The Leftovers of Planet Formation:
Small Body Populations of Our Solar System and Exoplanet Systems

by

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Abstract

The small body populations within a planetary system give information about the planet formation and migration history of the system. In our Solar System, we study these bodies (asteroids, comets, and trans-Neptunian objects), by directly observing them in reflected light. In other solar systems, dust traces the position of the planetesimal belts that produce it, and is observed as an excess above the stellar flux in the infrared. The dust is visible and not the planetesimals because of the much greater cross-sectional surface area of a swarm of dust particles. In this thesis, both leftover large planetesimals in our Solar System and dust around other stars are investigated.

Data from the Canada-France Ecliptic Plane Survey (CFEPS) are used to measure the absolute populations of trans-Neptunian objects (TNOs) in mean-motion resonances with Neptune, as well as constrain the internal orbital element distributions. Detection biases play a critical role because phase relationships with Neptune make object discovery more likely at certain longitudes. The plutinos (objects in the 3:2 resonance) are given particular attention because the presence of the secular Kozai resonance within the mean-motion resonance causes different detection biases that need to be accounted for to properly debias surveys that include detections of plutinos. Because the TNOs that are trapped in mean-motion resonances with Neptune were likely emplaced there during planet migration late in the giant planet formation process, the structure within and relative populations of the resonances should be a diagnostic of the timescale and method of giant planet migration.
Exoplanet systems that host several rocky planets are those that did not experience giant planet migration, and thus are likely to host planetesimal belts which should be detectable as debris disks. The *Kepler* Mission has detected a host of such systems, and we use data from the Wide-field Infrared Survey Explorer (WISE) Mission to search for debris disks around these stars. Though we tentatively detect more excesses toward these stars than would be expected, contamination from warm dust in the Milky Way Galaxy makes detection unreliable for these systems, and will have to await future infrared space telescopes.
Preface

The text of this dissertation includes modified reprints of previously published material as listed below.

Chapter 2 (published):


This paper presents an analysis of resonant TNOs detected in a large (five-year) observational program by Gladman et al. I contributed to this project by performing all of the modelling that was required to debias the observations and regain the orbital element distributions and true populations for the TNOs that were resonant. This involved planning and running many hundreds of simulations with varying parameters, comparing the resulting models to observed resonant objects, plotting the results, and doing statistical tests. The telescope time was originally proposed for by B. Gladman, J.-M. Petit, and J. Kavelaars. The observations for the survey were performed at the Canada-France-Hawaii Telescope during 2003-2008 by B. Gladman, J.-M. Petit, J. Kavelaars, R. L. Jones, J. Wm. Parker, and P. Nicholson. These observations were analyzed and orbits were fit by B. Gladman, R. L. Jones, and C. Van Laerhoven. In addition to writing my own code to analyze and display the results, I utilized the Survey Simulator code originally written by J.-M. Petit, J. Kavelaars, and B. Gladman. To test how
well the models fit the data, I modified statistical analysis code originally written by R. L. Jones. The bulk of the paper was written by B. Gladman, with editing by myself, J.-M. Petit, and J. Kavelaars. Section 2.3 and all of the figure captions were written by me, and all of the figures were made by me.

Chapter 3 (published):


I wrote the entire text, performed all of the modelling and simulated surveys, and made all of the figures. B. Gladman provided commentary, including the initial idea to separate this project from the larger resonant paper (Chapter 2). B. Gladman also helped my understanding of the complicated Kozai effect and dynamical disturbing function, as well as proofreading and editing.

Chapter 4 (published):


I had the idea for and formulated the project, wrote the entire text, wrote all of the code for analysis, and made all of the figures. B. Gladman provided understanding of the dynamical background, timescales, and interpretation of results, as well as providing feedback and proofreading.

Chapter 5 (submitted):


This was a follow-up to Chapter 4, using the slightly modified code from the first project with an expanded WISE dataset and more Kepler exoplanet hosts, as well as dealing with the fact that another paper had been published calling in
to question all of the debris disks claimed in Chapter 4. B. Gladman provided extensive guidance and advice on how to proceed, as well as many rounds of editing and proofreading.
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Glossary

ALMA  Atacama Large Millimeter Array
CFEPS  Canada-France Ecliptic Plane Survey
CFHT  Canada-France-Hawaii Telescope
COBE  Cosmic Background Explorer
DES  Deep Ecliptic Survey
DEBRIS  Disk Emission via a Bias-free Reconnaissance in the Infrared/Submillimetre
E-ELT  European Extremely Large Telescope
GMT  Giant Magellan Telescope
IAU  International Astronomical Union
IR  Infrared
IRAF  Image Reduction and Analysis Facility
IRAS  Infrared Astronomical Satellite
IPAC  Infrared Processing and Analysis Center
JCMT  James Clerk Maxwell Telescope
JFC  Jupiter Family Comet

JWST  James Webb Space Telescope

KOI  Kepler Object of Interest

LSST  Large Synoptic Survey Telescope

MPC  Minor Planet Center

NASA  National Aeronautics and Space Administration

PAH  Polycyclic Aromatic Hydrocarbon

Pan-STARRS  Panoramic Survey Telescope & Rapid Response System

PR  Poynting-Robertson

RV  Radial Velocity

SDSS  Sloan Digital Sky Survey

SED  Spectral Energy Distribution

SNR  Signal-to-Noise Ratio

TMT  Thirty Meter Telescope

TNO  Trans-Neptunian Object

2MASS  Two Micron All Sky Survey

VLT  Very Large Telescope

WISE  Wide-field Infrared Survey Explorer
AU - Astronomical Unit. Roughly the mean Earth-Sun distance. Defined as 149,597,870,700 metres, exactly.

pc - Parsec. The distance at which one AU subtends one arcsecond. 206,264.8 AU or $3.0856 \times 10^{16}$ m.

Jy - Jansky. A unit of flux density commonly used in radio, submillimeter, infrared, and x-ray astronomy. $1 \text{ Jy} = 10^{-26} \frac{\text{W}}{\text{m}^2\text{Hz}}$.

$L_{\text{dust}}/L_\star$ - Fractional luminosity, commonly used to measure the brightness of debris disks by comparing the luminosity of the dust disk $L_{\text{dust}}$ to the luminosity of the star $L_\star$.

μm - Micron. $10^{-6}$ meters.

$M_\oplus$ - The mass of the Earth. $5.97219 \times 10^{24}$ kg.

$M_\odot$ - The mass of the Sun. $1.9886 \times 10^{30}$ kg.

$R_\oplus$ - The radius of the Earth. 6,378.1 km.

$R_\odot$ - The radius of the Sun. 695,500 km.

$L_\odot$ - The luminosity of the Sun. $3.893 \times 10^{26}$ W.

a - Semimajor axis of an orbit. Defined as half of the major axis of the ellipse of the orbit.

e - Eccentricity, which describes how elliptical an orbit is. A perfect circle has $e = 0$, an ellipse has $0 < e < 1$, a parabolic orbit has $e = 1$, and a hyperbolic orbit has $e > 1$.

q - Pericenter distance. Pericenter (also called perihelion for orbits around the Sun in our Solar System) is the point of the elliptical orbit where the object is closest to the central body. $q$ can be calculated using $a(1 - e)$. 
**Q** - Apocenter distance. Apocenter (also called aphelion for orbits around the Sun in our Solar System) is the point of the elliptical orbit where the object is farthest to the central body. $Q$ can be calculated using $a(1 + e)$.

$i$ - The inclination of the plane of an orbit to a reference plane. For objects in our Solar System, the reference plane is the ecliptic plane, which is the plane through Earth’s orbit around the Sun.

$Ω$ - The longitude of the ascending node is the angle between the reference direction (this is the vernal equinox direction in the solar system) and where the orbit passes upward through the reference plane.

$ω$ - The argument of pericenter is the angle between where the orbit passes upward through the reference plane and the location of pericenter, the object’s closest approach to the central body.

$ɔ$ - The longitude of pericenter is the sum of $Ω$ and $ω$.

$M$ - The mean anomaly describes the average change in position of an object over time and is defined by $\frac{2\pi}{P}t_{peri}$, where $P$ is the period of the orbit, and $t_{peri}$ is the time since the object was last at pericenter.

$λ$ - The mean longitude is the sum of $Ω$, $ω$, and $M$. (Reader beware, $λ$ can sometimes mean wavelength).

**RA** - Right Ascension. Usually measured in hours, but can be measured in degrees. RA is like longitude, and gives the angle along the the celestial equator between an object and a reference direction, the direction of the Sun on the Spring Equinox.

**dec** - Declination. Like latitude, declination gives the angle of an object from the celestial equator along a great circle passing through the object and the celestial poles.
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Dedication

For my parents, with love.
Chapter 1

Introduction

This thesis involves theoretical dynamical modelling as well as optical and infrared observations of the small body populations of our Solar System and of a selection of nearby exoplanet systems. It is a compilation of four projects, all of which are related by the common theme of small body populations.

In our Solar System, we study small bodies in the form of asteroids, comets (minor planets with tails due to the release of material), and Trans-Neptunian Objects (TNOs) in the Kuiper belt. These objects are the leftovers from planet formation: either surviving planetesimals that were not accreted by the larger protoplanet cores or collisional fragments of these objects. We can directly image some of these small bodies through optical surveys. But to study the small bodies of other solar systems, we have to turn to more indirect means and even smaller bodies: dust grains.

Dust grains are detected around other stars because the dust grains are warmed by the star and then they re-radiate this energy primarily in the infrared. Because of the huge combined surface area of a cloud of very small (µm-scale) dust particles, this infrared flux can be measured as additional flux above that expected for the host star. These stars are said to possess debris disks, and hundreds of debris disks have been measured by infrared telescopes (see Section 1.3.1 and Wyatt, 2008). These dust grains are not stable over timescales similar to the age of the
system (Gyr timescales); very small grains ($\sim 0.1 \, \mu m$) are quickly blown away by radiation pressure from the star, while larger grains (>1 $\mu m$) gradually lose orbital energy from Poynting-Robertson (PR) drag and fall into the star. This destruction thus requires that these dust grains be replenished somehow. The widely accepted dust source is a collisional cascade, where planetesimals within a belt smash into each other and break into smaller and smaller pieces in the resulting collisions (see Section 1.3.3 and Wyatt, 2008). However, it is not well understood why this happens so strongly for some main sequence stars and not for others. In particular, only a small fraction ($\sim 1\%$) of stars similar to our Sun (main-sequence F, G, and K spectral type) have been found to possess detectable warm debris disks ($\sim 300 \, K$; e.g. Beichman et al., 2005a). Even these are very difficult to explain given the planetesimal collisional model (Rhee et al., 2008). These bright debris disks are either caused by rare events, such as planet-planet collisions or multiple planetesimal collisions, or the dust has survived in a steady-state over the lifetime of the star. However, both of these theories have problems: it is difficult to imagine that collisions catastrophic enough to produce the observed amounts of dust are happening 1% of the time around main-sequence stars, and because of PR drag removal of dust, to maintain steady-state, an unrealistic mass would be ground into dust over the lifetime of the star. There is still much that is not understood about the production of debris disks.

Debris disks are not made from the primordial nebula dust, but are secondary disks produced much later in the system’s life. Because of the short lifetime of dust grains in orbit around a star, the presence of a debris disk implies the presence of a planetesimal belt. Our Solar System possesses two such planetesimal belts: the asteroid belt and the Kuiper Belt (Figure 1.1). However, our dusty debris disk is extremely diffuse and difficult to study. Reflected light from the dust created by collisions in the asteroid belt can be observed with the naked eye in ideal conditions as the “zodiacal light” and “gegenschein”, and the Infrared (IR) emission has been measured by the Infrared Astronomical Satellite (IRAS) and the Cosmic Background Explorer (COBE).
(Greaves and Wyatt, 2010; Hahn et al., 2002; Nesvorný et al., 2011). The emission from dust created by the Kuiper belt has not been measured due to the much closer and brighter emission from the zodiacal light, but many predictions exist for how bright this dusty debris disk should be (e.g. Greaves et al., 2004b; Holmes et al., 2003; Kuchner and Stark, 2010; Liou and Zook, 1999; Moro-Martín and Malhotra, 2002; Stern, 1996; Vitense et al., 2012, see also Section 1.3.4). For comparison, the Spitzer Space Telescope was only capable of detecting debris disks $\sim 100$ times the fractional luminosity $L_{\text{dust}}/L_\ast$ of the zodiacal dust (Lawler et al., 2009).

This thesis will discuss the dynamical relationship between dust, the planetesimals that create that dust, and planets that cause gravitational perturbations from two very different perspectives: from within our Solar System, where we are able to measure the planetesimals directly (Chapters 2 and 3), and in many nearby solar systems, where we deduce the positions of planetesimal belts indirectly by measuring dust rings and belts (Chapters 4 and 5).

## 1.1 A Brief Overview of Planet Formation

The objects that remain in the Kuiper belt today are planetesimals and planetary cores that failed to accrete enough mass to become planets, or collisional fragments of those objects. To understand how they arrived in their present state, we must start at the beginning, with the planet formation process.

About 4.6 billion years ago, a gas cloud collapsed, starting the process to form our Sun and all the bodies in our Solar System (including human bodies). The collapse may have been initiated by a nearby supernova, which would have peppered the gas cloud with short-lived radioactive isotopes (Adams, 2010). Conservation of angular momentum caused the collapsing gas cloud to form a disk rather than all collapsing into the same point. The infant Sun formed in the center, the densest

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*aThere is evidence of short-lived radioisotopes from isotopic studies of meteorites. However, whether or not a supernova was what initiated the collapse is debated in the literature. The direct cause of the collapse is not important to planet formation.*
Figure 1.1: Orbital elements and sizes of catalogued minor planets: small black dots show the semimajor axis and $H$ magnitude of $\sim 600,000$ minor Solar System bodies that were catalogued in the MPC database as of February 2013. $H$ magnitudes are proportional to the log of the radius (for objects that reflect the same proportion of light), brighter (larger) objects have smaller $H$ magnitudes. The eight planets are shown as red circles. The two major planetesimal belts are very obvious concentrations of objects between the orbits of Jupiter and Mars (the asteroid belt) and exterior to Neptune’s orbit (the Kuiper belt). This plot suffers from observational biases (discussed further in Section 1.2): small objects (large $H$ magnitudes) are only detected closer to the Earth.
part of the collapsing cloud. From here, there are two different theories for how planets actually form within the disk: core accretion and gravitational instability.

In the core accretion model, solid dust grains begin to condense and settle out of the gas disk to the midplane \((\text{Lissauer, 1993; Safronov and Zvjagina, 1969})\). Dust particles begin sticking together in the dense midplane of the disk, growing to centimeter-size by static electricity, forming fluffy aggregates that compact and grow as they repeatedly collide \((\text{Dominik and Tielens, 1997})\). It is not well understood how the particles combine into kilometer-sized objects; coagulation is assumed to continue until kilometer-sized planetesimals are formed \((\text{Kenyon, 2002})\). Once the planetesimals reach this size within the gas disk, they have enough mass that interactions between planetesimals are likely to be mergers rather than catastrophic collisions \((\text{Lissauer, 1993})\). The merging continues until they reach lunar-to Mars-mass objects \((\sim 0.01–0.1 \, M_\oplus)\). At this mass, gravitational focusing becomes significant and the planetary cores accrete solid planetesimals, growing at a rate that depends on their radius, with the largest cores growing fastest, also called “runaway” growth \((\text{Chambers, 2001; Kokubo and Ida, 2002})\). If these planetary cores can grow large enough \((\sim 10 \, \text{Earth masses} \, M_\oplus)\) and quickly enough, before the young star has ionized and removed the gas portion of the disk \((\sim 10 \, \text{Myr after formation})\), they can become a gas giant by runaway accretion of all the gas in an annulus approximately the width of the protoplanet’s Hill radius \((\text{Lissauer et al., 2009})\). This is the radius in which the protoplanet’s gravitational influence dominates over the star’s:

\[
R_{\text{Hill}} = a \left( \frac{m}{3M_*} \right)^{1/3}
\]

(1.1)

where \(a\) is the semimajor axis of the protoplanet, \(m\) is the mass of the protoplanet, and \(M_*\) is the mass of the star.

The second model of planet formation is through gravitational instability, where a self-gravitating clump of gas and solids forms, probably due to turbulence in the dense midplane of the disk, and collapses to directly form an object anywhere from a planetesimal \((\text{e.g. Goldreich and Ward, 1973; Youdin and Shu, 2005})\).
2002) to a gas giant planet (Boss, 1997). This is analogous to the gravitational collapse of a star, but on a smaller scale.

Though this topic is still frequently debated, the core accretion model is generally more widely accepted, as it is difficult to produce objects as small as planets from gravitational collapse (mostly small stars and brown dwarfs are produced in simulations; Kratter et al., 2010). Also, recent spectroscopic evidence has been presented (Konopacky et al., 2013) that supports formation by core accretion for the planets around HR 8799, which are massive (∼5–10 times the mass of Jupiter) and widely removed from the star (∼15–70 AU; Marois et al., 2008, 2010), making them some of the best candidates for formation by gravitational collapse (Dodson-Robinson et al., 2009). For the purposes of this thesis, it doesn’t matter which planet formation method occurs, as the smallest bodies will form by coagulation and fragmentation in either case.

One important position in the protoplanetary disk is the “snow line” (also sometimes called the “frost line” or “ice line”). This is the distance from the young star where it is cool enough that ices (water, methane, ammonia, etc.) can condense into solids, and happens where the temperature drops below ∼170 K. This causes a factor of a few jump in the surface density, giving a size boost to objects that form outside this line, estimated to be ∼3–5 AU in our Solar System at the low pressure of the protoplanetary nebula (Ciesla and Cuzzi, 2006; Sasselov and Lecar, 2000). Giant planets are assumed to form outside this line because of the extra mass available for accretion, assisting them in reaching ∼10 $M_\oplus$ before the gas is removed.

Prior to the discovery of numerous Hot Jupiters (Jupiter-mass planets with orbital periods of only a few days), the conventional wisdom was that planets formed in their current positions. But at the ∼0.1 AU orbital distance of these Hot Jupiters, there is simply not enough mass in an annulus with that radius through a protoplanetary disk to form a Jupiter-mass planet. Presently, astronomers believe (Hubbard et al., 2002) that instead these planets formed further out and migrated to where we see them now. Simulations show that Jupiter-mass planets in dense
gas disks could launch spiral density waves that torque the planet and cause it to migrate inwards (i.e. D’Angelo et al., 2003), or if large enough, could open a gap in the disk and subsequently migrate inwards at a rate determined by the viscous accretion of the gas onto the star (i.e. Bryden et al., 1999).

It is now widely accepted that such migration of massive planets within protoplanetary disks is a common occurrence. A recently-proposed theory even suggests that in our Solar System, Jupiter migrated down to nearly the orbital distance of Mars before being trapped in an orbital resonance with inward-migrating Saturn, which forced both planets to migrate outwards again (called the “Grand Tack”; Walsh et al., 2011).

By the time the gas disperses, a few million years after the star’s formation, planet formation is mostly complete, though giant collisions between protoplanets and/or large planetesimals may still be frequent. The largest TNOs (along with asteroids and comets) are the planetesimals that are left over from the planet formation process, and can be thought of as time capsules from the beginning of the Solar System. But at this point, a solar system is not necessarily in its final stable state. Planets gravitationally perturb each other over time, slowly changing the orbits, sometimes to the point where large scale dynamical instabilities can occur. This next stage in the history of a solar system involves the migration of the giant planets. This is discussed for our Solar System in Section 1.2.3 and for exoplanet systems in Section 1.4.1

1.2 The Kuiper Belt: Our Solar System’s Outer Planetesimal Belt

The Kuiper belt is a planetesimal belt, populated by small bodies with orbits that cross or are completely external to the orbit of our Solar System’s outermost major planet, Neptune (hence the name “trans-Neptunian Objects”). Most of the known TNOs, the objects that inhabit the Kuiper belt, have semimajor axes $a$ spanning roughly 34–50 AU (1 AU is the semimajor axis of the Earth’s orbit around the Sun), exhibiting a wide range of orbital eccentricities $e$ and inclinations $i$. 

7
1.2.1 Discovery History

The first TNO to be discovered was Pluto in 1930 (Tombaugh, 1997). Several astronomers, including Gerard Kuiper, for whom the Kuiper belt is named (Kuiper, 1951), and Kenneth Edgeworth (which is why it is sometimes called the “Edgeworth-Kuiper Belt”; Edgeworth, 1949), have over the decades postulated that there should be a multitude of small objects outside the orbit of Neptune (Davies et al., 2008). The second TNO, (15760) 1992 QB₁ was not discovered until over six decades later, but in the two decades since then, well over one thousand have been discovered, confirming the hypothesis that the trans-Neptunian region is well-populated by minor bodies.

The MPC is responsible for collecting observations of minor Solar System bodies and computing orbits for these objects. Objects are assigned names after two nights of observations have been reported and the MPC is unable to match the orbit with any known body. The first part of the name is the four digit year of discovery, and the second part tells when during the year the discovery was made. For the first letter, each half-month period during the year is assigned a letter of the alphabet (skipping I and Z). The second letter is assigned so that the first object discovered in a half-month is given the letter A, the second B, all the way to Z (skipping I). The 26th object discovered in a half month is given the designation A₁, the 27th B₁, and so on. For example, the object 2009 MS₉ was the 237th object discovered in the time period June 16–30, 2009. After an object’s orbit is securely known (usually after two or more opposition observations), it is also given a number, and a permanent name may be suggested by the discoverer to the International Astronomical Union (IAU). Pluto, upon its status change from a planet to a dwarf planet, was assigned the number 134340, and should properly be referred to as (134340) Pluto.

The Neptune migration hypothesis of Malhotra (1993) (see Section 1.2.3) provided a mechanism to allow a significant number of TNOs to be in mean-motion.

¹Opposition is when the object is located directly opposite the Sun from our vantage point here on Earth, which is the optimum observing configuration.
orbital resonances with Neptune, that is, the orbital period of the TNO is a small integer ratio of the orbital period of Neptune (mean-motion resonances are discussed further in Section 1.2.2). We use the convention that a $j:k$ resonance means that $P/P_N = j/k$, where $P$ and $P_N$ are the orbital periods of the object and Neptune, respectively, and $j$ and $k$ are small integers. Pluto itself was found to be in the 3:2 mean-motion resonance with Neptune after a numerical orbital integration (Cohen and Hubbard, 1964). The second 3:2 resonant object, which as a class are now called “plutinos” (Jewitt and Luu, 1995), was not confirmed until 1994. Soon after, objects were found in the 4:3 and 5:3, and in 1998 the first 2:1 resonator was found (Davies et al., 2008). Now, hundreds of resonant objects are known, from the 1:1 resonance at $a = 30.1$ AU to the 27:4 resonance at $a = 108$ AU (Gladman et al., 2008; Lykawka and Mukai, 2007). Chapter 2 discusses resonant TNOs extensively.

One particularly significant TNO discovery (at least for Solar System nomenclature) was (136199) Eris, which was measured to be more massive than Pluto (Brown and Schaller, 2007) and in 2006 prompted the IAU to demote Pluto’s planetary status to that of a “dwarf planet”.

TNOs are visible only because of the sunlight they reflect. This means that there is a very sharp dependence between measured flux and distance from the Sun: The flux received by these objects from the Sun is already diminished by $d^{-2}_\odot$, where $d_\odot$ is the distance between the object and the Sun. The flux we measure reflected by the body is diminished by the albedo $^d$ of the body, and diminished further by $d^{-2}_{\oplus}$, where $d_{\oplus}$ is the distance between the object and the observer here on Earth. Because Kuiper belt objects are located at tens of AU from the Sun, and $d_{\odot}$ and $d_{\oplus}$ vary by at most 1 AU, one can approximate $d_\odot \simeq d_{\oplus}$, meaning that the measured flux is proportional to $d^{-4}_{\odot}$. Thus TNOs that are farther from the Sun are

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$^c$The reader should note that this convention is not standardized, and some authors use the opposite convention.

$^d$Albedo is a fractional measure of the light reflected by a body where 1 is a perfect mirror and 0 absorbs all incident light, the albedo of Kuiper belt objects ranges from a few percent to nearly 1.
much less likely to be detected, and introduces an important but well understood bias that is present in all observational surveys.

The flux or brightness of a TNO is usually measured as an apparent magnitude in a standard filter band, but for Solar System objects the brightness is often expressed as an absolute magnitude, $H$. Absolute magnitudes for Solar System objects are different than the absolute magnitude astronomers use for comparing stars and galaxies. The $H$ magnitude of a Solar System object is the magnitude it would have if it were located 1 AU from the Sun and we were observing it from the vantage point of the Sun. For reference, a 100 km diameter body with an albedo of 0.05 has an absolute magnitude $H_g$ of 9.16.

### 1.2.2 TNO Classification

TNOs are classified into different dynamical groups by performing a medium-length (~10 Myr) orbital integration of the position of the object and the four giant planets, and examining the results (Gladman et al., 2008). Secure classification requires the object’s orbital elements to be well-measured, which in turn requires consistent, long-term observation. Gladman et al. (2008) recommends observations over at least three oppositions. The precision of the orbit measurement can be taken into account by performing additional orbital integrations of “clones” of the object, using orbital elements that are $3\sigma$ away from the best-fit orbital elements. If the results of three integrations using the best-fit object, the highest-semimajor axis clone, and the lowest-semimajor axis clone all agree in their classification, the classification for that object is considered secure. If only two of three integrations agree, the classification is insecure and more observations of the object are necessary.

The first step in classification is deciding if an object is resonant or non-resonant. If the TNO is in the $j:k$ mean-motion resonance with Neptune, it will

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$^e$The $g$ subscript is because the absolute magnitude must be measured in a given optical band, in this case the $g$ band, which has an effective wavelength of 435 nm.
Figure 1.2: Orbital elements of a Solar System body. $\Omega$ is the longitude of the ascending node, $\omega$ is the argument of pericenter, and $f$ is the true anomaly. The mean anomaly $M$ is the time since the last pericenter times the mean motion: $2\pi/P$. For non-circular orbits, $f \neq M$ because of Kepler’s second law. $f$ can be expressed in terms of $M$ using an expansion in $e$: $f = M + 2e \sin M + \frac{5}{4}e^2 \sin 2M + \ldots$ The mean longitude is defined as $\lambda = \Omega + \omega + M$, and the longitude of pericenter is defined as $\varpi = \Omega + \omega$.

exhibit libration of the primary resonant angle for that resonance:

$$\phi_{jk} = j\lambda - k\lambda_N - (j - k)\varpi$$

(1.2)

where $\lambda$ is the mean longitude of the object, $\lambda_N$ is the mean longitude of Neptune, and $\varpi$ is the longitude of pericenter (see Figure 1.2). To understand what this angle represents physically, we use the simplest possible case of the 1:1 mean-motion resonance. For this case, $j = k = 1$ and Equation 1.2 becomes:

$$\phi_{11} = \lambda - \lambda_N$$

(1.3)

So $\phi_{11}$ can be thought of as the angle between the object and Neptune (as seen from the Sun), and over time, this angle will librate about a value. $\phi_{32}$ is harder to
translate physically than $\phi_{11}$. For the 3:2 resonance, Equation [1.2] becomes:

$$\phi_{32} = 3\lambda - 2\lambda_N - \varpi$$  \hspace{1cm} (1.4)

The easiest way to understand the angle $\phi_{32}$ is to examine the equation with $M = 0$ (as explained in the review by Gladman and Kavelaars, 2009). When $M = 0$ that means the resonant object is located at its perihelion so $\lambda = \varpi$. Then Equation [1.4] becomes:

$$\phi_{32} = 2(\varpi - \lambda_N)$$  \hspace{1cm} (1.5)

so $\phi_{32}$ is here twice the angle between the perihelion of the object and the position of Neptune, which can be equal to 90° or -90°. As with $\phi_{11}$, $\phi_{32}$ will librate about a fixed value. For all mean-motion resonances, libration of the resonant angle $\phi_{jk}$ will be clearly visible in the results of an orbital integration. The resonant TNOs are discussed extensively in Chapter 2 of this thesis.

Continuing to use the classification scheme of Gladman et al. (2008), the remaining non-resonant TNOs are divided by their current level of dynamical interaction with Neptune. Objects that interact strongly with Neptune and have their semimajor axis $a$ changed by more than $\sim 1.5$ AU over the course of the 10 Myr integration are considered Scattering Disk Objects. Detached Objects are those that have high enough eccentricity and semimajor axis that they are not able to directly interact with Neptune (meaning they are not Scattering; $e > 0.24$ and $a > 48$ AU, see Figure 1.3).

The non-resonant TNOs that are not Scattering or Detached are part of the Classical belt. The Classical belt contains three subcomponents as described by Petit et al. (2011). The “hot” component spans semimajor axis $a$ from 40–47 AU with a dynamically hot distribution of inclinations. The “stirred” component spans 42.4–47 AU and covers a much lower range of inclinations. The “kernel” is a

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The semimajor axis of Detached Objects must also be low enough that galactic tides are not relevant: $a < 2000$ AU.
Nomenclature of the outer Solar System is decided by examining the output of a 10 Myr orbital integration, and roughly follows the above pattern in semimajor axis $a$ and eccentricity $e$. Letters show the positions of the four major planets (Jupiter, Saturn, Uranus and Neptune), and SDO stands for Scattering Disk Object. Figure from Gladman et al. (2008).

Figure 1.3: Nomenclature of the outer Solar System is decided by examining the output of a 10 Myr orbital integration, and roughly follows the above pattern in semimajor axis $a$ and eccentricity $e$. Letters show the positions of the four major planets (Jupiter, Saturn, Uranus and Neptune), and SDO stands for Scattering Disk Object. Figure from Gladman et al. (2008).

Figure 1.3 shows a rough map showing the dynamical classification of the trans-Neptunian region.

Figure 1.3 shows a rough map showing the dynamical classification of the trans-Neptunian region.

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Figure credit: From The Solar System Beyond Neptune edited by M. A. Barucci, H. Boehnhardt, D. P. Cruikshank, and A. Morbidelli. ©2008 The Arizona Board of Regents. Reprinted by permission of the University of Arizona Press.
1.2.3 Giant Planet Migration in our Solar System

After the gas disk has dispersed, planet formation is mostly over for the gas giant planets, but rocky planets can continue to accrete a significant amount relative to their mass from the huge number of leftover planetesimals. Planetary migration can and does also occur at this point, but it is caused by scattering of planetesimals or interaction with other planets rather than interaction with a gas disk.

The outward migration of the giant planets, particularly Neptune, has left dynamical clues that astronomers are only starting to unravel. Several migration histories have been proposed, but all involve starting the four giant planets (Jupiter, Saturn, Uranus, and Neptune) at semimajor axes different from their current orbits and migrating outwards in the case of Saturn, Uranus, and Neptune, and inwards in the case of Jupiter.

The simplest planet formation scenarios have all the planets and planetesimals form on perfectly circular, co-planar orbits. However, this is far from what is observed in our Solar System, especially in the Kuiper belt. The wide distribution of eccentricities and inclinations that are observed in the Kuiper belt, especially those objects that are in orbital mean-motion resonances with Neptune, are best explained by past gravitational interactions with one or more planets.

Malhotra (1993) first proposed that Pluto could have been swept into its current high eccentricity, high inclination, resonant configuration with Neptune when Neptune migrated outwards through the Solar System. She proposes that Neptune’s outward migration is powered by scattering $\sim 10 M_\oplus$ of planetesimals inwards, where a much more powerful gravitational slingshot is provided by massive Jupiter and scatters the objects outwards into the Oort Cloud (from which Long Period Comets, having $a \simeq 20,000$ AU, now return). Scattering this much mass causes Jupiter’s orbit to migrate inwards by a few tenths of an AU, and Neptune’s to migrate outwards by a few AU. Using Pluto’s current orbital elements\textsuperscript{b} Malhotra calculated that if Pluto was initially on a circular orbit, Neptune would have had to pick up Pluto into its 3:2 resonance at $a = 25$ AU, 5 AU inwards of its

\textsuperscript{b}Pluto’s current orbital elements ($a$, $e$, $i$) are (39.26 AU, 0.24, 17.2°).
current semimajor axis, and Pluto would have started with $a = 32.8$ AU, 6.5 AU inwards.

This suggestion that the giant planets migrated to their current locations sparked many theories on the cause of the migration, each with slightly different outcomes for the Kuiper belt.

Several variations have been proposed closely based on the theory of Malhotra (1993): smooth outward migration by Neptune which “snowplows” TNOs (including Pluto) into mean-motion resonances (Chiang and Jordan, 2002; Hahn and Malhotra, 2005; Malhotra, 1995), thereby raising their eccentricities to the observed values.

“The Nice Model” (Levison et al., 2008a; Tsiganis et al., 2005) explains the current state of the Kuiper belt by invoking a dynamical instability: the four giant planets start in a much tighter configuration and slowly change position by gaining and losing angular momentum from accreting and perturbing Kuiper belt planetesimals. Saturn passes through a 2:1 mean-motion resonance with Jupiter, which causes the eccentricity of Saturn to vary much more strongly over much shorter time periods. Saturn has gravitational encounters with Uranus, which has gravitational encounters with Neptune, causing Neptune’s semimajor axis to jump outwards with a large eccentricity. The large eccentricity is then damped by perturbing the orbits of Kuiper belt objects.

Additional planets that are later ejected from the system have also been suggested. Gladman and Chan (2006) uses Earth- or Mars-mass rogue planets to perturb planetesimals and produce high inclination, high eccentricity, and high perihelion objects that are observed in the Kuiper belt, and Batygin et al. (2012) use the Nice Model framework with an additional ice giant planet that is ejected from the Solar System. Similar to the rogue planet theory, Petit et al. (1999) use large planetesimals within the Kuiper belt to excite the orbits of smaller TNOs.

Chapters 2 and 3 of this thesis discuss how observations can be used most effectively to precisely measure the true debiased distribution of objects within the
Kuiper belt’s resonances, which in turn helps astronomers reconstruct the migration history of our Solar System.

1.3 Debris Disks: Indirect Measurement of Extrasolar Planetesimal Belts

Debris disks are presently the only way to study planetesimal belts in other solar systems. The observable is actually small (µm-scale) dust, and not the planetesimal belt itself. Dust is heated by the star and re-emits the radiation at the equilibrium temperature set by the distance of the dust grain from the star and properties of the dust grain (emissivity, albedo, spectral features, etc.). The peak of the emitted radiation is in the infrared. Due to the large combined surface area of an annulus of heated dust particles, the host star will appear to have more infrared flux than would be predicted for that star’s distance and spectral type. Because the dust grains are destroyed in a time that is short compared to the age of the host star, they are thought to be replenished by planetesimal belts, and are thus used to measure the positions of these planetesimal belts around their host stars.

1.3.1 Debris Disk Observations

The first debris disk to be observed was around Vega, one of the brightest stars in the sky, using IRAS (Aumann et al., 1984). It was detected because Vega was much brighter in the infrared than it was predicted to be, showing an “infrared excess”. Infrared and submillimeter observations have become more sophisticated with each generation of space- and ground-based telescopes, allowing hundreds of stars with infrared excesses to be measured (Krivov, 2010). As of February 2013, 34 of the very brightest, closest debris disks have been directly imaged (according to http://www.circumstellardisks.org, see Figure 1.4). But most of the

Hundreds of known debris disks are too far away to be resolved, and are detected as an infrared excess in the Spectral Energy Distribution (SED) of the host star (Figure 1.5).

To measure the debris disks requires photometry or spectroscopy over a wide range of wavelengths, spanning from optical to infrared or longer wavelengths. Datapoints in optical and near-IR wavelengths\(^\text{1}\) are used to predict the flux density of the star in the infrared, using a stellar atmosphere model such as Kurucz (Castelli and Kurucz, 2004) or Phoenix (Allard et al., 2012). The measured flux in the infrared is then compared to the predicted flux. If there is an excess of flux at these wavelengths, a dusty debris disk may be the explanation.\(^\text{k}\)

Generally, more than one datapoint above the stellar photosphere prediction is recommended to calculate parameters of debris disks. Even with two datapoints, serious degeneracies can exist when modelling properties of the disk. Several photometric points (or a spectrum) are required to get an estimate of the temperature of the dust and its orbital distance. In many cases, the photometric points that lie above the stellar photosphere flux can be fit using a single-temperature ring of dust particles that emit like blackbodies. The peak wavelength of the dust emission ($\lambda_{\text{peak, dust}}$) can be used to estimate the temperature of the dust using Wien’s law:

$$T_{\text{dust}} = \frac{5100 \text{ K } \mu\text{m}}{\lambda_{\text{peak, dust}}} \quad (1.6)$$

The fractional disk luminosity $L_{\text{dust}}/L_*$ can be estimated using:

$$\frac{L_{\text{dust}}}{L_*} = \left( \frac{F_{\text{peak, dust}}}{F_{\text{peak, *}}} \right) \left( \frac{\lambda_{\text{peak, *}}}{\lambda_{\text{peak, dust}}} \right) \quad (1.7)$$

where $F_{\text{peak, dust}}$ and $F_{\text{peak, *}}$ are the peak flux or flux density values measured for the dust and for the star, respectively (Wyatt, 2008). Because of the range of wavelengths covered, the brightness of debris disks are usually measured as flux

\(^{1}\)Conventionally, near-IR means wavelengths less than about 3 $\mu$m.

\(^{k}\)There are other ways to add infrared flux that are not related to the circumstellar environment; see Chapter 5.
Figure 1.4: An assortment of directly-imaged debris disks using several different telescopes and wavelengths. Top left: Fomalhaut’s 130 AU radius debris ring in optical scattered light using the Hubble Space Telescope (blue; Kalas et al., 2005) and at 850 µm using ALMA (orange; Boley et al., 2012). Top right: Near-infrared $L'$ band (3.8 µm) image from the VLT of β Pictoris and its planet, with a semimajor axis of $\sim$10 AU (Lagrange et al., 2010). Center: 24 and 70 µm images of Vega using the Spitzer Space Telescope, showing Vega’s face-on debris disk extending out to 300–500 AU (Su et al., 2005). Bottom: Herschel Space Observatory images of HD 10647’s $\sim$200 AU radius debris disk at 70, 100, and 160 µm (Liseau et al., 2010).
Figure 1.5: Spectral Energy Distributions (SEDs) for different example debris disks. The absolute flux values assume a distance of 10 parsecs. The grey line shows the predicted stellar spectrum of a spectral type G2V star (the spectral type of our Sun). The thin coloured curves show the blackbody spectrum of dust rings at different temperatures (and corresponding different distances from the star, values shown in the figure), while the thick coloured curves show the total (star+dust) SED. Solid lines show disks with a fractional luminosity $f = L_{\text{dust}}/L_\ast = 10^{-3}$, dashed lines show $f = 10^{-5}$, and dotted lines (which are difficult to see) show $f = 10^{-7}$. Our Solar System is usually estimated to have $L_{\text{dust}}/L_\ast = 10^{-7}$ (e.g. Beichman et al., 2005b). Figure from Wyatt (2008).
density in Jy. The semimajor axis $a$ of a ring of dust particles can be estimated using:

$$a = \left( \frac{278 \text{ K}}{T_{\text{dust}}} \right)^2 \left( \frac{L_*}{L_\odot} \right)^{1/2}$$  \hspace{1cm} (1.8)

where $L_\odot$ and $L_*$ are the luminosity of our Sun and the star in question, yielding $a$ in AU (Wyatt, 2008). The surface area of the emitting dust particles can then be calculated:

$$\sigma_{\text{tot}} = 4\pi a^2 \frac{L_{\text{dust}}}{L_*}$$  \hspace{1cm} (1.9)

$\sigma_{\text{tot}}$ can then be used along with assumptions about dust grain size, density and emissivity to calculate the mass of dust around the host star.

### 1.3.2 The Evolution of a Protoplanetary Disk into a Debris Disk

Debris disks are not the remains of protoplanetary disks, but are secondary disks made from later grinding down the planetesimals that formed during the protoplanetary disk phase. Protoplanetary disks also commonly present themselves as infrared excesses around host stars. When an infrared excess is present around a star that is less than a few million years old, it is assumed to be a protoplanetary and not a debris disk. Figure 1.6 lends support to this assumption, as nearly 100% of stars in very young clusters ($\sim$1 Myr) show excesses. This fraction quickly drops, and by 10 Myr of age, few stars show excesses. This timescale is thought to be when the star becomes powerful enough to ionize and remove its gas disk, and planetesimal formation ceases (as discussed previously in Section 1.1).

At the end of the planet formation period, which may take a few tens of Myr, a solar system consists of the star, planets, and planetesimals. The planetesimals are located where planets did not form: outside the outermost planet (like the Kuiper belt), or in large gaps between planets (like the asteroid belt). Assuming that

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1 Jy = $10^{-26}$ W m$^{-2}$ Hz$^{-1}$. See glossary.

Measuring stellar age is generally difficult, but determining if a star is young or not (i.e. below or above a few million years in age) is fairly straightforward (e.g. Soderblom, 2010).
Figure 1.6: The fraction of stars in young named clusters with disks (that is, detectable infrared excesses), drops sharply with age. By 10 Myr, the fraction is very low. This corresponds to the timescale of gas removal by a young star. Figure from Wyatt (2008).

These planetesimals start on circular orbits, only a small perturbation is required to nudge some of them onto orbits which cross through the belt \( (e \simeq 0.001–0.01) \) (Wyatt, 2008), causing a collisional cascade where large planetesimals smash together, creating many smaller pieces, which in turn smash together, creating even smaller pieces, all the way down to the blowout limit, where radiation pressure from the star removes dust from the system \( (\sim 0.1 \mu m \text{ in radius}) \). The distribution of sizes is described by a power law:

\[
N(D) \propto D^q \ dD
\]  

where \( N(D) \) is the number of objects with diameters between \( D \) and \( D + dD \). For an infinite collisional cascade with strength independent of size in equilibrium,
q = -3.5 (Dohnanyi, 1969). This means that for any given size, there will be \( \sim 3000 \) times more objects that are a tenth the diameter. This slope means that both the mass and cross-sectional surface area is dominated by small objects.\(^n\)

Through these mutual collisions, the remaining planetesimals are ground down into dust, but on a timescale that may take billions of years or more.

### 1.3.3 Removal and Replenishment of Dust

In debris disks, the dust is believed to be physically close to a planetesimal belt because of PR drag. PR drag causes dust grains to spiral into the star on short (thousands to millions of years, depending on the semimajor axis) timescales, and causes dust to migrate inwards faster as it gets closer to the star. Because the dust grain is in motion, orbiting the star, in its reference frame, photons from the host star always appears to be coming from slightly in front of the dust grain. Similarly, in the reference frame of the star, the dust grain will emit photons non-uniformly, with more photons being emitted in the direction of the dust grain’s motion (see Figure 1.7). The result of PR drag is that the orbiting dust grains lose momentum and spiral into the star over very short timescales (Burns et al., 1979; Lawler and Gladman, 2012):

\[
 t_{PR} = 200 \text{ yr} \left( \frac{r_{\text{grain}}}{10 \ \mu m} \right) \left( \frac{a}{0.1 \text{ AU}} \right)^2
\]  

This scaling assumes the density of the dust grains is 3 g cm\(^{-3}\). For a typical dust grain radius \( r_{\text{grain}} = 10 \mu m \) with a semimajor axis \( a = 1 \text{ AU} \), the time to spiral into the star is 20,000 years. Dust grains are removed even more quickly if they are smaller, and if they are closer to the star.

\(^n\)A power-law size distribution (Equation 1.10) with \( q = -2 \) will have equal cross-sectional area in each size bin; \( q > -2 \) will have more cross-sectional surface area in the large objects, while \( q < -2 \) will have more in the small objects. A size distribution with index \( q = -3 \) will have equal mass in each size bin; \( q > -3 \) will have more mass in the large objects, while \( q < -3 \) will have more mass in the small objects.
Figure 1.7: PR drag: in the reference frame of the moving dust particle, photons from the Sun appear to be arriving from slightly in front of the dust particle. In the reference frame of the star, radiation is emitted by the particle more strongly in the direction of motion. Both of these effects result in loss of momentum by the dust particle. Adapted from Figure 2 in Burns et al. (1979).
This removal timescale is much shorter than the age of observed debris disk systems, thus something must be replenishing the dust. The most widely accepted explanation is that the dust is continuously replenished by a collisional cascade within a planetesimal belt (Wyatt, 2008).

However, the steady-state collisional cascade explanation does not work for many observed systems with large dust masses. To maintain the present dust masses for the age of the system (several Gyr) would require destruction of thousands of Earth masses of material. Wyatt et al. (2007) focus on a few massive debris disks with dust at < 10 AU that have been found around Sun-like stars. They find that there is a maximum dust luminosity that is possible at a given age due to a steady-state collisional cascade, proportional to \( t_{\text{age}}^{-1} \), where \( t_{\text{age}} \) is the age of the host star. Of the seven stars that were known at the time to possess warm debris disks, they find that four cannot be explained by a steady-state process due to their high dust masses, and must be caused by some transient event (Figure 1.8).

Many possible transient events have been suggested to explain these extremely bright debris disks. The most popular is a dynamical instability perturbing the orbits of many planetesimals and causing massive collisions (perhaps similar to the Nice Model instability in our Solar System) has been invoked to explain the too-bright systems shown in Figure 1.8: \( \eta \) Corvi (Lisse et al., 2012a), BD +20 307 (Song et al., 2005), HD 69830 (Beichman et al., 2005a), and HD 23514 (Rhee et al., 2008). For one of the more spectacular of these bright, warm systems (HD 23514), Rhee et al. (2008) calculate that the dust mass required to create the measured flux is equal to the mass of Earth’s Moon, and point out that the Moon-forming collision in our Solar System would have generated a similar amount of debris.

However, the transience of these extremely bright systems has not been measured directly. The brightness of the HD 69830 debris disk did not change by more than \( \sim 5\% \) within one year, and did not change by more than \( \sim 33\% \) within 24 years, despite the fact that the dust has an orbital distance of about 1 AU, and thus a ring of collisionally interacting planetesimals should change on timescales of
Figure 1.8: Fractional luminosity of known debris disks from Chen et al. (2006) compared to the age of the host stars. The solid line represents a decrease in the maximum debris disk luminosity proportional to $t^{-1}_{\text{age}}$, as predicted for steady-state debris disks by Wyatt et al. (2007). Systems above that line must have their debris disk be caused by a transient event. η Corvi, BD +20 307, and HD 69830 all have much brighter disks than predicted for their mature ages using the steady-state model. Figure from Lisse et al. (2012a).

about 1 year (Beichman et al., 2011). Another example of an extremely consistent dust brightness for a supposedly transient system was measured for HD 172555, which did not vary by more than 4% in 27 years, despite evidence for sub-micron dust grains that were originally believed to be blown out of the system by radiation pressure immediately after formation (Johnson et al., 2012).

There is still quite a bit that is not understood about the formation and survival of debris disks, and it is an active area of research.


1.3.4 Our Solar System as a Debris Disk

Just as our Solar System possesses two distinct planetesimal belts, the asteroid belt and Kuiper belt, it is predicted to have two separate populations of dust accompanying these two belts. Dust is produced by sublimation from comets and by collisions between objects. Because the two planetesimal belts are relatively diffuse, collisions are rare today.

Evidence for past collisions exists in the form of asteroid collisional families, which are groups of asteroids that have similar proper orbital elements and have been known for many decades (e.g. Hirayama, 1918). The initial family-forming event would have triggered a collisional cascade predicted to increase the number of collisions in the asteroid belt significantly for 2–30 Myr (Zappala et al., 1998). This would create a transient, brighter dust disk around the Sun.

Unlike the asteroid belt, where many collisional families are known, only one collisional family is suspected in the Kuiper belt (the Haumea family; Brown et al., 2007; Ragozzine and Brown, 2007). The much lower density of objects in the Kuiper belt makes collisions of this scale extremely unlikely (Levison et al., 2008b). This low density combined with the larger distance means that infrared light from dust from the Kuiper belt will be more diffuse than dust from the asteroid belt, making it very difficult to measure.

It is particularly difficult to measure IR emission from Kuiper Belt dust, which it is directly behind the brighter zodiacal emission from the asteroid belt from our perspective here on Earth. Many theoretical predictions exist for the SED of the Kuiper belt dust.

Figure 1.9 shows a prediction for the Kuiper belt SED from Moro-Martín and Malhotra (2002). They base their Kuiper belt dust model on numerical orbital intergrations of many test particles to find where the dust

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There are two types of orbital elements: proper and osculating. Osculating orbital elements are the orbital elements that a body possesses right now, which will change over time due to gravitational perturbations from other Solar System bodies. Proper orbital elements are found by running a several million year orbital integration and averaging the orbital elements over time, which removes secular periodic variations.
Figure 1.9: Predictions for the Kuiper belt SED based on a dynamical model of the Kuiper belt, seen from a distance of 30 pc. The solid lines show the results including the dynamical effects of the Solar System planets, while the dotted lines show results for no planets. Black shows a $3 \times 10^{-11} M_\odot$ disk only, blue shows the Sun’s SED plus a $3 \times 10^{-11} M_\odot$ disk, red shows the solar SED plus a disk ten times as massive, and green shows the solar SED plus a disk one hundred times as massive. Figure from Moro-Martín and Malhotra (2002).

Figure 1.10 shows another prediction for the Kuiper belt SED. Vitense et al. (2012) attempt to debias the MPC database to obtain a planetesimal distribution for the Kuiper belt. They then use collisional models as well as data from the Pioneer 10 Mission and New Horizons Mission to predict the dust distribution and produce an SED. The predicted SED peaks at 40–50 $\mu$m with a peak flux less

Figure credit: Vitense et al., A&A 540, A30, reproduced with permission ©ESO.
Figure 1.10: Predictions for the Kuiper belt SED using the MPC database as a starting point to model the Kuiper belt, seen from a distance of 10 pc. The dotted blue, dotted red, and solid black lines all roughly match, showing that inserting the giant planets and sublimation effects into their simulation did not significantly alter the predicted SED. The Sun’s SED is shown as a thin black line, several orders of magnitude higher in flux. Figure from Vitense et al. (2012).

than 0.5% of the solar flux at that wavelength. Their results support the idea that currently no telescope in existence would be able to detect a Solar System-analog debris disk around another star.

Figure 1.11 shows the measured SED from a nearby, Sun-like star, τ Ceti. In addition to being a similar spectral type to the Sun (G8V, while the Sun is G2V), it is a mature star, like our Sun. The debris disk was directly imaged at a wavelength of 850 µm using the James Clerk Maxwell Telescope (JCMT), and was measured to have a radius of approximately 55 AU, similar to the Kuiper belt. Greaves et al.⁴

⁴Figure credit: ©Monthly Notices of the Royal Astronomical Society.
Figure 1.11: SED for τ Ceti, a Sun-like star with a massive Kuiper belt analogue. The asterisks and diamonds are datapoints after subtraction of the stellar photosphere. Both the dotted and dashed curves are fit to the data using 60 K dust; the dashed line is fit using a distribution of dust sizes as predicted by a collisional cascade, while the dotted line uses modified-blackbody dust grains. The mass of dust required to produce this infrared excess is about an order of magnitude more than is predicted for our Solar System. Figure from Greaves et al. (2004b).

(2004b) use a collisional model to predict the mass in 10 km planetesimals that produce the dust disk, finding about an order of magnitude more mass is required than is predicted for our Solar System (1.2 $M_\oplus$ versus 0.1 $M_\oplus$). Despite its much larger mass, τ Ceti’s debris disks is one of the best analogues to our Solar System’s debris disk that is currently known.
1.4 A Relationship Between Planets and Debris Disks?

The research that appears in Chapters 4 and 5 was motivated by the question of whether or not the presence of a debris disk can be linked to properties of exoplanets in the same system. A few studies have already attempted to answer this question with available data.

Greaves et al. (2004a) used submillimeter wavelength observations to search eight Radial Velocity (RV)-discovered hot Jupiter host stars for debris disks, and reviewed many debris disk surveys for twenty other exoplanet hosts. They admitted that they are using a very small number of stars, but find a lower than expected proportion of exoplanet host stars that also have debris disks. They concluded that there is a weak anti-correlation between hosting an exoplanet and hosting a detectable debris disk.

Beichman et al. (2005b) was one of the first studies to have a (barely) statistically significant number of systems with and without known RV-detected exoplanets that were searched for excesses using the Spitzer Space Telescope. The 25 exoplanet-hosting stars they observed had one planet that was Jupiter to several times Jupiter mass, within a few AU of the star. They found that 24\% ± 10\% of these systems had debris disks detected at 70 μm, while the fraction of stars not known to host exoplanets from a similar Spitzer observation program (Bryden et al., 2006) was 10\% ± 4\%. They concluded that there was a slight correlation between the presence of debris disks and planets around the same star, exactly the opposite of Greaves et al. (2004a).

Moro-Martín et al. (2007) used data from a large Spitzer survey (Carpenter et al., 2008) and found that out of nine exoplanet-hosting stars, only one possessed a debris disk. Comparing with a carefully selected non-planet

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1Hot Jupiters are planets that have similar mass to Jupiter and orbit very close to their host stars. Because they have large masses, they produce a strong RV signal as they gravitationally tug on their host star, and because they are so close to their host star they orbit with periods sometimes as short as a few days, causing the RV signal to wobble quickly and be easy to detect.
hosting sample, they find that there is no correlation between hosting a debris disk and an exoplanet.

Bryden et al. (2009) and Kóspál et al. (2009) both used data from large Spitzer surveys and included about 150 known exoplanet hosts. The exoplanets had all been detected using RV, and were mainly hot Jupiters. Both papers used a control sample of stars without known exoplanets for comparison, and both papers concluded that there is no difference between the rates of debris disk occurrence in the two samples.

Dodson-Robinson et al. (2011) used spectroscopic data from Spitzer to search over 100 planet-hosting stars for excess, and found a similar rate of occurrence to non-planet hosting stars, concluding there is no difference.

As RV techniques have improved, smaller planets have been detected, down to Neptune-mass and even smaller. Wyatt et al. (2012) included these smaller exoplanets and data from the Disk Emission via a Bias-free Reconnaissance in the Infrared/Submillimetre (DEBRIS) Survey to look for correlations. They used a restricted sample of the nearest sixty Sun-like stars (from Phillips et al., 2010) and found that eleven have RV-discovered planets, six of which have planets that are Saturn-mass or lower. None of the five stars with larger than Saturn mass planets have debris disks, while four of the six smaller-exoplanet hosts do. They found that even though this is a fairly small sample, this division is unlikely to happen randomly, and conclude that stars that host exoplanets smaller than Saturn are more likely to host a debris disk than field stars.

As of January 2013, the Kepler mission has announced 2740 planet candidates, all discovered through transits. These planets are on average much smaller than the RV planets, but because the Kepler mission has only been collecting data since December 2009, only short orbital period planets have been announced so far. Nevertheless, this is a completely new population of planets. These are systems where we know there are several Earth masses of rocky material close to the star. That fact alone seems good reason to suspect the presence of planetesimal belts,

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3The mass of Saturn is about 95 times the mass of Earth.
but there are additional reasons why these systems have a better chance of hosting debris disks than other stars.

1.4.1 Debris Disks as a Marker for Dynamical Stability?
Planetesimal belts may be a diagnostic of stability in that the presence of a strong debris disk around a mature star implies a massive planetesimal belt survived on Gyr timescales, implying there could not have been any large dynamical instabilities or large-scale planet migration after the initial gas disk dissipated.

There is both observational and theoretical evidence that once Jupiter-mass planets are present in a system, they tend to migrate and disrupt any other planets or planetesimal belts that may be present in the system. Simulations by Raymond et al. (2011, 2012) predict that systems with giant planets on very close or eccentric orbits (which implies violent migration) will not show warm excesses; the migrating Jupiter-mass planets eject any planetesimals at asteroid-belt distances. The prevalence of hot Jupiter systems and the apparent ease of migration for large planets is supported by several large Spitzer surveys that found a much lower fraction of Sun-like stars with warm (asteroidal) excesses at 24 \( \mu m \) than cool (Kuiper belt analog) excesses at 70 \( \mu m \) (\( \sim 2\% \) at 24 \( \mu m \) and \( \sim 15\% \) at 70 \( \mu m \); Bryden et al., 2006; Carpenter et al., 2008; Trilling et al., 2008). Latham et al. (2011) find that among the Kepler-discovered systems, multiplanet systems do not host hot Jupiters, and are less likely to host a giant planet at all. This implies that these Kepler multiplanet systems are stable over Gyr timescales, perhaps meaning that there are intact planetesimal belts leftover from planet formation.

There is additional evidence that Kepler multiplanet systems may be the best place to look for planetesimal belts. Lissauer et al. (2011b) discuss the \( \sim 160 \) Kepler multiplanet systems known at the time, and point out that most of these systems are overstable. This stability is measured in terms of the mutual Hill radius:

\[
\Delta = \frac{a_o - a_i}{R_{\text{Hill}}} \tag{1.12}
\]
where \(a_o\) and \(a_i\) are the outer and inner planets’ semimajor axes, and \(R_{\text{Hill}}\) is defined in Equation 1.1. Gladman (1993) proves that whenever \(\Delta\) is greater than 3.5 for two equal-mass planets on low eccentricity orbits, those planets can never have their orbits cross. Because of the perturbations from other planets in the system, this criterion rises to \(\Delta = 9\) for multiplanet systems (Lissauer et al., 2011b). A large fraction of the known Kepler multiplanet systems to date are dynamically “filled to capacity,” with neighbouring planets having an average spacing of \(\Delta \approx 21\) (Fang and Margot, 2013). However, this tight spacing may still leave room for planetesimal belts in these systems that are known to be very good at forming rocky planets.

Chapters 4 and 5 present my attempt to investigate the presence of debris disks in exoplanet systems that have been detected by Kepler.

### 1.5 Datasets Used in this Thesis

The research contained in this thesis would not have been possible without extensive use of several large datasets, both public and proprietary.

#### 1.5.1 Solar System Observations

Chapters 2 and 3 make use of data from the Canada-France Ecliptic Plane Survey (CFEPS), an extensive photometric and astrometric survey covering 400 square degrees using the Canada-France-Hawaii Telescope (CFHT), carried out between 2003 and 2007 (Figure 1.12). The strategy for acquisition and tracking of the objects are described by Jones et al. (2006), and the results are published in Kavelaars et al. (2009), Petit et al. (2011), and Gladman et al. (2012).

As of February 2013, about 1400 TNOs are listed in the MPC database, about 20% of which are classified as plutinos (meaning they are in the 3:2 mean-motion resonance with Neptune). However, estimating the true number of objects of any given size and the relative number in different parts of the Kuiper belt is not straightforward, since most of these detections are from surveys that did not keep...
careful track of their observations or limitations, and many are serendipitous. In order to find the absolute numbers, a carefully planned large-scale survey needs to be conducted.

CFEPS was just such a large, well-calibrated survey, which covered about 400 square degrees (Gladman et al., 2012; Kavelaars et al., 2009; Petit et al., 2011). The observational survey was carried out using the CFHT between 2003 and 2007. Because pointings, magnitude limits, detection efficiencies, and tracking efficiencies were well known, the detections (and importantly, the non-detections) from the survey can be used to reconstruct the overall distributions and absolute numbers of the populations from which they are drawn (Figures 1.13–1.16).

I have used the results of this survey for two projects: one focusing on the populations and orbital element distributions of many mean-motion resonances within the TNOs (Chapter 2), and one focusing on the structure and subsequent
Figure 1.13: Orbital elements ($a$, $e$, and $i$) for 176 objects detected by CFEPS: 101 in the Classical Belt, 11 Detached objects, 56 resonant objects, and 8 Scattering objects. Top panel shows semimajor axis vs. eccentricity, bottom panel shows semimajor axis vs. inclination. Components are colour-coded: red are resonant objects, black and grey are classical, blue are detached, and green are scattering. Mean-motion resonances with CFEPS detections are noted with dotted lines.


**Figure 1.14:** Orbital elements \((a, e, \text{ and } i)\) for the CFEPS L7 debiased Kuiper belt model, for \(H_g < 9.16\). Top panel shows semimajor axis vs. eccentricity, bottom panel shows semimajor axis vs. inclination. The relative numbers within each component and resonance are correct for this size (roughly \(\sim100\) km objects). Components are colour-coded: red are resonant objects, black and grey are classical, blue are detached, and green are scattering. Mean-motion resonances are noted with dotted lines. The gaps in the inclination distribution near 40 AU are to simulate the destabilizing effect of the \(\nu_8\) resonance.
Figure 1.15: The CFEPS L7 debiased Kuiper belt model shown in the context of our Solar System, as seen from above. Components are colour-coded using the same colours as in Figure 1.14. This shows a face-down view, with green circles showing 30, 40, and 50 AU, and the position of Neptune shown by a blue circle. Each point represents one TNO with $H_g < 9.16$ (approximately equivalent to >100 km in diameter). The distribution is dominated by the dense main classical belt.
Figure 1.16: The CFEPS L7 debiased Kuiper belt model shown in the context of our Solar System, as seen from edge-on. Each point represents one TNO with $H_g < 8$. This edge-on view of the Solar System makes it easy to imagine the Kuiper belt as a debris disk. The slight tilt of the plane of the disk is due to the small inclination of Neptune ($\sim 1^\circ$) relative to the invariable plane of the solar system. Figure from http://www.cfeps.net.
detection biases within just the plutinos (objects in the 3:2 mean-motion resonance with Neptune; Chapter 3).

1.5.2 Extrasolar Observations

Chapters 4 and 5 make use of several public datasets.

The *Kepler* Mission, a space telescope that discovers exoplanets by detecting planetary transits of their host stars, provided visible photometry, stellar parameters, and planetary parameters. *Kepler* has been in operation since mid-2009, and is currently slated to be in operation until at least 2016. *Kepler* is currently monitoring the brightness about 200,000 stars in a 100 square degree field of view, and periodic, consistent dips are used to find planets (Borucki et al., 2003). The depth of the dip in brightness caused by a planet transit gives the ratio of the planet area to the stellar area, but in order to calculate the true size of the planet, the stellar radius needs to be known. Stellar parameters such as effective temperature, surface gravity, metallicity, and extinction\(^1\) are measured using model stellar spectrum fits to photometry that is available for millions of stars within the field of view (Brown et al., 2011). From these stellar parameters, stellar models are used to determine basic stellar parameters such as mass and radius. In addition to the stellar parameters, we also use the measured planetary parameters, mainly semimajor axis. All of these data are available in the released catalogues (Batalha et al., 2012; Borucki et al., 2011).

The Wide-field Infrared Survey Explorer (WISE) Mission is a space-based telescope that measured mid-infrared photometry for the entire sky at 3.5, 5, 12, and 22 \(\mu\)m (Wright et al., 2010). WISE is in a polar Sun-synchronous orbit around the Earth. Because of the orbit and pointing strategy of the telescope, every point on the sky was not visited the same number of times, resulting in different observational magnitude depths. Observations were carried out from December 2009 to October 2010, when the coolant ran out.

\[^1\]Extinction is a measure of how much absorbing dust lies along our line of sight to the star, and with some assumptions, gives a measure of the distance.
We also used visible photometry from the Sloan Digital Sky Survey (SDSS) (Abazajian et al., 2009), and near-infrared photometry from the Two Micron All Sky Survey (2MASS) (Skrutskie et al., 2006), both of which were ground-based surveys. 2MASS is an all-sky survey using the $J$, $H$, and $K_s$ photometric bands (centered on 1.25, 1.65, and 2.2 $\mu$m, respectively). It was carried out using ground-based telescopes, one in the northern hemisphere (at Mt. Hopkins Observatory in Arizona, USA) and one in the southern hemisphere (at Cerro-Tololo Observatory in Chile). SDSS covers about 35% of the sky, and was taken using one telescope at Apache Point Observatory in New Mexico, USA. It covers optical wavelengths in five filters: $u$ (0.35 $\mu$m), $g$ (0.48 $\mu$m), $r$ (0.62 $\mu$m), $i$ (0.76 $\mu$m), and $z$ (0.91 $\mu$m).

Chapter 5 makes use of the IRAS survey (Neugebauer et al., 1984). IRAS was a space-based infrared telescope that was launched and operated in 1983. It performed an all-sky survey at 12, 25, 60, and 100 $\mu$m. We use a reprocessed version of the original dataset (Miville-Deschênes and Lagache, 2005).

This research would not have been possible without so many fantastic publicly available datasets and databases for organization and retrieval of data. I have made extensive use of the Infrared Science Archive, which run by the National Aeronautics and Space Administration (NASA) and Caltech’s Infrared Processing and Analysis Center (IPAC), and the Barbara A. Mikulski Archive for Space Telescopes, which is run by the Space Telescope Science Institute.

### 1.6 Thesis Outline

Chapter 2 discusses resonant TNOs in our Solar System, and is based on Gladman et al. (2012). The orbital elements of CFEPS-detected objects that are in ten different mean-motion resonances are used to debias the survey and produce orbital element distributions and absolute populations for each resonance.

Chapter 3 focuses on the plutinos, and calculates the effects that the Kozai resonance within the 3:2 resonance has on the detection biases, based on Lawler and Gladman (2013b).
At this point, the focus shifts from our Solar System to extrasolar systems, and switches observing techniques from optical astrometry to infrared photometry.

Chapter 4 uses the WISE Preliminary Source Catalog to search for debris disks around Kepler planet-hosting stars (those that were announced as of November 2011), based on Lawler and Gladman (2012). A few preliminary detections are announced, and we conclude that the fraction of Kepler systems with disks is probably higher than for Sun-like field stars, but stress that this is based on preliminary data.

In Chapter 5, based on Lawler and Gladman (2013a), we expand the sample in Chapter 4 to include data from the full all-sky WISE data release, and include additional Kepler planets that had been announced as of February 2012. After inspecting IRAS data and re-measuring the WISE photometry, we conclude that most of these excesses are probably due to background dust in the Milky Way Galaxy, and are not due to circumstellar disks. Because of the distance and resulting flux of these Kepler exoplanet systems, debris disk detections in these systems will have to await future high-resolution space-based infrared telescopes.
Chapter 2

The Resonant Trans-Neptunian Populations

2.1 Introduction

The resonant TNOs are a set of Edgeworth-Kuiper Belt objects whose orbital elements are such that the perturbations of Neptune causes relatively large-amplitude (~1%) oscillations of the orbit on only $10^4$-year time scales (much faster than secular oscillations in the outer Solar System). A necessary, but not sufficient condition for a object to be in a mean-motion resonance is that its semimajor axis $a$ implies an orbital period $P$ which is a low-order integer ratio with Neptune with $P/P_N \simeq j/k$, where $j$ and $k$ are two small integers, in which case the object is said to be in the $j:k$ resonance with Neptune, whose period is $P_N$ and semimajor axis $a_N$. Kepler’s 3rd law then provides the resonant semimajor axis $a = a_N (P/P_N)^{2/3}$. Pluto was the first known resonant TNO, its presence in the 3:2 resonance at $a \simeq 39.5$ AU was discovered via direct numerical integration (Cohen and Hubbard, 1965). An important property of these resonances is

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that even resonant TNOs with eccentricities \( e \) so high that their perihelia \( q \) satisfy \( q = a(1 - e) < a_N \), and thus approach the Sun more closely than Neptune, are ‘phase protected’ by the resonance due to Neptune never being nearby when the TNO is at pericenter; in the case of Pluto, this phase protection means the planet actually gets closer to Uranus than Neptune (although Pluto’s orbit is especially rich in resonant behaviours; Milani et al., 1989).

A powerful idea is that the resonant TNOs were captured during an outward migration of Neptune in the distant past, although there exist several contexts. Malhotra (1993) proposed Pluto’s eccentricity had its origin due to capture into the 3:2 as the resonance swept over the initial heliocentric orbit of Pluto during Neptune’s outward migration; after capture, Neptune’s continued migration forced up the captured TNO’s \( e \) due to conservation of angular momentum. Hahn and Malhotra (2005) explored the sweep-up of resonant objects into a variety of resonances, showing how models match the observed-TNO distribution better if the resonances migrated into a primordial belt that has already been dynamically heated rather than the \( e \sim i \sim 0 \) case of a dynamically-cold planetesimal disk, although achieving an inclination \( i \) distribution as hot as the observed objects was difficult. Gomes (2003) showed that abundant large-\( i \) plutinos could be produced if the plutinos were trapped out of a scattering disk already having interacted with Neptune, rather than from a pre-existing cold belt. Chiang and Jordan (2002), Chiang et al. (2003), and Murray-Clay and Chiang (2005) simulated resonant capture, looking at the population of resonances after the migration phase, including studying how the relative populations of resonant ‘modes’ in the 2:1 resonance varied as a function of Neptune’s migration distance and rate. All these studies identified the problem that even though these models pump eccentricities via the capture process, they still favoured migration into a dynamically pre-heated disk and even then the inclination distribution of the trapped resonant objects is not sufficiently high. More recently, Levison et al. (2008a) explored the idea that the entire Kuiper Belt was ‘planted’ in its current location as particles
scattering off of Neptune during the late stages of planet formation are dropped to lower eccentricity while temporarily trapped in mean-motion resonance, and are then decoupled into the current Kuiper belt; in this model the resonant particles are simply those that remained in the resonances at the end of migration. This model has several desirable properties, although the production of a Kuiper Belt with the correct inclination distribution is a challenge (Petit et al., 2011). In this chapter, we will compare our measurements of how various resonances are populated with some published models.

2.1.1 Resonance Dynamics

We provide only a brief tutorial on TNO resonant dynamics; further introductory material can be found in Morbidelli et al. (1995), Malhotra (1996), Chiang and Jordan (2002), and Gladman and Kavelaars (2009).

Many TNOs are currently known to be in mean-motion resonances with Neptune, meaning that the TNO’s orbit is coupled to that of Neptune. Neptune’s mean longitude $\lambda_N$ (roughly its location around its orbit as measured from the J2000 ecliptic reference axis) is related to the TNO’s longitude $\lambda$ (its current position) and the longitude $\varpi$ of the TNO’s perihelion location (see Figure 1.2). Operationally, inhabiting the $j:k$ resonance can be diagnosed by confirming (in a numerical integration) that the resonant angle $\phi_{jk}$ (Equation 1.2) does not explore all values from 0 to $360^\circ$. The most common case (but not only possibility) for real resonant TNOs is that $\phi_{jk}$ oscillates (librates) around a mean $\langle \phi_{jk} \rangle = 180^\circ$ with some amplitude $L_{jk}$ (termed the libration amplitude). For example, a TNO in the 7:4 resonance with libration amplitude $L_{74} = 10^\circ$ means that $\phi_{74}$ oscillates (roughly sinusoidally) between 170 and 190°; such small amplitudes are rare in reality. Because $\lambda = \varpi + M$ where $M$ is the TNO’s mean anomaly, Equation (1.2)
forces that when the TNO is at perihelion ($\mathcal{M} = 0$),

$$\varpi - \lambda_N = \frac{1}{k} \phi_{jk}. \tag{2.1}$$

In our example of the 7:4 resonance, this means that the TNO’s pericenter is ‘leading’ ($\varpi - \lambda_N$) Neptune by $(180^\circ / 4) = 45^\circ$ for $\langle \phi_{74} \rangle = 180^\circ$; as $\phi_{74}$ oscillates by $\pm 10^\circ$, the perihelion longitude oscillates by $(10^\circ / 4) = 2.5^\circ$ relative to the 45° offset (see the first panel of Figure 2.1). Because $\phi_{74} = 540^\circ$, 900°, and 1260° (adding multiples of 360° to 180°) are all also valid, this results in perihelion longitudes for libration center to be $45^\circ$, $135^\circ$, $225^\circ$, and $315^\circ$ ahead of Neptune for $L_{74} = 0^\circ$ TNOs; essentially one can add $2\pi m / k$ for any integer $m$ to the right-hand side of (2.1). It is instructive to ‘trace the orbit’ of a single low-libration amplitude TNO in the co-rotating panels of Figure 2.1; any single particle for the $j:k$ resonance explores all $k$ perihelion concentrations after making $k$ orbits around the Sun. During that time Neptune will have made $j$ heliocentric orbits.

The two rightmost panels of Figure 2.1 illustrate the different generic case of the $n:1$ resonances, which can librate in more than one state of perihelion locking relative to Neptune (these are usually called different ‘islands’) despite the fact that $k = 1$ in Equation (2.1). Although these resonances still have ‘symmetric’ libration of the resonant argument $\phi_{n1}$ around an average value of $\langle \phi_{n1} \rangle = 180^\circ$, usually with very large amplitude, they can also exhibit ‘asymmetric libration’ around another $\langle \phi_{n1} \rangle$ which depends on the value of the orbital eccentricity (Beauge, 1994; Malhotra, 1996). Because these are $n:1$ resonances, the perihelion location of a given such particle is confined to one of the two sky longitudes (hence the term asymmetric); if the reader traces an asymmetric 3:1 orbit in Figure 2.1 they will see that it does not visit both perihelion clusters.
Figure 2.1: Toy models giving ecliptic projections (black dots) of resonant TNOs with $i \simeq 0^\circ$, $q \simeq 30$ AU, and libration amplitudes of $10^\circ$, to illustrate basic spatial TNO distribution induced by a given resonance. These patterns stay fixed in the frame that co-rotates with Neptune, whose position is indicated by the large blue dot; green reference circles show heliocentric distances of $d = 30$, 40, and 50 AU. Wedges show the ecliptic longitude range of the CFEPS blocks (labelled in Figure 2.5), and red squares show the locations of the real CFEPS TNOs in that resonance. For the 3:1 and 5:1, 10% of the model objects are in the symmetric libration island (with $50^\circ$ libration amplitudes) and 45% in each of the two asymmetric islands (with $10^\circ$ amplitudes).
The existence of confined pericenter locations for resonant TNOs has important implications for their observational study; surveys are most sensitive to resonant TNOs that can be at perihelion in the patch of sky being examined. Because the number of TNOs increases rapidly as one goes to fainter magnitudes (due to the size distribution being steep) and because most resonant TNOs occupy eccentric orbits ($e > 0.1$ or much larger), the number of detectable TNOs above the limit of a flux-limited survey is a strong function of longitude relative to Neptune. Essentially one becomes dominated by the hordes of smaller TNOs present near perihelion that become the majority of a detected sample. Although this is a generic effect of eccentric populations (Jones et al., 2006) it is more severe for the resonant populations than the main classical belt due to the usually lower eccentricities of the latter; we will illustrate this effect below with the plutino population. The longitude bias in $\varpi$ shown in Figure 2.1’s toy models are more extreme than reality because the libration amplitude distribution of the resonances is not concentrated towards zero. This introduces yet another effect: during the oscillation of the resonant argument more time passes with $\phi_{jk}$ near the extremes than at the libration center $\langle \phi_{jk} \rangle$ itself. As an example, using the 5:3 resonance (Figure 2.1), if one were looking $85^\circ$ ahead of Neptune, then the 5:3 resonators (from the nearby ‘average’ $\langle \phi_{53} \rangle = 60^\circ$ perihelion cluster) with $L_{53} \simeq 75^\circ$ ($k = 3$ times larger than the $25^\circ$ longitude difference, cf., Equation 2.1) will be favoured over other 5:3 resonators, if all other parameters are equal.

A similar detection bias is caused by orbital inclination $i$; as a TNO rises and falls in ecliptic latitude it will spend less time at latitudes near the ecliptic than at latitudes close to $\pm i$. This results in the true intrinsic TNO sky density of a given inclination peaking just below the latitude corresponding to the inclination, and the ecliptic being the least likely place to find any given high-$i$ TNO.

The real population in any given resonance is a superposition of all eccentricities, libration centers, and libration amplitudes. Conclusions about the distribution
of any of these parameters cannot be quantitative without detailed understanding of the longitude coverage and depth of the surveys in which they were found.

### 2.2 Resonant CFEPS Objects

The data acquisition of the CFEPS survey is described elsewhere (Jones et al., 2006; Kavelaars et al., 2009; Petit et al., 2011). The survey coverage was divided into ‘blocks’ of contiguous sky around the ecliptic, labelled L3f through L7a, where the number indicates the calendar year of the block’s “discovery” observations (2003 through 2007) and the letter is the common MPC format designation of the two-week chunk to the calendar year (thus, discovery observations of L5c were performed in the first half of February 2005). Objects discovered in the block are given internal designations like L5c11, indicating the eleventh TNO discovered in the L5c block.

This chapter models the resonant CFEPS TNOs that are characterized detections from the 3:2 (plutinos) and 5:2 resonances, three $n:3$ resonances (the 4:3, 5:3, and 7:3), the 5:4 and 7:4 resonance, and three $n:1$ resonances (the 2:1, 3:1, and 5:1). The orbital elements for the CFEPS TNOs in these resonances are given in Tables 2.1 and 2.2. In addition we give a 95%-confidence upper limit on the Neptune Trojan population from our non-detection of such an object. Other resonances had zero or one CFEPS TNOs in them, and we elected to not generate upper limits on their populations.

The discovery and tracking of these objects is discussed in Petit et al. (2011). Important for our purposes here is: (1) a wide range of ecliptic longitudes were surveyed with CFEPS, which means CFEPS was sensitive to objects with a large variety of libration amplitudes, (2) patches of sky away from the perihelion longitudes of the resonances were quantitatively characterized; the non-detection of resonant objects at those longitudes provides powerful constraints on the large-amplitude resonators, and (3) an extremely high fraction of the discoveries were

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*Characterized detections are those which have detection efficiencies >40% in their CFEPS discovery block, as defined in Jones et al. (2006).*
tracked, preventing loss of unusual objects, as described in Jones et al. (2010). As an example of this, CFEPS re-discovered TNOs actually inhabiting the rare 5:4 and 7:3 mean-motion resonances (L3y11, L3y07 and L5c19PD; see Table 2.2 caption) at on-sky positions \(\sim 1^\circ\) from the ephemeris that had been assigned based on an incorrect orbit computed from the short-arc discovery prior to 2003 (e.g., the TNOs had been lost before their resonant nature was recognized).

Tables 2.1 and 2.2 list the current barycentric \(a\), \(e\), and \(i\) J2000 osculating elements of each object and a determination of the resonant libration amplitude, which comes from the range of possible orbits as diagnosed in the method of Gladman et al. (2008). The resonance amplitudes listed should be interpreted as a range which encompasses nearly all (>99%) of the possible true values of the TNO’s libration amplitude. Because the CFEPS tracking strategy regularly provided off-opposition observations during the 3-opposition orbits, the libration amplitudes for the CFEPS sample are more precise than for the majority of the MPC sample given in Gladman et al. (2008) and Lykawka and Mukai (2007) because many of the objects in the MPC database have much sparser astrometric coverage.

In addition to mean-motion libration amplitudes, the Kozai resonance (see Section 2.3.3 and Chapter 3) is observed to function for two CFEPS plutinos (L4h07 and L4k01); Table 2.1 gives the amplitude and mean value of the argument of pericenter \(\omega\) (which is effectively the resonant angle). For objects in \(n:1\) resonances where there are symmetric and asymmetric libration islands, Table 2.2 identifies the mode and estimates of the libration center position and libration amplitude.

There are a few high-order resonances with CFEPS detections which we do not model here due to the fact that the resonant occupation is not yet secure. These include the 15:8, 17:9, and 12:5 mean-motion resonances, and are listed as insecure resonators in Petit et al. (2011).
Table 2.1: CFEPS 3:2 (plutinos) and 5:2 Resonators.

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Table 2.1 – Continued from previous page

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<td>5:2</td>
<td>91 ± 17</td>
<td>23.5</td>
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Notes. Characterized CFEPS resonators, with MPC (where available) designations. A ‘PD’ suffix indicates that the CFEPS team realized immediately that this was a previously-discovered TNO but which could now be used in our flux-calibrated analysis. All digits in the best-fit barycentric orbital $a/e/i$ are significant. $g$-band magnitudes are rounded to 0.1 mags, with exact values and errors given in Table 7 of Petit et al. (2011). Heliocentric distances $d$ and $H_g$ magnitudes are given at the first date of CFEPS detection. Libration amplitude is the best fit orbit’s value along with the range covering >99% of possible values given orbital uncertainties. For Kozai librators the libration center of $\omega$ and amplitude $A_\omega$ are given.
### Table 2.2: CFEPS TNOs in Resonances Other than 3:2 and 5:2

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<td>4:3</td>
<td>60 ± 20</td>
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<td>5.45</td>
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<td>~50</td>
<td>23.3</td>
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Continued on next page
Table 2.2 – Continued from previous page

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<th>Designations</th>
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<td>L4k16</td>
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<td>47.66</td>
<td>0.32262</td>
<td>5.732</td>
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<td>23 ± 5</td>
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<td>MPC&lt;sub&gt;W&lt;/sub&gt;</td>
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<td>8.024</td>
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<td>24</td>
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<td>5:1</td>
<td>~160</td>
<td>23.4</td>
<td>Insecure, symmetric</td>
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Notes. Characterized CFEPS and MPC (where available) designations are given; objects beginning with ‘K’ are from the CFEPS presurvey (Jones et al., 2006). All digits in the best-fit barycentric J2000 orbital $a/e/i$ are significant. Heliocentric distances $d$ at detection are rounded to 0.1 AU. $g$-band magnitudes are rounded to 0.1 mags, with exact values and errors given in Table 7 of Petit et al. (2011) or Jones et al. (2006) (the latter assuming $g - R = 0.8$). Libration amplitudes are the range covering >99% of possible true orbits. For $n:1$ resonances the libration island and mean-resonant argument are given. ‘Insecure’ indicates that this resonance occupation is not secure according to the Gladman et al. (2008) criterion. [MPC<sub>W</sub>] indicates the TNO was in MPC database with the wrong orbit; CFEPS re-found the objects (usually > 1° from predicted location) and CFEPS discovery and tracking observations improved the orbit to the listed values.
2.3 CFEPS Survey Simulation of a Resonant Population

We model the orbital distribution in each resonance with several goals. The orbital element distribution inside each resonance is represented either by a parametric model or, in the case of the $n:1$ resonances, a prescription based on the known dynamics of the resonance. In the case of a parametric model the functional forms chosen are ones which post-facto provide a reasonable match between the simulated and real CFEPS detections. In order to converge to our best models, candidate orbital distributions were tested as described in Kavelaars et al. (2009) and the best-matching models were determined; briefly, models for which one of the eccentricity $e$, inclination $i$, distance at detection $d$, apparent magnitude in $g$ band $m_g$, or libration amplitude $L$ cumulative distributions have an Anderson-Darling statistic which occurs by random <5% of the time are rejected. For example, we find that for most of the resonances the intrinsic eccentricity distribution can be satisfactorily represented by a probability distribution in the form of a Gaussian with center at eccentricity $e_c$ and half-width $e_w$ (rejecting negative eccentricities).

These parametric representations are entirely empirical, due to the fact that there is no physical model that provides a parametric form. However, because these functional forms provide rather satisfactory matches to the CFEPS detections, theoretical models of resonant TNO production will have to provide orbital parameter distributions that give roughly the same distribution as our intrinsic model, rather than values in the biased MPC sample. For example, we find the plutino eccentricity distribution is strongly peaked near $e_c = 0.18$ with narrow width; this intrinsic $e_c = 0.18$ peak is below the median plutino $e$ of 0.22 in the MPC database. Similarly, we find the median intrinsic plutino inclination to be $\approx 16^\circ$, whereas detected samples from ecliptic surveys (biased towards low-inclination

\footnote{The Anderson-Darling test provides a statistic that tests the goodness-of-fit between two cumulative distributions, similarly to the better-known Kolmogorov-Smirnov test. However, the Anderson-Darling test provides more sensitivity to the goodness-of-fit in the tails of the distributions.}
detections) have median inclinations $\sim 12^\circ$ both for CFEPS and the Deep Ecliptic Survey (DES) (Gulbis et al., 2010).

Our second goal is to produce debiased population estimates for each resonance, in order to compare the resonances to each other and to other Kuiper Belt components. For many resonances we lack sufficient detected numbers to explore the internal orbital distribution in detail, but can nevertheless provide calibrated absolute population estimates which should be accurate to a factor of a few, based on analytic expectations of the resonance’s internal structure.

The CFEPS Survey Simulator begins with synthetic objects having a range of $H_g$ magnitudes and with orbital elements that place them within a given resonance, correctly time weighted for their occupation of different regions of phase space. Due to differing structure, orbital elements for each resonance’s simulated objects are chosen differently; the procedure for each of the three groups of resonances are described in Section 2.3.1. As each synthetic object is created, the CFEPS pointings, magnitude limits, and tracking efficiencies are applied, to decide whether or not the object is detected. New synthetic objects are created and checked for detectability until a user-defined number of synthetic detections are acquired. If this desired number is equal to the number of CFEPS detections, the simulation provides an estimate of the intrinsic population of the resonance. If instead a cosmogonic model is available, a large number of synthetic detections may be requested, in order to build a well-sampled distribution of the orbital elements that the cosmogonic model predicts CFEPS should detect. The measurable parameters (eccentricity, inclination, discovery distance, apparent magnitude, and libration amplitude) of these synthetic detections are then compared statistically to the real detections to determine whether or not our distribution of synthetic detections from that model is rejectable.

### 2.3.1 Building the Simulated Populations for Each Resonance

In order to measure the CFEPS bias to get an estimate of a resonance’s true population, we select model objects by randomly drawing from a parametrization of
the orbital distribution for the given resonance, and assigning an $H_g$ magnitude from a power-law distribution. Each object is then run through the CFEPS survey simulator to decide whether or not it was detectable. This is repeated until a requested number of detections is reached; this number is usually either (a) $\sim 10^4$ to obtain a well-sampled distribution of orbits that would be detected if the model was correct or (b) the number of CFEPS detections to get an estimate of the true absolute population of that resonance. In case (a) the orbital distribution of the simulated detections is then statistically compared to that of the real detections to decide whether or not that model is reasonable.

The orbital elements for each object are chosen in a different order depending on which resonance the object is a member of. This is because of the differing internal constraints of each resonance. The plutinos have many detections, allowing a much more in-depth exploration of the possible orbital parameter distributions, as well as having a significant Kozai fraction (Section 2.3.3). The $n:1$ resonances have symmetric and asymmetric libration islands which must be populated (Section 2.3.4). Other remaining resonances have fewer detections, and thus the model need not be as complex and the orbital element distribution cannot be constrained as well. These selection processes is described below, in order of increasing complexity.

2.3.2 Simulating the 5:2, n:3, and n:4 Populations

Each of these resonances has between one and six CFEPS detections (Tables 2.1 and 2.2), allowing population estimates but no detailed modelling of orbital element distributions. The orbital elements and magnitudes of the synthetic objects in each of these resonances are chosen in the following order:

First the eccentricity is chosen randomly from a Gaussian distribution centered on the input parameter $e_c$ with a width $e_w$. Negative eccentricities and those that cause the object to approach the orbit of Uranus ($q \sim 22$ AU) are redrawn. The semi-major axis is then chosen. This is drawn randomly within 0.2 AU of the resonance center. Although in reality the resonances have semimajor axes bound-
aries that are \( e \) dependent, the effect on observability is so weak that given the numbers of detections and the fact that the \( e \) distribution is strongly peaked, this makes no difference to our current estimates.

The inclination is chosen independently of \( a \) and \( e \). We use the \( i \)-distribution parametrization where the probability of a given \( i \) is \( \propto \sin i \ e^{-i^2/2\sigma^2} \) as proposed by Brown (2001). This is a Gaussian distribution multiplied by \( \sin i \), which takes into account the fact that an isotropic distribution of orbits will have an inclination distribution proportional to \( \sin i \) because of geometry: there is less volume per inclination bin close to \( i = 0^\circ \) than close to \( i = 90^\circ \). The ascending node \( \Omega \) and mean anomaly \( M \) are chosen randomly from 0–360\(^\circ\).

The libration amplitude \( L \) for each TNO is chosen from a tent-shaped distribution based on the plutino libration amplitude distribution suggested by Lykawka and Mukai (2007). However our study of the plutinos leads us to use a slightly asymmetric shape (see Section 2.4.4). Our smallest libration-amplitude TNOs have \( L = 20^\circ \), the largest have \( L = 130^\circ \), and we put a peak in the libration amplitude distribution at 95\(^\circ \) (that is, the probability increases linearly from 20–95\(^\circ \) and then drops linearly to zero probability at \( L = 130^\circ \)).

After a libration amplitude is chosen, \( \phi_{jk} \) is chosen sinusoidally within the range allowed by the libration amplitude (that is, \( \phi_{jk} = L \sin \psi \) where \( \psi \) is a random phase). The argument of pericenter \( \omega \) is calculated via

\[
\phi_{jk} = j\lambda - k\lambda_N - (j - k) \varpi
\]  

Finally, the \( H_g \) magnitude is chosen from a power-law distribution \( 10^{\alpha H} \) with a maximum \( H_g \) of 11; because this is well below the CFEPS detection limit, our estimate has no dependence on the cutoff.

As each object is generated, its orbital elements and \( H_g \) are passed to the CFEPS Survey Simulator, which evaluates its detectability. If it falls within one of the CFEPS pointings and is bright enough, it becomes a synthetic detection. These detections include a certain fraction of objects that will be “lost” due to tracking losses in a magnitude dependent way (see Petit et al., 2011).
After the desired number of synthetic detections have been acquired, the distribution of synthetic detections and real detections are compared in five parameters: inclination, eccentricity, distance at detection, apparent magnitude, and libration amplitude, as discussed in Kavelaars et al. (2009). The Anderson-Darling statistic of the CFEPS detections relative to the simulated detections are calculated for each distribution and the probability of a departure as large or larger than the detected sample is determined by bootstrapping each sample. We consider a model rejectable when at least one of the five distributions has a bootstrapped probability of $< 0.05$.

For these resonances (the 5:2, n:3, and n:4 resonances) there are insufficient detections to constrain the orbital distribution directly, but this does not result in an important uncertainty in the population estimate. For example, modelling the 5:2 libration amplitude distribution as flat from 0–130° does not result in a rejectable model, like it did for the plutinos, but the $H_g < 9.16$ population only drops to 11,000 from 12,000, a change vastly smaller than the uncertainties, thus showing that our 5:2 population estimate is insensitive to the unknown libration-amplitude distribution. As a second example, changing the $e_w$ value for the 4:3 resonance from 0.06 to 0.10 (allowing easier-to-detect higher-\(e\) resonators to exist) drops the $H_g < 9.16$ best estimate from 800 (+1100,-600) to 640. We thus believe our Poisson uncertainties due to small numbers of detections dwarf the systematic errors for resonances other than the 3:2.

### 2.3.3 Simulating the Plutino Population

The model for TNOs in the 3:2 resonance is identical to the previous section, except that for this resonance we also force a fraction $f_{koz}$ of the objects to simultaneously be in the Kozai resonance. The presence of the Kozai resonance inside the 3:2 is well studied (Morbidelli et al., 1995; Nesvorný and Roig, 2000; Wan and Huang, 2007). While the Kozai resonance appears only at very large inclinations and eccentricities for TNOs outside mean-motion resonances (Thomas and Morbidelli, 1996), inside the 3:2 mean-motion resonance the preces-
sion rates rise enough that at moderate \((e \sim 0.25)\) and inclination \((i \sim 10^\circ - 25^\circ)\) the Kozai effect causes libration of the TNO’s argument of perihelion around \(\omega = 90^\circ\) or \(270^\circ\), which results in its perihelion direction being barred from the plane of the Solar System (see Chapter 3 for more details on the Kozai plutinos).

Two of the 24 CFEPS-detected plutinos are in the Kozai resonance, and the plutinos were already known to include a significant Kozai component (Lykawka and Mukai, 2007). The fraction of Kozai librators in the sample is one of our model input parameters. One effect on the detection of plutinos in an ecliptic survey like CFEPS is that Kozai librators are preferentially detected at larger distances than non-Kozais (Figure 2.2).

During model construction, each plutino is labelled as either a Kozai or non-Kozai resonator using the model’s value of \(f_{koz}\), with the goal being that the simulated detected fraction is satisfactorily in agreement with the true detection fraction of 2/24. If the plutino is not in the Kozai resonance, the orbital parameters are chosen as described in Section 2.3.2 above, with a slight change to the way the semi-major axes are chosen. Instead of just choosing them randomly within 0.2 AU of the center of the resonance, following Figure 7 of Tiscareno and Malhotra (2009), we narrow the resonance’s \(a\) width linearly to zero as \(e\) drops from 0.16 to 0.01; if the drawn \((a, e)\) pair falls outside this bound a new \(a\) and \(e\) are drawn.

For the fraction \(f_{koz}\) of the plutinos chosen to be Kozai resonators, the following procedure for choosing orbital elements is used:

First, a Hamiltonian level surface was generated, based on the calculations of Wan and Huang (2007). The libration trajectories in \((e, \omega)\) space are determined by the value of the \(z\)-component of the angular momentum, which is equivalently labelled by value of \(\cos I_{\text{max}}\) for the circular orbit of the same angular momentum.\(^5\) For our current purposes, we picked the single value of \(I_{\text{max}} = 23.5^\circ\) based on visual comparison with integrations of known plutino Kozai librators (Figure 2.3). With this fixed, a libration trajectory is picked at random, corresponding to Kozai libration amplitudes between \(20^\circ\) and \(80^\circ\). \(\omega\) is picked sinusoidally be-

\(^5\)Section 3.4 explains the meaning of the value \(I_{\text{max}}\) in much greater detail.
Figure 2.2: Model predictions for the detection distance distribution for non-Kozai plutinos, Kozai plutinos with $\omega$ libration amplitudes in the range 20–80°, and Kozai plutinos with libration amplitudes restricted to the range 20–25°. The cumulative distribution of the real [CFEPS] detections is also shown. Kozai plutinos, especially those with small libration amplitudes, are preferentially detected at larger distances.

between 90° and the maximum $\omega$ allowed by the chosen contour. The eccentricity is then found numerically using $\omega$, the chosen Hamiltonian trajectory, and equation (9) from Wan and Huang (2007). Then $i$ is calculated using conservation of the z-component of angular momentum ($L_z \propto \cos i \sqrt{1 - e^2}$). We then move half the Kozai librators to be around the $\langle \omega \rangle = 270°$ island by the transformation $\omega \rightarrow 360° - \omega$. 
Figure 2.3: The Hamiltonian phase space for the set of Kozai librators used in the CFEPS-L7 plutino model. Here eccentricity $e$ is the radial coordinate and the polar angle is $\omega$. This diagram’s set of contours corresponds to the angular momentum where the zero-eccentricity orbit has $I_{\text{max}} = 23.5^\circ$. Also shown (overlain) is the trajectory of a 10-Myr integration of a real Kozai plutino (numbered TNO 69986).

Next, the semimajor axis is chosen in the same manner as for the non-Kozai plutinos, and $M$ is chosen randomly. Libration amplitudes for $\phi_{32}$ are chosen in the same way as for the non-Kozais, again using the tent-shaped distribution. $\phi_{32}$ itself is chosen sinusoidally within the values allowed by the chosen libration amplitude, which allows the ascending node $\Omega$ to be calculated again using the primary resonant angle equation

$$\phi_{32} = 3\lambda - 2\lambda_N - \omega$$

(2.4)
because $\lambda = \omega + \Omega + M$ and $\varpi = \omega + \Omega$.

All the orbital elements have at this point been chosen, so after choosing a $H_g$ magnitude from the same power-law distribution, the object is completely defined and is sent to the survey simulator to evaluate whether or not it will be counted as a synthetic detection.

### 2.3.4 Simulating the n:1 Populations

Because the $n:1$ resonances possess several libration islands, the intrinsic orbital distribution must be picked in a more complex way before it is passed into the survey simulator. Compared to the plutinos and 5:2, we have far fewer CFEPS objects in these resonances than are needed to directly constrain their complex internal structure. Thus, our primary goal is to obtain a calibrated absolute population estimate based on the expected internal structure predicted by analytic studies of these resonances.

Because the effect on observability of the $a$-width of the resonance is tiny, we pick the semi-major axis for model $n:1$ resonant TNOs randomly within 0.2 AU of the resonance center. The eccentricity distribution is more complex because it is linked to the structure of the asymmetric islands. We have incorporated the main features of the resonance from studies of the structure and erosion (see, for example, Chiang and Jordan (2002) and Tiscareno and Malhotra (2009)). We define the symmetric fraction $f_s = 30\%$ for each $n:1$ resonance to be the fraction which are librating in the symmetric island, and as a working hypothesis take the remaining objects to be evenly divided between the two asymmetric libration islands. Symmetric librators have a resonant argument

$$\phi_{n1} = n\lambda - \lambda_N - (n - 1)\varpi$$

which librates around $\langle \phi_{n1} \rangle = 180^\circ$ with amplitudes $L_{n1}$ ranging from 125–165° (Figure 2.4), while the asymmetric librators have a more complex distribution. ‘Leading librators’ (to use the terminology of Chiang and Jordan, denoting orbits
whose pericenter directions are somewhat ahead of Neptune) are randomly given libration centers $\langle \phi_{n1} \rangle$ in the interval 65–110°, with libration amplitudes $L_{n1}$ from 10–75°, where we redraw if $L_{n1}$ is greater than a limit which linearly rises from $L_{n1} = 40°$ for $\langle \phi_{n1} \rangle = 65°$ to $L_{n1} = 75°$ for objects with 110° libration centers (Figure 2.4). This range sufficiently reproduces the main characteristics of analytic studies of the asymmetric islands (Beauge, 1994), of numerical results on the post-migration distribution (Chiang and Jordan, 2002), and of the known 2:1 detections. Half of the asymmetric librators then have their centers moved to the ‘trailing island’ via $\langle \phi_{n1} \rangle \rightarrow 360° - \langle \phi_{n1} \rangle$. Eccentricities for 2:1 resonators are drawn uniformly in the range 0.10–0.35 for symmetric librators or 0.10–0.40 for asymmetric librators (Chiang and Jordan, 2002). For the 3:1 the symmetric/asymmetric $e$ range is 0.25–0.50/0.25–0.55, and for the 5:1 they are 0.35–0.60/0.35–0.65. The dependence of the population estimates on the $e$ range chosen is small; if the 2:1 eccentricity distribution is changed to be uniform from 0–0.35 for all three islands, the model’s rejectability is not altered (Anderson-Darling $e$ match changes negligibly from 0.69 to 0.47) and the population rises from 3700 (+4400, -2400) to 5700 due to the greater preponderance of harder-to-detect low-$e$ 2:1 resonators in this alternate model. While this test is somewhat artificial because such low-$e$ twotinos are not abundantly present in Chiang and Jordan (2002) or Tiscareno and Malhotra (2009), even this large systematic change only alters the population estimate by half of our estimated uncertainty range.

Inclinations are chosen from a $\sin(i)$ times a Gaussian distribution (as for non-Kozai plutinos and for the other resonances). $\mathcal{M}$ and $\Omega$ are chosen randomly from 0–360°. $\phi_{n1}$ is chosen sinusoidally from within the range of possible libration amplitudes around the libration center. Then $\omega$ is calculated using the relation

$$\phi_{n1} = n\mathcal{M} + \Omega + \omega - \lambda_N$$

(2.6)

Finally, the $n:1$ object is assigned an $H_g$ magnitude in the same manner as for the other resonances, regardless of which libration island it is assigned to. It is then sent to the survey simulator.
The range of libration amplitudes, libration centers, and eccentricities chosen in our model for the symmetric and asymmetric islands in the 2:1 resonances. The 3:1 and 5:1 models are the same except for a differing range in eccentricities (see text). Real 2:1 detections are shown as red squares.

2.4 The Plutinos (3:2 Resonators)

The plutinos (3:2 mean-motion librators) are by far the largest sample in the flux-biased catalogues. This preponderance is partly due to the low semimajor axis, keeping heliocentric distances $d$ low, but detection of objects in $n:2$ resonances is also favoured over many other resonances because their perihelion sky densities are currently (due to Neptune’s position over the last two decades) larger at the high galactic latitudes that Kuiper Belt surveys have tended to favour. This well-known effect is illustrated in Figure 2.5 which shows the CFEPS survey block locations along with the CFEPS plutinos discovered (and tracked to obtain orbits with $\delta a/a < 10^{-4}$).
Figure 2.5: Ecliptic projection of the plutinos. The filled red squares are the 24 real detected plutinos, open blue triangles are 240 simulated detections, and tiny black dots show our model’s intrinsic plutino distribution. Neptune’s position is shown by the large blue dot. The CFEPS ‘blocks’ are shown as wedges covering the correct ecliptic longitude range, where the inner edges at ∼20 AU are set by the detection pipeline’s rate cut and the outer extent is at the distance where a $H_g = 7.5$ TNO would cease to be visible (larger TNOs are visible further away of course). The syntax L4jk means the L4j and L4k blocks are overlapping. The two ecliptic intersections with the galactic plane are roughly straight up and down in this diagram.
We began by improving the nominal CFEPS L3 plutino model, using the tripled sample size of 24 CFEPS detections (8 of which were part of the L3 plutino sample). To our surprise, the orbital distribution settled on by Kavelaars et al. (2009) from only eight characterized L3 plutinos remains a non-rejectable model despite tripling the sample, showcasing the ability of well-characterized surveys to constrain orbital distributions. Although we cannot reject the L3 plutino model, the 24 CFEPS plutinos now allow us to improve the details of the plutino model to explore other aspects of the resonance that were not accessible with a sample size of eight.

2.4.1 The Plutino Inclination Distribution

We find that an orbital-inclination probability distribution of the form

$$P(i) \propto \sin i e^{-i^2/2\sigma_{32}^2}$$ (2.7)

provides an acceptable representation of the intrinsic plutino inclination, with \(\sigma_{32} = 16^\circ\) giving the best match (first panel of Figure 2.6). The CFEPS 95% confidence intervals for this functional form range from \(12^\circ\) to \(24^\circ\). The lower end of this range overlaps with the estimates of \(\sigma_{32} = 10^{+3}_{-2}\) degrees (Brown, 2001) and \(\sigma_{32} = 11 \pm 2^\circ\) (Gulbis et al., 2010), which use a heavily-overlapping sample. As was the case in Kavelaars et al. (2009), the CFEPS survey continues to favour a significantly-hotter inclination distribution for the plutinos, with 4 of our 24 plutinos having \(i > 23^\circ\), whereas none of the 51 DES plutinos have \(i > 23^\circ\). We do not believe this is a sample-size problem, but rather an issue of preferential loss of the large-inclination detections in surveys that did not systematically acquire tracking observations 2–4 months after discovery in the initial opposition; Jones et al. (2010) illustrates how this bias enters Kuiper Belt surveys, regardless of the orbit fitting method used for the short-arc orbits. Since plutinos are often discovered at nearby 30–35 AU distances, their faster rate of motion makes accu-
rate determination of their orbits more critical than classical objects, and they are easier to lose at the next opposition.

Our plutino inclination distribution is quite similar to the inclination distribution of the ‘hot’ component of the classical Kuiper Belt, making it plausible that the plutinios and the hot classical belt are both captured populations whose inclination distribution neither affected their capture probability, nor was \(i\) critical for post-capture erosion over the Solar System’s age. We have shown that trying to use the same bimodal inclination for the plutinios as for the classical belt yields rejection at far more than 99% confidence.

We also explored a functional form of \(P_2(i) \propto \sin^2 i \exp \left( \frac{-r^2}{2\sigma_s^2} \right)\) for the plutinos, which is roughly a Maxwellian distribution for the velocity component perpendicular to the plane. This functional form also gives perfectly acceptable matches to the CFEPS detections, with a best match at \(\sigma_s = 11^\circ\) and an acceptable range (95% confidence) from \(\sigma_s = 8.5-13.5^\circ\). However, because this parameterization did not give a significantly better match, nor did it change the total population estimates by more than their uncertainties, for ease of comparisons with the literature we have elected to retain the \(\sin(i)\), rather than \(\sin^2(i)\), formulation.

We checked our plutino sample’s colours, tabulated in Petit et al. (2011), for a correlation with inclination or ‘size’ via \(H_g\) (Almeida et al., 2009; Murray-Clay and Schlichting, 2011), but find no significant correlation. We postulate that the size versus inclination correlation is an artifact of the survey depths that found them (with shallow wide-area surveys finding essentially all \(H < 5\) plutinios far from the ecliptic, whereas most fainter plutinios have been found in ecliptic surveys which don’t cover enough area to find the few \(H < 5\) TNOs near the ecliptic). Our plutino sample does not quite go deep enough (past \(H \sim 8.5\)) to have enough discrimination to see if the smaller plutinios become bluer; our colours are all uniformly blue.
Figure 2.6: Cumulative distributions of the 5 variables on which we perform statistical analysis, for the plutinos. Red squares show the distribution of the 24 detected CFEPS characterized plutinos. The dashed black line shows the distribution of the intrinsic plutino population from our favoured L7 model, and the thicker blue line shows resulting distribution of that model’s simulated detections. The number in each panel is the bootstrapped Anderson-Darling statistic, indicating the percentage of randomly-drawn samples from the simulated detection distribution that had worse Anderson-Darling values than the real detections. We reject the model if any parameter has a bootstrapped value <0.05 (meaning only 5% of randomly-drawn samples have a worse Anderson-Darling statistic than the real detections).
2.4.2 The Plutino Kozai Subcomponent

Two (8\%) of our 24 CFEPS plutinos (L4h07 and L4k01) are also in the Kozai resonance; their Kozai classifications are secure using the Gladman et al. (2008) nomenclature. The argument of pericenter for a Kozai plutino librates around $\omega = 90^\circ$ or $270^\circ$ due to the fact that the resonance affects the angular precession rate, so although Thomas and Morbidelli (1996) show that the Kozai effect in the non-resonant Kuiper Belt appears only at large $e$ and $i$, inside the 3:2 resonance the Kozai resonance can appear for even moderate-inclination plutinos (Morbidelli et al., 1995). The Kozai libration amplitude $A_\omega$ depends on the initial $e$, $i$, and $\omega$. The plutinos L4h07 and L4k01 both librate with a period of $\sim 4$–$5$ Myr, both are in the $\omega = 270^\circ$ Kozai island\(^h\), and have libration amplitudes of $A_\omega = 30^\circ$ and $50^\circ$, respectively.

With only two Kozai plutinos, the modeling we have done exceeds the level of detail needed to deal with the detections, but we present our efforts as a guide to the modeling that will be needed once characterized samples grow. We used the fourth-order averaged Hamiltonian given by Wan and Huang (2007) to provide a reasonable approximate dynamics for the Kozai plutinos in the CFEPS survey simulator (see Section 2.3.3 for details). Kozai-librating plutinos have coupled oscillation of $\omega$ and $e$ (and hence $i$ because the product $\cos i \sqrt{1 - e^2}$ is constant, proportional to the angular momentum’s z-component) that are determined by the value of $\cos I_{\text{max}}$ corresponding to the $e = 0$ trajectory with the same angular momentum. Looking at the full set of Kozai plutinos in the Gladman et al. (2008) compilation, we found that using a set of Kozai trajectories corresponding to the $I_{\text{max}} = 23.5^\circ$ Kozai Hamiltonian with different initial $e_{\text{min}}$ values provided a range of Kozai librations sufficient to model the current sample\(^i\).

Having this Kozai dynamics model, we proceeded to modify the L3 plutino model by introducing the Kozai fraction $f_{\text{koz}}$ parameter, which is the intrin-

\(^h\)We do not believe there is any statistical significance to both Kozai objects being in the same $\omega$ island; the MPC sample has roughly equal numbers in each island. Pluto itself is in the $90^\circ$ island.

\(^i\)Chapter 3 uses a more complex Kozai plutino model, with multiple values of $I_{\text{max}}$. 

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sic fraction of the plutinos that are also librating in the Kozai resonance. By running a one-parameter set of models, we find that an intrinsic Kozai fraction of $f_{koz} = 10\%$ gives the apparent CFEPS fraction of 8%; that is, given the longitude coverage of the CFEPS, there is a mild bias against the detection of Kozai librators. This $f_{koz} = 10\%$ fraction is similar to previous estimates (Chiang and Jordan, 2002; Nesvorný et al., 2000). Although not very constraining, our formal 95% confidence upper limit is $f_{koz} < 33\%$ so many more plutinos from characterized surveys will be required to accurately measure $f_{koz}$.

Tiscareno and Malhotra (2009) point out that because the Kozai plutinos are somewhat more stable than the average plutino, the Kozai fraction should have slowly grown with time. Dynamical simulations which attempt to create the plutino orbital structure must thus ‘erode’ their populations to the modern epoch and then state distributions of libration amplitude for both the 3:2 resonant argument and the Kozai libration amplitude, which may be matched to future de-biased surveys. LSST may provide enough resonant TNO detections (LSST Science Collaboration et al., 2009) to use these distributions as diagnostics.

### 2.4.3 The Plutino Size and Eccentricity Distribution

With 24 detections, we can now independently measure the standard $H$-magnitude distribution slope $\alpha$ in the formulation $N(< H) \propto 10^{\alpha H}$ for the plutinos. This is important because, as Kavelaars et al. (2009) showed, the distribution of plutino detection distances is a sensitive function of the combination of the $\alpha$, $e_c$, and $e_w$ parameters (where the latter two are the center and width of a Gaussian $e$ distribution). Detection biases favour finding larger-$e$ plutinos at small distances. This is simply understood; when a small-body population has a steeply increasing power-law size distribution, any flux-limited survey is very strongly biased towards detecting the hordes of smaller objects that come above the flux limit only at perihelion. Because of these considerations, surveys really only measure the slope of the size distribution which correspond to the $H$-magnitude range for
the population in question near perihelion; for CFEPS plutinos this means that we constrain the value of $\alpha$ for the range $H_g = 8−9$; smaller plutinos are undetectable and larger ones are too rare to be statistically constrained.

We proceeded to run a very large grid of models covering the plausible ranges of $\alpha$, $e_c$, and $e_w$, as preliminary explorations clearly showed these parameters were correlated. The results (Figure 2.7) give confidence regions for our plutino model, where the figure shows cuts in three perpendicular planes through the best-matching model, with $\alpha = 0.9$ for all plutinos, and $e_c = 0.18$ and $e_w = 0.06$ for the non-Kozai component (however, these parameters remain valid even if the Kozai sub-population’s dynamics is ignored in the modelling). For these experiments the inclination distribution and libration amplitude are kept fixed (and experiments showed they are only weakly coupled to the $\alpha$, $e_c$, and $e_w$ triad). A model is rejected if at least one of the distance, eccentricity, or magnitude distributions of the simulated detections disagree (via an Anderson-Darling statistical test) with the real CFEPS detections. We consider models outside the 5% contour rejected.

As Figure 2.7 shows, our 24-plutino sample is able to meaningfully constrain the properties of the plutino size and orbital distributions. The cumulative $e$, detection distance, and apparent $m_g$ distributions corresponding to our nominal model ($\alpha = 0.90$, $e_c = 0.18$, $e_w = 0.06$) were shown in Figure 2.6. As can be seen, there is the expected mild bias towards the detection of higher-$e$ plutinos. Much stronger is the remarkable bias seen in the (heliocentric) discovery distance $d$ distribution; 22 of the 24 CFEPS plutino were detected with $d < a_{3:2} = 39.4$ AU, even though any object on an eccentric orbit spends more than half its time further than its semimajor axis. As Figure 2.5 shows, CFEPS covered a large range of ecliptic longitudes and is thus extremely sensitive to the plutino distance distribution. It is not surprising that the most distant CFEPS plutino (L5c11) is roughly opposite to Neptune on the sky. However, the preponderance of low-$d$ detections demands steeper slopes for the magnitude distribution and large median eccentricities $e_c$. The median plutino $e$ in the Minor Planet Center from the Gladman et al.
Figure 2.7: Confidence regions in the plutinos orbital model parameter space. Three perpendicular slices through the \((\alpha, e_c, e_w)\) parameter space, showing the regions interior to which none of the cumulative distributions yield probabilities <5% or <1%. Note the coupling between the parameters; for example, smaller values of \(e_c\) are allowed only if the width \(e_w\) and the \(H\)-magnitude distribution \(\alpha\) both rise (or the detection distance distribution will fail as not being confined enough to small distances).
plutino compilation is 0.224; the hypothesis that the true median intrinsic 
e is this large or higher is ruled out at >99% confidence.

This analysis demonstrates a correlation between the acceptable values of \( \alpha \), \( e_c \), and \( e_w \). Somewhat shallower \( H_g \)-distributions (\( \alpha \) down to 0.6) are allowed within the 95% confidence range, but such a size distribution requires an \( e \) distribution peaked at larger values to maintain the dominance of small-\( d \) detections. While \( \alpha \) down to 0.6 is not formally rejected by CFEPS, slopes lower than this result in too large a fraction of distant detections. Figure 2.8 illustrates how going to models beyond the 95% confidence limit alters the \( d \) distribution dramatically.

Using \( \alpha \simeq 0.55 \) and the best possible \( e_c \) and \( e_w \) values still results in a rejectable detection-distance distribution and, unlike our strong suspicion in Kavelaars et al. (2009), we can now formally reject the suggestion in Hahn and Malhotra (2005) that the plutino size distribution is as shallow as \( \alpha = 0.54 \). On the other end, \( \alpha = 1.15 \) actually mildly improves the \( d \) distribution match, but such a model results in a \( g \)-magnitude distribution of the simulated detections being so strongly confined to magnitudes slightly brighter than 24 that this rejects the model at >95% confidence.

The ‘least rejectable’ model we have found has a size index \( \alpha = 0.9 \), corresponding to a diameter (\( D \)) distribution with differential \( \frac{dN}{dD} \propto D^{-5.5} \). Again, CFEPS measures this slope only in the \( H_g = 8-9 \) range which dominates the CFEPS plutino detections. It is interesting to compare this to the \( \alpha = 0.8 \) estimate from Petit et al. (2011) for the classical main-belt hot population, measured for the \( H_g = 7-8 \) range (the plutinos detections are dominated by physically smaller objects than the more distant main-belt detections). The 0.1 difference between the two estimates is not significant, given the uncertainties. Due to the similarity in inclination and size distributions, our working hypothesis is that the hot population and plutinos (and, as we shall see below, the other resonant populations) share a common origin.

The uncertainty in \( \alpha \) makes no significant difference to our plutino population estimate. If \( \alpha = 0.8 \) (instead of 0.9) our estimate for the \( H_g < 9.16 \) plutino
Figure 2.8: The cumulative plutino distance at detection distribution for a model with size distribution exponent $\alpha = 0.55$. Dotted black line is true heliocentric distance $d$ distribution, which would be detection biased by the CFEPS survey to the solid blue curve; red dots are the CFEPS plutino detections. For such a flat size distribution too many large TNOs exist at great distance to be detected, which is inconsistent with the concentration to small $d$ present in the CFEPS detections (this model is rejected at more than 99% confidence).

Population drops only $\sim8\%$, a difference which is much smaller than the current population uncertainties (see Section 2.5).
2.4.4 Plutino Libration Amplitudes

Libration amplitudes of the 3:2 resonant argument \( j \) vary for CFEPS plutinos from \( L_{32} = 28^\circ \) (L4j11) to \( 125^\circ \) for L4m02. Numerical simulations show that, in the present planetary configuration, plutino libration amplitudes \( L_{32} \) larger than about \( 125–130^\circ \) are unstable over the age of the Solar System (Nesvorný and Roig, 2000; Tiscareno and Malhotra, 2009). Any libration amplitudes \( > 130^\circ \) will be eroded away in the following 4 Gyr of evolution, but most smaller-amplitude librators will be stable. What cosmogonic processes set the distribution of the remaining stable libration amplitudes? Levison and Stern (1995) show libration amplitude distributions generated in a plutino population captured via gravitational scattering and then damping into the 3:2. Chiang and Jordan (2002) show different libration-amplitude distributions produced by sweep-up capture, depending on Neptune’s migration speed.

We first reconfirmed that a uniform \( L_{32} \) distribution from 0–130° was rejected (>98% confidence). This test also showed that CFEPS has a mild bias towards detecting plutinos with \( L_{32} < 100^\circ \) due to the longitude coverage. Note that this bias is not generic to all TNO surveys; it depends strongly on the longitude coverage and depths of the survey; the \( L_{32} \) panel of Figure 2.6 shows that CFEPS over-detects plutinos in the \( L_{32} = 50–100^\circ \) range relative to their true intrinsic fraction. However, the survey simulator allows us to remove this bias. Compared to the L3 plutino model (Kavelaars et al., 2009), we are now able to meaningfully constrain the libration-amplitude distribution. The L3 model used a symmetric triangle probability distribution motivated by the \( L_{32} \) compilation in Lykawka and Mukai (2007); that is, a probability that increases linearly from \( L_{32} = 0^\circ \) to a peak at \( 65^\circ \) and then decreases linearly to \( L_{32} = 130^\circ \). The L7 sample shows that this symmetric triangle is now a rejectable representation of the true distribution, producing too many low-libration amplitude plutinos. We decided to modify the model by changing the low-\( L_{32} \) start of the linear distribution

\footnote{Kozai plutinos still have their \( \phi_{32} \) argument librate, with the argument of pericenter librating roughly two orders of magnitude slower than \( \phi_{32} \).}
and its peak; the linear drop to the end of the probability distribution was retained. An end to the distribution just above the $125^\circ$ amplitude of L4m02 (which has the largest-known amplitude) is favoured by survey simulator analysis of the CFEPS detections. We found that a start of the linear probability distribution at $L_{32} = 20^\circ$ with a peak at $95^\circ$ provided the best ‘asymmetric triangle’ probability distribution. We tried expanding the range of libration amplitudes to different lower and upper limits while holding the peak of the $L_{32}$ distribution constant at $95^\circ$. The lower limits explored were $0^\circ$ or $20^\circ$, and the upper limits were $140^\circ$, $150^\circ$, $160^\circ$, and $170^\circ$. While none of these distributions were rejectable at 95% confidence, they provided poorer matches to the CFEPS data than our $20^\circ$ and $130^\circ$ nominal model for the lower and upper limits.

Only after arriving at this nominal model did we realize that the resulting asymmetric triangle is very similar to the libration amplitude distribution shown in Figure 6 of Nesvorný and Roig (2000), which estimates the $L_{32}$ distribution for surviving particles in the main core of the resonance after 4 Gyr of dynamical erosion, based on an assumed initial uniform covering of resonant phase space. We do not think that the de-biased CFEPS sample is able to constrain fine details of the current (and thus initial) libration-amplitude distribution, but it is clear that the mechanism which emplaced plutinos must be capable of populating small libration amplitudes efficiently.

### 2.5 The Population of Plutinos

Comparison with other resonant populations is discussed in Section 2.12, but we here put our plutino population estimate in the context of previous literature. CFEPS is sensitive essentially all the way down to $H_g = 9.16$ for plutinos, which corresponds to the frequently used 100-km reference diameter in the literature (for 5% albedo). The CFEPS estimate is

$$ N_{\text{plutinos}}(H_g < 9.16) = 13,000^{+6,000}_{-5,000} \ [95\% \ confidence]. \quad (2.8) $$
This can be compared to factor-of two estimate of 1,400 from Trujillo et al. (2001), which is the last published measurement independent of CFEPS, and the previous CFEPS L3 (Kavelaars et al., 2009) factor of two estimate of 6000 (scaled to \( H_g < 9.16 \) utilizing the \( \alpha = 0.72 \) slope, which now appears to be underestimated). The L3 plutino estimate is consistent with our current estimate, and remains discordant with Trujillo et al. (2001) for the same reasons given in Kavelaars et al. (2009). Table 2.3 lists both the median \( H_g < 9.16 \) estimate (which we adopt as standard for all our absolute resonant population estimates, being the limit to which [CFEPS] had high sensitivity) and an \( H_g < 8 \) estimate because this value is the CFEPS sensitivity limit in the classical belt, allowing comparison to that population. Due to the different size dependencies now being used, the Kavelaars et al. (2009) \( H_g < 10 \) estimate should be scaled to the \( H_g < 8 \) limit by dividing by \( 10^{2\alpha} = 10^{2(0.72)} \approx 30 \).

Given our current estimate (Petit et al., 2011) of the main classical belt having 130,000 \( H_g < 9.16 \) TNOs, the plutino population is thus \( \sim 10\% \) of the entire main classical-belt population at the 9.16 limit. Note that the L3 classical-belt estimate was only a restricted portion of the main-belt phase space, and the L7 model now essentially covers the entire non-resonant phase space from 40–47 AU. It is important to stress that the plutino/classical population ratio is \( H \)-mag dependent due to the steeper slope of the cold component of the main classical belt. Thus, for \( H_g < 8.0 \) the plutino/main-belt ratio is 15\%, in agreement with the estimate of Kavelaars et al. (2009).

### 2.6 The 5:2 Resonance

The dynamics of the 5:2 resonance are similar to that of the 3:2 in that low libration-amplitude TNOs in the 5:2 come to perihelion at a range of longitudes near \( \pm 90^\circ \) away from Neptune. The first real 5:2 resonators were recognized by Chiang et al. (2003). As usual, the libration amplitude \( L_{52} \) measures oscillations of the resonant angle \( (\phi_{52} = 5\lambda - 2\lambda_N - 3\omega) \) around a mean of 180\°. Thus, the detection biases are similar to plutinos, making population comparisons likely more
robust. Due to their larger semimajor axis near 55.4 AU, 5:2 \textsc{TNOs} spend a large fraction of their orbital period further away than even the most distant plutinos. Although Figure 2.1 shows that at a given ecliptic longitude low-$L_{52}$ \textsc{TNOs} could be found at several different discrete distances due to their phase behaviour, an eccentric orbit still massively biases the detections to be at the perihelia longitudes (constrained by the libration amplitude $L_{52}$ of $\phi_{52}$).

The five \textsc{CFEPS} 5:2 objects all have remarkably-high eccentricities (in the narrow range 0.38–0.42), inclinations from 2–23°, and $L_{52} = 44–91°$. Because the \textsc{MPC} has 5:2 resonators with $e < 0.38$, we think this concentration for $e \simeq 0.4$ is a statistical fluke; a similar situation occurred with the plutino discovery distances in Kavelaars et al. (2009) which disappeared in the current larger sample. Some 5:2 resonators with well-determined orbits in the \textsc{MPC} sample have eccentricities below $e \simeq 0.3$. With only 5 \textsc{CFEPS} detections we cannot place strong constraints on the internal orbital distribution, so we proceeded to build a model with a similar level of detail as the Kavelaars et al. (2009) plutino model. Luckily, the 5:2 lacks a Kozai sub-component (no known \textsc{TNO} librates in the 5:2, and we are unaware of any theoretical prediction indicating a non-negligible phase-space for Kozai inside the 5:2).

The inclination distribution is consistent with being the same as that for the plutinos; Table 2.3 lists the ‘least rejectable’ value of $\sigma = 14°$, but the large uncertainties mean identical inclination distributions for 3:2 and 5:2 \textsc{TNOs} is a plausible hypothesis, which we thus adopt. As for the plutinos, we ran a 3-dimensional $(\alpha, e_c, e_w)$ grid to set confidence intervals on these parameters. Unsurprisingly, this analysis did not meaningfully constrain $\alpha$ (which allowed the range 0.4–1.2 at 95% confidence, with a broad peak around $\alpha \sim 0.9$). We thus chose to use the plutino-determined value of $\alpha = 0.9$ for the 5:2 and all other \textit{H}-magnitude distributions for resonant populations which \textsc{CFEPS} had a detection.

The detected eccentricity distribution for 5:2 resonators is obviously different than for the plutinos; eccentricities up to $e \simeq 0.4$ exist, corresponding to $q \simeq 30$ AU. The prevalence of orbits with perihelion at Neptune might be taken as
firm evidence that this population was emplaced by scattering, but the detection biases also favour low $q$, so perhaps there are abundant low-$e$ 5:2 resonators that make up only a small fraction of a detectable sample.

We performed a similar grid search for acceptable parameters of an eccentricity distributions with a Gaussian center $e_c$ and width $e_w$. Figure 2.9 shows that the most-favoured model is indeed a narrow peak centered near $e_c = 0.4$. Like the plutinos, there is a coupling between acceptable values of the distribution’s width and center. It is possible that $e_c$ is much lower and the width $e_w$ higher. We find this result to be generic for all the Kuiper Belt resonances; it simply results from extreme bias towards detecting the abundant small TNOs near the perihelia of high-$e$ orbits. Compared to the 3:2, the quality of fit of lower $e_c$/higher $e_w$ pairs is not in as great a contrast with the quality of the most favoured case. We thus elected to base our nominal 5:2 model and population estimate (Table 2.3) on $(e_c, e_w) = (0.3, 0.1)$ (along the ridge) instead of the absolute peak where $(e_c, e_w) = (0.4, 0.04)$; the latter case has a 40% smaller population but cannot be correct given that two 5:2 resonators (2005 SD$_{378}$ and 84522) exist in the MPC sample with $e < 0.3$. If the real 5:2 population has many even-lower eccentricity orbits, the population will be somewhat larger than our nominal estimate; for example, even the rather extreme case of $(e_c, e_w) = (0.14, 0.18)$ yields a population 40% larger than our nominal estimate.

The nominal 5:2 model produces a population estimate of 12,000 5:2 resonators with $H_g < 9.16$ (Table 2.3). Although the 95% confidence limits range from 4,000–27,000, the favoured population is, perhaps surprisingly, essentially equal to that of the plutinos. This is an unexpected result, as it indicates that the detection bias against 5:2 is roughly a factor of 5 stronger, due to the larger values of $a$ and $e$, both of which result in the population being much less detectable than the plutinos. We will return to the cosmogonic implications of this in Section 2.12, having compiled population estimates for other resonances.

The ability to capture TNOs into the 5:2 via either sweeping up a pre-existing belt or capturing scattering TNOs into the resonance was discussed by
Figure 2.9: Confidence regions for the 5:2 parameter space, for $\alpha=0.9$, showing the eccentricity distribution’s range of acceptable $(e_c, e_w)$ parameters. Although CFEPS most favours a strongly-peaked distribution near $e = 0.4$, a region of lower $e_c$ centers with larger widths is acceptable inside the 5% limit. Even a Gaussian centered on $e_c = 0$ cannot yet be formally rejected due to the strong detection bias towards larger $e$.

Chiang et al. (2003). These authors showed that although resonance sweeping could capture into the 5:2, the observed orbital-element distribution and the apparent 5:2/2:1 detection ratio could only be explained if the resonances captured objects with a pre-excited $e$ and $i$ distribution. Creating most 5:2 TNOs by ‘resonance sticking’ out of a disk of TNOs scattering off Neptune (in the current planetary configuration) was argued to be untenable.

The Levison et al. (2008a) scenario of having the resonant populations trapped during a phase of outward migration can produce lower-$e$ and lower-$L_{52}$ TNOs after Neptune’s eccentricity is damped, and in one simulation produced a con-
centration of $e \sim 0.4$ resonators (although this simulation fails to produce other needed constraints like the inclination distribution). This scenario is promising as a general way to trap resonant populations out of an already-scattering population. In their comparison with 5:2 resonators from the most successful [Levison et al. (2008a)] run (for that resonance) with the MPC sample, the total range of $e$, $i$, and $L_{52}$ almost span the values of known MPC TNOs, but their comparison was not corrected for observational biases which favour low-$i$ and high-$e$ detections, which means that the run produces a simulated 5:2 population that has too many large-$e$ and low-$i$ orbits.

The following two sections deal with the $n$:3 and $n$:4 resonances. Some readers may wish to skip forward to Section 2.9’s discussion of the $n$:1 resonances and cosmogonic significance, especially on a first reading.

### 2.7 The n:3 Resonances

The $n$:3 resonances have 3 different ecliptic-longitude centers (Figure 2.1 shows examples) at which objects are currently coming to perihelion: one is opposite Neptune and the others are 60 degrees ahead and behind. A resonant-argument libration of amplitude $L$ then results in an angular deviation of $L/3$ on the sky of the perihelion-longitude location relative to these three centers, over the course of a full cycle of the resonant argument.

[CFEPS] has detections in the 4:3 (4 TNOs), 5:3 (6 TNOs) and 7:3 (2 TNOs). The detection biases for these n:3 resonances are similar in the sense that the [CFEPS] block locations will not favour one of these three over the other unless they have different libration amplitude distributions, for which we see no evidence. However, the smaller semimajor axis objects will be favoured due to fraction of time spent in the detection volume.

With a few detections per resonance, we have not attempted to model the internal structure of the resonances, but have made a simple generalization of the $n$:2 resonances. We keep the same non-symmetric triangle for the libration amplitude distribution as for the 3:2 (with no amplitudes above 130°). The eccen-
tricity width $e_w$ is also retained, but the central value $e_c$ was moved to correspond to $q \simeq 33 - 35$ AU. This is consistent with an idea that the resonant objects were largely trapped from a primordial Neptune-coupled population, but is not required by our data. Lower values of $e_c$ are allowed in the same sense as the discussion of the 5:2 resonance; detection biases sufficiently favour the high-$e$ TNOs that lower-$e_c$/higher-$e_w$ pairs are permissible (which would slightly raise the population estimates). We retained $\alpha = 0.9$ for these resonances.

We did not retain the $\sigma \simeq 16^\circ$ inclination width from the plutinos, as we found all three $n:3$ resonances favoured somewhat lower inclination widths (although the 95% confidence regions allow $\sigma = 16^\circ$). Table 2.3 lists the favoured $\sigma$ width for each resonance (where the 7:3 is extremely uncertain, so $\sigma = 10^\circ$ was used) along with the population estimates for $H_g < 9.16$ and $< 8.0$. The 4:3 population must be small ($< 2,000$ with $H_g < 9.16$, at 95% confidence). Although we have five 5:3 resonators and two 7:3 resonators, the bias against the 7:3 TNOs (which have larger $a$ and $e$ values) results in the true 5:3 and 7:3 being roughly equal (at $\sim 5$ times the 4:3 population).

2.7.1 The 4:3 Resonance

The 4:3 resonance at $a \simeq 36.5$ AU was studied by Nesvorný and Roig (2001), who showed that a resonance amplitude distribution like the 3:2 (of an asymmetric triangle with peak near $L_{43} = 80 - 90^\circ$) represented those 4:3 TNOs that survive over the age of the Solar System. Although not heavily explored, these authors provide some evidence that the stability of the resonance is not a strong function of inclination; if this is also true for the 5:3 and 7:3 resonance then confirmation of a colder inclination distribution for TNOs currently in the $n:3$ resonances would require a cosmogonic explanation (as opposed to being due to later dynamical depletion). Nesvorný and Roig (2001) calculate that, under a simple scenario of excitation of a primordial belt with initial surface density dropping as $r^{-2}$, the number of 4:3 resonators should be 0.77 that of the 3:2 population, whereas our estimate is 0.06, with 0.77 excluded at more than 95% confidence. We thus confirm that this sce-
nario is excluded, and any Kuiper Belt structure-formation scenario must result in a very weakly-populated 4:3 resonance in the present epoch.

### 2.7.2 The 5:3 Resonance

The 5:3 resonance at $a \approx 42.3$ AU has the curious attribute that it is almost precisely at the lower semimajor-axis limit of the low-inclination component in the main part of the classical belt. The instability for non-resonant TNOs is due to the $\nu_8$ secular resonance which removes low-$i$ TNOs just interior to $a = 42$ AU. The faster precession caused by the resonant argument for TNOs inside the 5:3 shields its members from the $\nu_8$’s effects, so the proximity of the 5:3 and beginning of the low-$i$ classical belt is likely just a coincidence, and not of cosmogonic significance.$^k$

Lykawka and Mukai (2007) and Gladman et al. (2008) list 11 TNOs from the MPC as librating in the 5:3. CFEPS detected six 5:3 resonators, two of which were discovered before 2003 (Table 2.2).

Melita and Brunini (2000) performed a numerical study of 5:3 resonators, showing that the interior of the resonance does not contain a simply-connected stable region, and that lower-$e$ orbits appeared more stable in a frequency-map analysis; comparison with real objects was difficult as there was only one 5:3 resonator (1999 JS) at the time; the objects plotted in Figure 3 of Melita and Brunini (2000) with $e < 0.15$ are non-resonant. Lykawka and Mukai (2007) also explored the 5:3 numerically and found that particles surviving the age of the Solar System were mostly concentrated in the region $0.09 < e < 0.27$ and $i < 20^\circ$, which is indeed the range occupied by the known 5:3 TNOs.

We note that the 5:3 eccentricities are much higher than the classical objects in surrounding semimajor axes (also obvious for the 7:4). It remains uncertain

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$k$Some mean-motion resonance can always be found close to any given point in the main Kuiper Belt...
nThere is a typo in Table 2 of Gladman et al. (2008) in the 5:3 entry for K03UT2S = 2003 US292, whose unpacked designation is mistakenly given as 2003 US96. After 2008, this TNO was numbered 143751.
if this is because these resonant TNOs were captured from a lower-\(e\) population and pumped to higher \(e\) by migration, or rather if both resonant and non-resonant objects existed with \(e\) up to 0.25 and then nearby classical object were eroded away over the age of the Solar System. The former scenario seems disfavoured when considering distant resonances like the 7:3, which do not appear to have the low-\(e\) members they might be expected from a sweep-up scenario into a pre-existing belt (although the selection bias against their detection is strong).

At the other extreme, the absence of 5:3 TNOs with \(e < 0.10\) might also be seen to argue against ‘sweep up’ migration (because low-\(e\) 5:3 objects could be swept up during the final stages from the classical belt). On the other hand, there is detection bias against the discovery of the lowest-\(e\) members and the Lykawka and Mukai (2007) integrations show that such low-\(e\) 5:3 resonators can leak out into the surrounding classical belt.

### 2.7.3 The 7:3 Resonance

The 7:3 mean-motion resonance (with \(a \approx 53.0\) AU) is little discussed in the literature due to being beyond the 2:1 resonance and being 4\(^{th}\) order (and thus nominally weaker). Lykawka and Mukai (2007) and Gladman et al. (2008) each list 3 TNOs in the 7:3 resonance; only 2 of the TNOs were shared (2002 GX\(_{32} = 95625\) and CFEPS L3y07 = 131696) at the time, but due to improved orbital information both 2004 DJ\(_{71}\) and 1999 CV\(_{118}\), and perhaps 1999HW\(_{11}\), are also likely 7:3 resonators. Additionally, the CFEPS object L5c19PD is a re-discovery of the lost object 2002 CZ\(_{218}\), whose orbit based on a 1-month arc was given to be \(a \approx 56.6\) AU and the ephemeris was about 0.5 degrees away from the prediction by the time of our 2005 discovery; CFEPS tracked the object a year before being able to establish the linkage to the short arc from 3 years earlier.

With only two CFEPS detections, we are unable to strongly constrain the parameters that govern the internal orbital distribution. We find (Table 2.3) that a model with inclination width like the other n:3 resonances of \(\sigma = 10^\circ\) and eccentricity width \(e_w = 0.6\) works acceptably as long as the \(e\) distribution is centered
on $e_c = 0.30$ so that perihelia in the $q = 30-35$ AU range are allowed. As before, lower $e_c$ coupled to larger $e_w$ cannot be excluded. This yields population estimate of 4,000 7:3 resonators to factor of three accuracy at 95% confidence, about a factor of three below the 3:2 and 5:2 populations.

2.8 The n:4 Resonances

The 5:4 and 7:4 resonances are little discussed in the literature, despite them both being populated. The Gladman et al. (2008) compilation lists nineteen 7:4 librators and three 5:4 resonators. The resonant argument forces pericenters to be in bands centered on $\pm 45^\circ$ and $\pm 135^\circ$ away from Neptune. Due to the proximity of two of these pericenter locations to the galactic plane, observational surveys have probably not covered these regions as well as they cover the pericenter longitudes of the $n:2$ resonances.

2.8.1 The 5:4 Resonance

CFEPS has only one 5:4 resonator and the DES survey (Elliot et al., 2005) a second, bringing the current total to five known objects. With $a \simeq 35$ AU, the 5:4 is the closest (in semimajor axis) exterior mean-motion resonance to Neptune that is known to be populated, but the proximity to Neptune makes the stable phase space restricted. Malhotra (1996) showed how the zone of stable libration amplitudes shrinks rapidly with increasing $e$; all known 5:4 librators have $e$ in the range 0.07–0.1. With one CFEPS detection we provide an estimated population of $N(H_g < 9.16) \sim 160$, with factor of five 95% confidence limits. Despite its relative uncertainty, it is clear that the 5:4 population is at least an order of magnitude less populated than the 3:2 or 5:2.

2.8.2 The 7:4 Resonance

The dynamics of the 7:4 resonance at $a \simeq 43.7$ AU were discussed by Lykawka and Mukai (2005), who showed that the maximum stable amplitudes
drops as eccentricities rise. These authors noted that the most dynamically-stable part of the resonance \( (e = 0.25 - 0.30 \text{ with } i = 0 - 5^\circ) \) appears unpopulated, despite it being easier to find TNOs with these eccentricities than the lower eccentricities of the known 7:4 resonators (the largest-\( e \) CFEPS 7:4 has \( e = 0.21 \), while the MPC’s orbit for 2003 QX\(_{91} \) has \( e = 0.25 \)). Dynamical simulations (Hahn and Malhotra, 2005; Levison et al., 2008a; Yeh and Chang, 2009) rarely show occupation of the \( e > 0.25 \) region, so the lack of \( e > 0.25 \) 7:4 TNOs seems in line with model results that this region was not populated during the Kuiper-belt sculpting process.

Examinations of the dynamical ‘clones’ of the nominal classifications show that the phase space of the resonance is extremely complex. Even relatively long-arc orbits show great variation in libration amplitude amongst the clones, and thus our tabulated libration amplitudes are only accurate to a factor of 2. A striking aspect of the CFEPS 7:4 detections is their preferentially-small inclinations. When fitting a \( \sin(i) \) times a Gaussian distribution, we reach the same conclusion as Gulbis et al. (2010) that the acceptable \( \sigma \) widths are considerably colder than for other Kuiper Belt sub-populations. Our 95% confidence range for the inclination width is 2.5–14°, with 5° being favoured, in good agreement with the Gulbis et al. (2010) result. Lykawka and Mukai (2005) had already shown that 7:4 resonators with \( i > 10^\circ \) are much less likely to survive the age of the Solar System; thus the colder inclination distribution cannot be taken to be a direct signature of the trapping process, although the preference for \( e < 0.25 \) may be such a test.

2.9 The \( n:1 \) Resonances

The \( n:1 \) resonances require more modelling care because of the presence of symmetric and asymmetric libration islands (see Beauge, 1994, and citations to it). That is, instead of the resonant argument oscillating symmetrically around 180°, there are three possible modes. The symmetric mode is centered on 180° but, unlike for the resonances discussed earlier, there is a lower limit for the symmetric libration amplitude because the asymmetric islands occupy the phase space where
low-amplitude libration occur. The asymmetric librators have libration centers that depend on the TNO’s orbital elements (especially its $e$) and have an upper bound to their libration amplitudes (Malhotra, 1996). Detailed modelling of the $n:1$ resonances would require much more information than the small number of CFEPS objects provide. We have thus chosen to use orbital models motivated by analytic studies of the resonances, where our adjustable parameters are confined only to the inclination distribution and the fraction $f_s$ of the TNOs that are in the symmetric mode. The population estimates thus have some dependence on the accuracy of the analytic studies. Although some $n:1$ librators also show evidence of simultaneously being in the Kozai resonance (Lykawka and Mukai, 2007), we simply do not have the numbers of detections to warrant modelling this as an additional sub-component; as for the the plutinos we expect that the population estimates are only very weakly ($<10\%$) dependent on the presence or absence of the Kozai sub-component.

### 2.9.1 The Twotinos

The name ‘twotino’ has been given to 2:1 resonant librators. Much has been made in the past of the population ratio of plutinos to twotinos, because this may be diagnostic of migration models (e.g. Chiang and Jordan, 2002; Jewitt et al., 1996; Malhotra, 1995). An important goal for us has thus been to provide an estimate of the twotino population ratio to both the 3:2 and 5:2 resonances (which we discuss in section 2.12). In addition, Chiang and Jordan (2002) showed that Neptune’s migration rate could affect the population ratios of one asymmetric island to the other.

The CFEPS sample provides 5 characterized twotinos (Table 2.2). Another twotino detected in the survey, U7a08 (Petit et al., 2011), is associated with the symmetric island but U7a08 is excluded from this resonant study because its faintness puts it below the 40% detection efficiency threshold which CFEPS felt it could reliably debias (“U” means un-characterized). Unfortunately this is the largest-inclination twotino ($i = 7.0^\circ$) in our sample.
We elected to use a Gaussian inclination width of $9^\circ$, which allows the CFEPS survey simulator to provide a large fraction of $i < 7^\circ$ detections, while simultaneously allowing the existence of larger-$i$ twotinos known in the MPC sample. We find that the inclination distribution of the twotinos must (at >95% confidence) be colder than for the 3:2 and 5:2 resonances. The lack of large-$i$ 2:1 librators in CFEPS is not statistically alarming, especially when one considers that one does not expect the inclination distribution today to be Gaussian: Nesvorný and Roig (2001) and Tiscareno and Malhotra (2009) show that long-term dynamical stability of the 2:1 is inclination dependent, with inclinations above $15^\circ$ being more unstable, especially for symmetric librators. Thus the colder inclination width does not necessarily provide cosmogonic information. We have verified that changing the inclination width by a factor of two generates only a factor of two variation in the population estimate.

Three of the four characterized asymmetric CFEPS twotinos occupy the island with $\langle \phi_{21} \rangle \simeq 290^\circ$ (sometimes called the ‘trailing’ island because the perihelion longitudes are ‘behind’ Neptune’s ecliptic longitude) while the fourth occupies the leading asymmetric island. We thus have an apparent (biased) measure of the ‘leading fraction’ $f_{L}^{\text{biased}}$ of 0.25. This is an interesting contrast to Chiang and Jordan (2002) who reported that all of the twotinos from the DES at that time inhabited the leading asymmetric island, and Murray-Clay and Chiang (2005) discuss the apparent leading/trailing ratio of 7/2 at that time, or $f_{L}^{\text{biased}} = 7/9 = 0.78$. It is clear that our trailing preponderance is due to the depth of the CFEPS L4j, L4k, L5i, and L5j blocks (Petit et al., 2011) which are well placed to find trailing twotinos, while the CFEPS coverage of the longitude where leading asymmetric twotinos come to perihelion is sparse (the Ls3s block was not especially deep). We used the CFEPS survey simulator to show that on average one-third of asymmetric twotinos detected by CFEPS would be in the leading island ($f_{L}^{\text{biased}} = 0.33$) due to our block depth and placement relative to galactic plane.

\text{m}The only secure twotino with $i > 15^\circ$ is 130391 = 2008 JG$_{81}$, with $i = 23.5^\circ$, which appears to be a symmetric librating
even if the true population was equally distributed \( f_L = 0.5 \) between the two islands. We hypothesize that the DES survey simply had the opposite selection effect. Murray-Clay and Chiang (2005) suggested calibrating the observational selection effects by using the 3:2 ratio, but the galactic plane confusion is not the same for the two resonances; due to Neptune’s position, ‘trailing’ plutinos are not as confused by the galactic plane as trailing asymmetric twotinos. Therefore precise measurement of a population asymmetry demands an absolutely-calibrated survey with well-understood detection efficiency differences for the two relevant portions of sky. To illustrate what limits can be set on the true value of \( f_L \) using the CFEPS calibration, we asked the question: How large would \( f_L \) have to be before 95% of the time CFEPS would find 2 or more leading detections (and thus rule out this value of \( f_L \))? The CFEPS calibration demands \( f_L < 0.85 \) at 95% confidence. For a lower bound, \( f_L > 0.03 \) is required (95% confidence) to allow the existence of at least one leading twotino detection in CFEPS. The 67% confidence range is \( 0.35 < f_L < 0.64 \) but we prefer to use the 95% range of \( f_L = 0.03 - 0.85 \) for the fraction of all asymmetric twotinos in the leading island. The \( f_L > 0.03 \) limit only requires that the trailing/leading ratio be less than 30, to be compared with ratios up to \( \sim 10 \) found by Murray-Clay and Chiang (2005) in simulations of asymmetric capture during Neptune migration. This weak observational constraint does not yet provide interesting rejection of cosmogonic theories, but a factor of several more twotinos in well-calibrated surveys has the potential to do so.

The symmetric librator K02O12 = 2002 PU\textsubscript{170} has libration amplitude \( L_{21} = 154 \pm 4^\circ \); over a full libration cycle its perihelion longitude can thus be found anywhere on the sky not within \( \sim 25^\circ \) of Neptune. The excluded CFEPS discovery U7a08 (not characterized due to its faintness) is an alternating ‘three-timing’ 2:1 object, meaning that during numerical evolution forward in time, its resonant argument switches between symmetric and asymmetric modes. This commonly-seen behaviour (Chiang and Jordan, 2002) does not invalidate a parametrization
of the 2:1 as having a ‘symmetric fraction’ \( f_s \) because it is reasonable to assume that this fraction is maintained in steady state.

With only five characterized 2:1 CFEPS detections, our orbital distribution is based on abundant theoretical understanding of the resonance’s dynamics, rather than an empirical model fit to our detections (which will have too many parameters to be constrained by our 5 detections). Instead, the range of libration centers, amplitudes and eccentricities (and correlations between them) are provided from analytic understanding and numerical explorations of the resonance (see Section 2.3.4). This model provides a non-rejectable match, leaving as the only remaining adjustable parameter the unknown fraction \( f_s \) that the symmetric librators make up of the twotino population.

The symmetric libration fraction is poorly measured. A few such objects are known (Lykawka and Mukai, 2007), but again because the selection effects are very different for symmetric versus asymmetric librators only a survey with well-characterized sky coverage can provide an estimate. With only 1 in 5 characterized detection in CFEPS being symmetric, we can only weakly constrain \( f_s \). Because the fraction of detected twotinos which are symmetric will depend on a survey’s longitude coverage, we can only determine it for our own survey; we did this by running a large suite of models to determine the detected fraction of symmetric detections as function of the intrinsic value and find that our 20\% apparent fraction implies \( f_s \simeq 0.3 \), which we adopt. The remaining 70\% of the twotinos are equally divided among the two asymmetric islands. Luckily our population estimate is only a weak function of \( f_s \); we determined that even if \( f_s \) were increased to 0.75 the total twotino population estimate rises only 25\% (again, this result will not be identical for a survey with different sky coverage).

We find a plutino/twotino ratio to be \( \sim 3–4 \), similar to the ratio estimated by Chiang and Jordan (2002). An important new result from CFEPS is the fact that the twotinos are less numerous than 5:2 librators, which will be discussed in Section 2.12.
2.9.2 The 3:1 Resonance

CFEPS detected two TNOs in the 3:1 resonance, one of which (U5j01PD) was below the 40% detection efficiency threshold. With these statistics we are unable to explore details of the TNO distribution inside the resonance’s structure; instead we provide a population estimate, which is likely only accurate to order-of-magnitude. The dynamics allows both symmetric and asymmetric librators; Malhotra (1996) shows the 3:1’s structure. Due to lack of constraint, we retain the symmetric fraction $f_S = 0.3$ used for the twotinos. We use an orbital-element distribution inside the resonance essentially the same as for the 2:1, excepting that the model eccentricities extend up to that necessary to reach $q \approx 30$ AU.

Chiang et al. (2003) and Hahn and Malhotra (2005) demonstrated that 3:1 librators could be produced in an outward migration scenario into a initially warm ($e \sim 0.1$) pre-existing belt. In both simulations the initial disk extends to at least 55 AU from the Sun, although it is not clear where the warm disk must extend to in order to enable 3:1 trapping. Levison et al. (2008a) do not provide information on the 3:1 (or more distant) resonances, confining their discussion to $a < 60$ AU.

The 3:1 librator L5j01PD = 2003 LG$_7$ = 136120 (Table 4 of Petit et al. (2011)) was discovered by the DES survey in 2003 (Elliot et al., 2005) and independently re-discovered by CFEPS in 2005. Despite observations in each and every of 5 sequential oppositions from 2003–2007, we are unable to securely determine if the object is a symmetric or asymmetric librator, although the symmetric case is favoured. Lykawka and Mukai (2007) classified the object as symmetric using the DES data from 2003 to 2006 inclusive (with amplitude $\approx 160^\circ$ for the best-fit orbit) but we find that asymmetric libration is still allowed for orbits consistent with the astrometry. This serves as another example of the need for abundant high-precision astrometry to determine the details of the resonance dynamics. Because this TNO has flux resulting in a detection efficiency below the 40% limit in the L5j block, we do not use it in our population model.

The characterized TNO L4v08, with similar 5-opposition coverage, may also be either a symmetric or asymmetric 3:1 librator, with the former slightly
favoured. The 2-night 2004 discovery of L4v08 was already in the MPC astrometric database with designation 2004 VU_{130}, with an orbit putting it at the $d = 49$ AU aphelion of an $a = 43.9$ AU classical-belt orbit. Note that L4v08 happens to be the most distant CFEPS resonant TNO at $d = 49.7$ AU; L5j01PD is at $d = 33$ AU. Because the 3:1 population must extend to $d \sim 90$ AU, given their $e = 0.4 - 0.5$ range, the fact that in both cases $d \ll a$ again illustrates the extreme pericenter detection bias caused by the eccentricities and steep size distribution.

Using a 3:1 model similar to the 2:1 model, our population estimate is 4,000 3:1 TNOs with $H_g < 9.16$, with factor of 3 error bars at 95% confidence. The resulting debiased CFEPS 2:1/3:1 ratio estimate of $\sim 1$ is not statistically distinguishable from the $\sim 3.5$ estimate in the 50-Myr migration simulation of Chiang et al. (2003), given our uncertainties at 95% confidence. However, the Chiang et al. (2003) simulation does not ‘erode’ the surviving resonant populations (given 50 Myr after migration start) for the age of the Solar system, which could change the ratio. Hahn and Malhotra (2005) show (see their Figure 6) that for their model’s emplaced populations the 2:1/3:1 ratio does not change much during erosion even if both populations drop mildly over 4 Gyr; however their 2:1/3:1 ratio is $\sim 10$, which is rejectable at $>95\%$ confidence. Both models plausibly demonstrate the production of 3:1 TNOs with $e > 0.4$ and $i$ up to $20^\circ$.

2.9.3 The 5:1 Resonance

Our sole 5:1 TNO (L3y02 = 2003 YQ_{179}) was provisionally classified as a detached object by Gladman et al. (2008) but flagged as being insecure and quite possibly resonant in the 5:1 (despite already having a 3-opposition orbit in 2008). Further tracking observations by our team have resulted in the now-improved orbit being (insecurely) classified as a resonant 5:1 orbit. The high-order resonances are so ‘thin’ in phase space, that we postulate other ‘detached’ TNOs are actually in high-order (meaning $j - k$ is large) mean-motion resonances as well. In this case the largest-$a$ orbit consistent with the astrometry is just outside the resonance; however, we are essentially sure that this object is the first identified 5:1 libator.
We note that the detached object 1999 CF$_{119}$, discovered by Trujillo et al. (2001), has a semimajor axis \(\sim 0.3\) AU beyond the 5:1 resonance, and the Gladman et al. (2008) analysis indicates the lowest-\(a\) plausible orbits are just barely beyond the resonant semimajor axis. A small systematic error in one opposition of the 4-opposition orbit might suffice to remove the nominal orbit from the resonance; we thus suggest additional observations.

With \(a = 88.38\) AU, \(e = 0.579\), and \(i = 20.1^\circ\), the detection biases against TNOs like L3y02 are extreme. We used a 5:1 model similar to the 3:1, with asymmetric and symmetric \((f_S = 0.30)\) librators, and an inclination width \(\sigma = 10^\circ\). Using the single detection, we estimate 8,000 TNOs with \(H_g < 9.16\) in the 5:1 resonance, an estimate which is only good to a factor of five given our lack of knowledge of the inclination distribution. In particular, if the inclination distribution is considerably hotter than the \(\sigma = 10^\circ\) value we have taken from the 2:1 (which seems likely given that L3y02 has an inclination twice that value), then the population estimate will rise. Even at the nominal \(i\)-width, the 95% confidence limits permit this resonance to actually be the most populated of all trans-Neptunian space.

\subsection*{2.10 Resonances with No CFEPS Detections}

In this section we discuss two low-order resonances in which CFEPS did not find any objects, and we use this non-detection to provide upper limits on the populations. Because of its extensive discussion in the literature, we calculate an upper limit for the Neptune trojans (1:1 resonance), and because CFEPS detected one object in each of the 3:1 and 5:1 resonances, we calculate an upper limit for the 4:1 resonance.

\subsubsection*{2.10.1 The Neptune Trojans}

The first Neptune trojan was identified by Chiang et al. (2003), and only \(\sim 7–8\) are currently known (Horner et al., 2012; Sheppard and Trujillo, 2010a). The CFEPS
survey did not discover a single Neptune trojan$. As the survey ran, we were very aware of the possibility of detecting 1:1 resonators, and confirm that this has nothing to do with possible detection biases in the survey. The pericenter longitudes of many of the known resonances overlap with the longitudes where Neptune trojans would spend their time, and CFEPS found resonant and scattering TNOs at distances even closer than those which Neptune trojans would approach; maximum eccentricities of the known trojan sample (Sheppard and Trujillo, 2010a) of $e \sim 0.05$ would have Neptune trojans approach no closer than $q \sim 28.5$ AU (further than the distance at which we discovered and tracked the plutino L4m02).

We are not alarmed by the lack of such a detection, because the fraction of TNOs which are Neptune trojans is very small. To quantify this, we built a strawman trojan model and ‘observed’ it through the CFEPS survey simulator. The model trojans had $a$ within 0.2 AU of 30.2 AU, $e$ uniform from 0 to 0.08, with ascending nodes and mean longitudes uniformly distributed. Libration amplitudes $L_{11}$ were chosen between 0–40°, with the relative number of objects having each libration amplitude increasing linearly from 0 to 40°. Half of the trojans were set to be trailing ($\langle \phi_{11} \rangle = 300^\circ$) rather than leading ($\langle \phi_{11} \rangle = 60^\circ$). The resonant argument $\phi_{11}$ was chosen with sinusoidal time weighting with amplitude $\pm L_{11}$ around $\langle \phi_{11} \rangle$, with $\omega$ then calculated to fulfill the resonant condition. Since the literature lacks the information needed to estimate the inclination distribution, we chose a hot population with a similar inclination distribution to the non-Kozai plutinos ($\sigma = 15^\circ$). The $H_g$-magnitude distribution was fixed with $\alpha = 0.8$, as estimated by Sheppard and Trujillo (2010b).

We used the simulator to determine the Trojan population that would give 3 or more CFEPS detections (on average); this provides the 95% confidence limit for

$^a$Although the MPC currently lists L4k09 = 2004 KV18 as a L5 trojan (Horner et al., 2012), the eccentricity of 0.184 is larger than numerically-determined stability limits (Nesvorný and Dones, 2002). Although ‘near’ the L5 cloud, the Gladman et al. (2008) analysis shows the object scatters heavily on time scales $<10$ Myr and thus Petit et al. (2011) reported L4k09 as a scattering TNO; even if on a very short time scale it may be temporarily near the L5 state. Near-Earth asteroids exhibit similar temporary co-orbital behaviour (Morais and Morbidelli, 2002).
Poisson statistics. This limit is

\[ N_{\text{trojans}}(H_g < 9.16) < 300 \ [95\% \text{ confidence}] , \]  

when stated for the same \( H_g \) value as the other resonances we study. Sheppard and Trujillo (2010a) estimate that there are \( \sim 400 \) Neptune trojans with radii \( > 40 \) km; assuming a 5% albedo, this corresponds to \( H_g \sim 9.6 \). Scaling our population upper limit using \( \alpha = 0.8 \) makes the CFEPS upper limit \( < 600 \) trojans with \( D > 80 \) km (95% confidence), indicating that the non-detection of a Neptune trojan in CFEPS is not statistically alarming given the 400-trojan estimate of Sheppard and Trujillo (2010a).

\[ \text{2.10.2 The 4:1 Resonance} \]

CFEPS did not detect any 4:1 resonators. Given that only one object was discovered in each of the 3:1 and 5:1 resonances, and there is only one object in the MPC database that is tentatively flagged as a 4:1 resonator (2003 LA7; Lykawka and Mukai, 2007), it is not worrying that none were detected in the 4:1 resonance. Here we provide an upper limit on the number.

We base our orbital distribution on the 3:1 and 5:1 distributions, with \( f_S = 0.30 \) and \( \sigma = 10^\circ \). To get a 95% confidence upper limit, we used the simulator to determine the 4:1 population that would give 3 or more CFEPS detections (on average). This gives

\[ N_{4:1}(H_g < 9.16) < 16,000 \ [95\% \text{ confidence}] . \]  

This upper limit is consistent with the population of the 4:1 being similar to the 3:1 and 5:1 populations.
2.11 CFEPS Comparison to a Cosmogonic Model

The CFEPS project has produced three data products, all of which can be accessed at [http://www.cfeps.net](http://www.cfeps.net). First, there is database of TNO photometry and astrometry for TNOs (characterized and non-characterized) seen in the survey. The characterized list is intimately linked to the second data product: the Survey Simulator, described below. Thirdly, one can obtain an orbital element distribution (called the L7 synthetic model) which is an empirically-determined orbital and $H$ distribution which, when passed through our Survey Simulator, provides a distribution of detections statistically indistinguishable from the CFEPS detections.

The true power of CFEPS is the ability to compare a proposed model (resulting from a cosmogonic simulation) to reality. In order to decide how well a proposed Kuiper Belt orbital distribution matches the CFEPS data, one must not just compare the Kuiper Belt model to the L7 synthetic model. This is because CFEPS (or any survey) will be biased toward or against detections in particular parts of orbital parameter space; a model seemingly different from the the L7 synthetic model may be biased when ‘viewed’ through the CFEPS pointing history and flux limits into an acceptable match. Similarly, models which appear to match some aspects of the L7 synthetic model may fail dramatically. The only quantitative way to compare a model to the Kuiper Belt via the CFEPS survey is to pass the model through the L7 Survey Simulator and compare the distribution of simulated to real detections. As an example of this process, we here examine the results of a cosmogonic simulation based on the Nice model of giant planet migration ([Levison et al., 2008a](#)), in order to compare the simulated plutinos with the CFEPS plutino orbital distribution. We chose this model because the plutino libration amplitudes were made available by the authors; providing such information is the state of the art in Kuiper Belt formation models and should become the norm.

We begin with the plutino orbital elements from the end of Run B of [Levison et al. (2008a)](#), which are those emplaced during the planet-migration pro-
cess and then survive 1 Gyr ‘erosion’ process to eliminate TNOs that did not have long-term stability on the time scale of the Solar System’s age. Because there are only 186 model surviving plutinos, we create new particles with very similar orbital elements by “smearing out” those of the existing particles; values of $a$, $e$, and $i$ for each new particle were randomly chosen within ±0.1 AU, 0.02, and 5° of the orbital elements of one of the original Nice model particles. We verified that this does not change the overall shape of the cumulative distributions for these orbital elements. Next, $\phi_{32}$ is chosen sinusoidally from within the values allowed by the known libration amplitude of the Nice model particle. The ascending node’s longitude $\Omega$ and mean anomaly $M$ are chosen randomly, leaving $\omega$ to be chosen to satisfy the resonance condition. Lastly, the particle’s $H$ magnitude is chosen from the same $\alpha = 0.9$ exponential distribution used for CFEPS plutinos. The CFEPS survey simulator then evaluates whether or not it was detected.

The process was repeated until 10,000 synthetic detections were generated, creating cumulative detection distributions (Figure 2.10) from which the probability of drawing the detected CFEPS sample is judged. The detected $e$’s and discovery distances provide statistically-acceptable matches to the CFEPS detections. In contrast, the hypotheses that the $i$ or $L_{32}$ libration amplitude distributions of the CFEPS detections could be drawn from this Nice model simulation both fail at >99.9% confidence. The $i$-distribution of the detections that would come from an intrinsic plutino distribution produced by the Nice model is far too cold, and the $L_{32}$ distribution contains too many large libration-amplitude objects.

Although this model is rejected, this style of model shows the forefront of what models must now provide in Kuiper Belt science. That is, a cosmogonic model should produce TNO orbital distributions for the entire Kuiper Belt, including resonant libration amplitudes and determination of Kozai resonance occupation. Comparison with the current TNO distribution can only really be performed if the cosmogonic simulation (which often focuses on events in early Solar System history) is dynamically eroded for the $\sim 4$ Gyr interval to bring it to the present day. The fact that the Levison et al. (2008a) simulations were eroded for 1 Gyr
Figure 2.10: Comparison between CFEPS plutino detections and simulated detections from the Nice model plutino distribution. Red squares are real CFEPS plutino detections, the dotted black line shows the intrinsic Nice model plutino distributions, and the blue line is the simulated detections after running this intrinsic population through the CFEPS survey simulator. The magnitude distribution is not shown; this was not provided in the Nice model data but we find using the same $H_g$ magnitude distribution as for the CFEPS plutinos produced an acceptable match (which is unsurprising given that the $\epsilon$ distribution is similar and $\alpha = 0.9$ was chosen to represent the CFEPS detections). While the eccentricity and discovery distance distributions match the CFEPS data reasonably well, the Anderson-Darling analysis indicates the CFEPS $i$ and $L_{32}$ distribution would occur <0.1% of the time.
instead of 4 Gyr might result in small changes to the libration amplitude distribution of the survivors, but is unlikely to resolve the major discrepancy given that Nesvorný and Roig (2000) and Tiscareno and Malhotra (2009) show that the distribution only changes appreciably with order of magnitude increases of timescale.

We note a large number of non-resonant particles surrounding the 3:2 (and some other resonances) with low $e$ at the end of the Nice model simulations. We presume these TNOs to be generated during the phase where the neptunian eccentricity is shrinking rapidly, which causes the resonance to narrow and ‘drop out’ formerly-resonant particles on either side of the resonance. We call these the ‘beards’ of the resonance in this model. This features should be preserved in the Kuiper Belt if the resonances had abundant low-$e$ particles in the resonances when Neptune’s $e$ dropped, but these beards are not obviously present in the real Kuiper-Belt distribution. We doubt this is a selection effect, but are unable to present a quantitative analysis with the current CFEPS sample size.

2.12 Resonant Populations

This work provides for the first time absolute population estimates for a large variety of trans-Neptunian resonances, allowing population comparisons to quantitatively de-biased data that takes into account the myriad of observational selection effects. While the ratio of various resonance populations have been identified as potentially diagnostic – for example, Jewitt et al. (1996) already mention using the 2:1/3:2 population ratio to constrain Neptune migrations via models like Malhotra (1995)– the debiasing of the selection effects for the two resonances has never been done to the level of detail presented here. Chiang and Jordan (2002) and Chiang et al. (2003) showed models producing population ratios of resonances to each other (for example, the 5:2 to 2:1) or of sub-islands inside the 2:1 to each other, but again lacked the ability to compare to a survey for which the longitude coverage could be quantitatively de-biased for selection effects. Hahn and Malhotra (2005) produced ratios between resonances and to the main belt from a model in the context of an outward Neptune migration into a pre-
Table 2.3: Resonance Populations

<table>
<thead>
<tr>
<th>Res.</th>
<th># of det.</th>
<th>$e_c$</th>
<th>$e_w$</th>
<th>$\sigma_i$</th>
<th>Median Pop. ($H_g &lt; 9.16$)</th>
<th>Median Pop. ($H_g &lt; 8$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3:2</td>
<td>24</td>
<td>0.18$^k$</td>
<td>0.06$^k$</td>
<td>16$^k$+$^8$</td>
<td>13,000$^{+6,000}_{-5,000}$</td>
<td>1,200$^{+600}_{-400}$</td>
</tr>
<tr>
<td>5:2</td>
<td>5</td>
<td>0.30</td>
<td>0.10</td>
<td>14$^k$+$^20$</td>
<td>12,000$^{+15,000}_{-8,000}$</td>
<td>1,100$^{+1,400}_{-700}$</td>
</tr>
<tr>
<td>4:3</td>
<td>4</td>
<td>0.12</td>
<td>0.06</td>
<td>8$^k$+$^3$</td>
<td>800$^{+1,200}_{-600}$</td>
<td>70$^{+100}_{-50}$</td>
</tr>
<tr>
<td>5:3</td>
<td>6</td>
<td>0.16</td>
<td>0.06</td>
<td>11$^k$+$^14$</td>
<td>5,000$^{+5,200}_{-3,000}$</td>
<td>450$^{+470}_{-290}$</td>
</tr>
<tr>
<td>7:3</td>
<td>2</td>
<td>0.30</td>
<td>0.06</td>
<td>~10</td>
<td>4,000$^{+8,000}_{-3,000}$</td>
<td>320$^{+760}_{-270}$</td>
</tr>
<tr>
<td>5:4</td>
<td>1</td>
<td>0.12</td>
<td>0.06</td>
<td>~10</td>
<td>160$^{+100}_{-140}$</td>
<td>10$^{+60}_{-9}$</td>
</tr>
<tr>
<td>7:4</td>
<td>5</td>
<td>0.12</td>
<td>0.06</td>
<td>5$^k$+$^9$</td>
<td>3,000$^{+4,000}_{-2,000}$</td>
<td>300$^{+400}_{-200}$</td>
</tr>
<tr>
<td>1:1</td>
<td>0</td>
<td>0.0-0.8</td>
<td>-</td>
<td>~15</td>
<td>&lt;300</td>
<td>&lt;40</td>
</tr>
<tr>
<td>2:1</td>
<td>5</td>
<td>0.1-0.4</td>
<td>-</td>
<td>~7$^k$+$^5$</td>
<td>3,700$^{+4,400}_{-2,400}$</td>
<td>340$^{+400}_{-220}$</td>
</tr>
<tr>
<td>3:1</td>
<td>1</td>
<td>0.25-0.55</td>
<td>-</td>
<td>~10</td>
<td>4,000$^{+9,000}_{-3,000}$</td>
<td>340$^{+800}_{-290}$</td>
</tr>
<tr>
<td>4:1</td>
<td>0</td>
<td>0.30-0.60</td>
<td>-</td>
<td>~10</td>
<td>&lt;16,000</td>
<td>&lt;1,400</td>
</tr>
<tr>
<td>5:1</td>
<td>1</td>
<td>0.35-0.65</td>
<td>-</td>
<td>~10</td>
<td>8,000$^{+34,000}_{-7,000}$</td>
<td>700$^{+3,000}_{-700}$</td>
</tr>
</tbody>
</table>

Notes. Principle parameters for the models of each mean-motion resonance. All resonances used $\alpha = 0.9$ for the $H_g$-magnitude distribution (that measured for the plutinos), except the 1:1 which used $\alpha = 0.8$. Uncertainties reflect 95% confidence ranges. Population estimates for $H_g < 9.16$ correspond to 100-km diameter (for nominal albedo), while $H_g < 8$ estimates are provided for comparison with the classical-belt population estimates of Petit et al. (2011). The $k$ subscript for the plutinos indicates that these are the parameters for the non-Kozai component.

Existing ‘warm’ ($e = 0.1$) belt, while Levison et al. (2008a) produced a model in which the Kuiper Belt was moved out to its current location; both of these models were forced to make comparisons to surveys that could account for biases in, at best, an approximate way.

Figure 2.11 shows the debiasing of CFEPS, transforming the resonant populations from their biased apparent fractions (left column) to their ‘true’ values (right column, a debiased sample from the models presented in Table 2.3). An evident result is that the distant resonances make up a much larger fraction of the total resonant population in reality than in the flux-biased sample. Although it is obvious the fraction of large-$a$ resonant TNOs (compared to low-$a$ ones) will be higher in
Figure 2.11: The apparent versus debiased resonant Kuiper belt in $a$, $e$, and $i$. The two left panels show the $(a, e)$ and $(a, i)$ distribution of the flux-limited CFEPS resonant detections from $a = 30 - 65$ AU. The right panels show the distribution of their debiased population, scaled so that the plutinos have 100 members. It is obvious that the true Kuiper belt has a higher fraction of larger-$a$, lower-$e$, and larger-$i$ members than the currently-detected sample. The absence of low-$e$ resonant TNOs with $a > 46$ AU is not absolutely required by our modelling due to the detection biases against them.
Figure 2.12: Population estimates for the 3:2, 5:2, and 2:1 resonances. Each histogram is a separately-normalized distribution of population estimates which yield the correct number of detections for that resonance. Vertical lines show the median of the population estimates for each resonance. Although the 2:1 and 5:2 histograms overlap, the probability that the 2:1 population is larger than the 5:2 population when both are randomly drawn from these distributions is \(<5\%\).

In reality than in the flux-biased sample, this effect has never been quantified. In particular it is obvious that beyond the 2:1 current surveys have just seen the ‘tip of the iceberg’ and the resonant populations contain many more large-\(i\) and/or low-\(e\) members than either CFEPS or the full MPC sample have yet exposed.

Population comparisons benefit from uncertainty estimates. In particular, the population ratios in the well-studied 3:2, 2:1, and 5:2 are desirable. To obtain a set of absolute population estimates, we drew particles at random from our model
orbital and $H$-magnitude distributions until obtaining the true number of CFEPS detections for a given resonance (Figure 2.12). There is an essentially Poisson distribution of plausible ‘true’ populations that will allow the observed number of detections, explaining the shape to the histograms in Figure 2.12; the median is reported in Table 2.3 along with the upper and lower limits which leave only 2.5% of the measurements in each tail. Although we use conservative 95% confidence regions (resulting in large stated uncertainties), CFEPS is for the first time able to provide measurements of the resonant populations that take into account the longitude coverage and relative depth of its survey patches.

One of the most striking results (Figure 2.12) is that the best-estimate populations for the important resonance trio 3:2/2:1/5:2 are in the ratio $\sim 4/1/4$. To our knowledge, this is in stark contrast with all previously-published models; those which obtain a weakly-populated 2:1 (relative to the 3:2) never simultaneously have a 5:2 population equal to that of the plutinos. The simulations of Chiang et al. (2003) showed a huge 2:1/5:2 ratio unless migration occurred into a hot disk which dropped the ratio to roughly 3/2 (to be compared to the 1/4 ratio we favor), with a plutino population even larger than the 2:1. These authors ruled out ‘resonance sticking’ of scattering TNOs as the dominant production method for 5:2 resonators due to the incorrect libration amplitude distribution (a conclusion we share based on the small $L_{52}$ amplitudes for the CFEPS detections). Hahn and Malhotra (2005)’s simulations into a warm primordial disk exhibit 3:2/2:1/5:2 ratios of about 2/5/1, and Levison et al. (2008a) produce 8/3/1. That is, all simulations to date produce fewer 5:2 resonators than twotinos by factors of several, whereas CFEPS indicates that the reverse is true (and rules out the 2:1 being more populated than the 5:2 at $>95\%$ confidence). This is thus an important new constraint on formation models.

This kind of constant also holds for more distant resonances. Both the 3:1 and 5:1 have best-estimate populations larger than the 2:1 (although uncertainties are large), indicating that large-$a$ resonant orbits must be efficiently populated by formation models. In models like Levison et al. (2008a), where the Kuiper belt is
transplanted out, this is difficult to do because large-\(a\) orbits are inefficiently generated. The hypothesis that these objects are instead swept into \(a > 50\) AU resonances from warm or hot populations located at these distances before planet migration is faced with the problem of explaining where all the non-resonant TNOs have gone.

This realization that the distant resonances are heavily populated opens the possibility that the current scattering population is dominantly being supplied by abundant resonant escapees. If true then the resonant reservoir would be the ultimate source of Jupiter Family Comets (JFCs), through the chain: resonant \(\Rightarrow\) scattering \(\Rightarrow\) Centaur \(\Rightarrow\) JFC. Due to their chaotic boundaries, the resonances provide a ‘leakier’ source of scattering TNOs than the classical belt and most TNOs escaping a resonance would immediately find themselves on Neptune-coupled orbits and begin scattering. In such a scenario the escape rate from all resonances would balance the loss of actively scattering objects to the Centaur population or ejection from the Solar System. The flaw in this scenario is that there seem to be too many scattering TNOs in the current epoch to permit them being anything other than the decaying remnant of a huge primordial population (Duncan and Levison, 1997). Petit et al. (2011) estimate (to order of magnitude) that there are currently \(\sim 5,000\) \(H_g < 9.16\) actively-scattering TNOs (with a clarification on the definition of this population); this is too large a fraction of the sum of the resonant populations in Table 2.3 to permit the scattering population to be in steady state. Volk and Malhotra (2008) call into question even the ‘decaying remnant’ scenario as the supply rate they estimate from the metastable Kuiper belt (mostly a mix of detached and resonant objects with \(a > 50\) AU and \(q > 33\) AU) into the Jupiter-Family comets seems too low given their extrapolation of observational estimates of the ‘excited’ \((i > 5^\circ)\) population in the 10–100 km size range. This analysis should be re-done however because Volk and Malhotra assumed that essentially all of today’s ‘excited’ TNOs (observed by various surveys) are scattered objects.

\(^{6}\)Horner and Lykawka (2010) suggested that the Neptune Trojans alone could be an important Centaur source, but it seems unlikely that the other (vastly more populated) resonances would not dominate the leakage supply.
contributing to the Centaur supply chain, while in fact the $a > 50$ AU population has a very non-negligible resonant component (Gladman et al., 2008), and Petit et al. (2011) and this chapter show that the actively scattering population is only a tiny fraction of the other ‘excited’ (resonant + hot classical + detached) populations.

The total resonant population is, however, also comparable to the Petit et al. (2011) estimate for the sum of the outer classical and detached populations (of $\sim 80,000$ with $H_g < 9.16$). This permits serious consideration of the hypothesis that most detached TNO population are resonant objects that were dropped out of resonance while the resonant objects were being emplaced, but must generate roughly equal numbers of resonant and non-resonant objects surviving to the present day.

### 2.13 Discussion

Given the available constraints from the structure and relative populations of various Kuiper Belt components, what can one conclude about the processes that emplaced these components? Based on our debiased understanding from CFEPS, we feel that the following constraints are of chief importance:

1. The resonant populations appear to be consistent with all being emplaced from a source population that lacked a cold component. (The differences between them can be plausibly explained by capture or subsequent erosion processes that are inclination-dependent.)

2. The inner classical belt and outer classical belt lack a cold inclination component (Petit et al., 2011), with only the main belt having both hot and cold components.

3. The sum of the resonant populations is $\sim 75\%$ that of the main belt, for $H_g < 8$.

4. The current ‘actively scattering’ disk is $\sim 5\%$ of the main belt population, with at least factor of two uncertainty.
Although we do not support it here with detailed simulations, we believe that the following scenario could explain the known structure.

A crucial feature is that the cold population is confined to the $a = 42 - 47$ AU region of the main belt, with a hotter $e$ distribution for $a > 44.4$ AU. We postulate the cold population could be primordial, with an initial outer edge at this 44.4 AU boundary. The plausible scenario consists of all the other Kuiper Belt populations (hot classical, including the inner and outer belts, detached objects, resonant objects, and the currently scattering objects) being planted into the belt via a mechanism similar to that described by Gomes (2003) and Levison et al. (2008a), in which a massive scattering disk is flung out by the migrating giant planets; resonant trapping of the scattering objects and subsequent dropout litters the hot classical population behind the slowly-advancing resonances (which are wide and powerful due to Neptune’s temporarily larger $e_N$). Neptune ‘jumps’ out several AU due to encounters with Uranus and both planets decouple due to damping of their eccentricities. Today’s resonant objects are those which were still trapped during the final stages of this process as $e_N \to 0$. Unlike the Levison et al. (2008a) model, we posit: (1) The scattering disk extends to very large $a$ already when Neptune ‘jumped’ out to nearly 30 AU. (2) This scattering disk was very hot; essentially the $\sigma_h \simeq 15^\circ$ width which all the non-cold populations share. How this happens is unknown. (3) The cold population is already in place; it is largely unaffected because the 2:1 resonance jumps to or beyond the 44.4 AU edge. Eccentric Neptune is able to dimly ‘stir’ the $a/e$ distribution of the cold disk, keeping most stirred perihelia $q < 44$ AU, before $e_N$ rapidly decays.

A critical constraint is to prevent TNOs from the cold population appearing in either the 3:2 or 2:1 resonance; this requires that after jumping to near $a = 30$ AU, any remaining small outward Neptune migration cannot allow the resonances to sweep through a cold population, because it would readily trap and preserve them (Hahn and Malhotra, 2005). Keeping the 2:1 free of low-$i$ TNOs can be accomplished by having the post-jump value of the resonant semimajor axis beyond the outer edge of the cold disk (say, landing in the 45–46 AU range before finishing...
outward migration by another ∼AU or so). The situation with the 3:2 is more complex because its lack of a cold component seemingly implies that by the time Neptune jumped, the semimajor axis range between the post-jump $a_{3:2} \sim 37$ AU and today’s value must have already been empty of cold objects. Although a primordial inner edge of the cold population is not impossible, the fact that the current $a = 42.4$ AU inner boundary of the cold population is at the border of the $\nu_8$ secular resonance allows a scenario in which this strongly-unstable secular resonance swept through the ∼37–39 AU region prior to any final small-distance Neptune migration; Holman and Wisdom (1993) show that the $\nu_8$ drives particles to Neptune encounters in only ∼30 Myr, which is comparable to the migration time scale for Neptune in Levison et al. (2008a). In this scenario, the primordial cold objects with $a < 42.4$ AU join the scattering TNOs, but make up only a tiny fraction of this population as they are ‘diluted’ if any of them are later re-planted into the Kuiper Belt. Unfortunately, the timing (and even migration direction) of the $\nu_8$ is unclear; Nagasawa and Ida (2000) show early and rapid migration of the $\nu_8$ inwards as the the protoplanet disk’s mass eroded, but their calculations did not include the probable outward migration of Neptune.

In our scenario one has an easy explanation for the differences in colours, size distribution, and binary fraction of the cold main-belt fraction; the cold belt was simply steeper, redder, and either formed more binaries or preserved a greater fraction of them, unlike the implanted components (Parker and Kavelaars, 2010). Although there is no direct observational timing constraint, this implantation scenario seems more natural if the disk is scattered very early in the Solar System’s history, without the ∼600 Myr delay proposed in the Nice Model (Gomes et al., 2005). In fact, our scenario does not stipulate where the ‘early’ scattering component comes from, although the most plausible source is it being perturbed out from the planetesimal-rich giant-planet region interior to 30 AU. At the time of Neptune’s jump, this early scattering population must extend to $a > 50$ AU in order to allow efficient trapping into the well-stocked distant resonances.
The mechanism that causes this early scattering population (which is the source for all hot Kuiper Belt populations) to have the needed inclination width of $\sigma \sim 15^\circ$ is unclear. Perhaps the giant planets somehow vertically heated the planetesimal belt before it was scattered out (although in general scattering will pump $a$ and $e$ at least as fast as $i$). Gomes (2003) manages to produce large-$i$ implantations from a source disk, although the more recent Levison et al. (2008a) study produced much colder implanted population. Perhaps other now-gone (‘rogue’) planets caused the initial vertical dispersion, although this too seems inefficient (Gladman and Chan, 2006). Very nearby stellar encounters could generate the inclinations by scattering objects (e.g. Kobayashi et al., 2005) but preserving the $\sigma \sim 2^\circ$ cold disk in a $\sim 44$ AU ring is a very strong constraint.

The following estimates of sub-populations are intended only to provide a coherent picture to a factor of 3 or so, with all population estimates for $H_g < 9.16$ (roughly $D > 100$ km). CFEPS estimates (Petit et al., 2011) that today’s scattering population is $\sim 10^4$. Assuming this is not currently in steady state re-supply from another source, Duncan and Levison (1997) estimate that this would require about $\sim 100$ times as many scattering objects $\sim 4$ Gyr ago; in a scenario where this disk goes to considerably smaller perihelion distances than the current $q \sim 35$ AU, the initial population would have been at least several times larger and we take $10^7$ initial $D > 100$-km scattering bodies. In an $\alpha \simeq 0.8$ size distribution most of the mass in in the small end, and the resulting $\sim 10$ $M_\oplus$ of bodies is comfortably smaller than the mass of the outer planets. We take this primordial scattering population to be the source of the high-$i$ populations. Levison et al. (2008a) estimate $\sim 0.5\%$ of such a primordial scattering gets trapped into non-resonant orbits, implying a hot classical population of $\sim 50,000$, which is comparable to the 35,000 estimated in Petit et al. (2011) when one realizes that it is only the hot main-belt population that is relevant (the cold population being pre-existing in our scenario). In addition, Levison et al. (2008a) report that the plutinos make up about 20% of the non-resonant objects implanted in the main belt, or about 10,000 objects, again reasonably in accordance with the CFEPS estimate of 13,000. This scenario is not
here supported by simulations, which would need to show that (1) the cold belt could survive the process, (2) the distant resonances can be efficiently filled, and (3) the Levison et al. (2008a) trapping fractions are not strongly affected by the hotter primordial scattering population that is required. In this scenario, gradual migration is a relatively unimportant process for the Kuiper Belt’s current structure.

Much of the excitement in Kuiper Belt studies comes from the vigorous interplay over the last two decades between observation and theory, and the steady stream of unexpected discoveries in both domains. Much work remains to be done. While there is evidently considerable room for future surveys to improve upon the CFEPS estimates, this can only be done with well-characterized surveys whose selection effects are rigorously monitored. In turn, the debaised orbital elements distributions will lead to much tighter constraints on models seeking to solve puzzles still present in our understanding of how the outer Solar System settled to its current state.
Chapter 3

Detection Biases for the Plutinos, Including the Kozai Resonance

3.1 Introduction

In this chapter we discuss Plutinos, which are TNOs that are in the 3:2 mean-motion resonance with Neptune. Plutinos are among the easier TNOs to observe; their semimajor axis of approximately 39.5 AU places them near the inner edge of the Kuiper Belt. Discovery is also helped by the high eccentricities many plutinos possess, causing close-in pericenters and lower apparent magnitudes, and thus brighter objects (this is an extremely strong effect since the TNOs are observed with reflected light, so their flux is proportional to distance$^{-4}$). Plutinos (and other resonant TNOs) can have lower perihelia than non-resonant TNOs because the mean-motion resonance protects them from close encounters with Neptune (e.g. Malhotra, 1996).

As of November 2012 there were 244 plutinos listed in the MPC database. However, only about 120 of these have high-quality multi-opposition orbits, allowing numerical orbital integrations (Gladman et al., 2008; Lykawka and Mukai).

This chapter is based on the following published work: S. M. Lawler and B. Gladman, Plutino Detection Biases, Including the Kozai Resonance, AJ 146, 6 (2013).
(2007) to prove that the objects are truly resonant (showing libration of the resonant angle; see Section 2.1.1), and not just located near the resonance phase space.

This paper will highlight the special selection effects caused by the Kozai resonance within the 3:2 resonance. “Kozai plutinos” are TNOs that are simultaneously in the 3:2 mean-motion resonance with Neptune and in the Kozai resonance. Pluto was the first known Kozai resonator (Williams and Benson, 1971).

What is now known as the Kozai resonance was first described for high-inclination asteroids by Kozai (1962). This effect occurs when a small body in orbit around a large central mass is perturbed by a third mass at high relative orbital inclination $i$. In the case of the trans-Neptunian region, the small bodies are the TNOs themselves, and the high relative inclination perturber is Neptune, and to a smaller degree the other planets. Any gravitational perturbations on the orbit of a small body cause $\omega$ to change. Most of the time, this causes $\omega$ to precess, but under the special conditions that the perturber is at high relative inclination, $\omega$ will oscillate instead: this is the signature of the Kozai resonance. The $\omega$ oscillations are accompanied by $e$ and $i$ oscillations with the same period (on the order of millions of years) that are opposite phase from each other: when $e$ is at its highest, $i$ is at its lowest, and vice-versa (Figure 3.1).

For non-resonant TNOs, Thomas and Morbidelli (1996) show that the Kozai resonance only occurs at extremely large eccentricity ($e > 0.9$). However, inside mean-motion resonances, the precession rates speed up and the Kozai resonance can appear at moderate $i$. This is how Pluto can show Kozai oscillations despite being at the relatively low inclination of 17°.

Lykawka and Mukai (2007) presented the largest collection of plutinos that has been analyzed for Kozai resonance, after combing the contents of the MPC database at the time. They found that 22 out of 100 plutinos were solidly in the Kozai resonance, with $\omega$ oscillating around 90° or 270°, and in one case around 0°. Unfortunately, the MPC database’s collection of detections from many different non-uniform surveys does not allow for easy debiasing. Because of this, it is very
Figure 3.1: A simplified orbital diagram of the Kozai effect. Solid lines show the orbits of two objects, similar to Neptune (blue) and Pluto (brown), with the sun in the center. Top two panels show a face-down view of the system, and bottom two panels show an edge-on view, highlighting the relative inclination of the two orbits. Over the course of a Kozai oscillation (which has a period of a few million years), the smaller, outer object’s orbit changes from a high eccentricity, low inclination state (left panels) to a low eccentricity, high inclination state (right panels).

difficult to measure the true fraction of Kozai versus non-Kozai plutinos from this dataset.

This manuscript was motivated by the results of CFEPS (Gladman et al., 2012; Jones et al., 2006; Kavelaars et al., 2009; Petit et al., 2011), which detected and then re-acquired nearly 200 TNOs, including 24 plutinos, to produce high-quality orbits and orbital classifications. Two of the 24 detected Plutinos in the survey were found to be in the Kozai resonance upon examination of a 10 Myr orbital integration. Because 8% of the CFEPS-discovered plutinos were Kozai plutinos, in order to properly debias CFEPS’s result to produce the absolute population and orbital element distribution of the plutinos, this Kozai component had to be included and properly modelled (as discussed in Section 2.4.2).
The Kozai plutinos are poorly studied theoretically, with almost no discussion of their relation to giant planet migration. Neptune’s mode of migration, either a violent dynamical reaction that scatters Neptune outward onto an eccentric orbit which damps (as in the “Nice model”; e.g. Levison et al., 2008a) or a smooth outward migration (e.g. Chiang and Jordan, 2002; Hahn and Malhotra, 2005), as well as the timescale of the migration should have an effect on various measurable parameters of the Kozai plutinos. Chiang and Jordan (2002) do follow the particles in their Neptune migration simulation to diagnose Kozai plutinos, but they give no discussion of how the various parameters change with different migration speeds. This chapter provides guidelines for which parameters to measure in a distribution of simulated Kozai plutinos that may be produced by a giant planet migration simulation, and provides predictions for where on the sky to find the highest density of Kozai plutinos, so that parameters of these real Kozai plutinos can be compared with theoretical predictions.

Though this paper only discusses plutinos in the Kozai resonance, Lykawka and Mukai (2007) also catalogued Kozai resonators in the 5:3, 7:4, and 2:1 mean-motion resonances. Other resonances can also exhibit Kozai oscillations in some portion of the resonant phase space.

This paper provides a basic understanding of the Kozai resonance within the 3:2, how the Kozai dynamics affect plutino observations, and introduces possible uses for this resonance in distinguishing between different giant planet migration models. Section 2.1.1 discusses in-depth the dynamical requirements for a TNO to be in a mean-motion resonance. Section 3.3 presents toy models of the 3:2 resonance (Sections 3.3.1 and 3.3.2), in order to demonstrate the effect of libration, and finally gives a realistic plutino distribution (Section 3.3.3). Section 3.4 discusses both the dynamics of the Kozai plutinos and the on-sky detection biases that result from the dynamical constraints placed on the Kozai plutinos. Section 3.5 gives a summary of how we simulate the plutino population. This simulation is used in Section 3.6 to show biases that observers will encounter on different parts of the sky in detecting Kozai and non-Kozai plutinos. Section 3.7 gives a discus-
sion of previous observations of Kozai plutinos and of theoretical predictions in
the literature. And finally, Section 3.8 discusses future observations that may help
constrain the Kozai fraction and the distribution of the orbital elements of Kozai
plutinos.

3.2 Resonant Dynamics

As discussed in Section 2.1.1, resonances are diagnosed by inspecting the evo-
lution of an object’s orbital elements during a numerical integration (Figure 3.2).
For our purposes in this chapter, the integration must be long enough that the Myr-
timescale Kozai oscillations are visible. This chapter only discusses the 3:2 res-
onance, so here we use the general resonant condition described by Equation 1.2
with \( j = 3 \) and \( k = 2 \):

\[
\phi_{32} = 3\lambda - 2\lambda_N - \varpi.
\]  

(3.1)

If the object in question is a plutino, \( \phi_{32} \) will be confined and will not take on all
values 0° to 360° over the course of the integration.

A feature of the Kozai resonance (discussed in detail in Section 3.4) is cou-
pled oscillation in \( \omega \), eccentricity \( e \), and inclination \( i \) (see Figure 3.2). \( e \) and \( i \) are
anti-correlated, and \( \omega \) oscillates around 90 or 270° (or temporarily around 0 or
180°; Lykawka and Mukai, 2007; Nesvorný and Roig, 2000). Figure 3.2 shows a
10 Myr orbital integration of the Kozai plutino 28978 Ixion, showing the char-
acteristic oscillations of \( e \), \( i \), and \( \omega \). For comparison, Figure 3.2 also shows a
non-Kozai plutino and a non-resonant TNO nearby in semimajor axis.
Figure 3.2: Orbital integrations of the Kozai plutino 28978 Ixion (left), the CFEPS-discovered non-Kozai plutino L4h15 (2004 HB79; center), and a non-resonant TNO (2004 PA112; right) for comparison. In both resonant cases, the resonant argument $\phi_{32}$ librates around 180° (lower panels) because of the 3:2 mean-motion resonance with Neptune. The Kozai plutino 28978 Ixion’s integration also shows oscillations in $\omega$ around 270° with coupled, anti-correlated oscillations in $e$ and $i$. The integrations of the non-Kozai plutino and non-resonant TNO both show oscillations in $e$ and $i$, but they are not coupled, and $\omega$ circulates. Note that oscillations in semimajor axis and $\phi_{32}$ happen on much faster timescales (few thousand years) than the Kozai oscillations (few million years), and that the oscillations in semimajor axis are much larger for the resonant TNOs than the non-resonant one.
3.3 Non-Kozai Plutinos

To orient the reader and make several important points, we first discuss toy models and then a realistic libration amplitude distribution for the plutinos, ignoring the Kozai component until Section 3.4.

3.3.1 0° Libration Amplitude Toy Model

Due to the plutino resonance condition (Equation 3.1), the location where the Plutinos can come to pericenter is restricted. This is what allows resonant TNOs to remain stable on timescales of the age of the solar system, despite having orbits that in some cases cross the orbit of Neptune. For plutinos, $\phi_{32}$ librates around 180° or -180° (there are two options because of the two places relative to Neptune where the plutino can come to pericenter). At pericenter, $M = 0$, and at that moment the definition of $\lambda$ ($\lambda = \Omega + \omega + M$) implies $\lambda = \Omega + \omega = \varpi$. For a plutino with libration amplitude $A_{32} = 0°$, $\phi_{32} = 180°$ always, and

$$180° = 3\varpi - 2\lambda_N - \varpi$$

$$\varpi = \lambda_N + 90°$$

(3.2)

So the plutino always comes to pericenter 90° away from Neptune’s position. This is shown in Figure 3.3. Because $\phi_{32} = -180°$ is also valid, another perihelion occurs with $\varpi = \lambda_N - 90°$. The two points on the sky where a $A_{32} = 0°$ plutino comes to pericenter (in the ecliptic plane, at $\lambda_N \pm 90°$), are very important in this paper, so to avoid confusion we will refer to these as the “orthoneptune points”.

3.3.2 95° Libration Amplitude Toy Model

Real plutinos possess non-zero libration amplitudes; $A_{32}$ for known plutinos with well-characterized orbits ranges between 20° and 130° (Section 2.5 and Lykawka and Mukai, 2007). These libration amplitudes lead to different selection effects: during each $\phi_{32}$ libration period the perihelion direction oscillates around
Figure 3.3: $0^\circ$ libration amplitude toy model, showing the orbit of a $0^\circ$ libration amplitude, $0^\circ$ inclination, $e = 0.24$ plutino in a frame that co-rotates with Neptune. The motion in the co-rotating frame is clockwise, except at perihelion, where this example’s $e$ is so high that it briefly moves counterclockwise. Pericenter occurs $90^\circ$ ahead and behind Neptune’s position. Dotted circles are heliocentric distances of 30, 40, and 50 AU. Neptune’s position is shown for June 1, 2004 (which was midway through the CFEPS survey); this is true for all subsequent plots.

the orthoneptune points roughly sinusoidally in time with amplitude $A_{32}/2$. Equation 3.1 shows that the maximum excursion from the orthoneptune points occurs when $\phi_{32}$ is at a maximum or minimum. This means that plutinos spend more time near the extrema allowed by their libration amplitudes, and are actually more likely to be detected there. We demonstrate this using a population of plutinos with $A_{32} = 95^\circ$ (see Figure 3.4 and caption). To avoid confusion, all the plutinos in this toy model have $0^\circ$ inclination to the ecliptic, and all have the same eccen-
tricity. $A_{32} = 95^\circ$ is chosen because it was found to be the most common plutino libration amplitude by CFEPS (Section 2.5).

### 3.3.3 A Realistic Plutino Distribution

In actuality, plutinos possess a range of libration amplitudes. Figure 3.5 shows an observationally-motivated distribution of plutinos, based on the debiased model from CFEPS (excluding the Kozai component, which is discussed in detail in Section 3.4, see Section 2.3.3). Chiang and Jordan (2002) and Malhotra (1996) also presented models of the plutino distribution. Malhotra (1996) discusses the dynamics of plutinos for given values of the libration amplitude, while Chiang and Jordan (2002) examined distributions of particles where $A_{32}$ was established in a cosmogonic simulation. In contrast, our distributions of $a$, $e$, $i$, and $\phi_{32}$ are determined by debiasing the CFEPS Survey.

While at first glance it appears that the ‘turnaround’ effect shown in Figure 3.4 is completely lost, this is not the case. Each plutino is still most likely to be detected at its maximum perihelion excursion from the orthoneptune points of $A_{32}/2$. So, a plutino with an $80^\circ$ libration amplitude is most likely to be detected $40^\circ$ away from the orthoneptune points, at $\lambda_N \pm 50^\circ$ or $\lambda_N \pm 130^\circ$, while a plutino with a $20^\circ$ libration amplitude is most likely to be detected $10^\circ$ away from the orthoneptune points, at $\lambda_N \pm 80^\circ$ or $\lambda_N \pm 100^\circ$.

Because CFEPS showed that plutinos with $A_{32} < 20^\circ$ are so rare as to be approximated as absent, two peaks in the detectability are visible, about $15^\circ$ on either side of the orthoneptune points. As Figure 3.6 shows, this means that the place on the sky to find the most plutinos is not $90^\circ$ away from Neptune. For the CFEPS L7 model of the true libration amplitude distribution (Section 2.3.3), the maximum on-sky detection rate (integrated over all libration amplitudes) happens about $\pm 15^\circ$ away from the orthoneptune points.
Figure 3.4: 95° libration amplitude plutino toy model. Top panel shows a population of plutinos with $A_{32} = 95^\circ$, $i = 0^\circ$, and $e = 0.24$ (see caption for Figure 3.3). The 95° libration amplitude means that the perihelion turnaround points correspond to orbits with perihelia 42.5° and 137.5° ahead and behind Neptune. The red box is shown at higher resolution in the middle panel, where red circles show simulated detections from a flux-limited all-sky survey. The lower panel shows that there are more detections per degree of ecliptic at the 'turnaround' point, which is where the libration causes the objects to come to pericenter farthest from the orthoneptune points. (The number of detections per bin is arbitrary.)
Figure 3.5: A realistic non-Kozai plutino model, with a distribution of orbital elements that matches the debiased plutino model from CFEPS (excluding the Kozai component, see Section 2.3.3). Top panel shows the distribution of these plutinos as seen from above the Solar System. The red box is shown at higher resolution in the bottom panel, where red circles show simulated detections from a flux-limited all-sky survey.
Figure 3.6: The detection density on the sky using a realistic distribution of non-Kozai plutinos, with red being higher density and blue being lower, with contours evenly spaced in detection density. Two peaks are visible on either side of the orthoneptune points, which are caused by the turnaround effect. At these peaks, the detection density is $\sim 30\%$ higher than at the orthoneptune points.

3.4 Kozai Plutino Dynamics

This section discusses the dynamics of objects that are simultaneously in the 3:2 mean-motion resonance with Neptune (plutinos) and in the Kozai resonance, and the effects these two simultaneous resonances have on the on-sky detectability.

The Kozai resonance can occur at much lower inclinations within mean-motion resonances than for non-resonant TNOs (Thomas and Morbidelli, 1996; Wan and Huang, 2007). The $\omega$ oscillation is unique to the Kozai resonance: the perturbations of the other solar system planets on a small body (resonant or not) normally cause $\omega$ to precess rather than librate.
Figure 3.7: 3 different Hamiltonian level surfaces for Kozai plutinos (constructed using the disturbing function from Wan and Huang, 2007), for different values of angular momentum. Left panel is $I_{\text{max}} = 14^\circ$, center is $I_{\text{max}} = 21^\circ$, and right is $I_{\text{max}} = 34^\circ$. These are polar plots, with $e$ as the radius and $\omega$ as the angle. Also shown on the center plot are the 10 Myr orbital integrations of the Kozai plutinos 1997 QJ$_4$ and 2002 VR$_{128}$, showing circulation around the $\omega = 90^\circ$ or $270^\circ$ islands over time; these plutinos were chosen because they have $I_{\text{max}} \simeq 21^\circ$. While the presence of other planets causes small changes in the Hamiltonian, one can see that the evolution is decently approximated.
The libration in $e$, $i$, and $\omega$ can be best understood using a contour plot of the Hamiltonian level surface of the disturbing function, which describes secular oscillation due to the three-body interaction. Example surfaces for the 4th order disturbing function for a Kozai plutino (Wan and Huang, 2007) are shown in Figure 3.7. These are polar plots, where the length of the vector gives $e$, and the angle from $0^\circ$ gives $\omega$. In the cases shown, only the contours that close around $90^\circ$ or $270^\circ$ correspond to Kozai oscillations. Each plutino that is also in the Kozai resonance is confined to a particular contour on one of these surfaces. Tracing one contour reveals how $e$ and $\omega$ vary during the course of a Kozai cycle, with the range in $\omega$ values describing the Kozai libration amplitude $A_\omega$ around the relevant libration center. Orbital inclination is calculated using $e$ and conservation of the $z$-component of angular momentum $L_z \propto \cos i \sqrt{1 - e^2}$, because

$$
\cos i \sqrt{1 - e^2} = \cos i_0 \sqrt{1 - e_0^2}
$$

for initial inclination $i_0$ and initial eccentricity $e_0$ at any time. Each surface plot has its own $L_z$ value, which is parameterized using $I_{\text{max}}$:

$$
\cos i_0 \sqrt{1 - e_0^2} = \cos I_{\text{max}}
$$

$I_{\text{max}}$ is the inclination required by conservation of angular momentum for $e=0$, which means the Kozai libration amplitude is also zero (i.e., no Kozai libration). Note that $I_{\text{max}}$ is not the maximum inclination that these plutinos will reach; in order for an object to have that inclination, it needs $e = 0$ and thus is not Kozai oscillating. Plutinos always have maximum inclination values attained during their Kozai oscillations that are less than $I_{\text{max}}$. $I_{\text{max}}$ is just a way to parameterize the level surfaces.

For a known Kozai plutino, the level surface can be chosen using the measured $e_0$ and $i_0$ values to calculate $I_{\text{max}}$, which gives the Hamiltonian level surface for this object. The measured $\omega_0$ value sets which contour the object is oscillating on,
and knowing the contour allows the Kozai libration amplitude $A_\omega$ to be calculated numerically.

### 3.4.1 On-Sky Detection Biases for Kozai Plutinos

A direct consequence of the Kozai resonance-caused oscillation of $\omega$ is that these objects always come to pericenter out of the ecliptic plane. Because these are plutinos, we start with the same resonant condition (equation 3.1), and for this illustration choose $A_{32} = 0^\circ$:

$$\phi_{32} = 3\lambda - 2\lambda_N - \omega$$

$$180^\circ = 3(\Omega + \omega + \mathcal{M}) - 2\lambda_N - (\Omega + \omega)$$

(3.5)

One can see from Figure 3.7 that when the Kozai libration amplitude is $0^\circ$, $\omega = 90^\circ$ or $270^\circ$. At pericenter, $\mathcal{M} = 0^\circ$. Substituting these values into the above equation yields $\Omega = \lambda_N$, indicating that the node of the plutino orbit is in the same direction as Neptune, and $\omega$ will be $90^\circ$ ahead or behind that position ($\Omega = \lambda_N + 180^\circ$ is also valid). The orbital plane of the plutino will be tilted out of the ecliptic plane by the inclination, with the line of nodes through Neptune’s position acting as the pivot. Most of the Kozai plutinos listed in the MPC database have orbital inclinations between $10^\circ$ and $30^\circ$ (Lykawka and Mukai, 2007), which means that when they are at pericenter, a $0^\circ$ Kozai libration amplitude plutino will be roughly $10^\circ$ $-30^\circ$ above or below the ecliptic plane.

Solar System objects are most easily detected at pericenter, when they are closest and thus brightest. Because the Kozai plutinos are forced to be out of the ecliptic at pericenter, they will be harder to detect in ecliptic surveys than non-Kozai plutinos (see Figures 3.8–3.10). This is an important bias that must be accounted for.
Figure 3.8: 95° Kozai libration amplitude ($A_\omega$) toy model. Top panel shows a toy model of the Kozai plutinos, where all objects have $A_{3/2} = 95^\circ$, using one contour from one level surface (meaning that all these Kozai plutinos have the same Kozai libration amplitude). Objects in the red box are shown at higher resolution in the bottom panel. Red circles show simulated detections from a flux-limited ecliptic survey. The objects are only detected where they cross through the ecliptic plane, which is never when the objects are at pericenter.
Figure 3.9: Distance at detection for Kozai and non-Kozai plutinos in a simulated, flux-limited, all-sky survey, using realistic distributions for both populations, showing the different detection biases for each. The color of each point shows the distance at which the plutino was detected (see legend in panels), with Kozai plutinos in the top panel and non-Kozai plutinos in the bottom panel. The non-Kozai plutinos are detected at a broad distance range in each of the two pericenter lobes. The distance at which the Kozai plutinos are detected is very much dependent on ecliptic latitude, with higher ecliptic latitudes detected at closer distances, and lower detected at greater distances. The large green dot marks Neptune’s location.
Figure 3.10: Averaged distance at detection for plutinos in a simulated, flux-limited, all-sky survey, using realistic distributions for both populations. The lower panel shows the distance and ecliptic latitude where each plutino was detected, using one of the pericenter lobes (plutinos between ecliptic longitudes of 12h–18h). Blue shows Kozai plutinos, red shows non-Kozai. The solid lines show the average in 0.5° bins of ecliptic latitude. The average detection distance for Kozai plutinos depends on ecliptic latitude, with those at low latitudes being at the farthest average heliocentric distance. Non-Kozai plutinos show no such trend.

3.5 Simulating the Plutinos

We simulate the population of plutinos by randomly drawing from specified distributions of orbital elements and magnitudes, then taking each simulated plutino and “observing” it using the CFEPS survey simulator (publicly available at http://www.cfeps.net). Figure 3.11 shows the simulated Plutino distribution in semimajor axis, eccentricity, and inclination. The non-Kozai plutino distribution we use here is identical to the CFEPS L7 model, while the Kozai plutino distribution here is more detailed than that used in Section 2.3.3.
Figure 3.11: Distribution of simulated plutinos in semimajor axis, eccentricity, and inclination. Red points are non-Kozai plutinos and blue points are plutinos in the Kozai resonance. The Kozai plutinos are constrained by the conservation of $L_z$ and our choice of $I_{\text{max}}$ values. This is especially obvious in the lower panel, where each Kozai curve corresponds to a particular $I_{\text{max}}$ value; over a Kozai libration cycle, objects never reach $e = 0$, and thus always have $i < I_{\text{max}}$.

The following describes how our code builds a population of plutinos with a realistic orbital element distribution, one simulated object at a time. The first step is to choose whether or not a given object is in the Kozai resonance or not.

An important parameter in these simulations is the true Kozai fraction $f_{\text{true}}^{\text{koz}}$, which is the true number of Kozai plutinos divided by the total number of plutinos. This is not necessarily the same as the observed Kozai fraction $f_{\text{obs}}^{\text{koz}}$, which is the total number of detected Kozai plutinos divided by the total number of detected
plutinos for a given survey. For most of our simulations, we use a true Kozai fraction $f_{\text{true}}^{\text{koz}} = 10\%$, based on the results of CFEPS. However, because Section 2.4.2 shows that Kozai fractions up to 33% cannot be ruled out at the 99% confidence level, some of our calculations are repeated for $f_{\text{true}}^{\text{koz}}$ values of 20% and 30%. Constraining the value of $f_{\text{true}}^{\text{koz}}$ will require many more well-characterized plutino detections than are currently available.

### 3.5.1 Non-Kozai Plutinos

For the plutinos which are \textit{not} also in Kozai (with percentage $100\% - f_{\text{true}}^{\text{koz}}$), the following procedure is followed to choose its orbital elements. This is the same as the best-fit non-Kozai plutino model from CFEPS (Section 2.3.3).

First, the eccentricity is chosen from a Gaussian probability distribution centered on 0.18 with a width of 0.06. Eccentricities large enough to approach the orbit of Uranus ($e > 0.22$) are not allowed. The semi-major axis is chosen from a simplified version of the stability tests of Tiscareno and Malhotra (2009). $a$ is chosen within 0.2 AU of 39.45 AU for objects with $e > 0.15$. The allowed range of $a$ values drops linearly as $e$ gets smaller, reaching a width of zero at $e = 0$ (see Figure 3.11). The inclination is then chosen from a probability distribution of the form

$$P(i) \propto \sin i \exp \left( -\frac{i^2}{2\sigma_{32}^2} \right)$$  \hspace{1cm} (3.6)

with $\sigma_{32} = 16^\circ$ (originally based on the inclination distribution postulated by Brown, 2001). The libration amplitude is chosen from an asymmetric “tent-shaped” probability distribution with a peak at 95°, and linearly decreasing probabilities to the lower and upper bounds, 20° and 130° respectively, where the probability drops to zero.

Lastly, the object’s absolute magnitude $H_g$ is chosen from an exponential distribution: $N(< H_g) \propto 10^{\alpha H_g}$, with $\alpha = 0.9$. The reader is cautioned that this $\alpha$ value can only be considered valid in the range of $H_g$ magnitudes where CFEPS
had many detections (approximately $8 < H_g < 9$ for the plutinos). Sensitivity to the size distribution is discussed further in Section 3.6.3.

### 3.5.2 Kozai Plutinos

For a Kozai plutino, a slightly different path is followed to choose its orbital elements.

First, the Hamiltonian level surface is chosen. Inspecting the results of Lykawka and Mukai (2007), we found which Hamiltonian level surface corresponded to each of their Kozai plutinos. To reflect this distribution of surfaces, we used level surfaces corresponding to $I_{\text{max}}$ of $14^\circ$, $16^\circ$, $17.5^\circ$, $20^\circ$, $21^\circ$, $21.3^\circ$, $21.6^\circ$, $22.5^\circ$, $24^\circ$, $26^\circ$, $28^\circ$, and $34^\circ$, in equal proportions. In reality, due to the historical dominance of ecliptic surveys and the bias against detecting large $i$ TNOs, there are probably a larger fraction of large $i$ Kozai librators; however, the currently available information does not justify more complex modeling.

Once the $I_{\text{max}}$ level surface is chosen, we pick a Kozai libration amplitude $A_\omega$ at random between $10^\circ$–$80^\circ$. $\omega$ is then chosen sinusoidally within the values allowed by $A_\omega$. Because of the banana-shape of the contours, there are two values of $e$ allowed for any given value of $\omega$ (see Figure 3.7). Given this value of $\omega$, the Wan and Huang (2007) disturbing function allows numerical determination of the two $e$ values that correspond to $\omega$ on the contour. Half of the time we choose the lower value of $e$, and half higher. The inclination $i$ is calculated using $\cos i \sqrt{1 - e^2} = \cos I_{\text{max}}$. Since this only covers the $90^\circ$ Kozai libration island, half of the objects are flipped $\omega$ of to $360^\circ$ minus the original $\omega$ value.

Lastly, the semi-major axis, the libration amplitude $A_{32}$, and the absolute H magnitude are all chosen following the same procedure as for the non-Kozai plutinos.
3.6 On-Sky Biases

We build up a population of synthetic plutinos, drawing orbital elements and magnitudes from the specified distributions as described above, and determine if each object is detected by a survey using the survey simulator code. Using the specified field coverage, magnitude efficiency, and tracking fraction, the code determines whether or not each object will be detected by the survey. Comparing the distributions of the drawn and simulator-detected objects gives an idea of the biases that are present in surveys that cover different areas of the sky to different magnitude depths. When the simulated detections are compared to the true detections in a real, well-characterized survey, this is a powerful tool to help in debiasing to regain the real intrinsic population’s orbital distribution.

Figure 3.12 shows an on-sky detection density map for all the plutinos, including the Kozai component. With the Kozai component included, it is still true that most plutinos are detected in broad clumps around the orteoneptune points, 90° away from Neptune. The reader will notice that the highest detection densities still occur in clumps in the ecliptic on either side of the orthoneptune points rather than exactly centered on the orthoneptune points. This is caused by the “turnaround” detection effect described in Section 3.3.3.

The Kozai plutinos are only visible (for this realistic model) as subtle density enhancements about 10° off the ecliptic, making the density contours in Figure 3.12 appear slightly more rectangular than in Figure 3.6. This rectangular shape is enhanced for higher values of \( f_{\text{true}} \). Note that there is no “spike” in detections at the ecliptic latitudes where the density of Kozai plutinos is highest; the detection densities are still dominated by the much greater numbers of non-Kozai plutinos.

To clarify what is happening for the Kozai population, the Kozai component is shown separately in Figure 3.13. The Kozai plutino detections are more confined in ecliptic latitude than the non-Kozai plutinos, with the highest detection densities happening about 10° above and below the ecliptic, in broad swaths surrounding
Figure 3.12: Relative density of detections on the sky in an all-sky survey using our plutino model, including both Kozai and non-Kozai components, for three different values of $f_{\text{true}}^{\text{koz}}$. Contours are evenly spaced in detection density, and contour values are the same in all three plots (absolute detection densities are arbitrary). The position of Neptune is shown by a green circle. The detection density becomes somewhat less concentrated to the ecliptic for increasing Kozai fraction.
Figure 3.13: Relative density of detections on the sky using an all-sky survey only using the Kozai component of our model. Contours are evenly spaced in detection density. The position of Neptune is shown by a green circle.

the orthoneptune points, with the central minimum again caused by the lack of $A_{32} < 20^\circ$ plutinos.

3.6.1 Ecliptic Latitude Distribution of Detections

Figure 3.14 presents the ecliptic latitudes of detected Plutinos in a simulated all-sky survey. The number of detections for all plutinos smoothly falls from $0^\circ$ ecliptic latitude on up to higher latitudes. Although the number of Kozai detections climbs as one rises to $\sim 15^\circ$ ecliptic latitude, they never hold more than about half the detections in a bin.

Schwamb et al. (2010) and Brown (2008) found a factor of $\sim 4$ spike in the number of detections in their surveys in the $11–13^\circ$ ecliptic latitude bin, which they attribute to potentially being caused by Kozai plutinos. However, our simulation makes this explanation implausible. Even when we go to the extreme and
relative fraction of detections $f^\text{true}_{\text{koz}} = 10\%$

relative fraction of detections $f^\text{true}_{\text{koz}} = 20\%$

relative fraction of detections $f^\text{true}_{\text{koz}} = 30\%$

**Figure 3.14:** Stacked histograms of the ecliptic latitude distribution of detected plutinos for different values of $f^\text{true}_{\text{koz}}$, using a magnitude-limited all-sky survey. The number of detections per bin is normalized to the maximum bin. Kozai plutinos are shown in red, non-Kozai plutinos in blue.

unrealistic case of only using the lowest $I_{\text{max}}$ value of 14° (which makes the Kozai plutinos as compact as possible in ecliptic latitude), and using the highest value of $f^\text{true}_{\text{koz}} = 30\%$, we still find that there is only a $\sim 20\%$ increase in the number of plutinos at ecliptic latitudes of 11–13°. Kozai plutinos do not explain this spike in detections, because it is impossible to confine the detections of Kozai librators to a narrow ecliptic latitude bin. The reported detection spike is likely a small-number statistics fluctuation.

### 3.6.2 $f_{\text{koz}}$ On-Sky

Figure 3.15 shows the ratio between the number of Kozai plutino detections to the total number of plutino detections in small bins on the sky, providing a local $f^\text{obs}_{\text{koz}}$ map. (This is essentially the ratio of Figure 3.13 to Figure 3.12). Figure 3.15
shows the range of $f_{\text{kzo}}^{\text{obs}}$ values that could be locally found at different positions on the sky, for $f_{\text{kzo}}^{\text{true}}$ of 10%, 20%, and 30%.

$f_{\text{kzo}}^{\text{obs}}$ values vary from 0% to nearly twice the $f_{\text{kzo}}^{\text{true}}$ values. The highest $f_{\text{kzo}}^{\text{obs}}$ values occur where the Kozai detection density is highest: above and below the ecliptic plane by about 12°.

### 3.6.3 Size Distribution Effects

The diameter distribution of TNOs is fit by a power law, usually parameterized as $N(> d) \propto d^{-Q}$, where $N(> d)$ is the number of objects larger than a diameter $d$, and $Q$ is the index of the power law. However, because only a few of the largest KBOs have had their diameters directly measured by occultation or resolved imaging, what is actually measured is the magnitude. To convert this to a diameter, an albedo must be measured or assumed. For this reason we discuss the size distribution in terms of absolute magnitude: $N(< H) \propto 10^{\alpha H}$. The values $Q$ and $\alpha$ are related: $\alpha = \frac{Q}{5}$.

The logarithmic slope $\alpha$ is known to be different depending on the size of the KBOs (e.g. Fraser and Kavelaars, 2009; Fuentes and Holman, 2008). For most of our calculations, we use the nominal CFEPS value for plutinos of $\alpha = 0.9$ (Section 2.4.3). But for comparison, Figure 3.16 shows the effect of different values of $\alpha$ on the detection density, and Figure 3.17 shows the effect of different $\alpha$ values on $f_{\text{kzo}}^{\text{obs}}$ at different places on the sky. Steeper slopes result in steeper detection density distributions, where the peak detection densities are much higher. This is because the relative importance of detecting the large number of small plutinos that are only visible at perihelion increases. Lower values of $\alpha$ result in shallower density distributions. This effect is noticeable, but the overall pattern of where on the sky the highest $f_{\text{kzo}}^{\text{obs}}$ values are remains the same.

### 3.6.4 Example Simulated Surveys

We perform a number of strawman simulated surveys to demonstrate the different values of apparent average $f_{\text{kzo}}^{\text{obs}}$ that result from different survey parameters and
Figure 3.15: The observed Kozai fraction $f_{\text{koz}}^{\text{obs}}$ (the number of Kozai plutino detections divided by the total number of plutino detections) in $2^\circ \times 6^\circ$ bins on the sky, using our best plutino model. The true Kozai fraction (for the entire plutino population) $f_{\text{koz}}^{\text{true}}$, is 10% in the top panel, 20% in the center, and 30% in the lower panel. The local $f_{\text{koz}}^{\text{obs}}$ varies widely, from nearly zero around Neptune to its highest values about $10^\circ$ off the ecliptic.
Figure 3.16: Relative density of plutino detections (including Kozai, with $f_{\text{koz}}^{\text{true}} = 10\%$) using three different values of $\alpha$, the logarithmic slope of the size distribution in absolute magnitude. Contours are evenly spaced in detection density, and the contours represent the same values in each panel. For steeper power laws, the density of detection also becomes steeper, with more detections at the turnaround points, and fewer detections 90° away.
Figure 3.17: $f_{\text{koz}}^{\text{obs}}$ at different points on the sky, using $f_{\text{koz}}^{\text{true}} = 10\%$, and three different slopes for the power-law absolute magnitude distribution ($\alpha$). Steeper slopes cause the $f_{\text{koz}}^{\text{obs}}$ value to vary more widely, reaching maximum values of almost 30\%, while the shallow slopes give the highest $f_{\text{koz}}^{\text{obs}}$ values of only about 20\%.
different $f_{\text{koz}}^{\text{true}}$ values. These are summarized in Table 3.1. For all of these simulated surveys, we ignore the extra confusion caused by the plane of the Milky Way, and assume that the entire area within each survey is observed uniformly and with perfect tracking efficiency (that is, all discoveries are tracked to yield high quality orbits).

Each simulated survey goes to a magnitude depth of $g = 24.9$. Varying the magnitude depth did not have any noticeable effect on the detection density or $f_{\text{koz}}$ values on the sky, due to the assumed exponential nature of the distribution.

First we perform an all-sky survey with a set limiting magnitude (Survey 1), which finds a higher Kozai fraction than reality, with $f_{\text{koz}}^{\text{obs}}$ being higher than $f_{\text{koz}}^{\text{true}}$. Survey 2, an ecliptic survey covering the entire ecliptic within $\pm 2.5^\circ$, unsurprisingly finds the opposite effect, with $f_{\text{koz}}^{\text{obs}}$ being lower than $f_{\text{koz}}^{\text{true}}$. This is due to the Kozai plutinos preferentially being detected out of the ecliptic plane, as discussed in Section 3.4.1.

Surveys 3 and 4 are $20^\circ \times 20^\circ$ surveys centered on the ecliptic. Survey 3 is centered $90^\circ$ away from Neptune, and Survey 4 is centered $75^\circ$ away from Neptune, near the plutinos’ peak in detectability. Surveys 5 and 6 are smaller, $2^\circ \times 2^\circ$ surveys, centered on the ecliptic $90^\circ$ and $75^\circ$ away from Neptune, respectively. These four surveys all find lower $f_{\text{koz}}^{\text{obs}}$ than $f_{\text{koz}}^{\text{true}}$, for the same reason as Survey 2.

Surveys 7 and 8 are the same as Surveys 3 and 4, except centered $10^\circ$ above the ecliptic. Similarly, Surveys 9 and 10 are the same as Surveys 5 and 6, raised to $10^\circ$ above the ecliptic. Because these surveys cover the range of the Kozai plutinos’ peak detection density, they all find higher $f_{\text{koz}}^{\text{obs}}$ than $f_{\text{koz}}^{\text{true}}$ values.

There is not a significant difference between the $f_{\text{koz}}^{\text{obs}}$ values measured by the surveys that are centered on $90^\circ$ from Neptune and those centered on $75^\circ$ from Neptune. There \textit{is} a difference in the relative number of detections, with overall more plutinos detected at $75^\circ$ from Neptune. However, because both the Kozai and non-Kozai plutinos have the same $A_{32}$ distribution in this model, the Kozai fraction does not vary significantly between these two positions on the sky.
<table>
<thead>
<tr>
<th>Survey Description</th>
<th>$f_{\text{true}}^{\text{koz}} = 10%$</th>
<th>$f_{\text{obs}}^{\text{koz}} = 20%$</th>
<th>$f_{\text{true}}^{\text{koz}} = 30%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 all sky</td>
<td>12 %</td>
<td>23 %</td>
<td>35 %</td>
</tr>
<tr>
<td>2 5° high, on ecliptic</td>
<td>7.5 %</td>
<td>15 %</td>
<td>23 %</td>
</tr>
<tr>
<td>3 20°×20° box, 90° from Neptune, on ecliptic</td>
<td>9.5 %</td>
<td>19 %</td>
<td>29 %</td>
</tr>
<tr>
<td>4 20°×20° box, 75° from Neptune, on ecliptic</td>
<td>10 %</td>
<td>18 %</td>
<td>28 %</td>
</tr>
<tr>
<td>5 2°×2° box, 90° from Neptune, on ecliptic</td>
<td>7.6 %</td>
<td>16 %</td>
<td>24 %</td>
</tr>
<tr>
<td>6 2°×2° box, 75° from Neptune, on ecliptic</td>
<td>7.4 %</td>
<td>16 %</td>
<td>23 %</td>
</tr>
<tr>
<td>7 20°×20° box, 90° from Neptune, 10° off ecliptic</td>
<td>13 %</td>
<td>25 %</td>
<td>37 %</td>
</tr>
<tr>
<td>8 20°×20° box, 75° from Neptune, 10° off ecliptic</td>
<td>13 %</td>
<td>24 %</td>
<td>36 %</td>
</tr>
<tr>
<td>9 2°×2° box, 90° from Neptune, 10° off ecliptic</td>
<td>14 %</td>
<td>26 %</td>
<td>37 %</td>
</tr>
<tr>
<td>10 2°×2° box, 75° from Neptune, 10° off ecliptic</td>
<td>14 %</td>
<td>27 %</td>
<td>38 %</td>
</tr>
</tbody>
</table>
These surveys demonstrate that very different values of $f_{\text{koz}}^{\text{obs}}$ can be measured depending on the on-sky location of the survey. Due to the different biases inherent in the distribution of Kozai plutinos versus non-Kozai plutinos, careful debiasing is required to calculate $f_{\text{koz}}^{\text{true}}$ from any survey, even one which covers the entire sky.

### 3.7 Comparison with Previous Literature

In the previous sections, we have discussed two quantities that can be measured for a survey or simulation that contains many well-characterized plutinos: $f_{\text{koz}}$ and the distribution of $I_{\text{max}}$ for the Kozai plutinos. Below, we discuss these quantities as measured by observational surveys and giant planet migration simulations. Only a few surveys are discussed here, as only a few previously published TNO surveys have rigorous enough tracking and characterization methods to classify plutinos as Kozai or non-Kozai in orbital integrations.

#### 3.7.1 The Kozai Fraction $f_{\text{koz}}$

The simplest quantity to measure in a survey or simulation that contains plutinos is $f_{\text{koz}}$, the fraction of plutinos that are in Kozai. However, one must be careful to note whether this is the true or apparent $f_{\text{koz}}$. Most surveys will have some bias, as shown in Table 3.1, causing $f_{\text{koz}}^{\text{obs}}$ to be different than $f_{\text{koz}}^{\text{true}}$.

Below we discuss the $f_{\text{koz}}$ results presented in several observational surveys and theoretical simulations. A summary is presented in Table 3.2.

**Observational Surveys: $f_{\text{koz}}^{\text{obs}}$**

CFEPS (Petit et al., 2011), being a well-calibrated survey, was able to provide both an apparent and a true $f_{\text{koz}}$, albeit with large uncertainty (Section 2.4.2). They find $f_{\text{koz}}^{\text{obs}}$ of $2/24 = 8\%$. After debiasing, this would require a value of $f_{\text{koz}}^{\text{true}}$ of 10\%. However, because CFEPS was confined to the ecliptic plane, it was not very sensitive to the high-inclination Kozai plutino population, and $f_{\text{koz}}^{\text{true}}$ up to...
Table 3.2: Measurements of $f_{koz}$ from the Literature

<table>
<thead>
<tr>
<th>Source</th>
<th>type</th>
<th>$f_{true}^{koz}$</th>
<th>$f_{obs}^{koz}$</th>
<th># plutinos</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gomes (2000)</td>
<td>Observational</td>
<td>-</td>
<td>26%</td>
<td>23</td>
</tr>
<tr>
<td>Nesvorný et al. (2000)</td>
<td>Observational</td>
<td>-</td>
<td>12%</td>
<td>33</td>
</tr>
<tr>
<td>Chiang and Jordan (2002)</td>
<td>Theoretical</td>
<td>20–30%</td>
<td>-</td>
<td>42</td>
</tr>
<tr>
<td>Hahn and Malhotra (2005)</td>
<td>Theoretical</td>
<td>19%</td>
<td>-</td>
<td>133</td>
</tr>
<tr>
<td>Lykawka and Mukai (2007)</td>
<td>Observational</td>
<td>-</td>
<td>22–30%</td>
<td>100</td>
</tr>
<tr>
<td>Levison et al. (2008a)</td>
<td>Theoretical</td>
<td>16%</td>
<td>-</td>
<td>186</td>
</tr>
<tr>
<td>Schwamb et al. (2010)</td>
<td>Observational</td>
<td>-</td>
<td>33%</td>
<td>6</td>
</tr>
<tr>
<td>Chapter 2</td>
<td>Observational</td>
<td>10%</td>
<td>8%</td>
<td>24</td>
</tr>
</tbody>
</table>

33% cannot be ruled out with 99% confidence due to the small number statistics of having only two detected Kozai plutinos.

The Deep Ecliptic Survey (Elliot et al., 2005), while finding a reported 51 plutinos, did not specifically label any of their discovered plutinos as Kozai, and thus is not discussed further (although some of their discoveries are in the biased Lykawka and Mukai (2007) compilation, discussed below).

A few papers have tried to use the entire MPC database as a survey. While this does provide many plutinos, the MPC contains the results of many surveys and even serendipitous discoveries, each with completely different and possibly unknown biases, since one doesn’t know where searches failed to detect plutinos. Debiasing $f_{obs}^{koz}$ to find $f_{true}^{koz}$ is impossible for these surveys.

Gomes (2000) and Nesvorný et al. (2000) performed similar large MPC database searches capable of classifying objects as Kozai or non-Kozai plutinos. Gomes (2000) examined the first 23 discovered Plutinos with observations for 2 or more oppositions. Though many of these classifications were provisional due to a lack of precise data, he found $f_{obs}^{koz}$ of $6/23 = 26\%$. Nesvorný et al. (2000) performed a similar analysis for the first 33 plutinos, finding that only 4 of them were in Kozai, giving $f_{obs}^{koz}$ of 12%, despite an overlapping sample.

Currently, the largest collection of well-classified plutinos was presented in Lykawka and Mukai (2007), with 100 plutinos from the MPC database. All of
these plutinos had at least 2 opposition observations, and 10 Myr orbital integrations were performed. They found that 22 plutinos are solidly in the Kozai resonance, with 8 more that are in the Kozai resonance for part of their integration. Thus, from their integrations they find $f_{\text{koz}}^{\text{obs}}$ of 22–30%.

Schwamb et al. (2010) completed a wide-field survey covering a large fraction of the sky ($\sim 12,000$ square degrees) within 30° of the ecliptic. This relatively shallow survey ($R \sim 21.5$) found 6 plutinos, two of which are in Kozai, giving $f_{\text{koz}}^{\text{obs}} = 33\%$, albeit with large Poisson uncertainty. The higher ecliptic latitudes included in this survey would make detecting the Kozai plutinos more likely, thus this higher apparent $f_{\text{koz}}^{\text{obs}}$ value is not surprising. Although this is the first large area survey which found and tracked plutinos and Kozai plutinos, a much larger number of plutino detections will be needed to accurately measure the Kozai fraction.

**Theoretical Simulations:**

Of the published simulations of giant planet migration, only Chiang and Jordan (2002) includes information on which plutino test particles ended up in Kozai. Future simulations should include this information, as it may prove a useful diagnostic. We also discuss the Kozai plutinos from Hahn and Malhotra (2005) and Levison et al. (2008a), because the authors provided us with the output orbits of these simulations and were able to complete the required analysis ourselves.

Chiang and Jordan (2002) studied a smooth outward migration of Neptune, with different migration times for Neptune to reach its current location. They discuss objects that are captured into the Kozai resonance within the 3:2 mean-motion resonance for their simulation where Neptune migrates with a damping half-life of $10^7$ years. Because of the shorter timescale of their simulations, their resonance classification isn’t as secure as in the other simulations discussed below. Out of 92 plutinos at the end of their simulation, they estimate that 42 will remain in the 3:2 resonance for the age of the solar system. Of these, 8–12 are in Kozai, giving $f_{\text{koz}}^{\text{true}}$ of 20–30%. They unfortunately do not discuss the effect that different
migration timescales have on the Kozai fraction, nor how \( f_{\text{koz}}^{\text{true}} \) might evolve over 4 Gyr.

Hahn and Malhotra (2005) and Levison et al. (2008a) provided enough data from the end of their theoretical migration simulations that we were able to continue the integrations for 10 Myr, long enough to diagnose if a plutino is in Kozai or not. Hahn and Malhotra (2005) used a smooth outward migration of Neptune, while Levison et al. (2008a) had Neptune on a large-eccentricity orbit that damps after interacting with the Kuiper Belt (motivated by the “Nice Model” scenario; Tsiganis et al., 2005).

For the Levison et al. (2008a) simulation, we were given the 10 Myr orbital integrations originally used to classify objects as resonant or non-resonant at the end of their 1 Gyr migration simulation (Run B). These integrations contain the osculating orbital elements at each timestep, and these were searched for oscillation of \( \omega \) around 90° or 270° to determine \( A_\omega \).

For the Hahn and Malhotra (2005) data, we were provided the osculating orbital elements of all test particles and the 4 giant planets at the end of their 4.5 Gyr giant planet migration simulation. However, these were divided into 100 separate simulations, each with different giant planet positions and different numbers of remaining test particles (most had ~50). Each of these were input into a slightly modified version of the orbital integrator SWIFT (Levison and Duncan, 1994), and 30 Myr orbital integrations were performed. We analyzed the remaining test particles for libration of \( \phi_{32} \), and then for oscillation of \( \omega \) around 90° or 270°.

Out of 133 plutinos in the Hahn and Malhotra (2005) simulation, 25 were in Kozai, giving \( f_{\text{koz}}^{\text{true}} = 19\% \). The Levison et al. (2008a) simulation provided 186 plutinos, of which 29 were in Kozai, giving \( f_{\text{koz}}^{\text{true}} = 16\% \).

These \( f_{\text{koz}} \) values all contain large uncertainties, and in our opinion, all roughly agree with each other at this point. As more plutinos are found by rapidly repeating all-sky surveys such as the Large Synoptic Survey Telescope (LSST), the value of \( f_{\text{koz}}^{\text{true}} \) should become precisely measurable as the survey characterization becomes well-determined.
Figure 3.18: $I_{\text{max}}$ and $A_\omega$ values for real and simulated Kozai plutinos. Scatterplot shows the $I_{\text{max}}$ values (which parameterize which Hamiltonian level surface the plutino is on), and the Kozai libration amplitudes $A_\omega$ (which parameterizes which contour of the level surface the plutino is on). Purple circles show simulated Kozai plutinos from Levison et al. (2008a) (LMVGT2008), green squares show real Kozai plutinos from the MPC database (L&M2007; Lykawka and Mukai, 2007), and red triangles show simulated Kozai plutinos from Hahn and Malhotra (2005) (H&M2005). Error bars for the MPC values are those reported by Lykawka and Mukai (2007), other error bars are estimated by eye from orbital integrations.

3.7.2 Distribution of Kozai Parameters

The two main parameters we use to describe the Kozai behavior of a given Kozai plutino are $I_{\text{max}}$ and the Kozai libration amplitude $A_\omega$. The distribution of $I_{\text{max}}$ tells about the range of $e$ and $i$ that are possible during a Kozai cycle. $I_{\text{max}}$ is a parameterization of which Hamiltonian level surface currently best describes the Kozai libration of that plutino. The Kozai libration amplitude $A_\omega$ measures which contour within the $I_{\text{max}}$ level surface the plutino is on, and is found from looking at the results of a 10–30 Myr diagnostic orbital integrations.

Because this is a distribution and not just a single value like $f_{\text{koz}}$, it is only instructive to analyse for the surveys and simulations with the largest number of
Figure 3.19: $i_{\text{max}}$ and $A_\omega$ cumulative distributions for real and simulated Kozai plutinos. See Figure 3.18 caption for symbols. The intrinsic distribution of thousands of simulated Kozai plutinos using our model is shown by a dotted black line. The results of running this distribution through a survey simulator using parameters for Surveys 1 (an all-sky survey), Survey 2 (an ecliptic survey), and Survey 3 (a $20^\circ \times 20^\circ$ survey centered $90^\circ$ from Neptune) are shown by solid lines. The models produce generically lower inclination objects, but seem to produce similar Kozai libration amplitudes to the MPC objects (keep in mind that the MPC objects are not debiased). Note that the intrinsic model matches the MPC sample quite well because, as explained in Section 3.5.2, the distribution of Kozai $i_{\text{max}}$ values was based on the objects in the MPC.
plutinos. We compare the $I_{\text{max}}$ distributions found in the theoretical giant planet migration simulations of Hahn and Malhotra (2005) and Levison et al. (2008a) with the MPC database analysis presented in Lykawka and Mukai (2007) (Figure 3.18), and with our own simulation (Figure 3.19).

When looking at Figure 3.19, it is important to keep in mind that we are comparing different kinds of distributions. The simulated Kozai plutinos from Levison et al. (2008a), Hahn and Malhotra (2005), and this paper are intrinsic distributions, that is, not observed by a biased survey. The MPC-detected Kozai plutinos (from Lykawka and Mukai, 2007) and the distributions resulting from the simulated surveys presented in this paper are biased. In the case of the MPC sample, which contains the results of many uncharacterized surveys, precise debiasing is impossible. The simulated surveys, however, are all based on our simulated plutino distribution, so we can see the effects of different types of surveys on the detected parameters. The all-sky survey (Survey 1) is slightly biased toward finding a higher proportion of higher $I_{\text{max}}$ objects than reality, while Surveys 2 and 3 are weakly biased toward finding a higher proportion of lower $I_{\text{max}}$ objects than reality. All three simulated surveys show little bias in the distribution of Kozai libration amplitudes.

The distribution of Kozai libration amplitudes is shown in the bottom panel of Figure 3.19. There is general agreement between libration amplitude distribution of the MPC Kozai plutinos and both giant planet migration simulations, however, the Hahn and Malhotra (2005) simulation finds generally lower libration amplitudes, while the Levison et al. (2008a) simulation finds generally higher. This is an area that could use more theoretical work, as different migration timescales and migration modes may cause different libration amplitude distributions.

It is obvious from the top panel of Figure 3.19 that the $I_{\text{max}}$ values are much lower in both of the simulations than in the MPC. Although the MPC distribution is not debiased, and thus may not reflect the true distribution of Kozai plutinos, the inclination distribution discrepancy between models and the true Kuiper Belt has been noticed before (Section 2.11). This is a generic problem with giant planet
migration simulations, and not unique to this Kozai problem: these simulations are not good at raising the inclinations of the captured resonant objects (noted by Chiang and Jordan, 2002, and others).

3.8 Conclusion

With the upcoming inauguration of such rapid-fire all-sky surveys as LSST (Jones et al., 2009; LSST Science Collaboration et al., 2009) and Panoramic Survey Telescope & Rapid Response System (Pan-STARRS) (Grav et al., 2011), which are expected to detect hundreds of new TNOs, we are entering an era when we have enough well-characterized plutinos to be able to debias and measure the value of $f_{\text{koz true}}$ with more precision than has been possible.

Little theoretical work has been done relating the value of $f_{\text{koz true}}$ to the migration timescale of Neptune, but this may be an important and helpful diagnostic. To our knowledge, no theoretical work has been done so far to understand how the $I_{\text{max}}$ distribution is set, and how it evolves over time. There are also other relationships that have not been explored, such as the relation between the libration amplitude of $\phi_{32}$ and the Kozai libration amplitude.

Our results allows optimization for observers planning targeted surveys. If the goal of the survey is to find as many plutinos as possible, the highest density on the sky is not exactly 90° away from Neptune, but about 15° on either side of $\lambda_{N} \pm 90°$. If the goal of the survey is to find as many Kozai plutinos as possible, the best places on the sky are about 12° above and below the ecliptic, and 15° on either side of the orthoneptune points. The value of $f_{\text{koz obs}}$ that is measured in a given survey can be significantly different from $f_{\text{koz true}}$, and careful debiasing is necessary to derive the true value. Parameters of the survey such as pointings, field depths, tracking efficiencies, and fields with no detections must all be characterized in order to properly debias the results (see Jones et al., 2010).
Chapter 4

Debris Disks in Kepler Exoplanet Systems

4.1 Introduction

The evolution of solid circumstellar material is a competition between the accretional processes that form planets (when speeds are low) and the violent collisional processes which in post-formation systems inexorably grind down the material into particles small enough that radiation forces can remove them from the system. The disappearance of IR excesses around forming stars (on 3–10 Myr time scales) occurs because the accretional processes win and sequester the huge mass of dust into a (relatively) tiny number of objects with very low surface area/mass ratios. In contrast, the observed debris disks (known around hundreds of solar-type stars) are thought to be from collisional dust production as velocities rise during the final stages of planet assembly (for relatively young stars), or triggered around older stars as dust is liberated in higher-speed collisions (Wyatt, 2008). It is these more mature systems where our understanding of the physical picture is arguably incomplete. Although only \(~2\%\) of nearby main-sequence

stars have massive detectable warm disks (>200K; Lawler et al., 2009), several of these disks require dramatic hypotheses to produce the large dust mass estimated. That is, estimates of the collisional grinding of small-body belts indicate that the systems in question should not still possess dust at anywhere near the observed level (Wyatt et al., 2007). In fact, Rhee et al. (2008) went so far as to liken the dust mass around BD +20 307 to a ‘miracle’ given collisional-model predictions.

Debris disks are detected and studied via the excess IR flux they produce in the system, caused by the dust re-radiating the absorbed stellar light. This dust emission can then dominate the stellar photosphere in the IR, where the dust’s blackbody peak is set by the equilibrium temperature at the orbital distance (for a narrow ring) or at an extended disk’s inner edge, because the warmest dust usually dominates the system’s dust emission. For ‘realistic’ (although uncertain) dust grain size distributions, cross-sectional area and thus thermal emission is dominated by small grains, but the system mass is dominated by the unconstrained larger bodies.

Known disk systems consist of two types. The most common are massive and cold, analogous to the Solar System’s Kuiper belt. A few hotter (T >200K) systems are known, which are massive cousins of our asteroid belt. An example is HD 69830, hosting 3 RV-discovered Neptune-mass planets inside 0.6 AU (Lovis et al., 2006) and a warm dust ring at ~1 AU, with an estimated 4x10^{-7} M_\oplus in small grains alone (Lisse et al., 2007). Production of this dust is problematic (Beichman et al., 2005a), with hypotheses ranging from comet swarms to supercomets to massive planetary collisions; it is difficult to judge the plausibility of these mechanisms given the rarity of these outcomes.

Dust grains in orbit around a star are quickly destroyed, and must be replenished by a fragmentation cascade, where asteroids within a belt collide and break into progressively smaller pieces (Wyatt, 2008). The lifetime of a debris disk will be the same as the lifetime of the largest bodies in the source population that are available to be pulverized into dust grains. Very small grains (≲1 \mu m) are quickly blown away by stellar radiation pressure. Larger grains suffer PR drag and spiral
into the star in much less than the system’s age; for dust very close to the star (~0.1 AU), orbits collapse in only hundreds of years.

In this paper we search for excess infrared emission, indicating the presence of a dusty debris disk, in systems found by the Kepler mission to possess transiting exoplanets. In Section 4.2 we define our sample and discuss our methodology for identifying excess candidates, and in Section 4.3 we provide additional details on the eight stars that have excesses. Section 4.4 contains a discussion of the problems associated with having this much dust so close to the host star, and finally Section 4.5 contains a summary of recent work and this study’s contribution relating debris disks to the presence of exoplanets.

4.2 WISE Data

WISE released preliminary survey data in April 2011, covering about half the sky in photometric bands centered on 3.4, 4.6, 12, and 22 μm (W1, W2, W3, and W4, respectively; Wright et al., 2010). Due to the pointing strategy of the telescope, the coverage on the sky is non-uniform. Many individual images are stacked to produce the preliminary survey data, and some areas of the sky were visited more often than others, providing deeper coverage in those areas.

4.2.1 Defining Our Sample

We use WISE data to search for dust emission around the 928 solar-type stars (F, G, and K0–7 spectral types; $T_{\text{eff}} = 4000–7200$ K) that have stellar data ($T_{\text{eff}}$, metallicity, and log($g$)) reported in the Kepler database as of November 2011 and are candidates to host one or more planets contained in the first Kepler data release (Borucki et al., 2011; Lissauer et al., 2011b). Although most of these transiting planet candidates remain to be independently confirmed, we will refer to them as planets for brevity. The false-positive rate for single planet systems is <10% (Morton and Johnson, 2011), and is much lower for multiplanet systems (Lissauer et al., 2011a; Ragozzine and Holman, 2010).
There are several M stars reported as having planets in the *Kepler* database, but we did not consider these for analysis because of known systematic problems with modeling the atmospheres of such cool stars (Beichman et al., 2006a; Sinclair et al., 2010). We also did not consider the few hotter (A-type) stars with known *Kepler* planets in order to keep to stars similar to our Sun.

The *WISE* preliminary data release covers about half the *Kepler* field, detecting 439 out of the 928 solar-type stars; we consider a target to be ‘detected’ in a given band if the *WISE* Source Catalog reports a Signal-to-Noise Ratio (SNR) greater than 3. (*WISE* does not report fluxes, but rather magnitudes and their corresponding magnitude uncertainty.) All *WISE* detections are within 1 arcsecond of the coordinates given by the *Kepler* team. The band wavelengths cover where hot dust emission would peak, which in these systems is also near the equilibrium temperatures of the known *Kepler* planets, most of which are located 0.01–0.4 AU from their host stars (Lissauer et al., 2011b).

134 of these stars were flagged by the *WISE* team as having contamination from nearby, bright stars in the form of a halo or diffraction spike, or flagged as extended, suggesting probable contamination by a background object. These stars were not considered further.

The remaining 325 stars are all detected in both *WISE’s* W1 (3.4 µm) and W2 (4.6 µm) bands, ~2/3 of these are also detected in W3 (12 µm), and only six are also detected in W4 (22 µm).

### 4.2.2 Finding Excesses in the WISE Data

To determine if the *WISE* data match the expected stellar photospheric spectrum or represent an excess of emission, we use Kurucz stellar atmosphere models (Castelli and Kurucz, 2004) chosen from a grid of stellar temperatures, surface gravities, and metallicities to closely match those given for each star in the *Kepler* online database (Brown et al., 2011). The Kurucz models are scaled using photometry from the SDSS (Abazajian et al., 2009) and the 2MASS (Skrutskie et al., 2006). These *Kepler* targets are more distant than most known debris disk hosts.
Figure 4.1: The excess significances for each band $(m_{\text{pred}} - m) / \delta$. Overplotted are Gaussians with width $\delta$ for reference. Small ($\sim 1 \delta$) systematics are still clearly present. Filled bars show what we accept as significant excesses. One system (KOI 379) shows excess in W1 and W2 above the $5 \delta$ level ($7 \delta$ in W1 and $5 \delta$ in W2) but is not considered to be significant because of an obvious large flux mismatch between the SDSS and 2MASS photometry, perhaps indicating stellar variability.

(hundreds of parsecs), so extinction must be taken into account. The Kepler database provides reddening measurements $E(B - V)$ for each star, which we scale for each photometric band (Rieke and Lebofsky, 1985) and use to correct the photometry. The Kurucz model is scaled to each star’s flux using the corrected photometry, and then convolved with the efficiency function over each WISE passband. The band zero points then provide expected magnitudes.
Figure 4.2: Histograms of each of the four stellar parameters reported by the Kepler team: reddening $E(B-V)$, effective stellar temperature $T_{\text{eff}}$, stellar surface gravity $\log(g)$, and metallicity. Filled bars show these values for the eight systems with significant excess in at least one WISE band. There appear to be no correlations between the presence of a significant excess and any of these parameters.

There are several sources of error in these measurements. To estimate magnitude uncertainties $\delta$, the given error bars for each datapoint ($\delta_W$) and the WISE absolute photometric errors (2.4, 2.8, 4.5, and 5.7% in W1–4, respectively; Wright et al., 2010) are added in quadrature. To assess the significance of each possible excess, the WISE magnitude ($m$) is subtracted from the predicted stellar magnitudes ($m_{\text{pred}}$), and this is divided by the uncertainty ($\delta$) in that band to give a dimensionless excess. Our sign choice results in positive excesses indicating that
the WISE flux is higher than the predicted flux. The uncertainties in the stellar measurements reported by the Kepler database are ±200 K in temperature and 0.4 dex in log($g$) (Brown et al., 2011). We estimated the contribution of these uncertainties to the significance of the WISE excesses by using stellar models from ($T_{\text{eff}}$, log($g$)) gridpoints that were varied by the error bars in each of these parameters, and we found that this resulted in $<1\sigma$ changes in the excess. In order to account for this without having a formal number to add to our errors, we adopt the fairly stringent requirement that in order for an excess to be considered significant, it must be more than 5 $\sigma$ above the photospheric magnitude.

Because of the large photometric aperture on the WISE measurements (8.25 arcseconds for W1–W3 and 16.5 arcseconds for W4), we visually inspected both the WISE and SDSS images for each of the stars we find to have an excess and found that there is not contamination from any nearby, bright stars. In addition, we checked to make sure there were not systematic offsets in the expected position of the stars with excess, indicating contamination by a nearby background source.

The distribution of excesses for each band are shown in Figure 4.1. They follow approximately Gaussian distributions, with very few datapoints falling above 5 $\sigma$ and none falling below -5 $\sigma$. Figure 4.1 makes it clear that the systematic error here actually underestimates the significance of the excesses; if anything we are being more conservative with our definition of what is a significant excess and what is not.

Figure 4.2 shows stellar properties for our sample as reported by the Kepler team: extinction, stellar temperature, surface gravity, and metallicity. No trends are seen between any of these data and the presence of a significant excess in any band.

### 4.3 Candidate Debris Disk Detections

The vast majority of stars we examined have WISE fluxes consistent with the photosphere (these are listed in Table 4.1 and a selection of these stars are shown in Figure 4.3). In our full sample, we find eight stars with an excess of 5 $\sigma$ or
higher in at least one band. Unsurprisingly, these excesses generally appear in the longest two wavelength bands. These stars are listed in Table 4.2, the WISE data for each of these systems are listed in Table 4.3, and we discuss each of them in Sections 4.3.1–4.3.3 below.

Many simple debris disk models have been made using blackbody dust grains (i.e. Beichman et al., 2006b; Hillenbrand et al., 2008; Lawler et al., 2009). We adopt 10 $\mu$m radius blackbody dust grains for an initial model, both for ease of comparison with previous literature and for simplicity given the lack of spectral data. These models are not attempting to find the highly unconstrained ‘best’ possible dust model due to the myriad of complications inherent in dust emission (composition, radial distribution, emission features, and grain size), but rather to prove that the presence of dust provides a physically plausible model that approximately reproduces the WISE measurements. A single temperature blackbody is scaled to fit the WISE photometry, giving $L_{\text{dust}}/L_*$, which is used to calculate the area of absorbing dust $\sigma_{\text{dust}}$ using $\sigma_{\text{dust}} = 4\pi a^2 L_{\text{dust}}/L_*$ (Wyatt, 2008). When a density and radius are assumed for the grains, this can be converted to the mass in dust. We used 3.3 g cm$^{-3}$ for silicate dust grains. The temperature changes the wavelength of the peak of the curve. The orbital distance of the dust is then calculated using the same assumptions as were used to calculate Kepler planet temperatures (albedo of 0.3, uniform surface temperature, and no atmospheric effects; Borucki et al., 2011).
Table 4.1: FGK Stars with No Significant (>5 \( \delta \)) Excess.

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Figure 4.3: Included here and in Figure 4.3 are eight stars with WISE data that are consistent with the stellar photosphere. This is all of the confirmed planet systems systems that currently (as of November 2011) have WISE data (none of these systems have significant excesses): KOI 7 = Kepler 4 (Borucki et al., 2010), KOI 10 = Kepler 8 (Borucki et al., 2010), KOI 20 = Kepler 12 (Fortney et al., 2011), KOI 72 = Kepler 10 (Batalha et al., 2011), KOI 84 = Kepler 19 (Ballard et al., 2011), KOI 97 = Kepler 7 (Borucki et al., 2010), and KOI 377 = Kepler 9 (Holman et al., 2010). KOI 1427 is included as an additional example, being one of the coolest stars in the sample. Blue curve represents the stellar spectrum model. Cyan diamonds and squares show the SDSS and 2MASS data respectively used to scale the stellar model, and the red triangle shows the Kepler visual magnitude estimate converted to flux. The filled symbols show the extinction-corrected data, and the empty symbols show the original measured values. Green filled circles show the WISE data, and the error bars are often smaller than the datapoints. The excess (in multiples of $\delta$) of the observed-predicted magnitudes are given for the stellar model above each WISE datapoint. At the top, triangles show the planetary thermal emission wavelength peak for reference.
Figure 4.4: See caption for Figure 4.3.
### Table 4.2: Debris Disk Candidates.

<table>
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</table>

*Assuming blackbody dust grains of radius 10 µm

*Assuming modified blackbody dust grains of radius 1 µm

*Planet mass is calculated according to \( M/M_\oplus = (R/R_\oplus)^{2.08} \) (Borucki et al., 2011)

*Spectral type is approximate and based on the stellar temperature provided by the Kepler team

*KOI 904 has two known planets, both of which are listed here

^Tdust is assumed rather than fit. See text for details on each system.
Table 4.3: WISE Data for Debris Disk Candidates.

<table>
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<tr>
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<td>0.83 ± 0.03</td>
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</table>

Fluxes and flux uncertainties are approximate, estimated from the given WISE magnitudes and magnitude errors.

SNR < 3

It is probable that smaller grains are present, and if they dominate in the size distribution of dust grains, the dust could be significantly farther out and produce the same fit. We show this uncertainty in Table 4.2, where in addition to the best-fitting 10 \( \mu \)m dust grain model we show the best fit for a disk made of 1 \( \mu \)m modified blackbody dust grains, where the emissivity of the grains drops proportional to \((\lambda/\lambda_0)^{-\beta}\), with \(\lambda_0\) equal to the grain size and \(\beta\) equal to 1 (Dent et al., 2000). This means that the spectral energy distribution resulting from these grains is the same as a blackbody for wavelengths shorter than 1 \( \mu \)m, and drops off more steeply than a blackbody at larger wavelengths. Smaller grains may be more realistic, as resolved disk studies have found larger disks than predicted by fitting a blackbody spectrum to the mid-IR excess (i.e. Krist et al., 2010). However, in the interest of picking a uniform model that can be compared with previous Spitzer studies, we use the 10 \( \mu \)m blackbody dust model for most of our calculations.

For our 10 \( \mu \)m dust grain model, the resulting blackbody curve is added to the stellar atmosphere model, and this is used to fit the WISE data. For most systems, the temperature and surface area of the emitting dust are both free parameters in the fit. Because in some systems there is only one datapoint that rises off the pho-
tosphere, the temperature cannot be determined uniquely even if an upper limit can be set. These systems are noted in Table 4.2 and discussed below. For all eight systems, the fit for temperature is fairly unconstrained; the longest wavelength datapoint is almost always still rising, allowing a large number of cooler blackbody curves from more massive dust disks to pass through the datapoints. Thus the temperatures we calculate should be thought of as upper limits, as a larger amount of somewhat cooler dust could also produce the observed emission.

Using this simple dust model, the dust masses needed to produce these excesses range from \(~10^{-8}–10^{-4}\) Earth masses, orders of magnitudes larger than the dust mass of our solar system (\(~4 \times 10^{-10} \, M_\oplus; \) Hahn et al., 2002), and comparable in mass to known debris disk systems (typically \(~10^{-8}–10^{-2} \, M_\oplus; \) Beichman et al., 2005a; Bryden et al., 2006; Hillenbrand et al., 2008; Rhee et al., 2008; Wyatt et al., 2007), but with higher fractional luminosities as shown in Figure 4.5.

We find that our eight excess systems can fit into three categories based on dust temperature (and thus dust location). For much of the literature, \(>200\) K dust is considered to be hot dust. Due to the large variation in debris disk temperatures and terminology, for this paper we define our debris disk categories as the following: hot (\(~1000\) K), warm (\(~300–500\) K), and cool (\(~100\) K). We now discuss each excess system in turn.
Figure 4.5: The derived masses, fractional luminosities, and temperatures of dust rings in our sample (using 10 $\mu$m radius grains) compared to other known systems. Red triangles show systems examined by Wyatt et al. (2007) (see also Beichman et al., 2005a, 2006b; Lisse et al., 2012a), green squares are from a Spitzer spectroscopic survey (Lawler et al., 2009), and green pentagons are from a Spitzer photometric survey at 70 $\mu$m (Hillenbrand et al., 2008). For comparison, the purple diamond shows zodiacal dust in our solar system (Hahn et al., 2002). All systems shown are mature, solar-type stars. Blue circles show the dust models presented in this paper. Because $M_{\text{grain}}$ is proportional to grain size $r_{\text{grain}}$, using smaller grains would result in lower total dust masses (see Table 4.2). While the masses shown here are similar to known systems, because of the warmer temperatures we find for these Kepler systems, the resulting fractional luminosities $L_{\text{dust}}/L_*$ are much higher.
4.3.1 Warm Dust Systems (∼300–500 K)

Because of its wavelength range, WISE is most effective at detecting dust temperatures between 300–500 K, similar to the handful of warm debris disks discussed in Wyatt et al. (2007). Figures 4.6 and 4.7 show the five systems that possess excesses consistent with emission by warm dust (for comparison with our solar system’s SED, see Figures 1.9–1.11). Not surprisingly, due to the low photospheric flux predicted at long wavelengths, none of these systems have W4 detections with SNR>3. The low SNR datapoints and upper limits that are plotted (but not included in our fit) give tantalizing hints of what the upcoming full WISE data release could reveal about the dust locations in these systems. The W4 datapoints may reveal that these systems are actually similar to the cool dust systems in dust mass and temperature (see Section 4.3.2). Only one of these five systems (KOI 1099, Figure 4.6) requires dust warmer than >300 K (see discussion below); in the other four systems (Figure 4.7) we may be seeing the Wien tail of a cooler, more massive dust ring.

With the currently available WISE data, we find that these disks are plausibly hotter than the majority of known debris disk systems, and our best fitting models place dust on orbits of a few tenths of an AU. This is perhaps not surprising, as it is similar to the orbital radii of the known planets in these systems, so the dust may be roughly co-located with several Earth masses of planetary material. Most systems in the literature known to host both a debris disk and one or more exoplanets have a large separation (tens of AU) between the orbit of the planet(s) and the debris disk (Bryden et al., 2009; Dodson-Robinson et al., 2011). One of the five systems in our sample with warm excesses has the modelled dust ring interior to the known planet (Figure 4.6), with four exterior (Figure 4.7). All planets are within a few tenths of an AU of the modelled debris disk, which may allow dynamical interactions between the planet and the disk.

The G5 star KOI 1099, shown in Figure 4.6, is our most promising candidate to host a debris disk, appearing to have excesses in all WISE bands. The weak excess in W1 combined with highly significant excesses in both the W2 and W3
Figure 4.6: SED for KOI 1099, our best candidate for a debris disk due to the strong excess in W2 and W3, and a weak excess in W1. See caption for Figure 4.3. The excess values (in multiples of the total magnitude uncertainty $\delta$) for the stellar atmosphere model predictions (not including the dust model) are shown above each WISE datapoint. Additionally, the purple dotted curve shows the blackbody spectrum of a thin ring of 10 $\mu$m-radius dust grains, and the solid red curve shows the star+dust spectrum. The best-fitting dust mass, temperature, and orbital distance are also shown. The unfilled WISE datapoint shows SNR $<3$ data that was not used in the fit, but gives a preview of where the upcoming WISE full data release may provide significant data.
Figure 4.7: SEDs for stars with warm excesses, at orbital distances similar to that of the known planets. See captions for Figures 4.3 and 4.6. Open green triangles show upper limits on the W4 data, while open circles show W4 data with SNR < 3.

bands demands dust emission of ~500 K using our nominal model. An excellent match to all photometry between 0.5–12 µm is provided by a confined dust ring at 0.14 AU, although more complex distributions are certainly possible. The 1 µm modified-blackbody alternate model results in a cooler temperature (~400 K) and larger orbital radius for the dust ring (0.22 AU). Either model results in a ring inside the 0.57 AU orbit of the known Neptune-mass planet (candidate), and may point to inspiral of dust past the planet from a more distant dust source. The W4 flux has SNR < 3 and so was not used in our fit, but if future WISE data release confirm the ~1.5 mJy flux, this points to an even greater dust mass, much of
which would be at larger distances from the star. Using the low SNR W4 datapoint along with the other WISE bands would not allow a single temperature fit, simply indicating that this system contains dust that is producing significant emission at more than one orbital distance.

The other four warm systems, shown in Figure 4.7 do not have formally significant W1 and W2 excesses. It is possible that these weak W1 and W2 excesses should be ignored, in which case the W3 excess is the start of a blackbody of a much more massive but colder disk. We evaluate this explicitly for the KOI 943 system, where there is a low-SNR W4 datapoint rather than just an upper limit.

KOI 469 is a G0 type star with a super-Neptune at 0.095 AU. Because only the W3 datapoint is off the photosphere (with only an upper limit in W4), for our best-fitting dust ring we fixed the temperature by eye and fit the data by scaling the mass of the ring. This gives a dust ring at 0.36 AU, outside the orbit of the known planet.

KOI 559 is a K0 type star with a \( \sim 2 M_\oplus \) planet at 0.05 AU. There is a very weak excess in W1 and W2, with a strong excess at W3 and only a W4 upper limit. The best-fitting dust ring sits at 0.23 AU, well outside this hot super-Earth’s orbit.

The G5 star KOI 871 has a Saturn-mass planet at 0.1 AU. The WISE data is similar to KOI 559, with a very weak excess in W1 and W2, a strong excess in W3, and a W4 upper limit. The best fitting dust ring sits at 0.27 AU, again outside the planet’s orbit.

The K0 star KOI 943 follows the usual pattern of weak excesses in W1 and W2, with a strong excess in W3. The best-fitting dust ring sits at 0.28 AU, outside the orbit of KOI 943’s super-Earth at 0.05 AU. The low-SNR W4 datapoint hints at a more massive, more distant dust ring that will perhaps be confirmed by the upcoming full WISE data release. Using the nominal 1.5 mJy W4 flux in the fit pulls down the temperature to about 150 K, corresponding to a ring at 1.5 AU with a mass of \( 10^{-4} M_\oplus \), similar to the cool dust systems described in Section 4.3.2.
The dust masses required for all of these systems are fairly low: $\sim 10^{-7}$–$10^{-6} \, M_\oplus$, approximately equivalent mass to grinding a 10 km asteroid into 10 $\mu$m dust grains.

### 4.3.2 Cool Dust Systems ($\sim$100 K)

These systems (Figure 4.8) more closely resemble classical debris disk systems that have been studied extensively at longer wavelengths by Spitzer, with dust located at several AU, well outside the orbits of the known planets (Bryden et al., 2009; Dodson-Robinson et al., 2011). We find two stars in our sample with excesses that are consistent with blackbody peaks at the W4 effective wavelength of 22 $\mu$m or longer, and strongly significant W4 detections.

**KOI 1020** is a G3 star, and is the only star which has a strong W4 excess and yet no W1–W3 excess. This would require the dust to be quite cool ($\lesssim$ 100 K). Due to the unconstrained temperature fit, we fixed $T_{\text{dust}}$ at 100 K, but it could be colder if a more massive disk is present. Even the 100 K temperature places the dust at about 10 AU, far outside the orbit of its hot Jupiter at 0.3 AU.

**KOI 1564** is a G4 star, and has weak excesses in each of the 3 shortest wavelength bands, with a large excess in W4. Due to the large error bars on the W3 and W4 data, there are quite a range of temperatures that fit, but our best dust model
Figure 4.9: SED for KOI 904: the only star with a strong excess in both W1 and W2, and the only multiplanet system in this sample to show an excess. See captions for Figure 4.3, 4.6 and 4.7.

has an orbital distance of 2.0 AU. KOI 1564’s known Neptune-mass planet orbits well inside this, at 0.28 AU. The future improvement in the error bars provided by the next WISE data release will certainly better constrain this belt.

Both of these stars require high dust masses ($\sim 10^{-4} M_\oplus$) to produce the large excesses observed in W4, similar to the masses calculated for Spitzer surveys of solar-type stars at 24 and 70 µm (Bryden et al., 2006; Hillenbrand et al., 2008), although KOI 1564’s disk may be among the hottest known with this mass or larger.
4.3.3 Hot Dust ($\sim 1000$ K) in a Multiplanet System

The K5 star KOI 904 (Figure 4.9) is the only multiplanet system to show an excess, as well as the only system with significant excess in the shortest wavelength band (3.4 $\mu$m). This alone requires extremely hot dust. Both of the planets are super-Earths, one at 0.03 AU and one at 0.16 AU, indicating that there is at least $\sim 15$ $M_\oplus$ of material near the star. Having two planets is significant, because it greatly reduces the possibility of a false positive exoplanet detection (Lissauer et al., 2011a; Ragozzine and Holman, 2010). Currently, there is not a detection in the W3 or W4 bands, only a low-SNR W3 point and a W4 upper limit, so the dust model is not well constrained. For lack of any other guidance on the temperature, we fixed it so the blackbody curve peaks near the W2 effective wavelength and found an acceptable fit. This results in very hot dust, close to the sublimation temperature of silicates. This exact temperature is uncertain, but must be quite hot (>1000 K) in order to fit both the W1 and W2 datapoints. But with currently only two datapoints, more detailed dust modelling is unwarranted.

This remains our most insecure excess; there is a small mismatch between the SDSS and 2MASS photometry which could be the result of stellar variability (although the star is not flagged as variable in the WISE database). This is among the cooler stars in the sample, and so there is the possibility of poor stellar atmosphere modelling, as for the M stars. However, other K5 and later stars appear in our sample without showing signs of excess in the WISE data (see Figure 4.2). The very hot dust is also a bit worrying, though it is not very different from the equilibrium temperature of the innermost known planet in this system (960 K), and dust temperatures in the range 500–1200 K have also been detected around white dwarfs (Chu et al., 2011).

If real, this very hot dust cannot be produced by planetesimals in orbit at this distance; as discussed in more detail in Section 4.4, both the collisional timescale and the Poynting-Robertson drag time guarantee that particles cannot stay in such close orbits to the star for more than hundreds to thousands of years.
Only a very small dust mass (\(\sim 10^{-8} \, M_\oplus\)) is required in this system because of the extremely hot temperatures. Dust this hot has only been observed with optical interferometers, and may actually be fairly common around solar-type stars (Defrère et al., 2011), but is difficult to observe. Excesses this hot may have been missed by Spitzer spectroscopic surveys, which assumed no excess was present at shorter wavelengths if no slope other than Rayleigh-Jeans was found in the short wavelength data (Beichman et al., 2006b; Lawler et al., 2009). This excess is perhaps similar to that found for HD 23514 (Rhee et al., 2008), which was also detected at short wavelengths only. The full WISE data release should provide a high SNR W3 datapoint; if it remains above the photosphere, the case for a tenuous, but very hot ring of small grains near the planets will be strengthened.

We note that as this paper went to press, the February 2012 Kepler data release (Batalha et al., 2012) provided 3 additional planets in this system, bringing the total number of planet candidates around KOI 904 to five, all within 0.2 AU.

### 4.4 Dust Production

For the systems with abundant dust interior to 1 AU, it is difficult to produce viable steady-state models with such a large mass of small grains remaining so close to their stars. The PR drag timescale for dust to spiral into the star is extremely short at these close distances:

\[
 t_{\text{PR}} = 200 \, \text{yr} \left( \frac{r_{\text{grain}}}{10 \, \mu\text{m}} \right) \left( \frac{a}{0.1 \, \text{AU}} \right)^2
\]

(4.1)

for 3 g cm\(^{-3}\) grains (Burns et al., 1979), giving only \(\sim 1200\) years for 10 \(\mu\)m grains at 0.25 AU (a typical orbital distance in Table 4.2). Inspiraling grains would not ultimately reach the star in any case, as they should vaporise at \(\sim 1000–1500\) K (depending on composition) once they get close enough for temperatures to rise this high. But even before reaching this sublimation temperature, the collisional lifetime may drop below the orbital timescale and dust will quickly be smashed into pieces smaller than the blowout limit.
This timescale \( t_{\text{coll}} \) for inter-grain collisions to produce fragments smaller than the blowout limit is very short at these orbital radii and number densities. For a completely coupled narrow ring of width \( 2ae \) and height \( 2ai \) where all grains have the same size \( r_{\text{grain}} \), semimajor axis \( a \), eccentricity \( e \), and inclination \( i \), \( t_{\text{coll}} \) can be approximated by the inverse of the collision frequency, \( n\sigma v \), where \( n \) is the number density of particles, \( \sigma \) is the cross-sectional area of one particle, and \( v \) is the relative speed between particles. This gives the scaling:

\[
t_{\text{coll}} \sim 0.3 \text{ yr} \left( \frac{e}{0.1} \right) \left( \frac{\rho}{3 \text{ g cm}^{-3}} \right) \left( \frac{10^{-6} M_{\oplus}}{M_{\text{grain}}} \right) \left( \frac{r_{\text{grain}}}{10 \mu\text{m}} \right) \left( \frac{a}{0.25 \text{ AU}} \right)^{3.5} \tag{4.2}
\]

where \( \rho \) is the density of a dust grain, \( M_{\text{grain}} \) is the total mass of the dust grains, and the approximation has been made that \( e \approx i \) (where \( i \) is in radians). With \( t_{\text{coll}} = 0.3 \text{ yr} \) for our typical warm systems, a steady state requires replenishing \( \sim 10^{-6} M_{\oplus} \) (or more) of dust every year, consuming \( > 1000 M_{\oplus} \) of material in 1 Gyr, which is unreasonably large.

Unrealistic mass-consumption rates implied by such steady-state models are worrying, and have been noted for several main-sequence, warm debris disk systems. The fractional luminosities \( L_{\text{dust}}/L_\ast \) we measure for Kepler systems with excesses also greatly exceed the maximum predicted by the steady-state collisional model of Wyatt et al. (2007) given these stars’ mature ages. The common logical conclusion is that dust masses at this level have to be transient. Several theories have been proposed to explain such systems, including comet swarms, a “super-comet”, or a recent collision (HD 69830; Beichman et al., 2005a), or a late-heavy bombardment-style dynamical instability forcing many icy bodies onto highly eccentric orbits, bringing them in close to the star where they collide or sublimate (\( \eta \) Corvi; Lisse et al., 2012a; Wyatt et al., 2007).

Another possible explanation for these massive debris disks may be that these stars are all young (<100 Myr) and still in the process of clearing their protoplanetary disks. This hypothesis was offered for three of the seven stars presented by Wyatt et al. (2007) with disks far too massive to be maintained by a steady-state
collisional cascade. We inspected Palomar Sky Survey images and found no evidence that any of our sample stars are associated with the open clusters in the Kepler field, reducing the probability that this explains these systems.

A further option is that the dust may be impact ejecta from asteroids or comets recently striking the extant exoplanets, similar to the production of giant planet rings in our Solar System (Burns et al., 1999); however this is challenging given the super-Earths’ large escape speeds, even if impact speeds are also high.

What is clear is that these dust grains cannot be co-located with a planetesimal belt that is producing them. A massive asteroid belt at 0.2 AU or closer is an unsatisfactory dust source because \( t_{\text{coll}} \) for the planetesimals is \(< 1 \) Gyr (Moldovan et al., 2010; Wyatt et al., 2007) and the source would now be gone. Dust also cannot simply spiral in to these close distances from a more distant source in steady-state. The high optical depths that we calculate must be present in these systems (ranging from \( \tau \sim 0.01 - 0.5 \)) are several orders of magnitude higher than the maximum allowed (Wyatt, 2005) in a steady-state inspiral scenario, meaning that the dust present in these quantities would destroy itself collisionally before PR drag has a chance to move it any significant distance.

Given the large dust masses and the small fraction of stars that possess these excesses, episodic dust production mechanisms provide the most attractive explanation. Dust could be produced by a catastrophic collision between asteroids located further from the star, and we are seeing the dust as it spirals in toward the star. However, the probability of occurrence required by this theory may not be high enough to explain the observed fraction of such systems. Dynamical instabilities forcing many bodies from the outer reaches of these solar systems onto eccentric orbits so they collide close to the star, perhaps similar in style to the Late-Heavy Bombardment in our Solar System, might be seen as an attractive theory. However, the fraction of systems observed to have 24 \( \mu \)m excesses observed by Spitzer appears to be much too high given the short lifetime of such an event (Booth et al., 2009).
4.5 Discussion

We find IR excesses around eight stars that also host planet candidates. The masses and luminosities of these excesses are all too large to be explained by a steady-state collisional cascade. Given that the WISE field coverage was not uniform in depth, and so many of these relatively faint stars suffered from contamination by nearby bright stars, it is challenging to say anything robust about the fraction of these planet-hosting stars that have excesses in our sample. Out of the 186 stars with W3 data of SNR > 3, five have significant >5 $\delta$ excesses, or $\sim 3\%$. At first glance, this fraction appears similar to the 2% of solar-type stars with warm excesses (Lawler et al., 2009; Wyatt et al., 2007).

However, the fractional luminosities of these Kepler systems with excesses are much higher than most known debris disks (Figure 4.5 and Moór et al., 2006). Because of the larger distances and thus lower apparent brightness of these Kepler systems as compared to most known debris disk systems, only the brightest debris disks would be detected by WISE. Presumably the eight systems presented here are the very highest luminosity debris disks in the sample, and many more fainter disks are present around these planet-hosting stars. This would indicate that the fraction of stars possessing debris disks and Kepler-detected exoplanets is actually much higher than the fraction of field stars with debris disks.

Surveys with Spitzer (Bryden et al., 2009; Dodson-Robinson et al., 2011) have found that hosting a radial velocity-detected exoplanet does not make a star more likely to host a debris disk, and a recent WISE survey of exoplanets discovered by ground-based transit searches (Krivov et al., 2011) has found that the fraction of these stars ($\sim 2\%$) to host warm debris disks (excesses in W3 and W4) is similar to that found by Wyatt et al. (2007).

The Kepler systems studied here are different from other disk-bearing exoplanet systems, whose planets are primarily discovered by the radial-velocity (RV) technique, and by ground-based transit searches. Exoplanets from the latter two techniques are mostly large and very close to their stars, with an average mass of $\sim 2$ Jupiter masses and $<20\%$ known to be in multiplanet systems (Wright et al.,...
The recently announced 1200 *Kepler* exoplanet candidates (Borucki et al., 2011) are in a different regime, with about 1/3 in multiplanet systems, and having many super-Earths (typically 2–6 Earth radii) well inside 1 AU. Among our excess systems, all but two of the planets are Neptune mass or smaller; only one is massive enough to be a hot Jupiter. The resulting correlation we find between a star hosting both small planets and a debris disk inside a few AU agrees with the planetary formation and dynamical evolution models presented by Raymond et al. (2012). This makes sense in a picture where migrating Jupiters clean out the terrestrial systems they pass through.

Explaining the large dust masses required to produce these excesses remains a huge challenge. Steady-state theories appear to fail completely, as the mass consumption rates required would involve destroying thousands of Earth masses of planetary material over the lifetime of these stars. Episodic events such as catastrophic collisions between bodies may provide an explanation, but these events must last long enough and occur frequently enough to result in $\sim 3\%$ of solar-type stars in this state at any given moment. Much theoretical work needs to be done here.

The upcoming *WISE* full-sky data release, discussed in Chapter 5, will access additional systems, decrease the error bars on existing measurements, and provide longer-wavelength measurements for some of these systems to see if there is dust at larger orbital distances. Spectral data on these candidates would be invaluable, but the host stars are faint. Future *Kepler* data releases will contain planets on more distant orbits, perhaps yielding planets in closer proximity to these new debris disks.
Chapter 5

Galactic Dust Limits Detection of Asteroid Belts in Kepler Exoplanet Systems

5.1 Introduction

This chapter expands the sample examined in Chapter 4, which used only the preliminary WISE release with limited depth and sky coverage. We here use the WISE full-sky survey, and include the systems hosting Kepler-discovered planet candidates announced as of February 2012. During the final stages of preparing this paper for publication, we became aware of a similar work by Kennedy and Wyatt (2012) that analyzed the entire Kepler Input Catalogue. We agree with their conclusion that about half of the Kepler field is too confused by background galactic dust to take the WISE fluxes at face value, and find that all of our excess hosts fall into this region of high confusion. We discuss this further in Section 5.4.

This chapter is based on the following work which has been submitted for publication: S. M. Lawler and B. Gladman, Galactic Dust Limits Searches for Asteroid Belts in Kepler Exoplanet Systems.
Our work differs from that of Kennedy and Wyatt (2012), who used a largely statistical approach on the extremely large dataset they utilized, in that we analyze the systems with high-quality WISE excesses individually, measuring photometry from the WISE images and measuring background fluxes provided by IRAS (Neugebauer et al., 1984) for each of these sources. Section 5.2 outlines our method to identify excesses in the WISE data and compares our results with previous papers that also performed this analysis. Section 5.3 provides strawman models of simple dust rings that would produce the measured excesses around host stars and looks at the presence of exoplanet candidates in these systems. Section 5.5 provides a detailed discussion of the background contamination that is present in the Kepler field of view. In section 5.4 we carefully analyze WISE and IRAS data for our six initially most promising excess sources. Using IRAS data, we observe possible background contamination, and arrive at blackbody dust temperatures of about 100 K, significantly hotter than galactic cirrus in general. Photometry of the WISE data for these six sources utilizing different sized apertures also shows strong evidence of contamination by background flux. Section 5.5.1 compares our results with other debris disk surveys, and after concluding that background contamination is too serious an issue to believe that these excess measurements are due to circumstellar dust, Section 5.5.2 discusses prospects for future observations of these Kepler exoplanet systems.

5.2 Our Sample

We examined the 1790 systems with planet candidates found by the Kepler mission as of February 2012 (Batalha et al., 2012). We searched the WISE database and found that 1734 of these have a corresponding WISE source within 1 arcsecond; 69 of these stars do not have stellar data (temperature, log\(g\), and metallicity) reported by the Kepler team. Because we use the Kepler team’s stellar data for all of the other systems, in the interest of keeping a uniform analysis procedure we discard these stars. Of the 1665 remaining systems, 452 were flagged by the WISE team as having photometric contamination such as a halo or diffraction
spike from a nearby, bright star. After discarding these contaminated stars, this leaves us with 1237 stars which we examined for excess emission.

5.2.1 WISE Data

WISE released an all-sky survey in April 2012, in mid-IR photometric bands centered on 3.4, 4.6, 12, and 22 µm (W1, W2, W3, and W4, respectively; Wright et al., 2010). Due to the pointing strategy of the telescope, the coverage on the sky is non-uniform. Many individual images are stacked to produce the preliminary survey data, and some areas of the sky were visited more often than others, providing deeper coverage in those areas.

The following outlines our procedure for finding excesses in the WISE all-sky catalog. However, it should be noted that this procedure ignores any possible contribution from diffuse galactic cirrus, and assumes that any detected excess is circumstellar (see Section 5.4).

5.2.2 Finding Excesses in the WISE Data

To determine whether or not a star has an excess of emission in the WISE bands above the stellar photosphere, we use the following procedure:

- We take the stellar parameters given by the Kepler team (temperature, log(g), metallicity) and use these to pick a model stellar spectrum that closely matches. We initially use both Phoenix (Allard et al., 2012) and Kurucz (Castelli and Kurucz, 2004) stellar atmosphere models to double check.

- Because these stars are hundreds to thousands of parsecs away, photometry from the Sloan Digital Sky Survey (SDSS; Abazajian et al., 2009) and the Two Micron All-Sky Survey (2MASS; Skrutskie et al., 2006) is corrected for extinction using $A_V$ values provided by the Kepler team.

- The stellar spectrum is scaled to match this extinction-corrected photometry.
The scaled stellar spectrum is convolved with the sensitivity function for each WISE passband, giving a predicted magnitude in each WISE band for the contribution of the stellar atmosphere.

The predicted and measured WISE magnitudes are converted to fluxes using the flux of a zero magnitude source given by the WISE team (Wright et al., 2010), assuming a Rayleigh-Jeans slope from 3–22 \( \mu m \).

The given magnitude error bars (\( \delta_W \)) are added in quadrature with the error on the WISE absolute photometric calibration (2.4, 2.8, 4.5, and 5.7% in W1–4, respectively; Wright et al., 2010), giving \( \delta \), which is used to calculate a SNR. This SNR is then used to calculate the uncertainty in flux \( \sigma_W \).

The significance of the difference between the predicted and measured fluxes is then evaluated: \( \frac{(F_{\text{pred}} - F_{\text{obs}})}{\sigma_W} \). Note that in Chapter 4, we used magnitude excesses; we use flux excesses here to be more consistent with the literature. Several of the stars which had large error bars have an excess below the threshold for significance when fluxes and not magnitudes are used to calculate the excess.

We adopted the relatively stringent requirement that the excess \( \frac{(F_{\text{pred}} - F_{\text{obs}})}{\sigma_W} \) must be greater than 4.5 using both Kurucz and Phoenix stellar spectrum models for the star to be considered to have a significant excess in a given band. There was no significant difference between the two models, except for M stars, which are known to be poorly matched by Kurucz models (e.g. Lawler et al., 2009). 4.5 was chosen as a cut simply because it produced a good number of systems to study more closely.

Lastly, a system with a candidate significant excess is visually scrutinized closely for signs of nearby stars, plainly visible background cirrus, or other obvious contamination.
5.2.3 Systems with Apparent Excess

Of the 1237 Kepler planet hosts examined, we found 39 stars that have a significant excess in at least one band and initially had no obvious signs of contamination. As found in Section 4.3, the vast majority of the sample shows no sign of excess, with the WISE measurements being very close to the magnitudes predicted by the stellar atmosphere models. The full list of systems we analyzed is available upon request.

Figure 5.1 shows the distributions of excesses \((F_{\text{obs}} - F_{\text{pred}})/\sigma_W\) for each band. It is apparent that the entire W2 distribution is offset so that it peaks about 1 \(\sigma\) below zero excess, and the peak of the W4 distribution appears to be offset toward higher excess values.

A plausible explanation for the W2 distribution offset is that the W2 bandpass (covering \(\sim 4-5.5\) \(\mu\)m) sits on top of a region of the stellar spectrum that contains a dense collection of absorption lines, some of which are poorly measured. This wavelength range in stellar spectra is hard to study because heavy telluric absorption makes ground-based observation in these wavelengths unfeasible (Civiš et al., 2012a,b). The W2 distribution we measure hints that the absorption line depths in the Phoenix model, which relies on theoretical hydrodynamic and radiative transfer calculations (Allard et al., 2012), are not deep enough. In any case, because we define an excess as 4.5 \(\sigma_W\) above an excess of 0, and not from the peak of the distribution, this means the excesses are actually more conservative because they are even higher above the distribution’s center. Because we do not use the W2 band to fit the stellar spectrum model, this offset has no effect on the analysis of excesses in the other bands.

The W4 excess distribution also does not match a Gaussian. This is because only the very brightest stars are detected in the W4 band, and some of these stars are above the detection limit because of the presence of an excess.

We divide these excess systems into four “tiers” based on the quality of the WISE data available:
Figure 5.1: Excess significances \((F_{\text{obs}} - F_{\text{pred}}) / \sigma_W\) in each band for all the stars in our sample that were not found to have contamination or confusion by background objects. A Gaussian with width 1 \(\sigma_W\) is shown for reference, and systems with significant excesses are shown in red. We call “excess detections” those with \(> 4.5 \sigma_W\) values above the photosphere; these systems were then the subject of further investigation as to the reality of the excess.
• **High Excess Tier**: The most significant excesses in the W3 and W4 bands. One of these two bands has a $>4.5 \sigma_W$ excess, and the other has a $>3.5 \sigma_W$ excess. Six stars fall into this category: KOI 423 (also known as Kepler 39; Bouchy et al., 2011), 464, 1894, 2130, 2158, and 2370.

• **Medium Excess Tier**: Has a significant ($>4.5 \sigma_W$) W3 excess, and a W4 value well above the photosphere, but because of large error bars on the W4 data, the W4 excess is $3.5 \sigma_W$ or less. Thirteen stars are in this tier: KOI 242, 418, 655, 700, 916, 1102 (also known as Kepler 24; Ford et al., 2012), 1686, 1760, 2082, 2180, 2194, 2358, and 2841.

• **Low Excess Tier**: Has a significant ($>4.5 \sigma_W$) W3 excess but only a W4 upper limit. (For low-SNR data, WISE provides upper limits on the measured magnitudes.) Seventeen stars are in this category: KOI 421, 559, 658, 758, 822, 842, 844, 887, 922, 1070, 1526, 1632, 1828, 1861, 2091, 2117, and 2531.

• **Short Wavelength Group**: Has a significant ($>4.5 \sigma_W$) W1 or W2 excess and only upper limits in W3 and W4. Only three stars fall into this category: KOI 840, 904 (also known as Kepler 55; Steffen et al., 2013), and 1099. Though these excesses are significant, because of the small error bars on the W1 and W2 WISE data, the fractional excesses are actually quite small, $(F_{\text{obs}} - F_{\text{pred}})/F_{\text{pred}} \sim 0.1–0.3$, and these excesses may very well be due to a small calibration error.

Table 5.1 lists *Kepler* data and *WISE* data for the six High Excess Tier candidates.
Table 5.1: Data for High Excess Tier Systems and Strawman Dust Models

<table>
<thead>
<tr>
<th>KOI</th>
<th>423 (^a)</th>
<th>464</th>
<th>1894</th>
<th>2130</th>
<th>2158</th>
<th>2370</th>
</tr>
</thead>
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<tr>
<td># planets</td>
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<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Kepler</td>
<td>9.42</td>
<td>2.63</td>
<td>6.73</td>
<td>16.29</td>
<td>1.66</td>
<td>1.61</td>
</tr>
<tr>
<td>planet r ([R_\oplus])</td>
<td>106.0</td>
<td>8.53</td>
<td>331.0</td>
<td>3</td>
<td>3</td>
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<tr>
<td>planet a [AU]</td>
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<td>0.05</td>
<td>0.287</td>
<td>0.075</td>
<td>0.106</td>
<td>0.053</td>
</tr>
<tr>
<td>planet r [AU]</td>
<td>2.3 ± 0.07</td>
<td>2.97 ± 0.09</td>
<td>13.0 ± 0.4</td>
<td>2.77 ± 0.09</td>
<td>10.4 ± 0.3</td>
<td>6.3 ± 0.2</td>
</tr>
<tr>
<td>W1 flux [mJy]</td>
<td>1.19 ± 0.04</td>
<td>1.56 ± 0.05</td>
<td>6.5 ± 0.2</td>
<td>1.44 ± 0.05</td>
<td>5.4 ± 0.2</td>
<td>3.3 ± 0.1</td>
</tr>
<tr>
<td>W2 flux [mJy]</td>
<td>1.61 ± 0.08</td>
<td>0.97 ± 0.09</td>
<td>2.6 ± 0.1</td>
<td>1.36 ± 0.11</td>
<td>1.7 ± 0.2</td>
<td>1.6 ± 0.2</td>
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<tr>
<td>W3 flux [mJy]</td>
<td>4.7</td>
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<td>5.2</td>
<td>10.0</td>
<td>3.9</td>
<td>5.0</td>
</tr>
<tr>
<td>frac. ex. [c]</td>
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</tr>
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<td>0.1</td>
<td>0.2</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>IRAS (\mu m) flux [mJy]</td>
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<td>1700</td>
<td>1500</td>
<td>2000</td>
<td>5600</td>
<td>5500</td>
</tr>
<tr>
<td>Temp. [K]</td>
<td>110 ± 30</td>
<td>130 ± 30</td>
<td>150 ± 30</td>
<td>160 ± 30</td>
<td>120 ± 30</td>
<td>120 ± 30</td>
</tr>
<tr>
<td>Putative</td>
<td>5.6 ± 1.5</td>
<td>3.1 ± 1.1</td>
<td>14.5 ± 2.6</td>
<td>0.9 ± 0.5</td>
<td>4.8 ± 1.9</td>
<td>7.8 ± 2.8</td>
</tr>
<tr>
<td>a [AU]</td>
<td>9.4 ± 2.2</td>
<td>10 ± 2</td>
<td>2.4 ± 0.4</td>
<td>25 ± 11</td>
<td>4.1</td>
<td>5.1 ± 2</td>
</tr>
<tr>
<td>L_{dust}/L_\star [10^{-3}]</td>
<td>600 ± 2500</td>
<td>200 ± 1000</td>
<td>900 ± 700</td>
<td>40 ± 100</td>
<td>200 ± 800</td>
<td>600 ± 1600</td>
</tr>
<tr>
<td>M_{dust} [10^{-6} M_\star]</td>
<td>340</td>
<td>370</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100 (\mu m) flux [mJy]</td>
<td>1900</td>
<td>1700</td>
<td>1500</td>
<td>2000</td>
<td>5600</td>
<td>5500</td>
</tr>
</tbody>
</table>

\(a\)Also known as Kepler 39. Bouchy et al. (2011) have confirmed KOI 423b using radial velocity, and found that the mass is 18.0 ± 0.92 M_{Jup}, or about 5700 M_{\oplus}, so the mass-radius relation of Borucki et al. (2011) must be used with care.

\(b\)Planet mass is calculated according to \(M/M_\oplus = (R/R_\oplus)^2.08\) (Borucki et al., 2011). Above \(\sim 5 R_\oplus\), masses are likely significantly underestimated.

\(c\)Fractional excess is \(\frac{F_{\text{obs}} - F_{\text{pred}}}{F_{\text{pred}}}\).

\(d\)Sky error is calculated using three different sky annuli and gives a rough measure of the error added by cirrus in the sky annulus. Magnitude errors can be roughly considered percentage errors in flux. See section 5.4.2

\(e\)IRAS background fluxes are estimated using the procedure outlined in Section 5.4.1.

\(f\)Calculated values using 1 \(\mu m\) radius modified-blackbody dust grains
5.2.4 Comparison with Previous Papers

None of the systems presented here as High Excess Tier excess systems were analyzed in Chapter 4, either because the system was not within the area of sky covered by the WISE Preliminary Data Release, or because the system was not yet announced as a planet-hosting star by the Kepler team. It is instructive to look back to see how the additional data from the WISE Full Sky Data Release has altered our previous conclusions.

Out of the eight systems identified as having significant excesses in Chapter 4, in the reanalysis presented here we confirm only three: KOI 559 (Low Excess Tier), KOI 904, and KOI 1099 (both Short Wavelength Group). The other five systems are no longer significant due to revisions in the WISE fluxes and error bars. KOI 904, 943, 1020, and 1099 all had at least one WISE band’s measurement revised from a \( \text{SNR} > 3 \) detection in the WISE Preliminary Catalogue to only an upper limit in the full-sky release (though KOI 904 and 1099 still have significant excesses in W1 or W2). KOI 469 and 1564 had their magnitudes revised fainter and error bars increased.

Ribas et al. (2012) performed a very similar analysis to that performed in Chapter 4. They flagged a few excesses that we rejected due to low significance, and they rejected a few of the stars we flagged as excesses (KOI 871, 943, 1020, and 1564) due to contamination. Both papers agreed that KOI 469, 559, and 1099 hosted excesses. However, Kennedy and Wyatt (2012) mention that KOI 469 is probably confused with a bright, moving background object in some of the WISE frames, so this system should not be counted as an excess host. Kennedy and Wyatt (2012) further point out that the remaining two stars are located in areas of high background flux (see Section 5.4).

WISE provides a fantastic full-sky dataset, but we lack control over the data reduction process, and are of course limited by revisions and errors that occur in the dataset. The WISE sensitivity so greatly exceeds ground-based telescopes that, in virtually all cases, additional data will be nearly impossible to obtain until another space-based infrared telescope is flown. Even Herschel’s sensitivity of \( \sim 4 \) mJy at
70 µm (Poglitsch et al., 2010) is too low to detect the predicted Rayleigh-Jeans tail of the brightest High Excess Tier system (KOI 2370, with ∼5 mJy at 20 µm).

5.3 Strawman Dust Models

Assuming that the excesses we observe are caused by circumstellar dust, we use simple single-temperature dust models to estimate, to order of magnitude, strawman value for the masses and orbital distances of the emitting dust in these systems needed to provide the measured flux excess. Though we believe the background galactic dust makes the detection of these disks completely unreliable (see Section 5.4), we provide these calculations to explore the plausibility of circumstellar dust as the cause of these excesses.

We compute disk properties using modified-blackbody 1 µm-radius dust grains, where the Raleigh-Jeans tail is attenuated at longer wavelengths due to the small dust grains’ inability to radiate effectively at these longer wavelengths. Table 5.1 shows the result of strawman models that best reproduce the measured WISE data for the six High Excess Tier systems, and Figures 5.2 and 5.3 shows SEDs for these systems. These SEDs are the integrated fluxes in the WISE beam and thus are valid regardless of the interpretation of the >10 µm flux’s origin.

Generally, the best-fitting dust would sit at approximately asteroid-belt temperatures (∼100–160 K), with masses ranging from roughly 4 to 6 orders of magnitude larger than the mass of our solar system’s zodiacal cloud (∼4 × 10⁻¹⁰ M⊕; Hahn et al., 2002). Unsurprisingly, the dust temperatures WISE is most sensitive to are those where the thermal peak is in the 10–30 micron range (about 100–300 K), so one would detect signatures at this temperature. In cases where there is only an upper limit for the W4 data, one could use this to place a lower limit on the dust temperature, and an upper limit on the semimajor axis, $L_{\text{dust}}/L_*$, and dust

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8Note that in Chapter 4 we used 10 µm-radius dust grains: there is no “standard” grain size used in the literature. More recent papers seem to favour using 1 µm dust grains, so we adopt that here. In any case, the calculated dust masses are only good to order-of-magnitude precision, so the grain size used is not terribly important for these calculations.
Figure 5.2: SEDs for the High Excess Tier systems (see also Figure 5.3).

We believe that all of the excesses presented here are due to contamination by galactic cirrus, and not by circumstellar dust, but these plots are presented to show how well circumstellar dust could be modelled in these systems, where there are statistically significant excesses in the W3 and W4 bands. The Phoenix stellar spectrum model is shown in blue (note that in Chapter 4 we used Kurucz models; here we choose Phoenix models simply because of the higher resolution offered). Yellow diamonds and squares show the SDSS and 2MASS data respectively used to scale the stellar model, and the yellow triangle shows the Kepler visual magnitude estimate converted to flux. The filled symbols show the extinction-corrected data, and the empty symbols show the original measured values. Green filled circles show the WISE data; the error bars are often smaller than the datapoints. The IR excess values (measured in multiples of the total uncertainty $\sigma_W$) above the stellar atmosphere model predictions (not including the dust model) are shown next to each WISE datapoint. The red dotted curve shows the modified-blackbody spectrum of a thin ring of 1 $\mu$m-radius dust grains, which has been scaled to match the IR excess, giving the amount of emitting dust that would be present. At the top, triangles show the planetary thermal emission wavelength peak for reference.
mass. These temperatures imply that the dust would be located \(\sim 1\text{–}6 \text{ AU}\) from
the stars.

### 5.3.1 Properties of Systems with IR Excess

Most of the six High Excess Tier planetary systems are similar, with a single known super-Earth or Neptune within a few tenths of an AU from the star, well inside the innermost possible dust ring allowed by the \textit{WISE} data. KOI 464 has two planets, both meeting the above criteria. We note that KOI 423, also known as Kepler 39, has been confirmed by radial velocity, and Kepler 39b is found to be nearly massive enough to be classified as a brown dwarf (Bouchy et al., 2011). KOI 1894 is different in several ways. It has the largest radius planet of any of the
High Excess Tier systems (slightly larger than Jupiter), and also would have the largest semimajor axis for its nominal dust ring.

The Medium Excess Tier systems follow the same pattern, with the planets well inside the candidate disk. One notable Medium Excess Tier system is KOI 1102, also called Kepler 24, which has had its two planets confirmed dynamically using transit timing variations (Ford et al., 2012).

Only upper limits on what would be the dust belts’ semimajor axes are available for the Low Excess Tier systems. These upper limits are all within a few AU of the host stars, and again well outside the orbits of the known planets.

The Short Wavelength Group systems only have detections in the W1 and W2 bands, and due to the small WISE error bars in these bands, though the excesses are $>4.5 \sigma_W$, the fractional excesses are quite small. A similar excess flux in W3 or W4 would not be detectable due to the lower sensitivity at longer wavelengths. These three systems would have to possess extremely hot (~1000 K) dust in order to produce excess emission at these wavelengths. Such hot dust is not unheard of, and has been observed around some main-sequence stars (Defrère et al., 2011), but it is much warmer than the typical debris disk. As discussed in Section 4.3, the collisional timescales for dust on these close orbits is extremely short: a few hundred years. Explaining the presence of dust this close to a host star is difficult, and an unlikely episodic event would have to be invoked. All three systems would have dust that is orbiting at about the same distance as their known planets. KOI 904, also called Kepler 55, is of particular interest, as it is currently known to possess five super-Earth candidates, packed tightly together, with two confirmed by transit timing variations (Steffen et al., 2013).

5.4 Contamination by Background Galactic Cirrus

Although this chapter gives estimates of possible debris disks surrounding several Kepler exoplanet hosts, due to galactic cirrus and the way the WISE fluxes are measured, the fluxes for all of the excess hosts are unreliable.
Just the locations of the excess hosts within the *Kepler* field are a good indication that suspicion is warranted. All of the excess hosts are located close to the galactic plane (a point also made by Kennedy and Wyatt, 2012), even though the distribution of *Kepler* planet-hosting stars is roughly constant across the field (Figure 5.4). Although this makes the ensemble of systems extremely suspect, it does not formally rule out any individual system.

The typical distances and galactic latitudes of these stars correspond to \( \sim 100 \) pc above the midplane of the Galaxy, similar to the scale height of the Galactic thin disk. Because the metallicities (the fraction of heavy elements within the atmosphere of the star) of thin disk stars are higher than thick disk stars, which are primarily at larger distances from the midplane, one could imagine a related selection effect causing more disks to appear close to the Galactic plane. While it has been shown that stars with higher metallicity are more likely to host a hot Jupiter (Fischer and Valenti, 2005), this correlation does not extend to debris disks. That is, the fraction of mature, main sequence stars that have a debris disk does not vary with host star metallicity (Greaves et al., 2006). There also does not appear to be a relation between host star metallicity and the presence of super-Earths and Neptunes (Buchhave et al., 2012), so there should not be a selection effect for higher metallicity stars in this sample of *Kepler* planet hosts. The fact that stars’ metallicities increase on average toward lower galactic latitudes likely has no bearing on the trend of excesses shown in Figure 5.4.

### 5.4.1 Galactic Dust in the WISE and IRAS Images

Figure 5.5 shows images of the fields immediately around the six High Excess Tier stars in the *WISE* W3 and W4 and *IRAS* 25 and 100 \( \mu \)m bands (IRAS images are from the Improved Reprocessing of the *IRAS* Survey; Miville-Deschênes and Lagache, 2005, downloaded from [http://skyview.gsfc.nasa.gov](http://skyview.gsfc.nasa.gov)). Each of the images shows structure in the nearby galactic cirrus background. In particular, several of the stars appear on top of “knots” of emission at 100 \( \mu \)m. While Kennedy and Wyatt (2012) use a statistical
Figure 5.4: Locations of *Kepler* targets analyzed in this chapter within the *Kepler* field of view. Red filled circles show High Excess Tier excesses, red squares show Medium Excess Tier excesses, empty squares show Low Excess Tier excesses, and empty triangles show Short Wavelength Group excesses. Galactic latitudes ($l$) are shown for reference. Contamination by Galactic cirrus is much more likely closer the Galactic plane, and most excesses hosts are located below a galactic latitude of 15°, though there is not a noticeable increase in the density of the *Kepler* targets (black dots).
analysis on the entire Kepler Input Catalog to make a cut in IRAS 100 µm flux, above which they do not believe excesses, we inspected IRAS images for each of our High Excess Tier sources.

We use the IRAS data as a measure of possible Galactic cirrus that is independent of WISE and contains longer wavelength data that may more easily show structure from cold Galactic dust. Because of the large IRAS pixel size compared to the WISE pixel size (pixel size ~1.5′ versus the WISE pixel size of ~1.375′′), we estimate the IRAS background at 25 and 100 µm by measuring the flux in the nine pixels immediately around the coordinates of the star of interest, which covers more than the area inside the WISE 70″ sky annulus outer boundary. These measurements are reported in Table 5.1. The 25 µm sky background ranges from 240–370 mJy, while the reported WISE fluxes are only a few mJy (~0.5–1% of the background flux) for each of these stars. A slight error in sky subtraction could easily result in a few mJy boosting of the derived aperture flux, which would result in an excess in our analysis.

Over the range of wavelengths (10–100 µm) covered by the images in Figure 5.5, it can be seen that the patterns of brighter and darker emission are correlated; that is, the variations on the sky are real and not caused by random noise. In particular, this is not infrared background produced by the WISE telescope itself. The temperature of galactic cirrus has been measured in large-scale, low-resolution maps based on long-wavelength observations, and in most portions of the Galactic disk, it is a fairly constant ~18–20 K (Schlegel et al., 1998; Schnee et al., 2005). If the dust emission was blackbody, one would not expect the dust to be easily visible in any but the W4 (22 µm) band, as the rapid Wien tail falloff should make the Galactic emission in even the W3 (12 µm) band negligible. However, the correlated structure visible in Figure 5.5 combined with additional evidence (see below) indicates that we are in fact observing the Galactic cirrus background in both the WISE W3 and W4 images.
Figure 5.5: WISE and IRAS images of excess hosts: 10'-wide WISE W3 images (top row) and W4 images (second row), 45'-wide IRAS 25 µm images (third row) and IRAS 100 µm images (bottom row). Images are centered on each of the six High Excess Tier stars, left to right: KOI 423, 464, 1894, 2130, 2158, and 2370. Cyan circles show the 50–70" sky annulus, and the red circles show the 8.25" aperture used for photometry in the W1–3 bands and the 16.5" aperture used for W4. As discussed in Section 5.4, the patchy structure of the galactic cirrus background leads to photometry for some sources that may not be reliable at better than the few percent level. Contrast is chosen to better show structure, and is not the same for all images.
5.4.2 Independent Measurements of WISE Photometry

The WISE documentation emphasizes that the photometry their pipeline provide is likely to be more accurate and more consistent than independently re-measuring the photometry. However, after analyzing the provided photometry in many different ways, it became clear that the next step to fully understand the measured excesses was to attempt to reproduce the photometric measurements on the WISE images ourselves.

To proceed, we first downloaded WISE images (from http://irsa.ipac.caltech.edu/) of the fields immediately around each of the six High Excess Tier stars, and performed aperture photometry centered on these sources. All of the sources are visible in the W1 and W2 images, which furnishes the star position for the low-SNR W3 and W4 images. Figure 5.6 shows our magnitude difference measurements using different aperture sizes (the photometric curve of growth), ranging from the 8.25” (6 pixel) aperture used for the WISE automated photometry in bands W1–3, to 20 pixels, larger than that used for W4 (12 pixels = 16.5”). We compare the photometry from the High Excess Tier stars with an average curve of growth measured from several stars within the Kepler field of view chosen to be uncrowded and relatively bright. Measuring a well-behaved point source using increasing aperture sizes will cause the measured flux to increase by smaller and smaller increments, until finally at large apertures, increasing the aperture size does not result in increased measured flux. None of the six High Excess Tier stars follow the stellar curve of growth; they increase in brightness more quickly with increasing aperture size, even just outside the 6 pixel aperture, which is additional evidence in favour of extended background contamination in the beam. (These disks are unresolved, of course). This analysis indicates that there are likely problems with the background subtraction for these stars.

To characterize the variations in the background, we also measured the photometry using three different sky annuli: one similar to the size used for the WISE automated photometry (53–70”), one smaller (27–53”), and one larger (70–85”).
**Figure 5.6:** Photometric curve of growth for the six High Excess Tier stars (red dotted lines) compared to an averaged photometric curve of growth measured from several uncontaminated stars (black solid lines at top). Different aperture sizes are on the x-axis. W3 data is shown in the top panel, W4 data in the bottom, and curves are labeled with KOI number on the right side of each panel (KOI 2130 does not appear in the bottom panel because of extremely poor photometry). Values are normalized to the magnitude at the smallest aperture, 6 pixels = 8.25′′, and use the WISE sky background annulus size (50–70′′). None of the six High Excess Tier stars follow the well-behaved curve of growth shown; all increase in brightness more rapidly, which is additional evidence in favour of background contamination.
We found that for nearly all of the High Excess Tier stars, there are large (few tenths of a magnitude) differences in the sky background, which results in a large difference in the measured photometry depending on which sky annulus is used for sky subtraction (see Table 5.1). The cause of the difference is patchy sky background on scales of several arcseconds. These large errors in magnitude measurement, resulting in tens of percent error in the flux calculation, are not taken into account in the reported WISE error bars. Incorporating these variations into our excess measurements renders all of the detections statistically insignificant.

We were able to reproduce the magnitude measurements reported by WISE for stars with fluxes well above the sky brightness. These six High Excess Tier sources, however, are only weakly above the sky background and we were unable to reproduce the reported WISE magnitudes, finding instead that these sources tend to be fainter than reported, further undermining the significance of these excesses.

As an example, we present in detail the measurements for KOI 2130, which has the most significant W3 excess shown in Table 5.1. Figure 5.7 shows the WISE W1–4 images centered on KOI 2130. We were unable to match the WISE-reported magnitudes after performing standard aperture photometry using the software package Image Reduction and Analysis Facility (IRAF) in the W3 and W4 image centered on the coordinates of the star in the W2 frame. In fact, the counts per pixel in a 12 pixel radius aperture around the star in the W4 band are lower than the counts per pixel in the 36–51 pixel sky annulus, thus no magnitude can be measured. By using slightly different sky apertures, one can measure a positive flux above the sky value. This implies that in the W4 band, one is not measuring the source at all, but rather a fluctuation above the average sky brightness that happens to be roughly centered on the position of the star, and was detected using the WISE automated photometry pipeline.

Though we were unable to reproduce the photometry for this source, we were, however, able to match the reported magnitudes in both the W3 and W4 bands for
Figure 5.7: 10′-wide WISE images centered on KOI 2130 (marked with green circle): W1 in top left panel, W2 in top right, W3 in bottom left, and W4 in bottom right. In the W4 image, blotchy structure is visible on scales similar to the size of the photometric aperture.
three brighter stars that are easily visible in the W4 image (lower right panel of Figure 5.7).

For KOI 2130, \textit{WISE} reports a W3 magnitude of 10.918 and a W4 magnitude of 8.583. In our re-analysis, we obtain a W3 magnitude of 12.11 (∼1.2 magnitudes fainter) and were unable to measure a W4 magnitude. The reported \textit{WISE} W3 magnitude converts to a W3 flux of 1.36 mJy and an excess of 10.0 in our initial analysis. However, the revised W3 magnitude we measure converts to a W3 flux of only 0.45 mJy, which with the calculated error bars results in an insignificant excess of 1.7. We conclude that although the \textit{WISE} team claims that magnitudes they report with SNR as low as 3 are valid, in reality it appears that low SNR values may not correspond to real point sources.

### 5.4.3 Calculated Dust Temperatures

It is initially puzzling that although the evidence points to these excesses being due to galactic cirrus, the best-fitting temperatures (based on the W3 and W4 flux only) are consistently near ∼150 K, higher than the ∼20 K expected for galactic cirrus (Schnee et al., 2005), and also higher than the ∼18–19 K temperature toward each of these stars that is reported in the Galactic dust temperature map of Schlegel et al. (1998). In fact, our fits all nominally exclude 20 K at >95% confidence.

Because we are only fitting for temperature using two datapoints, we are essentially using the ratio of the W3 to W4 flux as a proxy for temperature. We can calculate this ratio for a wide range of possible temperatures by convolving the W3 and W4 passbands with blackbodies of differing temperatures, and calculating the flux ratio for each. The result is shown in Figure 5.8.

We measure the W3 and W4 fluxes both for sources and for the background in areas with no sources across different parts of the \textit{Kepler} field, and find it to be very consistently 0.16, implying a blackbody temperature of ∼150 K. However, in reality, emission from the dust that makes up the galactic cirrus is simply poorly represented by a blackbody: emission features are present in the spectrum of the
galactic dust, adding flux in the wide W3 band and inflating the temperatures we calculate. Emission features at \( \sim 10 \) microns due to silicates are well known (e.g. Beichman et al., 2006b; Lisse et al., 2009; Rhee et al., 2008; Telesco and Knacke, 1991; Wooden et al., 1999), as are emission features due to Polycyclic Aromatic Hydrocarbons (PAHs) (e.g. Robitaille et al., 2012; Vogt et al., 2012). A model spectrum of the Milky Way Galaxy from Robitaille et al. (2012) is shown in Figure 5.9. It is obvious that in the \( \sim 10–20 \) \( \mu \text{m} \) range the emission is not at all similar to a blackbody, so the blackbody temperatures fit in the strawman circumstellar dust models are meaningless for this Galactic dust.
Figure 5.9: A model spectrum of the Milky Way Galaxy showing extensive emission features from PAHs in the 10–20 µm wavelength range. The WISE W3 and W4 approximate bandpasses are shown as yellow overlays. Different components of the spectrum are shown by different colour lines: yellow line is due to blackbody emission by “large” (>0.02 µm) dust grains, orange line is due to emission by very small (<0.02 µm) dust grains, red line shows emission from PAHs, and other colours show emission from stellar populations. The black line above the grey shaded region shows the total spectrum. The model clearly shows that blackbody dust is a very poor fit in the W3–W4 wavelength range. For comparison, the yellow line traces a roughly blackbody curve for large dust grains heated to ~20 K; it is obvious that in the 10–20 µm range this dust is overwhelmed by much brighter emission from PAHs, resulting in a W3/W4 ratio that is much higher than would be measured for a blackbody spectrum. Figure modified from Robitaille et al. (2012).
5.5 Discussion

We conclude from the above analysis that none of the excesses presented here are due to circumstellar dust. In the following sections we review results from surveys similar to ours and discuss prospects for future observations of these systems.

5.5.1 Comparison with Other Surveys

Due to the pointing strategy of WISE, where different parts of the sky have different depths of coverage, it is difficult to assess the statistics of detection. And because of the significant cirrus contamination, we are unable to evaluate the statistics of detection for the sample of Kepler systems surveyed here.

We would naively expect the fraction of these Kepler planet-hosting stars with excesses to be similar to the fraction of systems with “hot” (∼300 K) excesses found by Spitzer surveys, ∼1–2% of solar-type stars at ∼10μm (Beichman et al., 2006b; Lawler et al., 2009). A similar fraction is found by a WISE survey of nearby planet-hosting stars: 2–4% with excess (Krivov et al., 2011), and another WISE survey of nearby solar-type stars has found 1% with excess (Morales et al., 2012). Thus, ∼10 debris disk systems out of a sample of 1237 would not be unexpected.

The mass of circumstellar dust we calculate would need to be present in these Kepler systems to produce the observed excesses ranges between that of our solar system at the low end, to the average debris disk systems found using Spitzer on the high end (left panel of Figure 5.10; Hillenbrand et al., 2008; Lawler et al., 2009; Wyatt et al., 2007). However, the fractional luminosities $L_{dust}/L_*$ the excess fluxes would demand around the Kepler stars are generally 1–2 orders of magnitude higher (right panel of Figure 5.10). This is another clue that most or all of these excesses are due to background contamination and not to circumstellar dust.
Figure 5.10: The derived strawman masses, $L_{\text{dust}}/L_*$, and temperatures of dust rings in the putative sample (using 1 μm modified-blackbody grains) compared to other known systems. Blue symbols show the dust models presented in this paper as blue filled circles for High Excess Tier systems, empty circles for Medium Excess Tier, blue filled triangles for upper limits for Low Excess Tier systems, and empty triangles for approximate values for Short Wavelength Group systems. Red filled diamonds show massive dust systems examined by Wyatt et al. (2007); from left to right: η Cor (Lisse et al., 2012b), HD 69830 (Beichman et al., 2005a), HD 72905 (Beichman et al., 2006b). Empty diamonds show fairly young (≥100 Myr), massive dust systems; from left to right: HD 92945 (Golimowski et al., 2011), Fomalhaut (Boley et al., 2012), HD 107146 (Ardila et al., 2004), and BD +20 307 (Song et al., 2005; Weinberger et al., 2011), while green filled squares are from a Spitzer spectroscopic survey of solar-type stars (Lawler et al., 2009) and empty squares are from a Spitzer photometric survey of solar-type stars at 70 μm (Hillenbrand et al., 2008). For comparison, the purple star shows zodiacal dust in our solar system (Hahn et al., 2002). While the derived nominal masses would be very similar to known systems, the fractional luminosities are suspiciously higher than almost all known systems (with the exception of BD +20 307), making background contamination a more likely hypothesis than circumstellar dust.
If dust was the source of the IR flux in one of these systems, the dust would be located exterior to the known planets. This is unsurprising, as all of the planets announced by Kepler thus far are on orbits with periods of less than one year, which leaves little space between the star and planet, where complete collisional destruction of an asteroid belt would require orders of magnitude less time than the star’s age (e.g. Moldovan et al., 2010).

### 5.5.2 Future Observations

From the analysis presented here, we agree with the conclusion of Kennedy and Wyatt (2012) that the vast majority (likely all) of these excesses are due to contamination by Galactic dust.

Finding excesses due to circumstellar dust in Kepler systems will be difficult due to the high level of background contamination, and will almost certainly have to await the launch of a future infrared space telescope with better resolution. Even disks similar to the brightest candidate disks presented here would have predicted fluxes just below the limits of the Herschel Space Observatory, (approximately 4 mJy at 70 μm; Poglitsch et al., 2010) and with the typical distances of Kepler targets (hundreds to thousands of parsecs) these disks at 1–6 AU would be smaller than the (0.01”) resolution limit of the Atacama Large Millimeter Array (ALMA) in its highest-resolution baseline. However, should these systems have an exterior debris disk as well (a Kuiper Belt rather than asteroid belt analogue), then these systems could be quite bright at longer wavelengths where the colder disk’s emission would peak. The James Webb Space Telescope (JWST) (Clampin, 2011) is expected to have 0.1” resolution at these wavelengths, and superb sensitivity of about 1 μJy, which will help distinguish any possible debris disks in the Kepler field from the galactic cirrus background.
Chapter 6

Future Work

There is much that remains to be understood about the small bodies in our Solar System and other solar systems, from both a theoretical and observational perspective.

6.1 Solar System

Upcoming automated surveys such as LSST (LSST Science Collaboration et al., 2009) and expansions to Pan-STARRS (Grav et al., 2011) will find thousands of TNOs. LSST in particular is predicted to find about 20,000 TNOs (Jones et al., 2009). Because of the enormous coverage and rapid repeat time, tracking these objects and acquiring their orbits will be straightforward to do in a consistent manner. Of these TNOs, many will be in mean-motion resonances with Neptune: using the population estimates from Petit et al. (2011) and Chapter 2, roughly 30% of the discovered TNOs with diameters larger than 100 km will be resonant.

Chapter 2 constrained the absolute populations of many resonances, but only to “factor of a few” precision. More detections will lead to better constraints on the population and orbital element distribution within these resonances, which in turn gives information on the migration of Neptune that sculpted these resonances.
Careful modelling and debiasing of the LSST and Pan-STARRS resonant detections will allow the absolute populations within the resonances to be measured more precisely than was possible with the relatively small number of CFEPS detections. The ratio of the populations of different resonances gives clues about the giant planet migration history of the Solar System. The CFEPS observations require a 3:2/2:1/5:2 population ratio of $\sim 4/1/4$, which is not matched by any giant planet migration simulations in the literature, which all produce a different 3:2/2:1 ratio by a factor of a few (Chiang and Jordan, 2002; Hahn and Malhotra, 2005; Levison et al., 2008a) and produce many more objects in the 2:1 resonance than in the 5:2 (Chiang et al., 2003; Hahn and Malhotra, 2005; Levison et al., 2008a). The larger-$a$ resonances with very few detection (the 3:1 and 5:1 each have only one CFEPS detection) have very large error bars on their absolute populations, mainly due to the unconstrained $e$ distribution. Currently, the error bars on the 5:1 population allow it to be the most populated of all the mean-motion resonances, which would be interesting because most migration models do not predict this. More detections will bring down the error bars on the distribution and population considerably.

LSST will provide deeper images (limiting magnitude of $g = 25.0$; Jones et al., 2009) than CFEPS (limiting magnitude of $g = 23.5–24.4$; Petit et al., 2011). This will help to better constrain the size distribution of objects. It is already known that the slope of the logarithmic $H$ magnitude distribution ($\alpha$) changes for different sized TNOs (e.g. Fraser and Kavelaars, 2009; Fuentes and Holman, 2008), but recently it has been suggested that there is a “divot” in the size distribution (Shankman et al., 2013). For objects larger than 100 km in diameter, the original planetesimal size distribution at formation may still be preserved. With current CFEPS detections, we are completely insensitive to this, even for plutinos at perihelion, and were only able to measure $\alpha$ for a very narrow $H$ magnitude range ($7 \lesssim H \lesssim 9$).

All of the resonances will greatly benefit from additional confirmed members, particularly those where the structure is currently mostly based on theoretical pre-
dictions without much constraint from observations (the \( n:1 \) resonances) or uses a distribution taken from measurements of a different resonance (the 7:3 and 5:4). Only the plutinos had enough CFEPS detections to run a grid of models with different \( e_c, e_w, \) and \( \alpha \) and formally reject some parameter space. In order to perform this detailed analysis and obtain meaningful constraints, approximately 20 detections would be needed in each resonance, which LSST may be able to deliver. In particular, confirming the existence of a changing slope \( \alpha \) would be important because it implies that there are different collisional or formation populations in the different resonances. The inclination distribution widths are fairly well constrained, but more detections would bring down the error bars in all of the resonances. The 7:3, 5:4, 3:1, and 5:1 distributions in particular have only an assumed value because of only having one or two CFEPS detections, and additional detections would provide a better estimate of the inclination distribution, affecting the population estimates as well.

The \( n:1 \) resonances have different libration islands, which adds additional distributions relating \( \phi_{n1} \) to \( e \) that are currently based on theoretical predictions and is not independently constrained by observations. The relative number of objects in the different resonant islands within each \( n:1 \) resonance may be related to giant planet migration. Chiang and Jordan (2002) predict that there should be more twotinos in the trailing than the leading asymmetric island, and this contrast increases with faster outward migration of Neptune. There are currently not enough known twotinos from CFEPS (Section 2.9.1) or in the MPC database (Lykawka and Mukai, 2007) to be able to measure the fraction in the different asymmetric islands to any reasonable degree of precision, given the detection biases.

More detections will also provide Kozai resonators within mean-motion resonances besides the 3:2, which possessed the only Kozai resonators found by CFEPS. The Kozai resonance is predicted to be non-negligible within other mean-motion resonances (e.g. Nesvorný and Roig, 2001), and Kozai resonators have been measured within the 5:3, 7:4, and 2:1 resonances (Lykawka and Mukai,
As found for the 2:1 asymmetric fraction and predicted for the plutino Kozai fraction, it may be that the Kozai fraction within other resonances is another constraint on Neptune’s migration speed and/or eccentricity damping timescale.

Because of their relative proximity, many plutinos will be detected in future surveys, some of which will be in Kozai. The all-sky \( f_{\text{koz}}^{\text{obs}} \) maps presented in Chapter 3 can be adapted to debiased observations to find the true Kozai fraction \( f_{\text{koz}}^{\text{true}} \). In addition to the usual orbital parameter distributions, the Kozai libration amplitude \( A_\omega \) and Hamiltonian level surface (parameterized by \( I_{\text{max}} \)) must also be determined for the discovered objects. This requires high-quality orbits. It is completely unclear from current observations if there are any correlations between the Kozai parameters and any other orbital parameters, and there are no theoretical studies in the literature that address this.

A new generation of giant planet migration simulations need to be performed, as many questions raised in Chapters 2 and 3 can only be answered by detailed dynamical evolution simulations. These simulations need to track many test particles, and carefully classify the survivors at the end of the simulation into resonances, paying particular attention to the Kozai resonance within the mean-motion resonances and to different libration islands within the \( n:1 \) resonances. There needs to be enough test particles in these simulations that the distributions of different orbital parameters \( (a, e, i, L_{jk}, \phi_{n1} \) for the \( n:1 \) resonances, and \( A_\omega \) and \( I_{\text{max}} \) for Kozai librators) can be easily measured and compared with debiased observations. One simulation is not enough; many simulations need to be run with different parameters such as how quickly Neptune migrates outwards, how quickly Neptune’s eccentricity damps (if using a Nice Model-style simulation), and whether the test particles are dynamically cold, hot, or some combination, to name a few. These simulations will take many years of core-time to compute, but computer clusters are getting faster and new techniques for parallel computing and new technologies such as using graphics processing units (GPUs) should bring this within reach. Future simulations should address some of the questions raised in Chapter 3, such as whether any of the Kozai parameters are correlated...
with each other, and if the Kozai fraction is indicative of some cosmogonic parameter.

### 6.2 Exoplanets and Debris Disks

Much theoretical work remains in the area of debris disks. The dust production mechanisms are still not well understood: Collisional cascades can’t explain the brightest, warmest debris disks that are observed, and it is difficult to imagine transient phenomena that both release so much dust and are fairly common (affecting \( \sim 1\% \) of Sun-like stars).

Several recent studies examined the correlation between the presence of debris disks and exoplanets, and have generally concluded that the fraction of exoplanet-hosting stars with debris disks is no different than the fraction of random field stars with debris disks. (e.g. Bryden et al., 2009; Dodson-Robinson et al., 2011; Morales et al., 2012). Krivov et al. (2011) specifically examined transiting exoplanet host stars for debris disks, and also concluded that the fraction with disks was no higher. These studies were all carried out using exoplanets detected using the radial velocity technique, which are biased toward the largest mass, shortest period planets. The planets discovered by Kepler, while still biased toward short periods simply by virtue of the length of the mission, include many much smaller planets: super-Earths and Neptunes. Raymond et al. (2012) predicts that systems that have hot Jupiters, which are generally believed to have migrated from larger semimajor axes, will be “cleaned out” with no small planets or debris disks within \( \sim 1 \) AU, hot Jupiter-hosting systems may be in a different regime than stars that only host small planets. Systems with small planets, due to their long-term stability, may be the best place to look for debris disks.

While Chapter 5 concludes that warm disks around Kepler exoplanet-hosting stars are out of reach for current infrared telescopes, future missions may be able to provide the high-resolution infrared observations needed to disentangle circumstellar excesses from the background galactic cirrus. Other RV and transit surveys may start to find a significant number of multiplanet systems like Kepler has found
around closer, brighter stars. These will be more easily searched for debris disks using archival data from WISE and from other infrared telescopes.

Resolved disk studies and directly imaged planets are becoming increasingly feasible with new telescopes and instruments. It is important to directly image debris disks because just fitting a dust grain model to the SED of a debris disk system has many degeneracies depending on assumptions about the properties of the dust grains. For example, small (∼1 μm) grains absorb and emit photons at far infrared wavelengths inefficiently, so a dust ring made of these small grains would need to be located significantly closer to the host star than larger (∼10 μm) dust grains to produce a similar SED. By directly imaging the disk, one can better constrain the size and emissivity of the dust grains.

As of early 2013, ALMA has already provided some beautiful sub-arcsecond resolution images of two of the closest debris disks: Fomalhaut (Boley et al., 2012) and AU Mic (MacGregor et al., 2013). As more of the antennas come online in the coming months, the resolution and sensitivity will improve. The biggest limitation of ALMA is that in its highest sensitivity configuration it does not have its highest spatial resolution, and vice-versa, requiring creative observing strategies to image fainter, smaller disks (Ertel et al., 2012). Nonetheless, because it is an interferometer, ALMA has incredible resolution of 0.01" in its highest resolution configuration, which for bright disks, would allow direct imaging of disks larger than 10 AU in diameter within 10 pc.

Coronography has also vastly improved in recent years, both in the instrumental setup and in the data reduction process. Several systems with debris disks and exoplanets have been directly imaged using this technique, including HR 8799 (using the Hubble Space Telescope; Marois et al., 2008, 2010), Fomalhaut (using Keck Observatory; Kalas et al., 2008), and β Pic (using the VLT; Lagrange et al., 2010). Adaptive optics systems on some of the largest telescopes in the world are further improving the power of coronography to image debris disks at very small separations from their hosts stars (i.e. the Subaru Telescope and Keck Observatory; Currie et al., 2012a,b).
Within the next few years, several extremely large telescopes are planned. The Giant Magellan Telescope (GMT) will be located at Las Campanas Observatory in Chile, planned to be completed in 202. It will possess a 24.5 meter diameter mirror made up of seven 8.4 meter diameter segments, the first of which have been built already (Martin et al., 2012). The instruments are still being selected (Jacoby et al., 2012). The Thirty Meter Telescope (TMT), as the name implies, will have a thirty meter diameter mirror, which will provide incredible sensitivity because of the huge collecting area, as well as incredible resolving power (Silva, 2008). TMT will be located on Mauna Kea in Hawaii, with a planned first light in 2018. It is planned to have instrumental capabilities from the ultraviolet to the mid-infrared, including laser guide star adaptive optics. There is also a planned high-contrast imager that is specifically designed to directly image Earth-size planets around K and M stars (Simard et al., 2012). Even further in the future, the European Extremely Large Telescope (E-ELT) is still in the early planning stages, with a planned 39 meter diameter mirror which will be located on Cerro Armazones in Chile. Prototype mirror segments and instrumentation mockups are currently being tested, and a diffraction-limited imager currently in the design phase (McPherson et al., 2012). Due to their large collecting areas, all three of these telescopes will have incredible resolving power and detection limits, and adaptive optics systems will help mitigate the effects of the atmosphere. However, the atmosphere is quite opaque at mid- and far-infrared wavelengths, so to observe at these wavelengths, a telescope must be placed above the atmosphere.

The JWST (Clampin, 2011) is planned as a mid-infrared imager with a powerful coronograph setup. With its huge, 6.5 meter diameter mirror and space-based platform, it will have fantastic resolution and sensitivity. After its planned launch in 2018, it should be able to directly image debris disks within several tens of parsecs (Ertel et al., 2012), greatly expanding the number of debris disk systems able to be directly imaged. It will also be useful for more distant systems, like the Kepler systems examined in this thesis, because of its high spatial resolution compared to WISE. The background contamination problems discussed in Chap-
ter 5 will be lessened, because the higher resolution will allow photometry with sky apertures closer to the target star, decreasing the chance that the sky annulus will be lower than the background immediately around the star, and allowing more accurate photometry.

The *Kepler* systems which host multiple super-Earths are theoretically the best known candidates to host debris disks. This is both because there are several Earth masses of rocky material in the inner portions of these solar systems, and because multiple, tightly packed super-Earth systems are those which did not undergo potentially catastrophic giant planet migration (Raymond et al., 2012). The systems are tightly packed (Fang and Margot, 2013), but not so tightly that planetesimal belts between the planets are dynamically ruled out. The spatial resolution required to test this hypothesis should be available within the next decade.
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