New Constraints on Mercury’s Internal Magnetic Field

by

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B.Sc., The University of British Columbia, 2006

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

in

The Faculty of Graduate Studies

(Geophysics)

THE UNIVERSITY OF BRITISH COLUMBIA

(Vancouver)

March 2009

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Abstract

Three-component vector magnetic field observations from MESSENGER's first two flybys of Mercury have confirmed the presence of an internal field, along with external fields related to magnetospheric current systems. We use techniques in inverse theory to investigate structure in Mercury's internal magnetic field permitted by the Mariner 10 and MESSENGER flyby data, and structures recoverable while spacecraft is in orbit. We remove external fields predicted by a parameterized magnetospheric model from the flyby observations. We estimate noise contributions from long-wavelength uncertainties in the external field and from un-modeled short-wavelength features. Internal field models are parameterized to spherical harmonic degree and order 8, with regularization constraints applied to the power spectrum. The field is predominately dipolar but additional latitudinal and longitudinal structure is required to fit the data. Enhanced radial magnetic field in the region of the Mariner 10 and MESSENGER flybys latitudes is seen. Contributions to the internal field predicted by Aharonson and others for a long-wavelength crustal field are present (namely, the $g^0_1$, $g^2_3$, and $g^3_3$ spherical harmonic coefficients), but our hypothesis testing has shown that the field is dominated by the $g^0_1$ term rather than the proposed $g^3_3$ term. Further hypothesis testing has determined that the dipole tilt of Mercury is in the range of 3° to 13° with 13° as a strict upper bound. Observations from the upcoming MESSENGER flyby will provide additional, and critical, low-latitude coverage. Analyses of flyby data have shown that much of the recovery internal field depends successfully characterizing the external field signatures that are present in the data. Currently, the limited number of observations prevents us from constructing a reliable external field model, but these external field models will improve as data becomes more abundant. We also investigate recovery of three simulated core fields using synthetic data during MESSENGER's orbital phase, under the assumption that long-wavelength external fields can be modeled and removed. The results show excellent recovery of the dipole field and of field structure at mid-northern to high latitudes out to degree and order 10, providing encouraging results for future identification and characterization of core fields.
# Table of Contents

Abstract .................................................................................................................. ii  
Table of Contents .................................................................................................... iii  
List of Tables .......................................................................................................... v  
List of Figures ......................................................................................................... vi  
Acknowledgements ................................................................................................. vii  
Dedication ................................................................................................................. viii  

1 Introduction .......................................................................................................... 1  
1.1 Background ........................................................................................................ 1  
1.2 Mercury’s Magnetic Field Environment ......................................................... 3  
1.3 Studies of Mercury’s Internal Field ................................................................... 6  

2 Observations of Mercury’s Magnetic Field ....................................................... 8  

3 Magnetic Field Modeling Methods .................................................................. 14  
3.1 Introduction to Forward/Inverse Problems ...................................................... 14  
3.2 Our Problem ..................................................................................................... 15  
3.3 Approaches to Solving Inverse Problems ....................................................... 18  
3.3.1 Singular Value Decomposition ................................................................... 18  
3.3.2 Regularization Method ............................................................................... 21  

4 Analyses of Flyby Data ...................................................................................... 23  
4.1 Characterization of External Field .................................................................... 23  
4.2 Internal Field Models ....................................................................................... 27  
  4.2.1 Estimation with Singular Value Decomposition ...................................... 27  
  4.2.2 Estimation with Regularization ................................................................. 29
Table of Contents

5 Recovery of fields from orbit .................................. 38
  5.1 Simulated Orbital Data .................................. 39
  5.2 Effect of Regularization Norm .......................... 39
  5.3 Inversions of Different Core Field Models ............... 41

6 Summary and Future Work ..................................... 45
  6.1 Current Magnetic Field Models of Mercury ............... 45
  6.2 Hypothesis Testing and Implications of the Flyby Data Analyses ................................................. 47
  6.3 Future Work ............................................... 48
    6.3.1 Orbit Simulations .................................. 48
    6.3.2 External Field Characterization ..................... 49

Bibliography ....................................................... 52
List of Tables

3.1 Smoothing Norms for Magnetic Field Inversions ............ 22

4.1 Spherical Harmonic Coefficients for the SVD Solution with
   Degree 1 Internal and Degree 2 External Field ............ 29

4.2 Spherical Harmonic Coefficients for the SVD Solution with
   Degree 2 Internal and Degree 2 External Field ............ 29

4.3 Spherical Harmonic Coefficients for the TS04-Corrected Regu-
   larized Internal Solution ................................... 37
List of Figures

1.1 Earth's Magnetic Field Environment ........................................ 4
1.2 Mercury's Magnetic Field Environment .................................... 5

2.1 Observations of Mercury's Magnetic Field .............................. 9
2.2 Trajectories of Mariner 10 and MESSENGER Flybys ................. 10
2.3 Vector Magnetometer Data for Internal Field Modeling .......... 13

3.1 Data and Model Vector Spaces ............................................. 19

4.1 An Example of External Fields for a Polar and an Equatorial Flyby ................................................................. 25
4.2 External Field Predictions from Different Models for Mariner 10 and MESSENGER Flybys ................................................. 26
4.3 Prediction by Singular Value Decomposition with Degree 1 Internal and Degree 2 External Field .......................................................... 28
4.4 Prediction by Singular Value Decomposition with Degree 2 Internal and Degree 2 External Field .......................................................... 30
4.5 Selected model Predictions of Regularized Inversions with Trade-off Curve ................................................................. 34
4.6 Results for the Preferred Model ............................................. 35
4.7 Data Predictions from the Preferred Model ............................ 36

5.1 Effects of Different Regularization Norms on Simulated Orbit Data ................................................................. 40
5.2 Predictions from Regularized Inversions of Different Core Field Models with Simulated Orbit Data ................................................. 42
5.3 Difference Maps for Regularized Inversions of Different Core Field Models with Simulated Orbit Data ................................................. 44
Acknowledgements

First and foremost, I would like to thank Dr. Catherine L. Johnson and Dr. A. Mark Jellinek for the invaluable expertise they made available to me. I would also like to thank the members of my committee, Dr. Doug Oldenburg and Dr. Garry K. C. Clarke.

I would also like to acknowledge everyone on the MESSENGER team, particularly Haje Korth at the Johns Hopkins University Applied Physics Laboratory and Mike E. Purucker at NASA Goddard Space Flight Center for providing us with some of the external field models.

I would like to thank Dr. P. Surdas Mohit and Andrew J. Schaeffer for their help regarding Tex and BibTex.

I would also like to thank my current and former house-mates, Weezee Brown, Nick Hermes, Justin Hsia, Tom Matthews, Gord Robert, and Derek Townsend for de-stressing and character build up.
Dedication

I would like to dedicate this thesis to my parents, Stella and Haruhisa Uno, my sister Yumiko Uno, and all of my other family members for their encouragement and support throughout the years.
Chapter 1

Introduction

1.1 Background

Mercury has long held the interest of scientists from different planetary communities because of its unique characteristics, such as its inferred high metal to silicate ratio, the 3:2 spin-orbit ratio, and its heavily cratered surface. Our current observational knowledge of Mercury is mainly the result of earth-based radioastronomy (Pettengill and Dyce, 1965; Harmon and Slade, 1992; Harmon et al., 2001; Margot et al., 2007), and the three successful Mariner 10 flybys between 1974 and 1975 (Ness et al., 1974; Ness and Lepping, 1975; Ness et al., 1976; Ness, 1978; Lepping et al., 1979; Connerney and Ness, 1988). Despite the need for further exploration of Mercury, technological and cost limitations prevented new missions until the 2004 launch of the MErcury Surface, Space ENvironment, GEochemistry, and Ranging (MESSENGER) spacecraft. This thesis focuses on the structure and origin of Mercury's internal magnetic field, using the observations returned by Mariner 10 and MESSENGER.

The first ever flyby of the planet by Mariner 10 (M10-I) on 29th March, 1974, resulted in the surprising discovery that Mercury possesses a weak but spatially coherent internal magnetic field (Ness et al., 1974; Ness and Lepping, 1975). The second flyby focused on imaging of the surface with spacecraft altitudes too distant from the planet to detect its internal magnetic field, but the discovery of an internal magnetic field was used to motivate a third flyby of the planet by Mariner 10 (M10-III) on 16th March 1975. Analyses of the two magnetic field tracks showed that Mercury possesses a dominantly dipolar internal magnetic field that is tilted approximately 10° to 20° from its rotation axis, and has the same polarity as, but a strength of about 1%, of Earth's dipole field (Ness et al., 1976; Lepping et al., 1979; Connerney and Ness, 1988). The dominantly dipolar structure inferred for Mercury's internal magnetic field has been used to argue in favor of a dynamo process.

Prior to the Mariner 10 flybys, an active dynamo on Mercury was not expected, due to predictions of early solidification of the planet's core (e.g.
1.1. Background

Solomon, 1976; Schubert et al., 1988). Subsequent thermal evolution models indicate that a present-day liquid outer core could exist – its thickness depends, in particular, on the concentration of light elements, such as sulfur (Schubert et al., 1988; Conzelmann and Spohn, 1999; Hauck et al., 2004). In a more recent study, Earth-based radar measurements of the amplitude of Mercury’s forced librations suggest decoupling of the core and mantle, in turn favoring a liquid outer-core region (Margot et al., 2007). Measurement of forced librations can be predicted from moment of inertia, which differs significantly depending on whether the planet has a liquid or a solid core.

If by analogy with Earth, Mercury’s dynamo is driven by thermo-chemical convection (see review in Zuber et al., 2007), a difficulty arises in explaining the weak strength of the field. Most thermo-chemical dynamo models with Mercury’s inner to outer core size ratio predict a much stronger field than that is observed. This implies that much of the field generated in the core must remain in the core, if Mercury’s dynamo is powered by thermo-chemical convection similarly to the Earth. Various numerical dynamo simulations have sought to explain the weak field (Heimpel et al., 2005; Christensen, 2006; Stanley et al., 2007; Wicht et al., 2007; Chen et al., 2008; Christensen and Wicht, 2008); some make specific predictions for the geometry and time variations in the field. We discuss these in the context of current and future observations in Chapter 5 and 6. An alternative dynamo mechanism proposed to explain Mercury’s weak field involves feedback between an internal dynamo and external fields due to Chapman-Ferraro currents (Grosser et al., 2004; Glassmeier et al., 2007) on the magnetopause (the location of the pressure balance between the solar-wind and the internal magnetic field). Other unconventional mechanisms include a thermoelectric dynamo, which relies on the topographic fluctuations at the core-mantle-boundary to produce a thermo-electric motive force (Stevenson, 1987; Giampieri and Balogh, 2002).

Remanent crustal magnetic fields were not originally favored for Mercury, since strong crustal anomalies would be required to explain the observed signals, and because crustal fields are usually dominated by smaller-scale structure. However, mapping of the global magnetic field of Mars has revealed that crustal fields an order of magnitude stronger than on Earth are possible (Acuña et al., 1999). In addition, the spatial variations in solar insolation on Mercury could give rise to long-wavelength variations in the depth to the Curie isotherm (Aharonson et al., 2004) that in turn, predict long-wavelength structure in a field arising from crustal remanence.

The MESSENGER mission will address several questions key to understanding the formation and evolution of Mercury, including the nature and source of the magnetic field. The mission provides three flyby encounters:
1.2 Mercury's Magnetic Field Environment

M1 (14th January, 2008), M2 (6th October, 2008), and M3 (29th September, 2009), before going into orbit in March 2011. Due to the strong reflection of solar radiation from Mercury's surface, the proximity of the Sun, and the planet's unique orbital characteristics, the spacecraft orbit will be highly elliptical (periapsis altitude = 200 km, apoapsis altitude = 15200 km) with the closest approach latitude near 65°N (McAdams et al., 2007). Analyses of Mercury's internal magnetic field are expected to be challenging because of contributions to the observed magnetic field from electrical currents that arise from the interactions between the solar-wind and its internal field (Korth et al., 2004; Slavin et al., 2007). A brief overview of the magnetic field environment at Mercury is given below.

1.2 Mercury's Magnetic Field Environment

The interactions of a planet's large-scale internal magnetic field (e.g. dynamo or crustal) with the solar wind plasma and the interplanetary magnetic field (IMF), result in current systems that in turn generate magnetic fields exterior to the planet. In this study, we refer to such fields as external fields, and to fields produced below the surface of Mercury (dynamo, remanent, induced) as internal. The magnetic field environment around a planetary body is known as the magnetosphere (Gold, 1959). Magnetospheres are observed within our solar-system around several bodies including, Jupiter, Saturn, Neptune, Uranus, Mercury, and Earth. The present knowledge of these current systems and external magnetic fields are determined mainly from extensive spacecraft observations at the Earth.

Figure 1.1 is an illustration of the Earth's magnetosphere. Moving from the Sun toward the planet, the first point of interaction between the solar-wind and the internal magnetic field is the bow-shock - the boundary where the solar wind speed decreases from super-sonic to sub-sonic. Closer to the planet is the magnetopause defined as the pressure balance between the solar-wind and the internal magnetic field. The region between the bow-shock and magnetopause is the magnetosheath and the region within the magnetopause is called the magnetosphere. On the night side of the planet, the magnetosphere is extended forming the magnetotail or tail region. The polar cusps are two funnel shaped areas between the day- and night-side magnetic field where the solar wind can enter the planet's ionosphere. A prominent current system at Earth is the ring current. The ring current is generated through movement of trapped plasma particles within the magnetopause near the equator, interacting with the Earth's dipolar magnetic
1.2. Mercury's Magnetic Field Environment

Figure 1.1: A sketch of Earth's magnetic environment. Orange arrows indicate movement of the plasma particles. See text for the explanation for the different boundaries.

field to produce a net clockwise current (viewing from the north pole). The trapped charged plasma particles gyrate around the magnetic field lines of Earth and their gyration radius increases as they move away from the Earth (due to $\frac{1}{r}$ force dependence). Hence, when a given plasma particle returns closer to the Earth, the particle has shifted towards dusk. This results in an overall movement of electrons in the anti-clockwise direction, in turn producing a clockwise current.

Other currents dominate outside the ring current, for instance, Chapman-Ferraro currents centered and circulate anti-clockwise with respect to the northern polar cusps flows along the magnetopause boundary.

Owing to the weaker internal field strength and the stronger solar wind conditions (Slavin et al., 2008a), Mercury's magnetosphere is thought to be small, but dynamic compared to the Earth (Figure 1.2). The distance to the bow-shock, or the stand-off distance for the Earth is normally about $10 \, R_E$ (measured along the planet-sun line) where $R_E$ is the mean radius of Earth ($\approx 6371$ km). At Mercury it is about $0.5 \, R_M$ where $R_M$ is the mean radius of Mercury ($\approx 2440$ km). The closer stand-off distance makes the magnetic field environment strongly dependent on the local solar wind direction and the IMF. A northward IMF (i.e. aligned with the direction of the field lines due to the internal field) emphasizes the structure of the internal magnetic
1.2. Mercury's Magnetic Field Environment

Figure 1.2: A sketch of Mercury's magnetic environment. The stand off distance is much closer compared to the case for the Earth. Orange arrows indicate movement of the plasma particles. The purple arrows show the plasma particles that reach the surface of the planet through the cusps. See text for the explanation for the different boundaries.

Compression of the internal field on the day-side occurs, and depends on the solar wind strength. In contrast, a southward IMF (i.e. opposing direction of the internal dipole) suppresses the internal magnetic field particularly on the day-side to the point where the cusps are close to the equator (see Figure 5 of Slavin et al., 2007). Another consequence of the proximity of magnetopause and bow-shock is the absence of a closed ring current (Korth et al., 2004). Mercury is also the only body in our solar system with a magnetosphere, but without a conductive ionosphere. This absence of the ionosphere is credited as the reason for strong but short-lived energetic particle events that largely occur in the tail region that could be responsible for the contamination of M10-I data (Slavin et al., 2007).
1.3 Studies of Mercury’s Internal Field

Analyses of the Mariner 10 data focussed on investigations of dipole models for the internal field due to limited data coverage especially at low altitudes. These models yield a dipole moment of 230 to 350 $nT R_M^3$ (≈ 0.1% of Earth’s) depending on the chosen data segment and external field approximations (Ness et al., 1974; Ness and Lepping, 1975; Ness et al., 1976; Ness, 1978; Lepping et al., 1979; Connerney and Ness, 1988). The lower limit for the dipole moment decreases to about 165 $nT R_M^3$ if non-dipolar (higher-order) in a spherical harmonic description for the internal field (see Chapter 3) are included (see Ness, 1978; Lepping et al., 1979, and references therein). The dipole tilt relative to the rotation axis ranges from 10° to 20° with the polarity the same as at Earth (Lepping et al., 1979).

The recently-returned three-component vector magnetometer (MAG) data from the first two flybys of Mercury by the MESSENGER spacecraft confirm the presence of an internal magnetic field. After accounting for long-wavelength external fields, the Mariner 10 and MESSENGER data together yield an estimate for the dipole moment of 230 nT $R_M^3$ to 290 nT $R_M^3$, tilted 5° to 12° from the planet’s rotation axis (Anderson et al., 2008a,b; Johnson et al., 2009). The results from the two spacecraft are consistent with each other. However, the dipole solutions for the internal field leave unexplained residual signal within the magnetopause that potentially includes contributions from non-dipolar structure in the internal field, currently unmodeled external fields, or both.

Characterizing the spatial and temporal structure of Mercury’s magnetic field is crucial to identifying the source(s) of the field. Modeling results to date (Connerney and Ness, 1988; Anderson et al., 2008a) illustrate the difficulties of estimating the internal field with the limited data coverage provided by the existing and upcoming flybys. Although the spatial distribution of data will greatly improve after MESSENGER’s insertion into orbit around Mercury on March 2011, southern hemisphere coverage useful for internal field modeling will be restricted to low-latitude, high-altitude observations because of the highly elliptical orbit. Consequently, determination of the internal magnetic field particularly short-wavelength structure using global techniques will be challenging.

An additional problem at Mercury is the presence of fields due to magnetospheric currents. As discussed in the previous section, the combination of the weak internal field and the strong interplanetary magnetic field at Mercury, as compared with those at Earth, results in external fields that are relatively large in magnitude and that have an unknown spatial and tem-
poral spectrum. MESSENGER will be in orbit at Mercury during the next solar maximum, and hence accurate external field characterization will be critical to modeling of the internal field. Two approaches to the removal of the external fields are (1) the adaptation of terrestrial parameterized magnetospheric models to Mercury (Korth et al., 2004; Anderson et al., 2008a), and (2) simultaneous estimation of internal and external fields using spherical harmonics. Both approaches are considered here.

In this study, we discuss the application of techniques in inverse theory to the determination of Mercury's internal field structure. One particular goal is to characterize non-dipolar structure. An incomplete data coverage makes this exercise challenging, because of difficulty resolving short-wavelength structure. Accordingly, we construct models with specific smoothness constraints, mitigating the effects of incomplete data coverage while allowing examination of shorter-wavelength structure permitted by the observations. We analyze, four data sets collected during the Mariner 10 and MESSENGER flybys, and investigate the field structure that might be recoverable during MESSENGER's orbital phase. In Chapter 2, we summarize observations of the vector magnetic field from the Mariner 10 and MESSENGER flyby and describe the time interval over which the field was identified as primarily internal in origin (Lepping et al., 1979; Anderson et al., 2008a). We review the technical approach used (Chapter 3) and discuss its application to analyzing the flyby data (Chapter 4). In modeling the internal field, we use data parameterized for external field contributions. We assess sources of error in the observations and weight our data accordingly. We discuss the spatial and spectral structure in the internal field permitted by the data collected to date. In Chapter 5, we use simulations to investigate how well core fields might be recovered from orbit. We discuss, in turn, the results in the context of proposed diagnostics based on recent numerical dynamo models. The results to date are summarized in Chapter 6. We discuss tests for specific aspect of field structure that may provide useful diagnostics and constraints for numerical dynamo models. We then conclude with a summary of issues that must be addressed in future studies.
Chapter 2

Observations of Mercury’s Magnetic Field

Observations of the vector magnetic fields from the four flybys of Mercury are shown in Figure 2.1. The Mercury-Solar-Orbital (MSO) coordinate system is used in magnetospheric studies, since the morphology of the magnetosphere is mainly controlled by the outbound flow direction of the solar wind plasma. In the MSO coordinate system, the $x$-axis is taken as the planet-sun line where $x$ is positive sunward, the positive $z$-axis is the normal to the orbit plane, and the positive $y$-axis is dusk-ward. The magnetometer on board MESSENGER samples the field at 20 samples per second (Gold et al., 2001), which is then averaged with 6 s intervals to smooth the data for internal field analyses. The Mariner 10 data are similarly averaged with a 6 s window. The trajectories of the four flybys are shown in Figure 2.2.

The two flybys of Mariner 10 have contrasting trajectories (Figure 2.2). M10-I was an equatorial flyby with its closest approach (CA) over Mercury's eastern hemisphere at an altitude of 705 km (Ness et al., 1974), while the M10-III encounter was a polar flyby with its closest approach (CA) near the north-pole at a lower altitude of 327 km (Ness et al., 1976). The maximum total field strength observed was 98 nT and 400 nT for M10-I and M10-III, respectively. Magnetic field readings during the second half of the M10-I encounter (i.e., after CA) are very noisy and have been interpreted as resulting from some external field event, possibly a solar sub-storm (Ness et al., 1974). Although the two data sets are dissimilar in terms of their observation magnitudes, both M10-I and M10-III show signatures consistent with a dominantly dipolar internal field, such as the observations of peak field strength near closest approach and an approximate $1/r^3$ field strength decrease with distance. A detailed summary of the Mariner 10 observations is available in Lepping et al. (1979).

The MESSENGER flybys are both equatorial with very similar trajectories in MSO coordinates (Figure 2.2). From the planet’s perspective, however, they are in opposite hemispheres, with M1 over the eastern and M2 over the western hemisphere. Their closest approach altitudes were 201 km
Figure 2.1: The raw data from Mariner 10 flybys (M10-I and M10-III) and MESSENGER (M1 and M2) in Mercury-Solar-Orbital coordinate system (see text). The x-axis is time in UTC for date of the respective flyby. The solid line (red) is the x-component ($B_x$), dashed line (blue) is the y-component ($B_y$), and the dash-dot line (green) is the z-component ($B_z$). The grey envelopes are the positive and negative bounds of the total field, $|B|$. The vertical black lines indicate different boundary crossings: dotted line is the bow-shock (BS), dash-dot line is the magnetopause (MP), and the dashed line is the position of closest approach (CA). Note the different scaling of the y-axes. Distance of the spacecraft from the center of Mercury is given as the ratio $r/R_M$ along the x-axis also.
Chapter 2. Observations of Mercury’s Magnetic Field

Figure 2.2: Trajectory of M10-I (red), M10-III (green), M1 (blue), and M2 (black) flybys shown in MSO coordinate system: a) view from the North Pole, b) view from the Sun. The grey circle centered at the origin is Mercury. Solid circles are the bow-shock crossings, plusses are the magnetopause crossings, and the empty circles are the position of closest approach for each flybys. Axes are in units of Mercury radii ($R_M$). The direction of spacecraft’s motion is shown on each panel with the labels “INBOUND” and “OUTBOUND”. Note that in (b), the flybys are on the night-side of the planet.
and 200 km with a maximum total field strength of 159 nT and 156 nT for M1 and M2, respectively. The MESSENGER observations are consistent with the Mariner 10 data in that the field signatures suggest an internal dipolar field. The position of the maximum field strength occurs before CA for M1 and after for M2. This is however, an expected observation from the geometry of the flybys (separated by almost 180° in longitude), if the internal dipole is slightly tilted. The two flybys show similar magnetic field structure in general, although M2 shows more short-period variability than M1.

In Figure 2.2, the positions of bow shock and magnetopause observed for each flyby are also shown. The apparent shift in crossing times in Figure 2.1 arises mainly due to the different flyby geometries, since time spent within magnetopause depends on where the spacecraft enters and leaves the magnetosphere. After correcting for this effect, some differences in bow shock and magnetopause crossings remain (Figure 2.1 and 2.2). This may be due to the differences in solar wind and IMF conditions (Slavin et al., 2008b), and we do not discuss this observation further here. The magnetopause positions are on average about 0.5 RM from the planet’s surface at the sub-solar point (Lepping et al., 1979; Anderson et al., 2008a; Slavin et al., 2008a). Differences in bow shock and magnetopause crossings reflect the time variability of long-wavelength structures that arise from external current field systems. In addition, short-period features, within the magnetopause that have amplitudes of 10 to 20 nT are observed. These are possibly due to local plasma effects (Anderson et al., 2008a) that are of unknown origin. The duration of these signatures is in the order of 1 s to 10 s corresponding to wavelengths on the order of 10 km to 100 km (assuming spacecraft velocities near CA of ≈ 7 km/s). These signatures are not likely of internal origin, as a magnetometer is not able to resolve features with such wavelengths from altitudes 200 km and greater.

The data that are used for the internal field modeling are shown in (r, θ, φ) in Mercury-Body-Fixed coordinates (Figure 2.3). The MBF coordinate system is defined by having the positive x-axis as the line from the center of the planet to the direction of 0° longitude, the positive z-axis along the spin axis, and the y-axis completing the coordinate system. In other words, θ is colatitude and φ is East-longitude in MBF coordinates. Body-Fixed coordinates are used in internal field modeling, since the structure of the internal field is governed by this coordinate system. For the purpose of internal field modeling, we use observations taken within the magnetopause, since this is where the signal is mainly of internal origin. The exception is the M10-I flyby, where we only use the data between the incoming magne-
topause (20:37) and closest approach (20:46). As mentioned above the field during the outbound portion of this flyby was very noisy and likely not of internal origin (Ness et al., 1974).

All of the observations of Mercury's magnetic field available so far have confirmed the presence of a predominantly dipolar internal field. The data have also shown evidence of long- and short-wavelength signals that are likely of external origin. It is known that the internal field is governed by the MBF coordinate system, while the external field is represented best with a MSO coordinate system. The objective of this study is to characterize the structure of the internal field in order to determine its origin. However, an accurate representation of both long- and short-wavelength external fields are required to correctly determine the internal field. This is further complicated by the time variability that arises from changes in the external current systems, solar wind and IMF conditions, and the sparse observation geometry. The following chapter describes different approaches to model the magnetic field of a planet, with a focus on internal magnetic field modeling. The application of these approaches to Mercury's magnetic field, including the characterization of external fields and parametrization of the internal field, is discussed in Chapter 4.
Chapter 2. Observations of Mercury’s Magnetic Field

Figure 2.3: The portion of the raw data that are used for internal field modeling, plotted in Mercury-Body-Fixed (MBF) coordinate system (see text) versus time. Each column represents different flybys, and the rows correspond to $B_r$, $B_\theta$, and $B_\phi$. Note the different y-axis scaling for each panel. Time is in UTC for date of the respective flyby. Distance of the spacecraft from the center of Mercury is given as the ratio $r/H_M$ along the $x$-axis also.
Chapter 3

Magnetic Field Modeling Methods

Given our observations of Mercury’s magnetic field, we wish to extract information on the global characteristics of the internal field. We give an introduction to inverse problems below, followed by the specific model formulation for our magnetic field problem. We then outline some solution methods.

3.1 Introduction to Forward/Inverse Problems

A simple example of an inverse problem would be the practice of attempting to fit data to some function form. The most common and straightforward approach to solving an inverse problem is the least squares method, which minimizes the following equation,

\[ \Phi = \sum_{i=1}^{N} (d_i - d_i^{obs})^2. \]  

(3.1)

\( N \) is the total number of observations or data points, \( \Phi \) is the \( L_2 \)-norm data misfit, \( d_i^{obs} \) is the observed datum, and \( d_i \) is the \( i^{th} \) model prediction. No additional information about the data is required and the solution is purely a mathematical one. Hence, the predicted model need not make physical sense. This issue becomes more evident in real life situations where the data are sometimes scarce and accompanied by noise.

Additional information about the data or about the physical process described by the data can help overcome the problem of obtaining an unphysical model. This information/knowledge is often referred to as an \textit{a priori} constraint. For instance, if one were to carry out a surface magnetic survey of an area to produce a 3D subsurface conductivity map, it would be tremendously advantageous to have a drill core sample (\textit{a priori} information) from that area to constrain the range of possible conductivity values, possibly with depth.
3.2. Our Problem

An a priori constraint used frequently in geophysical problems is the smoothness constraint. One of the difficulties with a purely mathematical solution is that it sometimes overcomplicates the results in order to reduce the data misfit. This results in structure or roughness where data are absent. To avoid this a smoothness constraint can be imposed in addition to the requirement that the solution provides a good fit to the data.

Before undertaking an inverse problem, we must formulate the forward problem (which predicts the data for some known model coefficients). Assuming that the data are related to the model parameters by a linear process, the forward problem can be written in matrix form as,

\[ \vec{d} = G\vec{m} + \vec{e} \]  (3.2)

where \( \vec{d} \) is the data vector

\[ \vec{d} = [d_1, d_2, \ldots, d_M]^T, \]  (3.3)

\( \vec{m} \) is the model vector

\[ \vec{m} = [m_1, m_2, \ldots, m_N]^T, \]  (3.4)

\( G \) is the \( M \) by \( N \) matrix of coefficients derived from physics or from the geometry of the problem that is independent of both the model and the data, and \( \vec{e} \) is the error vector containing the error associated with each data point (Gubbins, 2004). Solving a forward problem is to produce a model prediction using (3.2) with some known model. In this case, the geometry of the problem (\( G \)) and the model (\( \vec{m} \)) are the knowns and the model predictions (\( \vec{d} \)) is the unknown. The inverse problem solves for model coefficients that can reproduce the data. So here, the geometry of the problem (\( G \)) and the data (\( \vec{d} \)) are the knowns and the model (\( \vec{m} \)) is the unknown. If the inverse problem was a simple 1D function, such as \( y = ax^2 + bx \), the model predictions or data vector would be the \( y \) values, the model vector will be \([a, b]^T\) and the rows of the \( G \) matrix will contain the values \( x_i^2 \) and \( x_i \) that correspond to the \( i_{th} \) data observation point.

3.2 Our Problem

In planetary science the magnetic field of a planet can be represented in spherical harmonics, which can be thought of as an analogue of a Fourier Transform on a sphere (see Bracewell, 1965; Gubbins, 2004). Instead of having different frequencies in the Fourier domain, spherical harmonics have
3.2. Our Problem

different spherical harmonic degrees and orders and are functions of differing
spatial complexity (defined on a sphere) that are orthogonal to each other.
They arise from the solution to Laplace’s equation,
\[
\nabla^2 \Psi = 0. \tag{3.5}
\]

Using separation of variables, the potential, \( \Psi \), for a magnetic field in a
source free region can be described in spherical coordinate system by the
following relation (Stacey, 1992),
\[
\Psi = \left( \frac{R_a}{\mu_0} \right) \sum_{l=1}^{\infty} \sum_{m=0}^{l} \left( \frac{R_a}{r} \right)^{(l+1)} (g_l^m \cos(m\phi) + h_l^m \sin(m\phi)) + \\
\left( \frac{r}{R_a} \right)^l (G_l^m \cos(m\phi) + H_l^m \sin(m\phi)) P_l^m(\cos \theta). \tag{3.6}
\]

The satellite magnetic field measurements are three component vector mea-
surements. This magnetic field is given by,
\[
\vec{B} = -\mu_0 \nabla \Psi \tag{3.7}
\]
\[
= -\mu_0 \left( \frac{\partial \Psi}{\partial r} - \frac{1}{r} \frac{\partial \Psi}{\partial \theta} + \frac{1}{r \sin \phi} \frac{\partial \Psi}{\partial \phi} \right) \tag{3.8}
\]
\[
= B_r \hat{r} + B_\theta \hat{\theta} + B_\phi \hat{\phi} \tag{3.9}
\]

where \((\hat{r}, \hat{\theta}, \hat{\phi})\) denotes the unit direction vector of each spherical coordinate.
For each of the components, this is further separated into two source con-
tributions of internal \((\vec{B}_{\text{int}})\), and external \((\vec{B}_{\text{ext}})\) origin. Hence, the internal
portion of the magnetic field is:
\[
B_{\text{int}} = \sum_{l=1}^{\infty} \sum_{m=0}^{l} (l+1) \left( \frac{R_a}{r} \right)^{(l+2)} (g_l^m \cos(m\phi) + h_l^m \sin(m\phi)) P_l^m(\cos \theta) \tag{3.10}
\]
\[
B_{\text{int}} = \sum_{l=1}^{\infty} \sum_{m=0}^{l} - \left( \frac{R_a}{r} \right)^{(l+2)} (g_l^m \cos(m\phi) + h_l^m \sin(m\phi)) \frac{\partial}{\partial \theta} [P_l^m(\cos \theta)] \tag{3.11}
\]
\[
B_{\text{int}} = \sum_{l=1}^{\infty} \sum_{m=0}^{l} \left( \frac{R_a}{r} \right)^{(l+2)} \left( g_l^m \sin(m\phi) - h_l^m \cos(m\phi) \right) \frac{m}{\sin \theta} P_l^m(\cos \theta) \tag{3.12}
\]
where $\vec{B}_{\text{ext}} = B_{\text{ext}} \hat{r} + B_{\text{ext}} \hat{\theta} + B_{\text{ext}} \hat{\phi}$ and the external contribution is:

$$B_{\text{ext}} = \sum_{l=1}^{\infty} \sum_{m=0}^{l} -l \left( \frac{r}{R_a} \right)^{(l-1)} [G_l^m \cos(m\phi) + H_l^m \sin(m\phi)] P_l^m(\cos \theta)$$ (3.13)

$$B_{\theta\text{ext}} = \sum_{l=1}^{\infty} \sum_{m=0}^{l} - \left( \frac{r}{R_a} \right)^{(l-1)}$$

$$[G_l^m \cos(m\phi) + H_l^m \sin(m\phi)] \frac{\partial}{\partial \theta} [P_l^m(\cos \theta)]$$ (3.14)

$$B_{\phi\text{ext}} = \sum_{l=1}^{\infty} \sum_{m=0}^{l} \left( \frac{r}{R_a} \right)^{(l-1)}$$

$$[G_l^m \sin(m\phi) - H_l^m \cos(m\phi)] \frac{m}{\sin \theta} P_l^m(\cos \theta)$$ (3.15)

where $\vec{B}_{\text{ext}} = B_{\text{ext}} \hat{r} + B_{\text{ext}} \hat{\theta} + B_{\text{ext}} \hat{\phi}$. It should be noted here that the external contribution, $\vec{B}_{\text{ext}}$, is not the equivalent to the physical external field, such as magnetic fields arising from the magnetopause currents, ring currents, and IMF. It is a mathematical representation of the possible external field contributions. As we shall see in Chapter 4, an alternative approach to modeling external fields is to explicitly describe the fields arising from the magnetospheric current systems to obtain a more physical representation. It should also be noted that the internal field, $\vec{B}_{\text{int}}$, can contain contributions from core, crustal fields, or both.

Thus, our forward problem involves the data vector $\vec{d}$ consisting of the three component vector measurements from the spacecraft, such that $\vec{B} = [B_r, B_\theta, B_\phi]^T$, and the model vector, $\vec{m}$, consisting of the spherical harmonic (or Gauss) coefficients in order of increasing degree and order so that $\vec{m} = [g_l^m, h_l^m, G_l^m, H_l^m]^T$. The $G$ matrix has rows corresponding to each data point and columns corresponding to each model coefficient. It should be noted that some model coefficients, such as $h_l^0$ and $H_l^0$ are omitted from the inversion, as they are equal to zero by definition (see equation(3.6)).
3.3 Approaches to Solving Inverse Problems

Assuming that there are no errors associated with the data and that the problem is over determined, by substituting (3.2) into (3.1), we obtain,

$$\Phi = \sum_{i=1}^{N} \left( d_i - \sum_{j=1}^{M} G_{ij} m_j \right)^2$$

(3.16)

By setting $d\Phi/dm = 0$ one obtains

$$G^T G \vec{m} = G^T \vec{d},$$

(3.17)

often called the normal equations (see Gubbins, 2004 for details). The least squares solution can be found by rearranging (3.17) to obtain the model parameter, $\vec{m}$.

$$\vec{m} = (G^T G)^{-1} G^T \vec{d}$$

(3.18)

Although the least squares solution is not used here, it is important to understand this concept as other methods, especially the regularized inversion, are built on this idea. Instead, we use singular value decomposition (SVD) as described below.

3.3.1 Singular Value Decomposition

We will not focus on the derivation of Singular Value Decomposition (SVD), as there are many publications on this topic (e.g. Menke, 1989; Parker, 1994). Instead, the usage and its advantages are explained here; these rely on the concept of vector spaces. When a linear problem is depicted as $\vec{d} = G \vec{m}$ (equation (3.2)), the model ($S(\vec{m})$) and data ($S(\vec{d})$) vector spaces can be broken into two portions; activated ($S_p$), and inactivated or null space ($S_0$) (Figure 3.1).

If the problem is underdetermined, equation (3.2) can only contain some, but not all, of the information about the model. This subspace of the model space is the activated portion, $S_p(\vec{m})$. The rest of the space is referred to as a null space, $S_0(\vec{m})$. The data contain no information about $S_0(\vec{m})$.

Conversely, if the problem is overdetermined, then $G \vec{m}$ may not span the entire data vector space, $S(\vec{d})$. Therefore, $\vec{m}$ provides no information about the data that lies in the inactivated portion of the subspace $S_0(\vec{d})$ (Menke, 1989).

The problem of unbalanced numbers of model parameters and data now becomes more apparent. For instance, if the problem is underdetermined, it
3.3. Approaches to Solving Inverse Problems

Figure 3.1: Relationship between model vector space ($S(\vec{m})$) and data vector space ($S(\vec{d})$). Subscript $p$ and 0 indicate active portion and the inactive portion (null space) of the vector space, respectively. The $G$ matrix indicates a forward modeling operation, while $G^{\text{inv}}$ is some inverse operation and not the actual inverse of the matrix.

is not possible to have enough linear combinations of the data to span the entire model space ($S(\vec{m})$). Instead the data only “illuminates” the active portion of the model space ($S_p(\vec{m})$). On the other hand, if the problem is overdetermined, there are not enough linear combinations of the model parameters to span the entire data space ($S(\vec{d})$). The model space “illuminates” only the active data subspace ($S_p(\vec{d})$). Hence, the data that lie in the inactive data space ($S_0(\vec{d})$) cannot be satisfied with any choice of model parameters (see Section 7.5 of Menke (1989)). Ideally, we would like to have an efficient solution that only uses the active portion of the data space.

Singular value decomposition overcomes this problem by the decomposition of the matrix $G$ into

$$G = UAV^T,$$  \hspace{1cm} (3.19)

where the matrix $G$ is broken down into three portions. $U$ is an $M \times M$ matrix containing the eigenvectors that span the data space $S(\vec{d})$ and similarly, the vector $V$ is a $N \times N$ matrix that contains the eigenvectors that span the model space $S(\vec{m})$. The matrix $A$ is a $M \times N$ semi-diagonal matrix containing the positive square roots of the eigenvalues or the singular values. The diagonal elements are usually arranged in decreasing order, and some eigenvalues may be zero. The eigenvectors with singular values corresponding to zero do not have any information spanning the vector spaces or
3.3. Approaches to Solving Inverse Problems

more precisely, they belong in the null space and are not be used to solve the inverse problem. With some manipulations the models solution can be described as,

\[ V_p V_p^T \tilde{m} = \tilde{m}_{est} = V_p \Lambda_p^{-1} U_p^T \tilde{d}, \]  

(3.20)

where \( \tilde{m}_{est} \) is called the natural solution, and is a solution that does not include the null portion of the model space. The subscript \( p \) indicates the combinations of eigenvectors and eigenvalues with non-zero singular values. \( V_p \Lambda_p^{-1} U_p^T \) is often referred to as the generalized natural inverse. It can be shown that the solution obtained by equation (3.20) is in effect the least squares solution and the minimum norm solution (see Menke, 1989; Parker, 1994; Gubbins, 2004), when the problem is overdetermined and underdetermined, respectively (Menke, 1989).

The generalized natural inverse can be also used to describe the covariance of the model coefficients. Assuming that the data variances are identical, the covariance matrix can be written as (Korth et al., 2004):

\[ \text{cov}(\tilde{m}) = \hat{\sigma}^2 U \Lambda^{-2} U^T, \]  

(3.21)

where the elements of \( \hat{\sigma} \) contains the variances associated with data. If the uncertainties are purely statistical, it can be shown that the diagonal elements of \( \text{cov}(\tilde{m}) \) are the statistical errors for the model coefficients (Korth et al., 2004). Additionally, the singular values, \( \Lambda \), can be used to calculate the condition number \( (\kappa) \), which indicates how well-conditioned the problem is numerically. It is defined the ratio of the largest eigenvalue to the smallest eigenvalue:

\[ \kappa = \frac{\Lambda_{1,1}}{\Lambda_{p,p}}, \]  

(3.22)

where subscripts indicate the row and column numbers, respectively. A large condition number implies that a small change in value of data results in a large perturbation in the model parameters, while a small condition number suggest a stable solution.

In our problem the true model, \( \tilde{m} \) is an infinite sum over spherical harmonic degree and order. Clearly, finding such a model is not practical or possible because of data resolution and computing resources. Typically, an SVD solution for a truncated model is found. The spherical harmonic truncation level \( l_{max} \) is determined by the resolution of the data, spacecraft altitude, and data quality. A practical problem arises: individual satellite tracks may resolve short wavelength structure (for instance of wavelength \( \lambda_{min} \)), however, the equivalent spherical harmonic terms \( (l_{max} = \frac{2\pi R}{\lambda_{min}}) \) may not be well resolved due to poor global coverage.
3.3. Approaches to Solving Inverse Problems

3.3.2 Regularization Method

The philosophy behind regularization is to further constrain the solution, using an \textit{a priori} constraint. To translate this into mathematical language, an additional term to the misfit function \( \Phi \) (equation (3.1)) must be introduced, so that factors other than the data misfit are added as criteria for the inversion. Commonly, the \textit{a priori} constraint involves minimizing some aspect of the model size or roughness. The misfit function \( \Phi \) has the form:

\[
\Phi = ||W \vec{d} - WG \vec{m}||^2 + \alpha ||R \vec{m}||^2,
\]

where \( \alpha \) is the Lagrange multiplier that governs the trade-off between data misfit and the model criterion, \( R \) is a diagonal matrix that implements the model constraint (e.g. smoothness), and \( W \) is a diagonal weight matrix. In geophysical inversion, \( W \) is usually takes the form

\[
W = diag(\sigma),
\]

where \( \sigma \) is the vector containing the standard deviations associated with each data point. Equation (3.23) is minimized to solve for the regularized model:

\[
\vec{m} = \left[ \alpha R^T R + (WG)^T WG \right]^{-1} (WG)^T W \vec{d}.
\]

Typically in global magnetic field studies in geomagnetism and paleomagnetism, the form of the \( R \) matrix is motivated by the spherical harmonic power spectrum (Lowes spectrum). This is expressed as

\[
P(l) = \left( \frac{R_a}{r} \right)^{2(l+2)} (l+1) \sum_m \left[ (g_l^m)^2 + (h_l^m)^2 \right].
\]

Regularization constraints used have been minimization of some aspect of the power spectrum over a spherical surface with radius \( R \). These include (a) minimizing the rms value of radial field strength, \( B_r \) over the CMB, (b) the horizontal derivative of the radial field strength, \( \nabla_H B_r \) over the planet's surface, or (c) energy of the magnetostatic field external to the planet, that have the form

\[
R_l^2 = \left( \frac{R_a}{R} \right)^{(2l+c_0)} f(l) \sum_m \left[ (g_l^m)^2 + (h_l^m)^2 \right].
\]

Each minimization constraints have different form for \( c_0 \) and \( f(l) \), which are listed in Table 3.1 along with the integral that is being minimized over the
3.3. Approaches to Solving Inverse Problems

surface R. All of the regularization norms are applied at the CMB, except for (b) as it is assumed that much of the internal field signatures originate from the core. Constraint (b) is applied at the surface of the planet in hopes to produce a smooth map of $B_r$ at the surface.

Table 3.1: Smoothing Norms for Magnetic Field Inversions

<table>
<thead>
<tr>
<th>Integral to Minimize</th>
<th>$f(l)$</th>
<th>$c_0$</th>
<th>$f(l)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\min B_r$</td>
<td>$\int B_r^2 d\Omega</td>
<td>_{r=R_c}$</td>
<td>4</td>
</tr>
<tr>
<td>$\min \nabla H B_r$</td>
<td>$\int (\nabla H B_r)^2 d\Omega</td>
<td>_{r=R_a}$</td>
<td>6</td>
</tr>
<tr>
<td>$\min \text{energy}$</td>
<td>$\int \frac{B^2}{\mu_0} dV</td>
<td>_{r&gt;R_a}$</td>
<td>1</td>
</tr>
</tbody>
</table>

It should be noted that the regularization constraints mentioned above can only be applied to the internal magnetic field. This is due to the fact that the form of the power spectra of the external field at Mercury is not known. There have been some attempts to regularize the external field (e.g. Holme and Bloxham (1996a)), however, they were only able to solve for a constant (degree 1) external field. Additionally, these studies were conducted with data from Uranus and Neptune, where the external fields are much less prominent and static compared to that of Mercury.
Chapter 4

Analyses of Flyby Data

The key to successfully modeling the Hermean internal field and subsequently determining its origin is the separation of internal and external field signals. Accurate modeling of external fields, however, is difficult given the sparse data coverage and likely temporal variability in these fields (Chapter 2).

4.1 Characterization of External Field

Currently, there are two approaches to modeling Mercury’s external field. In the first approach, magnetospheric models developed for Earth (Tsyganenko and Sitnov (2005) referred to as TS04) are used. TS04 is an empirical model of the Earth’s magnetosphere (Tsyganenko, 1995) where all the external field currents that are inferred from past observations are included. However, as mentioned in earlier (Section 1.2), there are contrasts between the magnetic environment of the Earth and Mercury. The adaptation of the Earth-based TS04 model to Mercury is achieved by fitting the position of the inbound magnetopause boundary and the flyby observation, assuming an internal axial dipole (Korth et al., 2004). In the applications of these models to Mercury, external fields resulting from magnetopause and magnetotail currents are included; those due to ring currents and field-aligned currents are excluded because we have little information on their existence at Mercury. In the second approach, internal and external fields are co-estimated, using equation (3.6) and singular value decomposition (SVD). External fields are modeled up to degree and order 2 and an internal field to degree and order 1.Experimentation with TS04-predicted fields has shown that at least degree and order 2 spherical harmonic field is required to represent the long-wavelength signature of the external field, assuming that the TS04 model sufficiently describes the external field. Both the TS04 and the internal/external coestimation approaches make the assumption that the external field geometry is primarily controlled by the dipolar component of the internal field. This is a valid assumption, since the major current systems are at a distance far enough from the planet that the higher spherical
harmonic degree contributions of the internal field have decayed (by $\frac{1}{r^{7/2}}$) significantly.

In the TS04 approach, the external fields are determined differently for each flyby in MSO coordinates (see Chapter 2) as follows (Anderson et al., 2008a): the IMF is taken from the average over an $\approx 10$ minute period near the bow-shock. This time interval is chosen in order to get some statistical measure of IMF as close in time to the flyby's closest approach as possible. An initial internal dipole moment is assumed. The solar wind and dipole moment are then adjusted iteratively such that the best fit to both the magnetic field data inside the magnetopause and the magnetopause crossing locations is obtained. The resulting model includes an internal axial dipole contribution and a contribution from the external current system. An example for such a field is shown for equatorial region and polar region is shown in Figure 4.1. The advantage of this approach is that the method estimates a common internal field but separate external fields for each flybys, and so allows for time variability in the external field. We note that TS04 model predictions were provided to us by Haje Korth at the Johns Hopkins University Applied Physics Laboratory.

The coestimation approach also allows us to solve for a time variable external field. This is done by solving for a degree 1 dipolar internal field with a degree 2 external field. Although the natural coordinate system for an external field is MSO, in this case, the choice of coordinate system does not matter, since the external field is solved for each flyby separately. An example of such an external field is shown for MIII and M1 in Figure 4.1 along with the data. The difficulty with this method is that the solution suffers from poor (large) condition numbers, as it has to determine global external field solutions, each with eight model parameters and a single three-parameter internal field, with a sparse data set. One way to overcome this problem is to reduce the number of model parameters by setting some of the external field coefficients to zero (i.e. omitting them) according to the track position. Purucker et al. (2009) uses this approach for the M1 and M2 flybys by excluding the $m = 0$ terms, on the basis that the flyby provides little latitudinal coverage. Another strategy to avoid poor condition numbers is to solve for a single internal dipolar field in MBF coordinates (see Chapter 2), and for one common external field in MSO. The condition of the problem improves significantly, however, this solution does not account for any time variability in the external field.

The different external fields obtained using the above approaches are plotted in Figure 4.2 for each flyby. It is not currently possible to evaluate which of the external field models best estimates the actual field signatures.
4.1. Characterization of External Field

Figure 4.1: An example of external fields for a polar flyby (MIII) and an equatorial flyby (M1) plotted in MBF versus time in UTC for date of the respective flyby. The data are plotted in solid black. Two external field models are plotted here. The red dashed line is the TS04 prediction, and the blue dashed line is the prediction from a spherical harmonic model, which solves for a common degree 1 internal field and a separate degree 2 external field for each flyby. The blue dash-dot line is the prediction of the internal field, and the blue solid line is the sum of internal and external field predictions from the spherical harmonic model. Distance of the spacecraft from the center of Mercury is given as the ratio $r_{\text{R}}$ along the $x$-axis also.
4.1. Characterization of External Field

Figure 4.2: A comparison of different external field predictions plotted in MBF. Time is in UTC for date of the respective flyby. Distance of the spacecraft from the center of Mercury is given as the ratio $\frac{r}{R_M}$ along the $x$-axis also. Each column represents different flybys, and the rows correspond to $B_r$, $B_\theta$, and $B_\phi$. The solid line is the spherical harmonic coestimation that determines a unique external field for each flyby, dash-dot line is the same model formulation as the solid line but with some spherical harmonic coefficients ($m = 0$ terms) omitted, dotted line is for a common degree 2 external field for all four flybys, and the dashed line is the TS04 prediction. As it can be seen from the amplitudes of the predictions, estimating the external field for each flybys individually from spherical harmonics (i.e. solid and dash-dot lines) usually results in an unstable solution. All models assume a degree 1 internal field (i.e. centered dipole).
4.2 Internal Field Models

The coestimation approach is limited by poor data coverage; the latter will improve during the orbital phase. The parameterized model has the advantage of including the physical current systems at Earth; however, how well the adapted model captures the longest wavelength external fields at Mercury is not known. Nonetheless, we assume that the two approaches provide a measure of the range of the long-wavelength external fields at Mercury. The variability among these models reinforces the point that much of the characterization of Mercury's internal field depends on the success of the treatment of external field signatures in the data.

4.2 Internal Field Models

4.2.1 Estimation with Singular Value Decomposition

The most straightforward approach to modeling magnetic field data is to coestimate the internal and external field with equation (3.6) using singular value decomposition. Below, we will see that while this approach can be used to investigate the dipolar (e.g. longest-wavelength) structure in the internal field, it cannot be used to determine the higher harmonic degree structures because of poor global data coverage.

The internal and external fields from the coestimation of degree and order one internal field and a degree and order two external field in MSO coordinates (from Section 4.1) are shown in Figure 4.3 and the model spherical harmonics coefficient are given in Table 4.1. The model has a dipole moment of 250 nT-Rₕ³, with a tilt of 5° and an azimuth of 126°. As can be seen in Figure 4.3, the sum of the internal and external field predictions does not match the data. The resulting model has an RMS misfit of 30 nT with a condition number of 7. This relatively large misfit level compared to the low condition number, along with the poor quality of the fit of the model prediction to the data indicates that additional structure in the data remains. We first investigated whether this structure could be due to unmodeled external fields. We conducted tests, in which we retained only degree and order 1 structure in the internal field, but increased the complexity (i.e. l_max) of the external fields. We found that significant improvement in data misfit was obtained at the expense of ill-conditioned solutions (at least l_max = 5 external fields) with unrealistic amplitudes in the predicted external fields. We then investigated increasing complexity in the internal field, while retaining degree and order 2 external fields.

A model with degree and order 2 for both internal and external fields is shown in Figure 4.4 and its model coefficients given in Table 4.2. The
4.2. Internal Field Models

Figure 4.3: An example of a fit of a spherical harmonic coestimation model where a common degree and order 1 internal and degree and order 2 external field is solved for all flybys. Each column represents different flybys, and the rows correspond to $B_r$, $B_\theta$, and $B_\phi$. Time is in UTC for date of the respective flyby. Distance of the spacecraft from the center of Mercury is given as the ratio $\frac{r}{R_M}$ along the $x$-axis also. The black solid line is the raw data, blue solid line is the sum of the internal and external field prediction, blue dashed line is the external field prediction, blue dash-dot line is the internal field prediction. There is a significant residual structure in the data not described by the predicted field suggesting that higher degree (and order) structure is required to satisfy the data. Table 4.1 shows the spherical harmonic model coefficients for this model.
4.2. Internal Field Models

resulting model has a lower dipole moment of 183 nT-Rm, a tilt of 6° and an azimuth of 150°. The RMS misfit level is reduced to 15 nT, while the condition number was increased to 12. The results suggest that structures of at least degree and order 2 are required to successfully model the internal field.

The difficulty of inverting for a higher degree and order model with a technique such as singular decomposition lies in the fact that the model is penalized only for the goodness of the fit to the data. The global field models that result from sparse data coverage can have unreasonable structure in unsampled areas. This is not only problematic for the analyses of potential field data from the flybys, but also for data that will be collected when the spacecraft is in orbit. This is the reason why it is crucial to use a technique such as regularization.

4.2.2 Estimation with Regularization

In this section we investigate regularized models for the internal field. We first correct all four data sets for external fields predicted by the TS04 model.

Table 4.1: Spherical Harmonic Coefficients for the SVD Solution with Degree 1 Internal and Degree 2 External Field

<table>
<thead>
<tr>
<th>l</th>
<th>m</th>
<th>g_l^m</th>
<th>h_l^m</th>
<th>g_l</th>
<th>h_l</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>-248.8</td>
<td>16.4</td>
<td>25.6</td>
<td>7.7</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>-2.9</td>
<td>-7.6</td>
<td>-15.4</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Table 4.2: Spherical Harmonic Coefficients for the SVD Solution with Degree 2 Internal and Degree 2 External Field

<table>
<thead>
<tr>
<th>l</th>
<th>m</th>
<th>g_l^m</th>
<th>h_l^m</th>
<th>g_l</th>
<th>h_l</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>-182.1</td>
<td>8.7</td>
<td>-3.6</td>
<td>-14.7</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>-107.7</td>
<td>-8.7</td>
<td>-9.2</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>10.0</td>
<td>5.5</td>
<td>-2.9</td>
<td>0.4</td>
</tr>
</tbody>
</table>
4.2. Internal Field Models

Figure 4.4: An example of a fit of a spherical harmonic coestimation model where a common degree and order 2 internal and degree and order 2 external field is solved for all flybys. Each column represents different flybys, and the rows correspond to $B_r$, $B_\theta$, and $B_\phi$. Time is in UTC for date of the respective flyby. Distance of the spacecraft from the center of Mercury is given as the ratio $r/R_M$ along the x-axis also. The black solid line is the raw data, blue solid line is the sum of the internal and external field prediction, blue dashed line is the external field prediction, blue dash-dot line is the internal field prediction. There is a significant residual structure in the data not described by the predicted field suggesting that higher degree (and order) structure is required to satisfy the data. Table 4.1 shows the spherical harmonic model coefficients for this model.
4.2. Internal Field Models

As discussed in Section 4.1, it is currently difficult to assess how best to model the external field. We prefer the TS04 approach because it is based on parameterization of known (terrestrial) current systems. Also as we have seen, SVD coestimation approaches that allow for time variations in the external fields result in solutions with poor condition numbers. However, we account for the range of external field solutions permitted by the data in our assignment of data errors.

Data errors

We consider data errors due to three sources: long-wavelength uncertainties in the external field, short-wavelength features that we do not attempt to model, and errors due to magnetometer attitude uncertainty (i.e. uncertainties in the orientation of the magnetometer instrument). We consider the instrument noise of 0.047 nT (Anderson et al., 2007) to be negligible. In addition to the MESSENGER flyby data, we use magnetic field data within the magnetopause from the first and third Mariner 10 flybys (Lepping et al., 1979). As in previous studies, we use only the inbound (quiet) portion of the data from the Mariner 10 first flyby.

Data from the MESSENGER flybys show short-wavelength perturbations, in particular depressions in the field amplitude of 10 to 20 nT prior to closest approach for M1 (Anderson et al., 2008a). Similar-amplitude features are seen in the M10-I, M10-III, and M2 flybys, and are presumably due to local plasma effects that cannot be represented with the formalism used here. These signatures are therefore treated formally as noise, and we assign a single value to this error contribution for all field components on all flybys, denoted by $\sigma_1$.

To estimate the long-wavelength error in the external field correction ($\sigma_2$), we take the mean of the absolute difference of two external field models – TS04 and the spherical harmonic model coestimated from all three flybys in MSO coordinates, since the other two spherical harmonic solutions are unstable (due to poor condition numbers). We compute a value of $\sigma_2$ separately for each field component for each flyby. The values for ($\sigma_2^{Br}, \sigma_2^{Bo}, \sigma_2^{Br} \sigma_2^{Bo}$) in nT are (24, 3, 14) for M10-I, (46, 43, 64) for M10-III, (18, 22, 2) for M1, and (18, 26, 20) for M2. Implicit in this calculation are the assumptions that the TS04 and coestimation approaches yield models that span the approximate range of external field solutions, and that the external and internal fields are uncorrelated. Both assumptions are likely incorrect, but the current data distribution and knowledge of the external fields are too limited to justify a more complex approach.
The two sources of errors are assumed to be independent, and we calculate a combined error associated with the $i^{th}$ data point from

$$e_i = \sqrt{\sigma_1^2 + \sigma_2^2}$$

(4.1)

Experimentation showed a choice of $\sigma_1 \approx 13$ nT to be reasonable.

For completeness we included errors due to attitude uncertainty, denoted by $\Psi$. Errors in the field components are no longer isotropic and uncorrelated, and the data covariance matrix is calculated from

$$C = (W^TW)^{-1} + \Psi^2\left(B^2I - BB^T\right)$$

(4.2)

where the diagonal matrix, $W$, is given by equation (3.24) and we use the maximum magnitude of the field along each profile for $B$ (Holme and Bloxham, 1995, 1996a). The matrix $W$ in equation (3.25) is then replaced by $C$. $\Psi$ is estimated to be $0.1^\circ$ for the MESSENGER data. We were unable to find the attitude uncertainty for the Mariner 10 data, so we used a value of $0.1^\circ$ for these flybys also. We tested attitude errors of up to $2^\circ$. The results of our inversions were unchanged, confirming that uncertainties in the external field are dominant and consistent with the conclusions in Holme and Bloxham (1996b).

**Internal field models**

We performed inversions using the different regularization norms described in Chapter 3; the resulting models were similar, and we report results from the minimum $B_r$ norm ((a) from Section 3.2.2). The effects of different norms are discussed in more detail using the simulations reported in Chapter 5. Since the data from all four flybys can be approximated to first order by the field due to a dipole aligned with Mercury's rotation axis (Anderson et al., 2008a), we further constrain the problem by minimizing the contributions to the model from all other terms. In other words the $l = 1$, $m = 0$ term is not regularized, and we set the first term in (3.27) to 0. This allows us to explicitly address the field structure other than an axial dipole that is permitted (or indeed required) by the data. Our inversions were performed out to spherical harmonic degree 8, to ensure that the solution is constrained by the regularization, rather than by the truncation level.

Figure 4.5 shows the relationship between data misfit and model norm, accompanied by three selected models to illustrate the effects of fitting the data to different tolerance levels. We show models for tolerance levels
4.2. Internal Field Models

$T=1.4$, $T=0.85$, and $T=0.80$ (see equation (3.25)), corresponding to root-mean-square (rms) misfits of 35 nT, 19 nT, and 18 nT, respectively. The models are shown in terms of the total field magnitude at 200 km altitude, the lowest altitude at which observations have been made to date. The model for $T=1.4$ (Figure 4.5a) is dominated by the axial dipole term, with little contribution from other spherical harmonics. The regularization constraint contributes heavily to this model and the field is almost completely dipolar. As the tolerance level is decreased, the model complexity increases as measured by the model norm. The model for $T=0.85$ (Figure 4.5b) is quite complex, and that at $T=0.8$ (only a 1 nT further decrease in rms misfit) shows significant structure in areas not sampled by any of the three flybys. At $T=0.8$ (Figure 4.5c), the regularization constraint has much less influence on the resulting model, which is no longer dominated by dipolar field structure. Models with a tolerance less than 1.0 result in completely unrealistic structure in fields at the core-mantle boundary (CMB).

Our preferred model has $T=1.06$ (95% confidence limit), corresponding to an rms misfit of 23 nT, and is shown on the trade-off curve in Figure 4.5d. A map of the total field at 200 km altitude is shown (Figure 4.6a). We also show the radial field, $B_r$ - a more common way of displaying field models, since $B_r$ over a spherical surface uniquely determines the field, whereas $|B|$ does not (Backus, 1970). We show $B_r$ evaluated at the CMB as we are mainly interested in the evaluation of our models in terms of core fields. The spherical harmonic coefficients are given in Table 4.3; however, the field morphology is of more interest than the actual values of the coefficients, since the latter are in part determined by the covariance inherent in the applied regularization. The field is dominantly dipolar, but substantial non-dipolar latitudinal ($m = 0$) and longitudinal ($m \neq 0$) structure can be seen in the field maps (see also Table 4.3). The model has a dipole moment of 220 nT R$^3_m$ with a tilt of 3° and azimuth (i.e., north magnetic pole longitude) of 9°. The spherical harmonic power spectrum, evaluated at the surface of Mercury, is shown in Figure 4.6c. The power spectrum at spherical harmonic degrees 4 and greater follows the form of the regularization constraint. Figure 4.6d shows the trace elements of the resolution matrix; these indicate the relative contribution of the data and regularization constraint to the model coefficient. A value of one indicates that the model coefficient is determined exclusively by the data (the case for our $l = 1$, $m = 0$ coefficient), a value of 0 indicates that the model coefficient is determined exclusively by the model norm. The apparent increase in resolution in the $l = m$ terms at and above $l = 3$ is an artifact due to a combination of the data coverage and aliasing effects. Model fits to each field component for each flyby are shown.
Figure 4.5: (a) - (c) Selected field models along the trade-off curve (d). Field models are shown as maps of $|B|$ at 200-km altitude in nT. Grid lines are every 60° in longitude and 45° in latitude. Maps are centered on 0° longitude. Locations of data from flybys are shown in black – M2 (thick solid line), M1 (solid line), M10-I (dashed-dotted line), M10-III (dashed line). The three models are for (a) $T=1.4$, (b) $T=0.85$, and (c) $T=0.80$ and are indicated by the red, green, and blue stars on the trade-off curve. The black star is for the preferred model (Figure 4.6).
4.2. Internal Field Models

Figure 4.6: Preferred model. (a) $|B|$ at 200 km, and (b) $B_r$ at the core-mantle boundary (CMB). Map format as in Figure 4.5. (c) Spherical harmonic degree power spectrum evaluated at the surface of Mercury (solid). The axial dipole term ($g_1^0$) is not included in the calculation since it is not regularized. Dashed line shows the functional form of the regularization constraint. (d) Trace elements of the resolution matrix, ordered in increasing $l$ and $m$: \{$g_1^0$, $g_1^1$, $h_1^1$, $g_2^0$, ...... , $h_8^8$\}.

in Figure 4.7.

The regularized inversion shows that to be able to fit the data to a 95% confidence level, non-dipolar internal field structures are required. This statement is also supported by the tests that we have conducted with singular value decomposition (SVD), as a degree 1 internal field did not provide sufficient long-wavelength structure to explain the data accurately. However, the regularized inversion shows an approximately 20% increase in the value of $g_1^0$, while the $g_2^0$ prediction have fallen by about 23% of that obtained with a SVD approach. This is expected because the regularized solution encourages power to be in the $g_1^0$ term. Our preferred model (Figure 4.6) shows area of increased $B_r$ at the CMB region at equatorial regions between 0 to 120 E° and around 30° N, 20° E. Despite these longitudinal and latitudinal structures, our preferred model predictions are not able to fully explain the
4.2. **Internal Field Models**

Figure 4.7: Predictions from preferred model (dashed), together with data corrected for TS04 external fields (solid), with error bounds (dotted) given by equation (3.2). All three field components are shown for each flyby. Time is UTC on the date of the respective flyby. Distance of the spacecraft from the center of Mercury is given as the ratio \( \frac{r}{R_M} \) along the \( x \)-axis also.
4.2. Internal Field Models

Table 4.3: Spherical Harmonic Coefficients for the TS04-Corrected Regularized Internal Solution

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structures present in the data (see Figure 4.7). In particular, the underestimation of $B_r$ near the equatorial regions (M1 and M2) are observed. This can be an indication of improperly modeled external fields. The model fits the M10-I flyby data the least well, showing a poor fit to all three components. We have observed this result in most models that fit all three data sets to an overall misfit level, irrespective of the modeling methods used. This is probably due to the fact that strong external field signals were present during the M10-I encounter. The issue of field signatures external to the planet are addressed further in section 6.2.3.
Chapter 5

Recovery of fields from orbit

Studies of the determination of Mercury’s internal field from MESSENGER orbit data have focused primarily on the recovery of a dipolar signal, potentially indicative of a core dynamo. Some investigations of the recovery of quadrupole components have, however, been carried out using different prescriptions for the external field (Korth et al., 2004; Kabin et al., 2008). In the Korth et al. (2004) approach, an SVD technique is used to estimate $l, m = 1$ and $l, m = 2$ contributions to the internal field. A difficulty in extending this approach to modeling higher-order structure during orbit is the incomplete data coverage. In more detail fitting the data with truncated solutions using global basis functions results in significant power in unsampled regions, as illustrated in the under-damped solution to flyby data shown in Figure 4.5d. Here we specifically address the question of the kinds of structure in the field that might be recoverable in orbit using regularized spherical harmonic models, assuming that the external fields can be modeled and then removed from the data.

We focus on core fields – those generated by dynamo action – both for practical reasons (the maximum resolvable signatures at periapse will be on the order of the height of the spacecraft above the core-mantle boundary, $\approx 850$ km, limiting inversions to those with $l \leq 20$), and because there has been interest in the literature in core field diagnostics that might be indicative of Mercury’s dynamo regime (Christensen, 2006; Glassmeier et al., 2007; Stanley et al., 2007; Christensen and Wicht, 2008). We investigate recovery of three core field models generated from numerical dynamo simulations (Stanley et al., 2007). These models have been chosen as purely illustrative. We first show how different regularization norms affect the resulting inversions assuming negligible noise in the observations. We next examine the recovery of field structure from models with different features at the CMB, for one choice of norm and including noise in our inversions.
5.1 Simulated Orbital Data

Orbit tracks for the period 19 March – 19 May 2011 were simulated using the MESSENGER orbit determinations (McAdams et al., 2007), as 60 days corresponds to one Mercury rotation, giving us global coverage in longitude. As reported by Korth et al. (2004), multiple coverages obtained after the initial 60 days will improve our predictions. Planetocentric latitude, longitude, and radial distance were determined at 60 s intervals along track. Near periapse (≈ 60°-70°N) this interval corresponds to an along-track sampling of 180 km, comparable to the spacecraft altitude, and less than the shortest-wavelength detectable core signals. We simulated data for different core field models, along the orbit tracks, retaining only data below 2100 km altitude, consistent with locations within the magnetopause during the near-polar M10-III flyby. The simulated data cover all longitudes and latitudes from 20° S to 80° N. A lower cut-off altitude reduces the profile length retained, affecting the most southerly latitude sampled, but does not affect recovery of the longest-wavelength (dipole) fields. We used spherical harmonic models 1, 5, and 7 from Stanley et al. (2007). The focus of that paper was to investigate the effect of inner core size on reverse flux spots at the CMB; we address this question briefly in the discussion, but note here that we simply chose three models that have quite different structure in $B_r$ at the CMB. Model 1 has a ratio of inner core to outer core of 0.35 (thick-shell geometry); models 5 and 7 have a radius ratio of 0.8 (thin-shell geometry); all three models are snapshots in time. The original models were scaled to the maximum absolute amplitude of the field and were also of different polarities, so for our analyses we scaled all the coefficients to a value of $g_1^0$ of -280 nT. Simulated data were calculated using expansions out to degree and order 21; higher degree and order structure was not observable even near periapse. Noise drawn from a Gaussian distribution with a prescribed standard deviation was added to the simulated observations before inversion. We did not consider long-wavelength uncertainties in the external fields in these simulations, on the grounds that knowledge of the external fields and their statistics will improve in orbit.

5.2 Effect of Regularization Norm

Inversions were performed using Model 7 of Stanley et al. (2007) to investigate the effect of different regularizations on the recovered field. As in the analyses of the flyby data the axial dipole term was not regularized. Noise
5.2. Effect of Regularization Norm

Figure 5.1: (a) $B_r$ in nT at the CMB expanded to $l, m = 21$ for Model 7 of Stanley et al. (2007). Map format as in Figure 3b. Reverse flux patches and features A – G are discussed in text. Recovered models, assuming negligible noise, using (b) grad($B_r$) norm, (c) $B_r$ norm, and (d) energy norm (see text). Color scales for input and recovered fields are the same.

drawn from a zero-mean gaussian distribution with standard deviation 1 nT was added; the low noise level was deliberately chosen so that we could investigate the effects of data distribution and the inversion technique. Inversions were carried out to $l = 20$ for three norms: (a) minimizing the rms value of $B_r$ over the CMB – we refer to this as the $B_r$ norm, (b) minimizing the rms value of the horizontal gradient of $B_r$ over the CMB – the grad($B_r$) norm; and (c) minimizing the energy of the magnetostatic field external to the planet – the energy norm (see Table 3.1 in Section 3.2.2). The functions $f(l)$ show that damping of the higher degree terms are least for the energy norm and greatest for the grad($B_r$) norm. The input field (Figure 5.1a) is dominantly dipolar but has complex structure with many reverse-polarity flux patches. These are regions with opposite polarity to the dipole polarity in the hemisphere in which they occur, such as features A, B, C, D, E (Figure 5.1a). In addition intense flux patches often occur in oppositely signed pairs, such as the group of features D, E, F and G (Figure 5.1a).
5.3 Inversions of Different Core Field Models

The recovered dipole moment was within 2% of the input value in all three inversions. Damping of short-wavelength structure is heaviest for the grad\(B_r\) norm (Figure 5.1b), as expected, intermediate for the min\(B_r\) norm (Figure 5.1c), and least for the minimum energy norm (Figure 5.1d). The field in the southern hemisphere is smoothed because of the absence of data south of 20\(^\circ\) S; in particular the intense reverse-flux patches, \(E\), in the input model are visible only as a broad, low-amplitude negative \(B_r\) feature. Substantial longitudinal structure in the field north of about 30\(^\circ\) N is resolved in the models, in particular when the minimum energy norm is used (Figure 5.1d). Regions of increased flux, \(C\) and \(D\), are resolved in all three models, due to their mid-northern-latitude location and their spatial extent. The lower-amplitude features \(A\) and \(B\) are difficult to resolve separately. The input model contains a strong octupolar component – approximately 50\% of the dipole term (Stanley et al., 2007). This is evident in all three inversions as the bands of alternating positive and negative \(B_r\) of about 45\(^\circ\) latitudinal extent.

5.3 Inversions of Different Core Field Models

On the basis of the results shown in Figure 5.1, we chose to use the minimum energy norm to investigate recovery of core fields for three models (Models 1, 5, and 7) of Stanley et al. (2007). Model 1 is a thick-shell model, while Models 5 and 7 are thin-shell dynamo models. The input field models are shown in Figure 5.2. The first model (Figure 5.2a) is dominantly dipolar and exhibits longitudinal structure at all latitudes, including long-wavelength structure as a function of longitude north of 45\(^\circ\) N. A pair of reverse flux patches is seen, close to the equator, at longitudes near 180\(^\circ\) W. This model is most Earth-like in its structure. Models 5 (Figure 5.2c) and 7 (Figure 5.2e) of Stanley et al. (2007) have strong octupolar components but are quite different in detail. Model 5 exhibits very little high-latitude structure, with an intense, large region of positive radial field at low northern latitudes, at about 120\(^\circ\) W. The features in Model 7 have been discussed in the previous section.

Our inversions included noise drawn from a zero-mean gaussian distribution with standard deviation 10 nT, motivated by the amplitude of the shorter-wavelength features in the flyby data. Smoothness imparted by the noise level rendered it unnecessary to go beyond \(l = 12\) in the inversions. Importantly, the resulting models show that in all cases the long-wavelength field structure is recovered. Amplitudes are reduced relative to the input
5.3. Inversions of Different Core Field Models

Figure 5.2: Input (left column) and recovered (right column) field models for three models from Stanley et al. (2007), shown in terms of $B_r$ in nT at the CMB. (a)–(b) Model 1; (c)–(d) Model 5, and (e)–(f) Model 7.
5.3. Inversions of Different Core Field Models

field, especially for shorter-spatial-scale features, a consequence of the regularization. In contrast, least squares solutions (not shown here) resulted in models with completely unrealistic structure (notably high-amplitude, short-wavelength features in the southern hemisphere) due to the combined effects of incomplete data distribution, the use of global basis functions, and downward continuation. The effect of the 10-fold increase in noise over the previous set of inversions is evident in the smoother field morphology seen in Figure 5.2f, compared with the model shown in Figure 5.1d. The difference between the input and recovered field models is shown in Figure 5.3. Model 1 is recovered well (amplitudes in the difference map are small compared with field amplitudes), with the exception of longitudinal structure within about 20° of the equator. Models 5 and 7 have more low-latitude and southern-hemisphere structure that is less well recovered. For all models, the amplitude of the field north of about 25° N is typically recovered to within 20%, and in the case of model 1, to within 10%.

The results from obtained simulated orbit data with different dynamo models are encouraging. In the case where long-wavelength external field signatures can be accounted for, our orbit simulations indicate that we are able to successfully overcome the difficulty of an elliptical orbit geometry. The models indicate that parameterization up to, and at least 10 degrees is possible. The different regularization constraint does not tend to affect the general outcome of the model, although the minimum energy norm seems to be best at recovering short-wavelength structures. Experimentation with dynamo simulation results from Stanley et al. (2007) has shown that it is possible to observe some of the larger and highly anomalous reverse flux patches. Further discussion of the result of the orbit simulation and its implication to future work are explored in Section 6.3.1.
5.3. Inversions of Different Core Field Models

Figure 5.3: Difference maps \( B_r \) in nT at the CMB for recovered minus input models for (a) Model 1, (b) Model 5, (c) Model 7.
Chapter 6

Summary and Future Work

In this chapter, the results from the previous chapters are summarized and their implications are discussed. We review dynamo simulations and crustal field model predictions for Mercury's internal field, in particular, predictions of such simulations for non-dipolar and non-axisymmetric (longitudinal) structure in the field. We discuss the results of hypothesis testing from the flyby analyses, and suggest types of diagnostics (based on modeling flyby and simulated orbit data) that might be useful for assessing predictions of numerical simulations. We then consider the results from the orbit simulations and provide a summary of possible future work to needed successfully model the internal field during orbit.

6.1 Current Magnetic Field Models of Mercury

As previously mentioned (Chapter 1), the dominantly dipolar structure of Mercury's internal magnetic field make dynamo processes a very favorable candidate to explain the observed field. However, there are several different dynamo processes that could explain the observed field. These include thermo-chemical (see review in Zuber et al., 2007), externally induced (Grosser et al., 2004; Glassmeier et al., 2007), and thermo-electric (Stevenson, 1987; Giampieri and Balogh, 2002) dynamos. Also, long-wavelength crustal magnetization is still a possibility (Aharonson et al., 2004). Short-wavelength crustal features are difficult to observe from flyby data; hence, any unambiguous detection of such signature will have to wait until the spacecraft is in orbit.

Although the magnetic field morphology and its long-wavelength structure support the hypothesis of a dynamo origin, the observed field strength is unexpectedly weak for a dynamo driven by thermo-chemical convection, such as that of the Earth. Two possibilities have been explored in recent numerical thermo-chemical dynamo simulations. In the first class of models, weak fields at and above the planet's surface are generated in thin-shell dynamos with a thin fluid outer-core region (Stanley et al., 2005). Stanley et al. (2007) further propose that regions of reverse polarity (as evidenced in the
6.1. Current Magnetic Field Models of Mercury

radial magnetic field at the core-mantle boundary) is confined to the areas near the tangent cylinder of the inner and outer-core boundary and suggest that mapping of such reverse polarity could provide a potential diagnostic for outer core geometry. A second class of models, those with a thick-shell geometry can generate weak fields (Heimpel et al., 2005; Christensen, 2006). In some of these models, the fields show strong non-dipolar components especially at the core-mantle-boundary. However, at spacecraft altitude, the presence of a stable conductive layer, due to stratification, at the top of the outer core region suppresses the signal from strongly time variant non-dipolar components (Christensen, 2006; Wicht et al., 2007; Christensen and Wicht, 2008). The field observed at Mercury’s surface is expected to be dominated by the weaker, more slowly varying dipolar component. The melting behavior of the iron-sulfur system may permit the formation of such a stably stratified layer (Chen et al., 2008). The stably stratified layer attenuates non-axisymmetric components (Christensen, 2006). These models predict large-scale, dominantly axisymmetric fields at spacecraft altitude with little secular variation on decadal time scales.

An externally induced dynamo field as proposed in Grosser et al. (2004); Glassmeier et al. (2007) or a thermo-electric dynamo (Stevenson, 1987; Giampieri and Balogh, 2002) would be a difficult one to verify from a data perspective. External field induced dynamo models suggest that there exist a feedback mechanism between the core field and the external current system, namely the Chapman-Ferraro currents (see Section 1.2). From an observation perspective the signal that is induced in the core from external fields is not distinguishable from another process, as it is still generated within the core of the planet. However, there are some promising numerical dynamo simulation studies (Gómez-Pérez et al., 2008) that might provide insight into obtaining such signals. Thermo-electric dynamos strongly rely on the topographic undulations of the CMB (Giampieri and Balogh, 2002). It is predicted that a thermo-electric dynamo would result in some correlation between the magnetic field and gravity field. To be able to confirm a thermo-electric process, a more detailed observations of the gravity field and topography than what is available currently are required, as we must be able to attribute the gravity signatures to being of CMB origin rather than shallower densities anomalies (e.g. from mantle or crust). Additionally, the study in Giampieri and Balogh (2002) assumes that the magnetospheric signals are completely removed from data.

The model of Aharonson et al. (2004) shows that the spatial variation in solar insolation on Mercury could give rise to long-wavelength variations in the depth of the Curie isotherm. If the magnetic carriers lie at this
6.2 Hypothesis Testing and Implications of the Flyby Data Analyses

Although it can be inferred that Mercury's internal magnetic field is dominated by an axial dipole, the sparse data coverage of the planet, together with the difficulty of distinguishing internal and external field signals, prevents us from obtaining robust constraints on the global field structure from analyses of flyby data alone. However, our regularized inversions for the internal field show that non-dipolar structure is required to fit the flyby data to within a 95% confidence limit, even after assigning conservative errors to the observations. The preferred field model (Figure 4.6) shows increased positive $B_\tau$ at the CMB at low latitudes in the longitude band 0 - 120° E. A region of increased radial flux is also seen at around 30° N, 20° E, in a region sampled by the M10-III trajectory. Both latitudinal and longitudinal structure in the field are required by the data. The dominant non-axial-dipole term in the field model is the axial quadrupole $g_{30}$ term, mainly driven by differences in the field amplitude at high and low latitudes that are larger than predicted by a dipole field alone.

The regularization approach allows for direct hypothesis testing of fields of different morphologies. We have specifically tested whether structure other than $g_{30}^0$ is required by the data. Motivated by the power in terms below $l = 4$ (Figure 4.6c), and the suggested possibility of long-wavelength structure in the crustal field (Aharonson et al., 2004), we also examined models in which the $g_{30}^0$, $g_{80}^3$, and $g_{33}^0$ terms were left undamped. These solutions can fit the data to the same average misfit level as solutions in which only the $g_{30}^0$ term is left undamped. However, the model prediction for the $B_\tau$ and $B_\theta$ components in the MESSENGER flybys did not mimic the long-wavelength structure of the data properly, and without applying further constraints the
6.3. Future Work

6.3.1 Orbit Simulations

The results of our simulations of field structure recoverable when MESSENGER is in orbit are encouraging in a number of ways. In the event that it is possible to account for long-wavelength fields, our simulations suggest that core field modeling using global basis functions (spherical harmonics) together with regularization constraints will enable reasonable recovery of field morphology in the northern hemisphere. It should be possible to construct field models conservatively to degree and order 10, and possibly higher. As shown in Chapter 4, examination of the power spectra, together with the resolution matrix, allow identification of the relative roles of the data and regularization constraint in determining the resulting model coefficients. In our orbit simulations (and in our analyses of the flyby data) the choice of regularization constraint was not critical to the results. The minimum energy norm gave the best recovery of shorter-wavelength features (see Chapter 5), and the dipole moment was well recovered in all cases. Clearly, field structure at latitudes within 20° or so of periapse will be best recovered, due to spacecraft altitudes.

Identification of reverse flux patches requires such features to be of the appropriate scale length. Reverse flux patches have been proposed as a tool to identify the location of Mercury’s tangent cylinder (and hence the ratio of inner to outer core radii), since in numerical simulations they are observed
6.3. Future Work

to occur at latitudes lower than the tangent cylinder location (Stanley et al., 2007). Radius ratios greater than 0.7 correspond to tangent cylinder latitudes lower than 45°. From a field modeling perspective alone, these thin-shell geometries will be difficult to diagnose because reverse flux patches will be confined to low latitudes where spacecraft altitudes are high. High-latitude regions of enhanced radial flux, such as are seen in the geomagnetic field (Bloxham et al., 1989), should be detectable in the northern hemisphere, if present. Particularly powerful during analyses of orbit data will be hypothesis testing. For example, explicit tests for models such as those that match the prediction of Aharonson et al. (2004), those with minimal non-axisymmetric power, and those with specific forms for the power spectrum can be designed.

6.3.2 External Field Characterization

As can be seen from analyses of the flyby data (Section 4.2.2), the key to successfully modeling the internal magnetic field of Mercury lies in the characterization of external fields. The solution obtained using current external field models and the weighting derived from them are not sufficient to accurately characterize the external field, as there are unexplained residual field structure when the data are compared to our predicted model (Figure 4.7). One possible method of improving the prediction of external fields is to compare the data from two similar track passes. For instance, M10-I and M1 have similar track positions (Figure 4.5a), therefore, any discrepancy between the two data sets must be explained either by unmodeled long-wavelength external field structure or secular variations. This can be applied to the third MESSENGER flyby data as well, since M2 and M3 will have similar track geometry.

Another drawback of current external field models are the variations among different predictions. We use differences among these predictions to weight our observations. Thus, large disagreement among external field models results in large weights in our inversions. As the number of tracks increases, this problem worsens because the condition numbers increase for the coestimation approach, in which the external fields are determined for each flyby individually. To determine a degree and order two internal field, the degrees of freedom of the problem increases by at least 6 (8 for the case where no coefficients are omitted) for each data set that is added. In contrast, the coestimation of a single external field would improve the condition number of the problem, but the time variability of the external field is ignored in these solutions. An external field model currently under de-
6.3. Future Work

Development is the paraboloid model (Alexeev et al., 2008) that could allow coestimation of internal and external fields, where external fields are parameterized, but not using spherical harmonics. This model is thought to be more appropriate than the TS04 model, and is also developed specifically for Mercury.

There are spectral techniques that might allow some identification between the internal and external field signals. Parker and O’Brien (1997) show that for any spacecraft observations, the sum of the squares of the spectral power of the along- and across-track component must equal to squares of the power of the vertical component for each frequency, if the potential source originates below the spacecraft orbit. This technique is referred to as the power sum rule, and has been used to estimate the crustal power spectrum from Magsat data (Lowe et al., 2001). One drawback is that power sum rule only holds for constant altitude data, due to continuation effects. However, the rule also states that the phase relationship between the horizontal components (along- and across-track) should be off by $\frac{\pi}{2}$ from the vertical component. This should still hold for variable altitude data, as phase relationships are independent of upward/downward continuation effects (personal communication: Parker). This phase relationship might not only address the problem of long-wavelength external field effects, but also distinguish crustal signals from local plasma effects, opening the possibilities of modeling short-wavelength crustal fields. Such modeling may be better accomplished using local basis functions.

Ultimately, the best approach to modeling the internal field would be to coestimate and regularize both the internal field and external field. This is currently not possible, as we lack a clear knowledge of the external field structure at Mercury. This will certainly improve as the observations become more abundant, but the problem of time variability will still remain. One solution to this problem might lie in the phase relationship that Parker and O’Brien (1997) have suggested, since this would allow us to group orbit tracks with similar characteristics and produce several models under different external field conditions.

Although currently modeling of the internal magnetic field faces obstacles stemming from sparse data and external field signatures, there are some encouraging studies of new models and techniques that might allow us to overcome such difficulties. Furthermore, the launch of the BepiColombo mission (possibly in 2012) by the European Space Agency (ESA) and Japan Aerospace Exploration Agency (JAXA) would improve the data coverage in the southern-hemisphere, which is critical to the identification of small-scale features, such as flux-patches. The problem of sparse data will always re-
6.3. Future Work

main somewhat of a problem for MESSENGER, but the characterization of the time variability of the external field should improve as we collect in-situ orbit data.
Bibliography


