COLLABORATIVE ASSESSMENT IN MIDDLE SCHOOL MATHEMATICS

by

CATHERINE ANNA LOUISE DICK

M.A., Michigan State University, 2005
B.Ed., Queen's University, 1999
B.A., Wilfrid Laurier University, 1998

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Abstract

This study examined the mathematical learning that grade 8 students demonstrated when they were given the opportunity to work collaboratively, with a teacher-assigned partner, on an in-class assessment. In addition to topic-specific concepts, skills, and procedures, mathematical learning also included more general abilities such as selecting strategies, developing plans, communicating ideas, and evaluating solutions. The primary sources of data for this study were the conversations and written papers of four “equal status” dyads as they worked on a problem-solving assessment in which they were encouraged to discuss their ideas and submit a joint solution. Analysis indicated that most dyads worked collaboratively throughout the task and that both students were relatively equal contributors to the joint solution. Therefore, while collaborative assessment reduced the ability to hold individual students accountable for what they had learned, it appeared to be an accurate reflection of most students’ mathematical knowledge and ability. One dyad, however, remained committed to working independently; the partners rarely discussed their ideas with each other and both students created their own solutions.

During their discussions, students who collaborated were more likely to discuss various calculations related to the problem, rather than discuss potential strategies or solutions. Students interacted comfortably and informally with each other and asked questions if they did not understand, but did not often critically challenge their partner’s suggestions or provide justification for their own ideas. As a result, students did not always make reasoned choices when approaching the problem or evaluate the appropriateness of their strategy or solution.

Preface

This research was approved by the Behavioural Research Ethics Board at the University of British Columbia. The ethics certificate, number H09-02843, is included in Appendix G.

Table of Contents

List of Figures

Acknowledgements

there are times when words do not suffice,
when ink upon the page does not convey
the meaning or sincerity of what i wish to say.

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1 Introduction

Within the purview of mathematics education, cooperative learning is an increasingly common classroom practice. Presented as a way to help students construct mathematical knowledge, participate more fully in the discipline, and provide opportunities to interact with others, it is a required component of many curricula (National Mathematics Advisory Panel, 2008; Qualifications and Curriculum Authority, 2007) and often encouraged as a recommended teaching practice (Ontario Ministry of Education, 2004). Cooperative learning is a broad category, to which a multitude of classroom practices can be assigned. From the five minute think-pair-share tasks designed to frontload learning to the well-known long-term group project in which emphasis is placed on students working together towards the creation of an end product, cooperative learning forefronts the social interactive nature of knowledge acquisition and of our society.

Collaborative learning, loosely defined as a partnership or group of people working together to accomplish a task or to increase understanding (Schmitz & Winskel, 2008), is often used synonymously with cooperative learning. However, I distinguish between the two and consider collaborative learning a smaller sub-category of cooperative learning. In this paper, I use the latter term to refer to any situation where students work with their peers, including activities such as 'jigsaw', where students maintain individual responsibilities within the group[1]. I reserve the term collaborative for circumstances in which the students work together throughout the entire task and are not held individually accountable for specific aspects (Damon & Phelps, 1989). It is collaborative learning, with its potential for the symbiotic development of new understandings, that most interests me and that I explore with this research.

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[1] When referring to the research of others, I use the term they most commonly employed. So, if a researcher termed their work cooperative, I referred to it as such, even if it fit my stricter definition of collaborative.

1.1 Rationale

Collaborative activities are most commonly used to help students develop specific skills or to provide students the opportunity to work with others. As such, there are many claims espousing collaboration's potential affective benefits including increases in student self-esteem and tolerance of others, and improvements in prosocial behaviours, attitude towards school, and general classroom environment (Slavin, 1991).

Though comparatively fewer studies examine the nature of discipline-specific learning afforded by a collaborative situation, this field of research is growing. At the secondary level, Goos and Galbraith (Goos & Galbraith, 1996; Goos, Galbraith, & Renshaw, 2002) have investigated the metacognitive strategies students employ when problem solving in groups, while Mercer (Mercer, 1996; Mercer & Sams, 2006) has investigated the quality of talk elementary students use when working together in a variety of subjects, including mathematics.

However, research on collaborative assessment, with an emphasis on ascertaining what students have learned and understood in relation to what they have been taught, is rare. Some research exists on collaborative assessment with post-secondary students (Berry & Nyman, 2002; Klecker, 2000; Lambiotte, Dansereau, Rocklin, & Fletcher, 1987; Zimbardo, Butler, & Wolfe, 2003), where its benefits and appropriateness were measured using summative test scores, researcher observations, and student self-reflection surveys. However, in-depth analyses of the learning that occurred during the assessments do not appear to exist. At the university level, collaborative assessment seems to benefit students in a variety of ways including increases in confidence, motivation, communication, and test scores (Berry & Nyman, 2002; Hancock, 2007), but some studies with elementary and middle school students question the effectiveness and benefits of collaborative work (Mercer, 2008a; Stacey, 1992; Webb, 1995).

Despite the potential benefits of collaboration, many of which I have experienced first-hand, I remain skeptical regarding its use in the classroom. From my personal perspective as a middle school teacher and as a collaborator in various situations, I am aware that effective collaboration is a delicate balance between attaining group and personal goals, between compromising and retaining individual beliefs, and between challenging and accepting the ideas of others. Tensions inevitably arise and can be difficult to navigate, especially for young teenagers.

With group projects, students (and parents) frequently want assurances that the workload will be divided equally, that a student's grade will not be affected negatively by lack of effort on the part of their group members, and that non-contributing students will not receive credit for work they did not do. As a teacher, I sympathise with these points, but am frustrated that they are driven by grade-focussed intentions. Collaborative assessment that takes place entirely within class time and under the supervision of the teacher seems likely to mitigate at least some of these concerns, while still affording students the benefits of working with their peers.

In addition, although I am comfortable with a classroom in which students talk, move about, and generally experience a fair amount of freedom, I am aware of the number of off-task conversations I have become accustomed to hearing. Juxtaposed against ever-present public cries for longer school days and a longer school year, and in light of the conflicting claims surrounding collaborative learning, I wonder about both the quality and quantity of learning that occurs when students work together. Yet, I also recognize that not all 'off-task' behaviours are disadvantageous, accept that few single classroom practices are either wholly positive or wholly negative, and readily admit that when collaboration 'works' the benefits to students and their learning are significant.

1.2 Purpose

As a teacher of collaborating students, I move from group to group, responding to questions and addressing concerns, but rarely have the opportunity to step back and observe the process as a whole. As a researcher, I have read widely on collaboration, assessment, problem solving, mathematics, and middle school students, but have not encountered a study that integrates all five components.

In this research, I intend to explore collaborative assessment in which individual accountability is not retained. Adhering to the belief that collaboration should enable students to 'do more' or 'do differently' than they could alone, I examine the mathematical learning that middle school students demonstrated when they had the opportunity to collaborate with a teacher-assigned partner on an in-class assessment. Similar to Damon and Phelps (1989), I am interested in exploring the mathematical learning that was demonstrated when equally (or similarly) matched students worked together on challenging problems. As such, the assessment on which students collaborated was a problem-solving type investigation, rather than a traditional mathematics test and mathematical learning included more general mathematical skills such as selecting strategies, developing plans, communicating ideas, and evaluating solutions in addition to the depth and breadth of concept-specific skills and procedures.

1.3 Research Question

What mathematical learning is demonstrated, verbally and on paper, during a mathematics assessment in which middle school students are given the opportunity to collaborate with a teacher assigned partner?

1.4 Significance

For the last twenty years, trends in education have reflected a move away from more traditional exercise-book questions and teacher-centred lecture methods towards an increased emphasis on student-centred problem solving (Goos, Galbraith, & Renshaw, 2002) and collaboration (Carter, Jones, & Rua, 2002). Although assessment philosophies are also changing in response and it is now commonly accepted that assessment serves multiple purposes (i.e. assessment of learning, assessment for learning, and assessment as learning), often simultaneously, people in many school communities are reluctant to use collaborative work as a summative assessment tool since they view collaboration as a form of cheating (Berry & Nyman, 2002) and believe that only independently-completed tasks accurately reflect what a student knows (Jensen, Moore, & Hatch, 2002) and can do. Consequently, many assessments remain strikingly similar to those of the past, which can be frustrating to teachers who are implementing more collaborative learning activities in their classroom (Suurtamm, 2004).

I feel that alternative assessment practices need to be explored in order to increase alignment between teaching and assessment and to enhance post-assessment student learning. Collaborative tasks present a reasonable approach for meeting both of these goals (Suurtamm, 2004; Webb, 1997). In addition, since there is a tendency to assess what is valued and, conversely, to value what is assessed, collaborative assessments have the potential to broaden the realm of mathematical learning considered important. By providing information on the mathematical learning students demonstrate during collaborative assessment, results from this research will help teachers better understand and justify what role, if any, collaborative assessment could play in their middle school classrooms

This study contributes to the current body of research on collaborative learning by specifically examining collaborative assessment in the middle school mathematics classroom. Although current research on collaborative assessment seems concentrated at the university level, I endeavour to investigate whether its use is also beneficial for younger students. Given the significant differences in maturity, schooling, life experiences, motivation and other factors between university and middle school students, it is possible that the two groups will respond differently to collaborative assessment. In addition, since much of the cooperative learning research at the elementary and high school level focuses on the social benefits of cooperation or on the controversial effects for high-ability learners, this study contributes to our understanding of the mathematical learning that is demonstrated during equal-status collaborative situations. Finally, this study looks in more detail at the types of collaboration and learning that are demonstrated during assessment, rather than focussing on final assessment scores.

2 Literature Review

2.1 Assessing Mathematical Learning

It is a premise of this study, and commonly accepted by many researchers, educators, parents, and students, that the main goal of assessment is to acquire information about students' learning that is valid, reliable, and meaningful (Klecker, 2000). Yet gathering information is only the first step in the assessment process and subsequent steps require the interpretation of the acquired information, as well as further action (Wiliam & Black, 1996). Commonly, many formal assessments, including the task in this research, are considered summative. Their primary intent is to determine what a student knows, particularly with respect to pre-specified outcomes, and further action includes assigning a mark or a grade. While this (questionably) provides useful information to teachers, parents, schools, and districts, such grade-oriented assessment has been shown to have limited impact on the improvement of student learning (Black, Harrison, Lee, Marshall, & Wiliam, 2004).

Conversely, formative assessment, also known as assessment for learning (AfL), stresses the importance of useful feedback, student engagement, and effortful improvement and has been shown to measurably improve student performance (Black, Harrison, Lee, Marshall, & Wiliam, 2004; Black & Wiliam, 2009; Wiliam & Black, 1996; Wiliam, Lee, Harrison, & Black, 2004). Although they are often perceived as separate and independent practices, it is possible for summative and formative assessment to coexist (Black, Harrison, Lee, Marshall, & Wiliam, 2004).

One identified component of AfL that is particularly relevant to this study is the use of peer assessment (Black, Harrison, Lee, Marshall, & Wiliam, 2004). Peer assessment is not customarily considered a component of collaborative assessment. However, it is implicit in the collaboration process since students' ideas and suggestions are continually open to exposure and judgment from the peer(s) with whom they are working, which may provide immediate feedback (to the individual and to the group) throughout the task.

Before information on student learning can be analysed or used for any purpose, though, it must first be collected. This seemingly straightforward task of determining what a student knows about the subject is complex and deeply influenced by one's beliefs regarding what it means to know, to learn, and to understand. In many classrooms, learning is conceived "as the individual mastering a predetermined body of knowledge and procedures" (Goos, Galbraith, & Renshaw, 1999, p. 37) and assessment is seen as a measure of "individual competence of students in their thinking skills and subject-matter knowledge and expertise" (Webb, 1995, p. 240).

However, from a social constructivist viewpoint, the learning that students are on the cusp of demonstrating independently is also valued. Therefore, what students can do with others, in addition to what they can do alone, is relevant. Consequently, from within this framework, educators "may view the lack of collaboration as a more serious defect than its inclusion" (Wineburg, 1997, p. 64) since the scaffolding provided by peers is perceived not as cheating, but as providing students the opportunity to show their highest capabilities.

Mathematics educators recognise that students are not simply receptacles of knowledge, but are active creators of meaning who are affected by their personal identities and their immediate and wider environments. For example, students who believe the work they are undertaking is meaningful and who feel confident and in control of their learning are more likely to be motivated by a desire to learn and understand the topics being presented (Seifert & O'Keefe, 2001), which means they may be more likely to persist with difficult tasks and to explore various mathematical approaches and connections without specific direction from the teacher. Particularly relevant to the middle school population is the observation that this type of motivation begins declining in grade 7 (Chouinard & Roy, 2008).

In addition, since learning is always contextualised, the learning a student demonstrates may vary with the environment in which it is requested. For example, people are frequently able to perform mathematical calculations in one situation that they are unable to perform in a different setting (Kieran, Foreman, & Sfard, 2001; Lave, Murtaugh, & de la Rocha, 1984; Nasir, Hand, & Taylor, 2008; Wineburg, 1997). As learning is not a simple matter of skill acquisition, but a more complex process that is affected by myriad factors, it is difficult to confidently assert when a student has achieved a specific learning goal.

Accepting that assessment is more complicated than simply having students 'show what they know' acknowledges that different assessments will lead to the demonstration of different types of knowledge. For example, a multiple choice test usually enables students to demonstrate factual recall, while a problem solving situation offers greater potential for students to demonstrate their ability to select, apply, and possibly evaluate their approach(es) to a problem. Therefore, the type of assessment and the environment in which it is performed are likely to co-influence the mathematical learning students demonstrate. In this study, I select a somewhat uncommon type of assessment (a single in-depth problem) and environment in which it is performed (with a partner), and examine the demonstrations of mathematical learning that ensue.

Conceptual understanding, identified by the United States' National Research Council (NRC) as one of the five components of mathematical proficiency, is defined as the "comprehension of mathematical concepts, operations, and relations" (Kilpatrick, Swafford, & Findell, 2001, p. 116). Students who possess a conceptual understanding of mathematics appreciate and draw upon the rich connections between various concepts. They view mathematics, not as a discrete collection of facts and algorithms, but as an interconnected web of ideas and practices. Conceptual understanding must be developed within, rather than handed to, the student. According to the NRC, it is a necessary, though not sufficient, component of successful mathematics learning.

To me, conceptual understanding, as defined by the NRC, appears reminiscent of Skemp's (1976) term relational understanding, which he used to indicate an understanding where one knows not only what to do, but why it makes sense to do so. He associated the alternative form of understanding, termed instrumental understanding, with rule following behaviours. Though instrumental understanding was, and still is, widely prevalent in mathematics classrooms and textbooks and is beneficial at times, Skemp argued that relational understanding is more conducive to the development of a positive attitude towards mathematics and is more useful in novel situations because of its flexibility.

In a related vein, Thompson, Philipp, Thompson, and Boyd (1994) proposed that students and teachers display either a conceptual orientation towards mathematics or a calculational one. Learners with a conceptual orientation recognise the contextual meaning of numerical values and mathematical procedures and desire to become skilled reasoners, rather than (or in addition to) skilled calculators. They explain their work by elucidating each variable's meaning within the context of the problem and by providing justification for the appropriateness of their chosen strategies. In contrast, learners with a calculational orientation aim primarily to 'get answers'. They focus on identifying and performing procedures and calculations and explain their solutions by describing, rather than justifying, what they have done. Just as Skemp favours the deeper why-oriented relational understanding, Thompson, Philipp, Thompson, and Boyd favour the more in-depth intricate conceptual orientation. As I explore the mathematical learning demonstrated by students in this study, I look for evidence of both procedural/ calculational and conceptual/ relational understandings and orientations. To identify each, I use the characteristics suggested by the NRC, Skemp, and Thompson, Philipp, Thompson, and Boyd.

2.2 Collaboration, Communication, and Understanding

Partnerships, like all forms of collaboration, "encourage students to discuss, debate, disagree, and ultimately to teach one another" (Slavin, 1991, p. 71). They provide students the opportunity to speak mathematically, which provides them the opportunity to think mathematically (Learner, 2001). Artzt and Armour-Thomas (1992) found that students who solve problems collaboratively tend to spontaneously verbalise their ideas, which exposes these ideas to critical examination from peers (Goos, Galbraith, & Renshaw, 2002). As a result, it becomes more likely that students will need to explain and justify their statements. The opportunity to communicate – to hypothesise, to discuss strategies, and to defend and justify their solutions – allows students to become participants, rather than just spectators and is paramount if students are to construct meaning, develop understandings of the relationships implicit in their mathematical knowledge (Steele, 2001), and become fluent in the mathematical discourses and cultures in which they participate. According to Vygotsky,

> students create their own knowledge and develop mathematical meanings as they learn to explain and justify their thinking to others. As they learn to speak the mathematical language, they transform their thinking of the mathematical concepts. The mathematical language comes from society, and thought (concept) comes from the individual. (as cited in Steele, 2001, pp. 404-405)

This view reinforces that meaning cannot simply be transferred from one person to another. Students who focus only on memorising individual definitions, procedures, and algorithms will not develop the internalized understanding required to appreciate the connections between these components and other mathematical experiences (Steele, 2001).

The level of collaboration afforded by a paired assessment situation provides a potential forum for students to think more critically than they would if working individually. The increased opportunity to grapple with ideas and prospective solutions may help students to define and communicate their own understandings more clearly. It is difficult to ascertain, however, the prerequisite precursors to successful collaboration. In general, studies on student collaboration focus on classrooms in which collaboration is valued and students are taught (explicitly or implicitly) to respect and value peer input. Though some researchers, including Webb (1997) and Lambiotte, Dansereau, Rocklin, and Fletcher (1987), recommend the direct teaching of collaboration skills, the specific levels and types of training and experience required for students to participate in effective collaboration are not clear.

Mercer (Mercer, 2008b; Mercer & Sams, 2006) argues that effective collaboration is a result of how successfully students critically examine their ideas during their quest for group consensus. His conclusions stem from large-scale in-depth studies with primary school children in England, including the SLANT (Spoken Language and New Technology) project and the Language, Thinking and ICT in the Primary Curriculum project. His earlier work, connected with SLANT (Mercer, 1996), focused on understanding how the dialogues between 9 and 10 year old students were affected by the teacher and by specifically selected computer-based activities. His later work focused on the effects of teacher intervention programmes designed to enhance student communication and reasoning. Reporting on video data gathered from one classroom of 10 and 11 year olds, Mercer (2008a) used excerpts from teacher-led lessons and from triads attempting to solve computer-based mathematics problems requiring a single numeric answer. A separate study (Mercer & Sams, 2006) specifically examined the effect of 'Thinking Together', a teacher intervention programme designed to improve Year 5 (9 – 10 year old) students' language, reasoning, and discussion skills, and the effect the resulting improvements would have during group tasks in mathematics. Data consisted of video recordings of teacher-led lessons and small group interactions, interviews with the teachers and students, and results from pre- and post- intervention tests measuring the students' mathematics knowledge and understanding.

As a result of his work, Mercer identifies three types of talk students exemplify when working together: disputational, cumulative, and exploratory. Disputational talk is characterized by the volley of ideas between partners. Instead of sharing ideas and jointly constructing solutions, students tend to offer assertions and counter-assertions and make individualized decisions regarding their work. In cumulative talk, students aim for consensus and joint understanding; they tend to build positively on each other's ideas and suggestions, but do so uncritically without challenging or justifying reasons and opinions. Exploratory talk, which Mercer (1996) claims is "most effective for solving problems through collaborative activity" (p. 370), is typified by its emphasis on all students reaching consensus through the joint construction of suggestions and reasoning that have been made visible and explicit through justification and exposure to questioning.

2.3 Collaborative Assessment in Schools

Providing students with a variety of collaborative strategies, such as exploratory talk, and numerous opportunities to become active participants in, rather than passive recipients of, mathematics, supports multiple assessment practices and purposes. As previously discussed, assessment can be conceived as a measurement of student learning (assessment of learning) or as an inquiry during which knowledge is constructed (assessment for learning). When the primary purpose of an assessment is learning, its validity is based upon how well the assessment promotes further valuable learning, rather than upon how well it measures what students have already learned (Hargreaves, 2007). Owing to the potential for the joint construction of knowledge, collaborative assessment is well-suited as an assessment for learning, which may explain why it is often used for project-based work. However, as investigated in this study, it can also be used as an assessment of learning.

As an assessment of learning, collaborative assessment takes many forms, each with its own degree of individual accountability. In some cases, such as the Connecticut Common Core of Learning Alternative Assessment in Science, students have the opportunity to work collaboratively on part of an activity before completing it individually (Webb, 1995; Webb, Nemer, & Zuniga, 2002). This is often seen in K-12 classrooms where students work with a partner on a science lab or brainstorm ideas for a story before completing the assignment individually. Such activities provide students many of the benefits of working with their peers, but maintain individual accountability. Similarly, in other situations, students have the option of working with a partner or group throughout the entire assessment. However, each student is responsible for submitting their own assessment (Klecker, 2000, 2003). Though individual accountability is maintained in the aforementioned circumstances, the assessments constitute collaborative work since students work together, rather than each taking responsibility for separate components of the task. In other cases, students complete the assessment collaboratively and submit a single assessment (Berry & Nyman, 2002; Hancock, 2007). As there is no option for individual work, students must agree on their responses. Though limited research seems to exist on this apparently uncommon type of collaborative assessment, most of it appears with university students. As the focus of this study, I aim to provide some preliminary findings regarding the use of this type of assessment, with pre-university students.

2.4 High Achieving Students within a Collaborative Setting

Though critics of collaboration may acknowledge that collaborative test scores tend to be higher than individual scores (Webb, 1993; Zimbardo, Butler, & Wolfe, 2003), they claim this occurs because the strongest student in the group carries and supports the weaker student(s). Because stronger students are often distributed amongst the groups, all students appear to do better than if they had been working individually. Webb's findings (1995) that a group's solution to a problem is sometimes no better than the solution provided by the strongest group member working alone further support this claim.

Critics are also concerned that collaboration disadvantages high-achieving students working in heterogeneous groups, a claim supported by Webb, Nemer, and Zungia's (2002) findings that some high-ability students working in high-ability groups performed better than their high ability peers working in mixed-ability groups. Other research, however, indicates that collaboration is equally beneficial for all students (Slavin, 1991), an idea corroborated by the high-achieving undergraduates who felt "they learned a great deal from their interactions with a [lower-achieving] colleague during the [collaborative] examinations" (Hancock, 2007, p. 224).

Though results are inconclusive, it appears that group work may benefit the group as a whole and the majority of its members as individuals, but it does not necessarily improve the performance of the strongest participant(s). In competitive classrooms, where student success is dependent upon the failure of others, this may be an important concern. However, in classrooms where success is considered collectively, as well as individually, this concern becomes less important (Epstein, 2007).

2.5 Zones of Proximal Development in Equal Status Partnerships

From a theoretical perspective, Vygotsky's zone of proximal development (ZPD) can be used to explain why high-achieving students often score higher when they work with other high achieving students, but do not do any better than they would have done had they been working alone or with lower achieving students. Learning occurs in a student's zone of proximal development, an area conceptualised "as a symbolic space involving individuals, their practices and the circumstances of their activity. . . . [Students] can be pulled into their ZPDs by a combination of the activity, the actors, and appropriate communication" (Lerman, 2001, p. 103).

In historical views of the zone of proximal development, the actors consist of an 'expert' who scaffolds learning for a 'novice'. This situation is created when a high-achieving student works with a lower achieving student; the novice's learning improves, but the expert's remains relatively stable. However, two 'equal status' partners can also work together to simultaneously broaden their ZPDs (Goos, Galbraith, & Renshaw, 2002). Equal status seems to refer to partnerships in which the students possess relatively equal levels of expertise and experience, with the implicit assumption that this will create a fairly equal balance of power between the two partners. As Forman describes, equal status partners are in the unique positions of being "able to coordinate their different perspectives on a problem in order to achieve progress" (as cited in, Goos, Galbraith, & Renshaw, 2002, pp. 195-196). Consequently, both partners can contribute differently, but (nearly) equivalently, giving the partnership the potential to demonstrate greater learning than either student could show individually (Goos, Galbraith, & Renshaw, 2002). In other words, when two high-achieving students work together an equal-status partnership develops whereby each student contributes to the other's learning. Hence, the group result is stronger than what either individual could have accomplished alone.

Equal-status relationships may be more likely to yield higher results in open-ended situations such as problem solving tasks because students develop a deep understanding of basic concepts as they experiment with and examine their own ideas and assumptions (Phelps & Damon, 1989). In such situations, Zimbardo, Butler, and Wolfe (2003) found that the accomplishments of the group exceeded the accomplishments of any single individual and collaborative test-taking yielded higher scores than what either student could produce individually. In part, these results may be possible because team members develop an appreciation of each other's strengths (Berry & Nyman, 2002) and can use these strengths to scaffold their own learning. Further, since solutions to open-ended tasks are less likely to depend upon a discrete set of knowledge-facts, the probability that one student tells the correct solution to the other(s) is reduced.

According to Granott (1993), collaboration in equal status partnerships is "characterised by shared activity, a common goal, continuous communication, and co-construction of understanding" (as cited in Goos, Galbraith, & Renshaw, 2002, p. 196). This collaboration seems evident during paired assessment at the university level, as students contributed equally to the test and worked collaboratively to generate solutions (Berry & Nyman, 2002; Ewald, 2005; Hancock, 2007). However, 'equal status' is a tenuous term. Taking Damon and Phelps' (1989) definition of equality as an engagement in which both students take suggestions and ideas from each other, as opposed to a situation where one student guides the other, pairing students on (perceived) mathematical capability does not necessarily lead to an equal status partnership since the role of other factors including students' abilities to work collaboratively, explain their solutions, and argue for answers they believe are correct are ignored. These other factors did not seem to affect collaboration during paired assessment at the university level (Berry & Nyman, 2002; Ewald, 2005; Hancock, 2007), but it is unclear if similar results will be seen with middle school students. Possibly, younger students will collaborate as effectively as older students. However, this cannot be assumed based on work with older students, since, as Mercer's work (discussed earlier) shows, younger students do not always possess the skills to communicate and reason effectively during group work. In addition, as will be discussed in the following section, students may not possess the same maturity levels as older students, which may leave them struggling.

2.6 Concerns Regarding Collaboration

In compulsory education, concerns exist regarding the negative peer influences that may be exerted during collaborative activities. At the elementary level, Webb (1997) is disquieted about the potential diffusion of responsibility that may occur if students allow other group members to take responsibility for completing all the work (which has been termed “social loafing”). In response, the students who have been doing all the work may stop doing so in order to avoid being taken advantage of (which has been termed the “sucker effect”). In either case, the final product is not likely to represent the collaborative effort of the entire group.

In addition, as Stacey (1992) found with 12 – 14 year olds, some groups have a tendency to favour simple solution strategies, whether or not they will lead to a correct answer. Her study was developed in response to data obtained during a larger project in which students in some classes took a problem solving test individually while students in other classes took it with a partner or small group. The test consisted six ‘real world’, non-routine problems which the students had 45 minutes to complete. Surprisingly, group performance was slightly, though not statistically significantly, worse than individual performance, which prompted further investigation. As a result, seven teacher-recommended high-ability triads were videotaped as they worked on problems that were similar to those in the test. Analysis showed that in most groups, at least one correct solution method and one incorrect solution method was proposed during the discussion. However, many proposed ideas were not discussed or acknowledged and some groups opted for simpler solution methods, which led to incorrect answers.

These scenarios may be especially likely if either or both of the partners are motivated by work-avoidance (Seifert & O'Keefe, 2001) and are more focussed on finishing the assignment rather than on understanding and applying the mathematics involved. While these effects were noticeably absent with university students in these studies, middle school students may be more susceptible to peer pressure and impulse decision making, which could render them more prevalent and powerful in a middle school classroom. The current study seeks to explore if that is the case.

2.7 A Review of Collaborative Assessment Studies with University Students

Within the domain of mathematics, Berry and Nyman (2002), who studied a university level ‘cooperatively-taught / cooperatively-tested’ mathematics modelling course, extol the many advantages afforded students working together. Based on student post-test survey responses, they found that team members engaged in discussion and explained their responses to each other during the test. The opportunity for students to discuss, explore, explain, and do mathematics, helps students come to see the subject as an active, rather than passive, discipline. Working with their peers also helps students to see that even ‘good’ mathematics students struggle with challenging problems (Zimbardo, Butler, & Wolfe, 2003) and need to put in considerable thought, time, and effort in the quest for an appropriate solution.

Lambiotte, Dansereau, Rocklin, and Fletcher (1987), also working with university students, explored the impact of collaborative learning (i.e. studying together) and collaborative testing in a pseudo-classroom environment. Even though students were assigned the partner with whom they would study for a reading passage test and/or take a reading passage test, they rated the cooperative test-taking as a favourable experience. In addition, students who studied together recalled information more accurately, whereas those who tested together recalled more (in terms of quantity) details.

Further work with college students in an introductory psychology class was completed by Zimbardo, Butler, and Wolfe (2003) who allowed students to work independently or to self-select the partners with whom they would take their test. Students working together earned higher test scores than those working alone and reported a variety of benefits including reduced test anxiety, more confidence, and an increased enjoyment of the topic. Since students knew their partner ahead of time, they could choose to split the studying between them. However, teams that did so seemed to lose the benefits that came from discussing and debating answers.

Hancock (2007) investigated the effect cooperative testing had on the motivation of graduate students in a research methods course. Students were paired in high performer/ low performer partnerships for three examinations (a multiple choice, a short answer, and an essay question). He found that paired students were more motivated than individual test-takers, achieved better exam scores, and felt competition within the class was reduced.

In summary, university participants responded positively to the opportunity to work collaboratively during assessments (Berry & Nyman, 2002; Lambiotte, Dansereau, Rocklin, & Fletcher, 1987; Zimbardo, Butler, & Wolfe, 2003) and claimed that it reduced test anxiety (Hancock, 2007; Zimbardo, Butler, & Wolfe, 2003). Given that many middle school students contend with mathematics angst, limited self-confidence, and general frustration with the discipline, these findings are encouraging and worthy of further investigation.

Although the research is limited, preliminary indications suggest the benefits are evident regardless of the type of partnership, form of test, or level of individual accountability. Therefore, tailoring collaborative assessments to the unique needs of younger students, as I have done with this research, may yield similar favourable outcomes.

2.8 Individual Learning and Accountability

In collaborative situations, both group and individual learning is valued. However, Webb (1995) and Slavin (1991) argue that individual learning is impeded in collaborative situations in which the emphasis is on a final goal, such as performing well on an assessment. According to Slavin, unless students are held individually accountable for their work they will learn that taking the time to give and receive assistance may reduce the group's overall performance.

Consequently, individual learning may be sacrificed for the greater benefit of the group. More directly,

> when the group task is to do something, . . . the participation of less able students may be seen as interferences rather than help. It may be easier in this circumstance for students to give each other answers than to explain concepts or skills to one another. (Slavin, 1991, p. 77)

In other words, in an assessment situation, there is the potential for partnerships to simply choose the best person for the task so as to ensure the highest mark. As a result, a situation may develop where the submitted assessment represents the work of one individual, rather than the collaborative effort of the dyad. Consequently, in this study, but also in everyday classroom practice, one motivation for selecting equal status partners and for providing an open-ended task is to avoid a situation where answers can be easily given from one student to another.

Various other strategies are sometimes employed to help increase or ensure individual accountability (and, hence, individual learning) is maximised. As discussed earlier, some teachers will require each student to submit their own assessment. Another strategy, used by Hancock (2007) and Klecker (2003), involves randomly assigning partners on the day of the exam, thus preventing students from ‘splitting the studying between them’.

Alternatively, teachers can choose to reduce the emphasis on individual accountability which may enable the development of a more collaborative and less competitive learning community. Contrary to Slavin’s (1991) and Webb’s (1995) expectations that individual (assigned) responsibility is a prerequisite for individual learning, university students “interacted extensively” (Hancock, 2007, p. 218) with their partners and worked collaboratively to generate solutions (Berry & Nyman, 2002), even when individual accountability was forsaken.

In my mind, the comparison between Slavin and Webb, on the one hand, and Klecker, Hancock, and Berry and Nyman, on the other is interesting. Slavin and Webb focussed their work with elementary students in the early to mid 1990s, whereas the other researchers worked with university students approximately ten to fifteen years later. Therefore, the researchers' approaches to individual accountability may differ as a result of cultural differences between the two decades and/or as a result of the differing ages of the students in their studies. As a researcher working with middle school students in 2010, I feel partially aligned with both sides. Like the latter researchers, I believe that individual accountability is not a prerequisite to individual learning. However, like Slavin and Webb, I appreciate that younger students sometimes need more direction and explicit responsibilities, which can be accomplished by assigning individual tasks. This study provides an opportunity to explore how middle school students respond to the autonomy they are afforded in a collaborative assessment situation where they are expected to jointly create understanding and mathematical solutions.

3 Research Design

For this research, I worked with a grade 8 class and their teacher. The teacher and I developed a classroom assessment that students completed during a single 80 minute lesson. This assessment was integrated into the volume and surface area unit that students were studying and was a required component of the course. During the assessment, students worked with a teacher-assigned partner. I video-taped four pairs of students as they worked on their assignment. I also collected the students' work, including the self-reflections they completed after the task. Data analysis focussed on the qualitative nature of student interaction and on the mathematical learning students demonstrated (both on paper in the submitted assignment and orally during the assessment itself) when they were given this opportunity to collaborate with a teacher-assigned peer.

3.1 Participants

Participants in this study came from a grade 8 mathematics class at a coeducational independent K – 12 school.

3.1.1 Recruitment of Participants

Initially, I contacted the headmaster of the independent school where I planned to complete my research. After he consented to the study, I contacted the mathematics teacher. Once she consented to the research, the mathematics teacher introduced the study to her class and distributed guardian consent forms and student assent forms, which she later collected. The consent and assent letters explained that the paired assessment task and the associated written reflection constituted part of the regular mathematics class. As such, all students would complete them with a teacher-assigned partner, whether or not they chose to participate in the research and consent to the use of their data. The letters further explained I would collect all consenting groups' written work, but only record some of the groups while they were working. After students had received this initial introduction to the study, I visited the classroom as a participant-observer. On the day of the assessment task, I verbally reaffirmed student assent.

3.1.2 The Teacher

I chose to work with this classroom teacher because I knew she supported and encouraged collaborative work (though not usually collaborative assessment) as well as the type of concept-driven problem-based assessment task I used in this research. She was an experienced teacher, though this was her first year teaching at this school. Unfortunately, on the day of the assessment the teacher was absent due to illness. However, the substitute teacher was willing to allow the research to proceed and consented to participate, as well.

3.1.3 The Students

Fourteen students (6 males, 8 females) consented to participate; I collected written work from all participating students and video recorded four of the partnerships, as they worked on the assessment. The classroom teacher paired the students the day before the day of the task. Her decisions were based on her professional knowledge of the students, acquired during the previous six months, rather than on their academic scores. After she briefly described to me her justification for the 'equal- status' pairing of each duo, I selected four partnerships to video record. My in-class observations of these students made during lessons prior to the assessment corroborate the teacher's interpretation of the students' behaviour and mathematical ability. My selection of which partnership to video was based on two criteria:

1) I chose partnerships that were equal status, in terms of both academic ability and social power within the classroom.
2) I chose a variety of equal status partnerships such that differences in mathematical competencies, friendship levels, and social behaviours were represented.

The partnerships I selected are described below.

Partnership 1: *Jesse*[**] *and Rebecca.* According to the teacher, Jesse and Rebecca are both considered fairly strong students who tend to do well in mathematics. Rebecca, an ESL student, is very quiet and rarely voices her views when working with others. However, she is comfortable with Jesse who listens to what she has to say, which is why they were partnered together.

Partnership 2: Melvin and Christina. Melvin and Christina are both considered weak in mathematics and frequently stay together after class for extra help. Though they seem to have difficulty understanding many concepts, they can both be earnest students. In some situations, Christina has a tendency to be distracted. However, in the past she has worked well with Melvin.

Partnership 3: Tidus and Alexandra. Like Jesse and Rebecca, both Tidus and Alexandra tend to do well in mathematics. They are both interested in understanding the concepts behind the procedure, voluntarily participate in discussions, and are relatively confident in their ability to ask questions and seek clarification.

Partnership 4: Rodriguez and Fergus. Rodriguez and Fergus are good friends who are easily distracted and who have a tendency to socialise with each other and with their classmates. They are fairly weak in mathematics and often rely on algorithmic procedures.

3.1.4 The Classroom Context

Throughout this research, the students were studying a three week unit on the volume and surface area of prisms, pyramids, and cylinders. The teacher expressed a desire to help the students develop conceptual understandings of the significant concepts involved with these topics, while also acknowledging the external expectations that students' success would be measured primarily by their ability to use the required formulae. To meet the required objectives of drawing and constructing nets for 3-D objects; determining the surface area of right rectangular prisms, right triangular prisms, and right cylinders; and developing and applying

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[**] This and all names are pseudonyms.

formulae for determining the volume of right prisms and right cylinders (Ministry of Education, 2008) she used a mix of tasks she designed herself and traditional textbook questions. (See Appendix A for a more complete description of the objectives.)

During the first lesson I observed, the students worked with partners to create all possible rectangular prisms that could be formed using 64 one-centimetre multilink cubes. As students worked on this task, they recorded the length, width, and height dimensions of each prism they found. This data served as an introduction to the idea that multiple objects could have the same volume and to the formula $V_{rectangular\ prism} = l \times w \times h = area\ of\ base \times h$.

During a different lesson, the students presented their ‘dream house’ projects which entailed designing their own personal dream space and calculating the cost of furnishing it including flooring and wall coverings. To complete this project successfully, students needed to use their knowledge of surface area in context and in conjunction with other mathematical understandings.

3.2 Method

3.2.1 Preparation for the Task

I spent three lessons in the classroom during the middle of the unit. My primary role during these lessons was that of participant-observer. I also answered students’ questions about the research and gave assistance if needed. My goal during these lessons was to develop an understanding of how mathematics was taught in the classroom and the types of skills, knowledge, and concepts that had been introduced. The observations also helped me to better understand the students and how they interacted with each other. This helped me to appreciate how and why the teacher paired the students for the assessment task and also enabled me to tailor the assessment task to the students. I kept a reflective journal of my observations.

During this time, I also worked with the teacher to select a relevant assessment task and related self-reflection. Decisions regarding the administration of the task were made in consultation with the teacher. This was important since the teacher possessed a clearer knowledge of various situational factors that could have affected how such an assessment would be perceived by students, parents, and school administrators. For example, we decided that all students would be required to work with a teacher-assigned partner, whether or not they participated in the research. Before the research began, I decided that students would work in dyads, not larger groups, since smaller groups tend to work more cooperatively (Alencar, de Oliveira Siqueira, & Yamamoto, 2008; Hamburger, Guyer, & Fox, 1975) and draw reluctant participants into discussions more frequently (Jorgensen, 1973). Also, I believe dyads are more manageable within the middle school classroom, will enable students to stay more focussed, and offer the greatest possibility for all students to participate actively. The teacher did not express any concern about the students working in dyads.

We also decided that students would submit a single assessment, as this provides an impetus for individuals to explain, justify, and defend their solutions in order to reach a consensus.

3.2.2 Selecting the Assessment Task

Although the type of assessment task used in this research was different from the conventional tests found in many mathematics classrooms, it was familiar to this classroom teacher. In lieu of numerous 'surface' questions that could be solved by remembering specific formulae or procedures, this task presented a problem that required students to explore a concept in more depth and to consider a variety of possible strategies, solutions, and factors. The goal was to provide a problem that was conducive to collaboration and to the creation of a learning space “rich in mutual discovery, reciprocal feedback, and frequent sharing of ideas” (Damon & Phelps, 1989, p. 13).

When developing the assessment task for this research, I felt it was important to work with the classroom teacher in order to ensure the task would be fully integrated into the classroom curriculum and would not be seen as an add-on or something extra that students must complete. In addition, since the purpose of this research is to explore paired assessments within the classroom, it was important that the teacher believe the selected task was appropriate for her students.

After speaking with the teacher and developing an understanding of her goals and plan for the unit, I prepared a number of possible tasks. Since the students were studying surface area and volume of three-dimensional shapes, this topic needed to constitute the main focus of the assessment. I began by selecting six possible problems which I gathered from past resources I had in my teaching files and from the support and resource materials available to IBO teachers on the International Baccalaureate Organization’s Online Curriculum Centre (http://occ.ibo.org/ibis/occ/guest/home.cfm). When selecting the possibilities, I looked for problems that would meet the following criteria:

- have multiple entry and exit points -- stronger students needed an opportunity to showcase their strengths and sophistication, while weaker students needed to be able to work on the problem (successfully) without teacher assistance
- take around 60 minutes to complete
- address a ‘big idea’ in mathematics related to volume and/or surface area
- provide a forum that would encourage discussion

Four problems became main contenders:

1. Enlarging a 3D shape. In this problem, students start by calculating the volume and/or surface area of a given cuboid. They are then directed to calculate its volume and/or surface area after it has been enlarged by a factor of two. Finally, students must develop and test a hypothesis that states a general rule relating the scale factor to the increase in volume and/or surface area (i.e. as a 3D shape is enlarged by a factor of n, the volume increases by n^3 and the surface area increases by n^2). I was drawn to this question because, in my experience, these relationships are ones that many students fail to appreciate, yet they are at the centre of many scientific explanations. Further, I felt this question provided the potential for the demonstration of a wide range of non-topic specific skills such as making an organised table, formulating justification for a hypothesis, and generating and applying a general rule. My concern with this problem was that it would not be a 'problem' for any students who already knew the relationship under investigation. While I could offer an alternative problem or focus these students on their application and explanation of their understanding of the relationship, I did consider this a drawback. The teacher and I were also concerned that many students did not have enough experience generating a general rule, which due to the required marking scheme would have been an expected part of this assessment.

2. Volume and/or surface area of the balloon. In this task, students are shown a photograph of a hot air balloon or a bouncy castle. Their task is to estimate the volume of air in the balloon/bouncy castle and/or the amount of fabric used in the object's construction. Students are directed to use the people in the photograph to give them an idea of scale. I felt this problem would enable students to explore the idea of composite shapes as well as to consider notions of accuracy and estimation. I was also drawn to this problem because it was very clear and students would be able to understand easily what was expected of them. In addition, this task offers various opportunities for students to show a sophisticated level of understanding of volume by allowing them to choose which shapes to use to make up the whole (i.e. estimating the volume of the hot air balloon using a rectangular prism, a cylinder, or an octagonal prism; recognising that different levels of accuracy are associated with different estimates).

3. Icing the Cake. In this question, students are given photographs of a container of icing and a round layer cake. They are given the height and radius of each object and asked to determine how many containers of icing are needed to make the cake. Like possibility number two, this question seemed easily understandable by all students and would encourage students to consider a variety of factors in addition to which formula to use. It offers the somewhat challenging task of determining the surface area of a cylinder and, since it addresses notions of conservation of volume, it encourages students to appreciate how information can be represented in different forms. There is also a unique interplay between volume and surface area in this question as students must look at the volume of icing, even though the icing essentially relates to the surface area of the cake.

4. The Cheese Problem. In this problem, students must determine how to slice a block of cheese, such that the exposed surface area is minimised. In many ways, it only requires students to perform the (relatively) simple tasks of determining the surface area of rectangular and triangular prisms. However, since some sides are exposed to air, while others remain protected by wax, students must visualise and determine which surfaces need to 'be counted'. One benefit of this problem is that, to complete it fully, students will need to employ the Pythagorean Theorem, which helps students see the ways various components of mathematics link together. I was concerned that this problem may be too lengthy for students to complete in the required time. This problem also helps students to understand that the relationship between volume and surface area is not always constant – even though all sliced pieces of cheese have the same volume, they do not have the same surface area.

3.2.3 The Assessment Task and Reflection

The task selected for this research was *Icing the Cake* (see Appendix B). Though the teacher and I seriously considered a modified version of *Enlarging a 3D shape* (see Appendix C), we were concerned that the students would need a lot of front-loading in order to grasp the concept of a general rule and that they would not be able to finish in a single period. In addition to being more accessible to the students, we felt *Icing the Cake* successfully integrated the concepts of volume, surface area, and nets and addressed the big idea of conservation of volume. In hindsight, and arguably of greater relevance to these students, it is apparent the significant concepts 'What is volume?' and 'What is surface area?' were also addressed.

We believed students would be able to partake in this assessment with limited guidance from the teacher. Struggling or hesitant dyads could begin by determining the volume of the icing can, a task similar to questions they had previously seen. Alternatively, pairs could begin by considering the amount of icing on the cake. Groups could successfully solve the problem in a relatively straight-forward manner by assuming the icing thickness was 1 cm, effectively making the volume of icing needed equal to the area of cake covered by icing. However, students looking to stretch themselves could attempt to accurately estimate the thickness of the icing using concepts of scale and ratio or opt to verify their results using an alternative method. We discussed the possibility of using a square cake, but felt that simplified the problem too much. Keeping the round cake afforded greater opportunity for discussion and challenge since cylinder nets tend to be trickier than cuboid nets, owing to the understanding that is needed to appreciate the role of the circumference of the base of the cake.

Before beginning this task, students had experience moving between 2D and 3D representations of cylinders and had calculated volume and surface area, but they had not encountered problems such as this where the 'surface area' had a volume. I expected most groups would solve this problem dividing the amount of icing needed by the volume of icing in the can. To find the amount of icing needed, I expected students to use surface area of a cylinder and area of a circle formulae. I believed that some groups would quickly attain a rough estimate and would have time to refine the amount of icing found on the cake.

During the task, students worked with a teacher-assigned partner. Students knew ahead of time with whom they were partnered and had worked with these partners on a previous task during this unit. When assigning partners, the teacher endeavoured to pair students in 'equal status' relationships (Goos, Galbraith, & Renshaw, 2002), in which both partners possessed similar levels of relevant knowledge and power. The teacher recognised that students with equal (or similar) mathematical backgrounds would not necessarily share power equally. Therefore, in addition to mathematical ability, she also considered pro-social skills, mathematics anxiety, friendship pairings, and English language confidence when choosing partners.

Though the students were familiar with and comfortable working in groups I (because the classroom teacher was absent) reviewed the established norms for working cooperatively before the assessment began. These included listening to each other and speaking respectfully. Partly to encourage positive social interactions, but also to encourage the deep level of mathematical thought that comes when students challenge and defend mathematical ideas (Maher, Powell, Weber, & Lee, 2006), I also encouraged students to actively discuss and debate the problem with their partner. During these pre-task instructions, I also ensured students understood that their marks for this assessment would be based on the mathematics they demonstrated on the paper they submitted, rather than on their discussion or how well they worked together.

During the assessment, paired students interacted freely with their partner, but were discouraged from discussing the problem with other groups. Students were able to use calculators, notes, and textbooks throughout the assessment. In addition, the substitute teacher and I offered some suggestions and guidance, though this was more to encourage students to reflect on their strategies rather than point out what to do. For example, we encouraged students to consider the thickness of the icing, the icing layers, and the reasonableness of their solutions. Immediately following the task, the students completed a brief guided reflection about the task, which took about ten minutes. Students understood that their responses were not anonymous and would be seen by the teacher, but not shared with other students. Students were encouraged to be honest, but kind. The reflection asked students to consider how they believed the assessment reflected their mathematical learning as well as to consider any situational factors that may have influenced the learning they demonstrated (see Appendix C reflection questions).

3.3 Data Collection

Data was collected from a variety of sources including video of the students working on the task, written work submitted by the students, and field notes. Video and written work was collected during a single 80 minute mathematics lesson near the end of the research, whereas the field notes were collected over a five week period during January and February. Figure 1 provides an overview of the data collected.

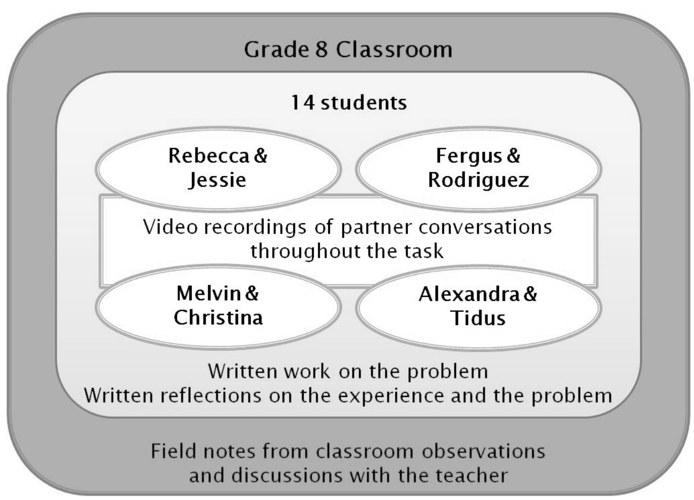

Figure 1: Overview of data collected.

3.3.1 Recordings of Partners Working on a Task

I video-taped four of the partnerships as they worked on the task, which enabled me to closely examine individual dyads, while still exploring a variety of equal status relationships (i.e. pairs with a strong/weak mathematics background; pairs with strong/weak collaborative skills; friendship pairings). Since the problem was a paper-and-pencil task, audio recording would have sufficed. However, the room was fairly loud and video recordings provided better sound quality. Video cameras were placed on the desks of four partnerships. The students were aware of the video cameras, but most groups tended to ignore them for the majority of the class. The exception to this was Rodriguez and Fergus who frequently and easily became distracted by the camera. As Fergus stated in his reflection, "Cameras make Rodriguez/ myself go crazy". These recordings provided an opportunity to examine the students' interactions throughout the entire time they were working on the problem, thereby allowing me to document the ways in which students interacted at different points in the problem solving process. Videotapes were only intended to record sound and images of the task on which students were working, not the students themselves. However, since some students were curious about the cameras, they occasionally filmed themselves, as well.

3.3.2 Assessments Submitted by Students

The completed assessments of all 14 participants were collected and photocopied before they had been marked by the teacher. These papers served as written documents of demonstrated learning. In addition, they offered insight into decisions that were made (i.e. consensus that was reached and then recorded on paper), allowed the teacher and researcher to see any individually attempted student work, and provided information regarding which students took responsibility for writing and how this affected the direction of the discussions and the work that was recorded on paper. Since this task was marked by the teacher and returned to the students, I only retained photocopies of the work. In the analysis, I only used the written work of the four dyads I had videotaped since, as I began to analyse the data, I realised that I wanted to focus on the interplay between the discussion and the written work.

3.3.3 Brief Written Reflections

At the end of the assessment, each student individually wrote a brief (approximately 10 minutes) guided reflection about the mathematical learning they felt they demonstrated during the task (See Appendix D). Student reflections served two purposes. First, they provided students an opportunity to self-assess their work, a practice which is encouraged at this school. Second, they enabled me to understand how students believe paired assessment affected the mathematical learning they demonstrated. This was important as the students' perceptions of demonstrated learning sometimes differs from adults' perceptions. Reflections were not shared with other students.

3.3.4 Field Notes from Class Observations and Discussions with Classroom Teacher

Classroom behaviours are influenced by the expectations, strategies, and philosophies of the teacher, which in turn affect myriad factors including students' beliefs regarding mathematics, cooperative work, alternative assessments, and the value of student contributions. Therefore, I felt it was important that I understand the context in which this research was conducted since it could impact the mathematical learning that students demonstrated during the paired assessment. To familiarize myself with the classroom context and the development of the mathematics unit for which the paired assessment was a culminating task, I observed the teacher and students on three occasions. I used a reflective journal to record my observations about the classroom learning environment. I also kept field notes of my formal and informal discussions with the teacher as we prepared for the assessment. Though I had intended to take field notes during the assessment itself, I did not because I was more occupied with the students than I had intended due to the teacher's absence. Instead, I recorded my observations in my reflective journal immediately following the lesson.

3.4 Data Analysis

To prepare the data for analysis, I examined each pair's written work and wrote a brief description of what I believed to be their strategies, approaches, and calculations. For Rodriguez and Fergus, who each completed their own written work, I analysed each paper separately. With the students' oral work, I transcribed the complete video recording of each dyad's discussion. After the initial transcription, I confirmed the data by listening to the recording in its entirety, comparing it to the transcription, and making changes when necessary. At times, I used the students' written work to help clarify their conversation and vice versa. I used the written transcriptions as the basis for data analysis. Occasionally, I re-examined the original recordings in order to verify a student's comment, intonation, or expression. For the interested reader, a complete transcript of one pair's dialogue and their written work are available in Appendices E and F.

Initially, I attempted to code the students' conversations based on the mathematical learning that they demonstrated during the paired assessment. The codes I developed were influenced by the students' conversations and by previous studies I had read (Denessen, Veenman, Dobbelsteen, & Van Schilt, 2008; Fuchs, Fuchs, Hamlet, & Karns, 1998; Powell, Francisco, & Maher, 2003). They included: suggesting a suitable idea; suggesting an inappropriate idea; expanding on an idea; questioning an idea; defending an idea; repeating what was being written down; and non-relevant comments. As I coded the transcripts, I continually revised my codes in response to the data. However, many comments were difficult to code and the micro-analysis made it difficult to appreciate the flow and themes of the students' conversations.

By the time I accepted that coding individual comments would not provide appropriate insight into the research questions, I had become thoroughly acquainted with the data. I had spent many hours attempting to determine students' intentions, follow their lines of reasoning, and decipher their mathematical approaches. As I began to focus on extrapolating emerging themes, I continued to read and reread the transcripts. I started to recognise the prevalence of certain events and topics including formula-centred discussions, the use of colloquial language, and students' willingness to ask and respond to questions. I had no strict criteria for what I considered prevalent, but in general, prevalent meant that it occurred often enough, between dyads or within a single dyad, to stand out to me. I made a list of these 'prevalent events'.

Based on loose connections I recognised between the prevalent events and the literature, I began to more clearly identify the emerging themes, which included the types of mathematical learning demonstrated (calculational and conceptual), the students' interactions with each other, and the partners' demonstrated ability to plan and check their solutions. I then focussed on each theme individually by reading the transcripts and written work with the intention of developing and clarifying the theme. For example, I read each transcript specifically looking for examples of calculational and conceptual understandings in order to learn more about when, why, how, with whom, and in which circumstances each one was likely to occur. When necessary, I referred to the students' written reflections and my field notes in order to augment my interpretation of the data.

At times, I returned to the literature to learn more about the existing research in an area, which I then used to help clarify my understanding of the data. For example, once I connected students' apparent lack of pre-planning their solutions to research I had read about students' use of the problem solving cycle, I sought various studies which explored novices' abilities to plan during problem-solving tasks. I used the main ideas presented in these studies to further inform and develop my understanding of the themes. In this way, I moved between the data, the themes, and the literature, using each to refine and inform the other, until I felt further work would be redundant (See Figure 2).

In addition to the thematic analysis, I estimated the amount of time each group spent on-task by roughly determining the number of minutes they spent working on the problem compared with the total length of time they took to complete the assessment (minus time taken for teacher instructions). I also roughly estimated each individual's quantitative verbal contribution to the problem by counting the number of words spoken by each student and expressing it as a percentage of the total number of words spoken by the partnership. The word counts for this second estimation only included on-task comments.

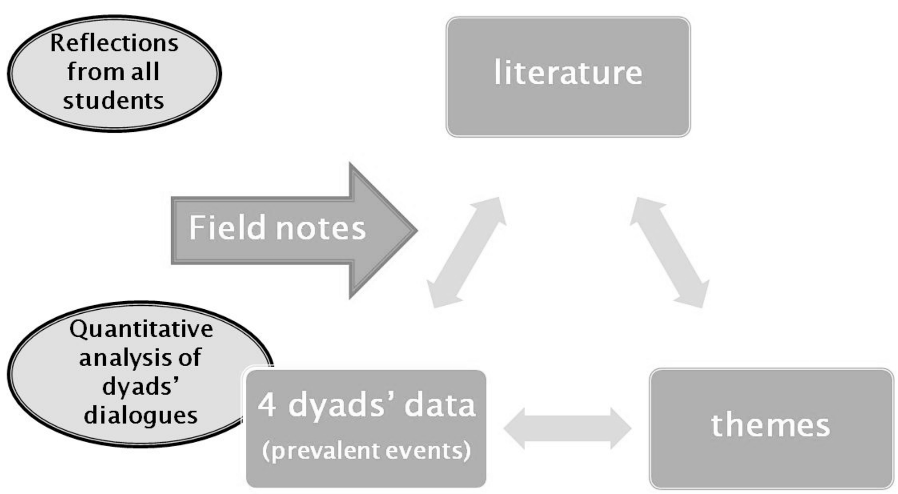

Figure 2: Overview of data analysis.

3.5 Ethical Concerns

This research was reviewed and approved by the Behavioural Research Ethics Board at UBC. In addition, the Headmaster of the school where this research was conducted gave his support and approval before I began. The teacher in the study provided free and informed consent and participated collaboratively with classroom decisions related to this research. The substitute teacher who was present on the day of the task also provided free and informed consent and was informed of the special nature of the lesson before he agreed to take the class.

Signed consent was collected from the parents or guardians of students and the students provided their signed and verbal assent. I maintained student confidentiality by ensuring that pseudonyms were used for participants and the school.

I protected the rights of students who did not wish to participate in this study by ensuring that videotapes only record pairs of consenting/assenting students and the task on which they are working. Students without consent/assent were not video-taped. Students who refrained from participating in the study were not excluded from participating in the paired assessment.

One of the concerns with asking middle-schools students to work cooperatively is that they have not always developed the social skills required to interact in a kind respectful manner with people who are not their friends. Throughout this research, I wanted to ensure that students did not become ostracized or targets for bullying because of collaboration. For this reason, the teacher assigned partners. In addition, since I worked with students who were familiar with group work, they had already had the experience of working with their classmates and mitigating difficulties as they arose. I also ensured that guidelines for collaborative work were discussed and reviewed. The teacher and I were prepared to pro-actively address conflicts if they arose, however, none did. Finally, student reflections were kept confidential and students were encouraged to be ‘honest, but kind’ when writing about their partners.

4 Findings

The aim of this research is to explore the mathematical learning that students demonstrate when they are given the opportunity to work collaboratively on an open-ended assessment task. Recognising that the learning demonstrated is always influenced by the context of the problem, as well as the dynamic between the participating students, I endeavour to acknowledge the uniqueness of each partnership while drawing out salient themes of a broader nature. I begin with a brief introduction to each pair of students and the strategies they used as they attempted to determine the number of cans of frosting that were required to ice the four-layer cake. I then concentrate in more depth on commonalities in the dyads' partner interactions and in the content and style of their discussions.

4.1 Melvin and Christina

Throughout the task, Melvin and Christina worked earnestly, if somewhat haltingly. They frequently commented that the task was difficult and they were unsure of what to do. Yet they seemed committed to trying hard, voicing comments such as, "We're trying to do it" and "I already tried my best". They had a tendency to hold brief off-topic discussions, but one of the students, usually Melvin, would quickly refocus their attention. It is unclear whether or not Melvin and Christina had a clear conceptual understanding of the problem since they spent most of their time attempting various calculations. By the end of lesson, they appeared tired and no longer had the energy to continue working on the problem.

The students began the task by determining the volume of the frosting can. This was quite challenging for them as each step in the calculation, $V = \pi r^2 h$, required considerable thought and discussion[2]. It took multiple attempts, using a calculator and referring to the textbook, for them to determine the values of r, r^2, πr^2, and, finally, $\pi r^2 h$. Once they had ascertained the

[2] Sections of this conversation are discussed in Section 4.6.

volume of the frosting can, they decided to calculate the volume of the cake, which went much more smoothly. These two calculations took approximately 20 minutes, after which Melvin and Christina declared, as I walked by, that they were stuck. We had a conversation (see Figure 3) that explored the meaning of volume and surface area and they acknowledged that determining the cake's surface area would be more helpful than determining its volume.

194. *Researcher: And what's the other volume that you calculated?*
195. *Melvin: This one.*
196. *R: The volume for the cake?*
197. *M: Yes.*
198. *R: And what's that going to tell me?*
199. *M: How much stuff can go inside the cake.*
200. *R: Yeah and is that what you want to know for the icing?*
201. *M: No . . .*
202. *R: I need to know the volume of that [icing can] because that tells me how much icing is in a can. But what do I want to know [about the cake]?*

. . .

207. *M: Oh, the surface area.*
208. *R: How do you know it's surface area?*
209. *M: Because you need the surface, like, the area around it.*

Figure 3: Volume or surface area?

Both students, however, were still unsure of how to use surface area to help them. Christina, who with the assistance of her father, had completed the previous night's homework on using nets to determine the surface area of cylinders, spent nearly ten minutes attempting to explain her understanding to Melvin and relating it to their problem. As can be seen from her drawing of the cake's net (see Figure 4), she was able to correctly diagram the cake's height of 20 cm and its diameter of 20 cm. The radius of the base is also labelled correctly as 10 cm. The two additional circles on each side of the rectangle were used to estimate the length of the rectangle using $2\pi r \approx 3d$, which the students had discussed in class[3].

---

[3] During a previous lesson, the classroom teacher taught the students to find the surface area of a cylinder using a two dimensional representation of the object. Students learned that the circumference of the base ($2 \pi r$) was equal to the length of the rectangle in the two-dimensional drawing and that they could estimate this measurement using $3d$, rather than $2 \pi r$. To illustrate this concept, the teacher repeated the two-dimensional representation of the base three times along the length of the rectangle. In Christina's net (see Figure 4), she has drawn this equivalence along both sides of the rectangle, hence the six circles, instead of the cylinder net's traditional two.

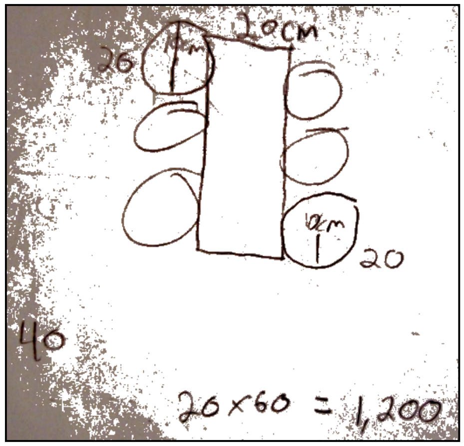

Figure 4: Christina's drawing of the cake's net.

In their ensuing discussion (see Figure 5), it is clear that despite Christina's confusion between radius and diameter [lines 254 – 256], she understood that the length of the rectangular portion of a cylinder's net can be estimated using three times the diameter of the base [line 257]. In addition, she recognised that the area of the bases were important to the total surface area of the cylinder [line 259]. However, she was less clear about how to determine the area of the bases [lines 259 and 261] or what to do with the measurements.

249. *Christina: Okay, dude, so listen, okay, so listen. You've got 10, right?*
250. *Melvin: Yup.*
251. *C: 3 tens, right.*
252. *M: Yup.*
253. *C: Right, you can fit 3 tens, so all 10s, right. And then, so all's first.*
254. *M: They're all 20.*
255. *C: They're all 20?*
256. *M: Yup, because 10 is half of it.*
257. *C: Oh, okay. Yeah, so they're all 20s. So, then you have three 20s and you get 60, right? So you go 60 multiplied by 20 and then you get the area of the rectangle . . .*
258. *M:. . . 1200 is the answer [to the surface area of the net].*
259. *C: Ok, no, no, it's not it yet, though. And then you— but then it's these two and then you— You got to find these two [indicating the circles representing the base]. These two together. If, ah, 4, ah, 40 because there's two 20s—*
260. *M: Ok.*
261. *C: Yeah, 40. So it's 20, I think it's times radius, is it 20? Oh it's 20. Okay, okay, 40 and then you get 40 and then you get 1200 and then you get 40 and then multiply—*
262. *M: What? . . .*

Figure 5: Christina attempts to explain how to find the cake's surface area.

The students spent the remainder of their time referring to their homework questions and attempting to use them as templates for determining the surface area of the cake. Despite their focus, however, they were unable to do more than Christina's initial drawing of the cake's net (see Figure 4).

Near the end of the task, Melvin and Christina discussed how they could have improved their answer. Christina's response, while acknowledging their difficulties, captured the unique blend of frustration and light-heartedness they demonstrated throughout this problem:

> "Oh, ah, [we could have improved] by knowing the formulas (laughing). We know some, it's just that we don't really know how to finish it, you know— We did it half way, but then we didn't know how to finish it. My head's hurting."

This comment also demonstrated the formula-driven calculation orientation motivating this group. Echoing Christina's thoughts, Melvin's written reflection comment, ". . . we forgot which formula to use and how to completely do them. If we had time to study we would have been able to remember them more" also emphasised the primary importance the students placed on remembering the formulae, rather than on understanding them and how they could relate to the problem.

4.2 Alexandra and Tidus

Alexandra and Tidus were focussed throughout most of the task, performed many calculations easily, and seemed to possess a conceptual understanding of how to approach the problem. For example, they understood that the icing covered the outside of the cake and the three layers, but not the bottom of the cake, and they knew that the number of cans needed would be equal to the amount of icing needed divided by the volume of icing in one can. As is explained in more detail in later sections of the data analysis, they frequently discussed various aspects of the problem, including the number of layers of icing (see Figure 30), the missing piece of cake (see Figure 29), and the meaning of the surface area of the cake (see Figure 9). In addition, they consistently rounded their numbers showing an implicit understanding of the estimation nature of this problem. However, they also had a tendency to make algorithmic errors with the formulae and their conceptual discussions often resulted in confusion and misunderstanding.

From their conversations both before and after their initial calculation (see Figure 6), it is clear that Alexandra and Tidus intended to begin the problem by finding the surface area of the cake. However, although their calculations were correct, they incorrectly used the formula for volume. They then used $V = [\pi]r^2h$ to find the volume of the frosting can, but forgot to multiply by π. Using these two calculated values – what they believed to be the surface area of the cake and the volume of the can – they estimated the number of cans needed by dividing surface area by volume to get an answer of 29 cans.

12. Alexandra: What are we trying to find out?
...
13. Tidus: The surface area.
14. A: The surface area.
****
43. T: So we found the surface area. Okay.
44. A: So this is for the cake.

Figure 6: Alexandra and Tidus want to find surface area of the cake.

After obtaining a preliminary answer of 29 cans, they focussed on the layers of icing in the cake, which involved determining how many layers of icing existed, as well as the area of a single layer. While both students agreed that they needed to find the area of the base of the cake (see Figure 7, lines 114, 118, and 126) in order to find the area of a layer, they had difficulty determining which formula to use, even with the help of the textbook (lines 127 - 131).

114. Alexandra: . . . We need to figure out the surface, the base area, do you know what I mean? No?
115. Tidus: Isn't pi r? Or, 2 pi r?
116. A: Did you find that— Isn't that the circumference?
117. T: Circumference? Circumference is all around, which is—

118. A: No, but we want to find the surface area of only the base. How do you find the surface area of a circle?
. . . (side conversation). . .
126. A: Okay. So, to find the surface area of a circle what do you—
127. T: I'll check my textbook.
128. A: Are we allowed?
129. T: I think it's 2 pi, it's pi r or is it 2 pi r — 2 pi r is diameter right?
130. A: No
131. T: (looking in text) Here, area of a circle. It's right here. Area of a circle is 2 pi r.

Figure 7: Alexandra and Tidus try to select the formula for the area of a circle.

It is possible part of this confusion arose because the students had different intentions and, therefore, lacked a common understanding of what they were doing or attempting to do. But, since their discussion became convoluted and challenging to follow (see Figure 8), this is speculation. Possibly, Tidus and/or Alexandra intended to calculate the area of the cake's side using $2\pi r h$, which could explain his references to the area of a rectangle [line 134] and height [lines 134, 148, and 158] and her references to the circumference [line 135]. Alternatively, Tidus' desire to multiply by height may indicate he planned to calculate volume, but had genuinely confused the formulae for area and circumference. Despite Alexandra's willingness to use circumference, she seemed confident they should be calculating the area of a circle [lines 155 and 157], and appeared to use $A_{base} = \pi^2 r = (10)(10) = 100\ cm^2$ [4] to do so. Tidus, on the other hand, seemed to calculate $A_{base} = r^2 = (10)(10) = 100\ cm^2$. Regardless of their undisclosed intentions and their seemingly different calculations, both students accepted that the area of a layer was equal to 100 cm^2 [lines 157 and 158]. Even though they were aware of their confusion [lines 138; 150 - 154] and Alexandra returned to the text and identified the appropriate formula as $A_{circle} = \pi r^2$ [line 154], they never followed through with this, partially because Tidus said they had already calculated πr^2 [line 156] (which they had, earlier) and partially because they

[4] Frequently in their oral discussion, and occasionally in their written work, students did not include any units or included incorrect units. For readability purposes, I have included the correct units with all calculations.

became preoccupied with discussing the missing piece of cake. Therefore, in later calculations, they used 100 cm^2 as the calculated area of the base.

134. *Tidus: . . . We already found the area of a circle is 2 pi r. Area of a rectangle is— Oh, so we didn't— No, so we just found the area of a circle because if we want to find out how much is its inside it's 2 pi r times height, so it's —*
135. *Alexandra: Yeah. 2 pi r is the circumference, right? Yeah, It's— so we have to do the circumference, 2 times pi, pi, pi times pi, right?*
136. *T: yeah*
137. *A: times radius, so what's the radius, the radius is 10*
138. *T: I'm really confused*
139. *A: So, pi —Where's your pi button on here (referring to calculator)? pi times pi, delete, pi times pi*
140. *T: Yup.*
141. *A: And then?*
142. *T: Umm, what is it?*
143. *A: So these—9.8*
144. *T: So just round it to 10*
145. *A: 10, okay and then so we go 10 times 10 . . . is 100*
146. *T: 10 squared, no never mind, never mind, never mind*
147. *A: No its 100*
148. *T: So it's 100 times height, didn't we just do that?*
149. *A: No we don't want to times height. . .because that would equal these two*
150. *T: Uh oh , no, no,*
151. *A Yeah, yeah we're about—*
152. *T: Ummm—*
153. *A: Let's see, we're having some troubles.*
154. *T: Ah.*
155. *A: Hold on. Okay. Area of a circle is pi r squared (from text). . .*
156. *T: . . . Look we already did that though,*
157. *A: Ok . . . so 100 is the area of the surface, surface area [top of the cake]*
158. *T: Yeah, it's 100 times 20 [height]*

Figure 8: Area of the base of the cake.

The conversation then focussed on the number of layers of frosting in the cake. Alexandra eventually convinced Tidus that their 'surface area' calculation included both the top and bottom bases of the cake. Since the bottom of the cake was not iced, they agreed to subtract the area of the base to account for this. They then added three times the area of the base, for the area of the three layers of icing, to determine the overall area of the cake to be iced ($A_{cake\ to\ be\ iced} = SA_{cylinder} - A_{bottom\ of\ cake} + 3(A_{one\ layer\ of\ icing})$). To calculate the number of cans of frosting needed, they divided the area of the cake to be iced by the volume of the frosting can. Like before, this gave an answer of 29 cans.

At this point, the teacher came by and expressed concern that 29 cans might be an unreasonable response. Alexandra and Tidus re-examined their work, determined they forgot to multiply by π when calculating the volume of the frosting can, and correctly recalculated the can's volume. Eventually they also correctly recalculated the area of the base of the cake using πr^2, which they then used to redetermine the total amount of icing needed for the cake. However, as before, this was a lengthy discussion laden process, fraught with more confusion and miscommunication (See Figure 9). Some of the confusion arose because, by this time, the students had agreed the area of the cake's base was 314 cm^2, yet Alexandra reverted to using their previously calculated value of 100 cm^2 [lines 349 and 365]. Finally, using the then-correct values, they recalculated the area of cake to be iced and the number of cans of frosting needed. Though they had successfully identified two of their previous miscalculations, they continued to calculate the surface area of a cylinder using the formula for volume.

349. *Alexandra: . . .Hold on. We have to minus 100 because we don't ice the bottom of the cake.*
350. *Tidus: (inaudible)*
351. *A: But that's still surface area.*
352. *T: 6280 is the surface area [of the whole cake].*
353. *A: Of the entire thing, right? Of this, right, so—*
354. *T: Subtract.*
355. *A: To ice it.*
356. *T: No, we have to subtract the bottom. Yeah, so we have to subtract 100.*
357. *A:Okay, so— But subtract [inaudible]. Yes.*
358. *T: No, the 314? Because the area of the circle is pi r squared.*
359. *A: Yeah.*
360. *T: So subtract. So 6280 subtract three fourteen so it's 6280—*
361. *A: Why pi? Why are we subtracting?*
362. *T: We're not subtracting pi,*
363. *A: We're subtracting.*
364. *T: We're subtracting the area of the circle.*
365. *A: Right, which is 100.*
366. *T: Yeah, times pi. Because 100 is only the radius, so to find the area—*
367. *A:Yeah the area because —all of it.*

368. *T: No, it's radius squared . . . 10 is the— [radius]*
369. *A: pi r squared— Yes, I see, I see the— it's— Yes, your right.*
370. *T: So instead now, it's radius equals to 10, squared*
371. *A: 10 centimetres*
372. *T: Centimetres squared. And then you have to multiply that by pi . . to find the area so it—*
373. *A: Then you minus that from 6280?*
374. *T: 5966 cm squared, right?*
375. *A:Yes.*

Figure 9: Alexandra and Tidus fluctuate between using 314 cm^2 and 100 cm^2.

4.3 Jesse and Rebecca

In this partnership, the two girls usually worked well together. In their reflections, they commented that they were able to "help each other" and "discuss it [the problem] together". Early on, they seemed to establish a system whereby Jesse did most of the recording and Rebecca did most of the calculator work. Though Jesse tended to be more verbal, Rebecca did voice disagreement later in the task including when she felt they needed to account for the missing piece of cake. At times Rebecca seemed to have a hard time understanding parts of the problem, but Jesse usually took the time to explain concepts (with varying degrees of success) and elicit Rebecca's opinion. Near the very end of the task, perhaps because there was less time remaining, Jesse stopped collaborating with Rebecca and began to work more independently.

Rebecca and Jesse started the problem by calculating the volume of a can and the cake. However, Jesse realized, somewhat spontaneously, that the cake's surface area would be a more useful measurement than its volume (See Figure 10).

55. *Jesse: Each [icing] can holds[699 cm^3]— right — Okay, then, what? How big is the cake, right?*
56. *Rebecca: The cake. The volume is 6280. The total volume.*
57. *J: Let me write that down. No, don't we need the outside?*
58. *R: What do you mean the outside?*
59. *J: We do surface area because you only ice the outside of the cake so we do surface area—Ahhhh (erasing)!*

Figure 10: Jesse realises that the cake's surface area is more useful than its volume.

They then calculated (correctly) the surface area of the cake using the side and top of the cake and used these two measurements to determine the number of cans of icing needed. Throughout this task, the girls revised their estimate of the required amount of icing. However, they always returned to the same formula – *volume of icing needed ÷ volume of the frosting can* – to determine the number of needed cans. At this point the girls felt they were finished, but realised they still had at least 20 minutes left to work on the problem. Rebecca's question, "What else can we do?" and Jesse's eventual proposal, "Let's do it with the whole cake, even the bottom," seemed to indicate that the girls understood that more could be done with this problem, even if they were unsure of what this 'more' included. Calculating the surface area of the cake, as though the bottom were iced, seemed to fulfill the need to 'do something' rather than serve as a logical next step. When I suggested they consider the thickness of the icing and the layers of icing instead of assuming the bottom of the cake was iced, they successfully determined the area of the three layers of icing, but had difficulty determining how to account for the icing's thickness. Though their work is unclear, it appears they interpreted the icing's thickness as an *r* value (see Figure 11). In their final written solution, they included the iced bottom of the cake,

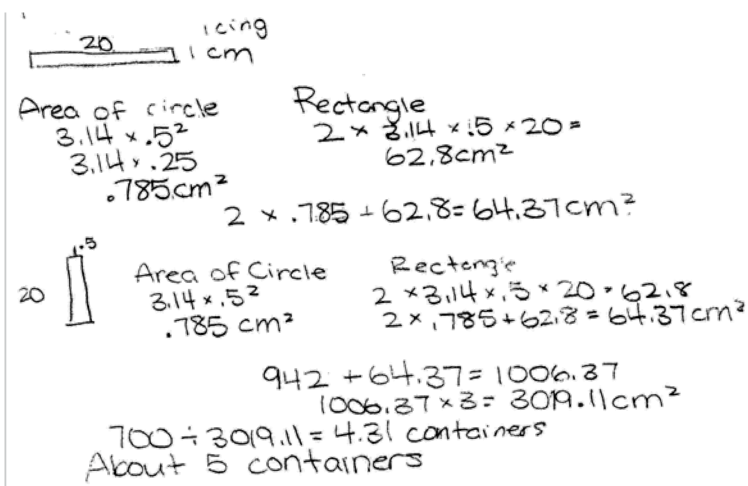

Figure 11: Rebecca and Jesse's thickness calculations.

which I speculate was an oversight on their part, since their initial calculation of the cake's surface area did not include an iced bottom.

4.4 Rodriguez and Fergus

Unlike the other three groups, Rodriguez and Fergus worked independently on most aspects of the problem, were easily distracted, spent half their time discussing mutual interests unrelated to the task, and submitted work that contained an assortment of errors related to the formulae used. Because they worked independently they spent more of their time writing, rather than discussing, their solutions. Each student submitted their own paper and made no attempt to ensure consistency with their partner's paper. The students seemed to understand that the intention was for them to work together (See Figure 12). However, they referred to their work as 'my answer' and 'your answer' and rarely discussed their strategies or calculations in any depth.

58. *Rodriguez: . . . Shouldn't we be doing this together, Fergus?*
59. *Fergus: What?*
60. *R: Shouldn't we be doing this together?*
61. *F: Yeah, okay.*

Figure 12: Shouldn't we be doing this together?

At the very beginning Fergus proposed that the number of frosting cans needed was equal to the diameter of the cake divided by the diameter of the can. Later, Rodriguez and Fergus mentioned both volume and surface area, but there was no sustained discussion regarding either. As Rodriguez worked on the problem, he developed a good conceptual understanding of it (see Figure 13 and 14), including both the layers and the thickness of the icing. His written work includes a descriptive explanation that is nearly identical to the very succinct one he offered Fergus:

Okay, so . . . we found the volume of the cake. But really the whole cake isn't filled with frosting. So what we have to do is find the surface area, and then half a centimetre around it in volume, to find the icing and then whether or not we want to make it a layer cake, . . . get it?

Figure 13: Rodriguez's expresses conceptual understanding of the problem.

However, he seems unable to follow through with his ideas, use appropriate calculations, or remember what the question is asking. His final answer appears to represent the volume of icing needed, rather than the required number of icing cans (see Figure 14).

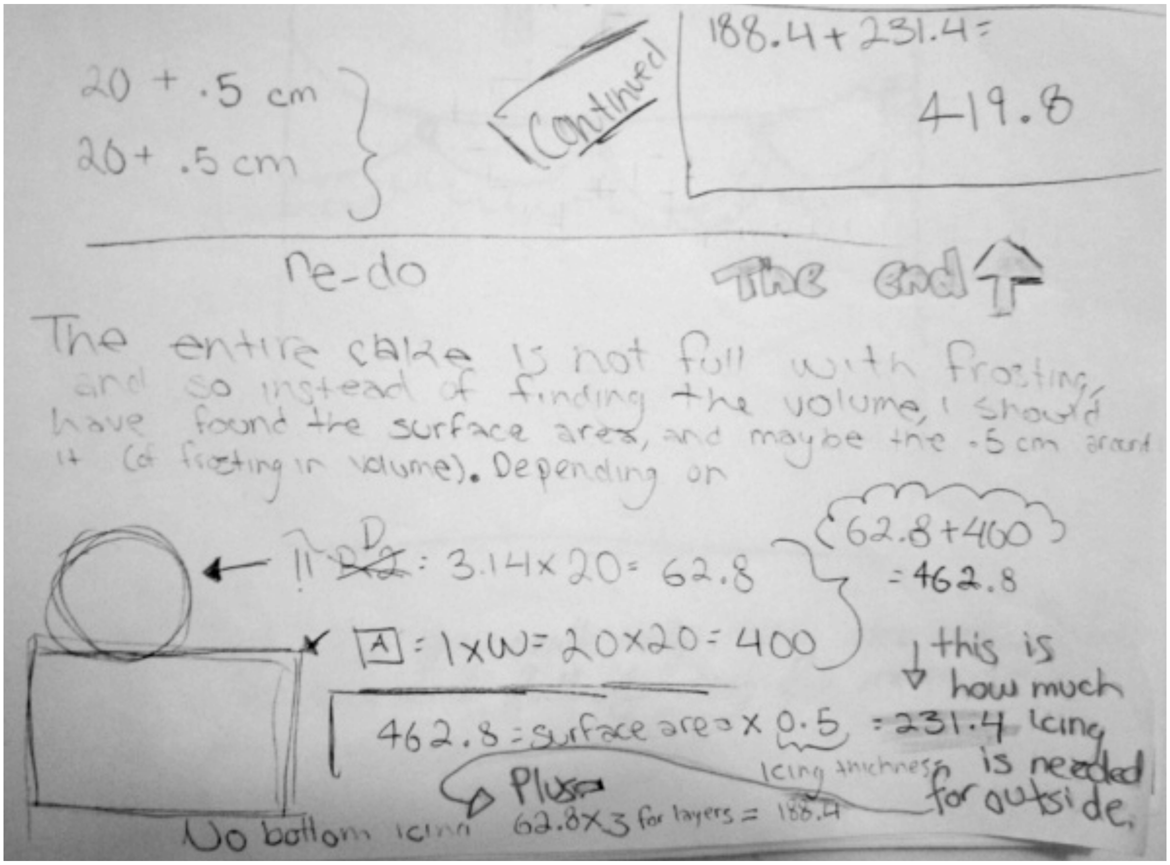

Figure 14: Rodriguez's written work.

Fergus’s written work (see Figure 15), on the other hand, shows a different understanding of the problem. Although it contains a sentence incorporating Rodriguez’s ideas about layers, thickness, and surface area, it appears his comprehension of why they were relevant was limited since, unlike Rodriguez, he does not seem to use them in his calculations or his diagrams. However, his final solution, the volume of the cake divided by volume of the can, shows an understanding of the need to determine the required number of icing cans. It should be noted that the typed comments in Figure 15 were added to aid the reader in deciphering Fergus’s writing,

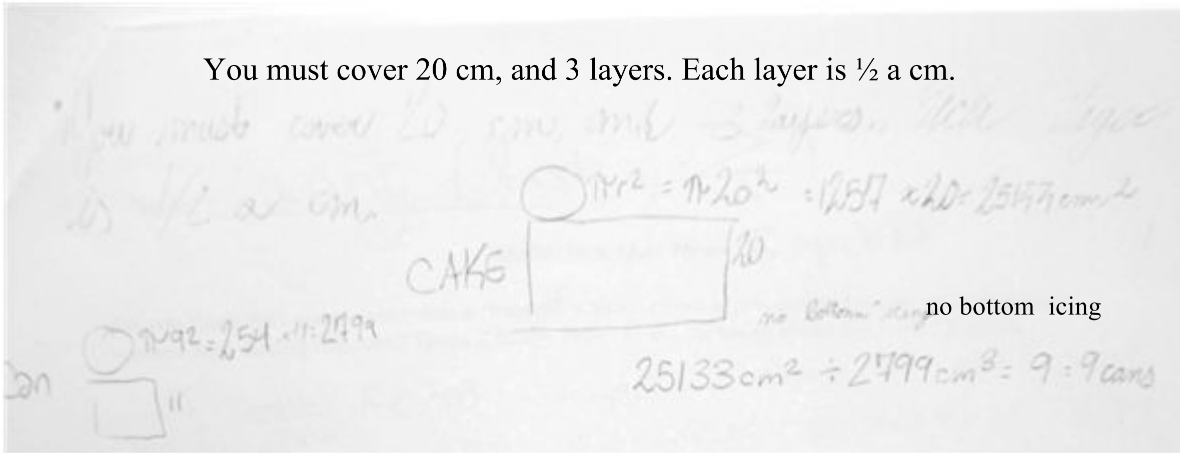

Figure 15: Fergus's written work that incorporated Rodriguez's ideas.

4.5 Interaction Between Partners

In this study, three of the four partnerships spent at least 80% of their time working on the assessment and engaged in collaborative discussion throughout this time. Though they were occasionally distracted by the video camera or held conversations about what they were doing after school, their main focus was the problem. In two of these groups, the talking time was split relatively equally between the two students. With both Tidus and Alexandra, who had a comparatively strongly grasp of the problem, and Christina and Melvin, who struggled to conceptualise it and to perform relatively basic calculations, the partners contributed fairly equally to the discussion, both in terms of quality and quantity. In Rebecca and Jesse's group, Jesse spoke almost three times as much as Rebecca. However, Rebecca's contributions were sometimes silent (i.e. she would show Jesse the answer on the calculator, rather than reading it aloud) and Jesse sometimes answered her own questions if Rebecca did not respond immediately. The written work submitted by each of these three groups reflected the outcomes of their discussions and joint decisions and was a reflection of both people's contributions.

On the other hand, one dyad, Rodriguez and Fergus, did not work collaboratively during their assignment. This group was unique in a myriad of ways: they spent only about 50% of their time on task and much of their on-task time was spent working silently and independently; they submitted two separate final solutions; they were the only group to suggest determining the number of cans by dividing the diameter of the cake by the diameter of the can; they submitted work with more calculation errors than the other groups; and, of all the students, Rodriguez seemed to have the strongest conceptual understanding of the relevance of the icing's thickness (see Figure 13). However, contrary to what might be expected from groups who fail to collaborate, neither student appeared unwilling to work on the problem or with their partner. Nor did their conversation resemble the "competitive and uncooperative argumentation" (Mercer, 2008a, p. 51) that Mercer terms disputational talk. Instead, it appeared that these two students were not interested in, or did not know how, to coordinate their mathematical efforts.

When students engaged in collaborative work, many of their conversations centred on the calculations they performed as they worked on the problem, rather than on strategies they were considering. Students often dictated calculations to their partner or repeated them aloud as they entered them into the calculator or wrote them down. It some cases, one student guided mathematical thinking while the other followed along or performed calculations as directed by their partner. As Jesse and Rebecca determined the volume of the frosting can (see Figure 16), it was, as usual, Jesse who took the lead, even when she was unsure of what to do (lines 3; 23-25). Though Jesse was the more vocal student, her leadership may have emerged because Rebecca had a difficult time expressing herself quickly [lines 4, 6, and 24]. In addition, Rebecca's quietude does not necessarily indicate she is dependent upon Jesse's instruction. Rebecca seems to calculate easily what is requested and assists Jesse at times [lines 6 and 24 - 26].

3. *Jesse: Okay, I'll calculate that – frosting!*
4. *Rebecca: First thing*
5. *J: How do you find the area, again (laughing), no how do you find the volume, no find the [base] area first, right?*
6. *R: Area of the circle— of the—*
7. *J: pi r squared times—, no plus—, no, how do you find the area of the circle,*

8. *R: Oh, um (pointing to paper)*
. . .
13. *J: (laughs) Okay, 3.14 , what's 4.5 times 4.5?*
14. *R: It's (working on the calculator) 20.25.*
. . .
19. *J: Okay, that is what?*
20. *R: (inaudible) equals 63 and [585].*
21. *J: cm.*
22. *R: cm.*
23. *J: cm^2. Okay, what's— what's the— how do you do volume again?*
24. *R: Well, it's, it's —*
25. *J: (interrupting) It's this [63.585] time height, right?*
26. *R: Yup.*
27. *J: 63.585 x 11 (as she writes).*
28. *R: Equals 699.435.*
29. *J: 699 point what (looking over at calculator)?*
30. *R: 699.435.*

Figure 16: Jesse takes the lead with finding the area.

In the other two collaborative groups, where both students were equally comfortable speaking, both partners tended to lead the discussion, at different times. In the first section of the excerpt below (See Figure 17), Tidus repeated and affirmed Alexandra's statements, while in the second section it is Alexandra who affirmed Tidus' statements. In these partnerships, the 'following' student also tended to verbally clarify or enhance the 'leader's' comments. For example, Tidus restated 4.5 × 4.5 as 4.5^2 [line 55], clarified that 11 indicated the height of the icing can [line 59], and provided the units [line 61], while Alexandra provided the units [line 327] and clarified the meaning of their calculation [line 329] in the second example.

54. *Alexandra: 4.5. So, this is frosting. 4.5 equals the radius. 4.5 times 4.5 equals—*
55. *Tidus: (working on the calculator) 4.5 squared— 20.25.*
56. *A: So you round it to 20.*
57. *T: You round it to 20.*
58. *A: 20 times 11.*
59. *T: Yup, times height, so 20 times 11, 2 hundred —*
60. *A: 220.*
61. *T: cm squared.*

****

320. *Tidus: Multiply that [63] by height [11].*
321. *Alexandra: that [63]?*
322. *T: Yup.*

323. A: Times 11.
324. T: Yup, so it's 63 times 11.
325. A: 63 times 11.
326. T: equals 693.
327. A: cm cubed.
328. T: That's volume.
329. A: Here, so this [the can] equals 693 cm cubed . . . so we found the volume of that [can].

Figure 17: Alexandra and Tidus take turns leading the discussion.

In some cases, there was no clear leader of the discussion. As Alexandra and Tidus identified the height and radius of the icing can, in preparation to calculate its volume (see Figure 18), both contributed equally meaningful pieces of information: Alexandra identified the height [line 305] and diameter [line 307] and Tidus provided the radius [lines 310 and 312].

305. Alexandra: . . . So this [height of the icing can] is 11.
306. Tidus: 11
307. A: And this [diameter of the icing can] is 9.
308. T: Yes.
309. A: Yup. So what's the—
310. T: We need to find the radius.
311. A: So
312. T: 4.5
313. A: So 9 divided by 2 is 4.5. . . .

Figure 18: Co-constructing the measurements needed to calculate volume.

When Melvin and Christina were determining the volume of the cake (see Figure 19), Melvin suggested the initial idea [line 132] and the calculation for determining the radius [line 136], but it was Christina who corrected Melvin's calculation of π times 10 [line 143] and, later, understood to multiply π by r^2 [line 145]. In addition, it is clear both students made their own calculations, rather than just echoing each other, since Christina responded with "Yeah, I *get* [emphasis added] 10," [line 139] rather than 'Yeah, I agree with your suggestion of 10,' when Melvin suggested the radius was 10 cm [line 138].

132. Melvin: [We found the] volume of the icing. Okay, now, find the volume of the cake, so we go, we have to do the radius.
133. Christina: Okay (pause) okay.
134. M: So it's—
135. C: 3.14. Wait, first I'll find the radius.

136. *M; Yup, so go 20 [diameter of can] divided by 2.*
137. *C: Yeah.*
138. *M: 10.*
139. *C: (working on the calculator) Yeah, I get 10 and then—*
140. *M: And then—*
141. *C: Yeah and then—*
142. *M: Pi, pi times 10.*
143. *C: No 10 times 10.*
144. *M: Oh yeah, yeah— 10 times 10.*
145. *C: 100, and then you multiply [pi] by a hundred. . .*

Figure 19: Christina and Melvin co-constructing the volume of the cake.

Throughout the discussions, students appeared comfortable working with and relying on their partners. There are many instances of one student asking for a specific piece of information that, had they been working alone, they probably could have accessed independently using a calculator or a textbook. The information sought was often of the ‘easily forgettable’ nature, such as a specific formula, hence Alexandra’s question “How do you find the surface area of a circle?”, or of the ‘calculation nature’ such as Jesse’s question “What’s 60 times 20?”.

Within their dyads, some students, especially Alexandra, frequently checked their work with their partner by phrasing suggestions and answers in a manner that required their partner to acknowledge that the solution or suggestion was correct and appropriate. “Should we round?” (Alexandra); “It’s this time height, right? (Jesse); “That’s eight, isn’t it?” (Alexandra); and “It’s 63.58, right?”(Christina) are examples of the different ways students sought confirmation from their partner. It is difficult to determine if students using this communication approach were unsure of themselves, and thus seeking encouragement, or if they were consciously attempting to ensure their partner remained aware of the work they were completing.

When students were confused, they were not reticent about asking their partner for assistance or clarification. At times their queries were articulate and specific such as “Are we getting the volume or the surface area?” (Rebecca) or “Why 4.5?” (Christina). However, the simple, yet rather vague, “What?” was also common. Whether the question required a simple response or a more complex one, students almost always took the time to help their partner.

When both students clearly and consistently contribute to the developing solution, it can be difficult to separate the joint construction of knowledge from either parallel working or from a novice/expert situation. Certainly, there were occasions when the students used each other to verify their work and when one student guided the other. However, within the collaborative groups, student roles appeared flexible and both partners shared and explained their understanding of the problem and knowledge of its related calculations. The students appeared to have developed a trust and confidence in their partners, as evidenced by the ease with which they asked questions, which may have freed them from the need to do everything themselves. As such, they were able to tread the same path, rather than individual ones, as they worked towards a solution. The co-construction of knowledge they demonstrated, whether during a formulaic calculation or during a conceptual discussion, was evident throughout their work and is better appreciated as a characteristic of the entire process rather than of a single excerpt.

4.6 Discussing Calculations

For the three partnerships who worked collaboratively on this problem, they maintained near-continuous dialogue throughout the task. As previously mentioned, much of their discussion centred on the calculations they were performing; comparatively fewer segments focussed on strategy or problem solving approach, though these will be discussed later in section 4.7. Calculation-focussed discussions afforded students the opportunity to reaffirm their use of selected formulae or values to themselves and to their partners, and to detect and correct errors in their partner’s work.

In many cases, students not only stated what they were doing, but often, consciously or unconsciously, provided justification by stating the connections between their calculations, the formula, and the values from the problem. Though there are numerous unfinished thoughts in the following dialogue (see Figure 20), Tidus and Alexandra seemed to follow each other's thinking as they calculated $\pi r^2 h$ (r = 10 cm; h = 10 cm). Tidus moved comfortably between $r^2 = 10^2 = 100$, but twice confirmed why the answer was 100 cm^2. His first justification, seemingly more for himself, occurred in line 29 with the statement "R squared is— so it's 100." The second justification, "because the radius squared is 10 [squared]—" [line 31], was a direct response to a question from Alexandra. In addition, both Alexandra and Tidus justified the decision to multiply by 20 by reaffirming that 20 cm was the height of the cake [lines 37 and 38].

25. *Tidus: . . . 10 squared*
26. *Alexandra: 100, no? So the diameter—*
27. *T: So the radius is equal to—*
28. *A: Why don't you just? Do you want me to? Don't you?*
29. *T: R squared is— So it's 100, so it's pi times 100 is— (working on calculator) Three hundred—*
30. *A: So, 100 times pi?*
31. *T: Pi, yeah, because the radius squared is 10 [squared]—*
32. *A: Equals*
33. *T: 314.15*
34. *A: 314.15. Should we round?*
35. *T: Times that [20]*
36. *A: Should we round?*
37. *T: Round it, yeah, so 314. Okay, times height, times 20 equals*
38. *A: 314 times 20, which is the height.*

Figure 20: Alexandra and Tidus justify their work, despite their numerous unfinished thoughts.

As Christina and Melvin tackled πr^2 (r = 4.5 cm), they struggled to interpret the formula and enter the values into the calculator (see Figure 21). It took multiple attempts before they determined the correct answer of 63.585 cm^2. Throughout the process, Christina verified that r^2 meant to multiply the radius value by itself [lines 81 and 105], which resulted in the calculation of 4.5 × 4.5 [lines 85, 93, and 105]. She and Melvin justified their use of 3.14 by referring back to the formula [lines 88 and 89].

81. *Christina: So multiply that by pi because it's r squared, squared, which means we have to multiply it twice by itself.*

. . .

85. *C: Okay, so you got 4.5, right? So if we get 4.5 (working on the calculator)— You get 4.5. You got to multiply it by 4.5. So we got 20.25. We got to multiply it, multiply it—*

. . .

88. *Melvin: Ahh, pi. Wait, no, because it's pi r squared, not just r squared—*
89. *C: Yeah, I know, but if it's pi — If it's pi r squared, this is telling you you've got to multiply. You've first got to multiply 3.14.*
90. *M: Ohh, I get it.*
91. *C: By this.*
92. *M: I get it.*
93. *C: And you get, wait, what did we have? We have 4.5 multiply by 3.14 by 4.5 and after you get, wait, wait, (going for calculator)— Whoa, we multiply— What the heck?*

…

103. *C: No, dude, we did something wrong here.*

…

105. *C: No, no, no, dude, because, because— We got to multiply the radius twice So we first go 4.5 multiplied by 4.5 and we get . . . 20.25 and then we multiply 20.25 multiplied by 3.14 and we get 63.585.*

Figure 21: Christina and Melvin justify their work.

Students' justifications were often part of the give and take of the conversation and the problem solving process and seemed to occur as students worked through their thought processes aloud. However, at times a student would supply an explanation (albeit incomplete at times) in response to a direct question such as when Christina thought they should multiply by 9 cm, the diameter, instead of 4.5 cm, the radius. Like most of the students' discussions, including Alexandra and Tidus' previous one with unfinished thoughts (see Figure 20), this conversation (see Figure 22) demonstrates the 'shorthand' that was often evident in their discussions; many concepts and statements were never fully stated, but were implied and assumed to be understood.

31. *Christina: Multiplied— Ahh— By 9,*
32. *Melvin: 4.5*
33. *C: Wait, why 4.5?*
34. *M: Because that's half of nine.*
35. *C: Ah, yeah, right.*

Figure 22: Melvin explains why the radius is 4.5 cm.

As students discussed their calculations, they often detected and corrected errors in their work. This was especially true for Melvin and Christina. In Figure 22, Melvin corrected

Christina's incorrect use of the diameter while [line 32], in Figure 23, Christina reminded Melvin that their calculation for volume was not complete until they had multiplied by the height of the frosting can [line 115 and 117].

114. *Melvin: That's [63.585] the volume of the icing [can]. So now we need to find the volume of this [cake], Christina.*
115. *Christina: Okay Melvin,— I know, but we still, we didn't finish it [volume of the icing can] yet.*
116. *M: Oh.*
117. *C: Because we only multiplied the radius and then we, and then we state the— that's not the —*
118. *M: Yeah, and then we find the —*
119. *C: Times it by 11 [height of the icing can].*

Figure 23: Christina points out the need to multiply by height.

In many ways, the students framed their mathematical learning around the formulae which were available to them. It is, therefore, understandable why many of their discussions focussed on their calculations. However, this approach limited the students' interpretations of the problem and their ability to focus on the meaning of the variables within each formula. In addition, since few students worked fluidly with $A = \pi r^2$, $SA = 2\pi r h$, and $V = \pi r^2 h$, their difficulties were further compounded.

4.7 Discussing Strategies and Approaches

During this task, students seldom discussed critically a general plan or considered different potential approaches to the problem. Rather, for any given part of the problem, the partners tended to accept, and then build upon, the first voiced suggestion or strategy. At times, the suggestion was appropriate. However, at other times the lack of questioning resulted in the acceptance of incorrect or inappropriate ideas (see Figure 24) such as calculating the volume of the cake (Melvin and Christina), using circumference to find the area of the icing layers (Alexandra and Tidus), and assuming that $r^2 = 2r = d$ (Rodriguez and Fergus).

132. *Melvin: . . . Okay, now find the volume of the cake.*
133. *Christina: Okay.*

*****

97. *Alexandra: I think that, now, no, we have to find— Is it the circumference? Of the base thing, like of the top and the bottom.*
98. *Tidus: Yeah.*

*****

62. *Rodriguez: . . .pi r squared times height. So 3.14 times, what's the radius?*
63. *Fergus: Ah.*
64. *R: Oh, but wait a sec, do the diameter instead of [r] times 2.*
65. *F: Okay.*

Figure 24: Students agree to their partner's suggestions, without question.

Because students often agreed with their partner's proposal to use a specific formula, procedure, or approach without questioning it, most ideas were not discussed, unless, after the calculations had been completed, the students believed that they had made a mistake. After Jesse and Rebecca determine the volume of the icing can, Jesse asked Rebecca for the cake's measurement (see Figure 10). Neither girl considered the appropriateness of this suggestion. However, it appeared to be an easy calculation for Rebecca and she quickly responded. It was only then that Jesse contextualized (perhaps intuitively?) the calculation, questioned the need to find the volume of the cake, and proposed that it was the 'outside' they needed (see Figure 25, line 59). Rebecca was initially confused by this new suggestion [line 60], but after a brief explanation [line 63], she accepted it [line 64].

59. *Jesse: We do surface area because you only ice the outside of the cake so we do surface area—Ahhhh (erasing)!*
60. *Rebecca: Are we getting the volume or the surface area?*
61. *J: The surface area.*
62. *R: What?*
63. *J: Surface area because you do the outside, surface area is—*
64. *R: Area, then.*

Figure 25: We need the surface area.

In most dialogues, the students did not offer a reason for their acquiescence, which makes it difficult to know why students readily accepted many of the ideas their partners proposed. Occasionally, students may have 'agreed' to something they did not fully understand (see Figure 26). When Melvin checked with Christina to see if she also determined the volume of the cake to equal 6280 cm^3, she affirmed his answer [lines 160 and 161]. However, when pressed, Christina was unable to explain why she agreed with him [lines 162 - 165]. It is also possible that students actually disagreed with some suggestions, but were hesitant to voice their concern. However, this seems unlikely to me given students' apparent comfort disagreeing with each other and asking for clarification, when needed. In my opinion, it seems more likely that the students did not consider the possibility that their initial idea might be incorrect.

160. *Melvin: 6280. Is that what you got?*
161. *Christina: (nodding yes)*
162. *M: Christina, what did you get for the formula?*
163. *C: What do you mean?*
164. *M: What did you do to get that number?*
165. *C: Oh, I—*

Figure 26: Christina agrees, but she may not be sure what she has agreed to.

There are two dialogues (see Figures 27 and 28) in which a student clearly explains why she accepted her partner's suggestion. Interestingly, both appear post-initial suggestion and post-contextualization of the answer. After calculating the volume of the frosting, Christina, who mistakenly interpreted the answer to represent the number of frosting cans needed, 'had a feeling' their answer was incorrect [Figure 27, line 51]. However, because she could not think of an alternative strategy, she conceded to Melvin's suggestion [Figure 27, line 53].

45. *Christina: Really, dude, you've got to be kidding me . . .Ahh! 699 cans to make one cake!*

. . .

51. *Christina: Melvin, I think we're doing it wrong, I just think, I have a feeling.*
52. *Melvin: Okay, what's your idea then?*
53. *C: Ahh, I already showed you my idea, but that's the area [of a triangle] so, let's go with this idea.*

Figure 27: Christina accepts Melvin's idea because she can't think of an alternative.

In the second case, Jesse made a reasoned decision to accept her partner's proposal. When determining the area of the cake's base, Jesse realised that she could take the time to verify the answer herself or she could accept Rebecca's assertion that it was correct.

73. *Jesse: . . . 314. Are you sure?*
74. *Rebecca: Yeah I'm sure.*
75. *J: Okay, then. . .*

Figure 28: Rebecca convinces Jesse that she is correct.

Though students rarely discussed their strategies unless difficulties became apparent, some students – mainly Alexandra and Tidus, but also Rebecca and Jesse – occasionally discussed conceptual aspects of the problem. This usually occurred when the partners disagreed, which placed them in a situation that required them to defend their ideas and produce acceptable justifications. Unlike the justifications students gave while performing calculations, students could not rely predominantly on the authority of the formula or isolated values within the problem. Instead, they also needed to rely on their perception of the problem within its context and the interaction between the context and the aforementioned mathematical factors. As Tidus tried to convince Alexandra that 1/6 of the cake had been cut away, it was her visual markings on the cake that finally enabled him to understand that he was incorrect (See Figure 29). (It is also worth noting that although Alexandra believed determining the size of the missing piece is irrelevant to their problem [line 161], she was drawn into the discussion by Tidus.)

160. *Tidus: Well, you know, but it's missing a chunk.*
161. *Alexandra: Oh, we can just pretend it's not there she [the researcher] said, wait, as if it was full*
162. *T: Because it's like 1/6.*
163. *A: Yeah.*
164. *T:Yeah, yeah (dividing the cake diagram into slices equal to the size of the missing piece).*
165. *A Are you positive?*
166. *T: Yeah, . . . , look at this, it's 1, 2, so there's 1 piece, 1 piece, then there's another piece right here and another piece right here (drawing on diagram).*
167. *A: That's eight, isn't it?*
168. *T: No, it's 1, 2, 3, 4, 5, 6.*
169. *A: 1, 2, 3, 4, 5, 6, 7, 8.*

. . .

172. *J: Okay.*

Figure 29: Tidus and Alexandra discuss the missing piece of cake.

As a disagreement progressed, the students' justifications necessarily became more sophisticated. In Figure 29, Alexandra relied on visual justification to support her statement. In another discussion (See Figure 30) these same students tried to determine whether to account for the icing in the layers by multiplying the base area of the cake by three (Alexandra's opinion) or four (Tidus' opinion). The students' justification progressed from repeating their initial statement to providing a well-reasoned articulated argument.

208. *Alexandra: No, you times it by 3 because there's 3 layers.*
209. *Tidus: Right. No, you times it by 4.*
210. *A: No, 3.*
211. *T: Because there's 1 layer, 2 layers, 3 layers, 4 layers that's how it works . . .*

And later

230. *T: No, it's times 4. Alexandra, look, see this is—. This top part it's 100. This layer's 200, this one's 300, and this layer's 400.*
231. *A: No, but, we already figured this [the top of the cake] out then we figured out the entire cake. We figured out the top, the cylinder part, and the bottom,*
232. *T: Oh.*

Figure 30: Alexandra and Tidus' justifications become more reasoned.

Detecting conceptual errors seemed difficult for students, though, and often required adult assistance. In this first case (See Figure 31), I have a discussion with Melvin and Christina about why they calculated the volume of the cake and what that meant in terms of the problem.

191. *Melvin: We're stuck here now. Okay, so we've gotten the two volumes for each thing. Now we're stuck.*
192. *Researcher: Okay, so you have the volume for this (the icing).*
193. *Christina: Yup.*
194. *R: And what's the other volume that you calculated?*
195. *M: This one (pointing to the cake).*
196. *R: The volume for the cake?*
197. *M: Yes.*
198. *R: And what's that going to tell me?*
199. *M: How much stuff can go inside the cake.*
200. *R: Yeah and is that what you want to know for the icing?*
201. *M: No, you only really want to know that one (can?).*
202. *R: I need to know the volume of that (icing can) because that tells me how much icing is in a can. But what do I want to know?*

. . .

207. M: Oh, the surface area.

Figure 31: What does the volume tell us?

In the following case (See Figure 32), the instructor spoke with Tidus and Alexandra about the reasonableness of their initial answer, inducing them to re-examine their work.

264. Tidus: . . . That's our estimation, we found 29.
265. Instructor: Oh, okay, so let's just pretend if it is 29 cans, right, think about the size of the cans. What's 9 by 11 roughly? That would look like— a can would be like?
266. Alexandra: Ahhhh.
267. I: About 11 cm high is about how high? About that high? And width is almost 9 cm so it's sort of like this, right? And if you need 28 cans about this size to make a cake that's only this high and this wide— You need 28 of those cans?
268. A: No.
269. I: Look at the icing on the cake, too, the picture that they give you. Is there icing all inside the whole cake?
. . .
277. T: So we need the volume of this (pointing to the can) and the surface area of this (pointing to the cake).

Figure 32: A discussion with the instructor causes Alexandra and Tidus to re-examine their work.

When an adult discussed aspects of the problem with a dyad, the adult was able to use their mathematical knowledge of the problem to tailor the discussion to meet the specific needs of a group. For example, when I realised Melvin and Christina had calculated the volume of the cake, I could guide the discussion to the differences between volume and surface area [Figure 31, lines 194-198]. When the instructor realised that Tidus and Alexandra believed 29 cans to be an appropriate estimate, he could help them understand why it might be unreasonable. In addition to guiding the discussion, the adults were likely to repeat [Figure 31, lines 200 and 202] or rephrase [Figure 32, lines 264 and 266] what they were saying and were willing to continue the conversation until they appeared to believe the students understood what they were saying.

4.8 Communication

It is possible for students to evaluate, question, and discuss their ideas effectively. In many circumstances, a student successfully explained a calculation or procedure to their partner.

When this happened, the dialogue served to clarify the partner's understanding of what was occurring or of conceptual aspects of the task. It also helped the students to establish a common ground upon which the further co-construction of knowledge could occur. However, there were also instances when a student's explanation did not seem helpful.

Sometimes confusion occurred if the students offering the explanation did not clearly articulate their knowledge. Christina understood that the length of the rectangle in a cylinder net could be approximated using three times the diameter of the cylinder's base ($2\pi r \approx 3d$). However, her explanation (see Figure 33), which lacked precision and clarity, left Melvin, and possibly any listener, very confused.

247. *Christina: . . . I'll just explain. We find these two, which is exactly these two, first, and after this— we got to— and after we find that we need to do the side. We have to find the area of this, so if there's three can fit in— so if three of the circles fit into one of side, then that means it's 6. So 6 multiplied by— yeah, so it's 6. So 3 times 3 equals 6, and then you multiply 6 by 6.*
248. *Melvin: I have no idea what you're saying.*

Figure 33: Christina's explanation is not clear to Melvin.

Another potential source of miscommunication was the students' use of informal language and 'shorthand'. As was appropriate for the situation, students related to each other in a relaxed manner, rather than a formal academic one. They frequently used 'this' and 'that' in place of specific values, concepts, or measurements. In addition, they only occasionally used units and summary statements to confirm the meaning of their calculations. However, given that students likely also relied on gestures, written work, shared contexts, and familiarity with each other, this style of communication was often successful and the students usually understood each other.

At times, however, the lack of precision may have indicated, or contributed to, confusion. Tidus and Alexandra had numerous conversations in which they discussed how to find the surface area of the cake. Even during their third attempt to find the area of cake's base (see

Figure 34), they believed they were communicating clearly and sharing information, even though it appears they were not. They began with πr^2 [line 223], an appropriate choice since the base of the cake was round, and both seemed content with their answer of 100 cm^2. However, unbeknownst to either of them, they appeared to arrive at this answer in different ways. Alexandra calculated $\pi^2 \times 10$ [lines 223 and 225], which could be interpreted as either $(\pi^2)^2$ or $\pi^2 r$, whereas Tidus seemed to think they calculated r^2 [line 226]. Interestingly, both agreed to the other's logic during the discussion. Tidus, in line 224, appeared to accept Alexandra's proposal of π^2 as part of the equation. Later, in line 227, Alexandra seemed to accept Tidus' assertion that they were calculating 10×10 in order to determine r^2 [line 226].

223. *Alexandra: Okay, let's do it again. Okay to find the surface area: pi r squared. So 3 times 3, or pi times pi, is 9.9. So we round that to 10.*
224. *Tidus: Oh right.*
225. *A: So then we squared that, so it's 10 times 10, which equals 10— [squared]*
226. *T: Right and 10 times 10 is the radius squared.*
227. *A: Yup.*
228. *T: So it's radius squared— so to find it, it's pi r squared.*
229. *A:* pi *r— So it's 100, pi r squared. Yep. . .*

Figure 34: Alexandra and Tidus use different approaches to find surface area.

In all mathematical forums, clear communication is important. However, in collaborative situations it is essential; its absence renders students unable to create, maintain, and develop a space of mutual understanding. Although mathematical communication necessarily encompasses the specialized lexicon, symbolic notation, and visual representations of the discipline, these devices are not integral to all rich mathematical discussions. For these students, their quotidian vocabulary and style of interaction did not prohibit discussion regarding the problem or their approach to solving it. Though I, as an outsider to the discussion, often struggled to make sense of what was being said, students' language, gestures, and actions seemed sufficient to create at least a partial shared understanding.

4.9 Summary of Findings

The mathematical learning demonstrated during the collaborative assessment has grounded these findings. In summary: (1) although many students collaborated throughout the task, their focus on calculations and their tendency to work the problem without planning or reflecting on their solutions, may have affected their success; (2) students commonly employed a relaxed, informal, accepting, and descriptive style of communication, which often enabled mathematical discussions that led to increased shared understanding; (3) student disagreement and adult questioning were sometimes the catalyst needed for students to justify their ideas or re-evaluate their solutions.

5 Discussion

The dual focus of this study was collaborative activity and mathematical learning within an assessment situation. In the discussion I examine both of these components and explore how these students' demonstrated behaviour can inform our understanding of collaborative assessment, including prospective paths for future research and practice in the area. In addition, I briefly discuss the potential benefits and one commonly quoted drawback of collaborative assessment and the roles equal status partnerships and classroom context played in this study. Though I have grounded this discussion in the current findings and in the related literature, it has been influenced by my experiences while teaching middle school mathematics.

As this exploratory study was designed to investigate the practice of collaborative assessment and its potential for use in the middle school mathematics classroom, I have tried to balance the need for a variety of student perspectives with the need for depth and close analysis of individuals' and pairs' experiences. Therefore, I chose to focus on the nuances that may have been overlooked in a larger study, while simultaneously anchoring my attention on mathematical learning rather than on mathematical affect or social factors within the classroom. Any conclusions drawn from this study are done so with the understanding that different students respond differently to similar situations and that each class creates a unique subculture, which influences how collaborative assessment is perceived and enacted.

5.1 Collaborative Activity

Though collaborative activity takes many forms, it is defined in this study by the engagement of students in a common task, with a joint goal that all participants strive towards together. Communication is essential to negotiating a shared understanding of the problem, critically analysing potential strategies and solutions, and overcoming differences that arise.

In this study, three of the four dyads consistently demonstrated behaviours characteristic of collaboration and produced work that was the combined effort of both partners. While all students were given the common task of determining the necessary number of cans of icing, the pairs who collaborated each produced a single jointly-constructed solution to the problem. At times their shared understanding was implied or nearly-implied, as evidenced by the conversations that took place in 'shorthand'. At other times the shared understanding was made more explicit and was created through discussion regarding the problem and the students' strategies for solving it (see Figure 9). In addition, there were occasions when students disagreed, but were able to develop a shared understanding because of their willingness to discuss their differing views and reach a consensus built not on compromise, but on reasoning (see Figure 30).

Essential to the pairs' collaboration was their ability to communicate. Necessarily, the students entered the task with a degree of common ground, including some understanding of volume, surface area, and the context of the problem. It is important to recognise the pre-existing commonalities since there can be no discussion in the complete absence of shared ground (Cobb, Yackel, & Wood, 1992). Discussion, though, is imperative to the expansion of the students' shared understanding. While more discussion does not directly extend the boundaries of common understanding, it does lead to more possibilities for extension. Similarly, lack of discussion reduces the opportunities available to extend these boundaries. For the three collaborative dyads in this study, their near continuous communication seemed to provide numerous opportunities to develop and enhance their collective understanding of the problem. At times, this enabled them to build on each other's suggestions and refine their approach. Thus, as Fernadez, Wegerif, Mercer, and Rojas-Drummond (2001) explain, for these pairs:

> the correct solution, then, is a joint achievement, generated by the collective thinking activity of the . . . participants. We might therefore say that we are observing a conversation situated in a joint Zone of Proximal Development, in which language is enabling them to provide mutual intellectual support or 'scaffolding'. (p. 51)

For the non-collaborative dyad, however, their lack of mathematical communication resulted in separate experiences; upon completion of the assessment their *common* understanding of the problem was similar to what it had been at the beginning of the task, as evidenced by the different solutions each student submitted. Had they followed instructions and submitted a single assignment, even after working independently, it would have been interesting to see the results. Would Rodriguez have taken the time to more fully explain his understanding of the problem to Fergus? Would Rodriguez have forgone the effort an explanation required and have accepted Fergus's solution? In either case, the results would have provided insight as to what can occur when students with a tendency towards, or preference for, individual work are denied the opportunity of completing their own assessment. As it is, this pair's response suggests that not all groups collaborate when provided the opportunity to do so.

Though the other groups collaborated more consistently than Rodriguez and Fergus, their interaction did not always lead to mathematical insight or success with the problem. For the most part, students in this class worked nicely together; they shared ideas, asked questions, and encouraged each other. As such, they most often demonstrated what Mercer terms cumulative talk, talk in which "speakers build positively but uncritically on what the other has said" (Mercer, 1996, p. 369). To me they appeared happy, engaged, and cooperative, though not necessarily as effective as possible.

Past research, though often vague, has acknowledged the need to instruct students how to work collaboratively. As I did in this study, instruction often focussed on directing students to respect each other, to participate, and to ask questions when needed. Most students in this study demonstrated each of these characteristics, yet the groups still struggled to collectively evolve their understanding of the problem and their strategies for approaching it. Given that these students already enjoyed pleasant working relationships with each other, this finding suggests that such instruction does not necessarily ensure effective collaboration amongst grade 8 students.

In examining students' talk, it is important to differentiate between quality and quantity. More does not necessarily equate to better: by some measures, American students talk more than their Japanese counterparts, but it is the Japanese students who are more likely to discuss, explain, and justify their mathematical reasoning (Sfard, Nesher, Streefland, Cobb, & Mason, 1998). Mercer's recent work (Mercer, 2008a, 2008b; Mercer & Sams, 2006) further supports the need to emphasise quality, rather than quantity, claiming "if teachers provide children with an explicit, practical introduction to the use of language for collective reasoning, then children learn better ways of thinking collectively and better ways of thinking alone" (Mercer & Sams, 2006, p. 525). Simply stated, Mercer believes that increasing the level of exploratory talk between students increases their ability to solve problems successfully. Exploratory talk is rare amongst British primary children (Mercer, 2008a), a phenomenon that seems to be partially replicated here, given that these students sporadically, and usually only in response to difficulty or dispute, questioned each other's suggestions or provided coherent reasoning for their own.

Interestingly, students in this study rarely displayed the type of talk Mercer refers to as disputational. At one point, Alexandra and Tidus engaged in a classic 'I'm right' / 'No, I'm right' debate regarding the number of icing layers but eventually, perhaps because they realised the counter-productiveness of the discussion, they switched to a more exploratory style of talk in which they offered justification for their reasoning (see Figure 30). In class discussions, students will provide more justification for their work when their ideas or solutions are questioned (Maher, Powell, Weber, & Lee, 2006); possibly, the same is true in a collaborative assessment situation.

An apparent advantage of exploratory talk is that student explanations, because of the working-together-on-a-common-task context in which they are obtained, are essentially different from teacher explanations. Supposedly, these explanations enable students to support each other and make further progress than they would have individually (Fernandez, Wegerif, Mercer, & Rojas-Drummond, 2001). In this study, students felt comfortable asking their partner direct questions and often benefited from the explanations provided. However, the benefits of student explanations may depend upon the student's ability to elucidate their reasoning and provide a coherent explanation. When students were confused, frustrated, or otherwise unable to articulate their thoughts (see Figure 33), the explanations did not appear immediately helpful, nor did they appear to further their understanding or the understanding of their partner. However, this is difficult to assess with this study since increased understanding is not always apparent forthwith; often, a comment or conversation is the trigger for learning that is not apparent until a (much) later point in time.

Effective student explanations are not necessarily qualitatively different from those which the teacher would have provided, however teacher explanations are likely imbued with more authority (Amit & Fried, 2005), which may change how students perceive them. For example, in this study, students would challenge a peer's explanation, but not an instructor's or researcher's. Similarly, when students questioned their partner there appeared an implicit acceptance that their partner may not know the answer and they would work through the solution together. However, when an adult was questioned, the expectation was that an answer would be forthcoming.

While the authority of the teacher did not fall under the initial scope of this study, it became relevant since both the teacher and I interacted with the students throughout the assessment. While the effect of this interaction is difficult to measure, I believe it is fair to conclude that it affected the mathematical learning demonstrated. For example, when Christina suggested to Melvin that they had made a mistake because 699 cans of frosting was an unreasonable answer (see Figure 27), Melvin acknowledged the statement, but neither student appeared to take it seriously and both students quickly gave up on the idea of finding an alternative strategy. However, when the instructor suggested to Alexandra and Tidus that 29 cans might be unreasonable (see Figure 32), both students accepted that they had made a mistake and became committed to finding it.

5.2 Approaches to Mathematical Learning

In this task, students initially seemed to approach the problem with a calculational, as opposed to conceptual, orientation (Thompson, Philipp, Thompson, & Boyd, 1994); they often selected and performed calculations without considering the context or applicability of a formula or procedure. For example, all groups determined the volume of cake, a step that was neither needed nor useful, given the context of the problem and the students' approach to solving it. In addition, the students seemed to focus on obtaining an answer, rather than on considering the meaning of the answer obtained, which may explain why 29 cans did not strike Alexandra and Tidus as an inappropriate solution and why Christina expressed concern that they were "doing it wrong" when a volume calculation correctly yielded a large value.

Students seemed both accustomed to and relatively successful working from a calculational orientation. They could often identify, discuss, and overcome calculational difficulties that arose and, at least in Melvin and Christina's case, they attributed their inability to solve the problem to their difficulty selecting and applying the appropriate formula(e), rather than to understanding the problem (see Section 4.1).

Though discussions regarding calculations and procedures appeared to dominate the conversations, students occasionally, and with varying degrees of success, displayed a conceptual orientation by exploring the problem, their calculations, and their answers within the context provided. For example, although Christina did not focus on the meaning of her selected formula, she did appreciate the meaning of her answers. Her concern with large numbers stemmed from her (incorrect) belief that the volume calculations indicated the number of icing cans needed and she readily appreciated the absurdity of requiring hundreds of cans “to make one little cake”. Students who modified their surface area calculations to account for the un-iced bottom of the cake were also demonstrating an understanding that the problem occurred within a given context and could not be solved by applying the standard surface-area-of-a-cylinder formula.

Students’ conceptual orientations developed as they worked on the problem, often in response to difficulties that arose. As opposed to using a conceptual orientation to develop a plan or strategy, it was usually subsequent to performing their calculations that students reflected upon their answer and considered the implication and meaning of their solution (see section 4.2 and Figure 10).

When conceptual difficulties arose students often struggled to navigate them successfully. Perhaps, the deep connected knowledge needed for conceptual understanding is difficult for novice learners to acquire which may explain why Christina finally abandoned her attempts to explain the connection between nets and surface area to Melvin and why Jesse had a difficult time explaining the icing thickness to Rebecca. Tidus and Alexandra successfully resolved their conceptual difficulty regarding how many ‘layers’ of icing existed, but it required a lengthy roundabout conversation (see Figure 30 for an excerpt).

With this research, there are various potential explanations for students’ apparent dominant calculational orientation including the pervasive ethos of school mathematics, the teacher’s approach to the discipline, and the students themselves.

School mathematics is often regarded as a field in which the one right answer can be obtained quickly by correctly following the necessary algorithm. As early as kindergarten, many students begin to accept that mathematics is comprised of specific processes and right answers (Anderson & Gold, 2006). For students in this study, these beliefs are also reinforced by a provincial curriculum that prepares students for a mandatory grade 10 exam in which students have an average of two minutes per question. It is reasonable to conceive that embedded systemic expectations, which emphasise solution over process have, and will continue to have, influence on the students’ approach to problem solving.

Similarly, it is important to recognise the impact of the classroom teacher. From our discussions and from the lessons I observed, I believe that the classroom teacher felt it was important for students to understand the concepts behind the formulae they used. She prepared activities that would allow students to explore and experiment with developing ideas and understandings. However, she also responded to the school culture, which tended to favour more procedural knowledge.

Alternatively, the students’ calculational orientations may be indicative not of external factors, but of their novice problem solving skills. Thinking conceptually about a problem requires forethought and planning, two strategies these students rarely engaged. Instead, students in this study usually seemed to pursue the first idea suggested. This is typical of novice problem solvers who, compared with experts, are more likely to begin a problem by plugging numbers into a formula rather than taking the time to understand it (Bransford, Brown, & Cocking, 2000). Also typical of novice problem solvers, these students appeared to try different operations on the numbers (e.g. by first calculating the volume of the cake, then its surface area) and did not consistently and independently conceptualise the problem within the context it was presented or consider the reasonableness of their answers (Muir, Beswick, & Williamson, 2008). A final difference between expert and novice problem solvers seems almost unworthy of mentioning since the observation that experts make better use of diagrams was based on a comparison between expert mathematicians and 'novice' undergraduates working on calculus problems (Stylianou & Silver, 2004). However, the novices' interpretations of diagrams are relevant given that in this study, students did not seem to appreciate that valid information could be communicated via the picture of the cake. For example, students did not automatically include the icing in the layers in their calculations, nor did they automatically accept that the bottom of the cake was not iced, even though these pieces of information were apparent in the picture.

It is unknown whether the students' novice problem-solving tendencies result from age, experience or, more likely, a combination of the two. In either case, with the exception of being *unwilling* to reflect on the appropriateness of their answers, these students seem remarkably similar to the grade 6 students Muir and Beswick (2005) asked to perform non-routine mathematical problems:

> Most students seemed not to monitor their progress, reflect on the appropriateness of the strategy they had chosen, or display any inclination to try an alternate strategy even when frustrated by their lack of progress. Students were similarly unwilling to reflect on the appropriateness of the answer they obtained or to attempt to confirm it using an alternate method. (p. 567)

Up to this point, I have discussed the conceptual and calculational orientations demonstrated by the students as though they were separable and dichotomous. However, they may not be as polarised as Thompson, Philipp, Thompson, and Boyd (1994) suggest. Conceiving answer getting and meaningful discussion as mutually compatible, as two sides of the same problem solving coin, may help to extrapolate their interwoven connections and to clarify how students approached this assessment task.

Conceptual understanding, owing to its rich contextual and interconnected nature, arguably requires more time, effort, and practice to master than any algorithm, which can be memorised. To be done well, students must think abstractly, see the mathematical possibilities within a given situation, and "link related concepts and methods in appropriate ways" (Kilpatrick, Swafford, & Findell, 2001, p. 119). In this task, one might expect students with a strongly developed sense of conceptual understanding to appreciate that icing has a thickness, and thus is three-dimensional, which means its volume can be determined. Similarly, one might expect students' two-dimensional representations of the surface area of the cake to include only one 'base' to account for the un-iced portion that is in contact with the plate.

However, it is difficult for students to successfully approach problems conceptually if they lack the co-requisite skills and methods such as selecting and applying an appropriate formula (for further discussion, see Wu, 1999). In North American style classrooms such as this, there is a tendency to dismiss the memorisation, skill work, and repetitive practice that, in other cultures, are considered prerequisites to developing a deep understanding of the underlying concepts (Leung, 2001). Consequently, students may not possess the necessary foundation on which to build the firm conceptual understandings beneficial to solving open-ended novel problems.

The intricate links between the conceptual and the calculational were clearly evidenced by the dyads in this research who struggled with this task as a result of their inability to perform calculations correctly. Most notable was Alexandra and Tidus whose confusion over πr^2 contributed to their difficulties regarding the amount of icing on the cake (see Figures 8 and 34) and Rodriguez whose inability to work with the task's variables and formulae potentially overshadowed his nuanced understanding of how to approach the problem conceptually (see Figures 13 and 14).

A polemic view of mathematical orientation induces a tendency to categorise actions and statements, rather than view them as development along a continuum. Though the students spent most of their time discussing calculations, it may be that, at times, they used the formula(e) to frame their understanding of the problem. For example, Christina and Melvin's surface area discussion clearly focussed on determining the necessary measurements and calculations (see Figure 5). However, it also demonstrated they understood that the iced area of the cake could be represented as a two-dimensional sketch, rather than the three-dimensional object in the photograph.

Similarly, the students' apparent lack of conceptual discussion does not necessarily indicate lack of conceptual thought. Although Jesse's statement, "So, you can divide that by that . . . ", was the most articulate explanation either of the girls offered for determining the number of frosting cans needed, this approach cannot be considered algorithmic given the unlikelihood that they had memorised a specific formula for determining the number of icing cans needed and instead were applying their conceptual understandings of division and amount to the problem. When students do not voice their conceptual understandings it is difficult to know if they considered the concepts too basic to require explanation or if they were not yet able to clearly describe the understanding they possessed (Kilpatrick, Swafford, & Findell, 2001).

5.3 Potential Benefits of Collaborative Assessment

Though this study did not specifically investigate the myriad potential benefits collaboration may afford students and researchers, it does offer some insight in these areas. Comments from the participants' self-reports corroborate claims from previous studies (Berry & Nyman, 2002; Hancock, 2007; Lambiotte, Dansereau, Rocklin, & Fletcher, 1987; Zimbardo, Butler, & Wolfe, 2003) that students respond positively to the opportunity to work together on assessments: all but two of the students in this study would opt to work with a partner again if given the opportunity. Like university students, these middle-schoolers appeared to appreciate the various contributions of their partners, which included providing an alternative view point, assisting with and double-checking calculations, and helping to get ideas flowing.

For the majority of students who responded favourably to collaborative assessment, it seemed to provide a pleasurable working environment, in addition to the opportunity to engage with and discuss the problem in new ways. Having a partner who could lend support and assistance seemed a positive benefit of this task. In addition, some students claimed to enjoy collaboration, even though they could not clearly explain why. As one student said, "It is just fun. I know that is vague, but it is true".

Though the benefits of collaboration are often framed in reference to the student, this format of work may also be helpful to teachers and researchers who are attempting to gain a more complete understanding of student knowledge and reasoning.

With written assessments, the teacher's view of student comprehension is limited to what can be gleaned from the final product. However, the brief one to two pages of (mostly) calculations that students submitted for this task did not reflect the complexity of their thought and did not enable the teacher to know which ideas were discussed and discarded, which ones reverberated throughout the task, which ones students struggled to grasp, and which ones students used fluidly. While it is unreasonable to expect the teacher to know what is taking place within each group at all times, listening to partial discussions may afford the opportunity to ascertain a more balanced view of the students' mathematical learning.

In addition, the student talk required for collaborative work afforded the researcher the unique opportunity to observe the spiralling of student ideas and the amount of time students spent formulating and refining these ideas. Although written work tended to be organised and linear, analysis of the discussion showed that students revisited ideas and calculations. Since some ideas developed as the task progressed, there were numerous points in the conversation that could be used to determine a student's level of understanding. Whereas written work may have indicated that a student did not understand a given concept, the discussion may have revealed a partial level of understanding or a particular misconception that was affecting a student's progress.

5.4 Potential Drawback of Collaborative Assessment

In theory, one potential drawback of collaborative assessment is that individual accountability is reduced or eliminated. Undoubtedly, when the students collaborated on the mathematical assessment in this study, teasing out their individual contributions would have been complex, time-consuming, and considerably subjective. Like many daily conversations, the students' discussions ebbed and flowed, meandering and non-linear, from topic to topic. Individuals' ideas were picked up, dropped, and then revisited, making it impossible to measure the impact of a single comment. Therefore, it would have been very difficult to assign individual marks based upon individual participation.

The primary dilemma with reduced individual accountability is usually one of perceived fairness: Is it fair for students in a group to receive the same mark or should an individual's score reflect their personal contribution (Hunter, 2006)? Therefore, some educators who readily acknowledge the benefits of collaboration are hesitant to allow it during assessment, especially if they feel they will face pressure from parents. When the principal of the school where I conducted the study heard about my research topic, this worry was his immediate response and his only concern.

Although, the fairness quandary can be sidestepped if summative individual tasks are used as assessments *of* learning while collaborative tasks are restricted to assessment *for* learning (Hargreaves, 2007), this study is premised on the stance that collaborative tasks can serve both assessment goals effectively. From the findings, it is clear that when students collaborate effectively a single score is a fair reflection of the joint work and ability of both students. In addition, I suggest that in the appropriate circumstances individual accountability may be less important to parents and students than some educators believe. For example, none of the students or parents in this study brought forth concerns regarding the collaborative nature of this assessment.

5.5 Equal–status Partnerships

One significant aspect of this study that may have affected how students collaborated was the formation of dyads based on equal-status partnerships. Teachers consider a variety of variables when matching students for group work. At times a teacher may pair students randomly or based on a common interest, need, or learning style. At other times, a teacher may pair students so their strengths complement each other (i.e. pairing a student who follows directions closely with one who approaches the problem in a more holistic manner). In this study the specific goal was to pair students so they were equally matched, thus reducing any power differential that might exist between the partners.

In most cases, this approach appeared advantageous and provided an appropriate milieu in which joint problem solving construction could develop. However, what is good for most (or even some) students is rarely the most appropriate option for everyone. Given the highly contextual and individual aspects of learning, it is worth examining the case in this study in which equally matched students did not work effectively together.

Rodriguez and Fernandez did not engage collaboratively on this task and the mathematical learning they demonstrated was a reflection of their individual efforts, which usurped any weak attempts either of them made at collaborating. However, even upon reflection, they appear well-matched on a variety of characteristics. The teacher perceived both students to be competent, but weak, in mathematics; this perception is at least partially supported by the students' written work on the problem. And, though Rodriguez actually had a fairly sophisticated understanding of the problem, he had difficulty applying this knowledge and effecting an appropriate solution. The students were friends who listened, and responded, to each other's comments. In addition, both students displayed a similar temperament, were easily distracted by near-by students, and appeared more interested in discussing common interests than in discussing the problem.

Despite the equal match, or possibly because of it, this group did not work effectively together; they did not engage collaboratively and the mathematical learning they demonstrated was a reflection of their individual efforts. While equally matched offers a variety of potential benefits, it also has potential drawbacks. One potential drawback of equally matching students is the possibility that some students would have been more focussed and demonstrated greater understanding if they had not been distracted by each other and had been given the opportunity to work independently or with another student who may have kept them more focussed. For example, unlike situations where the teacher pairs a student who is easily distracted with one who is more focussed, neither of these partners effectively regulated the group's off-task behaviour, which likely influenced the quality of their work and the mathematical learning they demonstrated. Even in equally-matched groups who are relatively focussed, there is the potential for one student to lead the other off-track, as Tidus did with Alexandra when he wanted to discuss the missing chunk of cake (See Figure 29). In the case of Rodriguez a more complimentary pairing may have been to partner him with a student possessing a strong background in calculational skills, thus allowing both students to benefit from the strengths of the other. However, for such a partnership to work, I suspect Rodriguez would need to communicate more openly with his partner. For example, although Fergus correctly understood the request to determine the number of icing cans needed, Rodriguez did not appropriate this notion in his final written solution.

5.6 The Classroom Context

Much has been written about the unique qualities of school mathematics and the contrived nature of many 'authentic' problems (see Boaler, 1994; Palm, 2007, 2008). I did not choose this problem because I believed it to be authentic; I selected it because I believed it would provide a relatable context, within which students could apply their understanding of volume and surface area. It was designed to allow for the application of recently studied concepts in an open-ended situation requiring the consideration of various variables.

However, students seemed to assume that a right answer and a right approach existed and were hindered by the school mathematics lens with which they consciously or unconsciously viewed the problem. Students questioned notions they may otherwise have considered obvious, such as whether or not to ice the bottom of the cake or include the layers of icing when estimating how much icing to buy. This diverted their focus and reduced the amount of time they had available to focus on other aspects of the problem. Their recent work with volume and surface area (correctly) informed their decisions to apply these formulae, yet it also seemed to prevent them from applying the formulae in novel and flexible ways.

5.7 Implications for Future Research and Practice

This study has established that it is possible for middle school mathematics students to work collaboratively on an in-class assessment task. In doing so, it has left me confident that collaborative assessment can adequately and concurrently function as both an assessment of learning (emphasis on what a student has already learned) and an assessment for learning (emphasis on what a student has yet to learn). As such, I can easily justify its use to myself and to others in my school communities. However, it has also established that effective collaboration requires more than a common task and a partner. I now better appreciate the diversity of student interactions that may occur during collaboration and, consequently, will be more likely to discuss with students the characteristics of effective collaboration as well as strategies for improving collaborative outcomes.

Further research that explores the quality of talk between partners, the factors affecting this talk, and the potential outcomes associated with different discourses may help students and teachers to maximise collaborative assessment time. In this study, it was difficult for students to detect conceptual misunderstandings without adult assistance (see Figures 31 and 32). Although its methods may need to be modified slightly for older students, direct instruction aimed at explicitly increasing exploratory talk (Mercer, 2008a) is one strategy worthy of investigation as it may help students and teachers conceive of collaboration as working together effectively, as opposed to working together nicely. In addition, teachers could emphasise the development of communication skills required to discuss a problem's concepts, as well as its calculations. By giving instruction that focuses on pre-planning and evaluating mathematical solutions and on acknowledging, addressing, and overcoming mathematical difficulties, students may acquire the collective freedom to analyse tasks from multiple perspectives, thus enabling them to deliberately select appropriate formulae based on how they understand the problem, rather than allowing the formulae they use determine how they conceive the problem.

Similarly, providing students the ability to develop and discuss their conceptual orientations proactively, rather than retroactively, may also be beneficial and may be especially relevant to middle school students since, in this study, the students showed an overwhelming tendency to accept their partner's suggestions without discussion (see Figures 24, 26, 27, and 28).

It is unknown if students can develop the skills needed to provide each other with the same level of direction and assistance that the teacher can provide. With this task, student pairs worked relatively independently, but adults occasionally highlighted specific considerations for individual groups, especially if they had clearly overlooked something. This served to redirect the students' attention, reduce frustration that would have become counter-productive, and/or push students to consider the problem differently. Although not all assessment situations are flexible in this regard, reducing this additional level of support may impact the difficulty level of questions that can be reasonably considered for collaborative assessment.

In addition, future research in the area of collaborative assessment needs to examine factors other than the quality of student talk. Studies that focus on students who seem prone to struggle with or dislike collaborative activities are needed to better understand how these students respond to such an assessment. From a mathematical stance, further research could investigate how changes in behaviour, such as developing an initial problem solving plan or using precise vocabulary to clearly communicate conceptual understandings, can affect students' abilities to collaborate effectively.

While single-time studies, such as this, provide some insight into collaborative assessment, longer studies in a variety of situations and with a variety of tasks are also needed. It is possible that experience changes student interactions; students who frequently engage in collaborative assessment may, though practice, become more skilled at justifying their opinions and questioning their partner's statements. Or, students who are consistently expected to participate in collaborative assessment may experience a decreased enjoyment of it, which may affect their willingness to invest fully.

In addition, considering different approaches to data collection and analysis may provide different insights. I believe an important consideration for any collaborative assessment study is the maintenance of ecological validity. However, more obtrusive methods of data collection, such as a researcher-observer or a greater number of video cameras that record not only student discussion, but also non-verbal communication and real time data of what the students write down, would provide a greater wealth of data on which to base claims and insights.

Collaborative activities are not the panacea many initially believed. To enhance students' mathematical understandings and their ability to communicate these understandings, collaborative assessment must be used appropriately. Further research will help practitioners and researchers understand how best it can be implemented and help assuage concerns regarding reduced individual accountability. However, curriculum change is challenging for teachers to implement (Tirosh & Graeber, 2003); beneficial suggestions to improve classroom practice often remain unacknowledged. Therefore, future research will need to explore not only what is required for collaborative assessment to be most effective, but also what is required for teachers to begin implementing it.

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Appendix A: Selected Grade 8 Shape and Space Objectives
(Ministry of Education, 2008, pp. 67-68)

C2 draw and construct nets for 3-D objects [C, CN, PS, V]	❑ match a given net to the 3-D object it represents ❑ construct a 3-D object from a given net ❑ draw nets for a given right circular cylinder, right rectangular prism, and right triangular prism, and verify by constructing the 3-D objects from the nets ❑ predict 3-D objects that can be created from a given net and verify the prediction
C3 determine the surface area of - right rectangular prisms - right triangular prisms - right cylinders to solve problems [C, CN, PS, R, V]	❑ explain, using examples, the relationship between the area of 2-D shapes and the surface area of a given 3-D object ❑ identify all the faces of a given prism, including right rectangular and right triangular prisms ❑ describe and apply strategies for determining the surface area of a given right rectangular or right triangular prism ❑ describe and apply strategies for determining the surface area of a given right cylinder ❑ solve a given problem involving surface area
C4 develop and apply formulas for determining the volume of right prisms and right cylinders [C, CN, PS, R, V]	❑ determine the volume of a given right prism, given the area of the base ❑ generalize and apply a rule for determining the volume of right cylinders ❑ explain the connection between the area of the base of a given right 3-D object and the formula for the volume of the object ❑ demonstrate that the orientation of a given 3-D object does not affect its volume ❑ apply a formula to solve a given problem involving the volume of a right cylinder or a right prism

[C]	Communication	[ME]	Mental Mathematics and Estimation	[PS]	Problem Solving	[T]	Technology
[CN]	Connections			[R]	Reasoning	[V]	Visualization

Appendix B: Icing the Cake

Diameter of can = 9 cm
Height of can = 11cm

Diameter of cake = 20 cm
Height of cake = 20 cm

1. Estimate how many cans of this frosting are needed to make this cake.
Justify your answer mathematically.

2. Do you think your answer is reasonable and makes sense? Explain.

3. Do you think your answer is accurate? Do you think it is ‘accurate enough’ for this problem? Explain.

4. How could you have made your answer more accurate?

Appendix C: Enlarging a 3D Shape

When a 3D shape is enlarged, both its dimensions and its volume increase.

But, the dimensions and the volume don't increase proportionately.

For example, if the dimensions are doubled, the volume is NOT doubled.

#1. If the dimensions of a 3D shape are doubled, what happens to the volume? Try to explain in words and using mathematical notation.

#2. If the dimensions are tripled, what happens to the volume? Try to explain in words and using mathematical notation.

#3. a) Find a rule that explains the general relationship between the increase in dimensions and the increase in volume. For example, if the dimensions are increased by a factor of x, what happens to the volume?

- Show and organize all of your work.
- How many (different) examples will you need to be convinced your rule works?
- Try to explain in words and using mathematical notation.

b) Explain WHY you think your rule works.

#4. Use your rule to answer the questions below:

a) A given object has a volume of $25cm^3$. Each of its dimensions is made 5 times bigger. What is the new volume?

b) The diameter of the Earth is 8,000 miles. The diameter of the Sun is 1,000,000 miles. How many times bigger in volume is the Sun compared with the Earth.

Appendix D: After Task Reflection Questions

- Do you think this assessment was a 'true reflection' of what you learned about volume and/or surface area during this unit? Does it fairly show what you know about the topic? Explain.
- Do you think working with a partner helped you to do better on this assignment? Why or why not?
- Do you think you helped your partner to do better on this assignment? Why or why not?
- If you could do this assessment over, would you still choose to work with a partner? Why or why not?
- What, if anything, did you learn (about mathematics or about something else) during this assessment?
- Is there anything else you'd like to share about this experience?

Appendix E: Transcript of Rebecca and Jesse's Dialogue

1. **Jesse**: Okay, diameter of a can. This is— oh— I'm not supposed to draw this am I?
2. **Instructor**: So if you need more scrap paper, just let me know. I'll give you a couple of pages to start.
3. **Jesse**: Okay, I'll calculate that – frosting!
4. **Rebecca:** First thing
5. J: How do you find the area, again (laughing), no how do you find the volume, no find the [base] area first, right?
6. R: Area of the circle— of the—
7. J: Pi r squared times—, no plus—, no, how do you find the area of the circle,
8. R: Oh, um. (pointing to paper)
9. J: Is that on? (points to camera)
10. R: Oh, um, um, yep.
11. J: Bah.
12. R: I can see you actually.
13. J: (laughs) Okay, 3.14. What's 4.5 times 4.5?
14. R: It's (working on the calculator) 20.25.
15. J: No, no, no (as R pushes the incorrect buttons on calc)
16. R: Oh, 4.
17. J: That's not a 4.
18. R: Oops, 20.25.
19. J: Okay, that is what?
20. R: (inaudible) equals 63 and [585].
21. J: cm.
22. R: cm.
23. J: cm^2. Okay, what's— what's the— how do you do volume again?
24. R: Well, it's, it's —
25. J: (interrupting) It's this [63.585] time height, right?
26. R: Yup.
27. J: 63.585 x 11 (as she writes).
28. R: Equals 699.435.
29. J: 699 point what (looking over at calculator)?
30. R: 699.435.
31. J: Now what? And this— what? Now we have to do cake. Oh my god!
32. R: So, um, we need to estimate.
33. J: Fine, okay, well that's about, 20, 20, 10.
34. R: So the r is 20 then? And, yeah—
35. J: That's like, like 10, 20, no 20, what 60?
36. R: What?
37. J: What's 60 times 20?
38. R: 60, oh, times 20, um 1200.
39. J: 1200 right? (spoken the same time as R gives answer) So it's about 2 cans of frosting – that's my estimate.
40. R: So now what?
41. J: What's your estimate? . . . What?
42. R: But this cake is cut in half so— anyway— (indicating the piece missing)
43. J: It's an estimate.

44. R: 1200
45. J: No, how many things of this. (tapping the paper, on the can)
46. R: Um.
47. J: I think it's two
48. R: 5 times 65
49. J: What? That's a lie.
50. R: I know, I know.
51. J: No, it's only like how many containers of icing are you needing for spreading like the outside of that.
52. R: (inaudible)
53. J: Is an estimate of nine a reasonable number?
54. Researcher: You're going to need to convince me mathematically that it makes sense.
55. J: Each [icing] can holds [699cm^3]— right — Okay, then, what? How big is the cake, right?
56. R: The cake. The volume is 6280. The total volume.
57. J: Let me write that down. No, don't we need the outside?
58. R: What do you mean the outside?
59. J: We do surface area because you only ice the outside of the cake so we do surface area—Ahhhh (erasing)!
60. R: Are we getting the volume or the surface area?
61. J: The surface area.
62. R: What?
63. J: Surface area because you do the outside, surface area is—
64. R: Area, then.
65. J: Pi r squared.
66. R: Pi r is— oh—
67. J: Isn't it just pi r squared? Yeah, just pi r squared.
68. R: (inaudible, as she gets text book)
69. J: 3.14 times (inaudible). Then what?
70. R: 100
71. J: Can I see— can I do it one more time?
72. R: Good, 300—
73. J: One more time. 314. Are you sure?
74. R: Yeah I'm sure.
75. J: Okay, then. That's the area for 2. It's 2 pi r h.
76. R: Yup.
77. J: So 2 times 3.14 times 10 times— times 20. . . What's that?
78. R: 1256
79. J: Okay, I'm going to do surface area is 1256 plus 314.
80. R: No, no, is the outer, no never mind.
81. J: I'll be right back. (leaves desk)
82. R: (takes paper and writes/ erases)
83. J: (returns to desk, inaudible)
84. R: Okay.
85. J: (inaudible) You erased it? Okay.
86. R: 1570. (showing J the calculator)
87. J: 600. Goes into that [1570] like two and a half times. So my estimate— what's your estimate?

88. R: My estimate?
89. J: How many cans of icing do you think we need?
90. R: Um . .. 2 cans.
91. J: I think 2 and a half because this [volume of the can] is like 700.
92. R: Yeah.
93. J: So that's 700 times 2 is 1400, but that's not what that means. Yeah— oh—
94. R: Well, isn't 2 like almost—
95. J: What?
96. R: 2 cans is almost like get the whole— (makes motion of the top of the cake)
97. J: Yeah, well almost. But that isn't— um, it's like two and a quarter.
98. R: But we forget to, um, do something.
99. J: Probably. What?
100. R: We cut the cake out so the surface area— we should—
101. J: We put the icing in the middle? I don't get it.
102. R: (drawing) Here . . . But then they cut it like a quarter of the cake so we don't have all the—
103. J: That's hard. I know (turning to another group). Do we have to cut out the piece of the cake? (Other group responds that they don't need to account for the missing piece.)
104. J: Okay.
105. R: We don't.
106. J: No. Okay, and then [question number] two because I said so— because — two because— what's 2 times 699?
107. R: (starts to read off numbers on the calc)

(Jesse gets up to leave, R picks up pencil and looking at the calc, starts writing)

108. J (coming back and sitting down): Okay, so, its 699. Oh. Okay. Pencil (taking pencil back from j and writing): Right? (typing into calculator) Oh, it's over, oh well.
109. R: Well almost.
110. J: Do you think your answer is accurate? Yes.
111. R: Yes, because we used (inaudible)
112. J: Okay, we don't have to do the cut out piece do we?
113. Researcher: No.
114. J: I have a question for you – it says do you think your answer is accurate, what does that mean?
115. Researcher: It means do you think your answer is right?
116. J: Kind of. . . it's accurate.
117. R: The area—we cover all the area of the—
118. J: (interrupting) Because it's more than enough. Yup . . . because you can't have two and a half cans, whatever.
119. R: I mean—
120. J: Because, because it covers—
121. R: Well, you almost need extra, you know.
122. J: Whoops.
123. R: You cover more than enough.
124. J: Enough what?
125. R: Enough, uh, the total surface area of the cake.
126. J: Whoops. (picking up eraser) How can you make your answer more accurate? Do you want to do that part? I don't want to do that part.
127. R: I can, I don't know.

128. J: Just put [question number] 4. Somewhere. Oh, I already have so you just put 4 and it would be like, um, 1570, yes?
129. R: Are we, like, do we copy down the question?
130. J: No, like, to make it more accurate we can divide 699.435 um into— yeah, yeah, equals— (R: writing) oops, wrong ways [writing the division statement reversed]
131. R: Um 4, 3, point 5
132. J: 2.354. Oh. (entering things on calculator) (pause) 2. 2446891. You don't have to write all of them down.
133. R: I know.
134. J: It's like 2 point, 2 point. Yeah, that's it, that's not a lot.
135. R: Yeah, and then.
136. J: Okay

(working and writing)

(unrelated conversation)

(teacher interruption)

137. J: We're done.

(unrelated conversation)

138. J: Here we go.
139. R: What else can we do?
140. J: It's [starts to spell out her name], recording our answers (show paper)
141. R: Let's see.
142. J: It's our awesome answers.
143. R: I know.
144. J: And then that this way. Okay, which way was it? Was it this way (putting camera back)?
145. R: Uhh.
146. J: It was this way. Oh, bah! (watching camera) Okay, I'm going to go ask (gets up).
147. Researcher: Have you looked at it on the back [where the assessment criteria are described]?
148. J: Yeah. Why isn't there any criterion B?
149. Researcher: Because Criterion B is looking at patterns and we aren't really looking at patterns here. So you've explained to [the teacher] what you've done? You're pretty confident in your answer?
150. J: Yeah.
151. Researcher: Yup, you've talked about how maybe you could have made it more accurate and maybe given a couple of suggestions for each of those, not just one?
152. J: For these [questions] ?
153. Researcher: For these ones. What would be not just one answer, but a complete answer?
154. R: Okay.
155. J: Okay #2 again. Um, 699 is pretty much 700 there—and then— I don't know— is it accurate because it covers more than enough? Doesn't that sound right? You're on the camera.
156. R: Me?
157. J: Yep. So you can divide that [1570] by that [699.435]and that equals [2.24]
158. R: (working on the calculator)
159. J: Use the calculator.
160. R: Just add a sentence. (writing)

161. J: Yup. If it was 3, though, that would be too much. Let's do it with the whole cake—yeah.
162. R: What are you doing?
163. J: Let's do it with the whole cake, even the bottom.
164. R: Okay (long pause)
165. J: 2 times—
166. R: 3.14
167. J: 3.14
168. R: Equals 1, 2, 5, 6
169. J: 1, 2, 5, 6? (looking on previous page's work) Oh you're right— equals—okay, you're turn. Then, it's the same.
170. R: (inaudible)
171. J: What's, um, 699? Yeah, what's 699 divide by—or, 700— or, no—
172. Researcher: Girls, I'm going to give you a little hint – have you thought about the thickness of the icing?
173. J: No.
174. Researcher: It might be something to think about— how that's going to affect your answer.
175. J: Okay. We should make icing this thick [indicating about a cm with her hands] then—
176. R: What she say?
177. J: What? I'm going to draw a cake here. That doesn't look like a cake. Voila! Looks like that's the cake.
178. R: Um . . .(taking the pencil)
179. J: Um,1 cm, um. If it was 1 cm thick though how would you do that?
180. R: Oh. You—oh, I know—
181. J: 1 cm
182. R: So here (takes pencil). From here to here is the icing so the icing is 1 cm thick so—write the end and then you use this to minus this, use this to minus this.
183. J: I'm confused—No, no, the icing is thick like um, it's um, it's not like flat. It's like that high, (showing with hands)
184. R: Can you say it again?
185. J: It's like, say this is the icing, it's higher so it's thicker (using the eraser as an example). It's not less icing. It's like if this is piece of the cake missing, the icing is like to here because it's thick (drawing on the diagram). It looks like a house (erasing). Ow, that hurt. What's the formula? We don't have the formula to get that then (to researcher).
186. Researcher: What do you mean? You have all the information to get the formula I bet.
187. J: But, um no. So like if we have cm thick icing around—she [R] didn't get what I meant though.
188. Researcher [speaking to R]: If this is my cake, how thick is the icing? Is it 1 cm? Is it 2 cm? Is it half a cm?
189. R: 1 cm.
190. Researcher: So how do I know how much I need?
191. R: Um, can we just guess the whole cake?
192. Researcher: Sure.
193. R: And not include the icing and then we get, um, the without the icing?
194. Researcher: You can pretend, if you want, this cake is whole.
195. J: Yeah, that's what we did.
196. Researcher: But keep in mind you might want to think about the inside layers, too.

197. J: Ohh
198. R: It's even harder.
199. J:Yeah. Ok, well, I told you layers, how many, 1, 2, 3.
200. R: 1, 2, 3.
201. J:Okay, these are layers, layers on the cake, layers so —(writing) —so what's the circle again?
202. R: Um.
203. J (writing) How's that? (inaudible) Because its 3 layers.
204. R: That means— (pointing to the paper).
205. J: Because its 3 layers.
206. R: So ok, then, what? (pointing)
207. J: 3.14 times 100 (writing).
208. R: (inaudible)
209. J: No, I'm— (writing) 900 that's a lot more. So that would be, this, right? (writing) Right? So that would be, um, 188, I mean 1884,
210. R: (inaudible)
211. J: What?
212. R: (inaudible)
213. J: 1884 plus 942 plus, equals, ugh, 2826, so that's the number— yeah (writing). That's a lot more [icing that is needed].
214. R: (inaudible)
215. J: What? If you do the top of the cake and the bottom of the cake—
216. R: Yeah.
217. J: And that's that.
218. R: But, but, this is just icing. Then how can— how can the icing, like, be the whole cake?
219. J: What?
220. R: How can the—how can the icing be for the whole cake?
221. J: No, no because I did layers in this one because of the layers— yeah (writing), 700 mL, of icing? Whatever. That means we need more than 4 cans of icing? (pause) Okay (talking to herself as she writing).
222. J: Estimate.
223. R: Um, 700 divided by—
224. J: It's the other way around [2826 ÷ 700, not 700 ÷ 2826]. That's my estimate.
225. R: Okay, let me see your estimate.
226. J: No, no, 700.
227. R: (shows calculator to J)
228. J: Oh I rock! What is it?
229. R: Um.
230. J: 4.037 cans, woo hoo, cans, you would need because—
231. R: Because
232. J: You, can't buy a part of a can.
233. R: Yup.
234. J: Oh my gosh (starts erasing).
235. R: You would need 5 cans.
236. J: Ok, 5. (talking to herself as she writes) Oh, I can't do this now. You can't buy part of a can (as she writes).
237. Researcher: You can do it, you know the information.
238. J: Well, we did it with the layers, but we don't know to do the thickness.

239. Researcher: Ok, what did you calculate for this?
240. J: Well this one we did— I don't know— we just did it as if it would pretty much be flat.
241. Researcher: Okay if it's flat, it will just pretty much look like that, right? Does that have any thickness?
242. J: No, well, kind of.
243. Researcher: Kind of. If I was being super accurate in my measurements, it would. What do you think is the smallest thickness you think that icing can be?
244. J: Probably 1 cm.
245. Researcher: Okay, you've told me now it's actually not totally flat, it's actually 1 cm high. Right? And I know that this is a cylinder also, right?
246. J: Oh, okay, come on help me. Say this is icing is, 1 cm high.
247. R: 535
248. J: No, but we're here.
249. R: Oh, the height?
250. J: No, that's the whole height because if you did the icing and it's a cm thick it would be like that.

(side conversation)

251. J: Okay (pause) is 2 times 3.14 times (writing) times (pause) I forget, 2 times 3,14 times 3.14 times 10 times 10. Oh my god.

(side conversation)

252. J: Okay, 2 times 3.14 times 10 times 20 is one thousand two hundred, I mean.
253. R: Yeah.
254. J: Yeah.
255. R: Yeah, 1256.
256. J: (writing) Okay, that is 2 times 3.14 plus 1256 (writing) this —what are you doing?
257. R: I don't know.
258. J: (helping R with calculator) Okay, 2 times (inaudible), what's that?
259. R: Um, 1262.28, that's—
260. J: Okay.

(side conversation)

261. J: . . . . That is only the top, that's 1, so that is 20, I just screwed up. (pause) Really, I don't know. I screwed up. I forgot to find the radius.
262. R: Right, the radius is 10.
263. J: (erasing) This is 20 and this is 1.5— I mean 0.5. This is so hard. Okay, what's 0.5 times 0.5? What's point 5 times point 5?
264. R: (inaudible, appears to be giving the answer)
265. J: And what's that times pi? 0.785—point 7. So, okay.

(teacher interruption)

266. J: Oh no, I screwed up. Argh. Times—
267. R: Times, um—
268. J: Do we have to do this?
269. R: We have to.
270. J: Ah.
271. R: (inaudible)
272. J: Okay, I'm going to try this (laughing).
273. R: Ok do this.
274. J: What? And then— and, no, no, just no. What? No.
275. R: (writing) What [pseudonym] do you want, anyways?

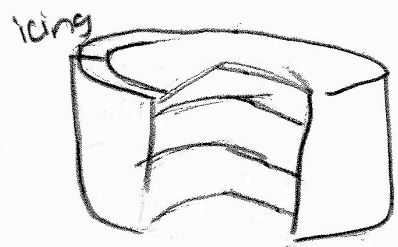
276. J: I don't know, just leave it blank.
277. R: Okay let's change the names, then.
278. J: What? No.
279. R: Oh, I don't know (distracted)
280. J: I need to be left alone again.
(pause)
281. J: Oh I know – (in answer to another student's question) Okay—oh, no.
282. R: Okay, your name. We need to staple.
283. J: Okay, wait, wait, what? No, no, no, wait.
284. R: Wait, just—
285. J: Ah, this isn't easy. What? No? What? What? I think we're wrong.
286. R: Um, no.
287. J: What I think— we—I think we have this part wrong.
288. Instructor: Oh, so is this all the work you've done so far?
289. J: Yeah, and this.
290. Instructor: Oh, of course.
291. J: We tried to do the thicker icing.
292. Instructor: Oh, so you've estimated it different thickness of the icing based if it's on the top or on the side. That's cool. That's pretty good if you, like, try to like—
293. J: It was annoying.
294. Instructor: It was annoying? It's kind of hard, I know. You can only estimate by the picture, right of how much icing you think was there. What did you come up with in the end? Or you're still not sure?
295. J: We answered these questions, but, but we haven't done—
296. Instructor: That's okay.
(side conversation)
297. R: No, can we do this.
298. J: Okay, I give up. A circle. How big was the circle? 314 plus, what's that? 942 plus 4 no, not point—that's not there (erasing)
299. R: Okay, are we finished?
300. J: No, do that times 3, times 3. Ah, okay, that means, um, from 700 divided by 3019.11 (writing).
301. R: 700 divided by 3019.
302. J: We did it [the division] wrong.
303. R: No, don't we—
304. J: That was first.
305. R: How did you (pointing to paper)—
306. J: That was 3019.11 divided by 700. We might have gotten it, but I don't know.
307. Researcher: You know there is not one right perfect answer.
308. R: 4.31, like estimate.
309. J: What?
310. R: Here.
311. J: Oh, rounded.
312. R: Yup, rounded so we finish.
313. J: No: this is still about the height. Hey, I'm done.

Appendix F: Jesse and Rebecca's Written Work

Name:
Name:

frosting container
$A = \pi r^2$
3.14×4.5^2
$3.14 \times 20.25 = 63.585 cm^2$
$V = \pi r^2 \times h$
$63.585 \times 11 = 699.435 cm^3$ each can has = 699.435 mL

Cake
Surface Area
πr^2
3.14×10^2
$3.14 \times 100 = 314 cm^2$



A of Rectangle

$2\pi r \times h$
$2 \times 3.14 \times 10 \times 20 = 1256 cm^2$
$S.A. = 1256 + 314 = 1570 cm^2$

1cm thick icing

$r - 1 = 9cm$

1. 2½ or 3

2. Because 2 × 699 = 1398 cm
So about 2½ is about 1570 cm

3. It's accurate because it covers more than enough of the total surface area of the cake

4. We can divide $\overline{699.435 cm3}\big)$ by $\sqrt{1570}$ cm^2 which equals 2.2446689 11 ≈ 2.24 by using a calculator to get the accurate answer It is the fastest and the easiest way we know now.

Icing the cake

Surface Area

$2 \times 314 + 1256 = 1884 cm^2$

Circle of layers

πr^2

$3.14 \times 10^2 = 314 \times 3 = 942 cm^2$

Icing top, bottom, sides, and layers

$1884 + 942 = 2826 cm^2$

Each can has about 700mL of icing

$700 \div 2826 =$

Estimate = 4.189

Accurate Answer = 4.037 cans

You would need 5 cans because you can't buy part of a can

Area of circle

$3.14 \times .5^2$

$3.14 \times .25$

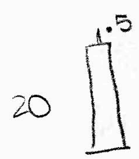
$.785 cm^2$

Rectangle

$2 \times 3.14 \times .5 \times 20 =$

$62.8 cm^2$

$2 \times .785 + 62.8 = 64.37 cm^2$



Area of Circle

$3.14 \times .5^2$

$.785 cm^2$

Rectangle

$2 \times 3.14 \times .5 \times 20 = 62.8$

$2 \times .785 + 62.8 = 64.37 cm^2$

$942 + 64.37 = 1006.37$

$1006.37 \times 3 = 3019.11 cm^2$

$700 \div 3019.11 = 4.31$ containers

About 5 containers

Appendix G: Research Ethics Board Certificate of Approval

The University of British Columbia
Office of Research Services
Behavioural Research Ethics Board
Suite 102, 6190 Agronomy Road,
Vancouver, B.C. V6T 1Z3

CERTIFICATE OF APPROVAL - FULL BOARD

PRINCIPAL INVESTIGATOR:	INSTITUTION / DEPARTMENT:	UBC BREB NUMBER:
Ann G. Anderson	UBC/Education/Curriculum and Pedagogy	H09-02843

INSTITUTION(S) WHERE RESEARCH WILL BE CARRIED OUT:

Institution	Site
N/A	N/A

Other locations where the research will be conducted:
one independent (grade 6-9 or K-12) school in the Lower mainland

CO-INVESTIGATOR(S):
Catherine A. Dick

SPONSORING AGENCIES:
N/A

PROJECT TITLE:
Paired Assessment in Middle School Mathematics

REB MEETING DATE:	CERTIFICATE EXPIRY DATE:
December 10, 2009	**December 10, 2010**

DOCUMENTS INCLUDED IN THIS APPROVAL:	DATE APPROVED:
	January 6, 2010

Document Name	Version	Date
Consent Forms:		
teacher consent v2 dec 16	2	December 16, 2009
Initial Contact letter and agency consent v2 dec 16	2	December 16, 2009
parental consent v3 dec 16	3	December 16, 2009
Assent Forms:		
Student assent version 1	1	December 16, 2009
Questionnaire, Questionnaire Cover Letter, Tests:		
Reflection prompts v2 nov 22	2	November 22, 2009
Interview prompts v2 nov 22	2	November 22, 2009
Sample Assessment Task v1 nov 22	1	November 22, 2009
Letter of Initial Contact:		
Initial Contact letter and agency consent v2 dec 16	2	December 16, 2009
Other Documents:		
Script for teachers introducing the consent form to students v1 nov 22	1	November 22, 2009
Script for verifying student assent v1 nov 22	1	November 22, 2009

The application for ethical review and the document(s) listed above have been reviewed and the procedures were found to be acceptable on ethical grounds for research involving human subjects.

Approval is issued on behalf of the Behavioural Research Ethics Board and signed electronically by one of the following:

Dr. M. Judith Lynam, Chair
Dr. Ken Craig, Chair
Dr. Jim Rupert, Associate Chair
Dr. Laurie Ford, Associate Chair
Dr. Anita Ho, Associate Chair