7.5 for the compressibility function regression is included here. The main file is:

• Ps_regression.m

Program J.1 Main Program: Ps_regression.m

```
% Author: Mohammed Alaqqad (2011)
% Description: This is a program that correlates the
% compressibility as a function of void fraction.
% The method used for the correlation is least
% square method (Linear regression).
% In the form Ps=m(voidg-void) ^n
% Clear History
clear all
% kappa(voidc, Pc(kPa), k1(80)-k2(53)-k3(25))
kappa = zeros(4, 2, 3);
kappa(:,:,1) = [0.538 \ 0.0;
              0.462 5.6;
              0.416 9.8;
              0.372 14.0];
kappa(:,:,2) = [0.537 \ 0.0;
              0.414 5.6;
              0.354 9.8;
              0.280 14.0];
kappa(:,:,3) = [0.536 0.0;
              0.397 5.6;
              0.316 9.8;
              0.250 14.0];
[npx npy npe] = size(kappa);
for i = 1:npe
   for j=2:npx
       xdata(j-1) = log(kappa(1,1,i)...
       - kappa(j,1,i));
       ydata(j-1) = log(kappa(j, 2, i) *1000);
   end
   p = polyfit(xdata, ydata, 1);
   fitP(i,:) = [p(1), exp(p(2))/1000, kappa(1,1,i)];
```

```
EpsI(i,:) = linspace(kappa(1,1,i),...
    kappa(end, 1, i), 100);
    psI(i,:) = exp(p(2)) * (kappa(1,1,i)...
    - EpsI(i,:)).^p(1);
end
hold on
box on
fitP
plot(kappa(:,1,3), kappa(:,2,3),'sk',kappa(:,1,2),...
kappa(:,2,2),'vk',kappa(:,1,1), kappa(:,2,1),'ok')
xlabel('Porosity \epsilon','Fontsize',16)
ylabel('P_s (kPa)', 'Fontsize', 16)
legend('\kappa = 25','\kappa = 53','\kappa = 80')
plot(EpsI(1,:), psI(1,:)/1000,'--k',EpsI(2,:),...
psI(2,:)/1000,'--k',EpsI(3,:), psI(3,:)/1000,'--k');
axis([0.2 0.7 0 18])
% use this command in all figures to change all
% legends and axis etc to this
% font size
set(qca,'fontsize',16)
mdata = fitP(:, 2)
kdata = [80 53 25]'
p = polyfit(log(kdata),mdata,1)
kI = linspace(kdata(1), kdata(end), 50);
%mI = polyval(p,log(kI));
mI = (0.0774 - 0.0129 \cdot \log(kI)) \cdot (-1.2)
figure(2)
plot(kdata, mdata, 'ok', kI, mI, '--k')
ylabel('m (kPa)','Fontsize',16)
xlabel('\kappa','Fontsize',16)
axis([20 86 50 120])
set(gca,'fontsize',16)
figure(3)
mI = (0.214 - 0.0357 \cdot log(kI)) \cdot (-1.7);
```

```
plot(kI,mI,'-k')
ylabel('m (kPa)','Fontsize',16)
xlabel('\kappa','Fontsize',16)
axis([20 86 50 130])
set(gca,'fontsize',16)
```

J.2 Pressure Drop Regression Code

The code written in MATLAB 7.5 for the pressure drop regression is included here. The main file is:

• Ph_regression.m

Program J.2 Main Program: Ph_regression.m

```
% Author: Mohammed Alaqqad (2011)
% Description: This is a program that correlates the
% pressure as a function of velocity and void
% fraction. The method used for the correlation is
% least square method (Multiple Linear regression).
% USER NEEDS TO SPECIFY/INPUT THE NECESSARY
% PARAMETERS AND FILENAME:-
% name
       : The name of the data set (Pressure Drop)
% datafile: The file name that the data store
% (pressuredrop.txt)
% Clear History
clear all
name='Pressure Drop';
datafile='dp_regression.txt';
% Clear Matlab command window
clc
disp('In progress...')
% Get data from text file
% column 1 column 2
                  column 3 column 4
% column 5
         column 6
% voidc
         U(mm/s) DP/L(Pa/m)
                           Ph (Pa)
00
    k
           Pc (kPa)
% fid =fopen(filename, permission)
fid = fopen(datafile,'r');
[x1 x2 x3 x4 x5 x6 x7]=textread...
```

```
(datafile,'%f%f%f%f%f%f%f%f');
status=fclose(fid);
% Determine number of data points
count=length(x1);
voidc=x1;
% Convert velocity from (mm/s) to (m/s)
u=x2./1000;
dp=x3;
Ph=x4;
% Convert Pc from (kPa) to (Pa)
kappa=x5;
Pc=x6.*1000;
%ne=x7;
ne=1.2;
% viscosity of liquid (Pa.s)
mu=0.000925;
% density of liquid (kg/m3)
ro=997.3;
% equivalent particle diameter (m)
d=0.00714;
% sphericity ()
sp=0.556;
delta=Ph./Pc;
voidg=0.54;
term1=(1+((delta./2).*((3-voidc)./(1-voidc)).*...
    ((voidg-voidc)./(voidc.*ne)))).*...
    (((1-voidc).^2)./(voidc.^3)).*u;
term2=(1+((delta./2).*((3-2.*voidc)./(1-voidc))...
    .*((voidg-voidc)./(voidc.*ne)))).*((1-voidc)...
    ./(voidc.^3)).*(u.^2);
% Create the x matrix from these vectors
% using a for loop
```

```
% The dependent variable goes into a n*1
% column vector.
for n=1:count
    x(n, 1) = term1(n);
    x(n, 2) = term 2(n);
    y(n, 1) = dp(n);
end
% Use the matrix division operation.
a=(x' * x) \setminus (x' * dp);
A=(a(1)*(d^2*sp^2))/mu;
B=(a(2) * (d*sp))/ro;
%Harkonen correction for r1 and r2
mu_h=0.000068;
ro_h=885;
r1_h=(A*mu_h)/(d^2*sp^2);
r2_h=(B*ro_h)/(d*sp);
%z=inv(x' *x);
% Calculate the values of the pressure drop
% predicted by the equation.
% Then calculate the difference between
% the experimental and predicted values
% dppred == dp predicted
% res == residuals
% diff == percent different
dppred =x*a;
res=(dp-dppred);
diff=abs(res./dp)*100;
% Statistical Analysis
% sum-of-squares terms
% sum of squares, error (SSE)
SSE=0; % initialize
```

```
sumres=0;
for n=1:count
    SSE=SSE+res(n) *res(n);
    sumres=sumres+res(n);
end
MeanErr=abs(sumres./count);
% Sum of squares, regression (SSR)
avedppred=0;
SSR = 0;
for n=1:count
    avedppred=avedppred+dppred(n);
end
avedppred=avedppred./count;
for n=1:count
    SSR=SSR +(dppred(n)-avedppred).^2;
end
% Sum of squares, total (SST)
avedp=0;
SST=0;
for n=1:count
    avedp=avedp+dp(n);
end
avedp=avedp./count;
for n=1:count
    SST=SST+(dp(n)-avedp).^ 2;
end
SST;
% R-squared
R2=1-(SSE./SST);
R22=SSR./SST;
% Adjusted R-squared
```

```
R2adj=1-(count-1) \cdot (1-R2) \cdot (count-2);
% Residual Mean Square (MSE)
% K = total number of predictors or number of
% independent variables
% n = sample size
n=count;
K=2;
MSE=SSE./(count-K-1);
% Standard Error of the Estimate (SEE)
SEE = sqrt (MSE);
% 95% Confidence Interval
[b, bint] = regress(dp, x);
c1=bint(1,2)-a(1);
c2=bint(2,2)-a(2);
c12=(c1*(d^2*sp^2))/mu;
c22=(c2*(d*sp))/ro;
c12_h = (c12 * mu_h) / (d^2 * sp^2);
c22_h=(c22*ro_h)/(d*sp);
%Sa1=sqrt(z(1,1)*MSE);
%Sa2=sqrt(z(2,2)*MSE);
%talpha_554=1.964255187;
%t1=talpha_554*Sa1;
%t2=talpha_554*Sa2;
%L1=a(1)-talpha_554*Sa1;
%U1=a(1)+talpha_554*Sa1;
%L2=a(2)-talpha_554*Sa2;
%U2=a(2)+talpha_554*Sa2;
% Create output report
disp('')
disp(' ========')
disp('Author:Mohammed Alaggad')
fprintf('Run at:%s\n', datestr(now))
fprintf('Title: Analysis of %s\n',name)
disp('Equation: y=a1*x1+a2*x2')
disp('Method: Multiple Linear Regression')
```

```
disp('')
disp('')
disp('')
disp('========')
fprintf('Total numbers of data points (n) = f n', count)
fprintf(' \ Durits for dP/L (Pa/m) and v (m/s)')
fprintf('\n The parameters r1 and r2 are\n')
%fprintf('a1=%9.2f\n', b(1))
%fprintf('a2=%9.2f\n\n',b(2))
fprintf('r1=%9.2f+-%9.2f\n',a(1),c1)
fprintf('r2=%9.2f+-%9.2f\n\n',a(2),c2)
fprintf('A=%9.2f+-%9.2f\n',A,c12)
fprintf('B=%9.2f+-%9.2f\n\n',B,c22)
fprintf('r1_h=%9.2f+-%9.2f\n',r1_h,c12_h)
fprintf('r2_h=%9.2f+-%9.2f\n\n',r2_h,c22_h)
fprintf('Void Velocity ExptdP PreddP Percent Diff Residuals\n')
fprintf('
               [mm/s] [Pa/m]
                             [Pa/m] \setminus n'
for n= 1 :count
fprintf('%4.3f %6.2f %5.2f %5.2f %5.1f %6.2f\n',...
   voidc(n), u(n) * 1000, dp(n), dppred(n) \dots
   ,diff(n),res(n) ./1000)
end
disp('')
disp('')
disp('STATISTICAL ANALYSIS')
disp('-----')
fprintf('Sum of Squares,Error(SSE)=%f\n',SSE)
fprintf('Mean of Residuals Error=%f\n',MeanErr)
fprintf('sum of Square, Total(SST) =%f\n', SST)
fprintf('Sum of Squares, Regression(SSR) =%f\n', SSR)
fprintf('Coefficient of Determination (R^2) = f n', R^2)
fprintf('Adjusted R-squared=%f\n',R2adj)
fprintf('Standard Error of Estimate(SEE) =%f\n', SEE)
% Save the data into text file
R=[u dp dppred];
save results.txt R -ASCII -TABS
```

```
% Plot the data
figure
plot(u.*1000,dp./1000,'*r',u.*1000,dppred./1000,'+k')
xlabel('Velocity m/s')
ylabel('Pressure drop Pa/m')
str =['Experimental Data'];
str1 =['Correlated Data'];
legend(str,str1,'Location','best')
%xmin=0; xmax=10; ymin=0; ymax=20;
%axis([xmin xmax ymin ymax])
```

```
fprintf('\n\n *****End of Analysis*****\n\n')
```

J.3 Pressure Drop Prediction Code

The codes written in MATLAB 7.5 for the pressure drop prediction are included here. The main files are:

- Main.m
- Valid_Pc0.m

Program J.3 Main Program: Main.m

```
% Author: Mohammed Alaqqad (2011)
% Description: This is a program that predict the
% pressure as a function of velocity and void
% fraction. The method used for the prediction is
% numerical solution using ode15s.
% USER NEEDS TO SPECIFY/INPUT THE NECESSARY
% PARAMETERS AND FILENAME:-
% name
       : The name of the data set (Pressure Drop)
% datafile: The file name that the data store
% (valid.txt)
8****
% Clear History
clear all
name='Pressure Drop';
datafile='valid.txt';
% Clear Matlab command window
clc
disp('In progress...')
% Get data from text file
% column 1 column 2 column 3
                           column 4
% v(mm/s)
        DP/L(Pa/m)
                   m(kPa)
                            n
% fid =fopen(filename, permission)
fid = fopen(datafile,'r');
[v dP m n]=textread(datafile,'%f%f%f%f');
v=v./1000; dp=dP.*1000; m=m.*1000;
status=fclose(fid);
```

```
% find the data for the three different kappa numbers
mexp = [115 \ 78 \ 67] \times 1000;
nexp = [1.3 \ 1.2 \ 1.2];
nexp = [1.2 \ 1.2 \ 1.2];
np = 100;
for i=1:3
    IDX = find(m == mexp(i));
    RNG = [IDX(1) IDX(end)];
    vfit = linspace(v(RNG(1)), v(RNG(end)), np);
    for j = 1:np
       dPdx(j)=Valid_Pc0([vfit(j),mexp(i),nexp(i)]);
    end
    figure(i)
    midP = round(mean(RNG));
    r1=3e4;
    r2=3.3e6;
    voidc=0.54;
    term1=(((1-voidc)^2)/(voidc^3)).*vfit;
    term2=((1-voidc)/(voidc^3)).*(vfit.^2);
    % without compressibility effects
    dp_model=r1*term1+r2*term2;
    plot(v(RNG(1):RNG(2))*1000, dP(RNG(1):RNG(2))...
        /1000,'ok',vfit*1000,dPdx/1000,'--k',...
        vfit*1000,dp_model/1000,'-k')
    xlabel('v (mm/s)','fontsize',16)
    ylabel('P_h/L (kPa/m)','fontsize',16)
    axis([0 10 0 1.5])
    legend('Measured','Predicted',...
    'No Compressibility Effects', 'location', 'best')
    % use this command in all figures to change all
    % legends and axis etc to this font size
    set(gca,'fontsize',16)
```

```
end
```

```
function phl=Valid_Pc0(D)
global epsg v m n r1 r2
v = D(1); m = D(2); n = D(3);
%v = 8/1000;
epsg = 0.54;
r1 = 3e4; r2 = 1.9e6; %r2 = 1.5e6;
L = 0.45;
[x,y] = ode15s(@porous,[L 0],0);
phl = y(end)/L;
function dy = porous(x, y)
global v r1 r2
dy(1) = -(r1*f1(y)*v + r2*f2(y)*v^2);
function yout = f1(y)
global epsg m n
if y<0, y=0; end
epsilon = epsg - (y/m)^{(1/n)};
yout = (1-epsilon)^2/(epsilon)^3;
function yout = f2(y)
global epsg m n
if y<0, y=0; end
epsilon = epsg - (y/m)^{(1/n)};
yout = (1-epsilon)/(epsilon)^3;
```

J.4 Axial Dispersion Minimization Code

The codes written in MATLAB 7.5 for the axial dispersion estimation are included here. The main files are:

- Main.m
- Importfile.m
- Inputdata.m
- Pdetest.m

Program J.5 Main Program: Main.m

```
% Author: Mohammed Alaqqad (2011)
% Description: This is a program that estimates the
% breakthrough curves for a convective-diffusion
% equation of the form
%
% C_T = 1/Pe∗C_XX - C_X [T,X] \in [0,1]
%
% T=0
       C = 0
% X=1
       (C-1) - C_X = 0
% X=0
       C_X = 0
% The method used for the estimation is minimization
% between the experimental data and the numerical
% solution using fminsearch and pdepe.
% clear history
clear all
% Clear Matlab command window
clc
disp('In progress...')
global Xe Te Ce Ci frames_c x_planes
% Get data from text file and modify
Inputdata
% Set initial values of Dax
Dax=1.8e-4;%m2/s
% Use the optimization function to find Dax
Dr=fminsearch(@Pdetest,Dax)
[rows columns]=size(Ce);
count= rows;
```

```
count2=columns*rows;
% Calculate the values of the concentration
% predicted by the equation.
% Then calculate the difference between the
% experimental and predicted values .
% Ce == C experimnetal
% Ci == C predicted
% res == residuals
% diff == percent different
res=(Ce-Ci);
diff=abs(res./Ce).*100;
% Statistical Analysis
% sum-of-squares terms
% sum of squares, error (SSE)
SSE=0; % initialize
sumres=0;
for n=1:count
    SSE=SSE+sum(res(n,:).*res(n,:));
    sumres=sumres+sum(res(n,:));
end
MeanErr=abs(sumres./count2);
% Sum of squares, regression (SSR)
aveCi=0;
SSR = 0;
for n=1:count
    aveCi=aveCi+sum(Ci(n,:));
end
aveCi=aveCi./count2;
for n=1:count
    SSR=SSR +sum((Ci(n,:)-aveCi).^2);
end
```

```
% Sum of squares, total (SST)
aveCe=0;
SST=0;
for n=1:count
    aveCe=aveCe+sum(Ce(n,:));
end
aveCe=aveCe./count2;
for n=1:count
    SST=SST+sum((Ce(n,:)-aveCe).^ 2);
end
SST;
% R-squared
R2=1-(SSE./SST);
R22=SSR./SST;
% Adjusted R-squared
R2adj=1-(count2-1).*(1-R2)./(count2-2);
% Residual Mean Square (MSE)
% K = total number of predictors or number of
% independent variables
% n = sample size
n=count2;
K=1;
MSE=SSE./(n-K-1);
% Standard Error of the Estimate (SEE)
SEE = sqrt(MSE);
% Create output report
disp('')
disp(' =========')
disp('Author:Mohammed Alaqqad')
fprintf('Run at:%s\n', datestr(now))
fprintf('Title: Analysis of %s\n','Axial Diffusion')
disp('Equation: 1D convective-disffusive model')
disp('Method: Solve and minimize')
```

```
disp('')
disp('')
disp('')
disp('=========')
fprintf('Total numbers of data points (n) = f n' \dots
    , count2)
fprintf(' \ D \ (m2/s)')
fprintf(' \ n \ The \ parameters \ Dax \ is \ n')
fprintf('Dax=%9.6f\n',Dr)
disp('')
disp('')
disp('STATISTICAL ANALYSIS')
disp('-----')
fprintf('Sum of Squares,Error(SSE)=%f\n',SSE)
fprintf('Mean of Residuals Error=%f\n',MeanErr)
fprintf('sum of Square, Total(SST) =%f\n', SST)
fprintf('Sum of Squares, Regression(SSR) =%f\n', SSR)
fprintf('Coefficient of Determination(R^2)=%f\n',R2)
fprintf('Adjusted R-squared=%f\n',R2adj)
fprintf('Standard Error of Estimate(SEE)=%f\n',SEE)
figure1=figure;
hold on
box on
plot(Te',Ce(:,2),'--k',Te',Ci(:,2),'-k')
legend('Measured', 'Regression', 'Location', 'best')
%title('Solution at X=0 to 1')
plot(Te',Ce,'--k',Te',Ci,'-k')
errorbar(Te(24),Ce(24,2),0.02,'k')
xlabel('Dimensionless Time T', 'fontsize',16)
ylabel('Dimensionless Concentration C','fontsize',16)
axis tight
set(gca,'fontsize',16)
% Create textbox
annotation(figure1,'textbox','String',{'P1'},...
    'FontName', 'Times New Roman', ...
   'FitBoxToText','off',...
```

```
'LineStyle', 'none',...
    'Position', [0.1798 0.67 0.03858 0.06616]);
% Create textbox
annotation(figure1,'textbox','String',{'P2'},...
    'FontName', 'Times New Roman', ...
    'FitBoxToText','off',...
    'LineStyle', 'none',...
    'Position', [0.1874 0.5613 0.03858 0.06616]);
% Create textbox
annotation(figure1,'textbox','String',{'P3'},...
    'FontName', 'Times New Roman', ...
    'FitBoxToText','off',...
    'LineStyle', 'none',...
    'Position', [0.1954 0.4744 0.03858 0.06616]);
% Create textbox
annotation(figure1,'textbox','String',{'P4'},...
    'FontName', 'Times New Roman', ...
    'FitBoxToText','off',...
    'LineStyle','none',...
    'Position', [0.2215 0.2671 0.03858 0.06616]);
% Create textbox
annotation(figure1,'textbox','String',{'P5'},...
    'FontName', 'Times New Roman',...
    'FitBoxToText','off',...
    'LineStyle', 'none',...
    'Position', [0.2075 0.4046 0.03858 0.06616]);
% Create textbox
annotation(figure1,'textbox','String',{'P6'},...
    'FontName', 'Times New Roman', ...
    'FitBoxToText','off',...
    'LineStyle', 'none',...
    'Position', [0.2116 0.3217 0.03858 0.06616]);
```

```
% Create textbox
annotation(figure1,'textbox','String',{'P7'},...
'FontName','Times New Roman',...
'FitBoxToText','off',...
'LineStyle','none',...
'Position',[0.2436 0.2022 0.03858 0.06616]);
```

Program J.6 Import File: Importfile.m

```
function Importfile(fileToRead1)
%IMPORTFILE (FILETOREAD1)
  Imports data from the specified file
%
00
  FILETOREAD1: file to read
% Import the file
rawData1 = importdata(fileToRead1);
% For some simple files (such as a CSV or JPEG
% files), IMPORTDATA might return a simple array.
% If so, generate a structure so that the output
% matches that from the Import Wizard.
[unused,name] = fileparts(fileToRead1); %#ok
newData1.(genvarname(name)) = rawData1;
% Create new variables in the base workspace from
% those fields.
vars = fieldnames(newData1);
for i = 1:length(vars)
    assignin('base', vars{i}, newDatal.(vars{i}));
end
```

Program J.7 Input and Modify Data: Inputdata.m

```
% Importing the data file where the first column
% contains the time data with rows equal to the
% number of frames.
% The second column till the end contains the
% concentration data where each column corresponds
% to one frame that contains the eight images
% sequentially in 2528 rows.
% Import data
Importfile('test.txt');
% return rows and columns
[Rows Columns] = size(test);
% number of frames (starts from second column)
frames=Columns-1;
% array of frames
for f=1:frames
    fr(f) = f;
end
% array fo time data
for f=1:frames
    t(f) = test(f, 1);
end
% extract only the concentration data
for c=2:frames+1
    data(:,c-1)=test(:,c);
end
% averaging data over z and y to get Cee(x,f)
x_planes=7;
start_plane=1;
```

```
end_plane=(x_planes-1)+start_plane;
adj=start_plane-1;
for f=1:frames
    for x=start_plane:end_plane
        x^2=x-adj;
        for i=1+316*(x-1):316*x;
             j=i-316 \star (x-1);
        m(j, x2, f) = data(i, f);
        end
        Cee(x2, f) = mean(m(:, x2, f));
    end
end
% normalizing the data after finding Co (initial)
% and Cf (final) for each pixel.
% average frames before fo and after ff
fo=20;
ff=250;
 for x=start_plane:end_plane
     x=x-adj;
     for f=1:fo
          dataCo(x, f) = Cee(x, f);
     end
     for f=ff:frames
          dataCf(x, f-(ff-1)) = Cee(x, f);
     end
     Co(x, 1) = mean(dataCo(x, :));
     Cf(x, 1) = mean(dataCf(x, :));
 end
 for f=1:frames
     Cen(:,f) = (Cee(:,f) - Co)./(Cf-Co);
 end
 % cropping the data
```

```
fco=44;
fcf=125;
frames_c=fcf-fco+1;
fr_c=[1:1:frames_c];
for f=1:(fcf-fco+1)
    Cec(:,f)=Cen(:,((f-1)+fco));
end
% flip the data
for x=start_plane:end_plane
    x=x-adj;
    Cer(x,:)=Cec(((x_planes+1)-x),:);
end
% transpose of data
Ce=Cec';
```

Program J.8 PDE Solution: Pdetest.m

```
function res=Pdetest(D)
global Pe Xm Tm Xe Te Ce Ci frames_c x_planes
% Input:
% Xf = grid size
% Tf = maximum time
% np = number of grid points
%D=1.8e-4;∖
void=0.538;
v = 0.00474/void; %m/s
L=0.450; %m
Pe=(v*L)/D;
%Pe=10;
ti=0; %s
xi=0; %m
tf=L/v; %s tow, L/(Q/A), A*L/Q, V/Q
xf=L; %m
Ti=ti*v/L;
Xi=xi/L;
Tf = (tf * v/L) * 3;
Xf=xf/L;
np=40;
m = 0;
X = linspace(Xi,Xf,np);
T = linspace(Ti,Tf,np);
Tm=T';
Xm=X;
```
```
Te=(linspace(Ti,Tf,frames_c))';
Xe=linspace(Xi,Xf,x_planes);
sol = pdepe(m,@pdex1pde,@pdex1ic,@pdex1bc,X,T);
% Extract the first solution component as C(T,X)
C = sol(:,:,1);
% Interpolate the solution to match our Xe and Te
Ci=interp2(Xm,Tm,C,Xe,Te);
size(Ci);
D
% Residuals
res=sum(sum(((Ci-Ce).^2)))
٥٥ ------
function [c, f, s] = pdex1pde(X, T, C, DCDX)
global Pe
c = 1;
f = (1/Pe) * DCDX;
s = -DCDX;
e _____
function CO = pdexlic(X)
C0 = 0;
                 _____
8 _____
function [pl,ql,pr,qr] = pdex1bc(Xl,Cl,Xr,Cr,T)
global Te Ce
Cleft=interp1(Te,Ce(:,1),T);
Cright=interp1(Te,Ce(:,end),T);
pl = Cl-Cleft;
ql=0;
pr= Cr-Cright;
qr = 0;
```

Appendix K

Experimental Data

K.1 Pressure Drop Data

Condition		p	polypropylene spheres		
Volume of col	umn (m^3)	8.	8.209×10^{-3}		
Mass of polyp	ropylene spł	neres (kg)) 4.	4284	
Bulk density (kg/m^3)		5.	39.48	
Polypropylene	spheres der	nsity (<i>kg</i> /	m^3) 90	05.00	
Porosity at no	flow (fractio	on)	0.	404	
	Flow rate	p_h	L	v	p_h/L
	(L/min)	(Pa) $(m$		(mm/s)	(Pa/m)
	0.00	0.00	0.450	0.00	0.00
	1.80	9.51	0.450	1.65	21.14
	3.25	19.95	0.450	2.97	44.33
	4.67	35.43	0.450	4.27	78.74
	5.76	54.65	0.450	5.26	121.44
	7.22	74.50	0.450	6.60	165.57
	8.57	95.65	0.450	7.83	212.56
	10.49	134.21	0.450	9.59	298.25
	12.02	160.21	0.450	10.98	356.01

204.20

230.61

13.79

14.30

 Table K.1: Pressure drop data of polypropylene spheres.

0.450

0.450

12.60

13.07

453.78

512.46

Condition	100% accepts
Kappa number	80
Compaction Pressure (<i>kPa</i>)	0.0
Volume of column (m^3)	$8.209 imes 10^{-3}$
Mass of O.D. cooked chips (kg)	0.8133
Initial cooked bulk density (kg/m^3)	99.08
Yield (%)	53.86
Basic chip density (kg/m^3)	398.27
Cooked chip density (kg/m^3)	214.51
Porosity at no flow (fraction)	0.538

Flow rate	p_h	dp_1	dp_2	dp_3	dp_4	L	V	p_h/L
(L/min)	(Pa)	(Pa)	(Pa)	(Pa)	(Pa)	(m)	(mm/s)	(Pa/m)
0.00	0.000	0.000	0.000	0.000	0.000	0.450	0.00	0.00
1.82	30.843	7.679	7.532	7.963	7.669	0.450	1.66	68.54
3.30	69.305	17.378	17.169	17.589	17.170	0.450	3.02	154.01
4.54	112.251	28.103	27.668	28.675	27.805	0.450	4.15	249.45
5.15	135.381	33.902	33.344	34.625	33.510	0.450	4.71	300.85
5.97	169.312	42.353	41.666	43.334	41.960	0.450	5.45	376.25
7.26	232.569	57.975	57.290	59.336	57.968	0.450	6.63	516.82
8.71	315.079	77.825	77.998	79.455	79.802	0.450	7.96	700.18
10.50	424.909	104.776	105.256	106.958	107.918	0.450	9.59	944.24

Table K.2: Pressure drop data of 100% accepts at $\kappa = 80$ and $p_c = 0.0 kPa$.

Condition	100% accepts		
Kappa number	80		
Compaction Pressure (<i>kPa</i>)	5.6		
Volume of column (m^3)	$8.209 imes10^{-3}$		
Mass of O.D. cooked chips (kg)	0.8133		
Initial cooked bulk density (kg/m^3)	99.08		
Yield (%)	53.86		
Basic chip density (kg/m^3)	398.27		
Cooked chip density (kg/m^3)	214.51		
Porosity at no flow (fraction)	0.462		

Flow rate	p_h	dp_1	dp_2	dp_3	dp_4	L	V	p_h/L
(L/min)	(Pa)	(Pa)	(Pa)	(Pa)	(Pa)	(m)	(mm/s)	(Pa/m)
0.00	0.000	0.000	0.000	0.000	0.000	0.386	0.00	0.00
1.79	33.334	4.188	9.818	9.473	9.854	0.386	1.63	86.36
3.36	79.962	10.047	23.325	23.063	23.527	0.386	3.07	207.15
4.61	129.646	16.290	37.756	37.521	38.080	0.386	4.21	335.87
5.26	162.766	20.451	47.270	47.284	47.760	0.386	4.81	421.67
5.91	194.347	24.419	56.351	56.558	57.020	0.386	5.40	503.49
7.47	282.346	35.476	81.510	82.620	82.741	0.386	6.82	731.47
8.82	370.377	46.537	106.696	108.723	108.420	0.386	8.06	959.53
10.65	505.173	63.474	145.728	147.644	148.327	0.386	9.73	1308.74

Table K.3: Pressure drop data of 100% accepts at $\kappa = 80$ and $p_c = 5.6$ kPa.

Condition	100% accepts
Kappa number	80
Compaction Pressure (<i>kPa</i>)	9.8
Volume of column (m^3)	$8.209 imes 10^{-3}$
Mass of O.D. cooked chips (kg)	0.8133
Initial cooked bulk density (kg/m^3)	99.08
Yield (%)	53.86
Basic chip density (kg/m^3)	398.27
Cooked chip density (kg/m^3)	214.51
Porosity at no flow (fraction)	0.416

Flow rate	p_h	dp_1	dp_2	dp_3	dp_4	L	V	p_h/L
(L/min)	(Pa)	(Pa)	(Pa)	(Pa)	(Pa)	(m)	(mm/s)	(Pa/m)
0.00	0.000	0.000	0.000	0.000	0.000	0.356	0.00	0.00
1.87	47.226	2.402	15.168	15.032	14.624	0.355	1.71	133.03
3.36	114.631	5.701	36.464	36.372	36.094	0.355	3.07	322.90
4.62	190.075	8.861	60.410	60.407	60.398	0.354	4.22	536.94
5.26	234.383	10.928	74.496	74.489	74.470	0.354	4.81	662.10
6.01	294.590	13.643	93.362	93.534	94.051	0.354	5.49	832.18
7.37	417.687	18.128	132.453	132.893	134.213	0.353	6.73	1183.25
8.72	550.894	22.336	174.874	175.661	178.023	0.352	7.97	1565.04
10.46	754.426	28.468	239.825	241.122	245.012	0.351	9.55	2149.36

Table K.4: Pressure drop data of 100% accepts at $\kappa = 80$ and $p_c = 9.8$ kPa.

Condition	100% accepts
Kappa number	80
Compaction Pressure (kPa)	14.0
Volume of column (m^3)	$8.209 imes 10^{-3}$
Mass of O.D. cooked chips (kg)	0.8133
Initial cooked bulk density (kg/m^3)	99.08
Yield (%)	53.86
Basic chip density (kg/m^3)	398.27
Cooked chip density (kg/m^3)	214.51
Porosity at no flow (fraction)	0.372

Flow rate	p_h	dp_1	dp_2	dp_3	dp_4	L	v	p_h/L
(L/min)	(Pa)	(Pa)	(Pa)	(Pa)	(Pa)	(m)	(mm/s)	(Pa/m)
0.00	0.000	_	0.000	0.000	0.000	0.331	0.00	0.00
1.86	79.442	_	25.556	26.884	27.003	0.331	1.70	240.01
3.54	205.104	_	65.942	69.446	69.716	0.331	3.23	619.65
4.65	317.405	_	102.064	107.453	107.888	0.331	4.25	958.93
5.26	390.914	_	125.658	132.383	132.873	0.331	4.81	1181.01
6.11	497.348	_	158.833	168.950	169.564	0.330	5.58	1507.11
7.47	684.651	_	218.651	232.577	233.423	0.330	6.83	2074.70
8.83	898.464	_	286.933	305.212	306.320	0.330	8.07	2722.62
10.69	1236.401	_	392.184	421.398	422.818	0.329	9.77	3758.06

Table K.5: Pressure drop data of 100% accepts at $\kappa = 80$ and $p_c = 14.0$ kPa.

Condition	100% accepts
Kappa number	53
Compaction Pressure (<i>kPa</i>)	0.0
Volume of column (m^3)	$8.209 imes 10^{-3}$
Mass of O.D. cooked chips (kg)	0.7614
Initial cooked bulk density (kg/m^3)	92.75
Yield (%)	50.35
Basic chip density (kg/m^3)	398.27
Cooked chip density (kg/m^3)	200.53
Porosity at no flow (fraction)	0.537

Flow rate	p_h	dp_1	dp_2	dp_3	dp_4	L	V	p_h/L
(L/min)	(Pa)	(Pa)	(Pa)	(Pa)	(Pa)	(m)	(mm/s)	(Pa/m)
0.00	0.000	0.000	0.000	0.000	0.000	0.450	0.00	0.00
1.89	24.177	6.228	6.522	5.699	5.728	0.450	1.73	53.73
3.41	57.128	14.311	14.965	13.741	14.110	0.450	3.12	126.95
4.41	89.274	21.882	22.960	21.718	22.715	0.450	4.03	198.39
5.21	118.457	28.820	30.240	28.955	30.442	0.450	4.76	263.24
6.01	152.776	36.758	38.590	37.590	39.838	0.450	5.49	339.50
7.65	219.116	53.251	54.789	54.465	56.611	0.450	6.99	486.92
8.99	303.817	72.296	75.184	75.839	80.498	0.450	8.21	675.15
10.85	365.225	90.484	90.707	91.621	92.412	0.450	9.91	811.61

Table K.6: Pressure drop data of 100% accepts at $\kappa = 53$ and $p_c = 0.0 kPa$.

Condition	100% accepts
Kappa number	53
Compaction Pressure (<i>kPa</i>)	5.6
Volume of column (m^3)	$8.209 imes 10^{-3}$
Mass of O.D. cooked chips (kg)	0.7614
Initial cooked bulk density (kg/m^3)	92.75
Yield (%)	50.35
Basic chip density (kg/m^3)	398.27
Cooked chip density (kg/m^3)	200.53
Porosity at no flow (fraction)	0.414

Flow rate	p_h	dp_1	dp_2	dp_3	dp_4	L	V	p_h/L
(L/min)	(Pa)	(Pa)	(Pa)	(Pa)	(Pa)	(m)	(mm/s)	(Pa/m)
0.00	0.000	0.000	0.000	0.000	0.000	0.355	0.00	0.00
1.81	50.663	2.355	16.087	16.103	16.118	0.354	1.65	143.12
3.35	121.512	5.304	38.657	38.736	38.815	0.353	3.06	344.23
4.23	187.812	7.538	59.555	60.091	60.627	0.352	3.87	533.56
5.26	286.276	9.716	90.606	92.187	93.767	0.350	4.80	817.93
6.05	372.320	10.562	118.113	120.586	123.059	0.348	5.53	1069.89
7.49	502.892	13.042	160.338	163.283	166.229	0.347	6.85	1449.26
8.80	614.494	12.795	197.684	200.566	203.449	0.345	8.04	1781.14
10.68	788.688	12.383	256.939	258.769	260.598	0.343	9.76	2299.38

Table K.7: Pressure drop data of 100% accepts at $\kappa = 53$ and $p_c = 5.6$ kPa.

Condition	100% accepts
Kappa number	53
Compaction Pressure (kPa)	9.8
Volume of column (m^3)	$8.209 imes10^{-3}$
Mass of O.D. cooked chips (kg)	0.7614
Initial cooked bulk density (kg/m^3)	92.75
Yield (%)	50.35
Basic chip density (kg/m^3)	398.27
Cooked chip density (kg/m^3)	200.53
Porosity at no flow (fraction)	0.354

Flow rate	p_h	dp_1	dp_2	dp ₃	dp_4	L	v	p_h/L
(L/min)	(Pa)	(Pa)	(Pa)	(Pa)	(Pa)	(m)	(mm/s)	(Pa/m)
0.00	0.000	_	0.000	0.000	0.000	0.322	0.00	0.00
1.75	103.937	—	32.645	35.907	35.384	0.322	1.60	322.79
2.71	190.417	_	59.833	65.882	64.702	0.321	2.48	593.20
3.46	272.618	_	84.123	94.857	93.638	0.320	3.16	851.93
4.60	435.773	_	133.658	151.794	150.322	0.320	4.21	1361.79
5.12	545.348	_	162.033	191.809	191.507	0.318	4.67	1714.93
6.11	726.022	_	215.351	255.895	254.776	0.317	5.58	2290.29
7.56	1053.386	_	306.981	373.804	372.601	0.315	6.91	3344.08
8.85	1358.608	_	391.578	484.856	482.174	0.313	8.09	4340.60
10.76	1907.267	_	529.375	688.874	689.018	0.310	9.83	6152.47

Table K.8: Pressure drop data of 100% accepts at $\kappa = 53$ and $p_c = 9.8$ kPa.

Condition	100% accepts
Kappa number	53
Compaction Pressure (kPa)	14.0
Volume of column (m^3)	$8.209 imes 10^{-3}$
Mass of O.D. cooked chips (kg)	0.7614
Initial cooked bulk density (kg/m^3)	92.75
Yield (%)	50.35
Basic chip density (kg/m^3)	398.27
Cooked chip density (kg/m^3)	200.53
Porosity at no flow (fraction)	0.280

Flow rate	p_h	dp_1	dp_2	dp ₃	dp_4	L	v	p_h/L
(L/min)	(Pa)	(Pa)	(Pa)	(Pa)	(Pa)	(m)	(mm/s)	(Pa/m)
0.00	0.000	_	0.000	0.000	0.000	0.289	0.00	0.00
1.86	313.704	—	69.484	122.095	122.125	0.289	1.70	1085.48
2.62	568.513	_	124.458	221.919	222.137	0.288	2.39	1974.00
3.49	887.160	_	194.259	346.229	346.671	0.288	3.19	3080.42
4.72	1430.260	_	309.338	560.042	560.880	0.287	4.31	4983.48
5.22	1735.799	_	370.680	682.024	683.095	0.286	4.77	6069.23
6.02	2341.460	_	493.594	923.155	924.711	0.285	5.50	8215.65
7.57	3292.519	_	675.716	1307.274	1309.529	0.283	6.91	11634.34
8.84	4104.871	_	819.172	1641.445	1644.254	0.281	8.07	14608.08
10.61	5184.375	_	1004.668	2088.250	2091.456	0.279	9.69	18581.99

Table K.9: Pressure drop data of 100% accepts at $\kappa = 53$ and $p_c = 14.0$ kPa.

Condition	100% accepts
Kappa number	25
Compaction Pressure (<i>kPa</i>)	0.0
Volume of column (m^3)	$8.209 imes 10^{-3}$
Mass of O.D. cooked chips (kg)	0.7143
Initial cooked bulk density (kg/m^3)	87.02
Yield (%)	47.14
Basic chip density (kg/m^3)	398.27
Cooked chip density (kg/m^3)	187.74
Porosity at no flow (fraction)	0.536

Flow rate	p_h	dp_1	dp_2	dp_3	dp_4	L	v	p_h/L
(L/min)	(Pa)	(Pa)	(Pa)	(Pa)	(Pa)	(m)	(mm/s)	(Pa/m)
0.00	0.000	0.000	0.000	0.000	0.000	0.450	0.00	0.00
1.69	21.018	5.750	5.151	5.105	5.012	0.450	1.55	46.71
3.31	55.310	14.258	13.737	13.697	13.617	0.450	3.02	122.91
4.59	90.150	22.422	22.562	22.573	22.594	0.450	4.20	200.33
5.09	107.360	26.710	26.867	26.879	26.903	0.450	4.65	238.58
6.11	143.125	34.929	35.960	36.039	36.197	0.450	5.59	318.06
7.37	194.336	47.222	48.869	48.996	49.249	0.450	6.73	431.86
8.73	253.038	61.156	63.700	63.895	64.287	0.450	7.97	562.31
10.52	338.593	84.778	84.621	84.609	84.585	0.450	9.62	752.43

Table K.10: Pressure drop data of 100% accepts at $\kappa = 25$ and $p_c = 0.0$ kPa.

Condition	100% accepts
Kappa number	25
Compaction Pressure (<i>kPa</i>)	5.6
Volume of column (m^3)	$8.209 imes 10^{-3}$
Mass of O.D. cooked chips (kg)	0.7143
Initial cooked bulk density (kg/m^3)	87.02
Yield (%)	47.14
Basic chip density (kg/m^3)	398.27
Cooked chip density (kg/m^3)	187.74
Porosity at no flow (fraction)	0.397

Flow rate	p_h	dp_1	dp_2	dp_3	dp_4	L	V	p_h/L
(L/min)	(Pa)	(Pa)	(Pa)	(Pa)	(Pa)	(m)	(mm/s)	(Pa/m)
0.00	0.000	0.000	0.000	0.000	0.000	0.346	0.00	0.00
1.79	49.0687	1.089	15.868	15.962	16.149	0.345	1.64	142.23
3.38	124.868	2.405	40.528	40.748	41.188	0.344	3.09	362.99
4.47	194.833	3.182	63.446	63.774	64.431	0.343	4.08	568.03
5.20	267.061	4.343	87.113	87.458	88.147	0.343	4.76	778.60
5.95	353.443	3.665	116.151	116.482	117.144	0.341	5.43	1036.49
7.52	548.924	4.067	181.118	181.494	182.246	0.340	6.87	1614.48
8.63	732.868	1.093	243.226	243.750	244.798	0.338	7.88	2168.25
10.30	1073.822	—	356.665	357.622	359.535	0.336	9.42	3195.90

Table K.11: Pressure drop data of 100% accepts at $\kappa = 25$ and $p_c = 5.6$ kPa.

Condition	100% accepts
Kappa number	25
Compaction Pressure (<i>kPa</i>)	9.8
Volume of column (m^3)	$8.209 imes 10^{-3}$
Mass of O.D. cooked chips (kg)	0.7143
Initial cooked bulk density (kg/m^3)	87.02
Yield (%)	47.14
Basic chip density (kg/m^3)	398.27
Cooked chip density (kg/m^3)	187.74
Porosity at no flow (fraction)	0.316

Flow rate	p_h	dp_1	dp_2	dp ₃	dp_4	L	V	p_h/L
(L/min)	(Pa)	(Pa)	(Pa)	(Pa)	(Pa)	(m)	(mm/s)	(Pa/m)
0.00	0.000	_	0.000	0.000	0.000	0.305	0.00	0.00
1.75	132.347	_	34.928	48.628	48.791	0.305	1.60	433.92
2.51	217.522	_	56.885	80.181	80.456	0.304	2.29	715.53
3.32	336.120	_	87.061	124.321	124.737	0.303	3.03	1109.31
4.59	557.653	_	144.440	206.263	206.951	0.303	4.19	1840.44
5.10	662.764	_	170.033	245.954	246.778	0.302	4.66	2194.58
6.02	879.482	_	223.443	327.472	328.567	0.301	5.50	2921.87
7.27	1258.020	_	313.275	471.594	473.150	0.299	6.65	4207.42
8.60	1779.281	_	434.018	671.528	673.736	0.297	7.86	5990.85
9.66	2458.271	_	573.885	940.743	943.643	0.293	8.83	8390.00

Table K.12: Pressure drop data of 100% accepts at $\kappa = 25$ and $p_c = 9.8$ kPa.

Condition	100% accepts
Kappa number	25
Compaction Pressure (<i>kPa</i>)	14.0
Volume of column (m^3)	$8.209 imes 10^{-3}$
Mass of O.D. cooked chips (kg)	0.7143
Initial cooked bulk density (kg/m^3)	87.02
Yield (%)	47.14
Basic chip density (kg/m^3)	398.27
Cooked chip density (kg/m^3)	187.74
Porosity at no flow (fraction)	0.250

Flow rate	p_h	dp_1	dp_2	dp_3	dp_4	L	v	p_h/L
(L/min)	(Pa)	(Pa)	(Pa)	(Pa)	(Pa)	(m)	(mm/s)	(Pa/m)
0.00	0.000	_	0.000	0.000	0.000	0.278	0.00	0.00
1.87	460.557	_	88.411	185.755	186.391	0.278	1.71	1656.68
2.49	707.437	_	133.707	286.384	287.346	0.277	2.28	2553.92
3.37	1140.454	_	215.504	461.723	463.227	0.277	3.08	4117.16
4.57	1850.151	_	344.055	751.869	754.228	0.276	4.17	6703.45
5.20	2408.650	_	440.570	982.591	985.488	0.275	4.75	8758.73
5.98	3129.978	_	553.488	1286.446	1290.044	0.273	5.47	11465.12
7.26	4484.264	_	764.191	1858.255	1861.818	0.271	6.63	16547.10
8.61	6524.210	_	1069.980	2725.389	2728.841	0.269	7.86	24253.57

Table K.13: Pressure drop data of 100% accepts at $\kappa = 25$ and $p_c = 14.0 \, kPa$.

Limiting Case	к	т	p_c	p_h	L	\mathcal{E}_{c}	δ	γ	D_e	Pe	r^2
		(kPa)	(kPa)	(kPa)	(m)				(m^2/s)		
$p_c \gg p_h$	80	115	5.6	0.163	0.386	0.462	0.028	0.008	9.09×10^{-3}	0.43	0.997
	80	115	9.8	0.234	0.351	0.408	0.023	0.011	$2.01 imes 10^{-2}$	0.20	0.999
	80	115	14.0	0.391	0.330	0.370	0.027	0.016	$3.78 imes 10^{-2}$	0.11	0.999
	53	78	5.6	0.286	0.343	0.414	0.049	0.017	$9.18 imes 10^{-3}$	0.42	0.997
	53	78	9.8	0.545	0.309	0.354	0.053	0.030	$3.31 imes 10^{-2}$	0.12	0.999
	53	78	14.0	1.736	0.277	0.280	0.110	0.078	$9.64 imes 10^{-2}$	0.05	0.999
	25	67	5.6	0.267	0.336	0.379	0.046	0.019	$3.40 imes 10^{-2}$	0.12	0.999
	25	67	9.8	0.663	0.291	0.283	0.063	0.040	$1.02 imes 10^{-1}$	0.05	1.000
	25	67	14.0	2.409	0.267	0.225	0.147	0.116	2.16×10^{-1}	0.03	0.999
$m \gg p_h$	80	115	0.0	0.135	0.450	0.538	1.000	0.007	$3.72 imes 10^{-3}$	1.06	0.989
	53	78	0.0	0.118	0.450	0.537	1.000	0.008	$3.17 imes 10^{-3}$	1.24	0.992
	25	67	0.0	0.107	0.450	0.536	1.000	0.009	3.25×10^{-3}	1.21	0.992

K.2 Axial Dispersion Data

Table K.14: A summary of the experimental conditions tested with estimatesof the axial dispersion coefficients with correlation coefficients r^2 for100% accepts wood chips.

Appendix L

Axial Dispersion Breakthrough

Curves



Figure L.1: Axial dispersion breakthrough curves for $\kappa = 80$ and $p_c = 0.0 kPa$.



Figure L.2: Axial dispersion breakthrough curves for $\kappa = 80$ and $p_c = 5.6 kPa$.



Figure L.3: Axial dispersion breakthrough curves for $\kappa = 80$ and $p_c = 9.8 kPa$.



Figure L.4: Axial dispersion breakthrough curves for $\kappa = 80$ and $p_c = 14.0 \, kPa$.



Figure L.5: Axial dispersion breakthrough curves for $\kappa = 53$ and $p_c = 0.0 kPa$.



Figure L.6: Axial dispersion breakthrough curves for $\kappa = 53$ and $p_c = 5.6 kPa$.



Figure L.7: Axial dispersion breakthrough curves for $\kappa = 53$ and $p_c = 9.8 kPa$.



Figure L.8: Axial dispersion breakthrough curves for $\kappa = 53$ and $p_c = 14.0 \, kPa$.



Figure L.9: Axial dispersion breakthrough curves for $\kappa = 25$ and $p_c = 0.0 kPa$.



Figure L.10: Axial dispersion breakthrough curves for $\kappa = 25$ and $p_c = 5.6 \, kPa$.



Figure L.11: Axial dispersion breakthrough curves for $\kappa = 25$ and $p_c = 9.8 \, kPa$.



Figure L.12: Axial dispersion breakthrough curves for $\kappa = 25$ and $p_c = 14.0 \ kPa$.

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