ACHIEVEMENT AND SELF-EFFICACY OF STUDENTS WITH ENGLISH AS A SECOND LANGUAGE BASED ON PROBLEM TYPE IN AN ENGLISH LANGUAGE-BASED MATHEMATICS CURRICULUM

by

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Abstract

Students who are learning English as a second language (ESL) have lower performance on mathematics problems based in language than students who are fully fluent in English. Students' performance on word-based mathematics problems is directly related to their English reading comprehension and language fluency (Abedi & Lord, 2001; Brown, 2005; Hofstetter, 2003). This places students who are not fully fluent in English at a disadvantage in the mathematics classroom. Students' self-efficacy beliefs also impacts their mathematics performance and motivation. The self-efficacy of students who are not fluent in English may be negatively impacted by their struggle with language. For this exploratory study, image-based mathematics problems were created to communicate problem solving questions with pictures instead of language or computational symbols. This problem format was investigated as a potential alternative to word-based or computation-based problems. Grade 6 students registered in ESL level 2, ESL level 4, and not registered in ESL, completed a mathematics task with four computation problems, four language-based problems, and four image-based problems. During a follow-up interview, students' solution strategies and thought processes were explored further. The results of this study indicated that the inclusion of wordless mathematics problems, such as image-based problems, assisted some of the students who were learning basic English interpersonal communication skills. As nonroutine problems, image-based mathematics also encouraged complex thought and mathematics understanding. Students in ESL Level 2 demonstrated higher self-efficacy beliefs on image-based problems than word problems.

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Dedication

To Matt, who always supports and encourages me.

Research Motivation

When I look critically at my research motivation, I am acutely aware that my early teaching experiences have impacted my research interests more than any other experience in my life. During my enrolment in the Teacher Education Program at the University of British Columbia in 2003, I completed my practicum in a grade 6 and 7 classroom. Since it was a middle-class school, I had little experience with students who spoke English as a second language (ESL). I graduated in the summer of 2004 with my Bachelor Degree in Education and a concentration in teaching the Intermediate grades.

My first teaching position was in a grade 4 classroom, and because my practicum experience had been in upper intermediate grades, I expected that I would have no problem with the grade 4 content that I would have to teach. I quickly learned that it was not as easy as I imagined. My primary difficulty was with the mathematics textbook that the school had recently acquired. One of the most embarrassing things for me to admit as a new teacher was that the word problem that a grade 4 student had just shown me in the math textbook had me completely stumped. Although this uncomfortable moment seemed to happen to me quite often in my first year of teaching, I felt that my knowledge of mathematics was not at issue. As an elementary student, I had been part of a gifted mathematics programs and had experience solving complex computation and word-based problems. Most of my confusion, as a teacher, with the grade 4 mathematics textbook occurred because the word-based mathematic problems were written with vague or confusing language, and the intent of the problem was unclear. The language used in the actual math problems was the issue for me and my students; it was not our understanding of how to interpret, apply and manipulate mathematical terms. Sometimes I was able to figure out a problem after looking the answer up in the teacher handbook and working backwards. At other times, when even this strategy could not help me, I would just tell the students to move on to the next question. Though I knew that it was the language and the wording of the mathematics problems that made them ambiguous and unclear, this situation made me feel that I was incompetent as a teacher, and I began to have serious questions about my own abilities. However, I also knew that if I was experiencing this level of frustration and anxiety, it was very possible that my students, who were still only learning the math concepts that I fully understood, were also experiencing the same self-doubts. The frustration that they were dealing with was worrisome to me, as it was enough to cause some of them to label themselves "stupid" at math, and express this to me. It concerned me that all of the children in the classroom were fully fluent in English, yet they were having such problems in mathematics because of a current focus on language in the mathematics curriculum. To help alleviate these feelings, so they would not have this negative view of their abilities, I began rewriting the textbook problems to help increase the clarity of the language and I added basic computation and manipulation questions to help reinforce students' learning. We increased the use of manipulatives and real world application of mathematics through language-based questions developed with grocery flyers, school supplies, and other applicable situations.

I wanted to help minimize the frustration in math, because the students' negative emotions and feelings did not just impact their willingness to learn math, but they also affected the students' emotional state. This in turn altered their focus, concentration, openness to learning, and willingness to be successful participants in the classroom. I was quite concerned that some of the students were beginning to experience negative self-efficacy. The students' self-efficacy beliefs, their perceptions of their own capabilities, are linked to their motivation, feelings, thought-processes and behaviours (Bandura, 1994). The students in my classroom were beginning to believe that their future attempts at problem solving with the textbook problems would also be unsuccessful because of their past experiences. Thankfully, creating an alternative language-based program for them helped to buoy their self-efficacy in the area of mathematics problem solving, and we finished the year using the textbook sparingly.

The following year, I taught in a grade 5 and 6 combined classroom in an inner-city school with a transient population, where approximately half of the students had an ESL designation. While the ESL students were not expected to have strong literacy skills because of their lack of language knowledge, many of the native English speakers in the class were also reading below grade level and struggling with reading comprehension. The students in this class had a significant number of language and literacy difficulties, and the mathematics textbooks that the school used typically assigned five to eight word problems per topic, very few numerical computation questions, and no picture-based questions. While I understood the benefits of a language-based curriculum as a way to deepen students' understanding of mathematics and increase their ability to apply and transform their mathematics understanding to real situations, it was hard to justify inundating students with written language-based problems when I was concerned that students were without the language and literacy skills necessary to participate in mathematics lessons, learning, and discussions, successfully. How was I meant to effectively teach these children mathematics concepts when they did not even process enough of the language and vocabulary to understand what they were being asked? How many of them could have performed better or felt more successful if the mathematics program had been based less on language and literacy, which were their areas of weakness, and instead focused more on numerical manipulation and mathematics exploration through hands-on learning, relevant situations, or picture-based images? For how many of the students did the language component and their resulting difficulty have a negative effect on their own feelings of mathematics capability and self-worth? These questions began to pose an interesting dilemma for me and made me passionate about finding a way to allow students to achieve maximum mathematics success in a culturally varied mathematics classroom while still experiencing meaningful mathematics exploration.

One of the students in my class that year was a refugee from a Middle Eastern country where her dad had been employed as a professor with a Ph.D. in mathematics. She had always received very high marks in math. Not only did she come to Canada and my classroom speaking very limited English, but she was also used to a completely different alphabet system. This impeded her initial attempts not only at acquiring the language but also at developing literacy skills because she first had to acquire English letter recognition skills. She had originally expected that when she came to Canada she would still be able to do well at mathematics, even with limited language skills. Like many other immigrant students, she expected math to focus more on numbers and computation and less on language abilities. She was surprised and dismayed by the inclusion of a significant amount of language in mathematics, and her father met with me many times because he was so upset about seeing her receive failing marks in math. Her problem with math was not because she had difficulty understanding computation or applying mathematics to problem-based situations; rather it was because of how heavily doused in language and literacy the mathematics curriculum has become. She was unable to answer many of the questions assigned to her in her math class because they were given in word problem format. In addition, her frustration about math was affecting her performance and attitude in my classroom during the rest of the day. She began to withdraw, and her motivation decreased. Though I did not teach grade 6 mathematics, I was able to negotiate with her mathematics teacher to let me give her an individualized program that minimized the need for English and reading comprehension, but still allowed her to investigate and explore the same mathematics as the rest of her peer group. This student's personal struggle really affected me and made me begin to take a critical look at the mathematics program as it is currently structured, and the successes and challenges that a language-based mathematics curriculum brings to a multicultural classroom, especially in areas with a high refugee or ESL population.

I feel that the education system should allow children to discover knowledge and teach them to realize their full potential. Though I was able to be successful in the education system as it was structured, I realize that one form of education is not necessarily best for all of the students in a classroom. These experiences have prompted me to look critically at shifts in the structure of education in the area of mathematics and how mathematics can be further developed to allow maximum success of the widest range of learners. As an educator, my students have opened my eyes to the important issues in a classroom, and I have come to believe that high self-efficacy and perceived success are very important qualities for students to possess. Through my research, I want to learn more about how I can make each day as successful as possible for them so that they will be passionate about education and learning. In this particular study, I will investigate a potential alternative to language-based mathematics which may allow students who are not fluent in English to interpret, apply and manipulate mathematics in a meaningful and significant way. Mathematics questions based purely on computation do not teach students to think deeply about mathematics as they can just follow a learned procedure to recite memorized facts in order to find the correct answer. These questions do not encourage students to discover how and why mathematics is used. Also, computation problems are not usually of comparable difficulty to language-based problems.

Other areas of the curriculum elicit insight and discovery on the part of the student. Literacy learning asks students to make inferences by "reading between the lines" by looking at the information presented, and developing ideas and judgements based on their perceptions. Science and social studies demand complex, investigative thought by the students in order to delve into the "whys" of the world. This same principle of discovery in learning should also apply to mathematics even when doing work in a textbook with a pencil and paper. Picture problems force students to be like detectives in mathematics, trying to find clues to determine the information given in the image and discover what is missing.

For this exploratory study, I created the image-based problems as a unique tool designed to remove language and minimize the computational components, such as numbers or mathematical symbols. Image-based mathematics problems are derived from word problems with similar problem outcomes and complexity but without the necessity of language. Similar to word problems, image-based problems have to be translated by the students into a mathematical context, interpreted to determine a solution strategy, and completed with accurate computation. The image-based problems created for the mathematics task involve multiple steps in a solution strategy and encourage analysis and independent thought on the part of the student.

Could creating mathematics questions that are image-based provide a suitable alternative to language-based problems while students are first learning English? Could image-based problems retain the meaningful and interpretive qualities that language-based problems often have? Image-based mathematics could allow students who are still in the process of learning English to have an opportunity to develop their mathematics knowledge while still meeting the requirements of the word-based curriculum (i.e. to provide students with situations in which they can interpret, apply, and manipulate mathematics). How would questions of this sort, which could allow students to continue their acquisition of mathematics knowledge, impact students' self-efficacy?

The research questions that guided this exploratory study are:

1. According to ESL students' self-reports, how is their mathematics ability and self-efficacy affected by the English used in the language-based mathematics curriculum?
2. How is mathematics achievement and self-efficacy different when mathematics problems are asked in a computation-based format, word-based format, or picture-based format for students in ESL level 2, ESL level 4, and those students not registered in ESL?

3. What is the impact on students' performance and self-efficacy when using the mathematics modification of image-based problems, instead of language- or computation-based mathematics problems?

Literature Review

Mathematics education for elementary school children in British Columbia has shifted recently to include a higher proportion of word-based problems instead of focusing primarily on computation and numeric problems. These word-based problems require a certain level of English proficiency, literacy, and reading comprehension (Hofstetter, 2003). According to Brenner et al. (1999), answering a word-based problem involves three steps: translation, interpretation, and planning. Translation refers to comprehension of the mathematics terms and their meaning, interpretation involves the application of the correct operations, and planning involves combining the operations with the numbers given and completing the question. I would suggest that another important factor in students' ability to translate the problem is also dependent on the proper interpretation of the wording and vocabulary used in the problem. Language-based problems which allow multiple interpretations, manipulation, and a variety of acceptable final answers can minimize the confusion that can arise from language understanding, and are considered to be best practice (National Council of Teachers of Mathematics, 1989; Baroody, 1998). However, these questions are not often the type of word problems found in mathematics textbooks being used in the classroom. In the average mathematics textbooks, students are expected to read the word-based problem, interpret what is being asked, translate the words into an equation, and solve the problem to find one correct answer. Having dealt with four different English word-based mathematics textbooks in the past three years, I have observed many students struggle in mathematics, not because of a lack of mathematics skill or comprehension, but due to inadequate English, literacy abilities, or reading comprehension skills. This has led me to question the validity of having a high proportion of word-based problems for all students in the current math curriculum, and I wonder how this expectation affects students who do not have the language or literacy skills necessary to succeed in this form of mathematics curriculum.

For the 2005-2006 school year, 10% of the students in the British Columbia public school system were designated as having English as a Second Language (ESL) (British Columbia Ministry of Education, 2006). The British Columbia Ministry of Education provides funding for ESL students for the first five years that they are registered in the provincial public school system (British Columbia Ministry of Education, 2002). That money is granted directly to the students' enrolling district and is intended to be used for specific English instruction for those students still acquiring the language. During the five years of funding, students are expected to move through five levels of ESL acquisition. Ideally, each level takes one year to complete so that the students are funded for every year that they are registered in ESL classes. Due to individual student learning and variation in ability, some students may take less time to complete the five levels and some students may need to take longer than the five years, however, any students requiring more time above and beyond their first five years of enrolment in the British Columbia school system receive no extra funding for language support (British Columbia Ministry of Education, 1999).

The Ministry of Education policy states that though the money is given directly to the school districts in order to provide ESL services, it is up to the individual districts to determine exactly how the money will be used (British Columbia Ministry of Education, 1999). In the Hargrove School District, in a suburb of a large city in British Columbia, 25% of the students in the public school system are registered as funded ESL learners and 60% of the students in the district do not have English as their first language (Carrigan, 2005). These students come from a variety of language backgrounds, each of which is attached to its own culture, traditions and beliefs. The district has organized a continuum of ESL levels 1 through 5 which provide varied amounts of support to ESL students based on what the district feels the language needs of students in each of the five levels should be. ESL levels 1 and 2 are designed to introduce students to the fundamentals of the English language and provide them with comprehension and fluency in basic conversational English. These students generally receive out-of-class support with a specialist teacher for approximately 2 to 3 hours a week (Carrigan, 2005). Students in ESL levels 3 through 5, which are intended to teach academic English, often go without any extra pullout support, or only sporadic meetings or limited in-class assistance with a specialist teacher during the school year (Carrigan, 2005). This lack of intensive support for ESL learners in the upper three levels of the ESL system is blamed primarily on a cut in funding which occurred approximately 6 years ago and dramatically decreased the specialized support that the district could provide to ESL students. Instead of acquiring language through focused instruction, students are expected to learn much of their English knowledge from interactions with peers, teachers and classroom material (Carrigan, 2005).

The focus of the present study investigates how students who speak English as a second language (ESL) are impacted by a language-based mathematics curriculum. Word-based problems are a useful tool in the mathematics classroom to help students investigate mathematics and deepen their understanding (Baroody, 1998). However, I am interested in investigating how English ability impacts a student's perception of his or her own mathematics ability and self-efficacy when she or he is placed in a language-based mathematics curriculum that is not taught in his or her first language, or language of fluency. How is mathematics ability and self-efficacy different when questions are asked in a specific format, such as in straight computation problem format, image-based problem format or word-based problem format? In what ways does mathematics ability and self-efficacy increase or decrease with increased English ability on each problem type, or are they independent of English ability?

This study investigated the experience of Chinese ESL students in British Columbia within the current problem- and language-based mathematics curriculum. For the purpose of this study I have limited my focus group to students from China. This is not to say that ESL students from other cultural backgrounds do not experience difficulty within the English mathematics curriculum, but because each culture may experience these difficulties differently, massing them all together into a single research project might not adequately convey their experiences. By limiting this study to Chinese immigrant students, I attempted to control one variable by investigating the experiences of a group of students who share a similar cultural background, community structure, and comparable experiences in mathematics instruction.

In the case of FEP students, studies have been carried out to investigate their performance on language-based mathematics questions. Kiplinger, Hang and Abedi (2000) point out that average FEP children's performance on language-based mathematics questions is between 10% and 30% lower than their performance on similar questions when they are presented in a numeric format. If this is the trend in the English-speaking population, what is the performance deficit of ESL students on comparable word and computation problems? If it is higher than the amount observed in English speaking children, then the impact of a language-based curriculum on ESL students is even more detrimental than the impact on FEP students. A higher deficit for ESL students than FEP students should cause educators to question if word problems are a suitable assessment of ESL students' mathematics abilities.

For the purposes of comparison, and due to limited studies investigating Canadian students' mathematics abilities compared with students in East Asian countries, I have chosen to include information about the United States as a representation of English-speaking Western classrooms (Klassen, 2004). The United States is comparable to Canada in its current focus on word-based mathematics problems and movement away from rote computational procedures and memorization of math facts (Mayer, Tajika, & Stanley, 1991). In studies of mathematics performance comparing East Asian students (those from China, Taiwan, and Japan) with students in the United States, computational skills and problem solving abilities were analyzed. In tests of mathematics achievement, students from China, Taiwan and Japan consistently outperform American students in computation and problem solving (Leung, 2005).

Studies such as Mayer, Tajika and Stanley (1991), Brenner, Herman and Ho, (1999) and Geary, Liu and Chen (1999) break down mathematics achievement into subgroups such as computation, representation, and problem solving abilities. Mayer et al. (1991) claim that each of these subgroups should be investigated separately and formulate the theory that when this is done, American students actually outperform Japanese students in the area of problem solving techniques. This claim of an American advantage is misleading, because it is only evident when the proportion of correct responses is examined at each ability level separately. For example, Japanese students who scored 11 or higher on a 15-question achievement test were also given an 18-question test of their problem solving abilities. This second test did not ask for the questions to be answered; rather, it investigated the ideas that students had about the steps they would take towards solving this problem. When the American students who received a score of 11 or higher on the achievement test took the problem solving test, they answered proportionally more questions correctly than their Japanese counterparts. This finding does not account for the fact that in the relatively similar sample sizes of American and Japanese children who completed the mathematics achievement test, only 5 American children achieved a score of 11 or above, while 77 Japanese children achieved this score. Geary et al. (1999) point out that this disparity between the groups might favour the American students taking the problem solving test in terms of IQ. The majority of American children in the achievement sample scored below 7 on their mathematics achievement tests. Therefore, the few who had scores of 11 or above significantly outperformed their peers. It could be argued that these students' high deviation from the norm is due to highly superior IQs and thinking abilities. Since the majority of Japanese children had scores of 11 or above, that sample is more likely to be indicative of a broad range of IQs. It could be argued that the American students taking the second test were a higher IQ population and were being inequitably compared to a sample of Japanese students with an average IQ base. The problem solving test may also be considered to be an inaccurate representation of the capabilities of the two sample groups, as it only asks students to devise a plan for how to solve a word-based problem and not for the actual computation to be carried out. However, the question needs to be raised: Can problem solving really be assessed without students completing the problem or is problem completion an integral step in problem solving? When carried out to completion, critical problem solving involves students assessing the validity of their answer and possibly adapting their approach to the question accordingly, which may result in students altering their problem solving methods (Baroody, 1998).

Brenner et al. (1999) conducted an investigation into problem solving abilities, but included full completion of the problem as part of the experimental design. Contrary to Mayer et al. (1991), Brenner et al. (1999) found that on representational forms of problem solving, where a problem had to be interpreted, modeled around previous knowledge, and strategies for solving the problem put in place with the question actually being completed, East Asian students scored 3 to 5 times higher than their American counterparts. Brenner et al. (1999) suggests that this is due to East Asian students' strong conceptual and abstract mathematics skills, which allow them to interpret and solve more difficult problems with higher accuracy. Also contrary to the findings of Mayer et al. (1991), Geary et al. (1999) finds that when questions involving problem solving are carried out to completion, American students of the same IQ and computational fluency as East Asian students have no advantage on the problem solving component of a question. Geary et al.(1999) further confirms that East Asian students are superior to Western students in computational abilities, and that this strength, coupled with the demonstrated equal ability in problem solving, allows them to have stronger overall mathematics achievement than their Western peers, regardless of problem type.

The National Council of Teachers of Mathematics reacted to the finding that American students did not perform as well in mathematics skill assessments as students in China, Singapore, and Japan by changing the Western mathematics curriculum so that it teaches mathematics through a language-based structure (Hook, Bishop, & Hook, 2007; Leung, 2005). They feel that this will assist students by helping them "become mathematical problem solvers and learn to communicate mathematically" (National Council of Teachers of Mathematics, 1989). This focus moves away from rote memorization of facts and procedures, and attempts to provide a fuller understanding of the derivation and manipulation of the strategies and systems used in mathematics. The desired outcome of the curriculum change is to provide students with a deeper and more complete understanding of the mathematics process, using language to investigate, discuss, and transform mathematical ideas (Salend & Hofstetter, 1996).

In studies that investigate the impact of language on mathematics performance in a Western English-speaking classroom, there is a strong consensus that students' performance on word-based problems is directly related to their English reading proficiency (Abedi & Lord, 2001; Brown, 2005; Hofstetter, 2003; Kiplinger, Haug, & Abedi, 2000). Students who struggle in reading, such as FEP students with poor reading comprehension skills or ESL students with limited English language knowledge, demonstrate a decrease in mathematics problem solving ability when the problem is in a language-based format (Kiplinger et al., 2000). Brown (2005) reveals an important correlation between ESL students' scores in reading and their achievement in mathematics due to the significant language components involved in both subjects as a result of the NCTM's curriculum change. ESL students' literacy abilities are greatly impacted by their developing English skills and their lack of vocabulary (Hofstetter, 2003; Brown, 2005). They generally are slower than FEP students at reading and reading comprehension because of this difficulty. Students who struggle with reading comprehension skills and vocabulary knowledge need extra time to complete word-based mathematics problems and often score lower on tests of this sort than their peers who have a thorough grasp of the language and stronger reading comprehension skills (Brown, 2005). This is not to say that when compared to FEP students, ESL students will always have lower mathematics scores in tests of their problem solving ability and lower reading comprehension scores in the English language classroom, but this is certainly the case as their English language fluency is developing.

Language modifications can be made to mathematics word problems in order to slightly benefit both ESL and FEP students who struggle with reading comprehension. One such modification is simplifying the language level by using familiar or simple language, active verbs instead of passive verbs, replacing conditional and relative clauses, and simplifying question phrases and long nominals to make them concise (Hofstetter, 2003). While both ESL students and FEP students are able to benefit from this modification, ESL students' mathematics achievement scores show a greater improvement than the mathematics achievement scores of FEP students who struggle with reading comprehension (Abedi & Lord, 2001). This may indicate that unfamiliar vocabulary and language difficulties are a larger component of ESL students' struggle in word-based mathematics than their ability to understand or complete the actual mathematics. If this were not true, simplifying language to increase reading comprehension should show similar increases in the mathematics achievement scores of both ESL and FEP students. It should be noted, however, that the improvements to mathematics problem solving achievement for either group is not considered to be statistically significant with language modification.

An alternative language modification is the inclusion of a glossary of terms which may be uncommon or unknown to students. This alternative also does not produce a statistically significant increase in ESL or FEP students' mathematics achievement scores. However, when provided with a glossary, ESL students do have slightly higher mathematics achievement scores than in situations when no modifications are provided or when mathematics is modified by language simplification (Kiplinger et al., 2000). When students were provided with a glossary on achievement-based tests, it actually increased the amount of reading that had to be done by the student in the same allotted time frame because now they were expected to read the mathematics problem and the extra information in the glossary. Without providing extra time for the students to complete the mathematics achievement test, glossary modification may be more difficult for ESL students than FEP students because their reading and comprehension rates are generally slower than FEP students (Kiplinger et al., 2000). Since the modifications of mathematics word-based problems using simplified language or a glossary only allow for modest increases in achievement, these alternatives are not the answer to the problem of an achievement disparity on mathematics word problems between ESL students and FEP students.

Regardless of the type of modification made to a mathematics problem, it is important to ensure that there is no change to the complexity or difficulty of the question, as was guaranteed by Kiplinger et al. (2000) and Hofstetter (2003). When language is simplified or a glossary is provided, if the mathematics difficulty remains unchanged between tests, FEP students with a satisfactory level of reading comprehension should perform equally well on modified and non-modified versions of the tests (Kiplinger et al., 2000).

For language-based mathematics to be as successful as possible in classrooms that combine students who are fully fluent in English with those who have limited English abilities, math problems must use English which students are familiar with (Brenner, Herman, & Ho, 1999). Key mathematics terms and academic level English should only be included in assessment situations after they have been clearly defined and used multiple times within the classroom and instructional setting through teacher-directed situations such as modeling and student-directed situations such as problem solving. Regardless of a student's first language, he or she is best able to answer mathematics word problems which use mathematics language which is in the same form as he or she has been introduced to in the classroom (Brenner, Herman, & Ho, 1999). For example, if the mathematics terms *difference* or *how many* are used in instruction, the same terms should be used in a similar context in word-based mathematics problems to allow students to use their existing language knowledge to interpret, translate and answer the question (Brenner, Herman, & Ho, 1999).

Due to students' existing language limitations, the vocabulary used in mathematics problems is of the utmost importance to the mathematics success of ESL students. When a student is presented with a word problem, he or she must interpret the English words used, deduce the intended meaning behind the words within the framework of the question, understand the mathematics terms used, apply the correct mathematical procedures and operations in the appropriate context, set up the question to integrate the operations and numerals, and carry out the mathematics to completion. The first two steps in this process involve language skills, while the last three focus primarily on the students' mathematics ability and knowledge within the context of the established mathematics problem. Mathematics educators have attempted to blur the distinction between manipulation and application of mathematics and language, or the communication and discussion of math, so that these two strands are not separate and distinct realms of education (Anderson, personal communication, January 21, 2008). For students who have limited language abilities, presenting mathematics problems in an image-based format, devoid of language, may be a way to allow students to understand and communicate mathematics knowledge without fully developed language capabilities.

Brown (2005) notes that higher socioeconomic status (SES) benefits ESL students attempting to learn English by allowing them to be exposed to a print rich environment and providing them with more pertinent English experiences, especially in the area of academic English. Conversational English, also called basic interpersonal communication skills (BISC), is often acquired by ESL students within two to three years of immersion, whereas academic English, or cognitive academic English proficiency (CALP), which includes the technical language necessary to read textbooks and technical writing with subject-specific vocabulary, often takes between five and seven years of immersion and specific instruction to attain (Cummins, 1997; Hofstetter, 2003). Brown (2005) finds that high SES ESL students score much lower on mathematics word-based problems than their high SES FEP counterparts, a finding that he attributes to ESL students' limited knowledge of English. Brown (2005) and Abedi and Lord (2001) further investigated the impact of SES on reading ability and emphasize that for both ESL and FEP students, high SES students have higher reading proficiency than their low SES counterparts. Because many mathematics problems in the current curriculum are word-based, this finding has an impact on mathematics also. Neither one of these studies delved specifically into the connection between self-efficacy and performance, but Brown (2005) does point out that for ESL students to continue to be motivated and succeed in mathematics, they must be given the opportunity to demonstrate the full extent of their abilities. The discrepancy between socioeconomic status, mathematical ability and literacy skills allows Brown (2005) to claim that a language-based assessment of mathematics ability is unfair for ESL students. A fair assessment system would allow high SES ESL students to perform on par with high SES FEP students, allowing for some lag in the scores of ESL students due to mathematics instruction which takes place in a language with which they are not yet fully proficient. Brown (2005) argues that, ideally, equal treatment of ESL students would allow them to show their mathematics ability, rather than be restrained by their incomplete English language proficiency. Presently, no matter how competent a student may be in mathematics, if his or her English knowledge is not advanced enough to complete word problems, his or her scores on mathematics assessment in a language-based curriculum will be lower than expected and the student's potential mathematics understanding and ability cannot be accurately assessed (Brown, 2005).

It is worth exploring how the language-based format used in the current mathematics curriculum impacts ESL students' assessed performance, since their performance appears to drop with the inclusion a high proportion of word-based problems. All cultures experience mathematics learning in a different fashion, and the way students are taught and assessed and the importance placed on specific skills may also differ (Gutstein, 2003). The effect that this shift of focus has on students who are new to the Western classroom is a topic that should be investigated in an effort to make the transition as uncomplicated and smooth as possible.

Self-efficacy, which is an individual's self-assessment of his or her likelihood of success on an upcoming event or situation, has its basis in Bandura's social cognitive theory. This theory argues that an individual makes judgments on his or her own ability to achieve a given result in relation to a specific task before that task is carried out (Bandura, 1986). An individual's judgment of self-efficacy is linked to his or her motivation, and is a predictor of future behaviour in the given area (Schunk & Gunn, 1986). Self-efficacy measures are not static throughout a person's life and are impacted positively and negatively by past performance, vicarious experiences, social persuasion, and emotional arousal (Klassen, 2004). For each event, individuals form an efficacy judgment as they undertake and complete further related tasks (Pajares & Kranzler, 1995). The outcomes of these efforts are then used to positively or negatively influence their self-efficacy in future tasks.

Bandura (1986) asserts that self-efficacy is task-specific; meaning that self-efficacy in word problems in mathematics will not necessarily be relevant or interchangeable with self-efficacy in relation to computation problems or image-based problems, thus they each need to be investigated separately. Pajares and Miller (1995) argue convincingly that with clearly defined self-efficacy measures, which are closely related to the math problems being solved, the predictive outcome of that relationship is increased. This makes a self-efficacy measure that is based specifically on the task being investigated (ex. image-based problems) a more reliable measure than a self-efficacy scale loosely based on the general topic being studied (ex. mathematics). A student's confidence in mathematics of his or her ability to solve given problems is incorporated as part of a self-efficacy measurement, along with the belief that he or she possesses the skills necessary to answer the questions (Pajares & Miller, 1994). This duality of self-efficacy is important when investigating the self-efficacy of ESL students, because it addresses their perceived ability when solving a language-based mathematics problem and their feelings about whether or not they possess the pertinent mathematics skills for completion. Self-efficacy is different from self-concept in that self-concept is a measure of an individual's perceived self-worth based on judgments of competence. Self-efficacy is only a measure of a student's reports of his or her own ability to be successful in a certain task and is not tied to any judgments of self-worth (Pajares & Kranzler, 1995).

This begs the question of how the inability to show the full extent of one's mathematics knowledge impacts a student's self-efficacy and perceptions of mathematics. When students have high self-efficacy in mathematics, they are more likely to be motivated to invest more time and energy into solving math problems in the future because their self-perceived measure of ability is higher (Pajares & Kranzler, 1995; Baroody, 1993). Students who experience problems in mathematics or have low self-efficacy tend to develop mathematics anxiety and a fear of mathematics-related situations (Beasley, Long, & Natali, 2001). Contrary to this, students who have higher self-efficacy are less likely to experience anxiety about their abilities in mathematics (Pajares & Kranzler, 1995). Math anxiety begins to develop as early as the primary grades in a student's mathematics education experience and, without early intervention geared specifically towards the anxiety and its root cause, it will continue to mount as a student is faced with mathematics situations (Baroody, 1993; Beasley *et al.*, 2001). In an attempt to decrease discomfort, over time individuals begin to avoid situations which increase their anxiety, such as optional mathematics courses in high school and college courses or professions which involve mathematics. This avoidance of advanced or upper level mathematics may have a detrimental effect on a student's future, as it seriously limits employment options (Beasley et al., 2001; Baroody, 1993).

Many of the studies carried out in the area of mathematics and language are done through quantitative methods, where mathematics abilities are measured with tests. However, previous studies have found a connection between self-efficacy and mathematics performance (Pajares & Miller, 1994; Pajares & Miller, 1995) and self-efficacy is shown to have as much impact on performance as a student's actual ability to solve a mathematics question (Pajares & Kranzler, 1995). Because of this, self-efficacy in mathematics should be of interest to researchers, in addition to written assessments of a students' ability. A shift in focus also necessitates a shift in research methodologies from primarily quantitative to a mix of quantitative and qualitative methodologies. In order to accurately investigate the impact of mathematics and language on self-efficacy, it is important to also investigate students' opinions and feelings on the subject from a qualitative perspective. The student's own perceptions which lead to his or her judgments of self-efficacy are very personal and cannot be uncovered through a written mathematics assessment. The researcher needs to provide the student with the time and opportunity to explain how and why he or she feels a certain way about a mathematics concept or problem type.

Many of the studies discussed in this paper assess students' self-efficacy using a five or eight point scale to measure their perceived ability on mathematics word-based problems. To delve further into the issue of how to positively influence a student's self-efficacy, we should examine the topic with more depth than simply interpreting a number as a comprehensive explanation for students' feelings and perceptions. Supporting a scaled measure with a qualitative analysis of students' self-efficacy would be beneficial in order to better understand why they might have higher or lower self-efficacy beliefs and how they feel it affects them. Abedi and Lord (2001) used some qualitative analysis in their study of the importance of language and language modification in mathematics testing for ESL and FEP students. They conducted a series of interviews with a small sample of the participants where the researchers asked students about their perceptions of certain math word problems and their ability to complete them. This interview step is important to allow students to inform the researcher of their preferences and why they have made certain selections and choices. Without the information about their feelings and opinions coming straight from the students, a researcher can only formulate an educated guess as to why students have specific preferences. Interviews also give students the opportunity to discuss how they feel their self-efficacy has developed and changed over time and how self-efficacy influences their mathematics achievement (Pajares & Kranzler, 1995).

In a quantitative analysis of self-efficacy measures in mathematics, Pajares and Kranzler (1995) discovered that 86% of students are overconfident in their mathematics abilities. These students demonstrate higher self-efficacy ratings than their actual abilities justify. Pajares and Kranzler (1995) suggest that this is beneficial because a marginally inflated self-efficacy measure actually motivates the student in the area of mathematics. A positive view of one's ability to perform well on a mathematics task also impacts the apparent effort that is required for a successful outcome (Chen, 2002). As students become more confident in their abilities, they report that their perceived effort on a mathematics task decreases even if the actual effort used does not. Perceived effort and motivation are negatively correlated so that a decrease in perceived effort actually increases students' motivation and willingness to learn further mathematics (Chen, 2002).

In a study comparing the self-efficacy of Asian-Canadian immigrant students and Anglo-Canadian students, Klassen (2004) reports that individuals from East Asian cultures have lower self-efficacy beliefs than their Western peers regardless of their superior success in assessments of mathematics ability (Whang & Hancock 1994). There has been no study carried out to specifically investigate the proportion of East Asian students who overestimate or underestimate their abilities in comparison to their actual assessed performance. This is an area that should be studied because of findings that link positive self-efficacy with increases in a student's performance, willingness to put forth greater effort, and level of persistence on difficult or novel math concepts (Brown, 2005). If East Asian students' self-efficacy in mathematics is already lower than their Western peers, how does this impact their motivation when their assessment in a Western classroom does not reflect their capabilities and is lower than they would expect?

Klassen (2004) investigated the impact that culture has on self-efficacy beliefs. He identifies that there are differences between individualist cultures, such as Canadian or American cultures, and collectivist cultures, such as those found in many Asian countries. In an individualist culture, the emphasis is on the individual, and his or her personal goals and ideals. In a collectivist culture, the focus is on the group as a whole and its members' duty within and for the group. Klassen's (2004) study of cross-cultural self-efficacy compared Indo-Canadian immigrant students (a collectivist culture) to Anglo-Canadian students (an individualist culture). For both collectivist cultures and individualist cultures, past performance on similar tasks has the strongest effect on students' mathematics self-efficacy beliefs. For Indo-Canadian immigrant students, the next most influential contributors to self-efficacy are students observing the abilities of individuals similar to themselves and the degree that their own parents value success in mathematics. In an individualistic culture, students' self-efficacy is influenced more strongly by emotional arousal, such as the fear of failure, than the influence of those around them (Klassen, 2004).

Future studies into cross-cultural self-efficacy could provide researchers and educators with beneficial information about how self-efficacy is formed and experienced within particular cultures (Klassen, 2004). It is necessary for self-efficacy to be looked at within a specific cultural group because it would be naive to believe that the self-efficacy experiences of all students enrolled in a Western mathematics classroom would be indicative of all of the populations and cultures that come together to comprise the classroom community. Klassen (2004) also points out that the factors which have the greatest influence on self-efficacy may change over time regardless of cultural background, as students are acculturated into Western individualistic culture and move away from adhering to the expectations of a collectivist culture. This is another area which would be worthwhile to study as the knowledge could increase ESL students' self-efficacy in the area of mathematics at particular stages of their education.

The purpose of this study is to investigate how the self-efficacy of students who are English language learners is impacted by curriculum and assessment practices which focus on learning through language-based word problems. Can a modification, such as using image-based mathematics, increase the mathematics performance of ESL students as they learn English? How does this modification impact an ESL student's self-efficacy belief? I compared the mathematics self-efficacy of students in ESL level 2 with that of students in ESL level 4 and of students who are not registered in ESL, who are considered by the British Columbia Ministry of Education and the Hargrove School District as fluent in English. Investigating potential alternatives to the current language-based mathematics program is important in order to increase the mathematics knowledge and learning of students who are limited by a word-based curriculum due to a lack of English language knowledge. During the transition to the English language classroom, educators should endeavour to provide ESL students with programs that will allow them to acquire the language but also as much of the curriculum knowledge as possible. If image-based mathematics can still provide meaningful mathematics experiences that allow students to manipulate, investigate, and explore mathematics without language barriers, it may be a useable alternative learning approach to allow ESL student to acquire much of the same mathematics learning as their peers who are fluent in English. The option would help minimize any gap in learning due to a period of language acquisition which might result in ESL students falling behind their FEP classmates. It is also important to investigate using image-based mathematics problems as a potential alternative for ESL students to increase their self-efficacy beliefs, because modifications which increase self-efficacy also help to increase students' present performance. The link between self-efficacy and motivation cannot be minimized because of the role that motivation plays in educational performance in the classroom. Increased motivation leads to increased effort and perseverance, which both help to contribute to increased performance. If motivation and, in turn, performance can be increased, this may allow the student to have a more positive view of school and learning (Pajares and Miller, 1995). The future increase in the likelihood that the student will pursue college classes or a career that is mathematically based also encourages investigation into the increase of self-efficacy beliefs (Pajares & Miller, 1995). If an improvement can be made in the area of mathematics self-efficacy, it would be wise to pursue it in hopes of increasing mathematics performance in elementary school and at all stages of life.

Methods

Problem Selection

After examining many of the problems found in resources currently used by teachers in grade 6 mathematics classrooms throughout the Hargrove School District, I selected the mathematics problems used on the mathematics task administered to grade 6 participants. I determined which resources, textbooks, and problems were being used in classrooms through observation in a variety of schools and through informal conversations with teachers and students about the resources and materials they were using and assignments being given. I found that, while there are many recently published textbooks available which are being used to varying degrees in classrooms, many teachers supplement these new resources with older textbooks and non-Ministry-approved resources, such as Scholastic computation and mathematics drill books.

Many of the resources found in classrooms contain mathematics problems that would not be considered by mathematics educators to be a reflection of currently accepted best-practice problems, because current best-practice problems involve students completing problems which involve multiple steps, manipulation of numbers, and complex thought about the mathematical processes involved (Baroody, 1998). Best-practice problems also involve answers which are not fixed so that the problems are open to interpretation by the students, thus allowing a variety of acceptable solutions (Baroody, 1998). The computation and word problems found in the mathematics task used in this study include problems that may be more similar to the ones in the classroom resources than to current best practice problems, because it was my intention that the mathematics task problems be an accurate reflection of current classroom practice in the Hargrove School District. In my attempt to include computation and word-based problems that are as close to best practice as possible, the problems that I selected involve multiple steps for students to work through. However, unlike best-practice problems, the problems included in the mathematics task are only open to a few interpretations, so that there are only one or two correct solutions to each problem. I made alterations to the problems discovered in classroom resources in order to maintain the language and problem format used in the original problems, but altered the questions enough so that they are different than those found in the classroom resources. This was done to ensure that there would be no chance of students having ever completed an identical mathematics problem before. The alterations included, but are not limited to, numbers, names, locations, and other identifying details found in the problems.

The mathematics skills and knowledge needed to complete the problems asked on the mathematics task do not directly correspond to the skills that are taught and acquired as a part of the grade 6 mathematics curriculum, as outlined by the Prescribed Learning Outcomes (PLOs) of the British Columbia Ministry of Education (British Columbia Ministry of Education, n.d.). The mathematics task used in this study, though administered to grade 6 students, generally includes mathematics concepts that are covered in the Learning Outcomes of earlier grade years. For example, Question #9, an image-based problem which involves the passengers on a bus, asks students to add and subtract single digit and two digit numbers and count up to 11, mathematics skills that students should have mastered years earlier in grades 2 to 3 and Kindergarten to grade 1, respectively (British Columbia Ministry of Education, 1996a; British Columbia Ministry of Education, 1996b). Questions with mathematics skills acquired in earlier grades are included in the mathematics task in computation, language-based and image-based problem formats because the mathematics task is not designed to assess students' acquisition of grade 6 learning requirements. Rather, the purpose of the mathematics task is to investigate how students process mathematics problems in computation, word-based and image-based problem formats, and what effect the format itself has on students and their opinion of mathematics. The British Columbia educational curriculum does not expect students to have mastered the learning outcomes of the PLOs by the end of the grade year, though they are expected to have a good understanding of them. The curriculum as designed creates a repetitive, circular format so that each of the learning outcomes taught in the previous years is revisited and built upon in later grades. I took this learning pattern into account as I created the mathematics task, as well as the fact that because students do not all learn at the same speed many of them might need the additional years to fully grasp a mathematical concept.

I was concerned that the inclusion of a high proportion of problems that require students to use mathematics that they have only been introduced to recently might impact or skew the results of the study. If only grade 6 level problems had been included in the mathematics task, when a student experiences difficulty with a specific problem, I would not know if it is caused by the problem format or the mathematics skills required of the student. The inclusion of mathematics problems that use skills that students should be adept at and familiar with because of years of practice makes it more likely that any difficulty encountered is the result of the problem itself.

While computation and word-based problems were created using the textbooks and classroom resources as models (see Appendix A and Appendix B respectively), I developed image-based problems as an original component, with the mathematics skills used in the computation and word-based problems as a guide (see Appendix C). The image-based problems used in the mathematics task were developed to meet the goal of producing problems devoid of language which minimize the use of numbers and computational symbols in the problem itself, while still clearly communicating the mathematics problem that needs to be solved. Mathematical symbols and numerical digits were generally avoided in an attempt to minimize the likelihood that the image-based problems would transform into computation questions that are simply illustrated. The images used in the problems are intended to be interpreted by the participant into a mathematics problem and carried out to completion.

Question marks are used in all of the image-based problems instead of an equal symbol to indicate to students which value they are meant to discover. Arrows are included in the problems as a method intended to direct students' attention to all areas of the problem, so that they would take into consideration all of the important visual information before attempting to provide a solution strategy and final answer. For example, image-based problem Question #2, which involves bananas intended to be evenly distributed between four monkeys, uses an arrow to show students that all of the bananas should be placed into the green bucket before students continue with the problem and distribute the bananas to the monkeys. Unlike the other image-based problems, this problem presents the need for a standard mathematics symbol to be used in order to make it clear and understandable to students. The monkeys shown in the problem are not all of equal size and as I was designing the problems, I was concerned that, because of this size discrepancy, students might not assign an equal number of bananas to each monkey, as intended. My suspicions were confirmed through conversations that I had with trial participants as they completed the mathematics task. These participants indicated that the monkeys' size difference made them wonder if there is a ratio or other factor by which to distribute the bananas to monkeys according to size. When the trial participants did not find this information about distribution, some of them asked whether they could randomly assign bananas to monkeys based on their assumptions of the representative size discrepancy and the total number of bananas available. In order to minimize this confusion, I placed an equal sign between the question marks above each monkey. I chose to use this representation because, without an equal symbol, even if the monkeys were the same size, the students could decide to allocate the bananas as they wished. The mathematical symbol placed between the question marks on the arrows pointing to each monkey was meant to indicate to students that the number of bananas allocated to each monkey should be equal regardless of size. Trials of the mathematics task that were conducted after an equal sign was added yielded results that were more closely matched to my desired outcome for the problem. Students in the second mathematics trial still took note of the size difference between the monkeys but assigned an equal number of bananas to each monkey when they noticed the equal sign. Adding a standard mathematics symbol in an effort to minimize students' confusion and decrease the possibility of multiple answers successfully directed the trial participants to the desired operation of division. This does raise the issue of future designs for image-based problems and the need to create images that are easily understandable to all students. This issue will be considered further when discussing implications for future research.

A question mark is also used to signify a combination of missing values in Question #1, the image-based problem involving sporting equipment. In this instance, the question mark is placed in the middle of the problem to indicate the placement of the unknown. Students need to infer that more than one of the objects needs to be purchased because the total cost is $82, shown on the cash register display, which is more than the cost of any one single item shown on the left of the shopping cart. This problem is presented to students in the form $x_1 + x_2 + \ldots + x_n = y$ where x_1, x_2, and x_n are unknown and y is the total paid. During trials of the mathematics task, students used a series of addition and subtraction problems to complete the problem. The expectations of this question were clear to participants, so its original format was maintained.

Image-based problem Question #3, a comparative measure question involving a jellyfish and a whale, requires students to complete multiple steps in order to determine the final solution. Students are asked to observe the number of segments for each creature and multiply or add by the indicated amount for two segments to find the total length of the creature. This question involves multiplication of decimals, though it can also be solved or verified through repeated addition of decimals, both of which are learning outcomes assigned to grade 5 (British Columbia Ministry of Education, 1996a). The whale length and jellyfish length are both within the acceptable size parameters for these ocean creatures in their natural habitats, which allows students to question whether or not their answers are realistic and valid. A third question mark, placed in the unoccupied space that remains when the two creatures are placed side by side, asks students to determine the difference in lengths between the two creatures. The answer to the size difference can be found by subtracting the length of the jellyfish from the length of the whale or using a guess-and-check addition strategy, though this would be more time-intensive. In trials of the mathematics task, all of the students understood the representative meaning of the three question marks.

The image-based problem, Question #4, involves people moving on and off a bus and requires students to follow the movement of passengers. This question involves simple counting, moving both directions on a number line, and working with addition and subtraction. In trials of the mathematics task, some students observed that the bus driver was still on the bus in the final frame, while others did not. I initially contemplated removing the bus driver from the image, but hesitated because image-based problems should still demand the concentration and keen observation that is necessary for computation- and word-based mathematics. I was interested in investigating the information that students notice, the distinction needed in images, and the information that students believe to be extraneous. In trials of this problem, some students followed the scenes sequentially, adding and subtracting as passengers moved on and off of the bus, and using guess-and-check to find an appropriate answer, while others took note of all of the passengers moving off the bus and subtracted this from the total number of passengers who boarded the bus to determine the number of passengers who were initially on the bus.

Mathematics problems based on language were created using textbooks currently used in classrooms as a guide, but are tailored to ensure that students use all basic mathematic operations such as subtraction, addition, division and multiplication, and that these operations are carried out with the same difficulty in the computation and image-based problems. Some number values used in the word-based problems are written in words as opposed to numerical format, because this limits students' ability to place all of the digits shown in the word problem into an equation and end up finding the correct solution without an understanding of the reason or rationale involved. This also forces the question to be further based on language knowledge, as opposed to digit recognition.

The computation questions selected for inclusion in the mathematics task were chosen in an attempt to minimize the "easy" or straightforward computation questions in which a series of operations are given that need to be completed in order to find a final answer. It should be noted that computation questions are usually not directly comparable to word-based problems or image-based problems in mathematics difficulty or sophistication because, unlike the other problem formats, computation problems do not require that students interpret any information in order to create a solution strategy or equation; it is given to them in the problem already. Due to this, computation problems often do not require that the student possess the same level of mathematics understanding, sophistication, or complex thought as the two other problem formats require.

Recognizing this discrepancy and my desire for the difficulty of each of the problem formats to be as similar as possible, I modified the computation problems I found in the classroom resources in such a way that the altered problems required students to manipulate the numbers and equation formats given in order to solve each problem. The computation problems used in the mathematics task did not simply ask students to regurgitate a memorized solution to a series of operations; rather, most of the problems were presented in a form which asked students to find a missing value(s) to make the given solution to the problem true. When they encountered a computation problem in a form which could not be directly solved, students needed to determine the strategy they should use in order to answer the problem, because this information was not provided for them. Optional strategies for many of these problems include guess-and-check or reordering the question to create an equation to find the missing value(s). The values that needed to be found by students were indicated on the mathematics task by an empty box inserted into the place of the missing value. Having the participants solve to find a missing value in a computation problem increased the difficulty of the problem and minimized the incidence of students answering in mechanical fashion by recalling memorized mathematics facts.

ESL Participant Group Selection

In the Hargrove School District, the ESL program has five levels that students learning English are registered in, based on their knowledge of the English language at each level. After consideration of the requirements of each level and the students' knowledge of English, I chose to include students in ESL level 2 and ESL level 4 in this study. These levels were selected because the students in ESL level 2 and ESL level 4 represent beginner and intermediate ESL levels respectively. My choice was based on my previous experience with ESL students and my understanding of students' progression through the ESL levels.

Students who are registered in ESL level 1 are generally in their first year of instruction in the English language. These students have usually arrived in Canada sometime after the start of the current school year and it is quite possible that some ESL level 1 students have only recently arrived in Canada. Many of the students in ESL level 1 come into the school system possessing little or no knowledge of the English language, and this might create many difficulties in the completion of the study. A major factor in my choice not to include students in ESL level 1 in this study was that, even though the study results may be more dramatic with these participants, I was unsure what the emotional effect of a mathematics task with language-based problems and an English interview would be on ESL level 1 participants. Also, due to the limited nature of this study, for valid results with ESL level 1 students, I would need an appropriate translator along for all interviews and at all times when language would be used for instruction or information between myself and the participants, and this was not possible within the confines of my study. If ESL level 1 students were included without a translator present, their English answers would likely be limited and their responses, though they could be written in Cantonese or Mandarin, might cause miscommunication or an information gap where key knowledge might be missed or left out. While it would be likely that students in ESL level 1 would be able to complete the mathematics task without a translator, their participation in the interview component would be limited, and I would not be able to have a complete grasp of their thoughts, feelings, and understanding of mathematics problems in language-based, computation-based and image-based formats.

Most of the students in ESL level 2 have been in an English language classroom in Canada for one or two years. ESL level 2 students are usually in their second year of language acquisition, so they have acquired some basic conversational English abilities and are working towards mastery of conversational English. To further develop their English knowledge, students in ESL level 2 still receive focused English instruction with a specialist teacher for a period of time each week. These students may experience difficulty with uncommon vocabulary or complex sentences in oral and written language, because they are still at a stage of general language acquisition. Students in ESL level 2 were selected to participate in the mathematics study because they have enough English knowledge to understand the English mathematics task instruction and interview questions, if it is supplemented with a written translation of simplified Chinese and a voice recording in both Cantonese and Mandarin. These aids were used to allow all ESL level 2 participants to complete the task and interview without unreasonable stress or discomfort. The ESL level 2 students' basic conversational English abilities enabled them to participate in the interview process with basic fluency, and allowed me to understand their statements, respond to their answers naturally, and know when to probe for further or related information.

Students in ESL level 3 were omitted from the study because, though they are beginning to learn academic English, the difference in English ability between ESL level 2 students and ESL level 3 students might not be significant enough to include both groups in this study.

Students in ESL level 4 were included in this study because according to the time standards set by the British Columbia Ministry of Education in their funding model, these students have acquired fluency in conversational English and are working towards fluency in academic English (BC Ministry of Education, 2002; Carrigan, 2005). ESL level 4 students generally have four years of instruction in the English language classroom with the first three years supplemented by specialized English instruction (Carrigan, 2005). The written instructions and oral directions on the mathematics task should not have been a challenge for students in ESL level 4. These students were able to participate fully in the English language interview process because they could understand questions posed in English and respond to the interview questions in English. Because every copy of the mathematics task had the instructions translated into Chinese, students in ESL 4 also had access to this language aid, though they should not have needed to use it. These students were also provided with a written and oral translation of the interview questions for extra support but none of the ESL level 4 students used them to supplement the oral interview.

Students with no ESL background were also included in the study to function as a control group. Students with no ESL background were selected according to the same criteria as the ESL students. They had to be enrolled in a grade 6 classroom, taking grade 6 math, and be of Chinese descent, having lived for at least some time in China. The cultural descent of the students is important to keep constant between the participant groups because self-efficacy in mathematics has been shown by Klassen (2004) to be culture-specific. This specificity is due to the fluctuation between cultures regarding the importance of schooling, mathematics, and their role within an individualist or collectivist community structure.

The students with no ESL experience have been exposed to the Western mathematics curriculum for the entirety of their time within the school system and were considered by the Hargrove School District and the Ministry of Education to be fluent in English within the first few years of their entrance into the school system. The impact that their cultural heritage may have on their judgments of school and mathematics could vary between the students based on their family's time in Canada and acculturation (Klassen, 2004). The departure from cultural heritage possible over time will likely happen for ESL students as they are immersed in Canadian culture, society and expectations. Any gradual change in self-efficacy due to acculturation or cultural influence cannot be teased apart from the impact that the mathematics classroom has on a student's self-efficacy, because self-efficacy is based on both previous and present experiences. Furthermore, there should be no attempt to take these two factors apart, because doing so would not be authentic or possible: a student's self-efficacy is based on the entirety of a student's mathematics experiences. The impact of cultural background and Western immersion does complicate my findings, because self-efficacy cannot be attributed singularly to the problem format of the mathematics task, and problem format can only be a contributing factor to a student's measure of self-efficacy. The difficulty encountered because of multiple factors contributing to a student's self-efficacy further justifies conducting interviews with the study participants. The interviews give the children the opportunity to share their opinions about the impact of problem format on their ability to solve the problem, and this may be one of the only ways to isolate the impact of problem format from other contributing factors.

In order to minimize the stress on the participants and allow all students to participate to the full extent of their abilities, I attempted to provide the students of limited English knowledge with the appropriate language translations needed to complete the mathematics task and the interview. As mentioned earlier, the instructions for the mathematics task were translated into written simplified Chinese on the second page of the task booklet, and were also provided to the students through an audio recording of the instructions in both Cantonese and Mandarin. The instructions were provided both orally and in written Chinese because of the awareness that many individuals who speak Mandarin or Cantonese are not fluent in the written forms of the language, especially not at such a young age. Unlike most of the other materials in this study, the language-based word problems on the mathematics task were not translated for students, because the goal of this study was to investigate the impact that mathematics based in English has on ESL students. To provide them with a translation would negate the study and be an unrealistic reflection of the challenges that ESL students face in the English-language classroom. If desired, the interview, which included a series of questions asked orally of the participant, was provided to participants in either Cantonese and/or Mandarin through the use of prerecorded questions in both of these languages. The question was asked first by the researcher and then played in the appropriate language directly afterwards. Students were encouraged to respond in either English or the language of their choice if they did not feel that they could fully explain their thoughts or opinions in English. This flexibility was communicated to all participants in all three languages at the beginning of each interview.

Mathematics Task Trials

Three formal and three informal trials were conducted to identify any potential problems with the mathematics tasks. The formal trials were carried out one-on-one with participants from grade six and seven classes. Participants provided feedback to the researcher and shared their solution strategies for each problem during completion. These trials were conducted over the course of two weeks, and changes were made to the mathematics task based on the input from trial participants.

Informal trials were conducted with children in grades 5, 6 and 7 who were available at school to provide input after completing their assigned school work. The informal trials were conducted after the formal trials and did not result in any changes being made to the mathematics task.

All of the participants in the formal and informal trials were of Chinese descent and were fluent in BISC.

School Selection and Approach

Every September, schools register their ESL students with the British Columbia Ministry of Education in order to receive funding for the students for the upcoming year. These numbers are reported on the BC Ministry of Education website as downloadable PDF files for public viewing. I downloaded the reports for all of the schools in the Hargrove School District and contacted the schools with 12 or more ESL students in grade 6. There were seven schools for the 2007-2008 school year who met this criteria. I contacted all seven schools by telephone to introduce myself and my study and arrange a meeting with the school principal to further explain the study. Of the seven schools contacted, five accepted the offer for a meeting and two declined. I contacted only one school with fewer than 12 ESL students in grade 6 to find more participants for the study.

The enthusiasm of the participating schools was mixed. One of the schools had no participant response, while another had only one student volunteer to participate. Two other schools provided me with six participants each. The fifth school provided me with five participants and the remaining three participants came from the sixth school.

Subjects

At the time of the study, all participants were enrolled in all grade 6 courses, including grade 6 mathematics. Table 1 shows the distribution of participants by gender and school.

Table 1
Participant Gender and School Affiliation

Student Group	Number of Participants Per School					Total
	A	B	C	D	E	
ESL Level 2						
Female	1	1	1	1		4
Male			2			2
ESL Level 4						
Female			2	2	1	5
Male		2	1			3
Not Registered in ESL						
Female		1		1	1	3
Male		2		1	1	4

Participants were between 11 and 12 years old and all of Chinese descent with varying English language abilities, having lived for at least some period of time in China with their families. Six of the participants were registered in ESL level 2, designating them as students who were still acquiring basic interpersonal communication skills (BISC) in English (Cummins, 1980; Carrigan, 2005). Eight participants were registered in ESL level 4 which designated them as students who had mastered BISC and were working towards mastery of academic English (cognitive academic English proficiency or CALP). Seven participants who were fully fluent in English, and not registered in ESL, were included in the study as a control group. These students had received at least six years of their educational instruction in English and have had full comprehension of the language since at least grade 3.

Materials

Children were asked to complete a mathematics task problem booklet consisting of 12 problems similar to those found in the grade 6 mathematics curriculum (see Appendix A, Appendix B, & Appendix C). Both the Mathematics Task Question Booklet and the Mathematics Task Work Booklet had covers with written instructions for the mathematics task (see Appendix D and Appendix E, respectively). The second pages of both of these booklets are the completion instructions written in simplified Chinese (see Appendix F). Four problems were computation-based; four were image-based problems that merge interpretation, computation and logic while minimizing the need for language; and four were language-based problems typical of the ones currently used in the provincially recommended textbook resources. Two questions of each problem type were easy, one of each was of medium difficulty, and the remaining problem was more difficult. The questions in each problem format were of comparable difficulty to one another and designed to assess the use of mathematics skills involving addition, subtraction, multiplication and division knowledge within each problem format. The skills needed to complete the mathematics task were based on the British Columbia PLOs describing the curriculum expectations for grade 6 students (BC Ministry of Education, 1996).

The order of the mathematics problems was randomized in each of the mathematics task problem booklets so that the computation, word-based and image-based problems were in no particular order. Students in each focus group (ESL level 2, ESL level 4, and those not registered ESL) were given one of eight problem booklets. No two students in the same focus group had a mathematics task problem booklet with the questions in the same order. Randomization of the problems was used to ensure that if success or difficulty is encountered repeatedly on a question, the observed outcome or pattern could not be attributed to the problem's placement in the mathematics task problem booklet or on the influence of a particular problem preceding it. The problems were not placed on the mathematics task in any order related to their difficulty and were also randomized so that the difficulty of the problems cannot be assumed by the student from their placement (ex. from easiest to hardest or vice versa). The questions were tested in trials on ESL and non-ESL students and amended before the actual mathematics task was administered to study participants.

Students participating in the mathematics task recorded their written work in a separate designated booklet. It provided one full page for each problem and an adjoining page allocated for a series of opinion questions about the mathematics problems (see Appendix G). After completing the written work for each question, students were asked to circle a word that could describe their perceived level of difficulty for the question: very easy, somewhat easy, somewhat hard or very hard. I chose to use words, rather than a number-based scale because numbers on a scale do not have a fixed meaning. For example, on a scale of 1 to 10, where 1 represents no difficulty and 10 represents extreme difficulty, one student may chose to assign moderate difficulty to a 7, while another might chose to assign the exact same difficulty to a 5. Giving the students words instead of numbers could help minimize this disparity. Taking the students' possible language difficulties into consideration, the English words were translated into simplified Chinese directly below to further reduce any confusion. Translation of terms in this instance was beneficial because students' responses to these questions of opinion should be as informed and complete as possible. Students were also asked in separate opinion questions to rate their confidence in their answer and their ability to answer a similar question.

Scoring the Mathematics Task

When marking the responses of participants on each of the three types of questions, they were assessed based on work shown, areas of error, and accurate answers. Before marking began, multiple possible solutions were written for each problem (ex. a solution to a problem that used repeat addition and an alternative solution to the same problem that was equally valid but used multiplication). Possible solutions were developed by the researcher through collaboration with the trial participants who tested the mathematics task before it was administered to the study participants.

Errors on the mathematics task were coded as one of four types: no answer given, computational error, solution strategy error or comprehension error. Those questions which have no marks written in the solution area were coded as "no answer". Some students made a "calculation error" during the completion of a question. These were coded as such when the solution strategy presented was viable and demonstrate that without the calculation error an accurate answer would have been reached. In other instances, the solution strategy used to answer the problem was not effective and could not result in an accurate answer. The coding for such responses was "solution strategy error" and was only used when the strategy could not satisfy the requirements of the problem. A lack of understanding based on the problem itself was also possible. In a "comprehension error", the student exhibited confusion about what the problem was asking, rather than what mathematics should be used. These errors were harder to deduce and had to be uncovered through consideration of the work shown and the oral responses provided by the participant.

The incidence of correct and incorrect answers, perceived difficulty, assessment of correct answers, and high or low self-efficacy reports were tabulated and recorded in groupings determined by English ability (ESL level 2, ESL level 4, or no ESL background).

Procedure

Mathematics Task

The mathematics task was given to participants after school hours in a small group setting with other participants from the same school. ESL designation was not a consideration when grouping participants for the assessment. The task was completed in a small group setting to alleviate any stress that students may have experienced in a one-on-one assessment with the researcher. Before beginning, the researcher explained to all of the participants that the problems that they would be solving are at a grade 6 mathematics skill level or lower, and that many of the problems were very similar to those found in textbooks that had been used at some point in grade 6 classrooms in British Columbia. Each student was given one Mathematics Task Question (Problem) Booklet, one Mathematics Task Work Booklet, and a pen. The directions for the Mathematics Task Problem Booklet and the Mathematics Task Work Booklet were given verbally, but were also written and translated into Mandarin and Cantonese and provided to each participant. The researcher explained that the Mathematics Task Question Booklet contained the mathematics problems only, and should not be written in. Participants were also advised that the Mathematics Task Work Booklet provided the space for them to record all of their work and answer opinion questions. Students were asked to complete each question to the best of their ability. They were encouraged not to leave a blank space in the event that they were unsure of an answer, but rather to make an educated guess as to what the problem was asking and how to formulate their response. Participants were strongly encouraged to show all of their thinking on the paper and asked not to erase any of their work. They were advised that the researcher wanted to see, not only the end result of their thinking, but also how they got to that answer. If participants thought that any of their work was incorrect, they were asked to simply cross out their mistake with a single line and make their corrections in the space beside their error. It was explained to them that these mistakes and the written work they complete before they find a final solution often provides valuable information to the researcher along with the final answer itself. After answering each mathematics problem, students were asked to make three judgments about it: to rate the difficulty of the questions; provide their belief that they have reached the correct answer; and their ability to successfully complete more questions just like it. Their self-perceived ability to correctly answer similar problems in the future was a measure of their self-efficacy beliefs relating to tasks of the same nature.

After the participants completed the mathematics task, they handed in both the Mathematics Task Problem Booklet and the Mathematics Task Work Booklet to the researcher so that correct answers could be marked and their work could be investigated for completion, errors, omissions, and demonstration of mathematics understanding.

Interview

The individual student interview and the mathematics task were not administered on the same day because twelve mathematics problems are enough to saturate most students after a day of regular school work. Since the interview portion of the investigation was important to understanding students' thought processes it was completed within one week of completion of the written mathematics test. Students met individually with myself as the researcher at the location of their parents' choice, such as their house or the public library. This was done to try to provide the students with the power of selecting an area that they would feel comfortable in, as suggested by Punch (2002). The interviews were audio taped and took approximately 0.5 hours to 1 hour to complete. It was comprised of a set series of interview questions (see Appendix H), discussion of the mathematics task, and time for participants to share their mathematics experiences and thoughts with the researcher. I introduced myself as a student at the University of British Columbia in an attempt to minimize the power imbalance that might have existed between me as an adult and the participant as a child (Punch, 2002). My teaching background and years of working with children provided me with the experience that I needed to establish rapport with participants before the interview began. This was done to try to encourage students to feel comfortable sharing their real opinions, rather than providing the answers that they felt that I wanted to hear, which has been reported as a problem in both child and adult research (Punch, 2002). I took time at the beginning of the interview to explain to each of the participants that his or her honest thoughts, ideas and insights about mathematics and mathematics instruction were important, informative and appreciated. This was done to try to set an open tone with the participants so they felt like their opinions were valued.

The interview was completed by the researcher asking the questions verbally, while providing students with an oral translation of the questions, so to minimize the language barrier. Students shared their responses verbally while the researcher audio taped their verbal responses and recorded notes about their answers and their nonverbal responses. If students were unable to successfully communicate their responses in English, they were given the opportunity to respond orally in the language of their choice, which could be translated later from the audio recording. The researcher gave students the time to explain their feelings about mathematics in general and compare their present math experiences with any they may have had in the past.

Before discussing their responses to the mathematics task, students were asked to describe their degree of success and enjoyment of computation problems, word problems, and picture problems, and asked to explain their choices with some discussion. This was asked early in the interview so that any discussion and questions asked about specific problems on the mathematics task would not impact students' judgments or responses about self-efficacy or enjoyment. The researcher then asked students questions related to the mathematics task that they had completed. Students were not told if their answers to the mathematics task were accurate because this may have swayed their initial responses regarding self-efficacy. It was intended that students' self-efficacy judgments be based on their regular classroom experience and not overly influenced by the mathematics task. The participants were shown an original unmarked copy of the mathematics task with their written work and question responses. The interviewer asked students about all the picture problems and only one or two computation problems and word problems chosen at random. This was done because the focus of this study was primarily on students' responses to picture problems, not the other forms of mathematics problems. They were asked to identify the question each problem was asking in their own words and to describe their solution strategies. Students were asked to elaborate on how the problems could be made clearer and asked why they felt they would or would not be able to complete similar questions in the future. The questions on the mathematics task were discussed with students in no particular order so that order would not influence students' opinions, thereby impacting the results of the interview. At the end of each interview, participants were asked if there was anything about the mathematics task that they wanted to share with the researcher. Each participant was thanked for his or her time.

Not all of the responses to the interview questions directly answered the main research questions being investigated. Some questions allowed me to develop a sense of the student's language background and his or her general view of mathematics. This paper discusses only the responses to questions that address English use in the mathematics classroom, self-efficacy, and performance on problem solving in different formats, as outlined earlier in the research questions for this study.

Results

The mathematics task and interview, administered to 21 grade six students in one of three designated ESL levels during the latter part of the school year, demonstrated some interesting trends and noteworthy observations. Due to limited sample size, these findings are not generalizable but serve as an indication of performance for the selected group. Trends observed in this group can provide a baseline for further investigation. The reader is reminded that the research questions being asked are:

1. According to ESL students' self-reports, how is their mathematics ability and self-efficacy affected by the English used in the language-based mathematics curriculum?
2. How is mathematics achievement and self-efficacy different when mathematics problems are asked in a computation-based format, word-based format or picture-based format for students in ESL level 2, ESL level 4, and those students not registered in ESL?
3. What is the impact on students' performance and self-efficacy when using the mathematics modification of image-based problems, instead of language- or computation-based mathematics problems?

I will share student judgments about English in the mathematics classroom, and discuss trends and differences seen between student approaches to the problems and solution strategies as individuals and groups. Finally, I will present the written and oral responses of students in ESL level 2, ESL level 4, and not registered in ESL to each of the picture problems collected through the mathematics task and subsequent interview. All students have been given pseudonyms to conceal their identity.

Due to the exploratory nature of the study, the results provide us with initial insights into the role image-based problems may play, and provides us with directions for future research into this issue.

English in the Mathematics Classroom

In order to investigate student's feelings about the use of English in a language-based mathematics curriculum, I asked each participant for his or her input. The question was raised within the first few minutes of each student's interview in order to minimize any influence that the interview itself might have on the student's opinions. There were varied responses to the question "What is your opinion about the amount of English used in the mathematics classroom?"

ESL Level 2 Response to the Use of English in the Mathematics Classroom

Among students registered in ESL level 2, responses showed an acute awareness of the prevalence of English in their mathematics classroom. Many of the students explained that they feel that as their English improves, the amount of difficulty caused by the language in mathematics decreases. Emily mentioned that she has difficulty with both the words and sentences used in mathematics. "Some sentence is hard to understand....Some, like, those questions, um, some has a really difficult words, so I can't understand."

Mikey mentioned that he has noticed the dominance of the English language in his textbook; "For the new mathematics textbook there are a lot of those word questions. There's only one or two question that [are] numbers or shapes and all the other questions are words, just words. So [the amount of English in the math classroom is] pretty much like 90%." Another student explained that the focus on words in the curriculum impacts her ability to participate in mathematics learning.

> **Liz:** I can't understand completely for the questions. Oh, I can guess.
> **Interviewer:** What's hard to understand?
> **Liz:** Words.
> **Interviewer:** Single words, when they're put together [in sentences] or both?
> **Liz:** Both.

ESL Level 4 Response to the Use of English in the Mathematics Classroom

Students in ESL level 4 occupy a unique position in the world of mathematics education for ESL learners. While they have acquired conversational English, permitting them access to the discussion and instruction in the mathematics classroom, they still do not have all of the technical language necessary for true fluency. These students are also able to reflect on a recent time when they were participants on the periphery in the mathematics class because of their limited knowledge of the English language.

When asked to respond regarding their opinion about the amount of English in the mathematics classroom, two participants stated there is not a lot and they do not put much thought into it. I was surprised by their comments, but both of these students later elaborated on this point and mentioned that their mathematics teachers focus on the completion of computation problems. This individual classroom focus could clarify why they are not concerned about the amount of English in the mathematics classroom. This was different than the majority of the participants in this study registered in ESL level 4. While one participant stated that she finds the language in mathematics "pretty easy," five of the respondents communicated that they still struggle with the language component of math, primarily due to word problems. Many also acknowledged that they struggle less with the vocabulary than they have in past years. "Like my first year in, in Canada was really hard 'cause, 'cause, like, some of the words I don't know, but like, simple questions, like 20 times 10 then I can, like, without word questions, then I can answer. But now I can answer both of them except for explaining. 'Cause I know how to explain in Mandarin, but I have to translate it to English. That's a little bit hard."

In addition, some students shared coping strategies they have developed. One student explained that her strategy for learning new math concepts is to bring the worksheets home to translate it into her first language and review the concepts with her mother so that she does not miss any important information. Another student shared that his trick is to locate the digits in the written questions to ensure that they are all somehow placed into the number sentence that he derives from the word problem.

Response of Students Not Registered in ESL to the Use of English in the Mathematics Classroom

Students in this study who are fluent in English recognized that mathematics instruction and problems are generally presented to students in a language-based format. Six of the seven students explained that they are comfortable working within this format, while Jimmy stated that he experiences some difficulty with these problems because he finds the wording hard to understand. "Some of the question they ask you are easy, um, but they put really hard words to sort of confuse you." Austin explained that the amount of English used in the mathematics classroom "doesn't really matter to me 'cause, like, I can understand it." Tracy recognized that ESL students have difficulty with the language component of mathematics, "For the ESL students in our class, they can do the math but sometimes they have to ask, um, fellow classmates what it means. And they all have this electronic dictionary thing [to help them]."

General Overview of All Three Problem Formats

Students completed four computation problems, four word problems, and four picture problems on the mathematics task. The results of the mathematics task were very surprising. I had expected students with no ESL designation to perform equally well on all three forms of mathematics, ESL level 4 students to perform slightly worse on word problems than the other two forms, and ESL level 2 students to struggle most dramatically with word problems. Not all of these assumptions were confirmed. I will provide basic information about the students' responses to computation problems and word problems so that these two mathematics formats, with which students are familiar, can be compared to picture problems, a unique mathematics format. Because picture problems are the mathematics modification being investigated in this study, I will elaborate on students' responses to picture problems later in detail as to ensure that the reader is able to understand the students' thought processes and solution strategies. Please note that Question #3 has three times the number of responses than the other problems because students had to provide answers for three separate components to satisfy the requirements of this picture problem.

ESL Level 2 Performance and Response on Mathematics Task by Problem Type

The six students in ESL level 2 made errors on every problem except Question #7 and Question #12, and only one error was made on Question #1 and Question #11 (see Table 2).

Table 2
Summary of ESL Level 2 Performance on Mathematics Task by Problem

	Problem Type											
	Picture				Word				Computation			
Error Type	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12
No Error	5	4	15	3	4	2	6	3	3	3	5	6
Calculation	0	0	2	0	1	0	0	2	0	0	1	0
Solution Strategy	0	2	1	0	1	4	0	1	2	0	0	0
No Answer	0	0	0	2	0	0	0	0	1	1	0	0
Comprehension	1	0	0	1	0	0	0	0	0	1	0	0
Unsolvable[a]	0	0	0	0	0	0	0	0	0	1	0	0

Note. The values represent the number of solutions coded with that error type. For all problems, except #3, n=6. For #3, n=18 because the problem asked for three separate values.

[a]A printing error occurred in the Mathematics Task Problem Booklet which made Problem #10 unsolvable for some students. This outcome was not recorded as an error on the part of the student.

Out of all of the computation problems, students had the most difficulty on Question #9 because two of the students felt that the question was "impossible" due to ineffective solution strategies. Another student felt that it would take too much time to solve it so he did not provide an answer.

Word problem Question #6 was difficult for two thirds of the participants. All four of the students with errors on this problem mentioned that they had difficulty with the language of the question, especially the last sentence "What number is the solution to the problem?" Shelley commented, "I don't really get this question so I couldn't answer this question, like, for sure." The students demonstrated attempts at placing all of the digits into a number sentence that uses subtraction and addition, though not necessarily in the order intended by the word problem.

Out of all of the picture problems, ESL level 2 students only demonstrated considerable difficulty on picture problem Question #4. During her interview, Shelley struggled to identify the question being asked by the image. Consequently, she was not able to complete the question effectively. Two other students explained that they were unsure about the appropriate solution strategy to use and chose not to answer the problem altogether. Both of these students also explained that if the question mark had been at the end of the problem they would have been more successful.

ESL Level 4 Performance and Response on Mathematics Task by Problem Type

The eight students registered in ESL level 4 presented many of the same difficulties and strengths as the students in ESL level 2, though their strengths and weaknesses appear to be more pronounced (see Table 3). Similar to ESL level 2 students, all ESL level 4 students accurately answered one word problem (Question #6) and one computation problem (Question #10).

There were only four errors made on computation problems. Two of the errors were answer omissions, one made because a student found Question #9 too difficult to answer, and the other omission on Question #11 for no obvious reason. Question #12 has two errors, one due to inaccurate subtraction and the other a solution strategy error because the student altered the order of operations in the problem so that it no longer led to an accurate answer.

Table 3
Summary of ESL Level 4 Performance on Mathematics Task by Problem

	Problem Type											
	Picture				Word				Computation			
Error Type	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12
No Error	5	4	14	7	2	8	6	4	7	4	7	6
Calculation	0	1	3	0	0	0	1	3	0	0	0	1
Solution Strategy	2	3	5	1	6	0	1	1	0	0	0	1
No Answer	0	0	2	0	0	0	0	0	1	0	1	0
Comprehension	1	0	0	0	0	0	0	0	0	0	0	0
Unsolvable[a]	0	0	0	0	0	0	0	0	0	4	0	0

Note. The values represent the number of solutions coded with that error type. For all problems, except #3, n=8. For #3, n=24 because the problem asked for three separate values.

[a]A printing error occurred in the Mathematics Task Problem Booklet which made Problem #10 unsolvable for some students. This outcome was not recorded as an error on the part of the student.

The students in ESL level 4 showed the most difficulty on word problem Question #5. Every error was the result of students subtracting the cost of supplies from the final profit instead of adding to it. The question asked "How many customers did they have if their profit was $84 after they paid $12 for their cleaning supplies?" In their interviews, many of the students who made this error explained that the fact that the people holding the carwash *paid* this money meant that it needed to be subtracted. Students also explained that they choose to carry out the subtraction of $12 before they determined the number of customers because of the apparent direction given by the use of the word "after." This indicated to them that the subtraction of $12 needed to be done before they could divide to find the number of customers. This particular error was made by ESL level 2 students in only a few instances. The presence of this error in responses provided by ESL level 4 seems to demonstrate that these students still experience language difficulty in mathematics word problems even though they are more fluent in English than ESL level 2 students. Some participants elaborated on their difficulties with the language during their interview, though others made no mention of it. Each of the students who answered Question #5 mistakenly were sure that they could answer another question like this, and half of them were completely sure of their answer.

Picture problems were more difficult for the participants in ESL level 4 than those in ESL level 2. The most difficulty was experienced on Question #2 and the least on Question #4. Most of the errors were made due to incorrect solution strategies. Picture problems will be discussed in more detail later in this chapter.

Performance and Response on Mathematics Task by Problem Type of Students Not Registered in ESL

The seven students not registered in ESL answered more computation problems correctly than word problems or picture problems (see Table 4). No errors were made on computation problems due to ineffective solution strategies. All seven participants were able to answer Question #10 correctly and only one error was made on each of the other three computation problems, two of these due to answers omitted by the same student.

The majority of the errors made on word problems were due to calculation errors with only three errors because of ineffective solution strategies.

Students not registered in ESL made twice as many errors on picture problems due to ineffective solution strategies than all other error types combined. Every student made a solution strategy error on Question #2. Six calculation errors were made on Question #3 and no other calculation errors were made on picture problems. Again, detailed responses to picture problems will be discussed in more detail later in this chapter.

Table 4
Summary of Performance of Students Not Registered in ESL on Mathematics Task by Problem

	Problem Type											
	Picture				Word				Computation			
Error Type	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12
No Error	5	0	11	4	4	6	5	4	6	7	6	6
Calculation	0	0	6	0	1	1	2	2	0	0	0	1
Solution Strategy	0	7	3	3	2	0	0	1	0	0	0	0
No Answer	0	0	1	0	0	0	0	0	1	0	1	0
Comprehension	2	0	0	0	0	0	0	0	0	0	0	0
Unsolvable[a]	0	0	0	0	0	0	0	0	0	0	0	0

Note. The values represent the number of solutions coded with that error type. For all problems, except #3, n=7. For #3, n=21 because the problem asked for three separate values.

[a]A printing error occurred in the Mathematics Task Problem Booklet which made Problem #10 unsolvable for some students. This outcome was not recorded as an error on the part of the student.

The high number of errors made on picture problems was not due to any obvious weakness in mathematics on the part of the non-ESL students. They demonstrated that they were more than adept at computational mathematics and were successful at word problems. These participants were used to being successful within the current format of the mathematics classroom and might have been unable or unwilling to deviate from the format that they were comfortable with: "We don't really use pictures, we just use words and numbers and I'm not really used to pictures." However, it can be argued that if students possess mathematical understanding, they should be able to apply their knowledge of mathematics to a wide variety of situations, including those problems presented without English language accompaniment.

Percentage of Errors Made by Each Participant Group by Problem Type

All participant groups made errors on each problem type: picture problems, word problems, and computation problems (see Table 5). Although students had no previous experience with picture problems, unlike computation and word problems, it seems "unfair" to directly compare their performance on picture problems to computation- and word-based problems. However, trends between the participant groups were present. The percentage of errors for each participant group and problem type is calculated by determining the number of errors due to solution omission, inaccurate computation, ineffective solution strategies or erroneous interpretation of the problem itself, and dividing the total number of errors by the number of each type of problem attempted by students (see Equation 1).

$$\text{Error calculation for each problem type} = \frac{\text{Total number of errors}}{\text{Total number of problems attempted}} \qquad (1)$$

The percentage of errors made by participants registered in ESL level 2 on picture problems and computation problems was the same, whereas the percentage of errors on word problems was slightly higher. The latter findings about word problems were expected as these students are not fluent in English and could be expected to struggle with language-based problems. ESL level 2 students made the fewest number of errors on picture problems when compared with the other two participant groups and made more errors on computation than the other two groups combined. It is possible that the ESL level 2 students had a mathematics weakness in the area of computation so their success rate in that area was below the other two groups. However, if this computation weakness did exist, it did not seem to have a large impact on ESL level 2 students' ability to complete word problems because their results were the same as ESL level 4 students and only slightly weaker than students not registered in ESL. The difficulty demonstrated by ESL level 2 students on computation did not affect their ability to complete picture problems when compared to the other two participant groups because their success rate was much higher than either group, especially those students not registered in ESL.

Table 5
Percentage of Errors Made by Each Participant Group by Problem Type

Participant Group	Errors by Problem Type (%)		
	Computation	Word	Picture
ESL Level 2	25.0	37.5	25.0
ESL Level 4	14.3	37.5	41.6
No ESL	10.7	35.7	64.3

Note. Errors types include: calculation, solution strategy, answer omission, and comprehension.

The participants in ESL level 4 made roughly the same number of errors on word problems and picture problems, though the percentage of errors on picture problems was slightly higher. They made considerably fewer errors on computation problems than either picture or word problems. This finding suggests that ESL level 4 students' difficulty on picture problems and word problems may not be linked to poor math skills but perhaps due to difficulty with other factors such as interpretation, comprehension, or solution strategy. When compared with participants who are not registered in ESL, the number of errors made on computation problems was roughly similar. When compared to the performance of ESL level 2 students, ESL level 4 students performed equally well on word problems, and better on computation problems. However, they performed worse on picture problems than students in ESL level 2.

Out of all three participant groups, students fully fluent in English performed the best on computation problems. Their performance on word problems was only slightly better than ESL level 2 and ESL level 4 students but their performance on picture problems was drastically below the performance of students in either ESL level. Students not registered in ESL made errors in over half of the picture problems which were approximately double the number of errors made by ESL level 2 students on these same picture problems.

Table 6
Percentage of Errors Made by Each Participant Group by Problem Type After Calculation Errors are Removed

Participant Group	Errors by Problem Type without Calculation Errors (%)		
	Computation	Word	Picture
ESL Level 2	20.8	25.0	19.4
ESL Level 4	10.7	25.0	31.3
No ESL	7.1	14.3	50.0

Note. Errors types include: solution strategy, answer omission, and comprehension.

In order to try and understand the number of errors made on mathematics problems due to misunderstanding the problem or misguided solution strategies, Table 6 shows percentage of errors made with calculation errors removed. The percentage of errors made due to inaccurate solution strategy or poor comprehension of the question is calculated by dividing the number of these errors by the total number of each problem attempted by students (see Equation 2). Omissions of a final answer are also included as this error category and not considered to be a calculation error.

$$\text{Error calculation for problem type} = \frac{\text{(Total number of errors} - \text{Number of calculation errors)}}{\text{Total number of problems attempted}} \quad (2)$$

The same sorts of trends were present when calculation errors were removed as when they were included. Students in ESL level 2 now performed slightly better on picture problems than computation problems and had the most difficulty with word problems. Students in ESL level 4 and those not registered in ESL still demonstrated the same trends as when calculation errors are included. Both of these groups performed better on computation problems than word problems and had the most difficulty on picture problems. Students registered in ESL level 2 and ESL level 4 still made the same number of errors on word problems as each other and fewer on computation problems than before calculation errors were removed. Students in ESL level 2 made double the number of errors on computation problems as ESL level 4 students and three times as many computation errors as students not registered in ESL. The percentage of errors made by students not registered in ESL on word problems dropped noticeably when calculation errors were removed. This suggests that their difficulty on word problems was superficial since a large part of the errors they made were simple calculation errors which could be easily remedied and their performance on computation problems, using their basic math skills, was strong.

Students in ESL level 2 made roughly the same number of errors on each problem type (within 6%). Students in ESL level 4 made over twice as many errors on word problems than computation problems and three times as many errors on picture problems as computation problems. Students who are fully fluent in English showed the most dramatic difference in ability on each problem type. They made twice as many errors on word problems than computation problems and seven times the errors on picture problems than computation problems. This suggests that their ability to work within multiple mathematics formats was not as well developed as those students registered in ESL level 4 and much less consistent than those students registered in ESL level 2.

Self-Efficacy by Problem Type

Self-efficacy, a student's judgment of his or her success on future tasks similar to ones already completed, was investigated for each problem format on the mathematics task by asking students how they would perform on the same sort of task in the future. Students were asked for their judgment of self-efficacy and to gauge their enjoyment of computation problems, word problems, and image-based problems.

ESL Level 2 Students' Self-Efficacy by Problem Type

When considering how well they would do on a page of number problems, word problems, and picture problems, students in ESL level 2 unanimously felt that they would be able to answer the highest percentage of computation problems correctly. The reason Alicia proposed for this expectation is "'cause it's easy. Every question is like the same way to do it." Another reason given by multiple participants was the importance of mathematics facts in China. One student explained, "Um, in China, math was more focused about adding and subtracting, dividing and timesing [*sic*]." Shelley also volunteered that her ability to succeed on number problems has decreased since her immersion in the Western classroom. "Because, um, like, in China the questions were more difficult, but when I came here for like more than one year I didn't do much, like, old questions like that. I probably wouldn't do as well as before."

All of the ESL level 2 students voiced concerns that their success on word problems would be significantly lower than number problems. Shelley stated, "Um, some English math questions, like, I don't get it, so yeah. Some sentence is hard to understand." When asked for an example, she elaborated that, "some, like those questions, um, some has a really difficult words so I can't understand." Another student continued this idea by explaining his struggle with word problems. "Some words I don't understand or I misunderstand some so it might get me wrong. Sometime the word combinations I don't understand." The other ESL level 2 students echoed this concern. They indicated that they also experience difficulty determining missing information, creating accurate translations, and having the necessary mathematics vocabulary available to them.

When asked about their enjoyment of word problems, five of the students explained that they do not enjoy working on word problems for a variety of reasons. Two students stated that their lack of enjoyment is simply because they do not enjoy mathematics. Three others implicated their incomplete understanding of the English language as their reason for not enjoying word problems. One student, Emily, stated that she enjoys word problems when they tell a story but does not enjoy them when they are used simply to elicit a series of mathematical operations.

After the ESL level 2 students gained experience completing the four picture problems on the mathematics task, when asked to consider how well they would do on future picture problems, their responses were mixed. Four students stated that in some instances picture problems confused them. Yet, five of the seven students thought that they would perform better on picture problems than word problems. They attributed this primarily to not having to translate any words in order to be successful on the problem. The two students who believed that they would likely perform worse on picture problems than word problems were the only two students who explained that they are used to problems in word format and are more likely to succeed in a form of mathematics more familiar to them. One student stated, "I'm too used to doing numbers and words, so I would do, like, pretty bad on [picture problems]....In China, in every country, you learn numbers and then you come to Canada, you learn words but right now you're learning a new kind of math that, like, new. And some symbols you don't understand or you don't know how to think about it so it really confuses you." The other student mentioned that while she understood two of the picture problems, the two that she did not understand were hard for her because "the picture look(s) weird and in school I always do word problems so...."

These two students explained that they are more comfortable with problems similar to their previous experiences, than new problem formats. This desire for familiarity was seen repeatedly, and to a greater extent in participants in ESL level 4, and even more so with participants not registered in ESL.

All of the students in ESL level 2 felt that if they were taught how to answer picture problems they would be highly successful on them relative to their level of success on word problems. "If I learned the picture problems compared to the word problems, I don't think I would misunderstand anything. Because every country uses pictures and pictures are easier to remember than words." Another student went on to say, "I think I can do good because pictures tell me, like, it doesn't tell me the words, but pictures tell me how many, like, what is like price or we have to add something and the picture tell me you have to get minus or times so that you have to just minus something."

Even though all of the ESL level 2 students felt they would be most successful at computation problems, they all ranked their enjoyment of picture problems on par or higher than both number and word-based problems. Three participants explained that picture problems were fun and enjoyable due to the visuals and different colours, but also warned that their enjoyment was jeopardized when the pictures were confusing.

ESL Level 4 Students' Self-Efficacy by Problem Type

Similar to the responses provided by ESL level 2 students, all of the ESL level 4 participants shared the same preference for computation-based mathematics problems over word or picture problems. All of the eight ESL level 4 students believed that they would be highly successful at number problems. Five students mentioned that their success would be amplified by the absence of words in computation problems. This demonstrates that even though ESL level 4 students are thought to be only one year away from English fluency, they still experience nervousness when they encounter English in mathematics. When Candy was asked how well she would do on number problems she responded, "good 'cause they're not in English so I won't have to worry if I understand it or not," while Brandon explained that number problems raise no concerns for him because there are "just numbers and no words to understand. [I] just look at the numbers and just write it out." All eight of the students in ESL level 4 shared that when they first came to Canada they would have done as well, or better than now, on computation problems due to the focus on computation problems and recall of math facts in China.

Five of the eight students replied positively that they enjoy number problems. As one student elaborated, "the numbers are really clear to you and then it's simple to do them, um, because you can easily see what the numbers are and then you just need to know what numbers are there to add, subtract, divide, and multiply." Most students agreed that their enjoyment of this problem format comes from a clear understanding of how to answer the problem and simply needing to recall mathematical principles.

ESL level 4 students demonstrated less apprehension than ESL level 2 students about their perceived level of success on word problems. All ESL level 4 students stated that their success would be fine, okay, or good on word problems. Michelle explained that she would do "fine" on word problems because "I learned pretty much English, so I could understand simple questions." One student explained that she believes "word problems are good. They help the ESL [students] learn words and then they, if you don't read them clearly, they can make the answer wrong for you, so it's tricky. So you have to be careful of, um, be careful of the numbers there. And then sometimes the word problems, they hide the numbers by making the numbers in words." Two students mentioned that they have actually practiced word problems outside of school hours to improve their performance on this problem format.

During their interviews, many ESL level 4 students spontaneously described moments which illustrated the struggles they have had with word problems in the past. One student shared that when she was first learning English she "asked a lot of questions like what does this word mean, what does this word mean? And sometimes, even though they explain it to me, I still won't understand." This same frustration was also touched upon by another participant as well.

> **Brandon:** I wouldn't get like, if I got one I would be really lucky because, like, when I first came I didn't know exactly English or what. Like, I didn't know any English actually, and then I couldn't understand anything except for 'okay.' Just one word. I don't know anything when I first came to Canada. When I was given word problems I didn't know what to do so then I told another person who knew, like, Chinese to tell the teacher I didn't know what to do and then the teacher took it away and gave me number problems instead.
> **Interviewer:** How did that make you feel?
> **Brandon:** Strange, because I was looking at this page with words on them and I didn't know what they meant, so yeah.
> **Interviewer:** And how did you feel about that [page of words] being math?
> **Brandon:** Pretty weird. It's such a jumble of letters together.

Compared to students registered in ESL level 2, students registered in ESL level 4 demonstrated increased self-efficacy on word-based problems and increased enjoyment of such problems as well. Three of the eight students stated that they find word problems enjoyable because of their increased familiarity with this problem format and the opportunity to "figure out new words for the definitions to improve…math skills" and grammar skills. Five of the eight participants believed that they would "sort of" enjoy word problems and explained that their enjoyment is hampered by difficulty understanding the vocabulary or sentence structure used in the problem.

When asked how well they would perform on picture problems, five of the eight participants in ESL level 4 believed that they would not perform as well on picture-based problems as word problems, while the other three participants felt that they would perform better on picture-based problems. Some students' hesitation regarding picture problems stemmed from a desire to experience minimal confusion in mathematics. As one student stated, "I would do average, because, um, the picture problems, they're, like, sometimes more confusing than the word problems because picture problems you have to, like, know what they're trying to tell you and word problems, they just tell you right away." Ricky was quick to question his understanding of picture problems and demonstrated discomfort when it was possible that his answers were wrong.

Despite ESL level 4 students feeling some confusion on picture problems, all of them, except Brandon, actually explained that they enjoyed picture problems more than word problems. The majority of the students expressed that they would enjoy completing picture problems because of the visual and mental stimulation, ease of comprehension, and assistance it gives to ESL learners. Nicole stated that when she first came to Canada she would have performed "like the same [on picture problems] as number problems. Because, like, pictures, they don't have any words and English to solve. I think the children in [Canada] will do better than me because the teachers in here taught in a more imaginative and creative way, but in China, it's just, like, textbooks."

Many students in ESL level 4 also explained that the multiplicity of question and answer possibilities, coupled with the onus on the student to determine the correct meaning, forced them to think logically and analytically, which they enjoyed. Matthew explained that, "It's more fun because picture problems actually give you those pictures and then you [are] trying to find out what they mean." Debby repeated many times that it is important for her to be actively engaged in mathematics, not simply repeating tasks over and over, and enjoyed picture problems because they challenged her and they forced her to think during each problem. "Well first of all you get to look at this pictures right. It's kind of fun doing that...It won't really boring me that much."

Some students did not share this sentiment however. Brandon's reason for decreased enjoyment of picture problems was directly related to his lack of exposure to them. He demonstrated a fear of being "incorrect" and a preference for the inclusion of language. Brandon had no knowledge of whether his answers on the picture problems on the mathematics task were "correct" or "incorrect," yet he already insisted that he is unable to do them. He stated that his enjoyment of picture problems was low because "I don't usually do picture problems that much." However, he believed that if he were to practice, he would "do good at them." Brandon actually grossly underestimated his performance on picture problems because he was able to interpret all of the picture problems accurately and answer them effectively.

Self-Efficacy by Problem Type of Students Not Registered in ESL

Students with no ESL designation demonstrated quite a different opinion about the three mathematics problem formats than ESL level 2 or ESL level 4 students. All seven participants felt that they would be successful completing computation problems because they knew the mathematics rules and "you don't have to read anything and it takes less time." Most of the students felt that they would get at least 90% of the problems "correct". Chris commented that "I'll get most of them right. A few wrong. Sometimes I forget some things or I forget to carry the one or something," indicating that the only difficulties he experienced are simple calculation errors, and as a consequence of carelessness, not comprehension.

When asked to explain their enjoyment of computation problems there was a myriad of responses. Two students enjoy computation problems, one commented that his enjoyment is linked to the fact that "they're easy to do. Just adding…you don't have to really think too much." Tracy stated that her enjoyment is dependent on the variety of computation problems given because simple, repetitive questions are "boring." One student did not know how much he would enjoy computation problems and another said that he would experience moderate enjoyment. Mark asserted that he would not enjoy number problems because they are math and he does not enjoy math. This was consistent with his responses for enjoyment of word problems and picture problems as well.

The students fully fluent in English relayed that their ability to be successful on a page of word problems is either 'okay' (four responses), 'good,' 'pretty good,' or 'very good'. The most striking difference between this group of participants and those enrolled in ESL was that only one student who is fully fluent in English seems to have concerns about the vocabulary and sentence structure posing a difficulty for him. He would be "sad" because he does not "like [word problems] at all. Um, find, um, I find them hard to understand." Two other students viewed the language used in word problems as a trick that they are usually capable of sussing out and not falling for.

Though all of the students with no ESL designation ranked their success on word problems from moderate to high, there was no consensus among the group about their enjoyment of such problems. Two students responded that they are happy receiving problems of this type, two felt that they would have moderate enjoyment with the potential for boredom, and the other three believed that they would not enjoy completing such problems. Mark commented that his lack of enjoyment is directly related to the fact that word problems are harder than number problems which make them less enjoyable.

Students with no ESL designation all had different opinions regarding their success on picture problems. Two students believed that they would do "bad," three felt they would do "okay" and two others responded with more optimistic judgments of "pretty well" and "pretty good." Tracy stated in her interview, "I can understand stuff from pictures and it uses the same math skills which I already know." The students who felt that they would have low or moderate success blamed this on finding picture problems confusing. "Like, sometimes they just put pictures there and then question marks and then, yeah, I don't get it."

> **Brandon:** This one, I thought it was just um, like this one, I didn't even know how you do it but then I used my fingers like this and went "One, two." I times it by two. I figure out [the jellyfish length] first and I times it by two and then that's it.
> **Interviewer:** But you didn't write it down because…?
> **Brandon:** Cause like I thought, I wouldn't get it right so then put some random thing there and then yeah.
> **Interviewer:** So you just put nothing?
> **Brandon:** Yeah, because I would have gotten it wrong anyway.

Brandon stated that his mathematics abilities on this picture problem were "really bad because it's all those picture things and then I did not like picture things at all so then I just didn't understand this one." He seemed to have strong negative feelings towards picture problems and applying mathematics to a different problem format, something also demonstrated by some of the other students, especially those not registered in ESL.

All of the students who thought that they would do poorly on a page of picture problems also stated that they would not enjoy completing those problems. Conversely, all of the students who felt that they would perform at least satisfactorily on picture problems would be "happy" to do picture problems, found them "fun," and considered their enjoyment of picture problems to be "better than [that of] word problems."

For picture problems, a new problem format, enjoyment for this group of students was directly related to their self-perceived level of success on future picture problems, or self-efficacy. This direct link was not obvious in participants enrolled in ESL level 2 or level 4. With the exception of a few students who were adamant that they were unable to understand picture problems and consequently would not enjoy them, the students in ESL level 2 or level 4 were less likely to base their enjoyment of picture problems directly on their self-efficacy.

Austin, a participant who has not been registered in ESL since grade 1, demonstrated a resistance to applying mathematics to a new format such as picture problems. He was used to interpreting and completing mathematics problems in a format that he is able to easily understand, such as numbers or English language-based problems. Because of this he seemed conditioned to only functioning in mathematics where his English language fluency allows him to succeed. When asked about how well he would do on picture problems he replied, "Bad!"

> **Austin:** I can't anticipate the pictures. If they're included with words then I will do very well but if they're just pictures, like, it would be very confusing.
> **Interviewer:** Why would it be confusing?
> **Austin:** There's no, like, question and then you, like, have to figure out the question on our own.

During Chris' interview, he communicated his frustration that picture problems had insufficient written information to determine the actual math questions from the picture alone. Even after he claimed that he was mostly certain about his answers and felt that picture problems were sort of easy, he reverted, stating that a subsequent question was "another picture so I don't know what to do. I wouldn't know what the correct answer would be."

Written and Oral Response to Individual Picture Problems

In order to understand how the unique mathematics format of picture problems impact students in their mathematics problem solving, each of the participant group's responses to the four picture problems will be discussed in some detail. Those students who provided the correct solution strategy will be discussed more briefly. Where students erred, explanations of their work and thought processes will be explored in greater detail.

ESL Level 2 Students' Responses to Question #1

All of the participants in ESL level 2, except one, answered Question #1 correctly. When asked to describe what was being asked in Question #1 with the sporting goods, shopping cart, and cash register, Alicia replied that it was not immediately clear to her what the image was asking.

> **Alicia:** [The question is:] What did I buy to put it in the cart and get the amount of money that's $82? But in the first time, I thought maybe they're asking you how much dollars that's more dollars or less dollars than $82.
> **Interviewer:** How did you decide to do this? [Points to the written work on the mathematics task.]
> **Alicia:** Because, um, because of the shopping cart. If they ask you [for the change] it might be, like, a different picture I think.

Shelley's attempt to answer Question #1 demonstrated that her confusion about the question led directly to her inability to answer the problem intended by the image (see Figure 1). She stated that "I thought [the cash register] was showing [that all of the items added up] was $82, so, um, or this could talk about if they paid more than this much, like they exchanged $82. This confuse me." Shelley's confusion about what the problem desired stopped her from determining a definitive solution strategy. It was unclear from her work and her interview whether she intended to have $82 as the change or as the amount difference (i.e. the difference between the cost of all of the items and the total displayed on the cash register). Rather than subtracting $82 from the five item total of $140, as is suggested by the "minus" symbol in her number sentence, Shelley added the two numbers resulting in a total of $222 which she stated she paid, placing herself into the position of the customer. This unclear solution strategy mimicked her confusion about what the question appeared to be asking and her resulting work did not satisfy the question the researcher intended as indicated by the placement of the question mark and arrows on the image.

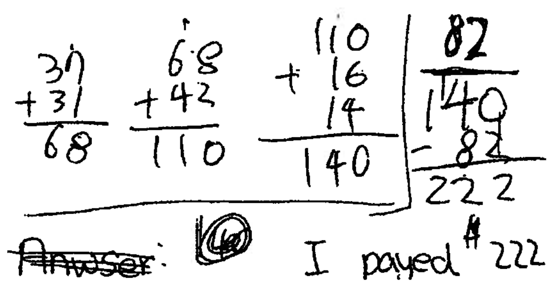


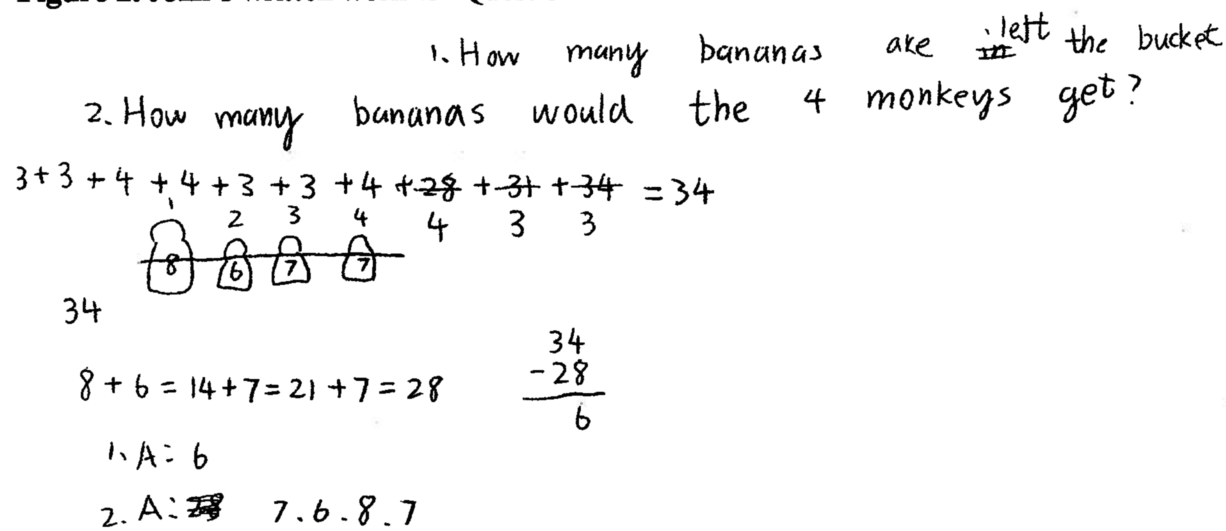
Figure 1. Shelley's written work for Question #1.

ESL Level 2 Students' Responses to Question #2

Students in all participant groups demonstrated metacognition, reflection on their own thought processes, while completing Question #2. During the interviews, many of the students across all participant groups explained that they debated two components of Question #2: the first being whether to distribute the bananas individually or by bunches, and the second being whether the bananas should be distributed to monkeys equally or distributed dependent on their size.

Shelley debated whether to divide individual bananas or bunches of bananas between the monkeys and she decided to answer what she considered the simpler of the two problems because "Um, I thought if you go to bunches, you can share too." Four of the six students ESL Level 2 students chose to divide the bananas equally, while the others divided the bananas according to their assumptions about the size difference between the monkeys.

Figure 2. John's written work for Question #2.



During the interview, John explained that he believed Question #2 was the most difficult picture problem primarily because of the confusion he experienced while determining what the question was asking (see Figure 2). His first attempt at interpreting the problem led him to believe that it was asking "how many bananas are in the bucket?" He stated that he realized that the bananas were to be distributed to the monkeys and he chose to assign bananas to monkeys based on their relative sizes. After he distributed the bananas to the monkeys, he found that he had bananas left over. In order to deal with the remaining bananas, John changed his interpretation of the problem to "How many are *left* [in] the bucket?" This fit more accurately with his selected solution strategy and allowed him to account for them, without creating a solution containing remainders. When questioned about his choice to assign bananas based on the size difference, John stated that "if they were supposed to [get the] same number the monkeys would be the same size." He explained that the visual cues given about size trump the instructions indicated by the mathematical symbols present.

Alicia indicated that she was not aware that the equal symbols placed between the question marks above each monkey were mathematical symbols. She explained that she simply assumed that they were placed there "as decoration." She asserted that the equal symbols should be placed in between the monkeys instead if they were meant to indicate that equal distribution should be used.

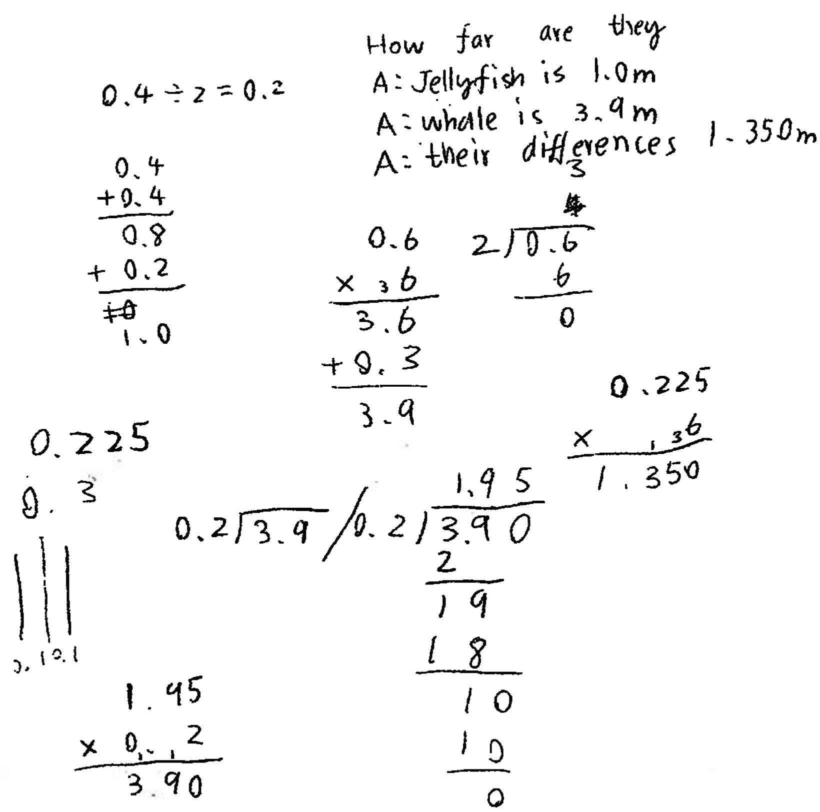
ESL Level 2 Students' Responses to Question #3

All of the students in ESL level 2 said they understood the question being asked by the images in Question #3. In her interview, Emily had difficulty explaining what the question was asking for with words, but was able to determine a solution that satisfies all three components. She achieved this by using her fingers to indicate the values for which she was solving. Four of the six students solved all three components of this problem accurately. Each of them determined the length of the whale and the jellyfish by multiplication and addition of the segments and subtracting the length of the jellyfish from the whale to find the difference in length. Mikey also used this solution strategy but made a multiplication error when determining the length of the whale resulting in an inaccurate answer for the difference in length.

John followed an effective solution strategy for the length of the whale and jellyfish and was able to accurately determine both of these values.

However, he made a solution strategy error when trying to determine the difference in length between the whale and the jellyfish. Using a series of mathematical operations, in a manner similar to a guess and check strategy, John concluded that the size difference is 1.350 metres (see Figure 3). John had previously determined that half of the whale's length is 1.95 metres and his measurement for the length difference was less than half the length of the whale. However, the picture shows the space left by the difference of their lengths is more than half the length of the whale.

Figure 3. John's written work for Question #3.



ESL Level 2 Students' Responses to Question #4

Three of the six students completed Question #4 with a correct solution strategy. All of these students understood that they must view the question with people moving in reverse. Therefore, as passengers get onto the bus, they subtracted the equivalent number, and when passengers departed from the bus, they added the same amount, resulting in accurate answers.

Of the other three students registered in ESL level 2, one student used an erroneous solution strategy and two students did not provide answers to the problem. Shelley's answer had a mistaken solution strategy. Viewing the movement of passengers on and off the bus enabled Shelley to determine that there had to be at least 11 passengers on the bus at some point in order for 11 passengers to disembark in the third to last frame. She selected 20 people as the number of people on the first bus, demonstrating that she was aware that this is a previously unknown variable. After working through the passenger movement, she stated that seven people would be left on the bus. Even though her answer does not satisfy the problem, it took some thought for Shelley, which, she stated, made her feel good about her mathematics ability.

Both Mikey and Emily chose not to provide an answer. In the interview, Mikey attributed this to a lack of understanding of the problem itself. He correctly identified that "I think maybe it was asking how many people on [the bus with the question mark]." Likewise, he described a viable strategy (guess and check), but was confused because he believed that such a strategy could lead to many possible outcomes depending on the number selected for the variable and he knew that he must have zero passengers remaining.

> "Because if it asking me how many people are on the bus. I don't know how many people on the bus in the first picture. And there like six people gets off, I still don't know how many people on the bus. So… maybe if there was like 10 people on the bus there will be only like four left but if it's a different number, there will be a different answer, so it's, like, pretty confusing."

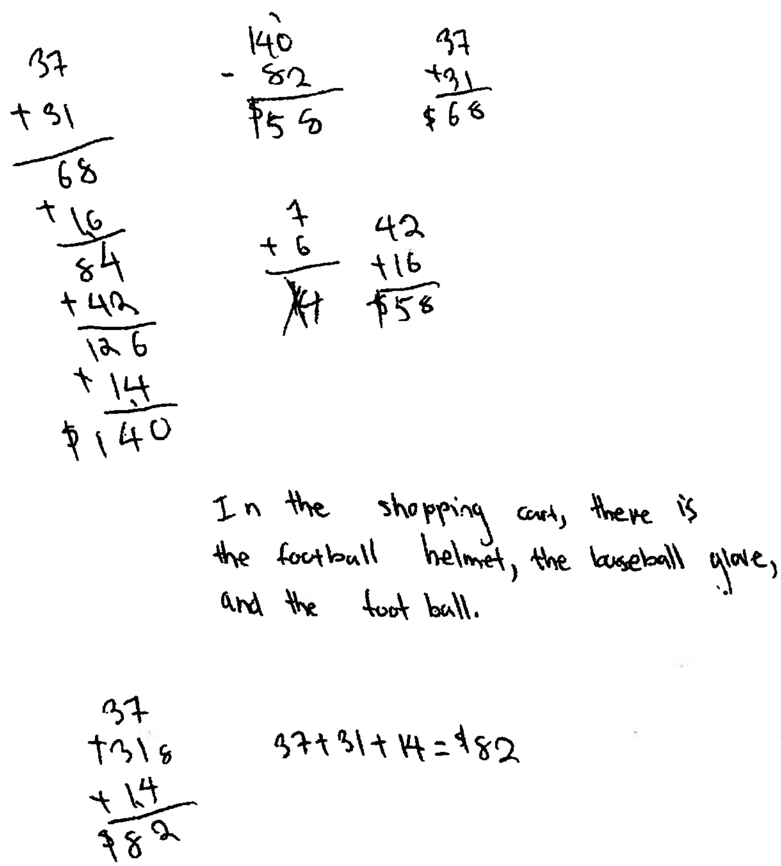
While Emily knew that she was to determine how many people begin as passengers on the bus, she was unsure of how to deal with 11 people leaving the bus in the third to last frame. She originally followed the problem through with the idea that the first six people to come off of the bus were the original passengers. When 11 people had to leave the bus, this number was larger that the number of people she thought was on the bus. This created a situation involving a negative number of people, which she did not know how to resolve.

All three students who made errors on Question #4 explained that if the direction of the question was reversed, so that the unknown number of passengers was in the final frame after the passenger movement, the question would be much easier. Similar to many other students with difficulty on this problem, Emily found the direction of the question challenging. "Um, because if you, if you like, draw it backwards, I would understand, like, it comes down and then goes up, like, the last one is [the bus with the question mark] and the first one is [the empty bus].” Before she reviewed her answer, she originally assumed that picture problems would be easy; however her difficulty on this problem led Emily to feel that “picture math is not as easy as I think.”

ESL Level 4 Students’ Responses to Question #1

Sam determined that the total cost of all of the items was $140 (see Figure 4). Rather than using guess and check to determine the three items purchased, he found the cost of the items not bought and used guess and check to eliminate those items that cost that amount, a faster process than the solution method used by most students. Once he determined the items not bought, he was able to deduce which items were purchased.

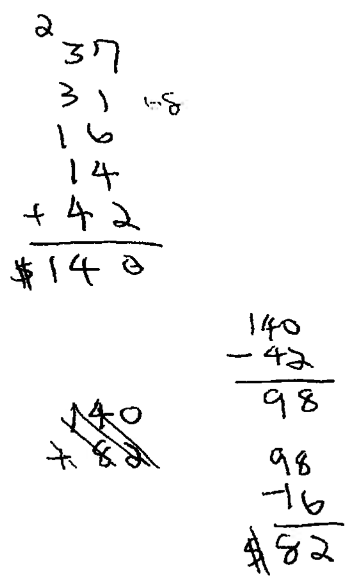
Figure 4. Sam’s written work for Question #1.



Similar to Shelley in ESL level 2, Nicole’s interpretation of the problem in Question #1 did not agree with the image and demonstrated difficulty comprehending the intentions of the image. However, Nicole explained that she believed that the customer wanted to buy all five items but only had $82 so she determined how much more money he or she needed in order to purchase all of the items. Such an interpretation does not seem to reflect the usual social meaning of an amount shown on a cash register in a purchasing situation. For this interpretation to be viable, the amount of $82 would have to be shown as belonging to a purchaser.

Candy and Ricky both employed inaccurate solution strategies to answer Question #1. Candy interpreted the total on the cash register as the change given, after all of the items were purchased. While this solution strategy included the numbers shown on the image, it did not actually account for the placement of the question mark in the image. The question mark in the picture problems was placed on the shopping cart to indicate that the solution desired from students should take the place of the question mark, inside the shopping cart. Candy's answer did not provide a solution at this point in the problem so it did not agree with the image represented. Ricky determined which items must be removed from the cart but only provided a partial solution because he did not communicate with pictures, numbers, or words which items were left in the cart which would satisfy the picture problem (see Figure 5).

Figure 5. Ricky's written work for Question #1.



ESL Level 4 Students' Responses to Question #2

Similar to the students in ESL level 2, students in ESL level 4 clearly reported during their interviews that they experienced metacognition when answering Question #2 due to the presence of bunches, and the size variance of the monkeys. They shared moments of questioning, rethinking, and wondering aloud. This was apparent in Matthew's reasoning for his solution to the problem, which did not contain an error. His interview demonstrated that he was quite perceptive about the visuals in the image such as the different numbers of bananas in the bunches and the equal symbols.

> **Matthew:** Um, I didn't really get this question. But now I think that it's asking me that if you put all these bananas into this bucket, how much bananas will there be? And if you share them among 4 monkeys equally, how much will each get?
> **Interviewer:** Why do the bananas go into bucket?
> **Matthew:** Because there's an arrow pointing to the bucket and there's a question mark on it so I thought it meant, like, how much bananas are the total.
> **Interviewer:** Why equally?
> **Matthew:** Because each monkey there is a question mark above it and there is a equal size to every question mark. That means all the numbers are the, must be the same, and that basically means like equally sharing between the monkeys.
> **Interviewer:** How did you decide whether to do bunches or bananas?
> **Matthew:** Because I see some bunches have four bananas instead. Some have three so if you share with bunches, I don't think that will be fair.

Although Matthew explained in his interview that he was dismayed by the presence of remainders while completing the problem, it did not stop him from making decisions about the problem that were in agreement with the images presented.

Three students made solution strategy errors, two of whom divided the bananas according to the size of the monkey. Like Michelle, Ricky became very uncomfortable when he realized that some bunches had three bananas and others had four. This made him question his decision to divide the bananas by bunches among the monkeys. Although he blamed his confusion about the question on this visual, he admitted that he noticed the different numbers of bananas during the interview only. Debby was aware of the equal symbol but chose not to place much importance on it. "Cause like at first I didn't get the monkey part and then I was thinking, like, what does the equal sign mean and then why is the monkey like, has, like, different sizes and then cause, like, I used guessing." In order to account for the presence of the equal sign in her answer, she placed equal symbols between the different amounts of bananas that the monkeys received. Candy allocated bananas based on an estimation of monkey size. When asked during the interview for more information about her logic, she insisted that she did not understand what the question wanted her to do, despite having earlier stated that she had to move the bananas to the bucket and then distribute them to the monkeys.

Though Sam determined an effective solution strategy and used equal division, he made a calculation error and had difficulty transferring the number of leftover bananas to a decimal, resulting in an incorrect solution.

ESL Level 4 Students' Responses to Question #3

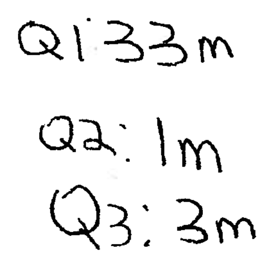
All eight ESL level 4 students correctly identified the objectives of Question #3, and were able to formulate an effective solution strategy for solving at least one of the whale and jellyfish lengths.

Three students used inaccurate solution strategies to find the length of the whale or jellyfish. When Debby explained her solution strategy, she stopped and gasped because she noticed that her answer to the length of the jellyfish was inaccurate. When completing the mathematics task, she did not notice that the 0.4 meter measurement was the length of two segments out of five on the jellyfish and thought that it represented the entire length. After realizing this error during the interview, she explained the correct solution strategy precisely.

> "I'll put 0.4 times 0.2. I mean plus. I mean 0.4 plus 0.4 plus 0.2. That would give me the length of the jellyfish. And then I'll use, and then I'll use, the length of the shark subtract by the length of the jellyfish and I get a difference."

Ricky explained his understanding that since two segments together are 0.6m in length, one segment would be half of this measure (see Figure 6). However, when he used these numbers to determine the total length of the whale, he did the calculation in his head and ended up with an incredibly inflated measurement of 33 metres.

Figure 6. Ricky's written work for Question #3.



Six students struggled with solving the difference in length between the whale and jellyfish. Three students used a typical subtraction solution strategy with calculation errors, two students devised alternate solution strategies, and one student opted not to answer.

Candy and Ricky both used estimation to solve for the difference in length. Candy used her fingers to estimate how many times the length of the jellyfish fits inside the empty space. She determined that 2.4 jellyfish would fit. Ricky was unable to explain the strategy he used to estimate the difference. He mentioned that he needed to find out "how long is the jellyfish and [the space] is" then quickly stated, "Actually, I don't understand the question, so I just guess like 'cause there's no like number helping us figure this out so I just go [3 metres]." In this response, he stated that he did not understand the question. However, by recording a measurement for the length difference, he demonstrated the he was fully aware of what the question was asking, but was unable to formulate the corresponding solution strategy.

Brandon did not record an answer for either the jellyfish length or the comparative difference in length. He insisted that he did determine the length of the jellyfish and simply did not record it. He explained that he chose not to record an answer for the difference in length between the two creatures because he was unsure about his answer and solution strategy. "I would have gotten it wrong anyway."

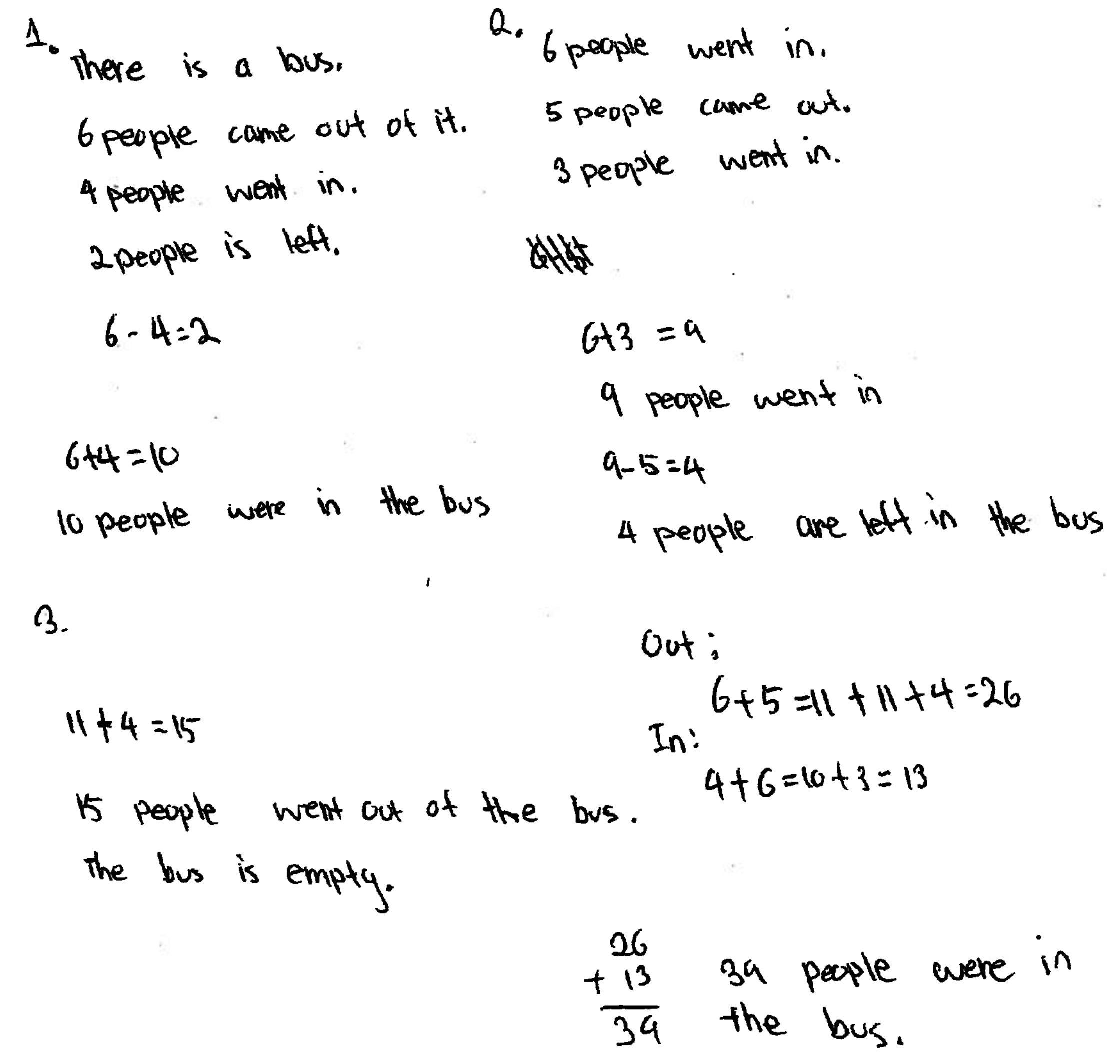
ESL Level 4 Students' Responses to Question #4

All students, except one, were successful answering Question #4. Matthew and Debby took the opportunity to apply their early knowledge of algebra to assist them in tackling this problem. They set the unknown number of passengers in the first bus to a variable and solved for that variable. Other students used guess and check, grouping positive and negative values, and making a numerical list as solution strategies.

When answering Question #4, Brandon demonstrated resistance to using mathematics in unfamiliar situations. He was able to correctly interpret the problem and he derived a correct solution to the problem. However, his difficulty with having to do the problem backwards resulted in him asserting that "I just, um, because I just didn't want to do another question like this, but because, like, it just made me all confused, like, make me mixed up and then….. I'm not used to working backwards."

Sam followed through the visual information given in the picture problem by writing a list of sentences describing the movement of the passengers on and off the bus (see Figure 7).

Figure 7. Sam's written work for Question #4.



1. There is a bus.
6 people came out of it.
4 people went in.
2 people is left.

6 - 4 = 2

6+4 = 10
10 people were in the bus

2. 6 people went in.
5 people came out.
3 people went in.

6+3 = 9
9 people went in
9-5 = 4
4 people are left in the bus

3.

11+4 = 15

15 people went out of the bus.
The bus is empty.

Out:
6+5 = 11 + 11 + 4 = 26
In:
4+6 = 10 + 3 = 13

26
+ 13
34

34 people were in the bus.

He erroneously added all of the passengers together, rather than assigning one direction of movement a positive value and the other a negative value. He did not verify his final solution by placing it into the position of the unknown and checking that the answer was zero. He stated that while he thought he knew "most of what the picture means," he "still [didn't] know how to solve it."

Responses to Question #1 of Students Fully Fluent in English

Only two students not registered in ESL made errors on Question #1, of which both were computational errors. Both students provided interpretations that were not in agreement with the image. Mark stated, "I think the persons is buying the things, the items and then look at the cash register and it says $82… Maybe it was save some money day." Mark's understanding of the significance of the price shown being less than the total of all of the items was that there was a discount provided to the purchaser. As a final answer, Mark found the difference between the items total and the price displayed on the cash register. This answer was creative and could fit with the social meaning of a smaller than expected price due to the presence of sale prices. However, similar to the answer given by Candy in ESL level 4, Mark's solution did not agree with the image shown in the picture problem. Chris, on the other hand, did not account for the total prices shown on the cash register. Chris believed that the question being asked was "How much was the total at the end. How much did the person use to buy all the equipment?" As a solution, he calculated the total cost of all of the items. His answer was isolated to the first two image blocks shown in the problem, the assortment of items, and the shopping cart. His solution did not consider that there was a price already displayed on the cash register, and the cost was never factored into his answer.

Responses to Question #2 of Students Fully Fluent in English

All of the students not registered in ESL made errors on Question #2. This was despite the fact that all of them, except Tracy, chose to distribute bananas equally to the monkeys. Austin made an error counting the number of bananas and, consequently, his answer was incorrect. Much like John in ESL level 2, Austin left the leftover bananas (of which he had one too many) in the bucket in order to deal with the remaining bananas. During his interview it was difficult to get Austin to state what he thought the picture problem was asking. In his initial response to this inquiry he provided many possible interpretations.

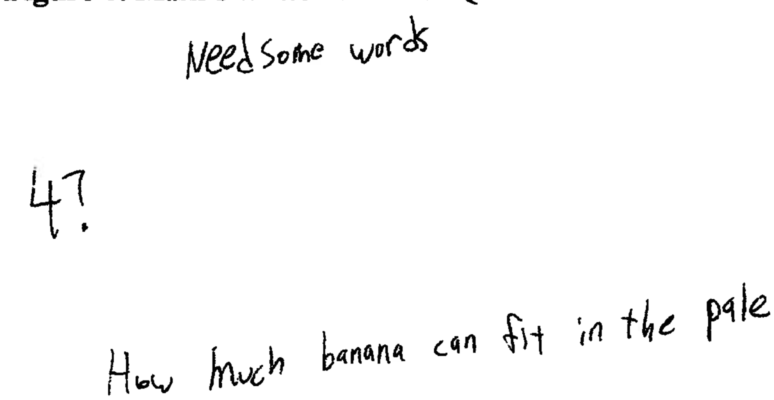
> "I had no idea what the question's asking and then it said, I think it means, like, split the bananas equally among the four guys, or monkeys, and then I kept on getting these other, like, wacko ideas so it was very confusing. First, it's like, how many, what the big guy gets more and the little guys get less, and then they should all be split equally, and then they're just a family, and then they just eat what they get."

While this demonstrated an awareness of the multiplicity of possible questions and solutions with picture problems, Austin hesitated to isolate which question he actually answered to complete the mathematics task.

> **Interviewer:** So what is [the picture problem] asking?
> **Austin:** I don't know.
> **Interviewer:** But you came up with an answer, so what did you answer?
> **Austin:** How can you split the bananas among the four monkey dudes? Yeah, it didn't really work.

During the interview, Mark clarified that while completing the mathematics task, he felt that the question being asked was how many bananas fit into the bucket (see Figure 8). The four

Figure 8. Mark's written work for Question #2.


Need Some words

47

How much banana can fit in the pale

recorded in his work indicated the number of monkeys shown and was not meant to be an answer to the question he stated. He explained that the question marks over the monkeys' heads were there to indicate that the monkeys were "thinking" or "wondering about something." He relayed that at this point in the mathematics task, he stopped writing anything down because he did not know what to do.

Jimmy chose to describe his solution strategy in words rather than with numbers to answer the problem (see Figure 9). The strategy that he described appears to be accurate but is coded as incomplete because no numerical solution is provided.

Figure 9. Jimmy's written work for Question #2.

I think you add up the monkeys, after add up all the banana's. Third, divide then evenly among the monkeys.

Tracy was the only student who did not choose to distribute the bananas equally to the monkeys. She explained that she ignored the presence of the equal symbols in the image when she completed the mathematics task. Upon reflection, during her interview she spontaneously stated that it might be correct to divide the bananas equally amongst the monkeys instead of the strategy that she chose to employ. When asked about her ability to complete another question of similar design, she explained that "maybe I would still try again but using a different kind of way of understanding the question." Her response indicates both reflection and metacognition during completion.

Responses to Question #3 of Students Fully Fluent in English

Five out of eight participants not registered in ESL accurately answered all three components of Question #3. They determined the answers of the two creatures' lengths through multiplication and addition, and found the difference in length through subtraction.

Figure 10. Steven's written work for Question #3.

Whole - .39 cm

Jellyfish - .1 cm

unknown - .28 cm - estimate

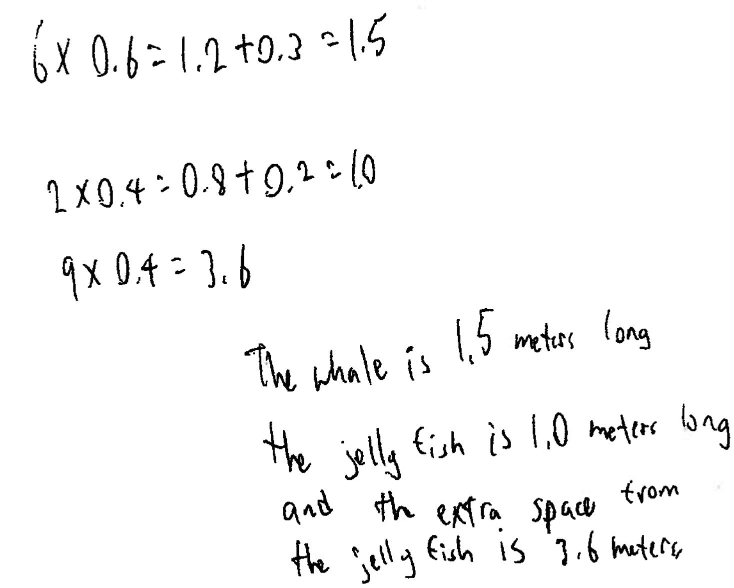
Three students made errors due to a combination of solution strategy and calculation challenges. Steven's errors on Question #3 were based on an inaccurate use of decimal, unit length, and solution strategy (see Figure 10). He skip-counted by 0.3 metres for the whale length and 0.2 metres for the jellyfish length. While the digits recorded were correct for both, the placement of the decimals and his conversion to centimetres was not accurate. Steven's strategy to find the difference in length between the two creatures was to skip count each of the whole segments shown in the whale image and then add what he estimated the length of the partial remaining space to be. He mentioned that Question #3 "made me think about like what to do with the problem. Because, like, well, the one part I didn't know was the gap here. Maybe just because of the part where there was, um, like, no length here and you have to like look around to find a clue, yeah." Steven did not choose to use subtraction as a viable strategy and instead used the picture clues, like segment length and a visual estimation of segment size, to attempt to discover a reasonable solution.

Mark used an incorrect strategy to determine the length of the jellyfish as he equated 0.4 metres with one segment instead of two. His strategy for determining the length difference between the two creatures was correct and by chance his answer also ended up being correct. This was due to the same resulting difference between his two earlier erroneous calculations.

Chris used a variety of strategies to answer Question #3 (see Figure 11). He wrote number sentences for all of the components based on his interpretation of the visual information. He incorrectly multiplied during his attempt to find the length of the whale. However, using the same solution strategy, he correctly determined the length of the jellyfish.

During his interview, Chris explained that he estimated the length of the space below the whale and counted the equivalent number of segments shown on the whale. He then multiplied the number of segments by the measurement of two segments in the jellyfish. After explaining this procedure, he mentioned that he felt that his answer for the length of the whale was incorrect because of what he already knew about these two creatures. "Like, the jellyfish is 1 metre and the whale is 1.5 metres so, um, I learned about whales before and they were supposed to be a lot bigger than a jellyfish." He used his background knowledge and illustrated his understanding of "a reasonable answer" to investigate his solution.

Figure 11. Chris's written work for Question #3.



Responses to Question #4 of Students Fully Fluent in English

Four students not registered in ESL, that answer Question #4 accurately, all mentioned during their interviews that they struggled with the question. Chris explained that it took him two attempts to answer Question #4. On his version of the booklet, Question #4 was the first picture problem and thus, this was the first problem of this form that he had ever encountered. He described that he looked at the problem for a few minutes and moved on to complete the other questions on the task, coming back to it to answer it after completing some of the other picture problems first. Chris initially chose to add all of the passengers together, regardless of their direction of travel, but he then concluded that the direction of travel should be included. He proceeded to use guess and check to randomly assign values to the unknown bus and completed the number sentence he had created. Chris commented that "once I figured out [how the get the answer] it was pretty easy." After explaining his solution strategy, Steven stated, "[This problem] made me feel pretty good about [my mathematics ability] 'cause at first I thought it would be a bit hard but I got through it."

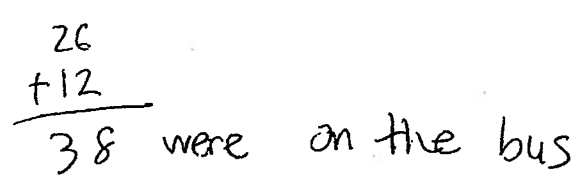
Austin was one of the few students who noticed the bus driver and the only student who actually included the driver in his solution.

Jimmy's answer to Question #4 was the single sentence shown in Figure 12. In the interview, he correctly identified the purpose of the picture problem. He stated that had there been a pattern in the number of people moving on and off the bus, he would have been more likely to answer the question correctly. As it stands, he correctly judged that his answer did not satisfy the problem. "Cause it was very complicated. I didn't understand the question." However, due to his clarity of understanding of the problem, he demonstrated and explained that his confusion was not with the problem itself, but with what solution strategy he should employ.

Figure 12. Jimmy's written work for Question #4.

Unfortunately, Flora showed little work for Question #4, which would help clarify her solution strategy (see Figure 13). In her interview she explained that she adds all of the people shown getting on or off of the bus regardless of their direction, an erroneous solution strategy.

Figure 13. Flora's written work for Question #4.



In his interview, Mark explained an accurate solution but in his work sample he did the intermediate steps in his head, instead of writing it on the mathematics task. Because of this, I am unaware whether his inaccurate final solution was due to an error in calculation or his solution strategy.

Clarity of Picture Problems

Student responses to the interview question "How can this picture problem be made clearer?" are shown in Table 7. This was asked after discussing a picture problem with the student. This question helped to discover more about how the modification to image-based problems impacted the students and their preference for certain forms of problem solving. Students' answers revealed the methods that they would choose to use to change the problems to increase their perceived performance and self-efficacy.

Table 7
Methods of Improving Picture Problems Suggested by Students

Participant Group	Improvement (%)			
	Picture	Numbers	Words	None
ESL Level 2	26.3	31.6	15.8	26.3
ESL Level 4	14.2	28.6	23.8	33.3
No ESL	5.3	21.1	63.2	10.5

The answers given by the participants fall into four general categories: clarification through pictures, clarification through numbers, clarification through words or no clarification is necessary. Students who believed that clarification would be best through pictures gave examples such as taking out unnecessary images, or adding or altering the images present in the picture problems. Some students commented that numbers would help to increase clarity. One student gave the example of having the number of passengers boarding and exiting the bus on

Question #4 written in each of the boxes so that counting was unnecessary. Another example provided to illustrate how clarity could be found through numbers would be to change the measurement given in the segments on Question #3 to show the length of one segment instead of two. The use of words was also suggested so that the question intended by the picture problem would be stated to supplement the image. Students also suggested that the entire problem could simply be transformed into a word problem. When asked how picture problems could be made clearer, some students also felt that picture problems needed no clarification as it was already clear to them.

Each group responded to this interview question for at least three of the picture problems. To get the percentage of improvement responses of each type, the number of responses by group to each specific form of clarification is divided by the total number of responses to picture problem clarification.

The student responses were again more similar between the groups of students still acquiring English and substantially different from the responses of those fluent in English. Students registered in either of the ESL levels were much more likely to say that there nothing needs to be done to make a picture problem clearer than those students not registered in ESL. Students with no ESL designation responded that their main method for making the picture problems clearer would be to add more words; the approach the ESL level 2 group least considered an option.

ESL level 2 students demonstrated no obvious preference for form of clarification. An equal number of students requested clarification through picture adjustments as stated that no clarification was needed. There were slightly more responses that favoured clarification through the addition of numbers or mathematical symbols and the fewest number of requests were for clarification through the addition of words or language. No one student requested the addition of words more than once in all of their responses to problem clarity. Since ESL level 2 students indicated during their interviews that they often find that the language in mathematics problem solving makes it difficult for them, it follows that they would be least likely to suggest this as an improvement.

ESL level 4 students showed a slight avoidance of clarification through image alteration, since fewer students requested this form of clarification over other forms. One third of the time, students in ESL level 4, with increased English comprehension, indicated that no improvements needed to be made to make picture problems more coherent. There did seem to be an increase in the number of times participants thought that language should be added over ESL level 2 students. However, this increase is only slight and is in agreement with the fact that these students have also acquired more English and may be increasingly more comfortable working within this educational format.

Students with no ESL designation overwhelmingly desired clarification through the use of words (73.9% of responses). This desire to include language in picture problems is quadruple and triple the preference of ESL level 2 and 4 students respectively. Only one response suggested that a picture-based method should be used for clarification. In only two instances students believed that the information shown in the image was sufficient.

Jimmy commented that it is important for picture problems to have words "so that [students] won't have a hard time comprehending the question so then they might be hard to understand if they, like, yeah. I only understand the words ones." When answering picture problems, students not registered in ESL exhibited a lack of confidence as evidenced by the small number of students who commented that picture problems were clear. On the other hand, students in ESL level 2 and ESL level 4 both felt that at least one quarter of the time the information presented in picture problems was clear enough.

Through their interviews, it was apparent that much of the reason students who are not in ESL desired clarification through language stemmed from wanting to be correct, and their desire to work with familiar mathematics problem solving formats. Tracy explained that while she thought that picture problems were fun, she also felt that when they were even a little bit unclear that she did not want to complete them. When I asked her why that was the case she told me, "Um, it's like you have risk. You get two answers for the questions. I mean, there would be two kinds of questions for this picture and, um, you could fall on one side or you could fall on the other side and you could get it right or you could get it wrong depending on what they're asking." Other students made statements similar to this. Tracy also explained that she needed there to be an "answer sheet" for the work that she completed because "if I did a whole sheet of math problems, I would like to know if I got them right or wrong." Language was the most guaranteed way for her to ensure that her answers were "correct."

Summary of Results

Image-based problems provided the participating students with a unique mathematics format which they had not previously encountered. The students in ESL level 2 were able to be the most successful completing problems of this format, though they did not have the strongest performance in either computation problems or word problems. The students in ESL level 4 were less successful than ESL level 2 students when completing picture problems, and the students not registered in ESL struggled significantly with this problem format. The students' ability to notice details, think logically, and apply their mathematics knowledge to this new format greatly impacted their rate of success when completing picture problems. The problem solving self-efficacy of some students in ESL level 2 was positively impacted by picture problems. Many of the ESL level 2 students explained that the English used in mathematics is difficult for them. Students in ESL level 4 also made comments about difficulty they have had with English in mathematics, though their self-efficacy on word problems is higher than the students in ESL level 2. Only one student not registered in ESL expressed any difficulty with the language component in mathematics. These students had much higher self-efficacy when completing word problems than picture problems.

Discussion

The present study endeavoured to determine the effects of image-based, wordless problems on students' perceptions of their ability to be successful when problem solving in mathematics and their performance on problems of this type. During interviews, ESL students repeatedly commented that they experience difficulty with the use of language in mathematics. This reaffirmed the need to investigate forms of problem solving that provide alternatives to language-based word problems. Overall, the results of this study indicate that picture problems may be valuable in creating more positive self-efficacy beliefs for some ESL learners and also increase their rate of problem solving success.

Because the current study had a very small sample size, the results found are not generalizable, but are indicative of the abilities of this particular participant group. Future research should be carried out with a larger sample size to strengthen the findings of this study.

Students in ESL were expected to struggle with word problems to some extent because of language difficulties, but none of the participant groups were expected to struggle significantly with picture problems. This surprise finding warrants attention and I will discuss this in greater detail to attempt to explain these results. Picture problems did not require language knowledge, though there was some degree of cultural fluency necessary. For example, picture problem Question #1 showed a shopping cart, cash register, and numbers on tags attached to sporting goods. In order to interpret this problem, students had to recognize these items, know how they work, and what the significance of each of these items would be in a shopping purchase. It was surprising that there is such a dramatic decrease in the percentage of picture problems successfully answered by students with more English knowledge. The relationship between language acquisition and decreased performance on picture problems, suggests that there were factors other than language which inhibited students' success on picture problems. This warrants further investigation.

Some students in ESL level 2 expressed a positive view of picture problems and provided comments about possible future experiences with picture problems which seemed to suggest that they would welcome this problem type. Many of these students demonstrated an acceptance of picture problems as an alternative problem solving format in mathematics. This acceptance was not as apparent in the responses of students in ESL level 4 and was essentially nonexistent in the responses of students who are fully fluent in English. Their rationale for these responses will be discussed further, and the implications for future research will be outlined.

Picture Problems as Nonroutine Problems

As a new problem format, picture problems presented students with situations in which they needed to apply their understanding of mathematics in new ways. Students were not taught any solution strategies before they encountered picture problems on the mathematics task. This ensured that students' responses to picture problems were a genuine reflection of their ability to transfer their learned mathematics knowledge to new situations (Mayer, 1998).

There are two types of problems that students can encounter when problem solving in mathematics: routine problems and nonroutine problems (Mayer, 1998). Routine problems are problems that are similar to ones that students have previously encountered. Students have background knowledge of possible solution methods which can be applied to provide success on the completion of routine problems (Mayer, 1998). For students in this study, computation problems and word problems were examples of routine problems (ESL students may not find word problems completely routine due to their lack of language fluency). Routine problems typically allow students to use passive learning behaviours because they do not have to construct or apply new knowledge (Mayer, 1998; Anthony, 1996). Passive learning includes the absorption of information, completion of single, fixed answer problems, and recycling of previously learned mathematics (Anthony, 1996). This format does not encourage meaning making or the application of mathematics in a way that would increase and deepen students' understanding of the material (Anthony, 1996).

Alternatively, nonroutine problems are question types which students have not yet learned how to solve (Mayer, 1998). The completion of nonroutine problems demands an understanding of mathematics application and knowledge of how, and when, to apply their mathematics skills to problems (Anthony, 1996; Mayer, 1998). Comments of students in this study indicated that picture problems were nonroutine for them and thus, confidence in their solution strategies or interpretations of the problem was affected.

The Use of Active Learning in the Solution of Picture Problems

Nonroutine problems, such as picture problems, encourage students to engage in active learning (Mayer, 1998). Active learning is the process of constructing new knowledge through the application of previous mathematics knowledge and skills (Anthony, 1996). This process engages students in their own learning and forces them to expend mental energy during the construction of a solution strategy (Anthony, 1996). Since deeper understanding of mathematics grows from students' construction of mathematics knowledge and active engagement in learning, nonroutine problems should be used to encourage this (Mayer, 1998).

The use of active learning was present in the problem solving skills demonstrated by the ESL students who participated in this study. Students in ESL level 2 and ESL level 4 explained that their overall expectation of their ability to succeed in mathematics has decreased since coming to Canada. Though they believe that they are becoming more comfortable with the English language, students explained that they still struggle with the language used in word problems and felt that it impacts their success on word problems. ESL students' beliefs are supported by the results of the mathematics task since ESL students demonstrated weaker performance on word problems than students who are fully fluent in English.

As difficult as word problems are for ESL students, their struggle with these problems may actually prove beneficial when students encounter nonroutine problems, such as picture-based problems. When ESL students complete word problems, they are forced to be more actively engaged in problem solving than students who are fully fluent in English. This active learning is the result of their limited successful experience with similar English language word problems. The likelihood that the ESL students are familiar with the word problems that they encounter is lower than for students who are fully fluent in English. ESL students cannot follow previously used strategies for completion and need to construct solution strategies for each word problem. Until word problems become routine or similar to ones they have seen in the past, ESL students need to develop solution strategies; this forces them to transfer previously learned mathematics to the new situation (Mayer, 1998). As these problems become more familiar to them, ESL students do not need to use active learning to the same extent and they may begin to rely more on passive learning (Anthony, 1996).

When students encountered picture problems on the mathematics task, they were introduced to problems that facilitated active learning. That is, to answer picture problems, students needed to analyse and investigate the problem, their solution, and their thinking. Since ESL level 2 students reported that they are still unable to fully understand word problems, they remain in a state of knowledge construction and active learning when completing such word problems. ESL level 2 students' recent experience with problems that are nonroutine may make them better equipped to apply active learning to picture problems as well. One possible explanation for their similar rates of errors on picture problems and word problems is that they apply active learning to both problem formats. Neither format is routine, as of yet.

ESL level 2 students demonstrated a quick reversion to active learning when completing picture problems. This strategy also suggests that their mathematics abilities are not being fully supported by, or accurately assessed with, problems solving questions that are posed in a word-based format (Kiplinger et al., 2000, Brown, 2005). Students should be able to complete problems which enable them to develop the deepest understanding possible (Anthony, 1996), but this cannot happen if ESL students are primarily encountering word problems when problem solving. This is supported by the comments made by the students during their interviews. Students' responses in all groups suggested that the use of picture problems may allow mathematics problem solving to be more equitable for ESL learners, especially those who have not yet mastered basic conversational English or acquired academic fluency.

Though ESL level 4 students have had more experience with word problems than ESL level 2 students, they reported that they still struggle with language. However, they also consistently mentioned that this problem solving format is becoming more familiar to them. These students have acquired the beginning of academic English, which includes mathematics language, with its specific usage and meaning in mathematics word problems. Their understanding of the nuances of the English used in word problems, and the practice that they have had applying this knowledge, are two factors stated by ESL level 4 students as beneficial to their success while solving mathematics word problems.

ESL level 4 students were starting to demonstrate a methodical approach to solving word problems that Puchalska and Semadeni (1987) call playing the "word problem game." This approach uses an awareness of solution methods for past word problems which can be applied to present problems to bring success. Previous knowledge of the routines and expectations of word problems can allow ESL level 4 students to participate less in active learning and knowledge construction and rely more on passive learning (Anthony, 1996). Consequently, when ESL level 4 students encountered nonroutine problems, they may have been less inclined to revert back to using active learning, thus completing picture problems with lower levels of success than ESL level 2 students.

Anthony (1996) explains that providing students with active learning situations and nonroutine problems does not guarantee that they will make a transition from passive learning to active learning. Instead, this transition is a decision that each student must make to take control of the construction of his or her own learning. Similar to Anthony (1996), Hegarty, Mayer, and Monk (1995) assert that students need a reason to make the shift from a problem solving strategy that takes less mental effort and engagement, to one that insists that they become active learners who accept mental challenges. According to Hegarty et al. (1995), students who engage in passive learning may use a direct translation approach to problem solving. With this approach, students look at the problem, glean the basic information such as key words or numbers, and place them into an equation. The errors made by students not registered in ESL suggest that they used such an approach.

Cobb, Wood, Yackel, and McNeal (as cited in Anthony, 1996) argue that direct translation and passive learning strategies, such as learning through the absorption of information, routine practice, and memorization are often sufficient for success in the typical mathematics classroom. However, this strategy often leads to these students becoming unsuccessful problem solvers when they are presented with more complex mathematics problems that require mental effort and the transfer of mathematics learning (Hegarty, Mayer, & Monk, 1995, Anthony, 1996). The students in the study who are fully fluent in English have likely had direct translation, a weaker mathematical approach, reinforced through past success in the classroom (Hegarty et al., 1995). During their interviews, students stated that they have had success on known problem formats in the past because they have been able to follow learned procedures for completion. Thus when completing tasks similar to previously learned problems, they were able to recycle this knowledge from previous tasks. The high rate of success on word problems and computation problems of students who are fully fluent in English may be explained by their successful use of this approach to problem solving on routine problems.

Conversely, students who are fully fluent in English had considerably more difficulty with picture problems than students in either of the other two participant groups. This could demonstrate reliance on the use of previously learned mathematics strategies, and may possibly indicate that students in this study who are not registered in ESL have a dependence on passive learning. I would hypothesize that their choice to use direct translation on word problems and computation problems may be a sign that they have rarely encountered situations where they have had to shift to active learning in order to be successful. This is a claim that would need further research to substantiate, but would be worth investigating, because few active learning situations may result in students having poorly developed problem solving strategies that use discovery, construction, and develop a deeper understanding of mathematics (Hegarty et al., 1995).

In this study, students who are not registered in ESL explained that they struggled when they were challenged with picture problems because it was a nonroutine problem format. They expressed difficulty constructing meaning for the problems and applying the appropriate mathematics. Their high rate of errors and interview statements stressed that they attributed the majority of their difficulty with picture problems to inexperience with this problem format. This response to picture problems suggests that the students in this study who are fully fluent in English did not make the transition to engaging in active learning when completing nonroutine problems. According to Hegarty et al. (1995), students who are used to experiencing high rates of success on familiar problem formats are less willing to solve nonroutine problems. Their avoidance of these tasks usually occurs because they are not able to employ a known algorithm to ensure their success (Hegarty et al., 1995). The students in this study also explained that mathematics problems should only have one “correct” answer. A single answer approach to problem solving is further indicative of the application of passive learning (Anthony, 1996). The lack of active learning and the desire to avoid failure that is demonstrated by the students who are fully fluent in English may account for their poor performance on picture problems.

The difficulty that students who are fully fluent in English demonstrated when applying their knowledge of mathematics to picture problems suggests that, like most students who are used to problem solving success, these students struggled with transferring their knowledge of mathematics to nonroutine problem solving tasks (Anthony, 1996; Kramarski, Mevarech, & Arami, 2002). When completing the mathematics task, these students did not demonstrate the "complex and flexible thought processes" necessary for successfully solving nonroutine problems (Hegarty et al., 1995). The poor performance of students who are fully fluent in English suggests that their ability to solve nonroutine problems was negatively impacted by their difficulty with knowledge transfer and also their use of passive learning.

The Role of Metacognition in the Completion of Picture Problems

The majority of the students in all participant groups in the present study made comments which indicate that picture problems encouraged them to mull over how to solve the problems, what strategies they should use, and when they should be applied. Many students also explained that they initially believed that picture problems could be solved with one method, but when considering their solution strategy, they reassessed those assumptions, and some of them altered their approach. The evidence provided through interviews about students' consideration of their own thought processes during the completion of picture problems suggests that these problems encouraged students to engage in consideration and analysis of their own thinking and learning, a process also known as metacognition (Swanson, 1990; Kramarski et al., 2002). Some students in this study commented that they would work through a problem, come to a new piece of information, and revaluate their initial procedure, adding in their new observations. Swanson

(1990) identifies that, regardless of their basic mathematics ability levels, those students who engage in metacognition are more likely to be successful in mathematics than those who do not engage in metacognition. For example, one student in this study commented that she would simply stop when she encountered information that was contrary to her initial completion strategy and might have indicated that she had made an error. On such problems, when she avoided delving into further thought in order to minimize her confusion or develop her understanding of the problem and possible solution strategies, her answers were incorrect.

Cognitive factors such as the knowledge of mathematics skills or the ability to accurately complete computation are not as important to successful problem solving completion as an understanding of one's own learning and knowledge construction (Mayer, 1998; Swanson, 1990). Following this argument, one might assume that all students who used metacognition are more able to be successful on picture problems, but this was not the result shown in the study. Rather, students in ESL level 2, ESL level 4, and those not registered in ESL, who used metacognition while completing the mathematics task, still showed a vast difference in their ability to correctly answer picture problems, which may be due to the level of metacognition that they engaged in (Swanson, 1990).

While metacognition was used more often during the completion of nonroutine problems than routine problems, the degree to which metacognition is used also seems to make a difference in the problem solving abilities of students in this study (Swanson, 1990). During their interviews, many of the students explained that they used different levels of metacognition while completing picture problems. Many of the students who are fully fluent in English or registered in ESL level 4 explained during their interviews that they used metacognition sparingly. They conveyed that they had an initial idea of the goals of the problem and a possible solution strategy but when they realized that there could also be another option for the problem and its solution, they second-guessed themselves. The possible multiplicity of solution strategies confused many of the students and created cognitive dissonance, where they were not sure what to think. Rather than working through this issue with mental effort, students who did not effectively use metacognition chose to ignore their thought-pattern discrepancy and answer the problem with their initial assumptions, or provide no answer at all. Because the individual resolution of internal metacognitive conflicts in mathematics helps to develop knowledge and facilitate the transfer of mathematics knowledge to new situations (Kramarski et al., 2002), these students' avoidance of cognitive dissonance may have been detrimental to their ability to complete nonroutine problems (Mayer, 1998). Many of these students chose to continue with their first assumptions of the problem even though they claimed to know that their solution strategy was incorrect, or they simply avoided completing the problem. Had these students been willing to engage more in their own process of knowledge construction, they would have invested more time and effort into accurate completion of the problem to the best of their abilities (Mayer, 1998; Anthony, 1996).

Conversely, when faced with a multiplicity of possible questions and solutions for a single picture problem, other students in the study, especially those in ESL level 2, chose to use their cognitive dissonance as motivation to reexamine the problem. In order to make their interpretation of the problem and their resulting solution strategies acceptable to themselves, these students used higher levels of metacognition. Many of these students commented on the amount of thought that they needed to use to complete picture problems and explained some of the inconsistencies that they encountered and their methods for solving these challenges. For example, one student explained that the first time she looked at the shopping cart problem (Question #1) she felt that all of the items should go into the cart, but realized that this interpretation was inconsistent with the information provided in the image, particularly the placement of the shopping cart in the image, and she changed her interpretation and solution strategy accordingly. Increased metacognition on the part of the student such as this is seen in the performance of these students in this study, results in increased mathematics achievement, especially on nonroutine problem types (Kramarski et al., 2002; Mayer, 1998).

The Role of Will in the Completion of Nonroutine Problems

According to Anthony (1996), metacognition is not the only factor which impacts students' success on nonroutine problems. A student's will also impacts his or her performance. "Will," as defined by Mayer (1998), is the culmination of a student's interest, self-efficacy, and motivation when solving nonroutine problems, such as picture problems. A more positive will to learn in problem solving situations, tends to lead to increased success during completion (Mayer, 1998). This aspect of problem solving is pertinent to this study because students in all participant groups made comments about the effort, determination, and desire that they had to complete the problems on the mathematics task.

The superior performance demonstrated by ESL level 2 students in this study on picture problems, may be positively impacted by their willingness to answer picture problems as frequently mentioned during their interviews. The students in ESL level 2 were most likely to view picture problems as having both individual interest and situational interest. Mayer (1998) defines individual interest as an individual's preference for, or positive feelings towards, an activity in general. Situational interest is more dependent on the task itself. Situational interest occurs when a student judges a specific task to be interesting (Mayer, 1998). ESL level 2 students' high opinion of both types of interest on picture problems predisposed them to putting more effort into thinking about, fully understanding, and working through the problems (Mayer, 1998). Mayer (1998) also suggests that there is a link between interest and the transfer of problem solving abilities. This is supported by the findings of this study, although ESL 2 students' high success on picture problems cannot solely be attributed to their higher levels of interest.

ESL level 2 students also demonstrated more positive self-efficacy beliefs about picture problem completion than students in the two other participant groups. More of the ESL level 2 students were optimistic about their ability to be successful on this problem format, than in either the ESL level 4 or non-ESL groups. Pintrich and DeGroot's study (as cited in Mayer, 1998) found that students with more positive self-efficacy judgements are also increasingly more likely to engage in active learning during mathematics problem solving. Most importantly, however, students' self-efficacy beliefs about success on similar problems in the future, directly impacts their present performance (Mayer, 1998). Students who expect success on problems before they solve them are likely to experience success. Conversely, students who expect to struggle with problems often experience greater difficulty (Mayer, 1998). This resulting difficulty arises because students who expect difficulty with problem solving tend to use only part of the information given, demonstrate uncertainty in their work and do not understand that multiple answers may be possible (Kramarski et al., 2002; Mayer, 1998).

Coupled together, the elements of will; namely interest and self-efficacy, played a large part in the problem solving success of students (Mayer, 1998). During their interviews, ESL level 2 students demonstrated higher levels of will than students in the other two participant groups. The increased success of ESL level 2 students on picture problems suggests that their positive will for such tasks did play a role in their success.

Through their interviews, students who are fully fluent in English demonstrated the lowest levels of interest and self-efficacy on picture problems, producing low overall will to complete problems of this type. These students also demonstrated the lowest performance on picture problems. Similar to the study carried out by Mayer (1998), this study seems to support the finding of a connection between students' will on problem solving and their success.

Trends in Solution Strategies

While problem interpretation allowed for multiple meanings for picture problems, they also allowed students to use a variety of strategies during completion. When answering word problems and computation problems on the mathematics task, students used (i) logical reasoning, (ii) writing number sentences, (iii) working backwards and (iv) guess and check as solution strategies. When completing picture problems on the mathematics task, students ma[illegible] use of (i) logical reasoning (ii) writing number sentences, (iii) working backwards, (iv) guess and check, (v) making a list, (vi) finding a pattern, and (vii) estimating. While the variety of solution strategies may have to do with the questions placed on the mathematics task, there [illegible] no reason that the same range of strategies could not have been used for computation and w[illegible] problems as alternate solution strategies. Picture problems on the mathematics task allowed students to manipulate a variety of strategies, especially in situations where they were unsu[illegible] how to answer the questions. When completing computation problems on the mathematics [illegible] students who did not know how to find an answer often left it blank. Though a few student[illegible] chose not to provide a solution to a picture problem they found challenging, many also ch[illegible] use a previously unused strategy, such as estimation, to attempt to answer the problem. Students' comments indicated that their views about the "openness" of picture problems [illegible] them feel that there was a range of acceptable solution strategies. Though students' altern[illegible] strategies did not always produce accurate solutions, the fact that they were willing to ap[illegible] other strategies suggests that picture problems may elicit more of a variety of solution str[illegible] than other types of problems. Future research in this area is needed.

Trends in Solution Justification

Students' justification of their interpretations for the picture problems and their subsequent solution strategies demonstrated an interesting, and unexpected, trend. Thr[illegible] interviews, it was apparent that many students created situations or stories to justify their interpretations of the image-based problems. The use of justification during nonroutine problem solving is intended to increase students' acceptance of their solution strategies (Kramarski et al., 2002). In this study, the instances of students' elaboration of picture problems suggest that these students were actively involved in sense making (Anthony, 1996). Their attempts to add details, explanations, and reasons to explain the problem situations, often led them to insert information into the picture problem that was not previously there.

For some of the students in this study, their envisioned scenario agreed with all of the details of the image, while others created situations that only accounted for part of the image. The situations that did not accurately account for all the information contained in the images, often resulted in solutions with errors because these situations omitted information that was important for a complete understanding of the problem. This was similar to the findings of Hegarty et al. (1995), that successful problem solvers use all of the details given in the problem to develop an understanding of the relation of all of the items (Kramarski et al., 2002). On the other hand, unsuccessful problem solvers tended to incorporate only the key features of the problems, or the information that can be easily observed, into their solution strategy (Hegarty et al., 1995; Kramarski et al., 2002). In this instance, much of the important, but not obvious, information about the problem was lost from the solution strategy.

In their explanations of their thought processes during the completion of picture problems, students in this study tended to place themselves inside of the problem as an active participant. For example, they may have included themselves as the customer buying the sporting goods in picture problem Question #1 or the individual who was in charge of banana distribution in picture problem Question #3. According to Anthony (1996), such a procedure of self-inclusion may encourage, but does not necessarily lead to, reflective thinking and metacognition to minimize cognitive dissonance and create a new, conflict free understanding of the mathematics problem.

Trends in Solution Communication

Students' written communication of their understanding and solutions to picture problems demonstrated a tendency to show their solutions with a wider variety of techniques than they would during the completion of computation problems or word problems. Students' written answers to computation and word problems on the mathematics task were limited to the use of words and numbers to communicate their solutions. When answering picture problems, students used words and numbers, but they also used illustrations to communicate their solutions. The use of illustrations was not present in any students' solutions for the other two problem types. This suggests that the picture problem format may allow for an increased use of visual representation in solution communication. This may have to do with the students' having a sense of openness in problem interpretation and transferring this same sense of openness to their solutions. Problems which allowed or encouraged the use of multiple solution representations such as numbers, words, and illustrations, may have allowed students to make use of their individual learning styles to help them better understand mathematics (Tang & Ginsberg, 1999). The use of illustrations by students were not isolated to students in any particular participant group, so it would seem that ~~this~~ preference for a visual descriptor of their solution was more based on an individual's learning style than their language ability.

Meaningful Learning in Mathematics

Active learning, metacognition, will, individual story development, and using rationale to justify picture problems, can all help to promote active student interaction with mathematics problems. This engagement in the process of mathematics problem solving helps to combat the lack of connection that students feel exists between themselves and mathematics (Puchalska & Semadeni, 1987; Baroody, 1993). As Baroody (1993) states, "Meaningful learning involves seeing or making connections. Unlike rote learned knowledge, meaningful knowledge cannot be imposed from without but must be constructed from within. Meaningful learning, then, is not simply a matter of passively absorbing information but entails actively making sense of the world." Many of the students in this study demonstrated that picture problems allowed them to explore mathematics through their creation of appropriate questions for the images, determination and application of a solution strategy, and rationalization of their chosen solution strategy. Indeed, many of these students readily rationalized their solutions aloud during the interview process.

Conclusion

"Maximizing mathematical learning involves fostering conceptual knowledge as well as procedural knowledge, encouraging the development of strategies and metacognitive knowledge and promoting a positive disposition" (Baroody, 1993). Image-based problems are wordless problems requiring interpretation of the problem and application of the appropriate mathematics using the visual information provided. As this study has begun to demonstrate, picture problems may provide students with the opportunities they need to engage in constructive mathematics knowledge acquisition. Image-based mathematics problems provided some of the students who struggle with language-based mathematics problems an opportunity to explore mathematics, while exposing them to mathematics that encourages active learning and engagement with the problems. The ESL level 2 students in this study, with limited English language knowledge, expressed difficulty with the use of English in language-based problem solving. Many of them also demonstrated more positive views about mathematics problem solving when completing picture problems. It is believed that such positive views of these problems and the students' ability to complete such problems could lead to increased judgements of self-efficacy, which would help encourage further mathematics learning and motivation (Pajares & Miller, 1995).

Implications for Educators

Most educators continue to use word problems primarily as the vehicle for problem solving in the classroom. Gerofsky (1996) suggests that this is because of our own inherent belief in the "good" of word problems and the established tradition of their use in the mathematics classroom. However, though word problems can be beneficial because of the discussion they create and the application of mathematics to new situations (Baroody, 1998), such problems do not have to be used in the classroom at the exclusion of all other forms of problem solving. Rather, opportunities for problem solving should be as plentiful as possible (Baroody, 1998).

One overarching goal of mathematics is to equip students with the skills necessary to use mathematics successfully in their daily lives as they experience the world around them (Baroody, 1993). The mathematics they will have to do spontaneously when shopping for groceries, calculating a measurement or estimating a difference will arise primarily through situations they encounter. It is likely that these problems will not be written in digits or words but will develop from each student's interpretation of the situation presented by his or her environment. The importance of students' being able to apply the appropriate mathematics skills and concepts to these situations as nonroutine problem solving cannot be understated. It seems plausible that picture problems, as a nonroutine form of problem solving, may be a problem solving method that can also be used to practice such thinking.

In light of the findings of this study, it is suggested that image-based problems might be used by practitioners as an alternative assessment tool to assess students' acquisition and internalization of mathematical concepts. The absence of words in picture problems may begin to permit this assessment regardless of the language of the educator or student. The use of image-based problems as a supplement to number problems and word problems seems to permit students to apply their knowledge of mathematics in new ways. For instance, rather than students knowing to subtract because of a computational symbol or learned word such as "difference," presenting students with picture problems encourages them to understand the mathematics involved and construct an appropriate solution without such cues.

As many students identified, there was more than one possible question and interpretation for each of the picture problems which would agree with the image presented. This variety of approaches and outcomes can be useful in the mathematics classroom to create dialogue among students about mathematics, as desired by the NCTM (1989). Also, the inclusion of images for interpretation allows those students not adept at reading comprehension or fluent in the language of instruction to participate in mathematics regardless of their limitations (Brown, 2005; Hofstetter, 2003). While it is desirable that students also be able to comprehend language-based problems, image-based problems can be used to provide problem solving experiences which do not demand language and allow students, especially those with limited English knowledge, to explore mathematics. The allowance created by multiple possible questions and solution strategies can help to encourage risk taking, as some students in this study explained (Baroody, 1993).

Implications for Research

In order to investigate the validity of the findings of this study, a similar study should be carried out on a larger scale with more participants and revised image-based mathematics problems. A study this size would allow the trends seen in this participant group to be further examined and would help to give more insight into the role that image-based mathematics should have in the mathematics classroom. One could investigate if there is a gender gap in the ability to interpret and solve picture problems or if there is an ideal age for the introduction of such problems. The ESL ability levels of the students needs to be determined as this appeared to have an impact on their performance on picture problems with ESL level 2 students performing better than ESL level 4 students on such problems. The ability of students in ESL level 1, ESL level 3, and ESL level 5 to successfully complete picture problems could also be investigated to determine which groups such problems are most beneficial for.

As was encountered with some of the questions on the mathematics task, the intention of the image is not always clear to students. While image-based problems should encourage active learning and discussion, some students found them confusing or frustrating in that they were unsure of the requirements of the problem. Further development of picture problems should attempt to make the problems as clear as possible, without unnecessary or confounding images. For example, Question #2, with the monkeys, could be redesigned to include monkeys of the same size and individual bananas to minimize students' confusion about the intention of the problem. Research should be done in this area to further understand students' "image-based literacy" to aid in the development of problems of the appropriate mathematics level using suitable images. However, while image-based problems are visual problems that students could work on individually, they are intended to be a form of dynamic assessment. Picture problems should encourage dialogue and allow students to develop mathematics understanding and knowledge transfer from working with problems which challenge them. In the context of multicultural classrooms, research also should be conducted to determine how to use images in a cross-cultural environment so that such problems can be as effective and applicable as possible to a wide variety of students.

Further research should also be done to delve into the unexpected response of FEP students to image-based problems. These students commented repeatedly that they were used to word problems and computation problems and that image-based problems were not clear to them. Their poor performance on image-based problems was unexpected as it was assumed that all participant groups would perform equally well on image-based problems which simply remove the necessity for language, providing all students with an equitable foundation. Why would students who are able to be very successful at computation problems and word-based problems not be able to transfer their understanding of mathematics to problems asked through visual images? Is their decreased performance impacted more by resistance to unfamiliar mathematics formats or a weakness due to an inability to apply their knowledge of mathematics to alternative situations?

In summary, this exploratory study demonstrated that picture problems provided the greatest benefit to those students registered in ESL level 2. These students were able to perform comparably well on picture problems as they were on computation- or language-based problems. Many of these same students also expressed more positive self-efficacy judgements when completing picture problems than language-based problems, which make up a large part of the current mathematics curriculum. Further research needs to be done to investigate the transferability of these results to classroom practice and the ideal extent of the inclusion of image-based problems into elementary classrooms.

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Appendix A. Computation problems used on the mathematics task.

Question 9

$$\begin{array}{r} 1\square 4 \\ \times \quad \square \\ \hline 1078 \end{array}$$

Question 10

$$11.3 + 24.2 \square 61.7 \square 42.4 \square 55.5 = 84.1$$

Question 11

$$\begin{array}{r} 58 \\ 73 \\ 140 \\ 263 \\ + \quad \square \\ \hline 571 \end{array}$$

Question 12

$$48/6+8-3= \square$$

Appendix B. Word problems used on the mathematics task.

Question 5

To raise money for their upcoming performance, the drama club decided to have a car wash. They charged $4 per car. How many customers did they have if their profit was $84 after they paid $12 for their cleaning supplies?

Question 6

To solve a problem, Jade needs to add 426 to the difference of 732 and 929. What number is the solution to the problem?

Question 7

How much change would you receive from a twenty dollar bill if you bought one book for $3.98, one for $9.98 and two for $2.99?

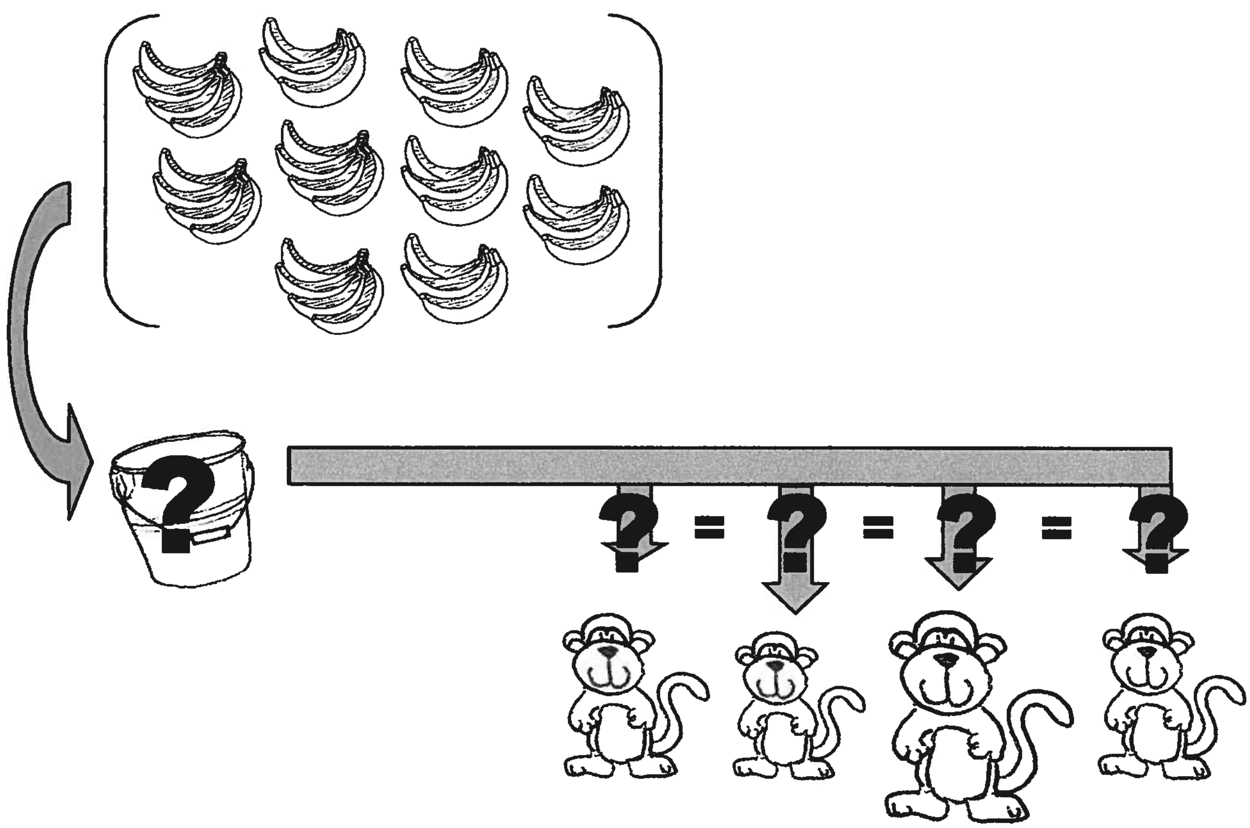
Question 8

Mitchell Elementary School donated nine cases of 20 cans of soup to the food bank. Byng Elementary School donated eight cases of 36 cans of soup to the food bank. The food bank would like to give out the soup to needy families in boxes that fit 12 cans each. How many boxes will they need to use?

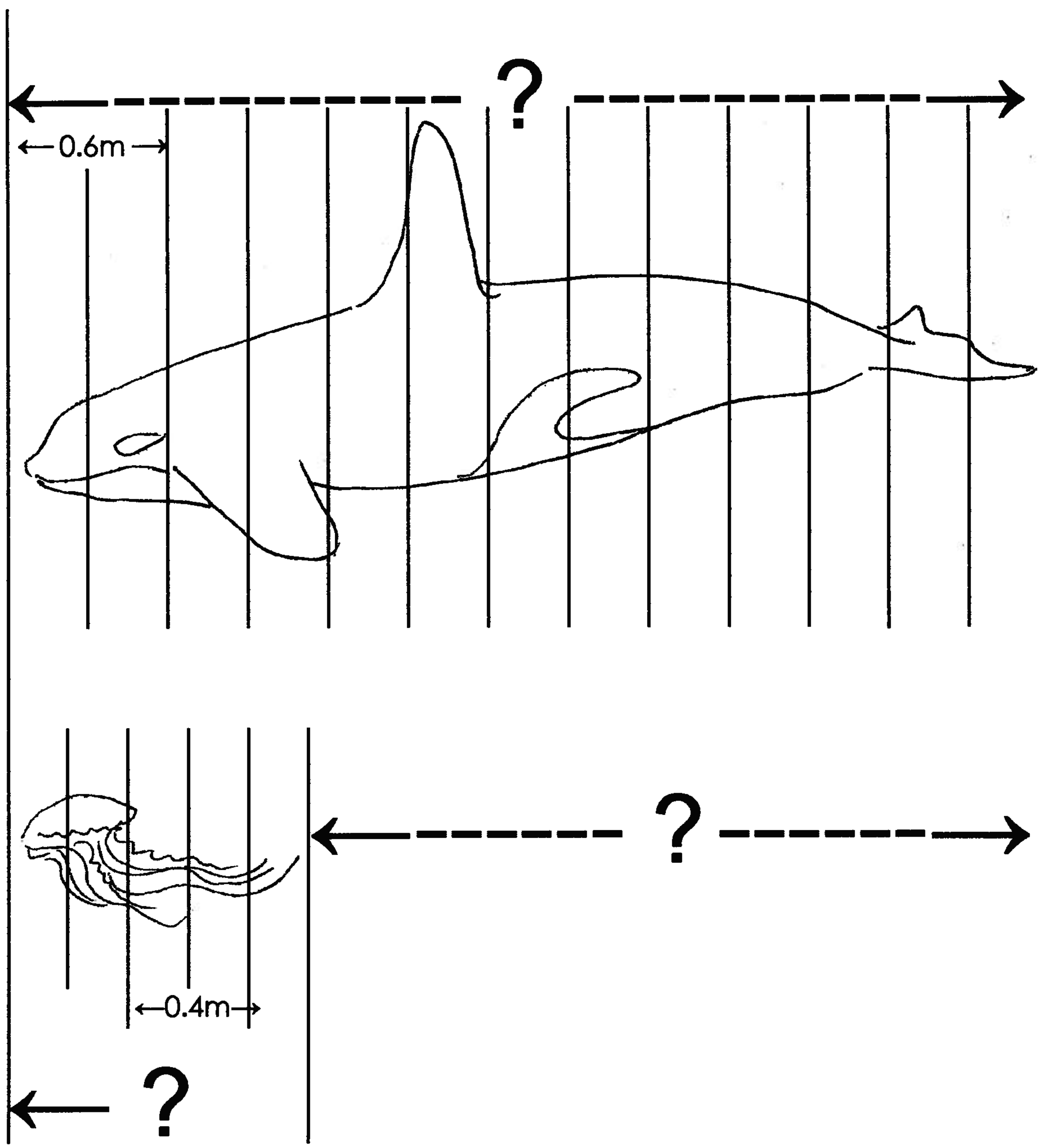
Appendix C. Image-based problems used on the mathematics task.

Question 1

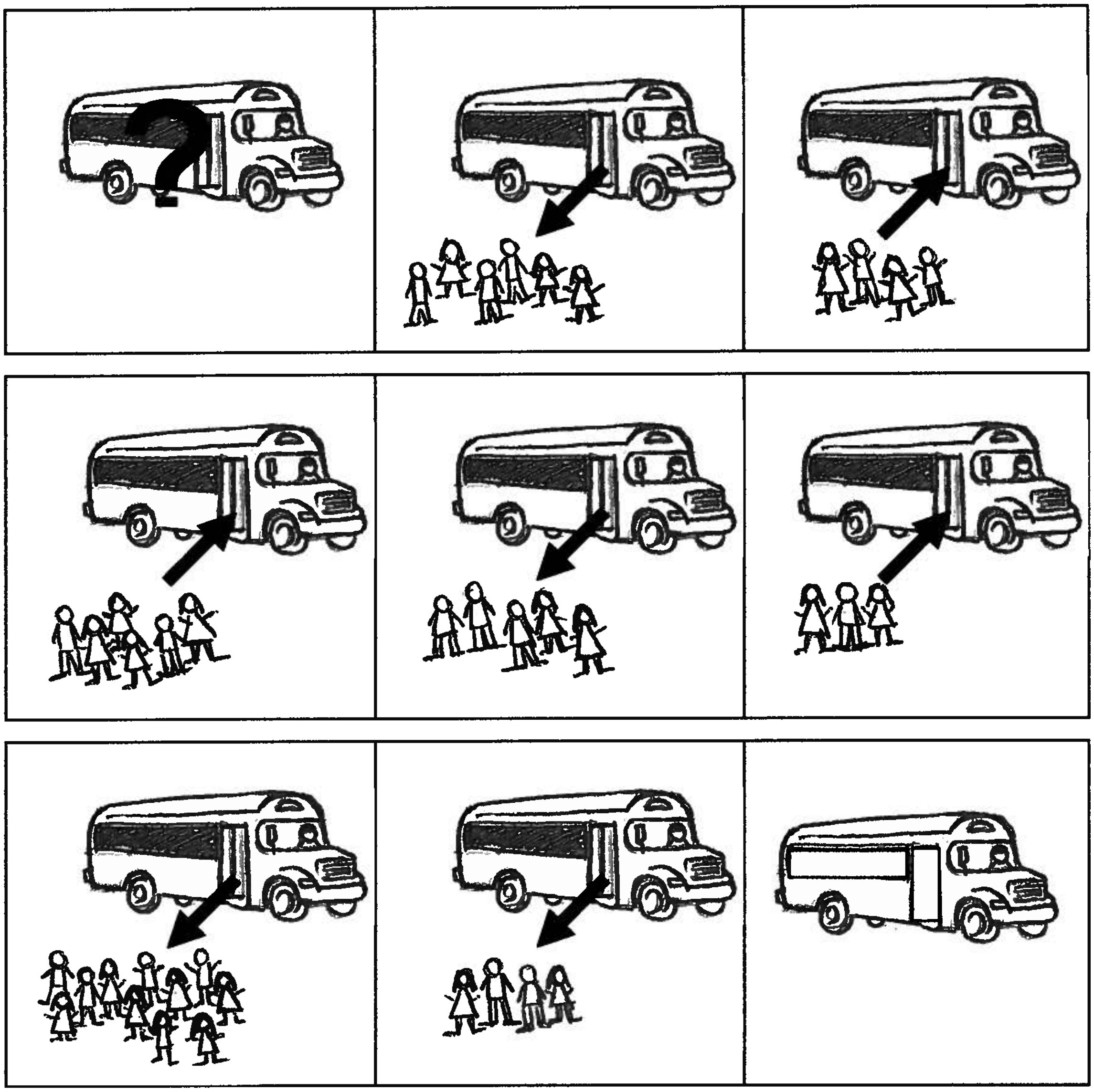
Question 2



Question 3



Question 4



Appendix D. Mathematics Task Question Booklet cover page.

Mathematics Task Question Booklet V3

INSTRUCTIONS

1. Please do all of your writing in the Mathematics Task Work Booklet, not in this book.
2. Do your best to answer every question.
3. Show all of your work. Do not erase any of your work. If you have made a mistake, please just cross it out with one single line and keep going.
4. After each question, write a note about any words, ideas or pictures that you did not know or understand.
5. After each question, answer the three questions at the end of that page, "How hard was this question?", "How sure are you that you have the best answer?" and "Could you answer another question like this one?" by circling the word that best describes how you feel.

Page 1

Appendix E. Mathematics Task Work Booklet cover page.

Mathematics Task Work Booklet

Student Number ______

Problem Book V_____

INSTRUCTIONS

1. Please do all of your writing in the mathematics task work booklet.

2. Do your best to answer every question.

3. Show all of your work. Do not erase any of your work. If you have made a mistake, please just cross it out with one single line and keep going.

4. After each question, write a note about any words, ideas or pictures that you did not know or understand.

5. After each question, answer the three questions at the end of that page, "How hard was this question?", "How sure are you that you have the best answer?" and "Could you answer another question like this one?" by circling the word that best describes how you feel.

Appendix F. Simplified Chinese interpretation of the instructions for the Mathematics Task Problem Booklet and the Mathematics Task Work Booklet.

指示

1. 写下你的答案，在其他书籍, Mathematics Work Booklet.

2. 对每一个答案,做你的最好的工作。

3. 显示您的所有工作。不要忘记你的任何工作。如果你有犯了一个错误，请你只交了出来单一路线继续下去。

4. 突出任何语言，思想，数学或图片，你不明白。

5. 之后每个数学问题，请你先回答：

   这个问题如何困难？

   如何肯定你有最好的答案呢？

   你是否可以回答另一个问题类似这样的一个呢？

   圈你的回答。

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Appendix G. Sample page for written work and question response.

Question #1

Show all of your work. Do not erase any of your work. If you have made a mistake, please just cross it out with one single line and keep going.

显示您的所有工作。不要忘记你的任何工作。如果你有犯了一个错误，请你只交了出来单一路线继续下去。

Question #1 cont

Write a note about any words, ideas or pictures that you do not know or understand.

How hard was this question?
这个问题如何困难？

Circle your answer

Very Easy 很容易	Sort of Easy 容易	Sort of Hard 有点硬	Very Hard 非常困难

How sure are you that you have the best answer?
如何肯定你有最好的答案呢？

Circle your answer

Not Sure at All Sure	Sort of Sure	Pretty Sure	Completely

Could you answer another question like this one?
你是否可以回答另一个问题类似这样的一个呢？

Circle your answer

Yes, Easily! 是的，谈何容易！	Maybe... 也许...	No, Probably Not 不，我不这么认为。

Appendix H. Interview Questions.

1. Tell me about which languages you speak and where you speak them?
2. How comfortable are you speaking English with me? In the classroom?
3. How important do you think your parents feel math is? Can you tell me about that?
4. Can you explain your school history to me? So, where you were to school, for how long and what languages you used there?
5. Tell me a little about how your math experiences in China and in Canada have been the same and different. How you feel about that?
6. What do you think about the amount of English used in a math classroom?
7. Now please think about math this year. How sure are you that you can do well in math? Why?

8a. Tell me about how well you think you would you do if you were given a page of number problems? Why? What would your enjoyment be?

8b. Tell me about how well you think you would you do if you were given a page of word-based math problems? Why? What would your enjoyment be?

8c. Tell me about how well you think you would do on a page of image-based math problems? Why? What would your enjoyment be?

9. Which of these question types would be good or bad for ESL learners and why?
10. What do you think it is important for me to know about the image-based problems before giving them to other students?

For random computation and word problems, and all picture problems, ask:

a. Can you tell me about how you solved this problem?
b. Why was this question (easy/hard/etc) for you?
c. I noticed on your math questions you said that your confidence on problem ____ is ____. Can you tell me a little about why you said that?

Appendix I. Behaviour Research Ethics Board Certificate of Approval.

The University of British Columbia
Office of Research Services
Behavioural Research Ethics Board
Suite 102, 6190 Agronomy Road, Vancouver, B.C. V6T 1Z3

CERTIFICATE OF APPROVAL - FULL BOARD

PRINCIPAL INVESTIGATOR:	INSTITUTION / DEPARTMENT:	UBC BREB NUMBER:
Ann G. Anderson	UBC/Education/Curriculum Studies	H08-00074

INSTITUTION(S) WHERE RESEARCH WILL BE CARRIED OUT:

Institution	Site
N/A	N/A

Other locations where the research will be conducted:
School District elementary schools

CO-INVESTIGATOR(S):
Amanda Jean Pel

SPONSORING AGENCIES:
N/A

PROJECT TITLE:
Achievement and Self-Efficacy of Students with English as a Second Language Based on Problem Type in an English Language Based Mathematics Curriculum

REB MEETING DATE:	CERTIFICATE EXPIRY DATE:
January 24, 2008	January 24, 2009

DOCUMENTS INCLUDED IN THIS APPROVAL:	DATE APPROVED:
	February 15, 2008

Document Name	Version	Date
Consent Forms:		
Principal Consent Form	2	August 2, 2008
Parental Consent Form	2	August 2, 2008
Assent Forms:		
Student Assent Form	N/A	November 1, 2008
Questionnaire, Questionnaire Cover Letter, Tests:		
Interview Questions	N/A	November 1, 2008
Sample Mathematics Task Problems	N/A	November 1, 2008

The application for ethical review and the document(s) listed above have been reviewed and the procedures were found to be acceptable on ethical grounds for research involving human subjects.

Approval is issued on behalf of the Behavioural Research Ethics Board and signed electronically by one of the following:

Dr. M. Judith Lynam, Chair
Dr. Ken Craig, Chair
Dr. Jim Rupert, Associate Chair
Dr. Laurie Ford, Associate Chair
Dr. Daniel Salhani, Associate Chair
Dr. Anita Ho, Associate Chair