Optimal Placement and Design of Passive Damper Connectors for Adjacent Structures

by

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Abstract

Passive coupling of adjacent structures is known to be an effective method to reduce undesirable vibrations and structural pounding effects. Past results have shown that reducing the number of dampers can considerably decrease implementation costs and does not significantly decrease the efficiency of the system. The main objective of this thesis is the optimal design of a limited number of dampers to minimize the inter-story drift. In this thesis, we present a bi-level optimization algorithm to find the optimal arrangement and mechanical properties of dampers placed between two adjacent buildings to minimize the maximum inter-story drift during (simulated) earthquake conditions.

Under the assumption of equal damping coefficients for all dampers, the optimal damping configuration is found via five different approaches: exhaustive search, inserting dampers, inserting floors, locations of maximum relative velocity, and a genetic algorithm. Through several numerical tests, efficiency and robustness of each optimization method is examined. It is shown that the inserting damper method is the most efficient and reliable method, particularly for tall structures. It is also found that, assuming equal damping coefficients for all dampers, increasing the number of dampers can exacerbate the dynamic response of the system.

Finding an efficient method to optimize dampers' locations, we focus on the optimization of the damping coefficients. Letting the dampers have varying damping coefficients, the optimization problem of damping coefficients is an n-dimensional optimization problem, whose objective function is provided via a simulation. Therefore, we use non-gradient based techniques for the inner-loop of the algorithm. We compare three different methods: a genetic algorithm (GA), a mesh adaptive direct search (MADS) algorithm, and the robust approximate gradient sampling (RAGS) algorithm. RAGS is a derivative free optimization (DFO) method that exploits the structure of the finite minimax problem. Using these techniques, we show that there exists a threshold on the number of dampers inserted with respect to the efficiency of the retrofitting system. Furthermore, we show that using a structured internal subroutine (such as RAGS) for the inner-loop of the bi-level problem greatly increases the efficiency of the retrofitting system.

Preface

Chapter 3 is based on research work conducted at UBC Okanagan. The original work has been published as:

Configuration optimization of dampers for adjacent buildings under seismic excitations, Kasra Bigdeli, Warren Hare, and Solomon Tesfamariam, Engineering Optimization, iFirst 2012, DOI:10.1080/0305215X.2012.654788.

Chapter 4 is based on the following research paper, which has been submitted to Optimization and Engineering:

Optimal Design of Damper Connectors for Adjacent Buildings, Kasra Bigdeli, Warren Hare, Julie Nutini, and Solomon Tesfamariam.

The "robust approximate gradient sampling" algorithm in chapter 4 is originally developed by Julie Nutini and it has been published as her MSc thesis, entitled:

A Derivative Free Approximate Gradient Sampling Algorithm for Finite Minimax Problems, MSc thesis, Julie Nutini, 2012, The University of British Columbia.

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Dedication

To my parents for their unconditional love and support…

1 Introduction

Every year thousands of earthquakes occur all over the world. Some of them are so weak that they are not felt; on the other hand, some strong earthquakes can destroy a city, damage infrastructures (e.g. bridge, buildings) and kill thousands of people. Building collapses are the most probable cause of death during an earthquake. Numerous research groups around the world are trying to improve building design approaches in order to increase the seismic stability of buildings. Consequently, every year design codes change and, buildings designed based on these newer guidelines are stronger. However, the majority of buildings are those which have been constructed based on older codes. These older buildings are more likely to get damaged during an earthquake. One solution is to destruct old buildings and reconstruct them based on newer design codes. Another solution is to increase seismic stability of existing buildings which is known as seismic retrofitting. Clearly, the former is not economically feasible because of the huge cost of construction. On the other hand, the latter can be done within a reasonable budget. The following section will review existing retrofitting techniques.

1.1 Seismic retrofitting methods

Three key factors for a good retrofitting method are efficiency, cost, and applicability to existing structures. Firstly, a good retrofitting method must be able to reduce the risk of damage to an acceptable level. Another important factor that must be considered is that a good retrofitting method must be economical. And finally, since these methods are to be used for existing structures, it is very crucial to take into consideration the amount of work needed to be done in order to employ these techniques.

Numerous methods have been introduced for seismic vibration control. Mainly, they can be categorized into four different groups:

1) passive methods,
2) active methods,
3) semi-active methods, and
4) hybrid methods.

Passive methods are those which do not use any external power [1]. They basically change mass, stiffness, or damping of the structure. Among those, one can refer to viscous dampers, friction dampers, metallic yield braces, fiber reinforced concrete, tuned mass dampers, and so on. Passive methods are generally effective, very economical, and since they do not need external power, they are reliable during an earthquake that may cause electricity loss.

On the other hand, active methods, such as active bracing and active mass damper [2], need external power. Active vibration control systems consist of three main units: sensors, decision making computer unit, and actuators. Sensors measure the dynamic behaviour of the building and transmit data to the computer where the required restoring force is calculated. Finally the computer transmits an appropriate signal to actuators to generate the required force. Since a computer and an external power provide the restoration force, active methods are generally more effective than passive methods. But they are less reliable and more expensive due to external power requirement and expensive equipment.

The third class of vibration control methods is called semi-active methods. They are very similar to active methods; but instead of actuators, in semi-active methods, restoring force is provided by changing mechanical properties of force generating devices such as variable stiffness springs [3], or magneto-rheological dampers [4]. Unlike active methods, semi-active methods need a very small external power which can be provided by batteries in most cases. As a result, they are less expensive and more reliable than active methods.

Finally, a combination of passive, active and semi-active methods working in parallel is called a hybrid method. For example, a hybrid mass damper system [5] is a common hybrid vibration control method which consists of a passive tuned mass damper and an actuator.

1.2 Coupled buildings method

A decent method to protect tall buildings from earthquake excitations is the coupled building method. The coupled building method makes use of the fact that dynamic responses of dissimilar structures are different under the same base excitation load. That is to say, under the same seismic load, relative displacement and velocity will change between two dissimilar adjacent structures. Therefore, if one connects two adjacent buildings using some linkages, the buildings will exert force on each other during a seismic event. The original concept of coupling adjacent buildings was first introduced by Klein et al. [6]. Since then, many researchers have been investigating various aspects of coupled buildings control method. Furthermore, many full scale applications are being constructed. For example, active coupling of buildings was employed to connect adjacent buildings in the Triton Square office project in Tokyo in 2001.

As mentioned earlier, three key factors for a good retrofitting are efficiency, cost and applicability to existing structures. Several theoretical and experimental studies have investigated the efficiency of coupled adjacent structures. Various devices, such as viscous dampers, magneto-rheological dampers, and so on, can be used to connect two adjacent buildings. It has been shown that these connectors can significantly reduce hazardous vibrations of the connected structures under earthquake load (e.g. see [7–9]). Besides mitigating hazardous vibrations, connecting two adjacent structures can reduce the chance of seismic pounding during an earthquake [10], [11]. In most metropolitan areas, limited land availability and high demand for residency and office buildings, leads to high-rise buildings constructed in close proximity. During an earthquake such adjacent buildings are prone to pounding. The damages observed from pounding during earthquakes are highly destructive and particularly frequent in dense urban centers [12]. For example, severe damage has been observed in the Mexico City earthquake (1985), the Loma Prieta earthquake (1989), the Kobe earthquake (1994), and the recent New Zealand earthquake (2011) (e.g. see [13–15]). Considering passive connectors, coupled building control is a very economical method. Note that not only passive devices, which are less expensive, can be used; but also if they are placed between two adjacent structures, one device can exert force on two structures which makes this control method even more economical. With regards to applicability to existing structures, coupling two adjacent buildings is a relatively easy task as it can be done with minimal damage to the interior of the structures since the damper connectors are installed between two buildings and the only part of the structure that might need reconstruction is the exterior of the building. Therefore installation time and work is much less than other passive methods such as installing damper bracing systems. All in all, the vibration control method of coupling adjacent structures meets all three main criteria of cost, efficiency and applicability.

1.3 Literature review

Since 40 years ago, when the original idea of coupling buildings was introduced [6], many researchers have been studying different control methods and devices for coupled buildings. Several theoretical and experimental studies have been done on various control strategies, passive connectors, active control algorithms, modeling and so on. In what follows, a literature review of coupled building vibration control method is presented. Methods for modeling of the coupled buildings, including single-degree-of-freedom (SDOF) and multi-degree-of-freedom (MDOF) methods are explained. In this section, different connecting devices including passive viscous dampers and magneto-rheological dampers are presented. Finally, the main concern of this thesis, optimization of coupled building control, is discussed.

1.3.1 Modeling

The first step in structural engineering studies is to formulate the mechanical behaviour of structures in terms of mathematical equations. No theoretical model perfectly matches the reality; however, some models are more accurate than others. Simpler methods usually lead to a closed-form solution for the problem. On the other hand, as the model becomes more accurate, the solution becomes more complicated and numerical methods must be employed to solve the model which increases the solution time. In this section, three popular structural models that have been widely used in coupled buildings control are presented: SDOF, MODF, and experimental models.

1.3.1.1 Single-degree-of-freedom

Single-degree-of-freedom (SDOF) model is the simplest way to model the mechanical behaviour of a structure. In this model, each building is modeled by a lumped mass which is connected to the ground by a set of springs and dampers. Two adjacent buildings are then connected via a connective device which can be a passive, semi-active, or active device. A schematic view of this model is shown in Figure 1-1. Although this model is the most basic representation of coupled buildings, it can provide some basic results on how two adjacent buildings and the linkage between them behave during a seismic event.

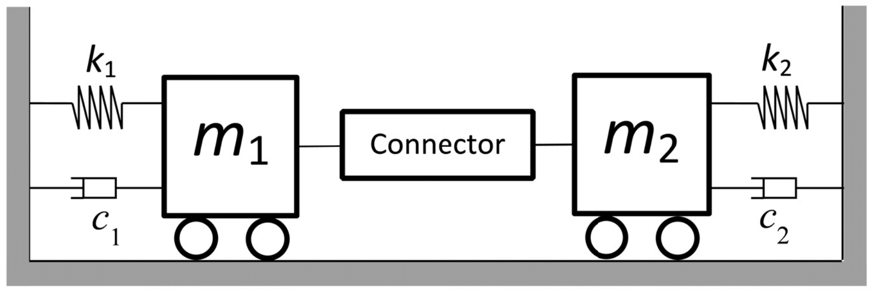

Figure 1-1: Single-degree-of-freedom model of coupled structures

Note that SDOF models are very limited in terms of research application. For example, since each building is modelled as a single mass, we are not able to investigate effects of distribution of damper connectors on the behaviour of the coupled buildings. Or effects of higher modes may not be readily observed through an SDOF model. Therefore, they are mostly used for preliminary studies.

For example, Basili et al. [11] studied optimal mechanical properties of nonlinear hysteric dampers connecting two SDOF structures. They presented explicit equations relating dissipation energy, relative displacement and relative acceleration to the mechanical properties of the dampers.

In [16], buildings are modeled as SDOF structures connected with friction dampers. They presented an analytical closed-form solution for dynamic responses of two adjacent buildings under seismic load.

Iwanami et al. [17] used an SDOF system to model each structure. The structures are connected to each other by a linkage that has damping and stiffness components. They presented an analytical solution for optimum parameters of damper/spring connectors to minimize the dynamic transmissibility during resonance.

In order to investigate the dynamic behaviour of series of adjacent buildings which are connected to each other with viscoelastic dampers, Kim et al. [18] modeled each building as a SDOF structure. Results from SDOF models are then verified by investigating seismic response of a 5-story and a 25-strory building.

In [19], two adjacent buildings are modeled as SDOF structures with damping and stiffness elements connecting the two buildings. A closed-form equation is presented for passive control vibration of the buildings. It is shown that the maximum absolute displacement transmissibility of each structure is reduced.

In order to compare various control methods, Zhu et al. [20] used SDOF models for adjacent structures. A closed-form analytical solution is presented for dynamic behaviour of the buildings. In their study, four different algorithms are compared: optimal passive, active, two different semi-active algorithms. The efficiency of their proposed method is confirmed via a time history analysis of El Centro 1940 ground excitation.

In [21], a methodology to find optimal parameters of Maxwell dampers is presented. Two buildings are modeled as SDOF structures and a closed-form analytical solution is derived for the required parameters of Maxwell dampers. The efficiency of their proposed method is investigated through a filtered white noise in the frequency domain, and El Centro 1940 earthquake record in the time domain.

As mentioned earlier, most studies on SDOF structures lead to a closed-form analytical solution. Although this type of modeling is not very realistic, closed-form solutions help researchers understand the effects of different parameters on the efficiency of their proposed method.

1.3.1.2 Multi-degree-of-freedom

The multi-degree-of-freedom (MDOF) model is a more accurate model to predict dynamic behaviour of buildings under seismic load. In a simple MDOF model, each floor is modeled by a lumped mass which is connected to upper and lower floors by a set of spring and damper. Heights of buildings do not need to match. Then adjacent buildings are connected to each other via connectors which connect (to) adjacent floors. A schematic view of this model is shown in Figure 1-2. Unlike the SDOF model, since the MDOF models consider each floor individually, various aspects such as the effects of dampers' distribution can be studied. Furthermore, effects of higher modes can be easily investigated. Still, some more complex effects, such as torsional motion or three-dimensional analysis, cannot be readily investigated through a simple MDOF model. Note that, to investigate these complex behaviours, more advanced models are available such as finite element (FE) models. In general, FE is a sub category of MDOF. Unlike simple MDOF models, in an FE model, each floor is modeled by several masses, springs and dampers; and this makes the model even more accurate. However, FE models are much more complex than MDOF models and the solution time for FE models is longer than simple MDOF models. Unless we are interested in complex behaviours such as relative displacement of two nodes on the same floor, MDOF models are usually accurate enough for most applications.

A very good example of MDOF models is a comprehensive study on the seismic response of damper connected structures done by Xu et al. [7]. In their study, two adjacent buildings are modeled as MDOF structures to which viscous dampers connected. Both time history analysis and frequency domain analysis are used to confirm the efficiency of their proposed method. It is worth noting that the simulation engine used in the present thesis is adopted from their study.

Basili et al. [22] proposed a methodology for optimal design of MDOF adjacent buildings connected by hysteretic dampers. First, a reduced order model is used to simplify the mathematical model. In order to find the optimal parameters, they use an analytical approach that they presented earlier for SDOF structures [11].

In [23], two MDOF structures are connected by MR dampers. Different control algorithms, including passive-off, passive-on and semi-active, are used and compared. Four different earthquake time history records are used to evaluate the control strategies.

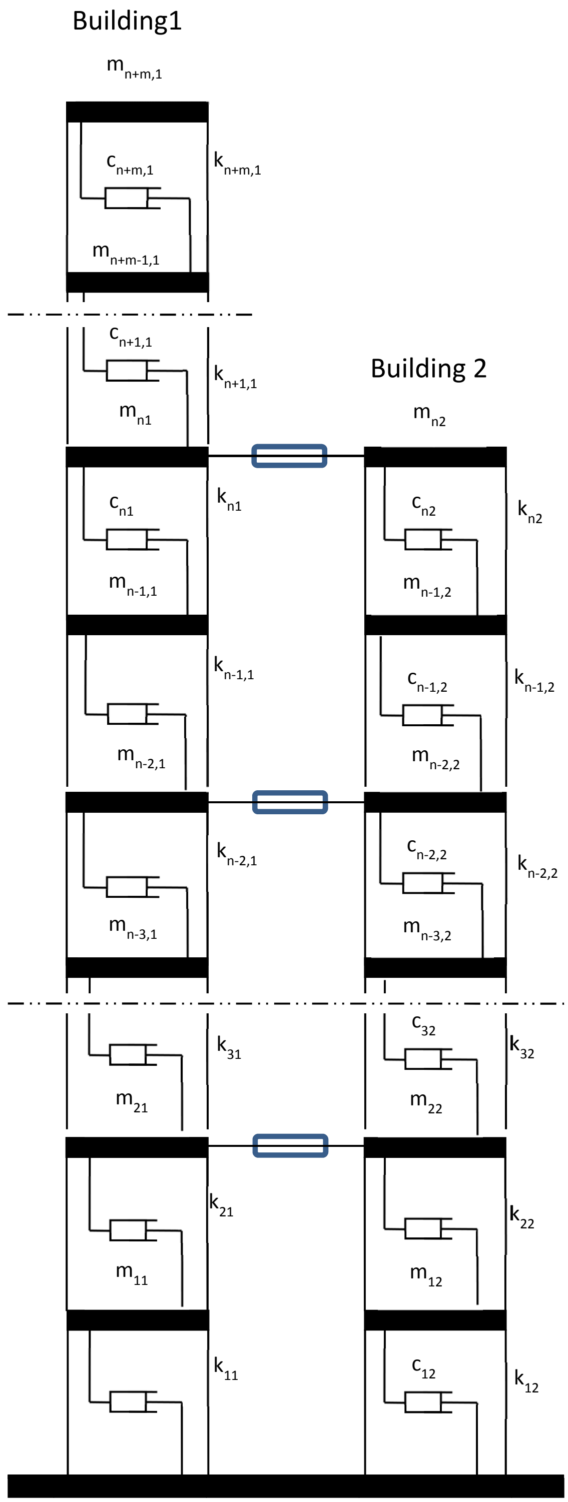

Figure 1-2: Multi-degree-of-freedom model of coupled structures

Ok et al. [24] presented an optimization procedure for optimal design of adjacent buildings connected by MR dampers. They use MDOF models and analyzed the building in the frequency domain under a stationary filtered white noise load.

In [9], MDOF structures are used in order to investigate the seismic response of two adjacent buildings connected by friction dampers. They used four different earthquake time history records to evaluate the control strategies. They showed that it is not necessary to connect all adjacent floors by dampers.

Cimellaro and Lopez-Garvia [25] presented a design approach for optimal passive vibration control of coupled MDOF structures. They first design an active controller; then by minimizing the difference between passive and active controller, they obtained optimal parameters for the passive controller.

In order to investigate the seismic response of coupled buildings with base isolators, Matsagar et al. [26] used MDOF structures and analyzed the system in the time domain under three different earthquake records.

In general, MDOF structures are the most common type of modeling used for seismic analysis of coupled buildings. Besides the aforementioned studies, many other studies use MDOF structures since MDOF models are more accurate than SDOF models, and at the same time are relatively simple.

1.3.1.3 Experimental models

Experimental models are designed to confirm the validity of mathematical models including SDOF and MDOF models. Although experimental models are usually much smaller than actual buildings (i.e. they are scaled down), they are still able to represent the dynamic behaviour of buildings under seismic loads. In experimental studies, a building model is designed and built. Then in order to simulate the base excitation due to seismic activities, the model is placed on a platform which is connected to an electrical motor. This setup is called a shake table model. Various shake tables are available commercially with different sizes, output load limits, and degrees of freedom including lateral, vertical, and rotational loads.

In a very early experimental study by Xu et al [27], two three-story buildings were designed and constructed. A series of experiments were conducted and the response of the system was recorded each time in order to find the optimal locations and the damping coefficients.

Christenson et al. [28] presented experimental results for two 2-story buildings which are connected by an active controller. The actuator is placed on the second floor and it consists of a servo motor (on the right) that is attached to a threaded rod and a bolt (on the left). More than 50% reduction is observed in peak values transfer function of rigidly connected structures.

In order to investigate the effects of the number and locations of the dampers, Yang et al. [29] conducted a series of experiments with different numbers and configurations of the dampers. They used a five-story steel frame building and a six-story steel frame building. They found that the use of more dampers does not necessarily result in a better vibration reduction.

Among other experimental studies on coupled buildings, one can refer to [4], [30–35].

1.3.2 Devices

In order to connect two adjacent buildings, we need to use some mechanical devices that can absorb energy. For this purpose, three main classes of connectors are available: passive, active, and semi-active. As mentioned earlier, passive devices are generally the cheapest. They usually require very little maintenance; and are reliable. Well-known passive devices are viscous dampers, metallic yield bracings, and hysteric dampers. On the other hand, active devices are expensive but they are the most effective ones. For example, an active mass damper is an active control device containing a huge mass which is connected to an actuator which can move the mass. Finally semi-active devices are passive devices that can be controlled. Among the most popular semi-active devices, one can mention magneto-rheological (MR) dampers and variable friction dampers.

1.3.2.1 Passive control

A very popular damping device is the viscous damper. Numerous studies have been done on seismic response analysis of adjacent buildings connected by viscous dampers.

In [36], a feasibility study for vibration control of coupled buildings is presented. Viscous dampers are used to connect two adjacent buildings. It is shown that inter-story drift and floor acceleration can be reduced by the use of viscous dampers.

An actual application of viscous damper connectors is discussed in [33]. A tall building is connected to its surrounding large podium structure. Forty viscous dampers are used in order to dissipate kinetic energy and reduce the vibration level of the building.

In [37] and [7], the seismic response of two adjacent buildings connected via a viscous damper along with a spring is investigated. They both used frequency domain solution in order to solve the problem. They showed that the floor shear force can be reduced by more than 50% if viscous dampers are placed between two adjacent buildings.

Another type of fluid dampers is the Maxwell model-defined fluid damper. Seismic responses of adjacent structures connected by Maxwell model-defined fluid dampers are investigated in [21], [38], [39].

As mentioned earlier, many experimental studies are done on the efficiency of passive fluid damper connectors, e.g. see [27], [29], [30], [35].

Non-linear hysteric dampers are effective devices that can be used to connect adjacent structures. For examples, seismic responses and optimal design of passive MR dampers are investigated in [24]. In their study, a stochastic linearization is used in order to estimate the dynamic response of the buildings.

Besides fluid dampers, friction dampers can also be used as passive connectors to dissipate kinetic energy of adjacent structures. Among them, one can refer to theoretical studies such as [9], [16] and experimental studies such as [4].

1.3.2.2 Semi-active control

Another class of control methods is called semi-active control. Of the different semi-active devices that can provide resisting force, MR dampers and variable friction dampers are the most popular ones for coupled building vibration control.

For example, in a recent study [23], the seismic response of adjacent buildings connected by MR dampers is investigated. Different control algorithms, including passive-off, passive-on, and semi-active algorithms, are implemented and compared.

Other studies on semi-active control of adjacent buildings are [8], [20], [23], [40–45].

1.3.2.3 Active control

The last class of control strategies is called active control. In this control method, an external power generates the required restoration force.

A comparison study between passive control and active control for adjacent buildings is done in [46], in which each building is modeled as a continuous beam. Using the reduction method, the continuous system is converted into an MDOF system. Under a filtered white noise load, the efficiency of each method is examined. It is worth mentioning that results from their study confirm that the optimal configuration of the dampers is not a convex function; therefore, a heuristic optimization method for dampers' configuration must be able to avoid getting trapped into local minima.

The efficiency of active control methods is also confirmed via experimental studies. For example, in [28], an active control approach based on H2/LQG is designed and tested. The experimental test is done on two 2-story buildings which are connected by an actuator at the top story.

For more information on the active control of coupled buildings, interested readers are referred to [20], [25], [40], [41], [45], [47], [48].

1.3.3 Optimization

Optimization is a critical part of every design approach. Engineers want to make the best use of their retrofitting techniques. Although several theoretical and experimental studies have investigated the mechanical behaviour of coupled buildings, only few papers have examined applications of modern optimization tools in designing coupled buildings. Most optimization studies done on coupling of buildings simplify the problem to an SDOF model (e.g. [11], [21], [38]). This simple model, in most cases, results in a closed-form solution. Some others assume that all dampers have similar mechanical properties (e.g. [7], [37], [39]); therefore they reduce the size of the optimization problem to a 1-dimensional problem. As an important outcome of this thesis, we will see that this is not a good assumption as it can prevent us from finding the global optimal design. In what follows, a general literature review on structural control optimization is presented. Then we focus on papers which considers the optimization problem of coupled buildings.

1.3.3.1 Structural control optimization

In this section, few sample works on structural control optimization are discussed and reviewed. Several studies have considered the optimization problem of placement of dampers.

For example, Wongprasert et al. in [49] used a genetic algorithm to find the optimal arrangement for a limited number of dampers. Damping coefficients for all dampers are assumed to be the same and fixed.

In a more comprehensive study [50], both the location and size are considered as design variables. A genetic algorithm is implemented in order to find the optimal size and configuration of dampers to provide a desired performance under seismic loads.

In [51], a couple of combinatorial optimization techniques were presented and examined. Various objective functions were introduced and used. It was shown that a uniform distribution of dampers is a sub-optimal arrangement for the dampers and combinatorial optimization methods must be used to find the optimal arrangement of dampers.

In [52], the goal is to minimize the summation of damping coefficients of supplemental damping elements added to the structure. The maximum drift is a constraint on the optimization problem. In other words, a solution is valid if the maximum drift is less than the allowable drift limit.

Lavan et al. [53] presented a methodology for a multi-objective evolutionary optimal seismic design for structures with supplemental energy dissipation devices.

A systematic procedure is proposed by Takewaki [54] in order to minimize the transfer function. A closed-form analytical formulation is derived, and the efficiency of the proposed method is discussed.

The problem of optimizing the placement of dampers is also investigated in [55], [56].

Unlike the optimal placement of dampers, the problem of determining optimal mechanical properties of dampers is a continuous optimization problem for which a closed-form solution might be achieved.

For example, Yamada [57] presented a closed-form analytic solution for the optimal design of a Maxwell fluid damper to connect two SDOF structures.

In [58], a minimax optimization is performed to optimize the design parameters of multi-degree-of freedom tuned-mass-dampers.

1.3.3.2 Coupled building control optimization

As mentioned earlier, many modern applications of structural control have been enhanced by the use of optimization tools. However, to the best of author's knowledge, before this thesis, no special optimization tool, except genetic algorithms, had been used to find the optimal design of coupled adjacent buildings using MDOF models.

Optimization studies on SDOF structures are rather simple and usually result in closed-form analytical solutions, e.g. see [11], [16], [19], [38].

On the other hand, optimization studies on MDOF structures are more complicated. For example, in order to determine the optimal design of adjacent structures connected by MR dampers, Ok et al. [24] used a genetic algorithm to find the optimal size and location of the dampers.

In order to reduce the size of the optimization problem, some papers assume that all dampers have the same mechanical properties [7], [37], [39]. Then by plotting the objective function versus the design parameters, optimal values are determined.

In [59], without presenting an optimization procedure, it is assumed that floors with higher relative velocity need a higher damping coefficient. It is also assumed that the optimal damping coefficients were functions of the relative velocity between the structures; the damping coefficients increased from a small value for the base floor to a large value for the top floor. In a similar study by Patel and Jangid [60], they proposed that floors with higher relative velocity are the best places to put dampers. However, results from this thesis invalidate this assumption.

Optimization of coupled structures has also been investigated through experimental studies such as [27], [29].

Among other studies on optimal design of coupled structures, one can refer to [9], [22], [25].

1.4 Motivation

As mentioned earlier, there is a need for a comprehensive optimization study of coupled buildings control. Optimization is a very crucial step in every engineering design as engineers always want to make the best use of their techniques. Optimization can also reduce the cost of implementation of retrofitting methods which attracts constructors' interest. For example, reducing the number of dampers without significant reduction of efficiency is a clear example how optimization can reduce the cost of implementation. It has been shown that we can reduce the number of dampers without any significant efficiency loss (e.g. see [29]). Furthermore, some studies showed that removing the assumption of equal mechanical properties of dampers can be effective (e.g. [59], [60]). The most comprehensive study made on this problem is the genetic algorithm optimization for MR dampers done by Ok et al. [24]. They used a genetic algorithm to solve the multi objective optimization problem of MR damper connectors. Still no deterministic procedure for finding the optimal arrangement is presented for a limited number of dampers.

All in all, to the best of the author's knowledge, no deterministic optimization technique is available for the design of damper connected buildings. Furthermore, in this thesis, different optimization methods, including discrete and continuous methods, are examined. Based on a bunch of numberical tests, the proposed methods are compared and the efficiency of each is investigated. The lack of optimization studies on coupled buildings control motivated us to investigate applications of modern optimization techniques on the control problem of coupled buildings.

1.5 Overview of this thesis

In this thesis, MDOF models are considered rather than simple SDOF models to observe the effects of non-uniform distribution of dampers. Buildings are modeled as lumped mass MDOF structures with linear springs and dampers. We use viscous dampers between two buildings as they are known to be effective and economical devices that can be used to connect adjacent structures. The dynamic analysis is done under a one dimensional ground excitation which is generated by the pseudo-excitation function of Kanai-Tajimi. Five discrete optimization algorithms, namely inserting dampers, inserting floors, genetic algorithms, maximum velocity, and exhaustive search, are introduced to find the optimal distribution for a limited number of dampers. In order to find the optimal design, we also need to find the optimal damping coefficients. For this purpose, we first show that the conventional assumption of equal mechanical properties for all dampers may result in a sub-optimal solution. Then three modern optimization techniques, namely mesh adaptive direct search method, robust approximate gradient sampling algorithm, and genetic algorithm, are compared to find the optimal damping coefficient of dampers. Finally, these two optimization problems are combined and solved as a comprehensive problem where configuration and damping coefficients are to be optimized.

The remainder of this thesis is organized as follows. In Chapter 2, the modeling of the system of coupled buildings is discussed. Assumptions and limitations of the current model is discussed and formulations are derived. Chapter 2 also discusses the methodology that we use for the simulation including the pseudo-excitation Kanai-Tajimi function, the frequency domain solution, the spectral analysis and the objective functions. In Chapter 3, the discrete optimization problem of optimal configuration for a limited number of dampers is studied. Chapter 4 expands Chapter 3 by adding a new set of design variables which are non-uniform damping coefficients. Finally, in Chapter 5, conclusions are made and trends for future work are presented.

2 Modeling

In order to investigate dynamic behaviour of coupled structures, the first step is to model coupled adjacent buildings. Modeling is to formulate mechanical behaviour of structures in terms of mathematical equations. Three major types of modeling are available: single-degree-of-freedom (SDOF), multi-degree-of-freedom (MDOF), and finite element (FE). Each of them has its own advantages and disadvantages. In general, the more accurate the model is, the more complexity will be involved in the model and its solution. Except for simple SDOF systems, closed-form solutions are almost impossible to achieve. For most cases, MDOF models provide enough information for researchers and engineers to predict the dynamic behaviour of buildings.

Beside the number of degrees of freedom, linearity and non-linearity are other factors that must be defined in the model. Linear models are simpler than non-linear models. However, only non-linear models can be used to examine plastic deformations of structures. In linear models, the structure is assumed to remain within its linear elastic limit. The basic assumption in linear models is that the relationship between force and displacement of each floor is a linear relationship. For example, if you double the excitation load and all other parameters remain unchanged, the response of the system, including displacement, velocity and acceleration, is also doubled.

Another factor that needs to be taken into consideration is the number of dimensions of the model. For example, in order to obtain an accurate response of a real structure, one may use 6-dimensional FE model which includes three dimensions for displacement and three dimensions for rotation. However, such a high level of accuracy is not always required. In order to investigate the effect of seismic retrofitting systems, structures usually are modelled via a 1D model which only includes the weaker side of the building. Since we know the weaker side of the building, we focus on that side.

2.1 Assumption and limitation

In order to introduce a model which accurately predicts the dynamic behaviour of coupled structures, we must make some necessary assumptions. First off, to ignore torsional effects, the buildings are assumed to be symmetric with their center of mass aligned in mid plane. The ground motion and dynamic response of the buildings are assumed to be unidirectional. Structural components are assumed to remain in linear elastic region during the seismic event. Therefore, no damage (including plastic joints) is assumed to occur. Ground excitation load is assumed to be exactly the same for two adjacent buildings. Any slight change in the mechanical behaviour of the soil is neglected. Buildings are connected via linear dampers in their weaker direction. Dampers are assumed to be perfectly linear. This means the force generated in the dampers is linearly dependent to the relative velocity of the two ends of the damper. Also, dampers are assumed to remain functional and undamaged throughout the seismic event.

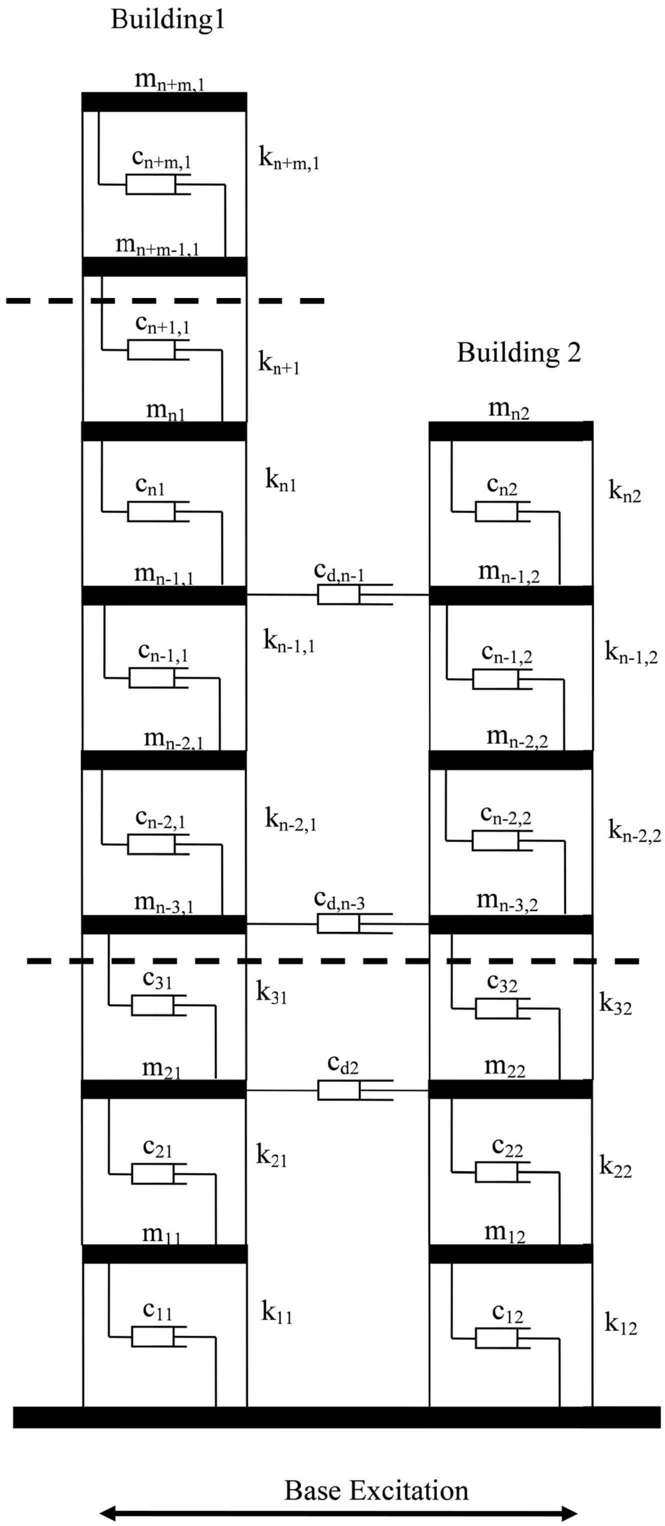

Figure 2-1: Schematic dynamic model of two adjacent buildings

2.2 Coupled structures model

This thesis uses a simple MDOF model in order to model the dynamic behaviour of the buildings. Each floor is modelled via a lumped mass which includes the mass of each floor as well as the mass of the walls connecting that floor to the upper floor. Then each mass is connected to the upper and lower mass with a set of linear springs and linear dampers. A linear spring is a mechanical component that generates a force which linearly depends on the relative displacement at the ends of the spring. And a linear damper is a mechanical component that generates a force which linearly depends on the relative velocity at the ends of the damper. After modeling each structure individually, we may now connect these two structures using linear dampers. In this thesis, we use a passive viscous damper as damper connectors.

2.3 Formulation

As shown in Figure 2-1, buildings 1 and 2 have $n + m$ and n stories, respectively. The mass, shear stiffness, and damping coefficients for the i^{th} story are m_{i1}, k_{i1}, and c_{i1} for building 1 and m_{i2}, k_{i2}, and c_{i2} for building 2. The damping coefficient and stiffness coefficient of the damper at the i^{th} floor are c_{di} and k_{di}, respectively. The dynamic model for both structures is taken to be a $2n + m$ degree of freedom system.

Let $x_{i1}(t)$ and $x_{i2}(t)$ be the displacement of the i^{th} floor of buildings 1 and 2 in the time domain, respectively. Consequently, $\dot{x}_{i1}(t)$ and $\dot{x}_{i2}(t)$ represent the velocity, and $\ddot{x}_{i1}(t)$ and $\ddot{x}_{i2}(t)$ represent the acceleration of the i^{th} floor of buildings 1 and 2, respectively. Now, we may write the governing equation of the system:

$$M\ddot{X}(t) + (C + C_d)\dot{X}(t) + (K + K_d)X(t) = MEg(t) \qquad \text{Eq 2-1}$$

where M, C and K are the given mass, damping and stiffness matrices of the buildings, C_d and K_d represent damping and stiffness matrices of the connectors, and X is the displacement vector of the system. Also, E is a vector with all elements equal to one, and $g(t)$ is the ground acceleration during the earthquake. The details are given as follows:

$$M = diag[m_{11},...,m_{(n+m)1}, m_{12},...,m_{n2}], \qquad \text{Eq 2-2}$$

$$K = \begin{bmatrix} \overline{K}_1 & 0 \\ 0 & \overline{K}_2 \end{bmatrix}, \qquad \text{Eq 2-3}$$

$$C = \begin{bmatrix} \overline{C}_1 & 0 \\ 0 & \overline{C}_2 \end{bmatrix}, \qquad \text{Eq 2-4}$$

$$\overline{K}_1 = \begin{bmatrix} k_{11}+k_{21} & -k_{21} & & & & \\ -k_{21} & k_{21}+k_{31} & -k_{31} & & 0 & \\ & & & \ddots & & \\ & 0 & & -k_{(n+m-1)1} & k_{(n+m-1)1}+k_{(n+m)1} & -k_{(n+m)1} \\ & & & & -k_{(n+m)1} & k_{(n+m)1} \end{bmatrix}, \qquad \text{Eq 2-5}$$

$$\overline{K}_2 = \begin{bmatrix} k_{12}+k_{22} & -k_{22} & & & & \\ -k_{22} & k_{22}+k_{32} & -k_{32} & & 0 & \\ & & & \ddots & & \\ & & 0 & -k_{(n-1)2} & k_{(n-1)2}+k_{(n)2} & -k_{(n)2} \\ & & & & -k_{(n)2} & k_{(n)2} \end{bmatrix},$$

Eq 2-6

$$\overline{C}_1 = \begin{bmatrix} c_{11}+c_{21} & -c_{21} & & & & \\ -c_{21} & c_{21}+c_{31} & -c_{31} & & 0 & \\ & & & \ddots & & \\ & & 0 & -c_{(n+m-1)1} & c_{(n+m-1)1}+c_{n+m)1} & -c_{(n+m)1} \\ & & & & -c_{(n+m)1} & c_{(n+m)1} \end{bmatrix}, \qquad \text{Eq 2-7}$$

$$\overline{C}_2 = \begin{bmatrix} c_{12}+c_{22} & -c_{22} & & & & \\ -c_{22} & c_{22}+c_{32} & -c_{32} & & 0 & \\ & & & \ddots & & \\ & & 0 & -c_{(n-1)2} & c_{(n-1)2}+c_{n)2} & -c_{(n)2} \\ & & & & -c_{(n)2} & c_{(n)2} \end{bmatrix}, \qquad \text{Eq 2-8}$$

$$K_d = \begin{bmatrix} \overline{K}_d & 0 & -\overline{K}_d \\ 0 & 0 & 0 \\ -\overline{K}_d & 0 & \overline{K}_d \end{bmatrix}, \qquad \text{Eq 2-9}$$

$$C_d = \begin{bmatrix} \overline{C}_d & 0 & -\overline{C}_d \\ 0 & 0 & 0 \\ -\overline{C}_d & 0 & \overline{C}_d \end{bmatrix}, \qquad \text{Eq 2-10}$$

$$\overline{C}_d = diag[c_{d1},...,c_{dn}], \qquad \text{Eq 2-11}$$

$$\overline{K}_d = diag[k_{d1},...,k_{dn}], \qquad \text{Eq 2-12}$$

$$X = [x_{11}, x_{21},..., x_{(n+m)1}, x_{12},..., x_{n2}]^T \qquad \text{Eq 2-13}$$

Using different numerical techniques, one may obtain the solution for the dynamic behaviour of the buildings for any given earthquake record, i.e. $g(t)$. The next step is to define a useful and reliable earthquake record that is to be used for optimization simulations.

2.4 Ground Motion

As seen in Eq 2-1, the dynamic response of the system can be obtained for a given earthquake record using the equation of the motion in the time domain. However, to find a general and reliable prediction for the dynamic behaviour of the buildings, we analyze the model in the pseudo-excitation frequency domain rather than the time domain, which is usually associated with one specific real earthquake record. Therefore, the seismic load must be defined in the frequency domain. In [7], it is shown that the model of coupled buildings can be analyzed by calculating the dynamic response of the buildings under a series of harmonic loads. In what follows, we explain the simulation procedure.

Assuming that the ground excitation is a stationary random process, the ground acceleration, $g(t)$, can be written as:

$$g(t) = \sqrt{S_g(\omega)}e^{i\omega t} \qquad \text{Eq 2-14}$$

where $S_g(\omega)$ is the spectral density function of the ground acceleration for a defined frequency of ω. In this study, a Kanai-Tajimi filtered white noise function (e.g. see [7], [21], [24]) is used for the spectral density function of the ground acceleration:

$$S_g(\omega) = \frac{1 + 4\zeta_g^2 \left(\frac{\omega}{\omega_g} \right)^2}{\left(1 - \left(\frac{\omega}{\omega_g} \right)^2 \right)^2 + 4\zeta_g^2 \left(\frac{\omega}{\omega_g} \right)^2} S_0 \qquad \text{Eq 2-15}$$

where ω_g, ζ_g and S_0 represent the dynamics characteristics and the intensity of earthquake and are chosen based on the geological characteristics of a specific zone. Eq 2-15 represents the acceleration transfer function of a mass which is connected to the ground with a set of linear springs and dampers. The typical shape of a Kanai-Tajimi spectrum is shown in Figure 2-2.

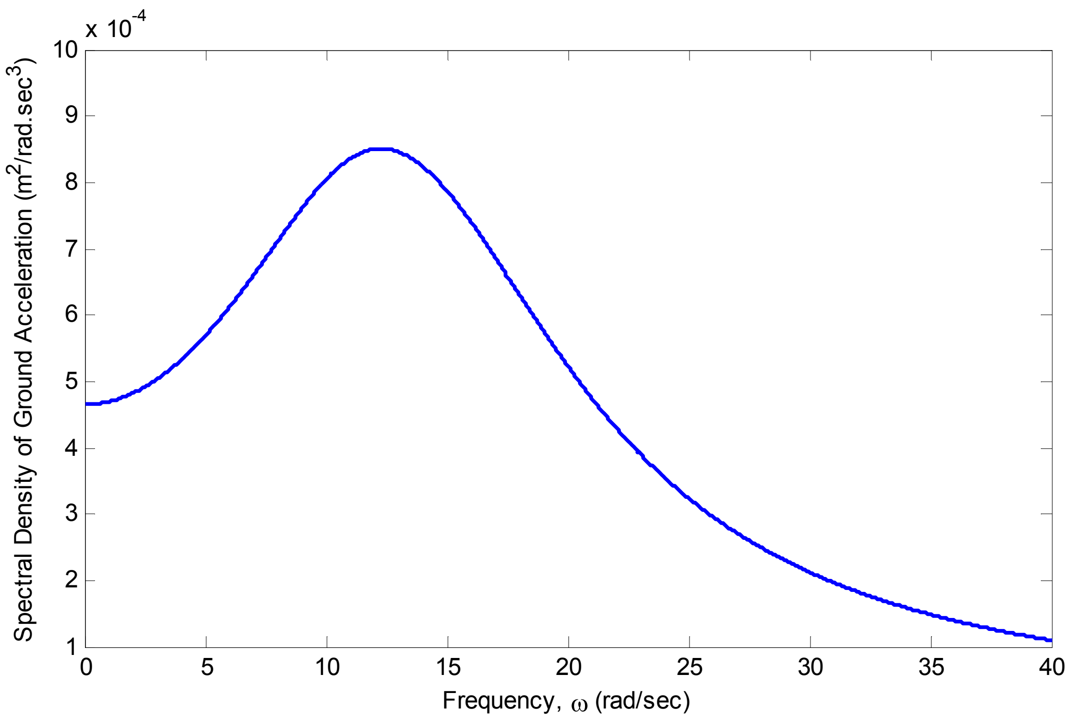

Figure 2-2: Kanai-Tajimi spectral density of ground acceleration

2.5 Solution Procedure

Considering ground excitation as a series of harmonic loads, one can rewrite the displacement vector in the frequency domain as

$$X(t) = X(\omega)e^{i\omega t}. \qquad \text{Eq 2-16}$$

Consequently, the velocity and acceleration vectors in the frequency domain are:

$$\dot{X}(t) = \frac{dX(t)}{dt} = i\omega X(\omega)e^{i\omega t}, \qquad \text{Eq 2-17}$$

$$\ddot{X}(t) = \frac{d^2X(t)}{dt^2} = -\omega^2 X(\omega)e^{i\omega t}. \qquad \text{Eq 2-18}$$

Substituting Eqs 2-16, 17, 18 and 2-14 into Eq 2-1, the governing equation in frequency domain is obtained,

$$-M\omega^2 X(\omega)e^{i\omega t} + (C + C_d)i\omega X(\omega)e^{i\omega t} + (K + K_d)X(\omega)e^{i\omega t} = -ME\sqrt{S_g(\omega)}e^{i\omega t}. \qquad \text{Eq 2-19}$$

Solving for the displacement vector, we find:

$$X(\omega) = \left[-M\omega^2 + (C + C_d)i\omega + (K + K_d)\right]^{-1} \times \left[-ME\sqrt{S_g(\omega)}\right]. \qquad \text{Eq 2-20}$$

Having developed a formula for S_g, Eq 2-15, it is now possible to numerically approximate the displacement vector in the frequency domain by numerically solving Eq 2-20. Therefore, the displacement, velocity and acceleration values are transformed from functions of the time into functions of the frequency ω, e.g. $x_{i1}(\omega)$, $x_{i2}(\omega), \dot{x}_{i1}(\omega)$, $\dot{x}_{i2}(\omega)$, $\ddot{x}_{i1}(\omega)$, and $\ddot{x}_{i2}(\omega)$. As before, the first index corresponds to the floor number and the second index corresponds to the building number. Since calculated parameters in the frequency domain take complex values, the squared magnitude, also known as the auto-spectral density, is used for optimization. For example, the auto-spectral density of displacement for the i^{th} floor of the building *b* is written as:

$$S_{xib}(\omega) = x_{ib}(\omega) \cdot conj\left(x_{ib}(\omega)\right). \qquad \text{Eq 2-21}$$

Subsequently, applying integration over the frequency domain will result in statistical parameters, which represent the standard deviations of displacement, velocity and acceleration of each floor. For instance, for the i^{th} floor of building *b*, its standard deviation of displacement response is

$$\sigma_{xib} = \left[\int_{-\infty}^{+\infty} S_{xib}(\omega)d\omega\right]^{1/2}. \qquad \text{Eq 2-22}$$

To do this, upper and lower limits of $\pm 20\ (rad/s)$ are imposed. Using a trapezoid rule approximation with a step size of 0.02 to the integral in Eq 2-22, one can easily calculate a good approximation of standard deviation of the response. It is worth noting that previous studies (e.g. [7], [47]) show that the effect of the frequencies greater than $20\ (rad/s)$ on the response of the structure is negligible. In fact, as we change the frequency from 0 rad/s to 20 rad/s, the magnitude of the transfer function of the system becomes 100 times smaller.

2.6 Objective function

The most important factor in optimization is the objective function that we would like to be minimized. In other words, one may want to minimize the cost; while another one wants to minimize the acceleration, or displacement. Particularly, for structural control optimization, many objective functions can be considered, such as maximum drift, maximum acceleration, cumulative drift, energy absorbed by the structure, damping ratio, cost of the retrofitting system, risk of damage and so on. Since throughout this thesis, we do not use the gradient, almost any objective function can be used. Note that, the derivative of the response with respect to design parameters, i.e. dampers' location and size, is not available. Therefore, we are not even able to use derivatives. In this thesis, two different objective functions are used:

- F_1) the summation of squared inter-story drift, and
- F_2) the maximum value of squared inter-story drift.

The mathematical formulation of each objective function is:

$$F_1 = \sum_{i=1..n+m} {\sigma_{di1}}^2 + \sum_{j=1..n} {\sigma_{dj2}}^2, \qquad \text{Eq 2-23}$$

$$F_2 = \max\left\{\max\{{\sigma_{di1}}^2 : i = 1..n+m\}, \max\{{\sigma_{di2}}^2 : i = 1..n\}\right\}, \qquad \text{Eq 2-24}$$

where σ_{dib} is the standard deviation of inter-story drifts, for i^{th} floor of building b, as:

$$\sigma_{dib} = \left[\int_{-\infty}^{+\infty} S_{dib}(\omega)d\omega\right]^{1/2} \qquad \text{Eq 2-25}$$

where S_{dib} is the auto-spectral density of the drift, and it can be calculated using the frequency domain response, as:

$$S_{dib}(\omega) = [x_{ib}(\omega) - x_{i-1,b}(\omega)] \bullet conj\left(x_{ib}(\omega) - x_{i-1,b}(\omega)\right) \qquad \text{Eq 2-26}$$

Note that in Eq 2-26, $x_{ib}(\omega)$ for i=0, representing relative displacement of the ground, is equal to zero. In Chapter 3, both objective functions are used and it is shown that the optimal design of the damper connectors depends on the objective function we choose. In Chapter 4, for the sake of brevity, we only use the maximum drift as the objective function.

In the next two chapters, different optimization techniques are considered to find the best configurations and damping coefficients of a limited number of dampers that minimize the desired objective function.

3 Configuration optimization

Past results (e.g. see [16], [29], [60]) have shown that reducing the number of dampers can considerably decrease the cost of implementation and does not significantly decrease the efficiency of the system. In this chapter, a couple of deterministic combinatorial optimization techniques are presented to determine the optimal location for a limited number of viscous dampers.

Heuristic combinatorial optimization methods have been shown to be effective to find optimal arrangement of viscous dampers for a single building [51]. Unlike random/evolutionary search methods, deterministic methods presented in this thesis always lead to a certain solution for each problem. In this chapter, as in [7], [37], [39], each floor can have one damper at most and all dampers are assumed to be similar. Considering effects of mechanical properties of dampers on the objective function, the final result is a bi-level optimization problem, where the objective function depends on the damping coefficient and damper location. In order to compute the "best" number of dampers to use, one can compute the optimal damping coefficient and damper location for each possible number of dampers, and then select the desired balance between cost and damage mitigation. Also, a comparison study is presented to verify the effectiveness of the presented methods.

This chapter introduces two optimization algorithms to determine the optimal configuration for a given number of dampers: a damper insertion heuristic and a floor insertion heuristic. For the purpose of comparison, this paper includes a non-heuristic approach, exhaustive search, as well as a genetic algorithm, and a fifth method, highest relative velocity heuristic, that is based on suggestions from [59], [60]. The exhaustive search is guaranteed to be globally optimal, but is extremely time consuming. Indeed, in the most difficult cases an exhaustive search is completely intractable. The damper insertion and floor insertion heuristics are considerably faster, but not guaranteed to return the globally optimal solution. However, numerical tests suggest that they are both effective. Results also show that the genetic approach can be considered as a reasonable option if convergence rate is more important than accuracy of the results. While for more accurate results, the damper insertion method is the best method among the presented methods, particularly for tall structures. The final technique (highest relative velocity heuristic) assumes that the best locations for viscous dampers are the floors that have the maximum relative velocity. This makes intuitive sense due to the fact that force generated in viscous dampers is a function of relative velocity. However, results presented in this study show that this arrangement does not generally result in optimal performance. It is also found that increasing the number of dampers does not necessarily increase the efficiency of the system. In fact increasing the number of dampers can exacerbate the dynamic response of the system.

In order to examine the efficiency of these techniques, various numerical examples are presented. For each method, the total computational effort is examined (both through the calculation of the number of simulations required and through CPU-time used for the numerical tests). Furthermore, a comparison between the resulting qualities of the final solution using each technique is presented.

3.1 Damping Coefficients Optimization

It is clear that the standard deviation of displacement (σ_{xib}) for a set of dampers is highly dependent on the mechanical properties of the damper connectors since damping and stiffness matrices of the connectors in Eq 2-20, C_d and K_d, are dependent on c_{di} and k_{di} (Eqs 2-1 and 2-9,10,11,12). Therefore, to determine the optimal effect of a given damper arrangement, it is necessary to determine the optimal damping and stiffness coefficients. Moreover, the optimal damping and stiffness coefficients for a set of dampers are dependent on both the number of dampers used and on the damper placement. Therefore, for each damper arrangement considered, the algorithm must optimize the damping and stiffness coefficients.

Results from [7], [37] show that the stiffness of the connectors does not change the objective function significantly as long as c_{di}'s are optimal and k_{di}'s are small. Moreover, the value of σ_{xib} will increase if k_{di} has a very large value, i.e., rigid connectors. Thus, it is assumed that k_{di}=0 for all d and i. This simplifies the problem while keeping the final result accurate.

Furthermore, results from [7], [10], [29], [37–39] indicate that it is reasonable to assume that all damping coefficients c_{di} are equal. Therefore, it is assumed that

$$c_{d1} = c_{d2} = \ldots = c_{dn}. \qquad \text{Eq 3-1}$$

This reduces the damping coefficient selection problem to a one-dimensional problem of computing c_{d1}. This is accomplished using a standard bisection method. As shown in Figure 3-1, the procedure starts with two ultimate points (practically $Cd_L = 0$ and $Cd_R = 10^{16}$) assuming that the optimal point is located between them.

Then the function is evaluated at a new point, say Cd_C, that lies between Cd_L and Cd_R. The bisection method then deletes one of the outer points (Cd_L or Cd_R) and replaces it with Cd_C. This narrows the search area and is repeated until the desired convergence tolerance (see Figure 3-2). Note that the dashed-line box in Figure 3-2 is the procedure that is presented in Figure 3-1. To increase the convergence rate, a couple of minor modifications are applied. First, results presented in previous studies (e.g. [7]) suggest to use logarithmic values of damping coefficient. Second, a Golden-ratio bisection method (see for example [61]) is recommended to improve the efficiency of the method.

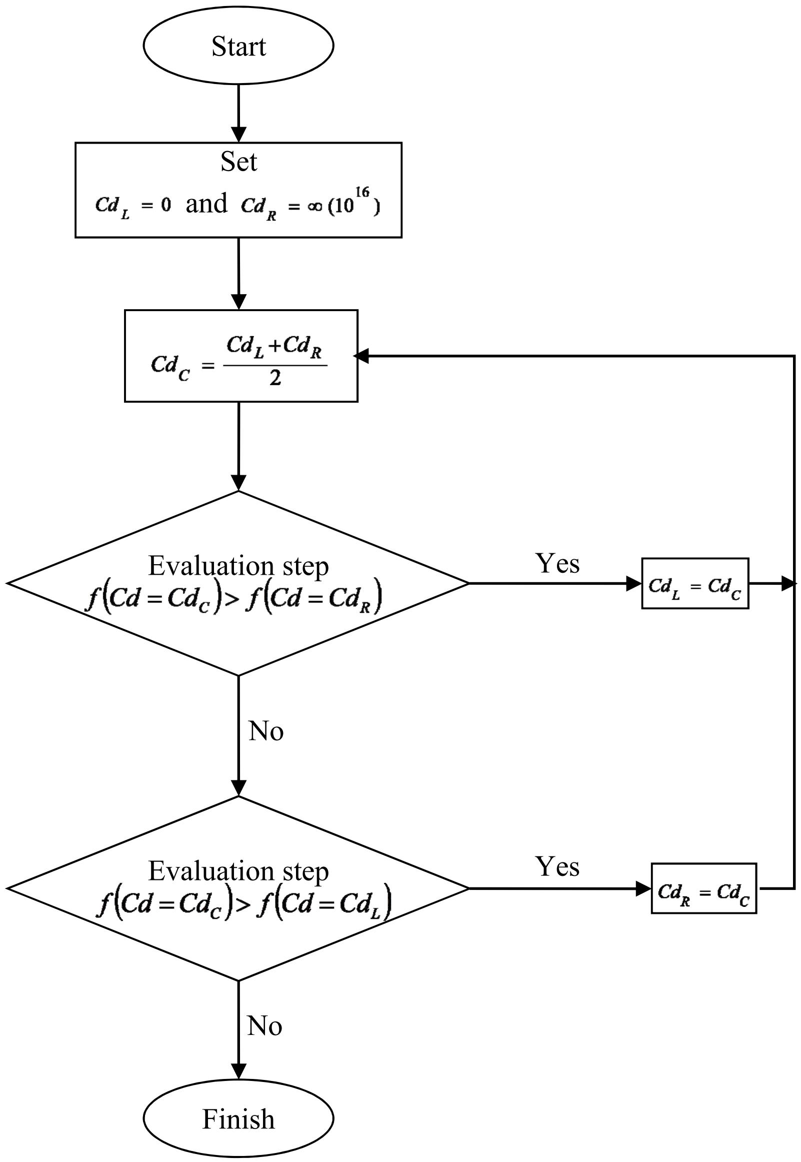

Figure 3-1: Set initial values for the bisection method

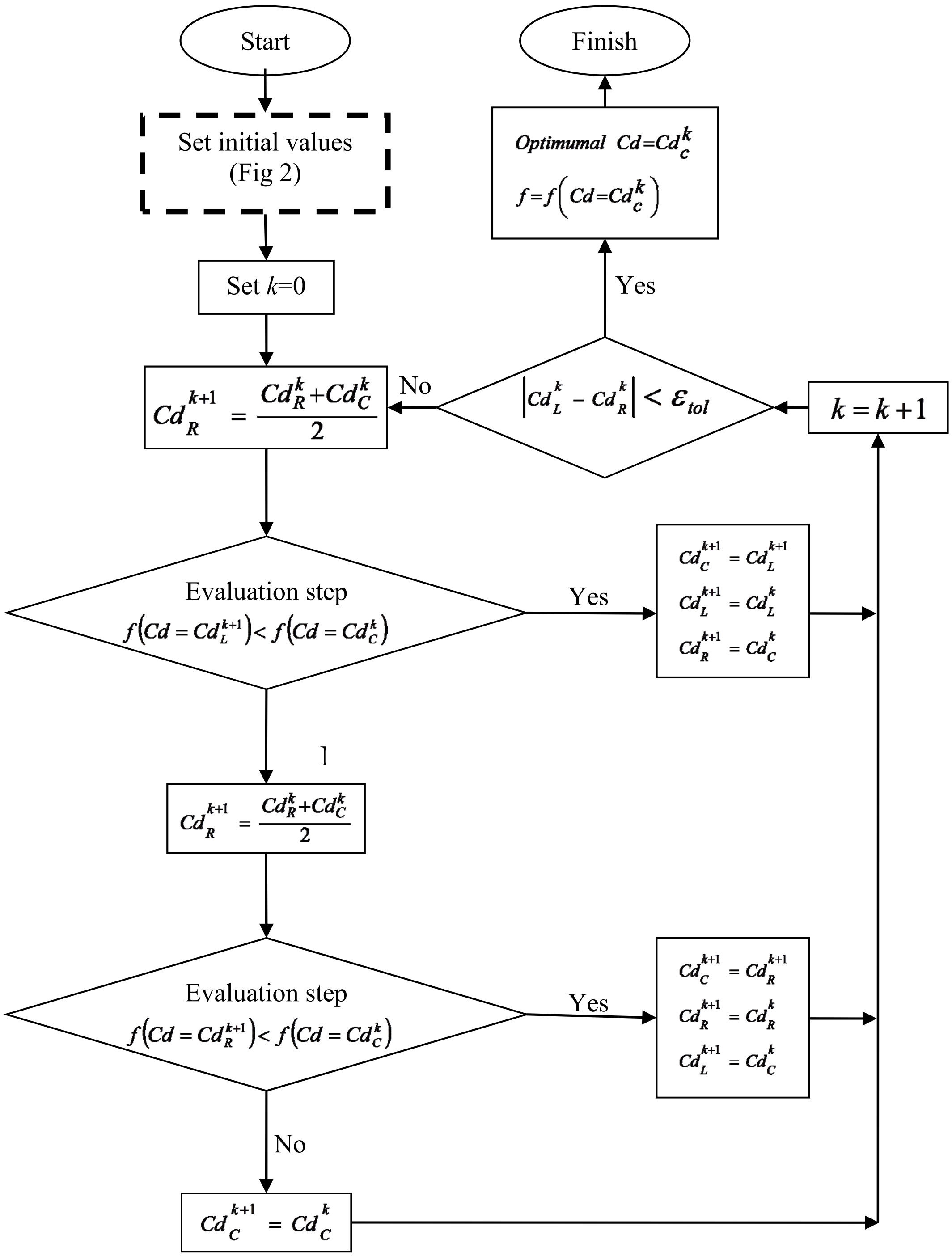

Figure 3-2: Optimizing damping coefficient of dampers using the bisection method

3.2 Damper Location Optimization

Assume two adjacent buildings with n and $n+m$ floors, where $m \geq 0$ and $n > 0$, are coupled using a limited number of dampers $n_d \leq n$. The goal in this section is to explore different methods to locate the optimal placement for these dampers. Five different optimal damper placement techniques are considered: i) exhaustive search approach, ii) inserting dampers, iii) inserting floors, iv) maximum velocity locations, and v) a genetic algorithm. Each damper placement technique is explained below.

3.2.1 Exhaustive search approach

The simplest, but the most time consuming, approach to determine the optimal damper location is to check all possible combinations of a limited number of dampers locations. First, a series of all possible combinations is created. Then all objective function values are compared. Based on the minimum objective function value, the corresponding configuration can easily be found. A schematic diagram of the exhaustive search approach is shown in Figure 3-3.

Clearly such an exhaustive search must return the global optimum. However, in this approach, the number of required simulations N_1 is equal to the number of all combinations:

$$N_1 = \binom{n}{n_d} = \frac{n!}{(n-n_d)!n_d!} \qquad \text{Eq 3-2}$$

where n is number of adjacent floors and n_d is the number of available dampers.

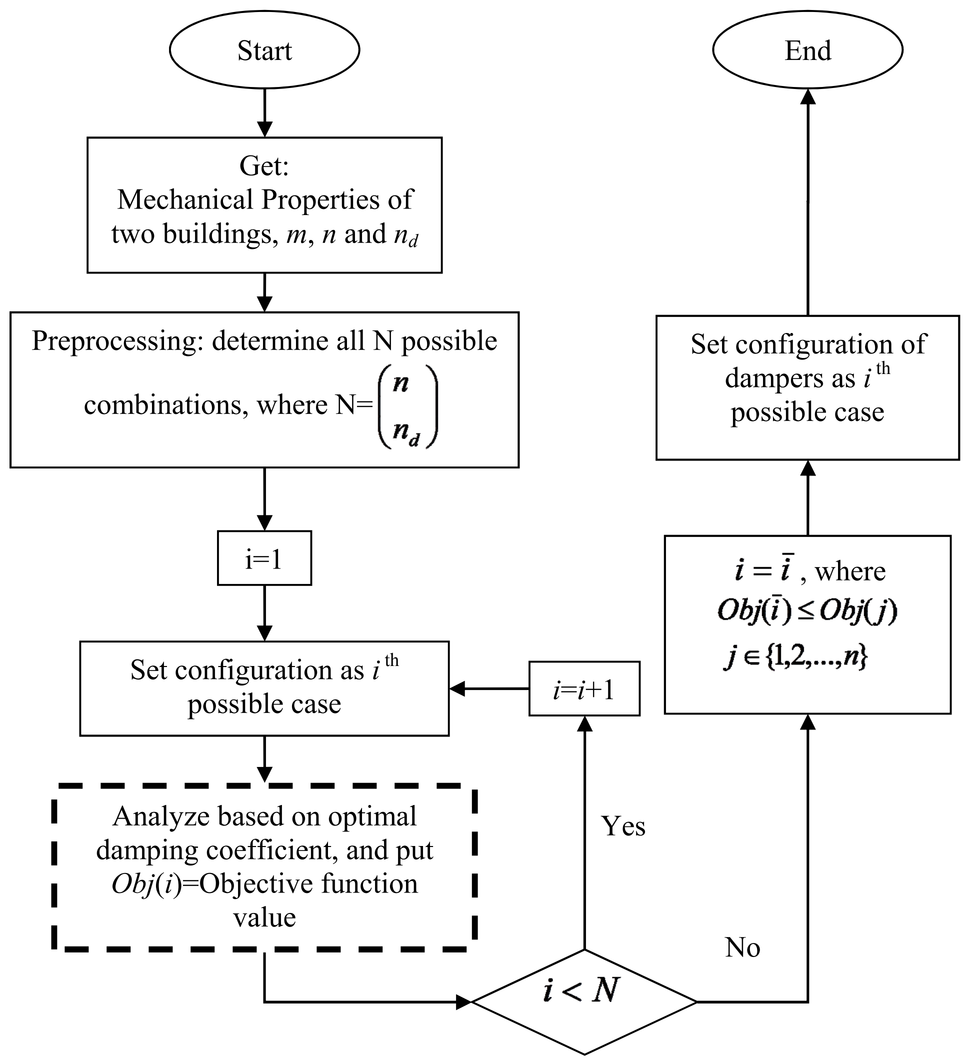

Figure 3-3: Flowchart for exhaustive search optimization approach

3.2.2 Inserting dampers method

Although the first method always results in the best possible solution for the problem, it can be time consuming when the number of possible combinations is large. For example, for the case of two 40-story buildings that are coupled using ten dampers the exhaustive search method will require evaluating the best objective function for 8.5×10^8 different dampers' configuration (approximately 250 years at 10 second per each dampers' configuration).

To reduce the number of required simulations, a simple heuristic method is to find the best location of each damper subsequently. First, two buildings with n and $n+m$ floors are assumed to be completely unconnected. Running n simulations, the best location to insert a single damper can be determined. Fixing a damper at this location, a similar procedure is performed to find the best location for the second damper. This procedure iterates until all available dampers are inserted to the building. A flowchart presenting this approach is depicted in Figure 3-4.

The number of required simulations N_2 can be calculated as:

$$\begin{aligned} N_2 &= n+(n-1)+....+(n-(n_d-1)) \\ &= nn_d + n_d - \left(\frac{n_d^2+n_d}{2}\right) \end{aligned} \qquad \text{Eq 3-3}$$

where, as before, n is number of adjacent floors and n_d is the number of available dampers.

3.2.3 Inserting floors method

This approach begins by reducing the number of adjacent floors (n) to the number of available dampers (n_d). Consequently, two buildings with n_d and n_d+m floors, which are connected on all adjacent floors, are constructed. Next, the algorithm locates the best place to insert a pair of adjacent floors without dampers and keep the objective function as low as possible. After inserting the first pair, two adjacent buildings with n_d+1 and n_d+1+m floors, which are connected in only n_d floors, have been constructed. An analogous procedure is carried out to find the best location for a second pair of unconnected adjacent floors. This iterative procedure continues until number of floors of the shorter building reaches n. A detailed scheme of this method is shown in Figure 3-5.

The number of required simulations can be calculated as:

$$N_3 = (n_d+1).(n-n_d) \qquad \text{Eq 3-4}$$

Note that, unlike the previous methods, in inserting floors the number of required simulations for this approach is not as closely related to the computational effort as pervious methods. In particular, in this approach, earlier simulations work with smaller structures while the size of system gradually grows until, at the last iteration, the system has the full-scale structure.

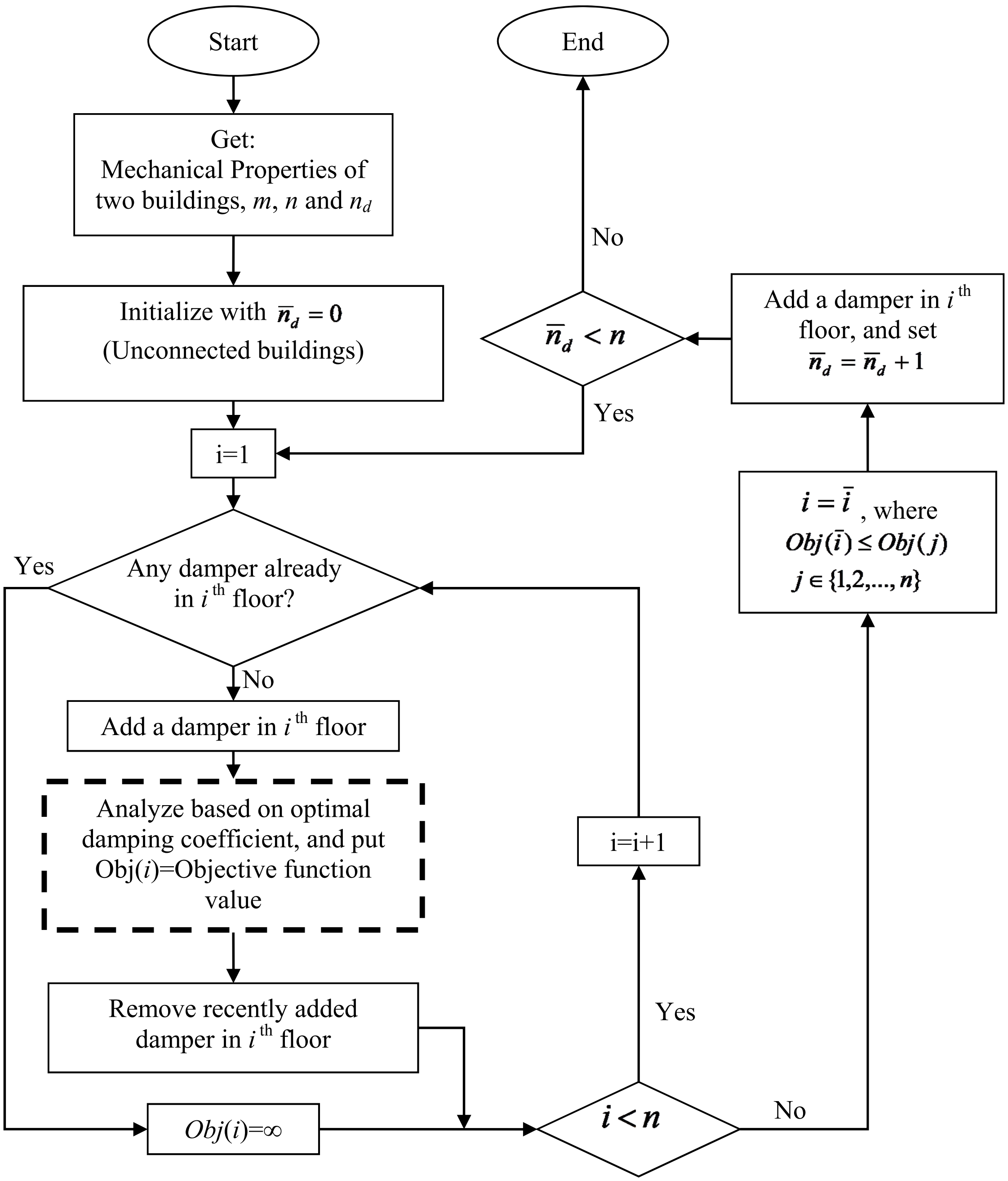

Figure 3-4: Flowchart for inserting dampers optimization approach

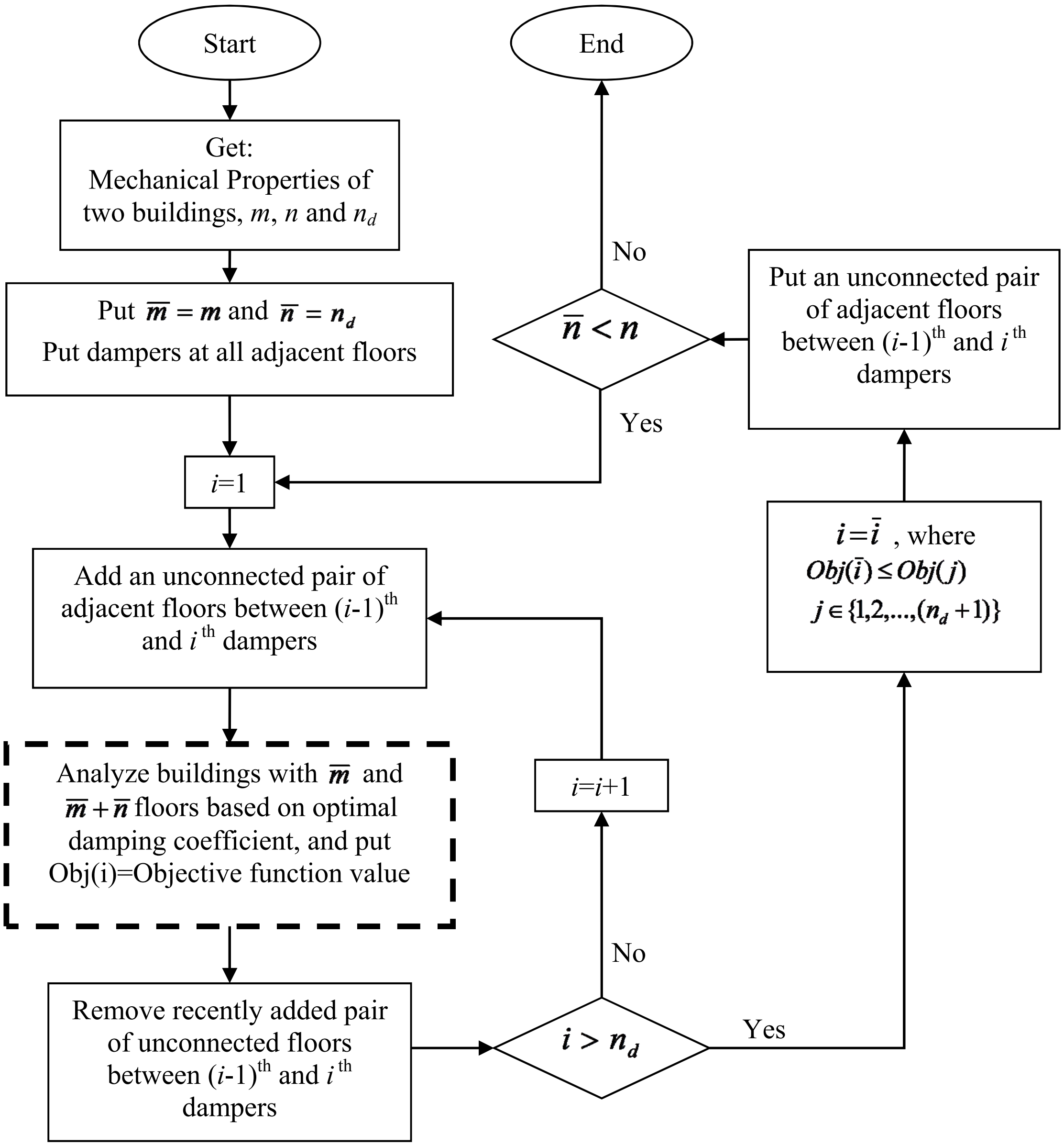

Figure 3-5: Flowchart for inserting floors optimization approach

3.2.4 Maximum velocity method

It is known that the force generated by a linear viscous damper is a linear function of the relative velocity across the damper. Also, the energy dissipated by a viscous damper is a linear function of the relative velocity. Thus adjacent floors with the highest relative velocity might be considered as the best locations to place dampers between two buildings. This method performs a single simulation on unconnected buildings and then places n_d dampers on the n_d floors with the highest relative velocities.

The clear advantage of this approach is that only one simulation is required, so computation time is extremely short. A primary drawback of this method is that the optimal arrangement does not depend on the objective function. Numerical results presented in this chapter show that this method generally does not lead to an optimal arrangement for our objective functions.

3.2.5 Genetic algorithm

Genetic algorithms are known to be an effective approach to solve many engineering optimization problems (see [24], [50], [62], [63] and references therein). Genetic algorithms are considered particularly useful when derivative information is not available. In this study, an optimization approach based on genetic algorithms is considered to find the best configuration of a limited number of dampers.

The implementation follows a standard Selection, Competition, Reproduction technique [64]. The first step is to generate the initial population of points. This is done using a procedure which picks random combinations of dampers and it continues until the desired population, i.e. number of points, is achieved. In accordance with a genetic algorithm, the next step is to evaluate the objective function at all points of the current population. Then, using a competition, some points survive and go to the next generation, while the others die. To make up for the dead points, a reproduction procedure will generate enough new points. The algorithm uses the survivors as the parents to reproduce children. Within the reproduction procedure, a mutation step randomly alters some points from one generation to the next generation. Finally, the objective function is evaluated at all points of the new generation. The procedure repeats until the stopping condition is satisfied.

3.3 Numerical tests

In order to illustrate the efficiency and accuracy of each method, various examples are studied. At the first stage, the time required for each presented method is examined. At the second stage, the quality of the solutions obtained by each method is examined. Before discussing the results, the test sets used in this paper are discussed.

3.3.1 Test problems

The test problems are generated by examining three distinct buildings of heights n=10, n=20, and n=40. Table 3-1 shows three different sets of mechanical properties of buildings used for numerical tests. For all sets, it is assumed that mechanical properties for all floors in each building are the same, i.e.:

$$\begin{aligned} m_{i1} &= m_{j1}, c_{i1} = c_{j1}, k_{i1} = k_{j1} \text{ for all } i,j \\ m_{i2} &= m_{j2}, c_{i2} = c_{j2}, k_{i2} = k_{j2} \text{ for all } i,j \end{aligned} \qquad \text{Eq 3-5}$$

Table 3-1: Mechanical properties of buildings

	Building (a)			Building (b)		
	m_a (kg)	k_a (N/m)	c_a (N.s/m)	m_b (kg)	k_b (N/m)	c_b (N.s/m)
Set I	1.29E+06	4.00E+09	1.00E+05	1.29E+06	2.00E+09	1.00E+05
Set II	2.60E+06	1.20E+10	2.40E+06	1.60E+06	1.20E+10	2.40E+06
Set III	4.80E+06	1.60E+10	1.20E+06	4.00E+06	2.30E+10	1.20E+06

It should also be noted that, for all numerical examples, ground acceleration parameters are considered as $S_0 = 4.65 \times 10^{-4}\, m^2 / rad.s^3$, $\omega_g = 15\, rad/s$, $\zeta_g = 0.6$, $\omega_k = 1.5 rad/s$, and $\zeta_k = 0.6$, i.e. the same as [7].

Table 3-2 tabulates the 8 different building height relations considered in this research. For each case the different building heights are given in f_a and f_b (denoting number of floors for buildings "a" and "b").

Table 3-2: Different sets for numerical tests

f_a	f_b	Case
10	10	Case 1
10	20	Case 2
20	10	Case 3
10	40	Case 4
40	10	Case 5
20	20	Case 6
20	40	Case 7
40	20	Case 8

Considering the three sets of mechanical properties (given in Table 3-1), this creates 24 adjacent building scenarios. For each building scenario, four possible numbers of dampers are considered: 1 damper, 25% of floors having dampers, 50% of floors having dampers and 75% of floors having dampers (rounding up). More precisely,

1. if the shorter building is 10 floors, then n_d=1, 3, 5 and 8,
2. if the shorter building is 20 floors, then n_d=1, 5, 10 and 15.

This yields 96 test problems. Solving each problem for objective function 1 and objective function 2 gives a grand total of 192 numerical tests.

For each problem, an optimal damper placement is determined using different techniques outlined in section 3.2. However, it should be noted that, due to problem size, the exhaustive search was only used on problems where the shorter building was 10 floors.

3.3.2 Solve time

Clearly, the time to apply each method is highly dependent on the number of simulations required to complete the method. Applying Eqs 3-2, 3-3, and 3-4, the number of simulations required for each method is computed. The number of required simulations versus number of available dampers is shown in Figure 3-6 to Figure 3-8. (The number of simulations required in the maximum velocity method is not plotted, as this number is always 1.)

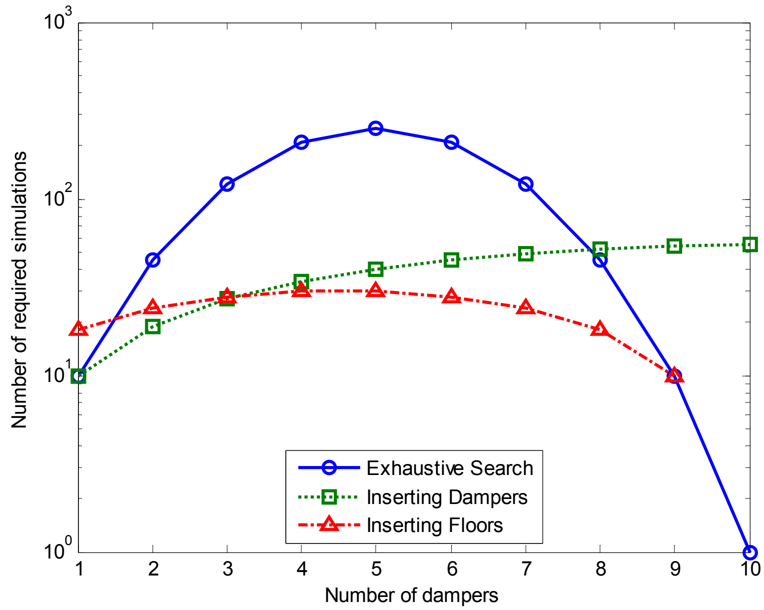

Figure 3-6: Number of required simulations for exhaustive search, inserting dampers, and inserting floors, for n=10

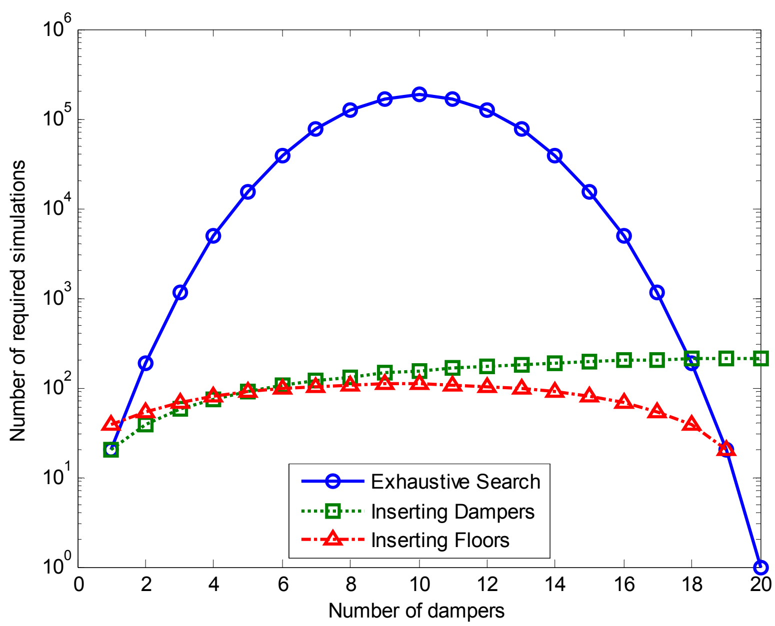

Figure 3-7: Number of required simulations for exhaustive search, inserting dampers, and inserting floors, when n=20

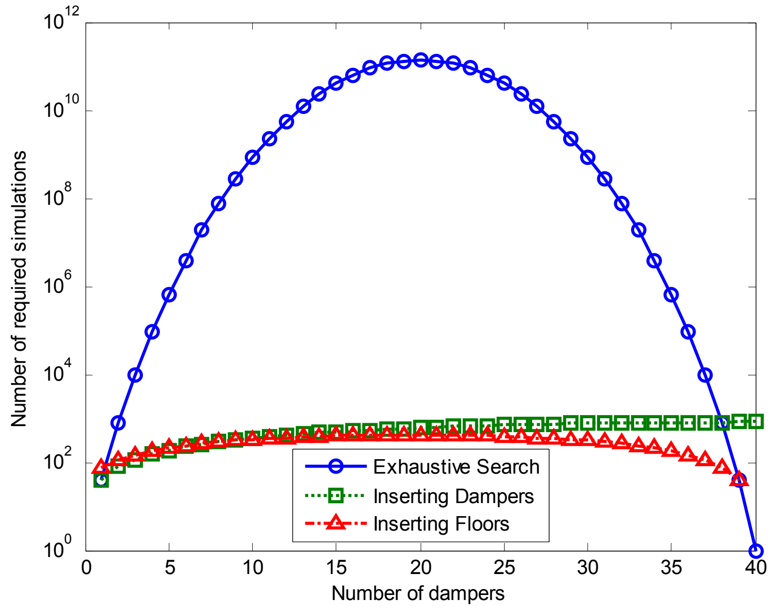

Figure 3-8: Number of required simulations for exhaustive search, inserting dampers, and inserting floors, when n=40

Obviously, the number of required simulations for the exhaustive search method is generally much higher than the other two methods. In fact, in many cases, it may be intractable to employ an exhaustive search to find the optimal configuration. For example, it is estimated that the time required for an exhaustive search to solve the problem of placing 20 dampers over 40 floors is about 40,000 years of CPU time; while the inserting floors method can solve the same problem in less than 2 hours. Even with a super-computer, say 100 times faster than common machines, the solution time of the exhaustive search method will be 400 years which confirms the impracticability of this method. Note that the number of required simulations for the genetic algorithm is not a fixed value for each case due to the fact that this method is an evolutionary method that uses some random numbers to generate the final result.

However, it should be noted that the required CPU-time associated with a method is not an exact formula of the number of simulations required. Since each simulation solves an optimization sub-problem (computing damping coefficient), the convergence rate of this sub-problem plays a role in the total CPU-time. This is even more pronounced in the inserting floors method, as this method starts with a small model, which needs a shorter time to complete, and concludes with a full size model, which needs a longer time to complete. As an example, the ratios of CPU time to number of simulations for case 5 and objective function F_1 are tabulated in Table 3-3. The results show that the CPU time per simulation for the inserting floors method is noticeably less than that for inserting dampers, and much more dependent on n_d.

Table 3-3: Sample of CPU time (sec) per simulation

n_d	ID	IF
1	18.4	13.3
3	16.7	14.5
5	17.1	14.5
8	17.1	15.7

3.3.3 Solution quality

In order to examine the performance and efficiency of the presented methods, 192 different problems are solved using four different approaches. It should be noted that the exhaustive search approach is not applied to the problems where the shorter building is 20 floors due to impractical CPU-times needed for this method to solve such problems. Detailed final results, including optimal arrangement and optimal damping coefficients are presented in Table A-1 to Table A-6. Also, results for solve time and final objective values for all problems can be found in Table A-7 to Table A-12.

In order to show the efficiency and applicability of each method, performance profiles [65] are computed and plotted in Figure 3-9 and Figure 3-10. Recall that performance profiles are designed to graphically compare both speed and robustness of algorithms across a test set [65]. This is done by plotting, for each algorithm, the percentage of problems that are solved within a factor of the best solve time. Mathematically, the *performance ratio* is first computed by

$$r_{p,a} = \frac{t_{p,a}}{\min\{t_{p,a}, a \in \mathrm{A}\}}, \mathit{for}\ p \in \mathrm{P} \qquad \text{Eq 3-6}$$

where P is the set of all problems, A is the set of algorithms, and $t_{p,a}$ is the solving time of algorithm *a* for problem *p*. The percentage of problems that are solved within a factor $\tau \in \Re$ of the faster algorithm is next computed by

$$\rho_a(\tau) = \frac{|\{p \in \mathrm{P} : r_{p,a} \le \tau\}|}{|\mathrm{P}|} \times 100 \ \ \% \qquad \text{Eq 3-7}$$

The percentage ρ_a of all problems solved by algorithm *a* at τ gives an overall assessment of the performance of algorithm *a*. High values of ρ_a near $\tau = 1$ represents fast solve times. High values of ρ_a, when τ is large, represent high success rates. For a good algorithm, the plot of $\rho_a(\tau)$ should therefore be found above the other algorithms. For a more detailed description of performance profiles can be found in [65].

To calculate the performance profile, one needs to define when a method "solves" a specific problem. In this paper, a method is considered as a "failed method" if the difference between the objective value obtained using that specific method and the best objective value obtained for that problem exceeds the defined allowable tolerance. Examining detailed results, it is found that the objective function does not significantly affect the solution efficiency; in particular, solution time for similar cases (but different objective function) is less than 10%. This is due to the fact that both objective functions are black box and no information, beforehand, is available about how they change with respect to design variables.

Results show that, for approximate results (Figure 3-9), the genetic algorithm is almost as fast as the damper insertion method. However, the same graph suggests that the genetic algorithm can only solve 84% of the problems; while the damper insertion method is successful for 99% of the problems. For more accurate results (Figure 3-10), inserting dampers consistently ranks as the best method in both speed and robustness. It is worth noting that, in theory, the exhaustive search method is able to solve all problems if we do not stop the algorithm until all possible combinations of dampers' arrangements are examined. If that is the case, the exhaustive search method finally reaches 100% in Figure 3-9 and Figure 3-10. However, in practice, it is impossible to let the algorithm work for 8,000 years. Considering the limit imposed on the solution time, exhaustive search is only able to solve 62% of the problems in Figure 3-9 and Figure 3-10.

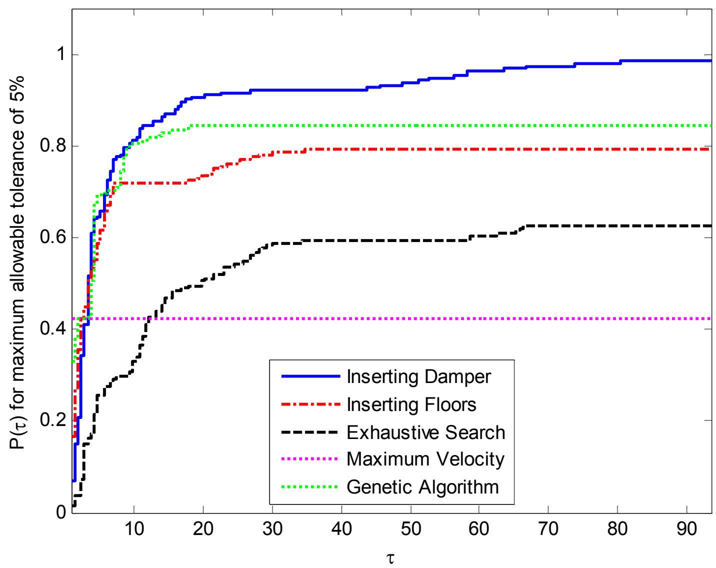

Figure 3-9: Performance profiles for inserting dampers, inserting floors, exhaustive search, maximum velocity and genetic algorithm, with maximum allowable tolerance of 5%

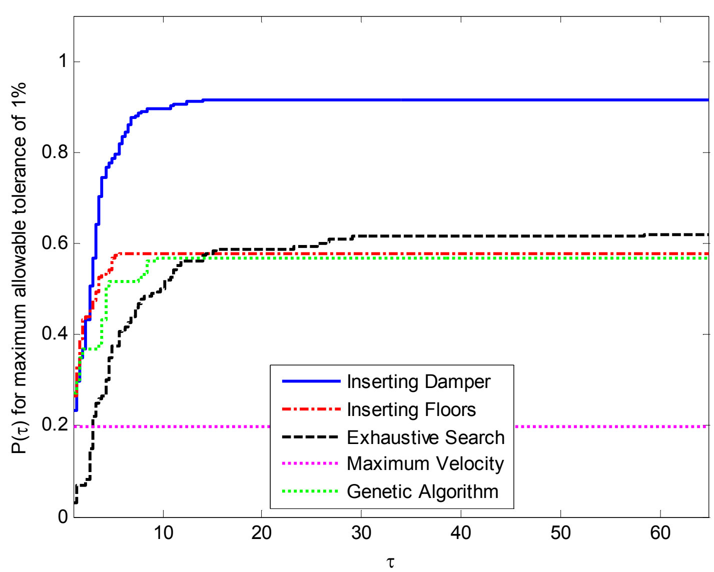

Figure 3-10: Performance profiles for inserting dampers, inserting floors, exhaustive search, maximum velocity and genetic algorithm, with maximum allowable tolerance of 1%

3.3.4 Number of dampers

Through determining the optimal arrangement for a limited number of dampers, one can investigate the effect of the number of installed dampers on the efficiency of the system. For the sake of comparison, the percentage of improvement, $\eta(n_d)$, is defined, as:

$$\eta(n_d) = \frac{obj\big|_1 - obj\big|_{n_d}}{obj\big|_1} \times 100\%, \qquad \text{Eq 3-8}$$

where $obj\big|_{n_d}$ is the objective value when the number of installed dampers is equal to n_d and $obj\big|_1$ is the objective value when only one damper is installed. For example, Figure 3-11 and Figure 3-12 show the percentage of improvement versus the number of dampers for the 8 adjacent structures resulting from mechanical property Set I (see Table 3-1). For these tests, it can be seen that increasing the number of available dampers does not necessarily improve efficiency of the system. Moreover, in many situations, increasing the number of dampers actually exacerbates the performance of the system. This inconsistency is due to the assumption that all dampers are assumed to have the same damping coefficient (see Eq 21). This assumption imposes an additional constraint to the problem that prevents finding the best damping coefficients for a given damper configuration.

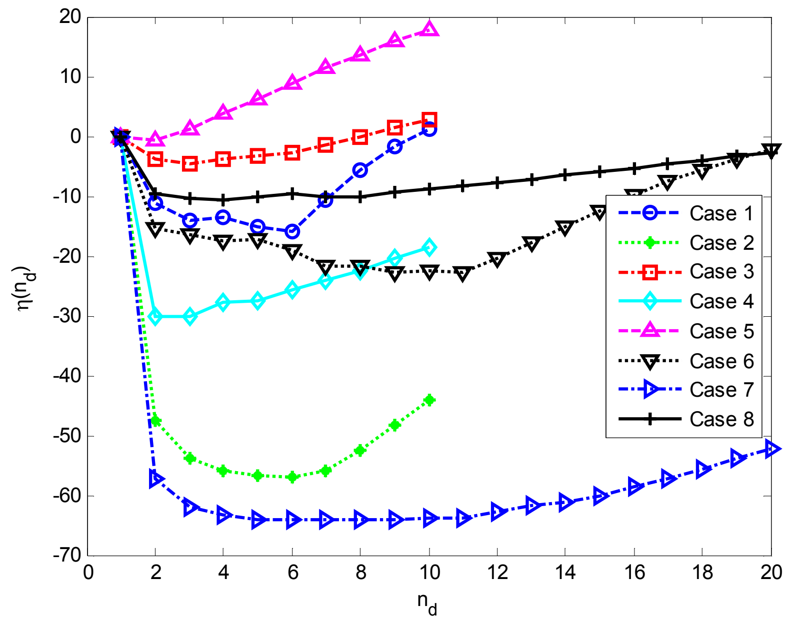

Figure 3-11: Reduction of response vs. number of dampers, F1 as objective function

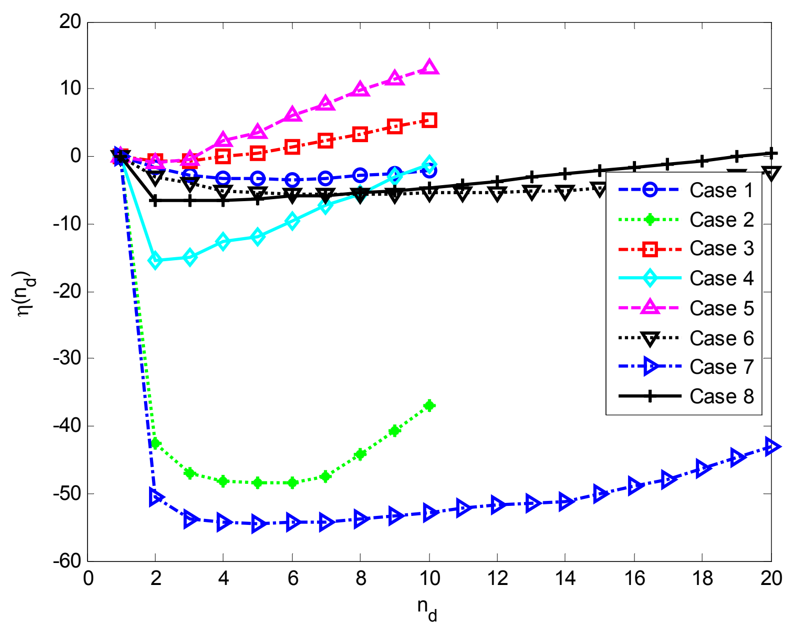

Figure 3-12: Reduction of response vs. number of dampers, F2 as objective function

3.4 Summary

This chapter considers how to determine an optimal design for the location of a limited number of viscous dampers to minimize inter-story drift. First, two buildings are modeled as a linear discrete system consisting of a lumped mass, a linear spring and a linear damper for each floor (see Chapter 2). The placement of dampers is next framed as the discrete optimization problem of minimizing the inter-story drift with two methods of measuring the total inter-story drift considered. The resulting problem is a bi-level optimization problem, where the inter-story drift for a given damper placement is found by minimizing with respect to the damper coefficients. This sub-problem is solved by use of a Golden Ratio bi-section method.

In this chapter, five approaches are presented to determine the optimal configuration for a limited number of dampers: exhaustive search, inserting dampers, inserting floors, maximum relative velocity, and genetic algorithm. The exhaustive search method is theoretically the most robust method; however, due to the required CPU-time, it is shown to be totally intractable in difficult cases. On the other hand, the maximum velocity method is the fastest method since it determines the best arrangement in a single simulation. However, the numerical tests show that this method does not lead to a reasonable final result. Another drawback of the maximum velocity method is that this method is not sensitive to the objective function. A more accurate method is the inserting floors method. This method is faster than the inserting dampers method; but it is not as accurate as the inserting dampers method. Also, the genetic algorithm method acts like the inserting floor method. The convergence rate is very fast; however, results show that it might not be a reliable method for more accurate results. All in all, the presented performance profiles suggest that the inserting dampers method is the most practical method, providing an excellent balance between speed and accuracy.

Furthermore, the sensitivity of the response to the number of installed dampers is examined through finding the objective function for the optimal arrangement for each specific number of dampers. Interestingly, the numerical results also show that increasing the number of dampers does not necessarily improve the efficiency of the system. In fact, increasing the number of dampers can even result in an increased inter-story drift. In Chapter 4, we will see that the reason for this is the assumption that all dampers are considered to have the same damping coefficient. Although this assumption has been widely used in many studies, the results show that this assumption is not a valid assumption and can lead to very inefficient damping values. Therefore, in order to find the optimal solution, it is necessary to remove this assumption and let dampers have different damping coefficient. This is done in the next chapter by the use of derivative free methods.

4 Global optimization

In the previous chapter, all dampers were assumed to be identical and have the same mechanical properties. Therefore a 1-dimensional optimization algorithm (the bisection method) was used to find the optimal damping coefficient. It was shown that, if all dampers have the same damping coefficients, increasing the number of dampers does not necessarily increase the dynamic stability of a structure. Moreover, under these conditions increasing the number of dampers may even exacerbate the dynamic behaviour of the buildings. Therefore, we need to remove this constraint, and solve the n-dimensional optimization problem of damping coefficients.

In this chapter, a bi-level optimization algorithm is designed to find the optimal arrangement and mechanical properties of dampers for an extensive set of test situations. We solve the outer-loop (optimal damping configuration) using the “inserting dampers" method, which was shown to be a very effective heuristic method in the previous chapter. Under the assumption that the dampers have varying damping coefficients, the inner-loop (optimal damping coefficients) is an n-dimensional continuous optimization problem. Since a series of simulations and different math operators are used to compute the objective value, we are limited to methods that do not require derivatives, that is, non-gradient based methods.

The research area of derivative free optimization (DFO) has progressed greatly in recent years. Derivative free methods are useful in situations when gradient information is computationally expensive, unreliable or unavailable. As the name suggests, these methods do not use derivatives; they only require function evaluations for a given set of input parameters. For a thorough introduction into several well-known DFO frameworks, see [66].

Genetic Algorithm (GA) is a popular and efficient method for engineering applications. GA has been widely used to solve many engineering problems. In this chapter, we use the stochastic based GA to have a baseline comparison to previous studies that have principally employed this method. Although popular, and generally effective, GA is heuristic in the sense that it does not enjoy mathematical convergence analysis. Hence we would like to examine the efficiency of DFO methods other than GA. Unlike GA, other DFO methods used in this chapter are proven to converge to an optimal point under certain conditions. In particular, we use the powerful mesh adaptive direct search (MADS) algorithm as implemented in MATLAB's global optimization toolbox. MADS is a derivative free directional direct-search method with corresponding convergence theory [67]. Finally, since the problem of finding the optimal damping coefficients takes the form of a finite minimax problem (minimizing the maximum inter-story drift), we test a novel robust approximate gradient sampling (RAGS) algorithm. RAGS is a novel robust approximate gradient sampling (RAGS) algorithm that is specifically designed for finite minimax problems [68]. A detailed description of each algorithm is given in the next section.

In comparing the efficiency and stability of these three methods, we find that the MADS algorithm generally produces the most robust results, while the RAGS algorithm produces highly comparable results in significantly less time. While the GA produces respectable results, it is generally not found to be competitive with the MADS and RAGS algorithms. In addition, we find optimal damping coefficients for 150 test problems and show that there exists a threshold for the number of dampers used, where no significant improvement is made if more dampers are installed in the retrofitting system. Finally, we show that this threshold is achieved faster for MADS and RAGS than for GA.

4.1 Damper Location Optimization

The algorithm used for the outer-loop is the inserting dampers method from the previous chapter, Section 3.2.2. This is a heuristic algorithm, which was shown to be the most effective among others at finding the optimal configuration of dampers. Therefore, in this chapter, the inserting dampers method is used to find the optimal configuration of limited number of dampers.

4.2 Damping Coefficients Optimization

The main purpose of the inner-loop of the optimization algorithm (the dashed box in Figure 3-4) is to find optimal damping coefficients of dampers for a fixed configuration of damper connectors.

As derivative information of the objective function is not available for the optimization problem, the inner-loop requires the use of a non-gradient based method to find the optimal damping coefficients for a fixed configuration of dampers. We consider 3 such methods herein.

4.2.1 Genetic algorithm

The genetic algorithm (GA) is a popular heuristic search method. It is shown to be an efficient method, particularly in engineering applications (see [24], [49], [50], [69] and references therein). A simple GA consists of several steps, including the generation of initial points, selection, competition and reproduction [64]. In this thesis, we use the GA in a standard form that is included in the MATLAB global optimization toolbox.

4.2.2 Mesh adaptive direct search

The mesh adaptive direct search (MADS) method [67] is a sub category of pattern search (PS) methods. In general, PS methods start with an initial point. The objective function is then evaluated at a set of points generated by a poll basis within a search radius of the initial point. If a point with a lower function value is found, then the algorithm updates the current point and loops. If a point with a lower function value is not found, then the search radius is reduced and the algorithm loops. Once a desired level of accuracy is achieved, the algorithm terminates. Specific to MADS, randomly rotated bases are used in each iteration to provide a more robust convergence.

4.2.3 Robust approximate gradient sampling

The robust approximate gradient sampling algorithm (RAGS algorithm) is a derivative free optimization algorithm that exploits the smooth substructure of the finite minimax problem,

$$\min_x F(x) \text{ where } F(x) = \max(f_i : i = 1, \ldots, N\}. \qquad \text{Eq 4-1}$$

In RAGS algorithm we assume each f_i is continuously differentiable, but no gradient information is available. Conventional DFO methods, like MADS, include a polling step in which the objective function is evaluated at certain points around the current point. Then they move toward a direction which leads to a decrease in objective function. This is fine; but it is likely to get stuck on sharp ridges, e.g. see Figure 4-1. In this figure, the straight dashed line is a sharp ridge, the dotted lines represent contour plot of a non-differentiable function, the optimal point is represented by a star, and the solid lines are the path of convergence for a DFO method.

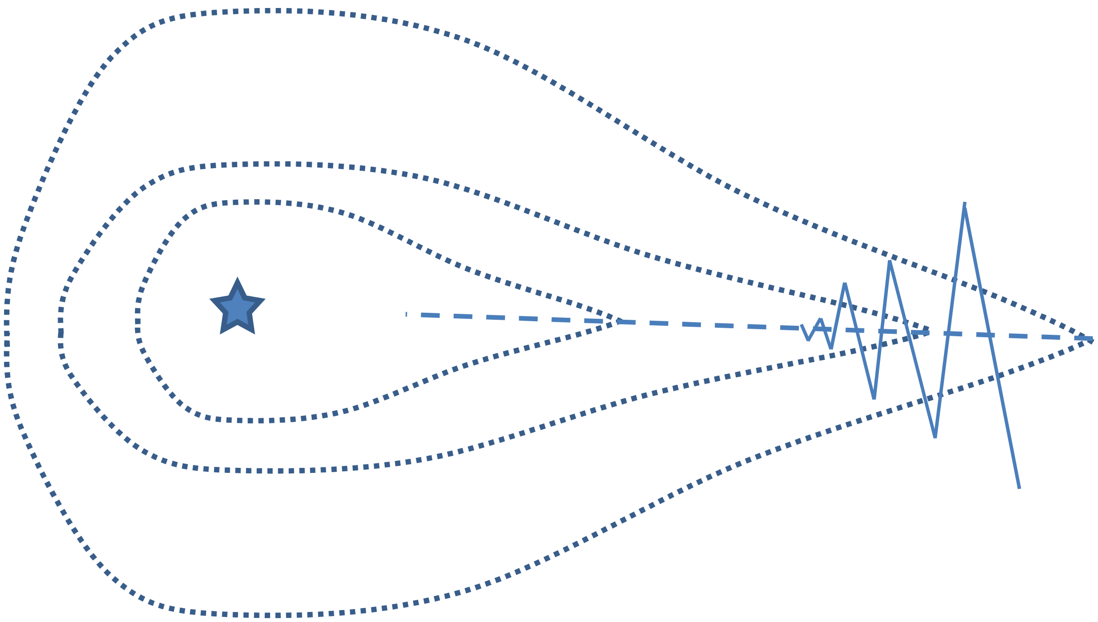

Figure 4-1: Convergence path of a conventional DFO method for a non-differentiable function

This becomes even more crucial for minimax problems as they are most likely to have many sharp non-differentiable ridges. RAGS uses a random sampling to generate an “approximate subdifferential” that reflects the shape of the function including sharp ridges. This can prevent the algorithm from getting stuck on sharp ridges. In other words, when the current iterate is close to a point of non-differentiability, RAGS algorithm is likely to move along non-differentiable ridges. The design and implementation of the RAGS algorithm is not within the scope of this thesis. See [68] for further information.

4.3 Numerical tests

In this section, we present results for various numerical problems for damper-connected structures. The number of required simulations and quality measures of the previously presented non-gradient based methods are compared.

4.3.1 Test problems

In order to compare the presented methods, three different sets of mechanical properties and five different sets of heights for the buildings are considered. These are provided in Table 3-1 and Table 3-2. Note that due to limited computational resources, only five heights (of total eight heights) are tested in this chapter.

In Table 3-2, f_a and f_b represent the number of floors for buildings a and b, respectively. For all numerical examples, to generate the ground excitation spectrum, we use the following values for the ground acceleration parameters used in Eq 2-15: $\omega_g = 15\, rad/s$, $\xi_g = 0.6$, and $S_0 = 4.65 \times 10^{-4} m^2/rad.s^3$ (These parameter values are the same as those used in [7], [37], [39]. For each of the 3 sets of mechanical properties, the number of dampers changes from 1 to 10. Therefore, incorporating all 5 building height combinations, a total of 150 test problems are generated, representing a wide range of situations. Each problem is solved via a combination of the inserting dampers method and a non-gradient based method (either GA, MADS or RAGS). Optimal arrangements and damping coefficients, as well as corresponding objective functions are determined.

4.3.2 Solution time and quality

In order to compare the presented methods, 150 test problems, which are defined in the above paragraph, are solved using the presented DFO methods. Note that these problems cover different sets of materials, different heights for the buildings, and different number of dampers; therefore it is expected that the results represent a wide range of problems.

Table B-1 to Table B-3 show the number of function calls required and the optimal objective function values obtained using various methods. For the sake of brevity, optimal design variables, including configurations of dampers and damping coefficients, are not included.

Note that in Table B-1 to Table B-3, instead of reporting solution times in seconds, the number of performed simulations is reported. Each simulation would take approximately 2 seconds. However, as test problems are sequentially stored in MATLAB's memory for the sake of solving one problem after the other, MATLAB's processing speed decreases, and the simulations for the later problems take longer than for the very first problems. All in all, the number of simulations can be considered as a solid alternative for solution time (e.g. see [70]).

Clearly, the rate of convergence is a crucial factor for any optimization algorithm. In Figure 4-2, Figure 4-3 and Figure 4-4, we present sample convergence rates for Building Height I, Material Sets I, II and III, when connecting all adjacent floors, i.e., 10 dampers. In each figure we plot the objective value (a) and minimum objective value (b) for each function call.

We see in Figure 4-2, Figure 4-3 and Figure 4-4, as GA is a stochastic based method, that GA evaluates the objective function at a wide range of points, resulting in a large range of function values. For the MADS algorithm, we see a similar variation in objective value range, with multiple spikes in the objective value as the number of function calls increases. We can see that RAGS converges quickly, with minimal objective value variation. This becomes even more evident in Figure 4-5, Figure 4-6, and Figure 4-7. Overall, we can see that, for these examples, RAGS converges much faster than GA and MADS. Furthermore, RAGS is shown to terminate with fewer function calls than MADS.

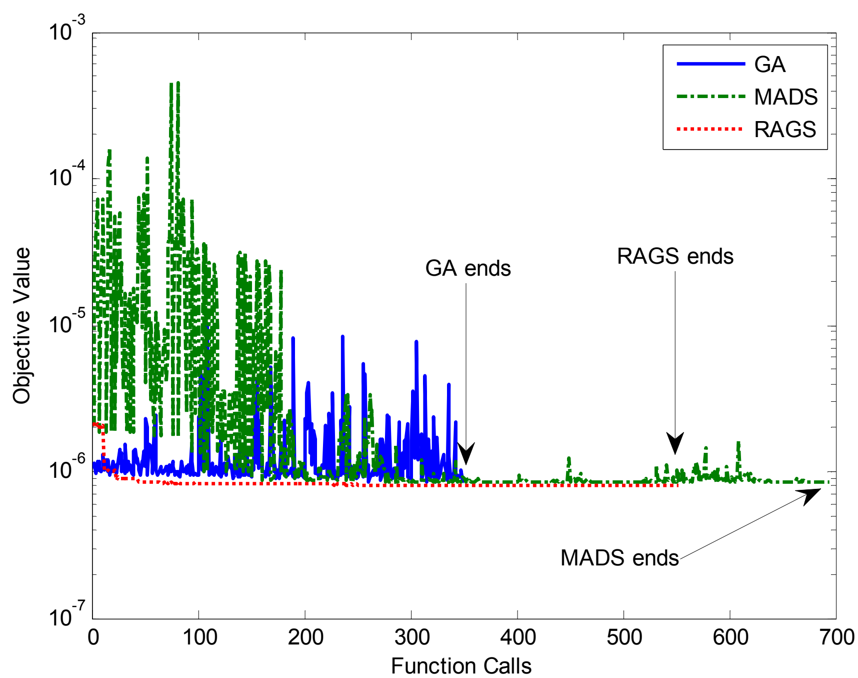

Figure 4-2: Objective value vs. function call for Material I and Height I

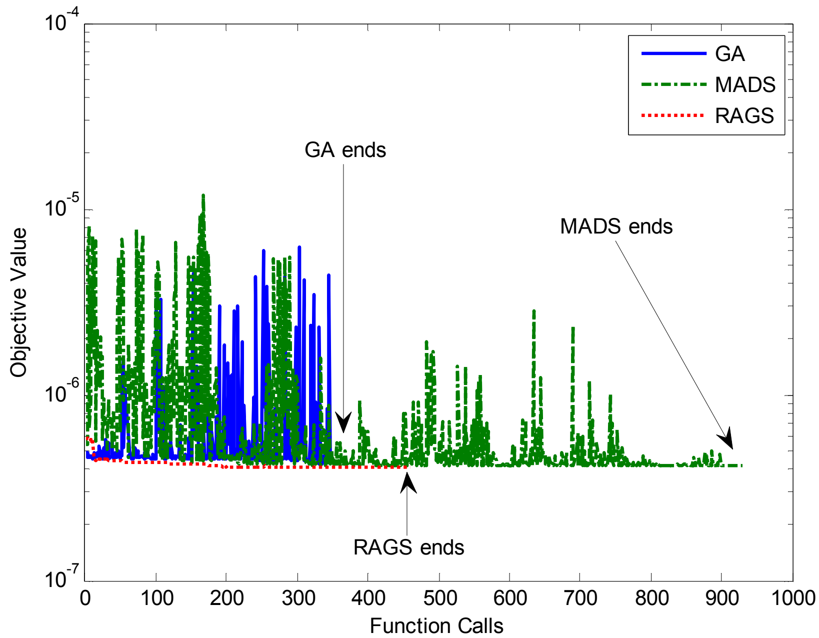

Figure 4-3: Objective value vs. function call for Material II and Height I

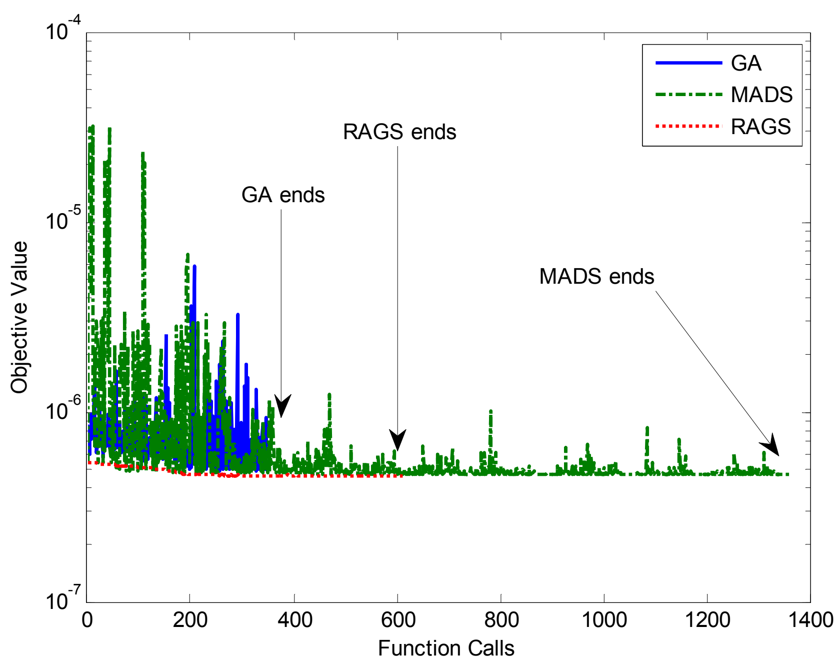

Figure 4-4: Objective value vs. function call for Material III and Height I

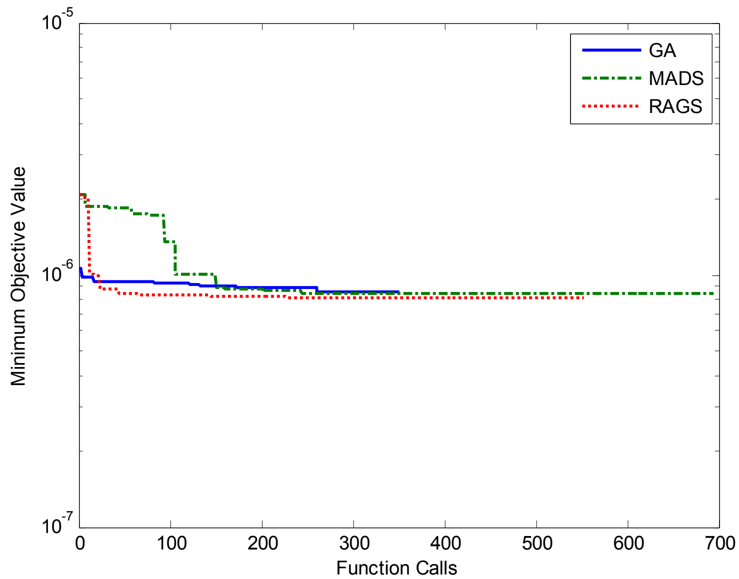

Figure 4-5: Convergence of the objective function vs. function calls for Material I and Height I

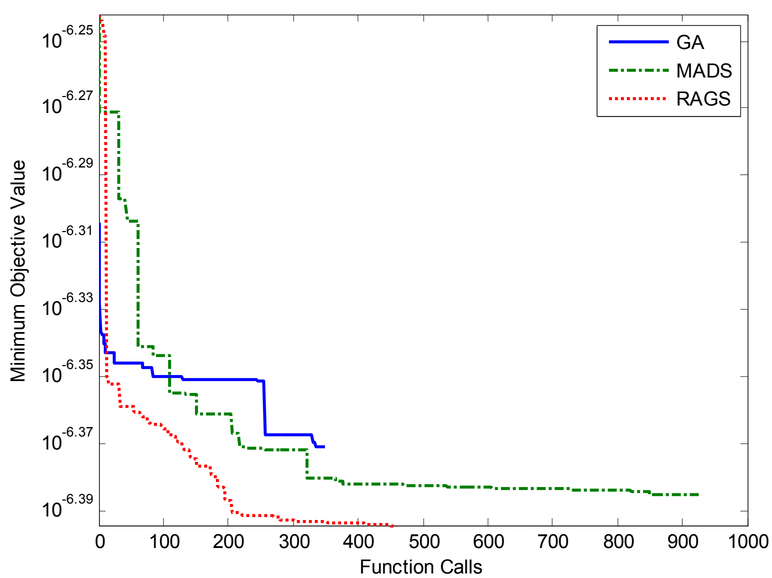

Figure 4-6: Convergence of the objective function vs. function calls for Material II and Height I

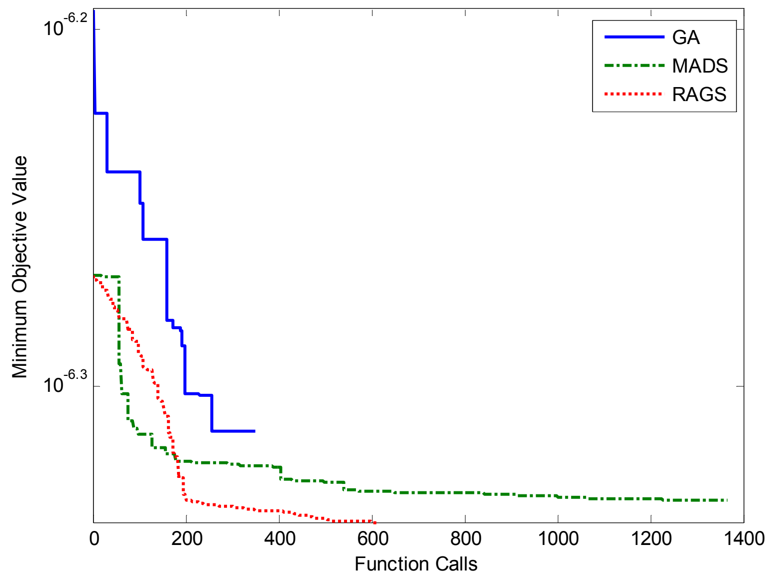

Figure 4-7: Convergence of the objective function vs. function calls for Material III and Height I

In order to investigate the overall performance of the presented methods, we use a performance profile [65] as explained in the previous chapter. As in the previous chapter, a method is considered as a "failed method" if the difference between the objective value obtained using the method in question and the best objective value obtained by any of the methods for that problem exceeds the defined allowable tolerance. Performance profiles for the presented methods are plotted in Figure 4-8, Figure 4-9, and Figure 4-10 for allowable tolerances of 5%, 2%, and 1%, respectively.

As seen in Figure 4-8, Figure 4-9, and Figure 4-10, RAGS converges to the optimal solution the fastest (in terms of function evaluations), whereas MADS has the highest probability of eventually returning the best found solution to a problem. This is possibly due to the randomness of the basis generated in MADS, which increases the solve time but reduces the chance of MADS getting trapped in local minima. Conversely, RAGS does not have any heuristics to differentiate between local minimum and global minima. It is worth mentioning that results from [46] show that the function of optimal location of dampers is not a convex function and it has several local minima.

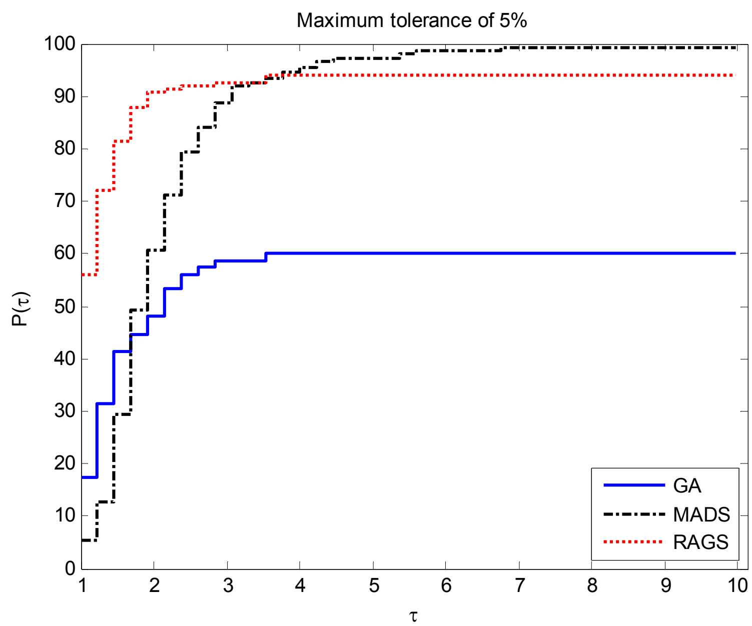

Figure 4-8: Performance profiles for GA, MADS, and RAGS, with maximum allowable tolerance of 5%

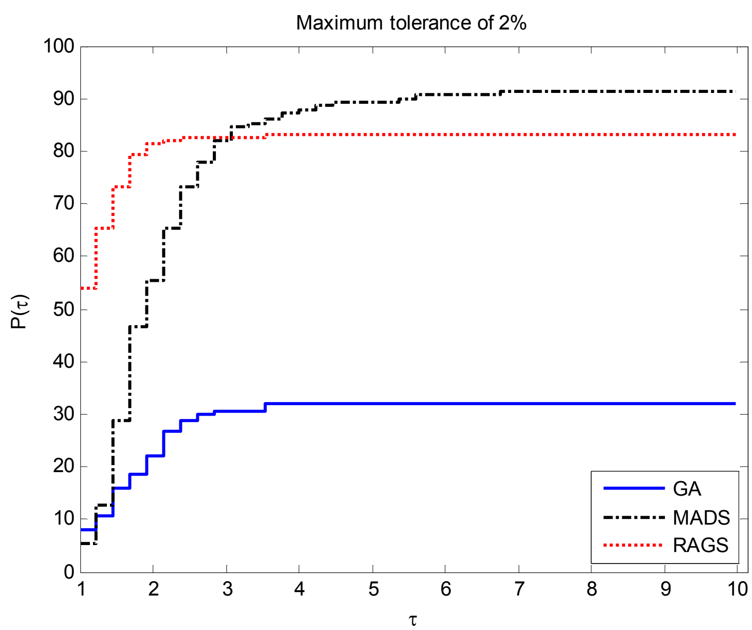

Figure 4-9: Performance profiles for GA, MADS, and RAGS, with maximum allowable tolerance of 2%

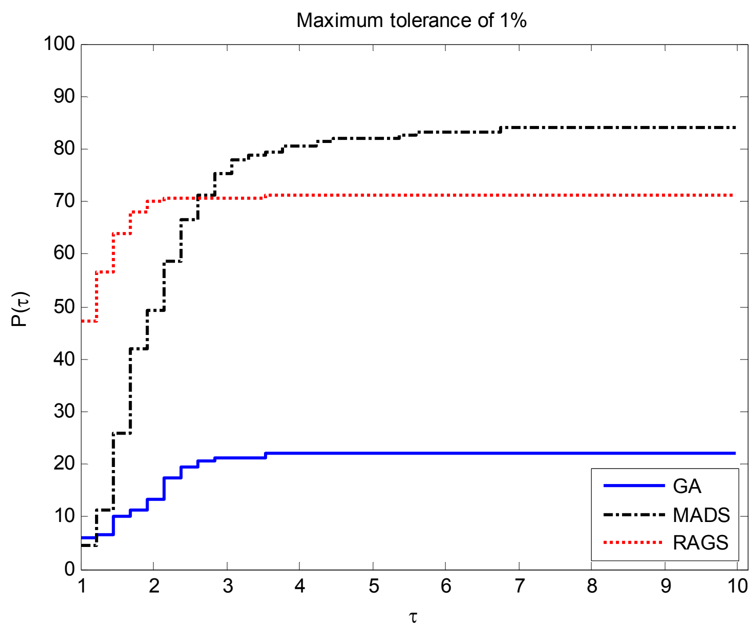

Figure 4-10: Performance profiles for GA, MADS, and RAGS, with maximum allowable tolerance of 1%

4.3.3 Number of dampers

In this section, we investigate effects of the number of dampers on the efficiency of the retrofitting system. As a multi-objective optimization study, fewer dampers and increased efficiency of the system are desired.

In Figure 4-11, Figure 4-12, and Figure 4-13, assuming Building Heights I, we plot the number of dampers against the objective value found for Material Sets I, II and III, respectively. It should be pointed out that, in order to make the difference more apparent, y-axis does not start from zero. In Figure 4-11, we see that the objective value decreases as the number of dampers are increased, until the insertion of the fifth damper for MADS, the sixth damper for RAGS and the seventh damper for GA. After this, no significant improvement is observed. In Figure 4-12, RAGS reaches its lowest objective value with five dampers, GA with six and MADS with nine. In Figure 4-13, we see a little more variation for all three algorithms, with significant objective value decrease until the insertion of the fifth damper for RAGS, the sixth damper for MADS and the eighth damper for GA.

This inspires us to consider how many dampers are required by each of the algorithms to find an optimal solution. The optimal solution is taken to be the overall lowest value found by all three algorithms. We use 15 test problems (3 sets of mechanical properties and 5 sets of building heights). The results for a solution found within 5% and 1% of the optimal solution are displayed in Figure 4-14 and Figure 4-15.

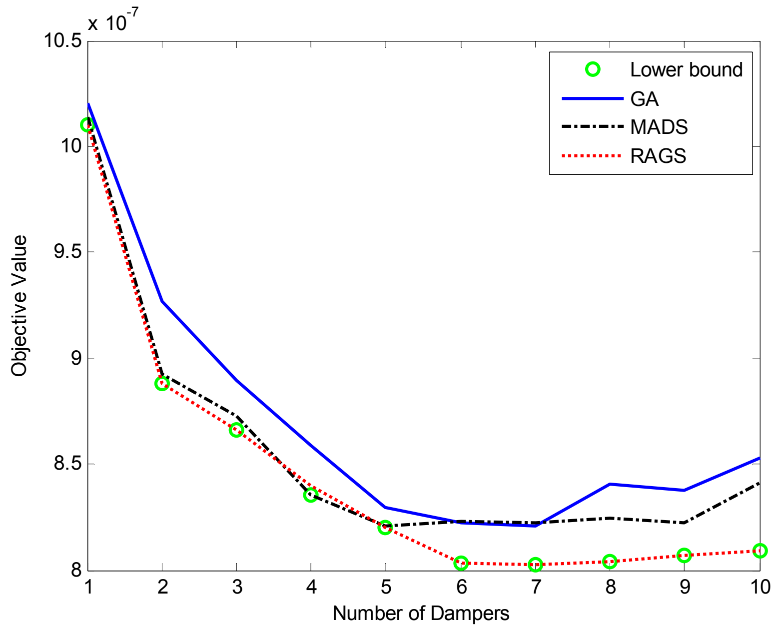

Figure 4-11: Objective value vs. number of installed dampers for Material I and Height I

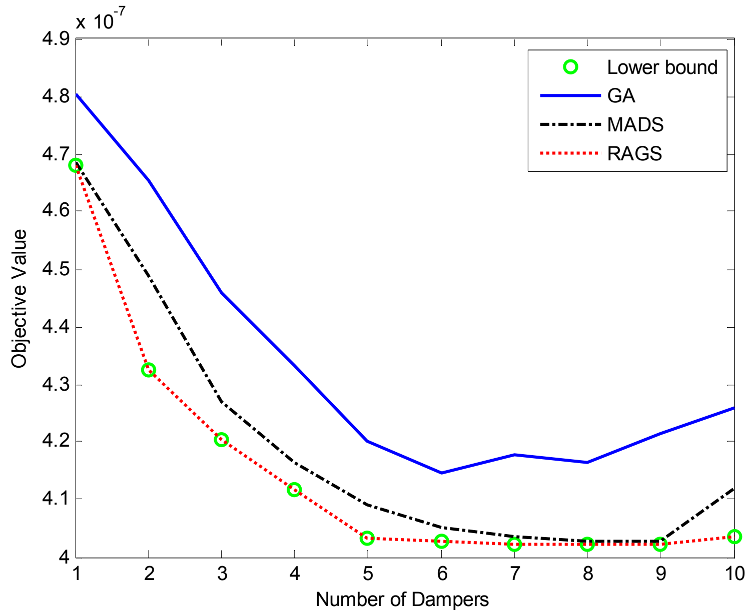

Figure 4-12: Objective value vs. number of installed dampers for Material II and Height I

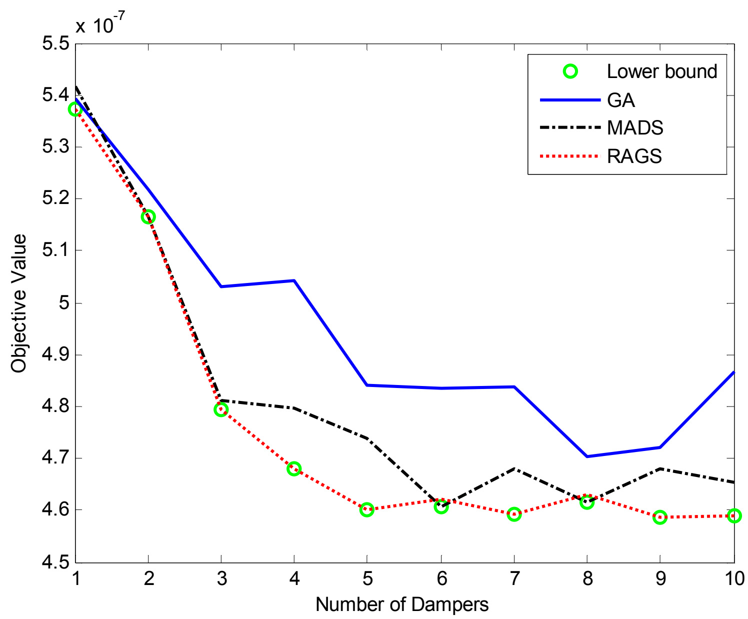

Figure 4-13: Objective value vs. number of installed dampers for Material III and Height I

In Figure 4-14, there is clearly a cluster of problems solved within a tolerance of 5% using 1-5 dampers. We see that MADS solves all 15 problems with 5 dampers or less and RAGS solves 14 of the 15 problems with 4 dampers or less. The problem that RAGS “did not solve” (DNS) was also one of the two problems “not solved” by GA. The DNS problem for RAGS found no additional function value decrease from its solution for 1 damper. Thus, we suspect that the initial iterate for the algorithm was close to a local minimum. For both problems that GA ”did not solve”, a tolerance of approximately 7% was found with respect to the optimal solution.

In Figure 4-15, we see a cluster of problems solved within a tolerance of 1% using 2-6 dampers. We note that the problem which required 9 dampers for MADS to solve was the same problem that RAGS (and GA) ”did not solve” in Figure 4-14. The problem that required 10 dampers for RAGS to solve was the other problem that GA ”did not solve” in Figure 4-14.

It is clear that using a structured internal subroutine for the inner-loop of the bi-level problem presented in this thesis greatly increases the efficiency of the retrofitting system with respect to the number of dampers used. Although the different methods find a wide range of objective values for each number of dampers, the curve of lower bound of the objective function is still a decreasing function. For example, in Figure 4-13, we observe a couple of bumps in the plot. At first glance, we may conclude that the results obtained by MADS indicate that by increasing the number of dampers, we decrease the efficiency of the system. This is due to the different convergence rates of the presented optimization algorithms for different numbers of dampers. In other words, although these methods are multi-dimensional, a not-so-good starting point can prevent them from finding the true optimal solution. A quick fix for this issue would be to set the initial point of the next problem (one more damper) to the optimal point of the previous problem, with the additional damper position (increased dimension) set to zero. This will guarantee that all graphs obtained by each algorithm will be decreasing. However, in order to have an unbiased comparison in the presented results, a unique initial point for design variables is used for all test problems.

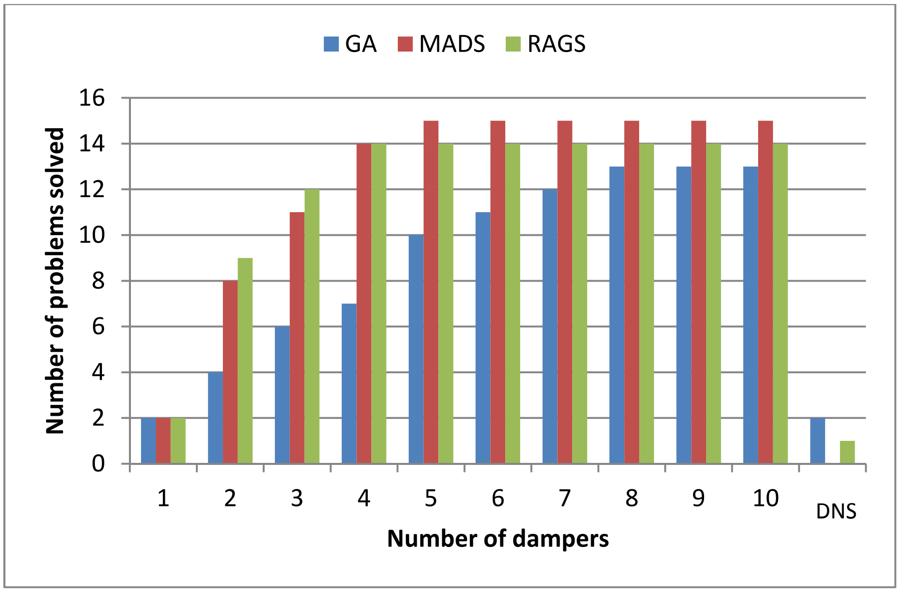

Figure 4-14: Cumulative number of dampers required with maximum tolerance of 5%

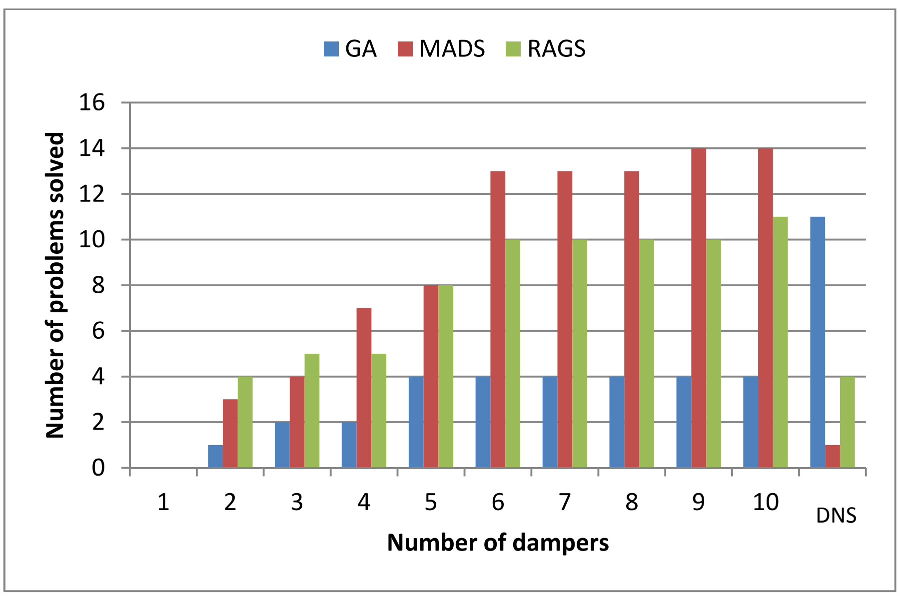

Figure 4-15: Cumulative number of dampers required with maximum tolerance of 1%

4.4 Summary

This chapter presents a comprehensive optimization procedure that can be used to design damper connected structures. In particular, two adjacent buildings are considered as lumped mass models connected to each other using discrete viscous dampers. A pseudo excitation formula is used to generate an earthquake load in a frequency domain. Considering a linear behaviour of the buildings, the dynamic response of the whole system is found. Using the dynamic response of the system, we calculated the desired objective function, i.e., the maximum inter-story drift.

The optimization procedure consists of two parts including discrete and continuous optimizations. An outer-loop (discrete optimization algorithm) finds the best configuration of a limited number of dampers between two buildings; an inner-loop (continuous optimization algorithm) finds the optimal damping coefficients of the dampers. We considered three different algorithms (GA, MADS and RAGS), which can be used for the continuous optimization problem. In order to compare speed and robustness of these non-gradient based methods, 150 test problems were generated and solved via these three methods. Results showed that RAGS is the fastest method, while MADS is the most robust method. The robustness of MADS likely comes from its heuristics to avoid getting trapped in local minima; (such heuristics present an obvious direction of future study for the RAGS algorithm).

5 Conclusion

This thesis considers how to determine an optimal design for the size (damping coefficient) and location of a limited number of viscous dampers to reduce damage from vibrations of adjacent structures during earthquake conditions. First, coupled buildings are modeled as a linear discrete multi-degree-of-freedom system consisting of a lumped mass, a linear spring and a linear damper for each floor. A pseudo excitation formula is used to generate an earthquake load in the frequency domain. Considering a linear behaviour of the buildings, the dynamic response of the whole system is found. Using the dynamic response of the system, we calculate the objective function. The optimization problem of damper connectors is done in two steps, location optimization and damping coefficients optimization. The placement of dampers is framed as the discrete optimization problem of minimizing the inter-story drift. On the other hand, determination of optimal damping coefficients is a continuous optimization problem. The resulting problem is a bi-level optimization problem, where the inter-story drift for a given damper placement is found by minimizing with respect to the damper coefficients. Based on a cluster of numerical tests, the proposed optimization approaches are compared and efficiency of them is examined. As an important outcome of this thesis, it is found that some methods are more robust and more efficient than other methods, such as genetic algorithm and maximum velocity, that had been used to find the optimal arrangement and mechanical properties of the damper connects.

In Chapter 3, we focus on the discrete optimization problem of dampers placement. Five approaches are presented to determine the optimal configuration for a limited number of dampers: exhaustive search, inserting dampers, inserting floors, maximum relative velocity, and genetic algorithm. The exhaustive search method is theoretically the most robust method; however, due to the required CPU-time, it is shown to be totally intractable in difficult cases. On the other hand, the maximum velocity method is the fastest method since it determines the best arrangement in a single simulation. However, the numerical tests show that this method does not lead to a reasonable final result. Another drawback of the maximum velocity method is that this method is not sensitive to the objective function. A more accurate method is the inserting floors method. This method is faster than but not as accurate as the inserting dampers method. Also, the genetic algorithm method acts like the inserting floor method. The convergence rate is very fast;

however, results show that it might not be a reliable method for more accurate results. All in all, the presented performance profiles suggest that the inserting dampers method is the most practical method, providing an excellent balance between speed and accuracy.

Furthermore, Chapter 3 examines the sensitivity of the response to the number of installed dampers through finding the objective function for the optimal arrangement for each specific number of dampers. Interestingly, the numerical results show that increasing the number of dampers does not necessarily improve the efficiency of the system; it can even exacerbate the dynamic behaviour of the buildings. The most likely reason for this is that, in Chapter 3, we assumed that all dampers have the same damping coefficient. Therefore the optimization problem of damping coefficients is a single variable problem which is solved by the use of a Golden Ratio bi-section method. Although this assumption has been widely used in many studies, presented results show that this assumption is not a valid assumption and can lead to very inefficient damping values.

In Chapter 4, we remove the assumption of equal damping coefficients. Therefore the resulting optimization problem of damping coefficients becomes a continuous optimization problem with n_d independent design variables where n_d is the number of dampers. Since a series of simulations are used to compute the objective value, we are limited to methods that do not require derivatives, that is, non-gradient based methods. In Chapter 4, we use three non-gradient based algorithms including genetic algorithm (GA), mesh adaptive search method (MADS), and robust approximate gradient method (RAGS). In order to compare speed and robustness of these non-gradient based methods, 150 test problems were generated and solved via these three methods. Results showed that RAGS is the fastest method, while MADS is the most robust method. The robustness of MADS likely comes from its heuristics to avoid getting trapped in local minima; (such heuristics present an obvious direction of future study for the RAGS algorithm). Nevertheless, it is apparent that by exploiting the structure of the finite minimax problem, RAGS converges to a high quality solution in a fraction of the time of MADS and GA.

Furthermore, Chapter 4 investigates the efficiency of the retrofitting system with respect to the number of dampers used. In Chapter 3, it was shown that when assuming equal damping coefficients, increasing the number of dampers may exacerbate the dynamic behaviour of the buildings. In Chapter 4, we removed the assumption of equal damping coefficients and found that there exists a threshold for the number of dampers and the resulting efficiency of the system. Additionally, for 15 test problems, we showed that MADS and RAGS use a smaller number of dampers to find optimal (or close to optimal) solutions than GA, reiterating the benefit of using a structured internal subroutine for the inner-loop of our bi-level problem.

5.1 Future work

This thesis is limited to linear behaviour of the structures, lumped mass models, and viscous dampers. In order to determine more accurate results, we may need to improve the model used in this thesis. For example, instead of the simple MDOF model, a comprehensive 2- or 3-dimensional finite element model is recommended for more accurate results. Since the linear model is not able to include damages and plastic hinges, a non-linear model can be used. In that case, non-linearity of some construction materials such as concrete can be included in the model which can improve the accuracy of the model. Finally, different damping devices may be considered. In this thesis, we considered a linear viscous damper with no stiffness element. However, we know that commercial dampers do not follow this linear relationship between force and velocity. Instead, a non-linear (polynominal) relationship can be used to calculate the force generated in commercial dampers. Furthermore, different types of dampers are commercially available other than viscous dampers. Among them, one can refer to friction dampers, metallic yield dampers, and Magnetorheological dampers. Mathematical models have been developed to represent the mechanical behaviour of different types of dampers and they can be replaced with the viscous damper model used in this thesis.

In terms of optimization techniques, we can categorize them into discrete and continuous optimization methods. Various discrete optimization techniques can be developed to find the optimal arrangement of dampers. For example, we may develop a robust version of inserting dampers method by checking the first damper after all dampers are inserted, as:

1- Apply the inserting dampers method and find the optimal configuration.
2- Set the first iteration: i=1.
3- Remove the i^{th} damper and calculate the objective value.
4- Put the damper back in the building, i=i+1 go to step 3, until all dampers are checked.
5- Sort dampers by the level of usefulness, the least useful one is the one which almost has no negative effect if one removes it.

6- Remove the least useful damper.
7- Search for the best place to put this damper.
8- Put the damper at the best location.
9- Follow a similar procedure for the next least useful damper.
10- Keep on until all dampers are relocated.

Furthermore, step 1 can be replaced with a random initial guess. These methods can be used to find the optimal arrangement of dampers. Note that, in general, discrete heuristic methods are not proven to result in the global optimal solution.

Besides the discrete optimization loop that is used to find the optimal configuration of the dampers, we use a derivative free optimization (DFO) method to find optimal damping coefficients of the dampers. In this thesis, we use three different DFO methods including GA, MADS, and RAGS. There are several other DFO methods which have been shown to be very effective in solving engineering problem. Among them, one can refer to the Nelder-Mead method, simulated annealing, and particle swarm optimization. It would be interesting to apply such methods and compare them in terms of efficiency and robustness. Another challenging issue in this thesis was the existence of local minima. Different techniques, such as multiple random restarts, may be used in order to avoid getting stuck in local minima.

All in all, it seems that the biggest issue is the solution time, particularly when we let dampers have different damping coefficients. Any change that decrease the solution time will be a very important improvement. A simple method is to use the assumption of equal damping coefficients to find the optimal configuration of dampers. Then, by applying a DFO method, we may find the optimal damping coefficients of the dampers. However, it is possible that the optimal configuration found under the assumption of equal damping coefficients prevents us from finding the global optimal solution. On the other hand, the difference between the global minimum and the sub-optimal solution found by this approach might be negligible. Hence, for a fair judgement, a convincing number of problems must be solved and results must be compared. Another method is to use a function to approximate the relationship between the damping coefficients and the height. For example, we may use a linear function, $C_d(i) = a \times i + b$, where $C_d(i)$ is the damping coeffient of the damper installed at the i^{th} floor. The function parameters, a, and b, must be determined by a DFO method. In other words, by doing this, we reduce the size of the optimization problem from n_d to 2, where n_d is the number of dampers.

Last, but not least, it should be noted that the basic optimization idea used in this thesis can be used for different structural problems. For example, instead of adjacent buildings, one can solve the problem of dampers placement for a single building; or even for a simple beam. The simple lumped mass model used in this study not only represents adjacent buildings, but also it can be used to model the mechanical behaviour of different structures such as lateral vibrations of two parallel beams and so on. The bi-level optimization method introduced and used in this thesis enables us to use similar methodology for finding the optimal location and size of actuators on different structures, such as aerospace structures, plates, shells, panels, etc.

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Appendices

Appendix A: Results for Optimization of Configuration of Dampers

Table A-1: Optimal design, mechanical properties of set 1

			Objective Function F1		Objective Function F2	
Scenario	nd	Method	Cd	Arrangement	Cd	Arrangement
Case 1	1	ID	3.01E+07	[0000100000]	2.58E+07	[0000100000]
		IF	3.01E+07	[0000100000]	1.57E+07	[0000000100]
		ES	3.02E+07	[0000100000]	2.58E+07	[0000100000]
		MV	1.17E+07	[0000000001]	1.22E+07	[0000000001]
		GA	3.02E+07	[0000100000]	2.58E+07	[0000100000]
	3	ID	1.83E+07	[1010100000]	7.33E+06	[0010100001]
		IF	1.45E+07	[0110010000]	9.01E+06	[0101000010]
		ES	1.83E+07	[1010100000]	8.35E+06	[0011000010]
		MV	4.49E+06	[0000000111]	4.45E+06	[0000000111]
		GA	1.37E+07	[1001001000]	9.47E+06	[0110000001]
	5	ID	1.06E+07	[1111100000]	5.93E+06	[0111100001]
		IF	1.14E+07	[1111010000]	5.55E+06	[0111010001]
		ES	1.14E+07	[1111010000]	7.80E+06	[1111000010]
		MV	3.25E+06	[0000011111]	3.06E+06	[0000011111]
		GA	1.06E+07	[1111000100]	8.22E+06	[1111000100]
	8	ID	4.49E+06	[1111111100]	3.36E+06	[1111110011]
		IF	4.49E+06	[1111111100]	3.36E+06	[1111110011]
		ES	4.49E+06	[1111111100]	3.37E+06	[1111110011]
		MV	2.67E+06	[0011111111]	2.42E+06	[0011111111]
		GA	4.06E+06	[1111110101]	3.57E+06	[1111111001]
Case 2	1	ID	1.26E+07	[0000000001]	1.15E+07	[0000000001]
		IF	1.90E+07	[0000001000]	1.63E+07	[0000001000]
		ES	1.26E+07	[0000000001]	1.16E+07	[0000000001]
		MV	1.26E+07	[0000000001]	1.16E+07	[0000000001]
		GA	1.26E+07	[0000000001]	1.16E+07	[0000000001]
	3	ID	3.21E+07	[0001100001]	4.07E+07	[0001100001]
		IF	3.82E+07	[0010010001]	4.28E+07	[0010100001]
		ES	3.22E+07	[0001100001]	4.09E+07	[0001100001]
		MV	7.44E+07	[0000000111]	7.78E+07	[0000000111]
		GA	4.60E+07	[0010000101]	5.10E+07	[0100010001]
	5	ID	2.58E+07	[0111100001]	3.43E+07	[0111100001]
		IF	2.89E+07	[1101100001]	4.07E+07	[1110010001]
		ES	2.58E+07	[0111100001]	3.43E+07	[0111100001]

		MV	3.32E+07	[0000011111]	3.64E+07	[0000011111]
		GA	3.32E+07	[1101001001]	3.43E+07	[0111100001]
	8	ID	2.44E+07	[1111111001]	3.09E+07	[1111111001]
		IF	2.44E+07	[1111111001]	3.09E+07	[1111111001]
		ES	2.45E+07	[1111111001]	3.12E+07	[1111111001]
		MV	1.93E+07	[0011111111]	2.25E+07	[0011111111]
		GA	2.45E+07	[1111111001]	2.37E+07	[1111110011]
Case 3	1	ID	1.74E+07	[0000000001]	1.43E+07	[0000000001]
		IF	1.74E+07	[0000000001]	1.43E+07	[0000000001]
		ES	1.75E+07	[0000000001]	1.43E+07	[0000000001]
		MV	1.75E+07	[0000000001]	1.43E+07	[0000000001]
		GA	1.75E+07	[0000000001]	1.43E+07	[0000000001]
	3	ID	1.37E+07	[1010000001]	1.31E+07	[1100000001]
		IF	1.37E+07	[1010000001]	7.15E+06	[0100000011]
		ES	1.37E+07	[1010000001]	1.30E+07	[1100000001]
		MV	6.46E+06	[0000000111]	5.23E+06	[0000000111]
		GA	1.37E+07	[1010000001]	9.17E+06	[1000010001]
	5	ID	7.91E+06	[1110000011]	6.59E+06	[1110000011]
		IF	7.91E+06	[1110000011]	6.59E+06	[1110000011]
		ES	7.91E+06	[1110000011]	6.52E+06	[1110000011]
		MV	4.49E+06	[0000011111]	3.66E+06	[0000011111]
		GA	7.65E+06	[1101000011]	6.52E+06	[1101000101]
	8	ID	4.92E+06	[1111100111]	3.94E+06	[1111100111]
		IF	4.92E+06	[1111100111]	3.63E+06	[1111001111]
		ES	4.92E+06	[1111100111]	3.92E+06	[1111100111]
		MV	3.82E+06	[0011111111]	2.99E+06	[0011111111]
		GA	5.21E+06	[1111110011]	3.76E+06	[1111010111]
Case 4	1	ID	3.85E+07	[0000000001]	3.52E+07	[0000000001]
		IF	5.60E+07	[0000000100]	4.65E+07	[0000000100]
		ES	3.88E+07	[0000000001]	3.50E+07	[0000000001]
		MV	3.88E+07	[0000000001]	3.50E+07	[0000000001]
		GA	3.88E+07	[0000000001]	3.50E+07	[0000000001]
	3	ID	4.12E+07	[1001000001]	4.49E+07	[1001000001]
		IF	4.12E+07	[1001000001]	4.49E+07	[1001000001]
		ES	4.12E+07	[1001000001]	4.45E+07	[1001000001]
		MV	1.90E+07	[0000000111]	1.53E+07	[0000000111]
		GA	3.89E+07	[0110000001]	4.45E+07	[1010000001]
	5	ID	2.44E+07	[1101000011]	2.60E+07	[1101000011]
		IF	2.49E+07	[1101000101]	2.94E+07	[1101001001]
		ES	2.41E+07	[1100100011]	2.67E+07	[1100100011]
		MV	1.85E+07	[0000011111]	1.26E+07	[0000011111]
		GA	2.28E+07	[1010010011]	2.58E+07	[1101000011]
	8	ID	1.45E+07	[1111100111]	1.49E+07	[1111010111]

		IF	1.44E+07	[1111010111]	1.49E+07	[1111010111]
		ES	1.45E+07	[1111100111]	1.50E+07	[1111100111]
		MV	1.21E+07	[0011111111]	1.10E+07	[0011111111]
		GA	1.42E+07	[1110101111]	1.49E+07	[1111010111]
Case 5	1	ID	2.67E+07	[0000000001]	2.22E+07	[0000000001]
		IF	2.67E+07	[0000000001]	2.22E+07	[0000000001]
		ES	2.67E+07	[0000000001]	2.22E+07	[0000000001]
		MV	2.67E+07	[0000000001]	2.22E+07	[0000000001]
		GA	2.67E+07	[0000000001]	2.22E+07	[0000000001]
	3	ID	2.51E+07	[1100000001]	2.14E+07	[1100000001]
		IF	1.80E+07	[0010100001]	1.66E+07	[0011000001]
		ES	2.50E+07	[1100000001]	2.14E+07	[1100000001]
		MV	9.94E+06	[0000000111]	8.46E+06	[0000000111]
		GA	1.38E+07	[0100000011]	1.00E+07	[0000010101]
	5	ID	1.28E+07	[1110000011]	1.09E+07	[1110000011]
		IF	1.36E+07	[1101001001]	1.23E+07	[1110001001]
		ES	1.27E+07	[1110000011]	1.09E+07	[1110000011]
		MV	6.96E+06	[0000011111]	5.95E+06	[0000011111]
		GA	1.16E+07	[1100100011]	7.38E+06	[0100100111]
	8	ID	7.04E+06	[1111001111]	6.08E+06	[1111001111]
		IF	7.78E+06	[1111011101]	6.76E+06	[1111011101]
		ES	7.01E+06	[1111001111]	6.07E+06	[1111001111]
		MV	5.81E+06	[0011111111]	4.97E+06	[0011111111]
		GA	6.65E+06	[1101101111]	5.88E+06	[1110101111]

Table A-2: Optimal design, mechanical properties of set 1

			Objective Function F1		Objective Function F2	
Scenario	nd	Method	Cd	Arrangement	Cd	Arrangement
Case 6	1	ID	3.04E+07	[00000000010000000000]	2.64E+07	[00000000010000000000]
		IF	3.04E+07	[00000000010000000000]	1.43E+07	[00000000000000000100]
		MV	1.24E+07	[00000000000000000001]	1.28E+07	[00000000000000000001]
		GA	3.05E+07	[00000000010000000000]	2.63E+07	[00000000010000000000]
	5	ID	1.24E+07	[11001010010000000000]	6.53E+06	[00011100010000000001]
		IF	1.09E+07	[00110101000100000000]	7.40E+06	[00111000100000000010]
		MV	2.89E+06	[00000000000000011111]	2.86E+06	[00000000000000011111]
		GA	9.94E+06	[01010001010010000000]	7.10E+06	[10001100010000000100]
	10	ID	6.31E+06	[11111111011000000000]	4.64E+06	[11111110110000000001]
		IF	6.76E+06	[11111110110010000000]	3.09E+06	[01111111000000000111]
		MV	1.81E+06	[00000000001111111111]	1.72E+06	[00000000001111111111]
		GA	4.38E+06	[01110111101001000100]	2.94E+06	[00110111101011000001]
	15	ID	2.93E+06	[11111111111111100000]	2.10E+06	[11111111111100000111]
		IF	2.81E+06	[11111111111110100100]	1.99E+06	[11111111111000001111]
		MV	1.50E+06	[00000111111111111111]	1.36E+06	[00000111111111111111]
		GA	2.74E+06	[11111111111101010100]	2.09E+06	[11111111111010001101]
Case 7	1	ID	1.21E+07	[00000000000000000001]	1.12E+07	[00000000000000000001]
		IF	2.91E+07	[00000000010000000000]	2.48E+07	[00000000010000000000]
		MV	1.55E+07	[00000000000000010000]	1.38E+07	[00000000000000010000]
		GA	1.21E+07	[00000000000000000001]	1.13E+07	[00000000000000000001]
	5	ID	2.37E+07	[00000011110000000001]	3.49E+07	[00100001011000000001]
		IF	2.89E+07	[01010001001000000001]	4.09E+07	[11000100001000000001]
		MV	3.66E+06	[00000000000001111100]	3.18E+06	[00000000000001111100]
		GA	2.81E+07	[01001001001000000001]	3.24E+07	[00000101101000000001]
	10	ID	2.02E+07	[11110111110000000001]	2.82E+07	[11110101111000000001]
		IF	1.54E+07	[11101010111000000011]	2.16E+07	[11110110000011000011]
		MV	1.62E+07	[00000000001111111111]	1.83E+07	[00000000001111111111]
		GA	1.45E+07	[00101011101101000011]	2.52E+07	[00111011110110000001]
	15	ID	1.23E+07	[11111111111110000011]	1.55E+07	[11111111111110000011]
		IF	1.10E+07	[11111011111110000111]	1.37E+07	[11111011111110000111]
		MV	9.98E+06	[00000111111111111111]	1.17E+07	[00000111111111111111]
		GA	1.11E+07	[11111011111100011011]	1.37E+07	[11101111111100101011]
Case 8	1	ID	1.84E+07	[00000000000000000001]	1.49E+07	[00000000000000000001]
		IF	1.49E+07	[00000000000000100000]	2.04E+07	[00000000000000100000]
		MV	1.83E+07	[00000000000000000001]	1.50E+07	[00000000000000000001]
		GA	1.83E+07	[00000000000000000001]	1.50E+07	[00000000000000000001]
	5	ID	1.24E+07	[11100100000000000001]	1.23E+07	[11101000000000000001]
		IF	8.66E+06	[01011000000000001001]	7.48E+06	[01101000000000010001]
		MV	4.20E+06	[00000000000000011111]	3.42E+06	[00000000000000011111]

	GA	7.91E+06	[01001010000000000101]	5.69E+06	[00100010001000001001]
10	ID	4.80E+06	[11111110000000000111]	4.17E+06	[11111110000000000111]
	IF	4.62E+06	[11111100001000000111]	3.26E+06	[11111000000100001111]
	MV	2.52E+06	[00000000001111111111]	2.02E+06	[00000000001111111111]
	GA	4.88E+06	[11111011000000100011]	3.28E+06	[10011010111000001011]
15	ID	2.91E+06	[11111111110000011111]	2.26E+06	[11111111100000111111]
	IF	2.88E+06	[11111111100100011111]	2.17E+06	[11111110010100111111]
	MV	2.11E+06	[00000111111111111111]	1.65E+06	[00000111111111111111]
	GA	2.83E+06	[11111101100110101111]	2.18E+06	[11111110100100111111]

Table A-3: Optimal design, mechanical properties of set 2

Scenario	nd	Method	Objective Function F1		Objective Function F2	
			Cd	Arrangement	Cd	Arrangement
Case 1	1	ID	5.33E+07	[0000100000]	2.81E+07	[0000001000]
		IF	5.33E+07	[0000100000]	2.81E+07	[0000001000]
		ES	5.33E+07	[0000100000]	2.81E+07	[0000001000]
		MV	2.30E+07	[0000000001]	2.10E+07	[0000000001]
		GA	5.33E+07	[0000100000]	2.81E+07	[0000001000]
	3	ID	4.36E+07	[1100100000]	1.05E+07	[0001001010]
		IF	3.96E+07	[1010100000]	1.13E+07	[0001010010]
		ES	4.36E+07	[1100100000]	1.13E+07	[0001010010]
		MV	8.43E+06	[0000000111]	7.48E+06	[0000000111]
		GA	3.73E+07	[1100010000]	1.21E+07	[0010010010]
	5	ID	2.50E+07	[1111100000]	7.55E+06	[0011011010]
		IF	2.00E+07	[1111001000]	7.17E+06	[0011010101]
		ES	2.00E+07	[1111001000]	8.71E+06	[0110101010]
		MV	5.84E+06	[0000011111]	5.10E+06	[0000011111]
		GA	1.31E+07	[1011010100]	8.71E+06	[0110101010]
	8	ID	7.72E+06	[1111111010]	5.60E+06	[1111011110]
		IF	7.72E+06	[1111111010]	4.89E+06	[0111111011]
		ES	7.72E+06	[1111111010]	4.89E+06	[0111111011]
		MV	4.89E+06	[0011111111]	4.13E+06	[0011111111]
		GA	7.36E+06	[1111110101]	5.05E+06	[1011111101]
Case 2	1	ID	6.24E+07	[0000000001]	5.21E+07	[0000000001]
		IF	6.24E+07	[0000000001]	5.21E+07	[0000000001]
		ES	6.24E+07	[0000000001]	5.21E+07	[0000000001]
		MV	6.24E+07	[0000000001]	5.21E+07	[0000000001]
		GA	6.24E+07	[0000000001]	5.21E+07	[0000000001]
	3	ID	3.13E+07	[0000110001]	4.81E+07	[1100000001]
		IF	2.74E+07	[0010000011]	2.70E+07	[1000000011]
		ES	3.13E+07	[0001010001]	4.81E+07	[1100000001]
		MV	2.40E+07	[0000000111]	1.93E+07	[0000000111]
		GA	3.18E+07	[0100001001]	3.62E+07	[0100100001]
	5	ID	2.04E+07	[1000110011]	2.45E+07	[1110000011]
		IF	1.92E+07	[0011010011]	2.45E+07	[1110000011]
		ES	2.07E+07	[1001100011]	2.45E+07	[1110000011]
		MV	1.74E+07	[0000011111]	1.35E+07	[0000011111]
		GA	2.07E+07	[1001100011]	1.82E+07	[0100110011]
	8	ID	1.39E+07	[1011110111]	1.36E+07	[1111001111]
		IF	1.42E+07	[1110110111]	1.36E+07	[1111001111]
		ES	1.41E+07	[1111001111]	1.36E+07	[1111001111]
		MV	1.30E+07	[0011111111]	1.13E+07	[0011111111]

		GA	1.41E+07	[1111001111]	1.36E+07	[1111001111]
Case 3	1	ID	8.03E+07	[0000000001]	6.57E+07	[0000000001]
		IF	8.03E+07	[0000000001]	6.57E+07	[0000000001]
		ES	8.03E+07	[0000000001]	6.57E+07	[0000000001]
		MV	8.03E+07	[0000000001]	6.57E+07	[0000000001]
		GA	8.03E+07	[0000000001]	6.57E+07	[0000000001]
	3	ID	9.23E+07	[0011000001]	7.29E+07	[0011000001]
		IF	9.43E+07	[0100100001]	7.15E+07	[0100100001]
		ES	9.23E+07	[0011000001]	7.29E+07	[0011000001]
		MV	3.46E+07	[0000000111]	2.60E+07	[0000000111]
		GA	8.61E+07	[0010010001]	7.53E+07	[0110000010]
	5	ID	7.89E+07	[1111000001]	7.01E+07	[1111000001]
		IF	7.61E+07	[1110100001]	6.65E+07	[1110100001]
		ES	7.89E+07	[1111000001]	7.01E+07	[1111000001]
		MV	4.48E+07	[0000011111]	2.03E+07	[0000011111]
		GA	4.94E+07	[0001110011]	3.81E+07	[1100100011]
	8	ID	4.17E+07	[1111110011]	3.26E+07	[1111110011]
		IF	4.17E+07	[1111110011]	3.26E+07	[1111110011]
		ES	4.17E+07	[1111110011]	3.26E+07	[1111110011]
		MV	3.36E+07	[0011111111]	2.00E+07	[0011111111]
		GA	4.37E+07	[1111110101]	3.26E+07	[1111110011]
Case 4	1	ID	5.42E+08	[0000100000]	1.86E+08	[0000000100]
		IF	4.31E+07	[0000000010]	1.62E+08	[0000000010]
		ES	5.42E+08	[0000100000]	1.86E+08	[0000000100]
		MV	2.63E+07	[0000000001]	1.41E+08	[0000000001]
		GA	5.42E+08	[0000100000]	1.86E+08	[0000000100]
	3	ID	1.89E+08	[0000110001]	1.96E+08	[0000100101]
		IF	3.04E+08	[0010000101]	1.64E+08	[0110000001]
		ES	1.98E+08	[0001010001]	1.73E+08	[0001010001]
		MV	3.60E+08	[0000000111]	1.92E+08	[0000000111]
		GA	2.22E+08	[0000101001]	2.13E+08	[0010000101]
	5	ID	1.39E+08	[0011110001]	1.27E+08	[0001110101]
		IF	2.09E+08	[1101001001]	1.62E+08	[1110000101]
		ES	1.49E+08	[0101110001]	1.34E+08	[0101110001]
		MV	1.89E+08	[0000011111]	1.34E+08	[0000011111]
		GA	1.37E+08	[0111100010]	1.39E+08	[0010111001]
	8	ID	1.37E+08	[1111111001]	1.05E+08	[1111110101]
		IF	1.37E+08	[1111111001]	1.05E+08	[1111110101]
		ES	1.37E+08	[1111111001]	1.13E+08	[1111111001]
		MV	1.13E+08	[0011111111]	8.74E+07	[0011111111]
		GA	1.46E+08	[1101111101]	1.13E+08	[1111111001]
Case 5	1	ID	2.79E+08	[0000000001]	1.86E+08	[0000000001]
		IF	2.79E+08	[0000000001]	1.86E+08	[0000000001]

	ES	2.79E+08	[0000000001]	1.86E+08	[0000000001]
	MV	2.79E+08	[0000000001]	1.86E+08	[0000000001]
	GA	2.79E+08	[0000000001]	1.86E+08	[0000000001]
3	ID	1.62E+08	[0100100001]	1.54E+08	[0100100001]
	IF	1.46E+08	[0010000101]	1.30E+08	[0010000101]
	ES	1.51E+08	[0010010001]	1.54E+08	[0100100001]
	MV	1.17E+08	[0000000111]	9.58E+07	[0000000111]
	GA	1.86E+08	[1000010010]	1.34E+08	[0001010001]
5	ID	1.26E+08	[1100110001]	1.21E+08	[1100110001]
	IF	1.04E+08	[1010100011]	9.58E+07	[1010100011]
	ES	1.32E+08	[1101010001]	1.25E+08	[1101010001]
	MV	7.93E+07	[0000011111]	7.01E+07	[0000011111]
	GA	1.13E+08	[1010010101]	1.35E+08	[1110100001]
8	ID	7.62E+07	[1111110011]	7.29E+07	[1111110011]
	IF	7.53E+07	[1111011011]	7.06E+07	[1111101011]
	ES	7.62E+07	[1111110011]	7.29E+07	[1111110011]
	MV	6.04E+07	[0011111111]	5.55E+07	[0011111111]
	GA	7.15E+07	[1101111011]	6.84E+07	[1110111011]

Table A-4: Optimal design, mechanical properties of set 2

			Objective Function F1		Objective Function F2	
Scenario	nd	Method	Cd	Arrangement	Cd	Arrangement
Case 6	1	ID	7.44E+07	[00000001000000000000]	5.21E+07	[00000000100000000000]
		IF	7.44E+07	[00000001000000000000]	2.75E+07	[00000000000000010000]
		MV	2.39E+07	[00000000000000000001]	2.21E+07	[00000000000000000001]
		GA	7.44E+07	[00000001000000000000]	5.21E+07	[00000000100000000000]
	5	ID	2.18E+07	[10010101000100000000]	1.21E+07	[00101100100000000010]
		IF	2.66E+07	[01111000000010000000]	1.13E+07	[00101010010000000010]
		MV	5.32E+06	[00000000000000011111]	4.76E+06	[00000000000000011111]
		GA	2.48E+07	[01101000110000000000]	1.19E+07	[10010010010000000010]
	10	ID	1.29E+07	[11111111100100000000]	7.93E+06	[11111110101000000010]
		IF	1.19E+07	[11111111100000100000]	4.94E+06	[00111111010000010011]
		MV	3.28E+06	[00000000001111111111]	2.82E+06	[00000000001111111111]
		GA	1.07E+07	[11111101101000100000]	6.00E+06	[11011101110000000110]
	15	ID	4.76E+06	[11111111111110010010]	3.60E+06	[11111111111100000111]
		IF	4.76E+06	[11111111111110010010]	3.42E+06	[11111111111000001111]
		MV	2.70E+06	[00000111111111111111]	2.30E+06	[00000111111111111111]
		GA	4.61E+06	[11111111101111100100]	3.76E+06	[11111111111101000110]
Case 7	1	ID	6.11E+07	[00000000000000000001]	5.27E+07	[00000000000000000001]
		IF	7.45E+07	[00000000000000100000]	8.03E+07	[00000000000000100000]
		MV	6.11E+07	[00000000000000000001]	5.27E+07	[00000000000000000001]
		GA	6.11E+07	[00000000000000000001]	5.27E+07	[00000000000000000001]
	5	ID	3.27E+07	[11100000000001000001]	2.07E+07	[11000000000001000011]
		IF	3.23E+07	[11010000000000100001]	3.05E+07	[11100000000000100001]
		MV	1.50E+07	[00000000000000011111]	1.21E+07	[00000000000000011111]
		GA	2.42E+07	[00110000000010010001]	1.61E+07	[00001000000010100011]
	10	ID	1.32E+07	[11110000000011110011]	1.07E+07	[11110000000011001111]
		IF	1.53E+07	[11111100000001010101]	1.20E+07	[11111000000001011011]
		MV	8.78E+06	[00000000001111111111]	7.42E+06	[00000000001111111111]
		GA	1.34E+07	[10011110000001111001]	8.67E+06	[00011000001110011111]
	15	ID	8.25E+06	[11111000001111111111]	7.24E+06	[11111100000111111111]
		IF	8.82E+06	[11111110010010111111]	7.57E+06	[11111110000011111111]
		MV	6.88E+06	[00000111111111111111]	6.00E+06	[00000111111111111111]
		GA	7.71E+06	[01111000011111111111]	7.33E+06	[11111001100011111111]
Case 8	1	ID	7.06E+07	[00000000000000000001]	6.04E+07	[00000000000000000001]
		IF	1.15E+08	[00000000000001000000]	6.36E+07	[00000000000000000010]
		MV	7.06E+07	[00000000000000000001]	6.04E+07	[00000000000000000001]
		GA	7.06E+07	[00000000000000000001]	6.04E+07	[00000000000000000001]
	5	ID	6.95E+07	[00001011001000000001]	6.24E+07	[01001010001000000001]
		IF	7.50E+07	[00110010001000000001]	3.69E+07	[00000100110000000011]
		MV	1.88E+07	[00000000000000011111]	1.48E+07	[00000000000000011111]

	GA	6.78E+07	[00010010010100000001]	6.36E+07	[01100001001000000001]
10	ID	5.21E+07	[11111111001000000001]	5.00E+07	[11111111001000000001]
	IF	3.95E+07	[11011011100010000011]	3.32E+07	[11111001011000000011]
	MV	2.75E+07	[00000000001111111111]	1.05E+07	[00000000001111111111]
	GA	4.57E+07	[11111000110101000001]	3.28E+07	[00111111101000001001]
15	ID	2.82E+07	[11111111111110000011]	2.07E+07	[11111111111100000111]
	IF	2.82E+07	[11111111111110000011]	2.07E+07	[11111111111100000111]
	MV	2.05E+07	[00000111111111111111]	1.05E+07	[00000111111111111111]
	GA	2.61E+07	[11111101101111010011]	2.25E+07	[11111111111100100011]

Table A-5: Optimal design, mechanical properties of set 3

			Objective Function F1		Objective Function F2	
Scenario	nd	Method	Cd	Arrangement	Cd	Arrangement
Case 1	1	ID	1.54E+08	[0001000000]	5.92E+07	[0000001000]
		IF	1.54E+08	[0001000000]	5.92E+07	[0000001000]
		ES	1.54E+08	[0001000000]	5.92E+07	[0000001000]
		MV	4.45E+07	[0000000001]	4.42E+07	[0000000001]
		GA	1.54E+08	[0001000000]	5.92E+07	[0000001000]
	3	ID	6.24E+07	[0101100000]	2.22E+07	[0001001010]
		IF	7.87E+07	[1010100000]	2.39E+07	[0001010010]
		ES	7.53E+07	[0110100000]	2.39E+07	[0001010010]
		MV	1.64E+07	[0000000111]	1.59E+07	[0000000111]
		GA	7.53E+07	[0110100000]	3.32E+07	[0101000010]
	5	ID	5.31E+07	[1111100000]	1.68E+07	[0011101010]
		IF	5.31E+07	[1111100000]	1.84E+07	[0110101010]
		ES	5.31E+07	[1111100000]	1.84E+07	[0110101010]
		MV	1.14E+07	[0000011111]	1.07E+07	[0000011111]
		GA	2.53E+07	[1011010010]	1.59E+07	[0011011010]
	8	ID	1.60E+07	[1111111100]	1.23E+07	[1111101110]
		IF	1.60E+07	[1111111100]	1.26E+07	[1111110101]
		ES	1.60E+07	[1111111100]	1.26E+07	[1111110101]
		MV	9.72E+06	[0011111111]	8.71E+06	[0011111111]
		GA	1.49E+07	[1111110110]	1.26E+07	[1111110101]
Case 2	1	ID	1.06E+08	[0000000001]	8.63E+07	[0000000001]
		IF	1.06E+08	[0000000001]	8.63E+07	[0000000001]
		ES	1.06E+08	[0000000001]	8.63E+07	[0000000001]
		MV	1.06E+08	[0000000001]	8.63E+07	[0000000001]
		GA	1.06E+08	[0000000001]	8.63E+07	[0000000001]
	3	ID	6.75E+07	[0010100001]	7.93E+07	[1100000001]
		IF	5.38E+07	[0010000011]	4.51E+07	[1000000011]
		ES	7.88E+07	[1001000001]	7.93E+07	[1100000001]
		MV	4.01E+07	[0000000111]	3.22E+07	[0000000111]
		GA	5.17E+07	[0001000011]	3.76E+07	[0000100011]
	5	ID	4.35E+07	[1010100011]	4.01E+07	[1110000011]
		IF	4.57E+07	[1101000011]	4.01E+07	[1110000011]
		ES	4.73E+07	[1110000011]	4.01E+07	[1110000011]
		MV	2.86E+07	[0000011111]	2.22E+07	[0000011111]
		GA	4.42E+07	[0110100101]	3.12E+07	[0110001011]
	8	ID	2.94E+07	[1111100111]	2.25E+07	[1111001111]
		IF	2.94E+07	[1111100111]	2.25E+07	[1111001111]
		ES	2.94E+07	[1111100111]	2.25E+07	[1111001111]
		MV	2.47E+07	[0011111111]	1.84E+07	[0011111111]

		GA	2.94E+07	[1111100111]	2.25E+07	[1111001111]
Case 3	1	ID	1.15E+08	[0000000001]	9.89E+07	[0000000001]
		IF	1.15E+08	[0000000001]	9.89E+07	[0000000001]
		ES	1.15E+08	[0000000001]	9.89E+07	[0000000001]
		MV	1.15E+08	[0000000001]	9.89E+07	[0000000001]
		GA	1.15E+08	[0000000001]	9.89E+07	[0000000001]
	3	ID	1.41E+08	[0011000001]	1.47E+08	[0011000001]
		IF	1.41E+08	[0100100001]	1.54E+08	[1000100001]
		ES	1.41E+08	[0011000001]	1.47E+08	[0011000001]
		MV	4.78E+07	[0000000111]	3.88E+07	[0000000111]
		GA	1.52E+08	[0110000001]	1.37E+08	[0000110001]
	5	ID	1.11E+08	[0111100001]	1.39E+08	[1111000001]
		IF	1.22E+08	[1110100001]	1.39E+08	[1110100001]
		ES	1.11E+08	[0111100001]	1.39E+08	[1111000001]
		MV	5.66E+07	[0000011111]	3.02E+07	[0000011111]
		GA	1.08E+08	[1110001001]	7.93E+07	[1001100011]
	8	ID	7.59E+07	[1111111001]	7.44E+07	[1111110011]
		IF	7.59E+07	[1111111001]	7.44E+07	[1111110011]
		ES	7.59E+07	[1111111001]	7.44E+07	[1111110011]
		MV	5.00E+07	[0011111111]	3.88E+07	[0011111111]
		GA	7.59E+07	[1111111001]	7.44E+07	[1111110011]
Case 4	1	ID	2.99E+07	[0000000001]	2.61E+07	[0000000001]
		IF	3.32E+07	[0000000010]	2.96E+07	[0000000010]
		ES	2.99E+07	[0000000001]	2.61E+07	[0000000001]
		MV	2.99E+07	[0000000001]	2.61E+07	[0000000001]
		GA	2.99E+07	[0000000001]	2.61E+07	[0000000001]
	3	ID	5.14E+08	[0010010001]	4.21E+08	[0001010001]
		IF	5.59E+08	[0100100001]	5.77E+08	[1001000001]
		ES	5.14E+08	[0010010001]	4.21E+08	[0001010001]
		MV	1.31E+09	[0000000111]	1.03E+09	[0000000111]
		GA	5.59E+08	[0100100001]	4.58E+08	[0010010001]
	5	ID	4.43E+08	[1010011001]	3.72E+08	[1001011001]
		IF	4.88E+08	[1101001001]	5.70E+08	[1110000101]
		ES	4.57E+08	[1100101001]	3.44E+08	[1010110001]
		MV	4.88E+08	[0000011111]	4.04E+08	[0000011111]
		GA	4.63E+08	[0010010111]	3.72E+08	[1100110001]
	8	ID	3.14E+08	[1111111001]	2.83E+08	[1111111001]
		IF	3.14E+08	[1111111001]	2.83E+08	[1111111001]
		ES	3.14E+08	[1111111001]	2.83E+08	[1111111001]
		MV	7.72E+06	[0011111111]	6.65E+06	[0011111111]
		GA	3.60E+08	[1111011101]	2.98E+08	[1110111101]
Case 5	1	ID	5.31E+08	[0000000100]	4.17E+08	[0000000100]
		IF	5.70E+08	[0000000010]	3.42E+08	[0000000010]

	ES	5.31E+08	[0000000100]	4.17E+08	[0000000100]
	MV	2.52E+08	[0000000001]	1.59E+08	[0000000001]
	GA	5.31E+08	[0000000100]	4.17E+08	[0000000100]
3	ID	2.94E+08	[0000100101]	2.79E+08	[0000100101]
	IF	3.35E+08	[1000100001]	3.31E+08	[1000100001]
	ES	3.24E+08	[0100010001]	3.14E+08	[0100010001]
	MV	3.88E+08	[0000000111]	3.46E+08	[0000000111]
	GA	3.24E+08	[0100010001]	2.92E+08	[0010010001]
5	ID	2.57E+08	[1100100101]	2.52E+08	[1100100101]
	IF	2.13E+08	[1010100011]	2.09E+08	[1010100011]
	ES	2.57E+08	[1100101001]	2.55E+08	[1100101001]
	MV	2.09E+08	[0000011111]	1.96E+08	[0000011111]
	GA	2.52E+08	[1101010001]	2.52E+08	[1101010001]
8	ID	1.56E+08	[1110110111]	1.52E+08	[1110110111]
	IF	1.56E+08	[1110110111]	1.57E+08	[1111011011]
	ES	1.57E+08	[1110111011]	1.56E+08	[1110111011]
	MV	1.36E+08	[0011111111]	1.34E+08	[0011111111]
	GA	1.56E+08	[1110110111]	1.74E+08	[1110111101]

Table A-6: Optimal design, mechanical properties of set 3

			Objective Function F1		Objective Function F2	
Scenario	nd	Method	Cd	Arrangement	Cd	Arrangement
Case 6	1	ID	1.56E+08	[00000001000000000000]	1.11E+08	[00000000100000000000]
		IF	1.56E+08	[00000001000000000000]	5.44E+07	[00000000000000001000]
		MV	4.81E+07	[00000000000000000001]	4.75E+07	[00000000000000000001]
		GA	1.56E+08	[00000001000000000000]	1.11E+08	[00000000100000000000]
	5	ID	4.61E+07	[00111001010000000000]	2.50E+07	[00011100100000000001]
		IF	4.61E+07	[00111010000100000000]	2.81E+07	[00110101000000000100]
		MV	1.08E+07	[00000000000000011111]	1.03E+07	[00000000000000011111]
		GA	4.70E+07	[00111000110000000000]	2.55E+07	[10001100010000000010]
	10	ID	2.69E+07	[11111111010100000000]	1.68E+07	[11111101110000000001]
		IF	2.70E+07	[11111111100010000000]	1.31E+07	[01111111010000000101]
		MV	6.76E+06	[00000000001111111111]	6.19E+06	[00000000001111111111]
		GA	1.81E+07	[01111111001100001000]	1.21E+07	[11001111100100001100]
	15	ID	1.02E+07	[11111111111111001000]	7.67E+06	[11111111111100000111]
		IF	9.98E+06	[11111111111110100010]	7.29E+06	[11111111111000001111]
		MV	5.51E+06	[00000111111111111111]	4.89E+06	[00000111111111111111]
		GA	9.32E+06	[11111111111011000101]	7.10E+06	[11111111100110011011]
Case 7	1	ID	1.08E+08	[00000000000000000001]	8.91E+07	[00000000000000000001]
		IF	1.65E+08	[00000000000000100000]	1.39E+08	[00000000000000100000]
		MV	1.08E+08	[00000000000000000001]	8.91E+07	[00000000000000000001]
		GA	1.08E+08	[00000000000000000001]	8.91E+07	[00000000000000000001]
	5	ID	6.55E+07	[11100000001000000001]	3.62E+07	[11000000000100000011]
		IF	5.75E+07	[01101000000001000001]	5.21E+07	[11100000000001000001]
		MV	2.53E+07	[00000000000000011111]	2.04E+07	[00000000000000011111]
		GA	7.61E+07	[11100100000000000001]	3.69E+07	[10010000001000000110]
	10	ID	2.94E+07	[11111100001000100011]	2.07E+07	[11111000000101000111]
		IF	2.89E+07	[11111100000010010011]	2.00E+07	[11111000000010010111]
		MV	1.58E+07	[00000000001111111111]	1.24E+07	[00000000001111111111]
		GA	2.62E+07	[11110010001010000111]	1.75E+07	[01110100001010001111]
	15	ID	1.72E+07	[11111110001110101111]	1.26E+07	[11111110000101111111]
		IF	1.75E+07	[11111111001010101111]	1.25E+07	[11111110000011111111]
		MV	1.33E+07	[00000111111111111111]	1.00E+07	[00000111111111111111]
		GA	1.76E+07	[11111111010100101111]	1.31E+07	[11111110001011101111]
Case 8	1	ID	1.13E+08	[00000000000000000001]	9.89E+07	[00000000000000000001]
		IF	1.76E+08	[00000000000001000000]	1.44E+08	[00000000000000100000]
		MV	1.13E+08	[00000000000000000001]	9.89E+07	[00000000000000000001]
		GA	1.13E+08	[00000000000000000001]	9.89E+07	[00000000000000000001]
	5	ID	1.13E+08	[00010010101000000001]	1.34E+08	[10001001001000000001]
		IF	1.20E+08	[00101001010000000001]	1.30E+08	[01001001001000000001]
		MV	2.96E+07	[00000000000000011111]	2.40E+07	[00000000000000011111]

	GA	1.10E+08	[00000101101000000001]	1.10E+08	[00001110000001000001]
10	ID	8.42E+07	[11110111101000000001]	6.92E+07	[11111001101000000011]
	IF	6.14E+07	[01110101111000000011]	6.92E+07	[11110110001100000011]
	MV	3.67E+07	[00000000001111111111]	1.75E+07	[00000000001111111111]
	GA	7.10E+07	[11110010011011000001]	9.77E+07	[11111011101000000001]
15	ID	4.60E+07	[11111111111110000011]	4.45E+07	[11111111111100000111]
	IF	4.60E+07	[11111111111110000011]	4.45E+07	[11111111111100000111]
	MV	3.07E+07	[00000111111111111111]	2.96E+07	[00000111111111111111]
	GA	3.90E+07	[11011111101111000111]	4.23E+07	[11101111111010100111]

Table A-7: Results for mechanical set 1 and objective function F1

		CPU time (sec)					Objective Value F1 x 1E7				
Scenario	nd	ID	IF	ES	MV	GA	ID	IF	ES	MV	GA
Case 1	1	49.0	78.9	48.2	4.8	92.2	10.1	10.1	10.1	12.4	10.1
	3	132.0	107.7	762.0	4.7	86.7	8.7	8.7	8.7	12.1	9.6
	5	197.5	126.8	1490.6	4.7	88.0	8.6	8.2	8.2	11.9	8.6
	8	248.5	82.1	257.2	4.1	84.6	9.5	9.5	9.5	10.6	9.7
Case 2	1	72.8	98.2	77.4	6.2	121.9	43.3	49.2	43.3	43.3	43.3
	3	192.8	175.1	893.5	5.8	117.6	20.0	21.2	20.0	42.4	27.2
	5	295.6	194.5	1834.9	5.4	175.2	18.7	19.3	18.7	29.0	21.7
	8	380.0	133.9	326.2	6.1	115.2	20.5	20.5	20.5	24.5	20.5
Case 3	1	73.3	99.6	73.2	5.8	123.2	15.9	15.9	15.9	15.9	15.9
	3	236.5	205.8	1072.8	6.9	163.7	15.1	15.1	15.1	16.2	15.1
	5	380.8	256.6	2132.6	6.1	151.9	15.4	15.4	15.4	16.4	15.5
	8	526.1	166.8	362.6	6.4	136.4	15.9	15.9	15.9	16.5	15.9
Case 4	1	175.0	230.8	192.6	10.8	224.9	81.9	85.3	81.9	81.9	81.9
	3	506.0	430.9	3264.3	10.3	250.1	57.3	57.3	57.3	77.2	59.4
	5	798.6	491.5	6216.6	14.2	271.1	59.4	60.9	59.3	69.1	60.1
	8	1065.7	394.5	1109.2	12.8	263.3	63.5	63.6	63.5	66.9	64.2
Case 5	1	183.8	239.9	203.0	10.3	209.7	59.4	59.4	59.4	59.4	59.4
	3	451.4	406.6	2202.3	10.3	207.7	60.1	65.1	60.1	63.4	61.5
	5	682.3	433.9	4332.7	10.3	200.8	63.1	67.0	63.1	66.6	64.1
	8	887.0	282.8	1096.7	10.3	205.2	67.5	70.3	67.5	69.4	68.0
Case 6	1	247.9	224.7	-	8.1	323.2	17.7	17.7	-	21.4	17.7
	5	1156.0	667.7	-	7.7	322.3	14.7	13.9	-	21.3	15.1
	10	1996.8	978.1	-	7.8	303.9	13.7	13.6	-	20.6	15.3
	15	2511.8	782.3	-	7.8	805.0	15.5	15.6	-	18.4	15.7
Case 7	1	427.1	522.9	-	16.2	658.6	85.2	107.0	-	179.9	85.2
	5	1981.4	1433.3	-	15.2	651.4	30.7	32.0	-	93.8	31.8
	10	3621.3	1969.0	-	15.2	1256.6	30.7	33.3	-	46.4	34.6
	15	4116.1	1566.8	-	14.3	999.6	34.0	35.6	-	41.1	37.4
Case 8	1	466.7	484.4	-	16.2	681.1	29.6	62.5	-	29.6	29.6
	5	2862.0	1745.2	-	15.2	766.9	26.6	27.1	-	30.4	27.1
	10	5704.5	2721.5	-	16.2	2843.1	27.0	27.2	-	29.5	27.3
	15	6486.4	2161.7	-	16.2	881.7	27.9	27.9	-	29.1	28.2

Table A-8: Results for mechanical set 1 and objective function F2

		CPU time (sec)					Objective Value F1 x 1E7				
Scenario	nd	ID	IF	ES	MV	GA	ID	IF	ES	MV	GA
Case 1	1	46.3	75.6	51.4	4.3	85.9	97.9	107.8	97.9	98.3	97.9
	3	122.8	101.3	578.7	4.3	87.0	95.1	94.9	94.8	98.9	95.1
	5	182.0	116.4	1270.0	4.6	83.3	94.6	94.7	94.5	100.0	94.7
	8	241.5	83.7	218.9	4.6	86.4	95.0	95.0	95.0	96.4	95.2
Case 2	1	76.2	97.4	75.0	6.1	124.8	525.9	625.0	525.9	525.9	525.9
	3	201.4	163.5	871.0	5.8	121.3	278.6	281.7	278.6	528.6	296.6
	5	290.0	195.0	1797.3	6.1	125.3	271.5	285.3	271.5	379.8	271.5
	8	371.8	124.5	318.8	6.2	115.5	292.8	292.8	292.8	334.0	301.4
Case 3	1	72.1	94.7	73.5	6.2	123.0	254.8	254.8	254.8	254.8	254.8
	3	203.2	162.7	880.6	5.9	124.8	252.8	255.7	252.8	259.9	258.7
	5	299.4	194.7	1837.1	5.9	122.7	256.2	256.2	256.2	266.1	260.0
	8	392.6	134.7	318.4	5.4	119.0	263.5	263.6	263.5	268.1	263.8
Case 4	1	160.7	236.7	191.1	11.1	199.5	1833.8	2061.3	1833.8	1833.8	1833.8
	3	423.5	394.8	2127.8	11.0	199.2	1558.3	1558.3	1558.3	1823.5	1574.8
	5	626.4	417.5	4139.4	10.3	299.3	1615.3	1676.2	1608.7	1797.4	1615.3
	8	828.4	284.8	904.8	9.1	194.3	1730.0	1730.0	1730.0	1805.9	1730.0
Case 5	1	167.7	218.9	204.2	9.7	199.6	1466.9	1466.9	1466.9	1466.9	1466.9
	3	439.5	376.4	2329.1	10.3	208.2	1459.5	1549.1	1459.5	1544.5	1578.4
	5	650.6	430.6	4675.2	10.4	389.8	1519.6	1581.2	1519.6	1602.7	1569.6
	8	865.7	295.2	865.9	10.3	212.6	1609.7	1667.1	1609.7	1652.6	1615.4
Case 6	1	238.3	195.9	-	8.2	323.9	321.1	336.5	-	323.9	321.1
	5	1108.2	730.8	-	7.7	462.9	303.8	303.7	-	333.8	305.9
	10	1900.0	937.5	-	7.7	314.9	303.6	305.5	-	339.8	307.4
	15	2380.0	788.9	-	8.2	487.7	306.0	306.9	-	320.0	307.0
Case 7	1	420.0	504.1	-	17.1	673.9	1997.7	2653.6	-	4059.5	1997.7
	5	1964.7	1345.5	-	15.2	655.9	910.6	930.5	-	2299.3	916.4
	10	3351.1	1893.6	-	15.2	654.5	943.7	1025.7	-	1236.2	1000.3
	15	4223.4	1595.0	-	16.3	654.7	995.5	1016.3	-	1137.1	1047.9
Case 8	1	436.4	506.9	-	17.1	679.5	928.6	1627.9	-	928.6	928.6
	5	2040.7	1346.0	-	16.1	1295.6	870.3	888.6	-	940.8	898.9
	10	4023.3	1896.7	-	15.3	641.5	885.9	893.6	-	938.7	908.6
	15	4573.1	1597.2	-	16.2	660.0	909.6	911.9	-	931.9	911.3

Table A-9: Results for mechanical set 2 and objective function F1

		CPU time (sec)					Objective Value F1 x 1E7				
Scenario	nd	ID	IF	ES	MV	GA	ID	IF	ES	MV	GA
Case 1	1	47.6	58.9	58.9	4.6	88.2	4.7	4.7	4.7	5.1	4.7
	3	130.5	88.9	572.0	4.6	128.0	4.3	4.3	4.3	4.9	4.3
	5	192.5	113.0	1187.9	4.6	85.9	4.3	4.3	4.3	4.7	4.4
	8	250.0	83.0	263.8	4.1	82.6	4.4	4.4	4.4	4.6	4.4
Case 2	1	80.3	107.3	76.3	6.1	127.1	2.4	2.4	2.4	2.4	2.4
	3	247.5	213.0	1051.6	6.1	144.8	2.4	2.4	2.4	2.4	2.4
	5	371.8	237.5	2271.4	6.9	152.1	2.4	2.4	2.4	2.4	2.4
	8	500.3	186.4	460.7	8.1	166.8	2.4	2.4	2.4	2.4	2.4
Case 3	1	77.8	87.6	74.3	6.9	125.1	5.0	5.0	5.0	5.0	5.0
	3	256.9	197.1	1038.2	6.3	128.5	3.3	3.5	3.3	4.8	3.5
	5	388.9	240.2	2465.1	10.2	163.0	3.2	3.2	3.2	4.5	3.7
	8	513.8	176.0	469.7	8.7	169.8	3.5	3.5	3.5	3.9	3.5
Case 4	1	184.5	254.9	202.8	12.8	221.7	38.6	44.2	38.6	47.4	38.6
	3	478.3	390.4	2649.0	9.7	183.3	28.4	31.0	28.3	37.6	28.6
	5	669.6	457.8	4925.3	9.1	253.4	27.2	28.0	27.2	31.2	31.5
	8	845.5	293.5	883.6	10.3	194.6	27.0	27.0	27.0	29.1	27.8
Case 5	1	191.6	242.0	212.9	10.3	235.7	19.3	19.3	19.3	19.3	19.3
	3	459.2	404.2	1958.1	9.7	183.4	12.3	13.0	12.2	15.6	14.9
	5	673.1	475.9	4816.5	9.1	196.9	12.4	12.7	12.4	13.8	12.7
	8	867.7	295.6	904.3	10.3	296.6	12.8	12.9	12.8	13.2	12.9
Case 6	1	252.7	203.4	-	8.5	324.0	7.1	7.1	-	7.9	7.1
	5	1155.5	619.0	-	8.1	469.8	6.4	6.2	-	7.7	6.4
	10	1975.3	872.3	-	7.6	793.6	6.3	6.3	-	7.7	6.4
	15	2490.3	729.1	-	8.1	309.4	6.7	6.7	-	7.2	6.7
Case 7	1	441.0	597.5	-	16.2	680.9	4.5	4.9	-	4.5	4.5
	5	2392.3	1707.1	-	17.1	1325.9	4.0	4.1	-	4.5	4.1
	10	4320.9	2646.8	-	19.0	932.3	4.1	4.1	-	4.1	4.1
	15	5523.5	2250.9	-	21.1	1400.8	4.1	4.1	-	4.2	4.1
Case 8	1	438.8	487.8	-	14.2	670.8	10.3	10.9	-	10.3	10.3
	5	2448.3	1547.1	-	17.1	827.8	6.1	6.2	-	10.0	6.2
	10	4398.2	2301.4	-	20.0	921.7	6.0	6.4	-	8.6	6.4
	15	5557.3	1897.6	-	20.3	1380.1	6.4	6.4	-	7.5	6.8

Table A-10: Results for mechanical set 2 and objective function F2

		CPU time (sec)					Objective Value F1 x 1E7				
Scenario	nd	ID	IF	ES	MV	GA	ID	IF	ES	MV	GA
Case 1	1	47.3	76.3	52.7	4.9	86.3	40.1	40.1	40.1	40.5	40.1
	3	129.0	106.1	604.3	4.6	83.7	39.8	39.8	39.8	40.1	39.8
	5	191.1	120.4	1332.3	4.6	83.7	39.7	39.7	39.7	39.8	39.7
	8	247.2	84.1	225.1	4.6	85.2	39.7	39.7	39.7	39.7	39.7
Case 2	1	75.9	101.7	69.8	6.2	117.5	41.5	41.5	41.5	41.5	41.5
	3	211.8	170.3	894.2	6.4	121.2	41.6	41.9	41.6	42.3	42.4
	5	316.0	191.6	1927.7	6.6	129.9	42.1	42.1	42.1	43.1	42.9
	8	409.2	146.8	328.2	5.9	123.8	43.2	43.2	43.2	43.9	43.2
Case 3	1	76.9	99.3	73.1	6.2	118.4	65.6	65.6	65.6	65.6	65.6
	3	204.4	165.0	887.0	6.2	120.4	55.9	57.3	55.9	65.9	63.4
	5	305.8	187.8	1786.6	5.8	115.4	55.4	56.1	55.4	65.4	61.1
	8	392.1	130.7	312.2	5.9	115.8	59.8	59.8	59.8	63.9	59.8
Case 4	1	146.7	226.3	181.7	8.0	171.0	1035.1	1045.2	1035.1	1068.5	1035.1
	3	395.7	366.6	2469.5	8.5	171.8	746.2	810.0	704.7	922.9	777.6
	5	594.2	435.0	4928.6	9.7	178.6	722.9	780.2	695.7	794.5	703.7
	8	789.4	263.9	813.4	9.8	195.8	724.1	724.1	707.8	750.0	707.8
Case 5	1	160.3	222.1	181.3	9.2	190.8	503.6	503.6	503.6	503.6	503.6
	3	442.2	382.7	2030.6	9.7	187.2	360.8	383.3	360.8	448.8	362.3
	5	654.2	438.6	4662.6	9.7	203.9	365.8	374.3	364.1	408.1	366.2
	8	855.0	288.3	826.8	10.9	206.6	377.5	380.0	377.5	392.9	382.2
Case 6	1	243.3	232.1	-	8.2	313.2	121.9	133.3	-	123.1	121.9
	5	1105.8	593.1	-	8.3	312.0	118.3	118.4	-	123.9	118.6
	10	1912.7	854.4	-	8.7	306.9	118.4	118.6	-	126.5	118.4
	15	2397.3	840.8	-	8.2	624.0	118.7	118.8	-	121.6	119.0
Case 7	1	436.0	509.6	-	17.1	654.3	144.7	168.2	-	144.7	144.7
	5	2011.4	1348.8	-	15.3	681.2	140.4	140.6	-	145.4	141.2
	10	3515.8	2005.8	-	16.3	648.3	141.7	142.3	-	143.3	142.9
	15	4412.5	1867.2	-	17.3	682.0	143.8	144.2	-	146.8	144.4
Case 8	1	436.7	499.1	-	16.1	648.1	257.1	261.4	-	257.1	257.1
	5	1933.2	1350.6	-	18.1	942.3	210.2	219.3	-	260.3	210.8
	10	3419.4	2143.3	-	13.3	634.0	216.0	215.9	-	248.3	221.0
	15	4233.3	1916.5	-	13.4	635.2	221.3	221.3	-	240.3	223.6

Table A-11: Results for mechanical set 3 and objective function F1

		CPU time (sec)					Objective Value F1 x 1E7				
Scenario	nd	ID	IF	ES	MV	GA	ID	IF	ES	MV	GA
Case 1	1	48.0	74.8	59.8	4.9	90.0	5.4	5.4	5.4	6.2	5.4
	3	135.5	137.2	600.2	4.6	133.0	5.0	4.9	4.9	5.9	4.9
	5	199.6	134.9	1516.1	4.9	88.9	4.8	4.8	4.8	5.7	5.2
	8	257.5	82.6	211.4	4.3	87.0	5.2	5.2	5.2	5.5	5.2
Case 2	1	69.8	96.4	73.0	5.8	120.9	4.6	4.6	4.6	4.6	4.6
	3	214.0	177.1	963.3	6.3	133.3	4.4	4.5	4.4	4.7	4.5
	5	344.2	260.9	2246.3	5.8	154.5	4.4	4.4	4.4	4.7	4.5
	8	497.4	205.2	473.3	6.2	166.9	4.5	4.5	4.5	4.7	4.5
Case 3	1	77.4	99.2	74.8	6.9	125.3	8.4	8.4	8.4	8.4	8.4
	3	224.1	179.2	1003.1	6.2	148.8	5.5	5.8	5.5	8.3	5.9
	5	339.7	200.0	2361.7	8.8	158.4	5.2	5.3	5.2	7.8	6.1
	8	447.0	152.4	446.5	9.0	232.3	5.8	5.8	5.8	6.8	5.8
Case 4	1	174.0	246.9	192.2	9.8	191.7	86.2	93.5	86.2	86.2	86.2
	3	436.8	419.4	2057.7	9.7	184.7	60.0	62.2	60.0	94.7	62.2
	5	658.7	465.2	5046.8	9.7	204.6	58.8	58.8	58.3	68.8	63.5
	8	1027.3	287.7	791.2	10.9	298.2	59.7	59.7	59.7	95.6	60.4
Case 5	1	206.3	237.0	202.3	8.5	230.6	61.9	68.7	61.9	97.6	61.9
	3	580.7	434.7	2358.8	10.9	203.2	25.5	24.9	24.5	35.5	24.5
	5	788.5	500.5	4399.2	9.1	190.9	25.5	26.2	24.9	28.4	25.2
	8	978.4	335.2	949.8	9.7	196.0	26.8	26.8	26.4	27.3	26.8
Case 6	1	230.9	229.7	-	8.2	328.3	8.3	8.3	-	9.8	8.3
	5	1175.5	657.2	-	8.2	338.7	7.3	7.2	-	9.7	7.3
	10	1989.2	904.2	-	8.1	475.4	7.2	7.2	-	9.7	7.6
	15	3079.2	816.3	-	8.2	326.2	7.9	7.9	-	8.8	8.0
Case 7	1	590.4	550.1	-	16.2	669.0	8.3	9.7	-	8.3	8.3
	5	3160.8	1796.1	-	17.1	845.2	7.6	7.6	-	8.6	7.7
	10	6004.7	2838.0	-	16.4	1445.8	7.7	7.7	-	8.0	7.7
	15	8066.9	2549.5	-	17.2	1457.6	7.8	7.8	-	8.0	7.8
Case 8	1	596.9	505.1	-	17.9	649.6	16.1	17.8	-	16.1	16.1
	5	3143.1	1459.4	-	16.2	841.1	9.4	9.4	-	16.1	9.5
	10	5618.2	2101.7	-	22.0	875.9	9.4	9.8	-	13.7	10.4
	15	7092.7	1882.5	-	20.2	884.2	10.0	10.0	-	11.9	10.6

Table A-12: Results for mechanical set 3 and objective function F2

		CPU time (sec)					Objective Value F1 x 1E7				
Scenario	nd	ID	IF	ES	MV	GA	ID	IF	ES	MV	GA
Case 1	1	55.4	51.5	59.4	4.6	84.8	50.2	50.2	50.2	50.7	50.2
	3	169.1	92.2	626.6	4.3	85.4	49.9	49.9	49.9	50.3	50.0
	5	253.5	113.5	1506.6	4.6	86.7	49.9	49.9	49.9	50.0	49.9
	8	340.8	78.8	240.8	4.9	81.8	49.9	49.9	49.9	49.9	49.9
Case 2	1	68.2	95.1	72.7	5.7	119.9	75.1	75.1	75.1	75.1	75.1
	3	186.5	168.0	883.3	6.2	125.3	75.0	75.6	75.0	76.4	76.2
	5	276.8	199.0	1929.4	5.9	125.3	76.0	76.0	76.0	77.9	76.9
	8	359.3	154.3	339.2	6.7	122.1	78.0	78.0	78.0	79.2	78.0
Case 3	1	69.5	96.0	73.8	5.8	119.0	108.9	108.9	108.9	108.9	108.9
	3	182.8	161.9	844.3	6.2	115.7	86.7	92.4	86.7	109.7	92.7
	5	264.6	184.9	2152.9	5.6	110.6	84.8	86.2	84.8	109.4	97.5
	8	344.9	124.3	302.8	5.5	110.5	94.3	94.3	94.3	106.0	94.3
Case 4	1	183.1	241.7	210.5	9.8	200.5	2074.8	2219.0	2074.8	2074.8	2074.8
	3	587.7	399.9	2138.8	9.7	196.1	1466.3	1679.2	1466.3	2364.2	1471.5
	5	856.5	457.1	5333.8	10.9	201.6	1476.9	1710.2	1472.7	1718.2	1473.4
	8	1049.8	299.8	907.0	10.3	198.2	1522.7	1522.7	1522.7	2260.9	1553.5
Case 5	1	197.9	226.4	192.6	8.5	191.6	1583.2	1697.0	1583.2	2251.7	1583.2
	3	473.4	391.9	2157.5	10.5	218.9	692.7	667.6	661.0	961.4	661.1
	5	795.6	441.2	4947.1	10.9	194.7	690.8	703.2	673.7	774.5	676.6
	8	1011.0	296.6	881.4	9.1	188.9	723.7	716.8	713.6	739.5	728.4
Case 6	1	230.1	208.9	-	8.3	315.8	153.0	162.6	-	155.9	153.0
	5	1051.7	596.8	-	7.7	322.4	148.9	148.8	-	158.1	149.2
	10	1827.7	876.6	-	8.3	474.8	148.8	148.8	-	163.2	149.7
	15	2298.9	816.8	-	7.3	337.5	149.8	149.9	-	154.7	150.7
Case 7	1	506.4	471.8	-	15.2	638.2	257.6	323.0	-	257.6	257.6
	5	2284.1	1389.6	-	15.2	668.8	254.0	254.7	-	267.5	259.3
	10	3967.9	2026.9	-	16.2	707.1	257.5	257.2	-	263.6	258.3
	15	4777.8	1681.6	-	14.4	991.5	261.1	261.0	-	265.4	262.0
Case 8	1	495.5	503.4	-	15.2	640.2	399.6	506.5	-	399.6	399.6
	5	2196.1	1314.5	-	16.6	1254.6	307.2	307.6	-	411.8	321.1
	10	3786.0	1832.3	-	15.5	636.1	319.6	322.7	-	394.5	319.8
	15	4658.7	1608.2	-	12.9	1887.4	329.6	329.6	-	373.7	340.5

Appendix B: Results for Optimization of Damping Coefficients

Table B-1: Number of function calls and objective values found for Material Set 1

		Objevtive value			Number of function calls		
Case	nd	GA	MADS	RAGS	GA	MADS	RAGS
1	1	1.02E-06	1.01E-06	1.01E-06	3.80E+02	2.72E+02	2.03E+02
	2	9.27E-07	8.92E-07	8.88E-07	1.03E+03	9.30E+02	8.37E+02
	3	8.89E-07	8.73E-07	8.66E-07	1.87E+03	1.85E+03	1.46E+03
	4	8.59E-07	8.36E-07	8.40E-07	2.85E+03	3.01E+03	2.30E+03
	5	8.30E-07	8.21E-07	8.20E-07	3.90E+03	4.45E+03	3.06E+03
	6	8.23E-07	8.23E-07	8.04E-07	4.95E+03	6.22E+03	3.89E+03
	7	8.21E-07	8.22E-07	8.02E-07	5.93E+03	8.19E+03	4.75E+03
	8	8.40E-07	8.25E-07	8.04E-07	6.77E+03	1.03E+04	5.60E+03
	9	8.38E-07	8.22E-07	8.07E-07	7.40E+03	1.13E+04	6.21E+03
	10	8.53E-07	8.41E-07	8.09E-07	3.50E+02	6.95E+02	5.52E+02
2	1	4.36E-06	4.33E-06	4.33E-06	3.80E+02	2.79E+02	1.70E+02
	2	1.79E-06	1.72E-06	1.72E-06	1.10E+03	8.73E+02	5.71E+02
	3	1.90E-06	1.72E-06	1.72E-06	2.00E+03	1.88E+03	1.66E+03
	4	1.87E-06	1.72E-06	1.71E-06	2.98E+03	3.68E+03	2.97E+03
	5	1.76E-06	1.72E-06	1.73E-06	4.06E+03	6.00E+03	4.36E+03
	6	1.77E-06	1.72E-06	1.73E-06	5.11E+03	9.00E+03	6.38E+03
	7	1.84E-06	1.71E-06	1.73E-06	6.16E+03	1.31E+04	9.19E+03
	8	1.88E-06	1.71E-06	1.74E-06	7.00E+03	1.66E+04	1.16E+04
	9	1.89E-06	1.73E-06	1.74E-06	7.63E+03	1.87E+04	1.35E+04
	10	1.96E-06	1.71E-06	1.77E-06	3.50E+02	1.51E+03	1.21E+03
3	1	1.67E-06	1.59E-06	1.59E-06	3.70E+02	2.66E+02	1.64E+02
	2	1.62E-06	1.53E-06	1.52E-06	1.02E+03	7.29E+02	7.24E+02
	3	1.56E-06	1.52E-06	1.51E-06	1.86E+03	1.37E+03	1.72E+03
	4	1.54E-06	1.51E-06	1.51E-06	2.84E+03	2.03E+03	2.81E+03
	5	1.54E-06	1.52E-06	1.52E-06	3.89E+03	2.93E+03	3.91E+03
	6	1.55E-06	1.52E-06	1.52E-06	4.94E+03	4.13E+03	5.23E+03
	7	1.54E-06	1.52E-06	1.52E-06	5.92E+03	5.78E+03	6.44E+03
	8	1.54E-06	1.52E-06	1.53E-06	6.76E+03	8.01E+03	7.79E+03
	9	1.54E-06	1.54E-06	1.53E-06	7.39E+03	9.67E+03	8.85E+03
	10	1.57E-06	1.52E-06	1.54E-06	3.50E+02	1.81E+03	5.13E+02
4	1	8.24E-06	8.21E-06	8.19E-06	4.05E+02	3.22E+02	1.54E+02
	2	5.87E-06	5.58E-06	5.56E-06	1.07E+03	1.05E+03	6.40E+02
	3	5.57E-06	5.48E-06	5.47E-06	1.92E+03	2.19E+03	1.25E+03
	4	5.57E-06	5.45E-06	5.44E-06	2.94E+03	3.96E+03	1.85E+03
	5	5.47E-06	5.44E-06	5.43E-06	3.99E+03	6.22E+03	2.60E+03

	6	5.46E-06	5.43E-06	5.42E-06		5.04E+03	9.05E+03	3.40E+03
	7	5.46E-06	5.42E-06	5.42E-06		6.02E+03	1.18E+04	4.17E+03
	8	5.44E-06	5.42E-06	5.43E-06		6.86E+03	1.45E+04	4.94E+03
	9	5.53E-06	5.42E-06	5.45E-06		7.49E+03	1.76E+04	5.67E+03
	10	5.66E-06	5.43E-06	5.46E-06		3.50E+02	1.93E+03	4.87E+02
5	1	5.96E-06	5.94E-06	5.94E-06		3.85E+02	3.12E+02	1.53E+02
	2	5.86E-06	5.86E-06	5.87E-06		1.05E+03	1.01E+03	7.56E+02
	3	5.94E-06	5.86E-06	5.87E-06		1.93E+03	2.10E+03	1.77E+03
	4	5.89E-06	5.86E-06	5.87E-06		2.93E+03	4.35E+03	3.42E+03
	5	6.17E-06	5.86E-06	5.88E-06		3.98E+03	7.13E+03	5.17E+03
	6	5.96E-06	5.86E-06	5.89E-06		5.03E+03	1.18E+04	7.05E+03
	7	6.22E-06	5.86E-06	5.89E-06		6.01E+03	1.61E+04	8.84E+03
	8	5.97E-06	5.86E-06	5.91E-06		6.85E+03	2.10E+04	1.05E+04
	9	6.14E-06	5.86E-06	5.92E-06		7.48E+03	2.58E+04	1.16E+04
	10	6.40E-06	5.86E-06	5.92E-06		3.50E+02	3.55E+03	8.20E+02

Table B-2: Number of function calls and objective values found for Material Set 2

		Objevtive value			Number of function calls		
Case	nd	GA	MADS	RAGS	GA	MADS	RAGS
1	1	4.81E-07	4.69E-07	4.68E-07	4.25E+02	2.98E+02	1.53E+02
	2	4.65E-07	4.49E-07	4.33E-07	1.06E+03	8.71E+02	5.75E+02
	3	4.46E-07	4.27E-07	4.21E-07	1.90E+03	1.88E+03	1.42E+03
	4	4.33E-07	4.17E-07	4.12E-07	2.88E+03	3.41E+03	2.26E+03
	5	4.20E-07	4.09E-07	4.03E-07	3.93E+03	5.12E+03	3.24E+03
	6	4.14E-07	4.05E-07	4.03E-07	4.98E+03	6.57E+03	4.50E+03
	7	4.18E-07	4.03E-07	4.02E-07	5.96E+03	8.00E+03	5.68E+03
	8	4.16E-07	4.03E-07	4.02E-07	6.80E+03	1.01E+04	6.50E+03
	9	4.21E-07	4.03E-07	4.02E-07	7.43E+03	1.19E+04	7.26E+03
	10	4.26E-07	4.12E-07	4.04E-07	3.50E+02	9.30E+02	4.60E+02
2	1	2.43E-07	2.43E-07	2.43E-07	3.55E+02	3.24E+02	1.96E+02
	2	2.41E-07	2.37E-07	2.36E-07	1.03E+03	1.05E+03	5.50E+02
	3	2.38E-07	2.37E-07	2.36E-07	1.90E+03	2.20E+03	1.04E+03
	4	2.37E-07	2.36E-07	2.36E-07	2.88E+03	3.46E+03	1.43E+03
	5	2.37E-07	2.37E-07	2.37E-07	3.93E+03	4.48E+03	1.89E+03
	6	2.37E-07	2.38E-07	2.37E-07	4.98E+03	5.28E+03	2.26E+03
	7	2.37E-07	2.36E-07	2.37E-07	5.96E+03	6.16E+03	2.81E+03
	8	2.37E-07	2.38E-07	2.38E-07	6.80E+03	7.18E+03	3.29E+03
	9	2.37E-07	2.40E-07	2.38E-07	7.43E+03	7.63E+03	3.78E+03
	10	2.42E-07	2.41E-07	2.38E-07	3.50E+02	1.72E+02	3.07E+02
3	1	5.35E-07	4.99E-07	4.99E-07	4.10E+02	3.08E+02	1.23E+02
	2	4.68E-07	3.32E-07	3.22E-07	1.06E+03	1.04E+03	6.11E+02
	3	3.82E-07	3.24E-07	3.19E-07	1.95E+03	2.13E+03	1.74E+03
	4	3.50E-07	3.21E-07	3.19E-07	2.95E+03	3.69E+03	3.28E+03
	5	3.42E-07	3.20E-07	3.18E-07	4.02E+03	5.12E+03	4.37E+03
	6	3.61E-07	3.13E-07	3.17E-07	5.07E+03	7.20E+03	5.78E+03
	7	3.33E-07	3.11E-07	3.21E-07	6.05E+03	9.16E+03	7.93E+03
	8	3.35E-07	3.15E-07	3.16E-07	6.93E+03	1.09E+04	1.05E+04
	9	3.41E-07	3.20E-07	3.15E-07	7.56E+03	1.27E+04	1.23E+04
	10	3.54E-07	3.24E-07	3.13E-07	3.50E+02	1.40E+03	1.20E+03
4	1	3.89E-06	3.86E-06	3.86E-06	3.90E+02	2.99E+02	4.09E+02
	2	3.50E-06	2.74E-06	2.74E-06	1.06E+03	9.54E+02	8.51E+02
	3	2.74E-06	2.68E-06	2.72E-06	1.95E+03	1.89E+03	1.66E+03
	4	2.75E-06	2.66E-06	2.70E-06	2.95E+03	3.38E+03	2.90E+03
	5	2.77E-06	2.64E-06	2.71E-06	4.00E+03	5.00E+03	4.04E+03
	6	2.73E-06	2.62E-06	2.70E-06	5.08E+03	7.17E+03	5.38E+03
	7	2.75E-06	2.62E-06	2.71E-06	6.06E+03	8.99E+03	6.51E+03
	8	2.79E-06	2.61E-06	2.70E-06	6.90E+03	1.09E+04	7.56E+03
	9	2.77E-06	2.61E-06	2.71E-06	7.53E+03	1.26E+04	8.55E+03

	10	2.78E-06	2.61E-06	2.71E-06		3.50E+02	9.81E+02	5.73E+02
5	1	2.06E-06	1.93E-06	1.93E-06		4.15E+02	2.99E+02	1.99E+02
	2	1.65E-06	1.24E-06	1.24E-06		1.09E+03	9.45E+02	6.99E+02
	3	1.52E-06	1.23E-06	1.23E-06		1.94E+03	2.05E+03	1.42E+03
	4	1.31E-06	1.22E-06	1.22E-06		2.94E+03	3.46E+03	2.28E+03
	5	1.22E-06	1.21E-06	1.22E-06		3.99E+03	6.59E+03	3.24E+03
	6	1.25E-06	1.21E-06	1.22E-06		5.04E+03	9.52E+03	4.25E+03
	7	1.26E-06	1.21E-06	1.22E-06		6.02E+03	1.22E+04	5.28E+03
	8	1.25E-06	1.21E-06	1.22E-06		6.86E+03	1.54E+04	6.13E+03
	9	1.27E-06	1.21E-06	1.22E-06		7.49E+03	1.89E+04	6.84E+03
	10	1.26E-06	1.21E-06	1.23E-06		3.50E+02	1.42E+03	4.21E+02

Table B-3: Number of function calls and objective values found for Material Set 3

Case	nd	Objevtive value			Number of function calls		
		GA	MADS	RAGS	GA	MADS	RAGS
1	1	5.39E-07	5.42E-07	5.37E-07	3.85E+02	2.93E+02	1.59E+02
	2	5.22E-07	5.16E-07	5.16E-07	1.02E+03	1.16E+03	6.35E+02
	3	5.03E-07	4.81E-07	4.79E-07	1.86E+03	2.41E+03	1.63E+03
	4	5.04E-07	4.80E-07	4.68E-07	2.84E+03	3.89E+03	2.82E+03
	5	4.84E-07	4.74E-07	4.60E-07	3.89E+03	5.85E+03	4.17E+03
	6	4.83E-07	4.61E-07	4.62E-07	4.94E+03	8.35E+03	5.53E+03
	7	4.84E-07	4.68E-07	4.59E-07	5.92E+03	1.01E+04	7.02E+03
	8	4.70E-07	4.62E-07	4.63E-07	6.76E+03	1.30E+04	7.93E+03
	9	4.72E-07	4.68E-07	4.59E-07	7.39E+03	1.42E+04	9.09E+03
	10	4.87E-07	4.65E-07	4.59E-07	3.50E+02	1.37E+03	6.10E+02
2	1	4.70E-07	4.64E-07	4.62E-07	3.95E+02	2.73E+02	1.17E+02
	2	4.41E-07	4.40E-07	4.54E-07	1.06E+03	1.13E+03	3.02E+02
	3	4.39E-07	4.40E-07	4.50E-07	1.91E+03	2.56E+03	8.36E+02
	4	4.40E-07	4.37E-07	4.43E-07	2.89E+03	4.56E+03	1.56E+03
	5	4.38E-07	4.35E-07	4.40E-07	3.94E+03	6.42E+03	3.54E+03
	6	4.38E-07	4.36E-07	4.39E-07	4.99E+03	8.29E+03	5.25E+03
	7	4.38E-07	4.38E-07	4.39E-07	5.97E+03	1.04E+04	6.73E+03
	8	4.40E-07	4.40E-07	4.38E-07	6.81E+03	1.31E+04	8.30E+03
	9	4.40E-07	4.41E-07	4.38E-07	7.44E+03	1.60E+04	9.42E+03
	10	4.50E-07	4.51E-07	4.38E-07	3.50E+02	1.04E+03	7.15E+02
3	1	8.98E-07	8.41E-07	8.41E-07	4.15E+02	2.90E+02	1.56E+02
	2	8.09E-07	4.97E-07	4.91E-07	1.08E+03	9.46E+02	7.02E+02
	3	6.49E-07	4.94E-07	4.91E-07	1.95E+03	1.93E+03	1.49E+03
	4	5.04E-07	4.93E-07	4.91E-07	2.99E+03	3.35E+03	2.57E+03
	5	5.28E-07	4.85E-07	5.00E-07	4.11E+03	5.34E+03	3.58E+03
	6	4.97E-07	4.95E-07	5.08E-07	5.22E+03	7.33E+03	5.25E+03
	7	5.11E-07	4.85E-07	5.09E-07	6.27E+03	8.94E+03	7.20E+03
	8	5.04E-07	4.90E-07	5.10E-07	7.19E+03	1.18E+04	9.60E+03
	9	5.14E-07	4.92E-07	5.09E-07	7.82E+03	1.36E+04	1.34E+04
	10	5.78E-07	4.88E-07	5.11E-07	3.50E+02	1.31E+03	9.55E+02
4	1	8.63E-06	8.64E-06	8.62E-06	4.25E+02	3.17E+02	2.02E+02
	2	8.65E-06	6.51E-06	8.63E-06	1.08E+03	9.89E+02	1.06E+03
	3	6.11E-06	6.10E-06	8.67E-06	1.95E+03	2.14E+03	2.20E+03
	4	6.17E-06	5.87E-06	8.70E-06	2.97E+03	4.03E+03	3.57E+03
	5	6.32E-06	5.84E-06	8.70E-06	4.02E+03	6.57E+03	5.95E+03
	6	8.73E-06	5.80E-06	8.73E-06	5.07E+03	1.01E+04	8.42E+03
	7	6.47E-06	5.81E-06	8.77E-06	6.15E+03	1.23E+04	1.08E+04
	8	6.35E-06	5.79E-06	8.78E-06	7.03E+03	1.60E+04	1.29E+04
	9	6.36E-06	5.72E-06	8.81E-06	7.93E+03	1.82E+04	1.46E+04

	10	6.19E-06	5.71E-06	8.82E-06	5.00E+02	1.19E+03	1.10E+03
5	1	6.85E-06	6.21E-06	6.19E-06	4.65E+02	2.98E+02	2.84E+02
	2	5.60E-06	3.20E-06	3.20E-06	1.13E+03	1.16E+03	8.13E+02
	3	3.60E-06	2.46E-06	2.47E-06	2.07E+03	2.33E+03	1.59E+03
	4	3.40E-06	2.41E-06	2.43E-06	3.11E+03	3.96E+03	2.44E+03
	5	2.68E-06	2.37E-06	2.41E-06	4.31E+03	6.95E+03	3.37E+03
	6	2.56E-06	2.36E-06	2.39E-06	5.42E+03	9.31E+03	4.48E+03
	7	2.41E-06	2.36E-06	2.38E-06	6.47E+03	1.28E+04	5.57E+03
	8	2.41E-06	2.35E-06	2.38E-06	7.35E+03	1.65E+04	6.60E+03
	9	2.57E-06	2.35E-06	2.38E-06	7.98E+03	1.93E+04	7.28E+03
	10	2.66E-06	2.34E-06	2.39E-06	3.50E+02	2.30E+03	3.67E+02